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ON THE PILOT-WAVE THEORY OF CLASSICAL, QUANTUM AND SUBQUANTUM PHYSICS

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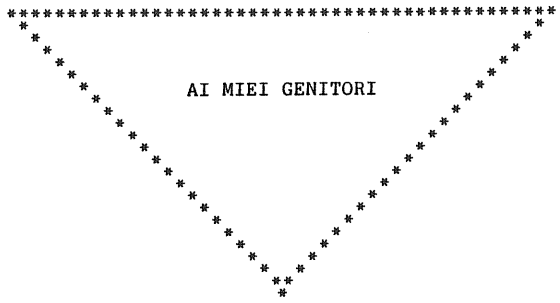
AI MIEI GENITORI

"I want you to go on to picture the enlightenment or ignorance of our human conditions somewhat as follows. Imagine an underground chamber, like a cave with an entrance open to the daylight and running a long way underground. In this chamber are men who have been prisoners there since they were children, their legs and necks being so fastened that they can only look straight ahead..... Behind them and above them a fire is burning...."

.....

"An odd picture and an odd sort of prisoner."

"They are drawn from life," I replied. "For, tell me, do you think our prisoners could see anything of themselves or their fellows except the shadows thrown by the fire on the wall of the cave opposite them?"

"How could they see anything else if they were prevented from moving their heads all their lives?"

.....

"Then if they were able to talk to eachother, would they not assume that the shadows they saw were real things?"

"Inevitably."

"And if the wall of their prison opposite them reflected sound, don't you think that they would suppose, whenever one of the passers-by on the road spoke, that the voice belonged to the shadow passing before them?"

"They would be bound to think so."

"And so they would believe that the shadows of the objects we mentioned were in all respects real."

"Yes, inevitably."

- Plato, The Republic, Book VII,
The Simile of the Cave

ABSTRACT

Classical Hamilton-Jacobi theory, suitably interpreted in terms of a fundamental and irreducible concept of "guiding field", is proposed as the basis of classical physics, independent of and more fundamental than Newtonian mechanics. The "pilot-wave" approach is then seen as a natural unified basis for classical, quantum, and subquantum physics, whereby all interactions are seen as forms of EPR-entanglement. We attempt to clarify the emergence of standard quantum theory as an equilibrium phenomenology, thereby accounting for its "conspiratorial" relation with relativity theory, as well as clarifying its peculiar features with regard to "measurement". We emphasise the phenomenological nature of classical language, and the essentially illusory nature of the classically-inspired theory of quantum "measurement" (the "Plato's Cave effect"). A subquantum statistical mechanics and H-theorem are developed, establishing $P=|\Psi|^2$ as an equilibrium distribution.

The pilot-wave theory is extended in its scope. A field theory of massive "particles", which explains these in terms of "field lumps", is given, as well as a Grassmann field theory of spin. The Lorentz absolute 3+1 view of special relativity is adopted, Lorentz symmetry being explained as a maximum-entropy equilibrium symmetry. This leads to an absolute 3+1 theory of electrodynamics in terms of a pure 3-vector potential, which avoids the troublesome time-component "non-dynamical" degree of freedom. A similar approach is applied to gravitation, which drops from the outset the non-dynamical time-components of the metric. Subquantum nonlocality is assumed to provide an absolute 3+1 slicing of spacetime. leading to a straightforward pilot-wave theory of quantum gravity, which describes the evolution of absolute 3-space geometry in absolute time. This approach overcomes the fundamental conceptual problems of standard quantum gravity and quantum cosmology. Cosmological implications are discussed.

The theory of measurement is extended to the subquantum domain, leading to the consideration of subquantum automata and enhanced parallel computation. It is shown how a subquantum automaton or "observer" could function as an essential part of a "system", in a manner outside the scope of standard quantum theory. It is also shown that the pilot-wave theory is in principle experimentally distinct from standard quantum theory, in particular for finite-ensembles and in the presence of residual disequilibrium or rare fluctuations.

Various other aspects of the theory are also discussed. An attempt is made throughout to view the theory in an appropriate historical and philosophical perspective.

CONTENTS

1. INTRODUCTION

Standard quantum theory generally makes only statistical assertions, giving no account of individual events. Further, it does not provide a description of a system in the absence of apparatus or observers. Related to this is the very remarkable fact that quantum theory uses classical concepts (such as energy and momentum) to discuss the nonclassical microworld - even though, since any macroscopic system, apparatus or observer is necessarily made of nonclassical atoms, the classical theory from which these concepts are taken can only be a phenomenological approximation. This reliance on classical concepts in quantum theory is especially striking in the theory of "measurement": The standard theory based on operators and their eigenvalues is inspired by the close analogy between the behaviour of these operators and that of the corresponding classical variables. This analogy makes it appear reasonable that, when the state vector is an eigenvector of the operator Q with eigenvalue q, the variable be said to "have the value q". The entire quantum theory of measurement has been inspired by this analogy with the language of classical physics, and by classical notions of interaction, in accordance with Bohr's view that the classical language is fundamental. However, excessive faith in this classically-inspired definition of "measurement" has contributed, as we shall see, to today's confused picture of the quantum world.

In addition to these unsatisfactory features, standard quantum theory contains a serious, and rather obvious, logical flaw, which has been stressed in particular by Bell[1]: The theory concerns "measurements" performed by an essentially classical "observer" on a "system" using an "apparatus". But the distinction between these three objects is fundamentally ambiguous, as was once admitted by Bohr[2]. While this ambiguity has no practical importance at present, it might do in the future, in particular in quantum cosmology (where everything is part of the system), and possibly in future experiments with nonclassical automata (see below). And in any case, a theory which is in principle ambiguous cannot be fundamental.

A natural way to avoid these difficulties is to consider a "realistic" and perhaps deterministic theory which is "universal", in the sense that it applies equally well (in principle) to any system, apparatus, or observer. Not only would such a theory provide a more detailed understanding of the quantum world: it should also show how our classical concepts emerge as an approximation, from a deeper level where such concepts (or some of them) are irrelevant.

A "realistic" theory is essentially one in which certain variables λ are regarded as having a definite existence, with no regard for what has been or could be measured by macroscopic human experimenters. And these variables, which may in

practice be "hidden" from human view, are regarded as in principle providing a deeper foundation for all of physical reality, and for the statistical predictions of quantum theory in particular. Such an approach to physics was of course widely taken for granted, at least since the Ionian philosophers of the sixth century B.C., and until comparatively recent times. However in the 1920s physicists, led in particular by Heisenberg and Bohr, abandoned this approach, mainly on philosophical grounds; apart from a few dissenters - notably Einstein and Schrödinger.

An attempt at a realistic interpretation of quantum theory was made by deBroglie in 1927, who proposed[3] the existence of definite electron trajectories with velocity given by the gradient of the phase of Schrödinger's wavefunction or "pilot-wave". However deBroglie abandoned this approach, apparently because of objections to the theory raised by Pauli. A similar approach was taken by Einstein who wrote and submitted a paper in 1927, associating definite electron trajectories with the wavefunction. However, Einstein realised that such a theory had to be nonlocal, and withdrew his paper[4] (which survives in the Einstein archives). Similar pilot-wave ideas had been considered by Einstein around 1922, according to which[4] particle-like light quanta were guided by "ghost fields" (Gespensterfelder) - the electromagnetic field, ideas which influenced Born's later interpretation of the wavefunction. And of course deBroglie's work of 1924 was a pilot-wave theory in which electron trajectories were guided by the law of "phase harmony" between a hypothetical internal oscillation and the external pilot-wave.

However in 1932 Von Neumann proved a mathematical theorem[5] which appeared to be a severe blow for realism; for according to Von Neumann this theorem showed that no realistic hidden variables theory, of any kind, could possibly exactly reproduce the statistical predictions of quantum theory, a claim which was widely accepted by physicists (and which is still quite commonly believed today).

Nevertheless in 1952 Bohm[6] elaborated the earlier pilot-wave ideas of Einstein and deBroglie into a systematic alternative formulation of quantum theory, and answered Pauli's objections. While Bohm unfortunately presented the theory in the inappropriate language of Newtonian mechanics, in terms of an unconvincing and inelegant "quantum potential", the complete equivalence to standard quantum theory was shown, assuming an appropriate initial probability distribution, despite Von Neumann's claim that such an equivalence was mathematically impossible.

The error in Von Neumann's reasoning was made clear by Bell[7] in 1964. Bell showed that Von Neumann's apparently reasonable axioms contained a covert assumption, which amounted to assuming that quantum measurements performed on a system directly reveal the value of a pre-existing system-quantity, the apparatus and experimental set-up playing no active role in determining the outcome. It is now known, and has been proved in various ways[8], that this cannot be so, that so-

called quantum "measurements" are actually "contextual": The outcomes of "measurements" cannot be accounted for simply by hidden variables in the system alone; the apparatus variables, and the apparatus-system interaction, must also play an active role in bringing about the result. Indeed one might today reverse Von Neumann's argument, and say that his theorem really indicates that hidden variables theories must be contextual, and that quantum "measurements" are not usually true measurements.

A further necessary property of hidden variables theories was demonstrated by Bell[9] in 1964, namely nonlocality. In order to account for EPR-correlations[10] between distant systems, whose joint wavefunction does not factorise, in particular for the case of a pair of spin-1/2 particles in the singlet state, Bell showed that the outcome of a spin "measurement" for one particle must depend instantaneously on the setting of the distant apparatus used to "measure" the other particle, as well as depending on the unknown hidden variables. (This might be regarded as a form of contextuality at a distance). This was shown by deriving an inequality, which expressed the maximum possible degree of statistical correlation which the particles could show, if there were no such nonlocal dependence, this maximum being surpassed by the statistical predictions of quantum theory (which have been experimentally confirmed[11]). That quantum theory is fundamentally nonlocal was also stressed by Einstein[12] in 1948.

The pilot-wave theory, being both realistic and in agreement with standard quantum theory, gives a clear picture of both contextuality and nonlocality. All variables in the pilot-wave theory, whether they belong to widely separated systems or to apparata, evolve in time by following the gradient of the phase of the total wavefunction. If the total wavefunction does not factorise, i.e. is "entangled", then the phase is not a separable function, implying a direct mutual dependence of the evolution of the variables. And such wavefunction entanglement generally occurs between system and apparatus during "measurement", thereby explaining the contribution of the apparatus to "measurement outcomes", and in EPR-type systems, thereby explaining the instantaneous connections implied by Bell's inequality. Contextuality and nonlocality are thus given a unified explanation, in terms of entanglement in the pilot-wave.

Despite its success, the pilot-wave theory has not been adopted as a definitive foundation for quantum physics, even by its originators (see below). For instance in 1954 Bohm and Vigier[13] introduced arbitrary background "fluid fluctuations" into the theory, in particular in order to explain how the initial probability distribution - required to reproduce quantum theory - came about, though also in order to generate testable departures from quantum theory (hardly a convincing motivation). And similar views have been more recently proposed by Bohm and Hiley[14]. In the author's opinion, such arbitrary departures from the original theory are unnecessary, if not unfortunate: If the latter is correctly interpreted and developed, it is quite capable of providing a coherent and complete foundation for all of contemporary physics, as we shall attempt to show.

Although Bohm in 1952 took a mechanical view, in terms of a nonlocal "quantum potential", the pilot-wave theory is radically different from Newtonian mechanics based on force and acceleration. The pilot-wave theory is much better regarded in terms of an abstract "guiding field" (pilot-wave) in configuration space, whose gradient determines the velocities of the system variables, this field being of the same nature as the phase function (or action) S of classical Hamilton-Jacobi theory. Indeed, as we shall see below, a very natural viewpoint is obtained if one begins with classical Hamilton-Jacobi theory, regarding this per se as a physical theory of classical physics, completely independent of Newtonian mechanics. By elevating the function S to a fundamental and irreducible status, classical physics becomes conceptually identical to the pilot-wave theory, and the latter is mathematically but a small step from the former. From this point of view, all interactions are regarded as effects of nonseparability, even classically: Classical "forces", and coupling between neighbouring field elements in classical field theory, are actually manifestations of EPR-entanglement in S . Thus, rather than explaining nonlocal EPR-entanglement effects in terms of a mechanical "quantum potential", we take the opposite view: Even classical "forces" are actually manifestations of such entanglement.

The pilot-wave theory is profoundly nonlocal (distant systems may be entangled), as well as contextual (system and apparatus may be entangled). And this nonlocality led Einstein to withdraw his pilot-wave paper of 1927. While nonlocality may have been reasonably rejected in 1927, there is now considerable evidence for its existence, suggesting that Einstein's judgement in 1927 was mistaken.

It is widely held that experimental violation of Bell's inequalities rules out "realism", because the implied nonlocality at the hidden variable ("subquantum") level would conflict with relativity theory. But it could be that relativity is not valid at that level. If one assumes this, one must then convincingly account for the validity of relativity, and of locality, at the statistical level of standard quantum theory.

This brings us to a further motivation for the study of a realistic theory: To explain the mysterious "conspiracy" between relativity and quantum theory. By this we mean the remarkable fact that, despite the nonlocality inherent in quantum theory, instantaneous communication at a distance is not possible in practice. Roughly speaking, the uncertainty principle seems to prevent one from making

practical use of quantum nonlocality for signalling. Why should the nonlocality be hidden in this manner? If the world contains an essential nonlocal element, why should this not show up directly, instead of having to be deduced indirectly via Bell's theorem? As Bell himself put it: "It is as if there is some kind of conspiracy, that something is going on behind the scenes which is not allowed to appear on the scenes."[15] This conspiracy is all the more remarkable when one considers an inequality derived by Roy and Singh[16], for hidden variable theories (not necessarily local ones) which have the constraint of no practical instantaneous signalling at the statistical level. Quantum theory precisely saturates the inequality, so that it only just avoids practical signalling at a distance. In other words, the conspiracy whereby uncertainty principle noise masks quantum nonlocality, so as to preserve relativity, just barely works.

We regard this extraordinary conspiracy, or "peaceful coexistence"[17], between relativity and quantum theory as being of fundamental significance, and as pointing to a deep-rooted connection between these two theories. In seeking a deeper understanding of relativity and quantum theory, one should surely be guided by the aim of explaining this conspiracy.

The apparent fine-tuning between uncertainty and nonlocality strongly suggests that quantum theory is really an equilibrium theory, in which the statistical distribution has arrived at some sort of balance, such that the underlying nonlocality is effectively hidden. Perhaps, then, the distribution $P=|\Psi|^2$ of quantum theory is not fundamental, but rather represents a special equilibrium, as was indeed suggested by Bohm.

This motivates a helpful analogy with classical statistical thermodynamics: Consider a box of gas at some definite temperature T, and a Maxwell demon which attempts to observe the molecular trajectories. As is well known, with the aid of a "trap-door", the demon would be able to decrease the entropy of the gas, if his temperature were lower than T. In practice, however, the demon will eventually come into thermal equilibrium with the gas, and will thereafter be unable to perform this task without generating an additional entropy which at least compensates for the said decrease. Our analogy with quantum theory considers a "subquantum demon" which observes the (nonlocally connected) hidden variables at the subquantum level, for some ensemble of quantum systems. Such a subquantum demon would presumably be able to use his knowledge for practical signalling at a distance. However, if the demon's activities are restricted by the equilibrium distribution $P=|\Psi|^2$ (i.e. by the uncertainty principle), then the demon would be unable to operate with sufficient precision, and nonlocal signalling would in practice be impossible.

This analogy suggests that $P=|\Psi|^2$ be heuristically regarded as a subquantum analogue of the classical thermodynamic "heat death", where all systems are in

equilibrium with eachother. In the classical heat death, no further macroscopic changes are possible (in the absence of a rare thermal fluctuation). In the "subquantum heat death", where all systems are subject to the same uncertainty noise, the underlying nonlocal connections may no longer be used for practical instantaneous signalling (in the absence of a rare fluctuation $P\neq|\Psi|^2$ away from equilibrium - see below).

To explain the above conspiracy, then, we suggest that such a subquantum analogue of Boltzmann's heat death has actually happened in our observed universe. A detailed theory of how this came about may be developed on the basis of the pilot-wave theory, explaining why locality works in practice, despite the fundamental nonlocality. The possibility of such an explanation for the above (otherwise mysterious) conspiracy is a major motivation for this work.

We have shown elsewhere[18],[19] how the above thermodynamic analogy, and a theory of the "subquantum heat death" $P\longrightarrow|\Psi|^2$, is concretely realised on the basis of the pilot-wave theory: We have demonstrated a subquantum H-theorem, which accounts for the relaxation $P\longrightarrow|\Psi|^2$. Further, we have shown that instantaneous signalling (and violation of the uncertainty principle) is possible if and only if $P\neq|\Psi|^2$. The close analogy between $P=|\Psi|^2$ and the classical heat death is then clear.

These results also lead to an appealing relation between the three "impossibility principles" of physics - the principle of signal-locality, the uncertainty principle, and the law of entropy increase: To signal nonlocally in the pilot-wave theory, one must circumvent the uncertainty principle, which requires a distribution $P\neq|\Psi|^2$. And to obtain such a distribution requires that one decrease the "subquantum entropy" introduced earlier[18] (and see below), this entropy being maximised for equilibrium $P=|\Psi|^2$.

In what follows we develop, as an appropriate conceptual setting for the pilot-wave theory, the "guiding field" approach to both classical and quantum physics. Some effort is made to conceptually clarify the emergence of standard quantum theory as an equilibrium phenomenology. The subquantum statistical mechanics initiated earlier is more fully developed, and the relaxation $P\longrightarrow|\Psi|^2$ is studied in more detail. The pilot-wave theory is also extended in its scope: The theory of fields, which has been developed by Bohm et al. for the massless case, is extended to massive fields, showing how apparent localised "particles" emerge as "field lumps"; and a Grassmann field theory of spin is given. The Lorentz absolute 3+1 view of special relativity is discussed. Relativistic Poincaré symmetries are seen as stemming from the invariance (in equilibrium) of an ensemble of field configurations, under an active deformation in absolute space and time at the subquantum level. An absolute 3+1 theory of electrodynamics, in terms of

a pure 3-vector potential, is given. On the basis of absolute 3+1 slicing of spacetime, at the nonlocal subquantum level, a theory of gravitation is developed, which overcomes the conceptual problems of standard quantum gravity. The theory is applied to quantum cosmology. Cosmological implications of a possible disequilibrium $P \neq |\Psi|^2$ in the remote past are briefly considered. The theory of measurement is extended to the subquantum domain, leading to the consideration of subquantum automata and enhanced parallel computation. It is shown how a subquantum automaton or "observer" could function as an essential part of the "system", in a manner outside the scope of standard quantum theory. It is also shown that the pilot-wave theory is in principle experimentally distinct from standard quantum theory, in particular for finite ensembles and in the presence of residual disequilibrium or rare fluctuations. Various other aspects of the theory are also discussed.

It is a thesis of this work that the pilot-wave theory deserves to be developed to its logical conclusions, without adding any arbitrary modifications. And the pilot-wave theory, if correctly interpreted, is a very natural and elegant extension of classical physics, having nothing artificial or arbitrary about it, contrary to what is commonly thought. Indeed the theory seems to be adequate, both conceptually and formally, as a foundation for all of presently known physics, including gravitation. Further, following the theory to its logical conclusions opens up new possibilities beyond the scope of standard quantum theory which, as we shall see below, could eventually become experimentally accessible.

2. PILOT-WAVE FORMULATION OF CLASSICAL, QUANTUM, AND SUBQUANTUM PHYSICS

2.1.1 Holistic "guiding field" description of individual systems

There generally exist alternative, and mathematically equivalent, formulations of the same physical laws, each offering particular advantages depending on the situation. The predominant "mechanical" view favours equations of motion, or field equations, containing the physical variables and space-time parameters, while the Lagrangian view favours a teleological principle of stationary action. The latter has generally been regarded as a convenient mathematical tool, rather than as representing a realistic physical viewpoint. A third view, the so-called Hamilton-Jacobi "method", is sometimes used as a mathematical tool. But apart from when Schrödinger constructed wave mechanics, it seems to have been ignored as a realistic physical theory.

One may ask: Which formulation would be most appropriate as a starting point for a subquantum theory? Which formulation could yield a "quantisation" procedure whose end result is the deterministic subquantum physics of an individual system, rather than (as is usual) the statistical quantum physics of an ensemble? Clearly, since the subquantum domain is nonlocal, one should begin with classical physics written in a language suited to nonlocality. And just such a language is provided, as we shall see, by classical Hamilton-Jacobi theory, suggesting that it is the appropriate starting point for a theory of subquantum physics.

Let us then take the view that Hamilton-Jacobi theory is an actual physical theory, conceptually and mathematically independent of the usual mechanical formulation. In this theory all physical variables (particles, fields, or the geometry of 3-space) are guided in their time evolution, not by mechanical laws or field equations, but rather by a multidimensional "guiding field" (or pilot-wave) S which has an autonomous existence in configuration space. Indeed we shall reverse the usual view and say that the mechanical concepts of "force", "momentum", and even "energy", arise from mere mathematical reformulations of this more fundamental physical theory - and such derivative mechanical concepts prove to be inappropriate for extension to the subquantum domain.

Consider first the elementary classical theory of n moving mass points (mass m), with trajectory X(t) in configuration space. The motion X(t) is determined by the guiding function, or pilot-wave, S via

$$m\frac{dX}{dt} = \nabla S \tag{2.1}$$

where S is itself governed by the Hamilton-Jacobi equation

$$\partial S/\partial t + (\nabla S)^2/2m + V = 0 \tag{2.2}$$

Here there is a definite actual path X(t), while the function S(X,t) is of course defined for all arguments X . The form of V=V(X,t) depends on the system under consideration. In the mechanical formulation V is referred to as the "potential energy". But in our view V is simply a characteristic of the system, related to the evolution of S.

From (2.1) and (2.2) it is clear that S is a scalar with respect to (time-independent) transformations $X \longrightarrow X'$ of configuration space coordinates, dX/dt and ∇S both being configuration space vectors: $S'(X',t)=S(X,t)$.

It should be noted that the above theory is not quite the same as standard Hamilton-Jacobi theory. For in the latter S is not only a function of X and t, but also of n "constant momenta". These latter arise from a canonical transformation to a vanishing Hamiltonian, performed in the usual derivation of the Hamilton-Jacobi equation. In our view, this is an artifact of regarding the mechanical formulation as primary. For in the mechanical formulation, the complete initial data are X(0) and $\dot{X}(0)$, and the latter half of this data lead to the appearance of n constant momenta on transformation to the Hamilton-Jacobi picture. But if instead one takes the Hamilton-Jacobi picture as fundamental, one's initial data are X(0) and S(X,0), where S(X,0) is given for all arguments X. The unknown constant momenta are then replaced mathematically by S(X,0). In this view then, the equations (2.1) and (2.2) are a complete representation of the classical motion, given the initial position X(0) and the initial "pilot-wave" S(X,0).

Not only do the constant momenta make no appearance; we shall see below that the concepts of momentum, force, and energy, play no fundamental role. The theory takes place in configuration space, predicting the rate of change of configuration space variables, and phase space plays no role. Among the many advantages gained by this approach, we may mention the mathematical simplicity of the subquantum H-theorem given below, based on a statistical mechanics in configuration space.

We shall avoid the consideration of generalised coordinates p and q, which tend to obscure the physical nature of the theory.

Consider now the notion of "interaction". How should this be defined? Only one definition makes sense: Two bodies A and B are "interacting" if and only if the trajectories $X_A(t)$ and $X_B(t)$ (which are components of X(t)) are not independent. This occurs if and only if S is nonseparable in the coordinates X_A and X_B, $S \neq S_A+S_B$. For the above case of particles, such nonseparability will generally arise only if $V \neq 0$. The definition is then equivalent to the usual mechanical one. (But as we shall see this equivalence does not hold for field theory).

Physically then, at the nonrelativistic level, the elliptical orbit of the Earth around the Sun arises not from a "force" emanating from the Sun, but rather from the entangled guiding field $S(X_{Earth},X_{Sun},t)$. And generally, we regard all

classical mechanical "forces" as large-scale manifestations of nonseparability, of a form identical to that associated with the Einstein-Podolsky-Rosen experiment. (For classical field theory, the interaction between neighbouring field elements is seen as a manifestation of nonseparability in S - see below).

Its fundamental holism is one of the reasons why Hamilton-Jacobi theory comes into its own as an approach to the subquantum domain. Another reason is the non-mechanical character of the theory, in this sense: The same function S determines the path taken for any initial point X(0). One might say that the future evolution of all possible initial points is "encoded" in the guiding field, this information being "read" from the gradient ∇S. (This "potentia" aspect of S is of course reminiscent of quantum theory).

Turning now to classical field theory, the evolution $\phi(x,t)$ of a massless scalar field at the point x in space is determined by (with c=1)

$$\frac{\partial \phi(x,t)}{\partial t} = \delta S[\phi,t]/\delta\phi \qquad (2.3)$$

where, for the massless case,

$$\partial S/\partial t + (1/2)\int dV[(\delta S/\delta\phi)^2 + (\nabla\phi)^2] = 0 \qquad (2.4)$$

Here $S=S[\phi(x),t]$, a functional of $\phi(x)$, has no connection to ordinary space x, and transformations of coordinates $x \longrightarrow x'$ leave S invariant: $\phi(x,t) \longrightarrow \phi'(x',t) = \phi(x,t)$ and $S'[\phi',t] = S[\phi,t]$. The equations (2.3) and (2.4) are to be solved subject to initial conditions $\phi(x,0)$ and $S[\phi,0]$, the initial pilot-wavefunctional being of course given for all arguments $\phi(x)$.

The notion of "interaction" now strictly speaking differs from the usual one, for the above would normally be considered to be a free field. But the rate of change $\dot{\phi}(x,t)$ will generally depend on field values at neighbouring points: Owing to the term $(\nabla\phi)^2$ in (2.4), the functional $S[\phi,t]$ cannot separate into a sum of functions $S_x(\phi(x),t)$, one for each point x. (The field system may be considered as the continuum limit of a system of "masses" with "displacements" $\propto \phi(x)$, coupled to nearest neighbours by "springs" with "potential energy" $\propto (\nabla\phi)^2$, making clear the nonseparability of the system). This nonseparability does not of course act across finite spatial distances, and field distributions propagate at finite speed. Nevertheless it effects an "interaction" between neighbouring field values. And it is amusing to note that this nonseparability is responsible for the continuity of classical fields. For what might have been a completely discontinuous field $\phi(x)$ is in fact held smoothly together by the entanglement of $S[\phi,t]$, which ensures that evolutions of field values at neighbouring points are not independent. Thus we regard the continuity of classical fields as a residue, in the classical approximation, of quantum nonseparability.

Indeed, such continuity contains a clear hint of nonlocality: If for instance $\phi(x_o)=0$, then for any $\varepsilon > 0$ there exists some $\delta > 0$ such that $|\phi(x)| < \varepsilon$ for all $|x-x_o| < \delta$, so that field values in a finite neighbourhood δ of x_o are not completely arbitrary. In the quantised theory, on the other hand, this continuity is lost, while the entanglement of S acts across finite spatial distances.

The quantum generalisation of the above classical theory was effectively carried out by Schrödinger[20] in 1926, the Hamilton-Jacobi equation for S being replaced by the Schrödinger equation for $\Psi = |\Psi| e^{(i/\hbar)S}$. This replacement may be seen as the generalisation of the classical pilot-wave theory to finite wavelengths. In the generalised theory, the evolution of physical variables is still given by (2.1) and (2.3). The only difference is that the guiding phase function S is now governed by the Schrödinger equation

$$i\hbar \partial\Psi/\partial t = \hat{H}\Psi \qquad (2.5)$$

where the classical $H(\partial S/\partial X, X)$ is replaced by $\hat{H}(-i\hbar\partial/\partial X, X)$, or $H(\delta S/\delta\phi, \phi) \longrightarrow \hat{H}(-i\hbar\delta/\delta\phi, \phi)$ for field theory. The function $S = \hbar \operatorname{Im} \ln \Psi$ itself obeys

$$\frac{\partial S}{\partial t} + \operatorname{Re}\left(\frac{\Psi^* \hat{H} \Psi}{|\Psi|^2}\right) = 0 \qquad (2.6)$$

which is often written, for example in the particle case, as

$$\frac{\partial S}{\partial t} + H(\partial S/\partial X, X) - \frac{\hbar^2}{2m}\frac{\nabla^2|\Psi|}{|\Psi|} = 0 \qquad (2.7)$$

with a rather inelegant "quantum potential" Q added to the classical Hamilton-Jacobi equation. It will however become clear below that this quasimechanical concept is inappropriate in the pilot-wave theory.

We emphasise that this generalised theory is conceptually exactly the same as the above Hamilton-Jacobi pilot-wave view of classical physics. Both theories regard all variables as being guided by S via (2.1) and (2.3), where the guiding function S (or Ψ) may be regarded as a fundamental and irreducible entity. The only difference between the two theories is in the equation governing S itself. This leads, of course, to major qualitative differences. In particular, the linearity of (2.5) ensures that S is generally nonseparable (since one is able to linearly superpose different products $\Psi_1(X_1)\Psi_2(X_2)$), yielding further nonlocal and contextual connections which are not present classically. But nevertheless the whole of quantum theory may be regarded as a simple and natural extension of classical Hamilton-Jacobi-based physics, i.e. of classical pilot-wave theory.

In contrast the quasimechanical "quantum potential" viewpoint has, unfortunately, created the widespread and erroneous impression that the pilot-wave theory is inelegant and artificial, with trajectories being arbitrarily "appended" to the wavefunction. This is indeed one of the ironies of history, since the Schrödinger equation itself actually arose historically from pilot-wave ideas, Schrödinger's work of 1926 being inspired by deBroglie's elementary pilot-wave theory of 1924 (and by Hamilton's analogy between classical mechanics and geometrical optics). Given this historical fact, it is then no wonder that the pilot-wave theory is so strongly suggested by the Schrödinger equation alone, and has been independently suggested by several workers (the local conservation of $|\Psi|^2$ in configuration space being strongly suggestive of trajectories).

2.1.2 Remarks on the "guiding field" concept

Historically, unlike deBroglie, Einstein, and Bohm, Schrödinger did not take the above "dual" view of physical variables being guided by Ψ, and instead took the view that particles were simply wavepackets. We shall see below that something similar to the Schrödinger view of "particles" does emerge from pilot-wave field theory, but there one still has the "dualism" of field variables on the one hand being guided by a wavefunctional on the other.

Indeed the view taken here, that the abstract notion of "guiding field" is fundamental, has generally not been taken by workers on the pilot-wave theory. DeBroglie saw Ψ as a field in 3-dimensional space, and assumed that the guiding condition (2.1) (or relativistic version thereof) arose as some sort of consistency condition for the motion of a singularity in a nonlinear field. Other mechanistic views have been proposed by Bohm[21], Bohm and Vigier[13], and Bohm and Hiley[14], which regard Ψ as representing some sort of background "fluctuating fluid". Further, as we have said, Bohm's systematic development of the pilot-wave theory in 1952 was presented in the unfortunate guise of a quasimechanical theory with a "quantum potential". We propose an abandonment of all such mechanical ideas, and suggest instead that the notion of guiding field be taken as fundamental and irreducible: The rate of change of all physical variables is given by the gradient or functional derivative of S, with no need for further explanation.

Of course a future theory will surely provide a deeper basis for the guiding field concept. But we suggest that, at the present time, it is adequate as it stands, and that the existing theory deserves to be developed to its logical conclusions. In particular, we suggest that attempts at an explanation in terms of conventional mechanical concepts are logically misguided. For as we shall see these latter concepts are more naturally seen as entirely derivative, arising phenomenologically from statistical equilibrium and in particular from the classical limit of equilibrium. Further, taking the view that the conventional mechanical concepts are merely phenomenological, it becomes illogical to object to the theory, as is often done, on the grounds that it does not respect these concepts

(such as energy-momentum conservation) at the subquantum level.

It seems worthwhile to propose in more detail what we hope to be an adequate philosophical background to the pilot-wave concept, not least because this concept is so widely regarded as unphysical or artificial, even by the originators of the theory.

Consider the widespread criticism according to which it is "unphysical" that Ψ guides the evolution of physical variables X(t) without itself being influenced by these variables. This criticism has two aspects: (i) That physics should not contain such "ghostly" constructs, and (ii) That the theory does not respect energy and momentum conservation at the subquantum level.

Regarding (i) we point out that there already exist similar constructs in physical theory, which "guide" without themselves being affected. One example is Maxwell's equations, which may reasonably be said to guide the behaviour of, say, a radio. Another example would be the "attracting states" of dynamical systems theory. Of course one could take the view that physical laws such as Maxwell's equations do not have a "real existence", that they are merely a convenient mathematical summary of the behaviour of physical systems. But then in this case one could take a similar view with regard to Ψ , i.e that it is a convenient summary of the real motion X(t), and there would again be no reason to expect Ψ to be influenced by X(t). In this latter viewpoint, the world consists purely of the evolving variables X(t), whose time evolution may be summarised mathematically by Ψ , which has an abstract existence in configuration space (in the sense that classical attractors have an abstract existence in phase space). One never directly "sees" Ψ , just as one never sees gravitational "force" in Newtonian mechanics. One sees only the variables X(t) and their movements. (This view also removes the supposed mystery that Ψ , like the classical S, has no "source").

Regarding (ii), it is of course true that the lack of a backreaction on Ψ prevents the definition of a conserved energy and momentum for individual systems. However the classical principles of energy and momentum conservation are important and powerful only if they are local principles, whereby the decrease of energy-momentum in a local region is seen as arising from a local outward flux. One may then expect that such conservation laws will not play a role in a fundamentally holistic theory. As discussed below, energy and momentum conservation may reasonably be seen as phenomenological aspects of equilibrium.

These issues recall the historical oscillation (since ancient times) between, roughly speaking, "mechanistic" and "dynamistic" explanations in physics[22]. Newton's concept of gravitational "attraction" or "force" was widely regarded by his contemporaries as an obscure (if not occult) idea, which should be "explained" in terms of Cartesian direct contact between bodies or fluids (presumably filling space). Faraday's introduction of "fields" as irreducible entities had a similar

reception, it being widely thought (even by Maxwell) that these should be explained in terms of the Newtonian mechanics of a fluid medium. Interestingly enough, the notion of guiding field Ψ , which arises so naturally from Hamilton-Jacobi theory and which is so appropriate for quantum physics, is once more regarded as an abstraction in need of a mechanistic explanation (in terms of conventional fields, particles, or fluids). Indeed the hesitant and tortuous development of the pilot-wave concept (beginning with Hamilton in the early nineteenth century) may be seen as a general reluctance to accept a nonmechanical realism: Einstein rejected his own early pilot-wave constructions owing to their nonlocal "spooky" nature. DeBroglie's approach is largely an attempt to retain conservation of energy and momentum, while Bohm's early path-breaking work was cast in an inappropriate mechanical language. More recently, Bohm and Hiley[23],[24] have introduced the concept of "active information", whereby the quantum potential Q is seen as an "information potential" which organises the activity of energy. This idea has influenced the viewpoint taken here, where Ψ itself is regarded as something similar to a "guiding field of active information". But again, Bohm and Hiley claim that this concept cannot be in itself a definitive explanation, that it rather must be explained in more conventional terms, in particular by a hypothetical complex structure in the interior of electrons and other particles (whose complexity, they suggest, would be "at least comparable to that of a radio"[24]). We suggest instead that conventional mechanism, which inspires such a view, is merely an equilibrium phenomenology, so that such proposals appear very unnatural. Further, as shown below, "particles" are best regarded as phenomenological "field lumps", so that one is ultimately really trying to explain how field variables ϕ are guided by the wavefunctional $\Psi[\phi,t]$. Given the generality and simplicity of the pilot-wave formalism, which for example enables one to write a greatly simplified theory of quantum electrodynamics and gravitation (see below), it is natural to simply regard the guiding field Ψ as a general and fundamental concept, in no need of any such mechanical explanation. Indeed even classical physics may, as we have seen, be regarded as just such a theory based on an "active information" field S . We stress that while deeper theories will surely one day arise, one should be careful not to assume that such theories will use conventional mechanistic concepts.

Another curious aspect of this controversy is that the prejudice against a non-mechanical realism, in particular a nonlocal one contrary to relativity theory, has led many to a preference for no realsim at all. Even more curious is the fact that those who regard "realism" as old-fashioned and naive are inclined to reject the pilot-wave theory on the grounds that it does not conform to traditional mechanical ideas.

This clinging to conventional mechanical concepts has not only hindered the

general understanding of quantum physics, but is also responsible for much of the confusion in the quantum theory of measurement. For the behaviour of creatures on the Earth's surface is in many respects approximately classical, and such creatures naturally gather and store information in the language most natural to them: That appropriate to the approximately classical macroscopic world. Unfortunately, on encountering microscopic phenomena, these creatures have continued to think in their phenomenological language, which is incommensurable with the more fundamental microworld. By defining an abstract "measurement theory" inspired by this classical language, these creatures have been led in various ways to deny the real existence of a microworld (from which they are built), to deny the logical principle of excluded middle ("quantum logic" and "quantum probability"), to introduce negative "quasiprobabilities", and even to assert that the universe is continually splitting into distinct copies. We shall see below how these assertions result from the uncautious application of classical language to nonclassical systems. This vindicates Einstein's prediction[25], in a conversation with Heisenberg in 1926, that "your theory will one day get you into hot water", because "when it comes to observation, you behave as if everything can be left as it was, that is, as if you could use the old descriptive language" (my emphasis).

We take the view that human beings, and physicists in particular, are in a situation resembling that of the prisoners in Plato's Cave, who mistake the shadows on the wall for reality. This reversal of the (still popular) "operational" philosophy suggests that our elementary mechanical concepts and so-called "operational terms" really derive from a deeper and perhaps simpler structure, to which such terms cannot be applied. And insistence on such application only creates the need for more and more "epicycles" (complementarity, quantum logic, negative probabilities, many worlds). These may be removed by an appropriate "Copernican revolution" against the still prevalent instrumental philosophy, according to which one should not speak of that which cannot be "directly observed". The flaw in this philosophy is that "direct observations" simply do not exist, since "observations" are impossible, and indeed meaningless, without some prior body of theory. For instance even today, nobody has "directly observed" an atom. One might point to the images generated by electron microscopes, but these are just patterns of light and shade on photographic paper. Their interpretation in terms of "atoms" depends on an extensive theoretical structure (involving deBroglie waves). And indeed this is true of even everyday "observations". As Einstein put it: "It is quite wrong to try founding a theory on observable magnitudes alone. In reality the very opposite happens."(My emphasis). "It is the theory which decides what we can observe.....we must know the natural laws at least in practical terms, before we can claim to have observed anything at all. Only theory..... enables us to deduce the underlying phenomena from our sense impressions".[25]

One could say, with Wittgenstein, that "the limits of our language are the limits of our world". But we interpret this not as indicating that the subquantum world is beyond us, but rather that our language must be changed in order to comprehend that world.

The work of Bell indicates that reality is holistic and "contextual", indicating the need for a change in our thinking, perhaps along the lines suggested by the pilot-wave theory. If instead one continues to study and discuss this reality in terms of classically-inspired language, it inevitably has an ambiguous and schizophrenic appearance, which in our view is entirely the fault of classical perception. (The illusions created by classical perception may be quite spectacular: For instance a single subquantum automaton will appear by normal standards of perception to be effectively multiple, imitating the behaviour of a multiple set of classical automata (see below)).

Our perspective with regard to the pilot-wave theory is somewhat in the spirit of the following remark by Schrödinger, in a letter to Einstein in 1950: After pointing out that the emphasis on accelerations by Galileo and Newton "seems to work no longer", Schrödinger wrote that "One must therefore go back 300 years and reflect on how one could have proceeded differently at that time, and how the whole subsequent development would then be modified".[26] This is precisely the idea behind our suggestion that the Hamilton-Jacobi theory of the early nineteenth century be reinterpreted as a fundamental physical theory, independent of Newtonian mechanics. In this context we note that Galileo did at one time contemplate a purely geometrical "dynamic foundation" for kinematics, while he introduced the abstract notion of "force" and stressed the role of acceleration only later[27]. These latter ideas were of course taken up by Newton and were later widely accepted. In our view, however, even at the classical Newtonian level, a falling body such as a cannon ball is actually following a Hamilton trajectory guided by the pilot-wave S, and we interpret the gravitational "force" as a form of entanglement in S between the body and the Earth.

It should be stressed that the variables X(t) are completely free to follow whatever Ψ dictates. There are no restrictions on X per se, and no preassigned (independently of Ψ) relations or interactions between different components of X. These interactions depend entirely on the form which Ψ happens to take. As stressed by Bohm[28], the essence of "mechanism" is the interaction between parts according to preassigned and fixed rules, and since there are no such rules in the pilot-wave theory we do not have a mechanical theory in the usual sense.

The pilot-wave theory is mathematically deterministic, and may reasonably be termed "causal" provided one is willing to identify Ψ as a "cause". If one wishes to think in terms of cause and effect one must then regard Ψ as being "real", and not merely as a summary of the motion X(t). But in what sense could Ψ be

regarded as a cause? A sound intuitive view is probably best obtained by thinking in terms of Aristotle's "formal cause", rather than in terms of the "efficient cause" of traditional classical causality. Thus, rather than conceiving Ψ as an efficient cause analogous to force in classical mechanics, one could think of it as a formal cause or "guiding form", somewhat in the spirit of Aristotle's view that "motion is the fulfilling of what exists potentially".[29] (Russell notes the incompatibility of this idea with the relativity of motion). One could say that the "potentia" in Ψ is physically realised by placing a variable at a given point (bearing in mind that there are no preassigned interactions between particles or field elements from which the motion could be seen as mechanically arising). This view is reminiscent of Spengler's notion of "destiny", which according to Spengler "still awaits its theoretical formulation".[30] Perhaps the pilot-wave theory should be seen as just such a formulation.

From the viewpoint of standard mechanism, the idea of Ψ as a "guiding form" may seem like an "unscientific vitalism", as the concept of "gravitational attraction" once seemed to be. (And even the introduction of the universal concept of "energy" by Helmholtz and others in the last century was widely greeted as being either "vitalist" or simply empty of physical content[22]). Of course, while the notion of formal cause may aid physical understanding, one can in principle avoid using this concept altogether by taking the pragmatic view that Ψ is simply a mathematical summary of the time evolution X(t). Though such a pragmatic view does not really do justice to the physics. For example the possibility of "storage" of "spooky" connections between very distant objects, where one object could be in the laboratory and the other in a box beneath the ocean, suggests that Ψ is "real"; in the same sense that the existence of travelling electromagnetic waves suggests that the electromagnetic field is "real" (rather than just being a convenient way of expressing forces between charges). The most succinct interpretation of Ψ, free of complications while still doing justice to its physical significance, is then surely as a field of formal cause in the Aristotlean sense. Thus one would be justified in referring to Ψ as an informative field, which "informs" the time evolution X(t).

2.2 The phenomenological emergence of standard quantum theory

We now discuss how the theory for individual (subquantum) systems reproduces the usual statistical quantum theory of ensembles. In particular we would like to show how the classical language and the standard theory of "measurement", together with locality, uncertainty, and energy-momentum conservation, are all emergent and phenomenological aspects of statistical equilibrium.

Consider first a real ensemble of n similar but independent systems, each represented by a point X in configuration space, where X generally varies over

the ensemble. Let each X be guided by the same wavefunction Ψ. (Thus if the ensemble is considered as a single super-system, then the total wavefunction is just a product of n wavefunctions Ψ). The actual fractional distribution P(X,t) will then be

$$P(X,t) = (1/n)\sum_{i=1}^{n} \delta(X-X_i(t)) \qquad (2.8)$$

where X_i is the value of X for system i, and $P(X,t)\Delta X$ gives the fraction of systems occupying ΔX. In the limit of large n, P(X,t) may for our purposes be replaced by a purely theoretical smooth function, again denoted just P(X,t), which in practical circumstances will behave like a probability distribution. The subquantum H-theorem[18] (see below) establishes that, for appropriate initial conditions at t=0, there exists a finite time interval (0,T) during which the coarse-grained subquantum entropy

$$\bar{S} = -\int dX\, \bar{P} \ln(\bar{P}/\overline{|\Psi|^2}) \qquad (2.9)$$

cannot decrease (where overbars indicate coarse-graining). Further, $\bar{S}$ is bounded above by zero and is maximised by $\bar{P} = \overline{|\Psi|^2}$. If X represents the configuration of some complicated system (such as a large number of interacting particles), we assume that in most physical circumstances $\bar{S}$ does actually reach its maximum, so that $P=|\Psi|^2$ is actually attained in a coarse-grained sense. It may then be shown that, if a single component of the multidimensional X is extracted to form an independent system, guided by a reduced wavefunction ψ, then its distribution will be just $\rho = |\psi|^2$. (Detailed discussion of this, and of coarse-graining, is deferred to Section 3).

Once one has the equilibrium distribution $|\Psi|^2$, the usual theory of "measurement" follows along the lines given by Bohm[6]. Before discussing this and other matters, we should stress that the equilibrium $\rho = |\psi|^2$ is only an approximation, in three senses: (i) If the "mixing" of P and $|\Psi|^2$ described by the H-theorem has occurred for only a finite time (perhaps since the big bang), then one expects that at a sufficiently small fine-grained level there will be a difference $P \neq |\Psi|^2$. (ii) The distribution $P=|\Psi|^2$ is purely a statistical result, in the same sense as is Maxwell's distribution of molecular speeds. The possibility is open for the existence of extremely rare large fluctuations $P \neq |\Psi|^2$, similar to those of classical statistical mechanics. (iii) The very concept of a smooth distribution P or ρ is limited, being strictly valid only in the purely theoretical limit of an infinite ensemble ($n \longrightarrow \infty$). This implies for example that in a laboratory consisting of a finite number of atoms, the actual distribution (say of electron positions) has the discrete form (2.8), so that one necessarily has some disequilibrium $\rho \neq |\psi|^2$ on a fine-grained level.

Thus the very concept of equilibrium, together with its properties such as locality, can never be exactly valid in real experience (an explicit example is given below). In the real world then, standard quantum theory can only be a good approximation, as is the case with classical statistical mechanics and thermodynamics. These points are important with regard to the possibility of distinguishing the pilot-wave theory from standard quantum theory, and are discussed further in Section 8.

2.2.1 The phenomenological emergence of standard "measurement" theory

Taking the distribution $|\psi|^2$ as given, let us briefly review the theory of measurement as given by Bohm, before turning to some subtleties which need clarification.

If the system "observable" Q has eigenvalues q, with "corresponding" apparatus eigenvalues A_q, then a standard quantum-theoretical "measurement" has occurred if the total wavefunction

$$\Psi = \sum_q c_q \psi_q \chi_{A_q} \qquad (2.10)$$

where ψ_q and χ_{A_q} are system and apparatus eigenfunctions respectively. Here distinct χ_{A_q} have essentially no overlap in configuration space. (In standard measurement theory distinct χ_{A_q} may be orthogonal in Hilbert space and yet still overlap in configuration space - e.g. if A_q is an energy. Nevertheless at a later point in the measurement chain a set of disjoint eigenfunctions in configuration space is always eventually encountered in a real experiment, so there is no loss of generality here). In the theory as given by Bohm, and discussed by Bell, the apparatus variable is treated as if it were simply "seen" by a macroscopic experimenter. This picture of how the Von Neumann chain ends is rather vague, and an attempt at greater precision will be made below. For the moment, following Bohm, if the experimenter "perceives an actual value" $A_q=A_q^{meas}$, he may deduce that the total configuration X (system + apparatus) lies somewhere in the region where $\chi_{A_q^{meas}} \neq 0$ (and of course $\Psi \neq 0$). Since the distinct χ_{A_q} do not overlap, this restricts the actual X to a single branch of the superposition (2.10), and we thereby deduce that "the value of Q is q^{meas}". And the associated probability is easily shown to be just $|c_q|^2$.

Remark: Given $P=|\Psi|^2$ it follows that X can never be in a finite region where $|\Psi|=0$, which would otherwise lead to difficulties in the definition of the guiding phase S. Without $P=|\Psi|^2$, such problems are avoided by assuming that in any case $|\Psi|\neq 0$ everywhere. (Except at infinity and at isolated points. In the latter case S is still well-defined by continuity.) This is reasonable since one expects that, for example, infinite potentials would be required to make $|\Psi|$ strictly vanish throughout a finite region.

The above presentation is incomplete in several respects: (i) At the fundamental level, the pilot-wave theory sees the whole world ("system", "apparatus", and "experimenter") as a single configuration space point guided by a univeral wavefunction. How does the system-apparatus-experimenter division arise? (ii) The theory deals simply with the path in configuration space and is entirely non-mechanical. Yet the above "measurement" theory chooses "observables", such as energy and momentum, by analogy with classical mechanical physics. Why? (iii) How exactly does a wavefunction χ_{A_q} lead to the experimenter "seeing" a value A_q? (iv) Indeed what does it mean, for a general Q, to say that "Q has the value q"? (v) What approximations (such as "classicality") must the experimenter satisfy? (vi) Without being, as Shimony has put it, "parasitic on standard quantum mechanics"[31], why should one ever, in the pilot-wave theory, define (2.10) as corresponding to "measurement"? (vii) Contextuality shows, as stressed by Bell, that the usual language of "measurement" is misleading, and indeed contextuality makes a mockery of the word "measurement" as currently used. Is a more logical view not possible?

In order to clarify this extremely confusing situation, the following elementary (but usually forgotten) point is crucial: In the pilot-wave theory, the entire mechanical "language of classical physics" derives from a deeper level and emerges merely as a phenomenological approximation. In particular the standard Von Neumann "measurement" scheme is merely a phenomenological book-keeping device, which is inspired by the classical limit of equilibrium, i.e. the domain of common human experience, and it must be seen as such if it is to be correctly understood. A clear point of view may then be arrived at by considering things from the deeper and more objective level, and seeing just how the phenomenological concepts arise.

Our task then, is to understand in physical terms just why human experimenters have come to regard the world, even at the quantum level, in phenomenological classically-inspired language (the "Plato's Cave effect"). This requires, in part, an understanding of the emergence of approximately classical behaviour for the experimenter and his macroscopic surroundings.

As a first step, consider the behaviour of a single particle. It is usually said that the classical limit emerges in the pilot-wave theory when the "quantum potential" Q is negligible. But things are not so simple. For instance if the wavefunction is a plane wave e^{ipx}, then Q=0 and the particle trajectory $\dot{X}=p/m$ appears to be "classical". But if the plane wave strikes a macroscopic two-slit screen, the nonclassicality of the system will become apparent. Conversely, if a particle is guided by an appropriately peaked packet, the behaviour of the system will appear to be classical on the macroscopic level. And yet within the packet,

near the actual location of the particle, the quantum potential Q may be by no means negligible, and the particle might even undergo a nonclassical jittering motion within the packet. So the condition "$Q \approx 0$" is actually a poor characterisation of the classical limit. A better one is that taken from standard quantum theory, based on Ehrenfest's theorem: "If the wavepacket is narrowly peaked in an appropriate manner, so that its centroid follows an approximately classical path and its spread remains small (over the relevant timescale), then the system behaves approximately classically". This may be adopted in the pilot-wave theory even for a single system, by adding the assumption that the actual position X is located somewhere near the bulk of the packet (which is virtually certain if the particle is extracted from an ensemble $P=|\Psi|^2$).

We note that this characterisation of the classical limit refers principally to the behaviour of the wavepacket, and only secondarily to the particle itself, which is merely stipulated to lie near the peak of the packet. That the actual variables play a somewhat secondary role is also true even at the quantum level. For at the quantum level, if a variable X is isolated (by "measurement") within a branch ψ_q of the wavefunction, then the precise value of X within ψ_q is never actually seen without performing a further division of ψ_q into sub-branches. And if this is done then the precise value of X within the subsequent sub-branch containing it is in turn never actually seen. The reason for this is that the uncertainty principle would otherwise be violated, since one would effectively be directly seeing the subquantum variable inside the wavepacket. That this does not happen is clear from (2.10): The nonoverlapping apparatus wavepackets $\chi_{A_q}(y)$ have a typical "uncertainty" width Δy in "pointer position" y, and a typical macroscopic separation $d >> \Delta y$. A "reading" $A_q = A_q^{meas}$ then takes place on the basis of an accuracy $\delta y << d$, but of course $\delta y \gtrsim \Delta y$. In other words the measurement takes place in terms of an accuracy δy in "pointer reading" y which is sufficient to distinguish alternative packets $\chi_{A_q}(y)$ and yet is not so small as to be within the uncertainty limit Δy . The important point we wish to make here is then that, while the precise value of the apparatus variable y does single out the realised packet, the precise value of y within the packet is never actually seen. Bell's statement[1], to the effect that the pilot-wave hidden-variables are not really hidden, might then be somewhat misleading. The precise values are hidden, and they merely single out the packet containing the system.

These seemingly pedantic points suggest the following important observation: The standard theory of "measurement" is really a set of rules for the evolution of Ψ and of its sub-branches in configuration space, and has little to do with the actual values of the physical variables X . Before showing how this is directly connected with the "Plato's Cave effect", and therefore with the confusion in the quantum theory of "measurement", let us first justify this point in more detail.

At first sight one might think that there is in general such a close correspondence between the motion of X and the evolution of the sub-branches of Ψ that the above distinction should be unimportant. But the following examples show that standard "measurement" theory may lead to completely erroneous conclusions as to the motion X(t).

Let an electron with coordinate x be fired at a two slit screen. On the immediate far side of the screen, we have an electron wavefunction $\psi_1(x)+\psi_2(x)$ emerging from the pair of slits, where at early times ψ_1 and ψ_2 do not overlap. Let us place a detector with internal variable r near slit 2, while for simplicity no detector is placed near slit 1. (The case with a detector near each slit is similar and yields the same conclusions). We say in the usual language that "there is an interaction V(x,r) between the electron and the detector", by which we really mean that there is a term V(x,r) in the Schrödinger equation, which in the classical limit is indeed "an interaction V(x,r) between electron and detector". In the language of standard quantum theory we say that the detector "remains in its original state D_1 if the electron passes slit 1", while it is "changed to state D_2 if the electron passes slit 2". Thus the initial wavefunction

$$(\psi_1(x) + \psi_2(x))D_1(r)$$

evolves into

$$\psi_1(x)D_1(r) + \psi_2(x)D_2(r) \qquad (2.11)$$

where during the interaction the wavefunction is of the form $\psi_1(x)D_1(r)+f(x,r)$. Note that throughout the interaction (assumed to take place sufficiently rapidly), the two branches do not overlap, since at early times ψ_1 and ψ_2 do not overlap. This implies that the actual variable r will move (i.e. $\dot{r} \neq 0$) if and only if the actual electron position x lies within the packet ψ_2 . Thus, if D_1 and D_2 are also nonoverlapping, so that they correspond to the final stage of a standard "measurement", the actual r will occupy the displaced packet D_2 if and only if the electron actually passed slit 2. For this case we have then correctly determined the path x(t) taken. But consider now the case where D_1 and D_2 do overlap. For instance if the "detector" is a single Hydrogen atom, D_1 and D_2 could be the first two energy eigenfunctions $D_1(r) \sim e^{-r/a_o}$, $D_2(r) \sim (2-r/a_o)e^{-r/2a_o}$ which overlap completely. (The detector could of course just as well be macroscopic, with for example D_1 and D_2 being two overlapping packets in r-space with very different macroscopic velocities). In standard quantum theory the state of the detector would be believed (and in a sense defined) to accurately provide "which-path" information, accessible at any later time by measuring this state. And the ortho-

gonality $\langle D_1|D_2\rangle = 0$ ensures that interference is lost, at later times when ψ_1 and ψ_2 overlap, the probability distribution for x being just $|\psi_1|^2+|\psi_2|^2$. But things are very different in the pilot-wave picture.

Certainly, at later times when ψ_1 and ψ_2 overlap, the probability distribution for x will indeed be just

$$\int dr\, |\psi_1 D_1 + \psi_2 D_2|^2 = |\psi_1|^2+|\psi_2|^2$$

since $\int dr\, D_1^* D_2 = 0$. However, the overlap between D_1 and D_2 implies that the two branches in (2.11) are not disjoint in configuration space, and so they <u>both</u> affect the motion x(t) from screen to backstop. Thus there is still "interference" at the subquantum level, even if this disappears upon averaging over r . More importantly, the outcome (E_1 or E_2) of a measurement of the detector's state need not have any relation at all to the path taken through slit 1 or 2.

To see this, first note again that the nonoverlapping of the two branches at early times implies that $\dot{r}\neq 0$ during the electron-detector interaction if and only if the actual x is in ψ_2 , i.e. if x(t) actually passes slit 2. This $\dot{r}\neq 0$ results in a displacement of r which, since D_1 and D_2 completely overlap, is noticeable only at the subquantum or sub-uncertainty level. At our level, we introduce a further apparatus with variable y, to "measure the energy state of the detector". Beginning with the wavefunction

$$(\psi_1(x)D_1(r) + \psi_2(x)D_2(r))\chi_1(y)$$

we have during the apparatus-detector interaction a wavefunction

$$\psi_1(x)D_1(r)\chi_1(y) + \psi_2(x)g(r,y)$$

which settles into

$$\psi_1(x)D_1(r)\chi_1(y) + \psi_2(x)D_2(r)\chi_2(y)$$

where χ_1 and χ_2 do not overlap. The pilot-wave picture now depends crucially on when the energy measurement is performed.

If this energy measurement is done at early times when ψ_1 and ψ_2 still do not overlap then, again, the apparatus coordinate y will move (and occupy the displaced packet χ_2) if and only if x occupies ψ_2 , yielding a correct determination of the slit passed by the electron. But this breaks down completely if, before the energy measurement is performed, the electron wavefunctions ψ_1 and ψ_2 are first allowed to spread and to overlap (towards the backstop). This is most easily seen at large times, where $\psi_1 \approx \psi_2$ far from the screen may be factored out, and the energy measurement essentially begins with

$$(D_1(r) + D_2(r))\chi_1(y)$$

and ends with

$$D_1(r)\chi_1(y) + D_2(r)\chi_2(y)$$

Here y either moves into the displaced packet χ_2 or it remains in χ_1 . Now if this outcome is to be determined by the slit passed by x(t), this determination can only take place via the value of r . But the outcome is <u>not</u> completely determined by the initial value of r : Since D_1 and D_2 overlap, the same initial r may clearly lead to either result, depending on the initial y and on the form of the apparatus-detector interaction (an example of "contextuality"). Thus the outcome will be E_1 or E_2 with probability ratio 1:1 , <u>regardless of which slit was passed</u>. If for example the electron passed slit 1, and yet we find y in χ_2 , this is due to the component D_2 of the detector wavefunction, where this component arose from Schrödinger-equation "interaction" with the <u>empty</u> packet ψ_2 . And this "ghost" branch of the wavefunction manages to affect y , owing to the overlap of ψ_1 and ψ_2 at large distances, which reoverlaps the branches of the total wavefunction <u>after</u> the supposed "which-path detection" has taken place.

In standard quantum theory, if the electron is finally found at a point x on the backstop, and the detector near slit 2 is then measured to have energy E_1 (or E_2), one would conclude that the electron "took path 1 (or 2)". According to the pilot-wave theory this conclusion is false. Thus the quantum theory of "measurement" need not be reliable if it deals with a total wavefunction whose branches reoverlap in configuration space in the above manner. This is to be expected, since the said theory is inspired by the classical limit of narrowly-peaked wavefunctions (see below).

In the above example, the finally "measured" energy actually tells nothing as to which slit was really passed. Conversely, as stressed by Bell[1], in a two-slit experiment where one makes no explicit attempt to measure the path, the pilot-wave theory predicts that particles arriving on the upper half of the backstop actually passed the upper slit. Though of course, in experiments performable at present, these claims of the pilot-wave theory cannot be checked by observation.

As a final example, we consider a "scattering experiment". To put the matter graphically, consider a stone of mass $\sim$ 10 grams whose wavefunction is very sharply peaked at x_o with a very small but nonzero tail extending up to and beyond ten metres from x_o. (We assume in any case that all wavefunctions have a nonzero tail extending to all finite points). It will very rarely happen that the stone is actually at ten metres from x_o. Now let an experimenter ("Dr. Johnson")

attempt to kick the stone. In equilibrium $P=|\Psi|^2$ it will usually happen that the experimenter's foot is deflected, corresponding to an apparent impact with a stone at x_o, and it will usually be that the stone was actually at x_o. But such deflection will occasionally occur with the stone actually at ten metres from x_o, since position measurements are always subject to error, and the error may happen to be many standard deviations. Of course such very rare "ghost" events are equally present in standard quantum theory, and do not conflict with standard measurement theory, being simply "errors". Nevertheless this example drives home the point that, from the viewpoint of the pilot-wave theory, the standard measurement scheme has really to do with the motion of wavepackets, and is only marginally related to the actual variables.

Such examples show that the standard measurement scheme cannot always be trusted to yield objective information, and must be regarded merely as a phenomenological book-keeping, consistent as long as precise access to the true variables is barred. At this point one could object that perhaps the fault lies with the path predicted by the pilot-wave theory, and not with standard measurement theory. In the absence of direct evidence we are of course unable to refute this. In support of the view taken here, however, one may cite, apart from the simplicity and naturalness of the pilot-wave theory, the following point: The pilot-wave theory gives a clear picture of how standard measurement theory arises phenomenologically at the macroscopic human level, and allows one to understand just how and why the very peculiar features of the standard theory arise (see below). This point is of course not a proof. But on the other hand it is perhaps the main reason which favours the abandonment of the Earth-centred astronomy based on epicycles, and the adoption of the heliocentric system. Continuing this analogy between standard quantum mechanics and Ptolemaic astronomy, we now outline the "Copernican" explanation for the very peculiar "epicycles" of standard measurement theory. Given the points made so far, this will be straightforward.

We would like to understand why human experimenters have constructed a classically-inspired "measurement" theory which has more to do with the evolution of Ψ than with the actual variables. This has come about in something like the following manner: As far as regards his manipulation of laboratory equipment, a human experimenter, together with his apparatus, behaves approximately classically. If the experimenter consists of variables ϕ guided by $\psi(\phi,t)$, it must be that ψ is narrowly peaked with centroid satisfying an approximately classical evolution. At the same time, the experimenter is oblivious to the precise actual values of ϕ within the "uncertainty" range $\Delta\phi$ of the packet ψ, and operates only to an accuracy $\gtrsim\Delta\phi$ (at most). Roughly speaking the experimenter functions on the level of accuracy of wavepackets and not directly at the hidden variable

level. (If his senses operated at the finer hidden variable level, he would directly perceive nonlocal connections, as well as having other remarkable characteristics - see Section 7). Note that if ψ consists of a superposition of non-overlapping narrowly-peaked packets, only one of these is relevant, this being the one containing the actual value of ϕ. (This is in contrast to the Everett many-worlds interpretation, where all packets would be regarded as equally real). In the pilot-wave theory, "reality" is by definition the actual value of ϕ, the wavefunction merely guiding its motion. The experience of the uniquely real experimenter rests on this unique actual value, even if he is sensitive to it only to an accuracy $>\Delta\phi$. As in the above discussion of quantum measurement, the actual ϕ singles out the realised packet, even though its precise value within the packet is irrelevant. (This might seem to argue in favour of abandoning the hidden-variable altogether, and retaining only the wavefunction, thereby leading to the many-worlds theory. But the situation is really the same here as in classical mechanics: An experimenter built from classical atoms has a reality which rests on the precise configuration ϕ of all his atoms, and yet his functioning is completely insensitive to ϕ beyond a certain level of accuracy). Just as the human ear is insensitive to Brownian motion "hiss", so the human experimenter is generally insensitive (as far as we know) to the hidden-variable level of his own constitution and of his surroundings. And this is the root of the "Plato's Cave effect". For at this gross level of accuracy, the experimenter's macroscopic world may be well represented by classical equations which are, as it were, the "shadow" of the underlying equations for the evolution of narrowly-peaked wavepackets. The experimenter, being unaware of this (at least prior to the 1920s), conceives the world in terms derived from this "shadow", these terms being just the classical notions of locally interacting forces, noncontextual variables, energy, and so forth. Thus the experimenter's basic concepts are from the beginning based on (narrow) wavepacket evolution only, and have little direct relation with the true variables themselves. Armed with these conceptions, the experimenter is then of course surprised and confused on entering the atomic domain. Indeed his experience is somewhat as if the wavelength of light were increased to say 10 cm., diffraction then causing havoc with any geometrical-optics attempt to make visual sense of one's everyday surroundings. Performing a two-slit experiment with electrons, for example, which divides and reoverlaps the narrow wavepackets on which the experimenter's worldview is based, he is led to believe that the electron "takes both paths" in some very obscure sense. For after some experience with the "non-geometrical-optics" level of finite wavepackets, the experimenter, for positivistic and other philosophical motives, decides in any case to retain his phenomenological macroscopic notions, regarding them as a fundamental "classical

level" in terms of which all experiments and "measurements" must be discussed (Heisenberg, Bohr). This can indeed work in a formal sense, and a definite mathematical theory may be constructed, for the following reason: Since the experimenter's classical concepts result from the behaviour of narrow wavepackets, and since the Schrödinger equation is linear, there does exist a formal mathematical analogy between the classical variables and the general mathematics of (even broad) wavepackets, this analogy being of course just the correspondence between classical variables and the linear operators on the Hilbert space associated mathematically with the linear wave equation. And the positivist experimenter constructs a vaguely plausible "measurement" theory on the basis of this formal correspondence. It is then no wonder that his "measurement" scheme is primarily concerned with the evolution of the wavefunction, the actual variables playing a secondary role.

To see how this formal analogy works, consider an electron in the post-"measurement" situation described by (2.10). If the electron observable Q is say an energy, the various eigenfunctions ψ_q may overlap, and yet only one of these subsequently affects the electron (owing to the nonoverlapping of the apparatus wavepackets). In this situation our experimenter (in "Plato's Cave") usually states that "Q has the value q". But what does this mean, and on what grounds does he say it? Since the ψ_qs overlap, the fact that the electron lies within one of these packets is not the important aspect of the post-"measurement" situation. The important point is that the electron is thereafter guided by only a single branch ψ_q. Thus we have the correspondence

$$\begin{array}{l}\text{"Q has the value q"}\\ \text{(Standard quantum theory)}\end{array} \quad \longleftrightarrow \quad \begin{array}{l}\text{"X is guided by } \psi_q\text{"}\\ \text{(Pilot-wave theory)}\end{array} \qquad (2.12)$$

But why does guidance by ψ_q lead the experimenter to say that "Q has the value q"? The reason is simply this: In the classical approximation on which the experimenter's language is based, guidance by a narrow packet which approximates ψ_q ("approximates" in the sense of Q-spectrum content) does indeed lead to Q having the value q in the sense of classical mechanics. (This is an example of the formal correspondence between classical variables and linear operators). While outside the classical limit the statement "Q has the value q" is, from the pilot-wave viewpoint, quite meaningless. It is merely a semantic convention adopted by our positivist experimenter, and this purely linguistic assignment of "values" is done purely in order to maintain an analogy with classical mechanics, and ultimately in order to satisfy the peculiar philosophy propounded by Heisenberg and Bohr. The entire language of "measurements", and of Hamiltonians formally

representing apparatus-system "interaction", is constructed according to a formal correspondence between linear operators and classical variables, in order to formally resemble the classical-mechanical "shadow in Plato's Cave". And we may see in the pilot-wave theory how the shadow arises, and why this formal parallel is possible.

From this perspective it is no surprise that the so-constructed theory of "measurement" is, while superficially consistent, quite baffling on closer analysis. Apart from the numerous perplexities which such experimenters have encountered since the 1920s, perhaps the most damning evidence against this whole enterprise is the necessary "contextuality" of the nonclassical level, which proves that the outcome of a quantum-theoretical "measurement" does not reveal the value of a pre-existing system-quantity, and indeed is not even determined by some such quantity or quantities, the experimental apparatus and set-up necessarily playing an active role in determining the value of this outcome. This means that, as stressed by Bell, quantum-theoretical "measurements" simply are not measurements in general, and should really be referred to as "experiments" of a particular kind, in which two systems ("apparatus" and "system") are coupled in a particular way. Acceptance of contextuality, which is clearly present in the pilot-wave theory, immediately disposes of the commonly accepted "epicycle" according to which the experimenter's choice of measurement somehow "creates reality". For example for a measurement of spin-1/2, where operators along different axes do not commute, the standard theory gives the impression that the reality of spin along an axis depends directly on the content of the experimenter's consciousness (his subjective choice of axis), an impression which disappears if one includes the active physical role of the apparatus (such as a Stern-Gerlach magnetic field) in bringing about the result.

For the case of two-slit interference it is the classical concepts of locality and noncontextuality which blind the experimenter to the possibility that the particle only traverses one slit, while nonlocally or "holistically" responding to the whole screen. The experimenter must of course somehow acknowledge the effect of the whole screen in producing the interference pattern, but his clinging to classical language and concepts forces him to deny the logical principle of excluded middle, and to assert that the particle "traverses both slits and neither". As pointed out by Koopman [32] and discussed by Ballentine[33], Feynman's claim[34] that standard probability theory cannot account for the two-slit experiment rests on a failure to appreciate the importance of conditional probabilities in this experiment, the crucial conditional statements (omitted by Feynman) being those regarding the arrangement of the whole apparatus (such as whether or not one of the slits is covered). Inclusion of these statements shows

that standard probability theory is satisfied, there being no need for "quantum probability" or "quantum logic".

A further "epicycle", namely negative joint probabilities, is produced if the experimenter attempts to account for the statistical predictions of quantum theory by means of a noncontextual joint distribution P(x,p) for position and momentum[35]. This distribution is found to be not positive definite, and the reason is surely simply this: "Measurement" of momentum is contextual, so that the distributions for position and momentum necessarily refer to different experimental arrangements (where these play an active role) i.e. to different physical situations. By mathematically uniting these into a single joint distribution, one creates negative probabilities, somewhat inversely to the situation in relativistic quantum field theory where, according to Bartlett[36], the appearance of negative probabilities is due to the "mathematical segregation of systems or states which physically exist only in combination".

Perhaps the most spectacular "epicycle" is the "many-worlds" theory, an alternative to the usual "double-think" regarding the two-slit experiment. Deutsch[37] has proposed an experiment, involving a self-measuring quantum computer, which would seem to support the many-worlds theory (assuming the experimental results to be as one would expect). But again, Deutsch's interpretation of this experiment relies on taking to extremes our faith in the classically-inspired definitions of "measurement", in terms of the operator formalism. Essentially , Deutsch considers an operator with eigenstates corresponding to the computer having formally "seen two worlds without recording which one", and gives an experiment whereby the final internal state of the computer is just such an eigenstate, after having performed an interference experiment with a spin (leaving the final spin in a superposition). In our view the computer's conclusion of having "seen two worlds" is an illusion created by taking the phenomenological Von Neumann scheme too seriously, somewhat as diffraction might lead to an apparent multiplicity from a geometrical-optics point of view. While the many-worlds view does in a sense avoid the "classical level" of the Copenhagen interpretation, in another sense it does just the opposite, and builds physics on the basis of an abstract formal language which is inspired ultimately from the classical approximation. Deutsch's parallel computer processing, also held to support many-worlds, is discussed in Section 7. Roughly speaking it could be said that a single holistic quantum computer, when analysed in local and noncontextual classical language, appears to be multiple.

2.2.2 General remarks

The assignment of a central role to the "observer" in standard quantum theory is often regarded, from a philosophical point of view, as having in a

sense undone the Copernican revolution which displaced Man from his central position in the Universe. The above view of the emergence of classical concepts, and of standard quantum theory, effectively reverses the situation once more, and sees today's experimenters as a modern version of the prisoners in Plato's Cave, misled by the illusory shadow of "classical language". In proposing this viewpoint, we do not by any means wish to claim that the pilot-wave theory, or indeed any theory, is a complete and final description of reality. Nor is our above discussion of an approximately classical experimenter intended to suggest that the pilot-wave theory is adequate to describe in all respects the behaviour of human beings. The relationship between "theory" and "reality", like the relationship between "mind" and "matter", has as yet not been fully clarified, and perhaps never can be. While on self-referential grounds, the possibility of human beings constructing a theory which fully describes their own behaviour might itself be questioned. We do suggest, however, that in experiments currently performed in physics, the actions of human beings such as the turning of a switch are better seen in "objective" terms. We further suggest that the present confusion in quantum theory (and also in the interpretation of the pilot-wave theory) may be eliminated by dropping the observer-centred viewpoint, and explicitly considering how the observer's classical concepts, together with his ideas with regard to "measurement", arise from a deeper "objective" level, as we have done in the above.

There is an intriguing parallel between Ptolemaic astronomy and standard quantum theory. In the second century B.C., Aristarchus of Samos devised a heliocentric model of the solar system, which in the words of Arthur Koestler "was rejected in favour of a monstrous system of astronomy [Ptolemy's], which strikes us today as an affront to human intelligence, and which reigned supreme for fifteen hundred years".[38] This occurred despite the fact that the ancients' astronomical data was just as accurate as that available to Copernicus (who actually relied on their data). According to Koestler the explanation is that, during the decline of Classical Greece, the craving for stability in a disintegrating culture led to a general turning away from realism. The ancients retreated into a "sublunary world", which supposedly had no common link with the "divine" heavenly bodies. With the rise of Platonism, visible and "imperfect" bodies were no longer of concern. The purpose of astronomical systems became merely to "save the appearances", i.e. to successfully predict the apparent motions in the sky. "Astronomy, after Aristotle, becomes an abstract sky-geometry, divorced from physical reality".[38] The introduction of "epicyclic" motions was done with no regard for whether or not these motions were true. All that mattered was that it "worked" as a calculating device (and indeed the predictions of the Ptolemaic system were very accurate). No one understood why it worked, and nobody seemed to care. Science became paralysed by

the "three fundamental conceits"[38] of Ptolemaic astronomy: (i) The dualism of the celestial and sublunary worlds, (ii) The immobile and central Earth, and (iii) The belief that all heavenly motion must be described in terms of "perfect" circles (dubbed "Plato's curse" by Koestler).

The eerie parallel with standard quantum theory is rather clear. In 1927 Louis deBroglie proposed the pilot-wave interpretation of Schrödinger's equation, but, as Bell put it, deBroglie's straightforward and natural ideas were "simply trampled on".[39] This was done in favour of a highly abstract, antirealist, and formal scheme, which enables one to successfully predict the outcome of laboratory experiments without providing a physical understanding. Now the antirealist tendency in the culture of post-World War I Europe, and in Germany in particular, in both the Arts and the Sciences, is a historical fact[40],[41], whose roots lie partly in the rise of the German Romantic movement in the latter half of the eighteenth century, when, as the Art historian Arnold Hauser put it, "German scientific style successively assumed that often vague, coquettish character, iridescent with half-expressed intimations, which differentiates it so sharply from the style of West European scientific language"[42] The parallel with ancient Greece has been stressed by Hauser, according to whom "Plato's theory of Ideas fulfils the same social function for Athens of the fourth century as German Idealism did for the nineteenth century; it furnishes the privileged minority with arguments against realism...."[43] And the Weimar Republic of the 1920s represented an unstable and disintegrating culture if ever there was one, whose widespread influence on contemporary thought has recently been stressed by the political philosopher Allan Bloom[44], who refers to the thought behind it as "the profound philosophical reflection [Nietsche, Heidegger] that broke with and buried the philosophic tradition, with the most ambiguous intellectual, moral and political consequences". Physicists, stimulated by Mach and encouraged by Einstein's subjectivist presentation of "relativity" theory, had by the 1920s retreated into the "sublunary" world of the immediate senses, claiming that a realistic representation of atomic behaviour was "purely metaphysical". As long as quantum theory could "save the appearances" at the laboratory level, this was considered to be "operationally adequate". And indeed, to do better was considered to be probably impossible and certainly philosophically unsound. There thus arose the extraordinary "three fundamental conceits" of standard quantum theory: (i) The dualism of the quantum and classical worlds, (ii) The central "observer", apparently outside the domain of physical law (i.e. "immobile"), and (iii) The belief that all experiments must be described in classical terms ("Bohr's curse").

The fate of deBroglie in 1927, like that of Aristarchus, seems indeed to have marked the beginning of another Dark Age in fundamental physics. According to Feynman[34] physicists have "given up" trying to make exact predictions, as if

they had capitulated after intense effort. But the fact that deBroglie's ideas were "simply trampled on" (at the 1927 Solvay Congress in Brussels) betrays the powerful irrationalism that gave shape to modern physics. To quote Koestler once more: "The medieval astronomers manipulated their epicyclic symbols as modern physics manipulates Schrödinger's wave equation.....Exact Science has ceased to be the Philosophy of Nature, and no longer has much inspiration to offer to the questing human mind.....At what point was the new version of Plato's curse uttered: "Thou shalt think in circles"?" The new version of "Plato's curse", we suggest, is "Bohr's curse" that "Thou shalt speak in classical terms", where such terms implicitly assume locality and noncontextuality.

2.2.3 The emergence of locality, uncertainty, and energy-momentum conservation

Locality: Consider two "boxes" A and B, separated by a large distance, each containing a single particle with coordinate X_A and X_B respectively. If each box has ground state ϕ_o and excited state ϕ_1, take the initial wavefunction to be entangled as

$$\psi_o(X_A,X_B) \sim \phi_o(X_A)\phi_1(X_B) + \phi_1(X_A)\phi_o(X_B)$$

Consider now an ensemble of such systems, with initial distribution $\rho_o(X_A,X_B)$. (The ensemble could consist of distinct box-plus-particle pairs, or equally of a single pair of boxes containing an ensemble of noninteracting particles guided by the same wavefunction). If the Hamiltonian of box B is suddenly altered, say by moving the walls, then entanglement implies that the actual values of X_A are nonlocally affected. But what about the box-A distribution $\rho_o(X_A)=\int dX_B\,\rho_o(X_A,X_B)$? Is this affected too? Standard quantum theory tells us that, in equilibrium, where $\rho_o(X_A,X_B)=|\psi_o|^2$, the partial distribution at A is not affected at a distance, and practical nonlocal signalling is not possible. However, any initial deviation $\rho_o \neq |\psi_o|^2$ from equilibrium leads to a breakdown of signal-locality, i.e. nonlocality becomes apparent at the statistical level, and the distribution at A is affected at a distance. This is easily seen as follows.

If we take ϕ_o and ϕ_1 to be real functions then X_A and X_B, and any distribution thereof, will be static if the Hamiltonian remains fixed. But a sudden change of box-B Hamiltonian from H_B to $H_B' \neq H_B$ leads, after a small time ϵ, to a change $\Delta\rho(X_A)$ in the distant distribution at A, given by (the calculations have been published elsewhere[19])

$$\Delta\rho(X_A) = -\epsilon^2/2m\,\frac{\partial}{\partial X_A}\left(a\int dX_B\, b\,\frac{(\rho_o - |\psi_o|^2)}{|\psi_o|^2}\right) \qquad (2.13)$$

(to leading order in ϵ) where

$$a(X_A) = \phi_1^2(X_A)\frac{\partial}{\partial X_A}(\phi_o(X_A)/\phi_1(X_A))$$

$$b(X_B) = \phi_1(X_B)\langle X_B|H_B'|\phi_o\rangle - \phi_o(X_B)\langle X_B|H_B'|\phi_1\rangle + (E_1-E_o)\phi_1(X_B)\phi_o(X_B)$$

If $H_B'=H_B$ then b vanishes and so does $\Delta\rho(X_A)$, as it must. But for $H_B'\neq H_B$ the distribution at A responds nonlocally to the sudden wall motion (say) at B.

Thus nonlocal signalling at the statistical level occurs if and only if $\rho_o \neq |\psi_o|^2$.

According to Bohm and Hiley[24] practical nonlocal signalling does not occur because the "quantum potential" Q is "too fragile and nonlinear". While in our view signal-locality is really a property of the equilibrium distribution $\rho=|\psi|^2$, for which nonlocal effects are "washed out" by "uncertainty noise", and has nothing to do with any property of Q per se. (For even in equilibrium, if ψ is entangled, there are subquantum nonlocal connections, so that Q does act nonlocally, and the effect of this only vanishes upon averaging over $\rho = |\psi|^2$).

We have noted that the smooth distribution $\rho=|\psi|^2$ is a purely theoretical limit for infinite ensembles. For any finite number n of systems, the actual distribution has the discrete form (2.8), so that $\rho\neq|\psi|^2$ on a fine-grained scale. This implies that, for any finite n, the right-hand-side of (2.13) cannot vanish for $H_B'\neq H_B$, and the nonlocal response $\Delta\rho(X_A)$ can never be strictly zero in real experience. Thus in any real (finite) laboratory, weak nonlocal influences must be occuring, for example between two separate large collections of atoms, and locality is never exactly valid.

Uncertainty: In equilibrium the statistical dispersions Δx in position and Δp in "measured" momentum satisfy $\Delta x\Delta p \geqslant \hbar/2$, while if $\rho\neq|\psi|^2$ this "uncertainty principle" is generally violated, as pointed out by Bohm[6]. We have given a simple explicit example of this elsewhere[19].

The "disturbance" aspect of the uncertainty principle arises from contextuality, i.e. from entanglement between the system and apparatus wavefunctions (which causes the unknown apparatus hidden variables to affect the "measured" outcome). Thus the nonlocal aspect of entanglement, which is masked in equilibrium by the uncertainty principle, is really rendered unobservable in equilibrium by the effects of entanglement itself. Roughly speaking, in equilibrium, quantum holism prevents itself from being seen directly.

Energy-momentum conservation: The autonomous (independent of X(t)) evolution of Ψ prevents the definition of a conserved energy or momentum. We argue that this is quite reasonable, as follows.

Firstly, the physical variables X(t) are subject to no preassigned mechanical rules of interaction. They are, as it were, "freely floating", ready to follow

whatever Ψ dictates. And this guidance takes place in configuration space, not phase space, determining the first time derivative of X(t). This profound difference from standard mechanics suggests that the usual mechanical energy-momentum conservation need not play a fundamental role. A second argument, already noted, is that energy-momentum conservation is powerful as a fundamental physical principle only if it is a local principle. Thus its absence at the nonlocal subquantum level is not unreasonable. Thirdly, standard quantum theory is in any case simply silent with regard to energy-momentum conservation for individual events (apart from when the state is an eigenstate of energy or momentum). For these reasons we propose that energy-momentum conservation may be reasonably abandoned as a fundamental principle, and regarded purely as a phenomenological property of equilibrium.

The actual (as opposed to "measured") value of momentum $m\dot{X}$, according to the pilot-wave theory, is $P_{pw}=\partial S/\partial X$. If one insists on defining an actual value for "energy", the natural choice is Bohm's

$$E_{pw} = (P_{pw})^2/2m + V + Q$$

Let us see how these compare with their counterparts in equilibrium quantum theory.

For a general distribution ρ, we define the mean values

$$\bar{E}_{pw} = \int dX\,\rho E_{pw}$$

$$\bar{P}_{pw} = \int dX\,\rho P_{pw}$$

while for equilibrium $\rho = |\psi|^2$ these means are, as is readily shown,

$$(\bar{E}_{pw})_{eq} = \langle H\rangle$$

$$(\bar{P}_{pw})_{eq} = \langle P\rangle$$

i.e. they are equal to the quantum-theoretical expectation values, as shown by Bohm[6]. However, even in equilibrium, the mean-square values are generally unequal,

$$(\overline{E^2_{pw}})_{eq} \neq \langle H^2\rangle$$

$$(\overline{P^2_{pw}})_{eq} \neq \langle P^2\rangle \qquad (2.14)$$

For instance it is easy to show that

$$\langle P^2\rangle = (\overline{P^2_{pw}})_{eq} + \int dX\,(\partial|\psi|/\partial X)^2$$

This means that the equilibrium probability distributions for E_{pw} , P_{pw} are generally not equal to the probability distributions for the "measured values" of the operators H and P. The physical reason for this is of course contextuality, and the inequality in (2.14) is a measure of this contextuality.

Using the continuity equation for ρ , together with $dE_{pw}/dt = \partial Q/\partial t$ and $\int dX\,|\psi|^2\,\partial Q/\partial t = 0$ one finds

$$d\bar{E}_{pw}/dt = d\langle H\rangle/dt + \int dX\,(\rho - |\psi|^2)\,\partial Q/\partial t \qquad (2.15)$$

while using $\partial S/\partial t + E_{pw} = 0$, with $\int dX\,|\psi|^2\,\partial Q/\partial X = 0$, it follows that

$$d\bar{P}_{pw}/dt = d\langle P\rangle/dt + \int dX\,(\rho - |\psi|^2)(-\partial V/\partial X - \partial Q/\partial X) \qquad (2.16)$$

where of course $d\langle H\rangle/dt = 0$ and

$$d\langle P\rangle/dt = \int dX\,|\psi|^2(-\partial V/\partial X)$$

Thus one recovers the usual energy-momentum conservation if and only if $\rho = |\psi|^2$, for which one has $d(\bar{E}_{pw})_{eq}/dt = 0$ and

$$d(\bar{P}_{pw})_{eq}/dt = \int dX\,|\psi|^2(-\partial V/\partial X - \partial Q/\partial X)$$

Given the emergence of standard physics in equilibrium $P=|\Psi|^2$, it is clearly important to study in detail how this equilibrium arises, a subject to which we now turn.

3. SUBQUANTUM STATISTICAL MECHANICS

At the fundamental level there exists a definite path X(t) guided by Ψ , and there is of course no mention of probabilities. In practice one usually deals with an ensemble P(X,t) of paths, each guided by the same Ψ . We must now derive equilibrium $P=|\Psi|^2$ for the ensemble, on the basis of the pair of deterministic equations $\Pi = \partial S/\partial X$, $i\partial\Psi/\partial t = \hat{H}\Psi$, in analogy with classical statistical mechanics.

First we shall derive a general H-theorem, which implies $P \longrightarrow |\Psi|^2$ in a coarse-grained sense provided the system is "sufficiently chaotic". After considering the definition of subquantum entropy and the statistical mechanics of equilibrium, we address the question of relaxation $P \longrightarrow |\Psi|^2$ for real systems.

The "probability" or "distribution" P may be conceived of in various ways, according to circumstances. For instance in Section 2.2 we defined P as the actual distribution of a real collection of n systems, P then being a sum of delta-functions. As $n \longrightarrow \infty$, this sum may be replaced by a theoretical smooth function representing the infinite ensemble (rather as one defines a smooth density in classical fluid dynamics). However one might also view P as a probability associated with a single system, where the system is considered to be an element of an imagined theoretical (and infinite) ensemble. For a system such as the entire universe such an ensemble is purely theoretical, while for say a box of gas one may realistically contemplate an at least very large ensemble. We need not restrict ourselves to any particular interpretation of P, since the mathematical theorems given below will be valid regardless. The H-theorem may then be applied to the whole universe, invoking a theoretical ensemble, or it may be applied to a real ensemble of sub-systems. This freedom in the definition of P reflects the fact that probabilities play no fundamental role. They are merely useful practical tools, whose definition may be varied according to convenience.

3.1 H-theorem

Consider an initial distribution P(X,0) of configurations X(0), each guided by the same Ψ , where $P(X,0) \neq |\Psi(X,0)|^2$. We take X to be the configuration of a complicated system, consisting of a large number N of interacting variables X_i , so that the path X(t) is generally very complicated. (For instance our system might be a box of N interacting particles). As is the case with the classical H-theorem, there is no simple and general specification of an "appropriately complex" system, though some comments on this are made below (Section 3.3). It is of course assumed that all regions where $|\Psi| \neq 0$ are accessible to at least some of the trajectories X(t).

By definition the distribution P(X,t) must satisfy the continuity equation

$$\partial P/\partial t + \nabla\cdot(\dot{X}P) = 0$$

while the Schrödinger equation implies that this is also satisfied by $|\Psi|^2$. As pointed out by Bohm[45], this implies that the ratio $f=P/|\Psi|^2$ is preserved along trajectories,

$$df/dt = \partial f/\partial t + \dot{X}\cdot\nabla f = 0$$

This means that initial deviations $P\neq|\Psi|^2$, or $f\neq 1$, are forever carried along the trajectories and can never disappear, which at first sight seems to imply that equilibrium can never be reached. But the situation here is exactly the same as in the classical statistical mechanics of an isolated system, where the phase-space probability density p is also preserved along the system trajectories (Liouville's theorem). In the classical case what happens is that p evolves in a highly complex "filamentary" manner over the energy surface so that, despite dp/dt=0, on a coarse-grained level p does become uniform. In the subquantum case P and $|\Psi|^2$ are so thoroughly mixed by $\dot{X}$ as to become indistinguishable on a coarse-grained level (this is rather like the classical stirring of two fluids).

Elsewhere[18] we introduced the quantity

$$H = \int dX\,|\Psi|^2 f \ln f = \int dX\,P \ln(P/|\Psi|^2) \qquad (3.1)$$

by analogy to the classical $\int d\Omega p \ln p$, replacing the phase-space volume element $d\Omega \longrightarrow |\Psi|^2 dX$ and $p \longrightarrow f$. The continuity equation and df/dt=0 together imply that dH/dt=0, i.e. the exact fine-grained H is constant, as occurs classically for an isolated system, reflecting the lack of fine-grained mixing. Dividing configuration-space into cells of volume δV, we define the coarse-grained quantities

$$\bar{P} = (1/\delta V)\int_{\delta V} dX\,P$$

$$\overline{|\Psi|^2} = (1/\delta V)\int_{\delta V} dX\,|\Psi|^2$$

(defining $\bar{P}$ and $\overline{|\Psi|^2}$ as constant in each cell), and a coarse-grained H

$$\bar{H} = \int dX\,\bar{P} \ln(\bar{P}/\overline{|\Psi|^2}) \qquad (3.2)$$

The classical H-theorem for an isolated system relies on the assumption $\bar{p}(0) = p(0)$ in phase-space, i.e. it is assumed that there is no fine-grained "microstructure" in the initial conditions[46]. Some such assumption is of course necessary owing to the time-reversibility of the theory, and the situation is similar in the subquantum case. We assume the initial conditions

$$\bar{P}_o = P_o\,, \quad \overline{|\Psi_o|^2} = |\Psi_o|^2 \qquad (3.3)$$

These will hold to high accuracy if P and $|\Psi|^2$ are initially almost constant over the volume δV (and to arbitrary accuracy as $\delta V \longrightarrow 0$ if P and $|\Psi|^2$ are smooth functions).

Defining $\tilde{f} = \bar{P}/\overline{|\Psi|^2}$, and using the facts that the exact H is constant in time and that $\tilde{f}$ is constant over δV, we have shown elsewhere[18] that

$$\bar{H}_o - \bar{H} = \int dX\,|\Psi|^2[f \ln(f/\tilde{f}) + \tilde{f} - f]$$

where $\bar{H}$, Ψ, f, $\tilde{f}$ denote values at any time $t\neq 0$. Since $x\ln(x/y) + y - x \geqslant 0$ for all x,y we have the subquantum H-theorem

$$\bar{H}(t) \leqslant \bar{H}(0) \quad \text{for all } t \qquad (3.4)$$

Note that $\bar{H}$ decreases on either side of t=0 (reflecting time-symmetry), so the conditions (3.3) imply that t=0 is a local maximum (as in the classical H-theorem[47]). The result (3.4) of course does not imply that $\bar{H}(t)$ decreases monotonically on $t>0$. But it does imply that there exists a time interval (0,T) during which $\bar{H}$ either decreases or remains constant, i.e. $d\bar{H}/dt \leqslant 0$ throughout (0,T). Further, it is easily shown[18] that $\bar{H} \geqslant 0$, so that $\bar{H}$ is bounded from below.

We assume our system to be such that $\bar{H}$ closely approaches its minimum value after some time t_{equil}, and that experimental observations take place at a time t_{now} satisfying

$$t_{equil} << t_{now} << t_{recur} \qquad (3.5)$$

where t_{recur} is the subquantum recurrence time (see Section 3.3). With these assumptions, and the fact that $\bar{H}$ is minimised if and only if $\bar{P} = \overline{|\Psi|^2}$ everywhere[18], the equilibrium $\bar{P}=\overline{|\Psi|^2}$ on the coarse-grained level is established. (Similar assumptions are of course required to establish the equilibrium $\bar{p}$ = uniform on the energy surface for the classical isolated system).

Now let a single variable X_1 be extracted from $X=(X_1,X_2,\ldots X_N)$ and prepared in a state ψ_q. (Here X includes the apparatus variables, where the apparatus is assumed to be constructed from material which is in equilibrium). This means that, given the observed value of the apparatus variable, the only nonvanishing part of Ψ is the branch $\sim\psi_q(X_1)\chi_{other}$, where χ_{other} represents the apparatus and other variables. If this is done with an ensemble $\bar{P}=\overline{|\Psi|^2}$ of X, the coarse-grained distribution of X_1 will be

$$\bar{\rho}(X_1,t) = \int dX_2 \ldots \int dX_N \, \bar{P} = \int dX_2 \ldots \int dX_N \, \overline{|\Psi|^2}$$

which, using the constancy of $\overline{|\Psi|^2}$ in each cell, may be recast as

$$\bar{\rho}(X_1,t) = (1/\delta V_1) \int_{\delta V_1} dX_1 \int dX_2 \ldots \int dX_N \, |\Psi|^2$$

(where $\delta V = \delta V_1 \ldots \delta V_N$) so that

$$\bar{\rho}(X_1,t) \propto (1/\delta V_1) \int_{\delta V_1} dX_1 \, |\psi_q(X_1)|^2 \int dX_2 \ldots \int dX_N \, |\chi_{other}|^2$$

or simply

$$\bar{\rho}(X_1,t) = \overline{|\psi_q(X_1)|^2} \qquad (3.6)$$

We thus recover the equilibrium probability distribution for a single extracted variable.

The result (3.6) might at first sight appear problematic. For ψ_q will often be a simple smooth function, such as the wavefunction of the electron in the ground state of Hydrogen (in which case $|\psi_q|^2 = |\psi_q|^2$ for $\delta V_1 << (1\ \mathring{A})^3$). The total configuration X is then at a point in configuration-space where the total wavefunction Ψ is $\sim \psi_q \chi_{other}$ which is smooth in the variable X_1. But $\rho(X_1,t)$ will be very "spiky", so that only on coarse-graining does one obtain a smooth function $\bar{\rho} = |\psi_q|^2$. The apparent paradox, then, is that in this region of configuration-space $|\Psi|^2$ is smooth while P is not (with respect to X_1). For how can this be so, if $|\Psi|^2$ and P are both "stirred" by the same velocity field $\dot{X}$ in the continuity equation? The answer is that $\dot{X}$ is related to the evolution of Ψ by the Schrödinger equation, as well as by the continuity equation, while no such additional relation exists between $\dot{X}$ and P. This has the effect that if $\dot{X}$ varies very rapidly across configuration-space, i.e. is a "spiky" function of X, then this leads to a highly filamentary and "chaotic" P while nevertheless $|\Psi|^2$ remains relatively smooth. The simple picture of P and $|\Psi|^2$ both being "stirred" by $\dot{X}$, in a manner analogous to the classical stirring of two fluids, is then somewhat misleading, and $|\Psi|^2$ generally remains smoother than P. (An explicit numerical model confirms this - see Section 3.3).

Remarks on alternative approaches to deriving equilibrium: We have developed the subquantum H-theorem by analogy with the classical treatment of isolated systems. It would be interesting to develop an alternative approach based on external perturbations, perhaps obviating the need for coarse-graining. With an external "dissipator", this might also lead to a fluctuation-dissipation theorem, which would enable one to calculate the relaxation rate from equilibrium fluctuations.

The classical theory may alternatively be developed using only a single system, by taking the equilibrium distribution to be a statement as to the fraction of time spent in each region of phase-space, an approach used in particular by Einstein. The subquantum analogue of this would be to try to prove that a single (complicated) system occupies the region dX for a fraction $|\Psi|^2 dX$ of the time. Unfortunately, the time-dependence of the measure $|\Psi|^2$ renders such a statement meaningless. And indeed a time-dependent measure is usually not considered in dynamical systems theory. One might try to avoid this problem by a change of variables such that the new measure is time-independent.

Finally, a highly controversial approach to deriving $P=|\Psi|^2$, which could readily be adapted to the pilot-wave theory, is that taken in the many-worlds interpretation[48]. However in this approach measurement of an infinite ensemble supposedly (almost) always yields the equilibrium distribution. This seems to confirm the commonly made criticism that the result $|\Psi|^2$ is really assumed at the outset. For in the pilot-wave theory, if one begins with an infinite nonequilibrium ensemble $P \neq |\Psi|^2$ at t=0, then measurements of X soon after t=0 will clearly yield a nonequilibrium distribution. Further, there is a time-dependence in the relaxation $P \longrightarrow |\Psi|^2$, which is absent in the many-worlds approach.

3.2 Subquantum entropy. Equilibrium fluctuations. Random instantaneous signals.

The significance of the subquantum entropy

$$S = -H = -\int dX \, P \ln(P/|\Psi|^2) \qquad (3.7)$$

deserves elaboration. This expression may seem peculiar in comparison with the classical entropy, but it becomes completely natural if one considers entropy in terms of "relative information".

The quantity

$$I = \int dX \, P(X) \ln[P(X)/M(X)]$$

is known as the "renormalised information" of P with respect to density M(X), or otherwise simply as the relative information of P with respect to M [49]. This quantity is widely used in mathematical statistics. It is the information for discrimination in favour of the hypothesis that the distribution is P(X), against the hypothesis that it is M(X).[50] For our purposes it is sufficient to note that it measures the amount of information contained in the distribution P(X), in comparison with M(X). We stress that the word "information" need not refer to anything subjective. One may take it to be just a measure of the mathematical complexity of the function in question, which is a perfectly objective property

if P represents a real ensemble.

Classically, the equilibrium distribution is uniform, p_{eq}=constant (on the energy surface). For a general p, the classical $\int d\Omega\, p \ln p$ then measures the information contained in p relative to p_{eq}.

For the subquantum case, the equilibrium distribution is $P_{eq}=|\Psi|^2$, and the expression $\int dX\, P \ln(P/|\Psi|^2)$ measures the information (negentropy) contained in an arbitrary P relative to that contained in $P_{eq}=|\Psi|^2$. Thus, measuring entropic information relative to the equilibrium state leads inevitably to the expression (3.7).

It is also useful to consider how S may be given a ln(number of ways) definition. To do this, at a fixed time t we transform the variables $X \longrightarrow X'$ such that the new volume element

$$dX' = |\Psi|^2 dX \tag{3.8}$$

and the new probability distribution is $P'= P/|\Psi|^2$ (so that $P'dX'=PdX$). The subquantum entropy is then

$$S = -\int dX' P' \ln P' \tag{3.9}$$

which may be given a ln(number of ways) definition as follows.

Consider a large but finite number n of systems, whose distribution in X' approaches P' in the limit $n \longrightarrow \infty$. For large n we have a fraction P'dX' of systems occupying dX'. We then define $W[P']$, the number of ways in which the n points in X'-space may be arranged without altering the distribution P'(X'), and the subquantum entropy is naturally defined by

$$S \propto \ln W[P'] \tag{3.10}$$

That this indeed leads to the subquantum entropy is seen upon calculation of $W[P']$. Dividing X'-space into ν equal cells of volume ε, let n_i be the number of systems occupying the i^{th} cell. The set $\{n_i\}$ is then equivalent to P', where $P'(X'_i)\varepsilon = n_i/n$ as $n \longrightarrow \infty$. The n points may then be arranged in $W[\{n_i\}]$ ways, without altering the set $\{n_i\}$, where

$$W[\{n_i\}] = n!/n_1!....n_\nu!$$

Stirling's formula, with $\sum_i n_i = n$, implies that

$$\ln W = \ln n! - \nu \ln(2\pi)^{1/2} + n - \sum_i n_i(1+1/2n_i)\ln n_i$$

For $n \longrightarrow \infty$, we may take $(1+1/2n_i) = 1$, so that

$$W[\{n_i\}] \propto \exp(-\sum_i n_i \ln n_i)$$

where the proportionality factor depends on n and ν but not on $\{n_i\}$. Since $n_i = nP'(X'_i)\varepsilon$, we then have

$$W[P'] \propto \exp[-\int dX' n P'(\ln P' + \ln(n\varepsilon))]$$

or, using $\int dX'P'=1$,

$$W[P'] \propto \exp[-n\int dX' P' \ln P']$$

The quantity $\ln W[P']$ gives us an entropy associated with the ensemble of n systems. Dividing this by n we obtain (3.9), which on transforming back to X-space yields the desired result (3.7) for S.

This ln(number of ways) definition of S enables it to be related to "equilibrium fluctuations". If n systems are extracted from an infinite equilibrium ensemble, their measured configurations will have a distribution $P_{meas} \neq |\Psi|^2$ while $P_{meas} \longrightarrow |\Psi|^2$ as $n \longrightarrow \infty$. Transforming again to X'-space and dividing it into cells, the probability $\Pi[P'_{meas}]$ of obtaining the distribution P'_{meas} is just proportional to the number of ways of arranging n points with distribution P'_{meas} (since in equilibrium the ensemble probability is uniform on X'-space). Thus

$$\Pi[P'_{meas}] \propto \exp(-\sum_i n_i \ln n_i)$$

which for large n may be written, transforming back to X-space,

$$\Pi[P_{meas}] \propto \exp[-n\int dX\, P_{meas} \ln(P_{meas}/|\Psi|^2)]$$

which is just

$$\Pi[P_{meas}] \propto (e^{S[P_{meas}]})^n$$

Since $S[P_{meas}] \leq 0$, being zero only for $P_{meas}=|\Psi|^2$, it is clear that for $n \longrightarrow \infty$ a distribution $\neq |\Psi|^2$ has vanishing probability, and the distribution $|\Psi|^2$ has probability $\Pi = 1$, as must of course be the case.

The subquantum entropy thus yields, by exponentiation, the probability for "fluctuations" $P_{meas} \neq |\Psi|^2$, which are always obtained for a finite ensemble. These fluctuations are of course expected on simple statistical grounds, and they are equally present in standard quantum theory. Nevertheless a very large (and extremely rare) fluctuation, analogous to the macroscopic rare fluctuations of classical statistical mechanics, may effectively give rise to nonequilibrium physics (see Section 8.3).

For an ensemble of pairs of systems with configurations X_1 and X_2, the total subquantum entropy separates into $S=S_1+S_2$ if and only if $P=P_1P_2$ and $\Psi=\Psi_1\Psi_2$, in which case the fluctuations $P_{1\ meas}\neq|\Psi_1|^2$ and $P_{2\ meas}\neq|\Psi_2|^2$ will be statistically independent, $\Pi=\Pi_1\Pi_2$. While if the two (perhaps distant) systems are entangled, $\Psi\neq\Psi_1\Psi_2$, the entropy will be nonseparable so that the fluctuations for each system will be correlated, $\Pi\neq\Pi_1\Pi_2$. As a simple example, consider an ensemble of EPR-correlated spin-1/2 particles, 1 and 2, in the singlet state. If out of 100 measurements 53 of the particles-1 are found with spin-up, then 47 of the particles-2 will be found with spin-up, the fluctuations being completely correlated. Again, such correlated fluctuations are equally present in standard quantum theory, and their probability is easily calculated by applying $P=|\Psi|^2$ to each outcome.

From the point of view of the pilot-wave theory, where nonlocality is only-just-hidden by equilibrium, these nonlocally-correlated equilibrium fluctuations may be interpreted as random instantaneous signals, analogous to the random entropic fluctuations of classical statistical mechanics. The attempt to make practical use of these random instantaneous signals is analogous to the attempt to extract useful work from classical thermal fluctuations. Occasional small-scale "violations" of the classical Second Law are usually of no practical value, and the Second Law holds in an average sense. However, from an extremely rare thermal fluctuation (such as the spontaneous boiling of the oceans) useful work may clearly be extracted. Similarly, extremely rare large-scale fluctuations $P\neq|\Psi|^2$ would give rise to macroscopic instantaneous signals, which however could only be controlled by a subquantum demon or automaton (see Section 8.3).

3.3 The approach to equilibrium

The subquantum H-theorem is a general result which tells us that, for appropriate initial conditions, the mixing $P\longrightarrow|\Psi|^2$ begins to occur on a coarse-grained level. The initial conditions are "appropriate" if they contain no fine-grained microstructure, which could lead to "unlikely" entropy-decreasing behaviour, as we have assumed in (3.3).

This general result does not in itself guarantee that equilibrium is actually reached. Nor does it indicate the magnitude of the relaxation time. The extent to which equilibrium is reached and the time taken for this to occur depend on the details of the system, as in classical statistical mechanics. It is clearly important that explicit models be studied in detail, not only as a point of principle, but also for the possible experimental and cosmological implications (see below). In this Section, some initial steps are taken in this direction.

We have, in multidimensional configuration-space, two "fluids" of density P and $|\Psi|^2$, each stirred by the same velocity field $\dot{X}$ which varies rapidly with X. How fast does $\bar{P}\longrightarrow\overline{|\Psi|^2}$? This clearly depends on the size of the coarse-graining

volume δV and on the form of the velocity field $\dot{X}$. (As we have already noted, $\dot{X}$ is related to $|\Psi|$ by the Schrödinger equation, so the "stirring" actually depends on the density $|\Psi|^2$ of one of the "fluids", and the analogy with Gibbs stirring is not entirely accurate).

Relaxation Time: Let us define a typical relaxation time τ, given an initial P_o, Ψ_o, and δV, where the relaxation $\bar{P}\longrightarrow\overline{|\Psi|^2}$ is equivalent to $\bar{H}\longrightarrow 0$. One might at first try taking $1/\tau=-\dot{\bar{H}}_o/\bar{H}_o$. However, with the initial conditions (3.3), the initial rate of change $\dot{\bar{H}}_o$ actually vanishes. The reasons for this are that $\bar{H}$ is at a local maximum at t=0 and that the dynamics is continuous and differentiable. Thus the time-curve of $\bar{H}$ is smoothly peaked at t=0, and $\dot{\bar{H}}_o$ must vanish. This is in contrast with the classical H-theorem where, owing to the suddenness of molecular collisions, the time-curve of $\bar{H}_{Class}$ may be sharply peaked and $\dot{\bar{H}}_{Class}$ may be discontinuous, so that $\dot{\bar{H}}_{Class}$ need not vanish near t=0.(47)

Let us show explicitly that $\dot{\bar{H}}_o=0$. From (3.7) it follows that

$$d\bar{H}/dt=\int dX\,[\partial\bar{P}/\partial t\ln(\bar{P}/\overline{|\Psi|^2})-\partial\overline{|\Psi|^2}/\partial t\,(\bar{P}/\overline{|\Psi|^2})]$$

or, using the continuity equation,

$$d\bar{H}/dt=\int dX\,[-\overline{\nabla\cdot(\dot{X}P)}\ln(\bar{P}/\overline{|\Psi|^2}+\overline{\nabla\cdot(\dot{X}|\Psi|^2)}(\bar{P}/\overline{|\Psi|^2})]$$

The constancy of $\bar{P}$ and $\overline{|\Psi|^2}$ in each coarse-graining cell implies that

$$\int dX[-\overline{\nabla\cdot(\dot{X}P)}\ln(\bar{P}/\overline{|\Psi|^2})]=\int dX\,[-\nabla\cdot(\dot{X}P)\ln(\bar{P}/\overline{|\Psi|^2})]$$

and similarly for the second term. Integration by parts then yields

$$d\bar{H}/dt=\int dX\,|\Psi|^2(f/\tilde{f}-1)\dot{X}\cdot\nabla\tilde{f}\qquad(3.11)$$

(where $f=P/|\Psi|^2$ and $\tilde{f}=\bar{P}/\overline{|\Psi|^2}$ as before). Since $f_o=\tilde{f}_o$ we indeed have $\dot{\bar{H}}_o=0$, though of course generally $\dot{\bar{H}}\neq 0$ for $t>0$.

Thus in order to define a relaxation time in terms of the initial quantities we must go to second-order time-derivatives. Let us define

$$1/\tau^2=-\ddot{\bar{H}}_o/\bar{H}_o\qquad(3.12)$$

and express $\ddot{\bar{H}}_o$ in terms of P_o, Ψ_o and δV. It will be seen that $\ddot{\bar{H}}_o\leqslant 0$.

From (3.11) we have, remembering $f_o=\tilde{f}_o$,

$$\ddot{\bar{H}}_o=\int dX\,|\Psi_o|^2(\dot{X}_o\cdot\nabla f_o)\,\partial(f/\tilde{f})/\partial t|_{t=0}$$

while the continuity equation together with $\partial f/\partial t+\dot{X}\cdot\nabla f=0$ implies

$$\partial(f/\tilde{f})/\partial t|_{t=0} = -(1/f_o)\dot{X}_o\cdot\nabla f_o + (1/|\Psi_o|^2 f_o)\overline{\nabla\cdot(\dot{X}_o|\Psi_o|^2 f_o)} - (1/|\Psi_o|^2)\overline{\nabla\cdot(\dot{X}_o|\Psi_o|^2)}$$

Using the fact that f_o and $|\Psi_o|^2$ are essentially constant over each cell, one then finds

$$\ddot{\bar{H}}_o = -\int dX(|\Psi_o|^2/f_o)(\dot{X}_o\cdot\nabla f_o)[(\dot{X}_o\cdot\nabla f_o) - \overline{(\dot{X}_o\cdot\nabla f_o)}]$$

which may be written

$$\ddot{\bar{H}}_o = -\int dX(|\Psi_o|^2/f_o)[\overline{(\dot{X}_o\cdot\nabla f_o)^2} - \overline{(\dot{X}_o\cdot\nabla f_o)}^2] \qquad (3.13)$$

where

$$\overline{(\dot{X}_o\cdot\nabla f_o)^2} - \overline{(\dot{X}_o\cdot\nabla f_o)}^2 = \mathrm{Var}(\dot{X}_o\cdot\nabla f_o) \qquad (3.14)$$

is the variance of $\dot{X}_o\cdot\nabla f_o$ over the coarse-graining cell. Since $\overline{G^2} \geqslant \bar{G}^2$ for any function G, we have $\ddot{\bar{H}}_o \leqslant 0$ as it must be. And of course if the coarse-graining overbars are dropped, one confirms that the exact H remains constant in time.

The result (3.13) may then be used to define the relaxation time τ given by (3.12). This relaxation time is governed by the variance (3.14): If $\dot{X}_o$ and ∇f_o vary greatly over the cell, and if $\dot{X}_o$ is in the direction of ∇f_o, then τ will be small.

Note that while we assume f_o to be essentially constant over each cell, this does not of course imply that ∇f_o vanishes, but only that $|\nabla f_o\cdot dX| << |f_o|$ for a displacement dX within a cell, as will be true for sufficiently small δV. Indeed the initial conditions (3.3) are satisfied to any desired accuracy ε, provided δV is everywhere less than some δ, assuming P_o and $|\Psi_o|^2$ to be smooth functions. In this rigorous sense the above is consistent.

Is there any general relationship between τ and the timescale $\Delta t \sim \hbar/\Delta E$ associated with the Schrödinger evolution of Ψ? It seems not, firstly because Δt depends only on Ψ, and secondly because roughly speaking $1/\Delta t$ is proportional to the $|\Psi|^2$-weighted "mean" of $\dot{X}$ while $1/\tau^2$ is proportional to the variance of $\dot{X}$ over the coarse-graining cells. And numerical calculations for a simple model, to which we now turn, show that τ may be a small fraction of Δt.

Numerical Calculations with a Simple Model: Consider the simplest possible model, where X consists of just a single variable, the position of a particle in a one-dimensional box with wavefunction ψ. Considering an ensemble distribution P of such systems, each guided by the same ψ, we study the evolution of an initial $P_o \neq |\psi_o|^2$. This simple model is a totally unrealistic setting for the H-theorem, a theorem which should be applied to an ensemble of complicated many-variable systems. Nevertheless the model is instructive.

A box with infinite barriers at X=0,L has energy eigenfunctions

$$\phi_n(X) = (2/L)^{1/2}\sin(n\pi X/L) \quad , \qquad n=1,2,3,....$$

with eigenvalues $E_n = (1/2)(\pi n/L)^2$ [units $m=\hbar=1$]. Our initial $\psi_o(X)$ is taken to be a superposition of the first M eigenfunctions

$$\psi_o(X) = \sum_{n=1}^{M} M^{-1/2} e^{i\theta_n}\phi_n(X)$$

with coefficients of equal modulus (so each energy state is equally likely in equilibrium). The phases θ_n are randomly chosen at t=0 (and of course fixed thereafter). For $t>0$ one then has

$$\psi(X,t) = \sum_{n=1}^{M} M^{-1/2}\phi_n(X)e^{i(\theta_n - E_n t)}$$

with trajectory

$$\dot{X}(t) = (\mathrm{Re}\psi\,\partial\,\mathrm{Im}\psi/\partial X - \mathrm{Im}\psi\,\partial\,\mathrm{Re}\psi/\partial X)/|\psi|^2 \qquad (3.15)$$

We begin with an initially uniform ensemble $P_o=1/L$ and calculate P at later times as follows: With L=100 we begin with 99 points X_o at X=1,2,....,99 and numerically calculate the trajectory X_t of each from (3.15). Each point, during its motion, carries with it the initial value of $f_o=P_o/|\psi_o|^2$, so that $P(X_t,t)$ at the 99 later points is easily determined from

$$P(X_t,t) = |\psi(X_t,t)|^2 P(X_o,0)/|\psi(X_o,0)|^2 \qquad (3.16)$$

(The lattice of points is unfortunately distorted by this method).

Note that, since the trajectories never cross eachother (the velocity field being single-valued), in this one-dimensional model the ordering of points along the line never changes.

The velocity field $\dot{X}$ is generally found to vary with X much more rapidly than does $|\psi|^2$. In particular, $\dot{X}$ shows very sharp spikes at the turning points of $|\psi|^2$.

For all $M \gtrsim 1$, ψ is periodic in time, with period $T=2\pi/E_1 \approx 12700$ in our units. For M=2 I have calculated the evolution for a whole period and found, as one might expect, a subquantum recurrence. All trajectories recur to their initial points after time T, and therefore so does any initial distribution P_o. For P_o= 1/100 the evolution of P is not particularly interesting, developing a single sharp peak several times higher than the maximum value of $|\psi|^2$. After time T/2 this peak begins to disappear as P recurs to P_o. From (3.16), it is clear that such sharp peaks in P originate from small values of $|\psi(X_o,0)|^2$.

It is clear that, speaking generally, a periodic flow X(t) must yield a periodic $|\psi|^2$ and P, yet the converse is not necessarily true. For instance a

rigid translation of $|\psi|^2 = \cos^2 X$ yields a periodic $|\psi|^2$ and yet X(t) never recurs. Nevertheless we expect that in most cases, if $|\psi|^2$ is periodic then so are the trajectories X(t), and we have subquantum recurrence. Certainly this is the case for our box.

For a superposition of M=10 energy states, P approaches $|\psi|^2$ to a degree which is quite good considering the extreme simplicity of the model, though any approach to $|\psi|^2$ is eventually unravelled again for this periodic system.

Some results for M=10 are sketched in Figures 1-4. The key features of interest are: (i) P changes drastically by t=40 even though $|\psi|^2$ has changed only a little ($\Delta t = \hbar/\Delta E \sim 60$ for M=10). There is as yet little correlation between P and $|\psi|^2$ (Figure 2). We expect that P will generally, also in more realistic models, change greatly over timescales less than Δt. (ii) P becomes very irregular and sharply peaked while $|\psi|^2$ remains relatively smooth, despite both being "stirred" by the same velocity field $\dot{X}$. As already noted, this is explained by $\dot{X}$ being tied to ψ by the Schrödinger equation. (iii) From $t \sim 80$ onwards, many of the sharp peaks of P coincide with the smooth peaks of $|\psi|^2$ (Figure 3 for t=120). While the coincidence of P with $|\psi|^2$ is by no means good, already in this extremely simple model an experimenter with a "blunt" measuring device (say with resolution $\delta X \approx 10$) would conclude that at t=120, on this approximate coarse-grained level, P and $|\psi|^2$ are very roughly equal over most of the box (see Figure 3). (iv) Several sharp peaks of P may form within a single smooth peak of $|\psi|^2$, as occurs by t=400 (Figure 4). This resolves the apparent paradox raised near the end of Section 3.1 for a single extracted variable: It is possible for the wavefunction to remain smooth while the initially smooth distribution becomes (relatively) highly irregular, the latter yielding the former on coarse-graining.

The "relaxation" time τ at t=0, defined by (3.12), may be evaluated numerically, given a coarse-graining length δX, for the above P_o and ψ_o . For $\delta X = 2$ one finds for M=10 that $\tau \sim 2$, so that $\tau/\Delta t \sim 10^{-2}$. (For M=20, $\tau \sim 0.005$ while $\Delta t \sim 15$ and $\tau/\Delta t \sim 10^{-4}$). Evidently τ merely indicates the timescale over which P changes significantly, this being a small fraction of Δt. The time t_{equil} for equilibrium to be reached is, at least in this simple case (where equilibrium is "reached" only in a very approximate sense), actually considerably larger than τ and more comparable with Δt.

This simple one-dimensional model, where the ordering of the points cannot change, serves an illustrative purpose only. Multidimensional models, with interactions, will presumably show a more realistic approach to equilibrium, though this remains to be studied.

Random Phase Conjecture: The derivation of the master equation of quantum statistical mechanics from the Schrödinger equation assumes that the phases of

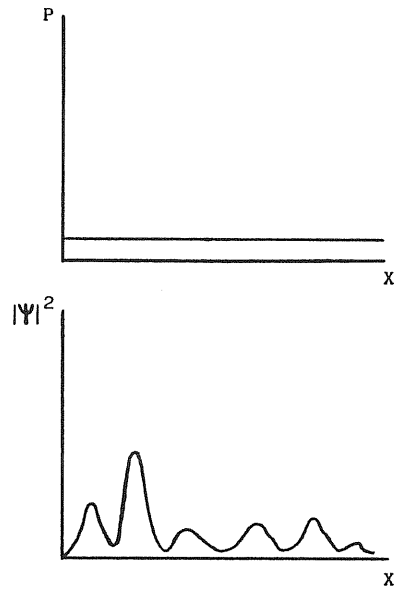

Fig.1: Initial conditions at t=0

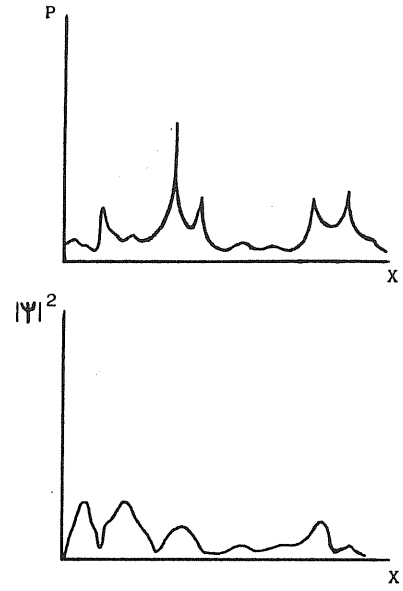

Fig.2: At t=40, P has drastically altered, despite the small change in $|\psi|^2$. As yet there is little coincidence between P and $|\psi|^2$.

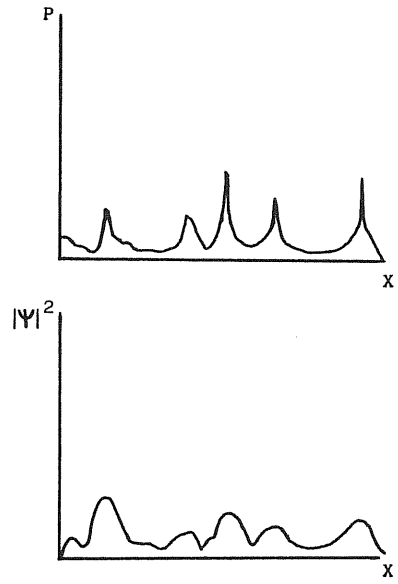

Fig.3: At t=120, strong coincidence between sharp peaks of P and smooth peaks of $|\psi|^2$.

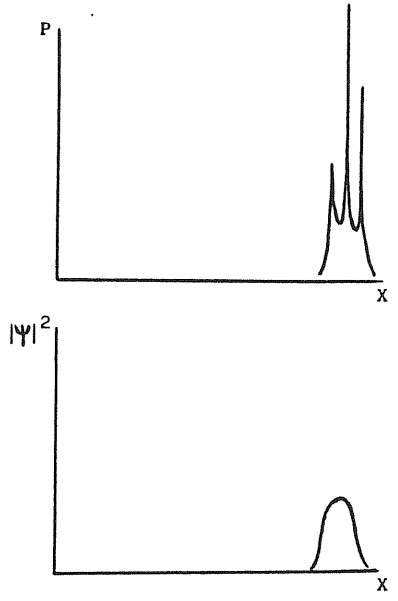

Fig.4: At t=400, in the region around X=84 sketched here, three sharp peaks coincide with a single peak of $|\psi|^2$.

the wavefunction (in the energy basis) at t=0 are chosen randomly. While quantum and subquantum statistical mechanics are of course very different, the following conjecture seems reasonable: That there exists a large class of many-variable systems, with interactions between the variables, for which if the initial wavefunction has randomly chosen phases in the energy basis, then the motion X(t) will be sufficiently irregular for coarse-grained equilibrium to be reached for an ensemble.

On Subquantum Recurrence: The classical Poincaré recurrence time for an isolated system is very sensitive to the initial starting point. Thus for an ensemble, at any time the probability distribution will be scrambled with respect to the initial distribution, and only a few points will have recurred. In the subquantum case there is a much stronger form of Poincaré recurrence for finite systems. For the discreteness of the energy spectrum implies that the wavefunction is quasiperiodic, i.e. returns arbitrarily closely to its initial value. Presumably this will usually imply that most of the trajectories will (quasi)recur as well. Almost all elements of the ensemble will then recur simultaneously, thereby unscrambling the probability distribution. A similar phenomenon occurs of course in quantum statistical mechanics, where ensemble averages for finite systems are quasiperiodic[51]. There are two ways of facing this situation, in both the quantum and subquantum cases: (i) Assume the recurrence time to be much larger than the equilibrium time, and that our observations take place somewhere inbetween. (ii) Work with infinite systems. The nondiscreteness of the energy spectrum then eliminates the quasiperiodicity.

4. FIELD THEORY OF "PARTICLES" AND SPIN

The pilot-wave theory of fields has so far been developed, by Bohm[6] and by Bohm, Hiley and Kaloyerou[24], for massless fields only, in particular for the electromagnetic and scalar cases. Bohm's 1952 treatment of the electromagnetic field uses the usual four-component $A^\mu = (\phi, A)$ written in the Coulomb gauge. In Section 5 we shall develop a more natural pilot-wave theory of electrodynamics based on an "absolute 3+1" version of the classical theory, without A^o. And in Section 6 we give an analogous pilot-wave theory of gravitation. In the present Section we are concerned with the case of massive fields, first scalar and then spin-1/2. In particular we wish to show how massive localised "particles" (with or without spin) may be naturally described with pilot-wave field theory. This is in contrast with the theory of electrons and positrons given by Bohm[52], based on particle trajectories, where positrons are treated as "holes" in the negative energy sea à la Dirac. Not only is the latter theory very inelegant, it also creates an unsatisfactory dualism: Particle description for the massive case versus field description for the massless case. Here we shall see that the field description may be applied to all cases.

A realistic field description of particles might at first sight seem untenable, for two reasons: (i) One may ask how a field distributed over all space can account for the highly localised massive particles seen in the laboratory. However, exactly the same query may be put to standard quantum field theory: For say the scalar case, the basic "observable" is surely the field operator $\hat{\phi}(x,t)$, whose eigenvalues are the set of definite field configurations $\phi(x)$, associated with eigenstates $|\phi(x)\rangle$. How do "particles" localised in space emerge? We show below how this happens, in terms of localised "field lumps", thereby clarifying both the pilot-wave and the standard quantum field theories. (ii) Fermion fields anticommute, a fact which according to Bohm et al.[24] makes it necessary to abandon field theory and use particle trajectories for the fermion case. However a pilot-wave field theory of fermions may be straightforwardly based on fields of anticommuting Grassmann numbers, these being an extension of the complex numbers, as given below.

It has been suggested that it is "premature"[24] to try to develop a coherent pilot-wave interpretation of relativistic quantum mechanics. We hope to make it clear in this and the subsequent two Sections that the pilot-wave theory of fields is not only as generally applicable as is standard canonical field theory, but is actually superior to it when it comes to quantising the electromagnetic and gravitational fields.

4.1 Field theory of massive "particles"

Let us consider the (real) massive scalar case. Restricting ourselves to equilibrium probability distributions, we may use the formalism of standard quantum field theory. (We work in the Schrödinger picture). The issue is this: Given the quantum field operator $\hat{\phi}(x)$, how does one account in terms of field configurations for the observed well-localised objects known as "particles"?

In the basis $|\phi(x)\rangle$ of definite field configurations, the state vector $|\Psi\rangle$ is just the wavefunctional $\Psi[\phi,t]$. To make contact with "localised particles", say for simplicity a single slowly-moving "particle", we need to construct a basis $|x\rangle$ representing approximately-definite particle positions. Given $|x\rangle$, if the state vector of our field system has the form

$$|\Psi\rangle = \int d^3x\, |x\rangle \psi(x) \tag{4.1}$$

then this will correspond to the nonrelativistic particle system with wavefunction $\psi(x)$. But as we shall see: (i) The $|x\rangle$ may be defined entirely in terms of fields, (ii) The strictly localised $|x\rangle$ are (as is well known) only a low energy approximation, (iii) The physics may be viewed entirely in terms of field configurations.

For $|x\rangle$ to exist we must have an associated "position operator" $\hat{X}$ defined in terms of the field operator $\hat{\phi}(x)$. The latter may always be expanded in terms of annihilation and creation operators as

$$\hat{\phi}(x) = \sum_p (2VE)^{-1/2}(\hat{a}_p e^{ip\cdot x} + \hat{a}_p^\dagger e^{-ip\cdot x})$$

where V is a normalisation volume and $E=(m^2+p^2)^{1/2}$. (Note: We do not use boldface type for 3-vectors. Here $p\cdot x = p^1x^1+p^2x^2+p^3x^3$). Restricting ourselves to low energy states we may take $E\approx m$, and define the low energy positive frequency field operator

$$\hat{\phi}_+(x) = \sum_p (2Vm)^{-1/2}\hat{a}_p e^{ip\cdot x} \tag{4.2}$$

and the operator

$$\hat{X} = 2m\int d^3x\, x\, \hat{\phi}_+^\dagger(x)\hat{\phi}_+(x) \tag{4.3}$$

The usual states $|p\rangle = a_p^\dagger|0\rangle$ may be used to define $|x\rangle$ by

$$|x\rangle = (2\pi)^{-3/2}\int d^3p\, e^{-ip\cdot x}|p\rangle \tag{4.4}$$

Writing $\hat{X}$ in terms of $\hat{a}_p$, $\hat{a}_p^\dagger$, and $|x\rangle$ in terms of $|p\rangle$, one easily shows that

$$\hat{X}|x\rangle = x|x\rangle \tag{4.5}$$

establishing $\hat{X}$ as a nonrelativistic position operator. Since

$$\hat{\phi}_+^\dagger(x)|0\rangle = (V/16\pi^3m)^{1/2}|x\rangle \tag{4.6}$$

the $\hat{\phi}_+^\dagger$ "creates a particle" at $|x\rangle$. (There is of course a considerable literature on position operators in relativistic quantum mechanics[53]. The simple approach just given suffices for what follows).

Given the basis $|x\rangle$, one may now contemplate a field state of the form (4.1), with wavefunctional

$$\Psi[\phi] = \int d^3x\, \langle\phi|x\rangle\psi(x) \tag{4.7}$$

where $\langle\phi|x\rangle$ is the amplitude for field configuration $\phi(x')$ in the state $|x\rangle$. This field state must somehow represent a nonrelativistic "particle" whose wavepacket $\psi(x)$ could be, for instance, a localised Gaussian. The question is: What does the probability distribution $|\Psi[\phi]|^2$ of field configurations look like, and how does one comprehend in terms of these field configurations the apparently well-localised "particle"? The answer, as we now show, is simply that the most likely field configuration is that corresponding to a "lump" of the form $\sim\psi(x)+\psi^*(x)$.

The wavefunctional for the vacuum is

$$\langle\phi|0\rangle = \Psi_0[\phi] \propto \exp[-(1/16\pi^3)\int d^3x\int d^3x'\,\phi(x)\phi(x')I(x-x')] \tag{4.8}$$

where

$$I(x-x') = \int d^3p(m^2+p^2)^{1/2}e^{ip\cdot(x-x')}$$

(this is just the generalisation of the well known result[54] for the massless case). The quantity I(x-x') is proportional to the second time-derivative of the Feynman propagator at equal times, and it is well-known[55] that the latter dies off exponentially $\sim e^{-m|x-x'|}$ for spacelike separations. This means that, in the vacuum, field values at x and x' are statistically uncorrelated for $|x-x'| \gtrsim 1/m$ (the Compton wavelength). And the most probable configuration in the vacuum is of course just $\phi=0$ everywhere.

We would now like an explicit expression, in terms of fields only, for the wavefunctional (4.7) for our single-particle state. From (4.6), and the fact that $\hat{a}_p|0\rangle = 0$, one may write

$$|x\rangle = (16\pi^3m/V)^{1/2}\hat{\phi}(x)|0\rangle$$

so that

$$\langle \phi | x \rangle \propto \phi(x) \Psi_o[\phi]$$

and so

$$\Psi[\phi] \propto \Psi_o[\phi] \int d^3x\, \phi(x) \psi(x) \qquad (4.9)$$

We thus have a probability distribution

$$P[\phi] \propto |\Psi_o[\phi]|^2 \left| \int d^3x\, \phi(x) \psi(x) \right|^2 \qquad (4.10)$$

(Analogous results have been obtained by Bohm et al.[24], for "wavepackets" in the massless case, approximating classical electromagnetic waves). Superimposed on the background vacuum fluctuations $|\Psi_o|^2$ we now have an additional statistical factor related to $\psi(x)$.

What will be the most probable field configuration? This is obtained by putting $\delta P[\phi]/\delta\phi = 0$, though the calculation is easier in momentum space. Retaining only momentum components such that $E \approx m$, we have

$$P[\tilde{\phi}(p)] \propto \exp[-(m/2) \int d^3p\, \tilde{\phi}(p) \tilde{\phi}(-p)] \left| \int d^3p\, \tilde{\phi}(p) \tilde{\psi}(-p) \right|^2$$

where

$$\tilde{\phi}(p) = (2\pi)^{-3/2} \int d^3x\, e^{-ip\cdot x} \phi(x)$$

and similarly for $\tilde{\psi}(p)$. The condition

$$\delta P[\tilde{\phi}(p)]/\delta\tilde{\phi}(p) = 0$$

then implies (remembering that $\tilde{\phi}^*(p) = \tilde{\phi}(-p)$)

$$\tilde{\phi}(p) = (2m|\alpha|^2)^{-1} (\alpha \tilde{\psi}^*(-p) + \alpha^* \tilde{\psi}(p))$$

for all p where

$$\alpha = \int d^3p\, \tilde{\phi}(p) \tilde{\psi}(-p) = \int d^3x\, \phi(x) \psi(x) \qquad (4.11)$$

The most probable field configuration is then

$$\phi(x) = \psi(x)/2m\alpha + \psi^*(x)/2m\alpha^* \qquad (4.12)$$

which is simply a linear combination of ψ and ψ^*. If ψ happens to be real, then so is α, and we have $\phi = \psi/m\alpha$. Multiplying by ψ and integrating over x then implies that $\alpha = 1/m\alpha$ or $\alpha = m^{-1/2}$. The most likely field configuration is then just

$$\phi(x) = \psi(x)/m^{1/2} \qquad (4.13)$$

It also follows from (4.10) that the ratio

$$P[\phi(x-a)]/P[\phi(x)] = \left| \int d^3x\, \phi(x-a) \psi(x) \right|^2 / \left| \int d^3x\, \phi(x) \psi(x) \right|^2$$

so the probability is small that ϕ deviates significantly from ψ.

Thus the field ϕ tends to "imitate" what we call the nonrelativistic wavefunction ψ. For the ground state wavefunction $\psi_o(x)$ of Hydrogen, with a spinless "electron", the most likely configuration of the electron field is $\phi = \psi_o/m^{1/2}$, so the "particle" is actually spread over a Bohr radius. And in general the most likely field is (being just a linear combination of ψ and ψ^*) spread over space like a wavepacket. If ψ is a well localised Gaussian of width Δx, corresponding to an approximately classical "particle", then the field will simply be a "lump" of size Δx.

We also note that at time t when the wavefunctional is

$$\Psi[\phi, t] \propto e^{-iE_o t} \Psi_o[\phi] \int d^3x\, \phi(x) \psi(x,t)$$

(where E_o is the vacuum energy) the field velocity $\dot{\phi} = \delta S/\delta\phi = \mathrm{Im}\, \delta \ln \Psi/\delta\phi$ is

$$\partial\phi(x,t)/\partial t = \mathrm{Im}[\psi(x,t)/\int d^3x\, \phi(x) \psi(x,t)]\, |_{\phi(x)=\phi(x,t)}$$

The field velocity at any point within the packet ψ therefore depends on the field throughout the packet. Our "particle" is then an entangled lump of field, moving on a chaotic vacuum background. (If ψ is real, then ϕ is static).

According to field theory, then, a "particle" is not a pointlike object lying somewhere within the nonrelativistic wavepacket ψ. It is rather a "field lump" of the form $\sim \psi + \psi^*$, where the actual field configuration lies, of course, somewhere within the wavefunctional $\Psi[\phi, t]$. The nonrelativistic pilot-wave theory of "particles", with trajectory $m\dot{X} = \partial S/\partial X$, is then incorrect. There are no such pointlike particles and no trajectories. There are only field variables with an evolution of the form $\dot{\phi} = \delta S/\delta\phi$, and the apparent "particles" are merely phenomenological field structures. Strictly speaking then, the H-theorem and other results of this work must be applied to field variables only. Nevertheless, as one often does in standard quantum theory, one may for simplicity use the nonrelativistic particle model for illustrative purposes.

In this view what appears classically as the motion of a permanent body through space is in actuality the propagation of a field disturbance: The disappearance of a field disturbance in one region, accompanied by the appearance of a similar disturbance in a nearby region, leads to the apparent "motion of matter".

One may of course consider "two-particle" states, built from $|p_1 p_2\rangle = a^\dagger_{p_1} a^\dagger_{p_2} |0\rangle$, with symmetric two-particle wavefunctions, and the likely field configuration will now consist of a pair of "lumps" representing the pair of "particles".

Finally, one may ask what happens during a "position measurement", from the field viewpoint. Consider first a nonrelativistic wavefunction consisting of a superposition of two distinct peaks, $\psi = \psi_1 + \psi_2$, localised near x_1 and x_2. The wavefunctional will still be just (4.7), and the most likely field is again $\sim \psi + \psi^*$, now yielding two lumps near x_1 and x_2. If one now performs a measurement of position, with an apparatus variable y, the total wavefunctional evolves into

$$\Psi[\phi,y] \propto \Psi_0[\phi][\chi_1(y)\int d^3x\,\phi(x)\psi_1(x) + \chi_2(y)\int d^3x\,\phi(x)\psi_2(x)]$$

where the apparatus wavefunctions χ_1 and χ_2 do not overlap. If y occupies say χ_1, then the second branch vanishes, and the most likely field is then just $\sim \psi_1 + \psi_1^*$, i.e. localised near x_1. The "measurement process" has thus constrained the field ϕ to evolve towards a narrow region peaked near x_1, the "measured position". But this "outcome" x_1 has no simple relation to the actual initial field configuration (which was distributed near both x_1 and x_2), and depends as much on the initial apparatus variables as it does on the initial ϕ. In other words the "particle position" is actually a contextual variable, and only the field variables themselves are noncontextual.

Thus in the two-slit experiment with a single "particle", the field is actually spread over both wavepackets emerging from the two slits. It is only upon "measurement" at the backstop that the field concentrates into a small region. This picture of particles somewhat resembles that of other interpretations of quantum theory, where the particle is in a sense spread over space until a measurement takes place. Though of course our fundamental variables - the fields, have unique and definite values at all times, in sharp distinction from other interpretations.

4.2 Grassmann field theory of spin

In the path-integral formulation of standard quantum field theory, operators do not appear, and one is obliged to make use of anticommuting c-numbers or Grassmann fields for the fermion case[56]. The situation is similar in pilot-wave field theory. We may represent fermions by an objective field of Grassmann numbers evolving in time, guided by a wavefunctional Ψ.

We shall concentrate on the case of massive spin-1/2. This is usually described by a four-component Dirac field ψ_α with Lagrangian density

$$\mathcal{L} = (i/2)[\bar\psi\gamma^\mu(\partial_\mu\psi) - (\partial_\mu\bar\psi)\gamma^\mu\psi] - m\bar\psi\psi$$

However, this is not appropriate for the pilot-wave theory, for the following reason: The momentum π canonically conjugate to ψ is $\pi = (i/2)\psi^\dagger$ so that the actual evolution of the "hidden variable" field will be

$$(i/2)\psi^\dagger(x,t) = \delta S/\delta\psi \qquad (4.14)$$

where S is the phase of the wavefunctional $\Psi = \Psi[\psi,\psi^\dagger,t]$. Thus, rather than telling us the rate of change of any given actual field, the functional derivative $\delta S/\delta\psi$ apparently tells us the actual field configuration. But (4.14) will imply an untenable restriction on the possible initial field. As a simple analogy, if the nonrelativistic particle trajectory were given by $X \propto \partial S/\partial X$, instead of $\dot X \propto \partial S/\partial X$, then for S=kX the only possible particle position would be $X \propto k$.

The origin of this difficulty is,of course, that the Dirac field obeys a wave-equation which is first order in time, in contrast with all other physical fields. To proceed, we must therefore use the Van der Waerden field, which is equivalent to the Dirac field, but whose wave-equation is second-order in time.

The Van der Waerden field[57] ϕ is a two-component complex field satisfying the wave-equation

$$\partial^2\phi/\partial t^2 - (\sigma\cdot\nabla)^2\phi + m^2\phi = 0 \qquad (4.15)$$

where σ^i are Pauli spin matrices. (All this discussion of wave-equations of course refers to the "prequantisation" level or, equivalently, to the operator level of quantum theory in the Heisenberg picture). If one defines[57]

$$\phi^L = \phi$$
$$\phi^R = (1/m)(-i\partial/\partial t + i\sigma\cdot\nabla)\phi$$

then

$$\psi = \begin{pmatrix} \phi^R - \phi^L \\ \phi^R + \phi^L \end{pmatrix}$$

obeys the Dirac equation

$$i\gamma^\mu\partial_\mu\psi - m\psi = 0$$

As shown by Feynman and Brown[57], the complex two-component ϕ is just as suitable as the Dirac field for the description of electrons and positrons, despite the common belief that four-component fields are necessary. For since a sin-

gle-component Klein-Gordon field accounts for both particle and antiparticle states, its two-component generalisation accounts for spin as well. As is well known, the historical reasons which led Dirac to devise a first-order wave-equation were erroneous. Further, from a general viewpoint, it is unsatisfactory to have one physical field obeying a first-order equation, in contrast with all the other known physical fields. And from the pilot-wave viewpoint, this first-order field is in any case actually untenable. We therefore propose an abandonment of the four-component Dirac field, viewing it as a historical curiosity, and shall work instead with two-component fields. As Feynman put it: "I was tempted to teach quantum electrodynamics with a two-component wavefunction [i.e. field]. The only difficulty is that you could not read any of the literature"[58]. But in the pilot-wave theory the two-component field is forced upon us.

The complex Grassmann fields ϕ_α , ϕ^*_α (α =1,2) obey anticommutation relations

$$\{\phi_\alpha(x), \phi_\beta(y)\} = \{\phi^*_\alpha(x), \phi^*_\beta(y)\} = \{\phi^*_\alpha(x), \phi_\beta(y)\} = 0 \qquad (4.16)$$

where $\{a,b\} = ab + ba$. We briefly review their mathematical theory (see Berezin[59] for details). By forming polynomials, these fields are the generators of an infinite dimensional Grassmann algebra. This algebra is a direct sum of two subspaces, "even" and "odd", consisting respectively of linear combinations of monomial elements of even and odd degree. An "even" element such as $\phi_\alpha(x)\,\phi_\beta(y)$ commutes with all other elements, and behaves like an ordinary complex number. Denoting a general element by f, the involutive mapping $f \longleftrightarrow f^*$ satisfies $(f_1 f_2)^* = f_2^* f_1^*$. Left and right differentiation are respectively defined by

$$\frac{\vec{\delta}(\eta(x)\eta(y))}{\delta\eta(z)} = \delta^3(x-z)\eta(y) - \delta^3(y-z)\eta(x)$$

$$(\eta(x)\eta(y))\frac{\overleftarrow{\delta}}{\delta\eta(z)} = \delta^3(y-z)\eta(x) - \delta^3(x-z)\eta(y)$$

where η represents any generator ϕ or ϕ^* . The following properties are used below.

$$\frac{\vec{\delta}}{\delta\phi^*}\left(f\frac{\overleftarrow{\delta}}{\delta\phi}\right) = \left(\frac{\vec{\delta} f}{\delta\phi^*}\right)\frac{\overleftarrow{\delta}}{\delta\phi}$$

$$\left(\frac{\vec{\delta} f}{\delta\phi^*}\right)^* = \frac{f^*\overleftarrow{\delta}}{\delta\phi}$$

which imply

$$\left[\frac{\vec{\delta}}{\delta\phi^*}\left(f\frac{\overleftarrow{\delta}}{\delta\phi}\right)\right]^* = \frac{\vec{\delta}}{\delta\phi^*}\left(f^*\frac{\overleftarrow{\delta}}{\delta\phi}\right)$$

If f_2 is an arbitrary element then

$$\frac{\vec{\delta}}{\delta\phi^*}(f_1 f_2) = \frac{\vec{\delta} f_1}{\delta\phi^*} f_2 \pm f_1 \frac{\vec{\delta} f_2}{\delta\phi^*}$$

$$(f_2 f_1)\frac{\overleftarrow{\delta}}{\delta\phi} = f_2\left(\frac{f_1\overleftarrow{\delta}}{\delta\phi}\right) \pm \left(\frac{f_2\overleftarrow{\delta}}{\delta\phi}\right) f_1$$

with + or - according as f_1 is even or odd respectively.

For a time-dependent Grassmann field, the time-derivatives $\dot{\phi}_\alpha$, $\dot{\phi}^*_\alpha$ are also included among the anticommuting generators.

For the pilot-wave theory of electrons and positrons, we take the Lagrangian density for the Van der Waerden field

$$\mathcal{L} = \dot{\phi}^*_\alpha \dot{\phi}_\alpha - (\sigma\cdot\nabla\phi)^*_\alpha(\sigma\cdot\nabla\phi)_\alpha - m^2\phi^*_\alpha\phi_\alpha$$

(where α is summed over) with canonical momenta

$$\left.\begin{aligned} \Pi^\alpha &= \frac{L\overleftarrow{\delta}}{\delta\dot{\phi}_\alpha} = \dot{\phi}^*_\alpha \\ \Pi^{*\alpha} &= \frac{\vec{\delta} L}{\delta\dot{\phi}^*_\alpha} = \dot{\phi}_\alpha \end{aligned}\right\} \qquad (4.17)$$

Our Hamiltonian density is then

$$\mathcal{H} = -\Pi^{*\alpha}\Pi^\alpha + (\sigma\cdot\nabla\phi)^*_\alpha(\sigma\cdot\nabla\phi)_\alpha + m^2\phi^*_\alpha\phi_\alpha \qquad (4.18)$$

Introducing the operators

$$\hat{\Pi}^\alpha = i\frac{\overleftarrow{\delta}}{\delta\phi_\alpha} \quad , \quad \hat{\Pi}^{*\alpha} = -i\frac{\vec{\delta}}{\delta\phi^*_\alpha}$$

we have the anticommutation relations

$$\left\{\phi_\alpha(x),\ i\frac{\overleftarrow{\delta}}{\delta\phi_\beta(x')}\right\} = i\delta^\beta_\alpha\delta^3(x-x')$$

acting on Ψ from the right, and

$$\left\{\phi^*_\alpha(x),\ -i\frac{\vec{\delta}}{\delta\phi^*_\beta(x')}\right\} = -i\delta^\beta_\alpha\delta^3(x-x')$$

acting on Ψ from the left.

We may then write a Schrödinger equation for $\Psi = \Psi[\phi_\alpha, \phi^*_\alpha, t]$

$$i\frac{\partial\Psi}{\partial t} = \int d^3x \left[-\frac{\vec{\delta}}{\delta\phi^*_\alpha}\left(\Psi\frac{\overleftarrow{\delta}}{\delta\phi_\alpha}\right) + (\sigma\cdot\nabla\phi)^*_\alpha(\sigma\cdot\nabla\phi)_\alpha\Psi + m^2\phi^*_\alpha\phi_\alpha\Psi \right] \quad (4.19)$$

where Ψ is of course an even element of the Grassmann algebra, as are $|\Psi|$ and the phase S.

Using the above properties of differentiation, (4.19) implies the continuity equation

$$\frac{\partial|\Psi|^2}{\partial t} = -\int d^3x \left[\frac{\vec{\delta}}{\delta\phi^*_\alpha}\left(|\Psi|^2\frac{S\overleftarrow{\delta}}{\delta\phi_\alpha}\right) + \left(|\Psi|^2\frac{\vec{\delta}S}{\delta\phi^*_\alpha}\right)\frac{\overleftarrow{\delta}}{\delta\phi_\alpha} \right]$$

We may then identify the evolution of the actual "hidden variable" fields

$$\dot{\phi}^*_\alpha = \frac{S\overleftarrow{\delta}}{\delta\phi_\alpha}$$
$$\dot{\phi}_\alpha = \frac{\vec{\delta}S}{\delta\phi^*_\alpha} \quad (4.20)$$

which are as one expects from (4.17).

The equations (4.19) and (4.20) define a pilot-wave theory of the free massive spin-1/2 field. As we have shown in the scalar case, localised "particles" emerge as phenomenological field "lumps" which imitate the nonrelativistic wave-function. Here such "lumps" may be thought of in terms of "even" quantities such as $\phi^*_\alpha\phi_\alpha$.

5. THEORY OF LORENTZ INVARIANCE AND ELECTRODYNAMICS

5.1.1 "Active" view of Lorentz invariance: an equilibrium symmetry

It has been suggested by Bell[1],[15], Popper[60], and Bohm et al.[23],[24], that in flat spacetime there exists a preferred frame singled out by quantum non-locality. While this frame is well defined at the hidden-variable level, it happens to be undetectable in practice at the statistical level of standard quantum theory, and hence in the classical approximation. Adopting this view, our aim here is to show how, in the context of the pilot-wave theory, this view may be made quite natural and plausible.

Let us first consider the purely classical level, where the preferred frame has absolute time t and coordinates x on absolute space. Lorentz invariance may be viewed in terms of an "active" symmetry (rather than in terms of passive coordinate transformations), a view which is essentially based on Lorentz's original (pre-Einsteinian) principle of "corresponding states"[61]. We shall first illustrate this view in terms of a free scalar field $\phi(x,t)$.

If the function $\phi(x,t)$ satisfies the wave-equation

$$\eta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu}\phi(x,t) = 0$$

then $\phi(x,t)$ is a physically allowed field configuration (in space and time). Given a definite ϕ, one may contemplate <u>active</u> transformations

$$\phi(x,t) \longrightarrow \phi'(x,t)$$

to new physically allowed configurations (the coordinate system (t,x) is fixed throughout). The general class of transformations $\phi \longrightarrow \phi'$ is vast, and ranges over all physically allowed configurations. However there exists a special class, the "field deformations", defined as follows: The new field configuration $\phi'(x,t)$ is constructed by taking the old field value ϕ at x^μ and moving it to the new point $x^{\mu'} = f(x^\mu)$, so that

$$\phi'(x^{\mu'}) = \phi(x^\mu)$$

This amounts to a redistribution of field values in absolute space and time, according to the active mapping $x^\mu \longrightarrow x^{\mu'}$. If we require the resulting mathematical configuration $\phi'(x,t)$ to be physically allowed, i.e. that

$$\eta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu}\phi'(x,t) = 0$$

then what are the possible f ? (Note that ϕ' is required to obey the "same law" as ϕ, i.e. the same field equation). The answer is simply that f must leave in-

variant the wave-operator:

$$\eta^{\mu\nu}\frac{\partial}{\partial x^{\mu'}}\frac{\partial}{\partial x^{\nu'}} = \eta^{\mu\nu}\frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial x^{\nu}}$$

Thus the allowed f correspond to translations, rotations, and Lorentz boosts of ϕ , these being the deformations which lead to new allowed configurations.

For a vector field w^{μ} , the field deformation

$$w^{\mu'}(x^{\lambda'}) = \frac{\partial x^{\mu'}}{\partial x^{\nu}} w^{\nu}(x^{\lambda})$$

consists of a mapping from x^{λ} to $x^{\lambda'}$ accompanied of course by a "rotation" of the 4-vector w^{ν} by the matrix $\partial x^{\mu'}/\partial x^{\nu}$, leading again to a new allowed configuration (if w^{ν} satisfies the wave-equation).

Generalising to general tensors, one may view the broad validity of Poincaré symmetry as arising from the ubiquitous presence in Nature of the wave-operator $\partial^2/\partial t^2 - \nabla^2$, whose symmetries are represented by the Poincaré group, these symmetries being maintained by the known interaction terms (excluding gravity - see below).

Now a "system" together with "apparatus" and "experimenter" may be regarded as a single super-system, represented for instance (for simplicity) by a scalar field configuration $\phi(x,t)$. A deformation $\phi \longrightarrow \phi'$, where $x^{\mu} \longrightarrow x^{\mu'}$ is a Lorentz transformation, then corresponds to imparting a velocity to the whole super-system, yielding a new "moving" configuration satisfying the same laws in the absolute frame (t,x). In this new physical situation, the "deformed" experimenter may if he wishes use coordinates $x^{\mu'}$, with the same equations , and his experience in terms of $x^{\mu'}$ is then identical to that of the original experimenter in terms of x^{μ} , so that he is unable to detect his own absolute motion. And of course the deformed experimenter will in practice naturally use the coordinates $x^{\mu'}$ (and of course the "rotated" $w^{\mu'}$ if one includes vector fields), and thereby "sees the same physical laws in the moving frame".

This situation was summarised by Lorentz as follows, in the context of electromagnetic theory: "If, in the system without translation, there is a state of motion in which, at a definite place, the components of P,D, and H are certain functions of the time, then the same system after it has been put in motion (and thereby deformed) can be the seat of a state of motion in which, at the corresponding place, the components of P',D', and H' are the same functions of the local time".(61) [The transformed t' is Lorentz's "local time"]. That this Lorentz interpretation, with a true rest-frame, is fully equivalent to Einstein's interpretation in terms of "special relativity", has been stressed in particular by Bell(1),(15) and by Popper(60).

However, at the purely classical level, the Lorentz view does seem artificial, since the true frame is not singled out by any physical process. Any inertial experimenter may consistently regard himself as being truly at rest, suggesting Einstein's view that there is no true rest. On the other hand, if one goes beyond the classical level, this unsatisfactory feature of the Lorentz view disappears. And indeed the Lorentz view becomes the most natural one, for the nonlocality of the subquantum level may define an absolute simultaneity, singling out the true rest-frame and the true time t (which defines a fundamental causal sequence). This happens by simply assuming that quantum nonlocality acts instantaneously across the true 3-space.

But if this is so, and there exists a physically preferred frame, then why does Lorentz invariance hold in standard quantum theory and in the classical limit? The answer according to the pilot-wave theory (at least as interpreted here) is that the statistical equilibrium $P=|\Psi|^2$ has symmetries, such as translational and Lorentz invariance, which the underlying subquantum physics does not possess, as one would expect for a statistical equilibrium. For instance, for a general distribution of gas molecules in a box, the velocity distribution is not rotationally invariant, and yet this is the case for the Maxwell equilibrium distribution. Similarly, in equilibrium $P=|\Psi|^2$ the actual ("hidden") variables of the ensemble may be altered in certain ways, without affecting the ensemble statistics.

As a simple example of such equilibrium symmetry, consider translational invariance for the vacuum. In the vacuum, $\Psi = \Psi_o e^{-iE_o t}$ where Ψ_o , given by (4.8), is real, so that any initial field configuration will remain static: $\phi(x,t)=\phi(x,0)=\phi(x)$. Now an arbitrary ensemble of such vacuum field configurations, with distribution $P[\phi(x)]$, is generally not translationally invariant: If each field configuration is deformed by $\phi(x) \longrightarrow \phi'(x)=\phi(x-a)$, the new ensemble will have distribution

$$P'[\phi'(x)] = P[\phi'(x+a)] \neq P[\phi'(x)]$$

and measurements performed on this new ensemble will yield different statistical results. If however $P[\phi]$ is such that $P[\phi(x+a)]=P[\phi(x)]$, then the new distribution will be statistically indistinguishable from the old, $P'[\phi'(x)]=P[\phi'(x)]$. In particular, this happens in equilibrium $P=|\Psi_o|^2$, since

$$|\Psi_o[\phi(x+a)]|^2 = |\Psi_o[\phi(x)]|^2$$

In equilibrium it is of course only the ensemble statistics which are translationally invariant; each individual field system is not.

Thus an individual vacuum system is not invariant under "active" translation of the field. While for an ensemble in statistical equilibrium, a translation of

the individual fields will be undetectable at the statistical level, the change being masked, roughly speaking, by uncertainty-principle "noise".

This example illustrates a general principle: Equilibrium statistics show symmetries which are not shared by the underlying theory. It is possible, in equilibrium, to actively change the values of the hidden-variables in ways which leave the statistics unaltered, the change being detectable only at the sub-uncertainty level. This possibility is, in our view, the root of the Poincaré symmetry group.

Lorentz invariance is also clearly broken by an individual vacuum system[24]. And generally, disequilibrium $P\neq|\Psi|^2$ will break both translational and Lorentz invariance at the statistical level. As we have shown in Section 2.2.3, disequilibrium leads to instantaneous signalling at the statistical level. One is then able, out of equilibrium, to detect the true rest-frame, this being simply the one where such signals occur purely across space.

While it is quite natural for equilibrium to show extra symmetries, one may ask just why the Poincaré group in particular should arise. The immediate reason, at least for free fields, is of course the appearance of the wave-operator $\partial^2/\partial t^2-\nabla^2$ at the equilibrium operator level (and therefore at the classical level); even the anticommuting spin-1/2 field obeys the wave-equation at the operator level. And the wave-equation arises because,at the fundamental level, the Hamiltonian is always of the quadratic form $-\delta^2/\delta\phi^2+(\nabla\phi)^2$, whether the field be scalar, spinor (see (4.18)), or vector (see below). But what about interacting fields? The known interaction terms all arise from gauge symmetries, and the total Hamiltonian is again of quadratic form: For example the term $|\nabla\phi|^2$ in the Hamiltonian for the charged scalar field is replaced, on introducing the electromagnetic interaction, by the term $|D\phi|^2$ where $D\phi$ is the gauge-covariant derivative (this quadratic term containing the "interaction" terms). This suggests that the Poincaré group is somehow related to gauge-symmetry. Of course the few fields found in Nature may in any case all be aspects of a single Poincaré-invariant field, explaining at a stroke the ubiquity of Poincaré invariance.

There is however one field in Nature which actually breaks Poincaré invariance even at the classical level, namely gravity: When space is curved, if an experimenter and his (finite) laboratory is for instance translated, the gravitational tidal forces will generally alter. Thus neither Lorentz nor translational invariance will hold in the "active" sense. It is only when space is perfectly flat (which will never occur in Nature), or has some other highly symmetric geometry, that such global symmetries can arise. Of course Poincaré symmetry will approximately hold in a small local region of curved space, though never exactly if the region is finite. Further, as stressed by Bohm and Hiley[14], for sufficiently small regions violent quantum fluctuations of the metric should eventually become dominant, again breaking local flatness (in the smooth sense of classical spacetime). From this viewpoint Lorentz invariance is in any case not as fundamental as it might seem, even at the statistical level, being approximately valid only in an intermediate region, on scales which are neither too small nor too large.

5.1.2 Remarks on special relativity

According to the above, both special relativity and standard quantum theory are equilibrium theories only. That these two theories are two sides of the same coin is in fact suggested by a remark in Einstein's 1905 relativity paper: "It is remarkable that the energy and the frequency of a light complex vary with the state of motion of the observer in accordance with the same law"[61]. No doubt Einstein had in mind his paper on light quanta of the same year. That special relativity is an equilibrium theory, analogous to classical thermodynamics, is also suggested by the manner in which it may be derived, like classical thermodynamics, from a few simple principles of an "operational" nature such as the constancy of the speed of light. Indeed Einstein himself regarded his approach to special relativity as analogous to basing classical thermodynamics on the impossibility of perpetual motion[62]. (For a discussion of the role played by thermodynamics in Einstein's early thought, see Klein[63]). It is satisfying that the pilot-wave theory, as developed here, bears out these hints. And, as mentioned in the Introduction, it is satisfying to see the "impossibility principle" of signal-locality, as well as that of the undetectability of uniform motion (the "principle of relativity"), arising statistically alongside standard quantum theory and the uncertainty principle (via a principle of subquantum entropy increase, which leads to equilibrium).

That locality and the relativity of motion emerge together is satisfying in another respect. For locality is closely related to the reductionist "particle" view of Nature, which sees the world as consisting of separate, localised, and permanent bodies or "parts". Now, psychologically speaking, the concept of a permanent body moving around in space would never have arisen were it not for translational invariance and the relativity of motion. For it is these which guarantee that a macroscopic body remains invariant, and thus permanent in identity, when translated or set in motion. The particulate view, and therefore the local vision of Nature, is thus suggested by the relativity of motion. It is then gratifying to see locality and relativity emerge together as related aspects of equilibrium. Both depend crucially on $P=|\Psi|^2$, and both break down at the fundamental level, where a translated "body" (consisting of field disturbances) actually differs, at the sub-uncertainty level, from the original, and all bodies are generally nonlocally connected at this level. The "building block" view of Nature, resting as it does on the twin pillars of locality and relativity, is then an illusion

peculiar to the present state of subquantum equilibrium or "heat death".

Quantum nonlocality defines an absolute 3+1 slicing of spacetime, leading us back to what is surely a more natural view: That the world is a single three-dimensional spatially-extended reality, this world being capable of change, parameterised by the "time" t. In contrast if special relativity were fundamental, so that there is no unique "present moment", then one cannot speak of an objective "present" world in space. The only sensible definition of "reality" would be as a four-dimensional world, as suggested of course by Minkowski. But such a view has the following very peculiar consequence: Considering the world-tube of an observer, such as the reader, laid out in spacetime, and regarding this 4-dimensional picture as "reality", then clearly all 3-dimensional spacelike slices of this world-tube are equally real. This means that the 3-dimensional observers at proper times τ_1, τ_2, τ_3, ..., are all equally real. There is nothing in the spacetime picture which singles out one of these observers as being more real than the others. Thus, though the reader may while reading this paper have the idea of being more real than his/her past or future, according to Minkowski this is a mistake; the reader at say t=1992 A.D. is no more real than the (3-dimensional) reader at t=1982 A.D. or 2002 A.D. The implication of Minkowski's interpretation of special relativity, then, is that there in reality exists an uncountable multiplicity of equally real 3-dimensional observers. This view is, like the many-worlds interpretation of quantum theory, logically possible of course. But it is surely better to take the view, if one can, that there exists a single real 3-dimensional world, capable of change, so that the reader at present is indeed the reader. And spacetime slicing by quantum nonlocality enables one to take this view.

As opposed to the absolute 3+1 view, one might try to graft subquantum nonlocality onto the background of conventional relativistic spacetime. The following remarks are intended to show that such attempts are physically unreasonable.

A fundamentally relativistic approach would of course lead to backwards-in-time signals at the subquantum level, though one might claim that such signals are not necessarily problematic, by arguing that (i) Distributions $P \neq |\Psi|^2$ may never become available, making such signals unobservable in practice, or (ii)Even if they should be observable, back-in-time signals need not by unphysical, or might be reinterpreted as "anti" signals propagating forward in time (à la Wheeler-Feynman).

Regarding (i): There are reasons to doubt that nonequilibrium will never be observed (see Section 8). But even if this is so, if one is interested in physics at the subquantum level, then it is quite irrelevant to point out that macroscopic beings may never, in practice, control the nonlocality at that level. For Bell's

theorem enables us to indirectly deduce the real presence of such nonlocality. And while our physical constitution may limit our capacity for "direct" observation, these limits should not play a fundamental role in the theory, since reasoning via theory effectively enlarges our capacity for "observation". And to declare as "meaningless" that which cannot be "directly observed" is simply an untenable philosophy, since all observations are theory-laden. (For example, despite the brevity of human life, geologists have deduced via theory that some stalactites found in caves have taken hundreds of thousands of years to form).

Considering (ii): It is possible, by means of currently-performable experiments, to generate arbitrarily large and unacceptable "conspiracies" in relativistic spacetime, at the subquantum level. For example, in the usual EPR-Bohm-Bell experiment, with a set of correlated photon pairs, let photon A remain on Earth while photon B moves towards the Sun. Setting aside the pilot-wave theory, Bell's theorem tells us that the angle θ_A of the polariser (set at time t_A) on Earth is not independent of the outcome Π_B at time $t_B=t_A$ for the polarisation measured at B near the Sun. Now let us assume that the angle θ_A is adjusted according to a fixed algorithm, whose input is for instance the mean atmospheric pressure at the Earth's surface (while θ_B is set by an experimenter near the Sun). According to special relativity, in addition to the Earth-Sun rest-frame S, there exists a supposedly physically equivalent frame S' in which the angle θ_A is adjusted a time $t'_A - t'_B > 0$ after measurement at B takes place, i.e. after the outcome Π_B is recorded, while nevertheless the values of θ_A and Π_B are not independent (for an ensemble). But this implies that, during the time-interval (t'_B, t'_A), the evolution of the Earth's atmosphere is, according to Bell's theorem, not independent of the past polarisation outcome at B. There would seem to be a "conspiracy" between the past photon measurement near the Sun and, not merely the future angle θ_A on Earth, but the future evolution of the entire Earth's atmosphere. For in S', the atmospheric evolution leading to the choice of θ_A at t'_A takes place after the time t'_B, thereby inextricably involving the atmosphere in the peculiar entanglement across time. (In contrast, since $t_A=t_B$ in S, one would simply say in S that the setting of θ_A, which was caused by a certain past atmospheric evolution, in turn instantaneously affected the outcome at B). Now one might argue that in a deterministic world, there is nothing wrong with such "conspiracies". However, one must do justice to the fact that such experiments may be performed apparently "at will", that they may be repeated, with ever more complex systems determining the choice of θ_A, thereby magnifying the "conspiracy" to arbitrarily large proportions. And not only could such conspiracies be created at will in any frame, according to special relativity; if such experiments are done in more than one rest-frame, then no frame will be free of these conspiracies. Of course it is logically conceivable in a deterministic world that the initial conditions

of the universe happen to be such that nobody ever performs such an experiment, so that such gross conspiracies are never created. For instance it could be that, whenever such an experiment is attempted, the experimenter involved happens to be struck by a bolt of lightning. But it is more probable that such experiments will be performed, and to explain the arbitrarily large conspiracies, a relativist could invoke arbitrarily large conspiracies in the initial conditions of the universe. But this seems rather like taking the logically consistent view that the fossil record in the Earth's crust was created intact in 4004 B.C. To avoid all this, it seems preferable to introduce a physically preferred time t, defining a fundamental causal sequence, whose associated rest-frame is that in which Bell-type entanglement occurs purely across space.

If practical instantaneous signalling ever becomes possible, then despite the preferred frame in which this is perfectly sensible, one may still feel uncomfortable with the idea that a moving observer will "see" the signal propagate apparently "back in time". But of course no experimenter ever directly "sees" the global time of his Lorentz frame. Rather, his collection of clocks distributed over space have to be set according to his choice. If he chooses Einstein's so-called "synchronisation" using light pulses, which are defined to always have speed c=1, then at absolute time t the moving clock at x will read the Lorentz "local time"

$$t' = (t-vx)/(1-v^2)^{1/2} \qquad (5.1)$$

and the clocks distributed along $x>0$ have been <u>set</u> to read progressively earlier times, by deliberate convention, so that an instantaneous signal propagating along +x appears to go "back in time". But this is in no way more mysterious than the familiar "jet lag" which occurs upon travelling rapidly from one time zone to another on the Earth's surface. If one adopts the convention of setting all clocks to midday when the Sun is seen to be at a certain point in the sky, then a jet passenger may to no-one's surprise travel "back in time".

If it is known, perhaps via nonequilibrium effects, that (t,x) is the preferred frame, then the uniformly moving experimenter need not adopt the Einstein synchronisation, even if he only has access to the classical level. If the moving experimenter wants his time to respect the fundamental causal sequence defined by t, he may set his clocks as follows: Each moving clock, as it passes a <u>nearby</u> stationary clock, at t=0 in the preferred frame, receives from the stationary clock a signal, which sets the moving clock to read $t'_{abs}=0$ (this set-up could be agreed upon beforehand). At later times $t>0$, the moving clocks at <u>all</u> positions x will read

$$t'_{abs} = t/(1-v^2)^{1/2} \qquad (5.2)$$

owing to Larmor time-dilation from the absolute motion of the clocks. In this manner, a surface t=constant corresponds to a surface t'_{abs}=constant, and the absolute simultaneity has been copied by the moving frame. Of course the speed of light in the moving frame will then no longer be the same in all directions. For example if a light pulse is emitted in all directions from the centre of Einstein's moving train carriage, then the pulse will reach the back of the carriage first, in both frames.

Alternatively, this system of absolutely synchronised moving clocks could be constructed simply by beginning with a synchronised set at absolute rest, and simultaneously (in the absolute frame) subjecting each clock to an equal (and gentle) acceleration to velocity v. (To obtain (5.2) the moving clock should be initially set at $t'_{abs}=0$ and switched on only at the end of the period of acceleration, which should be arranged to end at t=0). The resulting perfectly reasonable synchronisation yields the absolute (5.2) as opposed to the Lorentz-Einstein (5.1).

Of course one has to be sure that the "stationary" clocks are indeed at absolute rest, and to check this one needs access to the subquantum level. But once the absolute rest is known, it might be most reasonable and useful for the moving experimenter, even at the purely classical level, to set his own clocks in absolute synchronism, by these or other methods, and thereby avoid altogether the illusory "jet lag" effects which would otherwise arise should he encounter non-equilibrium instantaneous signals.

It should be noted, as stressed by Bell[1], that a single absolute frame is sufficient to describe all of physics, including the physical response of moving apparatus and experimenters - an elementary but often unappreciated point. With this in mind, once one has a sensible description at all levels - classical , quantum, and subquantum, - in the preferred frame (t,x), then the theory is <u>complete</u>. It follows from the classical (e.g. Maxwell) equations in terms of (t,x) that an absolutely moving clock is slowed by $(1-v^2)^{-1/2}$ while an absolutely moving rod is contracted by $(1-v^2)^{1/2}$. There is no need, in principle, to define time and space coordinates "for the moving experimenter", though this may be useful in practice. If one does do this, one may define mathematical "moving coordinates" in any convenient manner. For some purposes the Lorentz-Einstein (t',x') may prove useful, while for others the absolutely synchronised t'_{abs} might be better. A reasonable moving length coordinate, to accompany t'_{abs} is the usual

$$x' = (x-vt)/(1-v^2)^{1/2}$$

With this choice, if a light pulse is sent along +x between two moving clocks with

absolute separation ℓ, then this pulse will require an absolute time $\Delta t = \ell + v\,\Delta t$ or $\Delta t = \ell/(1-v)$ i.e $\Delta t'_{abs} = \ell/[(1-v)(1-v^2)^{1/2}]$, to travel the "length" $\Delta x' = \ell/(1-v^2)^{1/2}$, and the moving experimenter "sees" a light speed $\Delta x'/\Delta t'_{abs} = 1-v$. Alternatively the factor $(1-v^2)^{1/2}$ in x' may be obtained by defining the light speed to be 1-v. We stress once more that such mathematical definitions of "moving coordinates" are quite unnecessary at the fundamental level, where all processes, including "observations" by moving experimenters, take place unambiguously in absolute space and time.

In special relativity the global time is constructed by patching together, via light signals, many local times (one clock for each point of space). While the wholeness of quantum theory automatically generates a global time, which corresponds to the ticking of a single universal "clock": the entire universe.

We note the following remark by Einstein: "What really matters is not merely the greatest possible simplicity of the geometry alone, but rather the greatest possible simplicity of all of physics (inclusive of geometry)."[64] Given the nonlocality of quantum theory, it is the absolute 3+1 viewpoint which, in the author's opinion, offers the simplest description of all of contemporary physics.

Finally, we note that the "observer"-centred philosophy of physics, which emphasises "finding" or "observing" rather than "being", and which led to such confusion in quantum theory, became fashionable largely owing to the influence of Einstein's 1905 "relativity" paper. It is remarkable that an abandonment of this philosophy in quantum theory naturally leads to the abandonment of Einstein's theory of space and time (this philosophy being the radically new content of the 1905 work, in comparison with the earlier view of Lorentz and others).

Note added: A recent paper by Hardy[65] seems to demonstrate that, in any case, no Lorentz-invariant hidden variables theory can reproduce quantum mechanics.

5.2 Absolute 3+1 classical electrodynamics

Having abandoned Lorentz covariance at the fundamental level, we shall now recast classical electrodynamics into absolute 3+1 terms, i.e. in terms of a 3-vector potential A^i (i=1,2,3) evolving in absolute space and time. The "time-component" A^o will make no appearance whatever, even in the presence of charges. We regard the usual A^o as an unfortunate mathematical artifact arising from the usual insistence on Lorentz covariance (another "Plato's Cave effect"), an artifact which causes complications in standard quantum electrodynamics, such as the appearance of negative norm states for "scalar photons" associated with A^o (it being usually said that A^o is not a true dynamical variable, since its canonical momentum vanishes). By abandoning Lorentz covariance and dropping A^o from the

outset, these problems do not arise, and the pilot-wave theory of quantum electrodynamics is straightforward.

The classical 3-vector potential A is taken to obey the field equations, on absolute space and time,

$$\frac{\partial}{\partial t}\nabla\cdot A = -\rho \tag{5.3}$$

$$\nabla\times(\nabla\times A) + \frac{\partial^2 A}{\partial t^2} = j \tag{5.4}$$

in the presence of charge and current densities ρ and j.

The electric and magnetic fields, entering in the Lorentz force law, are defined by

$$E = -\frac{\partial A}{\partial t} \tag{5.5}$$

$$B = \nabla\times A \tag{5.6}$$

The E and B are invariant under gauge transformations of A,

$$A(x,t) \longrightarrow A'(x,t) = A(x,t) + \nabla\lambda(x) \tag{5.7}$$

where λ is time-independent (otherwise E would not be gauge-invariant). Thus we are restricted to purely spatial gauge-transformations, as expected in a 3+1 theory.

Equations (5.3)-(5.6) are equivalent to Maxwell's equations. From the definitions (5.5) and (5.6) follow the identities $\nabla\times E = -\dot{B}$ and $\nabla\cdot B = 0$, while (5.3) and (5.4) yield the remaining two Maxwell equations. Once one has the usual Maxwell equations one may of course introduce a four-component A^μ in the usual manner, and write a Lorentz covariant theory, reflecting the fact that the preferred frame is undetectable at the purely classical level. However, in preparation for subquantum electrodynamics, let us hold fast to the underlying 3+1 theory (at the classical level), in the preferred frame, as defined by the above equations.

Despite the clear equivalence to Maxwell's equations, one may be puzzled as to how the theory can account for Coulomb interactions, which are usually thought to be generated by A^o together with the longitudinal part $A_{||}$ of A. However $A_{||}$ alone is actually sufficient. Writing

$$A = A_\perp + A_{||}$$

where $\nabla\cdot A_\perp = \nabla\times A_{||} = 0$, the $A_{||}$ may be written as

$$A_{\parallel} = \nabla f$$

for some f . We then have $B = \nabla \times A_{\perp}$ and

$$E = -\dot{A}_{\perp} - \nabla \dot{f}$$

so that the $(\dot{f}, A_{\perp})$ replace the usual (A^o, A^i) . A static Coulomb field, for example, is then represented by $A_{\perp} = 0$ and $A_{\parallel} = t\nabla\phi_{Coul}$ (or $f = t\phi_{Coul}$) where ϕ_{Coul} is the usual 1/r Coulomb potential. Generally, writing $\phi = \dot{f}$, (5.3) implies

$$\nabla^2\phi = -\rho$$

The above theory is more elegant and natural in the holistic language of Hamilton-Jacobi classical pilot-wave theory. Let us consider the case $\rho = j = 0$. The 3-vector field A^i is guided by the classical pilot-wave $S = S[A^i,t]$ according to

$$\dot{A} = \delta S/\delta A \qquad (5.8)$$

where S satisfies the Hamilton-Jacobi equation

$$\partial S/\partial t + (1/2)\int d^3x[(\delta S/\delta A)^2 + B^2] = 0 \qquad (5.9)$$

(B is of course just shorthand for $\nabla \times A$). Gauge-invariance of the theory is secured by requiring S to be gauge-invariant, which implies that

$$\nabla\cdot(\delta S/\delta A) = 0 \qquad (5.10)$$

This follows simply by putting $\delta S=0$ for an (infinitesimal) gauge change $\delta A = \nabla\lambda(x)$. It is easily shown that the condition (5.10) on S is preserved in time by the Hamilton-Jacobi evolution (5.9), and that (5.9) and (5.10) imply the classical field equations (5.4) and (5.3) respectively (with $\rho = j = 0$).

5.3 Subquantum electrodynamics

The above classical theory is easily generalised to the quantum (or rather, subquantum) level. The actual 3-vector potential is of course still guided by (5.8), where now S is the phase of Ψ . For the free electromagnetic field, $\Psi = \Psi[A^i,t]$ satisfies

$$i\partial\Psi/\partial t = (1/2)\int d^3x[-\delta^2/\delta A^2 + B^2]\Psi \qquad (5.11)$$

(the canonical momentum $\dot{A}$ in the classical Hamiltonian being replaced by $-i\delta/\delta A$), together with the gauge-invariance condition

$$\nabla\cdot(\delta\Psi/\delta A) = 0 \qquad (5.12)$$

which is preserved by (5.11).

From (5.11) and (5.12) follow the Heisenberg-picture operator field equations corresponding to (5.4) and (5.3) respectively (with $\rho = j = 0$). In equilibrium the theory may of course be represented by Heisenberg operator equations. Since the operator field $\hat{A}$ is a 3-vector, the troublesome "scalar photons" associated with the "nondynamical variable" A^o make no appearance.

The full theory of subquantum electrodynamics of course includes interaction with the charged 2-component (anticommuting) spinor field of Section 4.2. For simplicity we consider here interaction with the charged scalar field only ("scalar electrodynamics").

First, the theory for the free charged scalar field. The guiding field $\Psi = \Psi[\phi,\phi^*,t]$ satisfies

$$i\partial\Psi/\partial t = \int d^3x[-\delta^2/\delta\phi\delta\phi^* + |\nabla\phi|^2 + m^2|\phi|^2]\Psi \qquad (5.13)$$

where, in the classical Hamiltonian, the canonical momenta $\pi = \dot{\phi}^*$, $\pi^* = \dot{\phi}$ are replaced by $\pi \longrightarrow -i\delta/\delta\phi$, $\pi^* \longrightarrow -i\delta/\delta\phi^*$. The continuity equation for $|\Psi|^2$, following from (5.13), shows that the actual fields ϕ , ϕ^* are guided by S via

$$\dot{\phi} = \delta S/\delta\phi^* \quad , \quad \dot{\phi}^* = \delta S/\delta\phi \qquad (5.14)$$

as expected. The theory is invariant under global gauge transformations

$$\phi \longrightarrow \phi' = \phi e^{ie\lambda} \quad , \quad \phi^* \longrightarrow \phi'^* = \phi^* e^{-ie\lambda} \qquad (5.15)$$

where λ = constant. Putting $\delta\Psi=0$ under such an (infinitesimal) transformation, it follows that Ψ must satisfy the condition

$$\int d^3x(\phi\frac{\delta\Psi}{\delta\phi} - \phi^*\frac{\delta\Psi}{\delta\phi^*}) = 0 \qquad (5.16)$$

which is preserved by (5.13). [From (5.13) it follows that

$$i\frac{\partial}{\partial t}[\phi(x)\frac{\delta\Psi}{\delta\phi(x)} - \phi^*(x)\frac{\delta\Psi}{\delta\phi^*(x)}] = [\phi^*(x)\nabla^2\phi(x) - \phi(x)\nabla^2\phi^*(x)]$$

which vanishes upon integration over x].

In the interacting theory, $\Psi = \Psi[\phi,\phi^*,A^i,t]$ is required to be invariant under local gauge transformations $\phi \longrightarrow \phi e^{ie\lambda}$, $\phi^* \longrightarrow \phi^* e^{-ie\lambda}$, $A \longrightarrow A - \nabla\lambda$, where $\lambda = \lambda(x)$. Putting $\delta\Psi = 0$ for infinitesimal $\lambda(x)$ implies the condition

$$\nabla\cdot(\delta\Psi/\delta A) = ie(\phi^*\frac{\delta\Psi}{\delta\phi^*} - \phi\frac{\delta\Psi}{\delta\phi}) \qquad (5.17)$$

Gauge-invariance of the total Hamiltonian of course requires replacement of $\nabla\phi$ by the gauge-covariant derivative

$$\nabla\phi \longrightarrow D\phi = \nabla\phi + ieA\phi \qquad (5.18)$$

so that

$$i\partial\Psi/\partial t = \int d^3x[-(1/2)\delta^2/\delta A^2 + B^2/2 - \delta^2/\delta\phi\delta\phi^* + m^2|\phi|^2 + |D\phi|^2]\Psi \qquad (5.19)$$

which preserves (5.17), and the actual fields are of course guided by S via (5.8) and (5.14). This defines our theory of subquantum electrodynamics.

At the operator level (5.17) and (5.19) of course lead to the operator equivalents of the classical field equations (5.3) and (5.4) respectively, with the usual Klein-Gordon charge and current density operators, as well as to the usual minimally-coupled Klein-Gordon equation (without A^o). We note that local charge conservation is, like the concept of "particle", only a feature of the operator level of statistical equilibrium.

While we write $\Psi=\Psi[\phi, \phi^*, A^i, t]$, Ψ is of course a function, not on the space of gauge-dependent fields, but on the space of equivalence classes of fields connected by gauge transformations. Equivalently, Ψ is a function of the abstract (gauge-independent) "Faraday geometry" associated with the electromagnetic interaction, where, as is well known, the vector potential may be regarded as an affine connection appearing in the gauge-covariant derivative (5.18). (Our 3-vector A yields a "curvature tensor" F_{ij} whose components are just the magnetic field). Thus Ψ is a function on an "electrodynamic superspace", a point of which is a Faraday geometry. And the time-evolution of the actual Faraday geometry is guided by S via (5.8) and (5.14).

We note how physically natural it is, in the holistic pilot-wave theory, to have a global geometry as a dynamical variable, rather than just local fields. This point is amplified in the theory of gravity, to which we now turn.

6. THEORY OF GRAVITATION AND COSMOLOGY

In the absence of gravitation, we have an absolute (flat) 3-space, and an absolute time t, singled out by quantum nonlocality. We now show how gravitation may be represented by a curvature of this absolute 3-space, rather than as a curvature of relativistic spacetime.

At first sight one might think that a curvature of 3-space only, with an absolute time, would yield too few degrees of freedom. But it is easily seen that in fact one is able to reproduce any classical gravitational field (apart from an obvious restriction on spacetime topology). Further, dropping the time-components ${}^{(4)}g_{o\mu}$ of the 4-metric actually eliminates precisely those "nondynamical" degrees of freedom which give so much trouble in standard quantum gravity. (This situation is similar to that of quantum electrodynamics, with ${}^{(4)}g_{o\mu}$ playing the role of A^o). Indeed, this "absolute 3+1" view of gravity enables one to straightforwardly quantise gravity using the pilot-wave theory, in a manner which immediately resolves the fundamental conceptual problems which beset standard canonical quantum gravity, and which leads to a conceptually clear and simple theory of quantum cosmology.

6.1 Absolute 3+1 classical gravitation

In flat space, if a clock moves a spatial distance ds in time dt, then the proper time $d\tau$ ticked is given by

$$d\tau^2 = dt^2 - ds^2 \qquad (6.1)$$

We have adopted the view that there is an absolute frame (or 3-space), and absolute time t, singled out by nonlocality at the subquantum level. And of course this view does reproduce ordinary special relativity at the classical level, where the true frame is undetectable.

Now let the absolute 3-space become curved,

$$ds^2 = g_{ij}dx^i dx^j \qquad (6.2)$$

with (time-dependent) metric $g_{ij}=g_{ij}(x^k,t)$, and arbitrary coordinates x^k. We assume that (6.1) continues to hold, so that a clock moving through curved space with absolute speed v=ds/dt is slowed by the factor $(1-v^2)^{1/2}$. And it is easily seen that this view does reproduce general relativistic spacetime at the classical level.

For beginning with a spacetime with line element

$$d\tau^2 = {}^{(4)}g_{\mu\nu}dx^\mu dx^\nu \qquad (6.3)$$

the 4-metric may always be written in the ADM form[66] (using 4-metric signature +---)

$$|{}^{(4)}g_{\mu\nu}| = \begin{vmatrix} N^2 - N_i N^i & -N_j \\ -N_i & -g_{ij} \end{vmatrix}$$

in terms of the lapse function N, the shift vector N^i, and a 3-metric g_{ij}. And if the topology is a simple product of timelike and spacelike manifolds, then coordinates may be chosen so that $N^i=0$ and $N=1$ everywhere, i.e. so that time and space are completely separated. The line element (6.3) is then reduced to the form (6.1) and (6.2), showing that our curved absolute 3-space, with appropriate 3-metric, can indeed represent an arbitrary general relativistic spacetime (with the obvious topological restriction).

We may thus describe gravity in terms of curved 3-space only, whose geometry evolves in a fixed absolute time t. And this true slicing of classical spacetime is generally singled out by nonlocality at the subquantum level. As in the case of special relativity, the true time is strictly speaking undetectable at the classical level: Given <u>any</u> initial spacelike slice, coordinates may be continued orthogonally off this surface, so that $N^i=0$, $N=1$, and there is strictly speaking no preferred initial slice at the classical level. (Though note that, given the initial slice, the subsequent slicing defined by t=constant is unique). In practice, however, there may be a preferred slicing even at the classical level, singled out by the global distribution of matter, which may define a cosmic "rest". A particular slicing may then be "preferred" in the sense of providing a simpler description of the physics, in the same sense that Newtonian time is "preferred" in Newtonian mechanics. Now such a preferred classical slicing does indeed exist for our observed universe at the cosmological level, and the question arises as to whether this agrees with the absolute slicing at the subquantum level. This seems possible, by virtue of nonlocality from $P\neq|\Psi|^2$ being operative in the early universe (see below), though it is of course unproblematic if these slicings happen to not coincide: The true 3-space must ultimately be determined at the nonlocal subquantum level.

The situation here should be distinguished from that in special relativity. Firstly, as we have just seen, it is possible that in practice the true slicing is visible even classically, on the cosmological scale. Secondly, and more generally, once the true slicing is known (perhaps via subquantum experiments), and if the absolute 3-geometry is given, then absolute motion through the 3-space will generally be detectable even classically, by an experimenter working in an arbitrarily small but <u>finite</u> region. For the variation of local tidal effects will depend on the motion of the experimenter through the true 3-space. (There is of course no contradiction here with the usual principle of equivalence, which emphasises the vanishing of tidal effects in the limit of an infinitely small region). This illustrates the fact, stressed by Fock[67] and DeWitt[68], that "general relativity" is really less "relativistic" than special relativity. It is only for very special gravitational fields that one has global "active" symmetries such as Lorentz and translational invariance. An arbitrarily small but finite apparatus displaced through absolute 3-space will generally feel tidal stresses which differ from those which it would have felt had it remained undisplaced. No such effects occur in special relativity. A general gravitational field thus actually breaks global Poincaré symmetry and allows one to define, <u>if one wishes</u>, a state of absolute rest. One is of course not obliged to do this at the purely classical level: Changing tidal stresses felt by a "moving" apparatus may be ascribed to the time-dependence of the 3-metric, the apparatus remaining "at rest". However, as we shall see, the view that classical spacetime is really curved 3-space evolving in absolute time is greatly favoured for the quantisation of gravity.

The absolute 3+1 viewpoint is of course quite contrary to the so-called "spirit of relativity". On the other hand it may be worth noting that, historically, the furor over "relativity" in the 1920s and 1930s was largely powered by the philosophical, social, and political connotations which this word carried at the time.[41]

To motivate further our view of classical gravitation, consider a quantised (massless) scalar field ϕ propagating on a classical background of <u>flat</u> space. Using arbitrary spatial coordinates x^k with metric g_{ij}, the Lagrangian density

$$\mathcal{L} = (1/2)g^{1/2}[\dot{\phi}^2 - g_{ij}\partial^i\phi\,\partial^j\phi]$$

and the canonical momentum $\pi = g^{1/2}\dot{\phi}$. And at the level of operators

$$[\hat{\phi}(x,t),\ \hat{\pi}(x',t)] = i\,\delta^3(x-x')$$

Replacing $\hat{\pi} \longrightarrow -i\delta/\delta\phi$ in the Hamiltonian density

$$\mathcal{H} = (1/2)g^{1/2}[\pi^2/g + g_{ij}\partial^i\phi\,\partial^j\phi]$$

the wavefunction $\Psi=\Psi[\phi,t]$ obeys

$$i\partial\Psi/\partial t = (1/2)\int d^3x\, g^{1/2}\,[-g^{-1}\delta^2/\delta\phi^2 + g_{ij}\partial^i\phi\,\partial^j\phi]\Psi \qquad (6.4)$$

Here Ψ is invariant under coordinate transformations $x \longrightarrow x'$,

$$\Psi'[\phi'(x'),t] = \Psi[\phi(x),t]$$

(where $\phi'(x')=\phi(x)$), and the functional derivative $\delta\Psi/\delta\phi$ is defined in the usual (coordinate-dependent) manner

$$\delta\Psi = \int d^3x\,(\delta\Psi/\delta\phi)\delta\phi$$

so that $g'^{-1}\delta^2\Psi'/\delta\phi'^2 = g^{-1}\delta^2\Psi/\delta\phi^2$ and (6.4) is indeed 3-covariant. Since $g'^{-1/2}\delta S'/\delta\phi' = g^{-1/2}\delta S/\delta\phi$, the actual ("hidden-variable") evolution $\dot{\phi} = g^{-1/2}\delta S/\delta\phi$ is of course also 3-covariant, this evolution being consistent with the continuity equation following from (6.4)

$$\partial|\Psi|^2/\partial t + \int d^3x \frac{\delta}{\delta\phi}\left(|\Psi|^2 g^{-1/2}\frac{\delta S}{\delta\phi}\right) = 0$$

The theory is thus 3-covariant, and the true 3-space is, as we have seen, singled out by quantum nonlocality: The actual field evolution $\dot{\phi} \propto \delta S/\delta\phi$ generally shows nonlocal connections across 3-space, distinguishing the true time t.

A glance at (6.4) suggests that it be adopted as the Schrödinger equation even when the (classical) 3-space is curved, the nonlocal connections in ϕ still acting across this true 3-space, and singling out the absolute time t. Indeed, the whole of the above formalism may be assumed to carry over intact to the case of curved 3-space. We thus have a 3-covariant pilot-wave theory of ϕ propagating on a classical background curved 3-space with metric $g_{ij}(x^k,t)$.

At the statistical level of equilibrium, (6.4) may be transformed into the Heisenberg picture, yielding the field equation, for the static case $g_{ij}=g_{ij}(x^k)$,

$$\ddot{\hat{\phi}} - g^{-1/2}\partial_i(g^{1/2}g^{ij}\partial_j\hat{\phi}) = 0$$

or

$$\ddot{\hat{\phi}} - g^{ij}\nabla_i\nabla_j\hat{\phi} = 0 \qquad (6.5)$$

where ∇_i is the 3-covariant derivative. In the Heisenberg picture the state-vector $|\Psi\rangle$ is fixed, independent of (absolute) time, and of the arbitrary spatial coordinates. And at this level the nonlocality in the actual ϕ is of course no longer directly visible.

Now observers working in equilibrium on the Earth's surface are not directly aware of subquantum nonlocality and absolute time. They therefore insist on using arbitrary 4-coordinates x^μ, and introduce the 4-metric ${}^{(4)}g_{\mu\nu}$ (where of course ${}^{(4)}g_{o\mu}=\delta_{o\mu}$ and ${}^{(4)}g_{ij}=-g_{ij}$ in absolute 3+1 coordinates). They then write (6.5) in 4-covariant form

$${}^{(4)}g^{\mu\nu}\nabla_\mu\nabla_\nu\hat{\phi} = 0$$

where ∇_μ is the 4-covariant derivative. This procedure is often useful mathematically. However it creates severe problems of principle for our observers. For we have seen that $|\Psi\rangle$ is a fixed global object, defined with respect to the absolute slicing and time t. Since these observers cannot distinguish t from any other t', they are unable to clearly define this global object. This problem becomes especially severe if the observer believes, as many do, that $|\Psi\rangle$ "collapses" upon measurement: Along which spacelike hypersurface does the collapse occur? Such problems are of course illusory in our view, being again "Plato's Cave effects" - which disappear at the deeper subquantum level.

It should be noted that the actual evolution $\dot{\phi} = g^{-1/2}\delta S/\delta\phi$ is generally sensitive to the entire 3-geometry, since S is generally nonseparable. Thus the usual "principle of equivalence", in the sense of the local indistinguishability of gravity and acceleration, breaks down at the subquantum level: Local measurements are generally sensitive, by virtue of nonlocality, to the geometry at a distance. On the other hand, in equilibrium, the metric in the operator equation (6.5) may be locally transformed away, so that the equivalence principle appears as another equilibrium symmetry, valid if and only if $P=|\Psi|^2$. Another way to see this is based on work by Candelas and Sciama[69] in the context of standard quantum field theory. They show, for an explicit example, that a detector falling freely in a classical gravitational field perceives the same spectrum of vacuum-fluctuations as does a similar free detector in Minkowski spacetime, thus upholding the equivalence principle. However, this result assumes that $P=|\Psi|^2$ for each field vacuum, and must break down for $P\neq|\Psi|^2$.

6.2 Subquantum gravitation

Given the above absolute 3+1 view, it is now straightforward to quantise gravity. Considering the case of the pure gravitational field, our dynamical object is the geometry of 3-space, which evolves in absolute time t. And this evolution is guided by Ψ in the usual manner. The 3-geometry itself is then connected by quantum nonlocality, in the sense that the evolution of the 3-metric in one region will usually depend instantaneously on the 3-metric everywhere else. In this manner, at the subquantum level, the effects of holistic guidance by Ψ distinguish and maintain the absolute slicing of spacetime.

The basis of pilot-wave quantum gravity is simply the following: Let Ψ be a 3-scalar function of the 3-geometry and of absolute time t. If one writes $\Psi = \Psi[g_{ij},t]$ and demands that this be invariant under change of spatial coordinates then one arrives, as is well known, at the condition

$$(\delta\Psi/\delta g_{ij})_{|j} = 0 \qquad (6.6)$$

where $|j$ is the 3-covariant derivative. (This follows by putting $\delta\Psi=0$ under a diffeomorphism which changes the metric by $g_{ij} \longrightarrow g_{ij} + \epsilon_{i|j} + \epsilon_{j|i}$). This condition is of course the analogue of the gauge-invariance condition (5.12) of pure electrodynamics. It ensures that Ψ is a function on superspace, the space of 3-geometries, and is actually independent of the coordinate-dependent part of the 3-metric g_{ij} .

Inserting $N^i=0$, $N=1$ into the usual Hamiltonian density of canonical quantum gravity, one obtains (with units $G=c=\hbar=1$)

$$\mathcal{H} = 16\pi G_{ijkl}\Pi^{ij}\Pi^{kl} - (1/16\pi)g^{1/2}R$$

Here Π^{ij} is the canonical momentum density, related to $\dot{g}_{ij}$ by the (invertible) equation

$$\dot{g}_{ij} = 32\pi G_{ijkl}\Pi^{kl}$$

where G_{ijkl} is the superspace metric

$$G_{ijkl} = (1/2)g^{-1/2}(g_{ik}g_{jl} + g_{il}g_{jk} - g_{ij}g_{kl})$$

Of course $g=\det g_{ij}$ and R is the 3-scalar curvature.

Replacing $\Pi^{ij} \longrightarrow -i\delta/\delta g_{ij}$ in the Hamiltonian leads to the Schrödinger equation for Ψ ,

$$i\partial\Psi/\partial t = \int d^3x[-16\pi\frac{\delta}{\delta g_{ij}}G_{ijkl}\frac{\delta}{\delta g_{kl}} - (1/16\pi)g^{1/2}R]\Psi \qquad (6.7)$$

which preserves the condition (6.6). An explicit operator ordering has been adopted in (6.7).

The classical relation between $\dot{g}_{ij}$ and Π^{kl} implies that the actual ("hidden variable") metric g_{ij} is guided by

$$\dot{g}_{ij} = 32\pi G_{ijkl}\frac{\delta S}{\delta g_{kl}} \qquad (6.8)$$

this being consistent with the continuity equation

$$\partial|\Psi|^2/\partial t + \int d^3x\frac{\delta}{\delta g_{ij}}(32\pi G_{ijkl}\frac{\delta S}{\delta g_{kl}}|\Psi|^2) = 0$$

which follows from (6.7).

The equations (6.6), (6.7) and (6.8) define our pilot-wave theory of (sub)quantum gravity. Before discussing the consequences, the following must be noted: In (6.7) we have not included the boundary term at spatial infinity in the

Hamiltonian, thus restricting ourselves to closed, or compact, 3-geometries. Now in the usual approach to quantum gravity, the noncompact case is complicated by the arbitrariness of the asymptotic hypersurface[70]. But since our 3-space is absolute, we have no such arbitrariness. We thus expect that the above theory will be adequate for the noncompact case as well, by simply adding the usual boundary term[70]

$$(1/16\pi)\int_{\infty} dS^k g^{1/2} g^{ij}(g_{ik,j} - g_{ij,k})$$

to the Hamiltonian in (6.7).

The quantum level again arises for an equilibrium ensemble $P=|\Psi|^2$ of 3-geometries, where the path (6.8) is no longer observable in practice. While (6.6) and (6.7) may be recast as Heisenberg-picture operator equations which define statistical predictions. It is well known that the operator equivalent of (6.6) implies $^{(4)}\hat{G}_{oj}=0$, while the dynamical operator equations arising from (6.7) imply $^{(4)}\hat{G}_{ij}=0$ (where the 4-dimensional Einstein tensor is written in our absolute 3+1 coordinates $x^\mu = (t,x^i)$ with $g_{o\mu} = \delta_{o\mu}$). The remaining component $^{(4)}\hat{G}_{oo}=0$ of the operator Einstein equations arises in the standard approach from the Wheeler-DeWitt equation $\hat{\mathcal{H}}\Psi=0$. Since our Ψ obeys a more general equation, there cannot be a general operator relation of this form. However if one assumes the wavefunction to satisfy $\langle\hat{\mathcal{H}}\rangle \equiv \int Dg\Psi^*\hat{\mathcal{H}}\Psi = 0$, where Dg is an appropriate probability measure on superspace, then one obtains, at the classical level, the c-number equation $^{(4)}G_{oo}=0$. (The case $\langle\hat{\mathcal{H}}\rangle \neq 0$ is discussed below).

Thus, we obtain $^{(4)}G_{oo}=0$ at the classical level only if Ψ happens to be such that $\langle\hat{\mathcal{H}}\rangle=0$. The physical reason for this is as follows: In the classical limit $\mathcal{H} = {}^{(4)}G_{oo}g^{1/2}/8\pi$, and $^{(4)}G_{oo}$ is equal to (one half of) the difference between the extrinsic and intrinsic curvature of space. Now the extrinsic curvature $K^{ij}K_{ij}-K^2$ (where $K_{ij} = -(1/2)\dot{g}_{ij}$) is essentially the square of the rate of change of the 3-metric, and in the pilot-wave theory the actual $\dot{g}_{ij}$ is determined by $\delta S/\delta g_{ij}$ i.e by Ψ . Thus a classical relation of the form $^{(4)}G_{oo}=0$ between $\dot{g}_{ij}$ and the intrinsic curvature R will only arise if Ψ obeys some special condition, which is clearly just $\langle\hat{\mathcal{H}}\rangle=0$.

The pilot-wave approach to gravity overcomes the following fundamental problems of the standard approach:

(i) Extraneous variables. The "nondynamical" variables $^{(4)}g_{o\mu}$ greatly complicate the standard approach. We dispose of these from the outset, by recognising the fundamental entity to be absolute 3-space evolving in absolute time.

(ii) Definition of time. Time is usually said to emerge in a phenomenological manner using quantum clocks[70], an approach which is not without difficulties. Here we have a unique time t, which is physically distinguishable at the level of

nonlocal hidden variables (by a "subquantum demon"). In the classical limit this time coincides with one of the classical time functions, as is clear from the way in which the classical Einstein equations emerge, this time-function probably being the one which is cosmologically preferred (see below). Outside the classical limit it should be stressed that the time t, like the hidden variables themselves, has a unique objective existence, defined in terms of the global and nonlocal hidden variable evolution, and conceived independently of practical macroscopic observations (this being after all the philosophy behind hidden variables theories).

One might object and claim that, outside the classical limit, one must necessarily introduce a "quantum clock" to define time, the presence of this clock having a non-negligible effect on the evolution of the system, this evolution being then parameterised by the "clock reading". While one may of course do this if one wishes, it must be emphasised that such a procedure is quite unnecessary for a fundamental definition of time. For in the pilot-wave theory we simply have an objective 3-metric evolving in an absolute time, the latter being perfectly well defined at the fundamental level. If one wishes to consider, for instance, the very early universe as it actually was in the past, it suffices to consider the objective variables and their evolution in that epoch. It is irrelevant to consider what would have happened had macroscopic observers or "clocks" been present; for while that may be an interesting exercise, as far as we know no such observers or clocks were actually present in the early universe. The description of that early epoch is therefore already complete without the "quantum clocks". In the present epoch, of course, one could envisage experiments performed by macroscopic experimenters, involving quantum or subquantum gravitational effects, and these experiments might involve nonclassical apparatus and "clocks". In this case it might be useful in practice to calculate, on the basis of the fundamental theory, the dependence of the "system" evolution on the "clock reading". But it should be stressed that this would be a purely phenomenological and highly arbitrary procedure, and that the fundamental time is that of the subquantum level.

(iii) Commutation relations. In the usual theory there is no unique hypersurface on which to define "equal-time" commutation relations, and the notion of spacelike separation is ambiguous. Whereas here two space points $x' \neq x$ at given t are by definition spacelike separated. And at the statistical level of operators, $\hat{g}_{ij}(x,t)$ represents an equilibrium ensemble of 3-geometries at time t, which may be regarded as an ensemble of functions $g_{ij}(x,t)$ on a fixed 3-manifold with coordinates x. Since the 3-manifold is completely separate from time, two points $x' \neq x$ at time t are clearly spacelike separated whatever the value of the 3-metric g_{ij}. Our Heisenberg-picture operators then obey the usual commutation relations

$$[\hat{g}_{ij}(x,t),\ \hat{\Pi}^{kl}(x',t)] = i\delta^{k}_{(i}\delta^{l}_{j)}\,\delta^{3}(x-x')$$

$$[\hat{g}_{ij}(x,t),\ \hat{g}_{kl}(x',t)] = [\hat{\Pi}^{ij}(x,t),\ \hat{\Pi}^{kl}(x',t)] = 0$$

at equal absolute time t.

(iv) Probability interpretation. In the usual approach based on the Wheeler-DeWitt equation, the probability interpretation is very obscure, and the space of solutions has no satisfactory norm. It is often said that, since the Wheeler-DeWitt equation imposes an infinity of constraints on Ψ, the existence of normalisable solutions is doubtful. Here however, (6.7) gives only a single constraint on Ψ at each time t. One then naturally adopts the usual probability density $|\Psi|^2$ in superspace, with the norm $\int \mathrm{D}g|\Psi|^2$. (Here, and throughout this work, we do not address the question of the rigorous mathematical definition or existence of such entities, and are only concerned with the fundamentals at a physical level).

(v) Initial-value formulation. Here the initial conditions take the simple form, just Ψ at t=0 (and the hidden variable 3-geometry with metric g_{ij} at t=0, or some distribution thereof). In contrast, attempts at an initial value formulation for the Wheeler-DeWitt equation have led to great confusion.

The assumption of a preferred time defined by quantum nonlocality thus resolves a number of fundamental difficulties which plague the interpretation and formulation of standard quantum gravity. This approach also leads to some new features, in particular at the cosmological level, to which we now turn.

6.3 Subquantum cosmology

(i) Disequilibrium near the big bang: An explanation for the observed cosmological rest-frame? In this theory nonlocality defines a state of absolute rest, and one might suspect that for our universe this state of rest coincides with that defined by the uniform cosmic microwave background. But how did this background become so uniform? It could be that this came about via ordinary processes bounded by the speed of light, as indicated by theories of inflation. On the other hand, this uniformity which defines the observed rest-frame might be directly linked to the fundamental source of absolute rest, i.e. quantum nonlocality, as follows: In principle one may contemplate distributions $P \neq |\Psi|^2$ near the big bang, which later relax to equilibrium via the H-theorem, perhaps on a timescale of the order of a Planck time. The resulting nonlocality at very early times might then be responsible for the uniformity seen today[19], though a detailed cosmological model incorporating this effect has yet to be developed. (This effect is of course completely outside the scope of standard quantum theory). If this theory is correct then the state of absolute rest does indeed coincide with the observed rest-

frame. And if quantum nonlocality ever becomes directly observable, it will be found to propagate along the hypersurface defined by the observed cosmological rest-frame. The possibility that the observed rest-frame defined by the microwave background may be related to some form of nonlocal physics at very early times has been advocated in particular by Sciama[71].

If indeed $P \neq |\Psi|^2$ near the big bang, with the relaxation $P \longrightarrow |\Psi|^2$ taking place soon after, then it could be said that quantum noise is a remnant of the big bang.

(ii) "Dark matter". If one includes nongravitational fields, with Hamiltonian density $\mathcal{H}_m$, then the "00" component of the classical field equations ${}^{(4)}G_{\mu\nu} = -8\pi T_{\mu\nu}$ (where $\mathcal{H}_m = g^{1/2}T_{oo}$) is obtained only if one imposes $\langle \hat{\mathcal{H}} + \hat{\mathcal{H}}_m \rangle = 0$. But the theory contains the possibility $\langle \hat{\mathcal{H}} + \mathcal{H}_m \rangle \equiv -g^{1/2}\tilde{\rho} \neq 0$ leading to ${}^{(4)}G_{\mu\nu} = -8\pi(T_{\mu\nu} + \tilde{T}_{\mu\nu})$ where $\tilde{T}_{\mu\nu} = \delta^o_\mu \delta^o_\nu \tilde{\rho}$ describes an effective pressure-free "energy" density or "dust", whose only interaction with ordinary matter is via gravity. We note that an observer at absolute rest measures an energy density $T_{oo}+\tilde{T}_{oo}$, and the frame in which $\tilde{T}_{\mu\nu}$ is a pure energy density distinguishes the absolute time at the classical level.

If $\tilde{\rho} \neq 0$, this may have cosmological implications. If $\tilde{\rho} > 0$ one effectively has a distribution of "cold dark matter" in addition to ordinary matter. And a positive $\tilde{\rho}$, with $\int d^3x\, g^{1/2}\tilde{\rho} > 0$, is possible since the gravitational Hamiltonian is not positive definite, according to the minisuperspace model given below. (This apparently confirms the suspicion that quantum gravity does not possess a ground state. The situation is then roughly speaking like that of Newtonian gravity, where gravitational potential energy has no lower bound).

(iii) Minisuperspace Friedmann universe. Let us apply the above theory to a minisuperspace model of quantum cosmology. Consider a closed Friedmann universe with expansion parameter a(t), containing a total nongravitational mass $M = \int d^3x\, g^{1/2}T_{oo}$. Following DeWitt[70] the Schrödinger equation (6.7) for $\Psi = \Psi(a,t)$ may be written

$$i\partial\Psi/\partial t = [(1/3\pi)\partial/\partial a\,(1/a)\partial/\partial a - (3\pi/4)a + M]\Psi = (\hat{H}+M)\Psi \qquad (6.9)$$

where the canonical momentum $\hat{\Pi}_a = -i\partial/\partial a$ is symmetric on $(0,\infty)$ if we impose $\Psi = 0$ at $a=0$ and $a=\infty$ (ignoring the issue of rigorous self-adjointness), while our operator ordering is as in (6.7). The actual hidden variable a(t) is governed by $\dot{a} = -(2/3\pi)(1/a)\partial S/\partial a$ (as is clear from the continuity equation following from (6.9)).

We note that Vink[72] has recently used this minisuperspace pilot-wave trajectory for a(t) to motivate a particular approach to the emergence of time and

of classical trajectories, in the context of the standard theory involving quantum clocks, general relativistic time, and the Wheeler-DeWitt equation. The approach taken here is of course very different.

The usual Wheeler-DeWitt equation leads to the vanishing of the right hand side of (6.9) which, as noted by DeWitt[70], restricts the mass content M to specific eigenvalues. This peculiarity does not occur in our approach, where M is arbitrary. Expanding Ψ in terms of eigenfunctions $(\hat{H}+M)\Psi_E = E\Psi_E$, we impose $\Psi_E(0) = \Psi_E(\infty) = 0$. The change of variables $x = 1/a^2$, $\phi_E = x\Psi_E$, then yields

$$-(1/2\mu)\partial^2\phi_E/\partial x^2 + V(x)\phi_E = 0$$

on the space $x > 0$, with boundary conditions $\phi_E(0) = 0$ and $\phi_E(\infty) = \lim_{a\to 0}(1/a^2)\Psi_E(a) =$ finite (since $(\hat{H}+M)\Psi_E = E\Psi_E$ implies that $\partial\Psi_E/\partial a = 0$ at $a=0$), where $\mu = 3\pi/8$ and

$$V(x) = 3\pi/4x^4 + (E-M)/x^{7/2}$$

This has no solution for $E \geq M$, while for $E < M$ the WKB method shows that

$$(E_n - M) \sim -(n+1/2)^{1/2}, \qquad n = 0,1,2,\ldots$$

so that $E_n < 0$ for sufficiently large n. Thus $\langle \hat{H}+M \rangle$ may indeed be negative, as stated above. Indeed in this minisuperspace model the gravitational Hamiltonian $\hat{H}$ is actually negative definite. Note that if we impose $\langle \hat{H}+M \rangle = 0$ on Ψ, this does not restrict M.

Using the Hamiltonian

$$\hat{H}_{total} = -(1/3\pi)\hat{\Pi}_a(1/\hat{a})\hat{\Pi}_a - (3\pi/4)\hat{a} + M$$

in the Heisenberg picture corresponding to (6.9), one readily derives the classical limit of quantum equilibrium

$$2(\ddot{a}/a) = -(\dot{a}/a)^2 - (1/a)^2$$

which is the usual "acceleration" equation for a pressure-free closed Friedmann universe. Further, writing $\langle \hat{H}_{total} \rangle = -\tilde{M}$ and using the classical limit $\Pi_a = -(3\pi/2)a\dot{a}$, one readily sees that

$$(\dot{a}/a)^2 = -(1/a)^2 + (8\pi/3)(\rho + \tilde{\rho})$$

where $\rho = M/2\pi^2a^3$ and $\tilde{\rho} = \tilde{M}/2\pi^2a^3$, yielding the "00" component of the classical Einstein equations with effective energy density $\rho + \tilde{\rho}$. The discussion of time and the emergence of the classical limit is clearly much simpler and clearer in this approach to quantum cosmology.

(iv) A mechanism for the big bang? So-called "operational" theories tend to declare as "meaningless" any attempt to discuss physics beyond the levels of measurement accessible to present technology. Thus any attempt to explain why an atomic decay occurred at a particular moment is said to be, according to standard quantum theory, "metaphysical". The combination of the two great operational theories, relativity and quantum theory, as applied to cosmology, has led to perhaps the most spectacular statement of this type: That it is "meaningless" to ask for an explanation for the enormous explosion which seems to have occurred some ten or twenty billion years ago, the big bang. For the treatment of the big bang as a quantum event makes it as incomprehensible as an atomic decay, while general relativity encourages the idea that "time itself began" at the big bang. But it seems extraordinary that the most cataclysmic event known to science should be regarded as having had no cause.

The philosophy behind the pilot-wave theory suggests otherwise. If singularities are avoided by quantum effects, one might in principle calculate back to an absolute time prior to the big bang. Indeed it is a tantalising task for subquantum cosmology to find a plausible explanation for the big bang event. The following seems suggestive: When an atom absorbs a light quantum, the pilot-wave theory shows[24] how, via nonlocality, the energy spread over space as an electromagnetic wave becomes suddenly concentrated into a small region centred on the atom, and absorbed, leading to a sudden "quantum jump". Could this sudden concentration of energy into a small region, which occurs all the time on the every day level, be somehow analogous to the mechanism behind the big bang?

It is a daunting task to distinguish alternative formulations of quantum theory at the laboratory level. But at the cosmological level the above pilot-wave theory is not only arguably much clearer in its application , it also leads to new physics as discussed above.

7. SUBQUANTUM MEASUREMENT, SUBQUANTUM AUTOMATA, AND PARALLEL PROCESSING

At the fundamental level all things consist of variables guided by Ψ . In certain situations, the ones generally known so far, one may apply an approximation in the pilot-wave theory, whereby the world breaks up into system + apparatus + classical experimenter, three entities which are fairly distinct and well-defined for practical purposes. In this approximation, the standard quantum theory of "measurement" emerges. And it is only in the context of this approximation that standard quantum theory may "speak". Generally, however, this neat break need not occur at all, and one may have more general situations which are outside the domain of standard quantum theory, but which may of course be described by the pilot-wave theory.

We have seen in Section 2.2.1 how standard quantum theory is constructed by experimenters who, with their everyday surroundings, operate in a manner which is insensitive to the sub-uncertainty level, leading them to adopt the usual "measurement" theory based on the analogy with classical physics, whereby linear operators are in formal correspondence with classical variables.

What would happen if an experimenter, or "intelligent" automaton, operated at the subquantum level? The automaton's senses, being directly receptive to the actual ("hidden") variables, would enable it for instance to directly perceive nonlocal connections. But how would such a "subquantum demon" define "measurement"? And how would it function?

We adopt the view that it is theory which decides what may be observed, and how it should be measured. And we assume that our subquantum demon understands and believes the pilot-wave theory.

A true measurement, at the subquantum level, would be for instance of the following form: If two variables X_1 and X_2 are coupled by a nonseparable guiding field $S(X_1,X_2,t)$, then the value of X_2 could be deduced from the relation

$$dX_1/dt = \partial S(X_1,X_2,t)/\partial X_1$$

by observation of the change in X_1 over a short time, thus effecting a measurement of X_2 . One might object that this ignores the issue of how to observe X_1 , but Newtonian mechanics, say, contains the same circularity, the value of X_2 being deduced from that of X_1 via Newton's laws. This is quite reasonable if the observer himself operates on and is directly sensitive to the level of the variables in question (so that X_1 could be directly read by his senses). It is of course assumed that S is known, perhaps from a previous "state" (i.e. wavefunction) preparation, just as one assumes in Newtonian mechanics that the forces are somehow known in practice.

We emphasise that, in the pilot-wave theory, the basic conception of true measurement must be of the above form, in terms of the actual variables, while "quantum measurements" are merely a phenomenological book-keeping used by macroscopic experimenters. It is only our gross constitution which prevents us from performing and using true measurements in practice.

As an illustration of subquantum measurement, let us consider the two-slit experiment from a subquantum viewpoint. Instead of firing a single particle at a two-slit screen, one might imagine firing our (small) subquantum demon at the screen, where the demon's memory operates at the subquantum level, while its centre-of-mass suffers the usual interference effects at the backstop. (Our demon is then both "system" and "observer"). For instance the demon might have an internal memory variable X_m, located somewhere within a packet ψ_m (initially centred on $X_m=0$) of width ΔX_m, with "bit element" 0 or 1 according to which half of the interval $(-\Delta X_m/2, \Delta X_m/2)$ the actual X_m occupies. One could then have interference at the backstop, while nevertheless the demon could later tell us which slit he traversed, as follows: Let X_m be initially near the centre of $(-\Delta X_m/2, \Delta X_m/2)$ (we assume the timescale of the experiment to be such that any natural spreading of ψ_m and consequent motion of X_m may be ignored). The demon with his memory is fired at the screen. Near each slit is applied an external electric field, that near slit 1 being in a direction opposite to that near slit 2. If the variable X_m is say the position of a charged particle, then the packet ψ_{m1} traversing slit 1 will (in appropriate conditions) be approximately simply displaced by $+\delta$, while the packet ψ_{m2} traversing slit 2 will be similarly displaced by $-\delta$. And the actual X_m will be approximately displaced by $\pm\delta$ according to the slit which is actually traversed by the demon. (This crude treatment is sufficient for illustration). The final wavefunction far from the screen will be of the form

$$\phi_1\psi_{m1}(X_m) + \phi_2\psi_{m2}(X_m)$$

If $\delta \gtrsim \Delta X_m$, then the memory variable has overstepped the bounds of the sub-uncertainty level, the displacement δ being capable of registration by a classical automaton. The packets ψ_{m1} and ψ_{m2} will not overlap, and there will be no interference at the backstop (the distribution being $|\phi_1|^2+|\phi_2|^2$). In this way the memory of a classical observer inevitably destroys interference by deactivating the empty nonoverlapping packet. On the other hand, if $\delta \approx \Delta X_m$, then ψ_{m1} and ψ_{m2} still overlap, and if $\delta < \Delta X_m$ then one has an interference distribution $\approx |\phi_1+\phi_2|^2$ at the backstop while nevertheless the information as to the slit traversed is recorded at the sub-uncertainty level, in the demon's memory. And assuming the demon to be sufficiently "intelligent", he could transmit this information to classical experimenters simply by arranging for the actual

value of X_m to determine the outcome of a macroscopic event. Or more simply, a classical experimenter could simply measure the (noncontextual) memory variable X_m, confident that the intelligent demon has performed his task correctly.

A subquantum automaton, then, not only directly perceives holistic connections, it can also perform measurements without destroying the effect of different overlapping branches of the wavefunction. This comes about by functioning at the sub-uncertainty level, so that the demon is roughly speaking operating "inside the wavefunction".

Let us briefly consider some further properties of subquantum automata.

Deutsch[73] has developed a theory of quantum computers, where these operate according to standard quantum theory. Briefly, a quantum computer has a wavefunction spanned in Hilbert space by a set of "computational basis states" which correspond to states of classical Turing machines. For appropriate unitary evolution of the wavefunction, one may effectively simulate several Turing machines, with different branches of the wavefunction performing different computations in the same time as that required for an analogous classical Turing machine to perform a single similar computation. (These parallel computations take place in "different universes" according to the many-worlds interpretation). However, while the final state of the quantum computer may contain the results of an arbitrarily large number N of separate computations, "unfortunately, at most one of these results is accessible in each universe"[73], limiting the usefulness of parallel quantum computation. This limitation could be overcome, however, if access to the subquantum level were possible.

For consider a quantum computer, augmented by a "subquantum sensor" which is able to directly read the actual value of a variable X_s attached to the device. Let the computer perform N separate computations, one in each of N branches of the wavefunction, as described by Deutsch, and let the results be coded into N' energy eigenvalues E'_i (i=1 to N') chosen from a set $\{E_i | i=0 \text{ to } \infty\}$ associated with the variable X_s, such that the final wavefunction guiding X_s is the freely evolving

$$\psi(X_s,t) = N'^{-1/2}\sum_{i=1}^{N'} \phi_{E'_i}(X_s)e^{-iE'_it} \qquad (7.1)$$

where $\phi_{E'_i}$ is the eigenfunction with eigenvalue E'_i. We assume that $\{\phi_{E_i} | i=0 \text{ to } \infty\}$ is a known set of functions built into the device, and that these functions all overlap with eachother (as occurs for instance with energy eigenfunctions for a box). The actual evolution $X_s(t)$ will be governed by (units "m=1")

$$\dot{X}_s(t) = \mathrm{Im}\left(\frac{1}{\Psi}\frac{\partial\Psi}{\partial X_s}\right) = \mathrm{Im}\left[\frac{\sum_{i=1}^{N'}\partial\phi_{E_i'}(X_s)/\partial X_s\, e^{-iE_i't}}{\sum_{i=1}^{N'}\phi_{E_i'}(X_s)e^{-iE_i't}}\right] \qquad (7.2)$$

and since none of the $\phi_{E_i'}$ vanish except at isolated points, they all contribute to the motion $X_s(t)$.

If the path $X_s(t)$ is read by the sensor over a finite time period, then the set of E_i' present in (7.1) may be deduced from (7.2) by Fourier analysis, thus yielding the result of all N computations. Since each computation could have been arbitrarily long, the time needed to decipher the N' values E_i' from the path $X_s(t)$ is usually negligible, of course. We may thus perform, and read the results of, N computations in essentially the same time τ required for an analogous classical Turing machine to perform a single similar computation.

Without the subquantum sensor, a quantum computer requires a mean running time of at least $N\tau$ to make all the results available to a user. However, by repeating appropriate measurements[73], a user will occasionally obtain the equivalent of a time $N\tau$ worth of classical processing, in only a time τ . One may then ask "where" these N computations were performed, suggesting Everett's "many-worlds" as an answer[73]. However from the viewpoint of the pilot-wave theory, the parallel computations are simply "performed" by the Schrödinger evolution of the wavefunction Ψ in configuration space, and the results are encoded in the path X(t) if the various branches overlap.

As we have stressed in Section 2.2.1 , the "observables" at the basis of classical and quantum theory are abstractions in the minds of classical experimenters, these abstractions deriving from the evolution of the wavefunction. Any theory of quantum computation is then necessarily, as is quantum "measurement", concerned primarily with the evolution of Ψ , the actual X(t) playing a marginal role. For instance, our example in Section 2.1.1 , which shows how a quantum "measurement" may give an erroneous result for the path taken in a two-slit experiment, shows equally how "quantum memory" may in fact have little relation to the actuality X(t). (As discussed in Section 2.1.1 , Deutsch's self-measuring quantum computer( ), which apparently "sees two worlds", in our view illustrates the breakdown of the phenomenological book-keeping device, known as quantum "measurement" theory, which has been constructed by classical experimenters).

The various branches of Ψ , containing different "computations", are regarded in the many-worlds interpretation as representing real parallel universes. While "reality" in the pilot-wave theory is by definition the variables X(t), these being merely guided by Ψ , and occupying a definite point in configuration space. Since classical language derives from narrowly peaked wavefunctions, the

various branches of Ψ appear, in the classically-inspired language of quantum "measurement", as "ghost worlds". And a subquantum demon or automaton would be sensitive to these "ghost worlds". (For instance each of the above N computations could be identified with a "ghost world history"). But we emphasise that these are not real parallel universes, and a subquantum demon familiar with the pilot-wave theory would not regard them as such. As a very crude analogy, one might consider a ship being guided by external radio waves[74]. Imagine that the ship is programmed to move orthogonally to the wave crests. Then while there is only one ship trajectory, the radio waves effectively contain a whole set of unactualised ship trajectories, which would have occurred had the ship started elsewhere. And if the radio waves undergo reflection or two-slit diffraction, and reoverlap near the ship, then the trajectory will become more complicated while of course remaining unique. Yet if an outside observer based his worldview on the behaviour of radio wavepackets, he might say that the ship is receiving instructions from "more than one world".

Quantum computation is a generalisation of classical computation. Given the classical theory the quantum generalisation follows compellingly, in a manner quite analogous to the way in which quantum "measurement" is inspired by classical theory, the memory and other Turing machine variables being promoted to operators. We have seen that the addition of a subquantum sensor enables one to make full use of the parallel processing power of quantum computers. But what about having a fully subquantum computer?

One might try the following approach: The processor and memory of a fully subquantum computer could be built from two-state systems, each consisting of a particle in a box with state "0" or "1" if the particle is on the left or right-hand-side respectively. (In view of the field theory view of "particles" in Section 4, one should really of course work with field variables, such as magnetic fluxes). In addition to these processor and memory variables X_p and X_m , one needs of course an "address" number X , which could be the integer part of the position of a further particle, free to move parallel to a string of our boxes (X then being Turing's "tape position"). So far $\{X_i\}=\{X_p,X_m,X\}$ could just as well be a classical-mechanical realisation of a Turing machine (given appropriate inter-particle interactions). However these variables are generally guided in the pilot-wave theory by

$$\dot{X}_i = \mathrm{Im}\,\frac{\partial}{\partial X_i}\ln\Psi(X_1,\ldots X_n,\ldots,t) \qquad (7.3)$$

and each $\dot{X}_i$ generally depends on all the other X_j $(j\neq i)$. If Ψ is appropriately narrowly peaked, corresponding to the classical limit, then one effectively has a classical computer. But what will happen for more general Ψ ? An arbitrary Ψ

would of course not result in a useful evolution of the X_j . Though presumably there do exist some nonclassical Ψ which would yield something interesting. The question is: Is there a useful and general specification of a subquantum automaton?

That there might not be is suggested by the following: The classical theory of computation, and its quantum generalisation, is inspired by local mechanistic physics, i.e. computers are usually devices constructed from parts which interact locally according to preassigned rules. Now not only locality, but also the usual idea of "mechanism", fails in the pilot-wave theory, where the "puppet" variables X are free to follow whatever Ψ dictates. It then seems that a general simple theory, analogous to Turing's for the classical case, may not be possible at the subquantum level; that any such theory would be restricted to a particular class of wavefunction Ψ (such as those yielding classical behaviour), in contrast to the situation with quantum computers. (For this reason we have not attempted a precise definition of subquantum automata). On the other hand, it might be that the holistic subquantum level offers entirely new possibilities. For while current theories of computation are inspired by local mechanism, an entirely new "holistic" approach to computation might be necessary in order to realise the possibilities inherent in the pilot-wave theory, leading to holistic computers which might be better termed "organa" than "automata". And such a development might throw light on theories of human and artificial intelligence: According to Pribram[75] the brain stores memories "holographically" (involving spatial Fourier transforms). This of course need not have anything directly to do with the subquantum level; holistic structures like holograms exist even at the purely classical level. Nevertheless a subquantum approach to computation might help stimulate a new line of thought, which might aid the understanding of human intelligence (even if the latter should prove to be an essentially classical phenomenon) and which might lead to a better theory of artificial intelligence.

Our present inability to control the subquantum level of course bars the deliberate construction of a subquantum automaton or "sensor". But could such a structure perhaps arise spontaneously, if not by chance then by a process of "evolution" analogous to that which has taken place on the Earth's surface apparently at the purely classical level? For the possibility of parallel processing and of nonlocal remote sensing would perhaps endow such a structure with an evolutionary advantage. Biological evolution requires classical disequilibrium, and has thus been possible because the classical heat death of the universe has not occurred, while the subquantum heat death of the universe apparently has, suggesting that a subquantum automaton cannot spontaneously evolve, such evolution presumably requiring disequilibrium $P\neq|\Psi|^2$. However, very small or rare disequilibrium $P\neq|\Psi|^2$, of which we are at present unaware, must in fact exist in our universe (see Section 8), so that if operation at the subquantum level should offer a sufficiently large evolutionary advantage, then it might be that this disequilibrium could lead to the development of some such automata.

It may seem gratuitous to speculate that perhaps the human mind operates to some extent at the subquantum level. Amusingly, such a suggestion could straightforwardly be tested, given the means to (gently) accelerate a human being to near light speed. For as we have seen, Lorentz symmetry and universal time dilation for moving systems are merely properties of equilibrium. Thus, if for instance conscious thought makes use of the subquantum level, then the thoughts of a rapidly moving human being should not be simply slowed by $(1-v^2)^{-1/2}$, as presumably the bodily processes would be.

8. DEVIATIONS FROM STANDARD QUANTUM THEORY

Our aim in this Section is to show that the pilot-wave theory is, contrary to widespread belief, in principle distinguishable from standard quantum theory. No practical test will be proposed here. However, once two theories are known to be in principle experimentally distinct, it is likely that a practical method for distinguishing them may eventually be found.

When one derives a well understood theory from a deeper but less familiar one, it is all too easy to convince oneself that the latter is completely equivalent to the former. And given the equilibrium distribution $P=|\Psi|^2$, the pilot-wave theory indeed seems to merely reproduce standard quantum theory. (Apart from in the theory of gravitation, in Section 6, where the pilot-wave theory naturally leads to the possibility of an effective "cold dark matter" in addition to ordinary matter). However this is not the whole story.

8.1 Finite-ensemble corrections

As pointed out in Section 2.2, the very concept of a smooth distribution $P=|\Psi|^2$ being realised in Nature is strictly valid only in the purely theoretical limit of an infinite ensemble. For a finite ensemble of n systems with configurations X_1 , X_2 ,..... X_n , the actual fractional distribution will be the sum of delta-functions in (2.8). For any finite n, the fraction of systems occupying the configuration space volume δV can only approximate $|\Psi|^2\delta V$, so that equilibrium can only approximately be attained. (Just as for instance the salinity of sea water can never be exactly uniform, owing to the particulate nature of salt). Thus any finite ensemble necessarily departs from equilibrium, so that in any finite region, such as a finite laboratory consisting of a finite number of atoms, equilibrium quantum and relativity theory can never be exactly valid.

One might object that even in a finite laboratory, by repeating measurements with even just a single system, one may generate an arbitrarily large "time-ensemble", so that an arbitrarily exact distribution $P=|\Psi|^2$ is possible, even in the absence of a real infinite ensemble (i.e. of an infinite collection of simultaneously present and separate similar objects). This is of course true, provided the same state preparation is repeated prior to each measurement. However, if we confine ourselves to a single moment, or finite period, of time, we may ask if the actual state of affairs in Nature (i.e. the actual momentary distribution of physical variables), in a finite region, may correspond exactly to $P=|\Psi|^2$. If the number of variables is finite, such exact correspondence is impossible, with the implication that properties which are specific to $P=|\Psi|^2$, such as locality, cannot be exactly valid at or during the said time in the said region.

Thus the distribution $P=|\Psi|^2$ can be a general result, actually realised in Nature, only for real infinite ensembles, where these may not exist in Nature at all (if the universe is finite), and which certainly do not arise in a finite region.

This elementary point implies that, in a finite region, equilibrium properties such as signal-locality, translational invariance, and Lorentz symmetry, cannot be exact. (This situation is similar to that of classical thermodynamics, which becomes exact in the purely theoretical "thermodynamic limit" of systems with infinite volume, but which is never exact for any finite system).

As an explicit example, noted already, the instantaneous change $\Delta\rho$ (given by (2.13)) in the distribution of particles in a box, owing to a sudden change in the Hamiltonian of a distant entangled box, can only strictly vanish if the initial joint distribution $\rho_o=|\psi_o|^2$ exactly, which is only possible for an infinite number of particles. It must then be concluded that in real experience there are always weak instantaneous signals connecting distant macroscopic bodies, and the magnitude of such signals may be straightforwardly estimated from (2.13). To claim that such signals would be masked by the uncertainty principle is of course untenable, since the uncertainty principle is itself only exactly valid in exact equilibrium. (Appeal to the uncertainty principle would of course be in any case irrelevant, should the instantaneous connections in question be between a distant macroscopic body and the "brain" of an intelligent subquantum automaton. This latter example, while extreme, nevertheless serves our purpose of showing how the pilot-wave theory in principle leads to physics beyond the scope of standard quantum theory).

Thus standard quantum theory and relativity are merely coarse-grained approximations, like classical thermodynamics, which become exact only in the theoretical "thermodynamic" limit $n \longrightarrow \infty$ of infinite ensembles (and in the approximation where experimenters behave classically). While these approximations cover a huge range of real experience, they cannot cover the whole of reality, and it is clear that, at some level of accuracy, they must break down (as must classical thermodynamics): Since no finite region of the universe can be in exact equilibrium $P=|\Psi|^2$, there must exist small corrections to these theories in real experience.

Apart from these small corrections, which certainly must exist, it might be suggested that finite-ensemble disequilibrium could lead to systematic effects whose net results could build up and become large over time. For instance there might exist a macroscopic quantum phenomenon, perhaps involving superconductivity, which is sensitive to the fine-grained disequilibrium $P\neq|\Psi|^2$ of, for instance, the finite number of electrons in a finite sample. (This possibility, and others, are presently under investigation).

It may also be worth remarking that, for finite n, an initial approximate equilibrium $P_o \approx |\Psi_o|^2$ need not necessarily remain so, the theorem of preservation of $P=|\Psi|^2$ in time strictly holding only if $P_o=|\Psi_o|^2$ exactly.

8.2 Residual disequilibrium

Besides the above irreducible finite-ensemble disequilibrium , there may exist an additional disequilibrium reflecting a lack of thorough mixing $P \longrightarrow |\Psi|^2$, a disequilibrium which may be present even for infinite ensembles (in an infinite universe). Assuming, for definiteness, that the relaxation $P \longrightarrow |\Psi|^2$ took place soon after the big bang, where a finite time has elapsed since that epoch, one expects that, below a sufficiently small coarse-graining volume δV, there is a residual disequilibrium $P \neq |\Psi|^2$. For example: Consider an infinite ensemble of Hydrogen atoms in the ground state, distributed over infinite space. In exact equilibrium, the actual fractional distribution of electron radial positions r (with respect to their nuclei) would be the smooth function $\rho_o(r) \propto \exp(-2r/a_o)$ where a_o is the Bohr radius. (Again, we use the nonrelativistic particle picture for illustration, despite our "field lump" view of "particles" in Section 4). However, the actual distribution might contain fine-grained irregularities, left over from disequilibrium at early times. For instance the actual distribution could be $\propto \exp(-2r/a_o)\cos^2 kr$ where $1/k \ll a_o$, this distribution approximating $\rho_o(r)$ only on a coarse-grained level $\gg 1/k$. And such irregularities would be detectable by sufficiently precise position measurements, performed on a sample which is sufficiently large for typical statistical fluctuations (for finite samples) to be small compared to the irregularities being sought. Any estimate of the magnitude of such residual disequilibrium, and of the relevant scale δV, must of course await a detailed model of $P \longrightarrow |\Psi|^2$ near the big bang. For the time being, one wonders to what accuracy the distribution $P=|\Psi|^2$, for instance in Hydrogen, has actually been tested so far.

At issue is the extent of occurrence of the subquantum heat death of the universe. We note the contrast with the classical case: A classical physicist familiar with Boltzmann's H-theorem could reasonably guess, a priori, that the observed universe should to good accuracy be in a state of classical heat death. Indeed, the fact that it manifestly is not was once claimed by Culverwell[76], a century ago, to be evidence against the validity of the H-theorem (a claim which Boltzmann countered with the suggestion that our universe has arisen from a rare fluctuation, which we happen to occupy since a (more likely) equilibrium universe could not contain observers). In contrast, the subquantum heat death, expected on the basis of the pilot-wave theory and the subquantum H-theorem, has in fact occurred to good accuracy at least for systems presently accessible to us. On the

other hand, there could conceivably exist distant regions of the universe which are still significantly out of equilibrium, due to inefficient mixing (for example).

Possible cosmological implications of large-scale disequilibrium, both in the present epoch and at early times, deserve investigation. For instance, distant matter which is far from equilibrium would obviously have nonstandard relative intensities in its emission spectrum, conceivably observable from Earth. And the possible effect of disequilibrium at early times has been noted in Section 6.3 .

It is thus not only conceivable that there exists a residual disequilibrium throughout the universe, left over from early times but too small to have been noticed so far; it could also be that there exist distant regions of the universe containing, say, clouds of Hydrogen gas which is far from equilibrium.(Inversely to the classical case, it could be that we happen to inhabit a region of the universe which is to good accuracy in subquantum equilibrium). Once disequilibrium were discovered, it could be put to practical use - for example for instantaneous signalling; just as one makes use of a chance concentration of gold in the Earth's crust. And as noted in Section 7, the presence of such disequilibrium implies the theoretical possibility of the spontaneous evolution of subquantum automata.

Let us consider more carefully how, for example, a cloud of gas in disequilibrium could be found and utilised. Imagine first of all that a detailed theory of the big bang, and of the accompanying relaxation $P \longrightarrow |\Psi|^2$, predicts a non-negligible probability δ for finding such a cloud somewhere within our local region of the universe. (The theory is presumed to be trustworthy on other grounds. Some such theoretical prediction is necessary, in order to distinguish from the case of rare fluctuations - see below). Imagine further that an experimenter, on drawing a random sample of atoms from a gas cloud, finds a distribution (say of electron positions) which is far from equilibrium. And assume the sample to be so large that the probability ε for such a nonequilibrium distribution to be drawn purely by chance, from a cloud actually in equilibrium, is $\varepsilon \ll \delta$. The experimenter must then conclude that the cloud as a whole is almost certainly out of equilibrium, and almost certainly has the distribution indicated by his sample; i.e. he must assume on statistical grounds that he has come across one of the disequilibrium clouds predicted by the theory which he believes. Assuming the rest of the cloud to have the said distribution, he may then make practical use of it. For instance he could use it for instantaneous signalling, by means of a pair of entangled boxes of atomic electrons drawn from the cloud, as in Section 2.2.3 : If the initial distribution $\rho_o \neq |\Psi_o|^2$ is known, the nonlocal response $\Delta\rho$ may be calculated from (2.13). In this way, various experimenters could use the cloud atoms for signalling to eachother at a distance.

It should be noted that all this could, strictly speaking, be accounted for

by standard quantum theory, which predicts a definite distribution $P=|\Psi|^2$ only for an infinite ensemble: For finite samples any distribution is in principle allowed, finite distributions $\approx |\Psi|^2$ being merely more likely. Since no real experiment can ever measure an infinite sample, any real (finite) sequence of results, however abnormal, may in principle be ascribed to "pure chance", including the above experimenter's apparent discovery and use of a disequilibrium cloud. Standard quantum theory could simply claim that the initial sample was found to be far from equilibrium by pure chance, and that the rest of the atoms continued to follow the dsitribution indicated by this sample again purely by chance. Even the experimenters' apparent success in "communicating" with eachother at a distance could be seen as pure chance: The distant atoms just happened to yield results corresponding to the intended message. Such an interpretation, while logically possible, is of course highly artificial. For such a cloud could in the future, in principle, become a resource used on an everyday basis for practical communication, and quantum theory would simply lose all explanatory power. And in any case, the probability of such a purely chance sequence occurring is fantastically small in comparison with the probability δ of finding a disequilibrium cloud predicted by the pilot-wave theory (as we have assumed); so that if such a cloud were actually found and successfully used, one would sensibly take this as clear evidence in favour of the pilot-wave theory against quantum theory.

We note also the contrast between pilot-wave and quantum theory: The experimenter's deduction above, on the basis of the pilot-wave theory, that the cloud as a whole is in disequilibrium (despite having measured only part of it), is only possible because the pilot-wave theory assumes a past causal history behind the outcomes, this history not necessarily being one which leads simply to $P=|\Psi|^2$. In contrast standard quantum theory assigns a probability $P=|\Psi|^2$ to all events, and admits of no causal history behind these.

8.3 Rare fluctuations

At the turn of the century, atomic theory was able to account for classical thermodynamics, and yet made no decisive new predictions, enabling many physicists to take a sceptical view as to the reality of atoms. Now one way to distinguish, in principle, between classical kinetic theory and classical thermodynamics, is to simply wait for an extremely long time: For according to the former, the oceans will occasionally spontaneously boil, while classical thermodynamics firmly predicts that this will never happen. And if such a rare thermal fluctuation did occur, human experimenters could clearly extract useful work from it.

The question then arises as to whether or not an extremely rare large scale fluctuation $P\neq|\Psi|^2$ could effectively lead to nonequilibrium physics. Indeed it could, though some care is required in the interpretation, and in the comparison

with standard quantum theory.

Consider an ensemble of systems, such as a cloud of Hydrogen atoms in some region of the universe, which is believed to have undergone thorough mixing. The actual distribution ρ of electron positions is then almost certainly $\approx |\Psi|^2$ (though it might not be). An experimenter extracts a random sample of atoms, and happens to find a distribution far from equilibrium (he may for example find all the electrons of his sample to be exceptionally near their nuclei). What should he conclude with regard to the distribution for the rest of the cloud? Unlike in the earlier case of residual disequilibrium, he should not suppose the rest of the cloud to be far from equilibrium. For while the probability is very small of drawing a far-from-equilibrium sample from an equilibrium distribution, the probability is nevertheless very much smaller that the whole cloud happens to be far from equilibrium, since we know it to have been thoroughly mixed. Indeed, thorough mixing implies that each atom has an independent probability distribution $|\Psi|^2$ associated with it, and any irregular sample drawn implies nothing as to the distribution of the remaining atoms, and must be ascribed to pure chance in the sampling. In contrast, in the case of residual disequilibrium, theory predicted a non-negligible probability of finding a badly mixed cloud, this probability being much larger than that of drawing a nonequilibrium sample from an equilibrium cloud, justifying the experimenter's opposite conclusion in that case.

To make clear the distinction between the case of residual disequilibrium, and the case of rare fluctuations despite thorough mixing, consider a simple analogy: One hundred boxes, each containing a thousand coins, are prepared, shaken, and then buried in random locations. For the case of "residual disequilibrium", let each box be initially prepared with all coins showing "heads", and let each box be shaken only once (and gently), and then buried. Suppose, by analysis of the physics of the (gentle) shaking process, one calculates the probability for each box to remain far from equilibrium, i.e. to remain with nearly all coins showing heads, to be 10^{-2}. An experimenter who knows the past history of the boxes finds one of them by chance, and assigns a probability 10^{-2} for it to be far from equilibrium. If he extracts, at random, one hundred coins, and finds them all showing heads, what does he conclude? The probability 10^{-2} that he has dug up a nonequilibrium box is fairly small, but nevertheless the probability $\sim 1/2^{100}$ of extracting one hundred heads from a 50:50 mix of one thousand heads/tails is far smaller, so the experimenter may be virtually certain that he has indeed come across one of the one-in-a-hundred nonequilibrium boxes predicted by theory, and that the remaining nine hundred coins are mostly heads. In contrast, for the case of "rare fluctuations", let each box be thoroughly shaken before burial. The same experimenter, who again knows this past history, upon finding

a box, assigns (for example) the probability $1/2^{1000}$ for all the coins to be heads. If he finds that a random sample of one hundred coins are all heads he will nevertheless assume that the rest of the box is almost certainly a roughly even mix, each coin being effectively independently tossed by the thorough shaking. Thus he would assume that his sample of one hundred heads was pure chance, i.e. he would assume the sample to have been a "rare fluctuation", and not that the whole box contains a rare fluctuation.

This example makes clear the essential role played by theory, and by knowledge of past history, in the application of probability theory and in particular in the proposal of a "best hypothesis" on statistical grounds applied to a sample. If a gas cloud is known, or believed, to have been thoroughly mixed, it could of course be that, by a chance fluctuation, it is in fact far from equilibrium. Nevertheless, our pilot-wave experimenter drawing samples from the cloud can never conclude anything with regard to the rest of the cloud, and will discover such disequilibrium only by actually measuring all (or most of) the atoms in the cloud. This implies that rare fluctuations cannot be straightforwardly put to practical use, since to become aware of them one must essentially measure the whole ensemble, and thereby disturb at least the wavefunction of each system. In contrast, in the classical analogy of a spontaneously boiling ocean, the disequilibrium may be confirmed by measurements of temperature throughout the ocean, without significantly disturbing the system.

Nevertheless, a rare fluctuation would spontaneously create effective non-equilibrium phenomena, such as large-scale nonlocal connections (the "random instantaneous signals" of Section 3.2), these phenomena being however uncontrollable by us. And they would be explained just as easily by standard quantum theory, which predicts exactly the same fluctuations for finite samples. However, even in this case, the pilot-wave theory contains a further possibility, completely outside the scope of quantum theory: In principle, in a region of the universe containing a large scale rare fluctuation, there could spontaneously evolve a race of subquantum automata, as discussed in Section 7, and they would be able to make practical use of the disequilibrium in which they find themselves (c.f. the example in Section 7 of an automaton operating "inside the wavefunction"). Further, such automata might send instantaneous signals on request from us, thus enabling us to indirectly make use of the disequilibrium, or otherwise they could simply communicate to us the actual disequilibrium distribution for a rare fluctuation gas cloud, enabling us to make direct use of it for ourselves. (Again, this extreme example serves the purpose of showing that the pilot-wave theory is in principle distinct from standard quantum theory).

The probability for a rare fluctuation, assuming thorough mixing, may be

readily calculated from $P=|\Psi|^2$. Consider for instance a universe containing 10^{80} Hydrogen atoms in the ground state. Let r_o be such that the probability, in equilibrium, of the electron being at a distance $r \leq r_o$ from its nucleus is $P(r \leq r_o) = 1/2$. Dividing the universe into $10^{80}/N$ localised regions of N atoms each, the probability p of finding a localised region with all N electrons such that $r \leq r_o$ is $p \sim (1/2^N)(10^{80}/N)$. For $N \sim 10^{23}$, p is vanishingly small. To obtain $p \sim 1$ requires $N \sim 200$. It is therefore likely that, in the said universe, there exists somewhere a localised bunch of ~ 200 atoms whose electron distribution is far from the expected $\approx |\Psi|^2$. Hardly an encouraging result. On the other hand, in an infinite universe, there will exist somewhere, with probability=1, arbitrarily large regions which are far from equilibrium.

In the classical case, the task of distinguishing between kinetic theory and thermodynamics was accomplished as follows: The fluctuations predicted by kinetic theory were eventually shown to lead to observable effects, such as Brownian motion, which contradict classical thermodynamics. One might then search for an analogous effect by which to test the pilot-wave theory. However, quantum theory is already a statistical theory which predicts fluctuation effects: Indeed quantum theory might be compared with the fluctuation formulas of classical statistical mechanics deprived of their kinetic/mechanical basis. To distinguish quantum and pilot-wave theory purely on the basis of rare fluctuations is then a much more difficult task, as we have seen in the above. (Though a systematic effect analogous to Brownian motion might arise from finite-ensemble disequilibrium, as hinted in Section 8.1).

9. FURTHER REMARKS

9.1 Connection to the path-integral formulation

In the path-integral formulation of standard quantum theory, the holistic nature of the quantum level is accounted for by summing over alternative paths, which roughly speaking effectively "sense" the whole environment. In the pilot-wave theory, in contrast, there is only one trajectory associated with each event, and this trajectory is itself directly sensitive to the whole environment. The question then naturally arises: What is the relation between the pilot-wave theory trajectory and the alternative Feynman paths? A simple and appealing relation in fact exists.

Consider a single nonrelativistic particle over an infinitesimal time interval $(0, \varepsilon)$. The wavefunction may be written in the path-integral form

$$\psi(X,\varepsilon) \propto \int dX_o \psi(X_o,0) \exp i[(1/2)m(\frac{X-X_o}{\varepsilon})^2 - V(\frac{X+X_o}{2})]\varepsilon \equiv \int dX_o \alpha(X_o,X)\psi(X_o,0)$$

so that the velocity $\dot{X}$ of the pilot-wave theory trajectory, at time ε and position X, is

$$\dot{X} = (1/m)\mathrm{Im}(\frac{1}{\psi}\frac{\partial\psi}{\partial X}) = \mathrm{Re}\frac{\int dX_o \alpha(X_o,X)[\frac{X-X_o}{\varepsilon} - \frac{\varepsilon}{2m}V'(\frac{X+X_o}{2})]\psi(X_o,0)}{\int dX_o \alpha(X_o,X)\psi(X_o,0)} \qquad (9.1)$$

where $\alpha(X_o,X)$ is the so-called "amplitude to go from X_o to X".

The interpretation of (9.1) is clear: The Feynman path velocity at time ε and position X is just the velocity $(X-X_o)/\varepsilon$ at time $\varepsilon/2$ (and position $(X+X_o)/2$) corrected by the action of the force $-V'(\frac{X+X_o}{2})$ over the remaining time interval $\varepsilon/2$. Summing over all initial points X_o, weighted by the amplitude $\alpha(X_o,X)\psi(X_o,0)$ yields the pilot-wave $\dot{X}$. Thus the velocity predicted by the pilot-wave theory is simply the amplitude-weighted "mean" of the Feynman path velocities.

9.2 Against subquantum stochasticity

The above connection to the path-integral formulation might, at first sight, suggest that the velocity predicted by the pilot-wave theory is merely an average over some underlying statistical distribution of velocities, perhaps suggesting the existence of stochastic "fluid fluctuations" at the subquantum level. However, the "amplitudes" $\alpha(X_o,X)$ are complex numbers and are not added to yield total probabilities according to the rules of probability theory. And as already mentioned, Koopman and Ballentine have shown the fallacy of "quantum probability". Thus, while the relation (9.1) is heuristically appealing as a "mean", there is

no justification for regarding it as a true mean value.

A relation analogous to (9.1), expressing the pilot-wave theory velocity as the "mean" of an underlying distribution, has been found by Sonego[77], in the context of a "hydrodynamical" formulation of quantum theory which makes use of joint position-momentum probability distributions which may become negative ("quasiprobabilities"). We have already argued in Section 2.2.1 that the attainment of negative values by such joint distributions is an "epicycle" which arises from ignoring the contextual nature of momentum "measurements", thus undermining the suggestion by Sonego, on the basis of the said relation, that trajectories predicted by the pilot-wave theory are merely average values.

The relation (9.1), and that found by Sonego, as well as the well known analogies between the Schrödinger and diffusion equations, may nevertheless seem vaguely suggestive of an underlying stochasticity at the subquantum level. For one may ask why it should be, if there is no such stochasticity, that such relations exist at all. This question may be answered as follows: As stressed throughout this work, classical concepts such as energy, momentum, and force, are to be regarded as mere equilibrium phenomenology, arising from the deeper pilot-wave theory. If one takes this view, it is then really not at all surprising that, if one views the situation the wrong way round, i.e. if one tries to explain quantum or pilot-wave theory on the basis of these derivative classical concepts, then one sees tantalising but physically obscure mathematical relations which seem to support such an enterprise.

The addition of background "fluid fluctuations" to the pilot-wave theory, by Bohm and Vigier[13] and Bohm and Hiley[14], was done partly in order to derive the equilibrium distribution $P=|\psi|^2$. This motivation is undermined by our subquantum H-theorem.

Attempts at a realistic explanation for quantum theory have often, almost by instinct, begun by assuming an underlying stochastic process. At least part of the reason for this is the contextuality of quantum theory which, if ignored, generates an apparent stochasticity.

For instance Von Neumann[78] argued that if there were a hidden variable explanation for quantum theory, then these hidden variables would have to fluctuate arbitrarily rapidly. His argument went essentially as follows: If a spin-1/2 particle with state $|+z\rangle$ (spin up the z-axis) is measured along x, one obtains either +x or -x with equal probability. If the same particle is then measured along z, one obtains either +z or -z with equal probability, and the particle appears to have no memory whatever of having previously had spin +z. And since this sequence of measurements could have been performed arbitrarily rapidly, it appears that the hidden variables determining the outcomes must be fluctuating

arbitrarily rapidly. The fallacy in this reasoning is of course that it ignnores contextuality: The "measurement" outcomes also depend on the apparatus, for instance on the orientation of the Stern-Gerlach magnetic field (this is clear in the pilot-wave theory). If the sequence of measurements is performed rapidly, the magnetic field orientation must be changed equally rapidly, with the consequent rapidly changing effect on the outcomes.

The illusory stochasticity in (9.1), and in the relation derived by Sonego, is of course also indirectly due to ignoring contextuality: For it is contextuality which shows the fallacious nature of "probability amplitudes" and of "quasi-probabilities".

9.3 Possible extensions of the pilot-wave theory

We have adopted the pilot-wave theory in its simplest possible form, with the trajectory $\pi = \partial S/\partial X$, without the addition of any arbitrary features such as background "fluid fluctuations"; and we have been able to derive $P=|\Psi|^2$ on this basis (with certain reasonable assumptions). This is analogous to the kinetic theory of gases of Maxwell and Boltzmann, which assumed the simplest possible form for the hypothetical atoms and molecules: colliding elastic spheres governed by Newtonian mechanics. The simple motion $\pi = \partial S/\partial X$ may turn out to be merely a crude model, as did the elastic spheres of kinetic theory. But one may reasonably hope that the main features are more or less correct - as is the case with the elastic spheres. It is surely best to adopt the simplest assumptions as to hypothetical elements of a theory, and to follow these to their logical conclusions. On this ground we maintain that the simple pilot-wave theory offers the best starting point for further research, and that modifications to it should be considered only for compelling theoretical or experimental reasons.

Experimental hints as to how the pilot-wave theory might break down are of course absent at the present time. Compelling theoretical reasons for modifying the theory might include the following: (i) As we have discussed, the guiding field Ψ may be pragmatically regarded simply as a convenient mathematical summary of the motion X(t), while a more illuminating theoretical interpretation sees Ψ as an "informative field". Either view might be taken as provisional, and a deeper explanation for Ψ might be sought. And a compelling explanation might naturally lead to corrections to the pilot-wave theory. However, as stressed already, in making such an attempt one should surely not assume that the deeper level is based on our phenomenological classical-mechanical concepts (as are "fluctuating fluids" for instance). (ii) A way to unify the various fields found in Nature might naturally suggest, or be suggested by, a modification of the simple pilot-wave theory. For instance the relation between gauge symmetries of fields and phase symmetries

of Ψ might be worthy of study. (iii) The subquantum H-theorem shows that evolution towards equilibrium may be accounted for by the simple pilot-wave theory, but nevertheless it might turn out, on studying concrete models, that some sort of perturbations to the path $\pi = \partial S/\partial X$ need to be added in order for an ensemble to actually reach equilibrium. This seems unlikely but might be the case. If so, such perturbations could be very small: For all known physical systems have had a long and violent astrophysical history, providing ample opportunity for "mixing" of P and $|\Psi|^2$. Given this past history, very small perturbations should be sufficient to ensure that equilibrium is reached, just as the celebrated "speck of dust" ensures that equilibrium is reached, after a sufficiently long time, for the radiation in a blackbody cavity. (The violent perturbations for single systems, suggested by Bohm and Hiley, should certainly be unnecessary). And the magnitude and form of the required perturbations, if any, might be suggested by detailed study of concrete models.

9.4 On the abandonment of "Cartesian order"

As stressed by Bohm[21],[79] and suggested by the insights of Bohr[80] (with regard to the role played by the whole experimental arrangement), quantum theory seems to point to a new kind of "order", very different in spirit from the usual "Cartesian coordinate order" of classical physics; a new order which roughly speaking resembles more that of the hologram than of the photographic plate (the "whole" versus the "sum-of-parts"). Does this mean, as suggested by Bohm, that the very concepts of space and time must be abandoned? While this might of course be the case, we point out that a change to a radically new kind of order need not be accompanied by an abandonment of older concepts; that a change of viewpoint with regard to these may be sufficient.

As an example, one might compare the two approaches to geometry, (i) Euclidean geometry as understood in ancient Greece, and (ii) The coordinate geometry initiated by Descartes. In the former, "extension" was associated with bodies rather than with space itself, and shapes such as triangles and spheres were abstractions from physical objects. As Einstein put it[81], the Euclidean approach does "not really deal with space as such". In contrast, in coordinate geometry, by introducing coordinates covering empty space, this space itself came to be regarded as the object of study. And the outlook provided by coordinate geometry was of course crucial for the subsequent development of mathematics and physics. (Further, at least according to Spengler[30], the difference in spirit behind these two systems parallels with the fundamental differences between the Classical and Western cultures). Here we have a case, then, where a highly significant change in "order" took place, not by throwing away the old concepts, but rather

by shifting our viewpoint with regard to them.

Similarly, a further shift of viewpoint with regard to space and matter may be a sufficient step away from the "Cartesian order". For historically, the order conceived by Descartes included the mechanical idea of bodies or fluids filling space and interacting by contiguous action. Cartesian order, then, does not only refer to a certain approach to space, but also (and perhaps especially) to a particular mechanical conception of matter. (The "coordinate grid" might be seen as a convenient means of describing such contiguous mechanical actions). This Cartesian conception was of course an inspiration for the development of classical mechanics (explaining both the resistance, especially in France, to Newton's "spooky" gravitational "attraction" at a distance, and the attempts to explain it in terms of the contiguous action of fluids, "vortices", etc.). And in classical field theory, the Cartesian order survives as the contiguous interaction of neighbouring field elements by local differential equations. In the pilot-wave theory, all this is changed: Mechanical principles of interaction between different parts, or between localised variables of any kind, are completely banished from physical three-dimensional space. This space remains purely as a "puppet theatre", the evolution of physical variables (and of the spatial geometry itself) being controlled from configuration space, by Ψ . And this banishment of mechanical actions from space is the essence of the nonlocal "spookiness" of EPR-entanglement. In this way, while retaining the concept of space (and of time), the old Cartesian order has nevertheless disappeared.

10. CONCLUSION

> When all is said and done, however, the number of original contributions to art made since 1925 is probably very small: it seems likely that the remainder of the 20th century will be needed to digest the innovations made in its first quarter. Anyone confronted with the bewildering profusion of present-day art may well consider this a surprising statement; we are always being persuaded that there has never been a more revolutionary period, never an age when art was more experimental. This remark, however, has been made about contemporary art for a great many years now - certainly since Manet exhibited at the Salon des Refusés, and probably since the time of the French Revolution and the rise of Romanticism.
>
> - Alan Bowness, Modern European Art (1972)

In 1927 Einstein wrote[82], in a letter to Hedwig Born, of the "tragic" invasion of irrationalism in physics at the time. If one believes that in the Europe of the 1920s, a highly questionable philosophy gave shape to modern physics, then it is natural to go back to the early 1920s to see how things looked before physics took the path which led to standard quantum theory. And one finds, following the lead of the early realist pilot-wave constructions of Einstein and deBroglie, in the first years of that critical decade, that the pilot-wave theory was the obvious and natural alternative path which could have been taken - in particular if nonlocality had been acceptable at the time.

Development of the pilot-wave theory leads to a rich harvest of insights, and to considerable simplification of the foundations of contemporary physics: The probability distributions of quantum theory may be derived, and need not be postulated. The conspiratorial relation between relativity and quantum theory is explained. Standard "measurement" theory may be derived as a phenomenological device, and its peculiarities explained. Contextuality and nonlocality are clearly comprehensible. There need be no distinction in principle between systems, apparatus or observers. One arrives at a unified view of classical, quantum and subquantum physics, based on the concept of guiding fields. The field theory understanding of "particles" is made sharper. One may write a simplified theory of electrodynamics. Poincaré invariance is explained in terms of equilibrium symmetry. The pilot-wave theory of gravitation and cosmology overcomes the fundamental conceptual problems of the usual approach, and suggests alternative approaches to certain cosmological problems. There are intriguing implications for the theory of computation. And the whole theory is based on just a few simple assumptions.

The pilot-wave approach suggests a revival of "realism", and in particular of the view that it is the task of physics to propose conceptual and mathematical

representations of a world held to exist behind the illusory veil of appearances (the philosophy suggested, for example, by Plato's simile of the prisoners in the cave). Just as Newtonian mechanics sees Aristotlean physicists as being misled by their confinement to a friction-dominated environment, so the pilot-wave theory sees quantum physicists as being misled by their confinement to an environment which is to high accuracy in equilibrium $P=|\Psi|^2$, and by their own limited sensitivity which is essentially classical. Is this "physics" or "metaphysics"? Two millenia after the first proposal of the atomic theory, history suggests that it may indeed be "physics"; though there are no clear divisions between physics, metaphysics, and philosophy, and any choice in favour of the pilot-wave theory must be made in part on philosophical grounds. At the same time it is of course incumbent upon proponents of the pilot-wave theory to find a practical means of testing it. As we have seen, the prospects of such a test are not entirely remote.

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