



SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI

SISSA Digital Library

What's the trouble with anthropic reasoning?

*Original*

What's the trouble with anthropic reasoning? / Trotta, R., Starkman, G.D.. - 878:(2006), pp. 323-329. (THE DARK SIDE OF THE UNIVERSE: 2nd International Conference on The Dark Side of the Universe DSU 2006 Madrid, SPAIN 2006) [10.1063/1.2409104].

*Availability:*

This version is available at: 20.500.11767/117315 since: 2021-01-07T08:36:56Z

*Publisher:*

*Published*

DOI:10.1063/1.2409104

*Terms of use:*

Testo definito dall'ateneo relativo alle clausole di concessione d'uso

*Publisher copyright*

AIP - American Institute of Physics

This version is available for education and non-commercial purposes.

note finali coverpage

(Article begins on next page)

# What's the trouble with anthropic reasoning?

Roberto Trotta\* and Glenn D. Starkman\*<sup>†</sup>

*\*Astrophysics Department, Oxford University, Denys Wilkinson Building, Keble Road, Oxford OX1 3RH, UK*

*<sup>†</sup>Department of Physics, Case Western Reserve University, Cleveland, OH 44106-7079, USA*

**Abstract.** Selection effects in cosmology are often invoked to “explain” why some of the fundamental constant of Nature, and in particular the cosmological constant, take on the value they do in our Universe. We briefly review this probabilistic “anthropic reasoning” and we argue that different (equally plausible) ways of assigning probabilities to candidate universes lead to totally different anthropic predictions, presenting an explicit example based on the total number of possible observations observers can carry out. We conclude that in absence of a fundamental motivation for selecting one weighting scheme over another the anthropic principle cannot be used to explain the value of  $\Lambda$ .

**Keywords:** Anthropic principle, selection effects, cosmological constant

**PACS:** 98.80.-k, 98.80.Es

## INTRODUCTION

The existence of observers' selection effects in cosmology might appear at first sight a mere tautology. Indeed, all of our observations of the Universe are (implicitly) conditional on the fact that we exist. A Universe without observers would never be measured for the simple fact that there would be no one around to make the observations.

We can easily conceive Universes where the laws of physics are such as not to allow the existence of sentient life. For example, a Universe without an arrow of time would be extremely hostile to the emergence of an organized complexity as required by the existence of intelligent observers, because the lack of causality would arguably prevent any meaningful anticipation of physical phenomena. Without going to such extremes, we might speculate about what the Universe would look like if the numerical value of physical constants were different from what is observed. In particular, a large number of so-called “anthropic coincidences” have been pointed out regarding our own Universe (see e.g. [1] and references therein): it appears that any small deviation from highly fine-tuned values of physical constants in our Universe would have catastrophic consequences for the emergence of life as we know it.

There are a few different viewpoints as to what meaning we should assign to the idea that physical constants (such as Newton's constant or the fine structure constant) could be different from what we observe. The traditional approach of physics has been to explain the laws of nature from fundamental principles, such as the existence and breaking of symmetries. In working out a final theory of everything, one might have hoped that the structure of physical reality would naturally emerge as the unique logically and mathematically consistent possibility. In this case, the value of natural constants would be uniquely determined by some deep, underlying principle. The imminent realization of

this positivistic hope seems to have receded as string theory has failed to deliver such a unique picture of fundamental physics. Indeed, the string landscape, with its large number of different vacua, points to a vast number of possible Universes, each with different properties, for example the values of fundamental constants. Opinions diverge as to the physical reality of such alternative realizations of the Universe. We might think of this ensemble as causally disconnected patches of this Universe, as separate sub-universes (a.k.a. the multiverse) or as a superposition of states (as in quantum cosmology). It seems to us that this problem is akin to the (unresolved) interpretation of measurement in quantum mechanics, where the physical reality of the many-worlds picture remains unclear. This has led some distinguished scientists (such as the late E.T. Jaynes) to cast doubts on the completeness of modern quantum theory.

## ANTHROPIC REASONING

One of the most outstanding problems in fundamental physics is to explain the incredibly low energy density of the vacuum compared to the characteristic Planck energy density – the cosmological constant problem. Lacking an explanation from fundamental arguments, much effort has been devoted to investigate the possibility of understanding the cosmological constant values in terms of selection effects. This goes usually under the name of “anthropic principle”, introduced by Brandon Carter [2] as a necessary limitation to the Copernican principle that we do not hold a special place in the Universe. The argument first given by Weinberg [3] is that in realizations of the Universe with too large a value of the cosmological constant structure formation cannot proceed, and hence life as we know it will not emerge. Although Weinberg’s original upper limit of the value of the cosmological constant energy density  $\rho_\Lambda$  is more than 2 orders of magnitude larger than what is observed, refined versions of this argument claim to successfully “predict”  $\rho_\Lambda$  comparable to what is actually observed, ie  $\rho_\Lambda/M_{\text{Pl}}^4 \approx 10^{-123}$  [4, 5, 6].

The rigorous translation of such selection effects in a probabilistic statement is far from trivial. Let us focus on the case where only the cosmological constant  $\Lambda$  is allowed to vary (see [7, 8, 9] for a discussion of how the situation changes when more parameters are varied). In a Bayesian language, we can write for the posterior probability for  $\Lambda$  given that intelligent life exist,

$$Pr(\Lambda|\text{life}) \propto Pr(\Lambda)Pr(\text{life}|\Lambda), \tag{1}$$

where  $Pr(\Lambda)$  is the prior probability distribution function (pdf) and  $Pr(\text{life}|\Lambda)$  is the likelihood for “life” given a certain value for  $\Lambda$ , thus encapsulating the selection effects. The proportionality constant (the “evidence”) is independent of  $\Lambda$  and can be ignored for the purpose of this discussion. Let us discuss the prior and likelihood in turn.

### The prior distribution

The whole point of the anthropic program is to go from a flat(ish) prior pdf – either describing insufficient knowledge (from the Bayesian perspective) or assumptions about

the frequency of realizations of a fundamental theory (frequentist) – to a strongly peaked posterior, hopefully centered around the observed value for  $\Lambda$ .

But to start with, how are we to assign the prior? Most of the literature has interpreted the prior in a frequentist sense, and calculated in various way the relative number of outcomes for a large number of realizations. However, the very concept of probability as a limiting frequency of outcomes, though natural when applied to repeatable experiments, is not obviously appropriate to describe the Universe as a whole. One way out of the problem that we have only one Universe to study is to make use of ergodic arguments in order to derive  $Pr(\Lambda)$  [9]. This is an approach whose validity remain unproven. A more radical point of view is the Multiverse scenario, according to which there is an infinite collection of, by definition inaccessible, universes. It is difficult to see how vastly increasing the number of universes could help determine the properties of the one universe we actually can observe, i.e. our own. This approach hardly seems economical in terms of explanatory power. From an operational point of view, the idea of a “random” distribution of values for  $\Lambda$  is meaningless unless a mechanism for the generation of the different values is also specified.

Some of these conceptual difficulties might be addressed by taking a fully Bayesian approach to the problem, and understanding the prior as an expression of our state of knowledge before we see the data, in this case represented by the observation that sentient life does exist in the Universe. Here, however, we are mainly concerned with issue arising from the choice of the likelihood function, to which now we turn our attention.

### **The likelihood function: dependence on reference class**

In order to compute  $Pr(\Lambda|\text{life})$  in Eq. (1), we need to specify exactly the meaning of “life”. This is a fundamental issue that all too often is glossed over, by simply using a more readily calculated surrogate – the physical number density of galaxies, or more precisely the collapsed gas fraction. One then assumes that the density of observers is proportional to this quantity. So if “life” really stands for “intelligent observers just like us”, we need to ask ourselves what *exactly* counts as an observer. Do future generations of humans count as separate observers or not – after all, we could pass on the information we gathered to them. Or perhaps, the whole human civilization ought to be counted as one single, collective observer? Do the ancient Egyptians count as separate observers? What if past observers die out, or if they forget previous measurements?

These considerations can be put in a more formal way by using the concept of a *reference class of observers*. The reference class contains all observers that are “just like you” in all relevant respects. It is clear that the resulting posterior pdf will depend on the chosen reference class, as shown by [10]. In other words, the choice of reference class is equivalent to giving different probabilistic weights to different realizations of the Universe (in a frequentist perspective). Here we present a specific example of this effect – we argue that there are many plausible weighting factors (or reference classes) for universes, and that the answers to questions such as the expected value of  $\Lambda$  depends enormously on the weighting. In [11] we introduced a weighting scheme based on the

maximal number of allowed observations (MANO) in a universe. This quantity is clearly relevant to the expected value of a constant, say  $\Lambda$ , since a value that allows more observations to be carried out will be measured more often. It also has the advantage of being independent of how one defines constant time hypersurfaces. As we show below, the resulting posterior pdf for  $\Lambda$  is peaked arbitrarily close to 0, thus giving a completely different result than the usual weighting by number density of galaxies.

## MAXIMUM NUMBER OF ALLOWED OBSERVATIONS

We wish to evaluate the probability that an observer will measure his or her universe to have a vacuum energy density no smaller than what we measure in our Universe. As the selection function for observing  $\Lambda$  in the different realizations we put forward the total number of observations that observers can potentially carry out over the entire life of that universe (called MANO for brevity, for “Maximum Allowed Number of Observations”). This maximum number is the product of two factors – the number of observers and the maximum number of observations that each observer can make. As argued above, there is a fundamental difficulty in determining the total number of observers in a given reference class, since we can neither compute nor measure it. However, in the limit where observers are rare (in a way we quantify below) the anthropic prediction for the probability of observing  $\Lambda$  will be independent of the density of observers. Below we focus on the second factor – the maximum number of observations that each observer can make – and offer some further comments about the many observers scenario in the next section.

### Rare observers scenario

For illustrative purposes and computability, we hold fixed all parameters of the universe other than the vacuum energy density, considering flat Lemaitre-Friedmann-Robertson-Walker universes with exactly the same matter and radiation contents and the same fluctuations as our own at the time of matter–radiation equality. This is a common setup in the literature. We consider only the case  $\Lambda > 0$ . This restriction can only increase the probability of observing  $\Lambda$  equal to or greater than the observed one, so we should interpret the probability we calculate as an upper limit. We refer to [11] for further details.

In a  $\Lambda > 0$  universe, the minimum temperature at which a system (*e.g.* an observer) can operate is the de Sitter temperature  $T_{dS} = \rho_{\Lambda}^{1/2}/(2\pi M_{\text{Pl}})$ . (Refrigerated subsystems can run cooler, but the energy consumption of the refrigeration more than compensates). As discussed in detail in [12] and [13], the maximum energy such an observer can collect is given by

$$E_{\text{max}} \simeq \frac{1}{8} \frac{4\pi}{3} [(\eta_{\infty} - \eta_{\star})a_{\star}]^3 \rho_m(a_{\star}) \quad (2)$$

where  $\eta_{\star}$  is the value of conformal time when the observer starts collecting energy and  $a(\eta_{\infty}) = \infty$ . The factor of  $\sim 1/8$  arises because we assume that the decision to collect

the energy is made by the observer at the origin, and must then be communicated out into space. During this time-consuming process, most ( $\sim 7/8$ ) of the volume currently within the apparent horizon is swept out of it by the accelerating expansion. (There is an additional suppression factor of  $\sim 1/8$  if one wishes to transport the energy back to the central location, rather than use it *in situ*) We will ignore the  $\mathcal{O}(1)$  geometric prefactors and focus on the functional dependence. In the “rare observers” scenario we assume that there is at most one observer within the comoving volume accessible to each from the time that they first become capable of making observations onward, otherwise there is a cut-off to the maximum collectible energy introduced by competition among observers (see below).

The number of thermodynamic processes (such as observations of  $\Lambda$ ) an observer can carry out is maximized if the observer saves up  $E_{\max}$  until the universe has reached the de Sitter temperature. Thus

$$N_{\max} \leq E_{\max}/k_B T_{dS}. \quad (3)$$

Following the arguments given above, we adopt  $N_{\max}$  as a probabilistic weight in the selection function, and since we have assumed a flat prior pdf in  $\Lambda$  we have that the posterior  $Pr(\Lambda|\text{life}) \propto N_{\max}$ .

The asymptotic limit for  $N_{\max}$  can be calculated analytically (see [11]), and one finds

$$Pr(R|\alpha_*) \propto \begin{cases} \frac{2}{3} (\alpha_0/\alpha_*)^3 R^{-2}, & R \gg 1, \\ 54R^{-1}, & R \ll 1. \end{cases} \quad (4)$$

Here we have introduced  $R$  as the ratio of the value of the cosmological constant in an hypothetical universe with respect to the value it takes in our own,  $R \equiv \Lambda/\Lambda_0$ . The quantity  $\alpha_*$  is the value of the scale factor when smart observers start collecting energy (normalized to matter–radiation equality), and it is conveniently expressed as

$$\alpha_* = \alpha_0 (3R)^{-1/3} \sinh \left( \ln(\sqrt{3} + 2) \sqrt{R} \tau \right)^{2/3}, \quad (5)$$

where  $\tau \equiv t_*/t_0$  is the physical time until observers smart enough to begin collecting energy arise, in units of 13.7 Gyrs, the age at which such observers (us, or our descendants) are known to have arisen in our Universe. However, the normalization integral in (4) diverges logarithmically, and is dominated by the minimum cut-off value,  $R_{\min}$ , if such exists. In the landscape scenario (see e.g. [14] and references therein), for instance, the number of vacua is estimated to be of order  $10^{500}$ , and therefore the corresponding minimum value of  $\Lambda$  can perhaps be taken to be  $\Lambda_{\min} \sim 10^{-500} M_{\text{Pl}}^4$ , or  $R_{\min} \sim 10^{-377}$ . One could of course go from (4) to a joint posterior for  $R$  and  $\tau$ , but this would involve the specification of a prior on  $\tau$ , thus introducing further uncertainty in the problem, given our ignorance about when smart observers are likely to arise. We prefer instead to evaluate the posterior probability of  $R > 1$ , equivalent to the probability of measuring  $\Lambda > \Lambda_0$ , for a few representative choices of  $\tau$ . With the above choice of cut-off value, and for a few values of  $\tau = 0.1, 1, 10$  we obtain a very small probability of observing a value of  $\Lambda$  as large or larger than in our Universe, and it falls further the longer it takes for intelligent observers to arise (ie, for larger  $\tau$ ). For  $\tau = 1$  we obtain a probability of  $9 \cdot 10^{-6}$ , which drops to  $4 \cdot 10^{-12}$  for  $\tau = 10$ . The situation is only marginally better in the

optimistic situation where intelligent observers evolve before one-tenth of the current age of the universe, since  $Pr(R > 1 | \tau = 0.1) = 5 \cdot 10^{-4}$ .

It is worth noting that the conclusion that low  $\Lambda$  is favored does not depend on the observer civilization foolishly squandering all of its resources on observing  $\Lambda$ . Rather, it requires only that civilizations spend a fraction of their resources doing so which does not depend on (or at least does not decrease with)  $\Lambda$ . One might wonder why a civilization would bother “observing”  $\Lambda$  more than once. First, since we have calculated the maximum number of thermodynamic processes, we must understand that “observing” should be rather broadly defined. In particular it would include “remembering” the cosmological constant (i.e. consulting permanent records), or communicating the value of the cosmological constant to other members of the civilization, including one’s descendants. Thus, it would actually be difficult for a civilization to stop “observing” the cosmological constant. Secondly, there is actual motivation for continuing to observe the rate of expansion to check to see if the dark energy density has changed, since this alone will allow one to take advantage of the decline in the de Sitter temperature to prolong the civilization’s existence.

### **Many observers scenario**

So far, we have worked exclusively in the rare observer limit – where each intelligent observer is free to collect all of the energy within their apparent horizon without competition from other observers. One might imagine that as the density of observers rose, one would mitigate the preference for low  $\Lambda$ , but the case is by no means so clear. If the observer density is high, then the observers will come into competition for the universe’s (or at least their Hubble volume’s) same scarce resources. Our own historical experience is that such competition never leads to negotiated agreement to use those resources as conservatively as possible. More likely is that the competition for resources will lead to some substantial fraction of those resources being squandered in warfare until only one of the observers remains. Moreover, unless they eliminate all possible competitors, observers will continue to spend their finite supply of energy at a rate exceeding that which would otherwise be necessary. What is clear is that given our inability to predict or measure either the density of intelligent observers or the way in which they would behave when they meet, our ability to use anthropic reasoning can only be further compromised.

Indeed, it is (not surprisingly) impossible to escape the psychology and sociology questions when the density of observers is high. No doubt one could perform some particular calculation of such a scenario – assuming, for example, that all the observers agree to use only their local resources and not to poach on each other, or by drawing inspiration from a game-theory approach about competitors evolving in an environment with limited resources. But any such assumptions would be rather strong, and it would be effectively impossible to test. The point is, that since such questions inevitably intrude, one cannot do a meaningful calculation. But certainly the anthropic prediction will depend on the answer to these effectively unknowable questions.

Finally, in our MANO approach we must specify what fraction of its available resources a civilization would devote to measuring the cosmological constant. We have

used here as measure the maximum possible number of observations, but the argument would be unchanged if civilizations used only a fixed fraction (however small) of that maximum. Of course a civilization would be stupid to spend all its energy in this task, and clearly the answer one gets in terms of the posterior pdf for  $\Lambda$  depends on the fraction of the energy consumption of the civilization as a function of time. We have assumed that the fraction is zero until the ambient temperature reaches the de Sitter temperature and then a constant (it need not be 100% – the probability distribution for  $\Lambda$  would remain unchanged if it were any other constant value.) Since we are not trying to prove that MANO is *the* correct way to weight universes, we make no attempt to justify that this is *the* correct functional dependence of the observation rate on time, just that it is a perfectly reasonable dependence. Other observation strategies – such as observe once and never observe again – may also be perfectly reasonable. One could also argue that instead of counting every observation one should count only once each observation in a causally connected region. But this is part of the problem – can we ever hope to understand, nay predict, the psychology of all intelligent civilizations from first principles, predict which psychology will produce the longest lasting (“dominant”) civilizations, and so infer the distribution of observation–strategies that would result? We fear not.

## CONCLUSIONS

We have argued that anthropic reasoning suffers from the problem that the peak of the selection function depends on the details of what exactly one chooses to condition upon – be it the number of observers, the fraction of baryons in halos or the total number of observations observers can carry out. A weighting scheme according to the maximum number of possible observations implies that the expected value of  $\Lambda$  is logarithmically close to its minimum allowed non–negative value (or is zero or negative), contrary to the usual result. In its usual formulation, the anthropic principle does not offer any motivation – from either fundamental particle physics or probability theory – to prefer one weighting scheme over another, and in particular one that does not lead to paradoxical or self–contradictory conclusions of the type described in [10]. Lacking either fundamental motivations for the required weighting, or other testable predictions, anthropic reasoning cannot be used to explain the value of the cosmological constant. We expect that similar statements apply to any conclusions that one would like to draw from anthropic reasoning.

## ACKNOWLEDGMENTS

RT is supported by the Royal Astronomical Society through the Sir Norman Lockyer Fellowship. GDS is supported in part by fellowships from the John Simon Guggenheim Memorial Foundation and Oxford’s Beecroft Institute for Particle Astrophysics and Cosmology, and by grants from the US DoE and NASA to CWRU’s particle astrophysics theory group. RT would like to thank the organizers of the Dark Side of the Universe 2006 conference for a very interesting meeting and the European Network of Theoretical Astroparticle Physics ILIAS/N6 under contract number RII3-CT-2004-506222 for

supporting his participation to this conference.

## REFERENCES

1. J. D. Barrow and F. J. Tipler, "The Anthropic Cosmological Principle", Oxford University Press (1988).
2. B. Carter, in "Confrontation of cosmological theories with observations", M. S. Longair (Ed), Reidel, Dordrecht (1974).
3. S. Weinberg, *Rev. Mod. Phys.* **61** (1989) 1;
4. A. Vilenkin, *Phys. Rev. Lett.* **74** (1995) 846 [arXiv:gr-qc/9406010].
5. H. Martel, P. R. Shapiro and S. Weinberg, *Astrophys. J.* **492** (1998) 29;
6. J. Garriga and A. Vilenkin, *Phys. Rev. D* **61** (2000) 083502.
7. A. Aguirre, *Phys. Rev. D* **64** (2001) 083508.
8. M. Tegmark and M. J. Rees, *Astrophys. J.* **499** (1998) 526
9. M. Tegmark, A. Aguirre, M. Rees and F. Wilczek, *Phys. Rev. D* **73** (2006) 023505.
10. R. Neal, Technical Report No 0607, Department of Statistics, University of Toronto, arXiv:math/0608592.
11. G. D. Starkman and R. Trotta, arXiv:astro-ph/0607227.
12. L. M. Krauss and G. D. Starkman, *Astrophys. J.* **531** (2000) 22;
13. L. M. Krauss and G. D. Starkman, arXiv:astro-ph/0404510.
14. A. Vilenkin, arXiv:hep-th/0602264.