Mass Inflation without Cauchy Horizons

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Mass inflation is a well established instability, conventionally associated to Cauchy horizons (which are also inner trapping horizons) of stationary geometries, leading to a divergent exponential buildup of energy. We show here that finite (but often large) exponential buildups of energy are present for dynamical geometries describing accreting black holes with slowly evolving inner trapping horizons, even in the absence of Cauchy horizons. The explicit evaluation of the adiabatic conditions behind these exponential buildups shows that this phenomenon is universally present for physically reasonable accreting conditions. This noneternal mass inflation does not require the introduction of global spacetime concepts. We also show that various known results in the literature are recovered in the limit in which the inner trapping horizon asymptotically approaches a Cauchy horizon. Our results imply that black hole geometries with nonextremal inner horizons, including the Kerr geometry in general relativity, and nonextremal regular black holes in theories beyond general relativity, can describe dynamical transients but not the long-lived end point of gravitational collapse.

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Introduction—The mass inflation instability is an integral part of our understanding of general relativity, playing a crucial role in destabilizing the Cauchy horizons associated with timelike singularities [1–3], as well as destabilizing the chronological horizons (a subclass of Cauchy horizons [4]) delimiting regions with closed timelike curves. The presence of such an exponential buildup of energy is considered a prerequisite to save causality in some general relativity solutions (such as the Kerr black hole), by making singular the boundary of the region violating causality and so, *de facto*, by excising the latter from the physical spacetime [5–8]. Hence, mass inflation plays a crucial role in the enforcement of both strong cosmic censorship [9,10] and chronology protection. So any possibility of evading mass inflation should be considered with deep suspicion.

Mass inflation is conventionally defined as an infinite divergent exponential buildup of energy [5–8]. The divergent behavior has been so far associated to stationary geometries, for which inner trapping horizons are always also Cauchy horizons. In all these cases, it is worth noticing that the (finite) exponential buildup of energy and curvature invariants generally leads to a breakdown of the effective description based on general relativity *before* any divergence is reached. Indeed, a more physical definition of mass inflation should rely only on the existence of a transient but large exponential buildup—until high enough curvatures (e.g., Planckian) are reached—regardless of the presence of any mathematical divergence. This is the novel perspective we adopt in this Letter.

This shift in perspective is motivated by recent results regarding mass inflation in regular black holes, in which a taming of the inner singularity is postulated on the basis of the idea that quantum gravity (regardless of the specific implementation) should provide a regular description of gravitational collapse [11–25] (see also the reviews [26–28]). It is by now well established that stationary regular black holes with inner horizons also display an initial exponential mass inflation phase [29–33]—at least unless the inner horizon is extremal [34,35]—that very rapidly brings the regular black hole into a regime where curvatures grow large (see also [36–40]).

In this Letter we show that (large but finite) exponential buildups of energy are also associated with slowly evolving inner horizons, with the results valid for Cauchy horizons recovered in the stationary limit. This provides a more physical realization of mass inflation implying that even noneternal black hole spacetimes endowed with a slowly evolving dynamical inner horizon will generically display a large but finite exponential buildup destabilizing the geometry in short timescales, which we call noneternal mass inflation.

Working with two different models of perturbations with pure radiation (also know as null dust [41]) sources, either of distributional or continuous nature, we will show that both models display noneternal mass inflation. The first model can be solved analytically, while we shall present numerical results for the second.

Adiabatic conditions—The ingredients necessary for mass inflation (namely, inner trapping horizons that are perturbed) can be defined also in the presence of rotation, and there is no indication that the latter can prevent the phenomenon [42–44]. Hence, for simplicity, we shall work in spherical symmetry, expecting that our results apply also for rotating geometries. Existing analytical and numerical studies support this expectation; see for instance [45,46] and [47,48] for rotating and charged black holes, respectively. An appropriate general parametrization is provided by generalized Eddington-Finkelstein coordinates in which the line element reads

$$\mathrm{d}s^2 = -e^{-2\Phi(v,r)}F(v,r)\mathrm{d}v^2 + 2e^{-\Phi(v,r)}\mathrm{d}r\mathrm{d}v + r^2\mathrm{d}\Omega^2, \quad (1)$$

where $\mathrm{d}\Omega^2$ is the line element on the unit 2-sphere. It is possible to recast this line element in terms of a non-null time coordinate t, but this form is best suited for our purposes. The Misner-Sharp mass [49,50], used below, is given by $M(v,r) = r(1-\partial_a r\partial^a r)/2 = r(1-g^{rr})/2$ [50,51].

We consider black holes with both outer and inner horizons in which the g_{vv} component of the metric vanishes (e.g., Reissner-Nordström or regular black holes). Without significant loss of generality, we can focus on the case with two horizons, the minimum number required by regularity at the origin [52]:

$$F(v,r) = e^{\Psi(v,r)}\left(1-\frac{r_{\mathrm{in}}(v)}{r}\right)\left(1-\frac{r_{\mathrm{out}}(v)}{r}\right). \quad (2)$$

Our analysis can be straightfowardly generalized to black holes with more than two horizons, such as the ones found in the presence of a cosmological constant [53,54].

The two models studied below include an outgoing null-dust thin shell. Outgoing null geodesics satisfy the equation

$$\frac{\mathrm{d}r(v)}{\mathrm{d}v} = \frac{e^{-\Phi(v,r)}F(v,r)}{2}, \quad (3)$$

which can be expanded around the position of the inner horizon, $r = r_{\mathrm{in}}(v)$, to first order

$$\frac{\mathrm{d}r(v)}{\mathrm{d}v} = \frac{e^{-\Phi(v,r_{\mathrm{in}}(v))}}{2}\left(\left.\frac{\partial F}{\partial r}\right|_{(v,r_{\mathrm{in}}(v))}[r(v)-r_{\mathrm{in}}(v)]\right)+\cdots. \quad (4)$$

We define the time-dependent surface gravity of the inner horizon, controlling the *peeling* of null rays around the latter, as [55]

$$\kappa_{\mathrm{in}}(v) = \frac{e^{-\Phi(v,r_{\mathrm{in}}(v))}}{2}\left.\frac{\partial F}{\partial r}\right|_{(v,r_{\mathrm{in}}(v))} = -|\kappa_{\mathrm{in}}(v)|. \quad (5)$$

This definition reduces to the usual surface gravity of a Killing horizon for stationary geometries [55].

Equation (4) can be recast as a differential equation for the difference $r(v) - r_{\mathrm{in}}(v)$ as

$$\frac{\mathrm{d}[r(v)-r_{\mathrm{in}}(v)]}{\mathrm{d}v} = -|\kappa_{\mathrm{in}}(v)|[r(v)-r_{\mathrm{in}}(v)] - \frac{\mathrm{d}r_{\mathrm{in}}(v)}{\mathrm{d}v}+\cdots. \quad (6)$$

In the stationary situation, r_{in} is constant in time and so is $|\kappa_{\mathrm{in}}|$, thus simplifying the equation above as terms proportional to derivatives of these quantities vanish identically. In more general situations, the derivative terms are negligible if the following adiabatic conditions are satisfied: (i) Adiabatic condition for radius of the inner horizon:

$$\left|\frac{\mathrm{d}r_{\mathrm{in}}(v)}{\mathrm{d}v}\right| \ll |\kappa_{\mathrm{in}}(v)||r(v)-r_{\mathrm{in}}(v)|. \quad (7)$$

(ii) Adiabatic condition for surface gravity of the inner horizon:

$$\left|\frac{\mathrm{d}\kappa_{\mathrm{in}}(v)}{\mathrm{d}v}\right| \ll |\kappa_{\mathrm{in}}(v)|^2. \quad (8)$$

These are conditions on the first and second derivatives of $r_{\mathrm{in}}(v)$ and thus can be violated or satisfied independently of each other. The first condition requires the specification of an outgoing null geodesic $r(v)$ for its evaluation. The second condition ensures the slow changing of the inner horizon and is equivalent to the condition for the outer horizon in [56,57], replacing the outer surface gravity by the inner surface gravity. Under these two conditions, we can write

$$r(v) \approx r_{\mathrm{in}}(v) + [r(v_0) - r_{\mathrm{in}}(v_0)]e^{-|\kappa_{\mathrm{in}}(v)|(v-v_0)}, \quad (9)$$

with the initial condition $r(v_0) \in (r_{\mathrm{in}}(v_0), r_{\mathrm{out}}(v_0))$. Inserting Eq. (9) in the adiabatic condition for the radius of the inner horizon, we obtain

$$\left|\frac{\mathrm{d}r_{\mathrm{in}}(v)}{\mathrm{d}v}\right| \ll |\kappa_{\mathrm{in}}(v)|[r(v_0) - r_{\mathrm{in}}(v_0)]e^{-|\kappa_{\mathrm{in}}(v)|(v-v_0)}. \quad (10)$$

This equation illustrates that, even if the adiabatic condition is always satisfied, the condition for slow variation of the inner horizon will always eventually cease to be valid as long as $\mathrm{d}r_{\mathrm{in}}/\mathrm{d}v$ is nonzero. This will happen at some time $v_\star$ that, given the assumed adiabatic evolution of the surface gravity, is approximately given by the explicit formula

$$v_\star \approx v_0 + \frac{1}{|\kappa_{\mathrm{in}}(v_\star)|}\ln\left\{\frac{|\kappa_{\mathrm{in}}(v_\star)|[r(v_0)-r_{\mathrm{in}}(v_0)]}{|\mathrm{d}r_{\mathrm{in}}/\mathrm{d}v|_{v_\star}}\right\}. \quad (11)$$

Note that the adiabatic conditions are defined in relation to the behavior of outgoing null shells around the inner horizon, without any reference to mass inflation. Connecting these conditions to mass inflation requires a separate treatment discussed below.

Analytical results—Aside from the outgoing null-dust thin shell, let us introduce an ingoing null-dust thin shell, which is a standard setup to discuss mass inflation [58,59]. One of the advantages of this setup is that the change of the metric coefficients due to the crossing of the shells can be determined geometrically, without specifying the field equations of the theory [29,37]. It is useful to parameterize the metric coefficients in terms of the Misner-Sharp mass $M_{\mathrm{f}}(v_\times, r_\times)$ in between the ingoing and outgoing null-dust shells after the crossing of the two shells at $r_\times = r(v_\times)$, which is given by [29,31,37]

$$M_{\mathrm{f}}(v_\times, r_\times) = M_{\mathrm{i}}(v_\times, r_\times) + M_{\mathrm{in}}(v_\times, r_\times) + M_{\mathrm{out}}(v_\times, r_\times) - \frac{2M_{\mathrm{in}}(v_\times, r_\times)M_{\mathrm{out}}(v_\times, r_\times)}{r_\times F_{\mathrm{i}}(v_\times, r_\times)}, \quad (12)$$

where $M_{\mathrm{i}}(v_\times, r_\times)$ is the mass in between the two shells prior to the crossing at $r = r_\times$ while $F_{\mathrm{i}}(v_\times, r_\times)$ is the metric function in between the two shells prior to the crossing which, using Eq. (2), we can write in terms of $(v_\times, r(v_\times))$ as

$$F_{\mathrm{i}}^\times = \frac{e^{\Psi(v_\times, r(v_\times))}}{r_{\mathrm{in}}(v_\times)}\left(1 - \frac{r_{\mathrm{out}}(v_\times)}{r_{\mathrm{in}}(v_\times)}\right)[r(v_\times) - r_{\mathrm{in}}(v_\times)]. \quad (13)$$

On the other hand, $M_{\mathrm{in}}(v_\times, r_\times)$ and $M_{\mathrm{out}}(v_\times, r_\times)$ measure the jump of the mass function across the ingoing and the outgoing shell. In the following, we simply assume that these quantities are proportional to the energy of the shells. The crossing time $v = v_\times$ can be chosen so that the quantity $r(v_\times) - r_{\mathrm{in}}(v_\times)$ can be as small as possible. Hence, the mass $M_{\mathrm{f}}(v_\times, r_\times)$ in Eq. (12) will typically grow large for generic perturbations. The exponential behavior characteristic of stationary situations is present whenever *both* of the aforementioned adiabatic conditions, Eqs. (7)–(8) are satisfied. However, the first adiabatic condition for the evolution of the inner horizon cannot be maintained indefinitely, in particular because the outgoing shell must eventually cross the inner horizon (see Fig. 1). Therefore, the minimum value of the function $F(v_\times, r(v_\times))$ that can be guaranteed to be reached exponentially using our argument, $F_{\mathrm{i}}^\star = F_{\mathrm{i}}(v_\star, r(v_\star))$, is given by

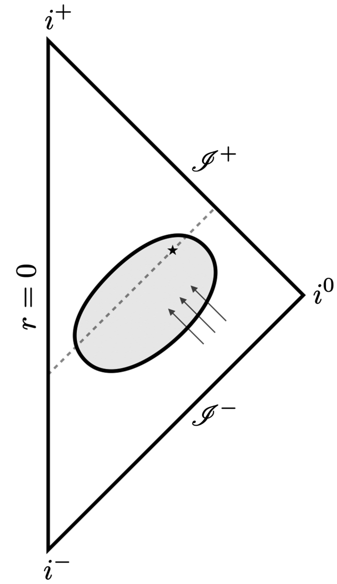


FIG. 1. Penrose diagram describing the formation and disappearance of a black hole with both outer and inner horizons, but not Cauchy horizons, marking the boundary of the (topologically closed, as envisaged by Frolov and others [13]) trapped region. The dashed line indicates the placement of an outgoing null-dust thin shell, which approaches the inner horizon exponentially if both adiabatic conditions are satisfied, until a certain critical point marked by the star symbol. The three arrows indicate ingoing perturbations, of which we have considered two different types, either an ingoing null-dust shell or a continuous stream of null dust. These perturbation setups are standard in the study of mass inflation.

$$F_{\mathrm{i}}^\star = \frac{e^{\Psi(v_\star, r_{\mathrm{in}}(v_\star))}}{r_{\mathrm{in}}(v_\star)}\left(1 - \frac{r_{\mathrm{out}}(v_\star)}{r_{\mathrm{in}}(v_\star)}\right)\frac{|\mathrm{d}r_{\mathrm{in}}/\mathrm{d}v|_{v_\star}}{|\kappa_{\mathrm{in}}(v_\star)|}, \quad (14)$$

where $v = v_\star$ indicates the time approximately given in Eq. (11).

Using Eqs. (12) and (14), the mass $M_{\mathrm{f}}(v_\star, r_\star)$ is guaranteed to display an exponential behavior in $v_\times$, up to a maximum value

$$M_{\max} \approx \frac{r_{\mathrm{in}}(v_\star)|\kappa_{\mathrm{in}}(v_\star)|}{|\mathrm{d}r_{\mathrm{in}}/\mathrm{d}v|_{v_\star}}\frac{2M_{\mathrm{in}}(v_\times, r_\times)M_{\mathrm{out}}(v_\times, r_\times)}{r_\times}. \quad (15)$$

Note that this can be factorized into the form $M_{\max} \simeq f_1(v_\star)f_2(v_\times, r_\times)$, with one function depending on the end of exponential mass inflation and the other only on the crossing of the null shells.

Equation (15) can be understood as the regularized version of $M_{\max} = \infty$ that is obtained in the static case, where the regulator comes from the exponential approximation ceasing to be valid due to the nonzero value of $|\mathrm{d}r_{\mathrm{in}}/\mathrm{d}v|_{v_\star}$. As a consistency check, for $|\mathrm{d}r_{\mathrm{in}}/\mathrm{d}v|_{v_\star} \to 0$ we recover the result $M_{\mathrm{max}} = \infty$.

Numerical results—Let us introduce a slightly different perturbation type wherein we maintain an outgoing null-dust shell but replace the ingoing null-dust shell with a continuous stream of null dust. This setup was originally examined by Ori [60] to investigate the instability of Reissner-Nordström black holes and has been applied later to more general situations [29,32,33]. The connection between the adiabatic conditions and mass inflation in this setup is more convoluted and will be demonstrated numerically.

The gluing conditions for two spherically symmetric geometries along a null-dust shell were discussed in [61]. Following the latter reference, we define the future-directed null normal to the shell as $n^\mu = \mathrm{d}x^\mu/\mathrm{d}r = (2e^\Phi/F, 1, 0, 0)$, choosing r as a common parameter for both geometries along the shell. The pressureless nature of the shell implies the continuity of $T_{\mu\nu}n^\mu n^\nu$. This constraint is invariant under reparametrizations of the null normal to the shell.

For concreteness, let us consider the Hayward metric [12],

$$\Phi_\pm(v, r) = 0, \quad F_\pm(v, r) = 1 - \frac{2r^2 m_\pm(v)}{r^3 + 2\ell^2 m_\pm(v)}, \tag{16}$$

where the + (−) index indicates that the corresponding quantity must be evaluated inside (outside) the outgoing null-dust shell (see Fig. 1). This metric has two horizons, a single extremal horizon or no horizons, depending on the relative value of $m_\pm(v)$ [11] and the regularization scale ℓ. We have checked that our results do not depend on the specific regular black hole considered, by considering alternatives such as the Bardeen metric [11]. This has to be expected as noneternal mass inflation is determined by the local geometric structure around the inner horizon, regardless of the specific geometry being considered.

The continuity of $T_{\mu\nu}n^\mu n^\nu$ is then equivalent to the following constraint for the Misner-Sharp mass:

$$\frac{1}{F_+}\frac{\partial F_+}{\partial v}\bigg|_{r=R(v)} = \frac{1}{F_-}\frac{\partial F_-}{\partial v}\bigg|_{r=R(v)}, \tag{17}$$

where $R(v)$ denotes the radius of the outgoing shell.

The functional form of $m_-(v)$ is loosely constrained to describe the situation depicted in Fig. 1, namely a geometry with no horizons at early and late times and an ingoing stream of radiation:

$$m_-(v) = [M_0 + \delta m(v)] I_{v_i,v_f}(v), \tag{18}$$

where $\delta m(v)$ is a nondecreasing function describing a perturbation of the mass due to accretion, M_0 the maximum mass of the background black hole, and $I_{v_i,v_f}(v) \leq 1$ an interpolating function vanishing for $v \ll v_i$ and $v \gg v_f$ and approximately constant for $v_i \ll v \ll v_f$. Any interpolating function (possibly of compact support) satisfying these requirements can be considered without changing our results below, and we will be choosing a specific realization using hyperbolic tangents, $I_{v_i,v_f}(v) = \{\tanh[s_1(v - v_i)] - \tanh[s_2(v - v_f)]\}/2N$, with a normalization factor N evaluated numerically.

The mass in the interior region, $M_+(v, R(v))$, can be obtained integrating Eq. (17) numerically. We perform this integration for a finite interval of time contained within the trapped region in which the adiabatic conditions are satisfied, both for the background geometry and the accretion flux. Inserting the relations $r_{\mathrm{in}} = \ell[1 + \mathcal{O}(\ell/m_-)]$ and $\kappa_{\mathrm{in}} = -\ell^{-1}[1 + \mathcal{O}(\ell/m_-)]$, valid for the Hayward metric (as well as most known metrics [31]), as well as Eq. (18), into the second adiabatic condition, we obtain

$$\left|\frac{\mathrm{d}m_-}{\mathrm{d}v}\right| \ll \frac{m_-^2}{\ell^2}[1 + \mathcal{O}(\ell/m_-)]. \tag{19}$$

This imposes constraints on the evolution of both background geometry and accretion flux. In the absence of accretion, the leading order in Eq. (19) around the maximum of I_{v_i,v_f} is

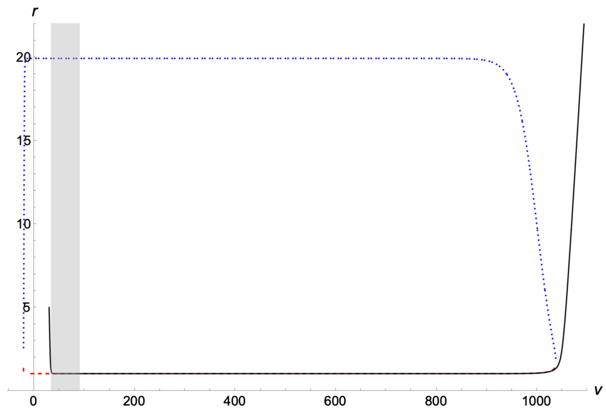
$$\left|\frac{\mathrm{d}I_{v_i,v_f}}{\mathrm{d}v}\right| \ll \frac{M_0}{\ell^2}. \tag{20}$$

The parameters $v_i \ll v_f$ can always be chosen so that this condition is satisfied for an arbitrarily long time interval. In such an interval, we introduce perturbations with shorter time variations, so that the leading order in Eq. (19) becomes

$$\left|\frac{\mathrm{d}\delta m}{\mathrm{d}v}\right| \ll \frac{M_0^2}{\ell^2}. \tag{21}$$

The left-hand side contains information about the amplitude $A_{\delta m}$ of the perturbation and its variation timescale $\tau_{\delta m}$. As $A_{\delta m} \ll M_0$ by construction, the condition $\tau_{\delta m} \gtrsim \ell^2/M_0$ is sufficient [note that $\ell/M_0 = \mathcal{O}(10^{-38})$ for a solar-mass black hole with Planckian regulator, but this constraint is still weak even when $\ell/M_0 = \mathcal{O}(1)$]. We thus conclude that noneternal mass inflation is generically present for fluctuating accretion conditions as long as the mild adiabatic conditions are satisfied, under which the results of the stationary case are recovered.

The above argument accommodates specific decaying profiles such as the Price law [62–64], ubiquitously used in previous works [29,32,33,36–39] but is much more general as it does not require a specific functional profile for $\delta m(v)$. To simplify comparison with previous works, while also illustrating the broader generality of our results, we show in



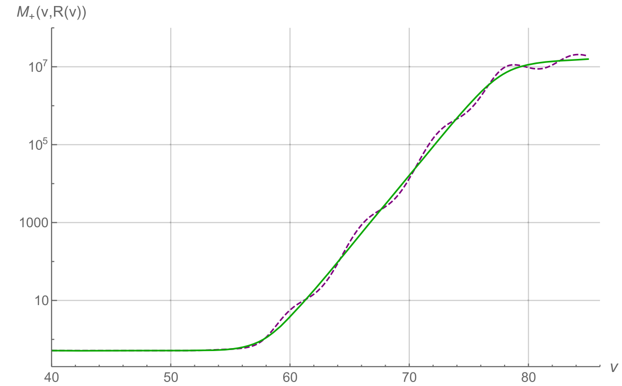


FIG. 2. *Left:* specific realization of Fig. 1 for the Misner-Sharp mass in Eq. (18) and parameters $M_0 = 10$, $\ell = 1$, $v_i = -20$, $v_f = 1000$, $s_1 = 1$, $s_2 = 1/40$. The radius of the outgoing shell is set initially at $R(v = 30) = 5$. Outer and inner horizons are indicated by the dotted blue line and the dashed red line, respectively, and the outgoing null-dust shell by the black solid line. The shaded region marks the interval of time for which $M_+(v, R(v))$ is plotted in the right-hand panel. *Right:* Misner-Sharp mass $M_+(v, R(v))$ in the region interior to the outgoing shell for an illustrative example $m(v) = -\beta[1 + \epsilon \cos(\omega v)/v]/v^p$ combining oscillatory and power-law behaviors, with $\beta = 1$, $\omega = 1$, $p = 2$, and $\epsilon = 0$ (solid green line) or $\epsilon = 1$ (dashed purple line). Oscillations satisfying the adiabatic conditions adhere, to leading order, to the purely exponential behavior displayed in the solid green line, which is the same as for a stationary regular black hole with the same parameters and perturbation profile. As in the stationary case (see Ref. [32]), the exponential buildup increases the value of the mass in a short interval of time until a critical time $v_0 \simeq 80$ in which $M_+(v_0, R(v_0)) \propto v_0^{p+1} M_0^2/\ell = \mathcal{O}(10^7)$.

Fig. 2 numerical results for a specific perturbation profile combining oscillatory and power-law behaviors.

Conclusions—We have shown that the exponential buildup characteristic of the mass inflation instability is not limited to stationary black hole spacetimes but extends to dynamical spacetimes, as long as the inner horizon is nonextremal and the geometry is evolving sufficiently slowly as encapsulated in two adiabatic conditions which become exact in the stationary limit, so allowing us to recover the standard results for Cauchy horizons.

Even if one or both adiabatic conditions eventually cease to be valid, this generically happens after this noneternal mass inflation has triggered a rapidly evolving phase resulting into the exponential growth of curvature invariants.

The analysis presented herein is restricted to spherical symmetry; nonetheless, the ingredients leading to noneternal mass inflation in our setting are present and known to generically lead to the same phenomenon, also in the presence of rotation (e.g., [45,46]). Hence, there is no reason to expect that inner horizons in rotating black holes would behave any differently.

It has been conjectured that mass inflation might lead to a singularity of null nature in the black hole interior without affecting the exterior geometry [65] (see also [45–48]). Note that the mass inflation instability and its large backreaction would still be present even close to a null singularity and that this conjecture assumes no resolution of the singularity by quantum gravitational effects. The latter could lead to different scenarios [52,66], including regular black holes with inner-extremal cores [34,35], Simpson-Visser cores (also called hidden wormholes) [67,68], or bouncing cores [36,69] that may result into a horizonless ultracompact object of the same family of the initial regular black hole [70].

The implications are striking: generic black holes with (nonextremal) inner horizons will always keep evolving in a timescale controlled by $1/\kappa_{\text{in}}$ and cannot be the end point of a stellar collapse. It is generally believed that astrophysical black holes are well described by a quasistationary Kerr metric, possibly with a regularized Planckian core. Our results challenge this expectation and show that determining the end point of stellar collapse is an inevitable open question.

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