



INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

DOCTORAL THESIS

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**Current correlators, supersymmetry  
breaking and holography**

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# List of PhD publications

This thesis summarizes part of my research during the Ph.D., that is contained in the following three publications

- R. Argurio, M. Bertolini, L. Di Pietro, F. Porri, D. Redigolo,  
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- R. Argurio, M. Bertolini, L. Di Pietro, F. Porri, D. Redigolo,  
“Exploring Holographic General Gauge Mediation,”  
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- R. Argurio, M. Bertolini, L. Di Pietro, F. Porri, D. Redigolo,  
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My activity has also focused on other topics in supersymmetric quantum field theory and holography, resulting in the following two publications

- M. Bertolini, L. Di Pietro, F. Porri,  
“Dynamical completions of generalized O’Raifeartaigh models,”  
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- M. Bertolini, L. Di Pietro, F. Porri,  
“Holographic R-symmetric Flows and the  $\tau_U$  conjecture”,  
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# Chapter 1

## Introduction

The AdS/CFT duality [1–3] is a conjectured equivalence between a certain gauge theory in  $D$  dimensions and a certain string theory formulated on a higher dimensional space-time. It then relates a quantum theory of gravity (string theory) to a quantum field theory which lives on a lower dimensional space. As such, it is a realization of a broader principle which goes under the name of holography [4, 5]. The AdS/CFT correspondence is a weak/strong duality, meaning that when one side is described in terms of a weakly coupled theory the dual theory is at strong coupling. In particular, when the field theory is weakly coupled it dually describes a strongly coupled gravitational theory. This is perhaps the best non-perturbative definition we have at present of a quantum theory of gravity. In the opposite regime, namely when the gravity side is at weak coupling, AdS/CFT provides us with an incredibly powerful tool to study properties of strongly coupled gauge theories in terms of classical gravitational backgrounds. This direction of the duality has been extensively exploited to study strongly coupled systems with applications in many different fields which range from condensed matter [6–8] and statistical physics [9] to the study of heavy ions collisions and quark-gluon plasma [10].

In this thesis we will make use of AdS/CFT to investigate supersymmetry breaking dynamics in four-dimensional strongly coupled field theories. The leading application we have in mind is a holographic model of Gauge Mediation (GM) where the strongly coupled field theory will be identified with the hidden sector responsible for supersymmetry breaking. However, the methods we will present in this thesis have wider applicability. In particular, we think that, with a bit more effort, they can be used to analyze string-derived supersymmetry breaking models, such as the one discussed in [11–14].

The primary objects AdS/CFT allows one to compute are correlators of gauge invariant operators. The technology needed to properly compute  $n$ -point functions in a strongly coupled field theory using AdS/CFT have been developed in [15–17]. Our aim in this

thesis is to apply such techniques to compute two-point functions of supermultiplets of gauge invariant operators in strongly coupled field theories. As we will argue, these objects can be effectively used as probes of the dynamics which breaks supersymmetry (and possible other kind of symmetries). A supermultiplet of operators is a set of operators that are related to one another by supersymmetry transformations. In four-dimensional  $\mathcal{N} = 1$  QFT a supermultiplet can be represented as a superfield (i.e. a function on superspace) subject to some *supercovariant* constraint. The most widely known example is perhaps the chiral superfield, a function of superspace coordinates  $(x, \theta, \bar{\theta})$  subject to the condition

$$\bar{D}_{\dot{\alpha}} \Phi(x, \theta, \bar{\theta}) = 0, \quad \bar{D}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} - i\theta^{\alpha} \sigma_{\alpha\dot{\alpha}}^m \partial_m. \quad (1.1)$$

This represents a supermultiplet made up of two complex scalar operators and one spin- $\frac{1}{2}$  operator,  $\Phi = \{\mathcal{O}, \Psi_{\alpha}, \mathcal{F}\}$ . Supersymmetry relates these three operators and it consequently imposes relations between their correlation functions, as long as the vacuum is supersymmetric. As an example, consider the case in which the chiral multiplet is made up of elementary, free fields. In such case the two-point functions are just free propagators

$$\frac{1}{p^2 - m^2} \sim \langle \mathcal{O} \mathcal{O}^* \rangle \sim \frac{\sigma^m p_m}{p^2} \langle \Psi \bar{\Psi} \rangle \sim \frac{1}{p^2} \langle \mathcal{F} \mathcal{F}^* \rangle, \quad (1.2)$$

supersymmetry fixes the masses of the three fields to be equal.

When supersymmetry is not realized in the vacuum or is it explicitly broken by some relevant perturbation, correlation functions of operators in a supermultiplet will no longer be related to one another. However, since supersymmetry breaking becomes less and less relevant as one approaches the UV regime, the constraints of supersymmetry will still hold, asymptotically, in the small distance (or equivalently large momentum) limit of the correlators. Moreover, one could also expect that the rate at which the supersymmetric behavior is recovered will differentiate an explicit breaking from a spontaneous one. In this sense, the study of two-point functions of operators belonging to a supermultiplet gives information on the dynamics of supersymmetry breaking. The discussion above is not specific to the chiral multiplet and in fact holds in general for any supermultiplet of operators.

In this thesis we will focus our attention on two particular supermultiplets. The multiplet containing conserved currents and the multiplet containing the stress-energy tensor, which in the following will be referred to as *current supermultiplet* and *supercurrent multiplet*, respectively. There are two main reasons why we have chosen these particular supermultiplets. First, they are (quite) universal: the supercurrent multiplet is defined in any supersymmetric QFT whereas the current supermultiplet only requires the existence of a preserved global internal symmetry, to be defined. The second main reason is that



the operators populating these multiplets have protected dimensions. This is in fact a crucial point in our holographic approach. An operator whose dimension is not protected usually gets a huge anomalous dimension at large 't Hooft coupling and this means that its holographic dual is not captured by the supergravity approximation we will be using. In fact, such operators typically correspond to massive stringy states, which get projected out by taking the  $\alpha' \rightarrow 0$  limit.

Any  $\mathcal{N} = 1$  supermultiplet in a four-dimensional CFT can be put in correspondence with an  $\mathcal{N} = 2$  supermultiplet in  $\mathbf{AdS}_5$  space-time. In fact, they are unitary irreducible representations of the same graded Lie algebra,  $SU(2, 2|1)$ . As we will discuss in Chapters 3 and 5, the current supermultiplet and supercurrent multiplet correspond, respectively, to a gauge vector multiplet and to the graviton multiplet. Following the AdS/CFT prescription, in order to compute two-point functions of the former, one has to consider fluctuations of the latter in the gravitational theory.

Our strategy will be the following. We will first choose a gravitational background, which can be either a solution to some supergravity theory or, in a more bottom-up approach, some ad-hoc assembled background. Such background corresponds, via holography, to our dual QFT in some definite vacuum state. We will then use the holographic prescription of [15–17] to compute two-point functions of the operators belonging to the supermultiplets we are interested in. From the outcome of these computations we will extract information about the dynamics of the dual strongly coupled theory.

The material of the thesis is organized as follows. In Chapter 2, after a brief review of AdS/CFT, we will introduce the holographic tools needed to compute two-point correlation functions from an asymptotically Anti de Sitter ( $\mathbf{AAdS}$ ) background. Since the discussion of the procedure will be kept on general lines, we will display in the last section some examples which concretely show how the holographic prescription works in practical situations. Chapter 3 is devoted to the supersymmetric multiplet of conserved currents. It is essentially divided in two main parts. In the first part the structure of the supermultiplet will be discussed at the level of four-dimensional QFT. We will show the content of the supermultiplet in terms of QFT operators, present a parametrization of two-point functions in terms of scalar form factors and then discuss the relations that supersymmetry imposes among them. The second part of Chapter 3 contains some considerations about the holographic description of the current supermultiplet in terms of  $\mathcal{N} = 2$  supergravity fields. We will also discuss how to compute the form factors relevant to this multiplet from a general  $\mathbf{AAdS}_5$  background using holographic techniques. In Chapter 4 we will display the results of [18, 19]. In particular, we will present models of gauge mediated supersymmetry breaking where the hidden sector is replaced by a dual gravitational background. Exploiting the formalism of General

Gauge Mediation (GGM) [20], the results of Chapter 3 will be then applied to compute the soft supersymmetry breaking terms generated by such holographic hidden sectors. Chapter 5 contains a discussion of the supermultiplet of the stress-energy tensor and is divided in two parts, in the same way as for Chapter 3. In the first part we will present the structure of the multiplet in a four-dimensional QFT and give an explicit parametrization of the various two-point correlators between the constituent operators. The last part is dedicated to the holographic description of the supermultiplet. We will discuss the AdS/CFT dictionary pertinent to this case and the holographic computation of two-point functions. This information will be then used in Chapter 6 where we will present the results of [21]. We will consider the simplest holographic setup one can think of, namely a five-dimensional hard wall background, and use holography to extract two-point functions. This will provide a holographic realization of a variety of different dynamical behaviors, including, e.g. a holographic description of the Goldstino mode. We end in Chapter 7 with a summary of our results and an outlook of possible future investigations. Finally, Appendices A and B gather conventions and notations used throughout this thesis.

## Chapter 2

# Two-point correlators from AdS/CFT

In this chapter we will review some details of the holographic correspondence between five-dimensional supergravity theories and four-dimensional quantum field theories. We will be mainly interested in displaying the tools needed for the computation of two-point correlation functions of QFT operators from holography. These tools will then be used in the forthcoming chapters to analyze the structure of two-point functions among operators belonging to supersymmetric multiplets.

### 2.1 Brief review of AdS/CFT

The statement of holography [4, 5] is that a certain quantum gravity theory in a  $(D+1)$ -dimensional space-time with a boundary, is equivalent to a quantum theory without gravity living on the  $D$ -dimensional boundary, and it was originally motivated by the search for a microscopic explanation to the area-law for the entropy of black holes. A precise formulation can be given if the gravity theory lives on a space-time that asymptotically has the geometry of  $\mathbf{AdS}_{D+1}$ . In this case, the space-time has a time-like conformal boundary which is conformally equivalent to a Minkowski (flat) space-time. In order for the dynamical problem to be well-defined in such space-times, the fields in the gravity theory must be assigned a fixed value on the boundary, for all times. This may sound strange compared to more usual evolution problems in flat space, that require initial values to be specified on space-like surface at fixed time, and then determine the behavior at subsequent times. However, the dependence of the gravity theory on these boundary values is actually at the core of the correspondence.

Indeed, for the reason just explained, the observables in the quantum gravity theory, and in particular the partition function, will be functionals of these boundary values which are fields defined on the boundary Minkowski space-time. On the other hand, a quantum field theory in a  $D$ -dimensional space also naturally defines functionals of  $D$ -dimensional fields. For instance, the generator of correlation functions is a functional of the external sources for QFT operators. The  $(D + 1)$ -dimensional theory may have an arbitrary field content, depending on the case, but one field in particular must be present, namely the  $(D + 1)$ -dimensional metric giving the graviton, whose boundary value is a metric in  $D$  dimensions. On the QFT side, an operator which is universally defined is the stress-energy tensor, hence the generator of correlation functions will always depend on its source, which is precisely a metric in  $D$  dimensions. Similarly, the holographic correspondence can be formulated as an identification between the two functionals defined in the two quantum theories [2, 3]

$$Z_{\text{grav}}[g_{mn}, J_a] = \left\langle e^{\int d^D x \sqrt{g}(g_{mn} T^{mn} + \Sigma_a J_a \mathcal{O}_a)} \right\rangle_{\text{QFT}}, \quad (2.1)$$

where  $m, n = 1, \dots, D$  and we have schematically indicated by  $\mathcal{O}_a$  the set of operators in the QFT and with  $J_a$  the corresponding sources. Already from this general formula we can derive some properties of the way the correspondence works: QFT correlators reflect the response of the gravity partition function to a change of the boundary conditions. For any gauge-invariant local operator  $\mathcal{O}_a$  in the quantum field theory, there is a corresponding field on the gravity side whose boundary value is the source  $J_a$ . For instance, a global symmetry of the quantum field theory entails a conserved-current operator, whose source is a gauge field, and there must exist a gauge boson in the  $(D + 1)$ -dimensional theory whose boundary value is the source of the current. Therefore, a global symmetry on the field theory side gets mapped to a gauge symmetry on the gravity side. In the same spirit, a space-time symmetry of the QFT corresponds to a diffeomorphism on the gravity side.

What explained so far is still rather abstract, both because neither of the sides of the correspondence has been specified, and also because neither of the two functionals is calculable without resorting to some approximation scheme (i.e. perturbation theory in some small parameter or semiclassical limit). The first example in which the correspondence has been made concrete, is that between type IIB string theory on  $\mathbf{AdS}_5 \times \mathbf{S}^5$ , with  $N$  units of  $F_5$  flux on  $\mathbf{S}^5$ , and  $\mathcal{N} = 4$ ,  $SU(N)$  super-Yang-Mills (SYM) theory. In this case the correspondence can be motivated starting from type IIB string theory in flat ten-dimensional space-time, with a stack of  $N$  parallel  $D3$ -branes [1]. The low-energy theory living on the stack of branes is  $\mathcal{N} = 4$ ,  $U(N)$  SYM theory. The additional  $U(1)$  in the gauge group is related to the overall position of the branes, and it decouples from the rest of the dynamics (moreover it can be disregarded in the large  $N$  limit that we are

going to consider). On the other hand, one can see the stack of branes as a black-brane solution in type IIB supergravity. Hence, in the low energy limit, taking into account the redshift caused by the localized objects, one is just left with the string modes which live in the near-horizon geometry of the black-brane solution, this geometry being exactly  $\mathbf{AdS}_5 \times \mathbf{S}^5$ . In this specific example, both sides of the correspondence come with dimensionless parameters which make the theory under control in some regime.

On the field theory side, we have the gauge coupling  $g_{YM}$ , associated to the usual perturbative expansion, and the number of colors  $N$ , associated to the large  $N$  expansion. All fields live in the adjoint representation of the gauge group  $SU(N)$ , i.e. they are  $N \times N$  matrices, and every gauge-invariant operator built out of such fields will have the form of a trace of products of matrices, or of products of such traces

$$\mathrm{Tr} [\Phi_1 \dots \Phi_n], \quad \mathrm{Tr} [\Phi_1 \dots \Phi_k] \mathrm{Tr} [\Phi_{k+1} \dots \Phi_m], \quad \dots \quad (2.2)$$

Therefore, gauge-invariant operators can be classified as single-trace, double-trace, and so on. In the large  $N$  limit with finite 't Hooft coupling  $\lambda = Ng_{YM}^2$ , correlators of single-trace operators factorize as products of one-point functions, so that the limit can be interpreted as a classical one (different from the usual, free-theory limit  $g_{YM} \rightarrow 0$ ). Moreover, insertions of multi-trace operators are suppressed in this limit. The diagrammatic expansion can be organized as a sum over surfaces of different topologies, weighted by a factor of  $N^\chi$ , where  $\chi$  is the Euler characteristic of the surface, so that the leading contribution comes from planar diagrams, and increasingly complex topologies give more and more negligible contribution. The surface is defined by the fact that the diagram can be drawn on it without self-intersections.

On the string theory side, the parameters are given by the string coupling constant  $g_s$  and by two dimensionless ratio  $R/\ell_s$  between the characteristic curvature radius of the background  $R$  and the string length  $\ell_s$ . The coupling  $g_s$  controls the loop expansion, which closely resembles the one we have just described for the field theory diagrams in the large  $N$  limit: higher loops corrections in the string amplitude imply higher genus of the corresponding world-sheet, and each diagram comes with a factor  $g_s^{-\chi}$ . This fact suggests that a sensible correspondence between parameters should map the small  $g_s$  expansion on one side with the large  $N$  expansion on the other side. Since the Yang-Mills interactions on the world-volume of the  $D3$ -branes is due to the zero-modes of open strings ending on them, one has the identification

$$g_{YM}^2 = 4\pi g_s. \quad (2.3)$$

Recalling that in the large  $N$  limit one keeps the 't Hooft coupling fixed, we can write

$$\frac{\lambda}{4\pi N} = g_s, \quad (2.4)$$

so that large  $N$  corresponds to small  $g_s$ , and indeed the two expansions are mapped into each other.

In the black-brane solution, the curvature radius  $R$  (i.e. the common radius of the five-sphere and of  $\mathbf{AdS}_5$ ) is fixed in terms of the string length and of the Ramond-Ramond flux by the relation

$$R^4 = 4\pi g_s N \ell_s^4, \quad (2.5)$$

which implies

$$\lambda = N g_{\text{YM}}^2 = \left(\frac{R}{\ell_s}\right)^4. \quad (2.6)$$

Here we see that when the gauge theory is in the perturbative regime,  $\lambda \ll 1$ , the geometry where strings propagate is highly curved, and it is not known how to calculate the complete spectrum of string excitations, much less how to quantize the theory. On the other hand, when the field theory is strongly coupled,  $\lambda \gg 1$ , the string length is negligible with respect to the typical scale of the geometry on which strings are propagating. In this regime, string theory should be captured by a field theory approximation, meaning that we can just keep the zero-modes and neglect higher excitations, whose mass-squared will be of order  $\ell_s^{-2}(1 + \mathcal{O}(\ell^2/R^2))$ . The resulting theory is type IIB supergravity on  $\mathbf{AdS}_5 \times \mathbf{S}^5$ . In this case, to leading order in  $g_s$ , the partition function on the gravity side can be evaluated by a saddle-point approximation, in terms of the on-shell action for the supergravity fields with the appropriate boundary conditions

$$Z_{\text{grav}}[g_{mn}, J_a] \approx e^{-S_{\text{supra}}^{\text{on-shell}}|_{G_{\mu\nu} \rightarrow g_{mn}, \mathcal{J}_a \rightarrow J_a}}, \quad (2.7)$$

where  $G_{\mu\nu}$  is the  $(D+1)$ -dimensional metric and  $\mathcal{J}_a$  indicates the supergravity field dual to a certain operator  $\mathcal{O}_a$ . Notice that, in the gravity theory, the answer will depend on which solution of the equations of motion we choose. In the dual field theory this ambiguity reflects the choice of the vacuum in which correlators are calculated.

To summarize, we first take the limit  $g_s \rightarrow 0$ ,  $N \rightarrow \infty$  with  $\lambda$  fixed. This leaves us with a free theory of strings propagating on  $\mathbf{AdS}_5 \times \mathbf{S}^5$  on the gravity side, and with a free theory (due to factorization) of matrices of infinite-size on the field theory side. Notice that the correspondence is telling us something very non-trivial at first glance, namely that the classical configuration which dominates the path integral of the field theory at large  $N$  is a theory of ten-dimensional strings. However, neither of the two theories, despite being free at leading order, is tractable for generic values of  $\lambda$ . In the

field theory, we know how to characterize the operators, their anomalous dimensions and OPE coefficients only when  $\lambda$  is small. In the string theory, we know the spectrum of excitations and their interactions only when  $\lambda$  is large. Therefore, the correspondence takes the form of a weak/strong duality between the two theories. The direction of the correspondence which is of interest for our applications is to use a supergravity action to calculate field-theory correlators at  $\lambda \gg 1$ .

Let us just mention that in the last decade a great advancement has been achieved in extending the test of the correspondence to finite values of  $\lambda$ , by using integrability techniques (see e.g. the review [22] and reference therein).

### 2.1.1 Generalizations of the correspondence

A natural question at this point is whether other examples of the holographic correspondence exist, and which of the features we described can have more general validity. The previous example was motivated by considering a stack of parallel  $D3$ -branes in flat ten-dimensional space-time: in this case, before considering backreaction, the six dimensions transverse to the world-volume of the branes are flat and homogeneous. It turns out that a first extension arises if one allows the existence of singularities at some point in the six transverse dimensions. If the branes are located at these special points, both the low-energy gauge theory living on their world-volume and their near horizon geometry get modified. Therefore, following the same logic we outlined in the previous section, one can derive a holographic correspondence between different pairs of theories [23–27]. For instance, if the geometry of the transverse dimensions is a Calabi-Yau cone over a compact five-dimensional Sasaki-Einstein manifold  $\mathbf{X}_5$ , the near horizon geometry of the branes located at the tip of the cone is  $\mathbf{AdS}_5 \times \mathbf{X}_5$ , and the number of conserved supercharges in both the dual theories is reduced in general from 32 to 8.

One can also consider a simplified version of the correspondence involving a five-dimensional gravity theory on  $\mathbf{AdS}_5$ . This can be motivated starting from type IIB supergravity on  $\mathbf{AdS}_5 \times \mathbf{X}_5$ , compactifying on  $\mathbf{X}_5$  and consistently truncating<sup>1</sup> the resulting theory so to keep only a finite number of Kaluza-Klein modes. When the compact manifold is  $\mathbf{S}^5$ , if one just keeps the lowest modes, the resulting theory is the maximally supersymmetric gravity theory on  $\mathbf{AdS}_5$ , namely  $\mathcal{N} = 8$  gauged supergravity [28, 29]. This theory, in turn, can be further truncated to less supersymmetric theories with reduced field content. In the dual field theory, a consistent truncation corresponds to restricting to a certain subset of operators closed under the OPE algebra. Another way to get less supersymmetric theories in five dimensions is to start with a more general

<sup>1</sup>By *consistently truncate* here we mean to set to zero a certain number of fields (infinite in this case) in a way that is consistent with the equations of motion.

Sasaki-Einstein manifold  $\mathbf{X}_5$  replacing the five-sphere, giving rise to a five-dimensional  $\mathcal{N} = 2$  gauged supergravity.

What we have briefly described until now are examples motivated by brane dynamics in string theory. However, a holographic correspondence is believed to exist in a broader class of theories. Indeed, nowadays the correspondence is often applied in a more general context, possibly in cases where only one of the two dual theories is known in detail. There are two basic necessary requirements a field theory should satisfy in order to admit a gravity dual [30]. First, a large  $N$  limit is necessary in order to get a weakly-coupled gravitational theory, and suppress quantum effects. The possibility to distinguish single-particle and multi-particle states in the weakly-coupled gravity theory is reflected in the classification of operators as single-trace or multiple-trace. Secondly, in order to be described in terms of a finite and possibly small number of fields with a local Lagrangian in the gravity dual, the field theory should have a large gap in the operator dimensions, with a finite set of operator with small dimensions which dominate the dynamics. In the case we discussed, the parameter  $\lambda$  provides such gap, by giving large anomalous dimension  $\sim \lambda^{1/4}$  to operators which are not protected by supersymmetry. Indeed, exactly the limit of large  $\lambda$  permits to neglect the tower of string excitations, keeping only the supergravity modes.

When the gravitational background is  $\mathbf{AdS}_5$ , whose isometry group,  $\text{SO}(4, 2)$ , coincides with the conformal group in four-dimensions, the dual field theory enjoys conformal symmetry, the dilations being mapped to isometries along the extra-dimension of the gravity theory. Since we want to describe theories which eventually break conformal symmetry and/or supersymmetry, it will be necessary to relax the homogeneity in the extra-dimension by adding scalar profiles to the geometry [31, 32]. As we are going to review below, from the field theory point of view, this amounts to perturb the interacting UV fixed point with (or switching on VEV's for) operators which are dual to the given non-trivial scalars in the five-dimensional background. Alternatively, the translational symmetry in the bulk coordinate perpendicular to the boundary can be broken “by hand” by truncating the geometry at some value along this radial direction. The gravity fields must then be assigned additional boundary conditions on the “wall” where the geometry ends. This class of models, going under the name of *hard wall* models, have the advantage of being easily calculable, but their interpretation in terms of the field theory is sometimes less transparent.

For our scope, instead of deriving the correspondence in a systematic way by starting with a brane construction and reducing consistently the resulting gravity theory, it will suffice to follow a more effective approach, by focusing on symmetry requirements. The four-dimensional field theories we would like to describe have  $\mathcal{N} = 1$  supersymmetry



(which can be eventually broken, but this does not affect the counting of supercharges) and therefore have four conserved supercharges. If we also require these theories to approach an interacting fixed point at high energy, supercharges are enhanced to eight in the deep UV. Therefore, the dual five-dimensional gravity theory must also have eight supercharges, making it an  $\mathcal{N} = 2^2$  supergravity theory. One expects that only half of them will be preserved by the solutions to this theory in order to be dual to non-conformal field theory vacua. Along this thesis, we will consider supergravity backgrounds which asymptotes to  $\mathbf{AdS}_5$  towards the conformal boundary. As we will see this reflects the requirement that the dual QFT approaches a non-trivial fixed point in the UV. Furthermore, in order to study correlators in a non-conformal regime we will consider both options previously outlined to break conformality in the gravity dual. Namely, *bottom-up* hard wall models and supergravity solutions with non-trivial profiles for scalar fields.

## 2.2 Asymptotically Anti de Sitter domain walls and AdS/CFT

In this section we want to describe in some detail the basic features that characterize the class of gravitational backgrounds we will be focusing on in the following chapters. We want our bulk geometry to fulfill two basic requirements. The first involves the symmetries of the supergravity solution and is essentially related to the fact that this should correspond to a Poincaré (Euclidean) preserving vacuum of the dual field theory. The second is a requirement about the asymptotics of the five-dimensional geometry which is needed in order for the holographic computations to be more under control.

Let us start by briefly reviewing how the four-dimensional conformal group naturally arises from the geometry of  $\mathbf{AdS}_5$ .

Anti de Sitter space-time is the maximally symmetric solution to Einstein's equations in a vacuum with negative cosmological constant<sup>3</sup>

$$R_{\mu\nu} - \frac{1}{2}(R - 2\Lambda)g_{\mu\nu} = 0, \quad \Lambda < 0. \quad (2.8)$$

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<sup>2</sup>Eight is the minimum amount of supercharges one can have in five dimensions, both in Lorentzian and Euclidean signature. The use of " $\mathcal{N} = 2$ " may thus sound awkward to a field theorist ear, albeit this is quite standard notation in the supergravity literature. This unusual nomenclature can be explained by the fact that in five dimensions the vector representation, carried by  $P_\mu$ , is contained in the anti-symmetric product of two spinor representations, carried by  $Q_\alpha$ . Consequently one cannot write down a supersymmetry algebra with just one spinor charge [33].

<sup>3</sup>For our conventions on curvatures tensors see Appendix A.

In Poincaré coordinates  $x^\mu = (x^m, z)$ ,  $m = 0, \dots, 3$ , the metric for the  $\mathbf{AdS}_5$  solution has the form

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{dx^m dx_m + dz^2}{z^2} L^2 \quad (2.9)$$

where  $dx^m dx_m$  is the Minkowski (Euclidean) line element,  $L = \sqrt{-\frac{6}{\Lambda}}$  is the  $\mathbf{AdS}_5$  curvature radius and  $z \in (0, +\infty)$  is the coordinate perpendicular to the boundary of the geometry at  $z = 0$ . The metric (2.9) actually does not extend to the boundary of  $\mathbf{AdS}_5$  since it is singular at  $z = 0$ . In order to define a boundary metric one has to pick a function of the coordinates  $f$  which is positive on the  $\mathbf{AdS}_5$  interior and has a first order zero on the boundary (e.g. one could choose  $f = z$ ). One can then replace  $ds^2$  with

$$\overline{ds}^2 = f^2 ds^2 \quad (2.10)$$

which is nonsingular at  $z = 0$ . The line element  $\overline{ds}^2$  restricts to the boundary of the manifold. Since there is no natural choice for  $f$ , this procedure does not yield a well-defined metric on the boundary but rather a conformal structure. In other words, the boundary metric is only well-defined up to a choice of the function  $f$  (i.e. up to conformal transformations).

The above argument shows that while  $\mathbf{AdS}_5$  has a metric invariant under  $\text{SO}(4, 2)$  (or  $\text{SO}(5, 1)$  in the Euclidean version), the boundary has only a conformal structure preserved by the action of  $\text{SO}(4, 2)$ . The  $x^m$  in (2.9) can be thought of as coordinates on the space where the dual field theory lives, and in order for such theory to be well-defined on the  $\mathbf{AdS}_5$  boundary it has to be insensitive to the particular choice of metric inside the class defined by the above mentioned conformal structure. This is in fact the defining property of a CFT. In particular, the transformation

$$(x^m, z) \rightarrow (e^{-\omega} x^m, e^{-\omega} z), \quad (2.11)$$

which leaves the metric (2.9) invariant, acts as a dilation in the boundary CFT and the holographic coordinate  $z$  is related to the (inverse of the) energy scale in the field theory.

The purely  $\mathbf{AdS}_5$  solution described above is dual to a four-dimensional CFT in a conformally invariant vacuum. The  $\mathbf{AdS}_5$  radius  $L$  is associated to the central charge  $c$  of the CFT via the relation [34, 35]

$$c = \frac{L^3 \pi^2}{\kappa_5^2}, \quad (2.12)$$

where the gravitational coupling is  $\kappa_5^2 = 8\pi G_5$ . Of course this does not specify the CFT completely, more information is needed for example about the spectrum of operator dimensions. Similarly, having an  $\mathbf{AdS}_5$  solution does not completely specify the gravity

theory one also need more information on this side such as, e.g., the spectrum of masses around the  $\mathbf{AdS}_5$  vacuum. We will clarify these statements with a concrete example.

Let us consider a massive scalar field  $\phi$  minimally coupled to the above  $\mathbf{AdS}_5$  background. The equation of motion for such field read

$$0 = (\square_{\mathbf{AdS}_5} - m^2) \phi = \frac{1}{L^2} (z^2 \partial_z^2 - 3z \partial_z + z^2 \square_4 - (Lm)^2) \phi \quad (2.13)$$

where  $\square_{\mathbf{AdS}_5}$  is the Laplace operator on  $\mathbf{AdS}_5$  and  $\square_4$  is the one in flat four-dimensional space-time. In order to find the leading behavior of the solution to the above equation, we plug in the ansatz  $\phi = z^\beta$  and work at leading order in  $z \rightarrow 0$ . We thus obtain an algebraic equation for  $\beta$ :

$$\beta(\beta - 4) = m^2 L^2 \quad \Rightarrow \quad \beta_\pm = 2 \pm \sqrt{4 + m^2 L^2}. \quad (2.14)$$

As usual for second order linear differential equations, this gives two independent solutions for the near-boundary behavior of the scalar field. The general behavior will be a linear combination of the two solutions with possibly  $x$ -dependent coefficients, namely

$$\begin{aligned} \phi(x, z) = & \phi_0^-(x) z^{\beta_-} + \dots \\ & + \phi_0^+(x) z^{\beta_+} + \dots \end{aligned} \quad (2.15)$$

where  $\dots$  stand for higher order terms in the  $z$ -expansion for each linearly independent solution. Since  $\beta_- \leq \beta_+$  in the following we will refer to  $\phi_0^-$  and  $\phi_0^+$  respectively as the leading and subleading mode of the near-boundary solution. According to the holographic prescription (2.7) the leading mode is identified with the source for the CFT operator  $\mathcal{O}_\phi$  dual to  $\phi$ , schematically

$$\mathcal{L}_{\text{CFT}} + \int d^4x \phi_0^- \mathcal{O}_\phi. \quad (2.16)$$

Recalling that the dilation (2.11) gives weight  $-1$  to space-time coordinates, one finds from (2.15) that under such transformation  $\phi_0^-$  has weight  $\beta_-$ . In order for the above perturbed Lagrangian to behave correctly under such scale transformations one then obtains that the correct weight of the operator  $\mathcal{O}_\phi$  is  $\Delta = 4 - \beta_- = \beta_+$ . This implies the relation

$$\Delta(\Delta - 4) = m^2 L^2 \quad (2.17)$$

which associates the conformal dimension of a scalar operator to the mass of the dual bulk scalar field. Repeating the same scaling argument for the subleading mode  $\phi_0^+$ , one finds the latter to have the right weight to be interpreted as the VEV of the same

operator,  $\langle \mathcal{O} \rangle$ . This result is one of the building block of  $\mathbf{AdS}_5$  holography and will be extensively used throughout this thesis.

Up to this point we just have analyzed the case of a scalar field fluctuating on a fixed  $\mathbf{AdS}_5$  background. This is relevant, as we will soon clarify, for the computation of correlation functions of scalar operators in the unperturbed CFT. However, for the purpose of describing a nonconformal QFT one has to do more. First of all one needs to consider more general solutions to gravity theories which differs from the  $\mathbf{AdS}_5$  one. For the aim of this thesis we want to focus on the holographic description of four-dimensional QFT's invariant under the Poincaré (Euclidean) subgroup of the conformal group  $SO(4, 2)$ . We thus come to our first requirement on the dual gravitational backgrounds. These should be solutions to some (super)gravity theory and should preserve at least an  $ISO(1, 3)$  ( $ISO(4)$ ) subgroup of the  $\mathbf{AdS}_5$  isometries. In general, such background will be supported by profiles for scalar fields along the fifth coordinate and, up to coordinates redefinitions, can be taken to have the form

$$ds^2 = \frac{F(\frac{z}{L})^2 dx^m dx_m + dz^2}{z^2} L^2$$

$$\Phi = \Phi(z) \tag{2.18}$$

where  $\Phi$  collectively denotes the scalar fields that have a nontrivial profile on the solution and  $F$  is a generic function of the fifth coordinate. These kind of geometries are usually called *domain wall* solutions and the dimensionless function  $F$  is referred to as the *warp factor* since it gives a measure of the deformation of the four-dimensional volume element. From now on we will restrict our analysis to these domain wall backgrounds.

As briefly mentioned in the previous section, in order for the holographic prescription to be under control, the bulk geometry must asymptote to that of  $\mathbf{AdS}_5$ . More specifically, it requires that the metric defines the same conformal structure on the boundary as in the  $\mathbf{AdS}_5$  case. Such geometries are usually referred to as Asymptotically Anti de Sitter or  $\mathbf{AAdS}$ .

Notice now that the domain wall metric in (2.18) satisfies this condition if the warp factor evaluated on the boundary  $z = 0$  is a constant. For simplicity we will choose this constant to be  $F(0) = 1$ . If we are sufficiently near to the boundary, or in other words for small value of the coordinate  $z$  (say  $z \ll L$ ), the  $\mathbf{AAdS}_5$  condition for a domain wall can then be written as

$$F(z) \underset{z \rightarrow 0}{\simeq} 1 + f_2 z^\alpha, \quad \alpha > 0 \tag{2.19}$$

where  $f_2$  is some constant coefficient. This is clearly a requirement on the metric. However, since the matter equations of motion will be coupled to those for the metric, one could expect that the  $\mathbf{AAdS}_5$  requirement would put some restriction on the scalar

fields as well. In fact this is exactly what is going to happen and can be shown in the following way.

Let us consider again a massive scalar field coupled to Einstein gravity<sup>4</sup>, with action

$$\mathcal{S} = \int d^5x \sqrt{-g} \left[ \frac{R}{2} - \Lambda - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 \right]. \quad (2.20)$$

The equations of motion for the metric and scalar field are

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (R - 2\Lambda) &= T_{\mu\nu} \\ \square \phi &= m^2 \phi \end{aligned} \quad (2.21)$$

where  $\square \phi = \frac{1}{\sqrt{-g}} \partial_\mu (g^{\mu\nu} \sqrt{-g} \partial_\nu \phi)$  and the matter stress-energy tensor is defined as

$$T_{\mu\nu} = -2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}} + g_{\mu\nu} \mathcal{L}_m \quad (2.22)$$

where  $\mathcal{L}_m$  is the lagrangian for the matter field. Since we want to look for a domain wall solution to the equations of motion we just put the ansatz (2.18) into the equations (2.21) and solve for  $F(z)$  and  $\phi(z)$ . For the present purpose we do not need a full solution but just its leading behavior near the boundary, so we further substitute

$$F(z) = 1 + f_2 z^\alpha, \quad \phi(z) = z^\beta \quad (2.23)$$

and solve the equations at first order in  $z \rightarrow 0$ . The leading order of the equation of motion for the scalar is the same as in the **AdS**<sub>5</sub> case, and from that one gets again the relation  $m^2 L^2 = \beta(\beta - 4)$ . Consider now the near boundary expansion of the  $zz$  component of Einstein's equations. One finds at zeroth order the relation  $\Lambda = -\frac{6}{L^2}$  that fixes the cosmological constant in terms of the dimensionful parameter  $L$ , and at first nontrivial order in  $z$  one obtains the equation

$$f_2 = -\frac{\beta(2\beta - 5)}{6\alpha(\alpha - 5)} (\phi_0)^2 z^{2\beta - \alpha}. \quad (2.24)$$

This gives us two relations. Consistency of the equation requires  $\alpha = 2\beta$  and, after substituting this back, one obtains  $f_2 = -\frac{\phi_0^2}{12}$ <sup>5</sup>. The former relation is important to understand the implications of **AAdS**<sub>5</sub> requirement that we are going to discuss momentarily. The latter shows the leading effect of the backreaction of the scalar profile on the geometry.

<sup>4</sup>For the time being we have set the gravitational coupling constant to  $\kappa_5^2 = 1$ .

<sup>5</sup>It is worth noticing that this relation among the leading coefficients of the warp factor and scalar field is universal, i.e. it does not depend on the form of the scalar potential.

Let us now discuss the meaning of the above result. Recalling the relation among the fall-off of the scalar  $\phi \sim z^\beta$  near the  $\mathbf{AdS}_5$  boundary and the dimension  $\Delta$  of the dual operator  $\mathcal{O}$  we have that (see eq. 2.15)

$$\begin{cases} \beta = 4 - \Delta & \text{if } \phi_0^- \neq 0 \\ \beta = \Delta & \text{if } \phi_0^- = 0 \end{cases} \quad (2.25)$$

and according to AdS/CFT the first case corresponds to deforming the CFT with the operator  $\mathcal{O}$ , whereas the second corresponds to switch on a VEV  $\langle \mathcal{O} \rangle \neq 0$  in the unperturbed CFT. As we have shown above, the  $\mathbf{AdS}_5$  condition on a domain wall solution implies that  $\alpha = 2\beta > 0$ . Combining this fact with the above argument we thus get that an  $\mathbf{AdS}_5$  domain wall can describe either a CFT perturbed by a relevant operator (i.e.  $\Delta < 4$ ) or a CFT in a vacuum with a nonzero VEV for an operator of arbitrarily high dimension. In the first case the relation among  $\Delta$  and the mass of the scalar implies that the bulk field must have negative mass squared, but as long as it satisfies the Breitenlohner-Freedman (BF) bound  $-4 \leq m^2$  this does not cause instabilities around  $\mathbf{AdS}_5$  [36].

### Summary

In this section we discussed the features of the gravitational backgrounds we will be focusing on in the rest of this thesis. These are  $\mathbf{AdS}_5$  domain wall supergravity solutions. The fields on such solutions have the following general forms and asymptotic expansions

$$\begin{aligned} ds^2 &= \frac{F(\frac{z}{L})^2 dx^m dx_m + dz^2}{z^2} L^2, & F(z) &\underset{z \rightarrow 0}{\simeq} 1 + z^{2\beta^*} \\ \Phi^I &= \Phi^I(z), & \Phi^I(z) &\underset{z \rightarrow 0}{\simeq} z^{\beta^I} \quad \beta^I > 0 \end{aligned} \quad (2.26)$$

where  $\beta^* = \min_I \{\beta^I\}$ . These corresponds on the dual side to either nonconformal QFT's, equivalently RG flows, obtained from CFT's deformed by relevant operators the least relevant of which has dimension  $\Delta = 4 - \beta^*$ ; or nonconformal vacua of a CFT where VEV's are turned on for some of the operators; or, more likely, a combination of the two. From now on the discussion will be restricted to this kind of backgrounds.

## 2.3 The holographic renormalization procedure

In Section 2.1 we made explicit the statement of the holographic correspondence through equation (2.1) which in our case simplifies to (2.7). Such equation says that the generating functional for connected diagrams in the QFT, as a functional of the operators

sources, equals the on-shell supergravity action, as a functional of the boundary conditions for the fields. In order to compute correlation functions in the strongly coupled field theories one thus has to differentiate the on-shell supergravity action with respect to the boundary conditions for the bulk fields. However, before actually doing this, one has to properly deal with divergences.

The left-hand side of (2.7) is not well-defined because it suffers from (at least) UV divergences, and so has to be for the right-hand side. In fact, UV divergences in the QFT are mapped to (what one would call) IR divergences in the gravity theory. This is a general phenomenon in gauge/gravity correspondence which goes under the name of *UV-IR connection* [37]. In a QFT these ambiguities are resolved by introducing a UV regulator and then choosing a prescription to subtract divergences. This renormalization scheme has better to be chosen in a proper way so not to spoil useful Ward identities and manifest invariance under the relevant symmetries of the problem. The same comment holds for any renormalization procedure one has to introduce on the dual gravity side. One such procedure for properly dealing with the problem of infinities in the context of holographic correspondence has been introduced in [15, 35, 38] for the case of purely **AdS** background and linear perturbations thereof, and then extended in [16, 17] to the nonconformal **AAdS** case. (See also [39–43] for earlier discussions about counterterms for **AdS** gravity.) This is nowadays a well-established procedure which goes under the name of *holographic renormalization*, which we are now going to review.

In the QFT the cancellation of UV divergences does not depend on IR information. Likewise the holographic renormalization procedure on the dual side should only depend on near-boundary data and not on the details of the bulk of the supergravity background, since short distance (UV) is the same as near-boundary on the gravity side. On the other hand, correlation functions capture the full dynamics of the QFT and cannot be determined only in terms of near-boundary data. In fact, the knowledge of the full supergravity solution will be required in order to compute correlation functions.

### 2.3.1 Outline of the procedure

A complete treatment of holographic renormalization is beyond the scope of this thesis. Here we will outline the general procedure one has to follow in order to regularize two-point correlation functions pointing out subtleties one can encounter in some cases relevant for the discussion in the forthcoming chapters.

The first step in this procedure is to write down a near-boundary expansion for the bulk fields dual to the operators whose correlation functions we are interested to compute.

Suppressing space-time and internal indices we have

$$\Phi(x, z) \underset{z \rightarrow 0}{\simeq} z^\beta (\Phi_{(0)}(x) + z^2 \Phi_{(2)}(x) + \dots) + z^\gamma (\log(z) \Psi_{(0)}(x) + \tilde{\Phi}_{(0)}(x) + \dots) \quad (2.27)$$

where  $\Phi(x, z)$  denotes all the fields in the gravity theory but the metric. For convenience we write the expansion for the metric separately as<sup>6 7</sup>

$$\begin{aligned} ds^2 &= \frac{dz^2}{z^2} + \frac{1}{z^2} \bar{g}_{mn}(x, z) dx^m dx^n \\ \bar{g} &\underset{z \rightarrow 0}{\simeq} \bar{g}_{(0)}(x) + z^2 \bar{g}_{(2)}(x) + z^4 \log(z) \bar{h}_{(4)}(x) + z^4 \bar{g}_{(4)}(x) + \dots \end{aligned} \quad (2.28)$$

Before moving to step two, some comments are in order. First of all the coefficients  $\beta$  and  $\gamma$  in (2.27) are related to the dimension  $\Delta$  of the dual operator. The precise relation for a scalar operator has been shown in the previous section but similar relations hold for operators of any spin [44]. For generic positive real values of  $\Delta$  the coefficients  $\beta$  and  $\gamma$  are real and the near-boundary expansion contains non-rational powers of  $z$ . However, in almost all examples discussed in the literature and in all of the applications we will discuss in this thesis  $\Delta \in \mathbb{N}(\frac{1}{2}\mathbb{N})$  for bosonic (fermionic) operators which implies  $\beta, \gamma \in \mathbb{N}(\frac{1}{2}\mathbb{N})$ . In these cases the expansion (2.27) solves the asymptotic equations of motion for the bulk fields. As for the near-boundary expansion of the metric, we have shown in the previous section that, at least for  $x$ -independent solutions, the backreaction of a scalar field affects the metric at order  $z^{2\beta}$  and thus (2.28) cannot hold in general. Nevertheless, if the above restriction on the coefficients  $\beta$  and  $\gamma$  holds then also the above ansatz for the metric solves the asymptotic Einstein's equations.

The second step in the procedure is to obtain the most general solution to the bulk equations of motion with fixed, but arbitrary, Dirichlet boundary conditions. In order to do this, one has to put the ansatz (2.27), (2.28) into the equations of motion and solve for the coefficients order by order in  $z \rightarrow 0$ . The coefficients  $\Phi_{(0)}(x)$  of the leading modes are left undetermined by this method and coincide with the arbitrary boundary conditions for the bulk fields. As we mentioned in the previous section they are interpreted as sources for the dual operators. All the other coefficients, but  $\tilde{\Phi}_{(0)}(x)$ , are thus uniquely determined as local functions of the sources  $\Phi_{(0)}(x)$ . The undetermined ones,  $\tilde{\Phi}_{(0)}(x)$ , are interpreted as the one-point functions of dual operators evaluated with sources  $\Phi_{(0)}(x)$  turned on. Finally the coefficients  $\Psi_{(0)}$ , which are also determined by the near-boundary analysis as local functions of the sources, are related to anomalous terms in conformal

<sup>6</sup>In this section we fix the asymptotic **AdS**<sub>5</sub> radius to  $L = 1$ . If needed, it could be restored by dimensional analysis.

<sup>7</sup>We suppress space-time indices of the metric not to clutter too much the notation.



Ward identities of the dual QFT<sup>8</sup>.

Once we have found the asymptotic behavior of the general solution to the equations of motion, we can plug it into the lagrangian and compute the on-shell action integrating in space-time. This turns out to be divergent, the divergences coming from the region near the boundary. To regularize the integral we thus restrict the integration domain to  $z \geq \epsilon$  for arbitrary positive  $\epsilon$ , and evaluate the boundary terms at  $z = \epsilon$

$$\mathcal{S}_{\text{reg}}[\Phi_{(0)}, \bar{g}_{(0)}; \epsilon] = \int_{z=\epsilon} d^4x \sqrt{-\bar{g}_{(0)}} \left[ \frac{f_{(0)}}{\epsilon^{2\alpha}} + \frac{f_{(2)}}{\epsilon^{2\alpha-2}} + \cdots + \log(\epsilon) f_{(2\alpha)} + \mathcal{O}(\epsilon^0) \right] \quad (2.30)$$

where  $\alpha$  is a positive number which depends on the dimension of the dual operator,  $f_{(2k)}$  are local functions of the sources  $\Phi_{(0)}$  and do not depend on the undetermined coefficients  $\tilde{\Phi}_{(0)}$ <sup>9</sup>. In order to subtract these divergences one proceeds by adding an appropriate counterterm action. The latter is required to be a generally covariant action for the bulk fields  $\Phi(x, z = \epsilon)$  and the induced metric  $\gamma_{mn} = g_{mn}(x, z = \epsilon)$  whose on-shell divergent part in  $\epsilon \rightarrow 0$  exactly equals the divergent part of  $\mathcal{S}_{\text{reg}}$ :

$$\text{divergent part of } \mathcal{S}_{\text{ct}}[\Phi(x, \epsilon), \gamma(x, \epsilon); \epsilon] = \text{divergent part of } \mathcal{S}_{\text{reg}}[\Phi_{(0)}, \bar{g}_{(0)}; \epsilon]. \quad (2.31)$$

The renormalized on-shell action is now defined as

$$\mathcal{S}_{\text{ren}}[\Phi_{(0)}, \tilde{\Phi}_{(0)}] = \lim_{\epsilon \rightarrow 0} (\mathcal{S}_{\text{reg}} - \mathcal{S}_{\text{ct}}) \quad (2.32)$$

where the limit is finite by definition, and the result is now a functional of the undetermined coefficients of the near-boundary analysis and is independent of the regulator.

Notice that the definition (2.31) leaves us the freedom to add terms that are finite as  $\epsilon \rightarrow 0$ . Such ambiguity, which reflects the scheme dependence in analogous QFT computations, is partially fixed by demanding that  $\mathcal{S}_{\text{ct}}$  must be covariant. This requirement is essential to assure that (2.32) yields correlation functions that respect Ward identities associated to space-time symmetries of the QFT. However we are still free to add finite covariant counterterms. One way to fix also this residual freedom is to require the action to behave covariantly with respect to other local symmetries of the gravity theory (e.g. internal symmetries when treating with gauge fields, or supersymmetry), this assures

<sup>8</sup>This can be intuitively seen as follows: under the dilation (2.11) the log-term shifts, and this contribute with an unexpected, i.e. anomalous, term to the transformation law of  $\tilde{\Phi}_{(0)}$

$$\tilde{\Phi}_{(0)} \rightarrow e^{-2\gamma\omega} \left( \tilde{\Phi}_{(0)} - \omega \Psi_{(0)} \right) \quad (2.29)$$

which corresponds to an anomalous contribution in the dilation Ward identities involving the one-point function of the dual operator.

<sup>9</sup>This is generically true but there can be exceptions, e.g. a scalar with  $m^2 = -2$

that the corresponding symmetries in QFT will be manifest in correlation functions thus computed.

The fourth and last step is to compute (two-point) correlation functions using the holographic prescription with our renormalized on-shell action. If we focus on a particular field  $\phi \in \Phi$ , the one-point function of the corresponding operator  $\mathcal{O}_\phi$  can be found differentiating (2.32) with respect to the source  $\phi_{(0)}$ . Explicit evaluation yields

$$\langle \mathcal{O}_\phi \rangle_{\Phi_{(0)}} = \frac{1}{\sqrt{-\tilde{g}_{(0)}}} \frac{\delta S_{\text{ren}}}{\delta \phi_{(0)}} \sim \tilde{\phi}_{(0)} + \text{local function of } \Phi_{(0)}, \quad (2.33)$$

where  $\langle \mathcal{O}_\phi \rangle_{\Phi_{(0)}}$  denotes the one-point function with the sources switched on. The coefficient in front of  $\tilde{\phi}_{(0)}$  depends on the theory under consideration but not on the subtraction scheme we used (i.e. it does not depend on the finite part of  $\mathcal{S}_{\text{ct}}$ ). The local function of  $\Phi_{(0)}$  yields contact terms in higher correlation functions and depends both on the theory under consideration and on the subtraction scheme. Differentiating the above one-point function with respect to the source and evaluating the result on the background value for the sources we obtain the two-point function

$$\langle \mathcal{O}_\phi \mathcal{O}_\phi \rangle \sim \left. \frac{\delta \tilde{\phi}_{(0)}}{\delta \phi_{(0)}} \right|_{\Phi_{(0)} = \Phi_{(0)}^{\text{bg}}} + \text{contact terms}. \quad (2.34)$$

We see that in order to compute n-point functions one needs to know the dependence of the coefficient  $\tilde{\phi}_{(0)}$  on the source  $\phi_{(0)}$ . As anticipated, this cannot be extracted from the near-boundary analysis but requires the knowledge of the exact solution of the full non-linear equations of motion. Once a regular solution is known, such dependence can be read off its asymptotic expansion. For the present purpose of computing two-point functions a great simplification occurs: as it is clear from (2.34), we just need the linear dependence of  $\tilde{\phi}_{(0)}$  on the source  $\phi_{(0)}$ , and this only requires us to solve the linearized equations of motion around the chosen background.

## 2.4 Holographic renormalization for scalars in AdS

The procedure outlined in the previous section is what one should follow to compute correlators of any (bosonic or fermionic) operator of interest. The details, however, depend on the particular form of the action for the gravity theory (e.g. masses and interaction terms). In this section we want to carry out the holographic renormalization procedure in a concrete example, namely that of a scalar with generic mass in  $\text{AdS}_5$ .

The (Euclidean) action for a massive scalar field minimally coupled to a fixed  $\text{AdS}_5$  background is given by

$$\mathcal{S} = \frac{1}{2} \int d^5x \sqrt{g} [g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2] \quad (2.35)$$

and the metric is as in (2.9). The main object we are interested in is the on-shell value of the action. Integrating by parts (2.35) and using the equations of motion (2.13), one obtains

$$\mathcal{S}^{o.s.} = -\frac{1}{2} \lim_{\epsilon \rightarrow 0} \int_{z=\epsilon} d^4x \epsilon^{-3} \phi \partial_z \phi = -\frac{1}{4} \lim_{\epsilon \rightarrow 0} \int_{z=\epsilon} d^4x \epsilon^{-3} \partial_z (\phi^2). \quad (2.36)$$

From the expression above one can already see that only terms up to and including  $z^4$  in the near-boundary expansion of  $\phi^2$  can contribute to the on-shell action.

The holographic renormalization procedure, and in particular the specific form of the counterterms, will depend on the mass of the scalar field. In order not to complicate too much the notation, in the following we will consider the cases  $m^2 = 0, -3, -4$  (in unit of the  $\text{AdS}_5$  radius) which, according to (2.14), correspond to CFT operators with  $\Delta = 4, 3, 2$ .

### 2.4.1 Massless scalar

According to the general recipe (2.27), the asymptotic expansion for the scalar field up to  $\mathcal{O}(z^4)$  can be written as

$$\phi(x, z) \underset{z \rightarrow 0}{\simeq} \phi_{(0)} + \phi_{(2)} z^2 + \left( \phi_{(4)} + \tilde{\phi}_{(0)} \right) z^4 + \psi_{(0)} \log(z) z^4, \quad (2.37)$$

where we used the fact that for a massless scalar  $\beta = 0$  and  $\gamma = 4$ . Notice that the two series merge at order  $z^4$ ; as a consequence, the coefficients  $\phi_{(2n)}$  with  $n \geq 2$  are redundant and can be consistently set to 0. These modes are in fact replaced by the logarithmic ones  $\psi_{(2n)}$ .

Inserting the expansion above into the equation of motion (2.13) one finds the following relations

$$\phi_{(2)} = -\frac{p^2}{4} \phi_{(0)}, \quad \psi_{(0)} = -\frac{p^4}{16}, \quad (2.38)$$

where we have performed a Fourier transform along the boundary coordinates and  $p$  denotes the four-dimensional momentum. As expected, the equation of motion leaves the coefficient  $\phi_{(0)}$  and  $\tilde{\phi}_{(0)}$  undetermined. These are then identified respectively with the source and the one-point function of the dual operator.

Plugging the solution back in (2.36) gives the regularized action in terms of the undetermined coefficients

$$\mathcal{S}_{\text{reg}}^{\text{o.s.}} = -\frac{1}{2} \int_{z=\epsilon} d^4x \left[ 2\epsilon^{-2} \phi_{(0)} \phi_{(2)} + 4 \log(\epsilon) \phi_{(0)} \psi_{(0)} + 2 \phi_{(2)}^2 + 4 \phi_{(0)} \tilde{\phi}_{(0)} + \psi_{(0)} \phi_{(0)} \right]. \quad (2.39)$$

Using relations (2.38) one can write the following covariant counterterm action which reproduces the divergences of  $\mathcal{S}_{\text{reg}}^{\text{o.s.}}$

$$\mathcal{S}_{\text{ct}} = \frac{1}{4} \int d^4x \sqrt{\gamma} \left[ \gamma^{mn} \partial_m \phi \partial_n \phi - \frac{1}{2} \gamma^{mn} \gamma^{rs} \partial_m \partial_n \phi \partial_r \partial_s \phi (\log(\epsilon) + \alpha) \right]. \quad (2.40)$$

Notice that the second counterterm, needed to remove the  $\log(\epsilon)$  divergence in (2.39), introduces an ambiguity. Indeed, rescaling the cut-off  $\epsilon$  shifts the log-counterterm by a finite contribution. We have introduced the real parameter  $\alpha$  to keep track of this ambiguity.

The renormalized action  $\mathcal{S}_{\text{ren}} = \lim_{\epsilon \rightarrow 0} (\mathcal{S}_{\text{reg}} + \mathcal{S}_{\text{ct}})$  in momentum space hence reads

$$\mathcal{S}_{\text{ren}} = \int \frac{d^4p}{(2\pi)^4} \left[ -2\phi_{(0)} \tilde{\phi}_{(0)} + \frac{3-4\alpha}{32} p^4 \phi_{(0)}^2 \right]. \quad (2.41)$$

Now that we have properly taken care of divergences, we can differentiate twice with respect to the source  $\phi_{(0)}$  and obtain the two-point function

$$\langle \mathcal{O}_4 \mathcal{O}_4 \rangle = - \frac{\delta \mathcal{S}_{\text{ren}}}{\delta \phi_{(0)} \delta \phi_{(0)}} \Big|_{\phi_{(0)}=0} = 4 \frac{\delta \tilde{\phi}_{(0)}}{\delta \phi_{(0)}} + \frac{4\alpha-3}{16} p^4. \quad (2.42)$$

This result shows that contributions proportional to  $p^4$  are scheme dependent (and indeed they can be subtracted by local covariant counterterms).

The evaluation of the two-point function requires the knowledge of the full solution of the equation of motion. In **AdS**<sub>5</sub> the general solution can be written in terms of modified Bessel functions [45] as

$$\phi(z, p) = z^2 (A(p) K_2(pz) + B(p) I_2(pz)), \quad (2.43)$$

where  $A(p)$  and  $B(p)$  are functions of the two undetermined coefficients  $\phi_{(0)}$ ,  $\tilde{\phi}_{(0)}$  and can be determined imposing that the solution above matches the expansion (2.37) near the boundary. Demanding the solution to be regular imposes  $B = 0$  and this gives us the dependence of  $\tilde{\phi}_{(0)}$  from  $\phi_{(0)}$ . One finds

$$\tilde{\phi}_{(0)} = -\frac{p^4}{64} (2 \log(p^2) + 4\gamma - 4 \log(2) - 3) \phi_{(0)}, \quad (2.44)$$

and using (2.42) one finally gets

$$\langle \mathcal{O}_4 \mathcal{O}_4 \rangle = -\frac{p^4}{8} \log(p^2) + \frac{\alpha - \gamma + \log(2)}{4} p^4. \quad (2.45)$$

### 2.4.2 Scalar with $m^2 = -3$

The logic is exactly the same as in the previous example so we will not repeat all steps here. The scalar is now dual to an operator with  $\Delta = 3$  and its near-boundary expansion is thus

$$\phi(x, z) \underset{z \rightarrow 0}{\simeq} \phi_{(0)} z + \left( \phi_{(2)} + \tilde{\phi}_{(0)} \right) z^3 + \psi_{(0)} \log(z) z^3 + \mathcal{O}(z^5). \quad (2.46)$$

As before, the modes  $\phi_{(2n)}$  with  $n \geq 1$  can be set to zero, while the equation of motion imposes

$$\psi_{(0)} = \frac{p^2}{2} \phi_{(0)}. \quad (2.47)$$

The on-shell regularized action now reads

$$\mathcal{S}_{\text{reg}} = -\frac{1}{2} \int_{z=\epsilon} d^4x \left[ \epsilon^{-2} \phi_{(0)}^2 + 4 \log(\epsilon) \phi_{(0)} \psi_{(0)} + 4 \phi_{(0)} \tilde{\phi}_{(0)} + \psi_{(0)} \phi_{(0)} \right], \quad (2.48)$$

and we introduce the following counterterms to subtract the divergences

$$\mathcal{S}_{\text{ct}} = \frac{1}{2} \int d^4x \sqrt{\gamma} \left[ \phi^2 - \gamma^{mn} \partial_m \phi \partial_n \phi (\log(\epsilon) + \alpha) \right]. \quad (2.49)$$

We obtain the renormalized result

$$\mathcal{S}_{\text{ren}} = \int \frac{d^4p}{(2\pi)^4} \left[ -\phi_{(0)} \tilde{\phi}_{(0)} + \frac{2\alpha - 1}{4} p^2 \phi_{(0)}^2 \right], \quad (2.50)$$

from which we extract the two-point function

$$\langle \mathcal{O}_3 \mathcal{O}_3 \rangle = 2 \frac{\delta \tilde{\phi}_{(0)}}{\delta \phi_{(0)}} + \frac{1 - 2\alpha}{2} p^2. \quad (2.51)$$

The regular solution can be written using Bessel functions as

$$\phi(z, p) = z^2 p \phi_{(0)} K_1(pz). \quad (2.52)$$

Expanding near the boundary we find

$$\tilde{\phi}_{(0)} = \frac{p^2}{4} (\log(p^2) + 2\gamma - 2\log(2) - 1) \phi_{(0)}, \quad (2.53)$$

and finally

$$\langle \mathcal{O}_3 \mathcal{O}_3 \rangle = \frac{p^2}{2} \log(p^2) + (\gamma - \log(2) - \alpha) p^2. \quad (2.54)$$

### 2.4.3 Scalar with $m^2 = -4$

In this case the scalar field saturate the BF bound. The dual operator has  $\Delta = 2$ . The near-boundary expansion in this case reads

$$\phi(x, z) \underset{z \rightarrow 0}{\simeq} \phi_{(0)} \log(z) z^2 + \tilde{\phi}_{(0)} z^2 + \mathcal{O}(z^4), \quad (2.55)$$

The logarithmic mode is now the leading term. For this reason we have identified its coefficient with the source, redefining  $\psi_{(0)} \rightarrow \phi_{(0)}$ . The  $\mathcal{O}(z^4)$  terms are not relevant in this case and we can neglect them. The on-shell regularized action is

$$\mathcal{S}_{\text{reg}} = -\frac{1}{2} \int_{z=\epsilon} d^4x \left[ 2 \log(\epsilon)^2 \phi_{(0)}^2 + \log(\epsilon) \phi_{(0)}^2 + 4 \log(\epsilon) \phi_{(0)} \tilde{\phi}_{(0)} + 2 \tilde{\phi}_{(0)}^2 + \phi_{(0)} \tilde{\phi}_{(0)} \right]. \quad (2.56)$$

The needed counterterms can be written in the form

$$\mathcal{S}_{\text{ct}} = \frac{1}{2} \int d^4x \sqrt{\gamma} \left[ 2 \phi^2 + \frac{\phi^2}{\log(\epsilon)} \right], \quad (2.57)$$

and the resulting renormalized action reads

$$\mathcal{S}_{\text{ren}} = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \phi_{(0)} \tilde{\phi}_{(0)}. \quad (2.58)$$

The regular solution can be written again using Bessel functions as

$$\phi(z, p) = -z^2 \phi_{(0)} K_0(pz), \quad (2.59)$$

Expanding near the boundary we find

$$\tilde{\phi}_{(0)} = (\log(p) + \gamma - \log(2)) \phi_{(0)}, \quad (2.60)$$

and finally

$$\langle \mathcal{O}_2 \mathcal{O}_2 \rangle = -\frac{1}{2} \log(p^2) - \gamma + \log(2). \quad (2.61)$$

The results (2.45), (2.54) and (2.61) are exactly what one would expect for two-point functions of CFT operators. In particular the scheme independent part has the correct scaling,  $p^{2\Delta-4}$ , and the typical log-behavior of CFT correlators. The formulas derived in this section will be used in the following chapters. In particular the BF scalar will

enter the holographic description of the current multiplet in Chapter 3, whereas the cases  $m^2 = 0, -3$  will be relevant for the discussion in Chapter 5.





## Chapter 3

# Current supermultiplet and holography

In a four-dimensional  $\mathcal{N} = 1$  supersymmetric QFT, the *current supermultiplet* is a multiplet of operators which contains conserved global currents. This chapter is devoted to the holographic study of two-point correlation functions among operators belonging to this particular supermultiplet. We will first review the structure and superspace description of the multiplet in QFT, and then present the holographic dictionary which connects the operators of the current supermultiplet to a specific  $\mathcal{N} = 2$  multiplet of five-dimensional supergravity fields. Finally, we will discuss the holographic prescription for computing two-point functions of these operators.

### 3.1 Field theory preliminaries

Throughout this section and whenever we will talk about four-dimensional supersymmetry and superspace, we will adhere to the conventions of [46]. In particular Grassmann odd superspace coordinates are denoted with Weyl spinors  $\theta_\alpha$  and  $\bar{\theta}_{\dot{\alpha}} = (\theta_\alpha)^*$  where undotted and dotted indices transform under the  $SU(2)_L$ , respectively  $SU(2)_R$ , subgroup of the Lorentz group  $SL(2, \mathbb{C})$ . The supersymmetry algebra is

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = -2i \sigma_{\alpha\dot{\alpha}}^\mu P_\mu \quad (3.1)$$

where  $P_\mu = -i \partial_\mu$  as a differential operator acting on (super)fields. We also have the algebra for supercovariant derivatives

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = 2i \sigma_{\alpha\dot{\alpha}}^\mu P_\mu. \quad (3.2)$$

In the following, we will focus our attention on the structure and operator content of a so called *linear superfield*. This is a real superfield satisfying the following differential equation in superspace

$$\begin{aligned}\mathcal{J}(x, \theta, \bar{\theta}) &= \mathcal{J}^*(x, \theta, \bar{\theta}) \\ D^2 \mathcal{J} &= \bar{D}^2 \mathcal{J} = 0,\end{aligned}\tag{3.3}$$

since the above condition puts restrictions on the  $x$ -dependence of some component operator, it should be understood as an on-shell condition. It is a well-known fact that linear superfields are associated to global internal symmetries in a supersymmetric field theory. We are now going to review how this correspondence works.

### 3.1.1 Supersymmetric Noether theorem

The association of linear multiplets to preserved global symmetries in a supersymmetric theory can be shown in a nice way which is analogous to the derivation of the standard Noether theorem, see e.g. [47]. Let us consider a supersymmetric action invariant under a certain global symmetry

$$\delta_\lambda \mathcal{S} = 0.\tag{3.4}$$

Let us suppose that the symmetry transformation is abelian (the generalization to the non-abelian case being straightforward) and  $\lambda$  is a real number parameterizing such transformation. By definition, the action should also be invariant under an arbitrary variation of the superfields when computed on a solution to the equations of motion (i.e. on-shell). We thus have

$$\delta_\Lambda \mathcal{S}|_{\text{on-shell}} = 0,\tag{3.5}$$

for any chiral superfield  $\Lambda(x, \theta)$ . When the chiral superfield is replaced by a real constant the above must hold for any superfield configuration (i.e. off-shell), (3.4). Recalling now that a chiral superfield which is also real reduces to a constant one can conclude that the off-shell variation of the action must be proportional to the imaginary part of  $\Lambda$ , and so

$$\delta_\Lambda \mathcal{S} = \int d^4x \int d^4\theta \, i (\Lambda - \Lambda^*) \mathcal{J}\tag{3.6}$$

for some real superfield  $\mathcal{J}(x, \theta, \bar{\theta})$ . Exploiting the chirality of  $\Lambda$  and the fact that  $d^4\theta \sim D^2 \bar{D}^2 \sim \bar{D}^2 D^2$  up to total derivatives, we can rewrite the variation as

$$\delta_\Lambda \mathcal{S} = \int d^4x \left( i D^2 \Lambda \bar{D}^2 \mathcal{J} - i \bar{D}^2 \Lambda^* D^2 \mathcal{J} \right).\tag{3.7}$$

Since the above variation must vanish on-shell for any chiral superfield  $\Lambda$  we find that  $\mathcal{J}$  is a linear superfield

$$D^2 \mathcal{J} = \bar{D}^2 \mathcal{J} = 0. \quad (3.8)$$

For the case of a non-abelian symmetry one just replaces the chiral superfield  $\Lambda$  with a matrix  $\Lambda_i^j$  whose entries are chiral superfields and which transforms in the adjoint representation of the symmetry group. In this case the superfield  $\mathcal{J}$  in (3.6) must also transform in the adjoint and following the same steps as before one arrives to

$$D^2 \mathcal{J}_i^j = \bar{D}^2 \mathcal{J}_i^j = 0. \quad (3.9)$$

The particular form of the current superfield depends on the details of the theory and of the symmetry transformations. However if the theory contains only chiral superfields and the transformation acts linearly, one can find a general formula. Consider a supersymmetric Lagrangian for  $N$  chiral superfields  $\Phi_i$ ,  $i = 1, \dots, N$

$$\mathcal{L} = \int d^4\theta K(\Phi, \Phi^*) + \int d^2\theta W(\Phi) + \int d^2\bar{\theta} W^*(\Phi^*) \quad (3.10)$$

and suppose this is invariant under the  $U(1)$  transformation

$$\delta_\lambda \Phi_i = i \lambda M_i^j \Phi_j \quad \delta_\lambda \Phi^{i*} = -i \lambda \Phi^{j*} M_j^i \quad (3.11)$$

with  $M$  a hermitian charge matrix and  $\lambda \in \mathbb{R}$ . Invariance of the above Lagrangian puts the following restriction on the Kahler potential and superpotential

$$\begin{aligned} \frac{\partial W}{\partial \Phi_i} M_i^j \Phi_j &= \Phi^{j*} M_j^i \frac{\partial W}{\partial \Phi^{i*}} = 0 \\ \frac{\partial K}{\partial \Phi_i} M_i^j \Phi_j &= \Phi^{j*} M_j^i \frac{\partial K}{\partial \Phi^{i*}}. \end{aligned} \quad (3.12)$$

We now promote the parameter  $\lambda$  to a chiral superfield  $\Lambda$  in the transformation law (3.11) and compute the variation of the action. Since the superpotential is a holomorphic function of  $\Phi$  it is automatically invariant also under this extended transformation. For the Kahler part we then have

$$\delta_\Lambda \mathcal{L} = \int d^4\theta i (\Lambda - \Lambda^*) \frac{\partial K}{\partial \Phi_i} M_i^j \Phi_j \quad (3.13)$$

and comparing with (3.6) we find

$$\mathcal{J} = \frac{\partial K}{\partial \Phi_i} M_i^j \Phi_j. \quad (3.14)$$

In the non-abelian case with chiral superfields in the fundamental of the symmetry group,

one simply consider  $\lambda M_i^j \rightarrow M_i^j$  as a hermitian matrix of infinitesimal parameters. Repeating then the same steps one finds

$$\mathcal{J}_j^i = \frac{\partial K}{\partial \Phi_i} \Phi_j. \quad (3.15)$$

### 3.1.2 Supermultiplet structure and two-point functions

Any superfield can be viewed as a multiplet of ordinary fields, or in the present case a multiplet of composite operators. Expanding the superfield (3.3) in  $\theta, \bar{\theta}$  one obtains

$$\begin{aligned} \mathcal{J}(x, \theta, \bar{\theta}) = & J(x) + i\theta j(x) - i\bar{\theta} \bar{j}(x) - \theta \sigma^m \bar{\theta} j_m(x) \\ & + \frac{1}{2} \theta \theta \bar{\theta} \bar{\sigma}^m \partial_m j(x) - \frac{1}{2} \bar{\theta} \bar{\theta} \theta \sigma^m \partial_m \bar{j}(x) - \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \square J(x) \end{aligned} \quad (3.16)$$

where  $J$  is real and  $\partial^m j_m = 0$ , as a consequence of the two conditions in (3.3). The operator  $j_m$  is conserved and can then be identified with the conserved current associated to the preserved symmetry by the usual Noether theorem. A conserved global current is a gauge invariant operator and its dimension is a protected quantity (e.g. it cannot change when going to strong coupling). As long as supersymmetry is preserved the same considerations must hold for all the operators belonging to the same supermultiplet. Namely, they are all gauge invariant and their dimensions are protected. A spin-1 conserved current, such as  $j_m$ , in four space-time dimensions has dimension equal to 3. Recalling that  $\theta$ -coordinates have mass dimension  $-\frac{1}{2}$ , we thus have that  $J$  is a scalar operator with dimension 2 and  $j_\alpha$  is a spin- $\frac{1}{2}$  operator of dimension  $\frac{5}{2}$ .

For concreteness, let us consider the example of the previous section but now with a canonical Kahler potential. In this case the linear superfield is  $\mathcal{J} = \Phi^{i*} M_i^j \Phi_j$  and using the expansion for a chiral superfield [46]

$$\Phi = \phi + \sqrt{2}\theta\psi + i\theta\sigma^m\bar{\theta}\partial_m\phi + \theta\theta F + \dots \quad (3.17)$$

one finds the following expressions for the component operators of  $\mathcal{J}$  in terms of elementary fields

$$J = \phi^{i*} M_i^j \phi_j, \quad (3.18a)$$

$$j_\alpha = -i\sqrt{2}\phi^{i*} M_i^j \psi_{j\alpha}, \quad (3.18b)$$

$$j_m = \left( i\partial_m \phi^{i*} \phi_j - i\phi^{i*} \partial_m \phi_j - \bar{\psi}^i \bar{\sigma}^m \psi_j \right) M_i^j, \quad (3.18c)$$

where  $M$  is the hermitian matrix containing the charges of the chiral superfields under the  $U(1)$  global symmetry. It is worth noticing that the real operator (3.18a) has dimension 2 and is protected, according to the previous argument<sup>1</sup>.

Using superspace methods one can easily work out the supersymmetry transformations of the current multiplet operators. These read

$$\delta J = i \epsilon j - i \bar{\epsilon} \bar{j}, \quad (3.19a)$$

$$\delta j_\alpha = i \sigma_{\alpha\dot{\beta}}^m \bar{\epsilon}^{\dot{\beta}} (j_m - i \partial_m J), \quad (3.19b)$$

$$\delta j_m = 2\epsilon \sigma_{mn} \partial^n j + 2\bar{\epsilon} \bar{\sigma}_{mn} \partial^n \bar{j}. \quad (3.19c)$$

We have gathered all the information about the operator content of the current supermultiplet and how the supersymmetry algebra is realized on this set of operators. We want now to analyze the structure of two-point correlation functions among the three operators  $J, j_\alpha, j_m$ . Lorentz invariance and current conservation imply that the current-current correlators have the following form in (Euclidean) momentum space

$$\langle j_m(p) j_n(-p) \rangle = - (p^2 \eta_{mn} - p_m p_n) C_1 \left( \frac{p^2}{M^2} \right) \quad (3.20a)$$

$$\langle j_\alpha(p) \bar{j}_{\dot{\beta}}(-p) \rangle = - \sigma_{\alpha\dot{\beta}}^m p_m C_{\frac{1}{2}} \left( \frac{p^2}{M^2} \right) \quad (3.20b)$$

$$\langle J(p) J(-p) \rangle = C_0 \left( \frac{p^2}{M^2} \right) \quad (3.20c)$$

$$\langle j_\alpha(p) j_\beta(-p) \rangle = \varepsilon_{\alpha\beta} M B \left( \frac{p^2}{M^2} \right) \quad (3.20d)$$

where  $C_1, C_{\frac{1}{2}}, C_0$  are three real *form factors*, whereas  $B$  can be complex. They are all dimensionless function of the ratio  $\frac{p^2}{M^2}$  ( $M$  indicating some typical scale) and their precise form depends on the details of the theory under consideration. All other two-point functions can be shown to vanish using symmetry arguments. Let us notice that the form of the correlators in (3.20) is also valid for a strongly coupled quantum field theory. What in general is not known and often uncalculable using field theory methods is the strong coupling limit of the model-dependent form factors.

Equations (3.20) have been deduced without the use of supersymmetry. As such, they hold on a non-supersymmetric vacuum as well (i.e. when supersymmetry is spontaneously broken). However, when the vacuum preserves supersymmetry one finds that

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<sup>1</sup>This is a really non-trivial fact. Real operators, like e.g. the Kahler potential, do not generically obey any non-renormalization theorem in  $\mathcal{N} = 1$  supersymmetry and usually acquire large anomalous dimensions in strongly coupled field theories.

the following variations vanish

$$\begin{aligned} 0 &= \langle \delta (J(p) j_\alpha(-p)) \rangle \\ 0 &= \langle \delta (j_m(p) j_\alpha(-p)) \rangle \end{aligned} \quad (3.21)$$

which imply, using (3.19), the following relations among form factors

$$C_0\left(\frac{p^2}{M^2}\right) = C_{\frac{1}{2}}\left(\frac{p^2}{M^2}\right) = C_1\left(\frac{p^2}{M^2}\right) \equiv C_{\text{susy}}\left(\frac{p^2}{M^2}\right), \quad B\left(\frac{p^2}{M^2}\right) = 0. \quad (3.22)$$

In a superconformal field theory (SCFT), assuming that the vacuum does not break conformal invariance, there is no parameter or coupling constant that can play the role of the scale  $M$ . In this case one then expects the form factor  $C_{\text{susy}}$  to have a logarithmic dependence on the momentum

$$C_{\text{scft}}\left(\frac{p^2}{\Lambda^2}\right) = c \log\left(\frac{\Lambda^2}{p^2}\right), \quad (3.23)$$

where  $\Lambda$  is a UV cut-off scale. The constant coefficient  $c$ , which is a central charge of the SCFT, is independent from the cut-off but depends on the amount of matter which is charged under the global symmetry associated to  $\mathcal{J}^2$ . Let us now focus on theories that are asymptotically superconformal in the UV (which is the relevant case for the forthcoming holographic discussion) but can spontaneously break supersymmetry in the IR. In this case the deviation from the superconformal behavior will become less and less important as we approach the large (Euclidean) momentum regime, meaning that the relations (3.22) should still be valid in the UV limit

$$\begin{aligned} \lim_{p^2 \rightarrow +\infty} C_0\left(\frac{p^2}{M^2}\right) &= \lim_{p^2 \rightarrow +\infty} C_{\frac{1}{2}}\left(\frac{p^2}{M^2}\right) = \lim_{p^2 \rightarrow +\infty} C_1\left(\frac{p^2}{M^2}\right) = c_{\text{uv}} \log\left(\frac{\Lambda^2}{p^2}\right), \\ \lim_{p^2 \rightarrow +\infty} B\left(\frac{p^2}{M^2}\right) &= 0, \end{aligned} \quad (3.25)$$

where now  $c_{\text{uv}}$  is the central charge computed in the unperturbed UV superconformal fixed point.

Before moving to the holographic part, let us add some comments about R-symmetry. This is a particular kind of continuous global symmetry that may or may not be present in supersymmetric theories. In four-dimensional  $\mathcal{N} = 1$  supersymmetry the R-symmetry

<sup>2</sup>In a superconformal theory the OPE of the conserved current satisfies some general constraints (see e.g. [48]). In particular, only the unit operator in the OPE of  $J(x)J(0)$  can have an expectation value, leading to

$$C_0(x) = C_{\frac{1}{2}}(x) = C_1(x) = \frac{\tau}{16\pi^4 x^4} \rightarrow C_0(p^2) = C_{\frac{1}{2}}(p^2) = C_1(p^2) = \frac{\tau}{16\pi^2} \ln\left(\frac{\Lambda^2}{k^2}\right). \quad (3.24)$$

The coefficient  $\tau$  gives the contribution of the CFT matter to the trace anomaly when the conserved current is coupled to an external gauge field.

group can be at most  $U(1)$  and acts on superspace coordinates in the following way<sup>3</sup>

$$\theta \rightarrow e^{i\alpha} \theta, \quad \bar{\theta} \rightarrow e^{-i\alpha} \bar{\theta}. \quad (3.26)$$

From the point of view of supersymmetry this is a (super)space-time symmetry, as opposed to internal symmetries, and has the same status of other space-time symmetries such as translations and Lorentz transformation. The supersymmetric Noether theorem does not apply in this case and there is no linear multiplet associated to an R-symmetry<sup>4</sup>. However, the presence of an R-symmetry imposes constraint on the form factors defined in (3.20), associated to some other (non-R)-symmetry present in the theory. The current superfield  $\mathcal{J}$  is real and cannot be charged under a  $U(1)$  group. Using (3.26) one can then show that if the theory has an R-symmetry, this must assign the following charges to the component operators

$$R(J) = 0, \quad R(j_\alpha) = -1, \quad R(j_m) = 0. \quad (3.27)$$

The two-point function (3.20d), and consequently the form factor  $B$ , carries R-charge  $-2$ . This means that also in presence of spontaneous supersymmetry breaking  $B = 0$  unless the R-symmetry is broken, either explicitly or spontaneously.

Let us summarize the main messages of this section. Two-point functions of a current supermultiplet can be parametrized in terms of four form factors  $C_0, C_{\frac{1}{2}}, C_1, B$  as in (3.20). We focus on supersymmetric QFT's that approaches a superconformal fixed point in the UV. The large momentum behavior of the form factors is then

$$\lim_{p^2 \rightarrow +\infty} C_0 = \lim_{p^2 \rightarrow +\infty} C_{\frac{1}{2}} = \lim_{p^2 \rightarrow +\infty} C_1 = c_{\text{uv}} \log \left( \frac{\Lambda^2}{p^2} \right), \quad (3.28)$$

$$\lim_{p^2 \rightarrow +\infty} B = 0.$$

We can then have in addition the following stronger conditions depending on the case:

- conformal invariance is broken and the vacuum is not supersymmetric. Just the weaker condition (3.28) holds for  $C$ 's and we can have two sub-cases for the form factor  $B$ 
  1. the R-symmetry is broken or is not there. Then nothing more than (3.28) holds in general;
  2. there is an R-symmetry and this is preserved by the vacuum  $\Rightarrow B = 0$  identically;

<sup>3</sup>The charge assignment on  $\theta$ 's (here  $+1$ ) is purely conventional.

<sup>4</sup>As we will review in Chapter 5, the R-symmetry current sits in another kind of supermultiplet together with the stress-energy tensor and the supercurrent.

- conformal invariance is broken (either explicitly or spontaneously) but the vacuum is supersymmetric, then

$$C_0 = C_{\frac{1}{2}} = C_1 \equiv C_{\text{susy}}, \quad B = 0; \quad (3.29)$$

- the theory is exactly superconformal, then the form factors have the form

$$C_0 = C_{\frac{1}{2}} = C_1 = c \log\left(\frac{\Lambda^2}{p^2}\right), \quad B = 0, \quad (3.30)$$

for constant  $c$ .

This analysis comprehends all the cases we will encounter in the next sections when dealing with holographic models.

## 3.2 Holography for the current supermultiplet

In this section we will focus on the holographic computation of two-point correlation functions for the current supermultiplet introduced in the previous section. We will present the operators/fields map pertinent to this multiplet and then use holographic renormalization techniques to compute the form factors defined in (3.20).

### 3.2.1 Holographic dictionary

Global symmetries of a QFT are mapped to local symmetries of the dual gravitational theory by AdS/CFT. In order to see this one can consider a U(1) gauge field living in the five-dimensional bulk. The action will be of the form

$$\mathcal{S}_{\text{gauge}} \sim \int d^4x \int_0^\infty dz \sqrt{-g} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}, \quad (3.31)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . Varying with respect to  $A_\mu$  and integrating by parts one obtains

$$\delta\mathcal{S}_{\text{gauge}} \sim - \int d^5x \partial_\rho (\sqrt{-g} g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma}) \delta A_\mu - \int_{z=0} d^4x \sqrt{-g} F^{zm} \delta A_m. \quad (3.32)$$

The first term gives Maxwell equations in a curved background. The second tells us that in order for the variational problem to be well-posed we need to specify either Neumann or Dirichlet conditions for the field  $A_m$  at the boundary  $z = 0$ . Let us choose, as usual,



the Dirichlet ones

$$\delta A_m(x, 0) = 0 \quad \Rightarrow \quad A_m(x, 0) = a_m(x) \quad (3.33)$$

for some fixed, but arbitrary, function  $a_m(x)$ . While Maxwell equations are manifestly gauge invariant, Dirichlet boundary conditions are only invariant under those transformations which become constant at the boundary:

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha, \quad \alpha(x, 0) = \lambda. \quad (3.34)$$

This in fact induces a  $U(1)$  global transformation with parameter  $\lambda$  on the boundary. So local internal symmetries in the bulk correspond to global internal symmetries on the boundary theory. Since the former are associated to massless gauge fields and the latter to conserved current operators, one expects these two to be one the dual of the other

$$j_m \xleftrightarrow{\text{AdS/CFT}} A_\mu. \quad (3.35)$$

The current supermultiplet described in the previous section contains, beside the conserved current  $j_m$ , a real scalar operator  $J$  of dimension 2 and a fermionic operator of dimension  $\frac{5}{2}$ . One then expects the five-dimensional gauge field to be part of an  $\mathcal{N} = 2$  supergravity multiplet which should also contain a spinor and a real scalar. This is in fact the field content of a *vector supermultiplet* in five dimensions

$$\mathcal{V} = \{A_\mu, \lambda^i, \rho\}, \quad (3.36)$$

where in  $\mathbf{AdS}_5$  the scalar  $\rho$  has  $m^2 = -4$  and the  $SU(2)$  symplectic-Majorana spinor  $\lambda^i$  which has  $m = \frac{1}{2}$ <sup>5</sup>. According to the AdS/CFT relations [44] which connect the mass  $m$  of a bulk field to the dimension  $\Delta_s$  of the dual spin- $s$  operator, the fields in the vector supermultiplet have the right masses to be interpreted as being dual to the operators in the current supermultiplet, see Table 3.1.

4D op.	$\Delta$	5D field	$\mathbf{AdS}_5$ mass
$J(x)$	$\Delta_0 = 2$	$\rho(z, x)$	$m_\rho^2 = \Delta_0(\Delta_0 - 4) = -4$
$j_\alpha(x)$	$\Delta_{1/2} = 5/2$	$\lambda(z, x)$	$ m_\lambda  = \Delta_{1/2} - 2 = 1/2$
$j_m(x)$	$\Delta_1 = 3$	$A_\mu(z, x)$	$m_A = (\Delta_1 - 2)^2 - 1 = 0$

TABLE 3.1: 4D  $\mathcal{N} = 1$  current multiplet and dual 5D  $\mathcal{N} = 2$  vector multiplet

<sup>5</sup>This is very different from flat space supersymmetry where all the fields in a supermultiplet share the same value for the mass, since  $P^2$  is a Casimir of the superalgebra. For supersymmetry on curved space-times this is no longer true. The masses of the component fields can still be related by supersymmetry, as in the  $\mathbf{AdS}$  case [49], but in general are different from one another.

### 3.2.2 Holographic renormalization for a vector multiplet

In this section we will try to say something general about holographic renormalization of two-point function for a current multiplet. In Section 2.3 we discussed the most general procedure which led to the introduction of a set of counter-terms. That procedure assured that divergences are removed from the on-shell action for an arbitrary solution to the supergravity equations of motion. However, such general procedure, requires to consider the on-shell boundary behavior for all bulk fields at once. This means a huge amount of redundant work if one is interested in a particular background solution rather than the most general one. For this reason, here and in the following chapters, we will pursue a more direct route trying to avoid useless calculations.

The counter-terms one has to add, although independent from the particular form of the solution, can depend on the details of the model (e.g. masses and interaction terms). However, not all of the terms in the supergravity Lagrangian will give rise to divergences when integrated near the boundary at  $z = 0$ . As we have shown in Section 2.3, for  $\mathbf{AdS}_5$  solutions the factor  $\sqrt{-g}$  in front of the Lagrangian diverges as  $z^{-4}$  at the boundary and so we do not need to keep track of terms which vanish faster than  $z^4$ . Since the fields we are going to consider goes to zero as  $z \rightarrow 0^6$ , this is equivalent to neglect higher order interactions in the Lagrangian.

supermultiplet	component fields	$\mathbf{AdS}_5$ masses
<i>gravity multiplet</i>	$\{g_{\mu\nu}, \Psi_\mu^i, A_\mu^R\}$	$\{0, \frac{3}{2}, 0\}$
<i>vector multiplet</i>	$\{A_\mu, \lambda^i, \rho\}$	$\{0, \frac{1}{2}, -4\}$
<i>hyper multiplet</i>	$\{\phi, \eta, \zeta^A\}$	$\{0, -3, \frac{3}{2}\}$

TABLE 3.2: Supermultiplet content of the theory. Spinor fields  $\Psi_\mu, \lambda, \zeta$  are  $SU(2)$  symplectic-Majorana, the index ‘ $i$ ’ transforms under the  $SU(2)_R$  R-symmetry group of  $\mathcal{N} = 2$  SUGRA, whereas ‘ $A$ ’ may transform under a different  $SU(2)$ . The masses are understood around the  $\mathbf{AdS}_5$  solution.

The field content of the  $\mathcal{N} = 2$  supergravity theory we are going to consider is summarized in Table 3.2. The graviton multiplet must be present in any supergravity and the vector multiplet is needed to describe the dual current multiplet. The additional matter hypermultiplet is needed to allow for non-trivial backgrounds solution (i.e. other than pure  $\mathbf{AdS}_5$ ). The fields that will neither be active in the background nor correspond to any current multiplet operators can be consistently truncated away from the theory (i.e. will be set to their on-shell value ( $= 0$ ) in the action). These are the gravitino  $\Psi_\mu$ , the

<sup>6</sup>This is true with the only exceptions of a gauge field and the leading mode of a massless scalar. However these are both switched off in the backgrounds we will be interested in.

graviphoton  $A_\mu^R$  and the *hyperino*  $\zeta$ . All in all our truncated theory have the following field content

$$\{g_{\mu\nu}, A_\mu, \lambda^i, \rho, \phi, \eta\} . \quad (3.37)$$

Let us start, for simplicity, by computing the renormalized on-shell action for a vector multiplet coupled to a purely  $\mathbf{AdS}_5$  background. This is a good approximation of a generic  $\mathbf{AAdS}_5$  background in the near-boundary region. (Later we will discuss what are the subtleties hidden by this approximation.) The (Euclidean) action in this case is given by<sup>7</sup>

$$\begin{aligned} \mathcal{S} = & \frac{1}{2} \int d^5x \sqrt{g} \left[ g^{\mu\nu} \partial_\mu \rho \partial_\nu \rho + \frac{1}{2} F^{\mu\nu} F_{\mu\nu} + \bar{\lambda} \gamma^\mu \mathcal{D}_\mu \lambda - \mathcal{D}_\mu \bar{\lambda} \gamma^\mu \lambda - \bar{\lambda} \lambda - 4\rho^2 + \dots \right] \\ & + \frac{1}{2} \int_{bdy} d^4x \sqrt{\gamma} \bar{\lambda} \lambda, \end{aligned} \quad (3.38)$$

where the ellipsis denotes higher order interactions among the fields. The derivatives are only covariant with respect to the  $\mathbf{AdS}_5$  metric, since the fields in the vector multiplet are uncharged under the U(1) group gauged by  $A_\mu$ . The field strength is defined as  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ .

The equations of motion from the Lagrangian (3.38) read

$$\begin{aligned} \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} g^{\mu\nu} \partial_\nu \rho) + 4\rho &= 0 \\ \partial_\tau (\sqrt{g} g^{\mu\tau} g^{\sigma\nu} F_{\mu\nu}) &= 0 \\ \gamma^\mu \left( \partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \right) \lambda - \frac{1}{2} \lambda &= 0, \end{aligned} \quad (3.39)$$

where, as already stated, we neglected possible interactions with background fields and corrections to the  $\mathbf{AdS}_5$  warp factor. We will put these corrections back into the game later, and see how they can affect the quadratic on-shell renormalized action. Integrating by parts (3.38) and using the above equations of motion one gets

$$\mathcal{S}_{\text{on-shell}} = -\frac{1}{2} \lim_{z \rightarrow 0} \int d^4x \sqrt{\gamma} \left[ \rho z \partial_z \rho + z \gamma^{mn} F_{zm} A_n - \bar{\lambda} \lambda \right], \quad (3.40)$$

which is a pure boundary term. This is always the case for the on-shell value of an action which is quadratic in the fields<sup>8</sup>. In order to compute the divergent part of the on-shell action we need the asymptotic behavior of the solutions to the equations of

<sup>7</sup>The boundary term in (3.38) is needed in order for the variational problem for the spinor field to be well-defined [50, 51].

<sup>8</sup>Notice also that the boundary terms only depend on the kinetic terms (and possibly other derivative interactions). Furthermore, since we are interested in two-point functions, we can always neglect term in the action which are more than quadratic in the “fluctuating” fields. As a consequence, the form (3.40) does not depend on the details of the model.

motion. The asymptotic expansion for the spinor becomes neater if we use a basis in which  $\gamma^5$  is diagonal and  $\lambda$  can be expressed as

$$\lambda = \begin{pmatrix} \chi \\ \bar{\xi} \end{pmatrix}, \quad (3.41)$$

where  $\xi$  and  $\chi$  are left-handed Weyl spinors from the point of view of the four-dimensional boundary. This also allows us to make contact with the dual quantum field theory. With this convention the spinor equation of motion splits into two coupled differential equations for the two Weyl components

$$\begin{cases} z\partial_z\chi + iz\sigma^m\partial_m\bar{\xi} - \frac{5}{2}\chi = 0 \\ -z\partial_z\bar{\xi} + iz\bar{\sigma}^m\partial_m\chi + \frac{3}{2}\bar{\xi} = 0. \end{cases} \quad (3.42)$$

The fields are now expanded near the boundary as

$$\rho(x, z) \underset{z \rightarrow 0}{\simeq} \rho_{(0)} z^2 \log(z) + \tilde{\rho}_{(0)} z^2 + \mathcal{O}(z^4) \quad (3.43a)$$

$$A_m(x, z) \underset{z \rightarrow 0}{\simeq} a_{m(0)} + a_{m(2)} z^2 \log(z) + \tilde{a}_{m(0)} z^2 + \mathcal{O}(z^4) \quad (3.43b)$$

$$A_z(x, z) \underset{z \rightarrow 0}{\simeq} b_{(0)} z \log(z) + \tilde{b}_{(0)} z + \mathcal{O}(z^3) \quad (3.43c)$$

$$\bar{\xi}(x, z) \underset{z \rightarrow 0}{\simeq} \bar{\xi}_{(0)} z^{\frac{3}{2}} + \mathcal{O}(z^{\frac{7}{2}}) \quad (3.43d)$$

$$\chi(x, z) \underset{z \rightarrow 0}{\simeq} \tilde{\chi}_{(0)} z^{\frac{5}{2}} + \chi_{(1)} z^{\frac{5}{2}} \log(z) + \mathcal{O}(z^{\frac{9}{2}}), \quad (3.43e)$$

where the coefficients are functions of the coordinates on the boundary. In particular  $\rho_{(0)}(x)$ ,  $a_{m(0)}(x)$ ,  $\xi_{(0)}^\alpha(x)$  are the sources for the dual current multiplet operators  $J(x)$ ,  $j_m(x)$ ,  $j_\alpha(x)$ , and  $\tilde{\rho}_{(0)}(x)$ ,  $\tilde{a}_{m(0)}(x)$ ,  $\tilde{\chi}_{(0)}^\alpha(x)$  are the corresponding one-point functions. The remaining coefficients become local functions of the sources once the equations of motion are imposed. Fourier transforming into momentum space we find

$$2a_{m(2)} - ip_m b_{(0)} = (p^2 \eta_{mn} - p_m p_n) a_{n(0)} \quad (3.44a)$$

$$a_{m(2)} + 2\tilde{a}_{m(0)} - ip_m \tilde{b}_{(0)} = (p^2 \eta_{mn} - p_m p_n) \left( \frac{1}{2} a_{n(0)} + \frac{2}{p^2} \tilde{a}_{n(0)} \right) \quad (3.44b)$$

$$\chi_{(1)} = -\sigma^m p_m \bar{\xi}_{(0)}. \quad (3.44c)$$

We notice that these relations are polynomial in the momentum  $p_m$ , but the factor of  $\frac{1}{p^2}$  in (3.44b). This latter is actually an artifact of covariance. Indeed, had we chosen some gauge fixing condition to remove redundant degrees of freedom the non-polynomial factor would not have been there but manifest covariance would have been lost. Substituting

the expansions (3.43) in the on-shell action one finds

$$\begin{aligned} \mathcal{S}_{\text{reg}} = -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \left\{ \log(\epsilon) \left[ 2\rho_{(0)}^2 \log(\epsilon) + \rho_{(0)}^2 + 4\rho_{(0)}\tilde{\rho}_{(0)} + a_{m(0)} (2a_{m(2)} - i p_m b_{(0)}) \right. \right. \\ \left. \left. - \xi_{(0)}\chi_{(1)} - \bar{\xi}_{(0)}\bar{\chi}_{(1)} \right] + a_{m(0)} \left( a_{m(2)} + 2\tilde{a}_{m(0)} - i p_m \tilde{b}_{(0)} \right) \right. \\ \left. + \rho_{(0)}\tilde{\rho}_{(0)} + 2\tilde{\rho}_{(0)}^2 - \tilde{\chi}_{(0)}\xi_{(0)} - \bar{\tilde{\chi}}_{(0)}\bar{\xi}_{(0)} + \mathcal{O}(\epsilon) \right\}. \end{aligned} \quad (3.45)$$

Using relations (3.44a) one can then rewrite the divergent terms as function of the sources. Divergences can be then subtracted by adding the following covariant counter-term action

$$\mathcal{S}_{\text{ct}} = \frac{1}{2} \int d^4 x \sqrt{\gamma} \left[ \left( 2 + \frac{1}{\log(\epsilon)} \right) \rho^2 - \frac{1}{2} \log(\epsilon) \gamma^{mr} \gamma^{ns} F_{mn} F_{rs} + 2 \log(\epsilon) \bar{\lambda} \gamma^\mu \partial_\mu \lambda \right]. \quad (3.46)$$

The counter-terms for the scalar contribute also to the finite part of the boundary action so we finally get the renormalized result

$$\begin{aligned} \mathcal{S}_{\text{ren}} = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \left[ \rho_{(0)}\tilde{\rho}_{(0)} - a_{m(0)} (p^2 \eta_{mn} - p_m p_n) \left( \frac{1}{2} a_{n(0)} + \frac{2}{p^2} \tilde{a}_{n(0)} \right) \right. \\ \left. + \tilde{\chi}_{(0)}\xi_{(0)} + \bar{\tilde{\chi}}_{(0)}\bar{\xi}_{(0)} \right]. \end{aligned} \quad (3.47)$$

We can finally compute the two-point functions differentiating twice the renormalized action with respect to the sources. In Euclidean convention the correct formula is

$$\langle \mathcal{O}_\phi(p) \mathcal{O}_\phi(-p) \rangle = -\frac{\delta^2 \mathcal{S}_{\text{ren}}}{\delta \phi_{(0)}(p) \delta \phi_{(0)}(-p)}, \quad (3.48)$$

and applying this to the above renormalized action, with factors of  $\kappa_5$  restored we obtain

$$\langle j_m(p) j_n(-p) \rangle = \frac{1}{2\kappa_5^2} \left( \Pi_{mn} + \frac{2}{p^2} \Pi_{ms} \frac{\delta \tilde{a}_s(0)}{\delta a_n(0)} + \frac{2}{p^2} \Pi_{ns} \frac{\delta \tilde{a}_s(0)}{\delta a_m(0)} \right) \quad (3.49a)$$

$$\langle j_\alpha(p) \bar{j}_{\dot{\beta}}(-p) \rangle = \frac{1}{2\kappa_5^2} \left( \frac{\delta \tilde{\chi}_{\alpha(0)}}{\delta \bar{\xi}_{(0)}^{\dot{\alpha}}} + \frac{\delta \bar{\tilde{\chi}}_{\dot{\alpha}(0)}}{\delta \xi_{(0)}^\alpha} \right) \quad (3.49b)$$

$$\langle J(p) J(-p) \rangle = -\frac{1}{\kappa_5^2} \frac{\delta \tilde{\rho}_{(0)}}{\delta \rho_{(0)}} \quad (3.49c)$$

$$\langle j_\alpha(p) j_\beta(-p) \rangle = \frac{1}{2\kappa_5^2} \left( \frac{\delta \tilde{\chi}_{\alpha(0)}}{\delta \xi_{(0)}^\beta} - \frac{\delta \tilde{\chi}_{\beta(0)}}{\delta \xi_{(0)}^\alpha} \right), \quad (3.49d)$$

where the functional derivatives are computed at vanishing sources. As expected, two-point correlators will be determined by the linear dependence of the VEV coefficients from the sources. This is determined by the bulk behavior of the solutions to the

equations of motion linearized around some given background. In the following chapter we will explicitly compute these correlators in a set of interesting background solutions.

In this chapter we have tried to say something general about the holographic renormalization procedure for computing two-point functions of a current supermultiplet. In order to be concrete, we have fixed the field content of the supergravity theory to be that of Table 3.2. In this way we have managed to arrive to the formulas in (3.49) which are valid for a large class of models. As anticipated, there are a couple of subtleties that have been obscured by our approximations and that we are now going to discuss.

The first approximation we have made was to neglect corrections in the  $\mathbf{AdS}_5$  metric. This approximation can be relaxed allowing the metric to be  $\mathbf{AAdS}_5$  with a warp factor of the form

$$F(z) \underset{z \rightarrow 0}{\simeq} 1 + f_2 z^{2\beta}, \quad (3.50)$$

where  $\beta$  depends on the near-boundary behavior of the scalar supporting the solution, as in (2.14). In our case this can be  $\beta = 4$  if the massless scalar  $\phi$  is the only active one<sup>9</sup>, or  $\beta = 1$  if the scalar  $\eta$  is switched on, see Table 3.2. The latter case is in principle the more problematic for the approximation we have done. Still, also in this case it does neither affect the divergent part nor the finite part of the regularized action (3.45). The other effect of a non-trivial warp factor is to modify the equations of motion, since now the metric in (3.39) contains a non-trivial warp factor. As a consequence, the relations among the coefficients of the asymptotic expansions can be modified. By inspection one can see that, as long as  $\beta \geq 1$ , the relations (3.44a) are not modified also in the presence of a non-constant  $F$ .

Let us now consider the second approximation we have made and briefly mentioned along the way. In deriving our results we have neglected possible interaction terms between the fluctuating fields and the background scalars. Assuming the matter is uncharged under the  $U(1)$  group associated to the vector multiplet, the possible interaction terms can only have the schematic form<sup>10</sup>

$$\lambda \lambda \rho^n \phi^m \eta^k \underset{z \rightarrow 0}{\simeq} z^{3+2n+4m+k} \quad \text{or} \quad \rho^{2+n} \phi^m \eta^k \underset{z \rightarrow 0}{\simeq} z^{4+2n+4m+k}, \quad (3.51)$$

where we used the fact that the correct fall-off for the leading mode of a spinor with mass  $|m| = \frac{1}{2}$  is  $\lambda \underset{z \rightarrow 0}{\simeq} z^{\frac{3}{2}}$  and the background scalars behave as  $\phi \underset{z \rightarrow 0}{\simeq} z^4$  (subleading mode) and  $\eta \underset{z \rightarrow 0}{\simeq} z$  (leading mode). The first of (3.51) is a Yukawa-like term that can

<sup>9</sup>For a background massless scalar we consider only the subleading mode  $z^4$  and not the leading one  $z^0$ .

<sup>10</sup>We assume that the configuration where the vector multiplet fields vanish is an extremum of the action for any background. We can always choose this to be the case via a field redefinition. As a consequence there cannot be terms in the lagrangian that are linear in the vector multiplet fields.

only contribute if  $3 + 2n + 4m + k \leq 4$  which then implies  $n = m = 0$  and  $k = 1$ . The second is a potential term that can never contribute to the renormalized action. Thus, in a general background, only terms of the form  $\lambda\lambda\eta$  can modify the near-boundary analysis. In particular, if the leading mode of the scalar  $\eta$  is turned on in the solution (i.e. if the background corresponds to a deformation by an operator of dimension 3, see (2.26)), then its interaction with the vector multiplet is no longer negligible in general. Analyzing the equations of motion one can see that the problems can come only from the spinor one. If interaction terms of the *Majorana-type*<sup>11</sup> form

$$\eta\bar{\lambda}\lambda^c \quad \text{or} \quad \partial_\mu\eta\bar{\lambda}\gamma^\mu\lambda^c \quad (3.52)$$

are present in the supergravity action, then the relation (3.44c) is modified to

$$\chi_{(1)} = -\sigma^m p_m \bar{\xi}_{(0)} + \xi_{(0)}\eta_{(0)}. \quad (3.53)$$

In this case the structure of the divergence in (3.45) slightly changes and one is forced to take into account also the field  $\eta$  in the holographic renormalization procedure. The precise consequence of this fact on the correlators will be discussed in the next chapter where we also choose a definite model for the supergravity action. For the moment let us just add some general comment on the fermionic correlators (3.49b) and (3.49d).

From the structure of the spinor equation of motion one can notice that the left-chiral mode  $\tilde{\chi}_{(0)}$ , which is only determined by the full bulk equation, will always have a non-trivial dependence on the leading mode of opposite chirality  $\bar{\xi}_{(0)}$ , ensuring a non-zero value for the two-point function  $C_{\frac{1}{2}}$ . On the other hand, already at this very general stage, we see that the only way to obtain a non-zero  $B$  is to have the mode  $\tilde{\chi}_{(0)}$  to depend also on the left-chiral source  $\xi_{(0)}$ . This is exactly the case if couplings like those in (3.52) are present. As we will see in the next chapter, these couplings can affect the correlators only if the scalar  $\eta$  is charged under an  $R$ -symmetry (that is mapped holographically to the  $U(1)$  symmetry gauged by the graviphoton). This result nicely reflects the fact that a non-zero  $B$  requires  $R$ -symmetry to be broken. We will see under which conditions non-trivial Majorana-type couplings of the bulk fermions can be produced.

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<sup>11</sup>Dirac-type terms  $\bar{\lambda}\lambda$  are subleading with respect to Majorana-type ones  $\bar{\lambda}\lambda^c$ . For conventions on five-dimensional spinors see Table B.1.





## Chapter 4

# Holographic general gauge mediation

In this chapter we want to present how the concepts explained in Chapter 3 can be applied to holographic models of gauge mediated supersymmetry breaking. As we will review below, in gauge mediation models much of the physical information is encoded in two-point functions of gauge-invariant operators belonging to a so-called hidden sector. This hidden sector is often strongly coupled at low energies and this renders the computation of correlators intractable from field theory methods. The idea of applying holographic techniques in this context has appeared quite recently in the literature (see e.g. [18, 19, 52–55]) and stemmed from the observation that strongly coupled correlators are the basic objects one can compute in AdS/CFT.

Here we will follow the strategy outlined in [19] and refer to it as the *Holographic General Gauge Mediation* (HGGM) program. In the first part of this chapter we will review the idea of gauge mediation and introduce the model-independent formalism of General Gauge Mediation (GGM). In the second part we will apply the holographic techniques reviewed in Chapter 3 to compute GGM correlators in various dual gravitational backgrounds.

### 4.1 General Gauge Mediation

We will now review the basic features and phenomenological motivations of gauge mediation models. For a complete review on the subject we refer the reader to [56] and references therein. In any viable Supersymmetric extension of the Standard Model (SSM) supersymmetry must be obviously broken. However, the supertrace theorem [57] implies

that if supersymmetry breaking is communicated to the SSM via tree-level renormalizable couplings then one will obtain superpartners which are lighter than experimental bounds. One of the way to circumvent the supertrace theorem and its consequences is to consider a hidden sector responsible for supersymmetry breaking which communicates to the SSM only via Standard Model gauge interactions. In such case one induces non-renormalizable kinetic terms in the effective theory at low energies, thus violating one of the assumptions of the supertrace theorem. This is what one usually calls a *gauge mediation scenario*.

There is a large zoology of gauge mediation models. These are usually classified according to the properties of the sector responsible for supersymmetry breaking. However, one can focus on the model-independent features of gauge mediation, namely the existence of a hidden sector which communicates to the visible sector only through SM gauge interactions. This led to the GGM [20] formulation which does not depend on the theory one uses as hidden sector.

The basic idea of GGM is to consider the limit in which the SM gauge couplings are turned off. In this limit the SM decouples from the hidden sector and the SM gauge group becomes a global symmetry of the latter. Consequently, as we discussed in the previous chapter, there is an associated linear multiplet  $\mathcal{J}$  in the theory which contains the conserved global currents and their superpartners. One then finds that all the data necessary to compute the soft spectrum (i.e. masses that softly break supersymmetry in the low energy effective theory) can be extracted from two-point correlation functions among linear multiplet operators. Or, more precisely, from the form factors defined in (3.20).

When SM gauge couplings are turned on the hidden sector couples to the SM sector. In superspace notation, at linear order in the gauge coupling  $g$ , the interaction between the two sectors can be written as<sup>1</sup>

$$2 \int d^4\theta g \mathcal{J} \mathcal{V} = g(DJ - \lambda j - \bar{\lambda} \bar{j} - A^m j_m), \quad (4.1)$$

where  $\mathcal{V}$  denotes the SM vector superfield whose components are: a gauge boson  $A_m$ , a gaugino  $\lambda$  and a real scalar  $D$ . Integrating out the whole hidden sector one obtains a low energy effective Lagrangian for the fields of the SSM. From that one can read the Majorana mass for the gaugino, implicitly given by the solution to the equation

$$\left[ \left(1 + g^2 C_{\frac{1}{2}}(p^2/M^2)\right)^2 p^2 + g^4 M^2 |B(p^2/M^2)|^2 \right]_{p^2=|M_{\tilde{g}}|^2} = 0, \quad (4.2)$$

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<sup>1</sup>generalization to a non-abelian, SM-like group is straightforward.

and soft masses for the sfermions given by

$$m_{\tilde{f}}^2 = g^2 C_f \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \left( \frac{1}{1 + g^2 C_0(p^2/M^2)} - \frac{4}{1 + g^2 C_{\frac{1}{2}}(p^2/M^2)} + \frac{3}{1 + g^2 C_1(p^2/M^2)} \right), \quad (4.3)$$

where the functions  $C_0$ ,  $C_{\frac{1}{2}}$ ,  $C_1$  and  $B$  are the same as in (3.20). To leading order in  $g$ , and restoring the full SM gauge group, the above relations reduce to

$$M_{\tilde{g},r} = g_r^2 M B^{(r)}(0) \quad (4.4)$$

$$m_{\tilde{f}}^2 = \sum_r g_r^4 C_f^r A_r$$

$$A_r = - \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \left( C_0^{(r)}(p^2/M^2) - 4C_{\frac{1}{2}}^{(r)}(p^2/M^2) + 3C_1^{(r)}(p^2/M^2) \right), \quad (4.5)$$

where  $r = 1, 2, 3$  labels, respectively, the factor  $U(1)$ ,  $SU(2)$ ,  $SU(3)$  of the SM gauge group, and  $C_f^r$  is the quadratic Casimir of the representation of  $f$  under the  $r^{\text{th}}$  gauge group.

All of the above formulas for the soft masses depends only on the form factors  $C_0$ ,  $C_{\frac{1}{2}}$ ,  $C_1$  and  $B$  which enters the current multiplet two-point correlators (3.20). However, the hidden sector is typically strongly coupled at low energies and the form factors are usually not computable using standard field theory methods. Nonetheless, when the hidden sector can be described holographically, one can resort to AdS/CFT techniques to compute two-point functions at strong coupling. In the following sections we will compute these quantities in concrete models where the hidden sector is replaced by a dual weakly curved supergravity background.

## 4.2 The supergravity model

The gravitational backgrounds we will be dealing with in the following sections can be understood as different solutions of a unique five-dimensional  $\mathcal{N} = 2$  supergravity theory. We are now going to describe the details of this theory.

As already emphasized, the five-dimensional gravity theory, besides the graviton multiplet, must contain at least one  $\mathcal{N} = 2$  vector multiplet, which is dual to the current multiplet of the boundary theory. Since here we want to pursue a “top-down” approach we would like our theory to be interpretable as coming from some sector of ten-dimensional type IIB supergravity. As a necessary condition, our  $\mathcal{N} = 2$  theory must be a consistent truncation of maximally gauged five-dimensional  $\mathcal{N} = 8$  supergravity. We will choose a truncation which includes, besides the graviton multiplet and one vector multiplet, one  $\mathcal{N} = 2$  hypermultiplet containing the (dimensional reduction

of the) ten-dimensional *dilaton*. This is usually called the *universal hypermultiplet*. In fact, enlarging the matter content to include a hypermultiplet is also necessary to the aim of finding non-AdS backgrounds. Therefore, the minimal five-dimensional theory one should consider consists of  $\mathcal{N} = 2$  supergravity coupled to a vector multiplet and a hypermultiplet, as described in Table 3.2.

In order to make our program concrete we consider a class of gauged supergravity theories studied in [58] which actually contains the minimal field content described above. We now briefly outline the main ingredients that specify our Lagrangian, whose form is dictated by the scalar manifold and the gauging. For further details we refer to [58, 59]. The scalars describe a non-linear sigma model with target space

$$\mathcal{M} = \text{O}(1, 1) \times \frac{\text{SU}(2, 1)}{\text{U}(2)}. \quad (4.6)$$

Because of supersymmetry the scalar manifold factorizes into a direct product of a very special manifold  $\mathcal{S} = \text{O}(1, 1)$  and a quaternionic manifold  $\mathcal{Q} = \frac{\text{SU}(2, 1)}{\text{U}(2)}$  spanned by the so-called universal hypermultiplet, which contains the axio-dilaton  $C_0 + i e^{-\phi}$ . The manifold  $\mathcal{S}$  is parametrized by the vector multiplet real scalar  $\rho$ , whereas  $\mathcal{Q}$  is parametrized by the four real hyperscalars  $q^X = \{\phi, C_0, \eta, \alpha\}$  with metric

$$\begin{aligned} ds^2 &= g_{XY} dq^X dq^Y \\ &= \frac{1}{2} \cosh^2(\eta) d\phi^2 + \frac{1}{2} \left( 2 \sinh^2(\eta) d\alpha + e^\phi \cosh^2(\eta) dC_0 \right)^2 + 2 d\eta^2 + 2 \sinh^2(\eta) d\alpha^2. \end{aligned} \quad (4.7)$$

where  $\eta \geq 0$  and  $\alpha \in [0, 2\pi]$ . The scalar  $\eta$  is sometimes called *squashing mode* since, within ten-dimensional compactifications, it is related to a squashing parameter of the internal compactification manifold. The isometries of this scalar manifold have a  $\text{U}(2)$  maximal compact subgroup acting on  $\mathcal{Q}$ . Since the theory contains two vectors, one in the gravity multiplet and the other one in the vector multiplet, the maximal subgroup we can gauge is a  $\text{U}(1) \times \text{U}(1)$  subgroup. In a minimal set up we choose to gauge just the  $\text{U}(1)$  corresponding to the shift symmetry

$$\alpha \rightarrow \alpha + c \quad (4.8)$$

of the above metric, which is a compact isometry because the scalar  $\alpha$  is a phase.

The  $\text{U}(1)$  which acts non-trivially on the scalar manifold is gauged by the graviphoton in the gravity multiplet. According to standard AdS/CFT [60], this gauge symmetry is then dual to the R-symmetry of the boundary theory. On the other hand, in our simplified setting the  $\text{U}(1)$  gauged by the vector belonging to the vector multiplet acts

trivially on all supergravity fields. Notice that the axio-dilaton is neutral under both  $U(1)$ 's while the complex scalar  $\eta e^{i\alpha}$  is charged under the symmetry (4.8) gauged in the bulk by the graviphoton. Therefore, a background with a non-trivial profile for the dilaton preserves the R-symmetry, while a non-trivial profile for  $\eta$  breaks it. For later reference let us notice that while the axio-dilaton is massless, and holographically dual to the hidden sector  $\text{Tr}(F_{mn})^2$  operator, the squashing mode  $\eta$  has  $m^2 = -3$  and it is dual to the hidden sector gaugino bilinear. Hence, the leading mode for this field at the boundary would provide an explicit mass to the hidden sector gauginos (hence an explicit R-symmetry breaking term), while a subleading term would correspond to a VEV for the gaugino bilinear (hence a spontaneous R-symmetry breaking term).

Starting from our five-dimensional Lagrangian, there are basically two steps one should perform:

- First, we should find a non-supersymmetric background configuration (corresponding to a supersymmetry breaking vacuum in the dual QFT) with just the metric and some of the hyperscalars turned on. In order to do this we will truncate the Lagrangian to the relevant field content (provided this is consistent with the full set of equations) and extract the equations of motion which the background must satisfy.
- Second, we need to extract the equations of motion for the vector multiplet linearized around the background we found. To this aim, we will perform a different truncation of the Lagrangian setting all fields but the vector multiplet to their background values, and retain only the couplings which are no more than quadratic in the vector multiplet fields.

We will now present the explicit form of these truncated Lagrangians.

### 4.2.1 Lagrangian for the background

Let us start by setting to zero the whole vector multiplet, as well as the gravitino, the graviphoton and the hyperino. The phase  $\alpha$  can be gauge-fixed to zero. The resulting truncated (Euclidean) action reads

$$\mathcal{S}_{\text{bg}} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g} \left[ -\frac{1}{2}R + \mathcal{L}_{\text{kin}} + U \right] \quad (4.9)$$

where the kinetic term is given in term of the scalar manifold metric (4.7) as  $\mathcal{L}_{\text{kin}} = \frac{1}{2}g_{XY}\partial_\mu q^X\partial^\mu q^Y$ , that is

$$\mathcal{L}_{\text{kin}} = \frac{1}{4} \left[ 4\partial_\mu\eta\partial^\mu\eta + \cosh^2(\eta)\partial_\mu\phi\partial^\mu\phi + e^{2\phi}\cosh^4(\eta)\partial_\mu C_0\partial^\mu C_0 \right]. \quad (4.10)$$

As a consequence of the gauging we have a non-trivial potential given by<sup>2</sup>

$$U = \frac{3}{4} (\cosh^2(2\eta) - 4\cosh(2\eta) - 5). \quad (4.11)$$

We end up with the following system of differential equations

$$R_{\mu\nu} = \frac{2}{3}U g_{\mu\nu} + 2 \left( \partial_\mu\eta\partial_\nu\eta + \frac{1}{4}\cosh^2(\eta)\partial_\mu\phi\partial_\nu\phi \right), \quad (4.12a)$$

$$\square\eta = \frac{1}{2}\frac{\partial U}{\partial\eta} + \frac{1}{8}\sinh(2\eta)\partial_\mu\phi\partial^\mu\phi, \quad (4.12b)$$

$$\square\phi = -2\tanh(\eta)\partial_\mu\eta\partial^\mu\phi, \quad (4.12c)$$

where we have also set  $C_0 = \text{const.}$  for simplicity.

We are interested in solutions which are **AAAdS**<sub>5</sub> domain walls, so we take the following ansatz for the metric

$$ds_5^2 = \frac{1}{z^2} (dz^2 + F^2(z)(dx^m)^2) \quad (4.13)$$

with  $F(z)$  approaching 1 as  $z \rightarrow 0$ . Therefore, the solution to the equations above determine the three unknown functions  $\phi$ ,  $\eta$  and  $F$  of the radial coordinate  $z$ .

In the case of unbroken R-symmetry,  $\eta = 0$ , the above system of equations reduces exactly to the one considered in [61], and admits both a supersymmetric **AdS**<sub>5</sub> solution with constant dilaton, as well as a singular dilaton-domain-wall solution [61, 62]. The latter breaks both conformal invariance and (all) supersymmetry. Another interesting background is one where also the charged scalar  $\eta$  has a non-trivial profile. We will consider all these examples in turn.

### 4.2.2 Quadratic Lagrangian for the vector multiplet

We now turn to the action describing the coupling of vector multiplet fluctuations to the background. To this end we fix  $F$ ,  $\phi$  and  $\eta$  to their (arbitrary for now)  $z$ -dependent background value into the full Lagrangian, and retain only those terms involving the vector multiplet up to second order. The resulting (Euclidean) action can be divided in

---

<sup>2</sup>With our choice of gauging the complex scalar  $\tau = C_0 + i e^{-\phi}$  can be identified with the ten-dimensional axio-dilaton. As expected the scalar potential does not depend on  $\tau$  because of the  $\text{SL}(2, \mathbb{R})$  symmetry inherited from type IIB supergravity.

two pieces

$$\mathcal{S}_{\text{quad}} = \int d^5x \sqrt{g} [\mathcal{L}_{\text{min}} + \mathcal{L}_{\text{int}}] . \quad (4.14)$$

The first one contains kinetic and mass terms for the fluctuations, and it is uniquely fixed, by the dimensions of the dual operators and their minimal coupling to the metric, to be the Lagrangian in (3.38). The second one contains interactions with the scalars  $\phi$  and  $\eta$  and takes the form

$$\begin{aligned} \mathcal{L}_{\text{int}} = & \frac{1}{2} \delta M^2 D^2 - \delta m_D \bar{\lambda} \lambda \\ & - \frac{1}{2} (m_M \bar{\lambda} \lambda^c + v_M \bar{\lambda} (\not{\phi} \eta) \lambda^c + \tilde{v}_M \bar{\lambda} (\not{\phi} \phi) \lambda^c + c.c.) , \end{aligned} \quad (4.15)$$

where the couplings are fixed by supersymmetry to be

$$\delta M^2 = 2 (\cosh^2(2\eta) - \cosh(2\eta)) , \quad \delta m_D = -\frac{1}{2} \sinh^2(\eta) \quad (4.16a)$$

$$m_M = i \sinh(\eta) , \quad v_M = -\frac{i}{\cosh(\eta)} , \quad \tilde{v}_M = \frac{i}{2} \sinh(\eta) . \quad (4.16b)$$

In the first line there are ( $z$ -dependent) shifts for scalar mass squared and fermion Dirac mass, whereas in the second line there are a Majorana mass for the fermion and additional Majorana-type couplings.

We wrote the couplings in a five-dimensional covariant manner, but one should bear in mind that  $\eta$  and  $\phi$  are background fields which actually can depend only on the fifth coordinate  $z$ , so that the additional terms are equivalent to four-dimensional covariant terms constructed with a  $\gamma^5$  matrix. Notice that all couplings in (4.16) vanish if  $\eta$  is identically zero in the background. This observation will be relevant later.

From the action (4.14) we get the equations of motion

$$(\square + 4 - \delta M^2) D = 0 , \quad (4.17a)$$

$$\frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma}) = 0 , \quad (4.17b)$$

$$(\not{D} - \frac{1}{2} - \delta m_D) \lambda - (m_M + v_M \not{\phi} \eta + \tilde{v}_M \not{\phi} \phi) \lambda^c = 0 , \quad (4.17c)$$

where

$$\not{D} = e_a^\mu \gamma^a \left( \partial_\mu + \frac{1}{4} \omega_\mu^{bc} \gamma_{bc} \right) . \quad (4.18)$$

As already noticed, the five-dimensional spinor is equal in form to a four-dimensional Dirac spinor and it is often useful to rewrite its equation of motion in terms of  $\gamma^5$

eigenstates, that is<sup>3</sup>

$$\lambda = \begin{pmatrix} \chi \\ \bar{\xi} \end{pmatrix}, \quad \bar{\lambda} = \left( \xi, \bar{\chi} \right), \quad \lambda^c = \begin{pmatrix} \xi \\ -\bar{\chi} \end{pmatrix}. \quad (4.19)$$

In terms of Weyl components  $\chi$  and  $\xi$ , eq. (4.17c) becomes

$$\left( z\partial_z - \frac{5}{2} + 2z\frac{F'}{F} - \delta m_D \right) \chi + i\frac{z}{F} \sigma^m \partial_m \bar{\xi} - (m_M + v_M z \eta' + \tilde{v}_M z \phi') \xi = 0, \quad (4.20a)$$

$$\left( z\partial_z - \frac{3}{2} + 2z\frac{F'}{F} + \delta m_D \right) \bar{\xi} - i\frac{z}{F} \bar{\sigma}^m \partial_m \chi - (m_M - v_M z \eta' - \tilde{v}_M z \phi') \bar{\chi} = 0, \quad (4.20b)$$

where primes denote derivatives with respect to the coordinate  $z$ . As can be seen from the equations above, when Majorana-type couplings are turned on, not only  $\bar{\xi}$  but also  $\xi$  appears in the equation for  $\chi$ , and vice-versa. As we concluded in the previous chapter, this is the only way to obtain  $B \neq 0$  (see eq. 3.49d) and consistently can only happen if we turn on a background for the scalar  $\eta$ .

### 4.2.3 Renormalized action with non-trivial $\eta$

The interactions in (4.15) have the form of the “problematic” couplings we discussed at the end of Section 3.2. We then expect that in backgrounds where the leading mode of  $\eta$  is switched on, the renormalized action (3.47) should be modified.

The scalar  $\eta$  has  $m^2 = -3$  as can be seen from (4.11), and therefore its leading and subleading boundary behaviors are

$$\eta \underset{z \rightarrow 0}{\simeq} \eta_{(0)} z + \tilde{\eta}_{(0)} z^3 + \dots \quad (4.21)$$

Whenever the leading mode  $\eta_{(0)}$  is non-zero in the background (a source term for the corresponding  $\Delta = 3$  boundary operator, the hidden gaugino bilinear), we found that the renormalized on-shell action (3.47) should be augmented by the following finite term

$$\mathcal{S}_{\text{ren}}^\eta = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \left[ i \eta_{(0)} (\xi_{(0)} \xi_{(0)} - \bar{\xi}_{(0)} \bar{\xi}_{(0)}) \right]. \quad (4.22)$$

Accordingly, the expression for the correlator (3.49d) is modified to

$$\langle j_\alpha(p) j_\beta(-p) \rangle_\eta = \frac{1}{2} \left( \frac{\delta \tilde{\chi}_{\alpha(0)}}{\delta \xi_{\beta(0)}} - \frac{\delta \tilde{\chi}_{\beta(0)}}{\delta \xi_{\alpha(0)}} + 2i \epsilon_{\alpha\beta} \eta_{(0)} \right). \quad (4.23)$$

<sup>3</sup>See Appendix B.1 for our conventions on 5d spinors.



As the results of the following sections will show, the corrected expression (4.23) is necessary to ensure that the fermionic correlator properly goes to zero at large momenta, as dictated by the condition (3.28). The local term (4.22) can be seen as a counterterm which we add to the boundary action in order to reabsorb an unwanted contact term in the correlator. This counterterm only depends on quantities that are held fixed in the variational principle.

Notice that if the  $\eta$  profile has a leading boundary behavior proportional to  $\tilde{\eta}_{(0)}$ , which is holographically dual to a purely dynamical generation of an R-symmetry breaking VEV, no modification in the renormalized boundary action occurs. Still, having  $\eta$  a non trivial profile,  $\tilde{\chi}_0$  would depend on  $\xi_{(0)}$ , and hence the correlator (3.49d) would be in general different from zero.

The origin of the finite counterterm (4.22) can be explained as follows. The Lagrangian for  $\eta$ , up to cubic order in this field reads

$$\mathcal{L}_\eta \sim -\eta \square \eta - 3\eta^2 + \frac{1}{2} \left( -i\eta \bar{\lambda} \lambda^c + i\bar{\lambda}(\not{\partial}\eta)\lambda^c - \frac{i}{2}\eta \bar{\lambda}(\not{\partial}\phi)\lambda^c + c.c. \right), \quad (4.24)$$

where we can actually neglect the third term inside the parenthesis, since the dilaton behaves as  $z\partial_z\phi \underset{z \rightarrow 0}{\simeq} z^4$  and cannot contribute to the finite part of the boundary action.

The key observation is that the following boundary term

$$\mathcal{S}_{\text{reg}}^\eta = \int_{z=\epsilon} \frac{d^4p}{(2\pi)^4} \frac{i}{2} \epsilon^{-4} [\eta(\xi\xi - \bar{\xi}\bar{\xi} - \chi\chi + \bar{\chi}\bar{\chi})]_{z=\epsilon}, \quad (4.25)$$

is obtained if one integrates by parts the derivative interaction term in (4.24). The term bilinear in  $\chi$  can never contribute to finite terms at the boundary since  $\chi \underset{z \rightarrow 0}{\simeq} z^{\frac{5}{2}}$ , but we notice that when  $\eta \sim \epsilon$  the term bilinear in  $\xi$  is actually finite, and is exactly the term (4.22).

One can easily verify that the action for  $\eta$  and  $\lambda$  with the interactions given in (4.24) vanishes on-shell up to quartic terms in those fields. Therefore, for the purpose of computing two-point functions of the vector multiplet, taking into account the holographic renormalization for  $\eta$  is equivalent to replacing the Lagrangian (4.24) by the boundary term obtained after integration by parts. This is exactly the finite counterterm (4.22).

### 4.3 Holographic GGM correlators from AdS

As a warm up, we want to compute the GGM two-point functions for a pure  $\mathbf{AdS}_5$  background, which solves the system (4.12) with  $\phi = \eta = 0$ . This exercise has several motivations. First of all it will enable us to verify that our machinery correctly reproduces what we expect for a superconformal field theory, namely equation (3.30). Secondly, the values for the correlators that we find in  $\mathbf{AdS}$  will be the reference to confront with, when considering other backgrounds. In particular, each correlator will have to asymptote to those of the pure  $\mathbf{AdS}_5$  case, at large momenta. Finally, the computations we are going to perform here will be used later, when conformal and supersymmetry breaking will be implemented by a hard wall in  $\mathbf{AdS}_5$ .

The pure  $\mathbf{AdS}_5$  solution is a trivial solution of our five-dimensional effective model. However, in order to fix the overall normalization of correlators, it is useful to uplift it to the  $\mathbf{AdS}_5 \times \mathbf{S}^5$  solution of ten-dimensional IIB supergravity<sup>4</sup>, which reads (see e.g. [61])

$$\begin{aligned} ds_{10}^2 &= \frac{L^2}{z^2} (dz^2 + dx^m dx_m) + L^2 d\Omega_5^2, \\ F_5 &= \frac{L^4}{\kappa_{10}} (\text{vol}(S^5) + \text{vol}(AdS_5)) \end{aligned} \quad (4.26)$$

where the flux quantization condition fixes  $\frac{2\pi^{7/2}L^4}{\kappa_{10}} = N\pi$ . The gravitational coupling constant in front of the five-dimensional action is given by

$$\frac{1}{\kappa_5^2} = \frac{\text{Vol}(S^5)}{\kappa_{10}^2} = \frac{N^2}{4\pi^2 L^3}, \quad (4.27)$$

where in the last equality we have used the flux quantization condition above.

In pure  $\mathbf{AdS}_5$  the solution to the equations of motion can be written in term of modified Bessel function [45]. The generic solution has the form<sup>5</sup>

$$\rho(z, p) = z^2 (c_1(p) I_0(pz) + c_2(p) K_0(pz)), \quad (4.28a)$$

$$A_m(z, p) = z (\alpha_{1m}(p) I_1(pz) + \alpha_{2m}(p) K_1(pz)), \quad (4.28b)$$

$$\bar{\xi}(z, p) = z^{5/2} (\bar{\theta}_1(p) I_1(pz) + \bar{\theta}_2(p) K_1(pz)), \quad (4.28c)$$

$$\chi = z^{5/2} \left( -\frac{\sigma^m p_m \bar{\theta}_1(p)}{p} I_0(pz) + \frac{\sigma^m p_m \bar{\theta}_2(p)}{p} K_0(pz) \right) \quad (4.28d)$$

<sup>4</sup>In this case the dual field theory is thus  $\mathcal{N} = 4$  SYM and the current multiplet is associated to some  $U(1) \subset SU(4)_R$

<sup>5</sup>For simplicity here and in the following we have fixed the gauge symmetry requiring  $A_z = 0$  and  $p^m A_m = 0$ .

Expanding these expressions near the boundary and comparing the result with (3.43) we get the relations

$$c_2(p) = -\rho_{(0)}(p), \quad \alpha_{m2} = p a_{m(0)}(p), \quad \bar{\theta}_2 = p \bar{\xi}_{(0)}(p), \quad (4.29)$$

which set the Dirichlet boundary conditions. This leaves us with three independent constants in (4.28). Those can be fixed demanding the solution to be regular in the AdS interior. We thus arrive at

$$\rho(z, p) = -z^2 K_0(pz) \rho_{(0)}(p), \quad (4.30a)$$

$$A_m(z, p) = z p K_1(pz) a_{m(0)}(p), \quad (4.30b)$$

$$\bar{\xi}(z, p) = z^{5/2} p K_1(pz) \bar{\xi}_{(0)}(p), \quad \chi = -z^{5/2} K_0(pz) \sigma^m p_m \bar{\xi}_{(0)}(p). \quad (4.30c)$$

Using again the asymptotic expansions we get the expressions for the subleading modes in terms of the leading ones

$$\tilde{\rho}_{(0)}(p) = \left[ -\frac{1}{2} \log \left( \frac{\Lambda^2}{p^2} \right) - \log 2 + \gamma \right] \rho_{(0)}(p), \quad (4.31)$$

$$\tilde{a}_{m(0)}(p) = \frac{p^2}{2} \left[ -\frac{1}{2} \log \left( \frac{\Lambda^2}{p^2} \right) - \log 2 + \gamma - \frac{1}{2} \right] a_{m(0)}(p), \quad (4.32)$$

$$\tilde{\bar{\xi}}_{(0)}(p) = \frac{p^2}{2} \left[ -\frac{1}{2} \log \left( \frac{\Lambda^2}{p^2} \right) - \log 2 + \gamma - \frac{1}{2} \right] \bar{\xi}_{(0)}(p), \quad (4.33)$$

$$\tilde{\chi}_{(0)}(p) = \left[ -\frac{1}{2} \log \left( \frac{\Lambda^2}{p^2} \right) - \log 2 + \gamma \right] \sigma^m p_m \bar{\xi}_{(0)}(p). \quad (4.34)$$

Finally, we substitute these expressions into (3.49) and obtain the two-point functions

$$\langle j_m(p) j_n(-p) \rangle = -\frac{N^2}{4\pi^2} \left( \eta_{mn} - \frac{p_m p_n}{p^2} \right) p^2 \left[ \frac{1}{2} \log \left( \frac{\Lambda^2}{p^2} \right) + \log 2 - \gamma \right]; \quad (4.35)$$

$$\langle j_\alpha(p) \bar{j}_{\dot{\alpha}}(-p) \rangle = \frac{N^2}{4\pi^2} \sigma^m p_m \left[ \frac{1}{2} \log \left( \frac{\Lambda^2}{p^2} \right) + \log 2 - \gamma \right] \quad (4.36)$$

$$\langle J(p) J(-p) \rangle = \frac{N^2}{4\pi^2} \left[ \frac{1}{2} \log \left( \frac{\Lambda^2}{p^2} \right) + \log 2 - \gamma \right]; \quad (4.37)$$

$$\langle j_\alpha(p) j_\beta(-p) \rangle = 0. \quad (4.38)$$

Our results are in agreement with CFT computations [63, 64]. Note that we can always subtract the constant contribution ( $\log 2 - \gamma$ ) to the two-point functions by means of finite counterterms which preserve the  $\mathcal{N} = 2$  supersymmetry of the bulk action, so these terms are inessential and will be ignored in what follows.

As expected for a supersymmetric background, we find that the relations (3.22) are satisfied, and thus that both gaugino and sfermion masses are identically zero. Moreover, in this case the dual field theory is exactly superconformal and the relation (3.23) is also

satisfied. In particular we find

$$C_{\text{sft}}\left(\frac{p^2}{\Lambda^2}\right) = \frac{N^2}{8\pi^2} \log\left(\frac{\Lambda^2}{p^2}\right) \quad (4.39)$$

which gives  $\tau = 2N^2$  (see footnote 2 of Chapter 3). As explained in [65], this coefficient gives the contribution of the CFT matter to the beta function associated to the gauging of U(1) global symmetry associated to  $j_m$  (which in this toy model represents the SM gauge group). We note that such a large number would be in contrast with keeping the SSM gauge couplings perturbative before unification. We will not comment on this further, besides saying that what we are really trying to extract from this holographic approach are qualitative features of correlators in strongly coupled hidden sectors, that we assume are a good approximation even outside the large  $N$  limit.

#### 4.4 Holographic GGM correlators from a dilaton-domain wall

In this section we do a step further and apply our machinery to a supersymmetry breaking background, which is also a solution of our five-dimensional model. In this case we keep a trivial profile for the squashing mode,  $\eta = 0$ , but allow for a non-trivial dilaton profile. This is known as the *dilaton-domain wall* solution. We will see how the IR behavior of the correlators will change drastically with respect to their conformal expressions found in the previous section.

The dilaton-domain wall is in fact a solution of the full ten-dimensional type IIB supergravity found in [61, 62]. This is a singular solution with a non-trivial background for the dilaton  $\phi$  which preserves the full SO(6) isometry group of the  $\mathbf{S}^5$  factor. Upon dimensional reduction on  $\mathbf{S}^5$  one gets the following five-dimensional background

$$ds_5^2 = \frac{1}{z^2} (dz^2 + \sqrt{1 - \left(\frac{z}{z_s}\right)^8} (dx^m)^2), \quad (4.40)$$

$$\phi(z) = \phi_\infty + \sqrt{6} \operatorname{arctanh} \left[ \left(\frac{z}{z_s}\right)^4 \right]. \quad (4.41)$$

The metric goes to  $\mathbf{AdS}_5$  at the boundary  $z \rightarrow 0$  and presents a naked singularity at  $z = z_s$ . Without loss of generality, we can set  $z_s = 1$  by adjusting one of the integration constants. At the singularity the dilaton diverges as well

$$\lim_{z \rightarrow 1} \phi(z) = \infty. \quad (4.42)$$

The presence of the naked singularity signals a breakdown of the supergravity approximation and therefore the holographic interpretation of this background as a well-defined field theory could be problematic. It appears that this particular singularity is physically acceptable according to the two criteria of [66] and [67]. Respectively, its scalar potential is bounded from above (it is exactly zero), and  $g_{tt}$  is monotonously decreasing towards the singularity. The reason this solution has had some bad reputation is due to the fact that it fails another criterion put forward in [66], namely that it has no generalization with a horizon.

A possible physical interpretation of this background was discussed in [61, 68]. It suffices here to say that it describes a vacuum of a theory which in the UV coincides with  $\mathcal{N} = 4$  SYM, where however a non-trivial VEV for  $\text{Tr}(F_{mn})^2$  is turned on triggering confinement and SUSY breaking. In the following we will probe some of its features by the explicit computation of the GGM correlators. This background is interesting for our program because it breaks, besides conformality, all the supersymmetries (as one can see from the supersymmetry transformation of the dilatino) but it preserves the full  $\text{SO}(6)$  R-symmetry group of  $\mathcal{N} = 4$ , so that we can consider an  $\mathcal{N} = 2$  vector multiplet gauging a  $\text{U}(1) \subset \text{SO}(6)$ .

The effective action at the linearized level for the  $\mathcal{N} = 2$  vector multiplet in the dilaton-domain wall is of the form (3.38), and the resulting equations of motion read

$$(\square_{\text{dw}} - 4)\rho = \left( z^2 \partial_z^2 - \left( \frac{3 + 5z^8}{1 - z^8} \right) z \partial_z + \frac{z^2 \square_4}{\sqrt{1 - z^8}} - 4 \right) \rho = 0, \quad (4.43a)$$

$$(\text{Max})_{\text{dw}} A_m \equiv \left( z^2 \partial_z^2 - \left( \frac{1 + 3z^8}{1 - z^8} \right) z \partial_z + \frac{z^2 \square_4}{\sqrt{1 - z^8}} \right) A_m = 0, \quad (4.43b)$$

$$(\not{D}_{\text{dw}} - \frac{1}{2})\lambda \equiv \left( z \gamma_z \partial_z - 2 \frac{1 + z^8}{1 - z^8} \gamma_z + \frac{z}{(1 - z^8)^{\frac{1}{4}}} \gamma^m \partial_m - \frac{1}{2} \right) \lambda = 0. \quad (4.43c)$$

We note that the **AdS** equations are modified by terms of  $\mathcal{O}(z^8)$  in a near boundary expansion.

The second order equations for the fluctuations of the supergravity fields can be solved once two boundary conditions are specified.<sup>6</sup> One boundary condition will always determine the leading term at the boundary, fixing the overall normalization of the solution. The second condition should be a regularity condition in the bulk. In the case under consideration this means to fix the behavior near the singular point  $z = 1$ .

<sup>6</sup>For the sake of the argument that follows, we can convert the two first order equations for the spinors  $\chi$  and  $\bar{\xi}$  into a single second order equation for  $\bar{\xi}$ .

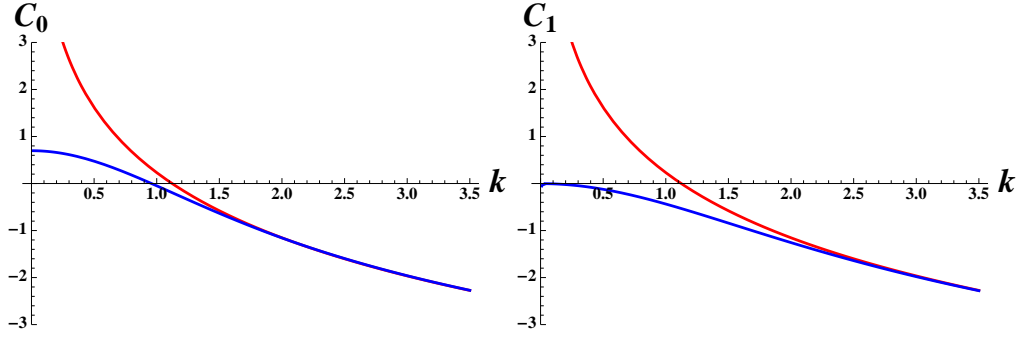


FIGURE 4.1:  $C_0$  and  $C_1$  as functions of the Euclidean momentum  $k$ : in red the **AdS** logarithm, in blue the dilaton-domain wall result.

Expanding equations (4.43) to leading order in  $1 - z \equiv y \rightarrow 0$  we get

$$(y^2 \partial_y^2 + y \partial_y) \rho = 0, \quad (4.44a)$$

$$(y^2 \partial_y^2 + \frac{1}{2} y \partial_y) A_m = 0, \quad (4.44b)$$

$$(y^2 \partial_y^2 + \frac{5}{4} y \partial_y - \frac{1}{8}) \bar{\xi} = 0. \quad (4.44c)$$

whose solutions are given in terms of two undetermined coefficients  $\alpha$  and  $\beta$  as

$$\rho \underset{y \rightarrow 0}{\simeq} \alpha_0 \log(y) + \beta_0, \quad (4.45a)$$

$$A_m \underset{y \rightarrow 0}{\simeq} \alpha_{m1} + \beta_{m1} y^{\frac{1}{2}}, \quad (4.45b)$$

$$\bar{\xi} \underset{y \rightarrow 0}{\simeq} \alpha_{\frac{1}{2}} y^{-\frac{1}{2}} + \beta_{\frac{1}{2}} y^{1/4}. \quad (4.45c)$$

The boundary-value problem is well-posed if we require, for all of the three fields, that a linear combination of  $\alpha$  and  $\beta$  vanishes<sup>7</sup>. A condition giving an unequivocal choice for all of the three fields is requiring that both the field and its derivative are finite at the singularity. This condition can be satisfied for all of the three fields and their first derivatives, except for the first derivative of the fermion, which will diverge in any case. We thus select the choice of parameters  $\alpha_0 = \beta_1 = \alpha_{\frac{1}{2}} = 0$ .

Once we specify the boundary conditions, a solution to equations (4.43) can be found numerically for any value of the parameter  $p$  corresponding to the four-dimensional momentum. By using the holographic formulas (3.49) we can then plot the  $C_s$  and  $B$  functions.

We show the plots for  $C_s$  in Figures 4.1 and 4.2 while  $B$  identically vanishes. In each graph we plot both the result for the supersymmetric **AdS**<sub>5</sub> case, as well that for the

<sup>7</sup>For instance  $\rho = 0$  or  $\partial \bar{\xi} = \text{const.}$  at the singularity are not suitable boundary conditions because they would kill both the coefficients.

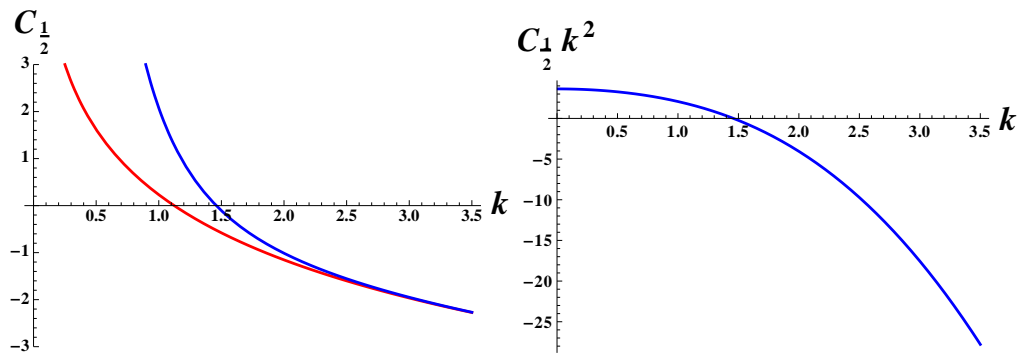


FIGURE 4.2: The plot on the left shows  $C_{1/2}$  as a function of the Euclidean momentum  $k$ : in red the **AdS** logarithm, in blue the dilaton-domain wall result. The plot on the right shows the  $1/k^2$  behavior at low momentum.

dilaton-domain wall solution. Notice that the three plots coincide for large momentum where they recover the **AdS**-behavior.

One of the interesting results of the plots is the  $\frac{1}{p^2}$  IR behavior of the fermionic correlator  $C_{\frac{1}{2}}$ . In Figure 4.2 we plot  $p^2 C_{\frac{1}{2}}$ , which clearly shows this form factor has a pole at zero momentum. This kind of behavior is related to the existence of massless excitations carrying the same quantum numbers of the corresponding current. For the fermionic current  $j_\alpha$ , this signals the existence of massless fermions, typically ‘t Hooft fermions, that compensate the global anomaly of the unbroken  $U(1)_R$ -symmetry [69]. Note, in passing, that imposing the “wrong” boundary condition for the vector field fluctuations, namely  $\alpha_1 = 0$ , we would have gotten a  $\frac{1}{p^2}$  pole also in  $C_1$ . This would have corresponded to a tree-level exchange of a massless Goldstone boson in the current two-point function, and would have implied the spontaneous breaking of the  $U(1)$ . Since the background preserves the full  $U(1) \times U(1)_R$  symmetry, this suggests that symmetry breaking can also be triggered by IR boundary conditions for the fields.

While we cannot prove that there are indeed R-charged ‘t Hooft fermions in our strongly coupled theory, and just observe that the holographic analysis suggests them to be there, it is useful to refer to the full ten-dimensional background to get some more confidence about our result. From such perspective there is a whole  $SU(4)$  symmetry which the background preserves. Hence, at every scale there must exist chiral fermions in the spectrum which reproduce UV global anomaly. The UV fixed point is  $\mathcal{N} = 4$  SYM, which has indeed a non-zero global anomaly for the  $SU(4)$  current. Our result suggests that (part of) the  $SU(4)$  anomaly is transmitted to the  $U(1)_R$  current. Let us emphasize that any other anomalous global symmetry would not provide a pole to the fermionic correlator  $C_{\frac{1}{2}}$ , which is neutral under any global symmetry but the R-symmetry. Hence, field theory expectations would suggest that when the R-symmetry is broken, R-charged

't Hooft fermions would not exist, and the pole in the fermionic correlator should vanish. We will come back to this point in the next section.

The Majorana gaugino mass, determined by  $B$  through (4.4), consistently vanishes because of unbroken R-symmetry. However, the pole in  $C_{\frac{1}{2}}$  provides a Dirac mass for the SSM gaugino. This is very similar to any other model of R-symmetric Dirac gaugino masses, except that the massless fermion in the adjoint that must couple bilinearly with the gaugino is here a composite fermion generated at strong coupling. The soft spectrum, in this situation, is very much reminiscent of that of gaugino mediation models. (See [70, 71] for a discussion of Dirac gaugino masses in General Gauge Mediation.)

Let us finally notice how different are the  $C_s$  in the dilaton-domain wall background with respect to the ones in  $\mathbf{AdS}_5$ , at large momentum. Numerically we find that

$$C_0 - 4C_{\frac{1}{2}} + 3C_1 \sim \mathcal{O}(p^{-8}), \quad p \rightarrow \infty. \quad (4.46)$$

This is due to the fact that the correction of the domain wall metric with respect to the  $\mathbf{AdS}_5$  one near the boundary is of  $\mathcal{O}(z^8)$ . Note that since the dilaton does not enter the equations for the vector multiplet fluctuation, its  $\mathcal{O}(z^4)$  behavior near the boundary does not influence the  $C_s$ . Another nice feature of the asymptotic behavior (4.46) is that it makes the integral (4.5) nicely convergent in the UV.

## 4.5 Holographic GGM correlators from $\eta$ /dilaton-domain wall

Let us discuss our third example, and look for a solution of equations (4.12) with a non-trivial profile for both the dilaton and the squashing mode. The latter breaks the R-symmetry so one should expect a very different behavior for the correlators.

In fact, in what follows we will only turn on a perturbative profile for the R-symmetry breaking scalar  $\eta$ , that is we consider only the linearized equation for  $\eta$  on the dilaton-domain wall background, and neglect the backreaction of such a profile on the dilaton and the metric. As we are going to show, this will still be enough to provide a drastic change in the holographic correlators (nicely matching, again, field theory expectations).

The linearized equation for  $\eta$  is most conveniently written and solved changing variables to  $r = -\log(z)$  (the boundary is now at  $r \rightarrow \infty$  and the singularity maps to  $r = 0$ ).



With this choice the dilaton-domain wall background has the form

$$\begin{aligned} ds^2 &= \left( dr^2 + \sqrt{2 \sinh(4r)} (dx^m)^2 \right), \\ \phi(r) &= \phi_\infty + \frac{\sqrt{3}}{2} \log \left( \frac{\cosh(2r)}{\sinh(2r)} \right). \end{aligned} \quad (4.47)$$

The linearized equation of motion for  $\eta$  on this background reads

$$\eta''(r) + 4 \coth(4r) \eta'(r) + 3 \eta(r) - \frac{3}{2(\sinh(4r))^2} \eta(r) = 0. \quad (4.48)$$

The general solution depends on two integration constants  $A$  and  $B$  and is given by

$$\begin{aligned} \eta(r) &= (e^{8r} - 1)^{\frac{\sqrt{6}}{8}} \left[ A e^r {}_2F_1 \left( \frac{2 + \sqrt{6}}{8}, \frac{4 + \sqrt{6}}{8}; \frac{3}{4}; e^{8r} \right) \right. \\ &\quad \left. + B e^{3r} {}_2F_1 \left( \frac{4 + \sqrt{6}}{8}, \frac{6 + \sqrt{6}}{8}; \frac{5}{4}; e^{8r} \right) \right], \end{aligned} \quad (4.49)$$

where  ${}_2F_1$  is a hypergeometric function.

Changing variables to the usual  $z = e^{-r}$  radial coordinate, one can verify that indeed this solution has the expected behavior (4.21) near the boundary, with  $\eta_{(0)}$  and  $\tilde{\eta}_{(0)}$  expressed as linear combinations of  $A$  and  $B$ . On the other hand, the  $r$  variable is useful for studying the equation near the singularity  $r \rightarrow 0$ . One finds the following behavior

$$\eta \underset{r \rightarrow 0}{\simeq} \alpha r^{\frac{\sqrt{6}}{8}} + \beta r^{-\frac{\sqrt{6}}{8}}, \quad (4.50)$$

with  $\alpha$  and  $\beta$  linear combinations of  $A$  and  $B$ . If one imposes the boundary condition at the singularity so to meet the criterion on the boundedness of the potential [66], that is  $\beta = 0$ , one finds a relation between  $A$  and  $B$  which imposes both  $\eta_{(0)}$  and  $\tilde{\eta}_{(0)}$  to be turned on at the boundary (indicating that R-symmetry is broken explicitly in the hidden sector). This implies that in doing the holographic renormalization procedure one should bear in mind the discussion in Section 4.2.3 and augment the boundary action by the term (4.22).

Plugging our results in the formulas for the holographic correlators (3.49), it is easy to see that  $C_0$  and  $C_1$  are unaffected. On the other hand, both fermionic correlators are modified. As shown in Figure 4.3, the correlator  $B$  has a non-trivial dependence on the momentum. Consistently with expectations, it reaches a finite value at zero momentum (hence providing non-vanishing Majorana mass to SSM gauginos), and falls off to zero at  $p \rightarrow \infty$ . On the other hand, the pole at  $p^2 = 0$  in  $C_{\frac{1}{2}}$  has now disappeared (see Figure 4.3). This is consistent with field theory intuition: R-symmetry being broken, 't Hooft fermions, if any, cannot couple to the  $j_\alpha$  current and provide zero momentum

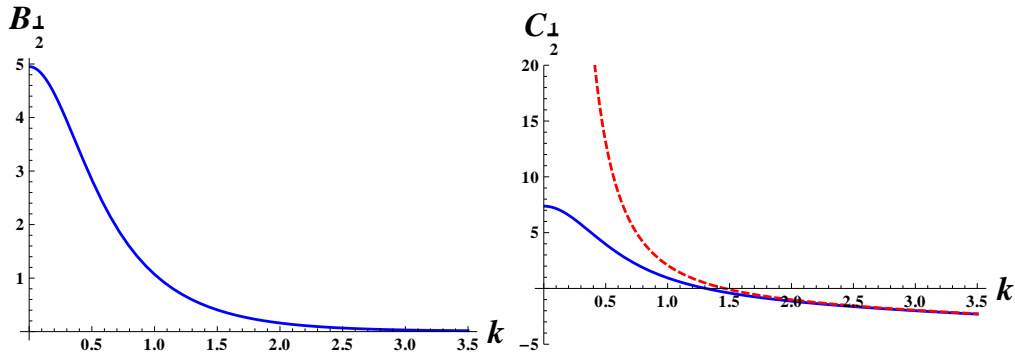


FIGURE 4.3:  $B$  and  $C_{1/2}$  as functions of the Euclidean momentum  $k$  in presence of a non-trivial profile for  $\eta$ . The plot on the left shows that a Majorana mass is generated in this case. On the right, the red dashed line shows the result for  $\eta = 0$ .

poles in  $C_{\frac{1}{2}}$ . We see the fact that as soon as  $\eta$  has a non-trivial profile the correlators  $B$  becomes non-vanishing and, *at the same time*, the pole in  $C_{\frac{1}{2}}$  vanishes, as a remarkable and non-trivial agreement with expectations from the field theory side.

**Summary** Before going on with hard wall models some comments are in order about the phenomenology described by the two top-down models we have presented so far.

**d-DW:** The dilaton-domain wall provides a scenario where the SM gauginos have R-symmetric Dirac masses. The contribution to sfermions masses is suppressed with respect to gaugino masses and the spectrum in this case is very similar to that of gaugino mediation models.

**d/ $\eta$ -DW:** The dilaton/ $\eta$ -domain wall solution generates Majorana masses for SM gauginos controlled by the parameter  $\tilde{\eta}_{(0)}$ , while sfermions masses are almost insensitive to  $\tilde{\eta}_{(0)}$ . Tuning this parameter one can obtain a spectrum which interpolates between a gaugino mediation scenario down to minimal gauge mediation. But in no case one can obtain sfermions masses larger than gauginos ones.

Both examples seem to rule out the possibility of having suppressed gaugino masses. At this stage, one could wonder if this is a generic result or if it depends on the specific backgrounds we have considered. However, it is difficult to answer such question within top-down models since there is a very poor number of explicit string-derived supersymmetry breaking solutions available in the literature. In the following sections we will try to answer the above question pursuing a bottom-up approach. Giving up any pretense of having full control on the UV completion, we will be able to gain more flexibility and possibly cover a larger portion of GGM parameter space.

## 4.6 Holographic GGM correlators from hard wall models

In the models of HGGM we discussed so far, the constraints of supergravity dictated the precise form of the interactions, and there was no free parameter left to play with. While this can be a welcome feature from the point of view of the predictivity of the model, it would be interesting to have more flexible, bottom-up examples of HGGM. Given such class of examples, one could hope to address interesting questions such as: how large a portion of the GGM parameter space can holographic models of gauge mediation cover? Are there any restrictions and/or preferred patterns? As already noticed, this kind of questions are difficult to answer within top-down models, given also the poor number of concrete and sufficiently explicit string theory supersymmetry breaking solutions available in the literature. On the contrary, within a bottom-up approach, at the price of loosing predictivity power and UV completeness, one can test HGGM with less background-dependent constraints.

In this chapter we will present the results in this direction obtained in [19]. We will consider supersymmetry breaking models which do not have necessarily a completion in string theory, but on the other hand allow for more flexibility and analytical power, enabling to try and answer the above questions. The simple backgrounds we will focus on are so-called hard wall (HW) models. This is just  $\mathbf{AdS}_5$  in which the geometry ends abruptly in the interior by putting a sharp IR cut-off at  $z = 1/\mu$ . This model was originally studied as a toy model of a confining gauge theory because it provides a holographic dual for theories with a gapped and discrete spectrum [72–74]. Our aim is to study the behavior of GGM two-point functions on this background.<sup>8</sup> Not surprisingly, the behavior of the correlators will depend strongly on the boundary conditions that one has to impose on the bulk fields at the IR cut-off.

In the case of a HW background the general solution of the equations of motion for the fluctuations in the vector multiplet is exactly the same as for pure  $\mathbf{AdS}$  (4.28) and depends on six integration constants (two for each field). Also the UV boundary conditions (4.29) remain the same, and can be simply understood as fixing the source of the boundary operator, leaving only three constants undetermined. The difference is that we are now solving the differential equations in the domain  $[0, 1/\mu]$ , and the regularity conditions are replaced by some IR boundary conditions at  $z = 1/\mu$ . These conditions can be solved for the three remaining constants, in order to fix the functional dependence of the subleading modes on the leading ones.

<sup>8</sup>Let us notice that our set up is reminiscent of extra dimensional scenarios like [75–77] in which, however, the physics of the  $4d$  hidden sector arises as a KK reduction of a  $5d$  theory in a slice of  $\mathbf{AdS}$ .

Expanding the solutions (4.28) near the UV boundary, one can easily find that the correlators are given by

$$C_0(p^2) = C^{\text{AdS}}(p^2) - 2 \frac{\delta c_1}{\delta \rho(0)}, \quad (4.51a)$$

$$C_1(p^2) = C^{\text{AdS}}(p^2) - \frac{1}{2p} \frac{\delta \alpha_{1m}}{\delta a_m(0)}, \quad (4.51b)$$

$$C_{\frac{1}{2}}(p^2) = C^{\text{AdS}}(p^2) - \left( \frac{1}{2p} \frac{\delta \bar{\theta}_1^{\dot{\alpha}}}{\delta \bar{\xi}_{(0)}^{\dot{\alpha}}} + c.c. \right), \quad (4.51c)$$

$$B(p^2) = - \frac{\sigma_{\alpha\dot{\alpha}}^m p_m}{p} \frac{\delta \bar{\theta}_1^{\dot{\alpha}}}{\delta \xi_{(0)\alpha}}, \quad (4.51d)$$

where the **AdS** result (4.39)

$$C^{\text{AdS}}(p^2) = \frac{1}{2} \log \left( \frac{\Lambda^2}{p^2} \right), \quad (4.52)$$

is obtained for  $c_1, \alpha_{1m}, \bar{\theta}_1$  set to zero (see Section 4.3).

#### 4.6.1 Homogeneous IR boundary conditions

We start by taking general homogeneous boundary conditions at the IR cut-off

$$(\rho(z, p) + \beta_0 z \partial_z \rho(z, p))|_{z=1/\mu} = 0, \quad (4.53)$$

$$(A_m(z, p) + \beta_1 z \partial_z A_m(z, p))|_{z=1/\mu} = 0, \quad (4.54)$$

$$(\bar{\xi}(z, p) + \beta_{1/2} z \partial_z \bar{\xi}(z, p))|_{z=1/\mu} = 0, \quad (4.55)$$

parametrized by three real coefficients  $\beta_s$ . As we will see, in order to cover all of GGM parameter space it will be necessary to turn on also inhomogeneous terms in the above equations, something we will do next.

As it befits coefficients computed with homogeneous boundary conditions, the coefficients  $c_1, \alpha_{1m}$  and  $\bar{\theta}_1$  in (4.51) are all proportional to the source terms. The resulting GGM functions are

$$C_0^{(h)}(p^2) = C^{\text{AdS}}(p^2) + 2 \frac{-(1 + 2\beta_0)K_0(x) + \beta_0 x K_1(x)}{(1 + 2\beta_0)I_0(x) + \beta_0 x I_1(x)}, \quad (4.56a)$$

$$C_1^{(h)}(p^2) = C^{\text{AdS}}(p^2) + 2 \frac{K_1(x) - \beta_1 x K_0(x)}{I_1(x) + \beta_1 x I_0(\frac{p}{\mu})}, \quad (4.56b)$$

$$C_{\frac{1}{2}}^{(h)}(p^2) = C^{\text{AdS}}(p^2) + 2 \frac{(2 + 3\beta_{1/2})K_1(x) - 2\beta_{1/2} x K_0(x)}{(2 + 3\beta_{1/2})I_1(x) + 2\beta_{1/2} x I_0(x)}, \quad (4.56c)$$

$$B^{(h)}(p^2) = 0, \quad (4.56d)$$

where  $x \equiv p/\mu$  and  $C^{\text{AdS}}$  is the result in pure **AdS** (4.52), and the superscript ( $h$ ) stands for homogeneous boundary conditions.

The analysis of the boundary condition-dependent soft spectrum emerging from the correlators (4.56) is postponed to Section 4.7. For future reference we would instead like to comment here on the behavior of the correlators in the IR and UV. Making use of the asymptotic expansion for  $x \ll 1$  of the Bessel functions, we find the correlators at low momentum to behave as

$$C_0^{(h)}(p^2) \underset{p \rightarrow 0}{\simeq} \log\left(\frac{\Lambda^2}{\mu^2}\right) + \frac{2\beta_0}{1+2\beta_0}, \quad (4.57a)$$

$$C_1^{(h)}(p^2) \underset{p \rightarrow 0}{\simeq} \frac{4}{1+2\beta_1} \frac{\mu^2}{p^2} + \log\left(\frac{\Lambda^2}{\mu^2}\right) - \frac{3+8\beta_1}{2(1+2\beta_1)^2}, \quad (4.57b)$$

$$C_{\frac{1}{2}}^{(h)}(p^2) \underset{p \rightarrow 0}{\simeq} 4 \frac{2+3\beta_{1/2}}{2+7\beta_{1/2}} \frac{\mu^2}{p^2} + \log\left(\frac{\Lambda^2}{\mu^2}\right) - \frac{(2+3\beta_{1/2})(6+25\beta_{1/2})}{2(2+7\beta_{1/2})^2}. \quad (4.57c)$$

As for the UV limit, using the large  $x$  behavior of Bessel functions, one can show that all the  $C_s^{(h)}$  functions approach the supersymmetric **AdS** value with exponential rate at large momentum

$$C_0^{(h)}(p^2) \sim C_{\frac{1}{2}}^{(h)}(p^2) \sim C_1^{(h)}(p^2) \underset{p \rightarrow \infty}{\simeq} C^{\text{AdS}}(p^2) - 2\pi e^{-\sqrt{\frac{p^2}{\mu^2}}}. \quad (4.58)$$

From the field theory point of view, the exponential restoration of supersymmetry in the UV suggests that supersymmetry breaking in a hidden sector described by a HW holographic model is induced by an operator of very large dimension.

Two additional remarks are in order at this point. The first is that one can of course compute the above functions also using the numerical approach pursued in [18], finding perfect agreement with the analytic computation above. The second comment is that the above functions can be continued to negative values of  $p^2$ . It is easy to convince oneself that they will then display an infinite sequence of poles on the negative  $p^2$  axis, corresponding to towers of glueball states for each spin sector. They return the same values that can be obtained through the more traditional holographic approach of computing glueball masses, i.e. finding normalizable fluctuations for each field.

#### 4.6.2 Inhomogeneous IR boundary conditions

Let us now consider the possibility of having inhomogeneous boundary conditions in the IR. We thus take general boundary conditions at the IR cut-off depending on three more

arbitrary terms, now

$$(\rho(z, p) + \beta_0 z \partial_z \rho(z, p))|_{z=1/\mu} = \Sigma_0(p), \quad (4.59a)$$

$$(A_m(z, p) + \beta_1 z \partial_z A_m(z, p))|_{z=1/\mu} = \Sigma_{m1}(p), \quad (4.59b)$$

$$(\bar{\xi}(z, p) + \beta_{1/2} z \partial_z \bar{\xi}(z, p))|_{z=1/\mu} = \bar{\Sigma}_{\frac{1}{2}}(p), \quad (4.59c)$$

where we have allowed for a non-trivial  $p$  dependence in the inhomogeneous terms  $\Sigma_s$ . We will see instantly that the arbitrariness actually amounts to four new constants.

The coefficients  $c_1$ ,  $\alpha_{m1}$  and  $\bar{\theta}_1$  in (4.51) will pick up an additional contribution, linear in the  $\Sigma_s$ . Since these coefficients enter the GGM correlation functions only through the first derivative with respect to the source, the inhomogeneous terms can contribute only if we allow them to be dependent on the source, with the result that the condition at  $z = 1/\mu$  involves both IR and UV boundary data of the fields. In particular, from equation (4.51d), a dependence of  $\bar{\Sigma}_{\frac{1}{2}}(p)$  on the source  $\xi_{(0)}$  can give a non-vanishing  $B$ , as opposed to the case of homogeneous boundary conditions (4.56d). Therefore, such a dependence would break the R-symmetry.

Since in any case only the first derivative enters equations (4.51), it is enough to let the  $\Sigma_s$  depend linearly on the sources  $\rho_{(0)}(p)$ ,  $a_{m(0)}(p)$  and  $\xi_{(0)}(p)$ . Taking into account Lorentz covariance, a reasonable choice is

$$\Sigma_0(p^2) = -\frac{1}{\mu^2} E_0 \rho_{(0)}(p), \quad (4.60a)$$

$$\Sigma_{m1}(p^2) = -E_1 a_{m(0)}(p), \quad (4.60b)$$

$$\bar{\Sigma}_{\frac{1}{2}}^{\dot{\alpha}}(p^2) = -\frac{1}{\mu^{3/2}} E_{\frac{1}{2}} \bar{\xi}_{(0)}^{\dot{\alpha}}(p) - H_{\frac{1}{2}} \frac{1}{\mu^{7/2}} \bar{\sigma}_m^{\dot{\alpha}\alpha} p^m \xi_{\alpha(0)}(p), \quad (4.60c)$$

where the  $E$ 's and  $H$  are coefficients which do not depend on the momentum. Hence we are left with four new parameters due to the inhomogeneous boundary conditions.

The GGM functions in this case take the form

$$C_0^{(nh)}(p^2) = C^{\text{AdS}}(p^2) + 2 \frac{-(1 + 2\beta_0) K_0(x) + \beta_0 x K_1(x) + E_0}{(1 + 2\beta_0) I_0(x) + \beta_0 x I_1(x)}, \quad (4.61a)$$

$$C_1^{(nh)}(p^2) = C^{\text{AdS}}(p^2) + 2 \frac{K_1(x) - \beta_1 x K_0(x) + \frac{1}{x} E_1}{I_1(x) + \beta_1 x I_0(x)}, \quad (4.61b)$$

$$C_{\frac{1}{2}}^{(nh)}(p^2) = C^{\text{AdS}}(p^2) + 2 \frac{(2 + 3\beta_{1/2}) K_1(x) - 2\beta_{1/2} x K_0(x) + \frac{2}{x} E_{\frac{1}{2}}}{(2 + 3\beta_{1/2}) I_1(x) + 2\beta_{1/2} x I_0(x)}, \quad (4.61c)$$

$$B^{(nh)}(p^2) = \frac{4x H_{\frac{1}{2}}}{(2 + 3\beta_{1/2}) I_1(x) + 2\beta_{1/2} x I_0(x)}, \quad (4.61d)$$

where  $x \equiv p/\mu$ . The result with homogeneous boundary condition is simply recovered by setting the  $E$ 's and  $H$  to zero.

Inhomogeneous terms contribute to the IR behavior as follows

$$C_0^{(nh)}(p^2) - C_0^{(h)}(p^2) \underset{p \rightarrow 0}{\simeq} \frac{2}{1 + 2\beta_0} E_0, \quad (4.62a)$$

$$C_1^{(nh)}(p^2) - C_1^{(h)}(p^2) \underset{p \rightarrow 0}{\simeq} \frac{4}{1 + 2\beta_1} \frac{\mu^2}{p^2} E_1, \quad (4.62b)$$

$$C_{\frac{1}{2}}^{(nh)}(p^2) - C_{\frac{1}{2}}^{(h)}(p^2) \underset{p \rightarrow 0}{\simeq} \frac{8}{2 + 7\beta_{1/2}} \frac{\mu^2}{p^2} E_{\frac{1}{2}}, \quad (4.62c)$$

$$B^{(nh)}(p^2) \underset{p \rightarrow 0}{\simeq} \frac{8}{2 + 7\beta_{1/2}} H_{\frac{1}{2}}. \quad (4.62d)$$

In particular, having  $H_{\frac{1}{2}} \neq 0$  we get now a non-zero Majorana mass for the gaugino. Indeed, the boundary condition (4.60c) explicitly breaks the R-symmetry.

As for the UV asymptotic, the large  $x$  behavior of the Bessel functions tells us that the exponential approach to the supersymmetric limit remains valid in this case, also for  $B(p^2)$  that asymptotes to 0. So we see that, consistently, the inhomogeneous boundary conditions do not modify the UV behavior.

## 4.7 Analysis of the soft spectrum in hard wall models

We now discuss the physical interpretation, in terms of soft supersymmetry breaking masses, of the  $C_s$  and  $B$  functions we have found in the hard wall models discussed in the previous section.

Let us start with a very basic requirement: since the correlators happen to have non-trivial denominators which depend on the momentum, we should exclude the possibility that tachyonic poles are developed. The denominators in (4.56), (4.61) are linear combinations of two Bessel functions evaluated at  $x = p/\mu$ . Studying their monotonicity properties and their limits for  $x \rightarrow 0$  and  $x \rightarrow \infty$  one can easily see that tachyonic poles are excluded if and only if the coefficients of the linear combination have the same sign. This condition results in the following inequalities

$$\{\beta_0 \leq -\frac{1}{2}\} \cup \{\beta_0 \geq 0\}, \{\beta_1 \geq 0\}, \{\beta_{1/2} \leq -\frac{2}{3}\} \cup \{\beta_{1/2} \geq 0\}. \quad (4.63)$$

The IR behavior of the  $C_s$  functions, in particular the expressions given in (4.57), show that the theory described holographically by the HW has a threshold  $\mu$  for the production of two particle states and possibly a certain number of massless poles which depends on

the choice of the boundary conditions. Below we analyze the cases of homogeneous and inhomogeneous boundary conditions in turn.

### 4.7.1 Homogeneous boundary conditions

For generic choices of  $\beta_s$  parameters, we see from equations (4.57) that  $C_1$  and  $C_{\frac{1}{2}}$  have poles at  $p^2 = 0$  while  $C_0$  does not. The interpretation of such poles is that they arise from the tree-level exchange of a massless state with the same quantum numbers of the corresponding operator. In  $C_1$ , this means that the global U(1) symmetry is spontaneously broken, the massless excitation being the associated Goldstone boson. If the symmetry is broken, we cannot identify it with the Standard Model gauge group, but rather with an extension thereof by some higgsed U(1)', a setting extensively studied in the literature (see for instance [78] and references therein).

The pole in  $C_{\frac{1}{2}}$  signals the existence of an R-charged massless fermion, neutral under the global U(1), which mixes with the fermionic partner of the current. The most natural interpretation of such a fermion in a strongly coupled theory is that of a 't Hooft fermion associated with a global anomaly of the unbroken U(1)<sub>R</sub>, as already discussed.

The consequence on the soft spectrum of poles in the correlators  $C_1$  and/or  $C_{\frac{1}{2}}$  was studied in [69, 71], and can be summarized as follows: the gaugino acquires a Dirac mass by mixing with the would-be massless fermion in  $C_{\frac{1}{2}}$  (recall that a Majorana mass is forbidden by the unbroken R-symmetry), and the integral giving the sfermion masses is dominated by the contribution of the poles. Comparing with the usual result in General Gauge Mediation without IR singularities, the sfermion soft mass is enhanced by a logarithm of the gauge coupling. Notice that the pole in  $C_1$  ( $C_{\frac{1}{2}}$ ) contributes with a negative (positive) sign, so that generically one can get a tachyonic contribution to the sfermion mass-squared. In formulae

$$m_{\tilde{g}} = gM_{\frac{1}{2}}, \quad (4.64)$$

$$m_{\tilde{f}}^2 \simeq \frac{g^4}{(4\pi)^2} \left( \log \frac{1}{g^2} \right) (4M_{\frac{1}{2}}^2 - 3M_1^2), \quad (4.65)$$

where  $g$  is the gauge coupling,  $m_{\tilde{g}}$  is the Dirac mass of the gaugino,  $m_{\tilde{f}}$  is the sfermion mass, and  $M_s^2$  is the residue of the massless pole  $C_s \simeq M_s^2/p^2$ . From eqs. (4.57b)–(4.57c) we see that in our model<sup>9</sup>

$$M_1^2 = 4\mu^2 \frac{1}{1 + 2\beta_1}, \quad M_{\frac{1}{2}}^2 = 4\mu^2 \frac{1 + \frac{3}{2}\beta_{1/2}}{1 + \frac{7}{2}\beta_{1/2}}. \quad (4.66)$$

<sup>9</sup>Here and in the following we tacitly assume that the prefactor  $N^2/8\pi^2$  can be set to unity.



Notice that in the tachyon-free range (4.63) the two residues are always positive. If we further impose the contribution to the sfermion mass-squared (4.65) to be positive, we get the additional inequality

$$\beta_1 \geq -\frac{1}{8} \frac{1 - \frac{9}{2}\beta_{1/2}}{1 + \frac{3}{2}\beta_{1/2}}. \quad (4.67)$$

We see from eqs. (4.64)–(4.65) that in this scenario the sfermions are somewhat lighter than the gaugino. This is typical of Dirac gaugino scenarios [70], though in our model the Dirac partner of the gaugino is a strongly coupled composite fermion.

**Tuning the  $\beta_s$  parameters** We now briefly mention different possibilities to evade the generic scenario presented above, which can be realized by choosing specific values for the  $\beta_s$  parameters.

1. As a first possibility, consider the case in which  $M_1^2 = M_{\frac{1}{2}}^2$ , that is

$$\beta_1 = \frac{\beta_{1/2}}{1 + \frac{3}{2}\beta_{1/2}}, \quad (4.68)$$

while  $\beta_0$  is kept generic. We are still in a scenario in which the global symmetry is spontaneously broken in the hidden sector, and the soft spectrum is described by the same formulae as before (notice however that the contribution to the sfermion mass-squared is positive, now). Nevertheless, in this case we can argue a different interpretation of the physics in the hidden sector, the reason stemming from a somehow surprising fact: the condition (4.68) that makes the two residues coincide, actually renders the whole  $C_1$  and  $C_{\frac{1}{2}}$  functions (4.56b) and (4.56c) equal for all values of  $p^2$ . As a consequence, one is led to interpret the massless fermion as the partner of the Goldstone boson associated to the broken global symmetry, rather than a 't Hooft fermion. Since  $C_0$  differs from  $C_1 = C_{\frac{1}{2}}$  for generic  $\beta_0$ , supersymmetry is still broken in the hidden sector, but mildly enough so not to lift the fermionic partner of the Goldstone boson.

2. As a subcase of 1, consider in addition to tune the  $\beta_0$  parameter to  $\beta_0 = -\frac{1}{2}$ . In this case the low momentum expansion (4.57a) is not valid, and by repeating the analysis one finds that also  $C_0$  develops a  $1/p^2$  pole, with residue  $M_0^2 = 4\mu^2$ . As explained in [69, 71], a pole in  $C_0$  is unphysical, unless the hidden sector breaks the global symmetry in a supersymmetric manner, so that  $C_0 = C_{\frac{1}{2}} = C_1$  and a massless Goldstone mode is present in all three functions<sup>10</sup>. Indeed, if we require  $M_0^2 = M_{\frac{1}{2}}^2 = M_1^2$ , that is  $\beta_1 = \beta_{1/2} = 0$ , we find from eqs. (4.56a)–(4.56c) that

<sup>10</sup>In the simple example of a U(1) broken by the VEV of a charged chiral superfield the pole in  $C_0$  is related to the modulus of the complex scalar.

this condition is sufficient to ensure  $C_0 = C_{\frac{1}{2}} = C_1$  for all values of  $p^2$ , supporting the interpretation of a supersymmetric global symmetry breaking in the hidden sector.

3. Finally,  $\beta_1$  and  $\beta_{1/2}$  can also be (independently) tuned in such a way to eliminate the massless pole in  $C_1$  and  $C_{\frac{1}{2}}$  respectively, the specific values being  $\beta_1 = \infty$ <sup>11</sup> and  $\beta_{1/2} = -2/3$ . If only one of the two parameters is tuned, the soft masses and the interpretation of the physics in the hidden sector remains the same as in the previous section, with the only difference that  $M_1^2$  or  $M_{\frac{1}{2}}^2$  are tuned to 0. It is therefore more interesting to consider the possibility that both parameters are tuned: in this case none of the  $C_s$  has an IR singularity and we are in a situation similar to ordinary GGM, as far as sfermion masses are concerned (the gaugino remains massless because the hidden sector does not break the R-symmetry). Since at large  $p$  all the  $C_s$  approach their supersymmetric value exponentially, the weighted sum  $-(C_0 - 4C_{\frac{1}{2}} + 3C_1)$  goes to zero at the same rate, so that we can determine the sign of the sfermion mass-squared by studying its IR limit. From eqs. (4.57a)–(4.57c) we see that the leading term, with the present values of  $\beta_1$  and  $\beta_{1/2}$ , is given by

$$-(C_0 - 4C_{\frac{1}{2}} + 3C_1) \underset{p \rightarrow 0}{\simeq} \simeq -\frac{2\beta_0}{1 + 2\beta_0}. \quad (4.69)$$

In the tachyon-free range (4.63) this expression is negative. Therefore, in this tuned scenario we find vanishing gaugino mass and tachyonic sfermion mass. We will see later that both this unwanted features can be overcome: one way, which is somehow more *ad-hoc*, consists in enlarging the parameter space by considering inhomogeneous boundary conditions; the other, which is more dynamical, consists in turning on a R-breaking scalar on top of the HW background. Most of what follows will therefore consist in improvements of this setting with tuned  $\beta_1$  and  $\beta_{1/2}$ .

### 4.7.2 Inhomogeneous boundary conditions

Let us proceed by considering the functions (4.61a)–(4.61c), which we obtained by adding source-dependent inhomogeneous terms in the boundary condition. Besides the  $\beta_s$ , we have now four additional real parameters to play with, namely the dimensionless  $E_s$  and the R-breaking parameter  $H_{\frac{1}{2}}$ , which has dimension of a mass.

For generic values of the parameters the situation is analogous to the one with homogeneous boundary conditions, so that the  $E_s$  parameters appear to be somehow redundant:

<sup>11</sup>A global parametrization which avoids infinities could be conveniently given in terms of angles  $\alpha_s$ , the change of variable being  $\beta_s = \text{tg}(\alpha_s)$ .

$C_1$  and  $C_{\frac{1}{2}}$  have a massless pole, while  $C_0$  has not. The major difference with respect to the previously considered case is that now  $H_{\frac{1}{2}}$  gives a non-zero Majorana mass to the gaugino,

$$m_{\tilde{g}} = \frac{8}{2 + 7\beta_{1/2}} H_{\frac{1}{2}}. \quad (4.70)$$

Since now R-symmetry is broken, the pole in  $C_{\frac{1}{2}}$  cannot be interpreted as due to a 't Hooft fermion, and it seems unphysical. In order to get more interesting and reasonable results, eliminating the poles at  $p^2 = 0$  in  $C_1^{(nh)}$  and  $C_{\frac{1}{2}}^{(nh)}$ , we can take  $E_1 = -1$  and  $E_{\frac{1}{2}} = -(1 + \frac{3}{2}\beta_{1/2})$ , see eqs. (4.62b) and (4.62c). As opposed to eq. (4.69), the IR limit of the weighted sum  $-(C_0 - 4C_{\frac{1}{2}} + 3C_1)$  depends now on four parameters, the  $\beta_s$  and  $E_0$ , so that one can easily obtain a positive mass-squared for the sfermions. For definiteness and for an easier comparison with eq. (4.69), consider taking  $\beta_1 = \infty$  and  $\beta_{1/2} = -2/3$ , so that

$$-(C_0 - 4C_{\frac{1}{2}} + 3C_1) \underset{p \rightarrow 0}{\simeq} \simeq -\frac{2\beta_0 + 2E_0}{1 + 2\beta_0}, \quad (4.71)$$

which can be positive if  $E_0 < -\beta_0$  (assuming a positive  $\beta_0$ ). The sfermion masses can then be even bigger than the Majorana gaugino mass if  $H_{\frac{1}{2}}$  is somewhat smaller than  $\sqrt{|E_0|}\mu$ .

The punchline of the above analysis is that tuning appropriately the boundary conditions, one can realize holographically any scenario between pure gaugino mediation [79–86] to minimal gauge mediation [87–89] as well as scenarios with suppressed gaugino masses [90–96] which would fit into a split supersymmetry scenario [97, 98]. Hence, HW models can actually cover all of GGM parameter space.

In fact, it is not entirely satisfactory that a necessary ingredient for all this amounts to introduce two parameters,  $H_{\frac{1}{2}}$  and  $E_0$ , which are directly proportional to gaugino and sfermions masses, respectively. This is reminiscent of minimal benchmark points. It would thus be desirable to try and obtain both Majorana gaugino masses and positive squared sfermions masses by enriching the dynamics in the bulk instead of introducing inhomogeneous terms in the IR boundary conditions. In the next section we will achieve this goal by turning on a linear profile for an R-charged scalar, as it was done in Section 4.5.

Let us finally mention that, as noticed in [99], a positive value for  $C_1 - C_0$  is a desirable feature, in that it helps raising the mass of the Higgs in gauge mediation scenarios. In our models, this is achieved by the same conditions which make the right hand side of (4.71) positive.

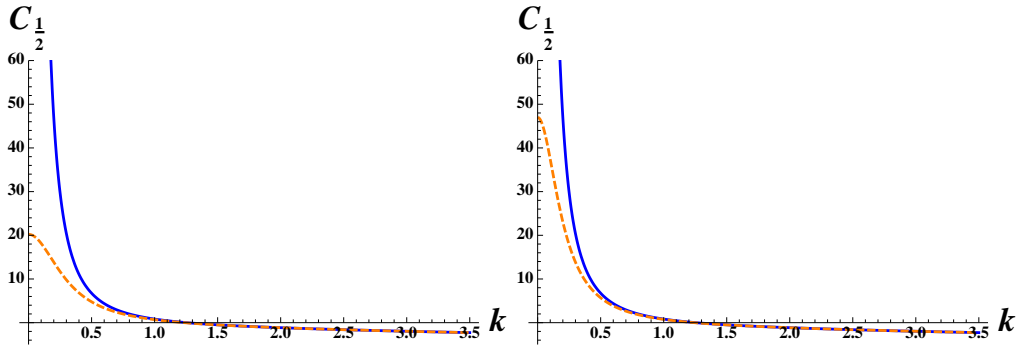


FIGURE 4.4:  $C_{\frac{1}{2}}$  as a function of the Euclidean momentum  $k$ . The solid blue line is for  $\eta = 0$ , the dashed one on the left figure is for  $\eta = 0.1 z^3$  and on the right figure for  $\eta = 0.1 z$ . In these plots  $\beta_{1/2} = 3$  and  $\mu = 1$ . Notice that turning on  $\eta$  the massless pole disappears.

## 4.8 Hard wall with R-symmetry breaking mode

In this section we would like to construct a simple scenario in which the R-symmetry is broken (and gaugino masses generated) dynamically. We will follow the same logic as in Section 4.5. As in the top-down model considered there, we will see that the dynamical breaking of R-symmetry implies automatically the absence of massless modes in  $C_{\frac{1}{2}}$ . Notice that this physical consistency condition had instead to be imposed by hand, in the previous section.

We introduce a new dynamical scalar field  $\eta$  in the bulk with  $m^2 = -3$ , and treat it as a linear fluctuation around the HW metric.

The action for  $\eta$  at the linearized level is completely determined by its mass while the precise values of its couplings with the vector multiplet can be taken from the  $\mathcal{N} = 2$  supergravity model of Section 4.2, based on the general results of [58, 59]

$$\mathcal{S}_{\text{kin}} = \frac{N^2}{4\pi^2} \int d^5x \sqrt{g} (g^{\mu\nu} \partial_\mu \eta \partial_\nu \eta - 3\eta^2), \quad (4.72)$$

$$\mathcal{S}_{\text{int}} = \frac{N^2}{4\pi^2} \int d^5x \frac{\sqrt{g}}{2} [(\eta + z\partial_z \eta)(\chi\chi + \bar{\chi}\bar{\chi}) + (\eta - z\partial_z \eta)(\xi\xi + \bar{\xi}\bar{\xi})]. \quad (4.73)$$

One might think that, in view of the possibility of constructing more general bottom-up models, it might be interesting to see what happens if we take arbitrary coefficients in the interactions term. On the other hand, asking for a gravity dual of a supersymmetric field theory (which then breaks supersymmetry spontaneously or by a soft deformation) puts severe constraints on the possible interactions. In fact, precisely the constraints dictated by supergravity. One can check that choices other than the interactions above do not give the right supersymmetric result in the UV.

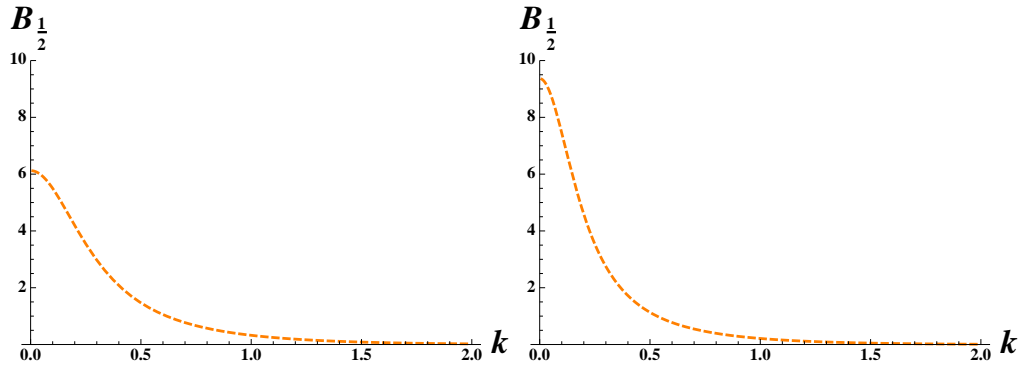


FIGURE 4.5:  $B$  as a function of the Euclidean momentum  $k$ . On the left  $\eta = 0.1 z^3$ , on the right  $\eta = 0.1 z$ . In these plots  $\beta_{1/2} = 3$  and  $\mu = 1$ . When  $\eta \neq 0$  a non-zero  $B$  is generated.

In order to preserve Poincaré invariance of the boundary theory, we demand the R-symmetry breaking mode  $\eta(z, x)$  to have a non-trivial profile in the vacuum which is independent on the boundary space-time directions. The most general solution to the resulting equations of motion for  $\eta$  without  $p$  dependence is

$$\eta(z) = z\eta_0 + z^3\tilde{\eta}_0, \quad (4.74)$$

where  $\eta_0$  and  $\tilde{\eta}_0$  are two arbitrary constants. These two constants can be fixed imposing, as usual, boundary conditions at  $z = 0$  and at the IR cut-off  $z = 1/\mu$ . This strictly amounts to considering them as free parameters, which we will do in the following.

The equations of motion for  $\lambda$  are modified by the presence of the extra contribution (4.73) and become

$$\begin{cases} (z\partial_z - \frac{5}{2})\chi + z\sigma^m k_m \bar{\xi} + (\eta - z\partial_z\eta)\xi = 0, \\ (-z\partial_z + \frac{3}{2})\bar{\xi} + z\bar{\sigma}^m k_m \chi + (\eta + z\partial_z\eta)\bar{\chi} = 0. \end{cases} \quad (4.75)$$

As discussed in Section 4.2.3, whenever  $\eta_{(0)} \neq 0$ , we have to modify the definition of the fermionic correlator defining  $B$  according to

$$\langle j_\alpha(p)j_\beta(-p) \rangle = \frac{\delta\tilde{\chi}_{(0)\alpha}}{\delta\xi_{(0)}^\beta} - \frac{\delta\tilde{\chi}_{(0)\beta}}{\delta\xi_{(0)}^\alpha} + 2\eta_{(0)}\epsilon_{\alpha\beta}, \quad (4.76)$$

while the expression for the non-chiral fermionic correlator remains unchanged.

We now need to solve eqs. (4.75) by imposing (homogeneous) boundary conditions in the IR (that for generic  $\beta_{1/2}$  would give a massless pole when  $\eta = 0$ ). Unfortunately, this cannot be done analytically, and we have to resort to numerics. Figures 4.4 and 4.5 contain our results.

It is remarkable to see that when the R-symmetry is broken by a scalar profile, the pole in  $C_{\frac{1}{2}}$  disappears automatically. We note that the sfermion mass-squared is driven positive by the fact that  $C_{\frac{1}{2}}$  is still quite large near  $p = 0$ , at least as far as  $\eta$  is a perturbation. If we stick to this model without playing with inhomogeneous boundary conditions in the IR, it can be seen that we are able to explore a smaller region of parameter space. (Possibly, a larger portion of parameter space can be reached by playing with  $\beta_{1/2}$ .)

While the above analysis is done numerically, it would be nicer to have some analytical control on (at least) the low momenta behavior of the correlators, to see, for instance, how the pole in  $C_{\frac{1}{2}}$  disappears when the R-charged scalar is turned on. This analysis turns out to be possible if we also take the coefficients  $\eta_{(0)}$  and  $\tilde{\eta}_{(0)}$  parametrically small, and we obtain

$$C_{\frac{1}{2}} \simeq \frac{1 + \frac{3}{2}\beta_{1/2}}{1 + \frac{7}{2}\beta_{1/2}} \frac{4\mu^2}{p^2 + 4M_{\eta_{(0)}, \tilde{\eta}_{(0)}}^2}, \quad (4.77)$$

$$B \simeq \frac{1 + \frac{3}{2}\beta_{1/2}}{1 + \frac{7}{2}\beta_{1/2}} \frac{8\mu^2 M_{\eta_{(0)}, \tilde{\eta}_{(0)}}}{p^2 + 4M_{\eta_{(0)}, \tilde{\eta}_{(0)}}^2}, \quad (4.78)$$

where

$$M_{\eta_{(0)}, \tilde{\eta}_{(0)}} = \eta_{(0)} + \frac{1 + \frac{11}{2}\beta_{1/2}}{1 + \frac{7}{2}\beta_{1/2}} \frac{\tilde{\eta}_{(0)}}{\mu^2}. \quad (4.79)$$

Notice that these formulas agree with the numerical plots in Figures 4.4 and 4.5.

### 4.8.1 The IR limit of correlation functions

In Section (4.8) we have shown how the presence of a non-trivial profile for an R-charged scalar field,  $\eta$ , while providing a non-vanishing value for the R-breaking fermionic correlator  $B$ , consistently removes the pole from the non-chiral fermionic correlator  $C_{1/2}$ . The analysis was done by numerical methods. Here we show that one can actually study the IR behavior of holographic correlators analytically.

We are interested in analyzing the behavior of the correlation functions for small  $p$ . More precisely, the relevant quantity is  $p/\mu \ll 1$ , where  $z = 1/\mu$  is the position of the IR wall, so that the limit can also be seen as moving the wall closer to the boundary. This suggests that if we just need to evaluate the behavior of the  $C_s$  functions at low momenta, i.e. (4.57a)–(4.57c), we can impose the IR boundary condition directly on the near-boundary expansion of the solutions, keeping only terms up to a mode high enough to match the order in  $p^2$  at which we need the  $C_s$ . Indeed, in previous sections we have seen that this limit is very easy to obtain when one has exact solutions, since it involves expanding Bessel functions near the origin, i.e. keeping only the near-boundary expansion.

Let us illustrate this procedure with  $C_0$  with homogeneous IR boundary conditions. We just need to substitute the near boundary expansion in the boundary conditions (4.53). We get

$$\frac{1}{\mu^2}(\rho_{(0)} \log(\Lambda/\mu) + \tilde{\rho}_{(0)}) + \beta_0 \frac{1}{\mu^2}(2\rho_{(0)} \log(\Lambda/\mu) + \rho_{(0)} + 2\tilde{\rho}_{(0)}) = 0 , \quad (4.80)$$

that is

$$\tilde{\rho}_{(0)} = -\rho_{(0)} \left( \log(\Lambda/\mu) + \frac{\beta_0}{1 + 2\beta_0} \right) . \quad (4.81)$$

Applying

$$C_0 = -2 \frac{\delta \tilde{\rho}_0}{\delta \rho_{(0)}} , \quad (4.82)$$

we obtain (4.57a) right away and effortlessly. In order to reproduce eqs. (4.57b) and (4.57c), the only added difficulty is that we have to go one order higher in the expansion, if interested in both the  $1/p^2$  pole and the finite term.

Notice that this procedure works because the equations of motion themselves are not modified with respect to the AdS ones. If we had  $\mathcal{O}(\mu)$  corrections to the metric (as in the example used in [18]), it would be impossible to take  $1/\mu$  small without introducing large corrections to the background metric and thus to the equations for the fluctuations.

The case of an AdS hard wall with a scalar profile turned is a particular case. In order to prove that the pole in  $C_{1/2}$  disappears when  $\eta = \eta_0 z + \tilde{\eta}_{(0)} z^3$  is turned on, we should take the limit  $p \rightarrow 0$  in such a way to keep terms of the form  $(p^2 + \eta_{(0)}^2)^{-1}$  or  $(p^2 + \mu^{-4} \tilde{\eta}_{(0)}^2)^{-1}$ . Therefore, the correct scaling is

$$\eta_{(0)}/\mu \sim \tilde{\eta}_{(0)}/\mu^3 \sim p/\mu = \epsilon \rightarrow 0 , \quad (4.83)$$

and we should focus on the order  $\epsilon^{-2}$  in the small  $\epsilon$  expansion of  $C_{1/2}$ . Keeping  $\eta$  small we also ensure that we can still use the AdS near boundary expansion for the fluctuations. The same kind of expansion can be done for the  $B$  correlator, with the difference that it starts from the  $\epsilon^{-1}$  order. In both cases, the leading terms in the  $\epsilon$  expansion receive a non-trivial contribution both from  $\eta_{(0)} \neq 0$  and from  $\tilde{\eta}_{(0)} \neq 0$  and they are determined by keeping the near-boundary expansion

$$\xi(z, x) = z^{3/2} \left[ \xi_{(0)} + \sum_{n=1}^{\infty} (\tilde{\xi}_{(2n)} + \xi_{(2n)} \log(z\Lambda)) z^{2n} \right] \quad (4.84)$$

up to  $n = 1$  and  $n = 2$ , respectively. The results for the order  $\epsilon^{-2}$  of  $C_{1/2}$  and the order  $\epsilon^{-1}$  in  $B$  are reported in (4.77)–(4.78). If one wants to go to the next order in  $\epsilon$ , which is order  $\epsilon^0$  for  $C_{1/2}$  and order  $\epsilon$  for  $B$ , one should keep terms up to  $n = 3$  in the near-boundary expansion. Let us stress that this  $\epsilon$  expansion is different from a simple

expansion for small momenta. For instance, the finite  $p = 0$  term will receive contribution from arbitrary high orders in  $\epsilon$ , which in turn would require to keep arbitrary high terms in the near boundary expansion. Nevertheless, as long as  $\eta_{(0)}$  and  $\tilde{\eta}_{(0)}$  are kept small, the approximations (4.77)–(4.78) give a reliable information about the finite value at  $p = 0$ , as can be checked with the numerical results plotted in Figures 4.4 and 4.5.

In the last part of this chapter we have computed GGM form factors in bottom-up hard wall models. These kind of models have allowed us to cover the entire GGM parameter space, although non-homogeneously. These results seem to suggest that holography itself does not put any restriction on the possible low energy dynamics in HGGM models, and can accommodate all possible scenarios, even if not all of them with the same genericity.



## Chapter 5

# Supercurrent multiplet and holography

In previous chapters we focused our attention on supermultiplets associated to conserved global currents, and computed correlators of operators belonging to such multiplets using holography. As we have shown in Chapter 4, our analysis can be applied to model of gauge mediation where the hidden sector is described through a dual gravitational background. In this chapter we will consider correlators of operators belonging to another supermultiplet, the *supercurrent multiplet*. This contains the stress-energy tensor and the supercurrent, i.e. the conserved current of supersymmetry, and as such is ubiquitous in a supersymmetric QFT. In addition, this multiplet contains an R-current, which, depending on the theory one is considering, gets identified with the superconformal R-current or some other R-current, which may or may not be conserved.

The universality of the supercurrent multiplet indicates that its correlators encode the very general features of a supersymmetric QFT. In particular, they are directly affected by the breaking of conformal invariance, R-symmetry and/or supersymmetry. For instance, when any of these symmetries is spontaneously broken, massless poles associated to the corresponding Goldstone modes appear in the relevant correlators. We will organize form factors in two distinct sets, one associated to the traceless part of the correlators, that computes the central charge at conformal fixed points, and another one which corresponds to the traces and is generated by the explicit breaking of conformal invariance.

The chapter is organized as follows. In the first part we will recall the structure of the supercurrent multiplet in four dimensions. As we will show, there are at least two different supermultiplet in which the stress-energy tensor and supercurrent can be embedded. These are known as the Ferrara-Zumino (FZ) multiplet [100] and the  $\mathcal{R}$

multiplet [101].<sup>1</sup> For both we will provide a complete parametrization of the two-point functions in terms of form factors, and derive the constraints imposed by supersymmetry and conformal symmetry. In the second part we will compute the two-point functions of the FZ multiplet using holography. We will consider the simplest holographic set-up one can think of, namely a five dimensional hard wall (HW) background. This will provide a holographic realization of a variety of different dynamical behaviors, including, e.g. a holographic description of the Goldstino mode.

## 5.1 Field theory preliminaries

In any supersymmetric field theory one can define an energy–momentum tensor  $T_{mn}$  and a supercurrent  $S_{m\alpha}$  (i.e. the Noether’s current associated to supersymmetry) which are both conserved on-shell. The supersymmetry current algebra

$$\{Q, \bar{Q}\} \sim P \quad (5.1)$$

intuitively shows that the supersymmetry variation of the supercurrent  $S_{m\alpha}$ , whose associated charge is  $Q$ , must contain the stress-energy tensor  $T_{mn}$ , whose associated charge is  $P$ . So the two operators must sit in the same supermultiplet. As it turns out, there are, at least, two different ways to accommodate  $T_{mn}$  and  $S_{m\alpha}$  into a supermultiplet, the most widely known being the Ferrara-Zumino (FZ) multiplet [100].

The FZ multiplet can be described<sup>2</sup> by a pair of superfields  $(\mathcal{J}_m, X)$  satisfying the relation

$$-2\bar{D}^{\dot{\alpha}}\sigma_{\alpha\dot{\alpha}}^m\mathcal{J}_m = D_{\alpha}X, \quad (5.2)$$

with  $\mathcal{J}_m$  being a real superfield,  $\mathcal{J}_m = \mathcal{J}_m^*$ , and  $X$  a chiral superfield,  $\bar{D}_{\dot{\alpha}}X = 0$ . From the defining equation above one can work out the component expression of these two superfields. They read

$$\mathcal{J}_m = j_m + \theta\left(S_m - \frac{1}{3}\sigma_m\bar{S}\right) + \bar{\theta}\left(\bar{S}_m + \frac{1}{3}\bar{\sigma}_m S\right) + \frac{i}{2}\theta\theta\partial_m x^* - \frac{i}{2}\bar{\theta}\bar{\theta}\partial_m x \quad (5.3)$$

$$+ \theta\sigma^n\bar{\theta}\left(2T_{mn} - \frac{2}{3}\eta_{mn}T + \frac{1}{2}\varepsilon_{mnr s}\partial^r j^s\right) + \dots \quad (5.4)$$

and

$$X = x + \frac{2}{3}\theta S + \theta\theta\left(\frac{2}{3}T + i\partial^m j_m\right) + \dots \quad (5.5)$$

<sup>1</sup>We will not consider situations in which none of the two supermultiplets can be defined, and one should resort to the so-called  $\mathcal{S}$  multiplet [102, 103]. See [103] for a detailed discussion.

<sup>2</sup>As in the previous chapters we adhere to the conventions of [46].

where ... stand for the supersymmetric completion of the superfield. We have defined the ‘trace’ operators  $T \equiv T^m_m$  and  $S_\alpha \equiv \sigma^m_{\alpha\dot{\alpha}} \bar{S}^{\dot{\alpha}}$ . All in all, the FZ multiplet contains a (in general non-conserved) current  $j_m$ , a symmetric and conserved  $T_{mn}$ , a conserved  $S_{m\alpha}$  and a complex scalar  $x$ . This makes a total of 12 bosonic + 12 fermionic operators.

From the component expressions above one can work out the supersymmetry transformations of the FZ multiplet. They read

$$\delta x = \frac{2}{3} \epsilon S, \quad (5.6a)$$

$$\delta j_m = \epsilon \left( S_m - \frac{1}{3} \sigma_m \bar{S} \right) + \bar{\epsilon} \left( \bar{S}_m + \frac{1}{3} \bar{\sigma}_m S \right), \quad (5.6b)$$

$$\delta S_{m\alpha} = 2i (\sigma_{mn} \epsilon)_\alpha \partial^n x^* + (\sigma^n \bar{\epsilon})_\alpha \left( 2T_{mn} + i \partial_n j_m - i \eta_{mn} \partial_\rho j^\rho + \frac{1}{2} \varepsilon_{mn\rho\lambda} \partial^\rho j^\lambda \right), \quad (5.6c)$$

$$\delta T_{mn} = -i \epsilon \sigma_{\rho(m} \partial^\rho S_n) + i \bar{\epsilon} \bar{\sigma}_{\rho(m} \partial^\rho \bar{S}_n), \quad (5.6d)$$

where the indices between round brackets are symmetrized with the combinatorial factor. We also list, below, the supersymmetry transformation for the trace operators of the FZ multiplet and the divergence of the current

$$\delta S = \epsilon (2T + 3i \partial_m j^m) + 3i \sigma^m \bar{\epsilon} \partial_m x, \quad (5.7a)$$

$$\delta T = \frac{i}{2} \epsilon \sigma^m \partial_m S + \frac{i}{2} \bar{\epsilon} \bar{\sigma}^m \partial_m \bar{S}, \quad (5.7b)$$

$$\delta (\partial_m j^m) = -\frac{1}{3} \epsilon \sigma^m \partial_m \bar{S} + \frac{1}{3} \bar{\epsilon} \bar{\sigma}^m \partial_m S. \quad (5.7c)$$

Notice that these last three variations plus (5.6a) close the algebra on their own (indeed, they make up the chiral multiplet  $X$  defined in (5.5)).

From the component expression (5.5) one can also see that whenever the superfield  $X$  vanishes the current  $j_m$  becomes conserved and all trace operators vanish. In this case the theory is superconformal and  $j_m$  becomes the always present (and conserved) superconformal R-current.

For theories with an R-symmetry, being it the superconformal one or any other, there exists an alternative supermultiplet accommodating the stress-energy tensor and the supercurrent, the so-called  $\mathcal{R}$  multiplet [101]. This is defined in terms of a pair of superfields  $(\mathcal{R}_m, \chi_\alpha)$  satisfying

$$-2 \bar{D}^{\dot{\alpha}} \sigma^m_{\alpha\dot{\alpha}} \mathcal{R}_m = \chi_\alpha, \quad (5.8)$$

where  $\mathcal{R}_m$  is a real superfield,  $\mathcal{R}_m = \mathcal{R}_m^*$ , and  $\chi_\alpha$  a chiral superfield,  $\bar{D}_{\dot{\alpha}} \chi_\alpha = 0$  which satisfies the identity  $\bar{D}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} - D^\alpha \chi_\alpha = 0$ ; this implies, in turn, that  $\partial^m \mathcal{R}_m = 0$ . From

the latter relation it follows that the lowest component of  $\mathcal{R}_m$  is indeed a conserved (R-)current. The component expression of the superfields making up the  $\mathcal{R}$  multiplet reads

$$\mathcal{R}_m = j_m + \theta S_m + \bar{\theta} \bar{S}_m + \theta \sigma^n \bar{\theta} \left( 2T_{mn} + \frac{1}{2} \varepsilon_{mnr s} (\partial^r j^s + C^{rs}) \right) + \dots \quad (5.9)$$

and

$$\chi_\alpha = -2S_\alpha - \left( 4\delta_\alpha^\beta T + 2i (\sigma^r \bar{\sigma}^t)_\alpha^\beta C_{rt} \right) \theta_\beta + 2i \theta \theta \sigma_{\alpha\dot{\alpha}}^n \partial_n \bar{S}^{\dot{\alpha}} + \dots \quad (5.10)$$

where again  $\dots$  stand for the supersymmetric completion, while  $C_{mn}$  is a closed two-form. The number of on-shell degrees of freedom is 12 bosonic + 12 fermionic, as for the FZ multiplet. For completeness, we also list the supersymmetry transformations of the fields belonging to the  $\mathcal{R}$

$$\delta j_m = \epsilon S_m + \bar{\epsilon} \bar{S}_m, \quad (5.11a)$$

$$\delta S_m = \sigma^n \bar{\epsilon} \left( i \partial_n j_m + 2T_{mn} + \frac{1}{2} \varepsilon_{mnr s} (\partial^r j^s + C^{rs}) \right), \quad (5.11b)$$

$$\delta T_{mn} = -i \epsilon \sigma_{r(m} \partial^r S_n + i \bar{\epsilon} \bar{\sigma}_{r(m} \partial^r \bar{S}_n), \quad (5.11c)$$

$$\delta C_{mn} = \epsilon \sigma_{[m} \partial_n] \bar{S} - \bar{\epsilon} \bar{\sigma}_{[m} \partial_n] S. \quad (5.11d)$$

In a theory where both the FZ and the  $\mathcal{R}$  multiplets can be defined, they are related by a shift transformation [103] (which acts as an improvement on  $T_{mn}$  and  $S_{m\alpha}$ ) defined as

$$\mathcal{R}_m = \mathcal{J}_m + \frac{1}{4} \bar{\sigma}_m^{\dot{\alpha}\alpha} [D_\alpha, \bar{D}_{\dot{\alpha}}] U, \quad X = -\frac{1}{2} \bar{D}^2 U, \quad \chi_\alpha = \frac{3}{2} \bar{D}^2 D_\alpha U, \quad (5.12)$$

where  $U$  is a real superfield.

While in this paper we will not be concerned with theories where the FZ multiplet cannot be defined [103, 104], it can sometime be interesting, provided an R-symmetry is present, to consider the  $\mathcal{R}$  multiplet, instead. Such a situation typically occurs in phenomenological models [105]. For this reason, we will also discuss  $\mathcal{R}$  multiplet correlators.

### 5.1.1 Parametrization of two-point functions

Let us start focusing on two-point functions of operators belonging to the FZ multiplet. One can use Poincaré invariance and conservation laws to fix completely the tensor structure of such correlators, and be left with a set of (model dependent) form factors.

In euclidean momentum-space, the real correlators can be parametrized as follows

$$\langle T_{mn}(p) T_{rs}(-p) \rangle = -\frac{1}{8} \Pi_{mnr s} C_2(p^2) - \frac{1}{8} \frac{m^2}{p^2} (\Pi_{mn} \Pi_{rs} - \Pi_{r(m} \Pi_{n)s}) F_2(p^2) \quad (5.13a)$$

$$\begin{aligned} \langle S_{m\alpha}(p) \bar{S}_{n\dot{\beta}}(-p) \rangle &= -(Y_{mn})_{\alpha\dot{\beta}}, C_{\frac{3}{2}}(p^2) - \frac{i}{2} m^2 \varepsilon_{mnr s} p^r \sigma_{\alpha\dot{\beta}}^s F_{\frac{3}{2}}(p^2) + \\ &+ M^4 (\sigma_m \bar{\sigma}^r \sigma_n)_{\alpha\dot{\beta}} \frac{2p_r}{p^2} \end{aligned} \quad (5.13b)$$

$$\langle j_m(p) j_n(-p) \rangle = -\Pi_{mn} C_{1R}(p^2) - \frac{1}{3} m^2 \eta_{mn} F_1(p^2) \quad (5.13c)$$

$$\langle x(p) x^*(-p) \rangle = \frac{2}{3} m^2 F_0(p^2) \quad (5.13d)$$

$$\langle j_r(p) T_{mn}(-p) \rangle = i p_r \Pi_{mn} I_3(p^2) \quad (5.13e)$$

where  $\Pi_{mn} \equiv p^2 \eta_{mn} - p_m p_n$  is the transverse projector, and we have defined the traceless tensor

$$\Pi_{mnr s} = \Pi_{mn} \Pi_{rs} - 3 \Pi_{r(m} \Pi_{n)s}, \quad (5.14)$$

and its fermionic analog (by trace of the supercurrent operator we mean the contraction with  $\sigma^m$ )

$$(Y_{mn})_{\alpha\dot{\beta}} = p_r \sigma_{\alpha\dot{\beta}}^r \Pi_{mn} + \frac{i}{2} p^2 \varepsilon_{mnr s} p^r \sigma_{\alpha\dot{\beta}}^s. \quad (5.15)$$

In some terms a mass scale  $m$  appears, which, as we will show below, is related to the explicit breaking of conformal invariance. Finally, a  $1/p^2$  pole appears in the supercurrent correlator when supersymmetry is spontaneously broken at some scale  $M$ , defined by  $\langle T_{mn} \rangle = -M^4 \eta_{mn}$ , signaling the presence of a Goldstino mode. Indeed, whenever supersymmetry is spontaneously broken, we have the Ward identity

$$\langle (\partial^m S_{m\alpha})(p) \bar{S}_{n\dot{\beta}}(-p) \rangle = -\langle \delta_\alpha \bar{S}_{n\dot{\beta}} \rangle, \quad (5.16)$$

where<sup>3</sup>

$$\langle \delta_\alpha \bar{S}_{m\dot{\beta}} \rangle = \langle \delta_{\dot{\beta}} S_{m\alpha} \rangle = i \sigma^n_{\alpha\dot{\beta}} \langle 2 T_{mn} \rangle \neq 0, \quad (5.17)$$

By substituting the parametrization (5.13b) of the supercurrent two-point function in (5.16), one easily sees that the above term provides the  $1/p^2$  pole contribution.

When appropriate, we have separated the structure of correlators in terms of a traceless and a trace part. The former is given by the functions  $C_2$ ,  $C_{\frac{3}{2}}$  and  $C_{1R}$ . Note that  $C_2$  determines the central charge  $c$  at a conformal fixed point. The form factors  $F_2$ ,  $F_{\frac{3}{2}}$ ,  $F_1$

<sup>3</sup>The additional factor of  $i$  with respect to the transformations in (5.6) arises when the correlators are continued in Euclidean space.

and  $F_0$  contribute instead to the trace operator correlators

$$\langle T(p) T(-p) \rangle = -\frac{3}{4} m^2 p^2 F_2(p^2) \quad (5.18a)$$

$$\langle \bar{S}_{\dot{\alpha}}(p) S_{\alpha}(-p) \rangle = 3 \sigma_{\alpha\dot{\alpha}}^n p_n m^2 F_{\frac{3}{2}}(p^2) + 32M^4 \frac{\sigma_{\alpha\dot{\alpha}}^n p_n}{p^2} \quad (5.18b)$$

$$p^m p^n \langle j_m(p) j_n(-p) \rangle = -\frac{1}{3} m^2 p^2 F_1(p^2) \quad (5.18c)$$

$$\langle x(p) x^*(-p) \rangle = \frac{2}{3} m^2 F_0(p^2). \quad (5.18d)$$

Additional non-trivial two-point functions, given in terms of complex form factors, are

$$\langle S_{m\alpha}(p) S_{n\beta}(-p) \rangle = m \varepsilon_{\alpha\beta} \Pi_{mn} G_{\frac{3}{2}}(p^2) - 2i m \varepsilon_{mnrst} p^r \sigma_{\alpha\beta}^{st} p_t \tilde{G}_{\frac{3}{2}}(p^2) \quad (5.19a)$$

$$\langle x(p)^* j_m(-p) \rangle = m p_m H_1(p^2) \quad (5.19b)$$

$$\langle x(p)^* T_{mn}(-p) \rangle = \frac{1}{2} m \Pi_{mn} H_2(p^2). \quad (5.19c)$$

All in all, two-point functions can be parametrized in terms of eight real and four complex form factors.

### 5.1.2 Supersymmetric relations among form factors

On a supersymmetry preserving vacuum, the supersymmetry algebra imposes the following relations among form factors

$$C_2 = C_{\frac{3}{2}} = C_{1R} \equiv C_{\text{susy}}, \quad F_2 = F_{\frac{3}{2}} = F_1 = F_0 \equiv F_{\text{susy}}, \quad I_3 = 0, \quad (5.20a)$$

$$H_2 = H_1 = G_{\frac{3}{2}} = \tilde{G}_{\frac{3}{2}} \equiv G_{\text{susy}}. \quad (5.20b)$$

Hence, when supersymmetry is preserved, one is left with just one complex,  $G_{\text{susy}}$ , and two real,  $C_{\text{susy}}, F_{\text{susy}}$ , independent form factors.

One might like to require conformal invariance on top of supersymmetry. The net effect on the form factors can be obtained by observing that in such case  $T = 0$  as an operator and hence, by supersymmetry,  $X = 0$ . Let us notice that one could perform a shift [103] in the superfields  $(\mathcal{J}_m, X)$  which leaves the definition (5.2) invariant. Here, choosing  $X$  to be exactly equal to zero, we are fixing this ambiguity. From now on we will always work within this assumption, i.e.  $X = 0$  at superconformal fixed points. The vanishing of  $X$  implies that

$$F_{\text{sft}} = 0. \quad (5.21)$$

As already observed, the vanishing of  $X$  also implies that the non-conserved part of the two-point function of  $j_m$  is projected out. Current conservation forces any correlator

carrying a net charge under the R-symmetry to vanish (notice that  $R(X) = 2$  and  $R(S_m) = -1$ ). Hence also all complex form factors vanish in this case

$$G_{\text{sft}} = 0. \quad (5.22)$$

Thus, in the superconformal case, only one (real) form factor survives,  $C_{\text{sft}}$ . When conformal invariance is unbroken its functional dependence on  $p^2$  is completely fixed up to an overall constant. This also shows that at a superconformal fixed point the central charge  $c$  completely determines the two-point functions of the supercurrent and of the R-current, besides that of the stress-energy tensor

$$C_{\text{sft}} = \frac{c}{3\pi^2} \log\left(\frac{\Lambda^2}{p^2}\right). \quad (5.23)$$

Equations (5.21) and (5.22) give also an a posteriori justification for the presence of a mass scale in the parametrization of the traceful part of real correlators and of the complex ones. Indeed, if the theory does not contain any scale, any correlator involving the mass scale  $m$  should vanish.

The most generic situation is obtained in a supersymmetry breaking vacuum, where both  $M$  and  $m$  are necessarily different from zero and the form factors are not related to one another anymore, in general. Notice that since  $T = 0$  is an operator identity in a conformal theory, in order to break supersymmetry spontaneously and get a non-vanishing vacuum energy, conformal invariance must be explicitly broken. In other words, one can never have a situation in which  $m = 0$  and  $M \neq 0$ .

### 5.1.3 Two-point functions for the R multiplet

We now comment on the structure of two-point functions for the  $\mathcal{R}$  multiplet. Correlators not involving  $C_{mn}$  have the same structure of those of the FZ multiplet (though the form factors will generically differ by contact terms). One crucial difference, though, is that now  $j_m$  is a conserved current and therefore  $F_1 = I_3 = 0$ .

As for correlators involving  $C_{mn}$ , the only non-vanishing ones are

$$\langle C_{mn}(p) C_{rs}(-p) \rangle = 3(\eta_{mr}p_n p_s - \eta_{nr}p_m p_s + \eta_{ns}p_m p_r - \eta_{ms}p_n p_r) m^2 E_0(p^2) \quad (5.24a)$$

$$\langle C_{mn}(p) j_r(-p) \rangle = \frac{i}{2} (\eta_{mr}p_n - \eta_{nr}p_m) m^2 E_1(p^2), \quad (5.24b)$$

where  $E_0$  and  $E_1$  are real form factors, and numerical coefficients have been chosen for later convenience.

Taking into account the supersymmetry transformations of the fields belonging to the  $\mathcal{R}$  multiplet, one finds that in a supersymmetric vacuum the following relations between form factors should hold

$$C_2 = C_{\frac{3}{2}} = C_{1R} \equiv C_{\text{susy}} \quad , \quad F_2 = F_{\frac{3}{2}} = E_1 = E_0 \equiv F_{\text{susy}} . \quad (5.25)$$

So, in this case, one is left with two independent real form factors,  $C_{\text{susy}}$  and  $F_{\text{susy}}$ . Notice the difference with respect to the FZ multiplet, for which the R-current is not conserved and, in turn, there can be a non-vanishing complex form factor in a supersymmetric vacuum, see equation (5.20b). For ease of notation, in (5.25) we have used the same letters adopted for the FZ multiplet for correlators involving  $T_{mn}$ ,  $S_{m\alpha}$  or  $j_m$ , but the explicit form of the  $F_s$  and  $C_s$  is a priori different.

For a superconformal theory, the  $\mathcal{R}$  and FZ multiplets can be chosen to coincide by selecting the superconformal R-current as the bottom component of  $\mathcal{R}_m$ <sup>4</sup>. In this case, one finds that  $F_{\text{scft}} = 0$ , while  $C_{\text{scft}} \neq 0$ , as for the FZ multiplet, and one is consistently left with only one real form factor. However, in the context of R-symmetric RG flows, there is another natural choice for the lowest component of  $\mathcal{R}_m$  at the UV fixed point, that is to select the R-symmetry preserved along the flow (let us assume for simplicity that it is unique). In this case, at the UV and IR fixed points one gets

$$F_{\text{scft}} = \frac{1}{3} \frac{p^2}{m^2} \frac{1}{(2\pi)^2} \tau_U^{UV, IR} \log \left( \frac{\Lambda^2}{p^2} \right) . \quad (5.27)$$

The quantities  $\tau_U^{UV}$  and  $\tau_U^{IR}$  have been studied in [106], where they were conjectured to satisfy the inequality  $\tau_U^{UV} > \tau_U^{IR}$ .

#### 5.1.4 Perturbation of the fixed point and non-conformal form factors

In the general parametrization of correlators given in (5.13), it has been stressed that some of them are generated only when conformal symmetry is explicitly broken. Here we will show that non-conformal form factors are in fact determined by correlators of the operator which perturbs the fixed point and starts the RG flow. We will do this for the FZ multiplet, and briefly comment on the analogous relations for the  $\mathcal{R}$  multiplet. The Lagrangian is that of a SCFT, perturbed by a relevant operator. As shown in [107],

<sup>4</sup>The  $\mathcal{R}$  multiplet is uniquely defined by its bottom component which is a conserved R-current. However, in a generic  $\mathcal{N} = 1$  QFT there is no unique choice of  $U(1)_R$  symmetry. Indeed, consider for example a theory with a global symmetry group  $U(1)_R \times U(1)$  with associated conserved currents  $j_m^R$  and  $j_m$ . Then any linear combination

$$\tilde{j}_R^m = j_m^R + \frac{p}{q} j_m \quad \text{with} \quad p, q \in \mathbb{Z} \quad (5.26)$$

defines a new conserved R-current. Each of these defines then a different  $\mathcal{R}$  multiplet.



the only possible relevant deformation is given by a superpotential, namely by a chiral operator  $\mathcal{O}$  of dimension  $\Delta$  with  $1 \leq \Delta < 3$

$$\bar{D}_{\dot{\alpha}}\mathcal{O} = 0, \quad \mathcal{O} = \phi_{\mathcal{O}} + \sqrt{2}\theta\psi_{\mathcal{O}} + \theta^2 F_{\mathcal{O}} + \dots \quad (5.28)$$

$$\mathcal{L} = \mathcal{L}_{SCFT} + m^{3-\Delta}F_{\mathcal{O}} + c.c.. \quad (5.29)$$

We can parametrize the real two-point functions of  $\mathcal{O}$  in terms of the following real form factors

$$\langle \phi_{\mathcal{O}}^*(p)\phi_{\mathcal{O}}(-p) \rangle = m^{2\Delta-4} Z_{\phi} \quad (5.30a)$$

$$\langle \bar{\psi}_{\mathcal{O}\dot{\alpha}}(p)\psi_{\mathcal{O}\alpha}(-p) \rangle = m^{2\Delta-4} \sigma_{\alpha\dot{\alpha}}^m p_m Z_{\psi} \quad (5.30b)$$

$$\langle F_{\mathcal{O}}^*(p)F_{\mathcal{O}}(-p) \rangle = -m^{2\Delta-4} p^2 Z_F, \quad (5.30c)$$

and the following complex form factors

$$\langle \phi_{\mathcal{O}}(p)\phi_{\mathcal{O}}(-p) \rangle = m^{2\Delta-4} Y_{\phi} \quad (5.31a)$$

$$\langle \psi_{\mathcal{O}\alpha}(p)\psi_{\mathcal{O}\beta}(-p) \rangle = m^{2\Delta-3} \epsilon_{\alpha\beta} Y_{\psi} \quad (5.31b)$$

$$\langle F_{\mathcal{O}}(p)F_{\mathcal{O}}(-p) \rangle = m^{2\Delta-4} p^2 Y_F \quad (5.31c)$$

$$\langle \phi_{\mathcal{O}}(p)F_{\mathcal{O}}(-p) \rangle = m^{2\Delta-3} Y_{\phi F} \quad (5.31d)$$

$$\langle \phi_{\mathcal{O}}^*(p)F_{\mathcal{O}}(-p) \rangle = m^{2\Delta-3} \tilde{Y}_{\phi F}. \quad (5.31e)$$

In a vacuum which preserves supersymmetry, the following relations hold

$$Z_{\phi} = Z_{\psi} = Z_F, \quad Y_{\psi} = Y_{\phi F}, \quad Y_{\phi} = Y_F = \tilde{Y}_{\phi F} = 0. \quad (5.32)$$

The relation between the chiral superfield  $X$  of the FZ multiplet and the operator  $\mathcal{O}$  reads

$$X = \frac{4}{3}(3 - \Delta) m^{3-\Delta} \mathcal{O}, \quad (5.33)$$

which implies the following relations between the correlators (up to possible contact terms, because the relation is only valid on-shell)

$$\langle T(p)T(-p) \rangle = 2(3 - \Delta)^2 m^{6-2\Delta} (\text{Re}\langle F_{\mathcal{O}}(p)F_{\mathcal{O}}(-p) \rangle + \langle F_{\mathcal{O}}^*(p)F_{\mathcal{O}}(-p) \rangle) \quad (5.34a)$$

$$\langle \bar{S}_{\dot{\alpha}}(p)S_{\beta}(-p) \rangle = 8(3 - \Delta)^2 m^{6-2\Delta} \langle \bar{\psi}_{\mathcal{O}\dot{\alpha}}(p)\psi_{\mathcal{O}\beta}(-p) \rangle \quad (5.34b)$$

$$\langle \partial j(p)\partial j(-p) \rangle = \frac{8}{9}(3 - \Delta)^2 m^{6-2\Delta} (-\text{Re}\langle F_{\mathcal{O}}(p)F_{\mathcal{O}}(-p) \rangle + \langle F_{\mathcal{O}}^*(p)F_{\mathcal{O}}(-p) \rangle) \quad (5.34c)$$

$$\langle x^*(p)x(-p) \rangle = \frac{16}{9}(3 - \Delta)^2 m^{6-2\Delta} \langle \phi_{\mathcal{O}}^*(p)\phi_{\mathcal{O}}(-p) \rangle. \quad (5.34d)$$

Comparing with equations (5.18a)-(5.18d), one gets for the FZ form factors

$$F_2 = \frac{8}{3} (3 - \Delta)^2 (Z_F - \text{Re}Y_F) \quad (5.35a)$$

$$F_{\frac{3}{2}} + \frac{32}{3} \frac{M^4}{m^2 p^2} = \frac{8}{3} (3 - \Delta)^2 Z_\psi \quad (5.35b)$$

$$F_1 = \frac{8}{3} (3 - \Delta)^2 (Z_F + \text{Re}Y_F) \quad (5.35c)$$

$$F_0 = \frac{8}{3} (3 - \Delta)^2 Z_\phi. \quad (5.35d)$$

In equation (5.35b) the additional term displaying the expected massless pole associated to the Goldstino is present, see equation (5.13b).

Let us also mention the case of the  $\mathcal{R}$  multiplet. In this case, the operator giving the superpotential perturbation is related on-shell to a real superfield  $\mathcal{O}_R$

$$\mathcal{O} = \bar{D}^2 \mathcal{O}_R. \quad (5.36)$$

The relation with the operator  $\chi_\alpha$  that contains the trace is

$$\chi_\alpha = -4 (3 - \Delta) m^{3-\Delta} \bar{D}^2 D_\alpha \mathcal{O}_R \quad (5.37)$$

and the non-conformal form factors in this case can be expressed in terms of those of the operator  $\mathcal{O}_R$ .

## 5.2 Holography for the FZ multiplet

In this section we will discuss the holographic computation of two-point functions of operators in the FZ multiplet. Before going through the details of holographic renormalization, we want to spend some words about the field/operator map relevant to this supermultiplet.

### 5.2.1 Holographic dictionary

Let us first consider the case in which the boundary theory is superconformal. In this case the superfield  $X$  vanishes (more properly it is redundant for the description of the FZ multiplet) and the FZ multiplet is described by the superfield  $\mathcal{J}_m$  only. Since we already know that the stress-energy tensor is dual to the bulk metric, supersymmetry tell us that the superconformal FZ multiplet must be dual to the five-dimensional graviton multiplet. The supercurrent is dual to the gravitino and the (conserved) superconformal R-current to the (massless) graviphoton.

Let us now consider a non-conformal QFT. In such case the number of degrees of freedom in the FZ multiplet increases, since now  $T_{mn}$  and  $S_m$  have non-trivial trace parts and the superconformal current is not conserved anymore. These non-conformal degrees of freedom are gathered in the superfield  $X$  (5.5). On the dual side, a non-conformal field theory corresponds to a supergravity solution which breaks diffeomorphisms along the radial direction and half of the supersymmetries. Thus, some of the gauge degrees of freedom contained in the graviton multiplet becomes physical because of a supersymmetric Higgs mechanism. In particular the graviton eats one Goldstone boson associated to radial translations, the gravitino eats the goldstino associated to half of the local supersymmetries and the graviphoton eats the Goldstone boson associated to the local five-dimensional R-symmetry. These higgsed gravitational degrees of freedom have the right properties to be dual to the non-conformal degrees of freedom contained in  $X$  and fit into an  $\mathcal{N} = 2$  hypermultiplet with a massless scalar and a fermion with  $|m| = 3/2$ , see Table 5.1.

4D multiplet	$\Delta$	5D multiplet	$\mathbf{AdS}_5$ masses
$\mathcal{J}_m \supset \{j_m, S_m, T_{mn}\}$	$\{3, 7/2, 4\}$	$\{A_\mu^R, \psi_\mu, g_{\mu\nu}\}$	$\{0, 3/2, 0\}$
$X \supset \{x, S, T + i\partial j\}$	$\{3, 7/2, 4\}$	$\{\eta, \zeta, \phi\}$	$\{-3, 3/2, 0\}$

TABLE 5.1: The 4D  $\mathcal{N} = 1$  FZ multiplet is made up of two superfields, correspondingly the dual description requires two 5D  $\mathcal{N} = 2$  supermultiplets: a graviton multiplet plus a hypermultiplet.

### 5.2.2 Holographic computation of traceless form factors

As explained above the supergravity fields dual to the traceless modes of the FZ multiplet are contained in the graviton multiplet and carry information about the traceless form factors  $C_s$ . Whereas, the information about non-conformal form factors,  $F_s$ , is contained in the remaining trace modes of the gravity fields whose equations of motion are entangled with those of the hypermultiplet fields.

Here we will focus on the holographic computation of the traceless form factors  $C_s(p^2)$  in an  $\mathbf{AdS}_5$  background. As we will show in the next chapter, these are indeed the only new ingredients we need for computing the quantities we will be interested in.

We will then consider a quadratic action describing free fluctuations of the supergravity bulk multiplet  $\{h_{\mu\nu}, \psi_\mu, A_\mu\}$  over an  $\mathbf{AdS}_5$  background. The  $\mathcal{N} = 2$  supergravity action reads

$$\mathcal{S} = \frac{N^2}{4\pi^2} \int d^5x \sqrt{g} \left( -\frac{1}{2}R - 6 + \bar{\psi}_\mu (\gamma^{\mu\nu\rho} D_\nu - \frac{3}{2}\gamma^{\mu\rho}) \psi_\rho + \frac{1}{4}g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right), \quad (5.38)$$

The overall constant is fixed in terms of the  $\mathbf{AdS}_5 \times \mathbf{S}^5$  ten-dimensional solution,  $\frac{1}{8\pi G_5} = \frac{N^2}{4\pi^2}$  with  $L = \alpha' = 1$ . Conventions on indices and curvatures tensors are collected in Appendix A. Supersymmetry in  $\mathbf{AdS}_5$  implies the gravitino has mass  $|m| = \frac{3}{2}$ . The  $\mathbf{AdS}_5$  background metric is

$$ds^2 = \frac{1}{z^2} \eta_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2} (dz^2 + \eta_{mn} dx^m dx^n) \quad (5.39)$$

and the graviton field  $h_{\mu\nu}$  is defined as the fluctuation around  $\eta_{\mu\nu}$ . As usual we can exploit bulk gauge freedom and consider fluctuations in the axial gauge  $A_z = h_{\mu z} = \psi_z = 0$ . Inspection of the  $\mathbf{AdS}_5$  equations of motion reveals that the transverse-traceless components of the bulk fields decouple from the rest and satisfy homogeneous ordinary differential equations which after Fourier-transforming from  $x_m$  to  $p_m$  read<sup>5</sup>

$$(z^2 \partial_z^2 - 3z \partial_z - z^2 p^2) h_{mn}^{tt}(z, p) = 0 \quad (5.41a)$$

$$\begin{cases} (z^2 \partial_z^2 - 4z \partial_z - z^2 p^2 + \frac{9}{4}) \xi_m^{tt}(z, p) = 0 \\ z \sigma^n p_n \bar{\chi}_m^{tt}(z, p) = (-z \partial_z + \frac{1}{2}) \xi_m^{tt}(z, p) \end{cases} \quad (5.41b)$$

$$(z^2 \partial_z^2 - z \partial_z - z^2 p^2) A_m^t(z, p) = 0, \quad (5.41c)$$

where  $h_m^{ttm} = \partial^m h_{mn}^{tt} = 0$ ,  $\gamma^m \psi_m^{tt} = \partial^m \psi_m^{tt} = 0$  and  $\partial^m A_m^t = 0$ . The remaining components of the fields are pure gauge in  $\mathbf{AdS}_5$  and can be gauge fixed to zero. We can then focus on the  $tt$  part of the bulk fields and disregard the rest. For ease of notation we will omit the  $tt$  superscript in the rest of the discussion.

Solutions to the above differential equations behave near  $z = 0$  as

$$h_{mn}(z, p) \underset{z \rightarrow 0}{\simeq} h_{(0)mn} + z^2 h_{(2)mn} + z^4 \log(z) h_{(4)mn} + z^4 \tilde{h}_{(0)mn} + \mathcal{O}(z^6) \quad (5.42a)$$

$$\begin{cases} \xi_m(z, p) \underset{z \rightarrow 0}{\simeq} z^{1/2} \left( \xi_{(0)m} + z^2 \xi_{(2)m} + z^4 \log(z) \xi_{(4)m} + z^4 \tilde{\xi}_{(4)m} + \mathcal{O}(z^6) \right) \\ \bar{\chi}_m(z, p) \underset{z \rightarrow 0}{\simeq} z^{3/2} \left( \bar{\chi}_{(0)m} + z^2 \log(z) \bar{\chi}_{(2)m} + z^2 \bar{\chi}_{(0)m} + \mathcal{O}(z^4) \right) \end{cases} \quad (5.42b)$$

$$A_m \underset{z \rightarrow 0}{\simeq} a_{(0)m} + z^2 \log(z) a_{(2)m} + z^2 \tilde{a}_{(0)m} + \mathcal{O}(z^4), \quad (5.42c)$$

<sup>5</sup>Notice that we have traded the first order equation of motion for a Dirac field with a second order equation of motion for one of its Weyl components plus a first order constraint for the other Weyl component, choosing, as in Section 3.2,

$$\psi_m = \begin{pmatrix} \xi_m \\ \bar{\chi}_m \end{pmatrix}. \quad (5.40)$$

where all coefficients are functions of the four-dimensional momentum  $p$ . The coefficients of the near-boundary expansion satisfy the following relations

$$h_{(2)mn}(p) = -\frac{p^2}{4}h_{(0)mn}(p) , \quad h_{(4)mn}(p) = -\frac{p^4}{16}h_{(0)mn}(p) , \quad (5.43a)$$

$$\xi_{(2)m} = -\frac{p^2}{4}\xi_{(0)m} , \quad \xi_{(4)m} = -\frac{p^4}{16}\xi_{(0)m} , \quad \tilde{\xi}_{(4)m} = -\frac{1}{4}\sigma^n p_n \bar{\chi}_{(0)m} + \frac{p^4}{64}\xi_{(0)m} ,$$

$$\bar{\chi}_{(0)m} = -\frac{1}{2}\bar{\sigma}^n p_n \xi_{(0)m} , \quad \bar{\chi}_{(2)m} = -\frac{p^2}{4}\bar{\sigma}^n p_n \xi_{(0)m} , \quad (5.43b)$$

$$a_{(2)m}(p) = \frac{p^2}{2}a_{(0)m}(p). \quad (5.43c)$$

The leading terms  $\{h_{(0)}^{mn}(p), \xi_{(0)}^m(p), a_{(0)}^m(p)\}$  are identified as the sources of the corresponding boundary operators  $\{T_{mn}(p), S_m(p), j_m(p)\}$ . Note that the scaling behavior at the boundary, which depends on the mass of the fluctuating field in  $\mathbf{AdS}_5$ , is the correct one to get a multiplet of operators of dimension  $\{4, 7/2, 3\}$  respectively. Also, having chosen a positive sign for gravitino mass term, the leading coefficient at the boundary has positive chirality. The undetermined sub-leading terms  $\{\tilde{h}_{(0)mn}(p), \tilde{\chi}_{(0)m}(p), \tilde{a}_{(0)m}(p)\}$  are associated to the one-point functions of the boundary operators, and their functional dependence on the sources will be determined by imposing boundary conditions in the bulk on the full solution.

The on-shell boundary action at the regularizing surface  $z = \epsilon$  is

$$\mathcal{S}_{\text{reg}} = \frac{N^2}{4\pi^2} \int_{z=\epsilon} \frac{d^4p}{(2\pi)^4} \left[ \frac{1}{4z^3} h^{mn} h'_{mn} - \frac{3}{2z^4} h^{mn} h_{mn} + 6 + \frac{1}{2z^4} (\xi^m \chi_m + \bar{\chi}^m \bar{\xi}_m) + \frac{1}{2z} A^m \partial_z A_m \right]. \quad (5.44)$$

Here the four-dimensional space-time indices are raised and lowered using the flat metric  $\eta_{mn}$  and we have added all the boundary terms that are needed to have a well defined variational principle [108–110].

The above action can be made finite by adding appropriate covariant counterterms at the regularizing surface [15, 16, 111]

$$\mathcal{S}_{\text{ct}} = \frac{N^2}{4\pi^2} \int_{z=\epsilon} \frac{d^4p}{(2\pi)^4} \sqrt{\gamma} \left[ 6 - R[\gamma] + (\log(\epsilon\Lambda) + \alpha_2) \frac{R^{mn} R_{mn}[\gamma]}{4} - \frac{i}{2} \bar{\psi}^m \gamma^n p_n \psi_m + \frac{i}{4} (\log(\epsilon\Lambda) + \alpha_{\frac{3}{2}}) \bar{\psi}^m p^2 \gamma^n p_n \psi_m + \frac{1}{4} (\log(\epsilon\Lambda) + \alpha_1) F^{mn} F_{mn} \right], \quad (5.45)$$

where now space-time indices are raised and lowered with the metric induced at the regularizing surface (as required by four-dimensional covariance)  $\gamma_{mn} = \frac{1}{\epsilon^2} (\eta_{mn} + h_{mn})$ .

The action is understood up to quadratic order in the fields. Notice that, as usual, counterterms needed to cancel  $\log(\epsilon)$  divergences introduce ambiguities. In the above action these are shown by the finite contributions proportional to the arbitrary coefficients  $\alpha_s$ . A choice of such finite counterterms defines a particular subtraction scheme. The resulting renormalized action  $\mathcal{S}_{\text{ren}} = \mathcal{S}_{\text{reg}} + \mathcal{S}_{\text{ct}}$  can be expressed purely in terms of the leading and subleading modes of the fluctuations

$$\mathcal{S}_{\text{ren}} = \frac{N^2}{4\pi^2} \int \frac{d^4p}{(2\pi)^4} \left[ \frac{1}{2} h_{(0)}^{mn} \tilde{h}_{(0)mn} + \frac{1}{2} (\xi_0^m \tilde{\chi}_{(0)m} + \bar{\xi}_0^m \bar{\chi}_{(0)m}) + \frac{1}{2} a_{(0)}^m \tilde{a}_{(0)m} \right. \\ \left. + \text{terms quadratic in the sources} \right]. \quad (5.46)$$

The operators of the boundary theory are defined through the AdS/CFT correspondence as the composite operators sourced by the leading modes of each fluctuation. This can be schematically represented by the interaction action

$$\mathcal{S}_{\text{int}}[h_{(0)}^{mn}, \xi_{(0)}^m, \bar{\xi}_{(0)}^m, a_{(0)}^m] = \int \frac{d^4p}{(2\pi)^4} \left[ \frac{1}{2} h_{(0)}^{mn} T_{mn} + \frac{1}{2} \sqrt{\frac{3}{2}} (\xi_{(0)}^m S_m + \bar{\xi}_{(0)}^m \bar{S}_m) + \sqrt{\frac{3}{2}} a_{(0)}^m j_m \right]. \quad (5.47)$$

where the relative coefficients between the different terms, normalized as to match the  $c = a$  charges of  $\mathcal{N} = 4$  SYM, are fixed by supersymmetry.

The corresponding two-point functions are then obtained differentiating twice the renormalized action with respect to the sources

$$\langle T_{mn}(p) T_{rs}(-p) \rangle_0 = \frac{N^2}{4\pi^2} \left[ 2 \frac{\delta \tilde{h}_{(0)mn}}{\delta h_{(0)}^{rs}} + 2 \frac{\delta \tilde{h}_{(0)rs}}{\delta h_{(0)}^{mn}} + \frac{(9 - 4\alpha_2)}{32} p^4 (\eta_{mr} \eta_{ns} + \eta_{ms} \eta_{nr}) \right] \quad (5.48a)$$

$$\langle S_\alpha^m(p) \bar{S}_{\dot{\alpha}}^n(-p) \rangle_0 = \frac{2N^2}{3\pi^2} \left[ \frac{1}{2} \left( \frac{\delta \tilde{\chi}_{(0)\alpha}^m}{\delta \bar{\xi}_{(0)n}^{\dot{\alpha}}} + c.c. \right) + \frac{1 - \alpha_3}{4} p^2 p_r \sigma_{\alpha\dot{\alpha}}^r \eta^{mn} \right] \quad (5.48b)$$

$$\langle S_\alpha^m(p) S_\beta^n(-p) \rangle_0 = \frac{2N^2}{3\pi^2} \left[ \frac{1}{2} \frac{\delta \tilde{\chi}_{(0)\alpha}^m}{\delta \xi_{(0)n}^\beta} - (\alpha \leftrightarrow \beta) \right] \quad (5.48c)$$

$$\langle j_m(p) j_n(p) \rangle = \frac{N^2}{6\pi^2} \left[ \frac{\delta \tilde{a}_{(0)m}}{\delta a_{(0)}^n} + \frac{\delta \tilde{a}_{(0)n}}{\delta a_{(0)}^m} + \frac{(1 - 2\alpha_1)}{2} p^2 \eta_{mn} \right]. \quad (5.48d)$$

As a final remark, it is worth noticing that because of our gauge fixing we are computing only a piece of the tensor structure of each correlator. The latter, after explicit evaluation of functional derivatives, are then promoted in a unique way to the full transverse-traceless tensor structures. For example in (5.48a) one simply replaces

$$p^4 (\eta_{mr} \eta_{ns} + \eta_{ms} \eta_{nr}) \rightarrow -\frac{2}{3} \Pi_{mnr s}. \quad (5.49)$$

Of course the same results would have been obtained without fixing the transverse gauge and reconstructing the full tensor structure at the level of the action.

In the following chapter, the results will be presented in a particular subtraction scheme in which all finite contributions that deviate from the pure logarithmic behavior in the superconformal case are reabsorbed by finite counterterms. In particular we choose  $\alpha_1 = 3 - \alpha_2 = -\frac{1}{2} + \alpha_{\frac{3}{2}} = \alpha$  to define a one parameter family of supersymmetric scheme choices and then we set  $\alpha = -\ln 2 + \gamma$ .

The results of this section, in particular equations (5.48), can be used for computing two-point functions of the stress-energy tensor  $T_{mn}$ , supercurrent  $S_m$  and superconformal R-symmetry current  $j_m$  in the QFT dual to either pure  $\mathbf{AdS}_5$  or HW backgrounds.





## Chapter 6

# Supercurrent correlators in hard wall backgrounds

In this chapter we will compute correlators of the supercurrent multiplet using holography. As we have done in Section 4.6 for the case of a current multiplet, here we will stick to the simplest possible set-up, namely a field theory whose gravity dual is Anti de Sitter space-time possibly cut-off by a hard wall in the bulk. This is a bottom-up model, which is however flexible enough to let us reproduce different dynamical situations.

The background is described by an **AdS** metric which can be written as

$$ds^2 = \frac{1}{z^2} (dz^2 + \eta_{mn} dx^m dx^n) , \quad (6.1)$$

understood to be extending from the boundary at  $z = 0$  to a cut-off at  $z = 1/\mu$ , which geometrically is indeed a hard wall. The boundary  $z = 0$  corresponds to the deep UV of the quantum field theory, while the cut-off  $z = 1/\mu$  represents the smallest scale in the IR, here given by  $\mu$ . Locally, for all values of  $z$  larger than the IR cut-off, the whole (conformal) isometry group of **AdS** is unbroken. Pure **AdS**<sub>5</sub> is recovered for  $\mu \rightarrow 0$ .

A hard wall is a (very simplified) model for a theory which flows from a UV conformal fixed point to a gapped phase in the IR, with spontaneously broken conformal symmetry [112, 113]. On the contrary, one recovers a fully conformal field theory when  $\mu \rightarrow 0$  and **AdS** space-time is no longer cut-off. Indeed, by considering the fluctuations of the graviton, the gravitino and the graviphoton, and applying the standard AdS/CFT machinery, we will see that one gets the correlators of a SCFT in unbroken and broken phases, for  $\mu = 0$  and  $\mu \neq 0$ , respectively. In particular, in the latter case, we will show that  $1/p^2$  poles arise in the form factors, corresponding to massless dilaton, dilatino and R-axion.

In theories where conformal symmetry is explicitly broken,  $X \neq 0$ . In this case, the graviton multiplet does not have enough degrees of freedom to describe, holographically, the FZ multiplet (in particular, one cannot generate non-trivial  $F_s$  form factors), and at least one hypermultiplet, dual to  $X$ , must be added.<sup>1</sup>

This agrees with the fact that specific non-trivial profiles of scalar fields are needed in order to describe, holographically, non-conformal theories, the scalar being dual to the operator perturbing the fixed point. One should then consider the backreacted solution for the coupled system given by the scalar and the metric (and possibly their supersymmetric partners). This implies that the HW is a too simple background to describe field theories in which conformal invariance is explicitly broken and, eventually, theories with spontaneously broken supersymmetry. Here we will take an effective approach, which consists in working at the lowest order in the relevant perturbation of the fixed point. The basic idea is that we start with the conformal theory in the non-conformal vacuum parametrized by the scale  $\mu$  of the IR wall, and then treat a perturbation with relevant coupling  $m$ , in an expansion in  $m/\mu$ . By means of the on-shell operator relation (5.33) and equations (5.34), this will allow us to recover the non-conformal form factors  $F_s$  at lowest order in this expansion, simply by considering fluctuations of the hypermultiplet on the un-backreacted HW background. This same shortcut approach will enable us to describe, holographically, supersymmetry breaking models and get, in particular, the expected Goldstino pole in  $\langle \bar{S}_{\dot{\alpha}} S_{\alpha} \rangle$ .

In what follows, we will always set our computations in the framework of  $\mathcal{N} = 2$  gauged supergravity, and exploit the holographic dictionary to compute correlators at the complete supermultiplet level, as in Chapters 3 and 4. This is a necessary ingredient in order to deal with strongly coupled supersymmetric QFT systematically, and have control on their (supersymmetry breaking) dynamics.

## 6.1 Unbroken conformal symmetry

We start by the most symmetric case, which amounts to considering fluctuations of the graviton supermultiplet on a pure  $\mathbf{AdS}_5$  background. We recall that this multiplet, in pure  $\mathbf{AdS}$ , consists of a massless graviton, a massless graviphoton and a gravitino with mass  $m = \frac{3}{2}$ . This is consistent with the fact that pure  $\mathbf{AdS}_5$  is dual to a four-dimensional SCFT. In this case the supercurrent multiplet indeed consists of a traceless conserved stress-energy tensor, a traceless conserved supercurrent and a conserved R-current.

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<sup>1</sup>Completely analogous statements can be made for the  $\mathcal{R}$  multiplet, where the extra fields sit in a vector multiplet dual to the real superfield  $U$  (or in a tensor multiplet dual to  $\chi_{\alpha}$ , in theories where the FZ multiplet is not defined).

In this simple set-up, we can restrain to fluctuations that are completely gauge-fixed, as in the previous section,  $h_{\mu z} = A_z = \psi_z = 0$ . We can furthermore consider transverse and traceless  $h_{mn}$ , transverse and  $\gamma$ -traceless  $\psi_m$ , and transverse  $A_m$ .

In a near-boundary expansion, fluctuations have two independent modes, one leading and one sub-leading, that determine the whole solution. Regularity conditions in the deep interior of **AdS** or boundary conditions at the HW then fix the dependence of the subleading mode in terms of the leading one. Two-point correlators are precisely given by this dependence, up to some local contact terms that can be set to zero in a suitable subtraction scheme (see formulas (5.48) and discussion thereafter).

The equations of motion for traceless transverse modes in pure **AdS** are (5.41). The general solution in terms of modified Bessel functions reads

$$A_m(z, p) = z (\alpha_{1m}(p) I_1(zp) + \alpha_{2m}(p) K_1(zp)) , \quad (6.2a)$$

$$h_{mn}(z, p) = z^2 (c_{1mn}(p) I_2(zp) + c_{2mn}(p) K_2(zp)) , \quad (6.2b)$$

$$\psi_m(z, p) = z^{5/2} (\theta_{1m}(p) I_2(zp) + \theta_{2m}(p) K_2(zp)) . \quad (6.2c)$$

Regularity condition in the bulk fixes  $\alpha_{1m} = c_{1mn} = \theta_{1m} = 0$ . Comparing the  $z \rightarrow 0$  expansion of the solutions above to (5.42) we obtain

$$\alpha_{2m}(p) = p a_{(0)m}(p) \quad c_{2mn}(p) = \frac{p^2}{2} h_{(0)mn}(p) \quad \theta_{2m}(p) = \frac{p^2}{2} \xi_{(0)m}(p) . \quad (6.3)$$

The regular solution in the pure **AdS** case is then

$$A_m(z, p) = zp a_{(0)m}(p) K_1(zp) , \quad (6.4a)$$

$$h_{mn}(z, p) = \frac{(zp)^2}{2} h_{(0)mn}(p) K_2(zp) , \quad (6.4b)$$

$$\psi_m(z, p) = z^{1/2} \frac{(zp)^2}{2} \xi_{(0)m}(p) K_2(zp) , \quad (6.4c)$$

from which one can extract the dependence of the subleading modes from the leading ones and compute the form factors using (5.48). The result is

$$C_2(p^2) = C_{\frac{3}{2}}(p^2) = C_{1R}(p^2) = C_{\text{AdS}}(p^2) = \frac{N^2}{12\pi^2} \log \left( \frac{\Lambda^2}{p^2} \right) , \quad (6.5)$$

where we have introduced  $\Lambda$  as a UV regulator, and there can be additional constant pieces according to the subtraction scheme (see discussion in Section 5.2). All other form factors vanish. These results are the expected ones for a superconformal field theory. In particular, the value for  $C_2$  is the well-known result [2] of the holographic derivation of the central charge of  $\mathcal{N} = 4$  SYM, for which  $c = a = \frac{N^2}{4}$  in the large  $N$

limit. What we have explicitly shown here is that the same central charge is recovered from the R-current correlator and from the supercurrent correlator, consistently with supersymmetry and equation (5.20a).

## 6.2 Spontaneously broken conformal symmetry

In order to reproduce a situation where the field theory has a vacuum where conformal symmetry is spontaneously broken, we consider a HW background where  $\mathbf{AdS}_5$  space-time is cut-off at  $z = 1/\mu$  and the scale  $\mu$  will be identified with the scale of the VEV that breaks the conformal symmetry. The HW is modeling a theory where such spontaneous breaking leads to a discrete spectrum, typical of a confining theory.

Differently from pure  $\mathbf{AdS}$ , the geometry now ends abruptly at the wall  $z = 1/\mu$ , and we have to impose there generic boundary conditions for the field fluctuations

$$(h_{mn}(z, p) + \beta_2 z \partial_z h_{mn}(z, p))|_{z=1/\mu} = 0 \quad (6.6a)$$

$$(\psi^m(z, p) + \beta_{\frac{3}{2}} z \partial_z \psi^m(z, p))|_{z=1/\mu} = 0 \quad (6.6b)$$

$$(A_m(z, p) + \beta_1 z \partial_z A_m(z, p))|_{z=1/\mu} = 0. \quad (6.6c)$$

The boundary conditions being homogeneous, it is obvious that they introduce only IR data to the theory, and no dependence on UV information. In principle, different boundary conditions will parametrize different ways in which conformal symmetry is spontaneously broken. Interestingly, we will actually see that consistency and unitarity of the resulting field theory will force us with a unique choice of boundary conditions.

Through the holographic renormalization procedure, the resulting form factors are

$$C_2(p^2) = C_{\text{AdS}}(p^2) + \frac{N^2}{6\pi^2} \frac{\beta_2 x K_1(x) - K_2(x)}{\beta_2 x I_1(x) + I_2(x)} \quad (6.7a)$$

$$C_{\frac{3}{2}}(p^2) = C_{\text{AdS}}(p^2) + \frac{N^2}{6\pi^2} \frac{2\beta_{\frac{3}{2}} x K_1(x) - \left(2 + \beta_{\frac{3}{2}}\right) K_2(x)}{\left(2 + \beta_{\frac{3}{2}}\right) I_2(x) + 2\beta_{\frac{3}{2}} x I_1(x)} \quad (6.7b)$$

$$C_{1R}(p^2) = C_{\text{AdS}}(p^2) + \frac{N^2}{6\pi^2} \frac{K_1(x) - \beta_1 x K_0(x)}{I_1(x) + \beta_1 \frac{p}{m} I_0(x)}, \quad (6.7c)$$

where  $C_{\text{AdS}}$  is the result in the pure  $\mathbf{AdS}$  case (6.5) and  $x = \frac{p}{\mu}$ .

The trademark of the HW model is that correlation functions approach their superconformal limit exponentially fast, at large momentum. On the other hand, in the deep infrared the physics is determined by the choice of boundary conditions and, in particular, correlators can develop massless poles for specific choices of  $\beta_s$ . By expanding the

above expression for  $p^2/\mu^2 \ll 1$  we get

$$C_2(p^2) \underset{p^2 \rightarrow 0}{\simeq} \frac{N^2}{6\pi^2} \left( -\frac{16}{1+4\beta_2} \frac{\mu^4}{p^4} + \frac{16(1+6\beta_2(1+\beta_2))}{3(1+4\beta_2)^2} \frac{\mu^2}{p^2} + \dots \right) \quad (6.8a)$$

$$C_{\frac{3}{2}}(p^2) \underset{p^2 \rightarrow 0}{\simeq} \frac{N^2}{6\pi^2} \left( -\frac{16(2+\beta_{\frac{3}{2}})}{(2+9\beta_{\frac{3}{2}})} \frac{\mu^4}{p^4} + \frac{16(4+\beta_{\frac{3}{2}}(28+37\beta_{\frac{3}{2}}))}{3(2+9\beta_{\frac{3}{2}})^2} \frac{\mu^2}{p^2} + \dots \right) \quad (6.8b)$$

$$C_{1R}(p^2) \underset{p^2 \rightarrow 0}{\simeq} \frac{N^2}{6\pi^2} \left( \frac{2}{1+2\beta_1} \frac{\mu^2}{p^2} + \dots \right). \quad (6.8c)$$

All these expressions have poles for generic values of the boundary conditions. The appearance of double-poles in  $C_2$  and  $C_{\frac{3}{2}}$  is a sign of non-unitarity in the dual field theory. Such double poles can (and have to) be canceled by a specific choice of boundary conditions, i.e.  $\beta_2 \rightarrow \infty$  and  $\beta_{\frac{3}{2}} = -2$ . This choice leaves us with form factors with only single poles, and makes also  $C_2(p^2)$  equal to  $C_{\frac{3}{2}}(p^2)$ . We then see that the only HW configuration which gives a dual QFT with a unitary spectrum has massless modes in both the stress-energy tensor and the supercurrent correlator, with positive residue. This shows that this configuration is mimicking a flow in which conformal symmetry is broken spontaneously, as advertised.

Since the theory is superconformal in the UV, supersymmetry cannot be broken along the flow because having a non-zero vacuum energy would contradict the operator identity  $T = 0$ , which remains true when conformal invariance is spontaneously broken. The  $C_{1R}$  form factor (which does not display double poles and hence does not have any unitarity problem) is hence dictated by supersymmetry to be equal to  $C_2$  and  $C_{\frac{3}{2}}$ , and this fixes the last parameter,  $\beta_1 = 0$ . This choice of boundary condition for  $A_m$  might be interpreted as the only one which corresponds to the correct superconformal R-current in the IR.

In summary, in the spontaneously broken conformal symmetry case we have

$$C_{\text{susy}}(p^2) = C_{\text{AdS}}(p^2) + \frac{N^2}{6\pi^2} \frac{K_1(\frac{p}{m})}{I_1(\frac{p}{m})} \underset{p^2 \rightarrow 0}{\simeq} \frac{N^2}{6\pi^2} \frac{m^2}{p^2} + \dots \quad (6.9)$$

The massless pole in the above form factor signals the presence of a supermultiplet of massless particles in the dual field theory: these are the dilaton for broken conformal symmetry [113], its superpartner the dilatino, and the R-axion, associated to the spontaneous breaking of the superconformal R-symmetry. The presence of these strongly coupled composite massless states nicely mirrors the same states that one finds in weakly coupled models [21]. Note, however, the difference in the rest of the spectrum. In weakly coupled models one usually finds a massless state and a continuum, after (possibly) a gap, while in the present case it is easy to see, by continuing the Bessel functions to negative values of  $p^2$ , that the spectrum is composed exclusively of discrete states.

### 6.3 Explicitly broken conformal symmetry

We now discuss the holographic version of a model with explicitly broken conformal invariance but preserved supersymmetry. We expect  $C_s$  form factors without massless poles, and non-vanishing  $F_s$  form factors.

We will consider the perturbation which breaks conformal invariance as given by a certain chiral operator  $\mathcal{O}$  in the superpotential, dual to a hypermultiplet in the gravity theory. As anticipated, even if only a fully backreacted solution with a non-trivial profile for the hyperscalars can fully encode breaking of conformality, here we will take a shortcut. Our approximation consists in considering only the lowest order effects in the expansion parameter  $m/\mu$ , where  $m$  is the scale of the perturbation, dual to the leading mode of the hyperscalar at the boundary, and  $\mu$  is the scale of the IR wall. The operator  $T$  and its supersymmetric partners have an explicit overall dependence on the scale  $m$ , reflecting the fact that they vanish in the limit  $m \rightarrow 0$ . This is nothing but equation (5.33). From which it is clear that to lowest order in  $m/\mu$  the correlators of the trace operators are determined by those of  $\mathcal{O}$  evaluated at  $m = 0$ , i.e. in the conformal theory. This expansion corresponds, via holography, to an expansion in the profile of the hyperscalar dual to the coupling  $m$ . This argument then shows that the  $F_s$  form factors can be obtained, to leading order, by simply fluctuating the hyperscalar dual to  $\mathcal{O}$  in the background without any scalar profile, i.e. the HW. The derivation of the precise relation between the correlators of  $\mathcal{O}$  and the form factor  $F_s$  are given in Section 5.1.4 (the relations are derived there without reference to a small  $m$  expansion, and therefore are valid independently from this limit). Note that, on the other hand, our crude approximation cannot capture the effect of the perturbation on the traceless part of two-point correlators. The dilaton, dilatino and axino should get a mass proportional to the scale  $m$  of explicit breaking of conformal invariance, and correspondingly in the small  $p^2$  limit the  $C_s$  should take the gapped form  $\sim (p^2 + m^2)^{-1}$ . We expect this correction to be visible only working at higher order in the scalar profile. Already at the second order, however, the backreaction starts to be relevant, and therefore no calculation in the simple HW background can show this effect.

Let us focus, for simplicity, on an operator with  $\Delta = 2$ . The relation between  $X$  and  $\mathcal{O}$  is in this case

$$X = \frac{4}{3} m \mathcal{O}. \quad (6.10)$$

From equations (5.35), we can read the relation between the  $F_s$  form factors and the form factors of the operators in the chiral multiplet  $\mathcal{O}$

$$F_2 = F_1 = \frac{8}{3} Z_F \quad , \quad F_{\frac{3}{2}} = \frac{8}{3} Z_\psi \quad , \quad F_0 = \frac{8}{3} Z_\phi. \quad (6.11)$$

Implementing the holographic machinery we get

$$Z_F(p^2) = Z_{\text{AdS}}(p^2) + \frac{N^2}{4\pi^2} \frac{(1 + \beta_1)K_1(x) - \beta_1 x K_0(x)}{(1 + \beta_1)I_1(x) + \beta_1 x I_0(x)} \quad (6.12a)$$

$$Z_\psi(p^2) = Z_{\text{AdS}}(p^2) + \frac{N^2}{4\pi^2} \frac{(2 + 3\beta_{\frac{1}{2}})K_1(x) - 2\beta_{\frac{1}{2}} x K_0(x)}{(2 + 3\beta_{\frac{1}{2}})I_1(x) + 2\beta_{\frac{1}{2}} x I_0(x)} \quad (6.12b)$$

$$Z_\phi(p^2) = Z_{\text{AdS}}(p^2) + \frac{N^2}{4\pi^2} \frac{-(1 + 2\beta_0)K_0(x) + \beta_0 x K_1(x)}{(1 + 2\beta_0)I_0(x) + \beta_0 x I_1(x)}, \quad (6.12c)$$

where  $Z_{\text{AdS}}(p^2)$  is the usual conformal form factor containing the  $\log \Lambda^2/p^2$  term. Note that the non-trivial parts of the form factors are very similar to the ones computed in Section 4.6 for a current supermultiplet, the dimensions of the corresponding operators being the same. The parameters  $\beta_1$ ,  $\beta_{\frac{1}{2}}$  and  $\beta_0$  are defined similarly as in (6.6a)–(6.6c), for the bulk fields of a hypermultiplet dual to  $\mathcal{O}$ .

The only choice of parameters making all form factors equal and with no massless poles is  $\beta_0 = 0$ ,  $\beta_1 = -1$ ,  $\beta_{\frac{1}{2}} = -\frac{2}{3}$  which gives

$$Z(p^2) = Z_{\text{AdS}}(p^2) - \frac{N^2}{4\pi^2} \frac{K_0(\frac{p}{\mu})}{I_0(\frac{p}{\mu})} \underset{p^2 \rightarrow 0}{\simeq} \frac{N^2}{8\pi^2} \left( \log \frac{\Lambda^2}{\mu^2} - \frac{p^2}{2\mu^2} + O(p^4) \right). \quad (6.13)$$

Through equations (5.35), this implies that all  $F_s$  form factors are non-vanishing, equal to one another, as expected, and gapped

$$F_2(p^2) = F_{\frac{3}{2}}(p^2) = F_1(p^2) = F_0(p^2) = \frac{N^2}{3\pi^2} \left( \log \frac{\Lambda^2}{p^2} - 2 \frac{K_0(\frac{p}{\mu})}{I_0(\frac{p}{\mu})} \right). \quad (6.14)$$

## 6.4 Spontaneously broken supersymmetry

We now consider the case of spontaneously broken supersymmetry. We remind that for this to be possible, conformal symmetry has to be explicitly broken. In a supersymmetry breaking vacuum we expect a Goldstino and, specifically, a massless pole in the supercurrent correlator. Using the relations we found in Section 5.1.4 as in the previous section, in particular equation (5.34b), this corresponds to a massless pole in the fermionic correlator  $\langle \bar{\psi}_O(p) \psi_O(-p) \rangle$ . Indeed, for any choice of the parameter  $\beta_{\frac{1}{2}}$  but the one discussed in the previous section, such a pole develops at low momenta

$$Z_\psi(p^2) \underset{p^2 \rightarrow 0}{\simeq} \frac{N^2}{4\pi^2} \frac{1 + \frac{3}{2}\beta_{\frac{1}{2}}\mu^2}{\frac{1}{2} + \frac{7}{4}\beta_{\frac{1}{2}}\mu^2} p^2 + \dots \quad (6.15)$$

Using (5.34b) we thus get, e.g. for  $\beta_{\frac{1}{2}} = 0$

$$\langle \bar{S}_{\dot{\alpha}}(p) S_{\alpha}(-p) \rangle = \sigma_{\alpha\dot{\alpha}}^m p_m \frac{N^2}{\pi^2} \frac{4m^2\mu^2}{p^2} + \dots \quad (6.16)$$

This massless fermionic state, a composite state of the strongly coupled gauge theory, is the Goldstino of spontaneously broken supersymmetry. We have thus provided a holographic realization of the Goldstino as the dual of the lowest lying excitation of the fermionic operator in  $\mathcal{O}$ . Note that here again we used the approximation of small  $m/\mu$ , and therefore the Goldstino propagator is expressed by the fermionic correlator evaluated in the conformal limit  $m = 0$ . The scale of supersymmetry breaking  $M$  can be read from the residue of the massless pole to be

$$M = \sqrt{m\mu}. \quad (6.17)$$

This approximate formula nicely reflects that the effect responsible for the breaking of supersymmetry are the boundary conditions at the IR wall ( $M = 0$  when  $\mu = 0$ ) and also that conformal symmetry must be explicitly broken to have a non-supersymmetric vacuum ( $M = 0$  when  $m = 0$ ).

In order to go beyond the lowest order in  $m/\mu$  and find a massless pole in the supercurrent correlator directly, we would need a backreacted geometry with scalar profiles that break supersymmetry by subleading modes (corresponding to the VEV of some F-term in the field theory). The latter would also be the only approach that would give us a non-vanishing one-point function  $\langle T_{mn} \rangle$ .

As a final remark, let us notice that there is in fact a special choice of parameters which, while keeping the massless pole in the fermionic correlator, makes all form factors equal, namely  $\beta_0 = -\frac{1}{2}$ ,  $\beta_1 = 0$ ,  $\beta_{\frac{1}{2}} = 0$ . This corresponds to a common  $Z$  form factor

$$Z(p^2) = \frac{N^2}{8\pi^2} \left( \log \frac{\Lambda^2}{p^2} + 2 \frac{K_1\left(\frac{p}{\mu}\right)}{I_1\left(\frac{p}{\mu}\right)} \right) \underset{p^2 \rightarrow 0}{\simeq} \frac{N^2}{2\pi^2} \frac{\mu^2}{p^2}. \quad (6.18)$$

This gives  $1/p^2$  poles at low momenta for all real correlators of operators in the FZ multiplet. While such result might be interpreted as a supersymmetric vacuum with a massless chiral superfield in an otherwise gapped spectrum, the most natural interpretation is in fact that the apparent spectrum degeneracy is just an accident of the specific model. This is reminiscent of a Polonyi model which, while breaking supersymmetry, has a massless supersymmetric spectrum as the Goldstino is matched with a pseudomodulus and an R-axion.



# Chapter 7

## Conclusions

In this thesis we have applied AdS/CFT to the study of supersymmetry breaking dynamics in four-dimensional strongly coupled QFT's. Our strategy has been to use holographic techniques to compute two-point correlation functions of operators belonging to supermultiplets by means of weakly coupled gravitational backgrounds. We have focused on two particular supermultiplets.

In Chapter 3 we have considered the current supermultiplet

$$\mathcal{J} = \{J, j_\alpha, j_m\}, \tag{7.1}$$

which contains a conserved current  $j_m$  and its superpartners. This multiplet corresponds, via AdS/CFT, to an  $\mathcal{N} = 2$  five-dimensional vector multiplet. On the gravity side, we have thus led to consider the holographic renormalization for a vector multiplet coupled to gravitational backgrounds. We have then computed two-point functions for the current supermultiplet in a set of concrete supergravity solutions as well as in bottom-up Hard Wall (HW) models.

As we have argued in Chapter 4, this framework can be directly applied to analyze General Gauge Mediation models where the strongly coupled hidden sector is replaced by its holographic dual. We dubbed this approach Holographic General Gauge Mediation. The results we have found can be summarized as follows.

**top-down** We have worked in the context of five-dimensional consistent truncations of type IIB string theory and focused our attention on supersymmetry breaking asymptotically **AdS** solutions. We have found that when R-symmetry is unbroken, Standard Model gauginos generically acquire a Dirac mass by coupling to composite fermions, which manifest themselves as massless poles in the fermionic

correlator  $\langle j\bar{j} \rangle$ . Sfermions have masses derived from an integral which converges very nicely in the UV, and are dominated by the pole of the fermionic correlator, providing a spectrum which is reminiscent of gaugino mediation models. On the contrary, for R-symmetry breaking backgrounds the pole disappears, while the R-breaking correlator  $\langle jj \rangle$  acquires a non-zero value, hence providing Majorana mass to SSM gauginos.

**bottom-up** We have used HW backgrounds as a prototype to see whether and how holographic hidden sectors can actually cover the whole GGM parameter space. We have found that for a generic choice of boundary conditions at the IR wall, the resulting low energy spectrum is that of mediation scenarios with extra, non-SM, gauge sectors, where  $Z'$ -like gauge bosons acquire a mass due to symmetry breaking in the hidden sector, and mediate supersymmetry breaking effects to the SM. Tuning some parameters one can eliminate the composite massless modes emerging in the hidden sector recovering more standard gauge mediation scenarios, and in fact cover all of GGM parameter space.

In Chapter 5 we have considered other kind of supermultiplets, namely supercurrent multiplets. We have mainly focused on a particular realization of this supermultiplet which is the Ferrara-Zumino (FZ) one

$$(\mathcal{J}_m, X) = \{T_{mn}, S_{m\alpha}, j_m^R, x\}, \quad (7.2)$$

which contains the stress-energy tensor, the supercurrent, the superconformal R-current and an auxiliary operator  $x$ . The gravity dual of this multiplet is constituted by the gravity multiplet and a hypermultiplet. The latter contains information about the ‘trace’ operators and should thus be considered whenever the dual theory is not conformal. For the holographic computations we have focused on pure  $\mathbf{AdS}_5$  and HW backgrounds. While the former case represents vacua preserving superconformal symmetry, the latter describes vacua where conformal symmetry is spontaneously broken, and massless poles associated to the corresponding Goldstone modes appear.

In order to describe non-conformal theories holographically, one should consider less trivial backgrounds, in which additional hypermultiplets, dual to superpotential perturbations, have non-trivial profiles, and as such backreact on the metric. Still, we have shown that working at the leading order in the perturbation, one can get non-trivial traceful contributions to the correlators by evaluating hypermultiplet two-point functions in the unperturbed, purely HW, background. This is just the leading contribution to non-conformal form factors, of course, but the only one the HW can capture. Finally, by considering non-supersymmetric IR boundary conditions for the hypermultiplet, we

have also been able to realize a holographic toy-model of spontaneous supersymmetry breaking, and to show that the supercurrent correlator has the expected massless pole corresponding to the Goldstino.

## Future directions

In the context of HGGM models, one possible direction is to consider more sophisticated top-down models. In this way one could relax some of the simplifying assumptions we had to make, and also have a broader range of examples that may allow to draw more general conclusions on the predictions of these holographic models. Particularly interesting extensions could consist in adding D7 branes to the background, or considering cascading backgrounds instead of the more manageable **AAdS** ones.

Another direction would be to try to extend the holographic hidden sectors beyond the strict definition of Gauge Mediation models, so to allow also for direct couplings of the hidden sector with the Higgs sector of the SM. If the Higgs couples linearly to a certain composite operator, correlators of the latter will determine the form of the soft terms. In holographic hidden sectors, the operator is mapped to a five-dimensional multiplet with the same quantum numbers of the Higgs, and the usual holographic prescription can be used to extract the relevant form factors, in complete analogy to what we have done for ordinary GGM.

As for the holographic analysis of the FZ multiplet, the holographic model we have used in this work, despite the virtue of being flexible and easily calculable, is not obtained as a solution of the supergravity equations of motion. One obvious future direction would be to work at the level of a consistent  $\mathcal{N} = 2$  truncation of  $\mathcal{N} = 8$  gauged five-dimensional supergravity, and consider backreacted backgrounds, such as (non-supersymmetric deformations of) those discussed in [31, 32, 58, 114, 115]. In such models, one would be able to compute holographically  $C_s$  and  $F_s$  form factors for non-conformal theories, to all orders in the relevant perturbation. Our approach could also be useful to analyze supersymmetry breaking models in the context of string theory, and possibly consider backgrounds which are not **AAdS**, as for example the one discussed in [11–14]. Indeed, two-point correlators can be effectively used as a probe of the dynamics which breaks supersymmetry, for instance by discriminating an explicit breaking from a spontaneous one. To this aim, a discerning result would be to obtain, via holography, the massless pole associated to the Goldstino.



# Appendix A

## Notations and conventions

We use Greek letter from the middle of the alphabet ( $\mu, \nu, \dots$ ) for five-dimensional curved space-time indices, flat indices are instead denoted with Roman letter from the beginning of the alphabet ( $a, b, \dots$ ). Letters from the middle of Roman alphabet ( $m, n, \dots$ ) are reserved for space-time indices in four dimensions, where we do not make any distinction between curved and flat indices since we always work with flat metric there.

For Lorentzian space-times, both in five and four dimensions, we use mostly plus signature  $(-, +, +, +, +)$ . In both Lorentzian and Euclidean signature the five-dimensional metric is denoted with the letter  $g$  and space-time coordinates with  $x^\mu = (x^m, x^5)$  so, e.g.

$$ds_5^2 = g_{\mu\nu} dx^\mu dx^\nu . \quad (\text{A.1})$$

For  $\mathbf{AAdS}_5$  spaces, when an explicit notation for the four-dimensional boundary metric is needed we use the symbol  $\bar{g}$ . For example in Poincaré coordinates ( $x^5 = z$ ) we write

$$ds_{\mathbf{AAdS}_5}^2 = \frac{1}{z^2} (\bar{g}_{mn} dx^m dx^n + dz^2) , \quad (\text{A.2})$$

notice that in this cases the metric induced on the boundary  $\gamma_{mn} = g_{mn}$  does not coincide with the boundary metric  $\bar{g}_{mn} = z^2 g_{mn}$ .

### A.1 General Relativity

We use the following conventions for curvature tensors. The Riemann tensor is defined as

$$R^\lambda{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\lambda_{\sigma\nu} - \partial_\nu \Gamma^\lambda_{\sigma\mu} + \Gamma^\lambda_{\rho\mu} \Gamma^\rho_{\sigma\nu} - \Gamma^\lambda_{\rho\nu} \Gamma^\rho_{\sigma\mu} , \quad (\text{A.3})$$

the Ricci tensor is then defined contracting the first and third indices

$$R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}, \quad (\text{A.4})$$

and the scalar curvature is as usual

$$R = g^{\mu\nu} R_{\mu\nu}. \quad (\text{A.5})$$

The expression of the spin connection in term of the vielbein can be found using the compatibility equation

$$0 = \nabla_{[\mu} e_{\nu]}{}^a = \partial_{[\mu} e_{\nu]}{}^a + \omega_{[\mu}{}^{ab} e_{\nu]b}, \quad (\text{A.6})$$

form which one obtains

$$\omega_\mu{}^{ab} = 2 e^{\nu[a} \partial_{[\mu} e_{\nu]}{}^{b]} + e_\mu{}^c e^{\rho b} e^{\nu a} \partial_{[\rho} e_{\nu]c}, \quad (\text{A.7})$$

where indices between square brackets are antisymmetrized with the combinatorial factor, i.e.  $t_{[\mu\nu]} \equiv \frac{t_{\mu\nu} - t_{\nu\mu}}{2}$ .

With our conventions the Einstein-Hilbert action has the following sign

$$\mathcal{S}_{EH} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} (R - 2\Lambda) \quad (\text{A.8})$$

where the cosmological constant term  $\Lambda$  is positive for positively curved solutions (i.e. is negative for **AdS**<sub>5</sub>). The coupling constant is  $\kappa_5^2 = \frac{8\pi G}{c^4}$  where  $G$  is the five-dimensional Newton constant. When we couple matter to gravity, we always rescale matter fields so that a factor of  $\kappa_5^{-2}$  can always be collected in front of the integral. For a canonically normalized scalar minimally coupled to (A.8) we thus have

$$\mathcal{S} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} [(R - 2\Lambda) - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2], \quad (\text{A.9})$$

with this conventions scalar fields are dimensionless.

In some parts of this thesis we work in Euclidean signature, in our conventions the gravitational action in this case reads

$$\mathcal{S}_{\text{eucl}} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g} [-R + 2\Lambda + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2]. \quad (\text{A.10})$$

## Appendix B

# Conventions on spinors

Here we collect some useful information about our conventions on five-dimensional spinors.

### B.1 Five-dimensional spinors

We take the metric in mostly plus signature  $\eta_{ab} = \{-1, 1, 1, 1, 1\}$ , the gamma matrices  $\gamma_a$  satisfy the Clifford algebra

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab}. \quad (\text{B.1})$$

Whenever an explicit form is needed we use the following unitary (Weyl) representation

$$\gamma^a = \left\{ \left( \begin{array}{cc} 0 & i\sigma^m \\ i\bar{\sigma}^m & 0 \end{array} \right), \left( \begin{array}{cc} \mathbb{I}_2 & 0 \\ 0 & -\mathbb{I}_2 \end{array} \right) \right\}, \quad (\text{B.2})$$

where the last is identified with  $\gamma^5$ . The  $2 \times 2$  matrices  $\sigma^m$  are hermitian and defined as in [46]

$$\begin{aligned} \sigma^m &= \{-\mathbb{I}, \sigma_r\}, \quad r = 1, 2, 3 \\ \bar{\sigma}^m &= \{-\mathbb{I}, -\sigma_r\} \\ \varepsilon \sigma^m \varepsilon^t &= (\bar{\sigma}^m)^t = (\bar{\sigma}^m)^*, \end{aligned} \quad (\text{B.3})$$

with  $\varepsilon = i\sigma_2$ .

The following relations hold in our representation

$$\begin{aligned}(\gamma^a)^t &= \{\gamma^0, -\gamma^1, \gamma^2, -\gamma^3, \gamma^5\}, \\(\gamma^a)^* &= \{-\gamma^0, -\gamma^1, \gamma^2, -\gamma^3, \gamma^5\}, \\(\gamma^a)^\dagger &= \{-\gamma^0, \gamma^1, \gamma^2, \gamma^3, \gamma^5\},\end{aligned}\tag{B.4}$$

$$(\gamma^a)^\dagger \gamma^a = \mathbb{I}_4, \quad \forall a\tag{B.5}$$

$$\gamma^0 \gamma^1 \gamma^2 \gamma^3 \gamma^5 = -i.\tag{B.6}$$

The element on the LHS of eq. (B.6) is a Casimir of the Clifford group. Any other unitary representation with same value for that Casimir must be unitarily equivalent to ours. In particular we can define three unitary matrices<sup>1</sup> relating the representation  $\gamma$  to the equivalent ones  $-\gamma^\dagger$ ,  $-\gamma^*$ ,  $\gamma^t$

$$\begin{aligned}A\gamma^a A^\dagger &= -(\gamma^a)^\dagger \\B\gamma^a B^\dagger &= -(\gamma^a)^* \\C\gamma^a C^\dagger &= (\gamma^a)^t.\end{aligned}\tag{B.7}$$

Given a Dirac spinor  $\lambda$  we can define the following *conjugate* spinors

$$\begin{aligned}\bar{\lambda} &= \lambda^\dagger A \\ \lambda^M &= \lambda^t C \\ \lambda^c &= B^\dagger \lambda^*,\end{aligned}\tag{B.8}$$

respectively called the Dirac-, Majorana- and charge-conjugate of the spinor  $\lambda$ <sup>2</sup>.

For concreteness we choose the following realization in terms of gamma matrices

$$A = i\gamma^0, \quad C = \gamma^1\gamma^3, \quad B = \gamma^5\gamma^2,\tag{B.9}$$

where we have  $C^{-1} = C^\dagger = C^t = -C$  and  $B^\dagger = C A$ . Notice that a *Majorana condition*, which consists in the identification  $\lambda^M = \bar{\lambda}$ , is inconsistent with the above algebra since  $B^* B = -\mathbb{I}$ .

<sup>1</sup>Actually only two of them are independent since, up to a phase,  $B = C A$ .

<sup>2</sup>Notice that  $\psi$  and  $\psi_C$  transform in the same way under  $\text{Spin}(1,4)$ , as well as  $\bar{\psi}$  and  $\psi^M$ .



From a four-dimensional point of view a Dirac spinor  $\lambda$  can be decomposed into two Weyl spinors using  $\gamma^5$ <sup>3</sup>

$$\psi = \begin{pmatrix} \chi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix}. \quad (\text{B.10})$$

Dotted and undotted indices are raised and lowered with  $\varepsilon^{12} = \varepsilon_{21} = \varepsilon^{i\dot{2}} = \varepsilon_{\dot{2}i} = 1$  acting from the left

$$\chi_\alpha = \varepsilon_{\alpha\beta} \chi^\beta \quad \chi^\alpha = \varepsilon^{\alpha\beta} \chi_\beta. \quad (\text{B.11})$$

The complex conjugation exchange dotted and undotted indices and also the order of spinors. In particular we have

$$(\chi\xi)^* = (\chi^\alpha \xi_\alpha)^* = \bar{\xi}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} = \bar{\xi} \bar{\chi}. \quad (\text{B.12})$$

### B.1.1 Symplectic-Majorana spinors

As we have already pointed out, the Majorana condition cannot be implemented in five dimensions, however if the spinor itself transforms in a symplectic representation of some flavor group one can implement a reality condition of a different kind. This is known as *symplectic Majorana* condition. For a spinor carrying the fundamental rep of  $SU(2)$   $\psi^i$  we can impose the symplectic-Majorana condition requiring

$$\bar{\psi}^i = (\psi_i)^\dagger A = (\psi^i)^t C \quad (\text{B.13})$$

where  $SU(2)$  indices are raised and lowered with the invariant tensor  $\varepsilon_{12} = \varepsilon^{12} = 1$  following the NW-SE convention, namely

$$\psi^i = \varepsilon^{ij} \psi_j, \quad \psi_i = \psi^j \varepsilon_{ji}. \quad (\text{B.14})$$

Notice that with this convention (B.13) can be rewritten as

$$(\psi^i)^c = \psi^i \quad (\text{B.15})$$

where now the charge-conjugate spinor is  $(\psi^i)^c = B^\dagger (\psi_i)^* = C (\bar{\psi}^i)^t$ .

For convenience we write down all these spinors in Weyl notation:

$$\psi^1 = \begin{pmatrix} \chi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix} \quad (\text{B.16})$$

---

<sup>3</sup>This is actually an *illegal* operation from the  $SO(1,4)$  point of view, however in a holographic context one of the four spatial dimensions is singled out and we can make this correspond to the one associated with  $\gamma^5$ .

and the explicit form of the charge-conjugation matrix reads

$$C = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}. \quad (\text{B.17})$$

Then it follows that

$$\psi^2 = \begin{pmatrix} \xi_\alpha \\ -\bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\psi}^1 = (\chi^\alpha, -\bar{\xi}_{\dot{\alpha}}), \quad \bar{\psi}^2 = (\xi^\alpha, \bar{\chi}_{\dot{\alpha}}). \quad (\text{B.18})$$

### B.1.2 From symplectic-Majorana to Dirac

Spinor bilinear	Weyl components	Dirac
$\bar{\psi}\psi$	0	$\bar{\lambda}\lambda + \bar{\lambda}^c\lambda^c$
$\bar{\psi}\sigma_1\psi$	$\chi\chi - \xi\xi + c.c.$	$-\bar{\lambda}\lambda^c + c.c.$
$\bar{\psi}\sigma_2\psi$	$-i(\chi\chi + \xi\xi) + c.c.$	$-i\bar{\lambda}\lambda^c + c.c.$
$\bar{\psi}\sigma_3\psi$	$-2\xi\chi + c.c.$	$-2\bar{\lambda}\lambda$
$\bar{\psi}\gamma^\mu\psi$	0	$\bar{\lambda}\gamma^\mu\lambda + \bar{\lambda}^c\gamma^\mu\lambda^c$
$i\bar{\psi}\sigma_1\gamma^m\psi$	$-2\chi\sigma^m\bar{\xi} + c.c.$	$-i\bar{\lambda}\gamma^m\lambda^c + c.c.$
$i\bar{\psi}\sigma_2\gamma^m\psi$	$2i\chi\sigma^m\bar{\xi} + c.c.$	$\bar{\lambda}\gamma^m\lambda^c + c.c.$
$i\bar{\psi}\sigma_3\gamma^m\psi$	$2\chi\sigma^m\bar{\chi} + 2\xi\sigma^m\bar{\xi}$	$-2i\bar{\lambda}\gamma^m\lambda$
$i\bar{\psi}\sigma_1\gamma_5\psi$	$i(\chi\chi - \xi\xi) + c.c.$	$-i\bar{\lambda}\gamma^5\lambda^c + c.c.$
$i\bar{\psi}\sigma_2\gamma_5\psi$	$\chi\chi + \xi\xi + c.c.$	$\bar{\lambda}\gamma^5\lambda^c + c.c.$
$i\bar{\psi}\sigma_3\gamma_5\psi$	$-2i\chi\xi + c.c.$	$-2i\bar{\lambda}\gamma^5\lambda$

TABLE B.1: Symplectic-Majorana bilinears expressed in Weyl and Dirac notations. In the first column SU(2) indices are contracted and not shown, contractions always follow the NW-SE rule and Pauli matrices have the index structure  $\sigma_i^j$

It is always possible to write a pair of symplectic-Majorana spinors in term of a single Dirac spinor (and its charge conjugate), although this hides the SU(2) symplectic structure it is sometimes useful to use a more familiar notation. If we define the spinor  $\lambda = \psi^1$  then it follows that

$$\begin{aligned} \lambda &= \psi_2, & \lambda^c &= -\psi_1 = \psi^2, \\ \bar{\lambda} &= \bar{\psi}^2, & \bar{\lambda}^c &= -\bar{\psi}^1 \end{aligned} \quad (\text{B.19})$$

where the charge conjugate is  $\lambda^c = C(\bar{\lambda})^t$ . Using the above relations one can easily translate any expression from symplectic-Majorana to Dirac notation. As an example

consider the kinetic term  $i\bar{\psi}^i\gamma^\mu\partial_\mu\psi_i$ , expanding the contraction we get

$$i\bar{\psi}^1\gamma^\mu\partial_\mu\psi_1 + i\bar{\psi}^2\gamma^\mu\partial_\mu\psi_2 \quad (\text{B.20})$$

then using (B.19)

$$i\bar{\psi}^i\gamma^\mu\partial_\mu\psi_i = i\bar{\lambda}^c\gamma^\mu\partial_\mu\lambda^c + i\bar{\lambda}\gamma^\mu\partial_\mu\lambda = i\bar{\lambda}\gamma^\mu\partial_\mu\lambda + c.c. \quad (\text{B.21})$$

other expressions are collected in Table B.1



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