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Addendum: Neutrino mass hierarchy determination using reactor antineutrinos

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Abstract: We update our study of neutrino mass hierarchy determination using a high statistics reactor $\bar{\nu}_e$ experiment in the light of the recent evidences of a relatively large non-zero value of θ_{13} from the Daya Bay and RENO experiments. We find that there are noticeable modifications in the results, which allow a relaxation in the detector's characteristics, such as the energy resolution and exposure, required to obtain a significant sensitivity to, or to determine, the neutrino mass hierarchy in such a reactor experiment.

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Contents

1 Introduction

Determining the type of neutrino mass spectrum, which can be with normal or inverted ordering (NO or IO) or hierarchy (see, e.g., [1]), is one of the most pressing and challenging problems of future research in neutrino physics. The recently measured relatively large value of the angle θ_{13} of the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) neutrino mixing matrix in the Daya Bay [2] and RENO [3] experiments[1] opens up the possibility of the neutrino mass hierarchy determination in an experiment with reactor $\bar{\nu}_e$. This possibility was discussed first in [6] and later was further investigated in [7–12] (see also [13]). It is based on the observation that for $\cos 2\theta_{12} \neq 0$ and $\sin\theta_{13} \neq 0$, θ_{12} being the solar neutrino mixing angle (see, e.g., [1]), the probabilities of $\bar{\nu}_e$ survival in the cases of NO (NH) and IO (IH) spectra differ [6, 14]: $P^{NH}(\bar{\nu}_e \to \bar{\nu}_e) \neq P^{IH}(\bar{\nu}_e \to \bar{\nu}_e)$, and $|P^{NH}(\bar{\nu}_e \to \bar{\nu}_e) - P^{IH}(\bar{\nu}_e \to \bar{\nu}_e)| \propto \sin^2 2\theta_{13} \cos 2\theta_{12}$. For sufficiently large $|\cos 2\theta_{12}|$ and $\sin^2\theta_{13}$ and a baseline of several tens of kilometers, this difference in the $\bar{\nu}_e$ oscillations leads, in principle, to an observable difference in the deformations of the spectrum of e^+ [6], produced in the inverse beta-decay reaction $\bar{\nu}_e + p \to e^+ + n$ by which the reactor $\bar{\nu}_e$ are detected.

In the present Addendum we re-evaluate the potential of the reactor $\bar{\nu}_e$ experiments for determination of the neutrino mass hierarchy using the Daya Bay and RENO data on θ_{13}. Such a re-evaluation is necessary since $\sin^2\theta_{13}$ was measured with a relatively high precision in the Daya Bay and RENO experiments and found to have a relatively large value. We expect the latter to lead to less demanding, than previously estimated, characteristics of the $\bar{\nu}_e$ detector, required for getting information about the type of the neutrino mass spectrum.

We perform the analysis uing the methods descibed in detail in [12]. We assume the experiment is performed with a KamLAND-like (see, e.g., [15]) 10 kT detector (planned, e.g., within the project Hanohano [16]), located at $L = 60$ km from a reactor $\bar{\nu}_e$ source, having a power of ~ 5 GW. As in [12] (see also [7]), the threshold of the visible energy used is set to $E_{\rm visth} = 1.0\,$MeV. As is well known, for the experimentally determined values

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[1] The angle θ_{13} was found to be different from zero, respectively at 5.2σ and 4.9σ in the Daya Bay and RENO experiments. Subsequently, the Double Chooz [4] and T2K [5] experiments reported 3.1σ and 3.2σ evidences for a nonzero value of θ_{13}.

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of the solar and atmospheric neutrino mass squared differences, which we give below, the optimal baseline of the experiment of interest is approximately 60 km (see, e.g., [6, 8, 9]). We present results also for the shorter non-optimal baseline of $L = 30$ km. For the reactor angle θ_{13}, we use the results of the Daya Bay experiment [2]:

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005\,, \quad 0.04 \le \sin^2 2\theta_{13} \le 0.14\,,\ 3\sigma\,. \tag{1.1}$$

In what concerns the other oscillation parameters which enter into the expressions for the reactor $\bar{\nu}_e$ survival probabilities in the cases of NO and IO spectra, the solar and atmospheric neutrino mass squared differences, $\Delta m^2_{\odot} \equiv \Delta m^2_{21}$ and $\Delta m^2_A \equiv \Delta m^2_{31} \cong \Delta m^2_{32}$, and the solar neutrino mixing angle, θ_{12}, we use the values obtained in the global analysis of the neutrino oscillation data, including the data from the Daya Bay and RENO experiments, performed in [18]. It follows from the results obtained in [18], in particular, that we have $\cos 2\theta_{12} \ge 0.28$ at 3σ .

Since the sensitivity to the neutrino mass hierarchy of a reactor $\bar{\nu}_e$ experiment depends critically on the value of the angle θ_{13}, we have redone our earlier analysis [12] taking into account the new data on $\sin\theta_{13}$, eq. (1.1), including the allowed 3σ interval of values. In the following section we present our updated analysis and results.

2 Updated χ^2-analysis of the sensitivity to the type of the neutrino mass spectrum

We perform a full χ^2 analysis of the hierarchy sensitivity of a medium-baseline reactor $\bar{\nu}_e$ experiment with a detector of the prototype of KamLAND, choosing the optimal baseline of 60 km unless otherwise stated. The hierarchy sensitivity is computed by simulating an "experimental" event spectrum for a fixed "true" hierarchy (we choose a normal hierarchy here, the difference being minimal if it is chosen to be the inverted one). A "theoretical" event spectrum is simulated with the other or "wrong" hierarchy. A standard Gaussian χ^2 is then obtained, which determines the confidence level at which the "wrong" hierarchy can be excluded.

Our rigorous analysis involves optimizing the event binning to give the best sensitivity while being compatible with constraints of detector resolution, marginalizing over the neutrino parameters $|\Delta m^2_{\rm atm}|$ and θ_{13}, and taking into account systematic and geo-neutrino uncertainties by the method of pulls (for further technical details of the analysis see [12]). We have checked in [12] that doing a marginalization over $\sin^2\theta_{12}$ and Δm^2_{21} over their present 3σ ranges of $\sin^2\theta_{12} = 0.26 - 0.36$ and $\Delta m^2_{21} = 7.0 \times 10^{-5}$ eV$^2 - 8.2 \times 10^{-5}$ eV^2 [18] does not significantly affect the results on hierarchy sensitivity, since they are relatively small variations. Hence, we have presented in [12] the final results with the values of $\sin^2\theta_{12}$ and Δm^2_{21} fixed at their best-fits of $\sin^2\theta_{12} = 0.31$ and $\Delta m^2_{21} = 7.6 \times 10^{-5}$ eV^2. We follow the same procedure here. [2]

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[2] We have made use also of the results found in [12] (see also [7–9]) that the inclusion of systematic and geo-neutrino uncertainties as well as of $\sim 1\%$ energy scale shrink/shift uncertainty (even if energy-dependent), has only a minimal effect on the neutrino mass hierarchy determination.

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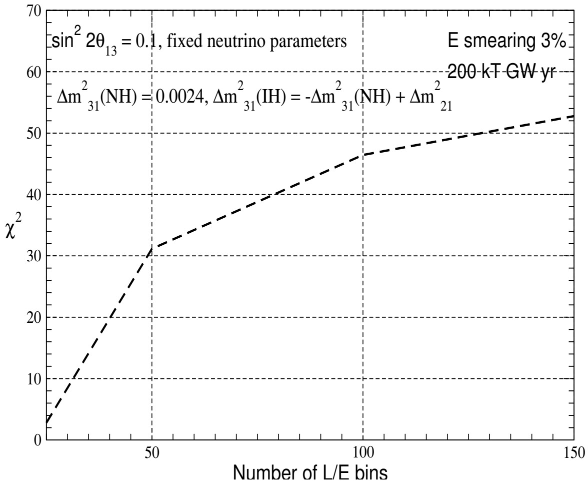

Figure 1. The hierarchy sensitivity $(\chi^2)_{\rm stat}$ as a function of the number of L/E bins, for fixed neutrino oscillation parameters, $\sin^2 2\theta_{13} = 0.1$ and detector's energy resolution of 3%, statistics of 200 kT GW yr, baseline of 60 km and different L/E binnings in the range L/E $= 5 - 32$ km/MeV.

We present results for different values of θ_{13}, the detector exposure and the energy resolution. As was done in [12], a prior term is added to take into account information from other experiments on parameter uncertainties. We find that the uncertainties in the values of $|\Delta m^2_{\rm atm}|$ and θ_{13} play a crucial role in the sensitivity to the neutrino mass hierarchy, and hence the reduction in the allowed range of θ_{13} as well as its increased value aid in hierarchy determination. We study the effect of the detector energy resolution, exposure, parameter marginalization and data binning using the new data on θ_{13}, eq. (1.1).

We consider the following error ranges for the two marginalized parameters: i) $|\Delta m^2_{31}|$ is allowed to vary in the range $2.3 \times 10^{-3} - 2.6 \times 10^{-3}\,\mathrm{eV}^2$, and ii) $\sin^2 2\theta_{13}$ is varied from 0.04 to 0.14, to be consistent with the 3σ range found in the Daya Bay experiment.

Figure 1 shows the behaviour of the χ^2 sensitivity with an increase in the bin number for fixed neutrino parameters and an exposure of 200 kT GW yr, using $\sin^2 2\theta_{13} = 0.1$, $\Delta m^2_{31}(NH) = 2.4 \times 10^{-3}$ eV^2, $\Delta_{31}(IH) = -\Delta_{31}(NH) + \Delta m^2_{21}$ and a detector resolution of 3%, for different numbers of L/E bins in the range L/E $= 5 - 32$ km/MeV. The sensitivity is seen to improve dramatically with an improvement in the fineness of division, and the binning is optimized at 150 L/E bins to derive the best possible sensitivity while being consistent with the detector resolution. For 150 (100) L/E bins, the bin width in energy in the case we are considering is about 68 (100) keV.

Table 1 lists the values of the hierarchy sensitivity $(\chi^2)^{\rm min}_{\rm stat}$ for different values of θ_{13} and the detector energy resolution, after a marginalization over the parameter ranges indicated above, for an exposure of 200 kT GW yr and a 150-bin analysis. The true θ_{13} values are chosen within the 3σ range allowed by the Daya Bay data. Prior experimental information regarding the other neutrino parameters is included in the analysis in the form of "priors", using the present 1σ error ranges of the respective parameters: $\sigma(|\Delta m^2_{\rm atm}|) = 5\% \times |\Delta m^2_{\rm atm}|^{\rm true}$ and $\sigma(\sin^2 2\theta_{13}) = 0.02$. Table 2 gives the values of the hierarchy sensitivity $[(\chi^2)^{\rm min}_{\rm stat}]_{\rm prior}$ for different values of θ_{13} and the detector energy resolution with a parameter marginalization including priors, for the same values of detector

$(\chi^2)^{\rm min}_{\rm stat}$	Energy resolution		
$\sin^2 2\theta_{13}^{\rm true}$	2%	3%	4%
0.07	6.21	4.99	3.81
0.1	12.91	10.41	7.90
0.12	18.80	15.10	11.48

Table 1. Values of $(\chi^2)^{\rm min}_{\rm stat}$ marginalized over the parameters θ_{13} and $|\Delta m^2_{31}|$, for $|\Delta m^2_{31}|^{\rm true} = 2.4 \times 10^{-3}$ eV^2, $\sigma(|\Delta m^2_{31}|) = 5\% \times |\Delta m^2_{31}|^{\rm true}$, $\sigma(\sin^2 2\theta_{13}) = 0.02$, three values of $\sin^2 2\theta_{13}^{\rm true}$ and three values of the detector energy resolution. The detector exposure used is 200 kT GW yr. The baseline is set to 60 km. The values of $(\chi^2)^{\rm min}_{\rm stat}$ are obtained in an analysis using 150 L/E bins in the range 5–32 km/MeV.

exposure and event binning. The slight improvement in the results with the inclusion of priors is enhanced if a lower prospective 1σ error of $\sigma(\sin^2 2\theta_{13}) = 0.01$ is considered. As recent reports from Daya Bay and RENO have shown, such an improvement in the precision of θ_{13} is not far out of reach of present experiments. Moreover, a combined analysis of the global data on the angle θ_{13} performed in [17] already yields $\sigma(\sin^2 2\theta_{13}) = 0.013$.

In table 3, we list the values of the hierarchy sensitivity $[(\chi^2)^{\rm min}_{\rm stat}]$ for $\sin^2 2\theta_{13}^{\rm true} = 0.07$ and 0.1, for 3 different values of the detector resolution and a scaling in the detector exposure. These results show the strong dependence of the sensitivity on the detector exposure. For example, a hierarchy sensitivity of nearly 3σ may be possible even for $\sin^2 2\theta_{13}^{\rm true} = 0.07$ and an energy resolution of 4%, with an exposure of 400 kT GW yr, and this would improve further with a higher detector mass/power.

To highlight the improved sensitivities possible even for smaller detector exposures when θ_{13} is close to the present best-fit value, we present in table 4 the hierarchy sensitivity $[(\chi^2)^{\rm min}_{\rm stat}]$ for $\sin^2 2\theta_{13}^{\rm true} = 0.1$ and 0.12 for lower detector exposures 100 kT GW yr and 150 kT GW yr with 3 different values of the detector's energy resolution. We note that even with an energy resolution of 4%, a 2σ sensitivity is achievable with a relatively low exposure of 100 kT GW yr for the indicated values of θ_{13}. With a better energy resolution of 2%, the sensitivity can go up to 3σ or even to a higher value.

In table 5 we list the values of hierarchy sensitivity obtained for two detector exposures and three values of detector resolution when the baseline is chosen to be 30 km instead of 60 km. This table shows that the sensitivities decrease for the indicated shorter baseline, i.e., when the baseline deviates significantly from the optimal one of 50–60 km. For example, with a baseline of 30 km, a resolution of 2% and an exposure of 200 kT GW yr would be required for a hierarchy sensitivity of 3σ if $\sin^2 2\theta_{13}^{\rm true} = 0.1$, while with a baseline of 60 km similar sensitivity is achievable with an exposure of 150 kT GW yr with the same detector resolution.[3]

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[3] The optimal baseline for hierarchy sensitivity lies in the region of maximization of the effect of the phase $\Delta m^2_{21} L/2E$ in the expression for the $\bar{\nu}_e$ survival probability. With the present error range of Δm^2_{21}, and the peak of the reactor $\bar{\nu}_e$ event rate spectrum at 3.6 MeV, this gives an optimal baseline range of 55 to 64 km. Hence, the hierarchy sensitivity becomes worse for baselines significantly shorter than the indicated range.

$[(\chi^2)^{\rm min}_{\rm stat}]_{\rm prior}$	Energy resolution		
$\sin^2 2\theta_{13}^{\rm true}$	2%	3%	4%
0.07	6.37	5.15	3.90
0.1	13.17	10.58	8.05
0.12	19.10	15.26	11.60

Table 2. The same as in table 1, but for $\sigma(\sin^2 2\theta_{13}) = 0.01$.

$(\chi^2)^{\rm min}_{\rm stat}$	$\sin^2 2\theta_{13}^{\rm true} = 0.07$			$\sin^2 2\theta_{13}^{\rm true} = 0.1$		
Detector exposure, kT GW yr	Energy resolution					
	2%	3%	4%	2%	3%	4%
200	6.21	4.99	3.81	12.91	10.41	7.90
400	12.40	9.98	7.60	25.80	20.80	15.78
600	18.61	14.95	11.71	38.70	31.20	23.50

Table 3. The same as in table 1, but for three values of the detector exposure and $\sin^2 2\theta_{13}^{\rm true} = 0.07;\ 0.1$.

$(\chi^2)^{\rm min}_{\rm stat}$	$\sin^2 2\theta_{13}^{\rm true} = 0.1$			$\sin^2 2\theta_{13}^{\rm true} = 0.12$		
Detector exposure, kT GW yr	Energy resolution					
	2%	3%	4%	2%	3%	4%
100	6.50	5.20	3.98	9.45	7.57	5.75
150	9.70	7.80	5.95	14.15	11.35	8.60

Table 4. Values of $(\chi^2)^{\rm min}_{\rm stat}$ marginalized over the parameters θ_{13} and $|\Delta m^2_{31}|$ for lower detector exposures (in kT GW yr), $\sin^2 2\theta_{13}^{\rm true} = 0.1$ and 0.12, for three values of the detector's energy resolution and a baseline of 60 km. The results are obtained in an analysis using 150 L/E bins in the range 5–32 km/MeV.

3 Conclusions

We find that the data on the parameter θ_{13} from Daya Bay experiment allow us to get information or determine the neutrino mass hierarchy with a greater efficiency, than was previously estimated, using a reactor $\bar{\nu}_e$ experiment: the stringent requirements of the detector's energy resolution and exposure obtained in the previous studies can be relaxed significantly. Since hierarchy sensitivity depends strongly on the the true value of θ_{13}, the energy resolution and the exposure, a relatively large value of $\sin^2 2\theta_{13}^{\rm true}$ close to the Daya Bay best fit of 0.092 makes it easier to achieve hierarchy determination using lower detector exposures and less demanding energy resolution.

$(\chi^2)^{\rm min}_{\rm stat}$ (30 km)	$\sin^2 2\theta_{13}^{\rm true} = 0.1$			$\sin^2 2\theta_{13}^{\rm true} = 0.12$		
Detector exposure, kT GW yr	Energy resolution					
	2%	3%	4%	2%	3%	4%
150	6.60	4.90	3.80	9.65	7.15	5.54
200	8.79	6.50	5.05	12.81	9.48	7.35

Table 5. Values of $(\chi^2)^{\rm min}_{\rm stat}$ marginalized over the parameters θ_{13} and $|\Delta m^2_{31}|$ for two values of detector exposures (in kT GW yr), for $\sin^2 2\theta_{13}^{\rm true} = 0.1$ and 0.12, three values of the detector's energy resolution and a baseline of 30 km. The results are obtained in an analysis using 150 L/E bins in the range 5–32 km/MeV.

For example, $(\chi^2)^{\rm min}_{\rm stat}$ for the "wrong" hierarchy improves from 3.5 (1.8σ sensitivity) for $\sin^2 2\theta_{13}^{\rm true} = 0.05$ (close to the Daya Bay 3σ lower limit), an energy resolution of 2% and a detector exposure of 200 kT GW yr, to 12.9 (a 3.6σ determination) for $\sin^2 2\theta_{13}^{\rm true} = 0.10$ (close to the Daya Bay best fit) for the same values of the resolution and exposure. With this value of $\sin^2 2\theta_{13}^{\rm true}$, even an energy resolution of 4% can give a sensitivity of nearly 3σ.

To summarise, for the values of θ_{13} from the interval allowed at 3σ by the Daya Bay data, a significant hierarchy sensitivity is possible even with a detector energy resolution of $\sigma \sim 4\%$ and an exposure of 200 kT GW yr. For $\sin^2 2\theta_{13}^{\rm true} = 0.10$ (0.12) and an energy resolution of $\sigma \sim 2\%$, a 3σ sensitivity to the neutrino mass hierarchy can be achieved with an exposure of 150 (100) kT GW yr. The indicated requirements on the detector specifications make the discussed reactor $\bar{\nu}_e$ experiment more feasible than the previous analyses have suggested.

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