Comment on "Regular evaporating black holes with stable cores"

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Regular black holes are generically unstable because of the classical phenomenon that goes by the name of "mass inflation" which destabilizes the inner horizon. In a recent article, [A. Bonanno, A.-P. Khosravi, and F. Saueressig, Phys. Rev. D **107**, 024005 (2023)], it is argued that semiclassical effects due to Hawking radiation can cure this instability, and some concerns are raised against the validity of previous analyses showing its existence in the first place. In this short comment, we explain our reservations regarding these recent claims, and reiterate the relevance of the mass inflation instability for regular black holes of astrophysical interest.

In a recent work [1], it is argued that the backreaction of Hawking evaporation is a relevant factor in the analysis of the stability properties of regular black holes. However, in this comment we show that this claim is based on an extrapolation of the mathematical model used beyond its regime of validity.

The model used in [1] is an extension of the well-known Ori model [2,3], used to study the instability of Reissner-Nordström black holes. This extension can be applied to study the behavior of different models of regular black hole. Once a specific regular black hole model is chosen, the following results follow [4] (see also [5] for a complementary discussion):

(i) Regardless of the model selected, there is an exponential mass inflation of the Misner-Sharp mass that continues until backreaction cannot be ignored, and thus the linear approximation breaks down (in complete parallelism with the Ori model [2]).

(ii) If one insists in using the (unphysical) linear approximation beyond this point, then the Misner–Sharp mass is always divergent but different background geometries can display different degrees of divergence (ranging from exponential to polynomial).

While from a mathematical perspective it is always possible to push the linear model to its ultimate consequences, such an exercise has clearly no physical relevance.

The authors of [1] call this polynomial behavior, in the models in which it appears, "late-time dynamics". For instance, in page 2 of [1], the authors write:

> In [30], it was established that this conclusion is premature though. While the extrapolation works for certain classes of static regular black holes, including the geometries proposed by Bardeen, the Hayward geometry and renormalization group improved black hole solutions are free from mass inflation. In these cases the mass function at the Cauchy horizon grows polynomially in time only and the resulting curvature singularity may be integrable. Technically, this behavior can be traced back to the presence of a late-time attractor in the evolution equation for the mass-function at the Cauchy horizon, rendering this quantity finite at asymptotically late times.

The statement above that some classes of regular black holes are free from mass inflation is not correct. As mentioned above and elaborated below, that the late-time behavior is not exponential does not mean that mass inflation has not taken place, and is in any case outside of the range of validity of the Ori model.

To make this concrete, let us consider the behavior of the Misner-Sharp mass explicitly. If the Misner-Sharp mass depends on the asymptotic mass linearly, i.e.,

$$M(v, r) = g_1(r)m(v) + g_2(r), \qquad (1)$$

then the behavior of the Misner-Sharp mass is always exponential,

$$M_+ \sim \frac{e^{|\kappa_-|v}}{v^{\gamma+1}}, \qquad (2)$$

where κ_- is the surface gravity of the inner horizon. Geometries in this class include the Reissner-Nordström black hole and the Bardeen regular black hole [6].

On the other hand, for Hayward's regular black hole [7], after the initial exponential phase, and beyond the regime of validity of the linear approximation, we get power-law behavior,

$$M_+ \propto |\kappa_-|\frac{v^{\gamma+1}}{\beta}. \qquad (3)$$

One could think that this different polynomial behavior could alleviate the instability, in particular when Hawking radiation is taken into account; this is what motivates the authors of [1]. However, by the end of the exponential phase the backreaction on the geometry is already very large, and the linear approximation at the core of the model can no longer be trusted. In the case of Hayward's metric, the transition between the exponential and polynomial phase occurs when the ratio, between the Misner–Sharp mass in the interior region M_+, and the initial mass m_0, is given by

$$\frac{M_+}{m_0} \sim \frac{v^{\gamma+1}}{6\beta}\frac{m_0}{\ell} \gg 1. \qquad (4)$$

Hence, the backreaction of perturbations cannot be ignored, and the linear model ceases to be valid. Any conclusions that result from the application of this linear model beyond its regime of validity are not reliable.

In particular, this applies to the analysis of the impact that Hawking evaporation has on this "late-time behavior" that is the driving motivation in [1]:

> The taming of the mass-inflation effect then suggests that the dynamics at the Cauchy horizon and the black hole evaporation process could happen on similar timescales. Thus, a more complete understanding of the actual dynamics mandates to take the Hawking evaporation process into account. Our work addresses this question for the first time. As our main result, we discover two classes of universal late-time behaviors whose

properties are dictated by simple structural properties of the mass function and the universality of the Hawking effect. The late-time attractors governing the dynamics either lead to a polynomial growth of the squared curvature tensors or even renders these quantities finite. Interestingly, the latter behavior appears for the case of Reissner-Nordström geometry once the Hawking radiation is included. The final state of the black hole evaporation process is still a cold remnant.

Note also that the authors of [1] are not properly appreciating the extremely important separation of scales between ringdown, the Price regime, and the truly enormous timescale associated with backreaction induced by Hawking radiation.

> We stress that, strictly speaking, the Ori model building on the Price tail behavior (10) is valid for asymptotically late times v only. Therefore, conclusions drawn from the model in a regime where v is small and the perturbation significant compared to the mass m_0 have to be interpreted with care and should be confirmed by an analysis of the full dynamics [36].

What the current authors call "late times" means late enough such that the Price law provides a good description of the system. It is true that at very early times after the black hole spacetime is perturbed Price's law cannot be applied, but after a timescale $v \sim \mathcal{O}(M)$ (corresponding to the ringdown time) it becomes the dominant one, while Hawking radiation is completely negligible for a much longer timescale $v \sim \mathcal{O}(M^3/M_{\text{Planck}}^2)$. Due to this vast separation of scales, there is a very long transient in which the exponential phase in the Ori model discussed in [8] is building up.

Let us also stress that while an idealized analysis of the role of Hawking radiation may be legitimate, still one should not neglect the cosmic microwave background (CMB) role when the analysis is aimed at testing the phenomenological relevance of regular black holes. Indeed, for a solar-mass black hole, Hawking radiation is subdominant even with respect to the incoming CMB radiation. Therefore, after the decay of the perturbations following Price's law, there will be an extremely long transient in which the dominant perturbation is given by incoming CMB radiation, not by outgoing Hawking radiation. For a solar-mass black hole, the order of magnitude of such a transient can be easily estimated to be around 300 billion years [9], and is hence even longer for supermassive black holes, which gives plenty of time for the exponential buildup and subsequent destabilization of the core. Nonetheless, still this should not overshadow the main message of this comment that the early exponential phase we discussed above is present even if the CMB is ignored.

For completeness, let us stress that our analysis should not be taken as a criticism to models predicting regular black holes as a possible piece in the resolution of the singularity problem. In fact, our analysis simply shows that if such solutions are generated, a small perturbation has a large backreaction on the geometry which, as a consequence, will then become nonstationary. The investigation concerning the end point of such evolution is a question that should be addressed on a case by case basis and which depends on the dynamical equations of the theory.

Possible scenarios at the moment entail an outcome in a regular black hole configuration with zero surface gravity at the inner horizon [8,10], the increase of the regularization length ℓ so to generate a horizonless configuration [11,12], a series of bounces that result into the formation of a horizonless configuration [13,14], or a bounce destroying the trapped region [15,16] (albeit in the latter case, the mechanism that would allow to reconcile the instability timescale with the long enough timescales needed to accommodate observations of astronomical black holes has not been identified). All of these possibilities lead to interesting phenomenological consequences which are worth exploring [17–22].

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