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Exploring Language Mechanisms: The Mass-Count Distinction and The Potts Neural Network

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Abstract

The aim of this thesis is to explore language mechanisms in two aspects. First, the statistical properties of syntax and semantics, and second, the neural mechanisms which could be of possible use in trying to understand how the brain learns those particular statistical properties. In the first part of the thesis (part A) we focus our attention on a detailed statistical study of the syntax and semantics of the mass-count distinction in nouns. We collected a database of how 1,434 nouns are used with respect to the mass-count distinction in six languages; additional informants characterised the semantics of the underlying concepts. Results indicate only weak correlations between semantics and syntactic usage. The classification rather than being bimodal, is a graded distribution and it is similar across languages, but syntactic classes do not map onto each other, nor do they reflect, beyond weak correlations, semantic attributes of the concepts. These findings are in line with the hypothesis that much of the mass/count syntax emerges from language- and even speaker-specific grammaticalisation. Further, in chapter 3 we test the ability of a simple neural network to learn the syntactic and semantic relations of nouns, in the hope that it may throw some light on the challenges in modelling the acquisition of the mass-count syntax. It is shown that even though a simple self-organising neural network is insufficient to learn a mapping implementing a syntactic-semantic link, it does however show that the network was able to extract the concept of 'count', and to some extent that of ‘mass’ as well, without any explicit definition, from both the syntactic and from the semantic data.

The second part of the thesis (part B) is dedicated to studying the properties of the Potts neural network. The Potts neural network with its adaptive dynamics represents a simplified model of cortical mechanisms. Among other cognitive phenomena, it intends to model language production by utilising the latching behaviour seen in the network. We expect that a model of language processing should robustly handle various syntactic- semantic correlations amongst the words of a language. With this aim, we test the effect on storage capacity of the Potts network when the memories stored in it share non trivial correlations. Increase in interference between stored memories due to correlations is studied along with modifications in learning rules to reduce the interference. We find that when strongly correlated memories are incorporated in the storage capacity definition, the network is able to regain its storage capacity for low sparsity. Strong correlations also affect the latching behaviour of the Potts network with the network unable to latch from one memory to another. However latching is shown to be restored by modifying the learning rule. Lastly, we look at another feature of the Potts neural network, the indication that it may exhibit spin-glass characteristics. The network is consistently shown to exhibit multiple stable degenerate energy states other than that of pure memories. This is tested for different degrees of correlations in patterns, low and high connectivity, and different levels of global and local noise. We state some of the implications that the spin-glass nature of the Potts neural network may have on language processing.

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In the loving memory of my father...

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1 General Overview

One of the epitomes of cognitive function is the ability to communicate through a *language*. The formal study of language has a long history with one of the earliest works dating back to 500 BCE when Panini studied Sanskrit. Humans are thought to be the only species that exhibit such a high level of complexity and structure in their communication, which far exceeds the few other species that show, to some extent, a developed communication ability like songbirds [van Heiningen et al 2009] and cetaceans like Killer Whales [Deecke et al 2009]. These species have a limited set of 'symbols' and rely mostly on repetition of sequences to communicate. The marked difference in the human languages is the possibility to combine symbols in different ways to convey different meanings, and possibly construct infinite set of unique sentences. It is still unclear why there is a sudden leap in human communication abilities despite sharing similar neural mechanisms to other species [Fisher, Marcus 2006].

Language processing can be investigated in several aspects, including A) The structure and rules of a language, which entails the study of syntax and semantics, and B) The encoding of those rules in the brain through neural mechanisms.

Several proposals have been made in the quest to explain language acquisition in humans. 'Generativism' was one of the earlier ideas in the 1980's and its initial formulation proposed that humans are born with biological constraints on their knowledge of linguistic principles and as the child grows, syntactic cues from the environment set certain features in the child's syntactic repertoire, thus bringing about complete language acquisition [Chomsky, 1980; Baker, 2002]. This view was challenged by the 'empiricism' idea which argues that in the light of lacking neuroscientific evidence to find a specific language acquisition device as proposed in generitivism, language is rather an emergent phenomenon developed through language use [Tomasello 2003; O'Grady 2008]. In relation to this, statistical models of language learning emerged, which suggest that a child can extract rules and structure from the statistics of the inputs it receives and thus is able to acquire the required knowledge to use the language [Saffran 2003; Lany, Saffran 2010].

Statistical models of language learning extend to connectionist models, which try to explore the mechanisms by which language can be encoded in the connections between neurons and make use of the general cognitive principles of learning. Several attempts have been made at modelling the brain mechanisms that would subserve language processing [SRN-Elman 1991; LISA-Hummel, Holyack 1997; Neural Blackboard Architecture-Velde, de Kamps 2006]. All these models provide a conceptual basis and requirements (like learning with distributed representations, self-organisation and the combinatorial property of syntax) for a neural architecture to support language processing but fall short in satisfying a realistic upward scaling of a natural language (eg. LISA) or require specifically pre-organised structure (eg. Neural Blackboard Architecture). An important aspect however is the ability of a neural network to learn from the statistics of a natural language. An attempt at artificially simulating statistical relations between words and syntactic categories was presented in BLISS [Pirmoradian, Treves 2011] and a neural network modelling cortical dynamics was tested in its ability to 'acquire' the statistical relationships. All such models show promise to some extent, however also highlight the enormous challenge and difficulty in approaching anywhere near the full requirements of a natural language.

In this thesis we focus our attention (in part A) on the statistical properties of 6 natural languages in the domain of the mass-count distinction in nouns. The mass-count distinction as explained in chapter 2 has been subject to intense debate for several decades and is particularly interesting to us, due to its perceived intuitive relation between syntax and semantics, which is also linked to the cognitive perception of nouns. We make a detailed cross linguistic study on the information obtained from the native speakers of the 6 languages and probe the statistical relationship between syntax and semantics of the mass-count nouns. Further, in chapter 3 we test the ability of a simple neural network to learn the syntactic and semantic relations of nouns, in the hope that it may throw some light on the challenges in modelling the acquisition of the mass-count syntax.

In part B we study properties of the Potts neural network, regarding its storage capacity and the spin glass phase. The Potts neural network is a simplified model of cortical dynamics and its dynamical behaviour exhibits some interesting features like latching between attractor states [Kropff, Treves 2006; Russo, Treves 2012]. The model was studied in its ability to produce sentences from BLISS [Pirmoradian, Treves 2012], however the correlations between words in a sentence was kept low to study the basic behaviour of the network. In chapter 4-section II we look at why correlations are important and a necessary requirement for a language processing model and then study the effects of increased correlations amongst stored memories in the Potts network. Lastly in section III of chapter 4 we look at an interesting observation, namely the spin glass phase of the Potts neural network. We describe what a spin glass phase is and look at the indications that the Potts neural network is operating in the spin glass phase. The possible implications of which are discussed in the conclusions.

Part A

Statistical study of natural languages: The Mass-Count Distinction

Chapter 2

A Statistical Investigation into the Cross-Linguistic Distribution of Mass and Count Nouns: Morphosyntactic and Semantic Perspectives

2.1 Introduction:

The mass/count distinction between nouns, in various languages, has been discussed in the linguistic literature since [Jespersen 1924], and has received considerable attention in particular in the last 35 years [see the bibliography in Bale & Barner 2011]. This distinction between mass and count nouns is a grammatical difference, which is reflected in the syntactic usage of the nouns in a natural language, if it makes the distinction at all (as has been often noted, not all language do; in the Chinese language family, for example, all nouns are mass). For example, in English, mass nouns are associated with quantifiers like little and much and require a measure classifier (kilos, boxes) when used with numerals; on the other hand, count nouns are associated with determiners like a(n), quantifiers like many/few or each, and can be used with numerals without a measure classifier.

These syntactic properties are intuitively correlated with semantic properties. Typical count nouns denote sets of individual entities, as in girl, horse, pen, while typical mass nouns denote ‘substances’ or ‘stuff’, for example, mud, sand, and water. It has often been noted that the correlation is not absolute, and that there are mass nouns which intuitively denote sets of individuals (e.g., furniture, cutlery, footwear). Nonetheless, the correlation seems non-arbitrary and there has been much discussion of this correlation in the linguistics literature as well as in the psycholinguistics literature [e.g., Soja et al. 1991, Prasada et al. 2002, Barner & Snedeker 2005, Bale & Barner 2009] and in the philosophical literature [e.g., Pelletier 2011 and references cited therein].

Within the semantics literature, a seminal attempt to ground the syntactic distinction semantically is [Link, 1983]. Link proposed that mass nouns are associated with homogeneity and cumulativity, while count nouns are associated with atomicity. Homogeneity, cumulativity, and atomicity are properties which can be associated with matter or with predicates. An object is atomic when it has a distinguishable smallest element which cannot be further divided without compromising the very nature of the object, and an atomic predicate denotes a set of atomic elements. Thus boy is an atomic predicate, since we can easily identify atomic boys, parts of which do not count as boys. Homogeneity is a property by which, when parts of an object are separated, each individual part holds the entire identity of the original object, and a homogeneous predicate is one which denotes entities (or quantities of matter) of this kind. For example, any part of something which is water is water, thus water is a homogeneous predicate. Cumulativity is the property that a predicate has if two distinct entities in its denotation can be combined together to make a single entity in the denotation of the same predicate. For example, if A is water and B is water, then A and B together are water. Cumulativity and homogeneity can be seen as different perspectives on the same phenomenon, though linguistic research has shown that the difference between them is important in certain contexts [see e.g., Landman & Rothstein 2012]. However, for our purposes, we can ignore these differences. The generalization emerging from [Link 1983] is that mass nouns are non-atomic and exhibit properties of being homogeneous and cumulative, whereas count nouns are atomic.

Link’s proposal has been hugely influential, giving a representation to the intuition that the syntactic expression of the mass/count distinction correlates with a real semantic or ontological contrast. Expressions of this intuition are widespread. Thus [Koptjevskaya-Tamm 2004] writes about the mass/count distinction: “In semantics, the difference is between denoting (or referring to) discrete entities with a well-defined shape and precise limits vs. homogeneous undifferentiated stuff without any certain shape or precise limits”.

Despite this ingrained intuition, it has been generally recognized that it is not possible to postulate a simple projection of the homogeneous/atomic or undifferentiated/discrete distinction onto mass/count syntax [seem e.g., some recent references such as Gillon 1992, Chierchia 1998, 2010, Barner & Snedeker 2005, Nicolas 2010, Rothstein 2010, Landman 2010, as well as Koptjevskaya-Tamm 2004]. There are various pieces of evidence which show this. In the first place, there are mass nouns which denote sets of atomic entities, such as furniture and kitchenware, and some of these have synonyms in the count domain as in the English pairs change/coin(s), footwear/shoe(s), carpeting/carpet(s) which denote roughly the same entities. Conversely, there are also count nouns such as fence and wall which show properties of homogeneity [Rothstein 2010]. Secondly, nouns stems may have both a count and mass realization in a single language, with the choice depending on context. In some cases, both count and mass usage are equally acceptable, as with stone and brick and hair in English. In other cases, one of the uses is considered non-normative, for example, when a count noun like dog is used as a mass noun in After the accident there was dog all over the road. Thirdly, items which are comparable in terms of lexical content do not have stable expressions cross-linguistically as either mass or count. The much cited examples is furniture, which is mass in English but count in French (meuble/s), while in Dutch and Hebrew, the comparable lexical item has both a mass and a count realization (Hebrew: count rehit/im vs. mass rihut, Dutch: count meuble/s vs. mass meubiliar).

The received wisdom therefore oscillates between these two perspectives, with much recent research trying to mediate between them, both capturing the basic generalization, while accounting for the variations both cross-linguistically and within a single language. [Chierchia 2010] suggests that the mass/count distinction is based on whether or not the noun is envisaged to have a set of stable atoms. [Rothstein 2010] argues that semantic atomicity is context dependent. [Pires de Oliveira & Rothstein 2011] argue that the mass/count alternation is a reflection of whether the noun relates to its denotata as a set of entities to be counted or as a set of quantities to be measured.

However, in the midst of all this discussion, certain basic facts remain unclear. In particular, how great is the cross-linguistic variation in mass/count syntax? Clear evidence that the syntactic mass/count distinction is not a projection of a semantic or ontological distinction has stayed at the level of the anecdotal, with discussion focusing on a few well known and well-worn examples [see, e.g., Chierchia 1998 and Pelletier 2010 for reviews]. As a consequence, most discussions of the basis of the mass/count distinction have been based on some explicit and some tacit assumptions, which have not been verified empirically. In particular, it is often assumed that the mass/count distinction is essentially binary, that is, that a noun is classified as mass or as count or as ambiguous. (This is explicit in accounts which assume that nouns are labeled as mass or count in the lexicon, and implicit in accounts such as [Borer 2005] which assume that noun roots are not classified lexically but naturally appear in either a count or a mass syntactic context.) Another, related, common assumption is that in a language with a mass/count distinction, most nouns are either mass or count, with the syntax reflecting the homogeneous/atomic distinction, and that cross-linguistic variation occurs in a lexically defined ‘gray area’ in the middle, which includes nouns which are not easily classifiable. But crucially, discussion of the facts of the matter has not gone far beyond the anecdotal. The semantics literature has discussed in great depth the syntactic properties of nouns like furniture and comparing it syntactically and semantically with its cross-linguistic counterparts, but despite very few more in-depth, but still narrow, studies [e.g., Wierzbicka 1988], we have little sense of how representative nouns like this actually are.

An answer to the question to what degree there is cross-linguistic variation in the expression of the mass/count distinction is essential to the discussion of its cognitive and semantic basis. If there is ultimately little cross-linguistic variation, then we are entitled to hypothesize that there may be some general strong correlation between properties of the denotata (e.g., as atomicity and homogeneity) and the grammatical distinction. In this case, the grammatical mass/count distinction may have a sound cognitive/perceptual foundation, and its semantic interpretation would reflect this. The task of linguistics would then be to characterise precisely the semantic basis of the grammatical distinction, to identifying ‘exceptional’ areas where the correlation does not hold and/or where cross-linguistic variation naturally appears, and to try and explain why these occur. This is an approach which has been exploited especially with respect to ‘furniture nouns’ which has been identified as ‘super-ordinates’ [Markman 1985] or functional artifacts [Grimm & Levin 2011]. On the other hand, if cross-linguistic variation is wide, then the basis for assuming that there is a correlation between cognitive/perceptual features and the grammatical distinction is considerably weakened. Then questions that linguistics should be asking will depend directly on the nature of the patterns, or lack of them, that an analysis of the cross-linguistic facts of the matter reveals. The lack of any quantitive data on the extent of cross-linguistic variation is thus highly problematic.

With the goal of remedying this lack of data and contributing to understanding the cognitive aspects of mass/count syntax and the relation between grammatical, semantic, and cognitive differentiation in this domain, we have conducted a statistical cross-linguistic empirical study based on a quantitative approach, and also a corpus study on the Browns section of the CHILDES database [MacWhinney 1995]. We hope with this to be able to begin to answer several basic questions: To what extent is the mass/count distinction a straight-forward reflection of the semantic properties of nouns? Is the variability across languages in any degree predictable, or is the grammatical division into mass and count arbitrary? Furthermore, is the division into mass and count absolute, or are some nouns 'more count' or 'more mass' than others? Do differences in the semantic explanations essentially arise due to the multi-dimensional nature of the semantic (as well as the syntactic) space? And if so, can the multi-dimensional aspect provide useful insights in the acquisition of mass/count syntax in humans?

Our study aims to go some way to providing empirically substantiated answers to these questions. We carried out a relatively large scale analysis of the mass/count classification of nouns cross linguistically. Count nouns are usually distinguished from mass nouns by a number of different syntactic properties, for example, co-occurrence with numerical expressions, co-occurrence with distributive quantifiers like each, and so on, but the specific tests vary from language to language. We focused on several issues:

(i) To what extent can mass/count syntax be predicted in language A on the basis of knowledge of language B?

(ii) To what extent is mass/count syntax a binary division (i.e. if a noun classifies as count on one test, what are the odds that it will classify as count on all tests)?

(iii) To what extent can mass/count syntax be predicted on the basis of real-world semantic properties?

2.2. Methods:

2.2.1. Data Collection:

A) Noun List:

Binary syntactic usage tables were compiled for a list of 1,434 common nouns in English, which included 650 abstract and 784 concrete nouns. The list was derived from a longer list of 1,500 very frequent English nouns, originally extracted from the CELEX database [see http://www.ldc.upenn.edu/Catalog/ CatalogEntry.jsp?catalogId=LDC96L14] for a different project, integrated with about 150 additional nouns often used in linguistics to study the mass/count domain, after translating the nouns into the five other languages included in our study, and eliminating over 200 nouns for which either the identification of the common semantic concept, or the syntactic classification in at least one language, as described below, were unclear or problematic. At the translation stage, each noun/concept was provided with a sample usage sentence, to disambiguate its potentially divergent meanings; thus trying to ensure that each language had the same semantic concept translated, for the same context, into a corresponding noun.

B) Usage Tables:

A set of yes/no questions was then prepared, in each language, to probe the usage of the nouns in the mass/count domain. The questions asked whether a noun from the list could be associated with a particular morphological or syntactic marker relevant in distinguishing mass/count properties. Some questions were designed to give positive properties of count nouns (e.g., can N be directly modified by a numeral?) and some to give positive properties of mass nouns (e.g., can the noun appear in the singular with measure expressions?). Since the mass/count distinction is marked by different syntactic properties cross-linguistically, the questions were dependent on the particular morphosyntactic expressions of mass/count contrast in each language. For example, in English we asked whether a noun could appear with the indefinite determiner a(n) but this was obviously an inappropriate question to ask in Hebrew where there is a null indefinite determiner. The questions in English are shown in Table 1 below.

The questions were answered by native speakers of each of the languages in our study. Thus each noun was associated, for each informant, with a string of binary digits, 1 indicating yes and 0 indicating no, reporting how that particular noun is used (or predominantly used) in the mass/count domain, by that informant. Such usage tables (a tiny portion of an English usage table is shown as Table 2.3 below) were compiled by Armenian, English, Hebrew, Hindi, Italian, and Marathi informants (at present, we have complete data for 16 informants; Armenian: AN, AR, GR, GY, RF; Italian: LE, FR, GS, RS, BG; Marathi: SN, TJ, SK; English: PN; Hebrew: HB; Hindi: MN). Although the choice of languages was ultimately determined by the available informants, the languages studied represent a spread across language families. The five Indo-European languages come from distinct branches: Germanic (English), Romance (Italian), Northern Indo-Aryan (Hindi), southern Indo-Aryan (Marathi), and Armenian, which constitutes a branch of its own. Hebrew comes from a distinct phylum, the Semitic family.

No.	Syntactic Questions
1.	Can the noun be used in bare form?
2.	Can the noun be used with a/an?
3.	Can the noun be pluralized (in a morphological distinct form)?
4.	Can it be used with numerals?
5.	Can the noun be used with every/each?
6.	Can the noun be used with many/few?
7.	Can the noun be used with much/little?
8.	Can the noun be used with not much?
9.	Can the noun be used with a lot of?
10.	Can the noun be used with a numeral modifier + plural on kind?
11.	Does the noun appear in the singular with a classifier or measure phrase?

Table 2.1: List of questions used in English to compile the usage table.

The questions probe whether a particular noun is associated with certain typical syntactic markers, important in English for the mass/count distinction. Similar questions were used for other languages, formulated according to the morphosyntactic properties of the languages in question. These are listed in tables A1–A5 in the Appendix.

C) Semantic Table:

A similar table was prepared by five informants (KM, RI, SL, SU, and TJ, four native Marathi and one Hindi speaker) using the English database to describe the properties of the denotations of the nouns in the list. These questions probed aspects of the denotations which were plausibly related to the more general semantic properties of atomicity, homogeneity and cumulativity discussed above. The questions asked (also supplied with an example to each, to clarify the meaning) are shown in Table 2.2. The questions were purposely formulated in informal terms, since we were interested in the correlation between mass/count syntax and what is often taken as the ‘intuitively obvious’ basis for the distinction. We will somewhat loosely refer to these as ‘semantic questions’.

No.	Semantic Questions
1.	Is it Alive irrespective of context?
2.	It is an Abstract Noun?
3.	Does it have a single Unit to represent itself ?
4.	Does it have a definite Boundary, visually or temporally?
5.	Does it have a stable Stationary shape (only if concrete)?
6.	Can it Flow freely (only if concrete)?
7.	Does it take the shape of a Container (only if concrete)?
8.	Can it be Mixed together indistinguishably (only if concrete)?
9.	Is the identity Degraded when a single unit is Divided (only if concrete)?
1(	Can it have an easily defined Temporal Unit (only if abstract)?
1:	Is it an Emotion /Mental process (only if abstract)?
1:	Can it have an easily defined Conceptual Unit (only if abstract)?

Table 2.2: Questions used to probe the semantic properties of the nouns.

The questions are based on the properties of atomicity, homogeneity and cumulativity, if nouns are concrete. For abstract nouns, the semantics is based on how easy it is to define a unit of the concept. The questions were asked without elaboration, with only a reference example; in the case of question 8, for example, applicable to concrete nouns: Can it be mixed together indistinguishably? [e.g., butter as opposed to man].

Both syntactic and semantic tables were then processed through the analysis described below.

2.2.2. Analysis:

Nouns in the syntactic usage table of a particular informant were clustered together according to the binary string associated with them. In this way, nouns which have the exact same binary string are grouped together, reflecting the fact that their mass/count syntactic behavior is (considered by that informant to be) the same. Thus each group formed in the usage table is identified with a unique binary string. Informants for each language of course group the nouns according to their own syntactic rules, hence the clusters formed in different languages inform us about mass/count phenomenology in that language. The same grouping procedure can be applied to the semantic table, generating ‘semantic classes’ (relative to the main features putatively underlying mass/count syntax across languages). The resulting distributions of nouns/concepts in syntactic or semantic classes were analyzed, with the measures described below, for both syntactic and semantic tables.

A) Hamming Distance Scale:

The data in the usage tables is in principle high-dimensional, containing distinct contributions from each of several syntactic markers. It is possible, however, that much of the relevant mass/count syntax might be organized along one main dimension. We consider the hypothesis that this most important dimension may be defined as the ‘distance’ from a pure count string, where nouns at different distances might be associated with characteristic combinations of syntactic markers (see Fig. 2.1 below).

Noun	Context	1	2	3	4	5	6	7	8	9	10	11
ability	Ability is more desirable than wealth.	1	1	1	1	1	1	1	1	1	0	0
accident	The crash was an accident, not intentional.	0	1	1	1	1	1	0	0	0	0	0
acid	Acid stains clothes.	1	1	1	1	1	1	1	1	1	1	1
act	The flood was an act of nature.	0	1	1	1	1	1	0	0	0	0	0
act of crime	Murder is always an act of crime.	0	1	1	1	1	1	0	0	0	0	0
activity	A favorite activity was spitting cherry stones.	1	1	1	1	1	1	1	1	1	1	0
actor	Any good actor can play Tarzan.	0	1	1	1	1	1	0	0	0	0	0

Table 2.3: A small section of the usage table for English as filled by a native informant. Numbers in the top row refer to the syntactic questions in Table 1.

To probe this potential organizing dimension, the high dimensional data is collapsed onto a single dimension. This is obtained by calculating the Hamming distance, or fraction of discordant elements, of each noun (i.e. of each syntactic group) from a bit string representing a pure count noun. A pure count string is one which has ‘yes’ answers for all count questions and ‘no’ answers for all mass questions. Hence a noun that has distance 0 from a pure count string is a proper count noun, whereas a noun with all its bits flipped with respect to a pure count string is a mass noun, and has a normalized distance of 1 from the pure count string. Such a noun has answers ‘no’ to all count questions and ‘yes’ to the mass questions. By plotting the distribution of nouns on this dimension we expect to be able to visualize the main mass/count structure, to relate easily with a linguistic interpretation. This measure does not strictly reflect the categorical nature of groups defined by a unique syntactic string, in the sense that all nouns with a syntactic string differing at 3 bits from the pure count string are clustered together, irrespective of which are the 3 syntactic markers for each noun. This allows for a coarser but perhaps more intuitive and linguistically more transparent comparison between languages than the mutual information measure discussed below, which is a fine-grained comparison between languages, taking into account all the existing dimensions.

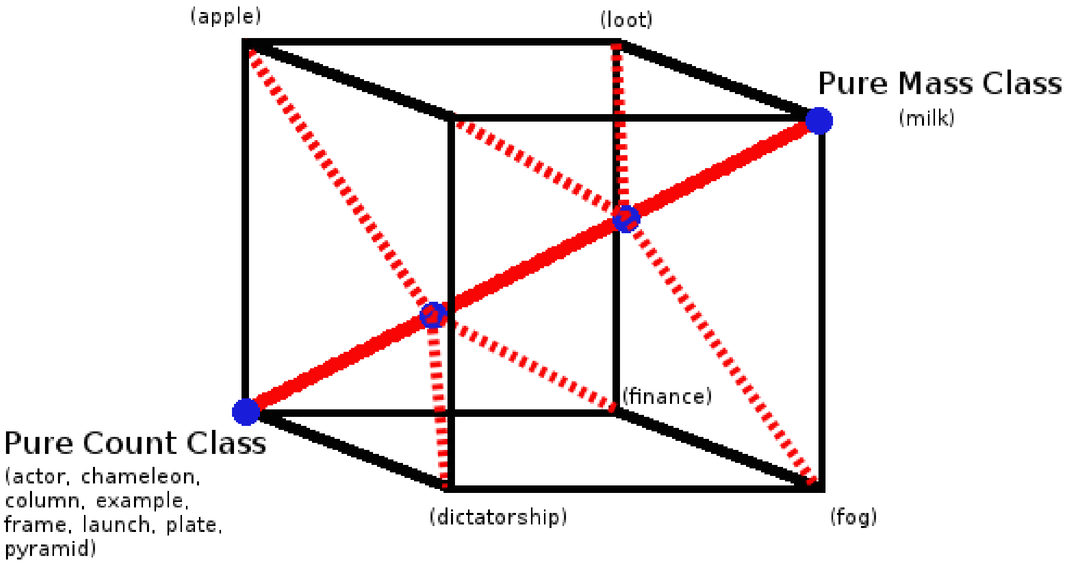


Figure 2.1: Schematic representation of the Hamming distance scale.

Nouns are located in an N-dimensional space (here only three dimensions are represented) and the Hamming Distance scale projects these points onto the mass/count dimension (red diagonal), going from the bit string of pure count to that of pure mass.

Agreement between two languages is estimated as a variance measure, $\langle x^2\rangle+\langle y^2\rangle-2\langle xy\rangle$ which is simply a sum of squares of the difference between the Hamming distances x and y of a noun from the pure count class, as found in the two languages concerned. This measure has a strict upper bound of 1, if Hamming distances are expressed as fractions of discordant bits, which is attained when each noun is either pure count in one language and pure mass in the other, or vice versa; clearly a rather implausible occurrence. A more natural reference value, although not strictly speaking an upper bound, can be estimated by calculating the variance measure between the Hamming distances in a language and those of randomly shuffled nouns in another language,

$\langle x^2\rangle + \langle y^2\rangle - 2\langle x\rangle\langle y\rangle$. The random shuffling simulates the case of a total absence of any relation between the position of the nouns along the main mass/count dimension in the two languages, while respecting the distribution of Hamming distances in each. Thus by comparing the actual value with the reference value, we can get an understanding of how the languages match each other in broadly classifying nouns on the main mass/count dimension. Each language however has different number of questions analyzing its mass/count structure and hence the Hamming distance space for a language is populated only at intervals of 1/Nth of a bit, where N is the number of questions in a language. To minimize the effect of different intervals we estimate a true minimum of variance between languages (which in an ideal case is 0) by calculating the variance between two languages when all the nouns are ordered in the same way in their position on the Hamming distance scale. We adjust the raw variance by simply subtracting the minimum variance for that pair, and then normalize it by dividing it by the (adjusted) effective maximum value as mentioned above.

B) Clustering and Information Measures:

Information theory provides us with useful tools to quantify aspects of the clustering observed in the data. The entropy of a variable, which can take a certain set of values, quantifies the uncertainty in predicting the value it can take in terms of its possible values and their probabilities. A variable which always takes a single value is perfectly predictable and has an entropy of 0 bits. A binary variable has an entropy of 1 bit when it has 50% probability to take either value, e.g. 1 or 0. We can apply this measure to the grouping structure formed around the mass/count distinction in the languages we study. In our case, the variable G is which group any given noun or concept has been associated to in a particular table, taking values 1,…,i,…,n, where n is the total number of groups observed in that table. The probability p(i) is determined for our purposes as the relative frequency of nouns/concepts assigned to group i. The entropy of the table is then calculated as:

$$H(G) = -\sum_{i=1}^{n} p(i)\log_2 p(i)$$

H(G) informs us about the overall syntactic variability expressed (by an informant) in a language, and can be regarded as the logarithm of an equivalent number of significant syntactic classes.

To make cross lingual comparisons, we quantify the extent to which the groups formed by informants in one language overlap with the groups formed by those in another. This amounts to defining equivalence classes, whereby two nouns are grouped together if and only if they are members of the same syntactic usage group in the two languages. For example, if the nouns water and wine are a part of the same group in language X and also fall in one group in language Y, whatever the syntactic usage questions that define groups in the two languages, they are members of the same equivalence class. For analyzing syntactic-semantic relations, language Y is replaced by the semantic table. To give a limiting case, if two languages were to behave exactly the same in classifying nouns in the mass/ count domain, the equivalence classes would coincide with the groups formed in the individual languages, reflecting the exact match between groups produced by language X and Y. At the other extreme, if two languages were to share no commonality, there would be no relation whatsoever between the groups in the two languages, and membership in a group in one language would not be informative about membership in the other language.

The mass/count similarity between X and Y can be quantified by the mutual information I(X;Y), a measure that quantifies the mutual dependence of two variables. If two variables share no common information then the mutual information between them is 0, which is the lower bound for I, whereas the upper bound on mutual information is the lower between the entropies of the two variables (the shared information between two variables cannot be more than the total information content in one variable, i.e. its entropy). Mutual information is calculated using the joint entropy of the two variables in question, which in our case is the entropy of the groups, by the relation

$$I(X;Y)=H(X)+H(Y)-H(X,Y)$$

which can be written also

$$I(X;Y)=\sum p(i,j)\log_2(\frac{p(i,j)}{p(i)p(j)})$$

and where H(X,Y) is the joint entropy of the two variables, at least equal to the higher of the two individual entropies. In the limit case in which the syntactic groups are identical, H(X)=H(Y)=H(X,Y)=I(X;Y), whereas in the opposite limit case, in which there is no relation whatsoever between the groups each table, p(i,j)=p(i)p(j), expressing independent assignments, and then H(X,Y)=H(X)+H(Y), so that I(X;Y)=0.

Mutual information measures suffer from a bias due to limited sampling [Panzeri & Treves 1996] related to the number of equivalence classes actually occupied compared to the total possible (2Nq1 × 2Nq2) classes, where Nq1 and Nq2 are the number of questions for the two languages in the pair. The correction to mutual information is estimated by calculating the mutual information between the pairs of languages when the nouns for one pair are randomly shuffled, thus simulating the lack of correlation between the two languages, and then averaging the value over 50 such shuffles. The correction is then subtracted from the raw value calculated for a pair.

C) Artificial Syntactic String Generation:

To test the importance of the mass/count dimension and its link with semantics, an artificial syntactic usage table was also generated, wherein the ‘yes/no’ decision to a syntactic question was decided by a stochastic algorithm based on the position of the noun on the main semantic mass/count dimension. This algorithm generates a 0 or 1 for each of a string of N ’pseudo-syntactic’ questions, one string per language, where N is the number of syntactic questions in that language. To do so, it uses two reference points, namely the syntactic pure count string for that language and the position of the noun concept along the semantic mass/count dimension, which is taken to be a language universal. The latter is quantified by the Hamming distance from the pure count semantic string, i.e. by the fraction d=D/N of semantic features that differ, for that concept, from those of the pure count. Each bit of the artificial string is then assigned, one by one, for a given noun, the value the bit has in the pure count string with probability (1–d), and the other value with probability d.

Syntactic questions, for this purpose, are empty of content, and simply refer to distinct bits of a pseudo-syntactic usage string. Such bits are determined, for a particular language, by the specific configuration of the pure count string for that language. If the noun is semantically close to a pure count then the probability to generate a syntactic pure count, or something close to it, is higher. The Hamming distances of the artificial strings from the pure count string have a certain distribution (a convolution with exponentials of the semantic Hamming distance distribution) which resembles that of the real syntactic strings, in most cases (except for Marathi, see below); while the position of each noun along the artificial syntactic mass/count dimension is strongly correlated with the position of the noun along the semantic mass/count dimension. The variance measure between the pseudo-usage table of any language and the semantics table provides us with a lower reference value for the variance itself, in contrast to the upper reference value obtained by random shuffling of the nouns. We are then able to better gauge the significance of the mass/count dimension and the importance of semantics with respect to the mass/count syntax. Also, the mutual information between natural usage tables and semantics can be compared to the mutual information between the pseudo-usage table and semantics, to allow a better estimate of what is the contribution of sheer semantics to the mass count syntax (by providing what for the mutual information scale is a more realistic upper value, see Fig. 12 below). The entropy for a particular language depends also on the number of questions used to investigate the mass/count syntax. By looking at the entropies of the artificial syntax we can see how the entropy measure scales with the number of questions.

2.2.3. Corpus Study of the Mass/Count Distinction in English:

Brown’s section of the CHILDES corpus was also used, in an additional component of the study, to obtain mass/count information about nouns occurring in a natural English language corpus. For this purpose all nouns were collected, in the adult-produced sentences of the corpus, which co-occurred with a set of predefined mass/count markers. The co-occurrence frequency of a noun and the set of mass/count markers was recorded and normalized to the total occurrence frequency of the noun. Thus, for each noun, there was a set of numbers which indicated the statistical distribution of syntactic markers for that noun. The markers that were used to measure co-occurrence frequency were a(n), every/each, pluralization, many, much, some + sing. N, and a lot of + sing. N. This study contains a total of 1,506,629 word tokens and 27,304 word types.

The usage table obtained from the CHILDES corpus was analyzed with multi-dimensional scaling, and the distribution of the nouns on the mass count dimension. Multi-dimensional scaling projects high dimensional data on a lower dimensional space while preserving the inter-data-point distance, allowing to visually identify structural information in the data. By analyzing the distribution and clusters in the projected space one can gain information about statistically important dimensions and markers. Moreover, the data from the CHILDES cor-pus was analyzed in terms of distribution of distances from the pure count class and of entropy measures, after binarising the table indicating the frequency of each marker. Thus, for example, if a noun was found at least once in plural form, this was taken as evidence that it could be pluralized; if found at least once with a or an, that it could take the indefinite article, and so on. In this way, the same analyses could be applied as for our database.

2.3. Results:

2.3.1. Individual Syntactic Rules and Semantic Attributes Do Not Match:

The starting point of our analysis is the observation that, at least in five out of the six languages we considered, roughly half the nouns in the sample can be easily classified as pure count nouns. The exact numbers in each language are Armenian: 1058, English: 693, Hebrew: 757, Hindi: 994, Italian: 863, and Marathi: 255. For example, in both Italian and English nouns like '*act', 'animal', 'box', 'country' (as the territory of a nation), 'house', 'meeting', 'person', 'shop', 'tribe', 'wave'; and 'accident', 'cell' (as in biology), 'loan', 'option', 'pile', 'question', 'rug', 'saint', 'survey', 'zoo'* were classified as count in all respects by our informants. In Marathi, while the first 10 examples were also classified as count, the second 10 tested positive on all count properties except one, usually the property of having a morphologically distinct plural form. Marathi appears to stand out from the group in other ways, as reported below. For all other languages, clearly the focus has to be on the remaining proportion of non-pure-count nouns. (see appendix A6, A7 for details about each question)

Among the informant responses, we observed cases of nouns that were regarded as pure count in English but cannot be normally used with numerals in Italian (*'back', 'forum', 'grin'*), or vice versa that test as pure counts in Italian but cannot be normally used with numerals in English (such as *'behavior'* or *'disgrace'*) Interestingly, when considering only usage with numerals and with distributive each/every (ogni in Italian), our informants classified as ‘count’ in English nouns the translation of which failed both tests in Italian: *'love', 'noon', 'youth'* have count usages in English, but not in Italian. The converse is also found: there are count nouns in Italian that, translated into English, failed both the numerals test and the test “can be used with each/every”: '*advice', 'blame', 'literature', 'trust', 'wood'*. There were cases where the impression of one of the authors was that his or her judgment might differ from the informants, or the informants disagreed among themselves. Since we are interested in this study in an overall quantitative analysis of cross-linguistic usage judgments, we did not subject these differences of judgments to in-depth linguistic analysis, but entered the judgments of the majority. We note that there were a significant number of such cases. Overall, there were only 116 nouns that were classified as pure count in all six languages, and still only 392 when excluding Marathi. We thus proceeded to a quantitative analysis, without further questioning the responses by the informants on a noun-by-noun basis.

For a quantitative analysis, we first assessed whether, in any of the languages in the database, a particular syntactic usage rule can be taken to reflect in a straightforward manner a particular semantic attribute of the noun. While in many cases the yes/no answer to a syntactic question turns out to be significantly or highly significantly correlated with a specific semantic attribute, we found no cases where the correspondence could be described as expressing a 'rule', even a rule with a few exceptions. To present quantitative results, we focused on cases where the semantic-syntactic correspondence was higher. The notion of high correspondence is somewhat arbitrary, because for example, one may contrast a case where among 10% of nouns with a particular semantic attribute, 90% admit a certain syntactic construct, with another case where those proportions are 30% and 70%. In our sample, the first 'quasi-rule' appears stricter, but it applies to only 129 nouns in the sample, whereas the second one, while laxer, applies to 301 nouns. For consistency with later analyses, we focus on relative (normalized) mutual information as a measure of correspondence, while reporting also the number of nouns for which syntax matches semantics. The relative mutual information measure ranges from 0 to 1 and it quantifies the degree to which the variability in the syntax, across nouns, reproduces that in the semantic attributes, both of which are quantified by entropy measures.

Language	++	+–	–+	—	H(Lang)	H(Sem)	MI(S,L)	Norm MI
Armenian	24	31	686	43	0.451	0.366	0.080	0.218
Italian	26	29	662	67	0.536	0.366	0.053	0.145
Marathi	25	30	559	170	0.819	0.366	0.020	0.054
English	29	26	668	61	0.503	0.366	0.046	0.126
Hebrew	29	26	682	47	0.447	0.366	0.055	0.150
Hindi	28	27	686	43	0.434	0.366	0.062	0.170

Table 2.4: A case of relatively high correspondence between a semantic attribute and a syntactic rule.

Semantic question 8, applied only to 784 concrete nouns, asked whether the noun denotes an entity (or individual quantity) that can be mixed with itself without changing properties. (This somewhat loosely phrased question makes reference to the homogeneity and cumulativity properties discussed in section 2.1, since it can be interpreted either as asking whether proper parts can be permuted without changing the nature of the object, or whether instantiations can be collected under the same description.) The syntactic question considered was whether the noun can be used with numerals, and it was present in all languages. The largest group of concrete nouns, in the –+ class, denote objects that are not homogeneous, and the nouns can be used with numerals. The relative proportion of nouns in each of the four classes, however, yield meager normalized information values, indicating that individual attributes are insufficient to inform correct usage of specific rules, even in this 'best case' example.

Table 2.4 shows that most concrete nouns in our database (729/784) denote entities that, according to our informants, cannot be 'mixed' while retaining their properties as instantiations of the noun. Most of these nouns can be counted in the sense that they can be preceded with numerals, across languages (with a somewhat less disproportionate bias in Marathi). Nevertheless, among the nouns for which the answer to question 8 was positive, i.e. that displayed properties of either cumulativity or homogeneity, roughly half can be used with numerals, again across languages, yielding rather low values of mutual information between semantics and syntax, as quantified in the last column of the table. Normalized MI (as mentioned in section 2.2.2 B) values are much closer to zero than to one.

Even though the correspondence with the particular semantic attribute of cumulativity is low, the results above suggest that there might be a high degree of correspondence among the syntactic usage with numerals across languages, at least when excluding Marathi. After all, across languages it is roughly half the nouns denoting entities which intuitively are cumulative, which can be used with numerals, and half which cannot. Is it roughly the same half?

Language pair	++	+–	–+	—	H1	H2	I(1:2)	Norm. MI
Arm–Ita	662	48	26	48	0.451	0.536	0.124	0.275
Arm–Mar	560	150	24	50	0.451	0.819	0.059	0.131
Arm–Eng	666	44	31	43	0.451	0.503	0.106	0.235
Arm–Heb	675	35	36	38	0.451	0.447	0.095	0.212
Arm–Hin	683	27	31	43	0.451	0.434	0.129	0.297
Ita–Mar	548	140	36	60	0.536	0.819	0.062	0.115
Ita–Eng	654	34	43	53	0.536	0.503	0.131	0.261
Ita–Heb	652	36	59	37	0.536	0.447	0.068	0.152
Ita–Hin	661	27	53	43	0.536	0.434	0.102	0.235
Mar–Eng	547	37	150	50	0.819	0.503	0.041	0.082
Mar–Heb	553	31	158	42	0.819	0.447	0.034	0.076
Mar–Hin	556	28	158	42	0.819	0.434	0.037	0.086
Eng–Heb	669	28	42	45	0.503	0.447	0.119	0.266
Eng–Hin	675	22	39	48	0.503	0.434	0.144	0.331
Heb–Hin	675	36	39	34	0.447	0.434	0.078	0.181

Table 2.5: The correspondence between languages is not higher.

In the same case of relatively high correspondence between a semantic attribute and a syntactic rule, entropy and mutual information between languages yield the relatively low normalized MI values listed in the fifth column, which indicate that a broadly applicable syntactic question ("Can the noun be used preceded by a numeral?") selects different subsets of nouns across different languages.

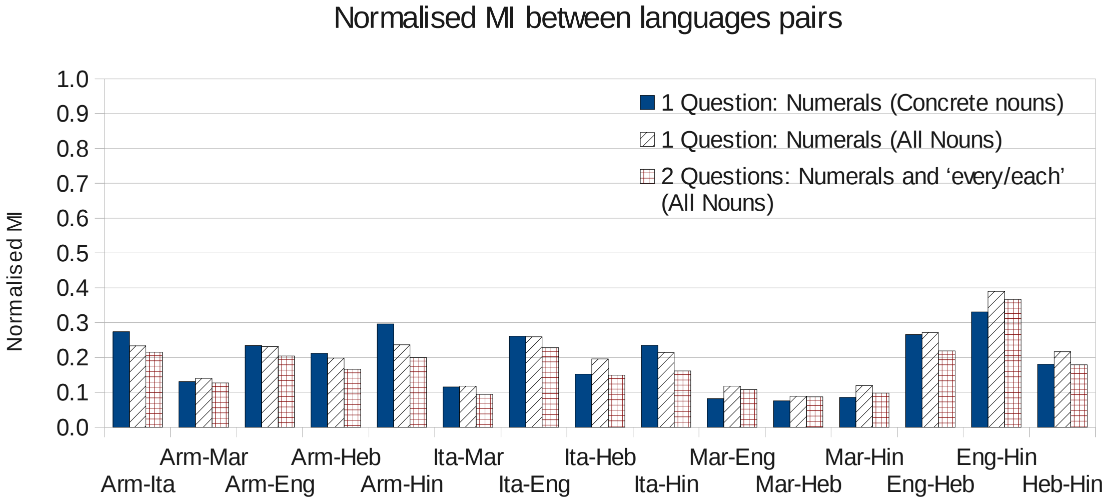


Figure 2.2: Agreement across languages remains low, however it is measured.

The solid bars show the normalized mutual information between pairs of languages for a single question, on usage with numerals, for concrete nouns only. The stippled bars are for the same measure over all nouns in the database, both concrete and abstract. The patterned bars are for pairs of questions, on both the use of numerals and that of distributive quantifiers such as each/every in English (see text).

Table 2.5 and Figure 2.2 show that the naïve expectation is not met by the data. The syntactic correspondence in the usability with numerals is weak across languages, even irrespective of any semantic attribute it may originate from. The congruence (number of concrete nouns in the same syntactic class when translated across languages) appears relatively high, because most nouns can be used with numerals anyway, but, properly quantified in terms of normalized mutual information, the degree of correspondence even excluding the special case of Marathi is roughly in the 15–30% range, with English and Hindi reaching a peak value of 33%. When considering all nouns in the database, including abstract nouns, the degree of correspondence does not change much (stippled bars in Fig. 2.2). Again excluding the special case of Marathi, it falls roughly in the 20–27% range, with English and Hindi reaching a peak value of 39%.

One may ask whether the low MI values with semantics, in Table 2.4 above, may be due to the lack of exact match between the semantic attribute considered and the specific syntactic rule. Similarly, one may ask whether the weak correspondence in the pattern of usage with numerals may also be due to the fact that numerals might point in different directions, so to speak, in the syntactic space of each distinct language, for example, atomicity vs. non-homogeneity. To approach these issues, we have begun by considering pairs of attributes, and pairs of syntactic rules. The degree of correspondence of each language with semantics does not change much, and in fact it tends to slightly decrease. For example, when asking whether the object denoted by the noun can flow freely, and also whether it is cumulative, and on the other hand whether the noun can be used with numerals and whether it can be used with distributive quantifiers like ‘each’ in English, we find that the normalized MI decreases with respect to the above analysis with one attribute and one rule, in all cases except for Marathi (data not shown). The decrease is entirely due to the increase in the entropy that appears in the denominator of the normalization (see Methods). In terms on non-normalized mutual information, instead, adding dimensions reveals perforce more variability.

Similarly, the match between languages, independently of semantic attributes, does not increase when considering two syntactic rules instead of one. Table 2.6 and Figure 2.2 report the data, this time for concrete and abstract nouns together, when considering the two syntactic rules above.

Table 2.6 shows that normalized mutual information values are low, all below 0.23 except for the English–Hindi match, even though ‘congruence’ values appear high. Congruence is the sum of the number of nouns that are used in the same way in both languages, with respect to the two syntactic constructs considered. Except for pairs including Marathi, between 70–81% of nouns are congruent across pairs. Yet mutual information is low because many of the congruent nouns are simply pure count nouns in either language, accepting both numerals and distributives, and their permanence in the largest class is not very informative about mass/count syntax in the other classes. As considering two questions rather than one does not affect results, it is interesting to ask what happens when considering all available questions together. We first focus on the main mass count dimension.

Language pair	H1	H2	I(1:2)	Norm. MI	Congruency
Arm–Ita	0.862	1.129	0.186	0.215	1119
Arm–Mar	0.862	1.427	0.109	0.127	825
Arm–Eng	0.862	0.872	0.176	0.204	1154
Arm–Heb	0.862	0.940	0.143	0.166	1152
Arm–Hin	0.862	1.242	0.172	0.200	1046
Ita–Mar	1.129	1.427	0.106	0.094	849
Ita–Eng	1.129	0.872	0.199	0.228	1132
Ita–Heb	1.129	0.940	0.141	0.150	1081
Ita–Hin	1.129	1.242	0.182	0.161	1011
Mar–Eng	1.427	0.872	0.094	0.108	882
Mar–Heb	1.427	0.940	0.082	0.087	826
Mar–Hin	1.427	1.242	0.122	0.098	812
Eng–Heb	0.872	0.940	0.191	0.219	1157
Eng–Hin	0.872	1.242	0.320	0.367	1099
Heb–Hin	0.940	1.242	0.169	0.179	1037

Table 2.6: Congruency and mutual information between languages.

The correspondence between languages is not higher when considering pairs of rules at a time. Here we considered whether a noun can be used with numerals, and whether it can be used with a distributive quantifier such as each/every in English.

2.3.2. Hamming Distance:

Plotting the data on the main mass/count dimension (Fig. 2.3) as the distance from the pure count string shows that a very high proportion of the nouns are at a distance zero from the pure count class (groups are labeled from 1 to N+1 at an increasing Hamming distance of a single bit, where N is the number of questions and group 1 represents pure count nouns). Overall there is an exponential-like decreasing trend in the group frequencies (see Appendix A9), as we go further from the pure count, for all languages but Marathi. Since this measure does not distinguish between different classes that are at the same distance from the pure count class but vary in the questions that define them, we use different colors in the bars to show the proportions of particular classes at that specific distance from the pure count class. The number of questions, N, is 9 for Armenian, 8 for Italian, 5 for Marathi, 11 for English, 9 for Hebrew, and 5 for Hindi. Since there are N+1 possible groups, we see that for Italian the 9th group is empty, whereas for Hebrew the last two groups are empty.

Distributions in Figure 2.3 seem to reflect the nature of the nouns as brought out by the questions used to investigate them. In the case of Marathi, the distribution is seen to have two groups of high frequency, at the distance of 1 bit from the pure count class and mass class, respectively. In each of these high frequency groups there is one class that accounts for most of the nouns. The class making up most of 5th group differs from the pure mass class in answering ‘no’ to the question regarding use of measure classifiers. Upon closer inspection, we find that, out of the 411 nouns that form the largest class in the 5th group, 332 are abstract nouns, hence answering ‘no’ to the measure classifier question. The question that differentiates, instead, the largest class in the 2nd group from the pure count class is ‘Pluralization with morphological change’, to which for nouns in the largest class the answer is ‘no’. Figure 2.4 shows the same distribution, only for Marathi, but restricted to the 650 abstract and to the 784 concrete nouns, respectively. Notice the changes in the frequencies of the 5th group for both concrete and abstract nouns, as compared to Figure 2.3. For other languages, the distributions restricted to concrete and to abstract nouns look similar to the overall distribution, with a quasi-exponential downward trend (not shown).

In summary, the distribution of mass/count syntactic properties is undoubtedly graded rather than binary, as might have been intuitively expected. Most common nouns are strictly count in nature, in five of the six languages considered, with mass features increasingly rarer as they approach the pure mass ideal. Marathi differs from the other languages, and it remains of be examined whether it is representative of several other natural languages not considered in this study.

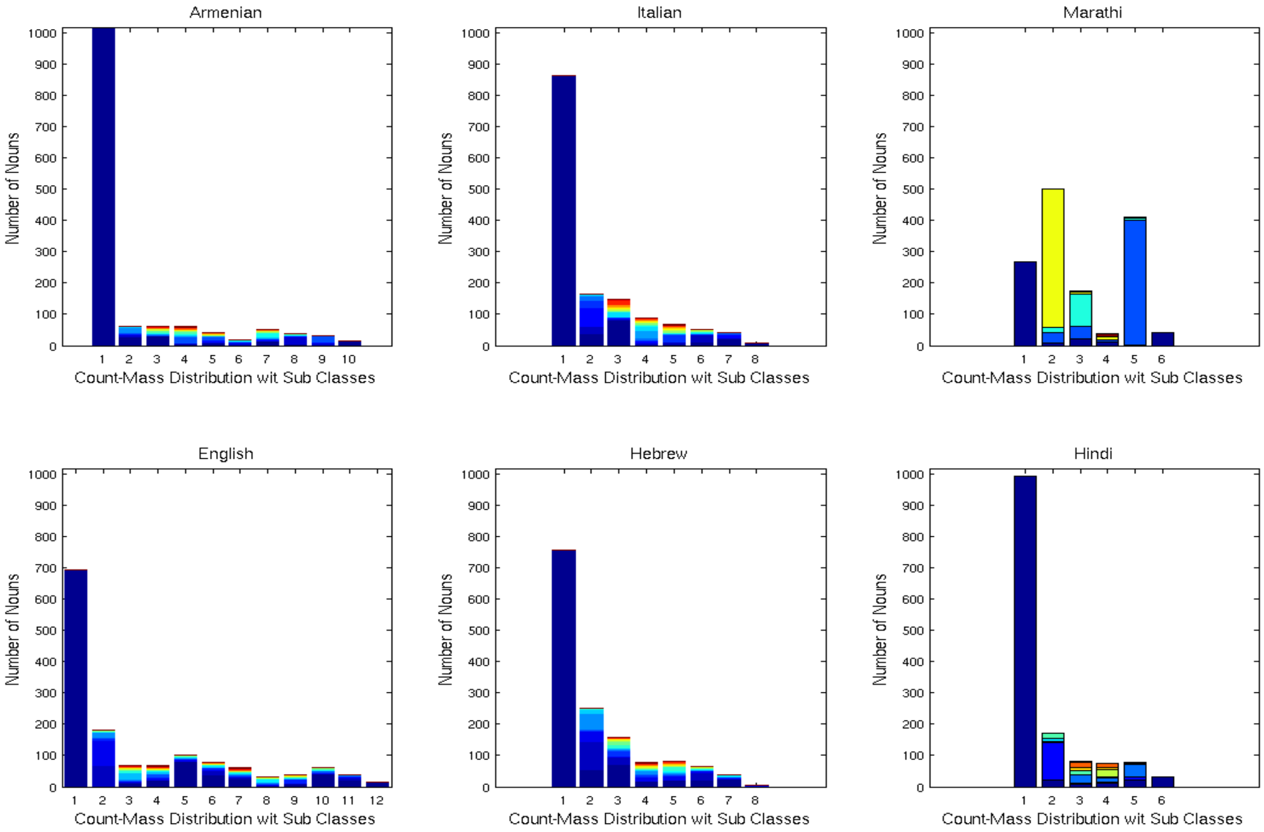


Figure 2.3: Distribution of nouns along the mass/count dimension. Each histogram reports the frequency of nouns in the database, for a particular language, at increasing distances from pure count usage (1) and towards pure mass usage (N+1), where N is the number of syntactic question for the language. Colors in the bars indicate the proportion of nouns in each of the syntactic classes occurring at the same Hamming distance from the pure count.

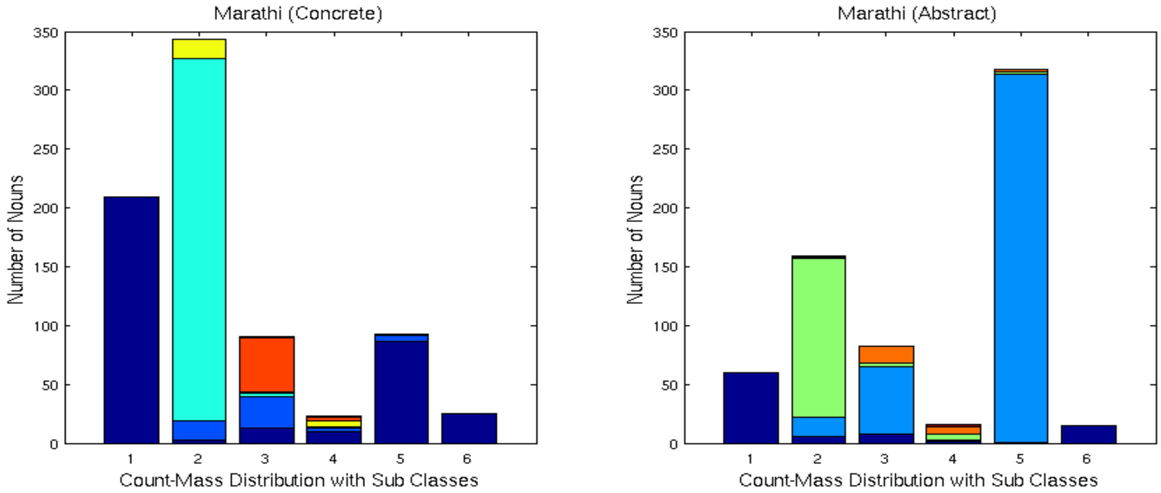


Figure 2.4: The distribution for Marathi, restricted to concrete and to abstract nouns. As the 5th group in the histogram in Figure 2.3 (top right) includes mostly abstract nouns, it dominates the abstract noun distribution (right), while it is considerably reduced for concrete nouns (left).

How distributed are the semantic features characterizing these same nouns? Figure 2.5 shows that, for concrete nouns, semantic features define a monotonically decreasing distribution from the pure count class, roughly similar to that observed for syntactic usage features in five out of six languages. Prima facie, this might suggest that concrete semantics might be the common source that influences the global structure of the mass/count classification, at least for concrete nouns. For abstract nouns, the two semantic questions considered leave most nouns in the ‘ambiguous group’ — in particular, in the class which includes abstract nouns without an easily definable conceptual unit or a temporal unit. Since the semantics of abstract nouns does not have as clear a definition as concrete nouns, it may not have a strong independent influence on the mass/ count syntax of abstract nouns.

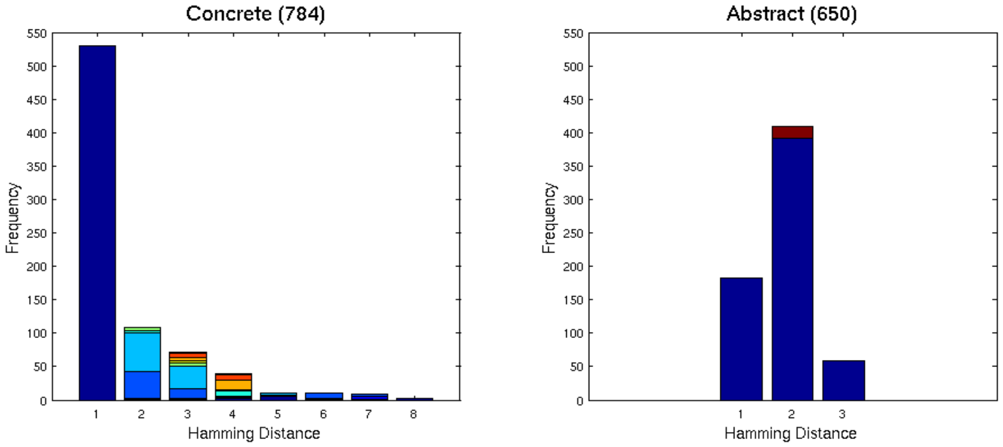


Figure 2.5: Distribution of nouns on the main mass/count dimension for semantics. Given the different applicable semantic questions, it is shown separately for concrete (left) and abstract nouns (right). Concrete nouns show an exponential like shape similar to most of those in Figure 2.3, whereas most abstract nouns are in the ambiguous group.

If semantics serves as the common source of the mass/count syntax for concrete nouns, we expect not only the distributions to look similar overall, but also to include individual nouns at similar positions along the main mass/count dimension, both when comparing semantics with syntax for each of the ‘well-behaved’ languages, and when comparing the syntax of two such languages. This can be assessed through our variance measure, which quantifies the overall difference between such positions.

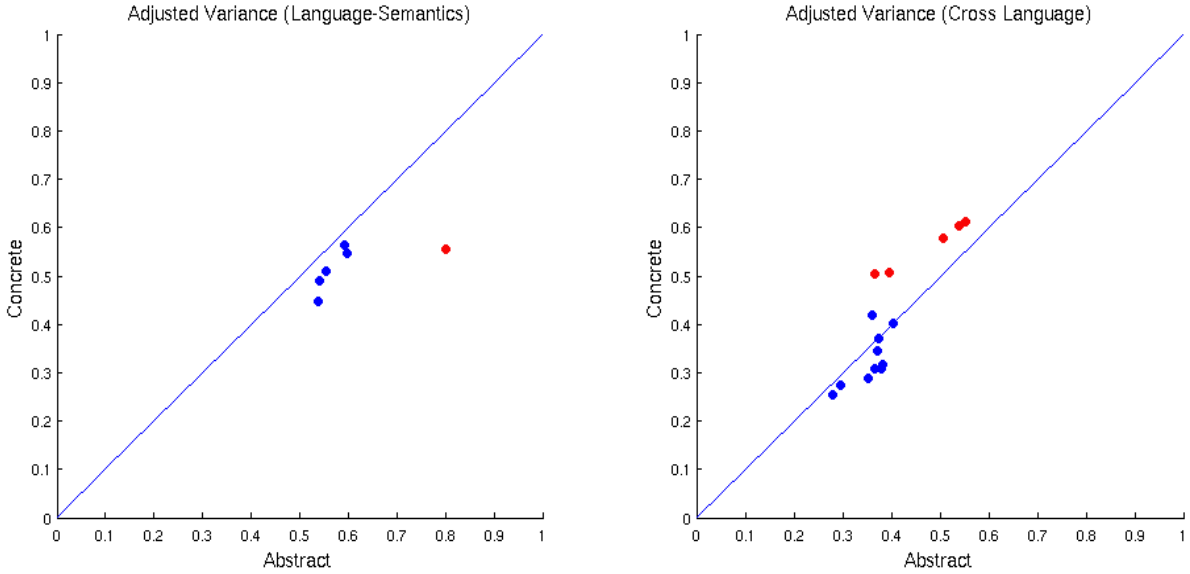


Figure 2.6: Scatter plots of variance values along the main mass/count dimension. The adjusted and normalized variance (see Methods) in the position of individual nouns is shown between semantics and syntax (left) and between syntax in pairs of languages (right). In each plot, the adjusted and normalized variance for concrete nouns is on the y-axis and for abstract nouns on the x-axis. Pairs that include Marathi are indicated in red.

This expectation is only weakly borne out by the data. Figure 2.6 shows our relative variance measure, which is normalized to range between zero (when individual nouns are identically ordered in terms of their distance from the pure count class, either in both of two languages or between semantics and the syntax of one language) and one (when the relative orders are completely unrelated to each other). For concrete nouns, Figure 2.6 (left) shows that relative variance from semantics hovers around 0.5, halfway to complete lack of any relationship. For abstract nouns, variance values from semantics are somewhat higher. Marathi is an outlier, with yet higher variance from semantics, for abstract nouns. It appears therefore that the relationship to the underlying semantics is not very strong, even when considered solely along the main mass/count dimension.

Between languages, the adjusted variance is less than 40% of the adjusted maximum for all the pairs except for those including Marathi, both for concrete and abstract nouns. Marathi is an outlier, with higher variances from semantics (for concrete nouns) and from other languages. Marathi has higher variance since it does not have the exponential-like distribution as the rest of the languages. The variance values calculated over the entire database tend to be, for each pair of languages, close to the average between the values calculated over concrete and over abstract nouns, separately (not shown). The fact that between languages variance values are relatively low, relative to those from semantics, may indicate that there is an overall agreement across languages in classifying the nouns in the mass/count domain. This is shown only a gross level, however, along the main mass/count dimension, since here we do not take into account the fine grained differences between the classes at the same distance from the pure count class.

2.3.3. Mutual Information along the Main Mass/Count Dimension:

The results from the analysis of the variance can be verified by considering an alternative measure of the correspondence in the classification, the mutual information. Along the main mass/count dimension, the mutual information can be calculated, e.g. between two languages, by grouping nouns at each Hamming distance from the pure count nouns, rather than each syntactically defined class. The mutual information (Table 2.7) ranges between zero (when there is no correspondence whatsoever in the groupings) and the minimum of the two entropy values, where entropy is calculated also by putting together all nouns in a group, i.e., at the same Hamming distance from pure count nouns.

Language	Entropy
*Armenian	1.63
*Italian	1.96
*Marathi	2.15
English	2.66
Hebrew	2.11
Hindi	1.54
*Semantics	(2.01) 1.58 (C) 1.24 (A)

*Table 2.7: Language–entropy relations. Entropy values along the main mass/count dimension in the six languages, and for semantics. The * sign indicates an 'average' over five informants (three for Marathi), taken by assigning to each question and each noun the yes/no answer chosen by the majority. For semantics, the overall value (in parenthesis) has little significance, because concrete nouns are assigned to eight distinct groups and abstract to only three, and combining them distributes the abstract nouns into the two extreme concrete groups and one central group.*

Figure 2.7 confirms, on the different quantitative scale of mutual information measures, the results obtained with the analysis of variance. The normalized mutual information with semantics is quite low, and lower for abstract than for concrete nouns, corresponding to higher variance values. It is at its lowest, 0.016, for abstract nouns in Marathi, which had the highest variance values. Between languages, mutual information is somewhat higher, and not markedly different between abstract and concrete nouns.

To better appreciate the significance of the relatively high variance values we measured, and of the relatively low MI values, we contrasted the values obtained between different languages with those obtained 'within' languages, i.e. measuring the correspondence between different informants of the same language. These data are available for five informants each for Armenian and Italian, and three for Marathi, and also five for the semantics classification. They give thus rise to 10 informant pairs in three cases and three pairs for Marathi.

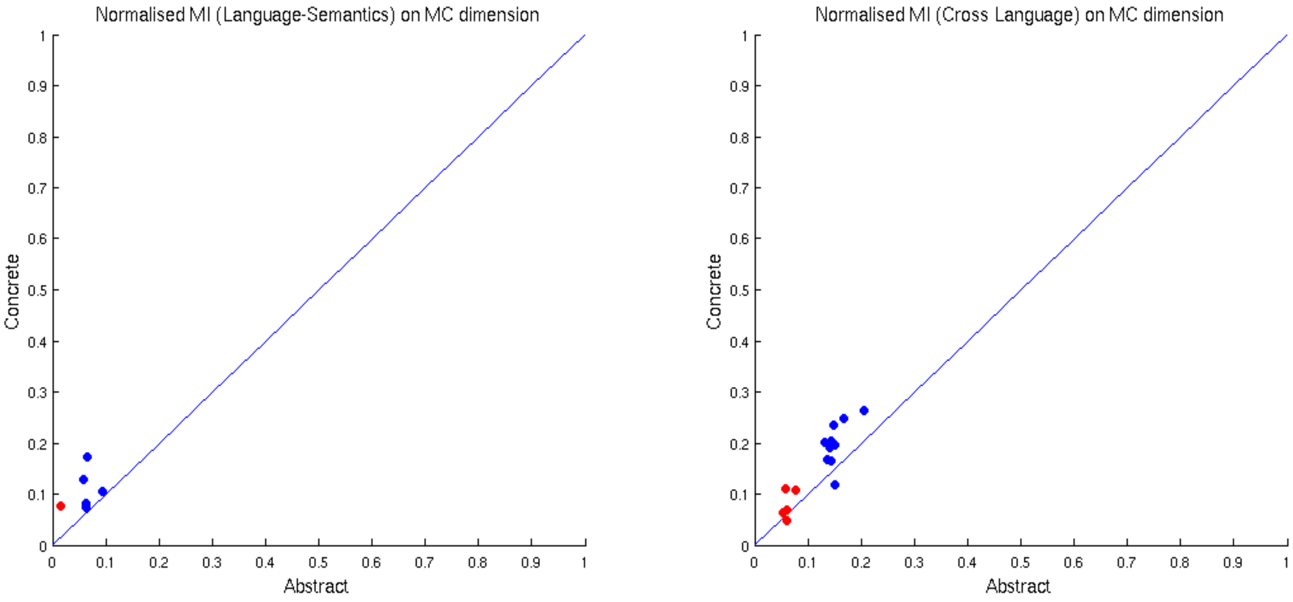


Figure 2.7: Scatter plots of mutual information values along the main mass/count dimension. The normalized mutual information (see Methods) between the groups of individual nouns is shown between semantics and syntax (left) and between the syntax of pairs of languages (right). In each plot, the normalized mutual information for concrete nouns is on the y-axis and for abstract nouns on the x-axis. Pairs that include Marathi are indicated in red.

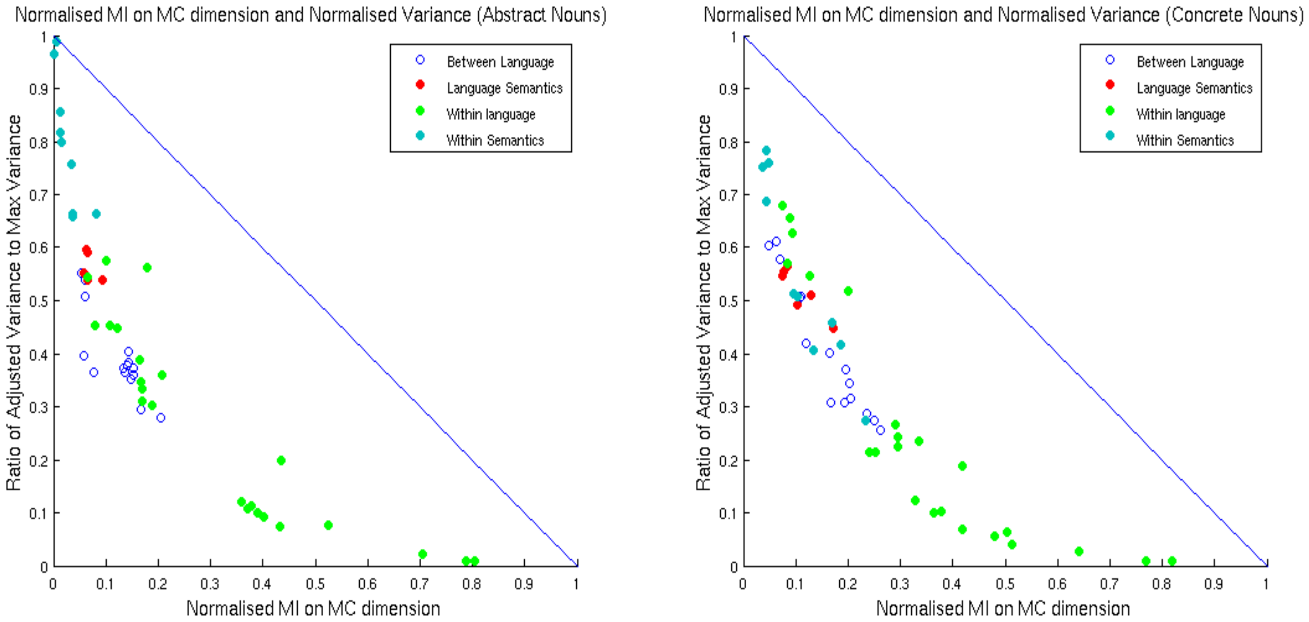


Figure 2.8: Scatter plots comparing variance with mutual information values.

The normalized mutual information (along the main mass/count dimension) is shown on the x-axis with the corresponding normalized variance value on the y-axis, for abstract nouns (left) on and for concrete nouns (right). Different colors denote data points between the syntax of pairs of languages (empty circles), between the semantics and syntax (red), within language (green) for 10 Armenian, 10 Italian, and three Marathi data points, and within different semantics informants (10 light blue data points).

Figure 2.8 shows, first of all, that the MI measure and the Variance measure are broadly equivalent. Their relation is (very roughly) Var ~ $(1–\text{MI})^4$. This occurs despite the different nature of the two measures: the mutual information is not sensitive to distance along the mass/count dimension, only to group membership, whereas variance has limited sensitivity to small differences in the exact classification of each noun, as long as its position on the mass/count dimension does not vary too much. Variance turns out to be a more informative measure with our data, which better span its 0–1 range, but mutual information can be easily generalized beyond the main mass/count dimension.

Second, the within language data show mostly more agreement (higher MI and lower Var) than the between language data. Exceptions are due to one Armenian informant (yielding four data points) and one Marathi informant (yielding two more data points) that differ sensibly in their syntactic judgment from the rest. The ‘average’ data for both Armenian and Marathi, however, due to the majority rule effectively disregards their peculiarities. Thus both measures overall indicate more agreement between informants of the same language than between languages, although this is very far from a clear cut all-or-none difference. Confronted with the requirement to answer yes or no to a set of binary questions, speakers of the same language vary substantially in their responses.

Third, the informants who contributed the semantic classification show the least agreement, particularly for abstract nouns. Even though there were just two questions to answer for abstract nouns, the responses to those two questions are effectively random, with the variance between informants close to its random reference value (in one case exceeding it), and the mutual information close to zero. This suggests that while the semantic properties that should inform the mass/count syntactic usage are already not that salient and self-evident for concrete nouns, they are completely irrelevant for abstract nouns.

2.3.4. Mutual Information across the Complete Syntactic Classification:

Mutual information is however higher when all the dimensions are considered (Fig. 2.11), even in relative terms, i.e. when taking into account that the entropy values are higher for the full classification (Table 2.8). Entropy values, as discussed in the Methods section, inform us about the logarithm of the equivalent number of significant classes found in the data. Table 2.8 shows that the entropies of the languages are in the range of 2–4 bits, which indicates the presence of something equivalent to 22–24 equipopulated classes of nouns (from slightly above 4 for Hindi to just below 16 for English). In a hypothetical case where there were just two significant classes of mass and count the entropy would have been in the range of 1 bit, in fact even less if the count class were, as it turns out to be in most cases, much more populated. This provides a quantitative estimate of the variability that exists in the mass/count classification, which is much higher than may have been intuitively expected.

Language	Entropy
*Armenian	2.29
*Italian	3.02
*Marathi	2.71
English	3.92
Hebrew	3.40
Hindi	2.12
*Semantics	(3.72) 2.94 (C) 2.34 (A)

Table 2.8: Entropy values for the full classification in the 2.6 languages, and for semantics.

*The * sign indicates an 'average' over five informants (three Marathi), taken by assigning to each question and each noun the yes/no answer chosen by the majority.*

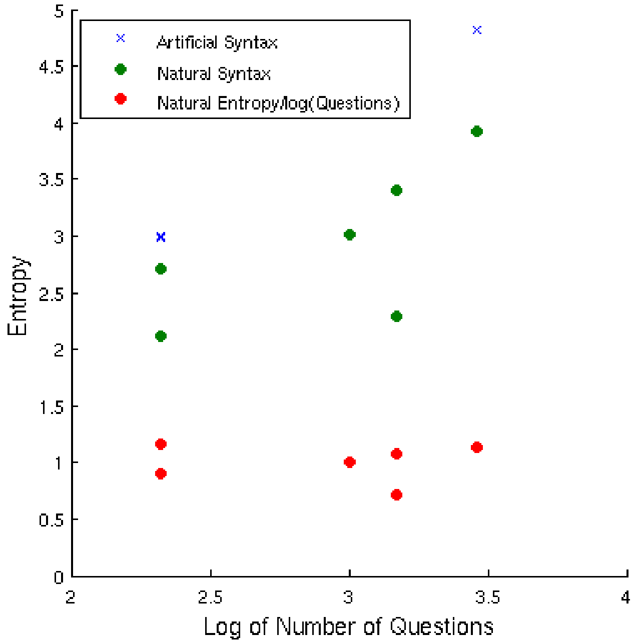


Figure 2.9: The entropy scales up with the number of questions. Both when calculated for natural syntax and for the artificial syntactic strings used as controls, entropy values turn out to be roughly proportional to the logarithm of number of questions, hence to yield almost the same value, around 1, when divided by that number.

It is important to note that the entropy and mutual information values obtained with our procedure are influenced by the number of questions used for each language. The scale of the entropy of the ‘artificial syntax’ depends solely on the number of questions, and we can see from Figure 2.9 how also the entropy values for natural syntax are strongly correlated with the logarithm of number of questions. Dividing the entropy of natural syntax (Table 2.8) by the logarithm of the number of question all the entropy values get together at around the 1 bit mark.

The limited agreement that there is, is somewhat stronger for concrete than for abstract nouns except for the 10 within Italian pairs. Figure 2.10 indicates that this holds within languages, between languages, and much more so when including semantics. As noted above in the case of measures restricted to the main mass/count dimension, the semantic classification of abstract nouns is so arbitrary that agreement among the five informants that filled the questionnaire is extremely low, and the correspondence of their majority response with any natural syntax is also low.

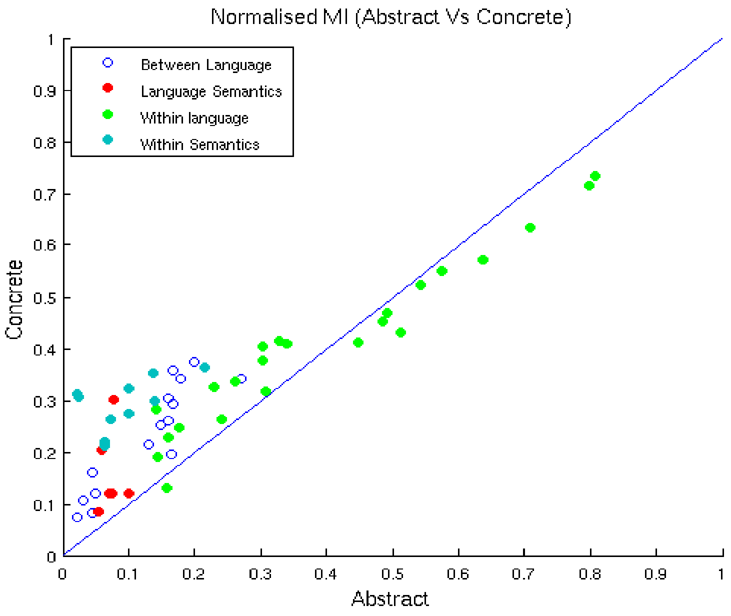


Figure 2.10: Scatter plots of mutual information values for abstract and concrete nouns. The normalized mutual information is shown for abstract nouns on the x-axis with the corresponding value for concrete nouns on the y-axis. Different colors denote data points between the syntax of pairs of languages (empty circles), between the semantics and syntax (red), within language (green) for 10 Armenian, 10 Italian, and three Marathi data points, and within different semantics informants (10 light blue data points).

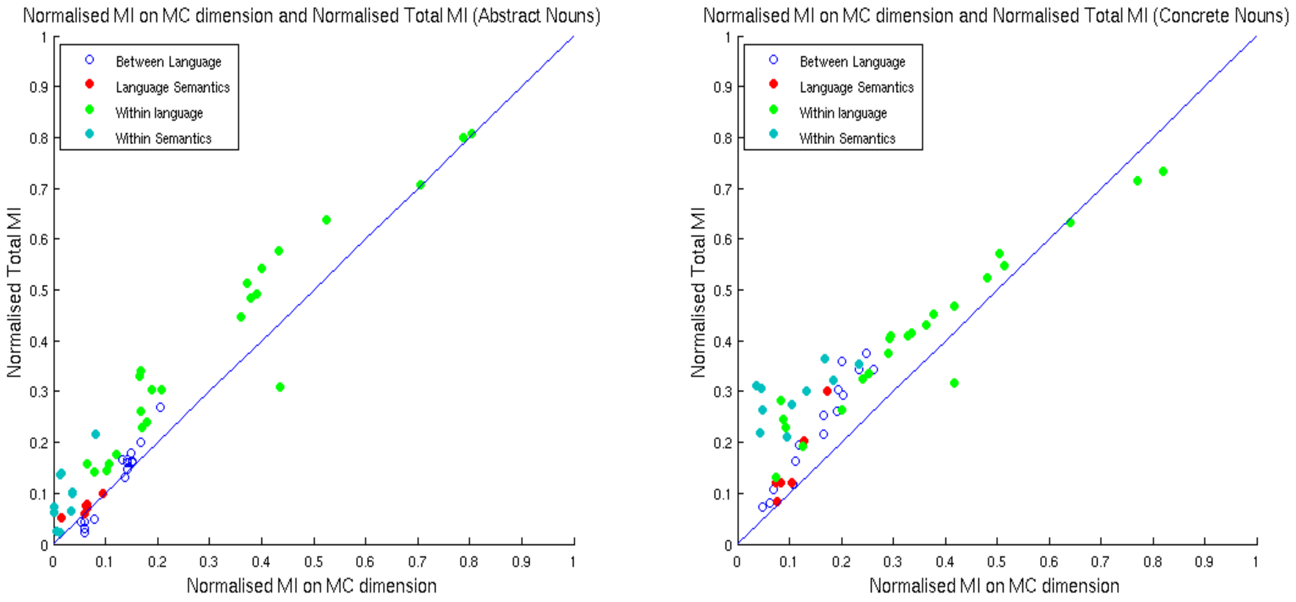


Figure 2.11: Scatter plots comparing mutual information on the MC dimension with total mutual information values.

The normalized mutual information along MC dimension is shown on the x-axis with the corresponding normalized mutual information including all dimensions on the y-axis, for abstract nouns (left) on and for concrete nouns (right). Different colors denote data points between the syntax of pairs of languages (empty circles), between the semantics and syntax (red), within language (green) for 10 Armenian, 10 Italian, and three Marathi data points, and within different semantics informants (10 light blue data points).

From Figure 2.11 we see that relative mutual information, when all the dimensions are taken together, is only slightly higher than when just the main mass/count dimension is considered, telling us that most of the variability is present along the main MC dimension. Again, abstract nouns show a larger variability between and within languages, and this difference is particularly strong within semantics. The source of the variability is most likely to be the degrees of freedom left in the syntactic or semantic classification task, applied to the abstract nouns. Even though the nouns and their meanings were disambiguated with a reference sentence, informants were still free to frame the sentences while deciding whether a particular marker can be used with a particular noun. Hence part of the variability may come as a result of the somewhat arbitrary determination of the exact meaning used by different informants when adapting their abstract cognitive categories to the classification of nouns, or of individual differences in the manipulation of context (Raymond et al. 2011).

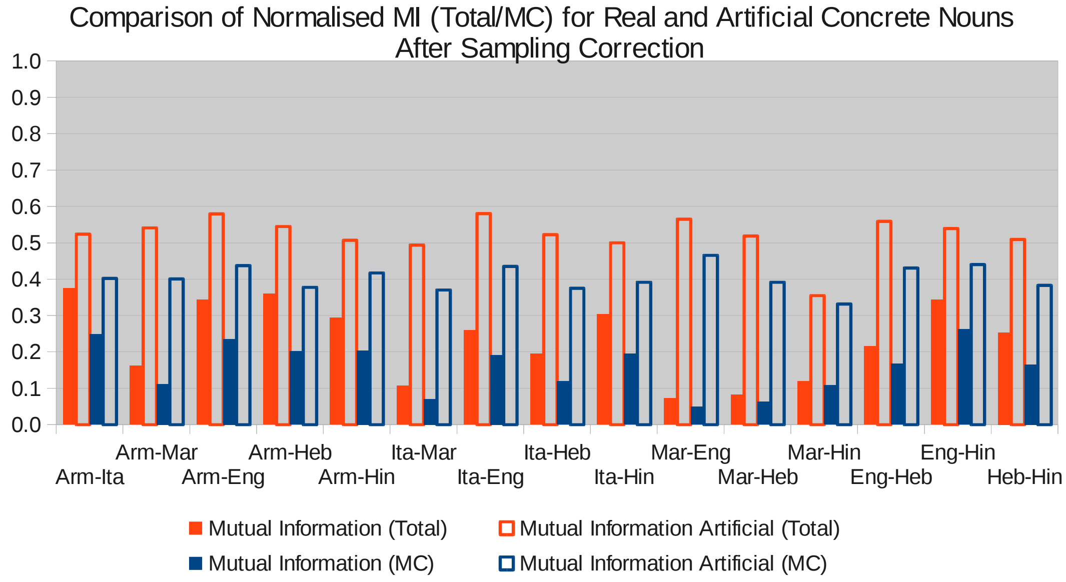


Figure 2.12: Mutual information between language pairs vs. artificially generated control values.

Normalized mutual information between language pairs (red solid) are in the 0.33–0.52 range, except for pairs including Marathi, for which they are around 0.2. These values can be contrasted with the higher values obtained by generating a pseudo usage table, based solely on semantic properties (red empty), as explained in Methods. A similar comparison is shown for the normalized mutual information but only on the MC dimension (blue-solid for real and blue empty for artificial).

Figure 2.12 tells us that there while relative values are higher than when computed only along the main mass/count dimension (Fig. 2.7), still there is little agreement across languages even on a finer scale, as the MI values are mostly less than half of the lower of the two entropies. Mutual information is a strict measure, wherein a single bit difference will put a noun in a different equivalence class and lower the mutual information. In contrast, however, artificial syntactic strings produced from the semantic ones, with the stochastic procedure outlined in Methods, share around 50% mutual information, relative to the lower of their entropy values. Artificial syntactic strings also ‘suffer’ from a sensitivity to single fluctuating bits, hence the contrast between their 50% agreement and the 20–30% (roughly) agreement of the real syntax tells us that real agreement is genuinely low, and it is not all due to using a bizarre measure. The low mutual information of the natural syntax suggests that there is considerable syntactic variability along different dimensions of the syntactic domain, although most of the variability is already in the main mass/count dimension, since when restricted along that dimension agreement is even lower (Fig. 2.11–2.12).

2.3.5. CHILDES Corpus Study:

With a method analogous to the Hamming distance measures, we analyze the Brown's section of the CHILDES corpus (only for the adult sentences) on the main mass/count dimension. We simply count the frequency of occurrence of a noun with mass markers out of the total occurrence of the noun in the corpus. There are 1551 nouns in this study out of which 522 nouns (151 are abstract and 371 concrete) are in common with the nouns used for the analyses above. Figure 2.13 plots the distribution of all the nouns on this main mass/count dimension. In a similar trend to Figure 2.3, we see the nouns to be distributed all across the spectrum from count to mass, with an overall decreasing trend in frequency going from count to mass (except for the pure mass class). Nouns with pure count usage are very many compared to the rest of the groups. We do find, however, a higher number of pure mass nouns in the corpus, as compared to the English syntactic data obtained from an informant.

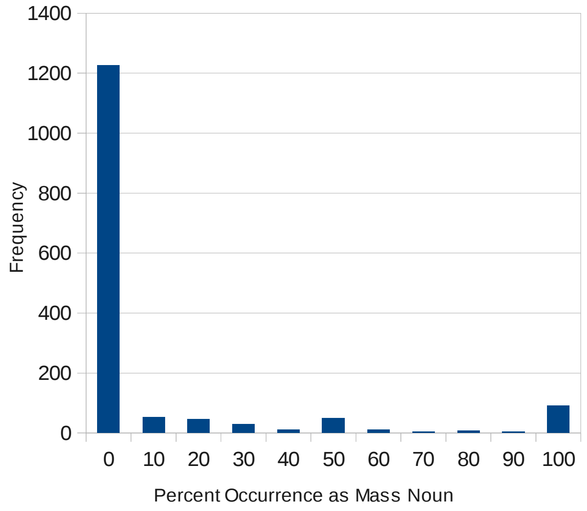


Figure 2.13: Distribution of nouns from the CHILDES corpus on the main mass/count dimension.

Count occurrences of nouns are very frequent as compared to mass occurrences, with nouns lying along the entire spectrum.

A multi-dimensional analysis of the corpus data brings forward four markers as salient, two count (‘a(n)’ and Pluralization) and two mass markers (bareness and ‘some + singular noun’). Nouns mostly lie along the vertices connecting these four markers. Figure 2.14 shows the most significant dimensions in terms of the co-occurrence frequencies found in the corpus, for example, along the edge connecting the vertices ‘a(n)’ and ‘pluralization’, close to the ‘a(n)’ vertex there are nouns that occur almost always with ‘a(n)’ but seldom in plural form, in the corpus, while close to the ‘pluralization’ vertex there are nouns with the opposite occurrence, with the rest of the nouns occurring in between these two extremes. All nouns along this edge are in any case classified as pure count nouns, in the first bin of Figure 2.13. The density of nouns along the count edge is much higher than along the ‘mass edge’ (defined by the properties of appearing in bare form, at one vertex, and appearing with ’some’ + singular noun at the other vertex). These four markers have the highest variance in their frequency of occurrence across the nouns in the corpus (Table 2.9).

bare	a/an	every/each	many	pluralization	much	some	a lot of
0.0485	0.1556	0.0044	0.0015	0.1177	0.0034	0.0275	0.0010

Table 2.9: Variance of the markers in the CHILDES corpus.

The variance of the markers we used to classify nouns in the Brown's section of the CHILDES corpus was calculated across its 1551 nouns, and the four markers with highest variance were used, a posteriori, to characterize the three most significant dimensions of mass/count variability, as independently generated by multi-dimensional scaling.

Finally, we contrast mass/count entropy values extracted from the corpus from those measured from the informant responses. To obtain entropy estimates from the CHILDES corpus, which can be used for the comparison, we first binarise the corpus co-occurrence frequency table, such that if a marker was found at least once with a noun, it was assigned the value of 1, and 0 otherwise. With this method, the total entropy of the corpus data was calculated to be 3.75 bits, as compared to the English informant entropy, which is 3.92. Since 522 nouns (151 abstract and 371 concrete) are common to the corpus and informant usage tables, we calculated the entropy on the MC dimension for them, too.

Informant entropy for concrete nouns on MC dimension	2.16
Corpus entropy for concrete nouns on the MC dimension	1.37
Informant entropy for abstract nouns on MC dimension	2.46
Corpus entropy for abstract nouns on the MC dimension	1.35

Table 2.10: The entropy values for nouns in both the database and the CHILDES corpus.

The entropy of the corpus on the MC dimension is lower than that of informants, perhaps due to the restricted contexts in which sentences can occur in a corpus, as opposed to the freedom of choice to the informants. The normalized mutual information, including all dimensions, between binarised corpus and informant data after sampling correction is 0.051 for concrete nouns and 0.001 for abstract nouns.

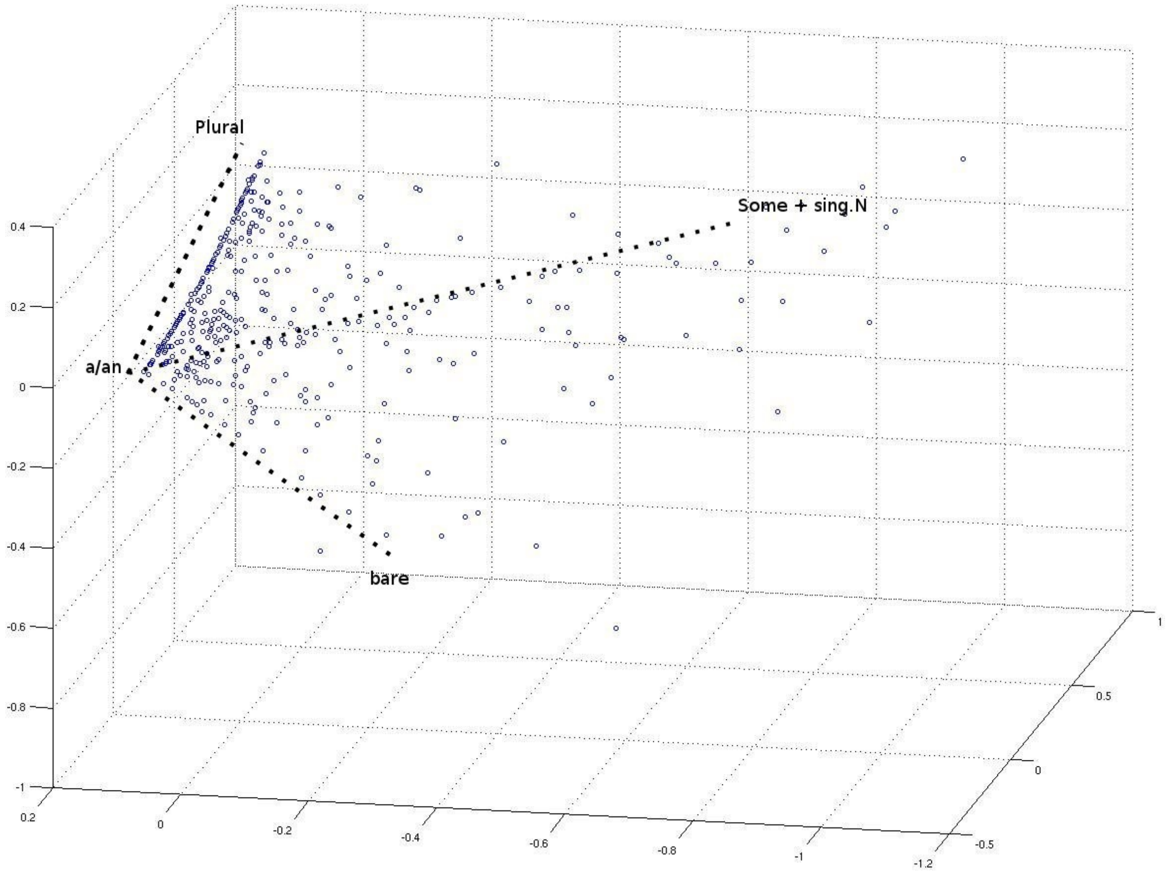


Figure 2.14: Visualization of the nouns in the Brown's section of CHILDES corpus in three dimensions, from multi-dimensional scaling.

2.4. Discussion:

This is to our knowledge the first wide scale examination of cross-linguistic variation in the expression of the mass count distinction, which attempts to investigate the question of the degree to which the distinction is driven by perceptual-semantic attributes. Previous discussions in terms of data have stayed more or less at the level of the anecdotal. Our major contributions to the discussion are to show that the relation between such universal perceptual-semantic attributes and syntactic usage in specific languages is very weak; as is the relation between languages: There is a core group of count nouns where semantic atomicity corresponds directly with count syntax, but beyond this there is indeed widespread cross-linguistic variation in whether or not a concept is expressed via count syntax. In our sample of 1,434 nouns, in the five languages excluding Marathi, approximately 50% were what we would call ‘robustly count’, however only 392 were robustly count cross linguistically. We have little to say about core mass nouns, of which there were few or none in our sample. This might conceivably be because of the way in which we chose our data base, rather than because of the inherently lower number of mass nouns in the languages. We leave it to other studies to identify a significant core group of mass nouns, cross-linguistically. However the frequency distribution from the native speakers is similar to the corpus obtained distributions.

We have made a number of observations which are relevant to the discussion of the mass/count distinction:

I. Semantic or ‘real world’ attributes do not lead in a straightforward manner to individual syntactic rules in the mass/count domain, hence we have to probe a potential mapping, for any given natural language, between semantic attributes and a constellation of multiple syntactic rules. The obvious alternation i.e. atomic vs. homogeneous does not predict mass vs. count morphosyntax. This provides solid statistical support for the theoretical discussion in [Gillon 1992; Chierchia 1998; Rothstein 2010], and many others.

II. When probing this domain with multiple syntactic usage alternatives, the distribution of 1,434 frequently occurring nouns in six natural languages is typically very far from binary. The largest single class of nouns in five of the six languages was the pure count prototype, i.e. the nouns classed ‘count’ by all syntactic probes. The rest are distributed in a graded fashion, with fewer and fewer nouns having more usage properties opposite to those of pure count nouns. Out of the 1,434 nouns, on average 873 were ‘pure’ count in a single language, range [693–1058], when excluding Marathi (where the figure was 255), but only 392 were 'pure count' in all other five ('typical') languages.

III. Outside of the pure count nouns, the correspondence between languages is weak, even when considering a single matching usage marker in each of the five non-exceptional 'typical' languages in the sample. In other words, learning what is a pure count noun, in any of these five languages, gave no significant clues as to the content of the pure count class in any of the other languages, beyond the 392 nouns which were pure count in all languages.

IV. Marathi differs from the other 'typical' languages in having a substantial fraction of nouns close to a pure mass prototype, particularly among abstract nouns, and a distribution closer to bimodal.

V. The semantic attributes that may be at the origin of the syntactic usage properties are distributed similarly, across concrete nouns, to the typical syntactic distribution, with most concrete nouns having 'count-like' attributes, and gradually decreasing proportions showing progressively more mass-like attributes (as seen from figures 2.3 and 2.5).

VI. Despite the overall similarity between distributions, of semantic attributes and of syntactic usage properties (in all languages tested except Marathi) the correspondence in position along the main mass/count dimensions between semantics and syntax is very weak, even for concrete nouns. Quantitatively, in terms of variance it is midway between fully matching and random, and in terms of mutual information it is close to random. The different range reflects the non-linearity of the MI measure, but both measures point at the weakness of the observed correlation.

VII. Similarly, the correspondence between languages is weak, whatever measure is used.

VIII. Taking into account the detailed attributes and syntactic rules, rather than only the main mass/count dimension, the correspondence remains weak.

IX. There is considerable variability also among informants of the same language; part of which may be due to the testing paradigm.

X. A similar distribution along the main mass/count dimension can be gauged from 1,551 nouns extracted from the adult section of the English-language CHILDES database, after a different analysis, namely in terms of graded rather than binary syntactic usage frequencies.

> The three main dimensions of syntactic variability of nouns in the CHILDES database describe an asymmetrically loaded pyramid: most nouns are countable, and simply vary in their plurality at each instance; many fewer nouns span the other two dimensions, characterized by an increasing frequency of use in bare form, and of use with some+singular form, both mass-like attributes.

First, we have provided solid empirical evidence that count syntax is not a direct reflection of atomicity in the denotation. Our initial aim to quantify the correlation (which we had presumed strong) between non-homogeneous nouns and count syntax could not reach beyond a core group of 392 nouns which pattern as pure count in all languages checked, excluding Marathi. This indicates a weak correspondence between perceptual/semantic and grammatical or morphosyntactic properties. Note that the 392 cross-linguistically count nouns included approximately 27% abstract nouns (284 concrete and 108 abstract), thus it is not even possible to argue that count syntax correlates directly with concrete atomic entities. Beyond this group of 392 nouns, the low level of mutual information between any two languages indicates language-specific grammaticalisation of the distinction. This means that it is no longer possible to assume a general correlation between atomicity and count, and homogeneity and non-count. A preliminary examination of the 284 items in the pure count group which are concrete rather than abstract indicates a high number of [+animate] nouns, in particular individuals of a certain profession (scientist, nurse, preacher, slave, spectator), nouns denoting buildings with a particular function (library, bank, apothecary) and nouns denoting artifacts that individuals stand in an one-to-one relation with (wallet, watch, handkerchief). All these predicates are atomic in an absolute sense, since they come in inherently individuable units, but they also frequently occur in contexts in which a particular instantiation of the predicate is perceptually salient. Thus it is plausible to posit that atomicity may be a necessary condition of a non-abstract noun being robustly count cross-linguistically. However, beyond this there are no straightforward generalizations.

The fact that these 284 nouns constitute between a third and a half of the robustly count nouns, in any particular language, indicate that beyond this weak generalization the grammaticalization of the correlation between atomicity and count differs from language to language. Furthermore, there are 108 abstract nouns in the pure count group, where the criteria for atomicity are by definition not well defined (since ‘atomicity’ is usually taken to express non-overlapping properties of matter). One can at this stage hypothesize potential criteria, for example, individuation via events: nightmare, appointment, and crash are all robustly count and non-concrete and atomic instantiations can be potentially be individuated via temporally located events. But this requires a notion of event individuation, itself problematic (see, e.g., Parsons 1990) and even then, leaves open the question of which event-types are ‘inherently atomic’ and which not. The conclusion for the linguist is that exploration of the basis of the mass/count distinction must be language particular, and will involve semantic features far beyond the homogeneous/atomic distinction.

We can draw a second theoretical implication from our results. We have seen that, for each language (again excluding Marathi), the approximately 50% of nouns which are not purely or robustly count in almost all cases cannot be characterized as ‘pure mass’. These nouns are located at varying distances from the pure count class, depending on how many non-count features they have. This could be taken as support for the view that mass/count syntax is imposed on a neutral root, that it is appropriate to talk of mass or count ‘usage’, and that essentially, noun roots are flexible and can appear in either context. This is the view taken in [Borer 2005], who claims that ‘being a count noun’ is an exoskeletal phenomenon, the result of count syntax being imposed on a neutral syntactic root. Our data, however, show that approximately 50% of the nouns in each language do show a consistent count pattern, and furthermore, as stressed above, beyond the first 392 nouns, the choice of which nouns are used consistently as counts is specified within a language (subject to some idiolectal variation) and not across languages. This suggests that count syntax is a lexical specification, and that beyond a core group, it is specified independently for each language.

A third point is that our data reveals cross-linguistically (again excluding Marathi) a large group of pure count nouns, and no comparable group of mass nouns. This may be taken to support the widely accepted view [e.g., Chierchia 1998, Borer 2005; Rothstein 2010] that mass syntax is the default case, and that count nouns are derived from mass nouns via some form of operation, which results in their sharing common properties. The degree to which Marathi differs from the other languages studied also forces us to realize that languages with a mass/count contrast may differ quite radically in how they implement it, and that the division of languages into those which have a count/mass distinction and those which do not tells us little about typological variation.

The overall conclusion is that the questions that linguists have been asking should be reformulated: Instead of looking for a general semantic characterization of the mass/count distinction which will explain the grammatical distribution cross-linguistically, linguists should be looking for language-specific patterns or generalizations, indicating that in a particular language, certain lexical classes are or are not grammaticalised as count. (For example, a cursory examination of the data indicates that Marathi is very restricted in allowing count syntax for abstract nouns.) If there are cross-linguistic generalizations, we might expect for them to have an implicational structure in the sense of [Greenberg 1963], i.e. we could look for patterns of the form: If lexical class C1 is pure count, then lexical class C2 is also pure count. But it is an open question whether we would find them at any significant level. We should avoid classifying nouns as ‘count’, ‘mass’ or ‘flexible’. In particular, our data show that non-robustly count nouns are flexible in different ways and to different degrees. What these ways and degrees are is still to be investigated.

If there is a general characterization of the mass/count distinction, then it probably is in terms of how the denotations of count (or mass) predicates are represented in the language, rather than in terms of any real-world feature. For example, [Rothstein 2010] suggests that count nouns denote entities which are indexed for the context in which they count as atomic. This leaves place for particular languages to rank features which contribute to contextual salience, or to give them different weights, which might then influence patterns in classifying nouns as count. Features which weigh heavily in their contribution to count syntax in all languages would result in the set of pure count nouns cross-linguistically. In any case, the set of robust count nouns and the lack of a set of robust mass nouns indicate that we are more likely to find a general semantic characterization of count nouns than of mass nouns.

At a deeper epistemological level, not only is mass count syntax largely left undetermined by semantic attributes, it is also mistaken to regard it as a binary or quasi-binary structure. The distribution of syntactic usage properties is very far from bimodal in five out of the six languages tested, in fact it has nothing to do with bimodality. One is led to think of this grammaticalisation as a graded self-organization process, operating within languages and to some extent within individual speakers, and driven only to a limited extent by universal attributes, and plausibly governed or at least constrained by language specific principles. However, at this stage we cannot tell to what degree the grammaticalisation is governed, beyond the universal semantic or perceptual principles that we have attempted to quantify, by language-specific principles of different nature, such as cultural factors, historical accidents, individual language acquisition history, even context dependence within individual speakers. What is already clear, however, is that a domain of grammar, that to the non-specialist may seem rather straightforward, in fact opens new vistas on the character of what are improperly called language ‘rules’.

Appendix A

The following tables are the equivalent of Table 2.1, for languages other than English.

No.	Syntactic Questions
1.	Can the noun be used with ‘a(n)’? (անորոշ գոյական +’մի’ հոդ)
2.	Number distinction: Can the noun be used with plural form? (հոգնակի թիվ)
3.	Can it be used in combination with numerals? (համադրում թվականների հետ)
4.	In combination with classifiers or measure phrases that manipulate number? (համադրում դասակարգիչների հետ)
5.	Can the noun be used with ‘every’/’each’? (Ամեն/յուրաքանչյուր)
6.	Can it be used with ‘(a) little’? (մի քիչ)
7.	Can it be used with ‘(a) few’? (մի քանի)
8.	In combination of ‘many’ + plural form of noun? (շատ + գոյականի հոգնակի թիվ)
9.	In combination of ‘much’ + singular form of noun? (շատ + գոյականի եզակի թիվ)

Table A1: List of questions used in Armenian to compile the usage table.

No.	Syntactic Questions
1.	Can the noun appear in the singular?
2.	Can the basic form appear with af (as in af yeled lo ‘ana, ‘not a single boy answered’)?
3.	Is there a plural form?
4.	Can the plural form of the noun appear with a number?
5.	Can the singular form of the noun appear after kol ‘every’?
6.	Can the singular form appear with kzat, me’at, harbe (‘a little, a little, a lot’)?
7.	Can the noun appear with tipa (literally ‘a drop’)?
8.	Can the noun appear with a classifier?
9.	Is it possible to say 10 + the singular form of the noun?

Table A2: List of questions used in Hebrew to compile the usage table.

No.	Syntactic Questions
1.	Can it be used with ‘many’/‘few’?
2.	Can it be pluralized?
3.	Can it be used with ‘every’?
4.	Can it be used with numerals?
5.	Can it be used ‘with a lot of’?

Table A3: List of questions used in Hindi to compile the usage table.

No.	Syntactic Questions
1.	Can the noun be in singular form with the indefinite article (un/o/a)?
2.	Can it appear (suitably pluralized) with a numeral (due, tre)?
3.	Can the noun appear with at least one singular indeterminate quantifier (molto/molta/un po' di)? Note: non molto should not be considered.
4.	Can the singular form be preceded by indefinite quantifier qualche?
5.	Can the singular form be preceded by exact quantifiers (chili di, litri di)?
6.	Can the singular form be preceded by non molto (‘not much’)?
7.	Can it have a plural form with a definite article (i, gli, le)?
8.	Can the plural form be preceded by exact quantifiers (chili di, litri di)?

Table A4: List of questions used in Italian.

No.	Syntactic Questions
1.	Can it appear with a numeral?
2.	Can it be used in combination with an exact quantifier (kilo, liter)?
3.	Can it be used with the article ek (‘a’)?
4.	Can it be pluralized?
5.	Does the morphology change when pluralized?

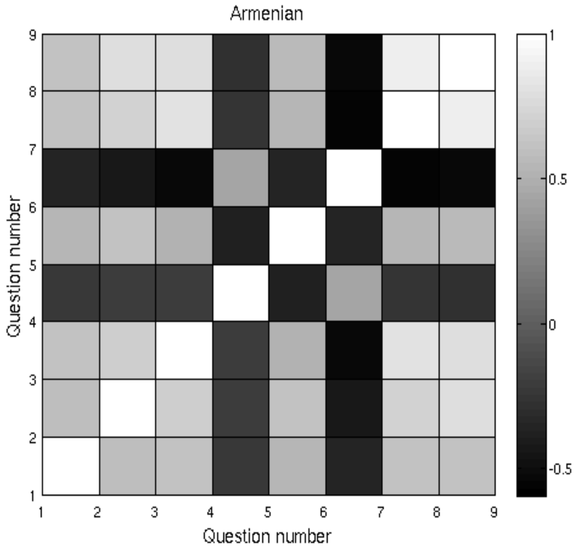
Table A5: List of questions used in Marathi.

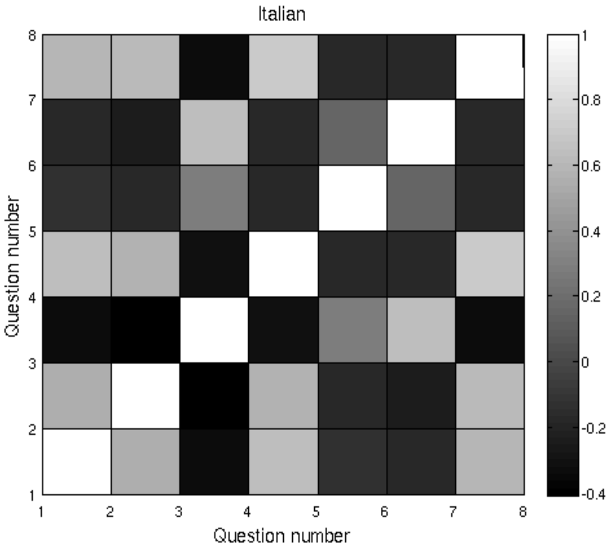
Note: The questions were posed to the informants in their respective languages, not in the English translation.

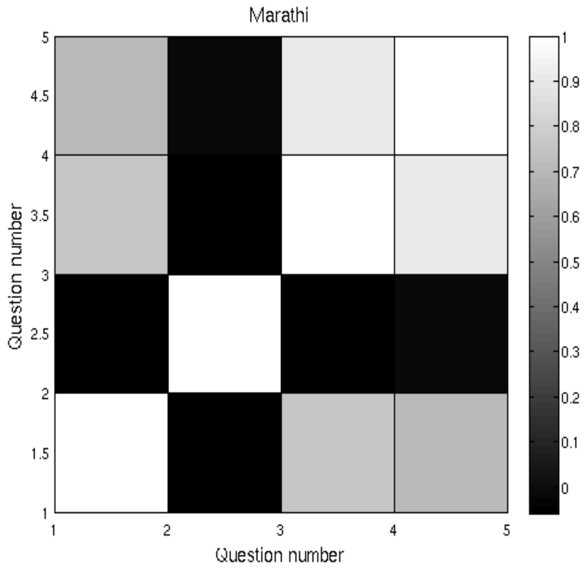
A6: Fraction of 'yes' answers to the questions above.

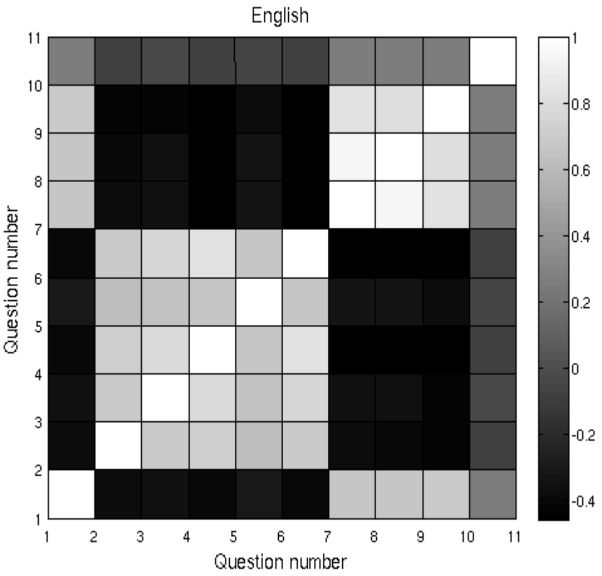
Question No.	Armenian	Italian	Marathi	English	Hebrew	Hindi	Semantics
1	0.90	0.89	0.57	0.37	0.99	0.91	0.11
2	0.90	0.81	0.09	0.87	0.77	0.90	0.45
3	0.83	0.27	0.67	0.87	0.88	0.78	0.54
4	0.05	0.85	0.66	0.83	0.84	0.86	0.52
5	0.94	0.08	0.23	0.85	0.91	0.83	0.44
6	0.18	0.16	-	0.85	0.30	-	0.04
7	0.85	0.88	-	0.30	0.16	-	0.04
8	0.85	0.01	-	0.29	0.08	-	0.04
9	0.14	-	-	0.34	0.05	-	0.43
10	-	-	-	0.19	-	-	0.14
11	-	-	-	0.12	-	-	0.22
12	-	-	-	-	-	-	0.41

A7: Correlation between question pairs.






Marathi
Question number
Question number
1
1.5
2
2.5
3
3.5
4
4.5
5
1
2
3
4
5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0


English
Question number
Question number
1
2
3
4
5
6
7
8
9
10
11
1
2
3
4
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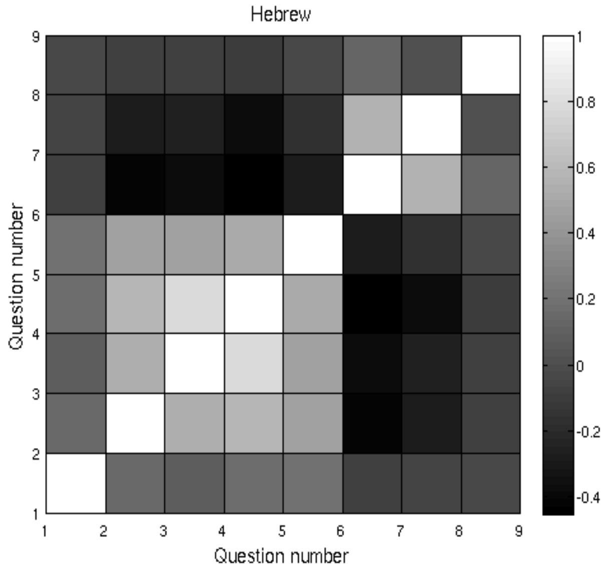
7
8
9
10
11
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4


Hebrew
Question number
Question number
1
2
3
4
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6
7
8
9
1
2
3
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0.6
0.4
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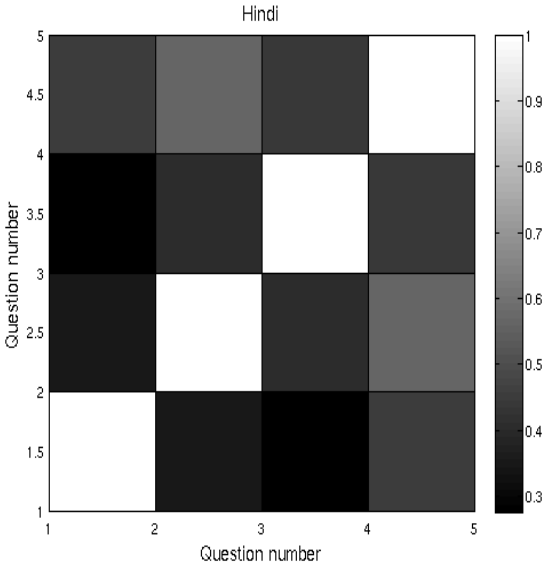
0
-0.2
-0.4

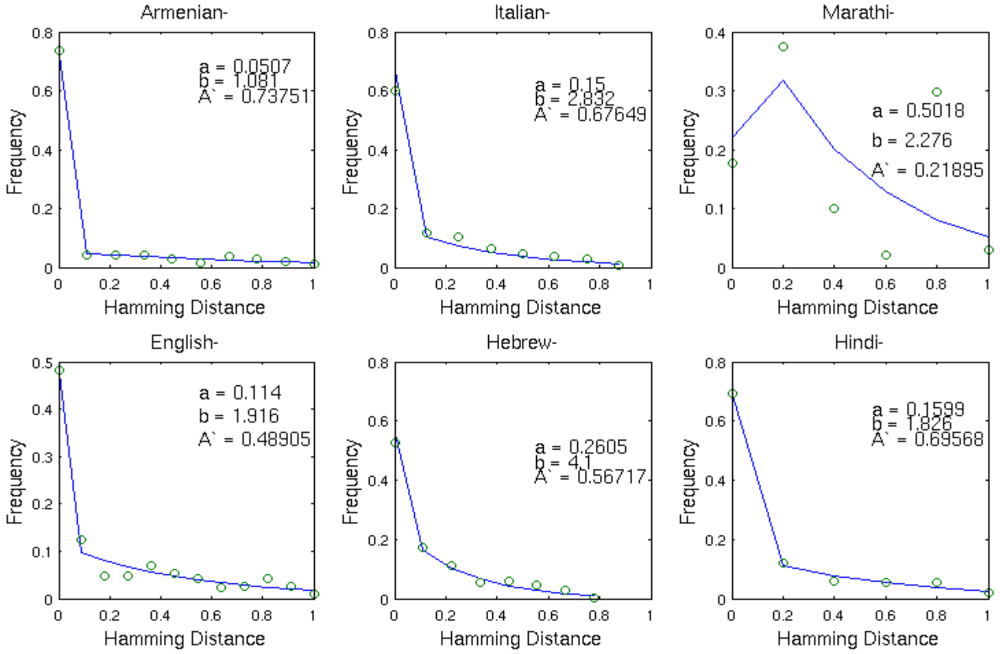

Hindi
Question number
Question number
1
1.5
2
2.5
3
3.5
4
4.5
5
1
2
3
4
5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3

A8: Subgroups at each hamming distance.

	Armenian			Italian			Marathi		
Group	# of Subgroups	Mean Size	Std	# of Subgroups	Mean Size	Std	# of Subgroups	Mean Size	Std
1	1	1058	0	1	863	0	1	269	0
2	7	8.7142857143	10.531132983	8	20.625	20.975751306	5	9	191.86140831
3	15	4	7.3775721907	19	7.8421052632	18.454939591	6	21	39.766401229
4	17	3.5294117647	4.302871818	17	5.2352941176	5.4946498042	6	12	4.5934736311
5	13	3.0769230769	3.2522181779	19	3.5263157895	4.6111991819	5	1	177.1205804
6	9	2.1111111111	2.260776661	13	4	6.5064070986	1	40	0
7	14	3.7142857143	4.7137861873	5	8	9.0829510623	-	-	-
8	9	4.2222222222	6.8698212818	3	3	3.4641016151	-	-	-
9	4	7.75	7.5443135318	-	-	-	-	-	-
10	1	15	0	-	-	-	-	-	-
	English			Hebrew			Hindi		
Group	# of Subgroups	Mean Size	Std	# of Subgroups	Mean Size	Std	# of Subgroups	Mean Size	Std
1	1	693	0	1	757	0	1	994	0
2	11	16.363636364	28.688927227	9	27.777777778	30.987004445	5	34.4	47.794351131
3	19	3.5263157895	4.9929775246	19	8.3684210526	16.197429585	10	8.2	7.4803446148
4	21	3.1904761905	4.8230596888	19	3.85	4.6822734019	10	7.6	7.3212020871
5	12	8.4166666667	22.589049372	19	4.3684210526	5.4488584566	5	15.8	16.783920877
6	17	4.5882352941	9.0488868344	12	5.3333333333	8.3047996345	1	31	0
7	21	2.8571428571	5.4155859728	9	4.2222222222	7.8386506775	-	-	-
8	15	2.2	2.0071301474	5	1.2	0.4472135955	-	-	-
9	16	2.4375	3.2035136959	-	-	-	-	-	-
10	12	5.1666666667	11.01101377	-	-	-	-	-	-
11	8	4.875	7.1999503967	-	-	-	-	-	-
12	1	15	0	-	-	-	-	-	-

A9: Exponential fit of the MC dimension distributions

Fit: $y = A' + a e^{-bx}$



Chapter 3

Attempts of a competitive neural network to decipher the mass-count information

3.1 Motivation and a brief summary:

3.1.1 Introduction:

In the previous chapters we looked at a detailed statistical investigation of the mass-count distinction with regards to its syntactic-semantic features, in this chapter and the ones following, we move our attention to the 'cognitive' aspects of language processing. Our aim is to study the properties of associative neural networks as a mechanism to sub serve syntactic processing in the brain. In the current chapter a competitive neural network is applied to the mass-count database to look at the self organising of nouns and markers in the neural space while later chapters involve the study of a recurrent associative network which attempts to model global cortical mechanisms in the brain.

The question of how the brain acquires language can be posed in terms of its ability to discover, from exposure to a corpus, the syntactic structure of a specific natural language and its relation with universal semantics. This has been a subject of study and of intense debate for the past few decades (Barner and Snedeker, 2005, Barner and Bale, 2009). Natural language acquisition appears to presuppose certain cognitive abilities like rule recognition, generalisation and compositionality. These high-level abstract concepts should be realized in the language domain and in specific sub-domains by general-purpose neural processing machinery, since there is no evidence for dedicated circuitry of a distinct type for each sub-domain nor, for that matter, for languages a whole [Tomasello 2003; Ogrady 2007]. How can rule recognition and generalization be implemented in standard, vanilla neural networks? To explore this issue, we focus our attention on a sub-domain of syntax, namely the syntax of the mass-count distinction. Following up on the results of statistical analysis of the mass-count distinction in 6 languages, with relation to its cross-linguistic syntactic and semantic properties, we now aim to study the learnability of those syntactic properties by a basic neural network model, with the distant goal of eventually understanding how such processes might be implemented in the brain.

We briefly summarise some of our main findings. As many linguistics studies have pointed out, a simplistic mapping between homogeneity and mass syntax and/or atomicity and count syntax on the whole would imply that the expressions denoting the same real world objects would be count or mass cross-linguistically. However, this is not the case seen in Kulkarni, Rothstein and Treves 2013, words with a similar interpretation may be associated with very different arrays of syntactic properties cross-linguistically. A noun which is associated with a count array in one language may not be associated with a count array in a different language. Furthermore, over a sample of 6 different languages we saw that there is no binary divide into mass/count nouns, but rather a continuum with a small group of nouns which are count with respect to all properties, and then a range of nouns which are more of less count depending on how many count properties they display. This makes the mass-count distinction an interesting linguistic phenomenon to model with its rich diversity and lack of simple intuitive correspondence between syntax-semantics. Acquisition of the correct syntactic usage of the mass-count nouns, by a child learning to speak, however should be brought about by the same neural mechanisms in the brain that bring out other cognitive functions. Thus the same neural principles of associative learning, proposed several decades ago, based on Hebbian plasticity of co-active neurons, should serve as a functional mechanism to acquire the knowledge of mass and count. We explore here, to what degree a standard neural framework with unsupervised learning, can be useful to gain knowledge of what is mass and count in a particular language.

3.1.2 Network modelling:

Our goal in the current study is to assess the learnability of syntactic and semantic features of the mass-count distinction using simple neural networks. Artificial neural networks have a long history as a method for neurally plausible cognitive modelling [Hopfield 1982; Elman, J, 1991;

Nyamapfene A. 2009], and can be endowed with properties including feature extraction, memory storage, pattern recognition, generalisation and fault tolerance. Understanding how humans might acquire the capacity for handling syntax in a specific sub-domain might start from encoding syntactic/semantic knowledge into a neural network, which self-organizes with a prescribed learning algorithm to recode that information in a neurally plausible format. That way one may draw parallels about governing principles in the brain that bring about the acquisition of syntax. Taking cues from biological neurons, most artificial neural networks employ 'Hebbian' plasticity rules, wherein the synaptic connection between two units is strengthened if they are activated nearly simultaneously, thus leading to associative learning of the conjunction or sequence of activations. (See chapter 4, section I, II for more on neural networks and learning rules). Here we consider a competitive network, a simple self-organising network which through 'unsupervised' learning may produce a useful form of recoding. A competitive network, under the right conditions, is able to discover patterns and clusters in a stimulus space and to train itself to correctly identify and group inputs that share a close resemblance to each other. A competitive network is particularly interesting in our case since much of linguistic information during language acquisition is rather 'discovered' than explicitly taught. Moreover, mass and count nouns have been shown to exhibit differential evoked potential responses, both with a syntactic and with a semantic stimulus [Chiarelli V, 2011]. We aim to study the performance of a simple competitive network in view of understanding how well can syntactic and semantic features of the nouns in our mass-count database be accommodated within a single network, thus exploring if the network can indeed achieve some rule-recognition that will allow it to successfully categorise nouns in the syntactic mass-count space.

3.2 The network:

A) Classification of nouns:

Our network consists of a single input and a single output layer. At the input layer each unit represents a syntactic feature ('numeral', 'a/an' etc) in case of the syntactic network or a semantic feature ('fixed shape', 'fluidity' etc) for the semantic network. We label the network as 'syntactic' or 'semantic' based on weather the network is classifying syntactic or semantic markers. The input layer is binary, and for each noun given as input a given unit can be active (activation value 1) to indicate that the feature can be attributed to the noun, or inactive (value 0) to indicate that it cannot.

Thus a single learning event for the network includes the application of a binary input string containing the syntactic or semantic information pertaining to a single noun, activity propagation to the output units, and modification of the synaptic weights according to the prescribed learning rule.

B) Classification of markers:

Similarly, instead of self-organizing an output representation of nouns, we explore the self-organization of syntactic features (‘markers’). Rather than an input noun with the features as components, we apply as input a single feature/marker, with the nouns as components, i.e. there are a few very long input string instead of many short strings.

On the output side, the number of units is variable, determined by the simulation requirements. Unlike the input units, outputs units are graded, taking continuous values in the range of 0 to 1.A competition amongst the output units based upon their activation levels decides the final output level of each unit.

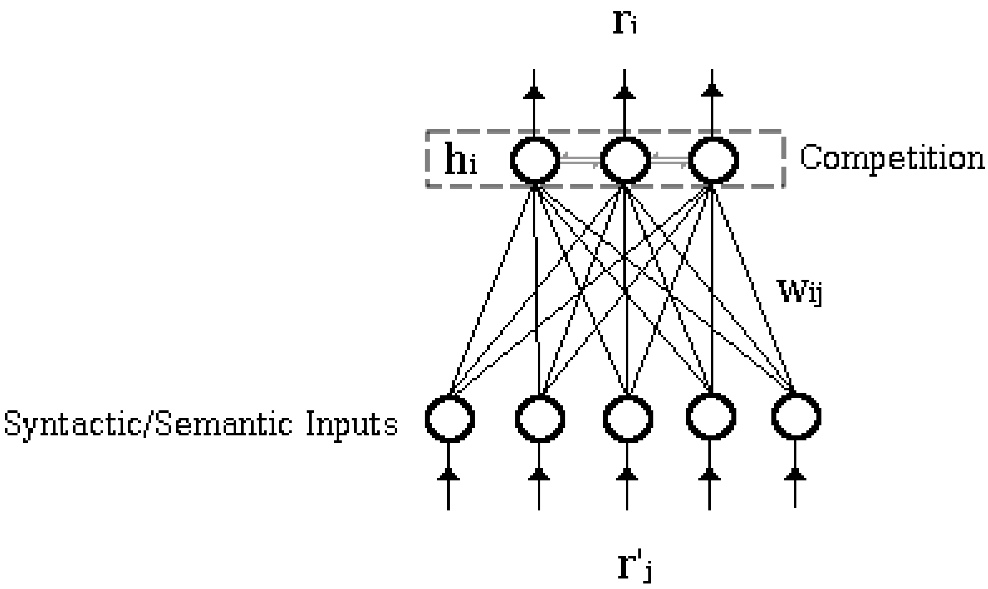


Figure 3.1. Schematic diagram of the artificial neural network, showing an input layer where units are binary strings containing syntactic/semantic information of nouns and an output layer where units compete with each other to produce graded firing rates based on connection weights and on competition.

We use a fully connected network, where each input unit j is connected to each output unit i with a synapse whose connection strength is given by w_{ij} . The training sequence is executed as follows:

An input is presented to the network and the activation h_i of each output unit i is calculated as

$$h_i = \sum_{j=1}^{N} r'_j w_{ij}$$

where N is the number of input units and r' is the input vector. The w_{ij} 's are initially set at random values, which randomly causes certain output units to have a higher activation and lower activation in others.

The final output firing of each unit r , is decided after setting up the competition between output units as

$$r_i = \frac{e^{\left(\frac{h_i}{T}\right)}}{\sum_i e^{\left(\frac{h_i}{T}\right)}}$$

Here T governs the strength of the competition, lowering T makes the competition stronger and as T approaches 0 it becomes ‘winner take all’, a case where only the unit which wins has a maximum firing rate while all other units are suppressed to be inactive; whereas the competition becomes softer as T is raised higher, allowing more graded output firing rates. Firing rates are automatically normalised in the range of 0 to 1 by this form of the output function (thus also allowing a probabilistic interpretation of the firing rates of the units).

In the next step we adjust the weights w_{ij} according to the Hebbian rule, taking into account the input and output firing rates of the units obtained in the previous step. The learning rule here is slightly modified from the standard Hebbian rule to incorporate normalisation of the weights during learning in a biologically plausible way. Normalising weights is important since it prevents a small fraction of connections becoming too strong and resulting in the same units winning the competition each time. The weights are adjusted at each presentation of an input as

$$\delta w_{ij} = k r_i \left(r'_j - w_{ij} \right)$$

k in the above equation controls the learning rate, i.e the size of increments in the weights as new input-output pairs are presented to the network. The change in weight is proportional to the input and output firing rates, however the second terms restricts a monotonous increase in weights by causing a decay proportional to the activity of the output unit and to its existing synaptic strength.

One training iteration includes presenting each noun in the list once and the above process is repeated for the desired number of iterations.

We now move towards the results section where we apply the above mentioned unsupervised cluster discovering process to the participant data in 6 languages and semantics.

3.3 Categorisation of markers:

As described in section 3.2 B, we present as input the syntactic markers used in the classification of the nouns. Here an input vector is comprised of N units, where N is the number of nouns (784 in case of concrete nouns and 650 in case of abstract nouns), for each of the syntactic markers. Thus an input includes information on how that particular marker is used over all the nouns. Each input vector is presented once in one iteration, for 100 such iterations, which is also when the synaptic weight matrix is observed not to change with further iterations. We use 3 output units, $N_{out}=3$, with T = 0.1 and *k* = 0.01 for all languages except, Marathi (T=0.1, k=0.001). After obtaining the output firing rates for each input marker at the end of the iterations, we calculate the correlogram, representing how correlated the output vectors are with each other, hence giving information about marker categorization. We show the mean correlograms over 100 distinct network simulations.

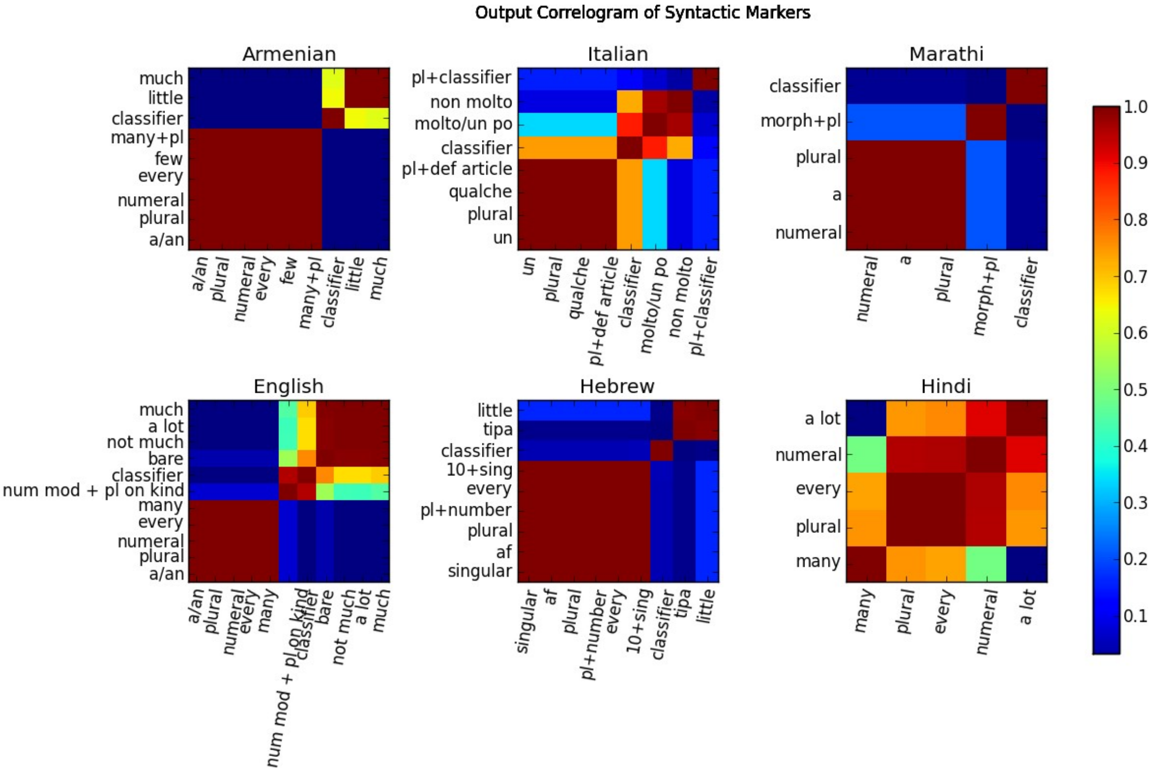


Figure 3.2. Correlograms for 784 concrete nouns in each of the 6 languages in our study. Dark blue regions represent complete lack of correlation (orthogonal vectors) while dark red regions represent congruent vectors.

The correlograms in Figure 3.2 allow us to visually identify markers that fall in the same category, as self-organized in the output of the network. High levels of correlation between two markers signify close proximity in the firing rates of the output units for that pair of markers, and are represented by light shades. For concrete nouns in Armenian, markers like ‘a/an’, ‘plural’, ‘numeral’, ‘few’, ‘every’ and ‘many+plural’ have a correlation of 1, thus occupying the same position in the output space of the network. These are markers that can be applied to count nouns and not to mass nouns. Instead, the typical mass markers of ‘measure classifier’ form an independent representation, whereas ‘little’ and ‘much’ share the same position in output space but distant from the count markers. Italian, Marathi, English and Hebrew follow the same Armenian line of grouping count markers together and having separate but nearby representation for mass markers, distant from the count markers. Hindi is different, as 4 of the 5 markers that were chosen appear to be ‘count’ in nature, but all show gradation within the broad count category.

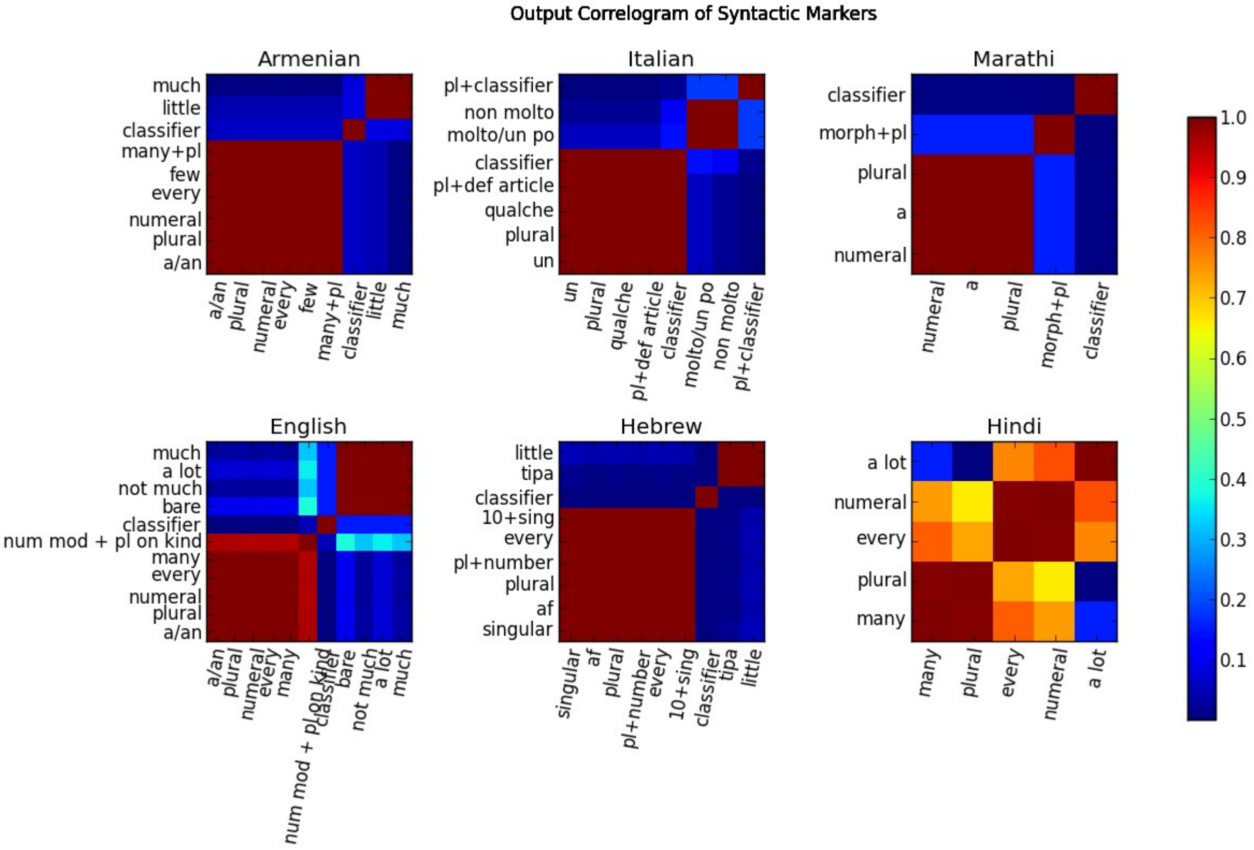


Figure 3.3. Correlograms, same as in figure 3.2, but for 650 abstract nouns. Note that markers are ordered in the same way as in figure 3.2.

Results are similar for abstract nouns except for Italian having fewer graded categorisation than for concrete nouns (figure 3.3) while for Hebrew ‘little’ and ‘tipa’ are co-incident.

The competitive network can be similarly tested on semantic features based on what value each feature assumes over all the nouns. As seen in figure 3.4, semantic features are neatly divided into mass and count features. Count features like ‘single unit’, ‘boundary’, ‘stable shape’ and ‘degradation’ all have a correlation of 1 with each other and 0 with mass features like ‘free flow’, ‘container shape’ and ‘mixing’. While ‘free flow’ forms a separate representation, ‘container shape’ and ‘mixing’ have the same output activation.

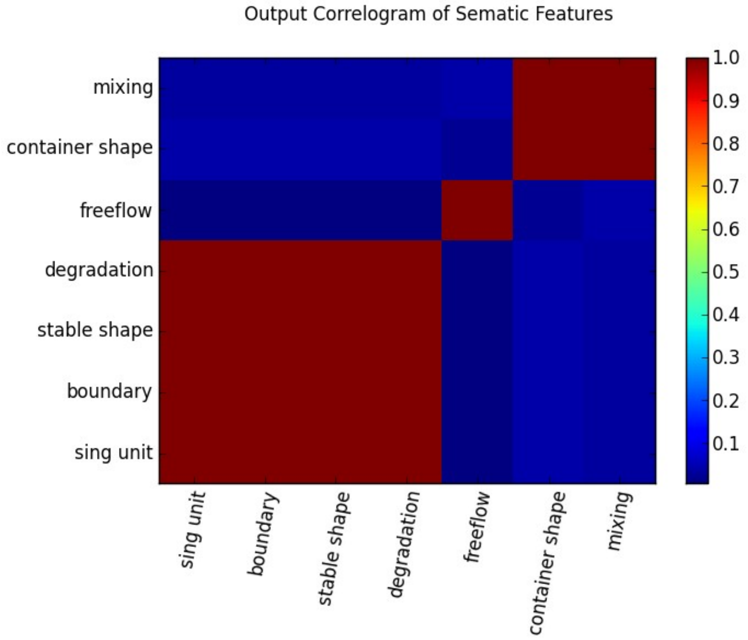


Figure 3.4. Correlogram of semantic markers for concrete nouns.

3.4 Categorization of nouns:

Similar to the process in section 3.1, we now present nouns as input to the network and visualize the activation of the output units. The input vector here consists of N units for each noun, equaling the number of markers for a language, hence containing information on how the noun is used over all the mass-count markers for that language. The parameters used are $N_{out}=3$ (the number of output neurons), T = 1, k = 0.01 and Iterations = 10 (except for Marathi, the parameters were: T=0.1 and k = 0.01). Figure 3.5 shows the position of the nouns in the 3-D output space, where each axis represents an output unit. Axes are selected such that x ,y and z, respectively, represent units in descending order of variance over the values of output activation they span. The colour of each point signifies where that noun (or cluster of nouns, since nouns classified as identical are co-incident) lies on the MC dimension as defined by the Hamming distance from the pure count string (see section 1.1 A). Red indicates a distance of 0, thus pure count, while yellow indicates a distance of 1, representing a 'mass noun'. (convergence of learning is shown in appendix B2)

Nouns are seen to approximately fall along a single line for all languages (a predominantly linear structure for English and Marathi), barring an outlier at 0 which represents inputs that are inactive for a noun. Moreover we can see a gradient from red to yellow, which implies that nouns, even though not completely faithful, to a great extent lie along a gradient from 'count' to 'mass'. We further visualise the distribution of nouns on this line, so as to assess the frequency of nouns in each cluster. The axis with maximum variance is selected and a histogram of the number of nouns in each cluster along this axis is plotted.

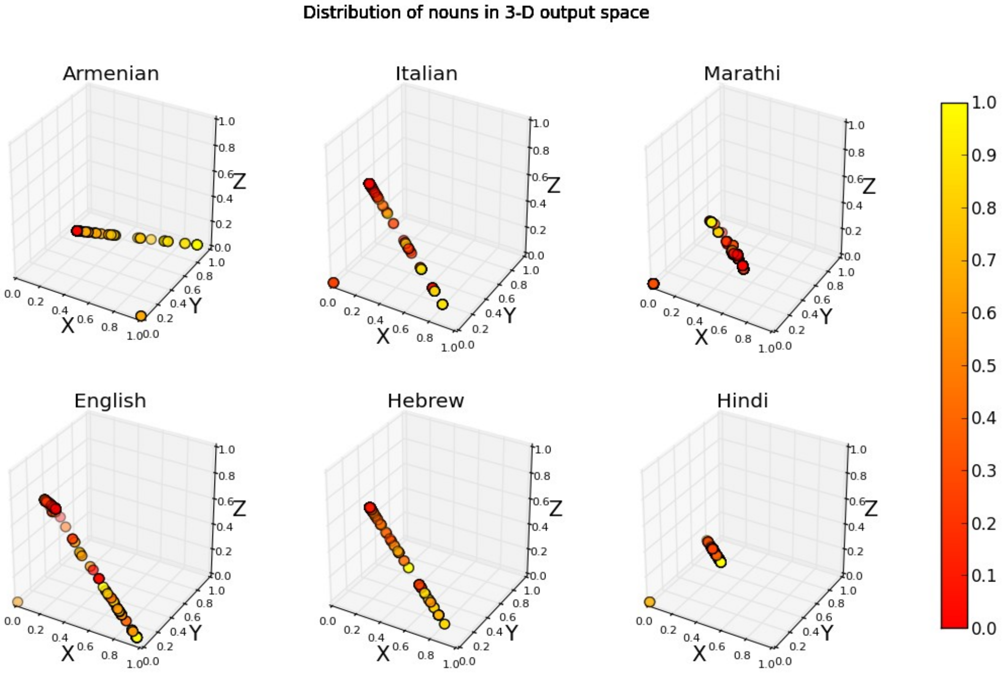


Figure 3.5. Position of 784 concrete nouns in the output space as defined by 3 output units in 6 languages. The gray scale indicates the Hamming distance of the noun on the MC dimension, from red = 'pure count' to yellow = 'pure mass'.

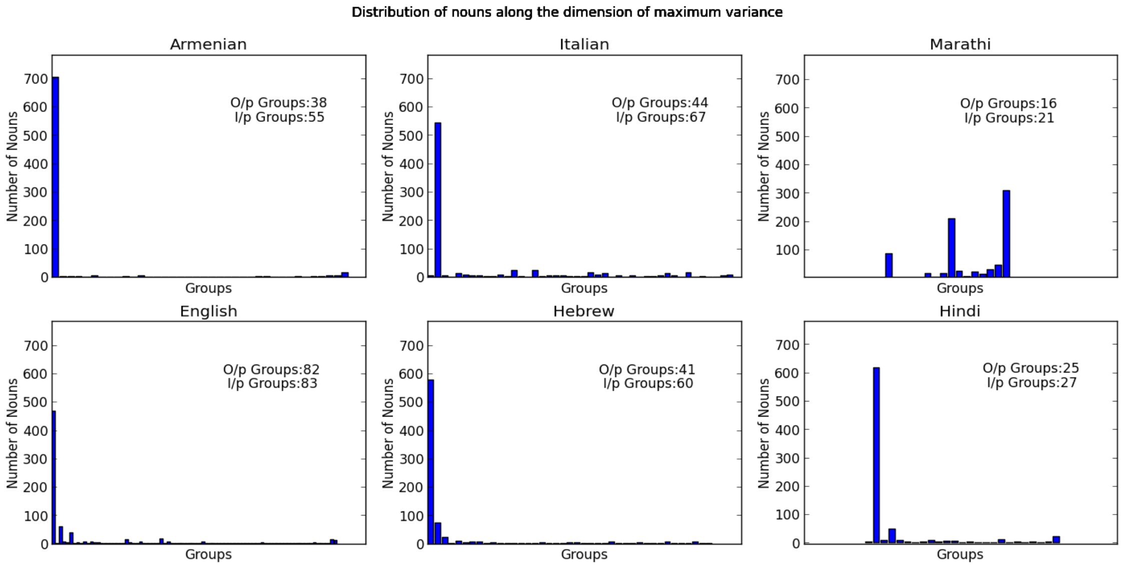


Figure 3.6. Histogram of nouns in reference to figure 3.5, along the axis of maximum variance.

Looking at the histogram of the linear alignment of clusters in the output space, we find that the cluster near the red end has the highest number of nouns, which is followed by smaller bars towards the mass end. Marathi is different, in having a significant cluster towards the yellow mass end too. Note that the axis of largest variance is inverted for Marathi with the count end on the higher side of X, this is because winning units are randomly selected.

It is interesting to note that a dimensionally reduced, entropy preserving representation of the mass-count nouns has a notional similarity to the concept of the MC dimension as in chapter 2 , figure 2.3. The MC dimension was introduced as a concept to better understand the mass-count division in terms of the 'pure count' string, but a competitive network with the appropriate parameters is able to bring about a roughly similar distribution without needing a prior ad-hoc definition.

Below are shown the same plots as in figure 3.5 and 3.6 but for 650 abstract nouns. Differences are seen in English : the group of clusters of red count nouns is more stretched in a separate direction, essentially having two separate lines for mass (yellow) and count (red) nouns.; Histograms on the dimension of maximum variance show a similar gradation from count to mass and the difference in Marathi with two significant peaks at the count and mass ends each.

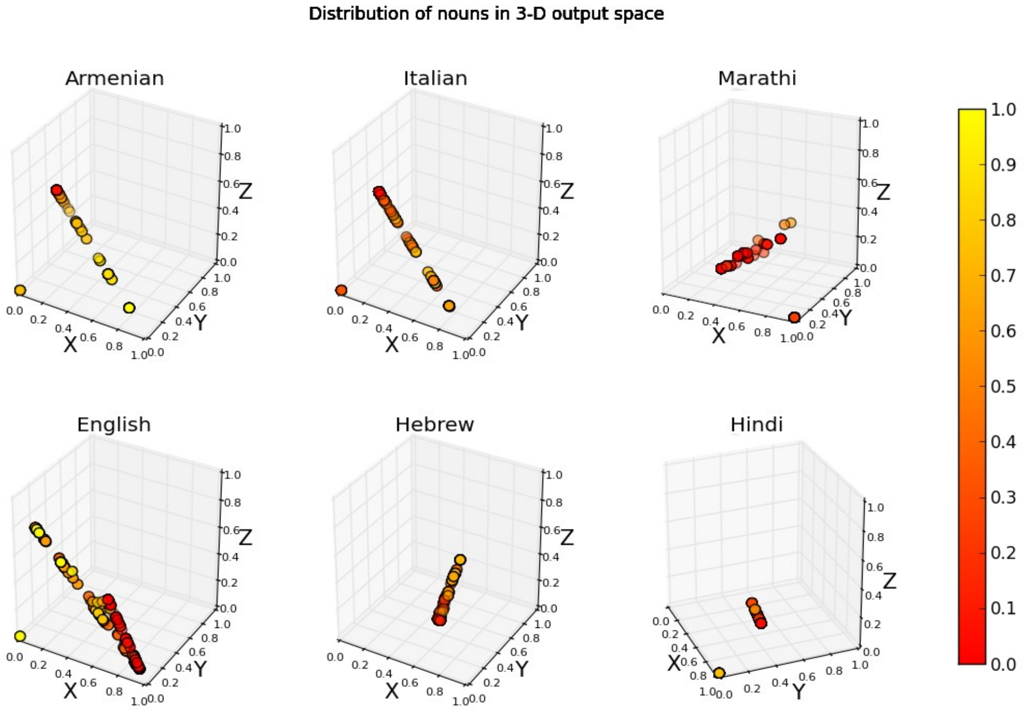


Figure 3.7. Position of 650 abstract nouns in the output space as defined by 3 output units in 6 languages. The gray scale indicates the Hamming distance of the noun on the MC dimension, from red = 'pure count' to yellow = 'pure mass'.

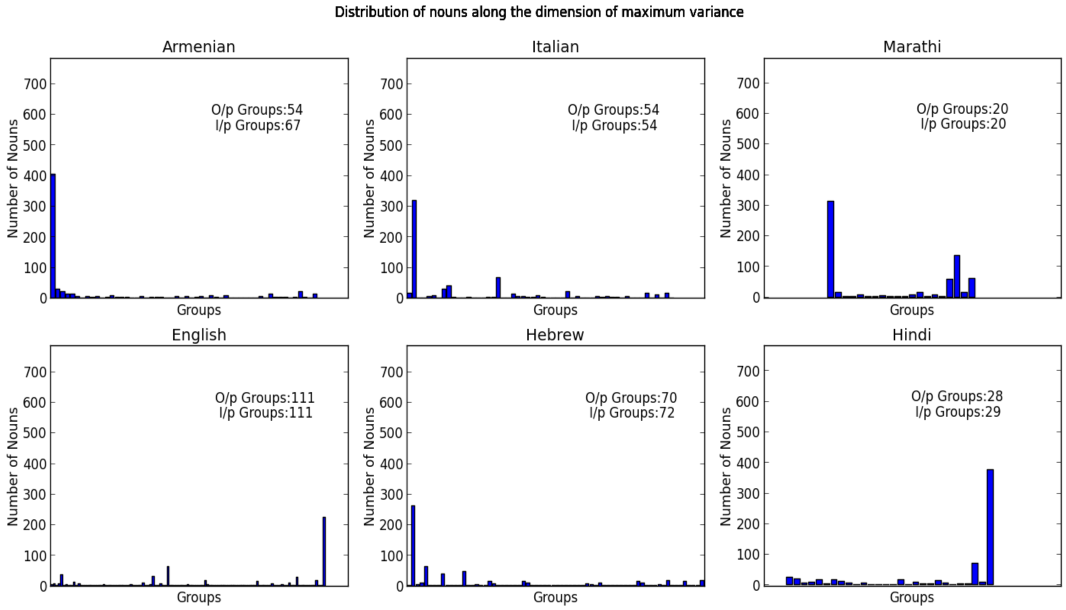


Figure 3.8. Histogram of nouns in reference to figure 3.7, along the axis of maximum variance.

3.5 The syntax-semantics interaction:

As we saw from the information theoretical analysis, the syntax and semantics of the mass-count distinction share only a weak direct link, in the core structure of the count class, chapter 2, figure 2.11. Thus acquiring the complete set of syntactic classes from semantic classes is not possible by any learning mechanism, due to a lack of a direct one-to-one correspondence. However it is improbable that syntax and semantics are independently learned without any mutual interaction during learning of mass-count concepts, and there is no evidence that either one is learnt before the other [David Nicolas, 1996]. From the classification of markers above, we see that broad categories of mass and count can indeed be extracted out of the data, interestingly for both syntax and semantics, thus rendering some semantic sense to the syntactic distinction. Classes of nouns however, formed from these markers do not reflect mass-count information in a straightforward manner between syntax and semantics. Hypothesising an underlying commonality of the mass-count divide between markers of syntax and semantics,we test the performance of the competitive network when syntax and semantics are simultaneously part of the input space during the learning phase, and test the correspondence between the syntactic and semantic classes after learning.

3.5.1 Quick summary of information measures:

We use mutual information as a measure to analyse the correspondence between two representations, encoded either in a syntactic network trained on input information about marker usage for the nouns in a particular language, or in a semantic network trained on information about the semantic properties of the nouns. Here we focus on systems that have undergone a slow process of self-organisation to categorise their inputs.

We first begin by calculating the entropy of the output of the network and the information it contains about the clustering of nouns in the input. Nouns are clustered together if they have exactly the same output firing rates. So in effect the output, labelled as O, contains n clusters $1 ... i ... n$, where each cluster contains nouns that are classified as identical by the network. Entropy is then defined as

$$H(O)=-\sum_{i=1}^{n} p(i)\log_2 p(i)$$

Where $p(i)$ is the 'probability' obtained as the relative frequency of the nouns in the cluster i .

To calculate mutual information between two representations X and Y we first obtain equivalence classes, i.e. groups where a particular set of nouns has been clustered into the same class in both X and Y. For example, if nouns like 'man' and 'dog' fall in the same cluster in X and are also found in the same cluster in Y, they belong to the same equivalence class. The joint entropy of X and Y, $H(X,Y)$ is then used to calculate the mutual information a

$$I(X;Y)=H(X)+H(Y)-H(X,Y)$$

Alternatively, one may consider a joint probability distribution table of nouns in X and Y from the equivalence classes, and calculate mutual information as

$$I(X;Y)=\sum_{i,j} p(i,j)\log_2\left(\frac{p(i,j)}{p(i)\,p(j)}\right)$$

Mutual information co-varies with the relevant individual entropies, so to facilitate comparisons we use normalised mutual information. When X and Y show completely unrelated clusters, $p(i,j)=p(i)p(j)$ the mutual information is 0, giving it a strict lower bound. It has a natural upper bound in the sense that the mutual information between X and Y can never be greater than the lower of the two entropies $H(X),H(Y)$. Thus for comparison purposes we define normalised entropy as $I(X;Y)/min(H(X),H(Y))$, which lies in the range of 0 to 1.

3.5.2 Processed Interacting Mutual Information:

First, to compare with previous results, we have calculated the baseline mutual information between syntax and semantics by providing only semantic information to the network, with no syntactic information during the learning phase. The mutual information was calculated between syntactic data and the output of the semantic competitive network. When no syntactic information is present at the inputs, the resulting mutual information is about equal to the mutual information between the syntactic and semantic data calculated using the procedure in chapter 2, section 2.2.2-B

A) The effect of competition strength:

The competitive network brings about a dimensional reduction from a high dimensional input space to a lower dimensional output space defined by the number of output units. Furthermore the strength of the competition affects how different input clusters collapse onto each other, depending on the distance between them. These processes reduce the entropy from its input value, which critically affects the available mutual information between two data sets, when measured at the output. We observed that the network failed to consistently learn when the number of output units, N, was less than 3 and that there was no noticeable change in information measures for N ≥ 3. Thus we set N =3 and varied the competition strength T to see the effect on the output entropy of the network. Figure 3.9 plots the ratio of the output entropy to input entropy of the network for various values of T. Since no information is added during processing, the output entropy is at most equal to the input entropy or less otherwise.

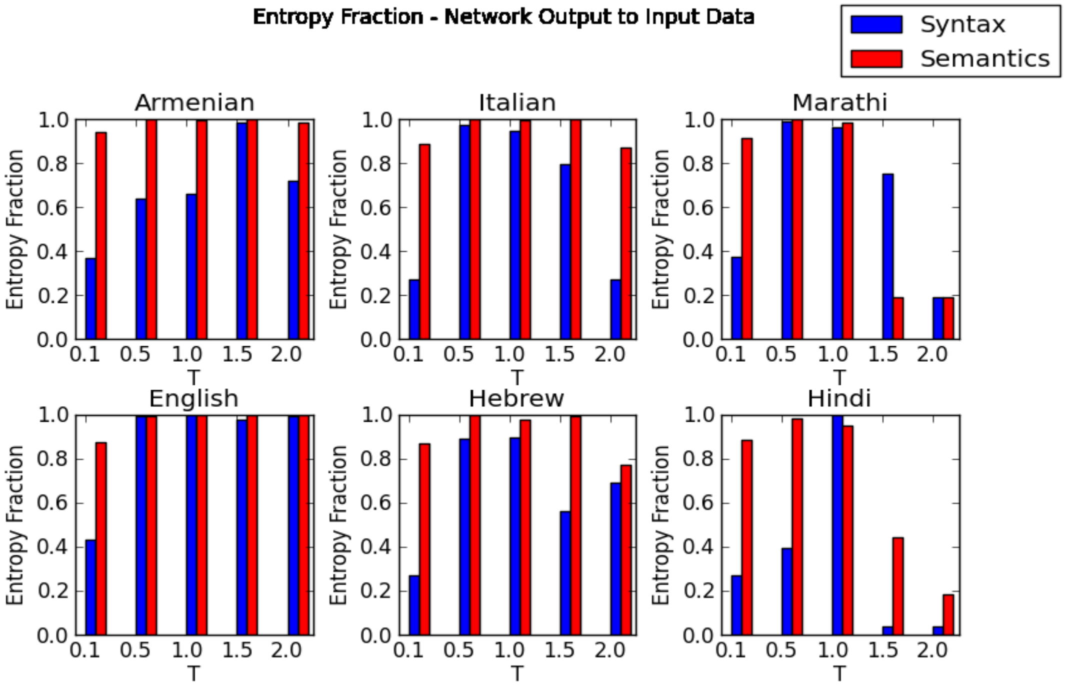


Figure 3.9. Variation of the output entropy to input entropy ratio for different values of the competition strength, T.

Figure 3.9 shows, at around T = 1, input and output entropies are comparable for all languages (except Armenian at 1.5), and tends to drop on either side (especially for syntax), thus we select T = 1 as optimal value of competition (when maximum information is retained after processing), to calculate mutual information between syntax and semantics.

B) Combined learning of syntax and semantics:

Syntactic information is then provided to the network in a partial manner, in a proportion γ, which signifies the fraction of input units of the syntactic segment of the input string that are set to the activation levels of the syntactic string of a particular language. $\gamma=0$ represents when none of the syntactic input units are receiving any information and are set to 0; while $\gamma=1$ implies all of the syntactic information is present; for in-between cases a fraction $1-\gamma$ units are randomly selected and set to 0. Thus we can vary the amount of syntactic information available to the network during learning and test the effect on the syntactic-semantic mutual information and weather the relevant syntactic and semantic classes are brought together in any systematic way. We train and test the network by providing the same proportion γ of syntactic inputs along with the semantic ones.

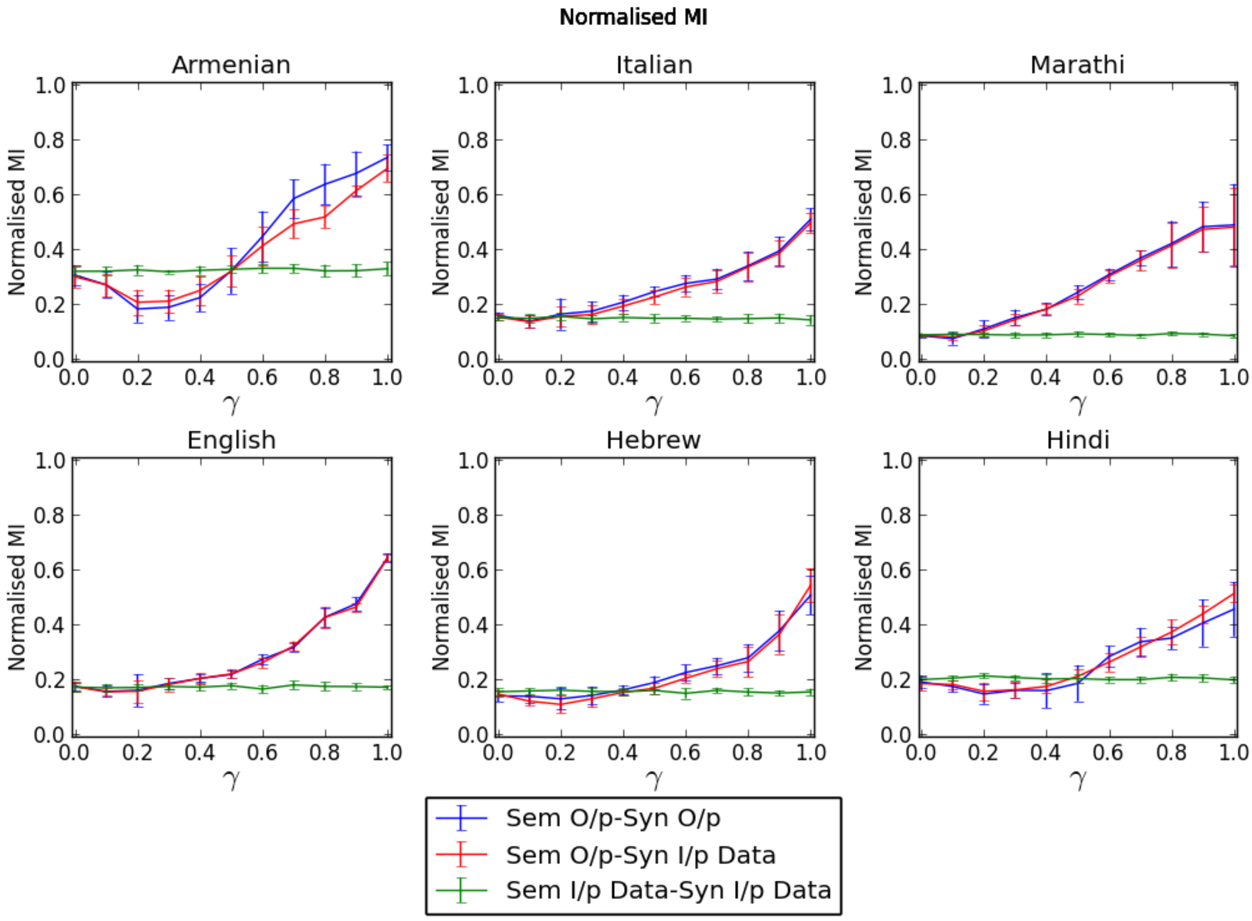


Figure 3.10. Mean normalised mutual information over 10 independent simulations at various γ *values for concrete nouns in 6 languages when N=3 output units, and 10 iterations.*

Figure 3.10 depicts the performance of the network when it is tested on the training set as the semantic inputs are incrementally supplied with syntactic information. The green curves represent the mutual information between unprocessed semantic and syntactic input, this is the baseline mutual information between semantic and syntactic data which remains flat, i.e. roughly independent of γ , the small variations on this flat line are due to the random selection of units that receive syntactic information at each run. The red curves represent the mutual information between the output of the network and unprocessed syntactic inputs while the blue curves plot the mutual information between processed semantic and syntactic outputs. The dashed and dotted curves tend to follow each other closely, implying that the syntactic competitive network results in a dimensionally reduced faithful representation of the syntactic input data. The mutual information rises above the baseline as γ increases above the 0.4-0.5 region for Armenian, English, Hebrew and Hindi, and above the 0.2 region for Italian and Marathi. Thus semantic classes tend to gradually realign, with increasing γ , in such a manner that they correspond more to the syntactic classes as compared to the baseline. This reorganisation is however very limited: at $\gamma=1$, the normalised mutual information for all languages is in the range of 0.5-0.6, which is around half way towards full agreement. Although interacting with syntax does help some reorganisation of the semantic classes, the divide between syntax and semantics is clear and almost half of the semantic information cannot be shared with syntax at $\gamma=1$.

The performance of the network is further limited by the fact that it cannot be driven by semantic units only, with no syntactic information during testing. A ‘syntactic context’ is necessary at the inputs for the network to result in a mutual information performance above baseline. When tested without syntactic information, or with only a partial amount, the drop in the normalised mutual information is significant, with only a tiny trace of learning shown by the network.

3.6 Discussion:

. This is a first attempt at assessing the learnability of the mass-count statistical data and rather an exploratory study towards a network modelling of the classification. By applying such a network to the mass-count information from participants we can draw a few inferences:

1. In most languages, syntactic markers tend to categorize ‘spontaneously’ between mass and count markers, lending validity to the intuitive perception of a quasi-binary distinction. This is not fully true, however, and particularly in Hindi the markers chosen show a graded distribution of mutual correlations. Hindi however is slightly different from the remaining languages, in such that the 4 out of 5 markers in Hindi, answer 'yes' for pure counts. Thus the network is driven towards a count-internal categorisation.

2. Nouns, instead, tend in most languages to distribute quite closely along a line which coincides with the main mass/count dimension introduced in our previous study (Kulkarni, Rothstein and Treves, 2013). Along this line, nouns are very crowded at the count end, and scattered all along towards the mass end. Their distribution is therefore graded rather than binary, with no emergence of a single ‘mass’ class, but rather of several non exclusive but distinct ways of a noun of being different from pure count. For example, in Armenian (Figure 8) nouns like ‘bird’ and ‘ship’ belong to the ‘pure count’ class while ‘troop ’and ‘lunch’ are in the 9th class away from the count class. On the mass end, nouns like ‘cotton’ and ‘milk’ are at the extreme mass end of the spectrum while ‘coffee’ and ‘wheat’ are more mass-like nouns but not at the pure mass end. The exception is English, where there are at least two clear non-equivalent dimensions of non-countability.

Both the above observations are interesting because the mass-count information in the categorisation arises on its own. The markers, in some cases, very cleanly segregate themselves into mass and count. The nouns are reduced to a one dimensional representation along a mass-count spectrum. Even though the network fails to associate specific syntactic markers with specific nouns based on the semantics, the network does develop a 'concept', if we may say, of what is the mass-count classification. The diversity and richness of this classification however, prevents a simple network to learn specific associations. Which brings us to the third observation,

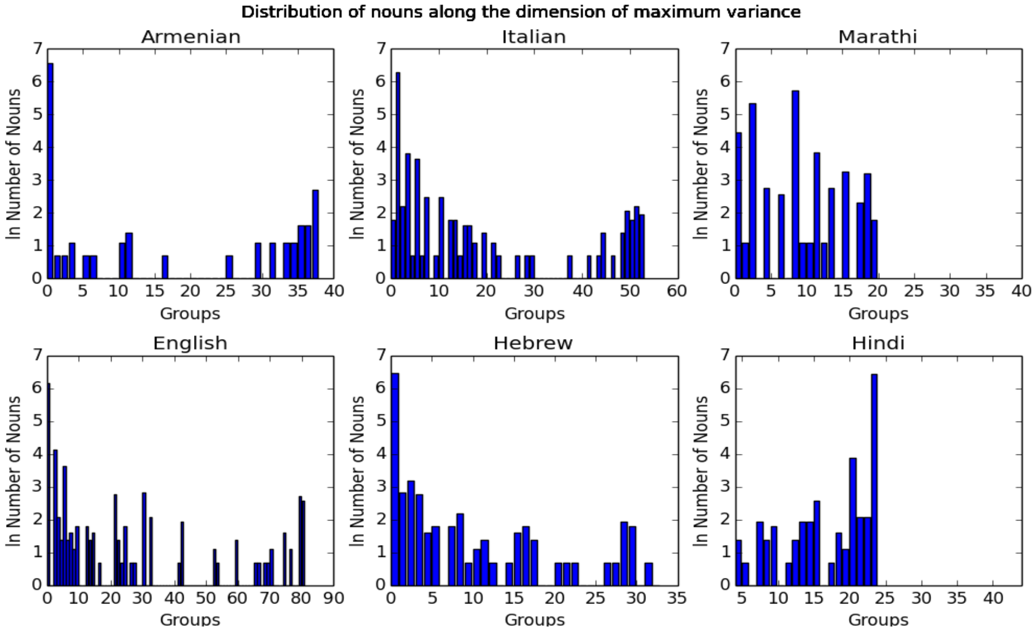
3. The lack of significant mutual information between semantics and syntax implies, as we have verified, that the latter cannot be extracted solely from the former. Further, when allowing the competitive network to self-organize on the basis of full semantics and partial syntactic inputs, and testing it with the full syntactic inputs, the mutual information obtained with the full syntactic usage distribution is only at most about half the corresponding entropy value. This occurs in fact only when the full syntax is given in the input also at training, and it indicates that giving also semantics information affects negatively rather than positively the performance of the network.

Overall, these observations indicate that the acquisition of the mass-count syntax by humans with neurally plausible mechanisms, involve more complex computations beyond what can be captured by a simple neural network. The observations do however,reinforce the conclusions of our earlier study that mass count syntax is far from a rigid binary rule, rather it appears as the flexible, speaker-specific usage of a variety of binary markers to a quantitatively and qualitatively graded repertoire of nouns, where being non-count can be expressed in many ways (see also, chapter 2, figure 2.8 and 2.11). Taking semantics into account helps speakers to a very limited extent in generating their own mass/count ‘dialect’.

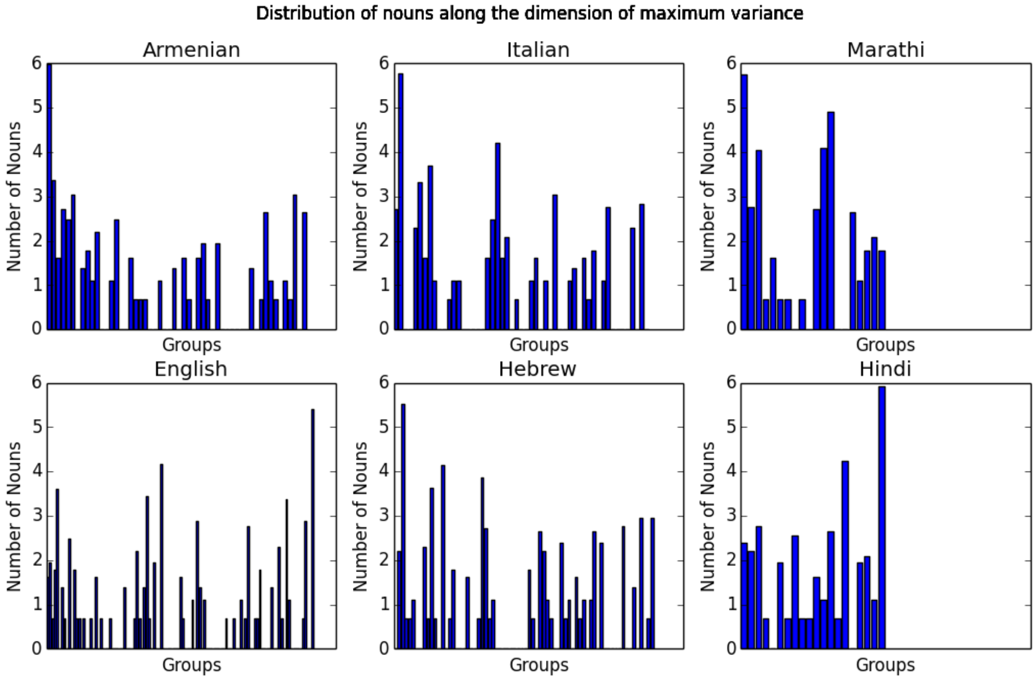
Appendix B

B1: Log plots of figures 3.6 and 3.8

a) Concrete Nouns

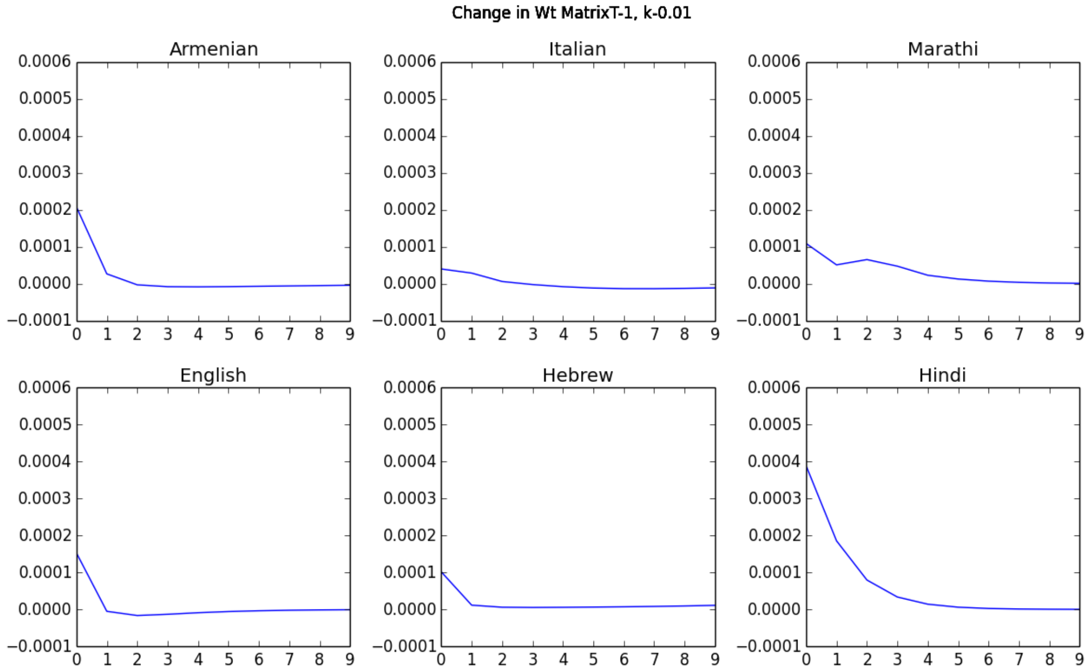


b) Abstract Nouns

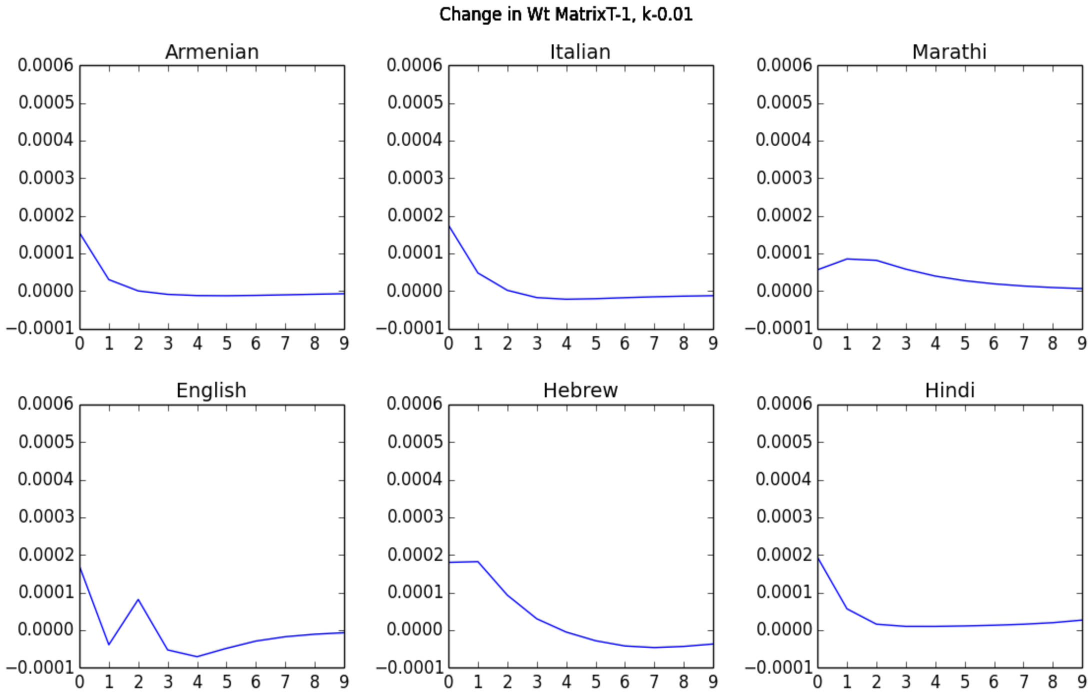


B2: Convergence of weight matrix during learning. (change in weight matrix against iterations)

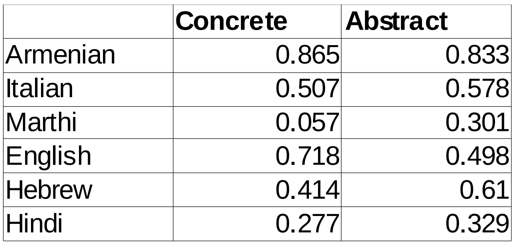
a) Concrete Nouns



b) Abstract Nouns



B3: Correlation coefficients of groups in figures 3.6 and 3.8 with MC dimension groups in Chapter 2, section 2.



	Concrete	**Abstract**
Armenian	0.865	0.833
Italian	0.507	0.578
Marthi	0.057	0.301
English	0.718	0.498
Hebrew	0.414	0.61
Hindi	0.277	0.329

Part B

Storing Correlated Memories and the Possible Spin Glass Nature of the Potts Neural Network

Chapter 4

Section I

The Potts Neural Network

A] Introduction:

In the previous chapter we have already seen the application of a neural network, namely the competitive neural network. In this chapter we look at a more complex network, and at some of its features in section II and III, which are particularly relevant to cognitive function including language acquisition. Section I summarises the basic concepts of neural networks and some past work in the field. First, let us have a quick recapitulation on neural networks.

Neural network models, to a great extent, are a product of the connectionism concept. Connectionism is founded on the idea that the core information necessary for cognitive function is stored in the neurons and in their synaptic connections in the brain. The connections are modifiable and respond to the exposure to various inputs, bringing about a 'learning process' in the network. One of the popular earlier models was the Parallel Distributed Processing model (PDP), [McClelland, Rumelhart 1986]. It consisted of having several independent parallel processing units called 'neurons' which were connected with a specific structure. Neurons were essentially variables taking a certain range of values decided by an output function and the structure of connection amongst them was given by a connectivity matrix of real numbers that represented the strength between two units. Much of the modern work on neural networks follows the same fundamental principles.

B) Types of neural networks:

Different classes of models can be defined from the structure of the connectivity and the details of how the neuron is modelled. One may model the detail the time dependent voltage-current response of a neuron [for eg. Izhikevich 2003], or use more abstract units which essentially only represent the firing activity of the neuron [an eg. Hopfield 1982]. We use the latter, since we are interested in the global behavior of a large number of interacting neurons, for which one can approximate the behavior of a neuron with its mean firing rate [Gertsner 1992a; Gertsner 1992b]. A synaptic connection between neurons is modelled by another variable J_{ij}, between neuron i and j, whose strength is changed according to Hebb's rule [Hebb 1949]: the change in connection strength, δJ_{ij}, in a learning event is proportional to both the pre and post synaptic activity of the two neurons in the event. Some other learning mechanisms like the discriminative learning, especially in the language domain [Bayen et al 2011] aim to achieve learning by the adjustment of conditional probabilities between 'language objects'. However we restrict ourselves to biologically inspired mechanisms like the Hebb's rule, rather than the powerful but maybe biologically implausible machine learning rules. Or at least, if one wishes to manifest a similar model in biologically inspired neurons, one must revert back to a Hebb like situation.

i) Different connectivity structures give rise to different types of neural networks. The very first neural networks were of the feed-forward type [Hornik 1989]. It consists of an input layer and an output layer which learns the association between 'representations' of inputs and outputs via modifiable connections between the two layers. It was found that adding another 'hidden layer' between the input and out layer, such type of network can potentially map any continuous input-output function. However, multilayer feed-forward networks predominantly operate with the back-propagation algorithm, in which the error between input and output travels backwards through the connections to adjust each connection strength. The biological plausibility of this method is unlikely. Since the network requires a training set of output patterns, this is a supervised learning algorithm.

ii) Another class of neural networks is when there is no 'teacher'; this is called the unsupervised network. We have seen an example of such a network in the previous chapter, one which uses competition between the output neurons during learning and classifies the inputs. These are self-organising networks which can form clusters and categorise inputs based on a similarity measure, without the requirement of an explicit target output. One may suppose that unsupervised networks form the first block of information processing where novel stimuli are categorised to be processed further.

iii) A third class of neural networks are the recurrent neural nets [Amit 1989], where neural activity is fed back to the same neurons by recurrent synaptic connections, thus forming a cycle of information flow. [Elman 1990, 1991] constructed one such type of a recurrent net called the 'Simple Recurrent Network' (SRN) and applied it to linguistic stimuli. Elman’s SRN had a modular structure and was a mixture of feed-forward and recurrent modules. His important finding was that the SRN could segment a continuous unbroken stream of words from a sentence into separate words that were clustered in grammatical categories of nouns and verbs.

Another class of recurrent nets is the autoassociatve network, where the inputs learn associations with themselves and thus facilitate a content addressable memory. The stored memories are attractors of the dynamics of the network and with a partial cue the network can can recover the entire memory of the object, for example, giving a cue like '*to e_r _s h_m_n*' may recover the entire phrase '*to err is human*'. Our model, to be discussed in Section C, falls into this category. It is of special interest to us due to its powerful computational abilities, which may help to replicate how some of the cognitive functions in our brains are brought about.

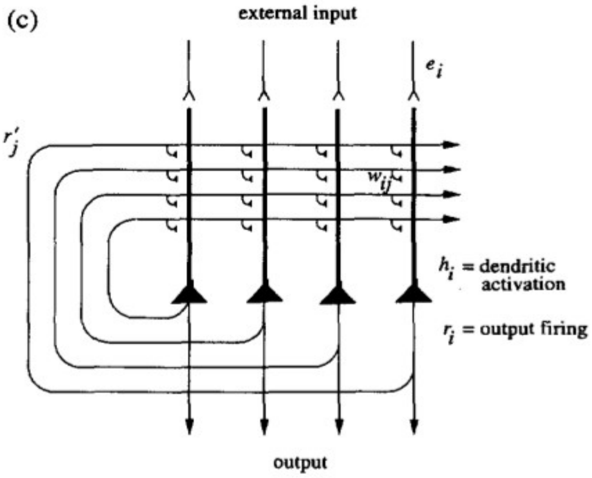


Figure A: A schematic representation of an autoassocitive network with recurrent connections. (reproduced from [Rolls, Treves 1999])

[Hopfield 1982] designed such a network from the Ising model of interacting spins which was mathematically treatable and hence led to an intense study, revealing the rich dynamics of such models which can serve as the foundation of cognitive function. The neuron was abstracted to two states of being active(+1) or inactive (-1) and N such neurons were connected by 'synapses', e.g. synapse J_{ij} between neuron i and j. The learning rule was prescribed in such a way that each pattern, ξ_i^μ, $i=1,2...N;\mu=1,2...p$, for N neurons and p memories to be stored, was situated at the bottom of a valley on the energy landscape, thus forming an attractor. The energy is defined as

$$H=-\frac{1}{2}\sum_{i,j\neq i}^{N} J_{ij}\xi_i^\mu \xi_j^\mu$$

This term is called the energy because for a suitable definition of the dynamics, a symmetric J, and in the absence of noise, a flipping of a spin will occur if either it keeps the energy of the system the same or it reduces it. Hence over many such spin flips the network naturally reaches a stable state which is the minimum of the energy function. The Potts model, which we study in section II and III, is in a sense an extension of the Hopfield model where the fundamental unit is not a model neuron and it is not restricted to the two states of an Ising spin, but it can be in more than two possible states.

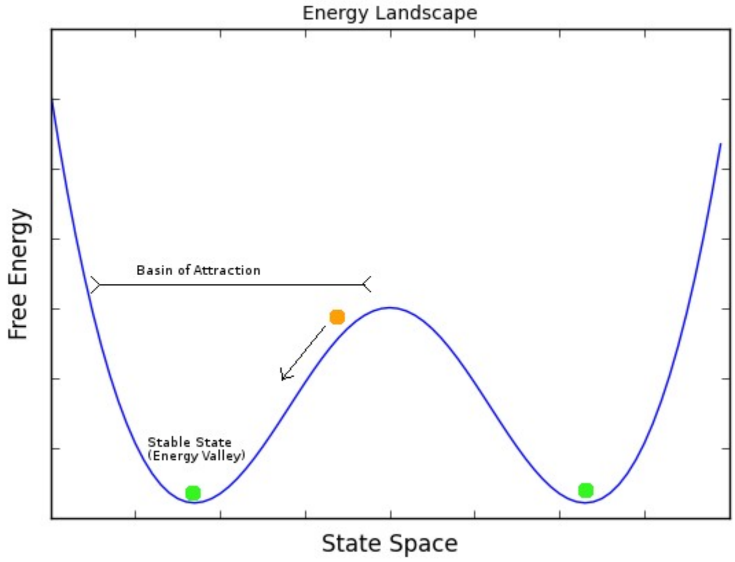


Figure B: Energy landscape depicting attractor dynamics.

C) The Potts neural network model for cortical dynamics:

The cortex is a large network of interconnected neurons and is thought to be composed of several sub-networks with dense recurrent connectivity. These sub-networks are also interconnected with each other, thus communicating information to and fro over the whole cortex. [review on functional connectivity: van den Heuvel, Pol 2010, Yeo *et al 2011*]. The Potts neural network model, the one described below, is an adaptation of the Potts network studied in [Kanter, 1988; Bolle et al, 1993]. Our model aims to present a simplified model of cortical mechanisms with adaptive dynamics. It is a global autoassociative network of interacting units, each unit representing a local sub-network.

a) *The Potts unit:* The model we study is a variant of the Potts neural network model [Kanter 1988] aimed at incorporating the time dependant dynamics of cortical mechanisms in the global attractor space [Treves 2005; Russo, Treves 2012]. A Potts unit, σ_i^k , represents the ith local cortical patch which is in the local attractor state k. Each unit can be in S possible states, hence the internal dynamics of the local patch are reduced to a simplified representation to depict only the local attractor states. At a given time, a single unit might be correlated to only one of its attractor states or partially correlated to more than one. With N such units we can represent a global pattern over the entire cortex, $\xi_{i,k}$ for $i=1,2\ldots N; 0\leqslant k\leqslant S$. s=0 is defined to be the inactive state of the unit. The total activity of a unit if fixed to 1, thus $\sum_{l=0}^{S}\sigma_i^l=1$. Several distinct combinations of states can produce different patterns, $\xi_{i,k}^{\mu}$, where μ is the pattern identity, $\mu=1,2\ldots p$.

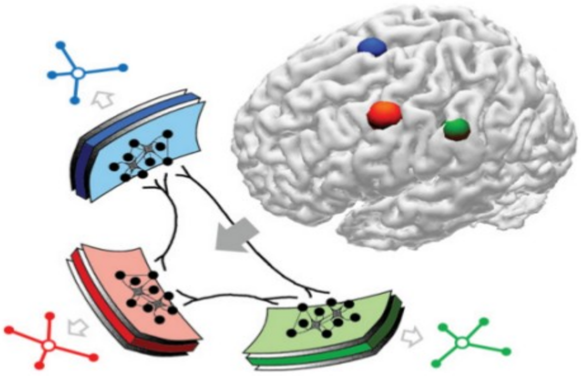


Figure C: Conceptual visualisation of the Potts model, with local patches represented by S state interconnected units, forming a global cortical network.

b) *The connections:* Local patches are connected with long range synapses which communicate the activity of unit i in state k to unit j in state l by an interaction term J_{ij}^{kl} . J forms a connectivity matrix which stores information of several patterns ξ^{μ} which are embedded in the network by a learning rule [Russo, Treves 2012], which is similar to the one in the Hopfield network but adapted to S-state units.

$$J_{ij}^{kl} = \frac{c_{ij}}{C_m a(1-\frac{a}{S})} \sum_{\mu=1}^{p} (\delta_{\xi_i^{\mu} k} - \frac{a}{S})(\delta_{\xi_j^{\mu} l} - \frac{a}{S}) \times (1-\delta_{k0})(1-\delta_{l0})$$

c_{ij} , is a binary number [0,1] which establishes the existence of a connection between unit i and j , c_{ij} is randomly distributed across the connections and is 1 with probability C_m/N , with $c_{ii}=0$, where C_m is the number of connections between units. a denotes the sparsity of patterns, so in effect a fraction Na units are active in a state $0<k\leqslant S$. The term a/S reduces bias due to random overlaps and is discussed in chapter 4, section II, 4.7.1. The last two product terms arise out of biological reasoning and make sure that learning occurs only on active states and that the 0 state is ineffective over J.

c) *Adaptive Dynamics:* Once the network has reached a stable attractor state, the active units are continuously 'firing', but in a biologically realistic scenario the units cannot go on firing continuously forever. Hence besides the constant baseline global threshold U , we introduce two thresholds that account for neuronal fatigue and slow inhibition [Horn, Usher, 1989]. They deplete the resources for the unit to fire and eventually destabilise the attractor. The first kind of threshold, θ_i^k , which affects only active units, σ_i^k , intends to model neural fatigue and short term depression

[Tsodyks, Markram, 1997] and has a time constant, τ_2 in the in order of tens of milliseconds. The second threshold, θ_i^0 , affects only the inactive state and models slow, delayed inhibition within a cortical patch and has a time constant τ_3 in the range of hundreds of milliseconds. The dynamics of the thresholds can be stated in the following way,

$$\tau_2 \frac{d\theta_i^k(t)}{dt} = \sigma_i^k - \theta_i^k \quad ; \quad \tau_3 \frac{d\theta_i^0(t)}{dt} = \sum_{k=1}^{S} \sigma_i^k - \theta_i^0$$

A third time constant, τ_1 , determines the rate of integration of the local fields over each unit, subject to the dynamic threshold. The local field is the summed inputs that a unit receives from all other units, given by $h_i^k = \sum_{j\neq i}^{N} \sum_{l=1}^{S} J_{ij}^{kl} \sigma_j^l + w(\sigma_i^k - \frac{1}{S}\sum_{l=1}^{S} \sigma_i^l)$. The second term is an addition, that models self-reinforcement in the local patch dynamics of an active unit, to make it converge towards an attractor. Hence the final inputs that a unit receives can be written as

$$\tau_1 \frac{dr_i^k(t)}{dt} = h_i^k(t) - \theta_i^k(t) - r_i^k(t)$$

with r_i^k being the input to the active states and θ_i^0 to the inactive states. The final state of the unit can be written as,

$$\sigma_i^k = \frac{e^{\beta r_i^k}}{\sum_{l=1}^{S} e^{\beta r_i^l} + e^{\beta(\theta_i^0 + U)}} \qquad \sigma_i^0 = \frac{e^{\beta(\theta_i^0 + U)}}{\sum_{l=1}^{S} e^{\beta r_i^l} + e^{\beta(\theta_i^0 + U)}}$$

where β represents the local patch inverse temperature thus modelling noise internal to the unit (more about this in chapter 4, section III, 4.13). The global dynamics of the network however, is noiseless.

D) Latching behavior:

The above equations completely describe the dynamics of the Potts neural network model with adaptive thresholds. The introduction of the adaptive thresholds generates some interesting and potentially useful features in terms of cognitive aspects. As the network reaches a stable attractor state, active units in the network start facing an increasing threshold, which destabilises them and the network starts falling out of the attractor. However it may happen that as the network falls out of the attractor it is pushed towards a new basin of attraction and enters into a new attractor. This process may continue endlessly in the right parameter phase space. In fact the network shows three phases of dynamical behaviour [Treves 2005; Russo, Treves 2012]: (a) No latching - the network falls out of the attractor and it decays into inactivity; (b) Finite latching - the network falls out of an attractor subsequently jumping into another one and this process continues for a finite duration; (c) Infinite latching - the sequence of jumping continues indefinitely.

It was observed that the jump from one attractor to another is facilitated if the two attractors are correlated.

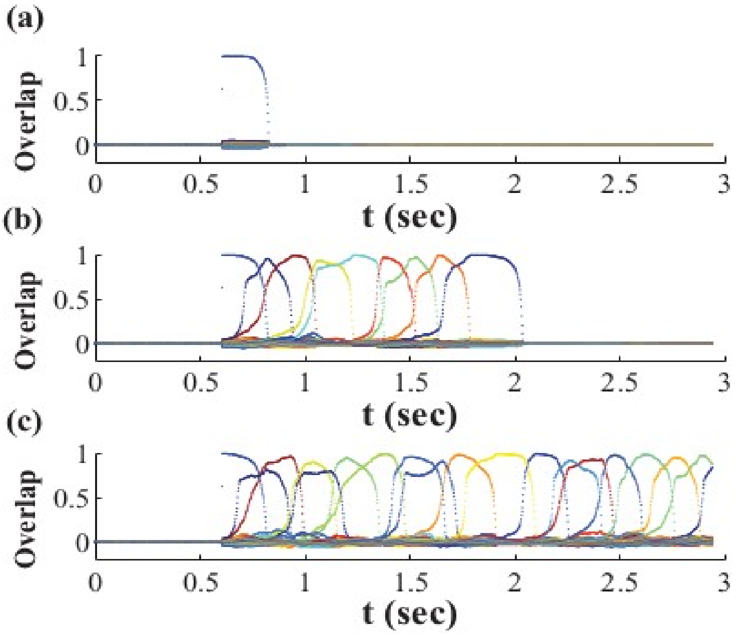


Figure D: Latching behavior of the Potts network. (a) No latching; (b) Finite latching; (c) Infinite latching. (reproduced from Russo, Treves 2012)

E) Application of the Potts neural network to language:

The latching phenomenon in the Potts network and particularly the fact that latching transitions are more probable amongst correlated patterns makes it a good model to start investigating its application to language. To produce a well formed sentence one has to make transitions from nouns to verbs to adjectives and so on depending on the sentence. Thus we can propose that the brain latches from one word to another in a structured manner, where each word is an attractor. A first attempt at applying the Potts network to language was presented by [Pirmoradian, 2012, PhD thesis]. An artificial language called BLISS (Basic Language Incorporating Syntax and Semantics) was generated with a medium level of complexity, although much less than a natural language, to be encoded into the Potts network. The learning rule was modified to incorporate hetero-associativity amongst consecutive words with their syntactic and semantic content. It was seen that upon giving an initial cue with a word the network spontaneously exhibited transitions to other words (Nouns or Verbs) in the network.

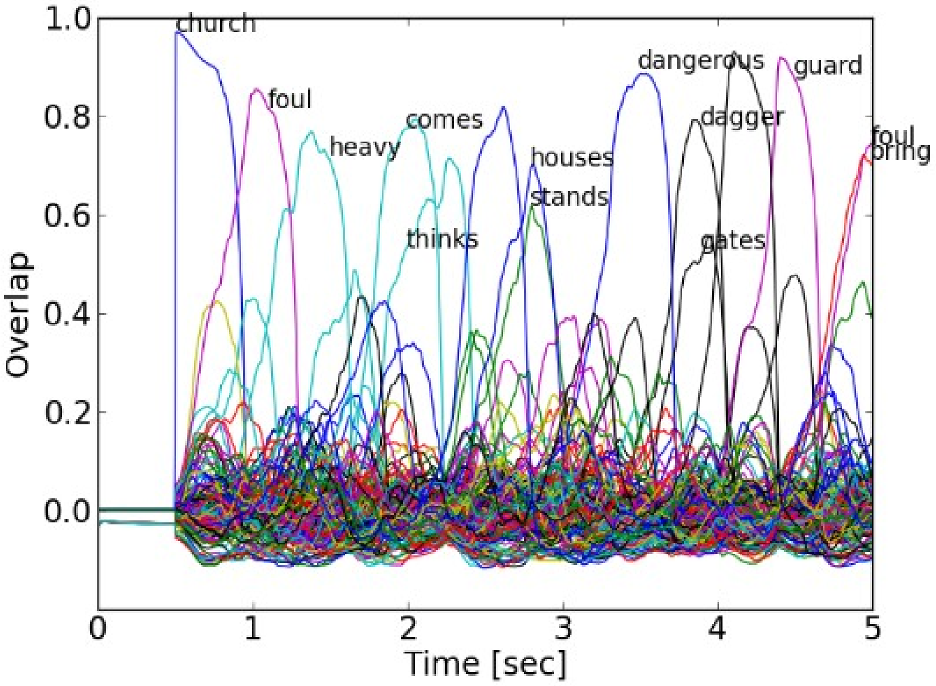


Figure E: Potts network applied to BLISS showing transitions amongts different nouns and verbs. (reproduced from Pirmoradian 2012)

The words encoded in BLISS had a near random correlation. However to build a realistic model of language production or acquisition, one needs to incorporate natural correlations between words/syntax/semantics which can be quite strong. By including correlations into word representation one soon runs into serious trouble as we shall now see in the section II.

Chapter 4

Section II

Storage of Correlated memories

4.1 Introduction:

After having an overview of the basics of a computational auto-associative model aiming to understand global cortical interactions in the preceding chapter, namely the Potts Spin neural network, let us look at some of its features that are crucial to understand if one intends to 'employ' it as a language processing model that can handle real world linguistic data. One important aspect of real world data is the natural correlations found amongst various entities. These correlations are not isolated just to the linguistic domain but are ever present in any natural representation of 'objects/entities' in their occurrence.

4.1.1 Importance of correlations in neuronal representations:

One clear and immediately occurring example to appreciate correlations is the semantic web of concepts. Here, each 'object' can be represented with a set of features that represent the physical or functional properties of that object. For example, some of the features might be, 'is living', 'has eyes', 'has wings', 'has wheels', 'is made of metal' and so on, and the list can be endless. Objects, then can possess or not possess these features, a 'dog' will possess features like 'is living', 'has eyes' etc. while 'a car' may entail some other features like 'has wheels'. Given such a representation one can obtain, in an N-dimensional space, the location of all objects and can study the structure and distribution of various objects. One such database is the McRae database of semantic features or the WordNet project of Princeton University. 'Dog' and 'Fox' will share many common features being highly correlated whereas 'Fox' and 'Fish' will be distant yet sufficiently correlated compared to 'Fox' and 'Car', which may be considered as uncorrelated objects. This however, is quite a simplistic representation of correlations; in reality, objects distant in feature correlation might be temporally or functionally correlated, making the picture quite complex. In the language domain, words are not only semantically correlated but also share syntactic correlations or pragmatic/contextual ones.

Various experimental findings implicate neuronal correlations, either as pairwise mean firing rates or temporal correlations and on the population scale in oscillatory rhythms [Salinas &

Sejnowski, 2001, de la Rocha *et al* 2007]. The interpretations of these correlations and how the brain makes use of them is however are a matter of debate.

In terms of our mass-count nouns, seen in chapters 2 and 3, we would intend to represent each noun and its marker encoded into the network as activation of neurons. The markers are either syntactic or semantic while each noun has a unique representation to identify itself from the rest. This first step itself requires us to represent the nouns and markers in such a way that they exhibit the natural correlations found amongst them, so that operations over them are meaningful and learning or acquiring ‘the rule' and links amongst them is efficient. Immediately, we then reach an important and long standing hurdle of storing correlated memories in the network. This has been a persistent challenge ever since the dawn of connectionist models of brain function. Most of the early quantitative work with artificial neural networks has focused on uncorrelated patterns of memory. They have provided a very important and useful insight into the functioning mechanisms of systems of interacting neurons and showed us the robustness and power of these systems to bring about computational operations that could form the base of cognitive activities in the brain. Such networks however fall short of satisfying expectations when one involves the use of correlated patterns. We use the word 'pattern' to mean a set of N elements representing N neurons/units, wherein each of the N elements indicates the firing activity of that neuron/unit. Together these elements encode the representation of a particular object in the network as a particular combination of firing activity over each unit, $\xi^{\mu}=\{x_{1,}x_{2.}..x_{N}\}$. In a standard Hopfield network x_i is either +1 or -1, while in the Potts network x_i can have S possible states.

4.1.2 A brief summary:

Early work on storage of correlated patterns was done in the 1980's by [Parga and Virasoro, 1986; Amit, Gutfreund and Sompolinsky, 1987; Gutfreund, 1988]. Various extensions of the Hopfield model were considered in its ability to store correlated patterns. The standard prescription for storing patterns is when the Ising spins in a particular pattern are randomly and independently distributed, made clear by the probability expression of a single unit being in one of the two states $P(\xi_i^{\mu})=\frac{1}{2}(\delta_{\xi_i^{\mu},-1}+\delta_{\xi_i^{\mu},1})$. Correlations were introduced in the patterns by adding a bias amongst patterns [AGS 1987, Tsodyks 1988] so as to violate the above mentioned probability, or by generating patterns in a systematic way by an algorithm that arranges patterns on an ultrametric tree [Parga, Virasoro 86, Gutfreund 88]. One sees that storing correlated patterns in an associative network greatly diminishes the storage capacity of the network when a standard Hebbian plasticity learning rule is used, however making modifications to the learning rule by adding a term to negate the bias or writing the learning rule in such a way so as to incorporate the hierarchy in the presented patterns during learning, can restore the storage capacity to some extent, i.e. the ability of the network, as an auto-associative attractor network, to retrieve a sufficient number of patterns. For our purposes the storage capacity is defined as the maximum number of patterns that can be stored in the network and then be successfully retrieved during recall, either through a complete or a partial cue. Various other attempts have been made by having different ways to bring about correlations between patterns. [Tamarit and Curado 1991] studied the retrieval behaviour of the Hopfield network when the patterns share a pairwise correlations such that patterns within a pair are correlated but those from a different pair do no overlap. The network was found to have two regimes; one where all the pairs (up to a certain level of storage limit) could be retrieved and another regime where only the pairs could be retrieved without distinguishing between the patterns within the pair. [Monasson 1993] studied spatial correlations between patterns, which the correlations between a pair of patterns decreased exponentially as a function of the distance between the specific units of the pair. It was found that the storage capacity increases for very weak spatial correlations however the information content in the network reduces. This is in line with previous work with biased patterns. [Lowe 1998] analysed the Hopfield network when patterns were generated from a Markov chain with correlations either in the semantic or spatial domain. It was shown that the storage capacity either scales as $\frac{N}{\gamma \, logN}$ for a strong definition of retrieval where every pattern is stable and retrieved with probability 1. In this definition the storage capacity increases with strength of semantic correlations but reduces with increasing spatial correlations. However, the storage capacity is scaled as αN as in the standard definition of [Hopfiled 1982]. The bound on α reduces with increasing strength of correlations in this definition. More recently, a slightly complex variation of the standard Hopfiled model was studied by [Agliari, *et al*, 2013] where the patterns are diluted, by having a portion of the inputs blank and correlations introduced by adding a bias to the non-blank inputs, similar to [AGS 87]. They found that having blank inputs allows space to add additional patterns later and have parallel retrievals.

4.1.3 On ultrametricity:

One important and significant difference in which our study differs from the previous ones is the use of non-ultrametric patterns. We shall see in detail in the next section, how patterns with variable correlations are generated in our network. In an ultrametric space, three arbitrarily selected points x, y and z have to follow both the triangle inequality (more specifically, if $d(x,y)$ denotes the distance measure between points x and y then, $d(x,y) \leqslant d(x,z) + d(y,z)$) and also the additional constraint $d(x,y) \leqslant max(d(x,z), d(y,z))$. In such a space patterns can be arranged in a 'family tree', where successive generations branch out from previous ones. In an ultrametric space one cannot have in-betweens, B cannot be between A and C. Either the nearest common forefather is common to A and B, hence they are the same distance from C, or the same argument can be made with respect to A and C. Thus they all lie at the vertices of isosceles triangles since they are at the same distance from the nearest common forefather. In reality, however, such an organisation of natural objects does not represent the true structure. Even more so in the linguistic domain, where words are hardly ever ultrametric, but rather have a spectrum of existence in the category space, when measured statistically. This means that a specific instantiation of a word in a specific category is probabilistic and depends on the multiple correlations along several syntactic or semantic dimensions. One can see as a small example in chapter 2 figure 2.14, how nouns in a corpus arrange themselves along a pyramid in the mass-count space. Our brains can easily store and process such correlations but the implementation of this structure in an associative network model is rather a tough task.

In the current chapter, we explore the ability of the Potts network to store correlated patterns in a non-ultrametic fashion and look at some variations in learning rules and their effect on the storage capacity. The storage capacity of the Potts network for uncorrelated patterns was extensively studied in [Kropff, Treves 2005]. The theoretical estimate of the storage capacity obtained here as a function of sparsity can be written as,

$$p_c = \frac{CS^2}{4a\ln\left(\frac{2S}{a\sqrt{\ln(S/a)}}\right)} \quad ...(e4.1)$$

where C is the dilution parameter that sets the average number of connections present between units, a is the sparsity and S are the number of active Potts States.

[Russo, Treves 2012] showed that this estimated storage capacity for correlated patterns is severely diminished. We extend the study and look at the correlations in detail and explore the reasons and how one may attempt to reduce the effect of correlations on synaptic connections between units to enhance the number of patterns that can be stored.

4.2 Correlations in patterns:

4.2.1 Pattern generation:

The pattern generation algorithm we use follows the procedure mentioned in [Treves 2005] as a two-step method to generate Potts network patterns whose degree of correlation can be tuned using two main parameters. In the first step, we generate a set of uncorrelated patterns, what we call as 'factors', which then facilitate the formation of final patterns by suggesting the states on each of the units. A competition amongst the units then finally decides which units will be active and in which particular state.

For each pattern that is generated, out of its N units having S possible states, each factor influences a distinct subset of the units. The influence of a factor on a particular unit suggesting a state decreases exponentially with an exponent ζ for successive factors, $\zeta \approx 0$ makes every factor almost equivalent , resulting in the production of uncorrelated patterns while $\zeta \approx 1$ makes only the first few factors influential, giving rise to patterns that are generated from a small group of factors and thus naturally sharing more common units in the same direction and producing highly correlated patterns.

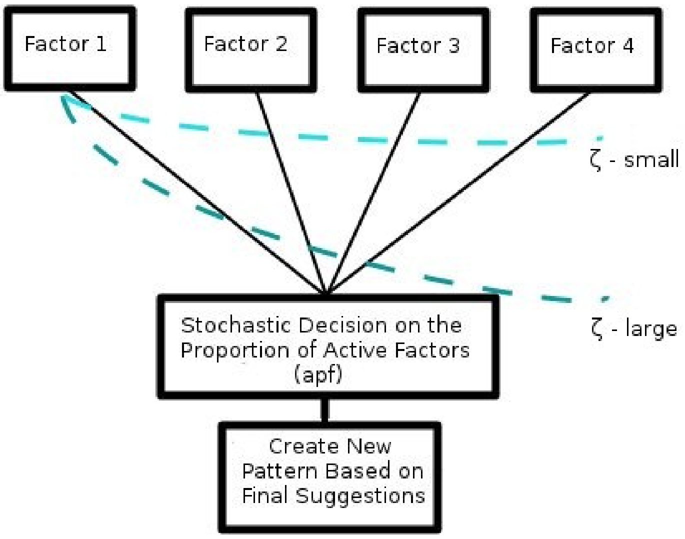

Factor 1
Factor 2
Factor 3
Factor 4
ζ - small
ζ - large
Stochastic Decision on the Proportion of Active Factors (apf)
Create New Pattern Based on Final Suggestions

Figure 4.1 Schematic representation of the pattern generation algorithm

Beside the strength of influence of a factor another parameter, a_{pf} , governs the probability that the factor will influence the units in the patterns. This probability is given by a random number in the range 0 to 1 and is set to 0 with a probability $1-a_{pf}$. In this way each factor tries to align a unit in one of S possible directions given its strength and the probability that it may align the unit. Once all the suggestions are made, the sparsity 'a' of the patterns is set by selecting a fraction N*a of the most active units, that is the units that have the strongest alignments in one of the S states, and those units are set in the state having the strongest 'suggestion field' while all other states are set to 0. Thus $\xi_i^k=1; \sum_{l=0}^{S} \xi_i^l=1$ (unit i has the strongest suggestion field in the direction k) and rest of the units, a fraction N(1-a) of them, are set in the inactive state $\xi_i^0=1$

This allows us to generate patterns with a varied degree of correlations, by tuning the parameters ζ and a_{pf} . The degree of correlation is measured by the fraction of units sharing the same active state as well as by the second measure, i.e the same fraction of units that are active in both patterns, in whichever state. Figure 4.2 a&b shows the effect of ζ, a_{pf} and sparsity a on the degree of correlation. The chance overlap between a pair of patterns, i.e. the number of shared active units in the same state, denoted as N_{as} ,can be estimated as Na^2/S , whereas the chance number of co-active units in any state, denoted as N_a , is Na^2 . We use these estimates as normalisation factors in figure 4.2 a&b to visualise the levels of correlations as a function of the three parameters affecting correlations.

4.2.2 Scatter of correlations:

Figure 4.2 shows the scatter of correlations, where each point is a normalised overlap between a pair of patterns for a total of 100 patterns, on the x-axis normalised fraction of units that are co-active in a pair and on the y-axis the normalised fraction of units which are active and in the same state. We use 3 levels of correlations depicted in the figures 4.2 a,b namely uncorrelated patterns (Random, $\zeta = 1\text{e-}6;\ a a_{pf} = 1\text{e-}6$), low level of correlations (Low, $\zeta = 1\text{e-}6;\ a_{pf} = 0.4$) and highly correlated patterns (High, $\zeta = 0.1;\ a_{pf} = 0.4$). Figure 4.2a shows the scatter plots for sparsity from 0.2 to 0.5, while figure 4.2b for sparsity from 0.6 to 0.9. As we can see, pairs of patterns have a high scatter on both the axes for highly correlated patterns indicating that they share many units aligned in the same direction, whereas for random patterns this overlap hovers around the probabilistic estimate normalised to 1. The case of low correlations lies in between these two extremes. The differences between the different levels of correlations are reduced as the sparsity a increases. As patterns become less and less sparse i.e as sparsity increases, more and more units become active in each pair of patterns which naturally increases the chance overlap between them. Thus increasing sparsity starts dominating the correlations between patterns rather than the correlations being controlled by factors.

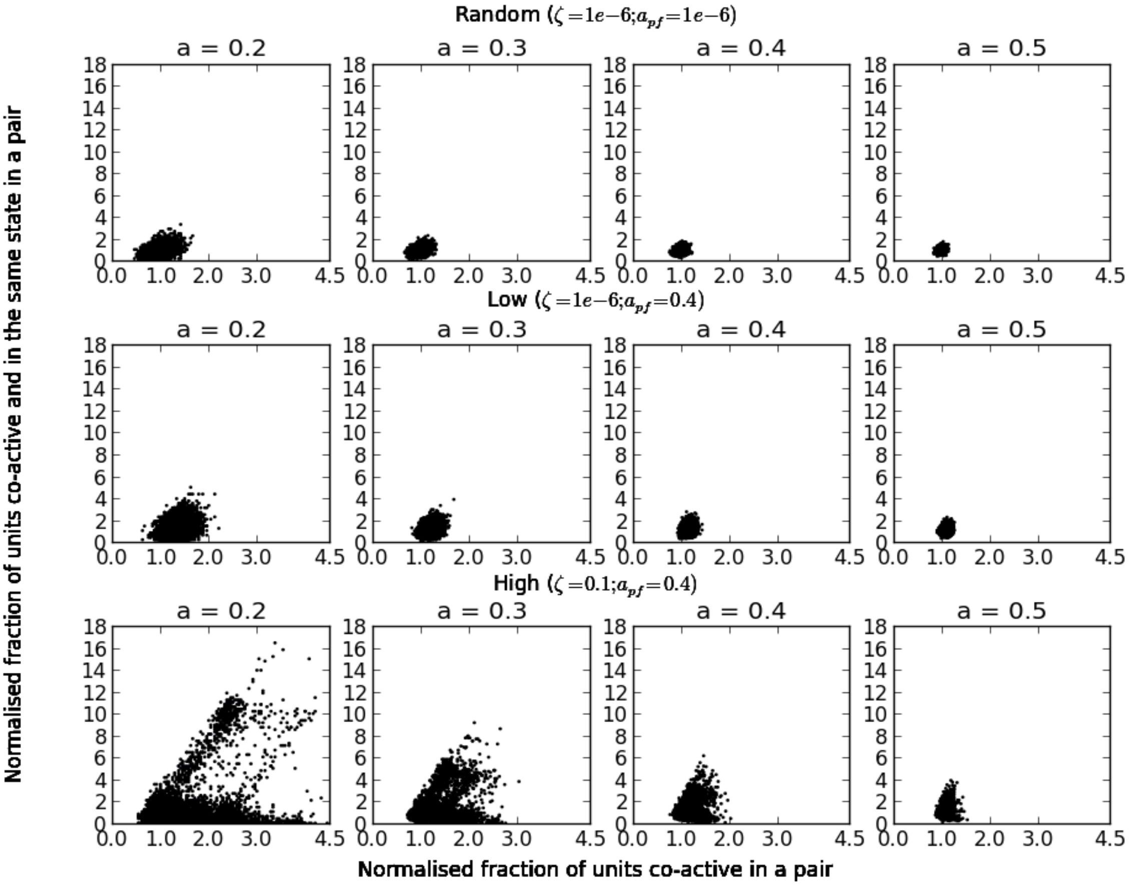


Figure 4.2a: Scatter Plots showing the spread of pairwise correlations amongst patterns for $a < 0.6$

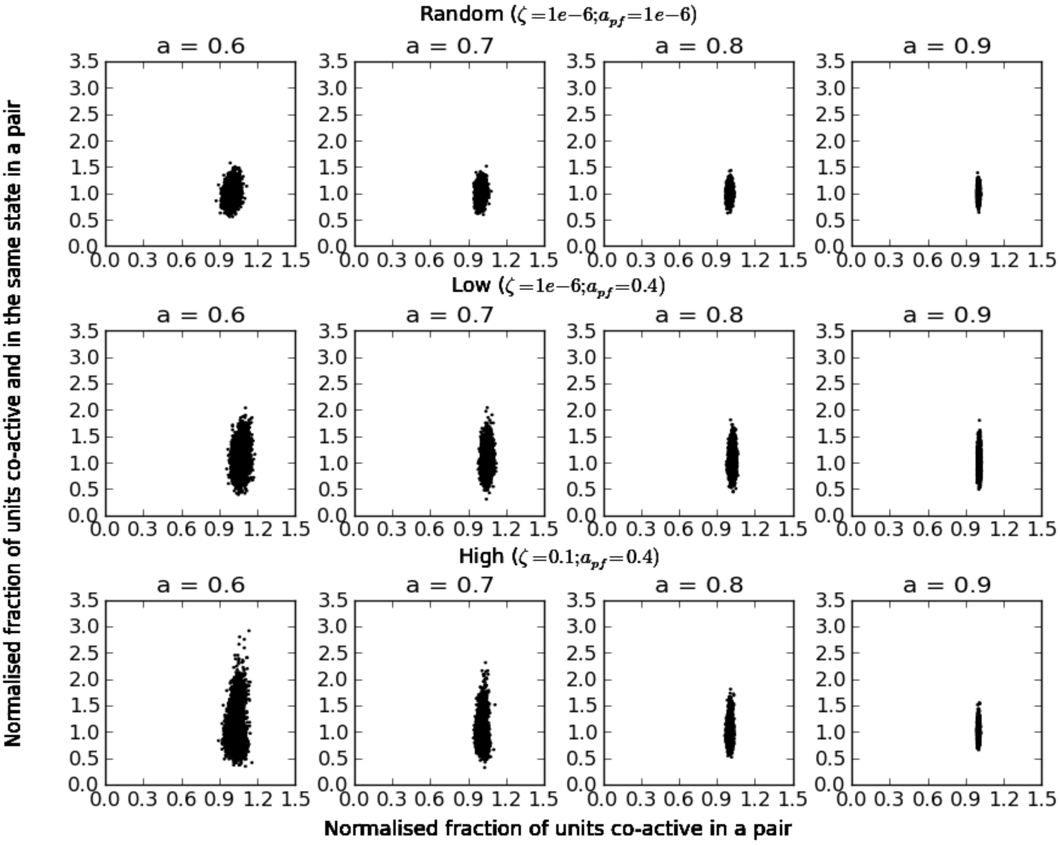


Figure 4.2: b Same pairwise correlation scatter as above but for $0.6 \leqslant a \leqslant 0.9$

4.3 Storage capacity:

We calculate the storage capacity of the Potts neural network through simulations by storing patterns of three different levels of correlations as mentioned in section 4.2. The final storage capacity is estimated by storing a varied number of patterns in the network, called as the storage load, 'p'. The network is tested for different p's from high p to low p and defining storage capacity at the highest p at which at least 0.5p of the patterns are successfully retrieved. The time varying thresholds θ_i^k and θ_i^0 , explained in section I-C, are kept constant since we want to investigate the asymptotic behaviour without adaptation. This procedure is repeated for a varied set of patterns and random connectivity between units for a constant $C_m=90$. In the absence of external noise the dynamics of the network is deterministic, hence for a given set of patterns, the success or failure in retrieval of a particular pattern remains unchanged and is determined by the interference (noise, as we shall see in the following section) from the remaining patterns stored in the network other than the one that is being retrieved. Thus any variance in the storage capacity, P_c (seen in figure 4.3), is largely due to the different sets of patterns being stored and to a lesser extent, due to the varying random connectivity between the units. Figure4.3 shows mean storage capacity over 10 independent runs with random connectivity for each value of sparsity. The network parameters, as described in chapter 4, section-I-C have the values of, $N=600$, $S=5$, $U=0.2$, $\tau_1=2$, $\tau_2=\infty$, $\tau_3=\infty$, $C_m=90$ and $\beta=11$, $w=0$. The network is allowed to relax for 500 time steps after the cue presentation. Same parameters are used throughout all the simulations with specific changes, if any, are mentioned in the description.

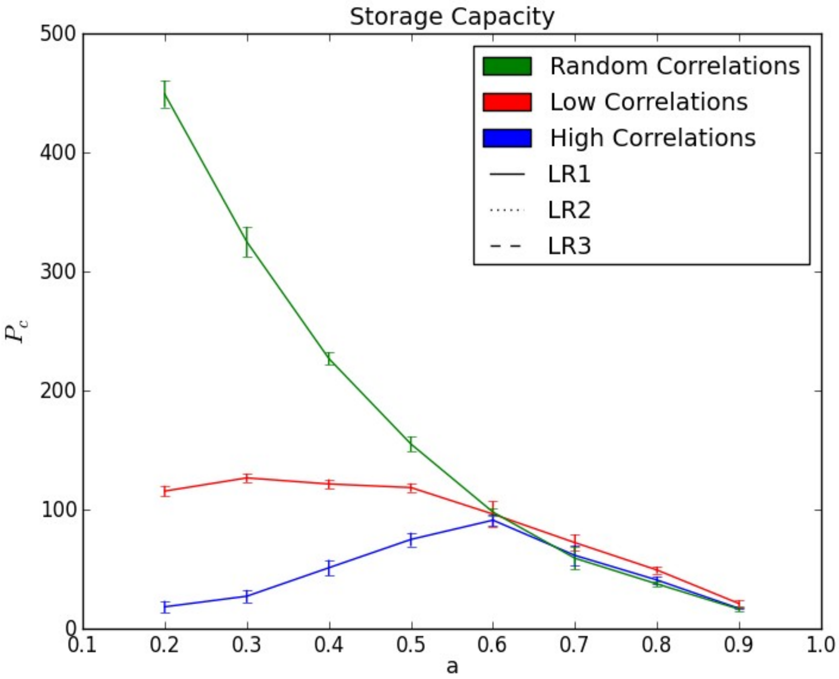


Figure 4.3: Storage Capacity as a function of sparsity for Random (green), Low (red) and high (blue) correlations amongst patterns.

The storage capacity is severely affected as the degree of correlations amongst the patterns increase (error bars show standard deviation over 10 independent runs). The difference is seen for

$a < 0.6$ after which specific correlations amongst patterns do not matter any more, and retrieval capacity is dominated by the effect of increasing sparsity. From figure 4.2b we see that the patterns become comparable in their scatter over the correlation measure for $a > 0.5$. In case of random patterns the storage capacity monotonically decreases as a function of sparsity as predicted by e4.1. For low correlations, instead, the storage capacity is greatly reduced and stays more or less constant, with a gradual decay up to $a = 0.5$ and then a more rapid decay as a increases beyond 0.5. For high correlations however the storage capacity is not a monotonic function of a, it rises initially till $a = 0.6$ and then decreases as with the random and low correlations. These simulations confirm the results of [Russo, Treves 2012]. We shall further study the effect of correlations looking at the noise profile and at different learning rules, in the following sections.

Let us recall the learning rule to store patterns in the synaptic connections mentioned in chapter 4 section I-C ,

$$J_{ij}^{kl} \;=\; \frac{c_{ij}}{Ca\left(1-\frac{a}{S}\right)} \sum_{\mu=1}^{p} \left(\delta_{\xi_i^\mu k} - \frac{a}{S}\right)\left(\delta_{\xi_j^\mu l} - \frac{a}{S}\right) \times \left(1-\delta_{k0}\right)\left(1-\delta_{l0}\right) \quad \text{...(e4.2)}$$

with this learning rule each pattern ξ^μ , is intended to be an attractor of the dynamics such that pushing the system in the vicinity of the ξ^μ with a partial cue, on every further updating of the unit (i,k) , the network state for each unit σ_i^k will eventually be driven towards the state $\xi_{i,k}^\mu$. Thus each ξ^μ forms a valley in the energy landscape and the system starting in the basin of attraction of a particular valley will 'fall' into the valley. But when patterns are correlated the boundaries between the valleys are unclear and the attractors merge with each other, hence giving a cue towards a particular attractor does not guarantee the system falling into the desired attractor thus resulting in a failure to retrieve; which in turn leads to the storage capacity getting lower as the level of correlations increase.

4.4 Signal and noise:

Another way to look at the effect of correlations is to calculate the signal and noise each unit receives on its field during retrieval. As seen in the section I-C, the field over each unit in a particular state while retrieving any pattern (considering w=0), say ξ^1 as an example, without loss of generality, is written as

$$h_i^k = \sum_{j\neq i}^{N}\sum_{l=1}^{S} J_{ij}^{kl}\xi_{j,l}^{1} \quad \text{...(e4.3)}$$

inserting equation (e4.2) in to (e4.3) we obtain,

$$h_i^k = Z'\sum_{\mu=1}^{p}\left(\delta_{\xi_i^\mu k}-\frac{a}{S}\right)\sum_{j\neq i}\sum_{l}\left(\delta_{\xi_j^\mu l}-\frac{a}{S}\right)\xi_{jl}^{1}$$

where Z' is the normalisation constant as in (e4.2)

(for the sake of simplicity we do not write the last term in (e4.2), denoting that learning occurs only over active states, but it is assumed that all calculations are done over active states).

We can split this equation into two terms, one denoting the component of the field which represents the signal on that unit while trying to retrieve the pattern ξ^1 and the other term which is the component that can be termed as noise, coming from all the other patterns, ξ^μ, $\mu\neq 1$ stored in the connectivity matrix J.

$$h_i^k = \underset{\textit{signal}}{C'\left(\delta_{\xi_i^1 k}-\frac{a}{S}\right)\sum_{j\neq i}\sum_{l}\left(\delta_{\xi_j^1 l}-\frac{a}{S}\right)\xi_{jl}^{1}} + \underset{\textit{noise}}{C'\sum_{\mu=2}^{p}\left(\delta_{\xi_i^\mu k}-\frac{a}{S}\right)\sum_{j\neq i}\sum_{l}\left(\delta_{\xi_j^\mu l}-\frac{a}{S}\right)\xi_{jl}^{1}} \quad \text{(e4.4)}$$

For the pattern to be retrieved successfully, the mean of the noise term has to be 0 and in the case where the patterns are randomly correlated the noise term is normally distributed centred around 0, with the standard deviation such that the signal term always has a greater magnitude than the noise term. Thus if the standard deviation is small enough so as to not interfere with the signal, the stored pattern can be successfully retrieved. However when patterns are correlated the noise term is not normally distributed any more but is rather a skewed normal distribution, as seen by the positive skewness measure in figure 4.4. The mean of the noise is still 0, but the positive asymmetry causes the noise to be of comparable or even of greater magnitude on the positive signal.

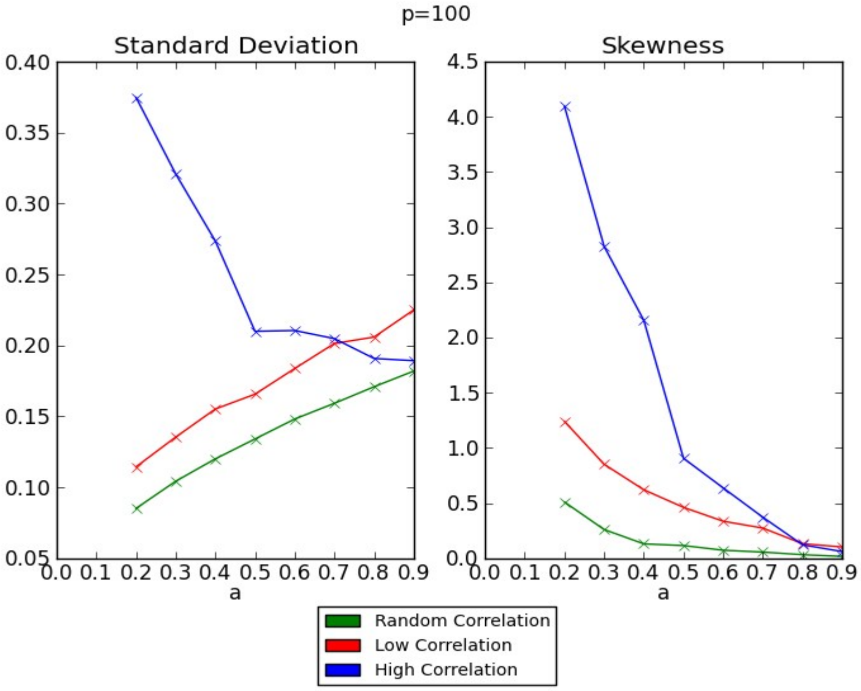


Figure 4.4: Standard deviation of the noise profile as a function of sparsity.

Figure 4.4 shows the standard deviation and the skewness of the noise term in (e4.4) as a function of sparsity a. Skewness here is the third standardised moment defined as $\frac{\mu_3}{\sigma^3}$, μ_3 is the third central moment and σ is the standard deviation. Thus if we denote the noise by $\kappa(k)$, taking values from $k_{1,} k_{2.} .. k_n$, then the skewness is given by

$$\gamma(\kappa)=\frac{\frac{1}{n}\sum_{i=1}^{n}(k_i-\bar{k})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(k_i-\bar{k})^2\right)^{3/2}}$$

For the high correlation case the noise term is highly skewed as compared to the low and random correlation, with random patterns having the least skewness. There is a monotonic decrease of skewness, almost as a slow exponential function for random and low correlations, and a sharp decrease for the highly correlated patterns, and they all converge as we approach $a=0.9$. Skewness affects the standard deviation of noise for highly correlated patterns in a significant way, as the standard deviation for $a<0.5$ is completely determined by the decreasing skewness. In the case of random and low correlation there is a monotonic rise in the standard deviation with increasing a.

This, at least in part, explains the curves in figure 4.3, for random and low correlations the skewness is low and increasing standard deviation with increasing a, decides the drop in storage capacity as a function of sparsity, because the signal increasingly gets weaker in comparison to noise. For high correlation in the region of $a<0.6$ the storage capacity is governed by the skewness and decreasing skewness of noise causes the increase in storage capacity due to better retrieval ability.

In Figure 4.5 we look at the standard deviation and skewness of the noise as a function of network load p, after fixing sparsity at $a=0.2$. Standard deviation rises linearly as p increases, and the slope is higher for high correlated patterns as compared to random and low correlations. An interesting observation is that the product of the standard deviation and skewness is almost constant as a function of p for the random and low correlations while it is more or less linearly increasing for high correlations, driven by the high slope of the standard deviation. Thus as we store more and more patterns in the network, the standard deviation increases, making the signal weaker and thus affecting the ability of the network to successfully retrieve a pattern.

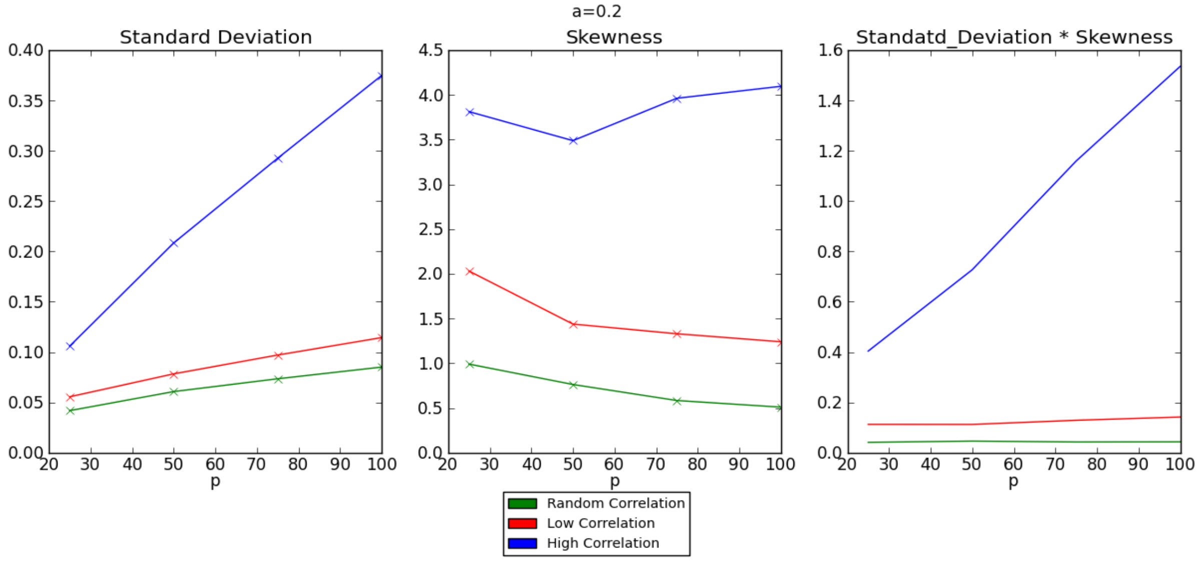


Figure 4.5: Standard deviation and noise as a function of storage load.

4.5 Retrieval behaviour above storage capacity:

In the previous section we saw how the noise has an increasing spread as correlation level increases and this affects the retrieval of a pattern. In the current section we look at how retrieval capacity is affected once the storage load on the network is above the storage capacity limit we saw in figure 4.3.

The network is loaded at $p(a)=P_c(a)+50$ for each specific sparsity a and each of the stored pattern is cued to test for retrieval. Since the network is operating above its storage capacity there is a higher fraction of patterns (more than half of the stored patterns) that fail to be retrieved successfully. We measure the correlation between the cued pattern and the pattern that has the highest overlap with the network state. When a pattern is cued and retrieved, besides the cued pattern there can be other patterns that have a certain degree of overlap with the network state. In a pure retrieval, the cued pattern has the highest overlap with the network and the remaining patterns have an overlap which hovers over the average value of overlap that a pattern may have by chance with the network state. There are three cases one can see when a pattern is cued and tested for retrieval:

(i) In the case of successful retrieval the cued pattern itself has the highest overlap with the network state and we instead measure the correlation of the cued pattern with the pattern that has the second highest overlap with the network state.

(ii) The cued pattern has failed and no other pattern is successfully retrieved beyond the threshold of retrieval (overlap with network state more than 0.7). We measure the correlation between the cued pattern and the pattern that is still below the threshold of retrieval but has the highest overlap amongst all the patterns

(iii) The cued pattern has failed and another pattern other than the cue has the highest overlap with the network state above the threshold of retrieval and we measure the correlation between these two patterns

Similar to figure 4.2a the three situations are plotted as a correlation scatter, where case (i) is represented in green, (ii) in red and (iii) in yellow in figure 4.6. We focus in the sparsity range $0.2 \leqslant a \leqslant 0.5$ which is more interesting in the biologically plausible sense, and because differences between patterns for different correlations are predominantly seen in this range of sparsity.

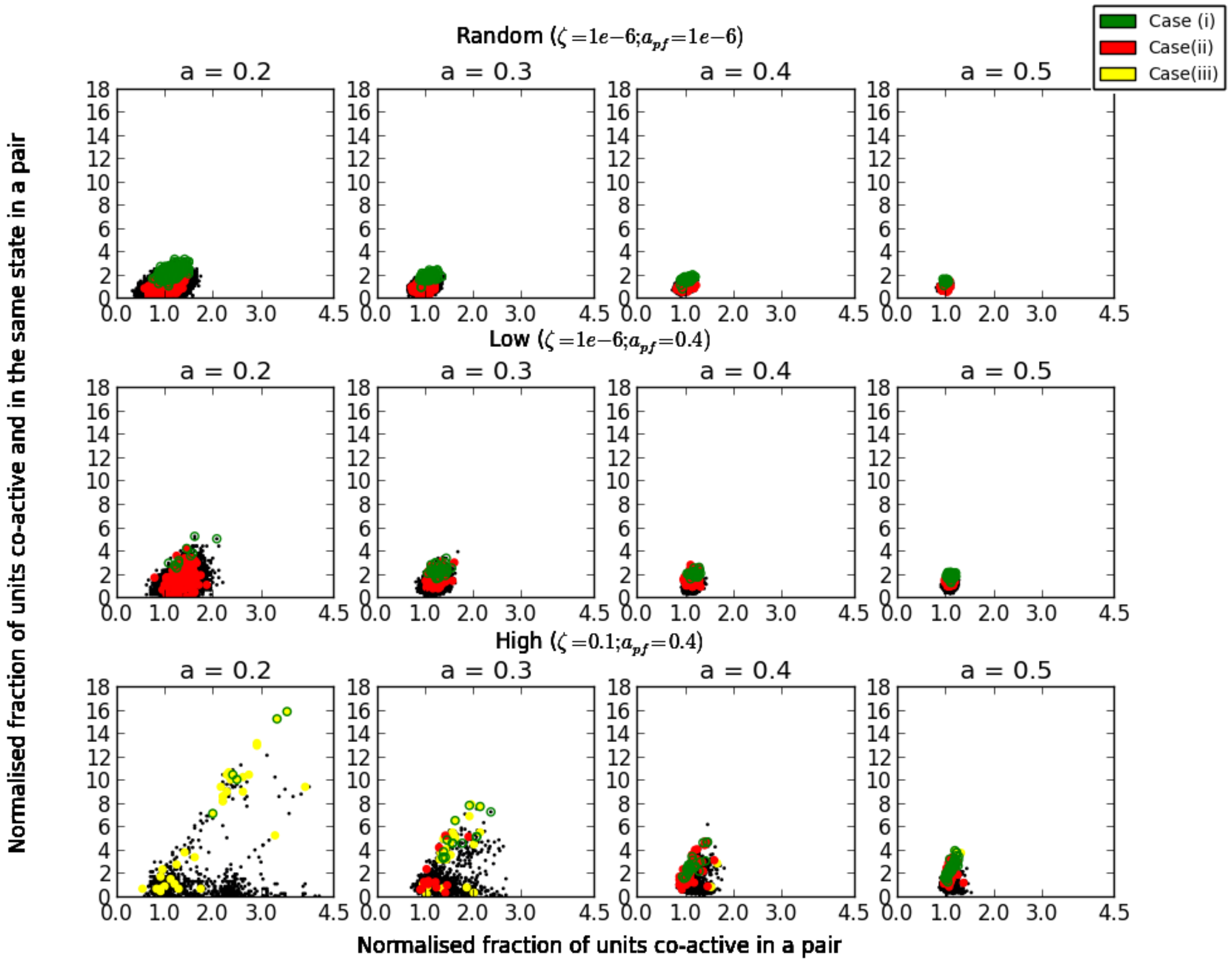


Figure 4.6: Correlation scatter same as figure 4.2a with an overlay of the three types of retrieval behaviours. case(i)-green, case(ii)-red and case(iii)-yellow

For random and low correlations, only two out of the three situations are seen (figure 4.6). The network can either successfully retrieve a stored pattern or fail all together with no other pattern being successfully retrieved. Surprisingly, patterns which have another pattern that shares a high correlation with it, in terms of N_{as}, are retrieved successfully, seen from the high concentration of 'green' points on the higher side of the scatter while the patterns which fail lie lower on the correlation scale. Table 4.1a,b & c summarises the mean normalised pairwise correlation, defined by N_{as} , between patterns in the three cases mentioned above

Table 4.1a: Mean Correlation, N_{as}, for Successful Pattern Pairs, case (i)

	$a=0.2$	$a=0.3$	$a=0.4$	$a=0.5$
Random Correlation	2.28	1.87	1.60	1.43
Low Correlation	3.58	2.79	1.98	1.77
High Correlation	11.70	5.10	2.69	2.44

Table 4.1b: Mean Correlation, N_{as}, for Failed Pattern Pairs of case (ii)

	$a=0.2$	$a=0.3$	$a=0.4$	$a=0.5$
Random Correlation	1.01	0.98	1.00	0.98
Low Correlation	1.69	1.68	1.58	1.49
High Correlation	-	1.84	2.38	1.83

Table 4.1c: Mean Correlation, N_{as} , for Failed Pattern Pairs of case (iii)

	$a=0.2$	$a=0.3$	$a=0.4$	$a=0.5$
Random Correlation	-	-	-	-
Low Correlation	-	-	-	-
High Correlation	6.69	3.86	1.79	2.18

From table 4.1a & b it is seen that patterns that are successfully retrieved have another pattern that is strongly correlated with it if compared to patterns that fail, which tend to be less correlated with other patterns.

Case (iii) is seen only in the highly correlated patterns, which also exhibits cases (i) and (ii), except at a=0.2 there are no pure failures (case (ii)) in the high correlation patterns. This implies that when a cued pattern in the high correlation fails, the network tends to successfully go in an attractor that shares a high correlation with the cued pattern, i.e., as seen from table 4.1c, the mean correlation between the cued pattern and the retrieved pattern is much above the normalised chance $N_{as}=1$

4.6 Storage capacity with correlated retrievals:

In the previous section we saw that in case of highly correlated patterns when a cued pattern fails to be retrieved, there is a tendency that another pattern that is strongly correlated to it will be successfully retrieved instead. Thus for patterns with high correlation, we test the storage capacity of the network by incorporating the correlations into the storage capacity definition and setting the successful retrieval condition such that the pattern that is retrieved successfully has a normalised correlation level above 1.5 with the cued pattern, $N_{as}(\xi^{cue}, \xi^{retrived})>1.5$.

Figure 4.7 overlays the correlated storage capacity (purple points) over the regular storage capacity (blue line) for the highly correlated patterns. Since the high correlations case shows all the three type of retrievals (cases (i),(ii) and (iii)), it is not guaranteed that each time there will be a correlated retrieval of type (iii) as seen from figure 4.6 , in which case then the network falls back to the case of a pure success or failure (cases (i) and (ii)), which correspond to the regular storage capacity. In fact, as seen from figure 4.6 for highly correlated patterns, the correlated retrievals fall as a increases and for $a > 0.5$ there are no correlated retrievals (not shown in figure 4.6, but seen in figure 4.7 through storage capacity). The number over each point, denoting a correlated retrieval in figure 4.7, shows the fraction of the simulations (10) when a case of correlated retrieval occurred, while the rest of the times the network reverted back to the regular storage capacity.

When correlations are incorporated in the definition of storage capacity, there is an enormous jump in the storage capacity for highly correlated patterns. However one must

remember that the attractors in this case are merged and high correlations imply that that there are several other patterns which share a significant number of units in the same state with the retrieved pattern, and have a non-trivial overlap with the network state.

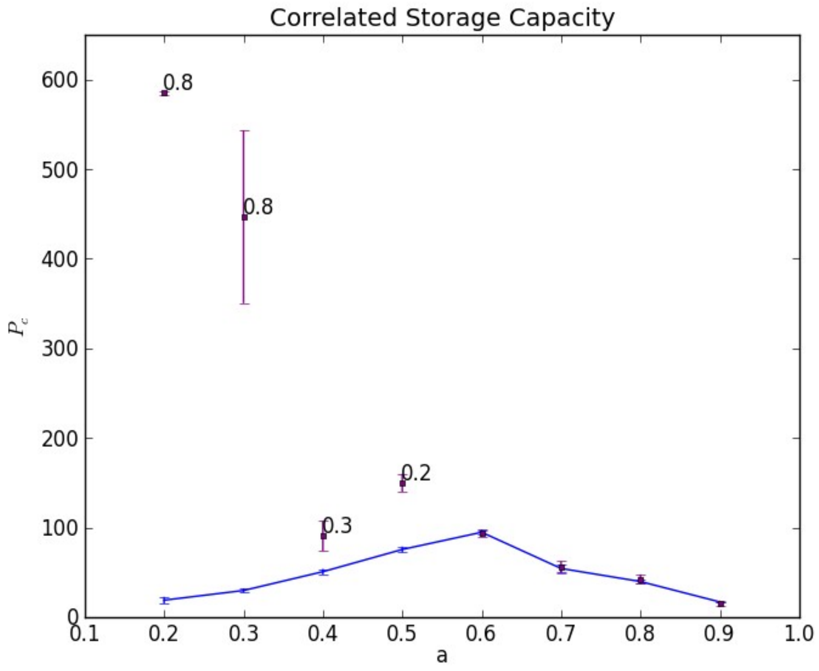


Figure 4.7: Storage Capacity when the strong correlations are incorporated in the definition of storage capacity

4.7 Modification of learning rules:

The storage capacity that we saw in the previous section was studied with the standard learning rule (e4.2), restated below

$$J_{ij}^{kl} = \frac{c_{ij}}{Ca(1-\frac{a}{S})}\sum_{\mu=1}^{p}(\delta_{\xi_i^\mu k}-\frac{a}{S})(\delta_{\xi_j^\mu l}-\frac{a}{S})\times(1-\delta_{k0})(1-\delta_{l0}) \quad \text{... (LR1)}$$

From the signal and noise analysis we saw that as more and more patterns are stored in the network there is an increasing interference amongst the patterns which limits the number of patterns that can be successfully stored and retrieved.

One aspect of this learning rule is the term $\frac{a}{S}$ which is subtracted from both the pre and post synaptic terms. A detailed analysis of the significance of including this term, for a standard Hopfield network, is done in [Amit, Gutfreund, Sompolinsky 1987]. Since the Hopfield model is only a two-state spin system, just subtracting a is required, whereas for an S state system the natural modification to this term is $\frac{a}{S}$. This term has an effect of reducing the effect of the bias in the patterns and making the noise term in (e4.4) to have a 0 mean and thus improving the storage capacity.

4.7.1 'Popularity' subtraction:

Subtraction of the bias term is effective when there are no systematic correlations between patterns and the overlaps between patterns are only at chance level. With the introduction of systematic correlations however, as we do with our pattern generation algorithm for low and high correlations, just subtraction of the $\frac{a}{S}$ term is not sufficient. In the case of a standard Hopfield model, this problem was investigated in [Kropff, Treves 2007]. Instead of subtracting a from the pre and post synaptic terms, they subtract what is called the 'popularity' of the neuron, defined as the average activation of that neuron over all the stored patterns. Here we extend that definition to the Potts units and write the 'popularity' term as

$$a_{ik}=\langle \xi_{ik} \rangle_{\mu} \quad ;where \ \ \langle a_{ik} \rangle = \frac{a}{S}$$

with this we write the modified learning rule as

$$J_{ij}^{kl}=A\sum_{\mu=1}^{p}(\delta_{\xi_i^{\mu}k}-a_{ik})(\delta_{\xi_j^{\mu}l}-a_{il})\times(1-\delta_{k0})(1-\delta_{l0}) \quad \text{...(LR2)}$$

Thus the reduction of bias while storing the patterns is more refined rather than using a single mean value of $\frac{a}{S}$, and it depends on the specific set of patterns being stored, thus affecting the storage capacity. We shall see the effect of this modification after the following section, where we look at another modification to the learning rule.

4.7.2 Constrained synaptic modification:

From the noise profile in section 4.4 we saw how the distribution of noise gets more and more positively skewed as correlations amongst patterns increase. This is in effect due to the high contribution of the positive terms in the summation of the noise over all the stored patterns: the positive contributions arise from the increased positive overlaps that result due the stronger correlations between patterns. Since the source of these excess positive contribution is in the synaptic connectivity matrix, J_{ij}^{kl}, we intend to mitigate this effect by modifying the synaptic strength between two units, on each successive presentation of a new pattern to be stored, proportionally to the existing strength of the synaptic connection between the units that the pattern modifies. The rule is written as,

$$J_{ij}^{kl}\{q\}=J_{ij}^{kl}\{q-1\}+(\delta_{\xi_i^{\mu}k}-a_{ik})(\delta_{\xi_j^{\mu}l}-a_{il})\,e^{-\alpha J_{ij}^{kl}\{q-1\}(\delta_{\xi_i^{\mu}k}-a_{ik})(\delta_{\xi_j^{\mu}l}-a_{il})} \quad \text{...(LR3)}$$
$$\textit{for the}\, q^{th}\, \textit{pattern}, q=1,2...\, p$$

Thus the change in the synaptic strength between units i and j is moderated by an exponential function whose exponent is a scaled product of the existing synaptic strength, $J_{ij}^{kl}\{q-1\}$, and the change due to the new inputs it receives, $(\delta_{\xi_i^\mu k}-a_{ik})(\delta_{\xi_j^\mu l}-a_{il})$. If the new inputs strongly affect a synapse that is already strong, then the influence of these inputs on that synapse drop exponentially. This drop can be controlled by the parameter α , setting $\alpha=0$ one gets back LR2. One must note that reducing the influence of new inputs also reduces, in fact, the information content about the new inputs, thus one cannot set $\alpha\geqslant 0$, otherwise the patterns will not be stored in the network. Through simulations we find that for highly correlated patterns the value of α which results in the desired effect of reducing the spread of noise and increasing storage capacity is $\alpha=1$ at $a=0.2$ while for $a>0.2$ and the low correlations case we see the intended effect at $\alpha=0.01$ for all a (figure 4.10) . This learning rule is local, biologically plausible and makes on-line modifications during learning.

4.7.3 Effect of modified learning rule on noise profile:

Similar to section 4.4 we look at how the standard deviation and skewness of the noise distribution is affected by the two modifications mentioned in the previous two sections. Figure 4.8 shows, overall, that LR2 (dotted line) and LR3 (dashed line) effectively reduce the standard deviation and skewness of the noise distribution compared to LR1 (solid line) for the high and low correlations cases, while there is no significant change in the case of random patterns.

For the high correlations case there is a marginal reduction in the standard deviation due to LR2 and a slightly greater reduction for LR3, except at $a=0.2$ where there is a stronger reduction in LR3. For the low correlations case the reduction in standard deviation is comparable for both LR2 and LR3, at a similar level lower than for LR1, while for the random patterns all three learning rules are comparable. If we look at the skewness, the pattern is similar for high correlations as mentioned above, while the skewness for the low correlations case is brought down to the level of random patterns.

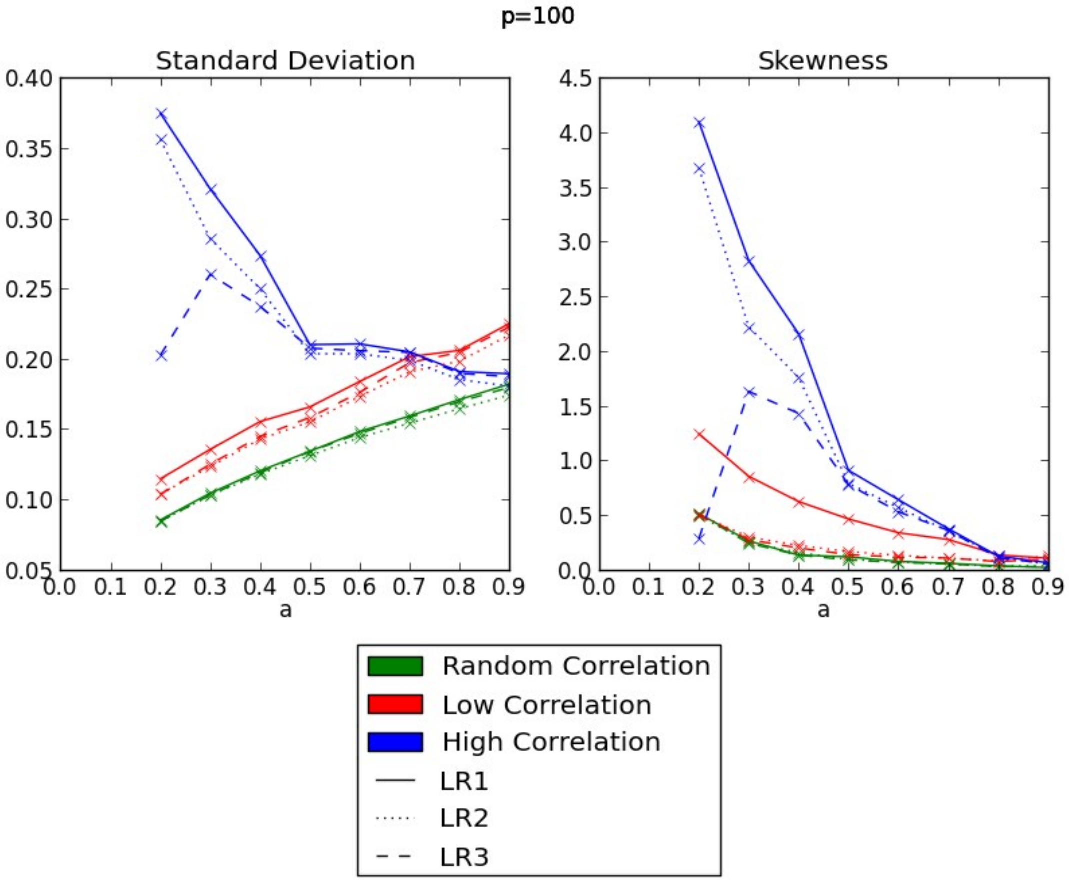


Figure 4.8: Reduction in standard deviation and skewness (as a function of sparsity) brought about by the modification of the learning rule, LR2 and LR3.

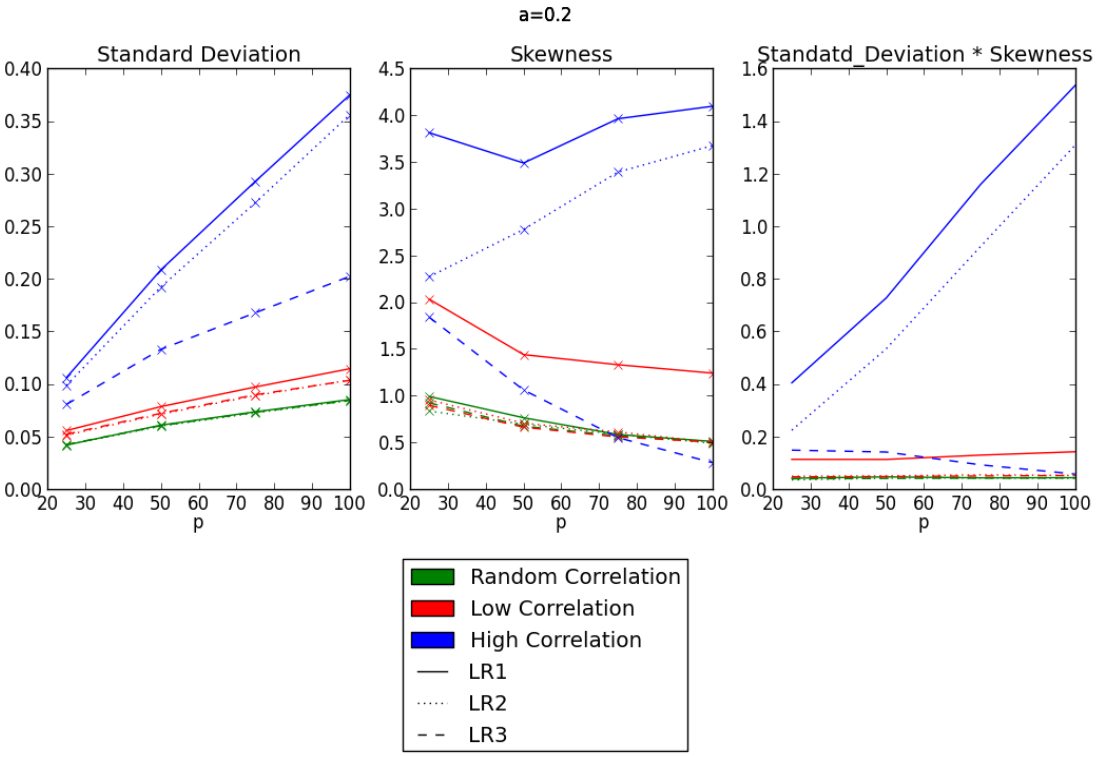


Figure 4.9: Reduction in standard deviation and skewness (as a function of storage load) brought about by the modification of the learning rule, LR2 and LR3.

A similar effect of reduction in standard deviation and skewness is seen as a function of network load p. LR3 however has a more pronounced effect in this case, as seen from the 3rd panel in figure 4.9, where the product of standard deviation and skewness which linearly rises for the high correlations case is made, instead, comparable to the low and random correlations cases by the application of LR3.

4.8 Storage capacity with modified learning rule:

From the results of section 4.7 we expect the storage capacity to be slightly higher than what we saw in figure 4.3, which is indeed the case as shown in figure 4.10 (pto).

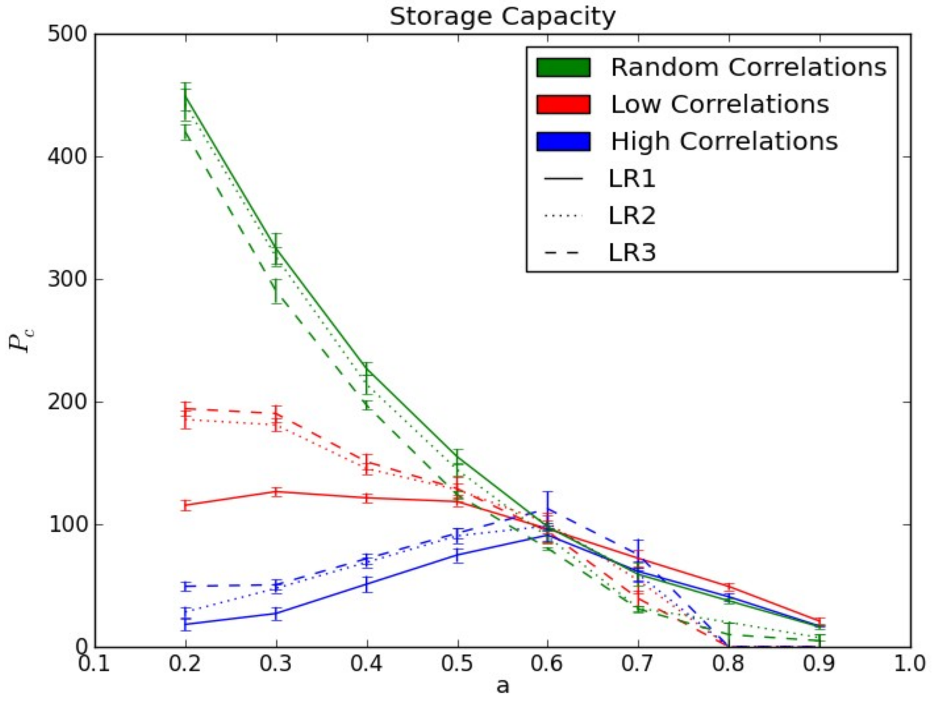


Figure 4.10: Storage capacity as a function of sparsity for the three learning rules LR1, LR2 and LR3 (see text)

Both LR2 and LR3 show an increase in the storage capacity for high and low correlations. The change is stronger in the case of low correlations which can be partly understood by the lowering of skewness for low correlations in figure 4.8. There is no big difference in the storage capacities between LR2 and LR3, with LR3 resulting in only a very marginal higher storage capacity as compared to LR2. A noticeable difference is seen at a=0.2 for the highly correlated case in LR3 which corresponds to the change seen in figure 4.8. None of the modifications however sufficiently reduce the interference due to correlations so as to increase the storage capacity to a comparable level with the random patterns case. LR3 in the case of random patterns, in fact, causes a slight lowering of the storage capacity, due to the fact that the correlations in the random cases are so low that any suppression of the inputs on the basis of correlations in turn results in the lack of information about the entire pattern. A similar effect is seen in the region of $a > 0.6$, where LR2 and LR3 result in lowering of storage capacity as compared to LR1. This is due to the subtraction of the popularity term, which becomes strong with increasing a and destroys information about the pattern.

4.9 A self-excitation trick to store more patterns:

We saw how interference from the stored patterns other than one being retrieved puts a limit to the maximum number of patterns that can be stored. This can be seen from the separation of the signal and noise terms in (e4.4). What happens if the signal is enhanced by an external input to the field, which makes it stronger than the interference from noise?

To recall (e4.3) , the field on each unit is given by

$$h_i^k = \sum_{j \neq i} \sum_{l=1}^{S} J_{ij}^{kl} \sigma_j^l + w \left(\sigma_i^k - \frac{1}{S} \sum_{l=1}^{S} \sigma_i^l \right)$$

here 'w' as explained chapter 4 section I-C, controls the self excitation of the unit and boosts the field in the direction of the signal. The results so far were obtained by setting w=0 and not having any self excitation. We now set w=0.8 thus making the signal stronger and looking at how the storage capacity is affect by this change.

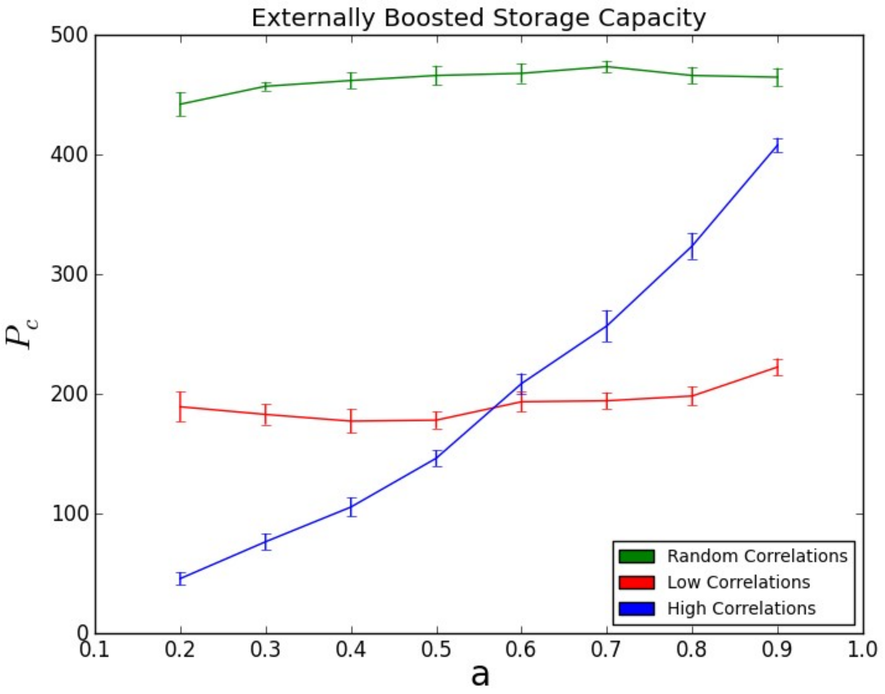


Figure 4.11: Storage capacity when signal is boosted by self reinforcement (w=0.8)

The effect of rising sparsity, a , which lowered the storage capacity, seems to be nullified in the case of random and low correlation patterns. The increasing spread of noise with increasing sparsity does not affect the boosted signal and the storage capacity remains more or less constant as a function sparsity at the level which corresponds to the higher storage capacity at $a=0.2$ in figure 4.10 for LR2 and LR3. Highly correlated patterns show a peculiar behaviour in the region $a>0.5$, there is a monotonic rise in the storage capacity, which becomes greater than the low correlated patterns and approaches the random patterns as a approaches 0.9. One may try to partly understand this behaviour from figure 4.4, for $a>0.5$ the standard deviation of noise for the high correlations case continues to drop, although at a much lower rate as compared to $a\leqslant 0.5$. This is in contrast to the high and low correlations case, for which the standard deviation rises with a over the entire range. In fact at a = 0.9 both the standard deviation and skewness for highly correlated patterns is close to the random patterns and lower than low correlation patterns. Thus self excitation has a higher impact on the high correlation patterns because of decreasing standard deviation and in the case of random and low correlations the effect of increasing standard deviation is balanced out by self excitation.

4.10 Discussion:

We presented an algorithm which generates patterns of various levels of correlations. These patterns are intended to represent the activity of local cortical patches and together represent the activity over the the entire cortex. Thus each pattern ξ^{μ} , is expected to be a stored memory which can be recalled when the right cue is presented. Since each unit in the pattern can be in several possible states and that there are several such units, a 'memory' of any particular object is thought be entailing its entire multidimensional aspects. Different objects can share various levels of correlations along several dimensions in the natural world. Thus it is important to study how correlations affect the storage of correlated memories in the Potts model.

Our long term aim is to store syntactic-semantic relations of words into the Potts network and we saw the first attempt at it in section I-E, [Primoradian 2012]. The words presented in that study shared a near random correlation but in future the aim is to introduce corpus-frequency based correlations amongst words.

Correlations severely limit the ability to store patterns in the Potts network. We saw that the signal becomes weaker in comparison to interference (noise) from other stored memories when correlations between the memories increase. Increasing noise drastically reduces the storage capacity for highly correlated patterns. By looking at the three different retrieval behaviours of the network for different level of correlations we decided to relax the definition of a successful retrieval and take into account the strong correlations amongst patterns. In doing so, we saw that the network could retrieve its maximum storage capacity around the level of random patterns but only for a low level of sparsity.

Modifying the learning rule based on the signal-noise analysis lead to a very marginal improvement in the networks ability to store correlated patterns. The increase in storage capacity due to LR2 and LR3 was more pronounced for the highly correlated patterns at $a=0.2$, with LR2 resulting in close to 1.4 times rise in the storage capacity while LR3 resulting in a rise of close to 2.7 times that of LR1. However, since the storage capacity is already so diminished for high correlations the increase too is small.

Storing correlated memories in a neural network, especially of the type as generated by our algorithm which intends to mimic natural correlations is a challenging task and certainly a crucial one in the context of language processing.

Chapter 4

Section III

Towards the Possibility of the Potts Glass Phase

4.11 On the glass phase of the Potts Neural network:

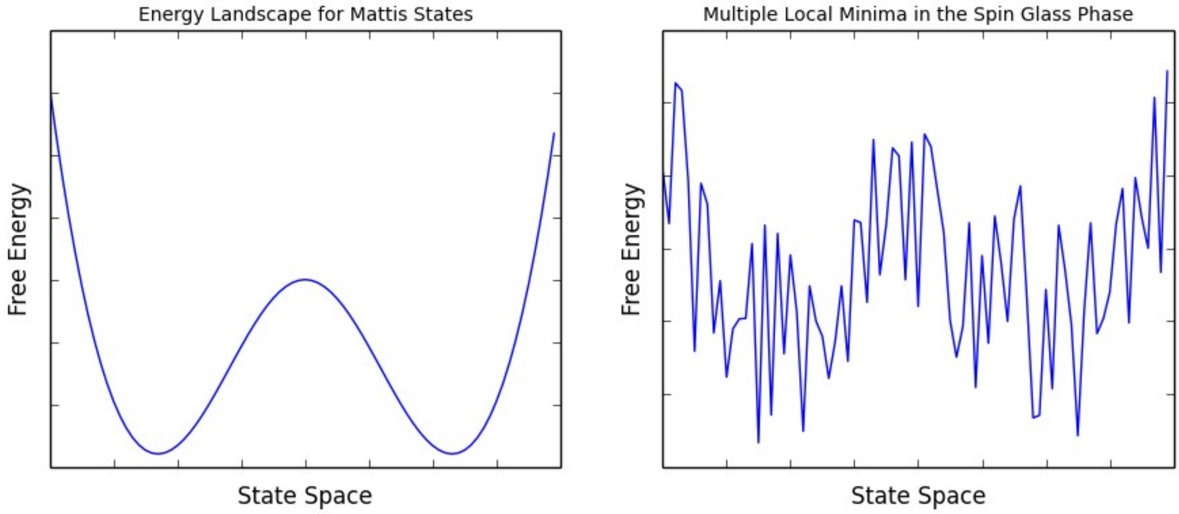
The Potts model, as described in section I-C, aims to model the interaction of several local modules in the cortex, where each module is represented by a Potts unit which can assume S different states. This is an N-body interaction system ($N \gg 1$), with infinite range interactions, which implies that a local unit i is not restricted to interact with only its neighbouring units but can interact with all other units in the system, with an interaction strength J_{ij} . Such systems, when the interactions are complex, or not following any simple ordering, are known to exhibit a rich variety of complex behaviour, one of which is the existence of a spin glass phase. A spin glass phase can be described as the typical low-excitation state of a system of interacting spins that exhibits both quenched disorder and frustration. In a relatively simpler case where each unit has only two possible spin states of + or – (Ising Spins), quenched disorder stands for a random distribution of 'ferromagnetic' and 'anti-ferromagnetic' interactions. This simply means that individual pairs of spins are either favoured to be aligned in the same direction or in the opposite direction, and such influences are 'quenched', i.e do not evolve with time. Thus on average, spin interactions cancel each other out. 'Frustration' in a spin system is when no spin arrangement of all the units completely satisfies all the couplings between the spins. The effect of unit i on j , determined by J_{ij} , contributes to the alignment of unit j, but unit j receives such orienting forces from many other units. In a frustrated system the spins get 'frozen' such that not all the interacting forces are satisfied by the final spin orientation of the unit j . This spin glass phase typically leads to multiple, possibly infinite different arrangements of spins and hence to a multitude of stable and metastable degenerate energy states.

Since it is a general property of disordered many body interacting systems, and we suppose the cortex, at least as a first order assumption, to fall under this category, it can be subject to investigation whether the cortex too, exhibits a spin glass like behaviour . The existence of frustration and multiple degenerate states makes it a particularly interesting problem when one thinks of rule learning and structured interactions between various domains in the brain.

4.11.1 Suggestions from the mean field models of the spin glass phase:

Spin Glass properties of interacting spins were originally studied in the scope of magnetic materials but soon the importance and application extended to various other N-body systems. [Sherrington and Kirkpatrick, 1975] proposed a mean field theory of spin glasses with infinite range interactions (SK model), which was an extension of the finite range Edward-Anderson model. An important mathematical tool was developed for this purpose called the 'Replica Trick', which makes possible to calculate the mean free energy of the system and hence further make a phase study analysis. Without going in details, the replica trick involves calculating the partition function, $Z=\sum_s e^{(\beta H_s)}$, of n replicas of the system and then take the limit $n\rightarrow 0$ using the identity, $\log Z=\lim_{n\rightarrow 0}\frac{Z^n-1}{n}$. The free energy is defined as $F=-T\ln Z$ (where T represents the temperature of the system). The SK model develops the solution in the replica symmetry assumption, where the n replicas are perfectly symmetric, however this leads to a negative entropy in the $n\rightarrow 0$ limit. As a solution [Parisi 1979, 1980] proposed a model by introducing an ansatz and breaking the replica symmetry which solves the negative entropy problem. Importantly, a notable result is that the low temperature solution shows a phase transition below a certain critical temperature, T_c , to the spin-glass phase of infinite degenerate states, in the system of infinite range interacting Ising spins.

[Gross, Sompolinsky 1985] developed the mean field theory of Potts glass where they found a continuous transition to two distinct glass phases for number of states $S \geqslant 3$, at low temperatures below the critical temperature T_c. The first one at $T < Tc$, (PG1), with infinite pure degenerate states which do not overlap with each other and the second one (PG2) at $T_2 < T_c$ where each of the pure state in PG1 splits into a hierarchical structure of partially overlapping states. For S >4 the transition becomes discontinuous.



4.11.2 Glass phase in the Potts neural network model:

Our model, which is specifically designed to address cortical dynamics, has a neural network structure and differs from the models mentioned above in three aspects,

a) The interaction term J_{ij} is a result of associative learning as determined by the specific patterns that are stored in the network. Importantly, the way in which we define the learning rule, the J's do not follow the full symmetry of $J_{ij}^{kl} = J_{ji}^{lk}$. Whereas in the models discussed above, J's are decided by a Gaussian probability function with a specific mean and standard deviation.

b) The Potts units in our model are graded and the final state σ_i^k is determined by a continuous transfer function applied to the fields, allowing a graded response in the units unlike the completely discrete units of above mentioned models.

c) An important distinction in the dynamics of our model is the definition of 'active' and 'inactive' states which have separate dynamics. Learning occurs only with the active states to remain in the biologically plausible scenario.

[Bolle *et al* 1992] studied the mean field solution of the S state Potts glass neural network model where the phase is investigated in Temperature-Storage capacity space. Though the model discussed by Bolle et al still differs from ours in all the three points mentioned above, nevertheless it sheds light on the spin glass phase of the Potts neural network and it reveals the rich behaviour of this class of networks. Among the findings of this study, they show that there exists a critical temperature T_g , below which the network exhibits a spin glass phase which consists of states that share no overlap with the stored states. Another transition occurs at $T_M < T_g$, where the states have a finite overlap with the stored patterns. Thus the stored patterns are local minima of the free energy and the network can function as an associative memory, and below T_M the stored patterns become a global minima of the network.

It is likely that there exists a spin glass phase in all such models of large interacting systems and in the limit that the brain can be modelled as a system of many interacting neurons/modules, this becomes an important characteristic which may influence cognitive function.

4.12 Indication of spin glass phase in the 'cortical' Potts model:

We explore the glassy nature of our model by examining the final steady state energy values that the network evolves into. In the usual attractor dynamics, the network settles at the bottom of the energy valley of a particular attractor which is designed to be one of the stored memory patterns. However in a spin glass situation the network can have multiple stable energy states brought about by the interactions of various units. We calculate the energy of the network state or pure pattern with the standard formulation with an addition for graded units,

$$E = -\frac{1}{2}\sum_{i,j\neq i}^{N}\sum_{k,l=1}^{S} J_{ij}^{kl}\sigma_i^k\sigma_j^l + \sum_{i}^{N}\sum_{k=1}^{S}\left(U\sigma_i^k + \frac{1}{\beta}\left(\sigma_i^k \ln\frac{\sigma_i^k}{\sigma_i^k+\sigma_i^0} + \sigma_i^0 \ln\frac{\sigma_i^0}{\sigma_i^k+\sigma_i^0}\right)\right)$$

where the first two terms denote the interaction energy and the contribution of the fixed threshold respectively, while the last term is the contribution due to the fact the Potts units are graded [Russo,Treves 2012].

We first look at the scatter of energy levels of the pure patterns and the stable network state when a pattern was cued to the network for retrieval. As in section 5.4 we separate the three different retrieval behaviour by the same colour code of green,red and yellow for case(i), case(ii) and case(iii) respectively. Figure 4.12 plots the energy of the pure pattern (triangle) for each of the pure pattern ξ^{μ} , $\mu=1,2...p$. p is set around the storage capacity level so the network can exhibit the different types of retrievals. Along with the energy of a pure pattern, the energy of the final network state when a particular pattern ξ^{μ} is cued to the network is represented with a cross.

We see that the steady network state energies do not coincide with the pure pattern energies and find a multitude of energy valleys to settle in. The network either settles in a region above the desired valley and yet have a successful retrieval in the retrieval type of case(i) or find an energy state below the pure pattern. This is an indication of a typical property of the spin glass phase, in which there are high number of metastable states due to the formation of multiple local minima, resulting from several possible spin interactions. One clear phenomenon seen is that whenever the network fails to retrieve, either case(ii) or case(iii), it almost certainly settles in a state lower than the cued pattern as seen clearly in figure 4.13. Thus it falls out of the desired attractor and finds a basin with a lower energy, leading to a failure in correct retrieval. This observation is consistent across the different levels of correlations amongst patterns. The failed network states are all on the same level for random patterns because this is the inactive state of the network and for unsuccessful retrievals the network decays into inactivity. Same is the situation for low correlation patterns but in this case the network decays into a mixed state with several patterns having a finite overlap with the network state. In the case of highly correlated patterns, there exist situations where the network settles into at the level of pure pattern energy level. An interesting observation in the case of random patterns is that the pure patterns which are lower in energy than the mean energy of patterns, are retrieved successfully while those above fail. This clear separation is not seen in the other two cases of correlated patterns.

We calculate the difference in the energy level of the network state and the energy of the pure pattern, thus positive values imply that the network could not reach the bottom of the valley

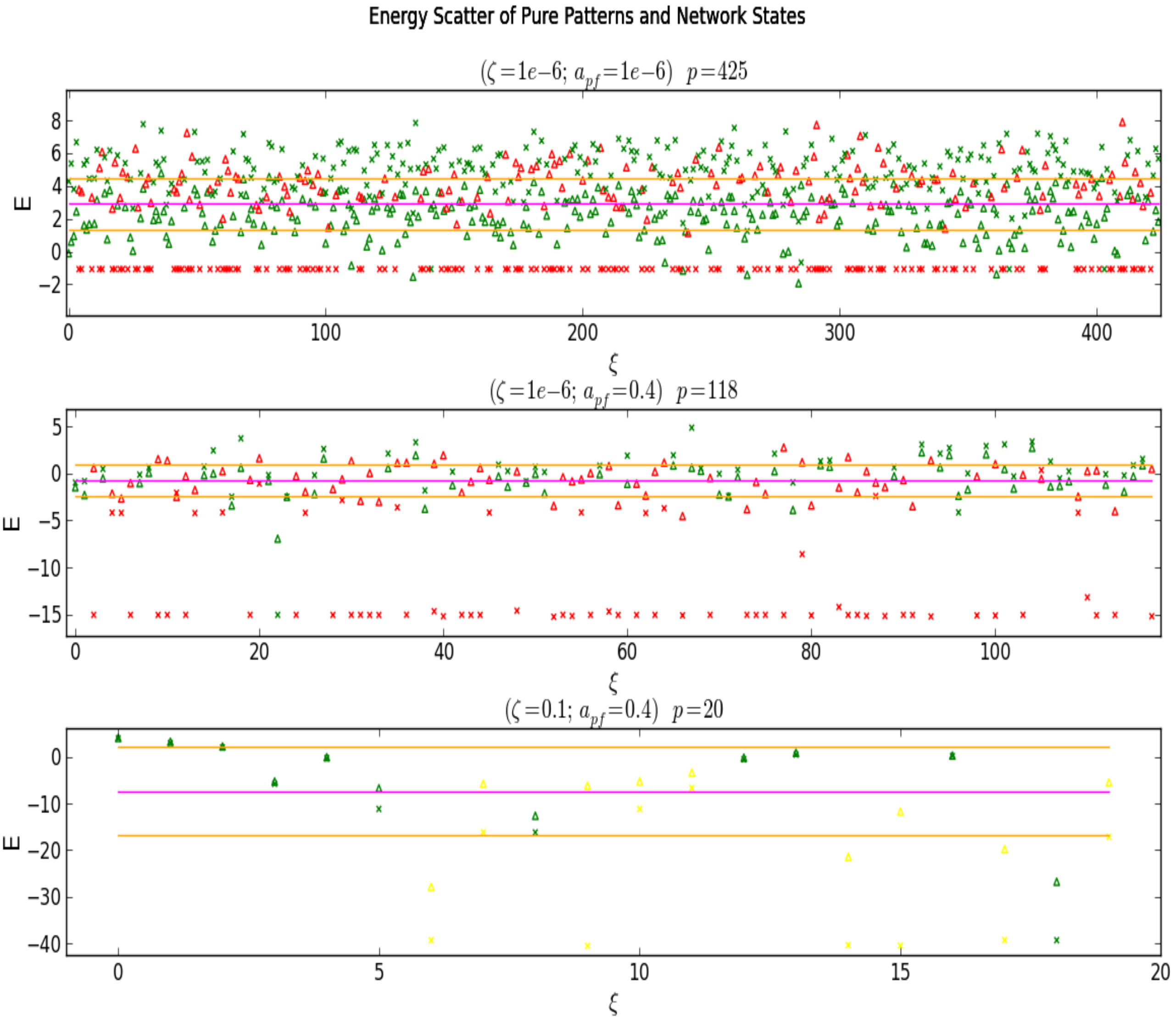


Figure 4.12: Energy states for pure patterns (triangles) and network state (crosses). Horizontal line depicts mean energy of pure patterns (magenta) with its standard deviation in orange. Random patterns (top panel), Low correlation patterns (middle panel) and high correlation patterns (bottom panel) Colour codes depict three types of retrievals green-case(i), red-case(ii) and yellow-case(iii)

and found a stable state above the cued pure pattern, whereas a negative value indicates the network was able to find an even lower energy level that the cued pattern. Figure 4.13 shows the mean energy difference for the three possible cases of the difference in energy between network state and pure pattern, namely being positive, negative or 0 (tolerance level of +/-0.5 of the energy difference) . The numbers around the bars indicate the fraction of patterns in a particular case (positive- above the bar, 0-beside the bar at 0 level, negative-below the bar) . The three types of retrieval behaviours are shown in their defined colours. Figure 4.13 makes it clear that failures always find a range of stable network states below the pure pattern while the majority of the successful states are when the network is trapped in the energy valley of the cued pattern but above the pure pattern energy level. One may speculate that the tendency of the network is to fall out of the energy valley towards an attractor with a deeper energy valley but probably frustrated frozen spins trap the network in the valley while for failures it manages to fall out.

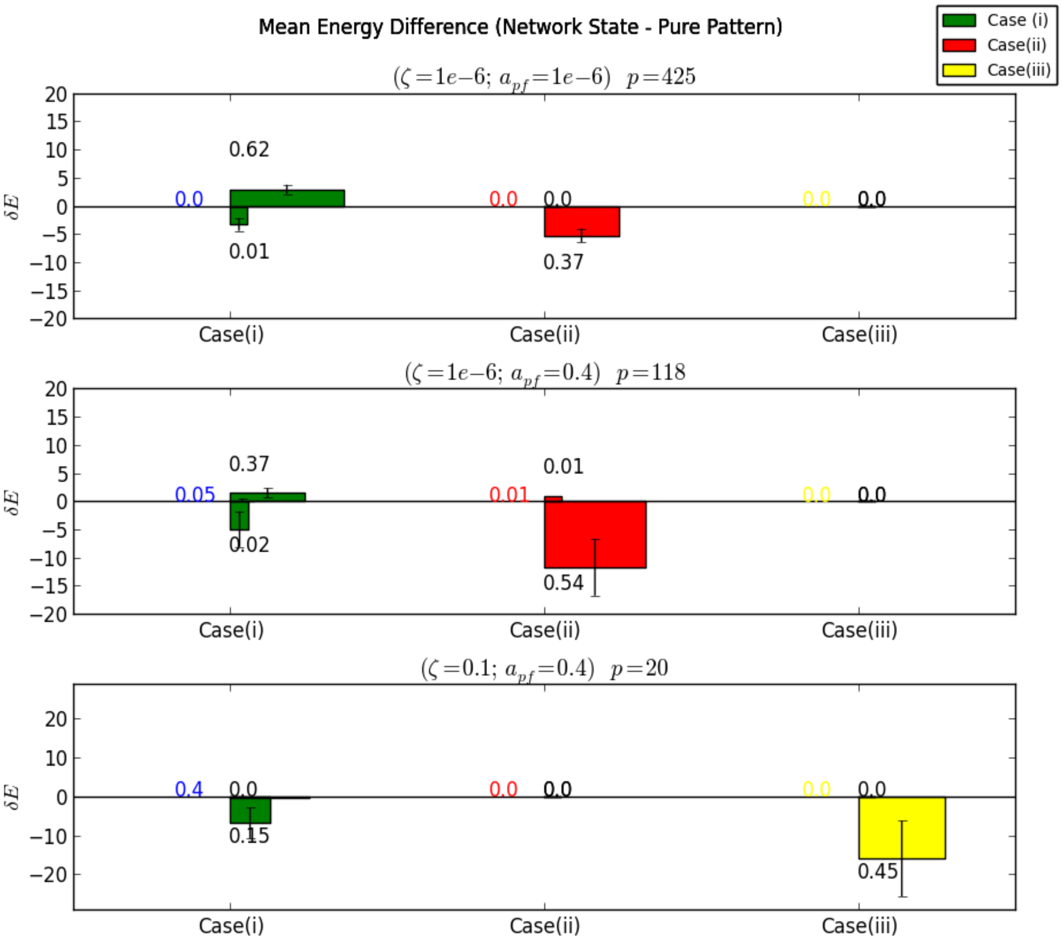


Figure 4.13: Mean difference in energy between network state and pure pattern. Numbers indicate fraction of patterns in each condition (see text) represented by the width of the bar.

4.13 Effect of local module temperature:

The parameter β intends to represent a local module inverse temperature and in effect controls the noise level within a unit over its various states. Low values of beta, implying high temperature would make the units very noisy and hence none of its states would be dominant. On the other hand at very low temperatures, high β, the state having the highest local field will essentially coincide with the final state of the unit while all other states will become 0, representing a noiseless situation. Thus at high β the Potts units become in practice non graded. We test how the above mentioned spin glass effects are affected at different local module temperatures. The results of the previous section were obtained at $\beta=11$, which represents a medium noise level which still allows the network to function as an associative memory. Now we test the extreme limits of very high temperature at $\beta=5$ and very low temperature, $\beta=100$, which makes the Potts units, in practice non graded.

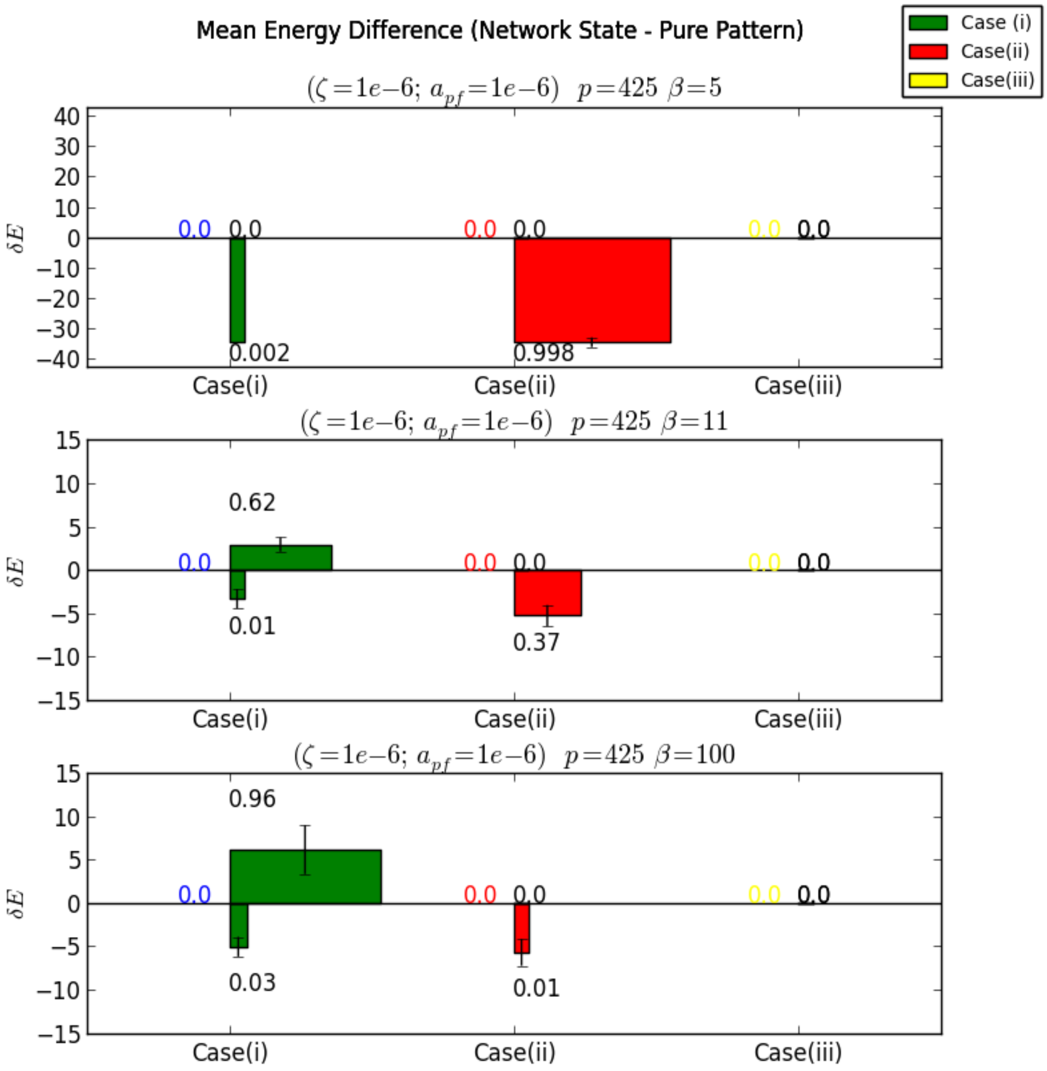


Figure 4.14: Mean difference in energy, same as figure 4.13 but for three different values of β - 5 (top), 11 (middle) and 100 (bottom) panel, in random patterns

At high temperature $\beta=5$, a retrieval becomes highly improbable since the units can not be kept in the state with the highest local field. This results in an almost total failure to retrieve with only one pattern retrieved, in which shows the network settles much below the pure pattern energy. In the noiseless case $\beta=100$,by contrast 99% of the patterns are successfully retrieved. Importantly the spin glass behaviour is present also when the units practically behave a non graded manner. A similar trend is seen for low and high correlation patterns (figures 4.15 and 4.16). As seen before (figure 4.13) the failures in high correlation patterns are of the type(iii).

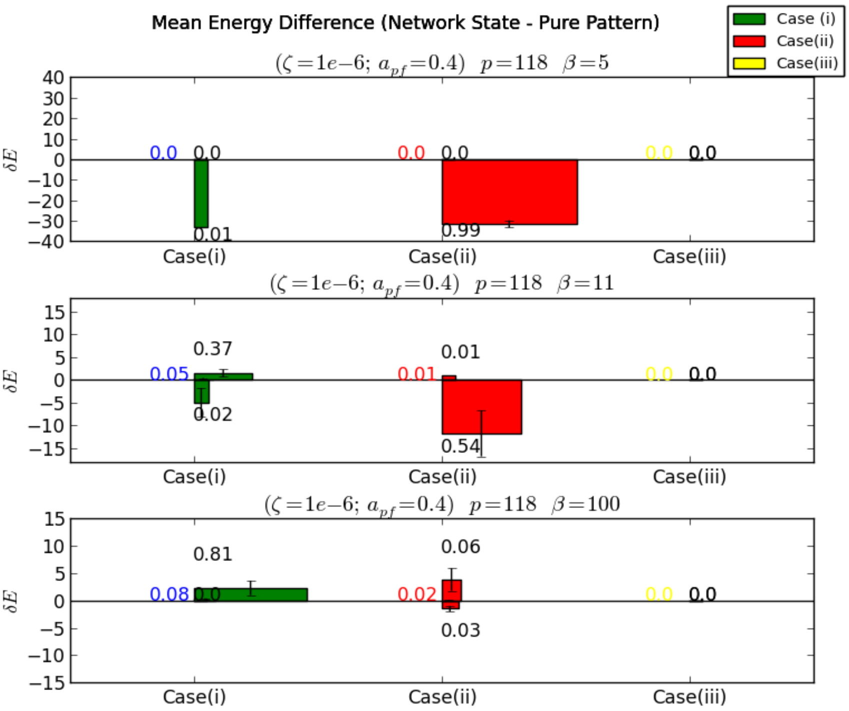


Figure 4.15: Mean energy difference for three levels of local noise, $\beta=5$ *(top),* $\beta=11$ *(middle) and* $\beta=100$ *(bottom), in low correlation patterns.*

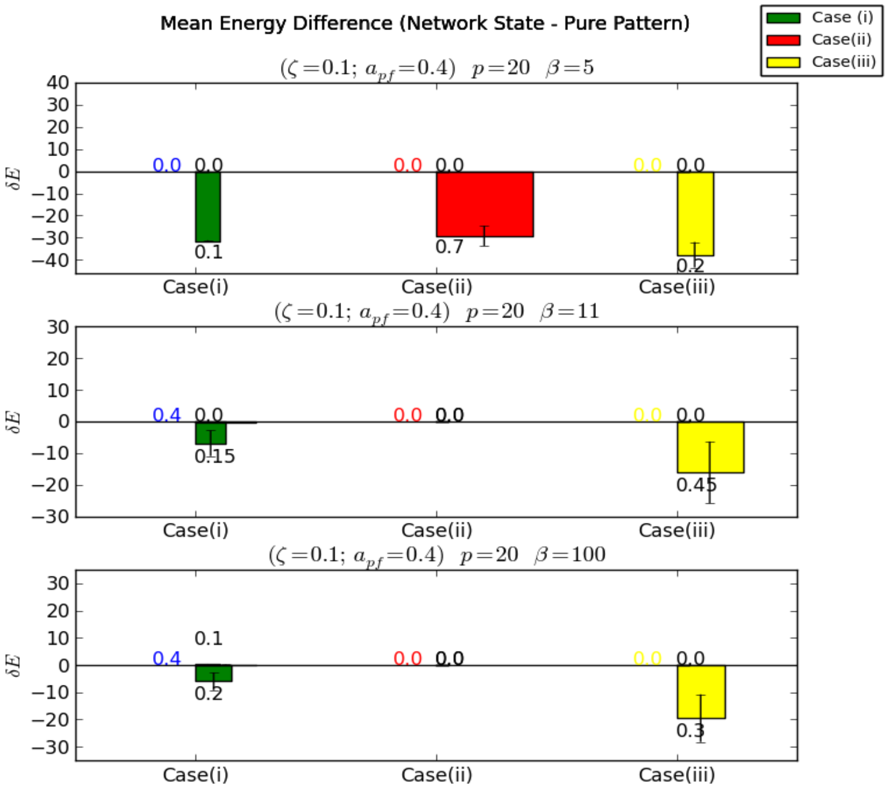


Figure 4.16: Mean energy difference for three levels of local noise, $\beta=5$ *(top),* $\beta=11$ *(middle) and* $\beta=100$ *(bottom), in high correlation patterns.*

4.14 Effect of network level noise:

Apart from the internal local module noise, we add a noise to the fields that globally affects all the units in a random manner. This maybe considered as having an effect analogous to a global temperature of the network. Noise is introduced as a random number, $0\leqslant\rho\leqslant1$, scaled by η that is added to each unit when it calculates local field on it.

$$h_i^k = \sum_{j\neq i}^{N}\sum_{l=1}^{S} J_{ij}^{kl}\xi_{j,l}^{1}+\eta\rho_i^k$$

Thus $\eta=0$ is the noiseless condition and as it increases the fields start to destabilise, causing them to be overridden by noise. The combination of the local and global noise has an interesting effect on the random patterns as we can see in figure 4.15.

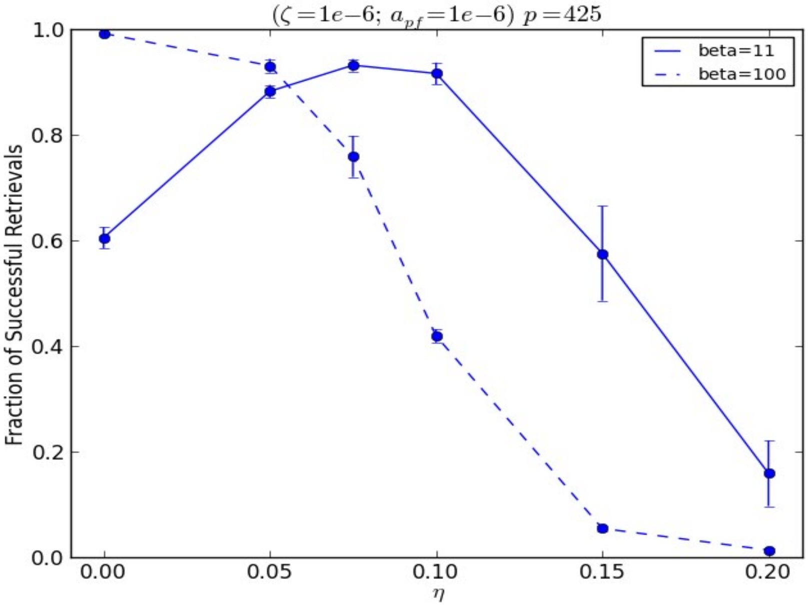


Figure 4.17: Fraction of patterns successfully retrieved as a function of external noise, η *. For two different values of* $\beta=11$ *(full),* $\beta=100$ *(dashed)*

When β is high, the fraction of patterns that is successfully retrieved, rapidly decays as noise increases, with a sigmoid like function. This fits the intuitive notion of how noise may affect retrievals, because it randomly reorients the units thus dominating over the inputs from the synaptic connections. However at $\beta=11$, when the units are graded there is a maximum in the fraction of successful retrievals at $\eta=0.075$, with a rapid decay for $\eta>0.1$. Overall the combination of both local and global noise results in a higher success rate for $\eta>0.05$. As one might expect, when both the internal and external noise levels are low ($\beta=100$, $\eta=0$), the network has the best retrieval rate.

As before we calculate the mean energy difference between network state and pure pattern for the global noise η in figure 4.18. but now we calculate the total mean instead of separating the positive and negative differences. In the case of high β, $\beta=100$, the mean difference in the energy is positive and increases monotonically with η. Hence the network finds it increasingly difficult to

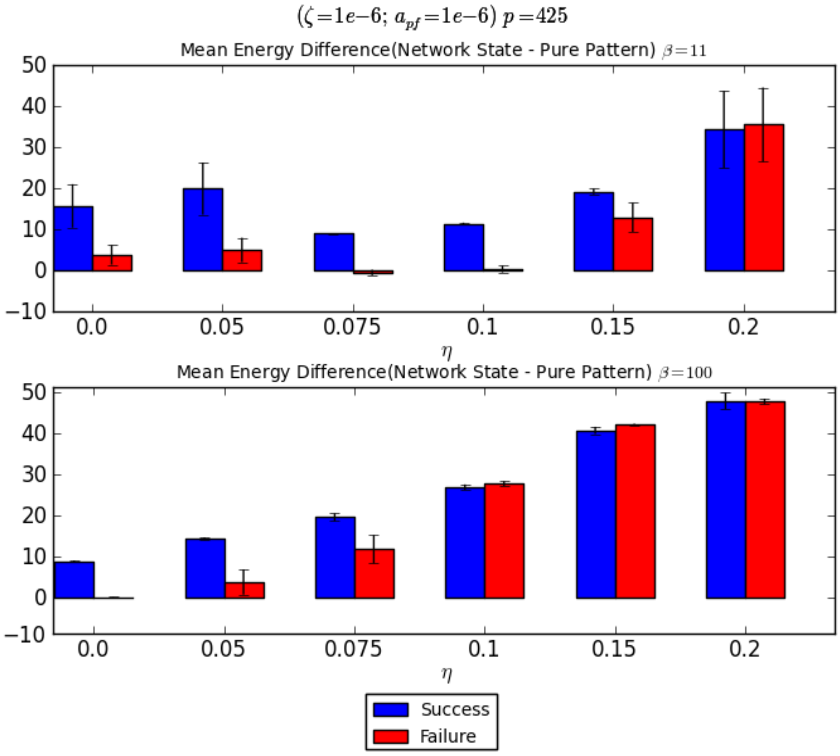


Figure 4.18: Influence of external noise η *on the total Mean difference in energy.*

reach the bottom of the energy valley when noise is increased. In the case of $\beta=11$, instead there seems to be a region around $\eta=0.075$, where the effects of the two types of noises prove beneficial for the network to reach closer to the energy valley of the pattern, seen as a reduction in mean energy difference. For $\eta<0.075$ the medium internal noise units ($\beta=11$) have a difficulty in reaching the energy bottom as compared to the noiseless units ($\beta=100$), thus resulting in lower fraction of successful retrievals in figure 4.17. Having graded units with a small global random noise helps in better retrieval of patterns as seen through the lower mean energy difference for $\beta=11$ as compared to $\beta=100$, in the region $\eta\geqslant0.075$.

This combined effect of global and local noise to cause an increase in successful retrievals is seen only for the random patterns and is absent in low correlation patterns, where the fraction of successful retrievals monotonically reduce with η (figure 4.19). The low correlation case seems to follow the intuitive expectation that low internal noise units $\beta=100$ will tend to have better retrieval ability and thus lead to higher number of patterns retrieved as compared to $\beta=11$.

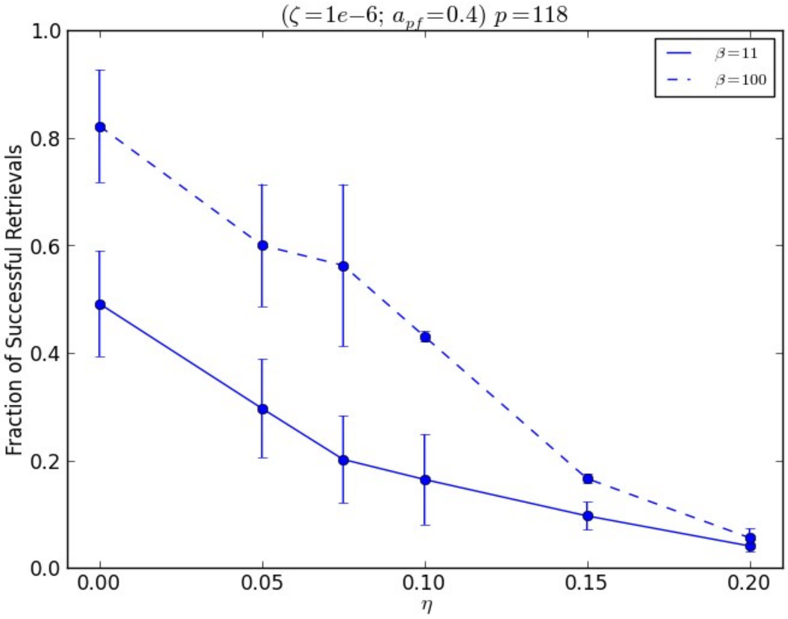


Figure 4.19: Fraction of patterns successfully retrieved as a function of external noise, η *. For two different values of* $\beta=11$ *(full),* $\beta=100$ *(dashed) in low correlation patterns.*

4.15 Behaviour at low S and full connectivity:

With cues from [Gross *et al* 1985] where they find a distinction in the phase transition at low S and S > 4, we reduce the number of states to $S=2$ to check if we find the same behaviour of the network as in the case of S=5 investigated before. In addition we test the behaviour when the network connectivity is increased to a fully connected network and eliminate effects of dilution, thus C_m which is 90 in previous cases is increased to 599 (N=600). In both cases the local inverse temperature β is set at mid-level of $\beta=11$.

Figure 4.20 confirms the spin glass nature of the 'cortical' Potts network in low S and also in a fully connected case. Apart from the fact that there is a higher fraction (0.11) of patterns in which the network reaches the bottom of the energy valley of the cued pattern for $S=2, C_m=90$, when compared to $S=5, C_m=90$, there are no major differences. For $C_m=599$ the network settles in a varied range of levels during successful retrievals below the cued pattern energy level thus finding a better configuration of spins which reduce the energy of the network and yet retrieve the intended pattern.

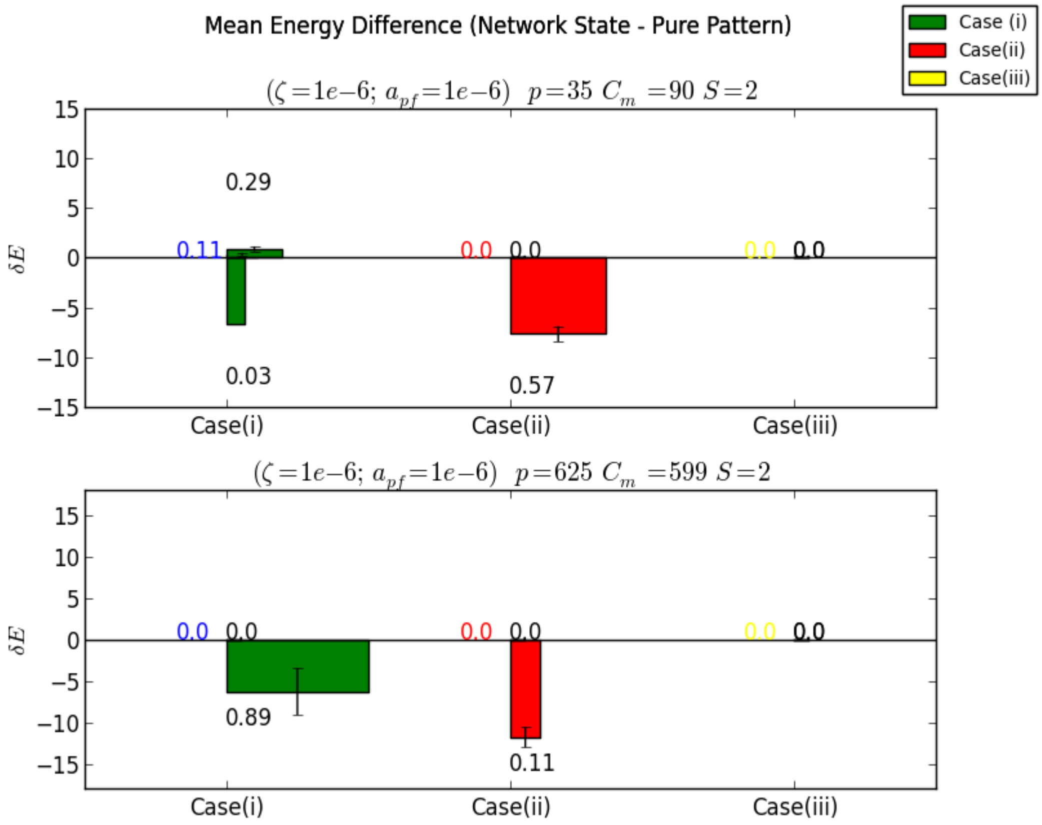


Figure 4.20: Mean difference in energy, same as in figure 4.13, but for $S=2$, $C_m=90$ *(top) and* $S=2$, $C_m=599$ *(bottom) panel*

4.16 Brief Discussion:

The human cortex is estimated to have 10^9 neurons [Herculano-Houzel S 2009] and an order of 10^{14} synapses. As described in section I-C we model the cortex as N interacting Potts units, each of which represents a patch of the cortex, that can be in S possible attractor states, resulting in a system of N interacting S state spins . It has been shown that at low temperatures a system of large N interacting spins exhibits a spin glass phase. A spin glass phase has multiple degenerate energy states and has a 'frustrated' spin configuration. This allows the network to settle in varied energy states with different spin configurations. If we assume that a simplified version of cortical dynamics can be modelled by the Potts neural network then it is possible that the cortex may function in a spin glass like manner.

We found that the Potts neural network does show signs of a spin glass like phase by allowing the network to evolve after an initial cue and update its unit as prescribed in section I-C. The network settles into multiple energy states which are either higher than the energy of the pure pattern or lower. In the case of successful retrievals, the network predominantly settles in a higher energy state as compared to the pure pattern energy but the fact the retrieval is successful implies that the network is in the correct energy valley. Since there is a difference in the two energy levels of the network state and pure pattern, it implies that not all the Potts spins are aligned with the pure pattern and some of them are 'stuck' in the attempt of the network to find a suitable energy level. Since whenever the network fails to retrieve it almost always settles in a state lower than the pure pattern, and these failed states are the lowest amongst all other states. Thus it is possible that the Potts spins in a successful state get 'frozen' in the networks attempts to settle in a lower global minima than the 'successful' higher local minima and hence leading to the observed positive energy differences between the network state and pure pattern.

Spin glass like characteristics that were seen with a diluted network connectivity, $C_m=90$ and medium internal noise $\beta=11$ at S=5, were also consistently observed in the low S=3 condition, full connectivity, $C_m=599$ and a near absent internal Potts unit noise $\beta=100$. Adding a small amount of external global noise combined with the medium level of internal noise seems to facilitate the network, to reach the energy valley bottom. This can be seen through the reduction in energy level difference between the network state and pure pattern. As a result it leads to a higher fraction of patterns that are retrieved successfully. One possibility is that a small external noise at , does not destabilise the unit but is sufficient enough so that it overrides the interaction forces that cause misalignment amongst units.

The effects of spin glass behavior in the Potts neural network and its application to cognitive process is yet unexplored but provides an exciting opportunity to study how structured transition between stored memories may occur in a dynamical scenario. Since spontaneous transitions between memories are largely driven by the energy landscape, the spin glass nature of the Potts network may influence the transitions. Structured transitions between memory 'objects' constitute a formal rule and hence the spin glass effects may play a role in language processing too.

5 A Report on the Effect of Learning Rules on Latching Dynamics

5.1 Latching behaviour for increased correlation between patterns

In chapter 4, section I-D we mentioned the dynamic behaviour of the Potts neural network, in which the network exhibits latching (spontaneous transition from one stored memory to another). Latching occurs due to the time varying thresholds (θ_i^k and θ_i^0), which destabilise the units once the network has reached a stable attractor state and thus eventually cause the network to fall out of the attractor. However, as the network is falling out of the attractor it may be driven towards another attractor which has its basin of attraction in the proximity of the previous attractor. The temporal scale of network dynamics is governed by three time constants,

(i) τ_1 – time constant in which the local field experienced by each unit in its active states is integrated, subject to the time varying threshold; (ii) τ_2 – the time constant with which θ_i^k , the threshold affecting only the active states, updates itself and (iii) τ_3 – the time constant for updating θ_i^0 , the threshold expressing the effect of slow inhibition. The specific dynamics are governed by the equations mentioned in chapter 4, section I-C.

An analysis of the storage capacity in chapter 4, section II was worked out considering the asymptotic steady state behaviour of the network. For that purpose τ_2 and τ_3 were effectively set to ∞ , thus removing the adaptive nature of the Potts units. We now consider the network in what is termed as the slow adaptive regime, referring to the situation in which neuronal dynamics is much faster than the threshold dynamics. This is obtained by setting $\tau_3 > \tau_2 \gg \tau_1$ for finite a τ_2 and . τ_3 [Russo, Treves 2012] studied the latching properties of the network using random patterns in the slow adaptive regime as a function of various parameters like the self-excitation term w , storage load p , local temperature T , connectivity dilution C_m and the time constants. One of the observations of interest was that the network tends to latch between patterns that have an above average correlation amongst them. However we have seen in section II that increasing the strength of the correlations drastically affects the behaviour of the network in terms of its stable retrieval ability of stored patterns. We now focus our attention on the dynamical behaviour of the network when the patterns are highly correlated and present a preliminary study with some immediate observations.

5.2 Effect on latching due to different learning rules:

The network is tested in the 'infinite latching' phase, which refers to the situation when the network does not stop latching for the entire duration of our simulation (5000 time steps). The asymptotic stability of a stored pattern during its retrieval and thus the critical storage capacity of the network has a limited impact on latching dynamics due to the non-stationary nature of retrieval in the latching phase. Latching is however, sensitive to the strength of correlations amongst patterns. We load the network with 200 patterns and record the latching transitions (consecutive pairs of patterns that have the highest overlap with the network state across a transition) using the three learning rules LR1, LR2 and LR3, described in section II-4.7. In the case of random patterns latching was seen to occur only for $w > 0$ [Russo, Treves 2012], however in the case of highly correlated patterns we see continuous and spontaneous activity at $w = 0$ (figure 5.2-(i),(ii)).

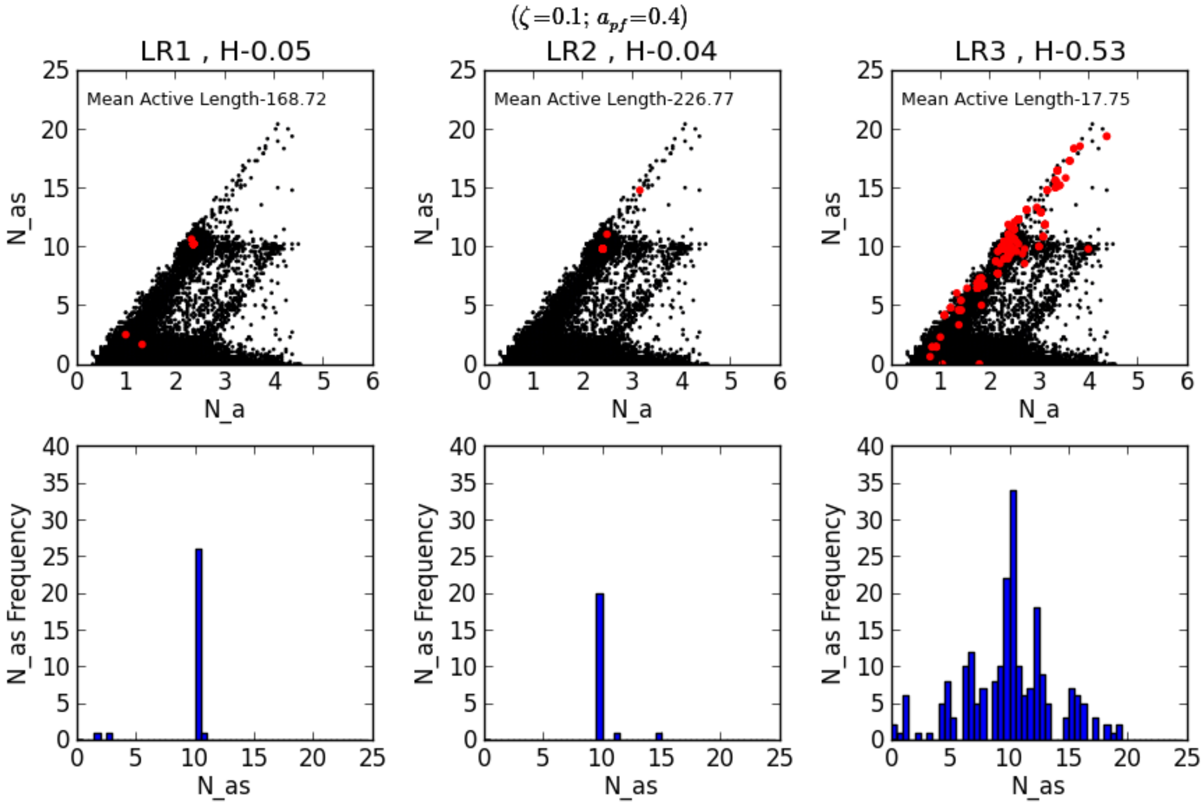


figure 5.1- Latching statistics for high correlation patterns using the learning rules LR1, LR2 and LR3. (see text for details)

When the patterns are highly correlated there does not seem to be 'proper' latching in the network (using LR1 and LR2) even though the network shows spontaneous activity. The network appears to be stuck in a mixed attractor (figure 5.2-(i) and (ii)) and keeps oscillating between highly correlated patterns without a transition into another pattern. However with the application of LR3 this cycle is broken, allowing the network to visit other patterns and exhibit 'proper' latching (figure 5.2-(iii)). The top row of figure 5.1 shows the correlation scatter of all the pairs of patterns (just as in figure 4.2a) but with an overlay of latching pairs (in red). The three columns represent plots for the three different learning rules. Most of the latching happens between pairs that share a correlation of around $N_{as}=10$ (2nd row figure 5.1) which is midway between no correlation ($N_{as}=0$) and the pairs with maximum correlation ($N_{as}=20$).

We calculate the entropy of latching, which quantifies the diversity of patterns visited during the latching sequence. Thus low entropy would indicate the network is stuck between a few patterns and an increase in entropy would imply that network is visiting more patterns during the latching sequence. The entropy was 0.05, 0.04 and 0.53 for LR1, LR2 and LR3 respectively. The mean active length - the time a pattern stays active with the highest overlap during latching is 168.72 and 226.77 time steps (in a total of 5000 time steps), for LR1 and LR2 respectively. This indicates that the highest active pattern, in the case of highly correlated patterns, shows resistance to decay and stays active for a longer period of time as compared to low and random correlations (figures C and D), for which the mean active length is in the range of 44 to 79 time steps. LR3 however, for highly correlated patterns, facilitates transitions to other patterns and brings down the mean active length to 17.75 time steps.

LR3, with its proportional suppression of high correlations during the storage of patterns, brings about a separation in the energy valleys of stored memories, which otherwise in the case of high correlations are merged into each other with strong overlaps. Thus we can restore the latching behaviour of the Potts network for highly correlated patterns.

Below, in Figures B (i), (ii) and (iii) in overlap of each pattern with the network state is plotted against time (shown for first 1500 time steps). As the network transits from one attractor to another, the respective patterns for those particular attractors have the highest overlap with the network. The overlap is measured as,

$$m_{\mu}=\frac{1}{Na(1-a/S)}\sum_{j\neq i}^{N}\sum_{l=1}^{S}(\delta_{\xi_j^{\mu},l}-a/S)\sigma_j^l$$

(all symbols have their usual meaning, as described in section I-C)

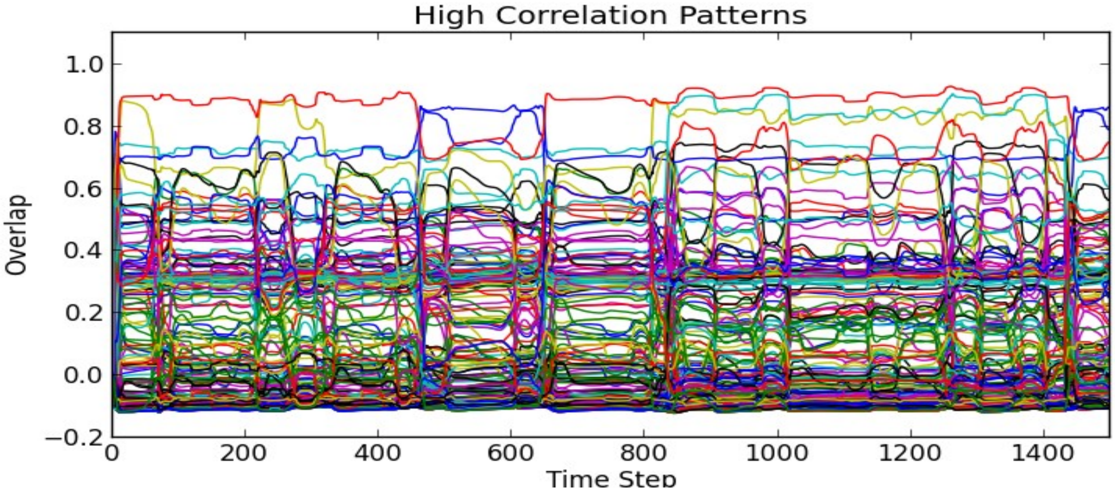


Figure 5.2-(i): Latching transitions using LR1. The network activity is mainly dominated by two patterns which do not show a complete decay

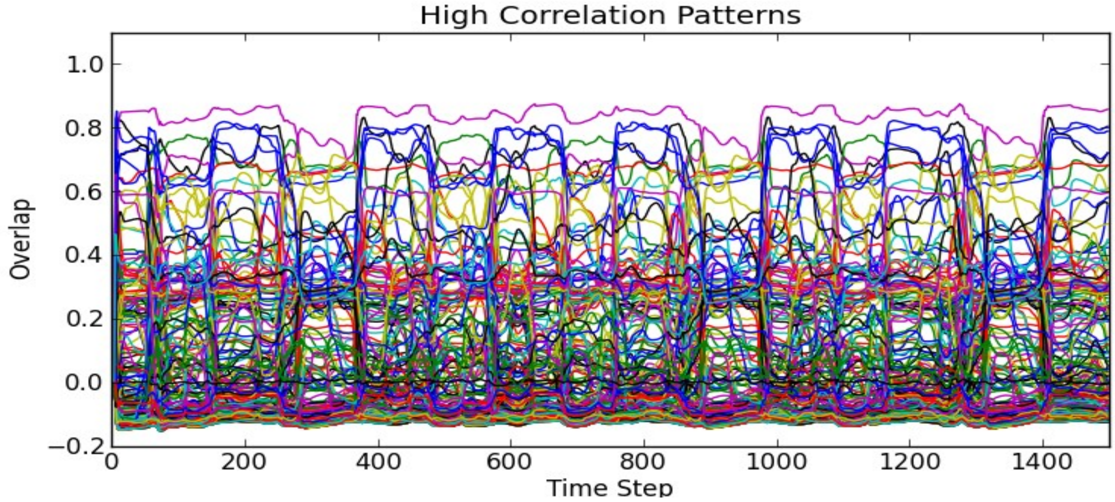


Figure 5.2-(ii): Latching transitions using LR2. The network activity is mainly dominated by a single pattern which does not show a complete decay

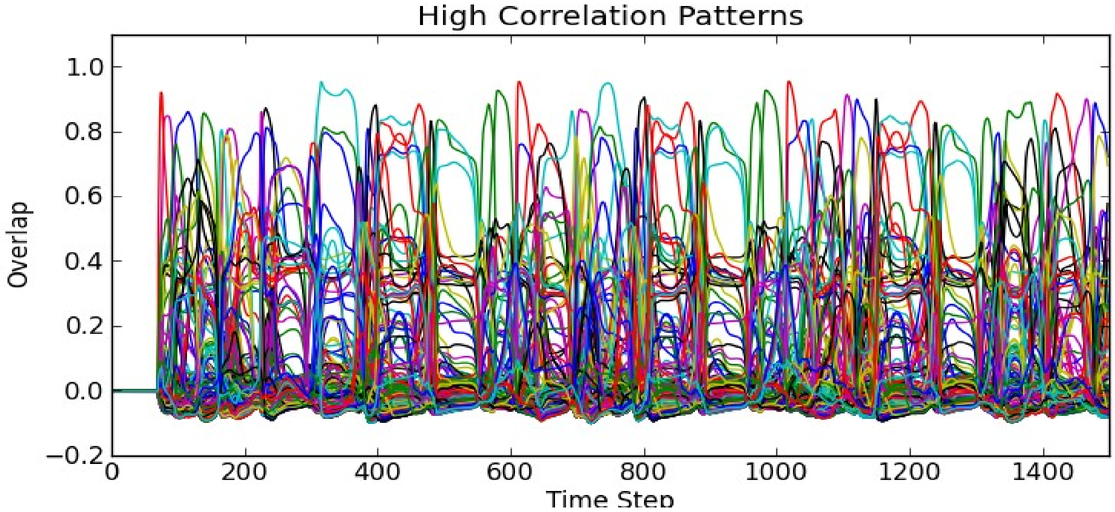


figure 5.2-(iii): Latching transitions using LR3 with $\alpha=1$ *. The network latches between various patterns.*

Rest of the parameters, as described in chapter 4, section-I-C have the values of, $N=600$, $S=5$, $a=0.2$, $U=0.2$, $\tau_1=2$, $\tau_2=10^2$, $\tau_3=10^6$, $C_m=90$ and $\beta=11$, $w=0$.

We now record the latching statistics, i.e. the same as in figure 5.1 but for low and random correlations, shown in figure 5.3 and D, with all the network parameters kept the same except w, which is set at $w=1$ in order to be in the infinite latching phase. The pattern generation parameters for low and random correlations are the same as in section II-4.2. The dynamical behaviour of the network for low correlations and random correlations is very similar to each other. The phase transitions in the $w-T$ space are seen to be similar too, although at different transition lines [Russo, Treves 2012].

From figures 5.3 and 5.4 we see that the three learning rules have only a marginal influence on latching dynamics when correlations are low.

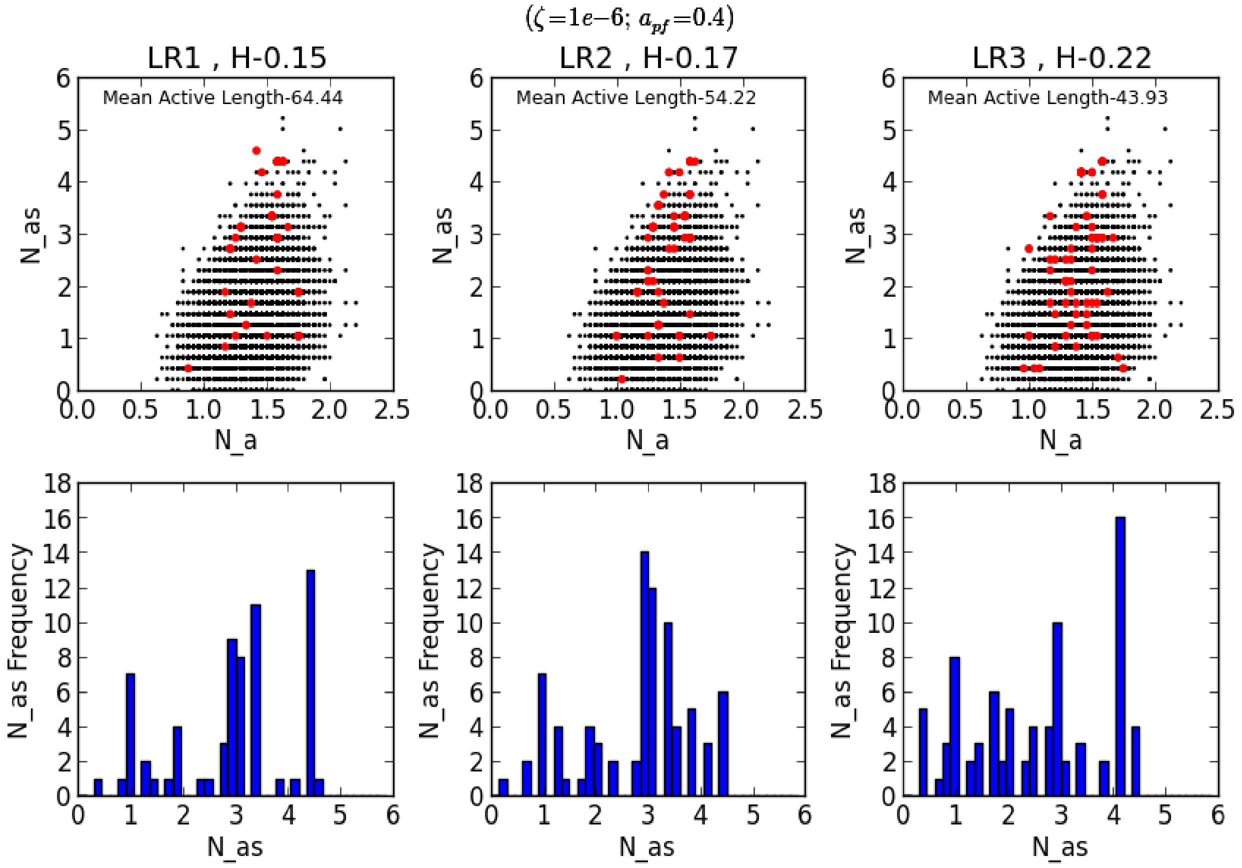


figure 5.3- Latching statistics for low correlation patterns using the learning rules LR1, LR2 and LR3.

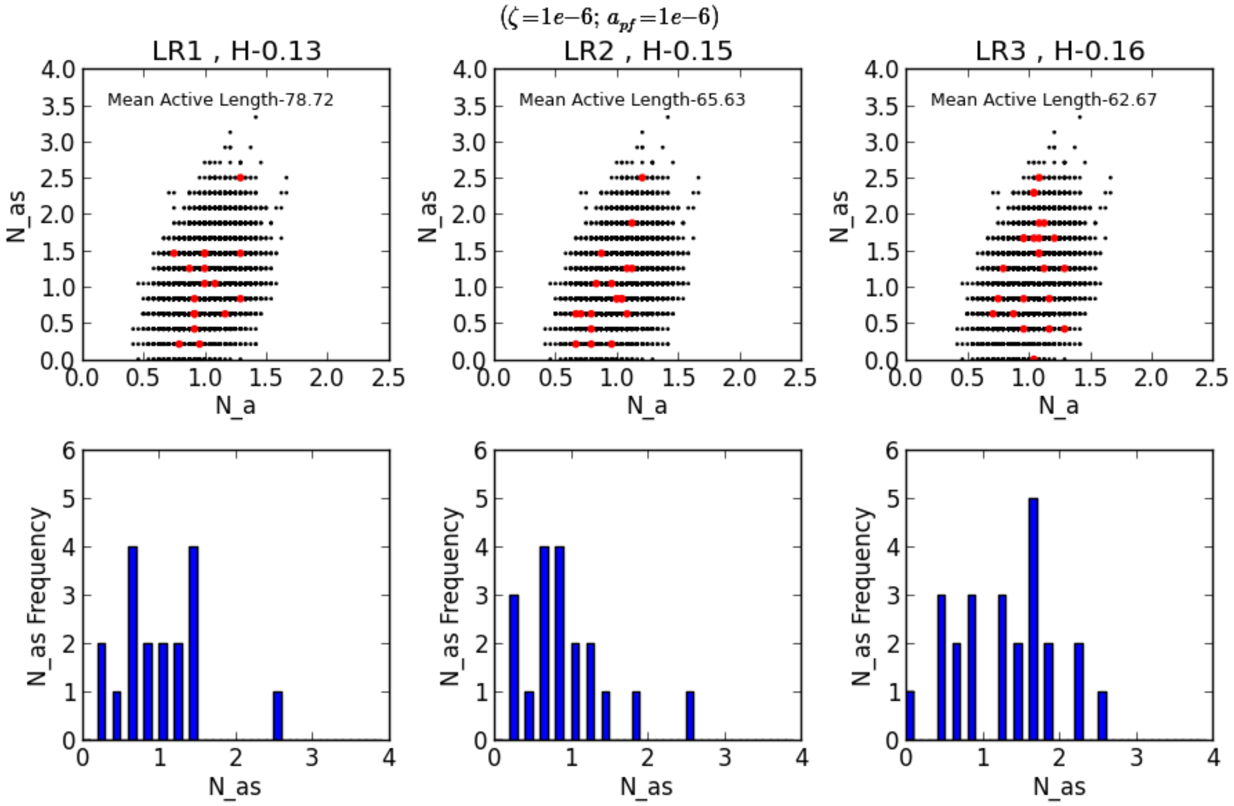


figure 5.4- Latching statistics for random correlation patterns using the learning rules LR1, LR2 and LR3.

Overall the entropy increases slightly from LR1 to LR3, thus indicating that more patterns become accessible to the network during latching. The entropy is highest for highly correlated patterns with LR3 at 0.53, and on an average it increases as correlations become stronger, from random patterns to highly correlated patterns. This may be the result of better access to other patterns, due to the fact that increased correlations cause overlaps between the basins of attraction of other attractors to increase. Increases in correlations also add a higher positive contribution to the local field h_i of each unit, when J_{ij}^{kl} is summed over all the remaining units and states. This has an analogous effect to w. Thus the network can exhibit spontaneous activity even at $w=0$ for highly correlated patterns, but this also causes the patters to have an increased stability against the time varying thresholds which hampers the latching process. LR3 suppresses this positive contribution to h_i which in turn facilitates latching.

As stressed in section II, correlations are crucial for a model of language processing and hence it is of much importance that the latching process is preserved when memories are highly correlated. We showed that the critical storage capacity has only a limited impact on the latching behaviour and the network does not lose its functional characteristics, in terms of dynamical behaviour while operating above its storage capacity and using LR3 for highly correlated patterns.

6 Conclusions

The broad aim of our study was to explore language mechanisms and in parallel investigate the properties of a general neural processing apparatus that may help sub serve those language mechanisms. Keeping the importance of statistical learning in mind, we focussed our investigation, in part A (chapters 2-3), on the statistical properties of the mass-count distinction between nouns, which neatly lies on the interface of syntax, semantics and cognitive perception. Language is mostly thought to be processed in the prefrontal cortex, which is also responsible for other high level cognitive phenomena [Gabrieli et al 1998; Bookheimer 2002; Bunge 2004], hence stressing the need to explore models that use general cortical mechanisms to address language processing. In Part B (chapter 4: sections I-III) we studied some of the features of a neural network, namely the Potts neural network, intended to model cortical processing in a simplified way, thus attempting to explore the feasibility of the model to serve the purpose of language processing.

The findings of the second chapter suggest that the mass count syntax is largely left undetermined by semantic attributes and that one cannot regard it as a binary or quasi-binary structure. The distribution of syntactic usage properties is very far from bimodal in five out of the six languages tested. One is led to think of this grammaticalisation as a graded self-organization process, operating within languages and to some extent within individual speakers, and driven only to a limited extent by universal attributes, and plausibly governed or at least constrained by language specific principles. However, at this stage we cannot tell to what degree the grammaticalisation is governed, beyond the universal semantic or perceptual principles that we have attempted to quantify, by language-specific principles of different nature, such as cultural factors, historical accidents, individual language acquisition history, even context dependence within individual speakers.

With rich multidimensional diversity in the syntax and semantics of the mass-count distinction and a low cross-linguistic and syntactic-semantic mutual information, it was shown that a simple self-organising neural network is insufficient to learn a mapping implementing a syntactic-semantic link. However the network was able to extract the concept of 'count', and to some extent that of ‘mass’ as well, without any explicit definition, from both the syntactic and from the semantic data. This categorisation was clear and sharper in the markers than in the nouns. Nouns on the other hand were aligned on a single dimension and showed a graded distribution over the mass-count spectrum. Thus it seems that even though there is a separation in the mass count divide within the markers, the application of them to the nouns is variable to a large extent and it is predominantly context dependent.

In chapter 4-section II, we looked at the ability of the Potts neural network to store correlated patterns. We expect a model of language processing to robustly handle various syntactic-semantic correlations amongst the words of a language. This seems to be a sticky problem and the question of negating the adverse effects on storage capacity due to increased correlations does not have an easy solution. By incorporating correlations in the definition of storage capacity we are able to regain the higher storage capacity seen for random patterns at low sparsity. During the retrieval of a pattern in the high correlations case, along with the cued pattern one or more patterns that are strongly correlated with the cue also have a high overlap with the network state. One may think of a scenario where another secondary network layer may be responsible for the fine tuning between highly correlated patterns to distinguish them and recover the desired pattern.

The possibility of a spin glass phase in the Potts neural network opens up an interesting window to explore its possible role in cognitive functions. Though this area has not been explored yet, we may think of a scenario where there are many interacting 'features' that bring about specific rules through their interactions. Language processing is an ideal candidate for this purpose. One of the ways to formally analyse linguistic structure is given by 'Principles and Parameters' [Chomsky 1986]. We refer only to the structural analysis of the syntax, in which the syntax of a natural language is proposed to have general principles (abstract grammar rules) accompanied with a set of binary parameters that describe the language. The spin glass phase has many stable degenerate energy states of frustrated spin interactions, thus a particular configuration of spins (which could represent parameters) may bring about a description of the syntactic rules in a natural language. Frustration would imply that a certain arrangement of some 'features/parameters' would enforce the remaining ones to be frozen in a particular state thus possibly shedding light on the correlations amongst syntactic rules. This also has an implication on the 'Poverty of Stimuli' argument [Chomsky 1980], which states that the entire grammar of a language is unlearnable purely from experience due to insufficient stimuli to a learning child. With frustrated interactions, the knowledge of a limited set of features may help to determine other features without explicitly experiencing them. The combinatorial possibilities of spin configuration of stable energy states could lead to the generative properties of syntax.

We have touched upon only a small aspect of language processing in this thesis. Not only are natural languages in themselves quite varied and complex in their nature but so is the question of how humans acquire and process them using general neural principles. This makes natural language processing the most fascinating, exciting and challenging aspects of the human brain.

Future Directions

The results that we have reported in chapter 2, have been purely statistical, that is to say, we have reported numbers with no discussion of any of the 1,434 items that make up of data base (where an item is a token from a particular language plus its particular feature values). An analysis of patterns within the data base is obviously the next stage in a linguistic analysis. This analysis will involve investigating whether there are recognizable patterns within the variation which are open to interpretation, whether there are lexical classes of nouns which function as classes cross linguistically, and if so how to characterize them. For example, advice, information, and evidence are strongly count in Hebrew and Italian, and mass in English. Do they behave as a class in other languages too? However, the results that we have so far already have theoretical implications relevant for continued research into the semantics and grammatical aspects of the mass/count distinction, and we conclude by specifying three of them.

Another important factor is the context dependence of the mass-count distinction. Our results establish that there is no independent semantic content in a noun that can completely determine its use in the mass-count domain. Thus particular instances of a noun, used in a sentence, as mass or count are influenced by the information that particular sentence conveys as a whole. Further investigation is needed in this regard beyond the anecdotal evidence.

In the corpus study (chapter 2, section 2.3.5) we studied the frequency of the nouns with markers, however to include effects of context one needs to involve higher order frequency relations of the nouns and markers with other words in a sentence. The artificial language, BLISS is a useful tool in this regard, to test higher order frequency effects in the mass-count domain. Subsequently, the Potts network can be tested exclusively in the mass-count domain in its ability to latch between correct nouns and markers based on higher order frequency correlations.

With that in mind, influence of high correlations on the latching transitions needs to be studied in detail. Although there is a tendency of the network to latch between correlated attractors, this remains true in the case of random patterns when the attractors are fairly separated. Latching behavior when the attractors are highly overlapping and share a wide basin of attraction, needs careful study as high correlations are a necessary requirement in order to model language processing.

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