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# A complete data-driven framework for the efficient solution of parametric shape design and optimisation in naval engineering problems

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## Abstract

In the reduced order modeling (ROM) framework, the solution of a parametric partial differential equation is approximated by combining the high-fidelity solutions of the problem at hand for several properly chosen configurations. Examples of the ROM application, in the naval field, can be found in [31, 24]. Mandatory ingredient for the ROM methods is the relation between the high-fidelity solutions and the parameters. Dealing with geometrical parameters, especially in the industrial context, this relation may be unknown and not trivial (simulations over hand morphed geometries) or very complex (high number of parameters or many nested morphing techniques). To overcome these scenarios, we propose in this contribution an efficient and complete data-driven framework involving ROM techniques for shape design and optimization, extending the pipeline presented in [7]. By applying the singular value decomposition (SVD) to the points coordinates defining the hull geometry — assuming the topology is unaltered by the deformation —, we are able to compute the optimal space which the deformed geometries belong to, hence using the modal coefficients as the new parameters we can reconstruct the parametric formulation of the domain. Finally the output of interest is approximated using the proper orthogonal decomposition with interpolation technique. To conclude, we apply this framework to a naval shape design problem where the bulbous bow is morphed to reduce the total resistance of the ship advancing in calm water.

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# 1 Introduction

The reduced basis method (RBM) [13, 21] is a well-spread technique for reduced order modeling, both in academia and in industry [24, 23, 31, 20], and consists in two phases: an offline phase that can be carried out on high performance computing facilities, and an online one that exploits the reduced dimensionality of the system to perform the parametric computation on portable devices. In the offline stage the reduced order space is created from full order complex simulations computed for certain values of the parameters. The selection of the reduced basis functions that span this new reduced space can be carried out by different techniques. In this work we employ the proper orthogonal decomposition (POD) [19, 3], which is based on the singular value decomposition (SVD), on the set of high-fidelity snapshots. After the creation of such space, in the online phase a new parametric solution is calculated as a linear combination of the precomputed reduced basis functions. The creation of a reduced order model is crucial in the shape optimisation context where the optimiser needs to compute several high-fidelity simulations.

Novelty of this work is the creation of a reduced order space containing the manifold of admissible shapes by applying POD over the sampled geometries, in order to reduce the parameter space dimension and to enhance the order reduction of the output fields. To generate the original design space we employ the free form deformation (FFD) method, a well-known shape parametrisation technique. Another approach for reduced order modeling enhanced by parameter space reduction technique can be found in [30] where they propose a coupling between POD-Galerkin methods and active subspaces. After the creation of the reduced space for the admissible shapes, we can exploit this new parametric formulation for the construction of the reduced space for the output fields, using the non-intrusive technique called POD with interpolation (PODI) [5, 6, 10] for the online computation of the coefficients of the linear combination. We would like to cite [16] where they present the concept of a shape manifold representing all the admissible shapes, independently of the original design parameters, and thus exploiting the intrinsic dimensionality of the problem.

This work is organised as follows: after the presentation of the general setting of the problem, there is a brief overview of the FFD method, then we illustrate how the parameter space reduction is performed, and we present PODI for the reduction of the high-fidelity snapshots. Finally the numerical results are presented with the conclusions and some perspective.

## 2 The problem

Let  $\Omega \subset \mathbb{R}^3$  be the reference hull domain. We define a parametric shape morphing function  $\mathcal{M}$  as follows

$$\mathcal{M}(\mathbf{x}; \boldsymbol{\mu}) : \mathbb{R}^3 \rightarrow \mathbb{R}^3, \quad (1)$$

which maps  $\Omega$  into the deformed domain  $\Omega(\boldsymbol{\mu})$  as  $\Omega(\boldsymbol{\mu}) = \mathcal{M}(\Omega; \boldsymbol{\mu})$ , where  $\boldsymbol{\mu} \in \mathbb{D} \subset \mathbb{R}^5$  represents the vector of the geometrical parameters.  $\mathbb{D}$  will be properly defined in Section 4. Such map  $\mathcal{M}$  can represent many different morphing techniques (not necessarily affine) such as free form deformation (FFD) [26], radial basis functions (RBF) interpolation [4, 17, 15], and the inverse distance weighting (IDW) interpolation [27, 33, 11, 2], for instance. In this work we use the FFD, presented in Section 3, to morph a bulbous bow of a benchmark hull. We chose the DTMB 5415 hull thanks to the vast amount of experimental data available in the literature, see for example [18]. In Figure 1 the domain  $\Omega$  and a particular of the bulbous bow we are going to parametrize and deform.

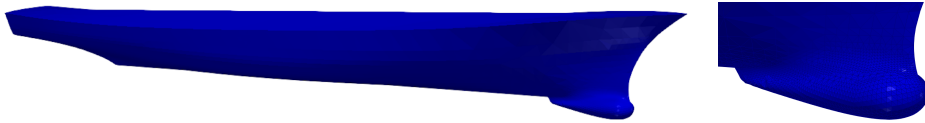


Figure 1: Complete hull domain representing the DTMB 5415 and, on the right, a zoom on the bulbous bow.

The pipeline is the following: using geometrical FFD parameters we generate several deformed hulls; then we apply the POD on the coordinates of the points describing the deformed geometries, and the new parameters will be the POD coefficients of the selected modes; after checking for possible linear dependencies between these coefficients, we sample the reduced parameter space producing new deformed hulls upon which we are going to actually perform CFD simulations. Regarding the full order model, we use the Reynolds-averaged Navier-Stokes equations to describe the incompressible and turbulent flow around the ship. The Froude number has been set to 0.2 and we chose the  $k-\omega$  SST model for the turbulence since it is one of the most popular benchmark for hydrodynamic analysis for industrial naval problems. In this way we reduce the parameter space, staying on the manifold of the admissible shapes, and reducing the burden of the output reduced space construction through PODI.

### 3 Free form deformation of the bulbous bow

Here we are going to properly define the deformation map  $\mathcal{M}$  introduced in Eq. (1), which we employed for this work, and that corresponds to the free form deformation (FFD) technique. The original formulation of the FFD can be found in [26], for more recent works in the context of reduced basis methods for shape optimization we cite [14, 22, 28]. It has also been applied to naval engineering problems in [7, 8, 32], while for an automotive case see [25].

The FFD map is the composition of three maps described in the following, while for a visual representation we refer to Figure 2:

- the function  $\psi$  maps the physical domain to the reference one where we construct the reference lattice of points, denoted with  $\mathbf{P}$  around the object to be morphed;
- the function  $T$  performs the actual deformation since it applies the displacements defined by  $\boldsymbol{\mu}_{\text{FFD}}$  to the lattice  $\mathbf{P}$ . It uses the B-splines or Bernstein polynomials tensor product to morph all the points inside the lattice of control points;
- finally we need to map back the deformed domain to the physical configuration through the map  $\psi^{-1}$ .

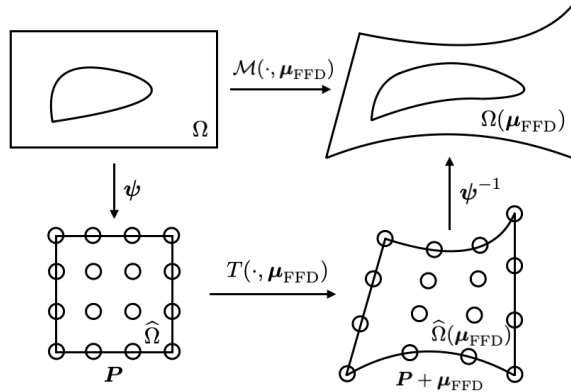


Figure 2: Sketch of the FFD map  $\mathcal{M}$  composition. The domain is mapped to a reference configuration, then the lattice of FFD control points induce the body deformation, and finally the morphed object is mapped back to the physical space.

So we can define the FFD map  $\mathcal{M}$  through the composition of the three maps presented above as

$$\mathcal{M}(\mathbf{x}, \boldsymbol{\mu}_{\text{FFD}}) := (\psi^{-1} \circ T \circ \psi)(\mathbf{x}, \boldsymbol{\mu}_{\text{FFD}}) \quad \forall \mathbf{x} \in \Omega. \quad (2)$$

In Figure 3 it is possible to see the actual lattice of points we used, in green, for a particular choice of the FFD parameters. For an actual implementation of this method in Python, along with other possible deformation methods, we refer to the open source package called PyGeM - Python Geometrical Morphing [1].

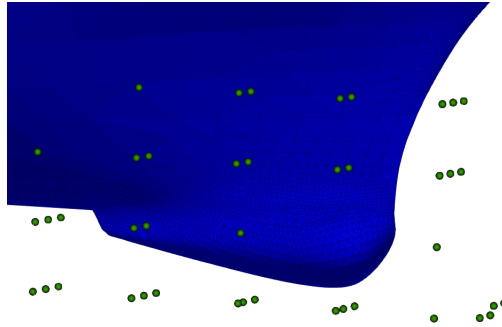


Figure 3: Example of FFD parametrisation and morphing of the DTMB 5415 hull. In green the lattice of control points that define the actual deformation.

## 4 Reduction of the parameter space through POD of the mesh coordinates

In order to reduce the parameter space dimension we apply the POD on a set of snapshots that depends on the FFD parameters. Each snapshot is the collection of all the coordinates of the points defining the *stl* file geometry. Since the generation of these snapshots does not depend on complex simulations but only on the particular FFD deformation, we are able to create a dataset with as many entries as we want. So we create a database of  $N_{\text{train}} = 1500$  geometrical parameters  $\boldsymbol{\mu}_{\text{FFD}} \in \mathbb{D} := [-0.3, 0.3]^5$  sampled with a uniform distribution. Moreover we create the corresponding database of mesh coordinates  $\mathbf{u}$  corresponding to these parameters, that is  $\Theta = [\mathbf{u}(\boldsymbol{\mu}_{\text{FFD},1}) | \dots | \mathbf{u}(\boldsymbol{\mu}_{\text{FFD},N_{\text{train}}})]$ . Then we perform the singular value decomposition (SVD) on  $\Theta$  in order to extract the matrix of POD modes:

$$\Theta = \Psi \Sigma \Phi^T, \quad (3)$$

where with  $\Psi$  and  $\Phi$  we denote the left and right singular vectors matrices of  $\Theta$  respectively, and with  $\Sigma$  the diagonal matrix containing the singular values in decreasing order. The columns of  $\Psi$ , denoted with  $\psi_i$ , are the so-called POD modes. We can thus express the approximated reduced mesh

with the first  $N$  modes as

$$\mathbf{u}^N = \sum_{i=1}^N \alpha_i \psi_i, \quad (4)$$

where  $\alpha_i$  are the so called POD coefficients. To compute them in matrix form we just use the database we created as follows

$$\boldsymbol{\alpha} = \boldsymbol{\Psi}^T \boldsymbol{\Theta}, \quad (5)$$

and then we truncate to the first  $N$  modes and coefficients.

After the selection of the number of POD modes required to have an accurate approximation of each geometry, we end up with the first reduction of the parameter space, that is with 3 POD coefficients  $\boldsymbol{\mu}_{\text{POD}} := \boldsymbol{\alpha} \in \mathbb{R}^3$ , we are able to represent all the possible deformations for  $\boldsymbol{\mu}_{\text{FFD}} \in \mathbb{D}$ . So we can express every geometry with 3 modes, but the coefficients can still be linearly dependent. We can investigate this dependence by plotting every component  $\boldsymbol{\mu}_{\text{POD}}^{(i)}$  against each other. As we can see from the plot on the left in Figure 4, we can approximate  $\boldsymbol{\mu}_{\text{POD}}^{(2)}$  with a linear regression given  $\boldsymbol{\mu}_{\text{POD}}^{(1)}$ . For what concerns  $\boldsymbol{\mu}_{\text{POD}}^{(3)}$ , we can constraint it to be inside the quadrilateral in Figure 4, on the right. So we are able to express every possible geometry described with the original 5 FFD parameters with only 2 new independent parameters. We can thus sample the full parameter space using a new reduced space, preserving the geometrical variability, and reducing the construction cost of the reduced output field space. This, as we are going to present, results in a faster optimization procedure.

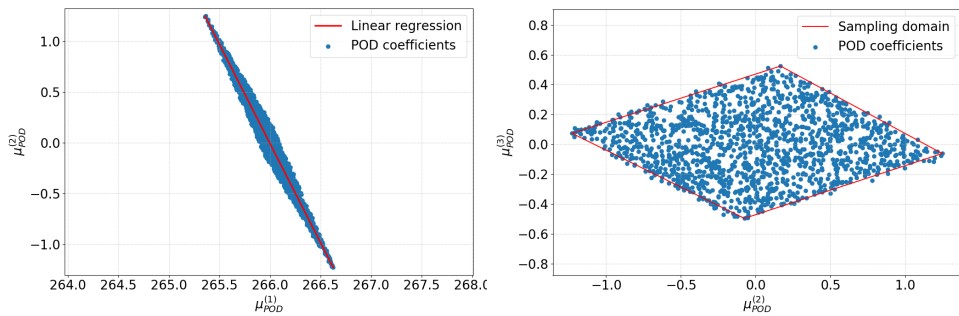


Figure 4: POD coefficients dependence. On the left we have  $\boldsymbol{\mu}_{\text{POD}}^{(2)}$  with respect to  $\boldsymbol{\mu}_{\text{POD}}^{(1)}$  in blue, and in red the linear regression to approximate one as a function of the other. On the right  $\boldsymbol{\mu}_{\text{POD}}^{(3)}$  against  $\boldsymbol{\mu}_{\text{POD}}^{(2)}$  in blue, and in red the boundaries defining the quadrilateral in which the sampling is performed.

## 5 Non-intrusive reduced order modeling by means of PODI

Proper orthogonal decomposition with interpolation is a non-intrusive data-driven method for reduced order modeling allowing an efficient approximation of the solution of parametric partial differential equations. As well as for the geometries, we collect in a database the high-fidelity solutions of several CFD simulations corresponding to different configurations, then we apply the POD algorithm to the solutions matrix — the matrix whose columns are the solutions — in order to extract the POD modes that span the optimal space which the solutions belong to. Thus the solutions can be projected onto the reduced space: we represent the high-fidelity solutions as linear combination of the POD modes. Similarly to Eq. (4), the modal coefficients of the  $i$ -th solution  $\mathbf{x}_i^{\text{PODI}}$  — also called the reduced solution — are obtained as:

$$\mathbf{x}_i^{\text{PODI}} = \mathbf{U}^T \mathbf{x}_i \quad \forall i \in \{1, \dots, M\} \quad (6)$$

where  $\mathbf{U}$  refers to the POD modes and  $M$  is the number of high-fidelity solutions. We call  $N$  the number of POD modes and  $\mathcal{N}$  the dimension of high-fidelity solutions then  $\mathbf{x}_i^{\text{PODI}} \in V^N$  and  $\mathbf{x}_i \in V^{\mathcal{N}}$ . Since in complex problems we have an high number of degrees of freedom, typically we have  $N \ll \mathcal{N}$ . The low-rank representation of the solutions allows to easily interpolate them, exploiting the relation between the reduced solutions and the input parameters: in this way, we can compute the modal coefficients for any new parametric point and project the reduced solution onto the high dimensional space for a real-time approximation of the truth solution. This technique is defined non-intrusive, since it relies only on the solutions, without requiring information about the physical system and the equations describing it. For this reason it is particularly suited for industrial problem, thanks to its capability to be coupled also with commercial solvers. The downside is the error introduced by the interpolation, depending by the method itself, and the requirement of solutions with the same dimensionality, that can be a problem if the computational grid is built from scratch for any new configuration. Possible solutions are the projection of the solution on a reference mesh [7], or to deform the grid using the laplacian diffusion [29]. Moreover, we cite [12, 25] for other examples of PODI applications. For this work, we employed the open source Python package EZyRB [9] as software to perform the data-driven model order reduction.

## 6 Numerical results

In this section we present the results for the application of the complete pipeline to the problem presented in Section 2.



First, we sample the full parameter space  $\mathbb{D}$  extracting  $N_{\text{POD}} = 100$  parameters to construct the reduce order model without any further reduction, and we identify this approach with the subscript “POD”. Then, as explained in Section 4, we compute the shape manifold with 1500 different deformations, and we extract the new coefficients describing the new reduced parameter space. We sample this 2-dimensional space uniformly and we collect  $N_{\text{POD}+\text{reduction}} = 80$  solution snapshots. We can compare the decay of the singular values of the snapshots matrix for the two approaches. In Figure 5 we can note how the proposed computational pipeline results in a faster decay and thus in a better approximation for a given number of POD modes.

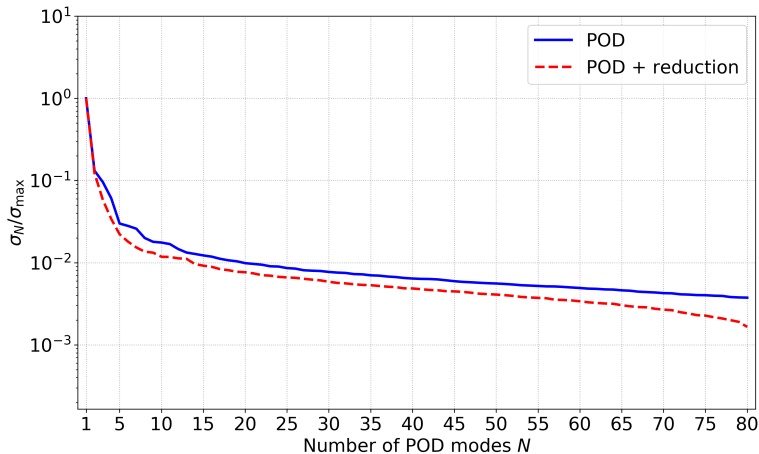


Figure 5: POD singular values decay as a function of the number of modes. The blue line corresponds to the original sampling in the full parameter space, while the red dotted line, which identifies the POD+reduction approach, corresponds to the sampling in the new reduced parameter space.

We underline that, despite the gain is not so big, the results do not involve further high-fidelity simulations. We only collected several different deformations at a negligible computational cost with respect to a single full order CFD simulation. Moreover the construction of the interpolator takes a huge advantage of the reduced parameter space since it counters the curse of dimensionality.

We can conclude that the proposed preprocessing step has sever benefits in terms of accuracy of the reduced order model at a small cost from a computational point of view.

## 7 Conclusions and perspectives

In this work we presented a complete data-driven numerical pipeline for shape optimization in naval engineering problems. The object was to find the optimal bulbous bow to minimize the total drag resistance of a hull advancing in calm water. First we parametrized and morphed the bulbous bow through the free form deformation method. Then we reduced the parameter space dimension approximating the shape manifold with the use of proper orthogonal decomposition and the investigation on linear dependence of the POD coefficients. We create the reduced order model sampling only the reduced two dimensional parameter space and with POD with interpolation we can compute in real time the outputs of interest for untried new parameters. Thus the optimizer can query the surrogate model and find the optimal shape.

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