

Multiple CP non-conserving mechanisms of $(\beta\beta)_{0\nu}$ -decay and nuclei with largely different nuclear matrix elements

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ABSTRACT: We investigate the possibility to discriminate between different pairs of CP non-conserving mechanisms inducing the neutrinoless double beta $(\beta\beta)_{0\nu}$ -decay by using data on $(\beta\beta)_{0\nu}$ -decay half-lives of nuclei with largely different nuclear matrix elements (NMEs). The mechanisms studied are: light Majorana neutrino exchange, heavy left-handed (LH) and heavy right-handed (RH) Majorana neutrino exchanges, lepton charge non-conserving couplings in SUSY theories with R -parity breaking giving rise to the “dominant gluino exchange” and the “squark-neutrino” mechanisms. The nuclei considered are ^{76}Ge , ^{82}Se , ^{100}Mo , ^{130}Te and ^{136}Xe . Four sets of nuclear matrix elements (NMEs) of the decays of these five nuclei, derived within the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA), were employed in our analysis. While for each of the five single mechanisms discussed, the NMEs for ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te differ relatively little, the relative difference between the NMEs of any two nuclei not exceeding 10%, the NMEs for ^{136}Xe differ significantly from those of ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te , being by a factor $\sim (1.3 - 2.5)$ smaller. This allows, in principle, to draw conclusions about

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the pair of non-interfering (interfering) mechanisms possibly inducing the $(\beta\beta)_{0\nu}$ -decay from data on the half-lives of ^{136}Xe and of at least one (two) more isotope(s) which can be, e.g., any of the four, ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te . Depending on the sets of mechanisms considered, the conclusion can be independent of, or can depend on, the NMEs used in the analysis. The implications of the EXO lower bound on the half-life of ^{136}Xe for the problem studied are also exploited.

KEYWORDS: Beyond Standard Model, Neutrino Physics, CP violation, Standard Model

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1 Introduction

If neutrinoless double beta $(\beta\beta)_{0\nu}$ -decay will be observed, it will be of fundamental importance to determine the mechanism which induces the decay. In [1] we have considered the possibility of several different mechanisms contributing to the neutrinoless double beta $(\beta\beta)_{0\nu}$ -decay amplitude in the general case of CP nonconservation.¹ The mechanisms discussed are the “standard” light Majorana neutrino exchange, exchange of heavy Majorana neutrinos coupled to (V-A) currents, exchange of heavy right-handed (RH) Majorana neutrinos coupled to (V+A) currents, lepton charge non-conserving couplings in SUSY theories with R -parity breaking.² Of the latter we have concentrated on the so-called “dominant gluino exchange” and “squark-neutrino” mechanisms. Each of these mechanisms is characterized by a specific fundamental lepton number violating (LNV) parameter η_κ , the index κ labeling the mechanism. The parameter η_κ will be complex, in general, if the mechanism κ does not conserve the CP symmetry. In [8–10] the authors analysed the possibility to identify the mechanisms of $(\beta\beta)_{0\nu}$ -decay using data on the half-lives of several isotopes. The indicated (and other) specific mechanisms were considered, assuming that only one of these mechanisms is triggering the decay.³ In [1] we have investigated in detail the cases of

¹The case of two CP conserving mechanisms generating the $(\beta\beta)_{0\nu}$ -decay was considered in [2].

²For a more detailed description of these mechanisms and references to the articles where they were originally proposed see [1] and, e.g., [3–6].

³In [11, 12] the authors derived a general effective $(\beta\beta)_{0\nu}$ -decay Lagrangian without specifying the mechanisms generating the different terms in the Lagrangian. Assuming that only one of the terms in the Lagrangian is operative in $(\beta\beta)_{0\nu}$ -decay and using the existing lower limit on the half-life of ^{76}Ge , in [12] constraints on the effective CP conserving couplings constant multiplying the different terms in the Lagrangian were obtained.

two “non-interfering” [7] and two “interfering” mechanisms generating the $(\beta\beta)_{0\nu}$ -decay,⁴ using as input hypothetical $(\beta\beta)_{0\nu}$ -decay half-lives of the three isotopes ^{76}Ge , ^{100}Mo and ^{130}Te . Four sets of nuclear matrix elements (NMEs) of the decays of these three nuclei, derived within the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA) [13–15], were utilized: they were calculated in [1] with two different nucleon-nucleon (NN) potentials (CD-Bonn and Argonne), “large” size single-particle spaces and for two values of the axial coupling constant $g_A = 1.25; 1.0$.

If the $(\beta\beta)_{0\nu}$ -decay is induced by two non-interfering mechanisms, which for concreteness were considered in [1] to be [7] the light left-handed (LH) and the heavy RH Majorana neutrino exchanges, one can determine the squares of the absolute values of the two LNV parameters, characterizing these mechanisms, $|\eta_\nu|^2$ and $|\eta_R|^2$, from data on the half-lives of two nuclear isotopes. This was done in [1] using as input all three possible pairs of half-lives of ^{76}Ge , ^{100}Mo and ^{130}Te , chosen from specific intervals and satisfying the existing experimental constraints. It was found that if the half-life of one of the three nuclei is measured, the requirement that $|\eta_\nu|^2 \geq 0$ and $|\eta_R|^2 \geq 0$ (“positivity condition”) constrains the other two half-lives (and the $(\beta\beta)_{0\nu}$ -decay half-life of any other $(\beta\beta)_{0\nu}$ -decaying isotope for that matter) to lie in specific intervals, determined by the measured half-life and the relevant NMEs and phase-space factors. This feature is common to all cases of two non-interfering mechanisms generating the $(\beta\beta)_{0\nu}$ -decay. The indicated specific half-life intervals for the various isotopes were shown to be stable with respect to the change of the NMEs (within the sets of NMEs employed) used to derive them. The intervals depend, in general, on the type of the two non-interfering mechanisms assumed to cause the $(\beta\beta)_{0\nu}$ -decay. However, these differences in the cases of all possible pairs of non-interfering mechanisms considered were found to be extremely small. Using the indicated difference to get information about the specific pair of non-interfering mechanisms possibly operative in $(\beta\beta)_{0\nu}$ -decay requires, in the cases studied in [1], an extremely high precision in the measurement of the $(\beta\beta)_{0\nu}$ -decay half-lives of the isotopes considered (^{76}Ge , ^{100}Mo and ^{130}Te), as well as an exceedingly small uncertainties in the knowledge of the $(\beta\beta)_{0\nu}$ -decay NMEs of these isotopes. The levels of precision required seem impossible to achieve in the foreseeable future. One of the consequences of this results is that if it will be possible to rule out one pair of the considered in [1] non-interfering mechanisms as the cause of $(\beta\beta)_{0\nu}$ -decay, most likely one will be able to rule out all of them.

The dependence of the physical solutions for $|\eta_\nu|^2$ and $|\eta_R|^2$ obtained on the NMEs used, was also studied in [1]. It was found that the solutions can exhibit a significant, or a relatively small, variation with the NMEs employed, depending on the hypothetical values of the half-lives of the two isotopes utilized as input for obtaining the solutions. This conclusion is valid for all other pairs of non-interfering mechanisms considered in [1].

⁴Two mechanisms contributing to the $(\beta\beta)_{0\nu}$ -decay amplitude can be non-interfering or interfering, depending on whether their interference term present in the $(\beta\beta)_{0\nu}$ -decay rate is suppressed (and negligible) or not. In the case of the five mechanisms considered in [1] and listed above, all pairs of mechanisms, which include the exchange of the heavy RH Majorana neutrinos coupled to (V+A) currents as one of the mechanisms, can be shown to be of the non-interfering type, while those that do not involve the exchange of heavy RH Majorana neutrinos belong to the interfering class.

In the case when two interfering mechanisms are responsible for the $(\beta\beta)_{0\nu}$ -decay, the squares of the absolute values of the two relevant parameters and the interference term parameter, which involves the cosine of an unknown relative phase α of the two fundamental parameters, can be uniquely determined, in principle, from data on the half-lives of three nuclei. We have analyzed in [1] in detail the case of light Majorana neutrino exchange and gluino exchange. In this case the parameters which are determined from data on the half-lives are $|\eta_\nu|^2$, $|\eta_{\lambda'}|^2$, $\eta_{\lambda'}$ being that of the gluino exchange, and $z = 2 \cos \alpha |\eta_\nu| |\eta_{\lambda'}|$. The physical solutions for these parameters have to satisfy the conditions $|\eta_\nu|^2 \geq 0$, $|\eta_{\lambda'}|^2 \geq 0$ and $-2|\eta_\nu| |\eta_{\lambda'}| \leq z \leq 2|\eta_\nu| |\eta_{\lambda'}|$. The latter condition implies that given the half-lives of two isotopes, T_1 and T_2 , the half-life of any third isotope T_3 is constrained to lie in a specific interval, if the mechanisms considered are indeed generating the $(\beta\beta)_{0\nu}$ -decay. If further the half-life of one isotope T_1 is known, for the interference to be constructive (destructive), the half-lives of any other pair of isotopes T_2 and T_3 , should belong to specific intervals. These intervals depend on whether the interference between the two contributions in the $(\beta\beta)_{0\nu}$ -decay rate is constructive or destructive. We have derived in [1] in analytic form the general conditions for i) constructive interference ($z > 0$), ii) destructive interference ($z < 0$), iii) $|\eta_\nu|^2 = 0$, $|\eta_{\lambda'}|^2 \neq 0$, iv) $|\eta_\nu|^2 \neq 0$, $|\eta_{\lambda'}|^2 = 0$ and v) $z = 0$, $|\eta_\nu|^2 \neq 0$, $|\eta_{\lambda'}|^2 \neq 0$. We have found that, given T_1 , a constructive interference is possible only if T_2 lies in a relatively narrow interval and T_3 has a value in extremely narrow intervals. Numerically the intervals for T_2 and T_3 are very similar to the intervals one obtains in the case of two non-interfering mechanisms (within the set considered in [1]). The intervals of values of T_2 and T_3 corresponding to destructive interference are very different from those corresponding to the cases of constructive interference and of the two non-interfering $(\beta\beta)_{0\nu}$ -decay mechanisms we have considered. Within the set of $(\beta\beta)_{0\nu}$ -decay mechanisms studied by us, this difference can allow to discriminate experimentally between the possibilities of the $(\beta\beta)_{0\nu}$ -decay being triggered by two “destructively interfering” mechanisms or by two “constructively interfering” or by two non-interfering mechanisms.

The “degeneracy” of the predictions of the pairs of non-interfering mechanisms of $(\beta\beta)_{0\nu}$ -decay for the interval of values of the half-life of a second nucleus, given the half-life of a different one from the three considered, ^{76}Ge , ^{100}Mo and ^{130}Te , is a direct consequence of a specific property of the NMEs of the three nuclei. Namely, for each of the five single mechanisms discussed (the light LH, heavy LH and heavy RH Majorana neutrino exchanges, the gluino exchange and the “squark-neutrino” mechanism), the NMEs for the three nuclei differ relatively little, the relative difference between the NMEs of any two nuclei not exceeding 10%:⁵ $|M_{i,\kappa} - M_{j,\kappa}| / (0.5|M_{i,\kappa} + M_{j,\kappa}|) \lesssim 0.1$, $i \neq j = 1, 2, 3 \equiv ^{76}\text{Ge}, ^{100}\text{Mo}, ^{130}\text{Te}$. This feature of the NMEs of ^{76}Ge , ^{100}Mo and ^{130}Te makes it impossible to discriminate experimentally not only between the four different pairs of non-interfering mechanisms, considered in [1], using data on the half-lives of ^{76}Ge , ^{100}Mo and ^{130}Te , but also between any of these four pairs and pairs of interfering mechanisms when the interference is constructive. The indicated “degeneracy” of the predictions of

⁵The general implications of this “degeneracy” of the NMEs of ^{76}Ge , ^{100}Mo and ^{130}Te for testing the mechanisms under discussion in the case of CP invariance were investigated in [16].

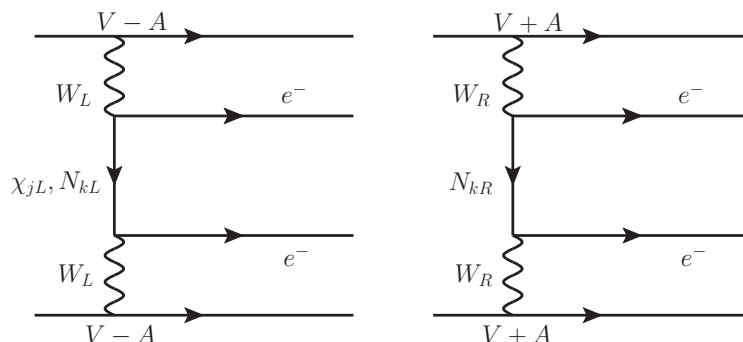


Figure 1. Feynman diagrams for the $(\beta\beta)_{0\nu}$ -decay, generated by the light and heavy LH Majorana neutrino exchange (left panel) and the heavy RH Majorana neutrino exchange (right panel).

different pairs of mechanisms possibly active in $(\beta\beta)_{0\nu}$ -decay can be lifted if one uses as input the half-lives of nuclei having largely different NMEs. One example of such a nucleus is ^{136}Xe , whose NMEs for the five mechanisms studied in [1], as we are going to show, differ significantly from those of ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te .

In the present article we investigate the potential of combining data on the half-lives of ^{136}Xe and of one or more of the four nuclei ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te , for discriminating between different pairs of non-interfering or interfering mechanisms of $(\beta\beta)_{0\nu}$ -decay. We consider the same five basic mechanisms used in the study performed in [1], namely, the “standard” light Majorana neutrino exchange, exchange of heavy Majorana neutrinos coupled to (V-A) currents, exchange of heavy right-handed (RH) Majorana neutrinos coupled to (V+A) currents, dominant gluino exchange and the squark-neutrino mechanism. The last two are related to lepton charge non-conserving couplings in SUSY theories with R -parity breaking.

The paper is organized as follows. In section 2 we give a brief overview on the possible mechanisms that can induce the $(\beta\beta)_{0\nu}$ -decay considered in this work. In section 3 we analyze the case of two mechanisms active in the decay in the non-interfering and in the interfering regime, combining the recent experimental results reported by EXO, while the section 4 contains our conclusions.

2 Mechanisms of $(\beta\beta)_{0\nu}$ -decay considered and nuclear matrix elements employed

The mechanisms of $(\beta\beta)_{0\nu}$ -decay we are going to consider in the present article are the same five mechanisms considered in [1]: i) the light Majorana neutrino exchange, characterized by the dimensionless LNV parameter $\eta_\nu = \langle m \rangle / m_e$, $\langle m \rangle$ and m_e being the $(\beta\beta)_{0\nu}$ -decay effective Majorana mass (see, e.g., [3–6]) and the electron mass, respectively; ii) exchange of heavy “left-handed” (LH) Majorana neutrinos coupled to (V-A) currents, characterized by the LNV parameter η_L ; iii) exchange of heavy “right-handed” (RH) Majorana neutrinos coupled to (V+A) currents, characterized by the LNV parameter η_R . We consider also two possible mechanisms in SUSY theories with R -parity non-conservation:

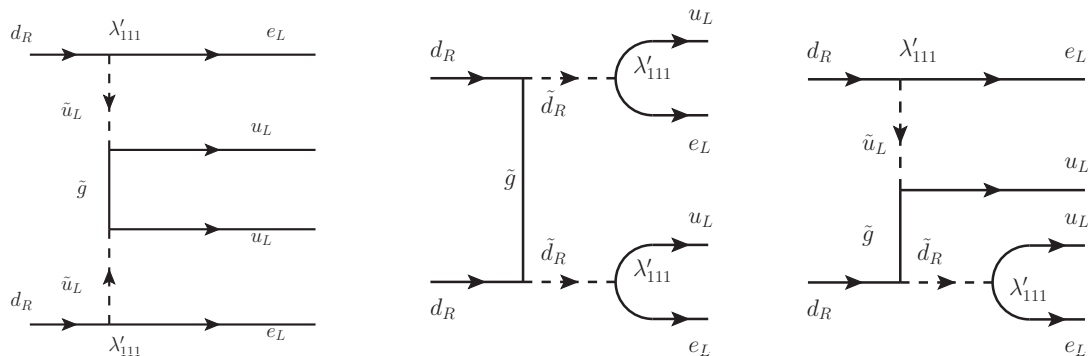


Figure 2. Feynman diagrams for $(\beta\beta)_{0\nu}$ -decay due to the gluino exchange mechanism.

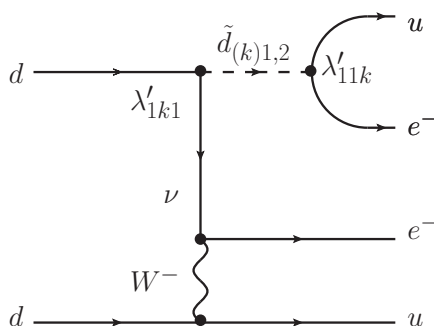


Figure 3. Feynman diagrams for $(\beta\beta)_{0\nu}$ -decay due to the squark-neutrino mechanism at the quark-level [17].

iv) the “gluino exchange” mechanisms, characterized by the LNV parameter $\eta_{\lambda'}$, and v) the “squark-neutrino” mechanism, characterized by the parameter $\eta_{\nu-q}$. All five LNV parameters we have introduced above η_ν , η_L , η_R , $\eta_{\lambda'}$ and $\eta_{\nu-q}$ are effective. The dependence of a given LNV effective parameter on the fundamental parameters of the theory in which the corresponding mechanism is possible/arises, is discussed in detail in [1]⁶ and we are not going to repeat this discussion here. Instead we will give just a graphical representation of each of the five mechanisms in terms of the corresponding leading order Feynman diagrams. For the light Majorana neutrino, heavy LH Majorana neutrino (N_{kL}) and heavy RH Majorana neutrino (N_{kR}) exchange mechanisms they are given in figure 1; for the gluino exchange and squark-neutrino mechanisms we show them in figures 2 and 3.

In the present analysis we employ four sets of nuclear matrix elements (NMEs) of the decays of the five nuclei of interest, ^{76}Ge , ^{82}Se , ^{100}Mo , ^{130}Te and ^{136}Xe , derived within the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA) [13–15]. The SRQRPA takes into account the Pauli exclusion principle and conserves the mean particle number in correlated ground state.

For each of the five nuclei, two choices of single-particle basis are considered. The intermediate size model space has 12 levels (oscillator shells $N=2-4$) for ^{76}Ge and ^{82}Se ,

⁶An extensive list of references to the original articles is also given in [1].

16 levels (oscillator shells $N=2-4$ plus the $f+h$ orbits from $N=5$) for ^{100}Mo and 18 levels (oscillator shells $N=3,4$ plus $f+h+p$ orbits from $N=5$) for ^{130}Te and ^{136}Xe . The large size single particle space contains 21 levels (oscillator shells $N=0-5$) for ^{76}Ge , ^{82}Se and ^{100}Mo , and 23 levels for ^{130}Te and ^{136}Xe ($N=1-5$ and i orbits from $N=6$). In comparison with previous studies [18–20], we omitted the small space model which is not sufficient to describe realistically the tensor part of the $(\beta\beta)_{0\nu}$ -decay nuclear matrix elements.

The single particle energies were obtained by using a Coulomb-corrected Woods-Saxon potential. Two-body G-matrix elements we derived from the Argonne and the Charge Dependent Bonn (CD-Bonn) one-boson exchange potential within the Brueckner theory. The schematic pairing interactions have been adjusted to fit the empirical pairing gaps [21]. The particle-particle and particle-hole channels of the G-matrix interaction of the nuclear Hamiltonian H are renormalized by introducing the parameters g_{pp} and g_{ph} , respectively. The calculations have been carried out for $g_{ph} = 1.0$. The particle-particle strength parameter g_{pp} of the SRQRPA is fixed by the data on the two-neutrino double beta decays [18–20, 22]. In the calculation of the $(\beta\beta)_{0\nu}$ -decay NMEs, the two-nucleon short-range correlations derived from same potential as residual interactions, namely from the Argonne or CD-Bonn potentials, were considered [23].

The calculated NMEs $M'_{\nu}{}^{0\nu}$, $M'_{N}{}^{0\nu}$, $M'_{\chi}{}^{0\nu}$ and $M'_{\bar{q}}{}^{0\nu}$ are listed in table 1. For ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te they are taken from ref. [1], while for ^{136}Xe the results are new [24]. We note that these NMEs are significantly smaller (by a factor 1.3 - 2.5) when compared with those for ^{76}Ge , ^{82}Se and ^{100}Mo . The reduction of the $0\nu\beta\beta$ -decay NMEs of the ^{136}Xe is explained by the closed neutron shell for this nucleus. A sharper Fermi surface leads to a reduction of this transition. This effect is clearly seen also in the case of $M'_{\nu}{}^{0\nu}$ of double magic nucleus ^{48}Ca [24].

By glancing the table 1 we see that a significant source of uncertainty is the value of the axial-vector coupling constant g_A and especially in the case of matrix elements $M'_{\chi}{}^{0\nu}$ and $M'_{\bar{q}}{}^{0\nu}$. Further, the NMEs associated with heavy neutrino exchange are sensitive also to the choice of the NN interaction, the CD-Bonn or Argonne potential. These types of realistic NN interaction differ mostly by the description of the short-range interactions. Although in table 1 we present results for NMEs of the nuclei of interest, calculated using both medium and large size single particle spaces within the SRQRPA method, in the numerical examples we are going to present further we will use the NMEs for a given nucleus, with the large size single particle space in both cases of Argonne and CD-Bonn potentials and for $g_A = 1.25; 1.00$ (i.e., altogether four NMEs).

3 $(\beta\beta)_{0\nu}$ -decay induced by two mechanisms

The observation of $(\beta\beta)_{0\nu}$ -decay of several different isotopes is crucial for obtaining information about the mechanism or mechanisms that induce the decay. In the analysis we are going to perform we will employ the lower bound obtained by the EXO collaboration on the $(\beta\beta)_{0\nu}$ -decay half-life of ^{136}Xe [25]:

$$T_{1/2}^{0\nu}(^{136}\text{Xe}) > 1.6 \times 10^{25}\text{y} \quad (90\% \text{ CL}). \quad (3.1)$$

Nuclear transition	$G^{0\nu}(E_0, Z)$ [y^{-1}]	NN pot.	m.s.	$ M_{\nu}^{0\nu} $		$ M_N^{0\nu} $		$ M_{\chi'}^{0\nu} $		$ M_{\tilde{q}}^{0\nu} $	
				$g_A =$		$g_A =$		$g_A =$		$g_A =$	
				1.0	1.25	1.0	1.25	1.0	1.25	1.0	1.25
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	$7.98 \cdot 10^{-15}$	Argonne	intm.	3.85	4.75	172.2	232.8	387.3	587.2	396.1	594.3
			large	4.39	5.44	196.4	264.9	461.1	699.6	476.2	717.8
		CD-Bonn	intm.	4.15	5.11	269.4	351.1	339.7	514.6	408.1	611.7
			large	4.69	5.82	317.3	411.5	392.8	595.6	482.7	727.6
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	$3.53 \cdot 10^{-14}$	Argonne	intm.	3.59	4.54	164.8	225.7	374.5	574.2	379.3	577.9
			large	4.18	5.29	193.1	262.9	454.9	697.7	465.1	710.2
		CD-Bonn	intm.	3.86	4.88	258.7	340.4	328.7	503.7	390.4	594.5
			large	4.48	5.66	312.4	408.4	388.0	594.4	471.8	719.9
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	$5.73 \cdot 10^{-14}$	Argonne	intm.	3.62	4.39	184.9	249.8	412.0	629.4	405.1	612.1
			large	3.91	4.79	191.8	259.8	450.4	690.3	449.0	682.6
		CD-Bonn	intm.	3.96	4.81	298.6	388.4	356.3	543.7	415.9	627.9
			large	4.20	5.15	310.5	404.3	384.4	588.6	454.8	690.5
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	$5.54 \cdot 10^{-14}$	Argonne	intm.	3.29	4.16	171.6	234.1	385.1	595.2	382.2	588.9
			large	3.34	4.18	176.5	239.7	405.5	626.0	403.1	620.4
		CD-Bonn	intm.	3.64	4.62	276.8	364.3	335.8	518.8	396.8	611.1
			large	3.74	4.70	293.8	384.5	350.1	540.3	416.3	640.7
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	$5.92 \cdot 10^{-14}$	Argonne	intm.	2.30	2.29	119.2	163.5	275.0	425.3	270.5	417.2
			large	2.19	2.75	117.1	159.7	276.7	428.0	271.0	418.0
		CD-Bonn	intm.	2.32	2.95	121.4	166.7	274.4	424.3	267.4	412.1
			large	2.61	3.36	125.4	172.1	297.2	460.0	297.0	458.8

Table 1. Nuclear matrix elements $M_{\nu}^{0\nu}$ (light neutrino mass mechanism), $M_N^{0\nu}$ (heavy neutrino mass mechanism), $M_{\chi'}^{0\nu}$ (trilinear R-parity breaking SUSY mechanism) and $M_{\tilde{q}}^{0\nu}$ (squark mixing mechanism) for the $0\nu\beta\beta$ -decays of ^{76}Ge , ^{82}Se , ^{100}Mo , ^{130}Te and ^{136}Xe within the Selfconsistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA). $G^{0\nu}(E_0, Z)$ is the phase-space factor. We notice that all NMEs given in table 1 are real and positive. The nuclear radius is $R = 1.1 \text{ fm } A^{1/3}$.

We use also the lower limits on the $(\beta\beta)_{0\nu}$ -decay half-lives of ^{76}Ge , ^{82}Se and ^{100}Mo , and of ^{130}Te reported by the Heidelberg-Moscow [26], NEMO3 [27] and CUORICINO [28] experiments, respectively, as well as as well the ^{76}Ge half-life reported in [29] (see also [30, 31]):

$$\begin{aligned}
 T_{1/2}^{0\nu}({}^{76}\text{Ge}) &> 1.9 \times 10^{25} \text{y} \quad [26], & T_{1/2}^{0\nu}({}^{82}\text{Se}) &> 3.6 \times 10^{23} \text{y} \quad [27], \\
 T_{1/2}^{0\nu}({}^{100}\text{Mo}) &> 1.1 \times 10^{24} \text{y} \quad [27], & T_{1/2}^{0\nu}({}^{130}\text{Te}) &> 3.0 \times 10^{24} \text{y} \quad [28]. \\
 T_{1/2}^{0\nu}({}^{76}\text{Ge}) &= 2.23_{-0.31}^{+0.44} \times 10^{25} \text{y} \quad [29].
 \end{aligned} \tag{3.2}$$

Following [1], we will consider two cases:

1. $(\beta\beta)_{0\nu}$ -decay induced by two mechanisms whose interference term in the $(\beta\beta)_{0\nu}$ -decay rate is negligible⁷ [7];

⁷This possibility is realized when, e.g., the electron currents (responsible for the emission of the two electrons in the final state), associated with the two mechanisms considered, have opposite chiralities.

2. $(\beta\beta)_{0\nu}$ -decay triggered by two CP non-conserving mechanisms whose interference term cannot be neglected.

In the case 1, given the two mechanisms A and B, the inverse of the $(\beta\beta)_{0\nu}$ -decay half-life for a given isotope (A_i, Z_i) reads:

$$\frac{1}{T_i G_i} = |\eta_A|^2 |M'_{i,A}|^2 + |\eta_B|^2 |M'_{i,B}|^2, \quad (3.3)$$

where the index i denotes the isotope. The values of the phase space factor $G_i^{0\nu}(E, Z)$, and of the NMEs $M'_{i,A}$ and $M'_{i,B}$ for the mechanisms we will consider and for the isotopes ^{76}Ge , ^{82}Se , ^{100}Mo , ^{130}Te and ^{136}Xe of interest, are listed in table 1. The LNV parameters are defined in section 2. If the two mechanisms A and B inducing the decay are interfering and CP non-conserving (case 2), the inverse of the $(\beta\beta)_{0\nu}$ -decay half-life of the isotope (A_i, Z_i) can be written as:

$$\frac{1}{T_{1/2,i}^{0\nu} G_i^{0\nu}(E, Z)} = |\eta_A|^2 |M'_{i,A}|^2 + |\eta_B|^2 |M'_{i,B}|^2 + 2 \cos \alpha |M'_{i,A}| |M'_{i,B}| |\eta_A| |\eta_B|. \quad (3.4)$$

Here α is the relative phase of η_A and η_B .

If the $(\beta\beta)_{0\nu}$ -decay is caused by two non-interfering mechanisms, the LNV parameters $|\eta_A|^2$ and $|\eta_B|^2$ characterizing the mechanisms, can be determined, in principle, from data on the $(\beta\beta)_{0\nu}$ -decay half-lives of two isotopes, i.e., by solving a system of two linear equations. In the case of two interfering CP non-conserving mechanisms, the values of the two parameters $|\eta_A|^2$ and $|\eta_B|^2$ and of the cosine of the relative phase α of η_A and η_B can be obtained from data on the $(\beta\beta)_{0\nu}$ -decay half-lives of three isotopes, i.e., by solving a system of three linear equations.

As was noticed and discussed in detail in [1], a very important role in identifying the physical solutions for $|\eta_A|^2$ and $|\eta_B|^2$ of the corresponding systems of two or three equations is played by the “positivity conditions” $|\eta_A|^2 \geq 0$ and $|\eta_B|^2 \geq 0$. If one of the two mechanisms inducing the $(\beta\beta)_{0\nu}$ -decay is the “standard” light neutrino exchange, additional important constraint on the positive solutions of the relevant systems of two or three linear equations can be provided by the upper limit on the absolute neutrino mass scale set by the Moscow and Mainz ^3H β -decay experiments [32, 33]: $m(\bar{\nu}_e) < 2.3 \text{ eV}$. In the case of $(\beta\beta)_{0\nu}$ -decay, this limit implies a similar limit on the effective Majorana mass⁸ $|\langle m \rangle| < 2.3 \text{ eV}$. The latter inequality translates into the following upper bound on the corresponding LNV dimensionless parameter $|\eta_\nu|^2 \equiv (|\langle m \rangle| / m_e)^2$:

$$|\eta_\nu|^2 \times 10^{12} < 21.2. \quad (3.5)$$

The KATRIN ^3H β -decay experiment [33], which is under preparation, is planned to have approximately a 10 times better sensitivity to $m(\bar{\nu}_e)$ than that achieved in the Moscow and Mainz experiments. If the designed sensitivity limit of $|\langle m \rangle| < 0.2 \text{ eV}$ (90% C.L.)

⁸We recall that for $m_{1,2,3} \gtrsim 0.1 \text{ eV}$ the neutrino mass spectrum is quasi-degenerate (QD), $m_1 \cong m_2 \cong m_3 \equiv m$, $m_j^2 \gg \Delta m_{21}^2, |\Delta m_{31}^2|$. In this case we have $m(\bar{\nu}_e) \cong m$ and $|\langle m \rangle| \lesssim m$.

will be obtained in the KATRIN experiment, it would imply the following rather stringent upper limit on $|\eta_\nu|^2$:

$$|\eta_\nu|^2 \times 10^{12} < 0.16. \quad (3.6)$$

In this work we will derive numerical results using the NMEs calculated with the large size single particle basis (“large basis”) and the Argonne potential (“Argonne NMEs”). We report also results obtained with NMEs calculated with the Charge Dependent Bonn (CD-Bonn) potential (“CD-Bonn NMEs”) and compared them with those derived with the Argonne NMEs.

3.1 Two non-interfering mechanisms

In this case the solutions for the corresponding two LNV parameters $|\eta_A|^2$ and $|\eta_B|^2$ obtained from data on the $(\beta\beta)_{0\nu}$ -decay half-lives of the two isotopes (A_i, Z_i) and (A_j, Z_j) , are given by [1]:

$$|\eta_A|^2 = \frac{|M'_{j,B}{}^{0\nu}|^2/T_i G_i - |M'_{i,B}{}^{0\nu}|^2/T_j G_j}{|M'_{i,A}{}^{0\nu}|^2|M'_{j,B}{}^{0\nu}|^2 - |M'_{i,B}{}^{0\nu}|^2|M'_{j,A}{}^{0\nu}|^2},$$

$$|\eta_B|^2 = \frac{|M'_{i,A}{}^{0\nu}|^2/T_j G_j - |M'_{j,A}{}^{0\nu}|^2/T_i G_i}{|M'_{i,A}{}^{0\nu}|^2|M'_{j,B}{}^{0\nu}|^2 - |M'_{i,B}{}^{0\nu}|^2|M'_{j,A}{}^{0\nu}|^2}.$$

It follows from eq. (3.7) that [1] if one of the two half-lives, say T_i , is fixed, the positivity conditions $|\eta_A|^2 \geq 0$ and $|\eta_B|^2 \geq 0$ can be satisfied only if T_j lies in a specific “positivity interval”. Choosing for convenience always $A_j < A_i$ we get for the positivity interval [1]:

$$\frac{G_i}{G_j} \frac{|M'_{i,B}{}^{0\nu}|^2}{|M'_{j,B}{}^{0\nu}|^2} T_i \leq T_j \leq \frac{G_i}{G_j} \frac{|M'_{i,A}{}^{0\nu}|^2}{|M'_{j,A}{}^{0\nu}|^2} T_i, \quad (3.7)$$

where we have used $|M'_{i,A}{}^{0\nu}|^2/|M'_{j,A}{}^{0\nu}|^2 > |M'_{i,B}{}^{0\nu}|^2/|M'_{j,B}{}^{0\nu}|^2$. In the case of $|M'_{1,A}{}^{0\nu}|^2/|M'_{2,A}{}^{0\nu}|^2 < |M'_{1,B}{}^{0\nu}|^2/|M'_{2,B}{}^{0\nu}|^2$, the interval of values of T_j under discussion is given by:

$$\frac{G_i}{G_j} \frac{|M'_{i,A}{}^{0\nu}|^2}{|M'_{j,A}{}^{0\nu}|^2} T_i \leq T_j \leq \frac{G_i}{G_j} \frac{|M'_{i,B}{}^{0\nu}|^2}{|M'_{j,B}{}^{0\nu}|^2} T_i. \quad (3.8)$$

Condition (3.7) is fulfilled, for instance, if A is the heavy right-handed (RH) Majorana neutrino exchange and B is the light Majorana neutrino exchange in the case of Argonne NMEs (see table 1). The inequality in eq. (3.7) (or (3.8)) has to be combined with the experimental lower bounds on the half-lives of the considered nuclei, $T_{i \min}^{exp}$. If, e.g., $T_{i \min}^{exp}$ is the lower bound of interest for the isotope (A_i, Z_i) , i.e., if $T_i \geq T_{i \min}^{exp}$, we get from eq. (3.7):

$$T_j \geq \frac{G_i}{G_j} \frac{|M'_{i,B}{}^{0\nu}|^2}{|M'_{j,B}{}^{0\nu}|^2} T_{i \min}^{exp}. \quad (3.9)$$

The lower limit in eq. (3.9) can be larger than the existing experimental lower bound on T_j . Indeed, suppose that $T_i \equiv T_{1/2}^{0\nu}(^{136}\text{Xe})$, $T_j \equiv T_{1/2}^{0\nu}(^{76}\text{Ge})$ and that the $(\beta\beta)_{0\nu}$ -decay is due by the standard light neutrino exchange and the heavy RH Majorana neutrino exchange. In

this case the positivity conditions for $|\eta_\nu|^2$ and $|\eta_R|^2$ imply for the Argonne and CD-Bonn NMEs corresponding to $g_A = 1.25$ (1.0):

$$1.90 (1.85) \leq \frac{T_{1/2}^{0\nu}(^{76}\text{Ge})}{T_{1/2}^{0\nu}(^{136}\text{Xe})} \leq 2.70 (2.64), \quad (\text{Argonne NMEs}); \quad (3.10)$$

$$1.30 (1.16) \leq \frac{T_{1/2}^{0\nu}(^{76}\text{Ge})}{T_{1/2}^{0\nu}(^{136}\text{Xe})} \leq 2.47 (2.30), \quad (\text{CD-Bonn NMEs}). \quad (3.11)$$

Using the EXO result, eq. (3.1), and the Argonne NMEs we get the lower bound on $T_{1/2}^{0\nu}(^{76}\text{Ge})$:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 3.03 (2.95) \times 10^{25} \text{ y}. \quad (3.12)$$

This lower bound is significantly bigger than the experimental lower bound on $T_{1/2}^{0\nu}(^{76}\text{Ge})$ quoted in eq. (3.2). If we use instead the CD-Bonn NMEs, the limit we obtain is close to the experimental lower bound on $T_{1/2}^{0\nu}(^{76}\text{Ge})$:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 2.08 (1.85) \times 10^{25} \text{ y}. \quad (3.13)$$

For illustrative purposes we show in figure 4 the solutions of equation (3.7) for $|\eta_\nu|^2$ and $|\eta_R|^2$ derived by fixing $T_{1/2}^{0\nu}(^{76}\text{Ge})$ to the best fit value claimed in [29], $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25}$ (see eq. (3.2)). As figure 4 shows, the positive (physical) solutions obtained using the Argonne NMEs are incompatible with the EXO result, eq. (3.1), and under the assumptions made and according to our oversimplified analysis, are ruled out. At the same time, the physical solutions obtained using the CD-Bonn NMEs are compatible with the EXO limit for values of $|\eta_\nu|^2$ and $|\eta_R|^2$ lying in a relatively narrow interval.

We consider next a second example of two non-interfering $(\beta\beta)_{0\nu}$ -decay mechanisms, i.e., $(\beta\beta)_{0\nu}$ -decay induced by the heavy RH Majorana neutrino exchange and the gluino exchange. Setting, as above, $T_i \equiv T_{1/2}^{0\nu}(^{136}\text{Xe})$ and $T_j \equiv T_{1/2}^{0\nu}(^{76}\text{Ge})$, we get for the positivity intervals using the Argonne or CD-Bonn NMEs corresponding to $g_A = 1.25$ (1.0):

$$2.70 (2.64) \leq \frac{T_{1/2}^{0\nu}(^{76}\text{Ge})}{T_{1/2}^{0\nu}(^{136}\text{Xe})} \leq 2.78 (2.67), \quad (\text{Argonne NMEs}); \quad (3.14)$$

$$1.30 (1.16) \leq \frac{T_{1/2}^{0\nu}(^{76}\text{Ge})}{T_{1/2}^{0\nu}(^{136}\text{Xe})} \leq 4.43 (4.25), \quad (\text{CD-Bonn NMEs}), \quad (3.15)$$

The lower bound on $T_{1/2}^{0\nu}(^{76}\text{Ge})$ following from the EXO limit in the case of the Argonne NMEs obtained with $g_A = 1.25$ (1.0) reads:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 4.31 (4.22) \times 10^{25} \text{ y}. \quad (3.16)$$

This lower limit is by a factor of 2.27 bigger than the experimental lower limit quoted in eq. (3.2). It is also incompatible with the 4σ range of values of $T_{1/2}^{0\nu}(^{76}\text{Ge})$ found in [29]. The lower bound obtained using the CD-Bonn NMEs is less stringent:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 2.08 (1.85) \times 10^{25} \text{ y}. \quad (3.17)$$

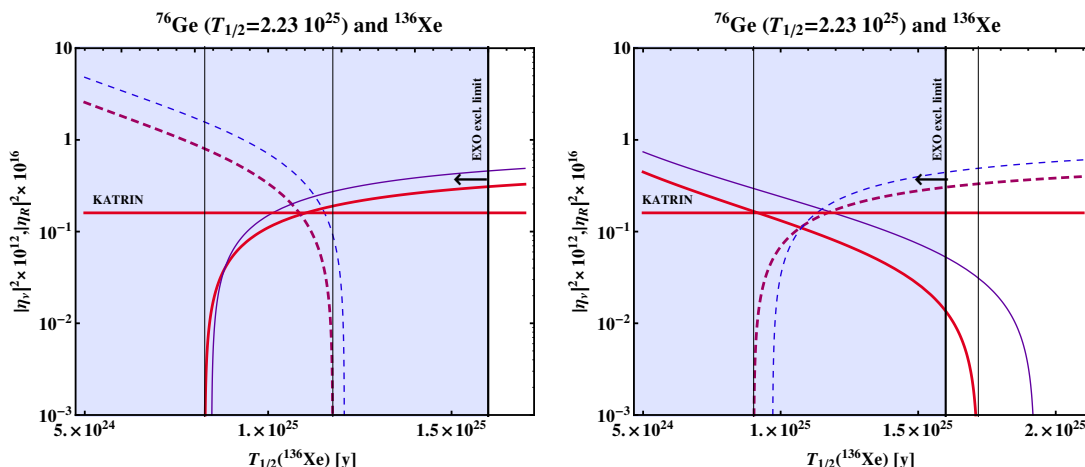


Figure 4. The values of $|\eta_\nu|^2$ (solid lines) and $|\eta_R|^2$ (dashed lines) obtained for $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25}$ y [29] as a function of $T_{1/2}^{0\nu}(^{136}\text{Xe})$, using the Argonne (left panel) and CD-Bonn (right panel) NMEs corresponding to $g_A = 1.25$ (thick lines) and $g_A = 1$ (thin lines). The region of physical (positive) solutions for $g_A = 1.25$ are delimited by the two vertical lines. The solid horizontal line corresponds to the prospective upper limit from the KATRIN experiment [33], while the thick solid vertical line indicates the EXO lower bound [25]. The gray areas correspond to excluded values of $|\eta_\nu|^2$ and $|\eta_R|^2$.

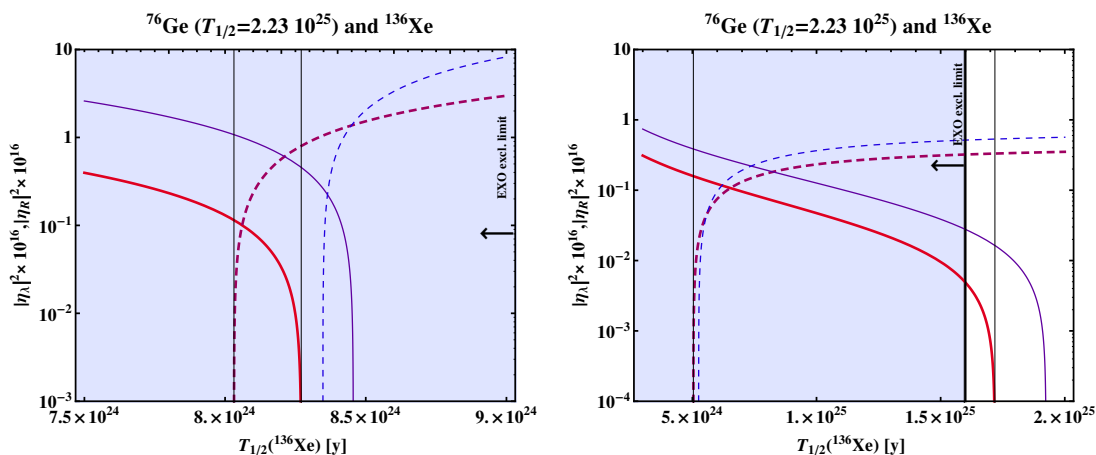


Figure 5. The same as in figure 4 but for the values of the rescaled parameters $|\eta_{\lambda'}|^2$ (solid lines) and $|\eta_R|^2$ (dashed lines).

In figure 5 we show that if $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23 \times 10^{25}$, the recent EXO lower limit allows positive (physical) solutions for the corresponding two LNV parameters $|\eta_{\lambda'}|^2$ and $|\eta_R|^2$ only for the CD-Bonn NMEs and for values of $|\eta_{\lambda'}|^2$ and $|\eta_R|^2$ lying in a very narrow interval.

We get similar results for the third pair of non-interfering mechanisms - the squark-neutrino exchange and the heavy RH neutrino exchange. Indeed, using the Argonne NMEs corresponding to $g_A = 1.25$ (1.0) we find for the positivity interval:

$$2.52 (2.40) \leq \frac{T_{1/2}^{0\nu}(^{76}\text{Ge})}{T_{1/2}^{0\nu}(^{136}\text{Xe})} \leq 2.70 (2.64), \quad (3.18)$$

The EXO lower bound in this case implies:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 4.03 (3.84) \times 10^{25} \text{ y}. \quad (3.19)$$

From the NMEs computed with the CD-Bonn potential we get

$$1.30 (1.16) \leq \frac{T_{1/2}^{0\nu}(^{76}\text{Ge})}{T_{1/2}^{0\nu}(^{136}\text{Xe})} \leq 2.95 (2.81), \quad (3.20)$$

and

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 2.08 (1.85) \times 10^{25} \text{ y}. \quad (3.21)$$

In the case the non-interfering LH and RH heavy Majorana neutrino exchanges, the NMEs for the two mechanisms coincide and the system of equation in (3.7) reduces to a relation between the half-lives of the two considered isotopes:

$$T_j = T_i \frac{G_i |M'_{i,N}|^2}{G_j |M'_{j,N}|^2}. \quad (3.22)$$

In this case the EXO lower bound implies the following lower limits on $T_j \equiv T_{1/2}^{0\nu}(^{76}\text{Ge})$ for the sets of NMEs we are considering for $g_A = 1.25$ (1.0):

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 4.31 (4.22) \times 10^{25} \text{ y} \quad (\text{Argonne NMEs}), \quad (3.23)$$

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 2.08 (1.85) \times 10^{25} \text{ y} \quad (\text{CD-Bonn NMEs}). \quad (3.24)$$

The range of positive solutions for the LNV parameters in equation (3.7) shifts towards larger values if T_i is increased. As we noticed in [1], if the experimentally determined interval of allowed values of the ratio T_j/T_i of the half-lives of the two isotopes considered, including all relevant uncertainties, lies outside the range of positive solutions for $|\eta_A|^2$ and $|\eta_B|^2$, one would be led to conclude that the $(\beta\beta)_{0\nu}$ -decay is not generated by the two mechanisms under discussion.

Assuming the half-lives of two isotopes, say, of ^{76}Ge and ^{136}Xe , $T_{1/2}^{0\nu}(^{76}\text{Ge}) \equiv T_1$ and $T_{1/2}^{0\nu}(^{136}\text{Xe}) \equiv T_2$, to be known, and $(\beta\beta)_{0\nu}$ -decay triggered by a pair of non-interfering mechanisms A and B , one can always use the physical solutions for $|\eta_{A,B}^{LFV}|^2(T_1, T_2)$, obtained using the two half-lives $T_{1,2}$ (in eq. (3.7)), to find the range of the half-life of a third isotope:

$$\frac{1}{T_3} = G_3 (|\eta_A(T_1, T_2)|^2 |M'_{3,A}|^2 + |\eta_B(T_1, T_2)|^2 |M'_{3,B}|^2), \quad (3.25)$$

In tables 2 and 3 we give numerical predictions based on this observation. Fixing the half-life of ^{76}Ge to $T_1 = 10^{26}$ y and assuming the ^{136}Xe half-life T_2 lies in an interval compatible with the existing constraints, the system of two equations is solved and the values of $|\eta_A|^2 > 0$ and $|\eta_B|^2 > 0$ thus obtained are used to get predictions for the half-life of a third isotope, in this case ^{82}Se , ^{100}Mo and ^{130}Te . The mechanisms considered are a) light and heavy RH Majorana neutrino exchanges (table 2) and b) gluino and heavy RH

<i>Argonne NMEs</i>	
Positive solutions	Predictions
$3.71(3.79) \cdot 10^{25} < T(^{136}\text{Xe}) < 5.27(5.42) \cdot 10^{25}$	$2.30(2.34) \cdot 10^{25} < T(^{82}\text{Se}) < 2.39(2.49) \cdot 10^{25}$
	$1.45(1.46) \cdot 10^{25} < T(^{100}\text{Mo}) < 1.80(1.76) \cdot 10^{25}$
	$1.76(1.78) \cdot 10^{25} < T(^{130}\text{Te}) < 2.44(2.49) \cdot 10^{25}$
<i>CD-Bonn NMEs</i>	
Positive solutions	Predictions
$4.04(4.35) \cdot 10^{25} < T(^{136}\text{Xe}) < 7.71(8.63) \cdot 10^{25}$	$2.30(2.33) \cdot 10^{25} < T(^{82}\text{Se}) < 2.39(2.48) \cdot 10^{25}$
	$1.44(1.45) \cdot 10^{25} < T(^{100}\text{Mo}) < 1.78(1.74) \cdot 10^{25}$
	$1.65(1.68) \cdot 10^{25} < T(^{130}\text{Te}) < 2.21(2.27) \cdot 10^{25}$

Table 2. Predictions using Argonne and CD-Bonn NMEs corresponding to $g_A=1.25$ ($g_A=1$ in parenthesis) in the case of two non-interfering mechanism: light and heavy RH Majorana neutrino exchanges. The physical solutions for $|\eta_\nu|^2$ and $|\eta_R|^2$ derived for given half-lives of ^{76}Ge and ^{136}Xe , are used to obtain predictions for the half-lives of ^{82}Se , ^{100}Mo and ^{130}Te . The ^{76}Ge half-life was set to $T(^{76}\text{Ge}) = 10^{26}$ yr, while the interval of values of the ^{136}Xe half-life was determine from the positivity conditions.

<i>Argonne NMEs</i>	
Positive solutions	Predictions
$3.60(3.74) \cdot 10^{25} < T(^{136}\text{Xe}) < 3.71(3.79) \cdot 10^{25}$	$2.27(2.32) \cdot 10^{25} < T(^{82}\text{Se}) < 2.30(2.34) \cdot 10^{25}$
	$1.43(1.459) \cdot 10^{25} < T(^{100}\text{Mo}) < 1.45(1.460) \cdot 10^{25}$
	$1.76(1.78) \cdot 10^{25} < T(^{130}\text{Te}) < 1.80(1.86) \cdot 10^{25}$
<i>CD-Bonn NMEs</i>	
Positive solutions	Predictions
$2.26(2.35) \cdot 10^{25} < T(^{136}\text{Xe}) < 7.71(8.63) \cdot 10^{25}$	$2.27(2.32) \cdot 10^{25} < T(^{82}\text{Se}) < 2.30(2.33) \cdot 10^{25}$
	$1.43(1.4542) \cdot 10^{25} < T(^{100}\text{Mo}) < 1.44(1.4543) \cdot 10^{25}$
	$1.65(1.68) \cdot 10^{25} < T(^{130}\text{Te}) < 1.75(1.81) \cdot 10^{25}$

Table 3. The same as in table 2 but for the gluino and RH heavy Majorana neutrino exchange mechanisms.

Majorana neutrino exchanges (table 3). It follows from the results shown in tables 2 and 3 that the intervals of allowed values of the half-lives of ^{82}Se , ^{100}Mo and ^{130}Te thus obtained i) are rather narrow,⁹ and ii) exhibit weak dependence on the NMEs used to derive them (within the sets of NMEs considered).

One can use eq. (3.25) and the lower bound, e.g., on $T_{1/2}^{0\nu}(^{136}\text{Xe})$ reported by the EXO experiment, to derive a lower bound on one of the half-lives involved in the study of two non-interfering mechanisms, say T_1 . Indeed, we can set $T_3 = T_{1/2}^{0\nu}(^{136}\text{Xe})$, in eq. (3.25), use the explicit form of the solutions for $|\eta_{A,B}^{LFV}|^2(T_1, T_2)$ and apply the existing EXO lower

⁹We note that the experimental lower bounds quoted in eq. (3.2) have to be taken into account since, in principle, they can further constrain the range of allowed values of $|\eta_A|^2$ and $|\eta_B|^2$ and of the half-life of the third isotope of interest.

(T_1, T_2)	Argonne $g_A=1.25$ (1.0)	CD-Bonn $g_A=1.25$ (1.0)
$^{130}\text{Te}-^{76}\text{Ge}$	$T(^{130}\text{Te})_i$ 7.40 (7.35) · 10^{24}	$T(^{130}\text{Te})_i$ 3.43 (3.11) · 10^{24}
$^{100}\text{Mo}-^{76}\text{Ge}$	$T(^{100}\text{Mo})_i$ 5.45 (5.19) · 10^{24}	$T(^{130}\text{Te})_i$ 3.00 (2.70) · 10^{24}
$^{82}\text{Se}-^{76}\text{Ge}$	$T(^{82}\text{Se})_i$ 7.25 (7.37) · 10^{24}	$T(^{82}\text{Se})_i$ 4.76 (4.32) · 10^{24}

Table 4. Lower bound on T_1 from eq. (3.29) using the EXO limit on $T_{1/2}^{0\nu}(^{136}\text{Xe})$, eq. (3.1), and the Argonne and CD-Bonn NMEs corresponding to $g_A=1.25$ ($g_A=1$), in the case of two non-interfering mechanisms — light and heavy RH Majorana neutrino exchanges. See text for details.

bound. We get:

$$\frac{1}{T_3} = \frac{D_1}{NT_1} + \frac{D_2}{NT_2} < \frac{1}{1.6 \times 10^{25} \text{ y}}, \quad (3.26)$$

where

$$D_1 = \frac{G_3}{G_1} \left(|M'_{2,A}|^2 |M'_{3,B}|^2 - |M'_{3,A}|^2 |M'_{2,B}|^2 \right),$$

$$D_2 = \frac{G_3}{G_2} \left(|M'_{3,A}|^2 |M'_{1,B}|^2 - |M'_{1,A}|^2 |M'_{3,B}|^2 \right),$$

and

$$N = |M'_{2,A}|^2 |M'_{1,B}|^2 - |M'_{1,A}|^2 |M'_{2,B}|^2. \quad (3.27)$$

Using further the positivity constraint given in eq. (3.7),

$$a T_1 \leq T_2 \leq b T_1, \quad (3.28)$$

where $b \equiv |M'_{1,A}|^2 / |M'_{2,A}|^2 > a \equiv |M'_{1,B}|^2 / |M'_{2,B}|^2$, we get¹⁰ the following lower limit on T_1 from eq. (3.26):

$$T_1 \geq T_3 \left(\frac{D_1}{N} + \frac{D_2}{bN} \right) > 1.6 \times 10^{25} \text{ y} \left(\frac{D_1}{N} + \frac{D_2}{bN} \right). \quad (3.29)$$

This lower bound on T_1 depends only on T_3 and on the NMEs $|M'_{i,A}|^2$ and $|M'_{j,B}|^2$, $i, j = 1, 2, 3$. We give examples of predictions based on the eq. (3.29) in tables 4 and 5. We notice that, for the NMEs used in the present study, the EXO lower limit on $T_{1/2}^{0\nu}(^{136}\text{Xe})$ sets a lower bound on the half-lives of the other isotopes considered by us that usually exceed their respective current experimental lower bounds.

3.2 Discriminating between different pairs of non-interfering mechanisms

The first thing to notice is that, as it follows from table 1, for each of the four different mechanisms of $(\beta\beta)_{0\nu}$ -decay considered, the relative difference between NMEs of the decays of ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te does not exceed approximately 10%: $(M'_{j,X} - M'_{i,X}) / (0.5(M'_{j,X} + M'_{i,X})) \lesssim 0.1$, where $i \neq j = ^{76}\text{Ge}, ^{82}\text{Se}, ^{100}\text{Mo}, ^{130}\text{Te}$, and

¹⁰The inequality in eq. (3.28) was derived assuming that $D_2/N > 0$. In the case of $D_2/N < 0$ one has to interchange a and b in it.

(T_1, T_2)	Argonne $g_A=1.25$ (1.0)	CD-Bonn $g_A=1.25$ (1.0)
$^{130}\text{Te}-^{76}\text{Ge}$	$T(^{130}\text{Te})_i 7.59 (7.53) \cdot 10^{24}$	$T(^{130}\text{Te})_i 3.43 (3.11) \cdot 10^{24}$
$^{100}\text{Mo}-^{76}\text{Ge}$	$T(^{100}\text{Mo})_i 6.25 (6.16) \cdot 10^{24}$	$T(^{130}\text{Te})_i 3.00 (2.70) \cdot 10^{24}$
$^{82}\text{Se}-^{76}\text{Ge}$	$T(^{82}\text{Se})_i 9.90 (9.87) \cdot 10^{24}$	$T(^{82}\text{Se})_i 4.76 (4.32) \cdot 10^{24}$

Table 5. The same as in table 4 for the gluino and heavy RH Majorana neutrino exchange mechanisms. See text for details.

X denotes any one of the four mechanisms discussed. As was shown in [1], this leads to degeneracies between the positivity intervals of values of the ratio of the half-lives of any two given of the indicated four isotopes, corresponding to the different pairs of mechanisms inducing the $(\beta\beta)_{0\nu}$ -decay. The degeneracies in question make it practically impossible to distinguish between the different pairs of $(\beta\beta)_{0\nu}$ -decay mechanisms, considered in [1] and in the present article, using data on the half-lives of two or more of the four nuclei ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te . At the same time, it is possible, in principle, to exclude them all using data on the half-lives of at least two of the indicated four nuclei [1].

In contrast, the NMEs for the $(\beta\beta)_{0\nu}$ -decay of ^{136}Xe , corresponding to each of the four different mechanisms we are considering are by a factor of $\sim (1.3 - 2.5)$ smaller than the $(\beta\beta)_{0\nu}$ -decay NMEs of the other four isotopes listed above: $(M'_{j,X}{}^{0\nu} - M'_{i,X}{}^{0\nu})/M'_{i,X}{}^{0\nu} \cong (0.3 - 1.5)$, where $i = ^{136}\text{Xe}$ and $j = ^{76}\text{Ge}, ^{82}\text{Se}, ^{100}\text{Mo}, ^{130}\text{Te}$ (see figures 6 and 7). As a consequence, using data on the half-life of ^{136}Xe as input in determining the positivity interval of values of the half-life of any second isotope lifts to a certain degree the degeneracy of the positivity intervals corresponding to different pairs of non-interfering mechanisms. This allows, in principle, to draw conclusions about the pair of mechanisms possibly inducing the $(\beta\beta)_{0\nu}$ -decay from data on the half-lives of ^{136}Xe and a second isotope which can be, e.g., any of the four considered above, ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te .

To be more specific, it follows from eqs. (3.10), (3.14), (3.18), (3.22) and table 1 that if the Argonne (CD-Bonn) NMEs derived for $g_A = 1.25$ (1.0) are correct, all four pairs of mechanisms of $(\beta\beta)_{0\nu}$ -decay discussed by us will be disfavored, or ruled out, if it is established experimentally that $R(^{76}\text{Ge}, ^{136}\text{Xe}) > 2.8$ (4.5) or that $R(^{76}\text{Ge}, ^{136}\text{Xe}) < 1.8$ (1.1), where $R(^{76}\text{Ge}, ^{136}\text{Xe}) \equiv T_{1/2}^{0\nu}(^{76}\text{Ge})/T_{1/2}^{0\nu}(^{136}\text{Xe})$. Further, assuming the validity of the Argonne NMEs, one would conclude that the light and heavy RH Majorana neutrino exchanges are the only possible pair of mechanisms operative in $(\beta\beta)_{0\nu}$ -decay if it is found experimentally that $1.9 \leq R(^{76}\text{Ge}, ^{136}\text{Xe}) < 2.4$. For $1.9 \leq R(^{76}\text{Ge}, ^{136}\text{Xe}) < 2.6$, i) the gluino and RH Majorana neutrino exchanges, and ii) the LH and RH heavy Majorana neutrino exchanges, will be disfavored or ruled out. One finds similar results using the CD-Bonn NMEs. The numbers we quote in this paragraph should be considered as illustrative only. In a realistic analysis one has to take into account the various relevant experimental and theoretical uncertainties.

We analyze next the possibility to discriminate between two pairs of non-interfering mechanisms triggering the $(\beta\beta)_{0\nu}$ -decay when the pairs share one mechanism. Given three different non-interfering mechanisms A , B and C , we can test the hypothesis of the $(\beta\beta)_{0\nu}$ -

decay induced by the pairs i) $A+B$ or ii) $C+B$, using the half-lives of the same two isotopes. As a consequence of the fact that B is common to both pairs of mechanisms, the numerators of the expressions for $|\eta_A|^2$ and $|\eta_C|^2$, as it follows from eq. (3.7), coincide. Correspondingly, using the half-lives of the same two isotopes would allow us to distinguish, in principle, between the cases i) and ii) if the denominators in the expressions for the solutions for $|\eta_A|^2$ and $|\eta_C|^2$ have opposite signs. Indeed, in this case the physical solutions for $|\eta_A|^2$ in the case i) and $|\eta_C|^2$ in the case ii) will lie either in the positivity intervals (3.7) and (3.8), respectively, or in the intervals (3.8) and (3.7). Thus, the positivity solution intervals for $|\eta_A|^2$ and $|\eta_C|^2$ would not overlap, except for the point corresponding to a value of the second isotope half-life where $\eta_A = \eta_C = 0$. This would allow, in principle, to discriminate between the two considered pairs of mechanisms.

It follows from the preceding discussion that in order to be possible to discriminate between the pairs $A + B$ and $C + B$ of non-interfering mechanisms of $(\beta\beta)_{0\nu}$ -decay, the following condition has to be fulfilled:

$$\frac{\text{Det} \begin{pmatrix} |M'_{i,A}|^2 & |M'_{i,B}|^2 \\ |M'_{j,A}|^2 & |M'_{j,B}|^2 \end{pmatrix}}{\text{Det} \begin{pmatrix} |M'_{i,C}|^2 & |M'_{i,B}|^2 \\ |M'_{j,C}|^2 & |M'_{j,B}|^2 \end{pmatrix}} = \frac{|M'_{i,A}|^2|M'_{j,B}|^2 - |M'_{i,B}|^2|M'_{j,A}|^2}{|M'_{i,C}|^2|M'_{j,B}|^2 - |M'_{i,B}|^2|M'_{j,C}|^2} < 0. \quad (3.30)$$

This condition is satisfied if one of the following two sets of inequalities holds:

$$I) \quad \frac{M'_{j,C} - M'_{i,C}}{M'_{i,C}} < \frac{M'_{j,B} - M'_{i,B}}{M'_{i,B}} < \frac{M'_{j,A} - M'_{i,A}}{M'_{i,A}}, \quad (3.31)$$

$$II) \quad \frac{M'_{j,A} - M'_{i,A}}{M'_{i,A}} < \frac{M'_{j,B} - M'_{i,B}}{M'_{i,B}} < \frac{M'_{j,C} - M'_{i,C}}{M'_{i,C}}. \quad (3.32)$$

One example of a possible application of the preceding results is provided by the mechanisms of light Majorana neutrino exchange (A), RH heavy Majorana neutrino exchange (B) and gluino exchange (C) and the Argonne NMEs. We are interested in studying cases involving ^{136}Xe since, as it was already discussed earlier, the NMEs of ^{136}Xe differ significantly from those of the lighter isotopes such as ^{76}Ge (see table 1). Indeed, as can be shown, it is possible, in principle, to discriminate between the two pairs $A + B$ and $C + B$ of the three mechanisms indicated above if we combine data on the half-life of ^{136}Xe with those on the half-life of one of the four isotopes ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te , and use the Argonne NMEs in the analysis. In this case the inequalities (3.31) are realized, as can be seen in figure 6, where we plot the relative differences $(M'_j - M'_i)/M'_i$ for the Argonne NMEs where the indices i and j refer respectively to ^{136}Xe and to one of the four isotopes ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te . In the case of the CD-Bonn NMEs (figure 7), the inequalities (3.31) or (3.32) do not hold for the pairs of mechanisms considered. The inequalities given in eq. (3.31) hold, as it follows from figure 7, if, e.g., the mechanisms A, B and C are respectively the heavy RH Majorana neutrino exchange, the light Majorana neutrino exchange and the gluino exchange.

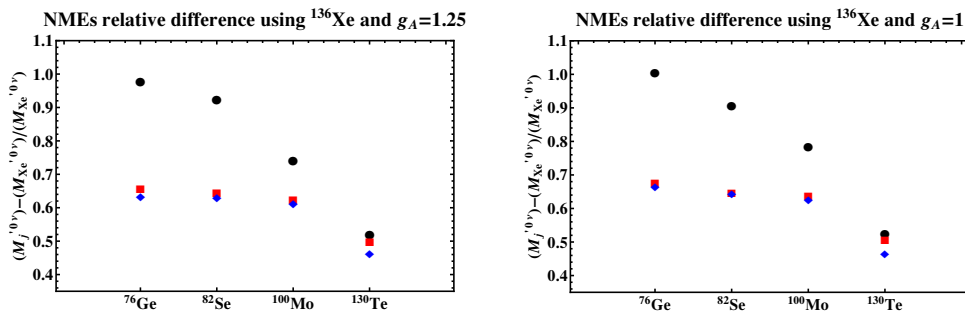


Figure 6. The relative differences between the Argonne NMEs $(M_j^{0\nu} - M_i^{0\nu})/M_i^{0\nu}$, where $i=^{136}\text{Xe}$ and $j = ^{76}\text{Ge}, ^{82}\text{Se}, ^{100}\text{Mo}, ^{130}\text{Te}$, for $g_A = 1.25$ (left panel) and $g_A = 1$ (right panel) and for three different non-interfering mechanisms: light Majorana neutrino exchange (circles), RH heavy Majorana neutrino exchange (squares) and gluino exchange (diamonds). See text for details.

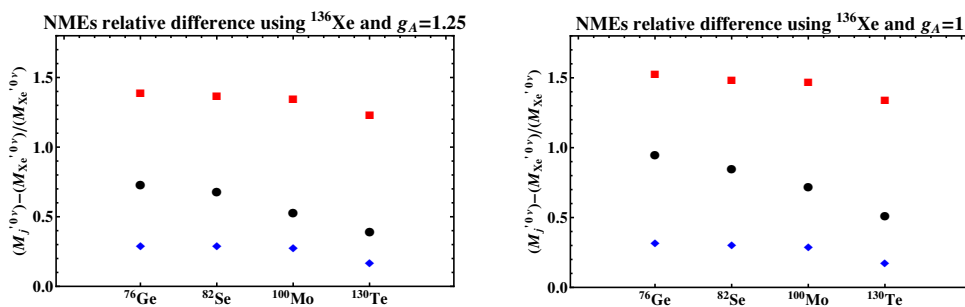


Figure 7. The same as in figure 6 for the CD-Bonn NMEs. See text for details.

The preceding considerations are illustrated graphically in figures 8 and 9. In figure 8 we use $T_i \equiv T_{1/2}^{0\nu}(^{76}\text{Ge})$ and $T_j \equiv T_{1/2}^{0\nu}(^{136}\text{Xe})$ and the Argonne (left panel) and CD-Bonn (right panel) NMEs for the decays of ^{76}Ge and ^{136}Xe to show the possibility of discriminating between the two pairs of non-interfering mechanisms considered earlier: i) light Majorana neutrino exchange and heavy RH Majorana neutrino exchange (RHN) and ii) heavy RH Majorana neutrino exchange and gluino exchange. The ^{76}Ge half-life is set to $T_i = 5 \times 10^{25}$ y, while that of ^{136}Xe , T_j , is allowed to vary in a certain interval. The solutions for the three LNV parameters corresponding to the three mechanisms considered, $|\eta_\nu|^2$, $|\eta_R|^2$ and $|\eta_{\lambda'}|^2$, obtained for the chosen value of T_i and interval of values of T_j , are shown as functions of T_j . As is clearly seen in the left panel of figure 8, if $|\eta_\nu|^2$, $|\eta_R|^2$ and $|\eta_{\lambda'}|^2$ are obtained using the Argonne NMEs, the intervals of values of T_j for which one obtains the physical positive solutions for $|\eta_\nu|^2$ and $|\eta_{\lambda'}|^2$, do not overlap. This makes it possible, in principle, to determine which of the two pairs of mechanisms considered (if any) is inducing the $(\beta\beta)_{0\nu}$ -decay. The same result does not hold if one uses the CD-Bonn NMEs in the analysis, as is illustrated in the right panel of figure 8. In this case none of the inequalities (3.31) and (3.32) is fulfilled, the intervals of values of T_j for which one obtains physical solutions for $|\eta_\nu|^2$ and $|\eta_{\lambda'}|^2$ overlap and the discrimination between the two pairs of mechanisms is problematic.

We show in figure 9 that the features of the solutions for $|\eta_\nu|^2$ and $|\eta_{\lambda'}|^2$ we have discussed above, which are related to the values of the relevant NMEs, do not change if

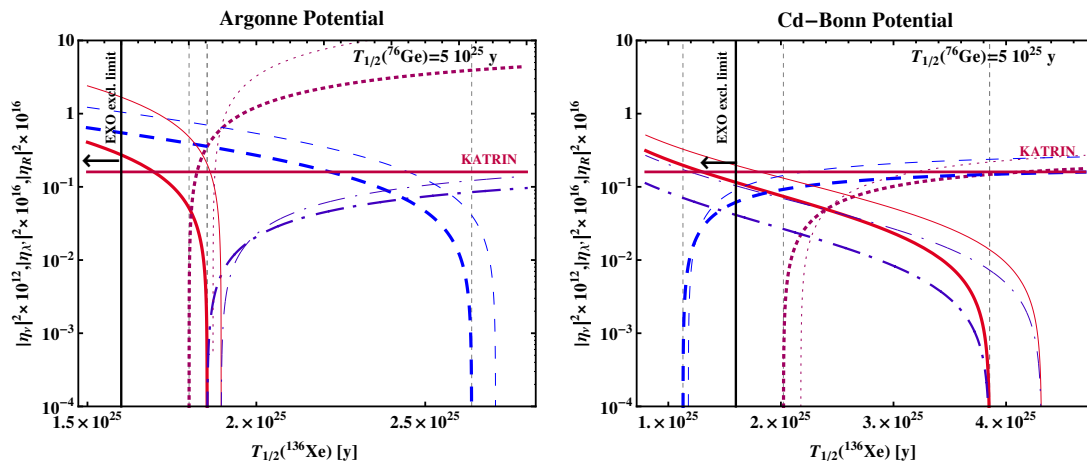


Figure 8. Solutions for the LNV parameters corresponding to two pairs of non-interfering mechanisms: i) $|\eta_\nu|^2$ and $|\eta_R|^2$ (dot-dashed and dashed lines) and ii) $|\eta_{\lambda'}|^2$ and $|\eta_R|^2$ (solid and dotted lines). The solutions are obtained by fixing $T_i = T_{1/2}^{0\nu}(^{76}\text{Ge}) = 5 \times 10^{25}$ y and letting free $T_j = T_{1/2}^{0\nu}(^{136}\text{Xe})$ and using the sets of Argonne (left panel) and CD-Bonn (right panel) NMEs calculated for $g_A = 1.25$ (thick lines) and $g_A = 1$ (thin lines). The range of positive solutions in the case of Argonne NMEs and $g_A = 1.25$ is delimited by the two vertical dashed lines. The horizontal solid line corresponds to the prospective upper limit $|\langle m \rangle| \leq 0.2$ eV [33]. The thick solid vertical line indicates the EXO lower limit on $T_{1/2}^{0\nu}(^{136}\text{Xe})$ [25]. See text for details.

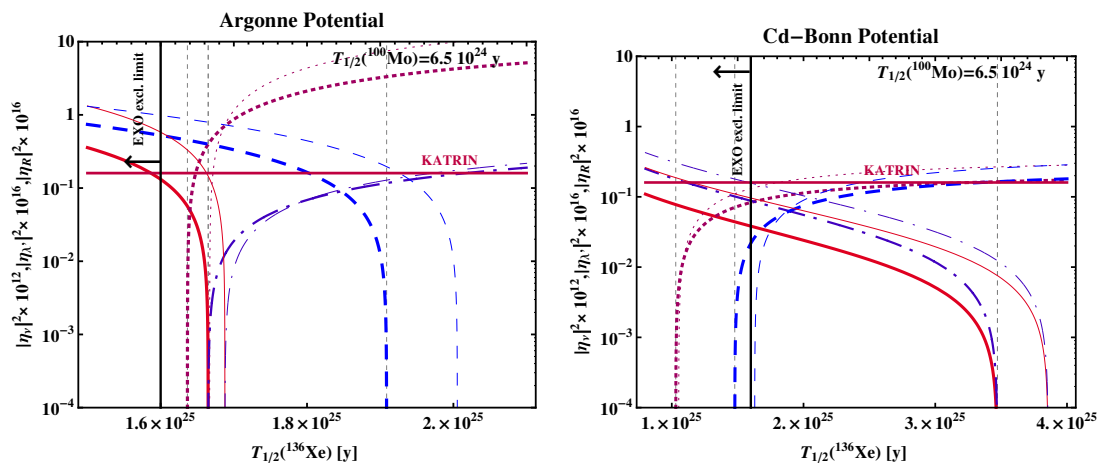


Figure 9. Solutions for the LNV parameters of two pairs of non-interfering $(\beta\beta)_{0\nu}$ -decay mechanisms i) $|\eta_\nu|^2$ and $|\eta_R|^2$ (dot-dashed and dashed lines) and ii) $|\eta_{\lambda'}|^2$ and $|\eta_R|^2$ (solid and dotted lines) obtained by fixing $T_i = T_{1/2}^{0\nu}(^{100}\text{Mo}) = 6.5 \times 10^{24}$ yr and letting free $T_j = T_{1/2}^{0\nu}(^{136}\text{Xe})$. The other notations are the same as in figure 8. See text for details.

one uses in the analysis the half-lives and NMEs of ^{136}Xe and of another lighter isotope instead of ^{76}Ge , namely, of ^{100}Mo .

3.3 Two interfering mechanisms

We analyze in the present section the possibility of $(\beta\beta)_{0\nu}$ -decay induced by two interfering CP-non-conserving mechanisms. This case is characterized by three parameters: the

absolute values and the relative phase of the two LNV parameters associated with the two mechanisms. They can be determined, in principle, from data on the half-lives of three isotopes, T_i , $i = 1, 2, 3$. Given $T_{1,2,3}$ and denoting by A and B the two mechanisms, one can set a system of three linear equations in three unknowns, the solution of which reads [1]:

$$|\eta_A|^2 = \frac{D_i}{D}, \quad |\eta_B|^2 = \frac{D_j}{D}, \quad z \equiv 2 \cos \alpha |\eta_A| |\eta_B| = \frac{D_k}{D}, \quad (3.33)$$

where D , D_i , D_j and D_k are the following determinants:

$$D = \begin{vmatrix} (M'_{i,A})^2 & (M'_{i,B})^2 & M'_{i,B} M'_{i,A} \\ (M'_{j,A})^2 & (M'_{j,B})^2 & M'_{j,B} M'_{j,A} \\ (M'_{k,A})^2 & (M'_{k,B})^2 & M'_{k,B} M'_{k,A} \end{vmatrix}, \quad D_i = \begin{vmatrix} i/T_i G_i & (M'_{i,B})^2 & M'_{i,B} M'_{i,A} \\ i/T_j G_j & (M'_{j,B})^2 & M'_{j,B} M'_{j,A} \\ i/T_k G_k & (M'_{k,B})^2 & M'_{k,B} M'_{k,A} \end{vmatrix}, \quad (3.34)$$

$$D_j = \begin{vmatrix} (M'_{i,A})^2 & i/T_i G_i & M'_{i,B} M'_{i,A} \\ (M'_{j,A})^2 & i/T_j G_j & M'_{j,B} M'_{j,A} \\ (M'_{k,A})^2 & i/T_k G_k & M'_{k,B} M'_{k,A} \end{vmatrix}, \quad D_k = \begin{vmatrix} (M'_{i,A})^2 & (M'_{i,B})^2 & i/T_i G_i \\ (M'_{j,A})^2 & (M'_{j,B})^2 & i/T_j G_j \\ (M'_{k,A})^2 & (M'_{k,B})^2 & i/T_k G_k \end{vmatrix}. \quad (3.35)$$

As in the case of two non-interfering mechanisms, the LNV parameters must be non-negative $|\eta_A|^2 \geq 0$ and $|\eta_B|^2 \geq 0$, and in addition the interference term must satisfy the following condition:

$$-2|\eta_A| |\eta_B| \leq 2 \cos \alpha |\eta_A| |\eta_B| \leq 2|\eta_A| |\eta_B|. \quad (3.36)$$

These conditions will be called from here on ‘‘positivity conditions’’.

Using the positivity conditions it is possible to determine the interval of positive solutions for one of the three half-life, e.g., T_k , if the values of the other two half-lives in the equations have been measured and are known. The condition on the interference term in equation (3.7) can considerably reduce the interval of values of T_k where $|\eta_A|^2 \geq 0$ and $|\eta_B|^2 \geq 0$. In table 6 we give examples of the constraints on T_k following from the positivity conditions for three different pairs of interfering mechanisms: light Majorana neutrino and supersymmetric gluino exchange; light Majorana neutrino exchange and heavy LH Majorana neutrino exchange; gluino exchange and heavy LH Majorana neutrino exchange. It follows from the results shown in table 6, in particular, that when $T(^{76}\text{Ge})$ is set to $T(^{76}\text{Ge}) = 2.23 \times 10^{25}; 10^{26}$ y, but $T(^{130}\text{Te})$ is close to the current experimental lower limit, the positivity constraint intervals of values of $T(^{136}\text{Xe})$ for the each of the three pairs of interfering mechanisms considered are incompatible with the EXO lower bound on $T(^{136}\text{Xe})$, eq. (3.1).

We consider next a case in which the half-life of ^{136}Xe is one of the two half-lives assumed to have been experimentally determined. The $(\beta\beta)_{0\nu}$ -decay is supposed to be triggered by light Majorana neutrino and gluino exchange mechanisms with LFV parameters $|\eta_\nu|^2$ and $|\eta_\chi|^2$. We use in the analysis the half-lives of ^{76}Ge , ^{136}Xe and ^{130}Te , which will be denoted for simplicity respectively as T_1 , T_2 and T_3 . Once the experimental bounds on T_i , $i = 1, 2, 3$, given in eq. (3.2), are taken into account, the conditions for destructive

$T_{1/2}^{0\nu}[y](\text{fixed})$	$T_{1/2}^{0\nu}[y](\text{fixed})$	Allowed Range
$T(Ge) = 2.23 \cdot 10^{25**}$	$T(Te) = 3 \cdot 10^{24}$	$2.95 \cdot 10^{24} \leq T(Xe) \leq 5.65 \cdot 10^{24}$
$T(Ge) = 10^{26**}$	$T(Te) = 3 \cdot 10^{24}$	$3.43 \cdot 10^{24} \leq T(Xe) \leq 4.66 \cdot 10^{24}$
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Te) = 3 \cdot 10^{25}$	$1.74 \cdot 10^{25} \leq T(Xe) \leq 1.66 \cdot 10^{26}$
$T(Ge) = 10^{26}$	$T(Te) = 3 \cdot 10^{25}$	$2.58 \cdot 10^{25} \leq T(Xe) \leq 6.90 \cdot 10^{25}$

$T_{1/2}^{0\nu}[y](\text{fixed})$	$T_{1/2}^{0\nu}[y](\text{fixed})$	Allowed Range
$T(Ge) = 2.23 \cdot 10^{25**}$	$T(Te) = 3 \cdot 10^{24}$	$4.93 \cdot 10^{24} \leq T(Xe) \leq 6.21 \cdot 10^{24}$
$T(Ge) = 10^{26**}$	$T(Te) = 3 \cdot 10^{24}$	$5.23 \cdot 10^{24} \leq T(Xe) \leq 5.83 \cdot 10^{24}$
$T(Ge) = 2.23 \cdot 10^{25}$	$T(Te) = 3 \cdot 10^{25}$	$3.95 \cdot 10^{25} \leq T(Xe) \leq 8.25 \cdot 10^{25}$
$T(Ge) = 10^{26}$	$T(Te) = 3 \cdot 10^{25}$	$4.68 \cdot 10^{25} \leq T(Xe) \leq 6.61 \cdot 10^{25}$

$T_{1/2}^{0\nu}[y](\text{fixed})$	$T_{1/2}^{0\nu}[y](\text{fixed})$	Allowed Range
$T(Ge) = 2.23 \cdot 10^{25**}$	$T(Te) = 3 \cdot 10^{24}$	$5.59 \cdot 10^{23} \leq T(Xe) \leq 1.26 \cdot 10^{25}$
$T(Ge) = 10^{26*}$	$T(Te) = 3 \cdot 10^{24}$	$1.21 \cdot 10^{24} \leq T(Xe) \leq 4.71 \cdot 10^{25}$
$T(Ge) = 2.23 \cdot 10^{25**}$	$T(Te) = 3 \cdot 10^{25}$	$1.05 \cdot 10^{24} \leq T(Xe) \leq 2.42 \cdot 10^{24}$
$T(Ge) = 10^{26*}$	$T(Te) = 3 \cdot 10^{25}$	$3.32 \cdot 10^{24} \leq T(Xe) \leq 2.16 \cdot 10^{25}$

Table 6. Ranges of the half-life of ^{136}Xe for different fixed values of the half-lives of ^{76}Ge and ^{130}Te in the case of three pairs of interfering mechanisms: light Majorana neutrino exchange and gluino exchange (upper table); light Majorana and heavy LH Majorana neutrino exchanges (middle table); gluino exchange and and heavy LH Majorana neutrino exchange (lower table). The results shown are obtained with the “large basis” $g_A = 1.25$ Argonne NMEs. One star (two stars) indicate that the EXO bound constrains further (rules out) the corresponding solution.

interference, i.e., for $\cos \alpha < 0$, are given by:

$$z < 0 : \begin{cases} 1.9 \times 10^{25} < T_1 \leq 1.90T_2, & T_3 \geq \frac{9.64T_1T_2}{16.32T_1 + 8.59T_2}; \\ 1.90T_2 < T_1 \leq 2.78T_2, & T_3 > \frac{3.82T_1T_2}{6.33T_1 + 3.66T_2}; \\ T_1 > 2.78T_2, & T_3 \geq \frac{7.33T_1T_2}{11.94T_1 + 7.61T_2}, \end{cases} \quad (3.37)$$

where we have used the “large basis” $g_A = 1.25$ Argonne NMEs (see table 1). The conditions for constructive interference read:

$$z > 0 : \begin{cases} 1.90T_2 < T_1 \leq 2.29T_2, & \frac{9.64T_1T_2}{16.32T_1 + 8.59T_2} \leq T_3 \leq \frac{3.82T_1T_2}{6.33T_1 + 3.66T_2}; \\ 2.29T_2 < T_1 < 2.78T_2, & \frac{7.33T_1T_2}{11.94T_1 + 7.61T_2} \leq T_3 \leq \frac{3.82T_1T_2}{6.33T_1 + 3.66T_2}. \end{cases} \quad (3.38)$$

If we set, e.g., the ^{76}Ge half-life to the value claimed in [29] $T_1 = 2.23 \times 10^{25}$ y, we find that only destructive interference between the contributions of the two mechanisms considered in the $(\beta\beta)_{0\nu}$ -decay rate, is possible. Numerically we get in this case

$$T_3 > \frac{3.44T_2}{5.82 + 1.37 \times 10^{-25}T_2}. \quad (3.39)$$

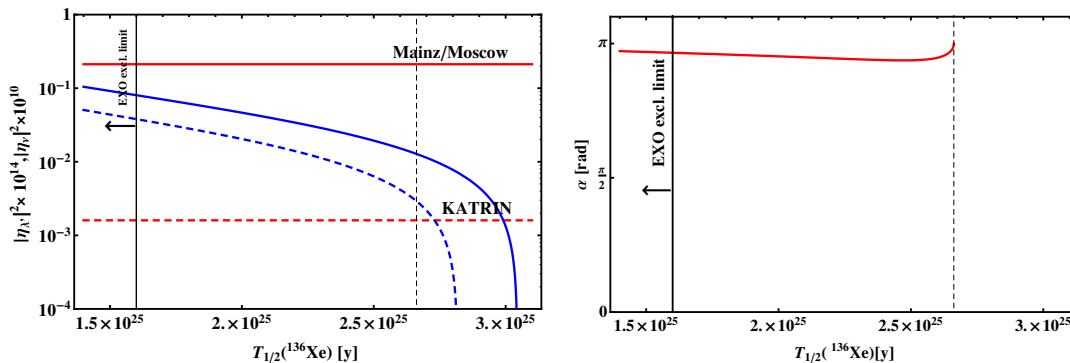


Figure 10. Left panel: the values of $|\eta_\nu|^2 \times 10^{10}$ (thick solid line) and $|\eta_{\nu'}|^2 \times 10^{14}$ (dotted line), obtained as solutions of the system of equations (3.4) for fixed values of $T(^{76}\text{Ge}) = 2.23 \times 10^{25}$ y and $T(^{130}\text{Te}) = 10^{25}$ y, and letting $T_{1/2}^{0\nu}(^{136}\text{Xe})$ free. The physical allowed regions correspond to the areas within the two vertical lines. Right panel: the values of the phase α in the allowed interval of values of $T_{1/2}^{0\nu}(^{136}\text{Xe})$, corresponding to physical solutions for $|\eta_\nu|^2$ and $|\eta_{\nu'}|^2$. In this case $\cos \alpha < 0$ and the interference is destructive. See text for details.

For $1.37 \times 10^{-25} T_2 \ll 5.82$ one finds:

$$T(^{130}\text{Te}) \gtrsim 0.59 T(^{136}\text{Xe}) \gtrsim 9.46 \times 10^{24} \text{ y}, \quad (3.40)$$

where the last inequality has been obtained using the EXO lower bound on $T(^{136}\text{Xe})$. Constructive interference is possible for the pair of interfering mechanisms under discussion only if $T(^{76}\text{Ge}) \gtrsim 3.033 \times 10^{25}$ y.

The possibilities of destructive and constructive interference are illustrated in figures 10 and 11, respectively. In these figures the physical allowed regions, determined through the positivity conditions, correspond to the areas within the two vertical lines (the solutions must be compatible also with the existing lower limits given in eq. (3.2)). For instance, using the Argonne “large basis” NMEs corresponding to $g_A = 1.25$ and setting $T(^{76}\text{Ge}) = 2.23 \times 10^{25}$ y and $T(^{130}\text{Te}) = 10^{25}$ y, positive solutions are allowed only in the interval $1.60 \times 10^{25} \leq T(^{136}\text{Xe}) \leq 2.66 \times 10^{25}$ y (figure 10). As can be seen in figures 10 and 11, a constructive interference is possible only if $T_2 \equiv T(^{136}\text{Xe})$ lies in a relatively narrow interval and $T_3 \equiv T(^{130}\text{Te})$ is determined through the conditions in eq. (3.38).

Next, we would like to illustrate the possibility to distinguish between two pairs of interfering mechanisms i) A+B and ii) B+C, which share one mechanism, namely B, from the data on the half-lives of three isotopes. In this case we can set two systems of three equations, each one in three unknowns. We will denote the corresponding LNV parameters as i) $|\eta_A|^2$, $|\eta_B|^2$ and ii) $|\eta_B|^2$ and $|\eta_C|^2$, while the interference parameters will be denoted as i) z and ii) z' . Fixing two of the three half-lives, say T_i and T_j , the possibility to discriminate between the mechanisms A and C relies on the dependence of $|\eta_A|^2$ and $|\eta_C|^2$ on the third half-life, T_k . Given T_i and T_j , it will be possible to discriminate between the mechanisms A and C if the two intervals of values of T_k where $|\eta_A|^2 > 0$ and $|\eta_C|^2 > 0$, do not overlap. If, instead, the two intervals partially overlap, complete discrimination would be impossible, but there would be a large interval of values of T_k (or equivalently, positive solutions values of the LNV parameters) that can be excluded using present or

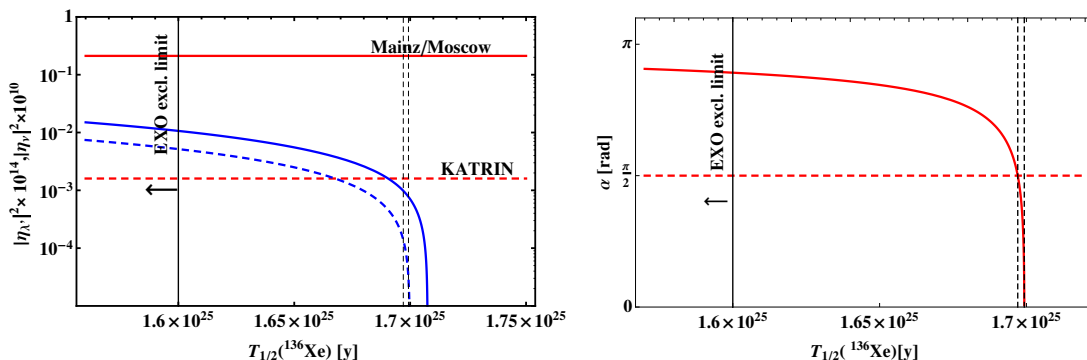


Figure 11. Left panel: the same as in figure 10 but for $T(^{76}\text{Ge}) = 3.5 \times 10^{25}$ y and $T(^{130}\text{Te}) = 8.0 \times 10^{25}$ y. The interval of values of $T_{1/2}^{0\nu}(^{130}\text{Xe})$ between i) the vertical solid and right dashed lines ii) the two vertical dashed lines, and iii) the vertical solid and left dashed lines, correspond respectively to i) physical (non-negative) solutions for $|\eta_\nu|^2$ and $|\eta_{\nu'}|^2$, ii) constructive interference ($z > 0$), and iii) destructive interference ($z < 0$). Right panel: the corresponding values of the phase α as a function of $T_{1/2}^{0\nu}(^{130}\text{Xe})$. Constructive interference is possible only for values of $T_{1/2}^{0\nu}(^{130}\text{Xe})$ between the two vertical dashed lines. See text for details.

future experimental data. In order to have non-overlapping positive solution intervals of T_K , corresponding to $|\eta_A|^2 > 0$ and $|\eta_C|^2 > 0$, the following inequality must hold:

$$\frac{(M'_{k,A}{}^{0\nu} M'_{i,B}{}^{0\nu} - M'_{i,A}{}^{0\nu} M'_{k,B}{}^{0\nu})(M'_{k,A}{}^{0\nu} M'_{j,B}{}^{0\nu} - M'_{j,A}{}^{0\nu} M'_{k,B}{}^{0\nu})}{(M'_{k,B}{}^{0\nu} M'_{i,C}{}^{0\nu} - M'_{i,B}{}^{0\nu} M'_{k,C}{}^{0\nu})(M'_{k,B}{}^{0\nu} M'_{j,C}{}^{0\nu} - M'_{j,B}{}^{0\nu} M'_{k,C}{}^{0\nu})} < 0. \quad (3.41)$$

The above condition can be satisfied only for certain sets of isotopes. Obviously, whether it is fulfilled or not depends on the values of the relevant NMEs. We will illustrate this on the example of an oversimplified analysis involving the light Majorana neutrino exchange, the heavy LH Majorana neutrino exchange and the gluino exchange as mechanisms A , B and C , respectively, and the half-lives of ^{76}Ge , ^{130}Te and ^{136}Xe : $T_1 \equiv T(^{76}\text{Ge})$, $T_2 \equiv T(^{130}\text{Te})$ and $T_3 \equiv T(^{136}\text{Xe})$. Fixing $T_1 = 2.23 \times 10^{25}$ y and $T_3 = 1.6 \times 10^{25}$ y (the EXO 90% C.L. lower limit), we obtain the results shown in figure 12. As it follows from figure 12, in the case of the Argonne NMEs (left panel), it is possible to discriminate between the standard light neutrino exchange and the gluino exchange mechanisms: the intervals of values of T_2 , where the positive solutions for the LNV parameters of the two pairs of interfering mechanisms considered occur, do not overlap. Further, the physical solutions for the two LNV parameters related to the gluino mechanism are excluded by the CUORICINO limit on $T(^{130}\text{Te})$ [28]. This result does not change with the increasing of T_3 . Thus, we are lead to conclude that for $T_3 \gtrsim 1.6 \times 10^{25}$ y and T_1 given by the value claimed in [29], of the two considered pairs of possible interfering $(\beta\beta)_{0\nu}$ -decay mechanisms, only the light and heavy LH Majorana neutrino exchanges can be generating the decay. The solution for $|\eta_\nu|^2$ must be compatible with the upper limit $|\langle m \rangle| \lesssim 2.3$ eV [32, 33], indicated with a solid horizontal line in figure 12. In the right panel of figure 12 we plot also the solutions obtained with the CD-Bonn NMEs. In this case is not possible to discriminate between the two considered pair of mechanisms since the condition in eq. (3.41) is not satisfied.

Another interesting example is the case in which A is the light Majorana neutrino exchange, B is the gluino exchange and C the heavy LH Majorana neutrino exchange,

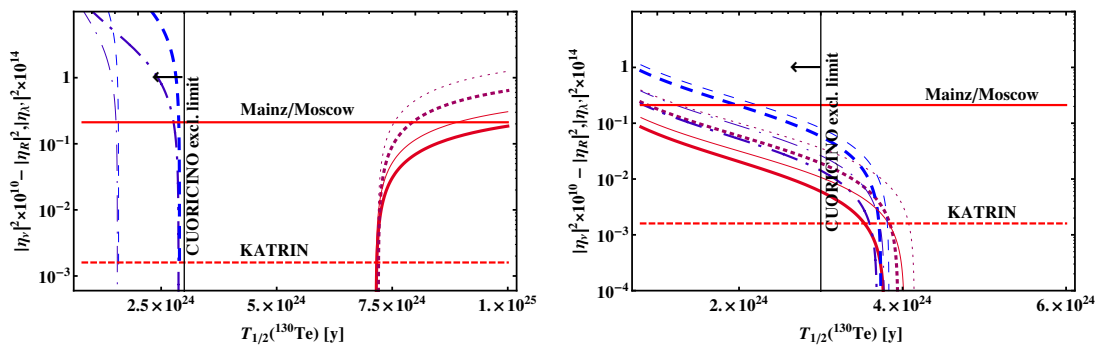


Figure 12. The parameters $|\eta_\nu|^2 \times 10^{10}$ (solid line) and $|\eta_L|^2 \times 10^{14}$ (dotted line) of the light and heavy LH Majorana neutrino exchange mechanisms, and $|\eta_{\lambda'}|^2 \times 10^{14}$ (dashed-dotted line) and $|\eta_L|^2 \times 10^{14}$ (dashed line) of the gluino and heavy LH Majorana neutrino exchange mechanisms, obtained from eq. (3.33) using the Argonne NMEs (left panel) and CD-Bonn NMEs (right panel), corresponding to $g_A = 1.25$ (thick lines) and $g_A = 1$ (thin lines), for $T_{1/2}^{0\nu}(^{76}Ge) = 2.23 \times 10^{25}$ y, $T_{1/2}^{0\nu}(^{136}Xe) = 1.60 \times 10^{25}$ y and letting $T_{1/2}^{0\nu}(^{130}Te)$ free. See text for details.

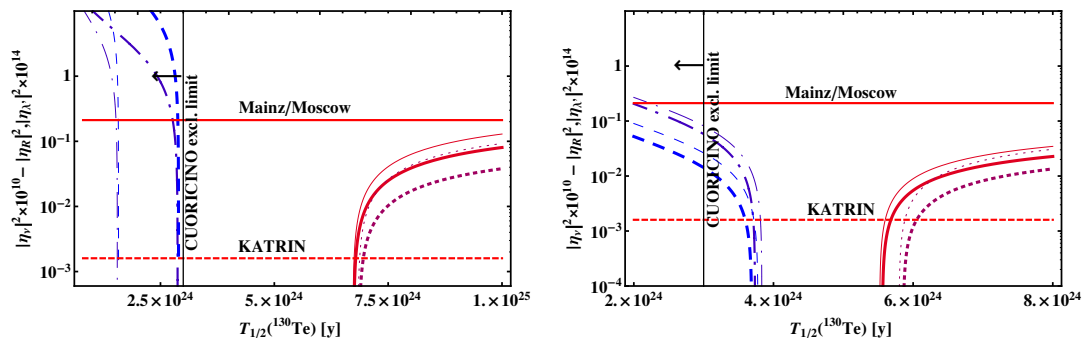


Figure 13. The same as in figure 12, but for i) $|\eta_\nu|^2 \times 10^{10}$ (thick solid line) and $|\eta_{\lambda'}|^2 \times 10^{14}$ (thick dotted line) of the light neutrino and gluino exchange mechanisms, and ii) $|\eta_L|^2 \times 10^{14}$ (thick dashed-dotted line) and $|\eta_{\lambda'}|^2 \times 10^{14}$ (thick dashed line) of the heavy LH Majorana neutrino and gluino exchange mechanisms, and using $T_{1/2}^{0\nu}(^{76}Ge) = 2.23 \times 10^{25}$ y and $T_{1/2}^{0\nu}(^{136}Xe) = 1.60 \times 10^{25}$ y. See text for details.

i.e., we try to discriminate between i) the light neutrino plus gluino exchange mechanisms, and ii) the heavy LH Majorana neutrino plus gluino exchange mechanisms. We fix, like in the previous case, the values for $T_1 = 2.23 \times 10^{25}$ y and $T_3 = 1.6 \times 10^{25}$ y. The results of this analysis are plotted in figure 13. Since the condition in eq. (3.41) is now satisfied for NMEs obtained either with the Argonne potential or with the CD-Bonn potential, in this case it is possible, in principle, to discriminate between the two pair of mechanisms independently of the set of NMEs used (within the sets considered by us). This result does not change with the increasing of T_3 . Hence, as far as T_1 is fixed to the value claimed in [29] and the limits in eq. (3.2) are satisfied, the two intervals of values of T_2 , in which the “positivity conditions” for i) $|\eta_\nu|^2$, $|\eta_{\lambda'}|^2$ and z , and for ii) $|\eta_{\lambda'}|^2$, $|\eta_N|^2$ and z' , are satisfied, are not overlapping (figure 13).

4 Conclusions and summary

We have investigated the possibility to discriminate between different pairs of CP non-conserving mechanisms inducing the neutrinoless double beta $(\beta\beta)_{0\nu}$ -decay by using data on $(\beta\beta)_{0\nu}$ -decay half-lives of nuclei with largely different nuclear matrix elements (NMEs). The mechanisms studied are: light Majorana neutrino exchange, heavy left-handed (LH) and heavy right-handed (RH) Majorana neutrino exchanges, lepton charge non-conserving couplings in SUSY theories with R -parity breaking giving rise to the “dominant gluino exchange” and the “squark-neutrino” mechanisms. Each of these mechanisms is characterized by a specific lepton number violating (LNV) parameter η_κ , where the index κ labels the mechanism. For the five mechanisms listed above we use the notations $\kappa = \nu, L, R, \lambda', \tilde{q}$, respectively. The parameter η_κ will be complex, in general, if the mechanism κ does not conserve the CP symmetry. The nuclei considered are ^{76}Ge , ^{82}Se , ^{100}Mo , ^{130}Te and ^{136}Xe . Four sets of nuclear matrix elements (NMEs) of the $(\beta\beta)_{0\nu}$ -decays of these five nuclei, derived within the Self-consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA), were employed in our analysis. They correspond to two types of nucleon-nucleon potentials - Argonne (“Argonne NMEs”) and CD-Bonn (“CD-Bonn NMEs”), and two values of the axial coupling constant $g_A = 1.25; 1.00$. Given the NMEs and the phase space factors of the decays, the half-life of a given nucleus depends on the parameters $|\eta_\kappa|^2$ of the mechanisms triggering the decay (eq. (3.7)).

In the present article we have considered in detail the cases of two non-interfering and two interfering mechanisms inducing the $(\beta\beta)_{0\nu}$ -decay. If two non-interfering mechanisms A and B cause the decay, the parameters $|\eta_A|^2$ and $|\eta_B|^2$ can be determined from data on the half-lives of two isotopes, T_1 and T_2 as solutions of a system of two linear equations. If the half-life of one isotope is known, say T_1 , the positivity condition which the solutions $|\eta_A|^2$ and $|\eta_B|^2$ must satisfy, $|\eta_A|^2 \geq 0$ and $|\eta_B|^2 \geq 0$, constrain the half-life of the second isotope T_2 (and the half-life of any other isotope for that matter) to lie in a specific interval [1]. If A and B are interfering mechanisms, $|\eta_A|^2$ and $|\eta_B|^2$ and the interference term parameter, $z_{AB} \equiv 2 \cos \alpha_{AB} |\eta_A \eta_B|$ which involves the cosine of an unknown relative phase α_{AB} of η_A and η_B , can be uniquely determined, in principle, from data on the half-lives of three nuclei, $T_{1,2,3}$. In this case, given the half-life of one isotope, say T_1 , the “positivity conditions” $|\eta_A|^2 \geq 0$, $|\eta_B|^2 \geq 0$ and $-1 \leq \cos \alpha_{AB} \leq 1$ constrain the half-life of a second isotope, say T_2 , to lie in a specific interval, and the half-life of a third one, T_3 , to lie in an interval which is determined by the value of T_1 and the interval of allowed values of T_2 .

For all possible pairs of non-interfering mechanisms we have considered (light, or heavy LH Majorana neutrino, and heavy RH Majorana neutrino exchanges; gluino, or squark-neutrino, and RH Majorana neutrino mechanisms), these “positivity condition” intervals of values of T_2 were shown in [1] to be essentially degenerate if T_1 and T_2 correspond to the half-lives of any pair of the four nuclei ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te . This is a consequence of the fact that for each of the five single mechanisms discussed, the NMEs for ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te differ relatively little [1, 16]: the relative difference between the NMEs of any two nuclei does not exceed 10%. One has similar degeneracy of “positivity condition” intervals T_2 and T_3 in the cases of two constructively interfering mechanisms (within the

set considered). These degeneracies might irreparably plague the interpretation of the $(\beta\beta)_{0\nu}$ -decay data if the process will be observed.

The NMEs for ^{136}Xe , results of calculations of which using the SRQRPA method are presented in the present article, differ significantly from those of ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te , being by a factor $\sim (1.3 - 2.5)$ smaller. As we have shown in the present article, this allows to lift to a certain degree the indicated degeneracies and to draw conclusions about the pair of non-interfering (interfering) mechanisms possibly inducing the $(\beta\beta)_{0\nu}$ -decay from data on the half-lives of ^{136}Xe and of at least one (two) more isotope(s) which can be, e.g., any of the four, ^{76}Ge , ^{82}Se , ^{100}Mo and ^{130}Te considered.

We have analyzed also the possibility to discriminate between two pairs of non-interfering (or interfering) $(\beta\beta)_{0\nu}$ -decay mechanisms when the pairs have one mechanism in common, i.e., between the mechanisms i) $A + B$ and ii) $C + B$, using the half-lives of the same two isotopes. We have derived the general conditions under which it would be possible, in principle, to identify which pair of mechanisms is inducing the decay (if any). We have shown that the conditions of interest are fulfilled, e.g., for the following two pairs of non-interfering mechanisms i) light neutrino exchange (A) and heavy RH Majorana neutrino exchange (B) and ii) gluino exchange (C) and heavy RH Majorana neutrino exchange (B), and for the following two pairs of interfering mechanisms i) light neutrino exchange (A) and heavy LH Majorana neutrino exchange (B) and ii) gluino exchange (C) and heavy LH Majorana neutrino exchange (B), if one uses the Argonne NMEs in the analysis. They are fulfilled for both the Argonne NMEs and CD-Bonn NMEs, e.g., for the following two pairs of interfering mechanisms i) light neutrino exchange (A) and gluino exchange (B), and ii) heavy LH Majorana neutrino exchange (C) and gluino exchange (B).

We have also exploited the implications of the EXO lower bound on the half-life of ^{136}Xe for the problem studied. We have shown, in particular, that for all four pairs of non-interfering mechanisms considered and the Argonne NMEs, the half-life of ^{76}Ge claimed in [29] is incompatible with the EXO lower bound on the half-life of ^{136}Xe [25]. If we use the CD-Bonn NMEs instead, we find that the result half-life of ^{76}Ge claimed in [29] is compatible with the EXO lower bound on the half-life of ^{136}Xe for values of the corresponding LNV parameters lying in extremely narrow intervals.

To summarize, the results obtained in the present article show that using the $(\beta\beta)_{0\nu}$ -decay half-lives of nuclei with largely different nuclear matrix elements would help resolving the problem of identifying the mechanisms triggering the decay.

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