Scuola Internazionale Superiore di Studi Avanzati - Trieste

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Phenomenological Aspects of Supersymmetry Breaking

Thesis submitted for the degree of
Doctor Philosophiæ

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Trieste, September 2010

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Chapter 1

Introduction

1.1 Beyond the The Standard Model

The Standard Model (SM) of particle physics is a renormalizable Quantum Field Theory (QFT) which describes the electromagnetic, weak and strong interactions. It is based on the local gauge invariance under the symmetry group $G_{\rm SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

The SM is today a widely and accurately tested theory. A huge amount of data has been accumulated over the years for a large number of observables and the knowledge of some of them has been pushed to below the permill precision level. However the experimental verification of the Model is not completed.

The original gauge symmetry $G_{\rm SM}$ has to be broken spontaneously to $SU(3)_C \otimes U(1)_Q$. In the SM this is achieved through the Higgs mechanism. As the remnant of this process the theory predicts the existence of a single scalar particle: the Higgs boson. Its discovery and the study of its properties are one of the main goals of the Large Hadron Collider (LHC).

Despite its extraordinary success, the Standard Model is believed to be only the low energy (long distance) limit of a more fundamental theory. There are many reasons to think that New Physics (NP) has to be present at a scale $\Lambda_{\rm NP}$.

First of all the SM does not describe the gravitational interactions. At least at energy of the order the Planck scale $M_{\rm PL}$ we know that the effects of gravity are no longer negligible and a unified treatment in a single theory of gravity and gauge interactions might be well beyond the realm of QFT (at present superstrings is the best candidate to play the role of such a theory).

Beyond the fact that quantum gravity is not included in the SM, there are other empirical evidences in favor of NP. It is now established that neutrinos oscillates, so they have a mass (at least two of them). In the SM the neutrino are massless.

In addition, while the SM is no doubt successful in reproducing data at the energy explored by accelerators, it badly fails to explain fundamental phenomena of astrophysical-cosmological interest. In particular the SM cannot explain the observed matter/antimatter asymmetry in the Universe and has no viable candidate for Dark Matter (DM).

Other phenomenological hints for NP are given by the quantum numbers of the matter fields and the values of gauge couplings. The quantum numbers of the SM fields can be naturally explained in a Grand Unified Theory (GUT). For example, the fact that all the observed electric charges appear in quantized units which are multiples of the down quark charge can be predicted in a GUT. Another indication in favor of a GUT is the gauge coupling unification. The renormalization group running of the three gauge couplings in the SM has been found to nearly, but not quite, meet at the same point. However, if the supersymmetric extension of the SM is used instead, the match becomes much more accurate. In this case, the coupling constants of the strong and electroweak interactions meet at the a scale $\Lambda_{GUT} \approx 10^{16}\,\mathrm{GeV}$.

Beyond these experimental evidences and indications for NP, there are also theoretical/aesthetic questions which are not answered by the the SM. Why are there three generations of fermions? Why we observe this peculiar pattern of fermion masses and mixings? Why the CP violation in the strong sector is so small?

The conclusions of this section is that the Standard Model must be considered as an Effective Field Theory (EFT) valid up to the scale Λ_{NP}. If we accept this picture and we assume that $\Lambda_{\mathrm{NP}} \gg M_Z$ then the SM is affected by the naturalness problem of the Higgs boson mass, which is considered to be the main theoretical problem in constructing theory beyond the SM and which is discussed in the next session.

1.2 The naturalness problem of the Higgs mass and its stabilization

The Higgs sector of the SM is described by the scalar potential

$$V(H) = V_0 - \mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2 \tag{1.1}$$

where H is the Higgs doublet.

Since the dimension 2 term in the Higgs Lagrangian is not protected by any symmetry similar to the chiral one in the fermion case, nothing prevents the occurrence of powerlike divergences when computing loop corrections to the Higgs mass.

The main correction to the Higgs mass is given by the top quark loop:

$$\delta m_H^2 = \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 \Lambda^2 = (0.3\Lambda)^2 \tag{1.2}$$

where Λ is a cutoff employed as ultraviolet regulator. We note that from the strictly technical point of view of renormalization, the quadratic divergence in (1.2) is just like any other, and can be removed in the usual way: by redefining constants and fields in the bare Lagrangian. The problem arises when one gives to the cutoff the meaning of a threshold for NP: $\Lambda = \Lambda_{\rm NP}$.

With this consideration on the cutoff scale, it is clear to see that, if for the correction δm_H is allowed a value in the range of hundreds GeV, *i.e.* comparable with the expected experimental value of the Higgs mass m_H, then quite stringent upper bound on $\Lambda_{\rm NP}$ is implied, $\Lambda_{\rm NP} \lesssim 1\,\text{TeV}$. If instead we insist to have a large scale for $\Lambda_{\rm NP}$ we can still get the Higgs mass in the correct phenomenological range, but we have to invoke an unappealing and unnatural fine tuning between the bare Higgs mass and its correction δm_H.

The problem of finding a consistent and phenomenologically acceptable extension of the SM has been a central issue in the last few decades. Many ideas have been introduced and developed to solve in particular the problem represented by (1.2), *i.e.* the problem of having vastly different scale like $M_{\rm PL}$ (or $\Lambda_{\rm NP}$) and the ElectroWeak (EW) scale.

A very elegant route is to invoke a new symmetry, SUperSYmmetry (SUSY). SUSY is a symmetry that turns fermions into bosons and viceversa, and predicts, for example, that for any fermion in the SM the presence of a scalar partner. What is relevant in this section is that SUSY entails the cancellation of the quadratic divergence in (1.2), caused by the top-quark loop, with a similar divergences brought about by another loop diagram with the scalar partner of the top itself, namely the s-top. By this mechanism the correction (1.2) goes into

$$\delta m_H^2 = \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 \Lambda_{\rm NP}^2 \quad \stackrel{\rm MSSM}{\longrightarrow} \quad \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 m_{\tilde{t}}^2 \ln \frac{\Lambda_{\rm NP}^2}{m_{\tilde{t}}^2} \tag{1.3}$$

where $m_{\tilde{t}}$ is the mass of the stop. The quadratic divergence is replaced by the log-like one, allowing δm_H to remain in the hundreds GeV range even with $\Lambda_{\rm NP}$ at the Planck scale, provided that the stop mass remains in the TeV range. SUSY in this case has solved the hardest part of the naturalness problem, namely the stability of the Higgs mass respect radiative corrections. Hence it is technically possible to maintain m_H and $\Lambda_{\rm NP}$ separated, but on the other hand the question why these scales are so different is not addressed by SUSY.

1.3 The Minimal Supersymmetric Standard Model

The minimal implementation of SUSY in a phenomenological viable theory is the Minimal Supersymmetric Standard Model (MSSM). The are several theoretical and phenomenological motivations to construct a supersymmetric extension of the SM.

SUSY is a symmetry that connects particles of different spin. This means that the generators of this kind of symmetry have non trivial transformation properties respect to the Poincarè group and in order to evade the Coleman-Mandula theorem [1] we have to promote the Lie algebra to a graded Lie algebra. Haag, Loupuszanski and Sohnius proved in [2] the remarkable result that the possible symmetries of a consistent 4-dimensional QFT do not only consist of internal and Poincaré symmetries, but can also include SUSY as a nontrivial extension of the Poincaré algebra. This significantly generalized the Coleman-Mandula theorem. One of the important results is that the fermionic part of the Lie superalgebra has to have spin-1/2 (spin 3/2 or higher are ruled out), in this ways they showed that SUSY is the unique way to treat in a unified way fermions and bosons.

Another very interesting point is that if we promote the global SUSY to a local symmetry, we obtain a SUperGRAvity (SUGRA) theory which is regarded as the EFT of superstring theory.

Moving instead to low energy SUSY, a clear solution to the hierarchy problem is not the only virtue of SUSY. Another remarkable feature is that there is a candidate for the DM, provided by the so called Lightest Superymmetric Particle (LSP), usually identified with the neutralino or the gravitino. The stability of the LSP can be guaranteed as a consequence of an R-parity conservation. The latter symmetry is an elegant way for removing from the MSSM operators that would violate B and L number conservation and that would lead to catastrophic phenomenological consequences, such as a proton lifetime that is too short. Now, SM particles have positive R-parity and their SUSY partners are required to have negative R-parity, so the conservation of the this quantum number automatically guarantees that the LSP not decays.

An additional striking feature of the MSSM is related to the gauge coupling unification. In the SM gauge coupling unification does not really work. In the MSSM, thanks to the extra degrees of freedom charged under the SM gauge interactions, it is possible to achieve unification and the predicted unification scale can be high enough to evade the bounds required by the non observation of the proton decay.

With all these motivations, we are now ready to write down the MSSM. Its matter content is specified in table 1.1, the bosonic and fermionic fields which are connected by SUSY are combined together in the same supermultiplet.

The MSSM is minimal in the sense that is obtained adding to SM fields their superparteners with the minor modification to add another Higgs doublet. This extra doublet is needed for two main reasons. The first one is related to the cancelation of the gauge anomalies: with only one chiral superfield containing the

Superfield	Fermions	Bosons	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\hat{G}$	$\tilde{G}$	G	8	1	0
$\hat{W}$	$\tilde{W}$	W	1	3	0
$\hat{B}$	$\tilde{B}$	B	1	1	0
$\hat{q}_i$	q_i	$\tilde{q}_i$	3	2	$\frac{1}{6}$
$\hat{u}_i^c$	u_i^c	$\tilde{u}_i^c$	$\overline{3}$	1	$-\frac{2}{3}$
$\hat{d}_i^c$	d_i^c	$\tilde{d}_i^c$	$\overline{3}$	1	$\frac{1}{3}$
$\hat{l}_i$	l_i	$\tilde{l}_i$	1	2	$-\frac{1}{2}$
$\hat{e}_i^c$	e_i^c	$\tilde{e}_i^c$	1	1	1
$\hat{h}_u$	$\tilde{h}_u$	h_u	1	2	$+\frac{1}{2}$
$\hat{h}_d$	$\tilde{h}_d$	h_d	1	2	$-\frac{1}{2}$

Table 1.1: Matter content of the MSSM

Higgs doublet it is not possible to cancel the gauge anomalies induced by the extra higgsino. A second reason is due to the fact that the superpotential has to be an analytic function of the chiral superfields: with only one Higgs doublet is not possible to construct Yukawa terms for all the fermions in the SM.

The MSSM is minimal also respect to the gauge interactions because the gauge group is the same as in the SM.

The most general superpotential which we can write according to gauge and R-parity invariance is

$$W = y^u_{ij}\hat{q}_i\hat{u}^c_j\hat{h}_u + y^d_{ij}\hat{q}_i\hat{d}^c_j\hat{h}_d + y^e_{ij}\hat{l}_i\hat{e}^c_j\hat{h}_d + \mu\hat{h}_u\hat{h}_d \tag{1.4}$$

where y^u, y^d and y^e are the Yukawa coulings and μ is a supersymmetric dimensionfull term.

In absence of SUSY breaking, the masses of the SM particles and their superpartners are identical. This is clearly not realistic; there is no selectron with mass 511 keV, nor is there a smuon with mass 106 MeV, etc. Indeed, untill now, no superpartners have been discovered yet. The conclusion is that if SUSY is a feature of the underlying laws of nature, it is certainly broken.

The simplest approach to model building with SUSY is to add soft-breaking terms to the effective Lagrangian so that squarks, sleptons and gauginos have sufficiently large masses that they have not yet been observed. The soft terms are by definition, all such terms that explicitly break SUSY but they do not spoil the cancellation of the quadratic divergences.

The soft sector of the MSSM is given by the sum of several contributions:

$$\mathcal{L}_{\rm soft} = \mathcal{L}_{\rm gaugino} + \mathcal{L}_{\rm softH} + \mathcal{L}_{\rm bilinear} + \mathcal{L}_{\rm trilinear} \tag{1.5}$$

We have soft masses for the gauginos

$$-\mathcal{L}_{\rm gaugino} = \frac{1}{2}\left(M_1\tilde{B}\tilde{B} + M_2\tilde{W}\tilde{W} + M_3\tilde{G}\tilde{G} + \text{c.c.}\right) \tag{1.6}$$

soft contributions to the Higgs sector

$$-\mathcal{L}_{\rm softH} = \tilde{m}_u^2 h_u^\dagger h_u + \tilde{m}_d^2 h_d^\dagger h_d - (B\mu h_u h_d + \text{c.c.}) \tag{1.7}$$

bilinear soft contributions for the scalar in the matter chiral superfields

$$\begin{aligned}-\mathcal{L}_{\rm bilienar} = \; & \tilde{q}_i^\dagger \left(\tilde{m}_q^2\right)_{ij} \tilde{q}_j + \tilde{u}_i^{c\dagger} (\tilde{m}_{u^c}^2)_{ij}\, \tilde{u}_j^c + \tilde{d}_i^{c\dagger} (\tilde{m}_{d^c}^2)_{ij}\, \tilde{d}_j^c + \\ & \tilde{l}_i^\dagger \left(\tilde{m}_l^2\right)_{ij} \tilde{l}_j + \tilde{e}_i^{c\dagger} \left(\tilde{m}_{e^c}^2\right)_{ij} \tilde{e}_j^c \end{aligned} \tag{1.8}$$

and holomorphic trilinear soft terms

$$\mathcal{L}_{\rm trilienar} = \left(A_{ij}^u \tilde{q}_i \tilde{u}_j^c h_u + A_{ij}^d \tilde{q}_i \tilde{d}_j^c h_d + A_{ij}^e \tilde{l}_i \tilde{e}_j^c h_d + \text{c.c.}\right) . \tag{1.9}$$

1.4 Supersymmetry breaking

The MSSM is an effective field theory and a microscopic theory of SUSY breaking would explain the origin of the soft terms present in the model.

To understand the origin of the soft scale m_{soft}, the two main quantities that we would like to know are the mass scale of SUSY breaking and the scale of the transmission of SUSY breaking. In a large class of models, SUSY breaking occurs in a sector of the theory where a chiral superfield Z, singlet under the SM, acquires an $\langle F\rangle$ term Vacuum Expectation Value (VEV). Then the SUSY breaking is communicated to the MSSM superfields at the scale M.

Let us briefly discuss the two most studied mediation mechanisms. The first one is through gravitational interactions. In this case we have to construct a supergravity (SUGRA) theory. There supersymmetry is extended to a local symmetry giving rise to a theory of gravity where the graviton field comes together with his spin 3/2 superpartner, the gravitino. SUGRA gets then spontaneously broken by $\langle F\rangle$. The low energy effective Lagrangian contains non renormalizable terms suppressed by powers of $M_{\rm PL}$ implied by supergravity. These terms couple Z to visible fields and soft terms will be generated. Dimensional analysis gives approximately

$$m_{soft} \sim \frac{\langle F\rangle}{M_{\rm PL}} \tag{1.10}$$

For this to give a realistic spectrum and to be good for the naturalness problem, we need $\sqrt{\langle F\rangle} \sim 10^{10} - 10^{11}\,\text{GeV}$. Much similarly to the breaking of a bosonic internal symmetry, when SUSY is broken a massless Goldstone fermion, the goldstino, is generated. In the case of SUGRA this fermion is eaten by the gravitino which acquires a mass

$$m_{3/2} \sim \frac{\langle F \rangle}{M_{\rm PL}} \tag{1.11}$$

and interactions that scale like $\frac{1}{\langle F \rangle}$. The gravitino couple with particles of the same multiplet in such a way that the coupling will be proportional to the mass splitting of the multiplet. In SUGRA scenarios the gravitino has thus a mass comparable with $m_{\rm soft}$. A good point of this scenario is the fact that we can have a natural explanation for the size of the μ term. A problematic point is that, in general, we expect soft terms completely anarchical in the flavor space if we do not impose a flavor symmetry.

Another way to obtain a phenomenologically viable pattern of SUSY breaking is known as Gauge Mediated Supersymmetry Breaking (GMSB). Here the hidden sector that develops the $\langle F \rangle$ term is coupled at tree level with a messenger sector which has SM interactions and can transmit SUSY breaking to the visible world via loop effects. Thus the range of the soft masses will be

$$m_{soft} \sim \frac{\alpha}{4\pi} \frac{\langle F \rangle}{M_{\rm mess}} \tag{1.12}$$

where $M_{\rm mess}$ is the average mass of the messenger fields and α is a SM gauge coupling. In this case the messenger scale $M_{\rm mess}$ is not fixed and we can obtain the SUSY breaking with $\sqrt{\langle F \rangle}$ as lower as 10^4 GeV. In this scenario the gravitino is always the LSP. The gravitino interactions are stronger with respect to the SUGRA case, because of the lower value of $\langle F \rangle$.

The soft terms generated thought this mechanism are flavor universal and the FCNC contribution from the induced low energy MSSM are under control. In this scheme instead we suffer from the so called $\mu/B\mu$ problem, namely a mechanism able to generate a mu-term typically generates a significantly larger $B\mu$ term, which worsen the naturalness of the framework.

1.5 Supersymmetry and Cosmology

The evidence for the dark matter is supported by several effects: galactic rotation curves, the Cosmic Microwave Background (CMB), the Big Bang Nucleosynthesis (BBN), the Bullet cluster, weak and strong gravitational lensing, etc. Current data imply that the dark matter is five times more abundant than ordinary matter and accounts for about a quarter of the Universe energy density. More precisely, these data constrain the energy densities of the Universe in Baryons (B), non-baryonic

Dark Matter (DM), and dark energy Λ [3] to be

$$\Omega_B = 0.0456 \pm 0.0016 \tag{1.13}$$
$$\Omega_{DM} = 0.227 \pm 0.014 \tag{1.14}$$
$$\Omega_\Lambda = 0.728 \pm 0.015 \tag{1.15}$$

where Ω_B, Ω_{DM} and Ω_Λ are the ratios between respectively the baryon, dark matter and dark energy density over the critical density. As can be seen above, the fraction of energy density carried by the ordinary matter (baryonic) is only a small fraction of the total amount.

If in one hand cosmology provides strong evidence for new particle physics, on the other hand it cannot strictly constrain the microscopic properties of dark matter and dark energy. Theoretical insights from particle physics are therefore required, both to suggest candidates for dark matter and dark energy and to identify experiments and observations that may confirm or exclude these speculations. Weak-scale supersymmetry, as we saw in the previous sections, is at present the most well-motivated framework for new particle physics. More than that, it naturally provides dark matter candidates with approximately the right relic density. This fact provides a strong, fundamental, and completely independent motivation for supersymmetric theories. For these reasons, the implications of supersymmetry for cosmology, and vice versa, deserve serious consideration.

Regarding the dark energy, unfortunately, in most of the models of supersymmetry, the vacuum energy is set by the SUSY breaking scale, which is many orders of magnitude larger than the observed value of the vacuum energy. From this starting point, Λ must be fine-tuned to the scale of eV^4.

On the other hand, supersymmetry offer a viable candidate for the DM: the LSP. In particular in the MSSM the neutral candidate to be the LSP are

Spin 0 Scalars:	$\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$
Spin 1/2 fermion:	$\tilde{B}, \tilde{W}, \tilde{h}_u, \tilde{h}_u$
Spin 3/2 fermion:	Gravitino

The neutral spin 1/2 fermions mix to form four mass eigenstates χ_i, the neutralinos, and the lightest of these is χ_1.

The sneutrinos are not good dark matter candidates, as both their annihilation and scattering cross sections are large, and so they are under abundant or excluded by null results from direct detection experiments.

The lightest neutralino belong to the class of the Weakly Interacting Massive Particle (WIMP). In this case the DM may be produced in a simple and predictive manner as a thermal relic of the Big Bang. The lightest neutralino can annihilate through a wide variety of Feynman diagrams. Which of these diagrams dominate the process of thermal freeze-out in the early universe depends on the composition of the lightest neutralino, and on the masses and mixings of the exchanged particles. Since so many different diagrams can potentially contribute to neutralino annihilation, the resulting relic density depends on a large number of supersymmetric parameters. It is then clear that a correct comprehension of the SUSY breaking mechanism and its mediation is very important to understand the neutralino cosmology.

The connection between SUSY breaking and cosmology is even stronger in the case of the gravitno dark matter. Indeed, the cosmology strongly depends on the gravitino mass $m_{3/2}$ that is also directly related to the scale of supersymmetry breaking:

$$m_{3/2} = \frac{\langle F \rangle}{\sqrt{3} M_{\rm PL}} \tag{1.16}$$

Long lived gravitinos can pose problems for cosmology. In particular, gravitinos may be overproduced in the early universe if the temperature of the reheating epoch is not sufficiently low and the decays of the Next to Lightest Supersymmetric Particle (NLSP) into gravitino, can spoil the predictions of primordial elements. In several scenarios, however, these problems can be solved.

1.6 Outline of the Thesis

Unlike the supersymmetry-preserving part of the MSSM Lagrangian, the soft terms in $\mathcal{L}_{\rm soft}$ introduce many new parameters that where not present in the Standard Model. In particular there are 105 extra new parameters respect to the ordinary Standard Model. Thus, in principle, the supersymmetry breaking terms seem to introduce a great amount of arbitrariness in the Lagrangian.

Fortunately, there is already good experimental evidence that some mechanism governs the soft supersymmetry breaking Lagrangian. Indeed most of the $\mathcal{L}_{\rm soft}$ parameter space is ruled out by Flavor Physics experiments. In this respect Flavor Physics can be regarded as a very important probe for the mechanism of supersymmetry breaking.

In this thesis work some aspects of phenomenological interest regarding SUSY breaking and its connection with flavor physics are discussed in two specific contexts: the scenario with the hierarchical soft terms and in Tree level Gauge Mediation (TGM) which is a specific model of SUSY breaking where the flavor problem is solved.

In Chapter 2 we study the framework of hierarchical soft terms [4], in which the first two generations of squarks and sleptons are heavier than the rest of the supersymmetric spectrum. This scenario can arise in several models of SUSY breaking and gives distinctive predictions for the pattern of flavor violations, which we compare to the case of nearly degenerate squarks.

In Chapter 3 we present the study of a new possible mechanism to communicate SUSY breaking: through the exchange of a new gauge vector superfield at the tree level [5, 6]. Remarkably, this possibility, which seems to be the easiest, has not been explored in the literature untill now. The idea is presented trough the study of an SO(10) GUT model. In this case the constraints imposed by the flavor physics are under control. Indeed, the gauge messenger is flavor blind and the resulting soft masses are flavor universal.

As we saw in the previous section, there is also a deep connection between cosmology and supersymmetry breaking, in this case indeed the gravitino is the LSP with a mass of around 15 GeV, the cosmological implications of this scenario are also discussed.

The general aspects of Tree level Gauge Mediation (TGM) are investigated in Chapter 4.

Chapter 2

Hierarchical Sfermions

2.1 The flavor problem in the MSSM

In the SM there are 3 physical mixing angles between different quark families and 1 CP violating phase. These 4 parameters are able to accommodate all the experimental data in the flavor physics. It is interesting and also pedagogical to compute the number of physical parameters in the SM and compare to case of the MSSM.

Let us consider a Lagrangian $\mathcal{L}_G$ which is invariant under a global symmetry group G. We add to $\mathcal{L}_G$ another term $\delta\mathcal{L}$, and now in the new theory $\mathcal{L}_H \equiv \mathcal{L}_G + \delta\mathcal{L}$ the global symmetry G is explicitly broken to a subgroup H.

The number of new physical parameters $N_{\delta\mathcal{L}}$ introduced by $\delta\mathcal{L}$ is given by

$$N_{\delta\mathcal{L}} = N_{\text{general}} - N_{\text{broken}} \tag{2.1}$$

where N_{general} is the number of all the parameters in $\delta\mathcal{L}$, and N_{broken} is the number of broken generators of the coset space G/H.

If we exclude the Yukawa sector, the Lagrangian of the SM contains 6 physical parameters: 3 gauge couplings, 2 parameters (μ and λ) entering the Higgs potential (1.1) and the QCD vacuum angle θ_{QCD}. At this level the global symmetry of the Lagrangian is (apart from the $SU(3)_C \otimes SU(2)_L$ factors):

$$G \equiv U(3)_q \otimes U(3)_{u^c} \otimes U(3)_{d^c} \otimes U(3)_l \otimes U(3)_{e^c} \otimes U(1)_H \tag{2.2}$$

and the index under each group factor specifies the fields on which the group act. G contains as subgroup also $U(1)_Y$. The gauge symmetries are broken only in the vacuum and they cannot been used to reduce the number of physical parameters. With a proper choice of the generators, the effects of removing the $U(1)_Y$ from G corresponds to dropping the $U(1)_H$ factor and for the rest of our analysis we assume $G = U(3)^5$.

Let us now add the Yukawa terms:

$$\mathcal{L}_{\text{Yukawa}} = y^u_{ij} q_i u^c_j H + y^d_{ij} q_i d^c_j \tilde{H} + y^e_{ij} l_i e^c_j \tilde{H} + \text{c.c.} \tag{2.3}$$

where $\tilde{H} = -i\sigma_2 H$. We have introduced 54 new parameters because each y_u, y_d and y_e is a 3×3 complex matrix. The symmetry is reduced from G to

$$H = U(1)_Q \otimes U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau\,. \tag{2.4}$$

These symmetries imply the conservations of the Baryon number and the lepton number separately for each family. In the end we have

$$\begin{aligned} N_{\text{broken}} &= 45 - 4 = 41 \qquad && \#\text{ generators of } U(3)^5 - \#\text{ generators of } U(1)^4 \\ N_{\mathcal{L}_{\text{Yukawa}}} &= 54 - 41 = 13 && N_{\text{general}} - N_{\text{broken}} \end{aligned}$$

we can identify the 13 physical parameters as the 9 fermion masses and the 4 parameters entering the CKM matrix. Adding also the previous 6 parameters, we conclude that the physical content of the SM is encoded in 19 parameters.

We can now discuss the case of the MSSM. The starting Lagragian is $\mathcal{L}_{\text{kinetic}}$, which is described by a canonical Kähler $K = \Phi^\dagger e^{2gV}\Phi$ and gauge kinetic function $W_\alpha W^\alpha$. Φ is the set of all the chiral superfields in the MSSM: $\Phi \equiv (\hat{q}_i, \hat{u}^c_i, \hat{d}^c_i, \hat{l}_i, \hat{e}^c_i, \hat{h}_u, \hat{h}_d)$. $\mathcal{L}_{\text{kinetic}}$ contains 4 parameters: 3 gauge couplings and the QCD vacuum angle. The global symmetry of the Lagrangian is (aside from the $SU(3)_C \otimes SU(2)_L$ factors):

$$G \equiv U(3)_{\hat{q}} \otimes U(3)_{\hat{u}^c} \otimes U(3)_{\hat{d}^c} \otimes U(3)_{\hat{l}} \otimes U(3)_{\hat{e}^c} \otimes U(1)_{\hat{h}_u} \otimes U(1)_{\hat{h}_d} \otimes U(1)_R\,. \tag{2.5}$$

apart from $U(1)_R$, the index under each group factor specifies the superfields on which the group act. $U(1)_R$ is an internal symmetry which does not commute with the SUSY generators.

We can perform a change of basis among all the $U(1)$ generators of G and trade $U(1)_{\hat{h}_u} \otimes U(1)_{\hat{h}_d}$ for the Peccei-Quinn symmetry $U(1)_{\text{PQ}}$ and the SM hypercharge $U(1)_Y$. As in the SM case the $U(1)_Y$ is not relevant for what follows. So the global symmetry we want to consider is

$$G = U(3)^5 \otimes U(1)_{PQ} \otimes U(1)_R \tag{2.6}$$

Let us now add to $\mathcal{L}_{\text{kinetic}}$ all the contributions in the MSSM that do not carry flavor indices $\mathcal{L}_{\text{unflavored}}$:

$$\mathcal{L}_{\text{unflavored}} \equiv \left(\int d^2\theta\, \mu \hat{h}_u \hat{h}_d + \text{c.c.} \right) + \mathcal{L}_{\text{gaugino}} + \mathcal{L}_{\text{softH}} \tag{2.7}$$

where $\mathcal{L}_{\text{gaugino}}$ and $\mathcal{L}_{\text{softH}}$ are defined in the previous chapter. In $\mathcal{L}_{\text{unflavored}}$ there are 12 parameters (5 complex parameters from $M_1, M_2, M_3, \mu, B\mu$ and 2 real parameters $\tilde{m}^2_{\hat{h}_u}, \tilde{m}^2_{\hat{h}_d}$). The $U(1)_R$ symmetry is now explicitly broken by the gaugino masses and the $U(1)_{\text{PQ}}$ by the μ and the $B\mu$ terms. Therefore we have two broken generators and we can eliminate two physical parameters obtaining $N_{\mathcal{L}_{\text{unflavored}}} = 12 - 2 = 10$ and usually one takes M_3 and $B\mu$ real and positive.

The global symmetry group of $\mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{unflavored}}$ is $U(3)^5$. To complete the MSSM we have to add the following Lagrangian:

$$\mathcal{L}_{\text{flavor}} = \left[\int d^2\theta \left(y^u_{ij}\hat{q}_i\hat{u}^c_j\hat{h}_u + y^d_{ij}\hat{q}_i\hat{d}^c_j\hat{h}_d + y^e_{ij}\hat{l}_i\hat{e}^c_j\hat{h}_d\right) + \text{c.c}\right] + \mathcal{L}_{\text{bilinear}} + \mathcal{L}_{\text{bilinear}} \tag{2.8}$$

this sector introduces a large number of parameters. There are 6 complex matrices $y^u, y^d, y^e, A^u, A^d, A^e$ and 5 hermitian matrices $\tilde{m}^2_q, \tilde{m}^2_{u^c}, \tilde{m}^2_{d^c}, \tilde{m}^2_l, \tilde{m}^2_{e^c}$. In total we have $N_{general} = 6 \cdot 18 + 5 \cdot 9 = 153$ parameters. The global symmetry $U(3)^5$ is reduced to $U(1)_Q \otimes U(1)_L$, as in the MSSM only the total lepton number is conserved and we can violate the flavor also in the lepton sector. Hence for the flavor sector of the MSSM

$$\begin{aligned} N_{\text{broken}} &= 45 - 2 = 43 && \#\text{ generators of } U(3)^5 - \#\text{ generators of } U(1)^2 \\ N_{\mathcal{L}_{\text{flavor}}} &= 153 - 43 = 110 && N_{\text{general}} - N_{\text{broken}} \end{aligned}$$

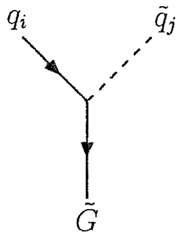
we can identify the 110 physical parameters as the 30 fermion and sfermion masses, 36 mixing angles and 41 CP violating phases. To distinguish among phases and mixing angles we used the result in [7, 8]. To complete the counting of the MSSM parameters we have to add 5 physical parameters in the Higgs sector, 5 parameters related to the gaugino masses and again 3 gauge couplings and 1 strong CP phase. The total is 124.

The flavor problem can be formulated in the following way: a huge part of this parameter space is ruled out by experimental constraints because the SUSY contribution to FCNC and CP violation processes are too large.

The most natural solution to solve the flavor problem by itself is simply to decouple the heavy particles from the model. Obviously the price to pay in this case is a large amount of fine tuning. In this respect the flavor problem can be seen as an aspect of the naturalness problem.

If we assume that sparticle masses are in the range of hundreds of GeV and a generic flavor structure for the soft terms, the contribution of SUSY to the FCNC and CP violation processes can naturally be several orders of magnitude larger than the SM one. This happens, for example, in processes like $\mu \to e\gamma$ or in $K^0 - \bar{K}^0$ mixing. All of the relevant data in the flavor sector is described to good accuracy by the SM, therefore a SUSY contribution to the physical observables has to be strongly suppressed with respect to the SM.

To understand the possible mechanisms that can suppress the SUSY contributions, let us discuss a specific example: the gluino contributions to a $\Delta F = 1$ process. For simplicity we will consider fermions and sfermions of one chirality. The relevant flavor violating vertex is given by Figure 2.1.



$$= -ig_s \mathcal{W}_{ij}$$

Figure 2.1

Where $\mathcal{W}_{ij}$ is the unitary matrix which connects the quark with flavor i and the j-th squark eigenstates $(i, j = 1, 2, 3)$ and g_s is the $SU(3)_C$ gauge coupling.

The MSSM amplitude $\mathcal{A}_{\text{SUSY}}$ for a quark transition $q_i \to q_j$ is

$$\mathcal{A}_{\text{SUSY}} \propto \frac{1}{M_3^2} \sum_{k=1}^{3} \mathcal{W}_{ik} f(\frac{m_k^2}{M_3^2}) \mathcal{W}_{kj}^\dagger \tag{2.9}$$

where m_k, M_3 are respectively the mass of the k-th sfermion and the gluino mass, f is a loop function which depends on the specific FC process.

We have three possible mechanisms that can suppress this amplitude with respect to the SM one:

- Degeneracy
- Alignement
- Decoupling

2.1.1 Degeneracy

If the squarks in the loop have the same mass then $m_i^2 = \tilde{m}^2$ for $i = 1, 2, 3$. Using the unitarity of the matrix $\mathcal{W}$ we get a Kronecker δ on the flavor space, this means that we cannot change flavor in the transition:

$$\mathcal{A}_{\text{SUSY}} \propto \frac{1}{M_3^2} f(\frac{m^2}{M_3^2}) \sum_{k=1}^{3} \mathcal{W}_{ik} \mathcal{W}_{kj}^\dagger = \frac{1}{M_3^2} f(\frac{m^2}{M_3^2}) \delta_{ij} \tag{2.10}$$

The complete degeneracy of the sfermions corresponds to the assumption of universal soft terms [9].

In general it is not easy to obtain a degenerate spectrum at the EW scale. This is because once the soft terms are generated at the messenger scale M_{mess} then the radiative corrections tend to split the sfermion masses. In particular the large Yukawa coupling for the third generation will make the third family of sfermions lighter. So in general one can expect that this mechanism is more operative for the sfermions of the first two families if we started with a universal soft term at the scale M_{mess}.

To understand the effects of the SUSY contributions, usually in literature, it is assumed a degenerate spectrum. This assumption is very useful in order to reduce the relevant number of parameters. A distortion from exact universality can be encoded in a model independent way adding small off diagonal contribution $\delta\tilde{m}^2_{ij}$ to the sfermions squared mass matrix. The flavor transition is now triggered by this off diagonal contribution and a small expansion parameter is given by the so called mass insertion: $\delta_{ij} = \delta\tilde{m}^2_{ij}/\tilde{m}^2$.

2.1.2 Alignment

The mixing matrix $\mathcal{W}$ is given by the product of two unitary matrices U and $\mathcal{W}'$, which diagonalize respectively the quark mass matrix and the squark mass matrix squared. So the exact alignment corresponds to the following condition:

$$\mathcal{W}_{ij} = (U\mathcal{W}')_{ij} = \delta_{ij} \tag{2.11}$$

In other words the assumption is that quark and squark mass matrices are nearly simultaneously diagonalized by a supersymmetric field rotation, either in the down or in the up sector [11]. The bounds from the kaon system severely constrain the case in which $\tilde{m}^2_q$ is aligned along the up direction. The bounds on D^0–$\bar{D}^0$ mixing give important constraints on the alignment along the down direction [12]. Correlations between quark and squark mass matrices leading to alignment are possible in models where some approximate flavor symmetry determines the form of Yukawa couplings and soft terms [11, 13]. Flavor alignment does not imply mass degeneracy of squarks. Thus, in this case the situation is exactly reversed with respect to the case of degeneracy. The suppression of flavor violating processes is due to the small squark mixing angles, while squark masses can be widely different.

2.1.3 Decoupling

The decoupling is the assumption that the particles in the loop are very heavy. For example in (2.9) if we take the limit of a very large squark mass we get that the argument in the function f is going to infinity, and in this limit f goes to 0. If the gluino is very heavy respect to the squark the loop function f approaches a finite value, but the factor $\frac{1}{M_3^2}$ suppresses the amplitudes $\mathcal{A}_{\text{SUSY}}$.

The flavor structure of the first and second generation squarks is tightly constrained by K physics. On the other hand, the upper bounds on the masses of the first two generations of squarks are much looser than for the other supersymmetric particles. Therefore one can relax the flavor constraints, without compromising naturalness, by taking the first two generations of squarks much heavier than the third [14, 15, 16, 17]. As discussed in more detail later, this procedure alleviates, but does not completely solve, the flavor problem and a further suppression mechanism for the first two generations must be present. However, it is not difficult to conceive the existence of such a mechanism which operates if, for instance, the soft terms respect an approximate U(2) symmetry acting on the first two generations [16, 18]. In the case of hierarchy, the small expansion parameter describing the flavor violation is the mismatch between the third-generation quarks identified by the Yukawa coupling and the third-generation squarks identified by the light eigenstates of the soft-term mass matrix. This small mismatch can be related to the hierarchy of scales present in the squark mass matrix and to CKM angles. However, for the phenomenological implications we are interested in this chapter, we do not have to specify any such relation and we can work in an effective theory where the first two generations of squarks have been integrated out. Their only remnant in the effective theory is the small mismatch between third-generation quarks and squarks.

In this next sections, we will revisit the properties of hierarchical soft terms, concentrating especially on their implications in flavor physics. We will show how the hypothesis of hierarchy predicts correlations between $\Delta F = 1$ and $\Delta F = 2$ processes which are different from the correlations found in scenarios with degeneracy. We will present the bounds on the expansion parameters of the hierarchy case and compare them with the case of degeneracy. As a particularly interesting example we will study the phase of B_s mixing, for which there are some claims [19, 20, 21, 22] that experiments have measured an excess with respect to the SM prediction. We will show that the case of hierarchy is compatible with much larger phases of B_s mixing than the case of degeneracy, and thus a hierarchical squark spectrum has more room to explain the alleged effect.

2.2 Hierarchical Soft Terms and Naturalness

The hypothesis of hierarchical soft terms states that the first two generations of squarks and sleptons are much heavier than the rest of the supersymmetric particles, assumed to lie near the electroweak scale. We will denote by $\tilde{m}_h$ the mass of the heavy squarks and sleptons and by $\tilde{m}_\ell$ the mass scale of the other "light" supersymmetric particles. The original motivation of this hypothesis [17] is that $\tilde{m}_h$ is more weakly bound by naturalness arguments than other supersymmetric parameters, because its radiative effect on the Higgs mass parameter m_H^2 is rather moderate. The leading effect comes from a one-loop renormalization of m_H^2 proportional to an induced hypercharge Fayet-Iliopoulos term

$$\mathrm{Tr}(Y\tilde{m}^2) = \mathrm{Tr}(\tilde{m}_Q^2 + \tilde{m}_D^2 - 2\tilde{m}_U^2 - \tilde{m}_L^2 + \tilde{m}_E^2). \tag{2.12}$$

Assuming that soft terms are generated at the GUT scale, this term leads to a naturalness bound on $\tilde{m}_h$ just below the TeV scale [15]. Nevertheless, the term in eq. (2.12) vanishes if, at some energy scale, scalar masses are universal or satisfy a GUT condition where hypercharge is embedded in a non-abelian group. Since the term in eq. (2.12) is only multiplicatively renormalized, it will remain zero at any scale.

If the Fayet-Iliopoulos term vanishes, then the leading renormalization of m_H^2 proportional to $\tilde{m}_h^2$ comes from two-loop effects. In Fig. 2.2 we show an upper bound on $\tilde{m}_h$, assuming that first and second generation scalars are degenerate at a matching scale $M_{\rm susy}$, where we start the renormalization group flow. The bound corresponds to an upper limit $\Delta < 10$ on the fine-tuning parameter Δ [24], which is optimistic in the light of the present naturalness status of the supersymmetric SM. Still, multi-TeV squarks are allowed by naturalness. It is also possible to reach values of $\tilde{m}_h$ in the range of 10 TeV, but only if soft terms are generated at a very low scale $M_{\rm susy}$.

Another bound on the hierarchy of soft terms comes from the requirement that $\tilde{m}_h$ does not drive the squared masses of third-generation squarks to negative values, through its two-loop renormalization-group effect [25]. This bound, although weaker than the previous one, is independent of naturalness arguments. Assuming complete degeneracy of the heavy states with mass $\tilde{m}_h$ and of the light states with mass $\tilde{m}_\ell$, the condition that color remains unbroken imposes $\tilde{m}_h/\tilde{m}_\ell \lesssim 15$, if $M_{\rm susy}$ is close to the GUT scale. In the case of low $M_{\rm susy}$, where the effect is due to two-loop threshold effects not log enhanced, the bound becomes $\tilde{m}_h/\tilde{m}_\ell \lesssim 25$. However, these bounds can be avoided by choosing appropriate boundary conditions of the soft terms at the scale $M_{\rm susy}$. For instance, all sfermions could be heavy at $M_{\rm susy}$, but Yukawa effects could dynamically bring the third generation to be light [26]. It is also possible to introduce new states that approximately cancel the two-loop renormalization-group contribution to $\tilde{m}_\ell^2$ proportional to $\tilde{m}_h^2$ and maintain the stability of the soft-term hierarchy against large radiative corrections [27].

These upper bounds on $\tilde{m}_h$ have to be compared with the lower limits coming from flavor-violating effects in the K system. Assuming that the heavy squark sector is neither degenerate nor aligned, we find the bound[1]

$$\tilde{m}_h > 35\,\mathrm{TeV} \tag{2.13}$$

[1]These numbers are based on the analysis presented in Section 2.6. The effect of QCD corrections for heavy squarks has been considered in ref. [28].

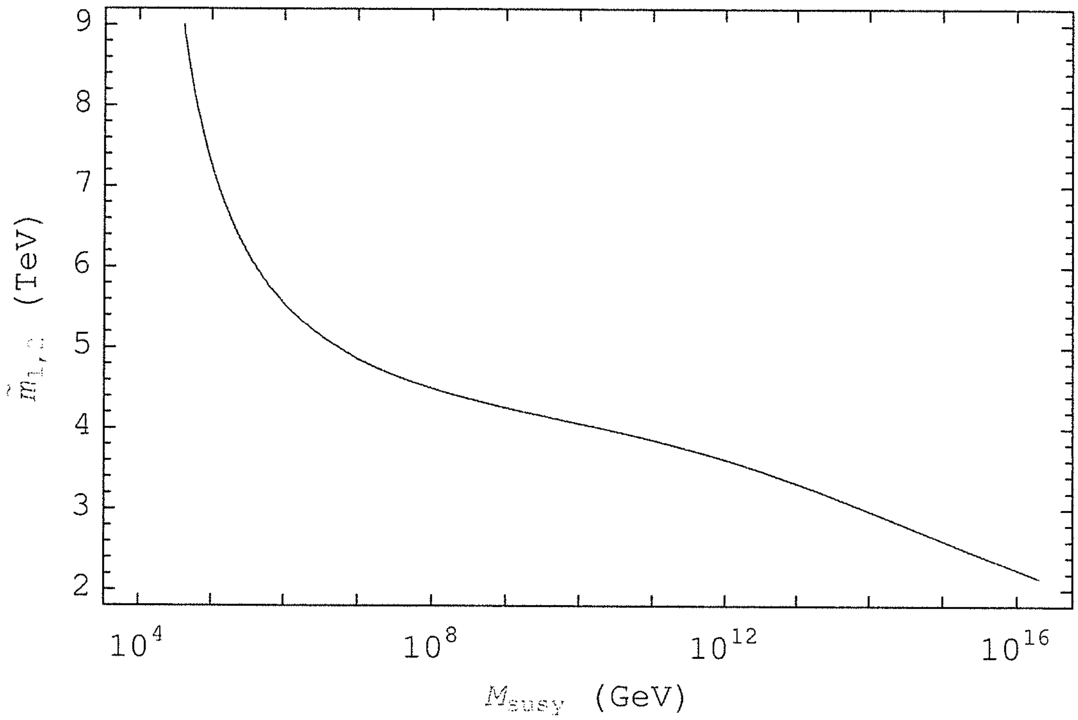


Figure 2.2: Upper bound on $\tilde{m}_h$, assuming that first and second generation scalars are degenerate at a matching scale $M_{\rm susy}$. The bound corresponds to an upper limit $\Delta < 10$ on the fine-tuning parameter.

from the real part of the $\Delta S = 2$ transition, and

$$\tilde{m}_h > 800\,\mathrm{TeV} \tag{2.14}$$

from ϵ_K.

This shows that the hypothesis of hierarchical soft terms is not sufficient to solve the flavor problem, unless one is willing to give up naturalness, in the spirit of Split Supersymmetry [29], assuming that the first two generations of sfermions are directly coupled to the supersymmetry-breaking sector. More concretely, we can retain naturalness and rely on a scheme for suppressing the flavor transitions in the heavy sector, as can be achieved by an approximate U(2) symmetry acting on the first two generations. In this respect, the hierarchical structure of soft terms can be a useful way of parametrizing supersymmetric theories which, for model-dependent reasons, have a certain separation of scales in the scalar sector. Moreover, hierarchical soft terms are interesting because they make specific predictions in flavor physics controlled by relatively few parameters related to physical quantities, like the mass hierarchy. As we will show, hierarchical soft terms offer a well-defined benchmark to be compared with the new experimental results in flavor physics.

2.3 Hierarchy versus Degeneracy in $\Delta F = 1$ and $\Delta F = 2$ Processes

Let us first consider the gluino contribution to a $\Delta F = 1$ process in the left-handed down quark sector, $d_i^L \to d_j^L$, neglecting for simplicity chirality changes. The amplitude of such a process is proportional to

$$A(\Delta F = 1) \equiv f\left(\frac{\mathcal{M}_{\tilde{D}}^2}{M_3^2}\right)_{d_i^L d_j^L} = \mathcal{W}_{d_i^L \tilde{D}_I} f\left(\frac{m_{\tilde{D}_I}^2}{M_3^2}\right) \mathcal{W}^*_{d_j^L \tilde{D}_I}. \tag{2.15}$$

Here f is a loop function, M_3 is the gluino mass and $\mathcal{W}$ is the unitary matrix diagonalizing the 6×6 down squark squared mass matrix $\mathcal{M}_{\tilde{D}}^2$ in a basis in which the down quark mass matrix is diagonal. We can simplify eq. (2.15) by using a perturbative expansion in the small off-diagonal entries of the squark mass matrix. It is often sufficient to keep the first order in the expansion. However, the second order can become important and even dominate in the case of 1–2 transitions, depending on the relative size of the 12 expansion parameter compared to the product of the 13 and 23 ones, and on the relative sizes of the sfermion masses. One important example of the case in which the second order dominates is the hierarchy case discussed below, in which the first order is suppressed because of the heaviness of the sfermions of the first two families. Then, eq. (2.15) becomes [30]

$$f\left(\frac{\mathcal{M}_D^2}{M_3^2}\right)_{d_i^L d_j^L} = \frac{\tilde{m}^2}{M_3^2} f\left(x_{d_i^L}, x_{d_j^L}\right) \delta_{ij}^{LL}, \tag{2.16}$$

where $x_i \equiv m_i^2/M_3^2$, $\delta_{ij}^{LL} \equiv (\mathcal{M}_D^2)_{d_i^L d_j^L}/\tilde{m}^2$, and

$$f(x,y) = \frac{f(x) - f(y)}{x - y}. \tag{2.17}$$

The "mass insertion" δ_{ij}^{LL} is the expansion parameter and we have normalized it to a mass $\tilde{m}$ which can be chosen to be a typical scale of squark masses. This parameter effectively accounts (at first order) for the flavor transition.

The "degenerate" case is obtained in the limit in which the squark masses in the loop function coincide,

$$m_{\tilde{d}_i^L}^2 = m_{\tilde{d}_j^L}^2 \equiv \tilde{m}^2. \tag{2.18}$$

With this assumption, we obtain

$$f\left(\frac{\mathcal{M}_D^2}{M_3^2}\right)_{d_i^L d_j^L} = x f^{(1)}(x)\, \delta_{ij}^{LL}, \qquad \text{(degenerate case)} \tag{2.19}$$

where $x = \tilde{m}^2/M_3^2$ and $f^{(n)}$ is the n-th derivative of the function. The δ parameters are in this case normalized to the universal scalar mass $\tilde{m}^2$.

In the "hierarchical" limit, the contribution to the loop function in eq. (2.15) from the heavy squarks is negligible. Therefore eq. (2.15) becomes

$$f\left(\frac{\mathcal{M}_D^2}{M_3^2}\right)_{d_i^L d_j^L} = f(x)\, \hat{\delta}_{ij}^{LL}. \qquad \text{(hierarchical case)} \tag{2.20}$$

Here $x = \tilde{m}^2/M_3^2$ as before, where now $\tilde{m}^2$ is interpreted as the third-generation squark mass. We have defined $\hat{\delta}_{ij}^{LL} \equiv \mathcal{W}_{d_i^L \tilde{b}_L} \mathcal{W}^*_{d_j^L \tilde{b}_L}$. Note that $\hat{\delta}_{a3}^{LL} \approx -(\mathcal{M}_D^2)_{d_a^L d_3^L}/\tilde{m}_a^2$, so that $\hat{\delta}_{a3}^{LL}$ is again a normalized mass insertion. Also, $\hat{\delta}_{12}^{LL} = \hat{\delta}_{13}^{LL}(\hat{\delta}_{23}^{LL})^*$. Eq. (2.20) can also be obtained from an extension of eq. (2.16) to the second order in δ.

Equations (2.19) and (2.20) show that for $\delta = \hat{\delta}$ the difference between the two schemes, the degenerate and the hierarchical one, is given by the order one difference between a function and its derivative. However, this $\mathcal{O}(1)$ difference becomes larger when we consider $\Delta F = 2$ processes and turns out to affect the predicted correlation between $\Delta F = 1$ and $\Delta F = 2$. In fact, let us now consider the gluino contribution to a $\Delta F = 2$ $d_i^L \leftrightarrow d_j^L$ process. The amplitude is proportional to

$$A(\Delta F = 2) \equiv \mathcal{W}_{d_i^L \tilde{D}_I} \mathcal{W}_{d_i^L \tilde{D}_J}\, g\left(\frac{m_{\tilde{D}_I}^2}{M_3^2}, \frac{m_{\tilde{D}_J}^2}{M_3^2}\right) \mathcal{W}^*_{d_j^L \tilde{D}_I} \mathcal{W}^*_{d_j^L \tilde{D}_J}, \tag{2.21}$$

where the loop function $g(x, y)$ is of the form[2]

$$g(x, y) = \frac{g(x) - g(y)}{x - y}. \tag{2.22}$$

Expanding in the small off-diagonal elements of the squark mass matrix and assuming, as in the case of $\Delta F = 1$, the dominance of 2×2 transitions, we obtain that eq. (2.21) can be written as

$$A(\Delta F = 2) = \frac{\tilde{m}^4}{M_3^4} \hat{g}(x_{\tilde{d}_i^L}, x_{\tilde{d}_j^L})(\delta_{ij}^{LL})^2, \tag{2.23}$$

$$\hat{g}(x, y) = \frac{g(x, x) - 2g(x, y) + g(y, y)}{(x - y)^2}. \tag{2.24}$$

Thus, eq. (2.21) becomes

$$A(\Delta F = 2) = \begin{cases} \frac{x^2}{3!} g^{(3)}(x)(\delta_{ij}^{LL})^2 & \text{(degenerate case)} \\ g^{(1)}(x)(\hat{\delta}_{ij}^{LL})^2 & \text{(hierarchical case)}. \end{cases} \tag{2.25}$$

Therefore, if $\tilde{m}^2$ is the same in the two cases we find that the amplitudes for $\Delta F = 1$ and $\Delta F = 2$ processes satisfy the relation

$$\left.\frac{A(\Delta F = 2)}{[A(\Delta F = 1)]^2}\right|_{\text{degenerate}} = \frac{g^{(3)}}{6g^{(1)}} \left(\frac{f}{f^{(1)}}\right)^2 \left.\frac{A(\Delta F = 2)}{[A(\Delta F = 1)]^2}\right|_{\text{hierarchical}}. \tag{2.26}$$

This result is independent of the values of the mass insertions in the two cases. Partly due to the different factorials involved, the ratio $R = (g^{(3)}/6g^{(1)})(f/f^{(1)})^2$ is typically small, easily $\mathcal{O}(10^{-1})$ for $x = 1$. As a consequence, the bounds on the $\Delta F = 2$ processes inferred from $\Delta F = 1$, or viceversa, may be significantly different in the two frameworks. The factor R is shown in Fig. 2.3 as a function of $x = \tilde{m}^2/M_3^2$. The loop functions entering the factor R plotted in Fig. 2.3 are the ones entering the coefficients of the LL insertions in the B_s–$\bar{B}_s$ oscillation amplitude and in the $B \to X_s\gamma$ decay amplitude.

Another interesting point has to do with the relation between the $s \leftrightarrow d$, $b \leftrightarrow d$, $b \leftrightarrow s$ $\Delta F = 2$ processes. In the degenerate case, such processes are proportional (for given chiralities and charge of the gaugino involved) to the a priori independent

[2] This decomposition follows from the form of the loop integral

$$g(x, y) = \int dk \frac{G(k)}{(k^2 - x)(k^2 - y)} = \frac{1}{x - y} \int dk\; G(k) \left(\frac{1}{k^2 - x} - \frac{1}{k^2 - y}\right) \equiv \frac{g(x) - g(y)}{x - y}.$$

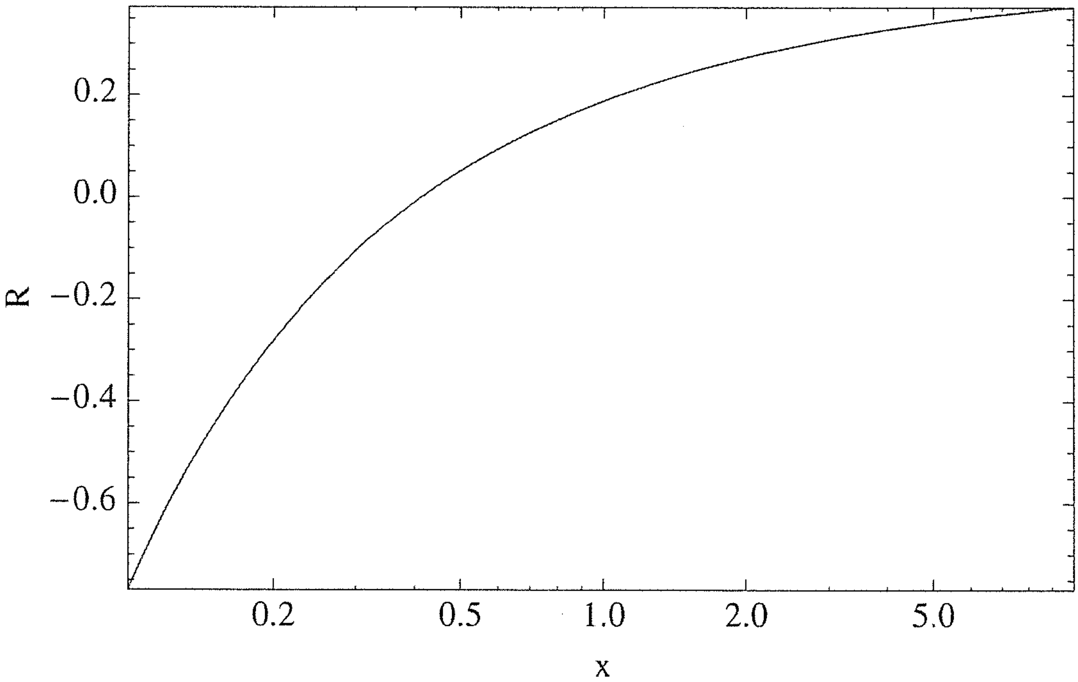


Figure 2.3: Dependence of the factor $R = (g^{(3)}/6g^{(1)})(f/f^{(1)})^2$ on $x = \tilde{m}^2/M_3^2$. The loop functions enter the coefficients of the LL insertions in the B_s–$\bar{B}_s$ oscillation amplitude and in the $B \to X_s\gamma$ decay amplitude.

three quantities δ_{sd}^2, δ_{bd}^2, δ_{bs}^2. A partial correlation among the three processes could in principle be generated by higher order contributions to the $s \leftrightarrow d$ transitions, *e.g.* the ones proportional to $\delta_{sb}^2\delta_{bd}^2$. However, such contributions turn out to be always small. This is because of the limits on the two factors δ_{bs} and δ_{bd} and because the four-insertions $\delta_{sb}^2\delta_{bd}^2$ contribution is proportional to $(x^4/5!)\,g^{(5)}$, *i.e.* it is suppressed by the factor $5! = 120$. On the other hand, in the hierarchical case, a correlation does arise because $\hat{\delta}_{ds} = \hat{\delta}_{db}\hat{\delta}^*_{sb}/|\mathcal{W}_{b\tilde{b}}|^2 \approx \hat{\delta}_{db}\hat{\delta}^*_{sb}$. Moreover, the higher-order contribution proportional to $\hat{\delta}^2_{db}\hat{\delta}^{*2}_{sb}$ is now proportional to $g^{(1)}(x)$, with no factorials involved.

2.4 The Flavor Structure for Hierarchical Soft Terms

In this Section we define the setting of hierarchical soft terms in greater detail. In order to obtain the expressions for the amplitude of a generic flavor process in the hierarchical case, it suffices to consider the case of a one-variable loop function, as in $\Delta F = 1$ transitions. The generalization to more variables is straightforward. Let us then consider an amplitude whose dependence on sfermion masses comes through

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Ai,Bj} = \mathcal{W}_{Ai,I}\, f\left(\frac{\tilde{m}_I^2}{M^2}\right)\mathcal{W}^*_{Bj,I}. \tag{2.27}$$

In the expression above, M is the mass of the relevant gaugino, $\mathcal{M}^2$ is the 6×6 sfermion squared mass matrix in the up squark, down squark, or charged slepton sector (the extension to sneutrinos is again straightforward), written in a basis in which the corresponding fermion mass matrix is diagonal and positive. The amplitude corresponds to a flavor transition between two fermions with chirality $A, B = L, R$ of families $i, j = 1, 2, 3$, $i \neq j$ and $\mathcal{W}$ is the unitary matrix diagonalizing $\mathcal{M}^2$, so that $\mathcal{W}_{Ai,I}$ is the mixing between the fermion "Ai" and the I-th sfermion mass eigenstate, $I = 1\ldots 6$.

According to our assumption, 4 out of the 6 squarks are much heavier than the others or the gaugino mass. Their contribution to the loop function is then suppressed by the light-to-heavy ratio of squared masses $\tilde{m}_l^2/\tilde{m}_h^2$ (at least, up to logarithms) with respect to the contribution from third-generation squarks. However, for flavor transitions between quarks of the first two families, the exchange of third-generation squarks comes at the price of mixing angles, also suppressed by powers of the heavy mass scale $\tilde{m}_h^2$. Nevertheless, as shown in the Appendix, the contribution of heavy squarks to eq. (2.27) is subdominant, as long as some GIM mechanism is operative in the first two generation squark sector. Since this must be the case in order to evade the strong constraints from ϵ_K, we can neglect the effect of the heavy squarks in the summation of eq. (2.27). Alternatively, the assumption of neglecting the heavy-state exchange is justified when the first two generations of squarks are completely decoupled and the flavor mixing of the third-generation squarks are determined by quark rotation angles (see Appendix).

We are then left with two light squarks with masses $\tilde{m}_{\ell_\alpha}$ and mixings $\mathcal{W}_{Ai,\alpha}$, where $\alpha = 1, 2$ is the index of the light eigenstates. This gives a total of 2+20 real parameters. However, since the mixings always appear in the combination $\mathcal{W}_{Ai,\alpha}\mathcal{W}^*_{Bj,\alpha}$, the overall phases of the mixing parameters (for any value of α) do not affect eq. (2.27) and the number of effective parameters is 2+18.

This is still more general than needed. In fact, the decoupling of the first two sfermion families leads (under certain assumptions) to two additional constraints, as discussed in the Appendix. First, in the limit in which the 4 heavy sfermions decouple, the 2×2 matrix $\mathcal{W}_{A3,\alpha}$ that diagonalizes the 2×2 third-family sfermion mass matrix is approximately unitary. It is then always possible to describe it in terms of an angle $0 \leq \theta \leq \pi/2$ and a phase ϕ. The angle θ corresponds to the usual mixing angle between the two chiral components of third-generation squarks. Second, the chirality-changing mixing is subdominant with respect to the chirality-conserving one, except within the third family. This means that the leading effect in any chirality-changing transition comes from the combination of a chirality-conserving one times a θ-angle rotation.

We are then left with 4 parameters describing the third generation squarks ($\tilde{m}_{\ell_\alpha}$, θ, ϕ) and the four complex chirality-conserving "insertions" $\hat{\delta}^{LL}_{i3}$, $\hat{\delta}^{RR}_{i3}$, $i = 1, 2$ defined as follows:

$$\hat{\delta}^{LL}_{i3} \equiv \sum_{\alpha=1,2} \mathcal{W}_{Li,\alpha}\mathcal{W}^*_{L3,\alpha} \qquad \hat{\delta}^{LL}_{3i} = \hat{\delta}^{LL*}_{i3} \tag{2.28a}$$

$$\hat{\delta}^{RR}_{i3} \equiv \sum_{\alpha=1,2} \mathcal{W}_{Ri,\alpha}\mathcal{W}^*_{R3,\alpha} \qquad \hat{\delta}^{RR}_{3i} = \hat{\delta}^{RR*}_{i3}. \tag{2.28b}$$

Using the expression of the matrix $\mathcal{W}$ derived in the Appendix, at first order in the insertion $\hat{\delta}$, eq. (2.27) becomes

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Li,L3} = \left[\cos^2\theta\, f\left(\frac{\tilde{m}^2_{\ell_1}}{M^2}\right) + \sin^2\theta\, f\left(\frac{\tilde{m}^2_{\ell_2}}{M^2}\right)\right]\hat{\delta}^{LL}_{i3} \tag{2.29a}$$

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Li,Lj} = \left[\cos^2\theta\, f\left(\frac{\tilde{m}^2_{\ell_1}}{M^2}\right) + \sin^2\theta\, f\left(\frac{\tilde{m}^2_{\ell_2}}{M^2}\right)\right]\hat{\delta}^{LL}_{i3}\hat{\delta}^{LL*}_{j3} \tag{2.29b}$$

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Li,R3} = \sin\theta\cos\theta e^{i\phi}\left[f\left(\frac{\tilde{m}^2_{\ell_1}}{M^2}\right) - f\left(\frac{\tilde{m}^2_{\ell_2}}{M^2}\right)\right]\hat{\delta}^{LL}_{i3} \tag{2.29c}$$

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Li,Rj} = \sin\theta\cos\theta e^{i\phi}\left[f\left(\frac{\tilde{m}^2_{\ell_1}}{M^2}\right) - f\left(\frac{\tilde{m}^2_{\ell_2}}{M^2}\right)\right]\hat{\delta}^{LL}_{i3}\hat{\delta}^{RR*}_{j3} \tag{2.29d}$$

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{L3,Ri} = \sin\theta\cos\theta e^{i\phi}\left[f\left(\frac{\tilde{m}^2_{\ell_1}}{M^2}\right) - f\left(\frac{\tilde{m}^2_{\ell_2}}{M^2}\right)\right]\hat{\delta}^{RR*}_{i3} \tag{2.29e}$$

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Ri,R3} = \left[\cos^2\theta\, f\left(\frac{\tilde{m}^2_{\ell_2}}{M^2}\right) + \sin^2\theta\, f\left(\frac{\tilde{m}^2_{\ell_1}}{M^2}\right)\right]\hat{\delta}^{RR}_{i3} \tag{2.29f}$$

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Ri,Rj} = \left[\cos^2\theta\, f\left(\frac{\tilde{m}^2_{\ell_2}}{M^2}\right) + \sin^2\theta\, f\left(\frac{\tilde{m}^2_{\ell_1}}{M^2}\right)\right]\hat{\delta}^{RR}_{i3}\hat{\delta}^{RR*}_{j3} \tag{2.29g}$$

The remaining transition elements cam be obtain from the relation $f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Bj,Ai} = \left(f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Ai,Bj}\right)^*$ and $i,j = 1,2,3$.

Equations (2.29) further simplify if the mixing angle θ is small, as in the case of the down squark sector in the moderate $\tan\beta$ regime. By taking, for simplicity, equal masses for the third generation squarks, $\tilde{m}_{\ell_1} \approx \tilde{m}_{\ell_2} \equiv \tilde{m}$, we obtain $(i,j = 1,2,3)$

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Ai,Aj} = f(x)\,\hat{\delta}^{AA}_{ij} \qquad A = L, R \tag{2.30a}$$

$$f\left(\frac{\mathcal{M}^2}{M^2}\right)_{Ai,Bj} = x\, f^{(1)}(x)\,\hat{\delta}^{AB}_{ij} \qquad A \neq B \tag{2.30b}$$

where $x = \tilde{m}^2/M^2$ and we have defined $(i,j = 1,2)$

$$\hat{\delta}^{AA}_{ij} \equiv \hat{\delta}^{AA}_{i3}\hat{\delta}^{AA*}_{j3} \tag{2.31a}$$

$$\hat{\delta}^{AB}_{i3} \equiv \frac{\mathcal{M}^2_{A3,B3}}{\tilde{m}^2}\hat{\delta}^{AA}_{i3} \tag{2.31b}$$

$$\hat{\delta}^{AB}_{ij} \equiv \frac{\mathcal{M}^2_{A3,B3}}{\tilde{m}^2}\hat{\delta}^{AA}_{i3}\hat{\delta}^{BB*}_{j3}. \tag{2.31c}$$

Here we have written $e^{i\phi}\sin\theta$ as $\mathcal{M}^2_{L3,R3}/(\tilde{m}^2_{\tilde{t}_1} - \tilde{m}^2_{\tilde{t}_2})$. Equations (2.31) express two important results of the flavor structure of hierarchical soft terms. The flavor transition between the first two generations ($\hat{\delta}^{LL}_{ij}$) is determined by the product of the transitions involving the third generation ($\hat{\delta}^{LL}_{i3}\hat{\delta}^{LL*}_{j3}$). The chiral-violating flavor transitions ($\hat{\delta}^{LR}_{ij}$ and $\hat{\delta}^{LR}_{i3}$) are determined by the product of chiral-conserving flavor transitions and the chiral violation in the third family ($\mathcal{M}^2_{L3,R3}/\tilde{m}^2$).

2.5 Effective Hamiltonian

Let us briefly recall the procedure to calculate the Effective Hamiltonian (EH) for a given process. One has to go through the following steps:

- calculate the amplitude between quark and gluon states of definite momenta in the full theory;

- choose a basis of local operators for the effective theory and calculate their matrix elements between the same states used in the previous step;

- determine the coefficients of the operators in the EH by matching the full theory with the effective one.

The matching is given by the following relation:

$$\langle f|S^{\text{full}}|i\rangle = -i\sum_j C_j\langle f|O_j|i\rangle \ , \tag{2.32}$$

where $\langle f|S^{\text{full}}|i\rangle$ is the S matrix element computed in the full theory, C_i are the Wilson coefficients and O_i the operators of the EH:

$$\mathcal{H}_{\text{eff}} = \sum_i C_i O_i \tag{2.33}$$

Let us now specialize to the case of $\Delta S = 1$ and $\Delta S = 2$ processes. The relevant formulae for the flavor transition in the other sectors can be easily derived from the $\Delta S = 1, 2$ case.

We will derive the formulae the gluino/squark contribution in the hierarchical scenario, in particular in the case of equations (2.30).

2.5.1 $\Delta S = 1$

A complete basis for the $\Delta S = 1$ EH is

$$
\begin{aligned}
O_3 &= (\bar{d}_L^\alpha \gamma^\mu s_L^\alpha) \sum_{q=u,d,s} (\bar{q}_L^\beta \gamma_\mu q_L^\beta) \,, \\
O_4 &= (\bar{d}_L^\alpha \gamma^\mu s_L^\beta) \sum_{q=u,d,s} (\bar{q}_L^\beta \gamma_\mu q_L^\alpha) \,, \\
O_5 &= (\bar{d}_L^\alpha \gamma^\mu s_L^\alpha) \sum_{q=u,d,s} (\bar{q}_R^\beta \gamma_\mu q_R^\beta) \,, \\
O_6 &= (\bar{d}_L^\alpha \gamma^\mu s_L^\beta) \sum_{q=u,d,s} (\bar{q}_R^\beta \gamma_\mu q_R^\alpha) \,, \\
O_7 &= \frac{Q_d e}{8\pi^2} m_s \bar{d}_L^\alpha \sigma^{\mu\nu} s_R^\alpha F_{\mu\nu} \,, \\
O_8 &= \frac{g}{8\pi^2} m_s \bar{d}_L^\alpha \sigma^{\mu\nu} t^A_{\alpha\beta} s_R^\beta G^A_{\mu\nu} \,,
\end{aligned}
\tag{2.34}
$$

plus the operators $\tilde{O}_i$ obtained from O_i by the exchange $L \leftrightarrow R$. Here $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, α and β are colour indices, g and e are the strong and electromagnetic couplings, $Q_d = -\frac{1}{3}$ and m_s is the mass of the strange quark.

The matching scale can be identified with the gluino or the squark mass. The Wilson coefficients at that scale are:

$$
\begin{aligned}
C_3 &= \frac{\alpha_s^2}{m_{\tilde{g}}^2} \left(\hat{\delta}_{12}^d\right)_{LL} \left(-\frac{1}{9} \mathrm{B}_1^H(x) - \frac{5}{9} \mathrm{B}_2^H(x) - \frac{1}{18} \mathrm{P}_1^H(x) - \frac{1}{2} \mathrm{P}_2^H(x) \right) , \\
C_4 &= \frac{\alpha_s^2}{m_{\tilde{g}}^2} \left(\hat{\delta}_{12}^d\right)_{LL} \left(-\frac{7}{3} \mathrm{B}_1^H(x) + \frac{1}{3} \mathrm{B}_2^H(x) + \frac{1}{6} \mathrm{P}_1^H(x) + \frac{3}{2} \mathrm{P}_2^H(x) \right) , \\
C_5 &= \frac{\alpha_s^2}{m_{\tilde{g}}^2} \left(\hat{\delta}_{12}^d\right)_{LL} \left(\frac{10}{9} \mathrm{B}_1^H(x) + \frac{1}{18} \mathrm{B}_2^H(x) - \frac{1}{18} \mathrm{P}_1^H(x) - \frac{1}{2} \mathrm{P}_2^H(x) \right) , \\
C_6 &= \frac{\alpha_s^2}{m_{\tilde{g}}^2} \left(\hat{\delta}_{12}^d\right)_{LL} \left(-\frac{2}{3} \mathrm{B}_1^H(x) + \frac{7}{6} \mathrm{B}_2^H(x) + \frac{1}{6} \mathrm{P}_1^H(x) + \frac{3}{2} \mathrm{P}_2^H(x) \right) , \\
C_7 &= \frac{\alpha_s \pi}{m_{\tilde{g}}^2} \left[\left(\hat{\delta}_{12}^d\right)_{LL} \frac{8}{3} \mathrm{M}_3^H(x) + \left(\hat{\delta}_{12}^d\right)_{LR} \frac{m_{\tilde{g}}}{m_s} \frac{8}{3} \mathrm{M}_1^H(x) \right] , \\
C_8 &= \frac{\alpha_s \pi}{m_{\tilde{g}}^2} \left[\left(\hat{\delta}_{12}^d\right)_{LL} \left(-\frac{1}{3} \mathrm{M}_3^H(x) - 3 \mathrm{M}_4^H(x) \right) \right. \\
&\quad \left. + \left(\hat{\delta}_{12}^d\right)_{LR} \frac{m_{\tilde{g}}}{m_s} \left(-\frac{1}{3} \mathrm{M}_1^H(x) - 3 \mathrm{M}_2^H(x) \right) \right] ,
\end{aligned}
\tag{2.35}
$$

where the coefficients $\tilde{C}_i$ can be obtained from the C_i just by the exchange $L \leftrightarrow R$.

And the explicit expressions for the loop functions are:

$$B_1^H(x) = \frac{1 - x^2 + 2x\log x}{8(x-1)^3} \tag{2.36a}$$

$$B_2^H(x) = \frac{2 - 2x + (1+x)\log x}{2(x-1)^3} \tag{2.36b}$$

$$P_1^H(x) = \frac{-11 + 18x - 9x^2 + 2x^3 - 6\log x}{36(x-1)^4} \tag{2.36c}$$

$$P_2^H(x) = \frac{-7 + 36x - 45x^2 + 16x^3 + 6(3-2x)x^2\log x}{36(x-1)^4} \tag{2.36d}$$

$$M_1^H(x) = \frac{1 - x^2 + 2x\log x}{2(x-1)^3} \tag{2.36e}$$

$$M_2^H(x) = \frac{1 - 4x + 3x^2 - 2x^2\log x}{2(x-1)^3} \tag{2.36f}$$

$$M_3^H(x) = \frac{-2 + 3x - 6x^2 + x^3 + 6x\log x}{12(x-1)^4} \tag{2.36g}$$

$$M_4^H(x) = \frac{-1 + 6x - 3x^2 - 2x^3 + 6x^2\log x}{12(x-1)^4} \tag{2.36h}$$

2.5.2 $\Delta F = 2$

We quote the results for the $\Delta S = 2$ case. The operator basis is

$$Q_1 = \bar{d}_L^\alpha \gamma_\mu s_L^\alpha \bar{d}_L^\beta \gamma^\mu s_L^\beta\,, \tag{2.37a}$$

$$Q_2 = \bar{d}_R^\alpha s_L^\alpha \bar{d}_R^\beta s_L^\beta\,, \tag{2.37b}$$

$$Q_3 = \bar{d}_R^\alpha s_L^\beta \bar{d}_R^\beta s_L^\alpha\,, \tag{2.37c}$$

$$Q_4 = \bar{d}_R^\alpha s_L^\alpha \bar{d}_L^\beta s_R^\beta\,, \tag{2.37d}$$

$$Q_5 = \bar{d}_R^\alpha s_L^\beta \bar{d}_L^\beta s_R^\alpha\,, \tag{2.37e}$$

plus the operators $\tilde{Q}_{1,2,3}$ obtained from the $Q_{1,2,3}$ by the exchange $L \leftrightarrow R$. Here $q_{R,L} = [(1 \pm \gamma_5)/2]q$, and α and β are colour indices.

From the matching we obtain:

$$
\begin{aligned}
H &= \frac{\alpha_s^2}{216 m_{\tilde{g}}^2}\Bigg\{\left(\hat{\delta}_{12}^d\right)_{LL}^2 \left(24\, Q_1\, j^{(1)}(x) + 66\, Q_1\, k^{(1)}(x)\right) \\
&+ \left(\hat{\delta}_{12}^d\right)_{RR}^2 \left(24\, \tilde{Q}_1\, j^{(1)}(x) + 66\, \tilde{Q}_1\, k^{(1)}(x)\right) \\
&+ \left(\hat{\delta}_{12}^d\right)_{LL} \left(\hat{\delta}_{12}^d\right)_{RR} \left(504\, Q_4\, j^{(1)}(x) - 72\, Q_4\, k^{(1)}(x)\right. \\
&\left. + 24\, Q_5\, j^{(1)}(x) + 120\, Q_5\, k^{(1)}(x)\right) \\
&+ \left(\hat{\delta}_{12}^d\right)_{RL}^2 \left(204\, Q_2\, \frac{x^2 j^{(3)}(x)}{6} - 36\, Q_3\, \frac{x^2 j^{(3)}(x)}{6}\right) \\
&+ \left(\hat{\delta}_{12}^d\right)_{LR}^2 \left(204\, \tilde{Q}_2\, \frac{x^2 j^{(3)}(x)}{6} - 36\, \tilde{Q}_3\, \frac{x^2 j^{(3)}(x)}{6}\right) \\
&+ \left(\hat{\delta}_{12}^d\right)_{LR} \left(\hat{\delta}_{12}^d\right)_{RL} \left(-132\, Q_4\, \frac{x^2 k^{(3)}(x)}{6} - 180\, Q_5\, \frac{x^2 k^{(3)}(x)}{6}\right)\Bigg\}, \qquad (2.38)
\end{aligned}
$$

where the functions $k(x)$ and $j(x)$ are

$$
k(x) = \frac{x^2 \log x - x + 1}{(x-1)^2} \qquad (2.39a)
$$

$$
j(x) = \frac{x \log x - x + 1}{(x-1)^2}. \qquad (2.39b)
$$

2.6 Bounds on Flavor-Violating Parameters

We now illustrate the bounds on the flavor-violating parameters $\hat{\delta}$ and δ in the hierarchical and degenerate cases, respectively. An early analysis of the hierarchical case was presented in ref. [31]. Our results for the LL insertions are summarized in Table 2.1. For definitess, here and below we set the A-terms to zero and we consider the case $\tilde{m} = M_3 = \mu$, with $\tilde{m}$ normalized to 350 GeV. This choice allows a direct comparison with several results in the literature and is appropriate for the sbottom mass. For simplicity we use the same value for the stop mass, relevant in the case of D^0–$\bar{D}^0$ oscillations, although that is barely compatible with the Higgs mass bound. For sufficiently large $\tan\beta$, the leading chiral flip in the sbottom sector comes from $\mu v \tan\beta$. The limits on the RR insertions are the same, except the one from BR($B \to X_s\gamma$), which is much weaker. This is because the contribution of the LL insertion to the $B \to X_s\gamma$ amplitude interferes with the SM one, while the RR contribution does not.

The bounds have been computed by constructing two-dimensional likelihood functions in the Re δ–Im δ planes. Such functions have been obtained using a standard bayesian approach. The real and imaginary parts of the insertions are varied with flat distributions and the input parameters, summarized in Table 2.2, are varied according to their distributions. The likelihood function is then constructed from a fit of the relevant experimental values, also shown in Table 2.2. The expressions for the supersymmetry contributions to the Wilson coefficients in terms of the hierarchical insertions have been obtained from [32, 33]. They have been used at the scale $\tilde{m}$ and then runned at lower scales according to [33, 34, 35].

The bounds on $s \leftrightarrow d$ transitions are obtained using the constraints from the kaon mass difference Δm_K and the kaon mixing CP-violation parameter ϵ_K. Because of the large theoretical uncertainty on the long-distance part of Δm_K, the absolute value of the supersymmetry contribution to Δm_K has been allowed to be as large as its experimental value, with a flat probability distribution. For each parameter δ (degenerate or hierarchical, LL or RR) a combined two-dimensional likelihood function is first built in the $\mathrm{Re}\,\delta$–$\mathrm{Im}\,\delta$ plane. The likelihood for $\sqrt{|\,\mathrm{Re}(\delta^2)|}$ (or $\sqrt{|\,\mathrm{Im}(\delta^2)|}$) is then obtained as the section along the $\sqrt{|\,\mathrm{Im}(\delta^2)|} = 0$ (or $\sqrt{|\,\mathrm{Re}(\delta^2)|} = 0$) direction and is used to determine the 95% CL limits shown in Table 2.1. The limit from Δm_K is compatible with the limit in [36], whereas the limit from ϵ_K is stronger. This is because the allowed range for the supersymmetric contribution to ϵ_K is now smaller, in particular it is not anymore allowed to take values as large as the SM contribution.

The bounds on $b \leftrightarrow d$ transitions are obtained using the constraint from the B_d^0–$\bar{B}_d^0$ system mass difference Δm_{B_d} and on the phase of the corresponding amplitude. Again, a two-dimensional likelihood is constructed. The corresponding 95% CL and 68% CL regions in the $\mathrm{Re}\,\delta$–$\mathrm{Im}\,\delta$ plane are shown in Fig. 2.4. The bounds on $\mathrm{Re}\,\delta$ ($\mathrm{Im}\,\delta$) in Table 2.1 are obtained from the one-dimensional section of the two-dimensional likelihood corresponding to $\mathrm{Im}\,\delta = 0$ ($\mathrm{Re}\,\delta = 0$). Choosing $\mathrm{Im}\,\delta = 0$ makes the limit on $\mathrm{Re}\,\delta$ in Table 2.1 much stronger than the size of the allowed region in the Figure. The corresponding constraint in Table 2.1 should therefore be considered as optimistic. Fig. 2.4 also shows that the point $\hat{\delta}_{db}^{LL} = 0$ ($\delta_{db}^{LL} = 0$) is excluded at more than 1σ. This is a consequence of the mild deviation from the SM or MFV hypothesis observed in $b \leftrightarrow d$ transitions (see e.g. [20]).

In the case of $b \leftrightarrow s$ transitions, the constraints we have considered are the mass difference Δm_{B_s} and the $B \to X_s\gamma$ branching ratio. We have used ref. [37] to compute the SM contribution to $\mathrm{BR}(B \to X_s\gamma)$. We have constructed two separate likelihoods because of the different $\tan\beta$ dependence of the two constraints. In fact, the Δm_{B_s} constraint is $\tan\beta$ independent, while the $B \to X_s\gamma$ constraint has a linear dependence on $\tan\beta$ for moderately large $\tan\beta$.[3] The 95% CL con-

[3]The reason is that the leading contribution to $\mathrm{BR}(B \to X_s\gamma)$ comes from the product of an LL insertion times an LR transition between sbottom states, which grows linearly with $\tan\beta$. At large $\tan\beta$, this dominates over the amplitude where the chiral transition occurs in the bottom

$D_0 - \bar{D}_0$ mixing

$\left\|\hat{\delta}^{LL}_{ut}\hat{\delta}^{LL*}_{ct}\right\| < 8.0 \times 10^{-3}\left(\frac{m_{\tilde{t}}}{350\ \text{GeV}}\right)$	$\left\|\delta^{LL}_{uc}\right\| < 3.4 \times 10^{-2}\left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$

$B \to X_s\gamma$

$\left\|\text{Re}\left(\hat{\delta}^{LL}_{sb}\right)\right\| < 2.2 \times 10^{-2}\left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$	$\left\|\text{Re}\left(\delta^{LL}_{sb}\right)\right\| < 3.8 \times 10^{-2}\left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$
$\left\|\text{Im}\left(\hat{\delta}^{LL}_{sb}\right)\right\| < 6.7 \times 10^{-2}\left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$	$\left\|\text{Im}\left(\delta^{LL}_{sb}\right)\right\| < 1.1 \times 10^{-1}\left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)^2\left(\frac{10}{\tan\beta}\right)$

Δm_{B_s}

$\left\|\text{Re}\left(\hat{\delta}^{LL}_{sb}\right)\right\| < 9.4 \times 10^{-2}\left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\left\|\text{Re}\left(\delta^{LL}_{sb}\right)\right\| < 4.0 \times 10^{-1}\left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$
$\left\|\text{Im}\left(\hat{\delta}^{LL}_{sb}\right)\right\| < 7.2 \times 10^{-2}\left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\left\|\text{Im}\left(\delta^{LL}_{sb}\right)\right\| < 3.1 \times 10^{-1}\left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$

B^0_d–$\bar{B}^0_d$ mixing

$\left\|\text{Re}\left(\hat{\delta}^{LL}_{db}\right)\right\| < 4.3 \times 10^{-3}\left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\left\|\text{Re}\left(\delta^{LL}_{db}\right)\right\| < 1.8 \times 10^{-2}\left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$
$\left\|\text{Im}\left(\hat{\delta}^{LL}_{db}\right)\right\| < 7.3 \times 10^{-3}\left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\left\|\text{Im}\left(\delta^{LL}_{db}\right)\right\| < 3.1 \times 10^{-2}\left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$

Δm_K

$\sqrt{\left\|\text{Re}\left(\hat{\delta}^{LL}_{db}\hat{\delta}^{LL*}_{sb}\right)^2\right\|} < 1.0 \times 10^{-2}\left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\sqrt{\left\|\text{Re}\left(\delta^{LL}_{ds}\right)^2\right\|} < 4.2 \times 10^{-2}\left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$

ϵ_K

$\sqrt{\left\|\text{Im}\left(\hat{\delta}^{LL}_{db}\hat{\delta}^{LL*}_{sb}\right)^2\right\|} < 4.4 \times 10^{-4}\left(\frac{m_{\tilde{b}}}{350\ \text{GeV}}\right)$	$\sqrt{\left\|\text{Im}\left(\delta^{LL}_{ds}\right)^2\right\|} < 1.8 \times 10^{-3}\left(\frac{m_{\tilde{q}}}{350\ \text{GeV}}\right)$

Table 2.1: Bounds on the LL insertions in the hierarchical and degenerate cases. The limits on the RR insertions are the same, except the one from BR($B \to X_s\gamma$), which is much weaker. The bounds are obtained at the 95% CL from one-dimensional distributions defined as explained in the text.

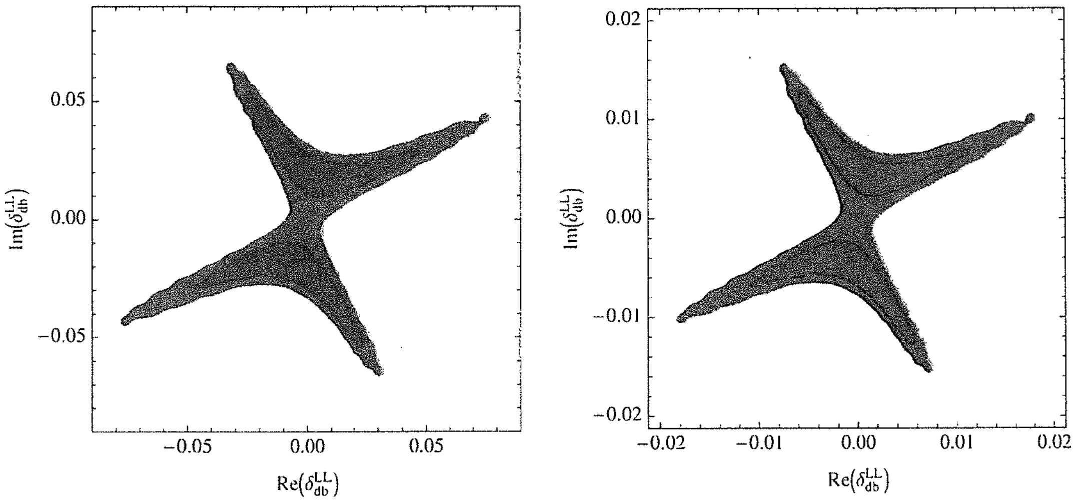


Figure 2.4: 95% CL (light shading) and 68% CL (dark shading) bounds on the real and imaginary parts of δ^{LL}_{db} (left, blue) and $\hat{\delta}^{LL}_{db}$ (right, red) from the measurements of Δm_{B_d} for $\tilde{m} = M_3 = \mu = 350\,\mathrm{GeV}$.

tours corresponding to the two constraints are shown in Fig. 2.5 for $\tan\beta = 10$. As mentioned, the $B \to X_s\gamma$ constraint is relevant for the LL insertions, whose contribution interferes with the SM one, but not for the RR insertions. The bounds on $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$ in Table 2.1 are obtained as in the case of $b \leftrightarrow d$ transitions. Because of the "holes" in the two-dimensional likelihood function shown in Fig. 2.5, the one-dimensional likelihood for $\mathrm{Im}(\delta)$ corresponding to $\mathrm{Re}(\delta) = 0$ has three almost disconnected parts. We calculated the bounds in Table 2.1 by using the central part of the likelihood only. A comment on this procedure is in order. It is of course possible to obtain the one-dimensional likelihood for $\mathrm{Im}(\delta)$ by a proper projection of the two-dimensional one. However, this would not take into account the fact that in the region at largest $|\mathrm{Im}(\delta)|$ the agreement of the SM with data, $\Delta m_{B_s} \sim 2|A_s^{\mathrm{SM}}|$, is reproduced through an accidental cancellation: $\Delta m_{B_s} = 2|A_s^{\mathrm{SM}} + A_s^{\mathrm{NP}} e^{2i\phi_s^{\mathrm{NP}}}|$, where $A_s^{\mathrm{NP}} e^{2i\phi_s^{\mathrm{NP}}} \sim -2A_s^{\mathrm{SM}}$. Our recipe "empirically" discards such possibilities, and it seems appropriate for the purpose of calculating the bounds in Table 2.1.

Finally, we show in Fig. 2.6 the bound on the $c \leftrightarrow u$ transitions obtained from D^0–$\bar{D}^0$ mixing. The theoretical prediction for the SM contribution to the mixing amplitude is affected by a large uncertainty due to long-distance contributions and it is assumed to lie in the interval $(-0.02, 0.02)\,\mathrm{ps}^{-1}$ [38], with flat probability distribution. We translate in this case the likelihood in a bound on $|\delta|$ by con-

quark line.

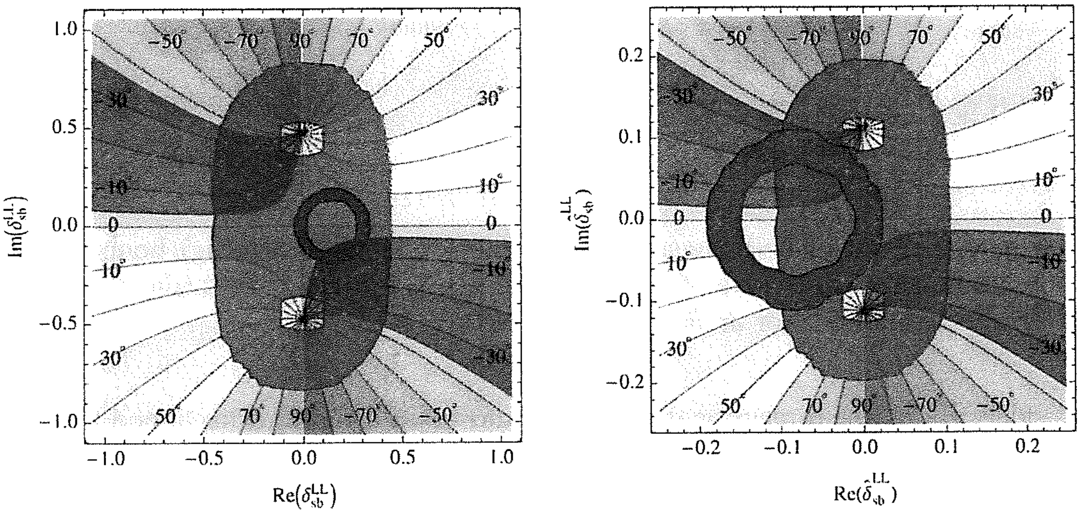


Figure 2.5: 95% CL bounds on the real and imaginary parts of δ_{sb}^{LL} (left, blue) and $\hat{\delta}_{sb}^{LL}$ (right, red) from the measurements of Δm_{B_s} (lighter shading) and BR($B \to X_s\gamma$) (darker shading) for $\tilde{m} = M_3 = \mu = 350\,\text{GeV}$ and $\tan\beta = 10$. Switching the sign of μ approximately corresponds to switching the sign of Re(δ_{sb}^{LL}) and Re($\hat{\delta}_{sb}^{LL}$) in the two figures. In the background, the contour lines of the phase ϕ_{B_s} are shown. The darker regions correspond to the 90% CL range presently favoured by the experiment [21]. The axis of the two figures are chosen in such a way that the contour lines are the same for the degenerate and hierarchical cases.

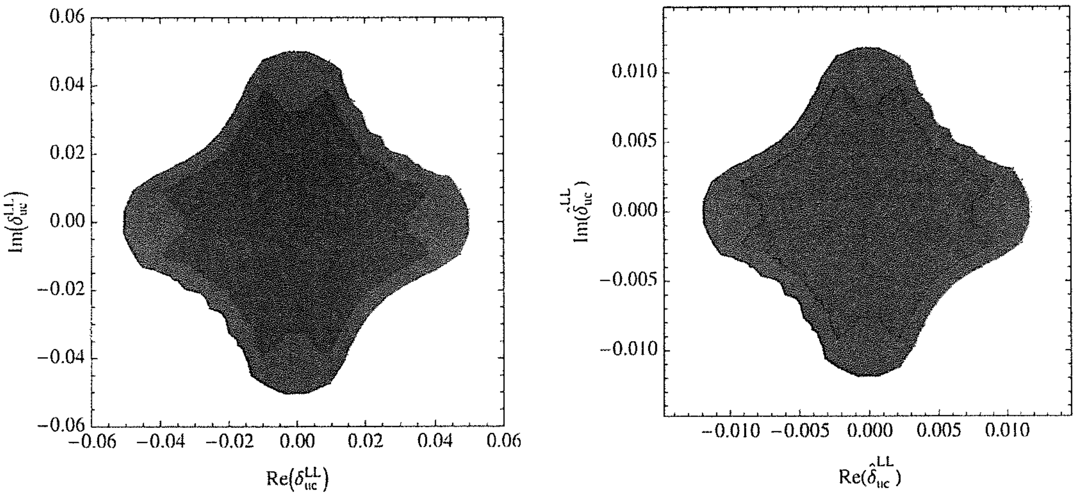


Figure 2.6: 95% CL (light shading) and 68% CL (dark shading) bounds on the real and imaginary parts of δ^{LL}_{uc} (left, blue) and $\hat\delta^{LL}_{uc} \equiv \hat\delta^{LL}_{ut}\hat\delta^{LL*}_{ct}$ (right, red) from D^0–$\bar D^0$ oscillations for $\tilde m = M_3 = \mu = 350\,\text{GeV}$.

sidering the one-dimensional section of the two-dimensional likelihood along the $|\,\text{Re}(\delta)| = |\,\text{Im}(\delta)|$ line.

In the hierarchical case, the bound from the $s \leftrightarrow d$ transitions apply to the product $\hat\delta^{LL}_{db}\hat\delta^{LL*}_{sb} \equiv \hat\delta^{LL}_{ds}$. It is therefore possible to compare that bound with the indirect one obtained from the constraints on $\hat\delta^{LL}_{sb}$ and $\hat\delta^{LL}_{db}$. It turns out that the combined bound is stronger than the direct one in the case of Δm_K but not in the case of ϵ_K.

If the parameters $\hat\delta$ are related to the hierarchy according to the relation $\hat\delta \sim \tilde m^2_\ell/\tilde m^2_h$, from the results in Table 2.1 we obtain a lower bound on the heavy mass scale

$$\tilde m_h \gtrsim \left(\frac{\tilde m_\ell}{350\text{ GeV}}\right)^{1/2} 5\text{ TeV}. \tag{2.40}$$

As discussed in the Appendix, it is plausible to expect that, independently of the value of the hierarchy $\tilde m_\ell/\tilde m_h$, the size of the parameters $\hat\delta^{LL}_{sb}$, $\hat\delta^{LL}_{db}$ cannot be smaller than the corresponding CKM angles, $|V_{td}|$, $|V_{ts}|$ respectively. Thus, it is particularly interesting to probe experimentally flavor processes up to the level of $|\hat\delta^{LL}_{db}| \approx 8 \times 10^{-3}$, $|\hat\delta^{LL}_{sb}| \approx 4 \times 10^{-2}$ and $|\hat\delta^{LL}_{ds}| = |\hat\delta^{LL}_{db}\hat\delta^{LL*}_{sb}| \approx 3 \times 10^{-4}$. The present constraints on the $b \leftrightarrow d$ transitions and on ϵ_K are at the edge of probing this region. An interesting conclusion is that hierarchical soft terms predict that new-physics effects in $b \leftrightarrow s$ transitions can be expected just beyond the present experimental sensitivity.

Parameter	Value	Gaussian (σ)	Uniform ($\frac{\Delta}{2}$)	Reference
$\lvert\varepsilon_K\rvert$	2.229×10^{-3}	0.012×10^{-3}	–	[39]
Δm_K (ps^{-1})	5.292×10^{-3}	0.009×10^{-3}	–	[39]
BR($B \to X_s\gamma$)	3.55×10^{-4}	0.26×10^{-4}	–	[40]
Δm_{B_s} (ps^{-1})	17.77	0.12	–	[39]
Δm_{B_d} (ps^{-1})	0.507	0.005	–	[39]
$\phi_{B_d}[^\circ]$	-4.1	2.1	–	[34]
$\lvert M_{12}^D\rvert$ (ps^{-1})	7.7×10^{-3}	2.5×10^{-3}	–	[38]
$\bar{\rho}$	0.167	0.051	–	[41]
$\bar{\eta}$	0.386	0.035	–	[41]
λ	0.2255	0.010	–	[39]
$\lvert V_{cb}\rvert$	41.2×10^{-3}	1.1×10^{-3}	–	[39]
F_K (GeV)	0.160	–	–	[39]
F_{B_d} (MeV)	189	27	–	[42]
$F_{B_s}\sqrt{B_s}$ (MeV)	262	35	–	[42]
F_D (MeV)	201	3	17	[43]
$\hat{B}_K$	0.79	0.04	0.08	[43]
B_1^B	0.88	0.04	0.10	[43]
η_{cc}	0.47	0.04	–	[44]
η_{ct}	0.5765	0.0065	–	[44]
η_{tt}	1.43	0.23	–	[44]

Table 2.2: Main inputs used in the numerical analysis.

2.7 The Phase of the B_s Mixing

Let us now discuss the implications for the phase of the B_s mixing. In the hierarchical scenario, the new-physics effects in $b \leftrightarrow s$ transitions are particularly promising. We have already pointed out that the value of $\hat{\delta}_{bs}^{LL}$ might be not so far from saturating the bound in Table 2.1. On top of that, a value of the insertion parameter close to its $\Delta B = 1$ bound gives rise to effects in $\Delta B = 2$ observables that are more pronounced in the hierarchical than in the degenerate case. The reason goes back to eq. (2.26). For most values of $\tan\beta$, the bound on the insertions is mainly due to the $B \to X_s\gamma$ constraint. Its translation into a constraint on $\Delta B = 2$ observables such as Δm_{B_s} or the phase ϕ_{B_s} of the B_s–$\bar{B}_s$ mixing depends on the scenario we consider. Eq. (2.26) shows that for $(g^{(3)}/g^{(1)})(f/f^{(1)})^2 \sim 1$ the bound on $\Delta B = 2$ observables is expected to be looser in the hierarchical case. This is confirmed by the relative size of the $\Delta B = 1$ and $\Delta B = 2$ constraints in Fig. 2.5.

The previous considerations have interesting implications on the possible size of new-physics effects in the phase of B_s mixing. The B_s–$\bar{B}_s$ mixing amplitude in the presence of new physics can be parameterized as

$$\langle B_s|H_{\rm eff}^{\rm full}|\bar{B}_s\rangle = C_{B_s}e^{2i\phi_{B_s}}\langle B_s|H_{\rm eff}^{\rm SM}|\bar{B}_s\rangle, \tag{2.41}$$

where $H_{\rm eff}^{\rm full} = H_{\rm eff}^{\rm SM}+H_{\rm eff}^{\rm NP}$, $\langle B_s|H_{\rm eff}^{\rm SM}|\bar{B}_s\rangle = A_s^{\rm SM}e^{-2i\beta_s}$, $\langle B_s|H_{\rm eff}^{\rm NP}|\bar{B}_s\rangle = A_s^{\rm NP}e^{2i(\phi_s^{\rm NP}-\beta_s)}$, and $\beta_s = \arg(-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)) = 0.018 \pm 0.001$. Recent measurements from the CDF [46] and D0 [47] collaborations have shown a mild tension between the experimental value $\phi_{B_s} \sim -20°$ (for the allowed region closer to the origin) and its SM prediction, $\phi_{B_s} = 0°$, at the $2.5\,\sigma$ level [19, 20, 21]. In the supersymmetric scenarios under consideration, the value of the phase ϕ_{B_s} can be read from the contour lines in Fig. 2.5. The lines have been obtained by fixing all the relevant parameters to their central values. They converge in the two points corresponding to a vanishing total amplitude $A_s^{\rm SM} + A_s^{\rm NP}e^{2i\phi_s^{\rm NP}}$. The figure shows that in the region allowed by both the BR$(B \to X_s\gamma)$ and Δm_{B_s} constraints, the phase reaches larger values in the hierarchical case. This is apparent in Fig. 2.7, where the expectation for ϕ_{B_s} in the two scenarios has been shown in the form of an histogram (for a fixed value of $\tan\beta = 10$). The hierarchical case allows values of the phase ϕ_{B_s} about three times larger than in the degenerate case, in agreement with the generic expectation from eq. (2.26). The range of ϕ_{B_s} presently favored by the experiment is shown in Fig. 2.5.

Recently the DØ Collaboration reported a measurement of the like-sign dimuon charge asymmetry in semileptonic b decay [22] which significantly deviates from the SM prediction [23] at $3\,\sigma$ level. It would be very interesting to perform a more complete and update analysis in the context of hierarchical sfermions.

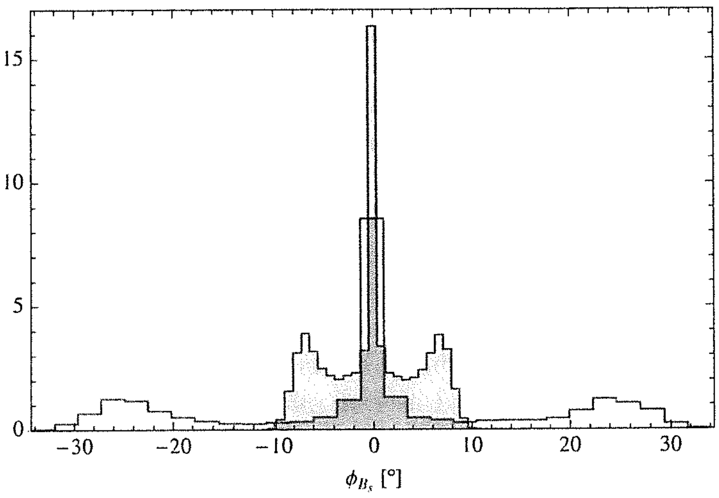


Figure 2.7: Expected distribution of the phase ϕ_{Bs}, as determined by the BR($B \to X_s\gamma$) and Δm_{B_s} constraints in the degenerate (blue) and hierarchical (red), for $\tan\beta = 10$.

2.8 Summary

Hierarchical soft terms describe a class of supersymmetric theories which is characterized by the existence of two separated mass scales: a large mass $\tilde{m}_h$ for the first two generations of squarks and sleptons and a smaller mass $\tilde{m}_\ell$, of electroweak-scale size, for the rest of the spectrum. A certain hierarchy of the ratio $\tilde{m}_h/\tilde{m}_\ell$ is not incompatible with naturalness, and it is welcome to relax constraints from K^0-$\bar{K}^0$ mixing and ϵ_K.

This class of theories includes radical proposals in which $\tilde{m}_h$ is in the range of hundreds of TeV, fully addressing the supersymmetric flavor problem at the price of a certain amount of unnaturalness. However, the pattern of hierarchical soft terms is also useful to describe less extreme scenarios in which there is a more modest mass separation in the squark sector, nevertheless sufficient to make the degeneracy assumption a poor starting point.

Hierarchical soft terms make well-defined and interesting predictions in flavor physics. Flavor-violating effects in the down sector are described by four complex numbers: $\hat{\delta}^{LL}_{db}$, $\hat{\delta}^{LL}_{sb}$, $\hat{\delta}^{RR}_{db}$, $\hat{\delta}^{RR}_{sb}$. There are fewer free parameters than in the ordinary case of degenerate squarks, mostly because the $d \leftrightarrow s$ transition is determined by the product of $d \leftrightarrow b$ and $b \leftrightarrow s$ transitions. Also, under certain assumptions, flavor and chiral violating transitions are specified in terms of $\hat{\delta}$ and of the same parameters that describe squark mixing in the third generation. Another interesting peculiarity is the correlation between $\Delta F = 1$ and $\Delta F = 2$ transitions, which is characteristic of the hierarchical soft term pattern and distinct from the one derived in the case of degeneracy.

In this chapter we have analyzed how present experiments constrain the parameters $\hat{\delta}$. The limits are derived by calculating the likelihood function for new-physics effects and combining the different experimental data and theory parameters with their relative errors. We have also applied the same procedure to the case of degeneracy, revisiting the limits on the mass insertion parameters δ.

For a degenerate spectrum, the mass insertions δ are the appropriate way to parametrize new flavor-violating effects. The coefficients δ describe the small deviations from universality but, lacking the knowledge of a complete theory of soft terms, they can only be treated as free parameters and do not provide information on the required experimental sensitivity to discover new-physics effects. The analogous quantities in the hierarchical scheme, $\hat{\delta}$, are related either to the $\tilde{m}_\ell/\tilde{m}_h$ hierarchy or to CKM angles, because of the special assumptions made on the pattern of soft terms. Therefore the quantities $\hat{\delta}$ are associated to physical parameters and they provide a defined target for the required experimental sensitivity. In particular, we expect that each $\hat{\delta}_{i3}$ is larger than the maximum between $\tilde{m}_\ell^2/\tilde{m}_h^2$ and the CKM elements V_{3i}^*. The results obtained in Table 2.1 show that present experiments have not yet probed $u \leftrightarrow c$ transitions at the level required by $\hat{\delta}_{i3} = V_{3i}^*$, and have only marginally tested the case of $d \leftrightarrow s$ and $d \leftrightarrow b$ transitions. On the other hand, experiments have begun to explore the crucial range of values for $\hat{\delta}_{sb}$ in $s \leftrightarrow b$ transitions. In this respect, it is tantalizing that there are claims for a deviation from the SM predictions in the phase of B_s mixing, ϕ_{B_s} [19, 20, 21, 22]. Hierarchical soft terms could account for such new-physics effect, compatibly with the other constraints in the b-s system. Actually we have proved that, because of the correlation between $\Delta F = 1$ and $\Delta F = 2$ transitions, hierarchical soft terms can lead to larger values of ϕ_{B_s} than degenerate ones, for an equal value of $\tan\beta$. Independently of the reliability of the alleged anomaly in ϕ_{B_s}, the hypothesis of hierarchical soft terms represents an interesting benchmark to confront experimental searches in flavor physics.

Chapter 3

Tree Level Gauge Mediation

If supersymmetry plays a role in the physics near the electroweak scale, then one of the most pressing question is how supersymmetry breaking is mediated to the superfields of the MSSM.

It is actually not possible to construct a realistic model of spontaneously broken SUSY where the supersymmetry breaking arises solely as a consequence of the interactions of the particles of the MSSM. A more viable scheme requires a theory consisting of at least two distinct sectors: a hidden sector consisting of particles that are completely neutral with respect to the Standard Model gauge group, and a visible sector consisting of the particles of the MSSM. Supersymmetry breaking is assumed to originate in the hidden sector, and its effects are transmitted to the MSSM by some mechanism (often involving the mediation by particles that comprise an additional messenger sector).

Several schemes have been investigated [48, 49, 50, 51]. Below the scale M at which supersymmetry breaking is communicated, sfermion masses are typically described by the model-independent effective lagrangian operator

$$\int d\theta^2 d\bar{\theta}^2 \, \frac{Z^\dagger Z Q^\dagger Q}{M^2}, \tag{3.1}$$

where Z is a Standard Model (SM) singlet hidden chiral superfield whose F-term vev breaks supersymmetry, $\langle Z \rangle = F\theta^2$, and Q is a generic light, observable chiral superfield, for example an MSSM one. Such an effective, supersymmetric description holds if $F \ll M^2$. Different models are characterized by different origins for the above operator.

Surprisingly enough, a simple and attractive possibility has been neglected in the almost three decades of phenomenological studies of supersymmetry: the possibility that the operator in eq. (3.1) arises from the renormalizable, tree level exchange of heavy vector superfields. This is the communication mechanism that we call tree level gauge mediation (TGM) [5]. Besides its simplicity (compare for example with the cumbersome set of two loop diagrams generating the operator in eq. (3.1) in ordinary, loop gauge mediation), this framework is also motivated by the necessary presence of superheavy vector fields in Grand Unified Theories (GUTs). On the verge of the LHC era, we believe it is worth filling this lacuna and spell out the consequences of TGM, also in the light of its peculiar predictions.

One may wonder how such a simple possibility could have been missed. The reason might be that well known arguments seem to prevent it. The main obstacle is represented by the supertrace formula [52] and its consequences are investigated in the next section.

Another potential problem is represented by the fact that gaugino masses arise at the loop level and are therefore potentially suppressed with respect to the sfermion masses by a large loop factor, thus pushing the sfermions out of the reach of the LHC and introducing a significant fine-tuning in the determination of the Higgs mass. We will see in the explicit $SO(10)$ model of this chapter and also in Section 4.2.2 a number of gaugino mass enhancement factors that can compensate fully or partially that loop factor.

3.1 The Supertrace formula

In the context of the tree level, renormalizable spontaneously broken supersymmetric theory underlying Fig. 4.1, we must have

$$\operatorname{Str}\mathcal{M}^2 = -2gD_a\operatorname{Tr}(T_a), \tag{3.2}$$

for the supertrace of the squared masses of the fields in the model. Eq. (3.2) holds separately for each set of conserved quantum numbers [9]. If the action of the gauge generators on the full set of chiral superfields, T_a, is traceless, as in the case we are going to consider, the supertrace vanishes. This represents a potential phenomenological problem. Still, TGM leads to a viable spectrum, as we will see.

We can see the potential problem at two different levels. First, eq. (3.2) holds in particular when applied to all fields with the quantum numbers of the SM fermions. Let us consider then the case of the MSSM. In this case the fields with the quantum numbers of the SM fermions are the SM fermions themselves, f, and their supersymmetric partners, the sfermions $\tilde{f}$. From $\operatorname{Str}\mathcal{M}^2_{f,\tilde{f}} = 0$ we then conclude that the sum of the squared masses of fermions and sfermions should coincide. This is in clear contradiction with the experimental bounds on the sfermion masses, giving $\operatorname{Str}\mathcal{M}^2_{f,\tilde{f}} > 0$. This is however far from being the end of the story, as any realistic complete theory of supersymmetry breaking is likely to involve additional fields on top of the MSSM ones. This is the case of our TGM framework, where the positive contribution to the supertrace from the MSSM fermions and sfermions is compensated by an opposite contribution from extra fields with quantum numbers within the ones of the SM fermions, $\mathrm{Str}\,\mathcal{M}^2_{\text{extra}} < 0$, so that $\mathrm{Str}\,\mathcal{M}^2_{f,\bar{f}} + \mathrm{Str}\,\mathcal{M}^2_{\text{extra}} = 0$, in agreement with the supertrace formula. The extra chiral superfields will get heavy supersymmetric mass terms. Their negative contribution to the supertrace is due to the fact that their scalar components get negative $\mathcal{O}\,(\text{TeV})$ soft masses, which however represent only negligible corrections to their much larger, positive supersymmetric mass term. As we will see, this can be obtained without ad hoc model building efforts.

The supertrace formula has stronger implications than the ones outlined above, which should also be addressed. Let us consider the fields with the $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\text{em}}$ quantum numbers of the d quarks only. The latter set will contain at least the three down-type SM quarks and their scalar partners and possibly the extra fields we need to compensate the supertrace formula. Let us project the supertrace formula in the flavor space of down-type fields along the direction corresponding to the lightest d-quark mass eigenstate, the down quark. Note that when restricted to a given set of quantum numbers, the trace on the right-hand side of eq. (3.2) can be non-vanishing. Assume now that the only U(1) factor in the gauge group is the SM hypercharge, $\mathrm{U}(1)_Y$. We then obtain [9, 53]

$$m^2_{\tilde{d}} \leq m^2_d - \frac{1}{3} g' D_Y, \tag{3.3}$$

where $\tilde{d}$ is the lightest d-sfermion mass eigenstate, m_d is the down quark mass, $m_d \sim 5\,\text{MeV}$, g' is the hypercharge gauge coupling and D_Y the hypercharge D-term. Eq. (3.3) represents a serious phenomenological problem, even in the presence of the extra fields invoked above. If $D_Y = 0$, in fact, eq. (3.3) would force a down sfermion mass to be smaller than about 5 MeV, in contrast with the lowest experimental limits of a few hundreds GeV. If $D_Y > 0$, the constraint would be even stronger. If $D_Y < 0$, the constraint would be loosened, but one could repeat the argument for the up quarks and squarks. For which the D_Y contribution to the relation analogous to eq. (3.3) would have opposite sign, leading to an even stronger bound for the lightest up squark. In order to bypass this problem, an extra U(1) factor, giving the same sign on both down and up fields, is needed. Such an extra U(1) factor is present "by definition" in the TGM scheme. It is the U(1) factor associated to the heavy vector exchange in Fig. 4.1 (as Z is a SM singlet, the heavy vector must also be a SM singlet). We therefore have all the ingredients needed to overcome the potential problem set by the supertrace formula. As we will see, those ingredients naturally combine in phenomenologically viable schemes.

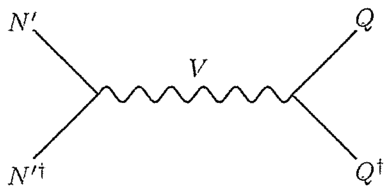


Figure 3.1: Tree level gauge mediation supergraph inducing a soft mass for the sfermion $\tilde{Q}$.

3.2 A GUT SO(10) Model

Before presenting the model, let us motivate its gauge structure and field content. Our aim is to identify the supersymmetry breaking messengers with heavy vector superfields corresponding to broken generators, X, of a simple grand unified group, as illustrated in Fig. 3.1. There, N' is a SM singlet superfield whose F-term breaks supersymmetry, $\langle N' \rangle = F\,\theta^2$ (the prime is there just for consistency with the notations used below). As N' has to couple to the heavy vector V associated to the broken generator X, N' must belong to a non-trivial multiplet of the unified group. Q represents a generic MSSM superfield. In the effective theory below M_{GUT}, the diagram in Fig. 3.1 induces a non-renormalizable contribution $-2g^2 X_N X_Q (Q^\dagger Q N'^\dagger N')/M_V^2$ to the Kähler potential, analogous to the ones of effective supergravity, but flavour universal ($X_{N,Q}$ are the X-charges of N', Q, M_V is the vector mass). A sfermion mass $\tilde{m}_Q^2 = 2g^2 X_N X_Q (F/M_V)^2$ is then generated. In the full theory at M_{GUT}, as we have seen in the previous section, everything takes place at the renormalizable level. In fact, the sfermion masses arise because N' couples to the broken generator X. As a consequence, its F-term generates a non-vanishing vev for the corresponding D-term:

$$\langle D_X \rangle = -2gX_N \left(\frac{F}{M_V} \right)^2, \tag{3.4}$$

which in turn induces the soft mass

$$\tilde{m}_Q^2 = -gX_Q \langle D_X \rangle = 2g^2 X_N X_Q \left(\frac{F}{M_V} \right)^2 \tag{3.5}$$

for the sfermion $\tilde{Q}$. Note that there is actually no dependence on the gauge coupling (and X-charge normalization) because the vector squared mass M_V^2 is also proportional to g^2 (and two X-charges).

Such a scheme requires specific gauge structures and field contents. First of all, the heavy vector field V in Fig. 3.1 must be a SM singlet, as N' is. Then, SU(5) does not provide viable candidates for the gauge messenger V and the minimal option is identifying the broken generator with the SU(5) singlet generator X of SO(10). As for the SM singlet N' whose F-term breaks supersymmetry, it must belong to a non-trivial SO(10) multiplet such that N' has a non-vanishing charge under X. Limiting ourselves to representations with dimension $d < 126$, the only possibility is that N' be the singlet component of a spinorial representation, 16 or $\overline{16}$. We also need a $16 + \overline{16}$ participating to SO(10) breaking at the GUT scale. At least two $16 + \overline{16}$ are then required, one getting a vev along the scalar component and the other along the F-term component. Gauge invariance, in fact, prevents from using a single $\langle N' \rangle = M + F\theta^2$, with both $M \neq 0$ and $F \neq 0$. This is an important difference with respect to standard gauge mediation. Finally, the standard embedding of a whole MSSM family into a 16 of SO(10) would not work, as it would lead to negative sfermion masses for some of the sfermions. That is why we distribute the matter fields in three 16 and three 10 of SO(10).

Having motivated some of its features, we now illustrate a minimal model satisfying the above requirements. The gauge group is SO(10). The matter fields (negative R-parity) are three $16_i = (\bar{5}^{16}_i, 10^{16}_i, 1^{16}_i)$ and three $10_i = (5^{10}_i, \bar{5}^{10}_i)$, $i = 1, 2, 3$, where the SU(5) decomposition is also indicated. Supersymmetry and SO(10) breaking to SU(5) are provided by $16 = (\bar{5}^{16}, 10^{16}, N)$, $\overline{16} = (5^{16}, \overline{10}^{16}, \bar{N})$, $16' = (\bar{5}'^{16}, 10'^{16}, N')$, $\overline{16}' = (5'^{16}, \overline{10}'^{16}, \overline{N}')$ (positive R-parity), with

$$\langle N' \rangle = F\theta^2 \qquad \langle \overline{N}' \rangle = 0 \qquad \langle N \rangle = M \qquad \langle \overline{N} \rangle = M, \tag{3.6}$$

$\sqrt{F} \ll M \sim M_{\text{GUT}}$. The D-term condition forces $|\langle N \rangle| = |\langle \overline{N} \rangle|$ and the phases of all the vevs can be taken positive without loss of generality. The MSSM up Higgs h_u is embedded in a $10 = (5^{10}, \bar{5}^{10})$ of SO(10), while the down Higgs h_d is a mixture of the doublets in the 10 and the 16,

$$10 = h_u + c_d h_d + \text{heavy}, \quad 16 = s_d h_d + \text{heavy}, \tag{3.7}$$

where $c_d = \cos\theta_d$, $s_d = \sin\theta_d$ and $0 < \theta_d < \pi/2$ parametrizes the mixing in the down Higgs sector[1]. We have checked that it is possible to generate such vevs, break SU(5) to the SM, achieve doublet-triplet splitting and Higgs mixing as above, and give mass to all the extra fields with an appropriate superpotential W_{vev} involving additional SO(10) representations.

[1]The most general viable Higgs embedding in this minimal model is described by the three parameters determining the up Higgs component in the 10 and the down Higgs component in the 10 and in the 16.

At this point we are in the condition of calculating the sfermion masses induced by integrating out the heavy vector fields:

$$\tilde{m}_Q^2 = \frac{X_Q}{2X_N} m^2, \qquad m \equiv \frac{F}{M}. \tag{3.8}$$

In the normalization we use for X, $X_N = 5$. In order to determine the X charge of the SM fermions we need to specify their embedding in the matter fields $16_i + 10_i$. We do that by first writing the most general R-parity conserving superpotential, except a possible mass term for the 10_i, as

$$W = \frac{y_{ij}}{2} 16_i 16_j 10 + h_{ij} 16_i 10_j 16 + h'_{ij} 16_i 10_j 16' + W_{\text{vev}} + W_{\text{NR}}, \tag{3.9}$$

where $W_{\text{vev}} = W_{\text{vev}}(16, \overline{16}, 10, \ldots)$ does not involve the matter fields and takes care of the vevs, the doublet triplet splitting, and the Higgs mixing, and W_{NR} contains non-renormalizable contributions to the superpotential needed in order to account for the measured ratios of down quark and charged lepton masses (we will ignore such issue here).

We can now see that the vev of the 16 gives rise to the mass term $h_{ij} M \bar{5}_i^{16} 5_j^{10}$, which makes the $\bar{5}_i^{16}$ and 5_j^{10} heavy. Only the MSSM superfield content survives at the electroweak scale (assuming the three singlets in the 16_i get mass e.g. from non-renormalizable interactions with the $\overline{16}$). Moreover, the three MSSM families turn out to be embedded in the three 10_i^{16}, with $X = 1$ and in the three $\bar{5}_i^{10}$, with $X = 2$. We can then go back to eq. (3.8) and obtain

$$\tilde{m}_q^2 = \tilde{m}_{u^c}^2 = \tilde{m}_{e^c}^2 = \tilde{m}_{10}^2 = \frac{1}{10} m^2, \quad \tilde{m}_l^2 = \tilde{m}_{d^c}^2 = \tilde{m}_{\bar{5}}^2 = \frac{1}{5} m^2 \tag{3.10}$$

$$m_{h_u}^2 = -\frac{1}{5} m^2, \quad m_{h_d}^2 = \frac{2c_d^2 - 3s_d^2}{10} m^2 \tag{3.11}$$

at the GUT scale. The result in eq. (3.10) is quite general, as it only depends on the choice of the gauge group and on the embedding of the three MSSM families in the $10_i^{16} + \bar{5}_i^{10}$. We note a few interesting features of this result.

- All the sfermion masses turn out to be positive. This is because the negative X charges (which must be there as X is traceless) happen to be associated to the fields that get an heavy supersymmetric mass.

- The sfermions masses are flavour universal, thus solving the supersymmetric flavour problem.

- The sfermions masses belonging to the 10 and $\bar{5}$ of SU(5) are related by

$$\tilde{m}_{q,u^c,e^c}^2 = \frac{1}{2} \tilde{m}_{l,d^c}^2 \tag{3.12}$$

at the GUT scale, a peculiar prediction that allows to distinguish this model from mSugra, gauge mediation, and other models of supersymmetry breaking.

Note also that the up Higgs squared mass is negative to start with, whereas $m^2_{h_d}$ is positive for $s_d < \sqrt{2/5}$. The negative value of the up Higgs squared mass means that the electroweak symmetry is broken at the tree level and the usual radiative breaking mechanism is not needed. In the presence of negative Higgs squared masses at the GUT scale, there is the potential risk that the Higgs potential develops a deep minimum along its flat direction $\tan\beta = 1$, if $m^2_{h_u} + m^2_{h_d} + 2|\mu|^2 < 2|B\mu|$ at the GUT scale or below. Of course, a negative value of $m^2_{h_u}$ (and/or $m^2_{h_d}$) does not necessarily mean that the above condition is satisfied. Moreover, in most of the parameter space, the presence of a local electroweak symmetry breaking minimum at low energy (which requires $m^2_{h_u} + m^2_{h_d} + 2|\mu|^2 > 2|B\mu|$ around the weak scale) guarantees that no deeper minima develop at higher scales.

In passing, the SM fermion masses are given (at the renormalizable level and before running the Yukawas to low energy), by

$$m^U_{ij} = y_{ij} v_u \qquad m^E_{ij} = \sin\theta_d h_{ij} v_d \qquad m^D_{ij} = \sin\theta_d h^T_{ij} v_d. \tag{3.13}$$

Despite the SO(10) structure, the up quark matrix is not correlated to the down quark and charged lepton masses, which allows to accommodate the stronger mass hierarchy observed in the up quark sector. Notice that the heavy $\bar{5}^{16}_i$ and 5^{10}_j mass matrix, $h_{ij}M$, turns out to be proportional to the charged lepton mass matrix, up to non-renormalizable corrections from $W_{\rm NR}$. In the context of type-II see-saw, this can lead to a predictive model of leptogenesis [55, 56].

Let us now consider gaugino masses. While the tree-level prediction for the sfermion masses, eq. (3.10), only depends on the choice of the unified gauge group and the MSSM embedding, gaugino masses arise at one loop, as in standard gauge mediation, and depend on the superpotential parameters. The chiral multiplets $\bar{5}^{16}_i$ and 5^{10}_j get an heavy supersymmetric mass $h_{ij}M$ and their scalar components get a supersymmetry breaking mass $h'_{ij}F$. They play the role of three pairs of chiral messengers in standard gauge mediation and give rise to one loop gaugino masses. The contribution of each messenger arises at a different scale. In the one loop approximation for the RGE running, the total gaugino masses at lower scales can be calculated by running effective GUT-scale gaugino masses given by

$$M_a = \frac{\alpha}{4\pi}\,\mathrm{Tr}(h'h^{-1})\,m \equiv M_{1/2}, \quad a = 1, 2, 3, \tag{3.14}$$

where α is the unified coupling. A possible contribution from loops involving the heavy vectors vanishes (at the F/M level) in this simple model. The sfermion masses also get the usual two-loop contributions.

Let us compare gaugino and sfermion masses. Particularly interesting is the ratio $\tilde{m}_t/M_2$. In fact, the W-ino mass M_2 is at present bounded to be heavier than about 100 GeV, while $\tilde{m}_t$ enters the radiative corrections to the Higgs mass. Therefore, the ratio $\tilde{m}_t/M_2$ should not be too large in order not to increase the fine-tuning and not to push the stops and the other sfermions out of the LHC reach. From

$$\left.\frac{M_2}{\tilde{m}_t}\right|_{M_{\rm GUT}} = \frac{3\sqrt{10}}{(4\pi)^2}\lambda, \quad \lambda = \frac{g^2\,{\rm Tr}(h'h^{-1})}{3} \tag{3.15}$$

we see first of all that the loop factor separating $\tilde{m}_t$ and M_2 is partially compensated by a combination of numerical factors: $(4\pi)^2 \sim 100$ (leading to $\tilde{m}_t \gtrsim 10\,{\rm TeV}$ for $\lambda = 1$) becomes $(4\pi)^2/(3\sqrt{10}) \sim 10$ (leading to $\tilde{m}_t \gtrsim 1\,{\rm TeV}$ for $\lambda = 1$). Note that the factor $\sqrt{10}$ is related to the ratio of X charges in eq. (3.8) and the factor 3 corresponds to the number of families (${\rm Tr}(h'h^{-1}) = 3$ for $h = h'$). A largish value of the factor λ can then further reduce the hierarchy and even make $M_2 \sim \tilde{m}_t$, if needed. Both $\mathcal{O}(1)$ and large values of λ are in fact not difficult to obtain depending on the overall size and flavour structure of h and h' (we remind that h is related to the down quark Yukawa matrix and has a hierarchical structure, with two eigenvalues certainly small and the third one, related to the bottom Yukawa, also allowed to be small, depending on θ_d and $\tan\beta$).

Reducing the hierarchy between gaugino and sfermion masses correspondingly reduces the hierarchy between the two-loop contributions to sfermion masses from standard gauge mediation and the tree level values in eq. (3.8). To quantify the relative importance of the two contributions, let us consider the basis in the messenger flavour space in which the matrix h is diagonal and positive, the limit in which h' is also diagonal in that basis, and let us call h_i, h'_i, $i = 1,2,3$ their eigenvalues. Neglecting the running between the GUT scale and the mass of the relevant messengers[2], the sfermion masses are given, at the high scale, by

$$\tilde{m}^2_Q = (\tilde{m}^2_Q)_{\rm tree} + 2\,\eta\, c_Q M^2_{1/2}, \qquad \eta = \frac{\sum(h'_i/h_i)^2}{(\sum_i h'_i/h_i)^2} \geq \frac{1}{3}, \tag{3.16}$$

where $(\tilde{m}^2_Q)_{\rm tree}$ is the tree level value given in eqs. (3.10,3.11) and c_Q is the total SM quadratic casimir of the sfermion $\tilde{Q}$ (or Higgs Q):

Q	q_i	u^c_i	d^c_i	l_i	e^c_i	h_u	h_d
c_Q	21/10	8/5	7/5	9/10	3/5	9/10	9/10

(3.17)

[2]The relevant messengers are the ones with the largest h'_i/h_i. If the most relevant messenger is the third family one, the effect of the running that we are neglecting is not too large. The third family messenger mass is in fact given by $h_3 M = m_b/(v\cos\beta\sin\theta_d)M$ (m_b is the bottom mass, $v = 174\,{\rm GeV}$), not too far (in logarithmic scale) from $M \sim M_{\rm GUT}$. Still, we expect the messengers to be lighter enough than the GUT scale in such a way that only the SM casimirs (and not the GUT ones) are relevant.

If the contribution of a single messenger dominates gaugino masses, $\eta \approx 1$. In the numerical example we will consider, the relative size of the two loop contribution to sfermion masses ranges from 2% to 10%.

Additional, subleading contributions to sfermion masses can arise from different sources. One-loop contributions from an induced $U(1)_X$ Fayet-Iliopoulos term [57] only arise if h' is non-diagonal in the basis where h is diagonal and $|h'_{ij}| \neq |h'_{ji}|$. Moreover, they are suppressed (typically negligible) because $U(1)_X$ is broken above the scale of the loop messengers. Another contribution could come from gravity effects. Since in our scenario the messenger scale is expected to be around the GUT scale, the gravity mediated contribution to the spectrum, although subleading, could be relevant for flavour physics, as it could in principle be strongly flavour violating. In order to quantify this effect, let us assume that the gravity contribution to an arbitrary entry of the squared mass matrix of the sfermions in the 10 of SU(5) is given by $\tilde{m}^2_{\text{grav}} = F^2/M_{\text{P}}^2$, where $M_{\text{P}} = 2.4 \cdot 10^{18}\,\text{GeV}$ is the reduced Planck mass. The conservative bound $\tilde{m}^2_{\text{grav}} < 2 \cdot 10^{-3}\, \tilde{m}^2_{10}$, which guarantees that all FCNC effects are under control, then translates in the following bound on the messenger scale:

$$M < 3 \cdot 10^{16}\,\text{GeV}. \tag{3.18}$$

If the messenger scale is higher, we are in a hybrid framework from the flavour point of view [58]. Finally, another potentially relevant source of flavour non-universality might come from one loop contributions to sfermion masses arising from the superpotential Yukawa interactions in eq. (3.9), once the (necessary) presence of mass terms for the components of the 16 and 16′ are taken into account. Such effects are certainly under control if the matrix h', as h, has a hierarchical structure and is approximately aligned to h.

Let us now consider the A-terms. The latter are generated at one loop by the Yukawa interactions in eq. (3.9), with no contribution from gauge interactions. Assuming for simplicity that the matrices h' and y are diagonal in the same basis in which h is, we have

$$A_{l_i,d^c_i} = -\frac{1}{4\pi^2}\frac{h'_i}{h_i}\left(h_i^2 + h_i'^2\right) m \tag{3.19a}$$

$$A_{q_i,u^c_i,e^c_i} = -\frac{1}{(4\pi)^2}\frac{h'_i}{h_i}\left(3(h_i^2 + h_i'^2) + 2y_i^2\right) m \tag{3.19b}$$

at the messenger scale. The A-terms above are defined in such away that they give rise to soft trilinear terms in the Lagrangian in the form $\mathcal{L} \supset -\sum_Q A_Q \tilde{Q}(\partial W(\tilde{Q}))/(\partial Q)$. By comparing with the expression for the gaugino masses, we conclude that only the A-terms of the third family have a chance to be sizable at the messenger scale, unless the h' matrix is not hierarchical. Within the simplified diagonal flavour structure we are considering, we can compare the A-terms in eqs. (3.19) with the gaugino masses in eq. (4.21). The gaugino masses are in this case proportional to $\sum_i h'_i/h_i$. Depending on which of the three terms dominates in the sum, the largest A-terms can be comparable or smaller than the gaugino masses. The (necessary) presence of mass terms for the components of the 16 and 16' can generate additional, model-dependent, contributions. In any case, sizable contributions to the A-terms will be generated as usual by the RGE evolution proportional to the gaugino masses.

Next, we comment on the μ problem. Relating the μ-term to supersymmetry breaking is, not surprisingly, a highly model-dependent issue, due to the various possibilities of implementing supersymmetry breaking and embedding the Higgs fields in SO(10). We point out, however, a simple possibility in which both the F-term, $\langle N' \rangle = F\theta^2$ and μ originate from the same parameter $m \sim$ TeV in the superpotential: $W \supseteq mN'\overline{N}$.

Once $\overline{N}$ is forced to get its vev $\langle \overline{N} \rangle = M \sim M_{\text{GUT}}$, N' acquires an F-term $F = mM$ (so that m is indeed the parameter introduced in eq. (3.8)). In our setup, N' and $\overline{N}$ are part of the SO(10) multiplets $16'$ and $\overline{16}$ respectively. A μ term related to the supersymmetry breaking scale $\mu \sim m$ is then therefore generated if h_u has a component in $\overline{16}$ and h_d has a component in $16'$. Such a situation can be achieved with an appropriate superpotential. Contrary to standard gauge mediation, there is no μ-$B\mu$ problem here, as $B\mu/\mu$ is not enhanced by an inverse loop factor. $B\mu$ can be generated at the tree level, for example as in [54], or it can be generated by the RGE evolution.

We now illustrate an example of low energy spectra that can be obtained in our framework. We neglect the (small, for our purposes) effect of the intermediate scale $\bar{5}_i^{16}$ and 5_j^{10} and use the MSSM RGE equations, as implemented in `Suspect 2.41` [59], with boundary conditions at high energy as in eqs. (3.10,3.11,3.16), the A-terms set to zero, and $\eta = 1$. We assume the messenger mass to coincide with the GUT scale, $M = M_{\text{GUT}}$. The overall normalization of the unified gaugino masses $M_{1/2}$ can be considered as a free parameter due to the presence of the factor $\text{Tr}(h'h^{-1})$ in eq. (4.21), or equivalently of the factor λ in eq. (4.61). As the size of the parameters μ and $B\mu$ is model dependent, we consider them as free parameters as well and recover them as usual in terms of M_Z and $\tan\beta$. Under the above assumptions, the parameters that specify the model are: m, θ_d, $M_{1/2}$, $\tan\beta$ and the sign of μ.

Table 3.2 shows the low-energy spectrum corresponding to $\theta_d = \pi/6$, $\tan\beta = 30$ and $\text{sign}(\mu) = +$. The common gaugino mass is $M_{1/2} = 150\,\text{GeV}$, near the minimal value allowed at present by chargino direct searches. The value of m is near the minimal value allowed by the bound $m_h > 114\,\text{GeV}$. This spectrum corresponds to

Higgs:	m_{h^0}	114
	m_{H^0}	1543
	m_A	1543
	$m_{H^\pm}$	1545
Gluinos:	$M_{\tilde{g}}$	448
Neutralinos:	$m_{\chi_1^0}$	62
	$m_{\chi_2^0}$	124
	$m_{\chi_3^0}$	1414
	$m_{\chi_4^0}$	1415
Charginos:	$m_{\chi_1^\pm}$	124
	$m_{\chi_2^\pm}$	1416
Squarks:	$m_{\tilde{u}_L}$	1092
	$m_{\tilde{u}_R}$	1027
	$m_{\tilde{d}_L}$	1095
	$m_{\tilde{d}_R}$	1494
	$m_{\tilde{t}_1}$	1007
	$m_{\tilde{t}_2}$	1038
	$m_{\tilde{b}_1}$	1069
	$m_{\tilde{b}_2}$	1435
Sleptons:	$m_{\tilde{e}_L}$	1420
	$m_{\tilde{e}_R}$	1091
	$m_{\tilde{\tau}_1}$	992
	$m_{\tilde{\tau}_2}$	1387
	$m_{\tilde{\nu}_e}$	1418
	$m_{\tilde{\nu}_\tau}$	1382

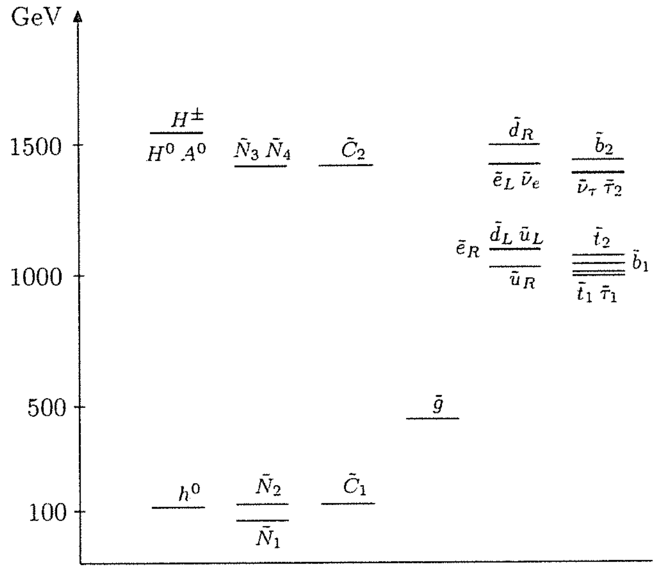


Figure 3.2: An example of spectrum, corresponding to $m = 3.2\,\text{TeV}$, $M_{1/2} = 150\,\text{GeV}$, $\theta_d = \pi/6$, $\tan\beta = 30$ and $\text{sign}(\mu) = +$, $A = 0$, $\eta = 1$. All the masses are in GeV, the first two families have an approximately equal mass.

$\lambda = 2.5$. Given the (moderate) hierarchy between $M_{1/2}$ and the sfermion masses, the sfermion RGEs are not significantly affected by the gaugino masses and the sfermion mass relations characterizing the model, eq. (3.10), survive, to some extent, at low energy. The relative size of the two-loop contributions to sfermion masses in eq. (3.16) range from 2% to 10%.

Finally, we comment about cosmology. As in loop gauge mediation, the LSP is the gravitino, if the messenger mass is consistent with eq. (3.18). In fact, the supersymmetry breaking parameter is given by

$$\sqrt{F} \approx 0.8 \cdot 10^{10}\,\mathrm{GeV}\left(\frac{\tilde{m}_{10}}{\mathrm{TeV}}\frac{M}{2\cdot 10^{16}\,\mathrm{GeV}}\right)^{1/2} \tag{3.20}$$

and the gravitino mass by

$$m_{3/2} = \frac{F}{\sqrt{3}M_{\mathrm{P}}} \approx 15\,\mathrm{GeV}\left(\frac{\tilde{m}_{10}}{\mathrm{TeV}}\frac{M}{2\cdot 10^{16}\,\mathrm{GeV}}\right), \tag{3.21}$$

where $\tilde{m}_{10}$ is the tree-level mass of the sfermions in the 10 of SU(5) at the GUT scale. Note that F and the gravitino mass are smaller than in loop gauge mediation, for a given messenger scale M, because of the absence of a loop factor in eqs. (3.20,3.21). For a stable (on the age of the universe timescale) gravitino with a mass as large as in eq. (3.21), a dilution mechanism such as inflation is necessary in order for its energy density not to exceed the dark matter one. The upper bound on the reheating temperature T_R depends on the gravitino and the gaugino masses [60]. The thermal contribution to the gravitino energy density, for a reheating temperature around 10^9 GeV is given by

$$\Omega_{\tilde{G}}^{\mathrm{TP}} h^2 \approx 6\times 10^{-2}\left(\frac{T_{RH}}{10^9\,\mathrm{GeV}}\right)\left(\frac{15\,\mathrm{GeV}}{m_{3/2}}\right)\left(\frac{M_{1/2}}{150\,\mathrm{GeV}}\right)^2 . \tag{3.22}$$

For the spectrum in Table 3.2, the bound $\Omega_{\tilde{G}}^{\mathrm{TP}} h^2 \leq \Omega_{\mathrm{DM}} h^2 = 0.11$ translates in $T_R < 2\cdot 10^9\,\mathrm{GeV}$.

We then have to take care of the decays of the NLSP into the gravitino, which might spoil big bang nucleosynthesis (BBN) unless it is fast enough. The fate of BBN depends on what the NLSP is. In the bulk of the parameter space we expect the NLSP to be the lightest neutralino or a stau. In the example in Table 3.2, the NLSP is essentially a Bino. For $m_{3/2} \sim 15\,\mathrm{GeV}$, the decay of a Bino NLSP through its coupling to the Goldstino component of the gravitino is way too slow (one would need $m_{3/2} < 100\,\mathrm{MeV}$ in order not to spoil BBN [61]). A Bino NLSP therefore requires a much faster decay channel. The latter can be provided by a tiny amount of R-parity violation [62]. Such a possibility is also consistent with thermal leptogenesis and gravitino dark matter. The other possibility is that the NLSP is a stau. In this case, all the BBN constraints can be satisfied if the lifetime of the stau is $\tau_{\tilde{\tau}} \approx 48\pi m_{3/2}^2 M_P^2 / m_{\tilde{\tau}}^5 \lesssim 6 \cdot 10^3\, s$ [63]. This is a viable possibility, which however requires large $\lambda = \mathcal{O}(100)$ and sizable gaugino masses. For such large values of λ, radiative contributions to sfermion masses (from RGEs and the standard gauge mediation contribution) dominate over the tree level one, the spectrum approaches the usual loop gauge mediated one, and the peculiar relation between sfermion masses at the messenger scale gets hidden.

In conclusion, we have considered what is perhaps the simplest way to communicate supersymmetry breaking: through a tree level renormalizable exchange of a gauge (GUT) messenger, as in Fig. 3.1. We showed that this possibility is viable, despite the well known arguments associated to the supertrace formula. Besides offering new model-building avenues, this scheme solves the supersymmetric FCNC problem and, in its simplest implementation, leads to peculiar relations among sfermion masses that can be tested at the LHC.

Chapter 4

General Aspects of TGM

A minimal model of tree-level gauge mediation has been presented in [5], solving the supersymmetric flavor problem and predicting the ratio of different sfermion masses to be different from mSugra and other schemes. In this chapter we would like to take a broader point of view and study the general implementation of TGM. This will allow to establish the general properties of TGM, to set up the guidelines for model building and to identify what are the hypotheses under which the peculiar predictions on soft masses of the minimal model hold. Moreover, we would like to present a few new approaches to the μ-problem, both in the context of well known (Giudice-Masiero, NMSSM) and new solutions. In particular, in Section 4.1 we will discuss what are the conditions under which heavy vector superfields can act as tree-level messengers of supersymmetry breaking and obtain a general expression for the tree level contribution to the supersymmetry breaking lagrangian, in particular to the sfermion soft masses. In Section 4.2, we will consider the one-loop contributions to soft masses, concentrating mostly on gaugino masses and the enhancement factors compensating their loop suppression. In Section 4.3 we consider the possibility to obtain a phenomenologically viable model from the general formalism previously introduced. We will see that clear model building guidelines emerge, leading to peculiar predictions for the pattern of MSSM sfermion masses and we will identify the assumptions underlying such predictions. In Section 4.4, we will discuss a few new approaches to the μ-problem, before summarizing in Section 4.5.

Related to this chapter there are also two Appendixes. In Appendix (6.2), we outline the procedure to integrate out vector superfields and address a few minor issues, such as the generalization to the non-abelian case and the role of gauge invariance in a consistent supersymmetric generalization of the expansion in the number of derivatives. In Appendix (6.3), we provide an example of a superpotential achieving supersymmetry breaking, SO(10) breaking to the SM, ensuring that only the MSSM fields survive at lower energy (in particular providing doublet-triplet splitting) and solving the μ-problem. Such a superpotential is not aimed at being simple or realistic, but it represents a useful existence proof.

4.1 Tree level soft terms

In this Section we discuss the conditions under which heavy vector superfields can act as tree-level messengers of supersymmetry breaking in the context of a generic, renormalizable, $N = 1$ globally supersymmetric gauge theory in four dimensions. Then we recover the general expression for the tree level contribution to the sfermion soft masses. We discuss their origin both in the context of the full, renormalizable theory, and in an effective theory approach.

We start from a lagrangian described by a canonical Kähler $K = \Phi^\dagger e^{2gV}\Phi$ and gauge kinetic function and by a generic superpotential $W(\Phi)$ function of the chiral superfields $\Phi \equiv (\Phi_1 \dots \Phi_n)$, with no Fayet-Iliopoulos term. We follow the conventions in [64]. We will denote by ϕ_i, ψ_i, F_i the scalar, spinor, and auxiliary component of Φ_i and by v^μ_a, λ_a, D_a the vector, spinor, and auxiliary component of V_a. The gauge group G (assumed for simplicity to be simple with a single gauge coupling g) is broken by the scalar component vev $\phi_0 = \langle \phi \rangle$ to the subgroup H at a scale $M_V \sim g|\phi_0| \gg M_Z$, at which the theory is approximately supersymmetric. In the phenomenological applications we have in mind, H contains the SM gauge group G_{SM}, G is a grand-unified group (for example SO(10) or E_6), and the breaking scale is of the order of the GUT scale. Correspondingly, the vector superfields split into light and heavy ones, associated to the orthonormalized generators T^l_a and T^h_b respectively: $V = V^l_a T^l_a + V^h_b T^h_b$, $a = 1 \dots N_l$, $b = 1 \dots N_h$.

The heavy vector superfields acquire a squared mass matrix given by

$$(M^2_{V0})_{ab} = g^2 \phi_0^\dagger \{T^h_a, T^h_b\} \phi_0. \tag{4.1}$$

We choose the basis of heavy generators T^h_a in such a way that the above mass matrix is diagonal,

$$(M^2_{V0})_{ab} = M^2_{V_a} \delta_{ab}. \tag{4.2}$$

The heavy vector superfields become massive by eating up a corresponding number of Goldstone chiral superfields. It is then convenient to split the chiral superfields as follows

$$\Phi = \phi_0 + \Phi' + \Phi^G, \quad \Phi^G = \sqrt{2}\, g \frac{\Phi^G_a}{M_{V_a}} T^h_a \phi_0, \quad \Phi' = \Phi'_i b_i, \tag{4.3}$$

where Φ^G_a, $a = 1 \dots N_h$ are the Goldstone superfields associated to the generators T^h_a and $b_i = (b^i_1 \dots b^i_n)$, $i = 1 \dots n - N_h$ is an orthonormal basis in the space of the "physical" chiral fields Φ', $b^\dagger_i T_a \phi_0 = 0$. In the supersymmetric limit, ϕ_0 is orthogonal to Φ^G and Φ^G does not mix with the physical superfields. The physical components of the massive vector superfield V_a are v_a^μ, λ_a, ψ_a^G, $\mathrm{Re}(\phi_a^G)/\sqrt{2}$, all with mass M_{V_a}. The imaginary part of ϕ_a^G, the Goldstone boson, becomes as usual the longitudinal component of the massive gauge boson v_a^μ and the spinors ψ_a^G and λ_a pair up in a Dirac mass term. This spectrum can be split by supersymmetry breaking corrections, as we will see in Section 4.2.1.

As for the physical chiral superfields Φ_i', their supersymmetric mass matrix is given by

$$M_{ij}^0 = \frac{\partial^2 W}{\partial\Phi_i'\partial\Phi_j'}(\phi_0). \tag{4.4}$$

Again, we choose the basis b_i in such a way that the above mass matrix is diagonal and positive,

$$M_{ij}^0 = M_i\delta_{ij}, \quad M_i \geq 0. \tag{4.5}$$

The scalar and fermion components of Φ' can be split by supersymmetry breaking corrections, which can also induce a mixing with the scalar and fermion components of the heavy vector superfields.

Supersymmetry is supposed to be broken at a much lower scale than M_V, where some of the fields Φ' get an F-term, $\langle\Phi'\rangle = F_0\theta^2$, $M_Z^2 \ll |F_0| \ll M_V^2$. As a consequence, ϕ_0 satisfies with good approximation the F-term and D-term conditions at the scale M_V, $\partial_i W(\phi_0) = 0 + \mathcal{O}(|F_0|)$ and $\phi_0^\dagger T_a \phi_0 = 0 + \mathcal{O}(|F_0/M_V|^2)$ (see eq. (4.6)) for each i, a.

The F-terms induce a non vanishing vev for the D-terms D_a^h of the heavy vector superfields. The stationary condition for the scalar potential V, $\partial V/\partial\phi_i = 0$, together with the gauge invariance of the superpotential give

$$\langle D_a^h\rangle = -2g\frac{F_0^\dagger T_a^h F_0}{M_{V_a}^2}, \tag{4.6}$$

with the light D-terms still vanishing. Clearly, only generators T_a^h that are singlets under the unbroken group H can contribute to such D-term vevs. Note also the condition

$$F_0^\dagger T_a \phi_0 = 0, \tag{4.7}$$

which implies that the Goldstone superfields Φ^G do not get F-term vevs (the D-term condition implies that in the supersymmetric limit they do not get scalar vev either). The latter relation also follows from the gauge invariance of the superpotential. In turn, the D-terms above give rise to tree level soft masses for the scalar components ϕ'_i of the chiral superfields Φ'_i

$$V \supset \frac{1}{2}D^2 \supset -g\,\phi'^{\dagger}T^h_a\langle D^h_a\rangle = (\tilde{m}^2_{ij})_D\phi'^{\dagger}_i\phi'_j \tag{4.8}$$

$$(\tilde{m}^2_{ij})_D = 2g^2(T^h_a)_{ij}\frac{F_0^\dagger T^h_a F_0}{M^2_{V_a}}, \tag{4.9}$$

provided that both F_0 and the scalars ϕ' are charged under the (broken) gauge interaction associated to T^h_a and provided that T^h_a is a singlet under H.

The complete list of tree level soft terms obtained from the gauge dynamics can be more conveniently recovered in the effective theory below M_V. Before discussing it, let us observe that this theory must necessarily satisfy the supertrace formula $\mathrm{Str}(\mathcal{M}^2) = 0$. In the case of the soft terms in eq. (4.9), this simply follows here from $\mathrm{Tr}\,T^h_a = 0$. In particular, the tracelessness condition implies that positive soft masses are accompanied by negative ones in eq. (4.8). This is a potential phenomenological problem, which has long been considered as an obstacle to models in which supersymmetry breaking terms are generated, as here, at the tree, renormalizable level. However, it has recently been shown [5] that such a potential problem can be easily solved by adding a large positive supersymmetric mass term to the chiral superfields whose tachyonic nature would be problematic.

As mentioned, the generation of the sfermion masses can be conveniently seen in the effective theory below M_V, where the heavy vector and the Goldstone chiral superfields have been integrated out. In this theory, the chiral degrees of freedom are the Φ'. The gauge group is H and it is unbroken (we neglect electroweak symmetry breaking). As a consequence, there is no D-term contribution to supersymmetry breaking. The scalar masses arise in this context from F-terms vevs through an effective Kähler operator, as we will see in a moment.

The vector superfields can be integrated out by solving the equations of motion $\partial K/\partial V^h_a = 0$ [65, 66]. In Appendix 6.2 we illustrate the details of such a procedure in a general case, we explicitly write the resulting effective theory at the leading order, and we make a few general remarks on the approximations involved in using $\partial K/\partial V^h_a = 0$ and on the role of gauge invariance in a consistent supersymmetric generalization of the expansion in the number of derivatives [65]. For the present purposes, we are only interested in the terms in the effective lagrangian relevant to (sizable) soft supersymmetry breaking. Those are the ones following from the effective tree level contribution to the Kähler potential in eq. (6.19a):

$$\delta K^0_{\text{eff}} = -\frac{g^2}{M^2_{V_a}}(\Phi'^\dagger T^h_a\Phi')(\Phi'^\dagger T^h_a\Phi'), \tag{4.10}$$

where we remind that Φ' has no vev in its scalar component. The operator in eq. (4.10) can be seen to arise from the diagram on the left-hand side in Fig. 4.1.

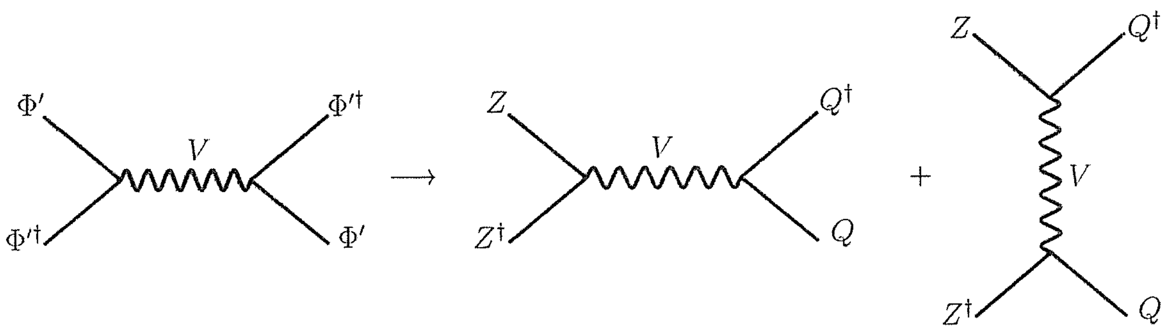


Figure 4.1: Tree level gauge mediation supergraph generating the operator in eq. (4.10) when integrating out the heavy vector superfield messengers.

As mentioned, the only possible source of supersymmetry breaking in the effective theory are the F-term vevs of the chiral superfields Φ'. We remind that such F-term vevs must belong to non-trivial representations of the full group G, in order to play a role in TGM. The only terms in the lagrangian containing such F-term vevs, at the tree level and up to second order in F_0, $F_0^\dagger$, and $1/M_{V_a}$, arise from the superpotential and from the operator in eq. (4.10):

$$-\mathcal{L}_{\text{soft}}^{\text{tree}} = -F_{0i}\frac{\partial\hat{W}}{\partial\Phi_i} - 2g^2\frac{(F_0^\dagger T_a^h\psi')(\phi'^\dagger T_a^h\psi')}{M_{V_a}^2} + \text{h.c.} + 2g^2\frac{(F_0^\dagger T_a^h F_0)(\phi'^\dagger T_a^h\phi')}{M_{V_a}^2} + 2g^2\frac{(\phi'^\dagger T_a^h F_0)(F_0^\dagger T_a^h\phi')}{M_{V_a}^2} - F_0^\dagger F_0, \quad (4.11)$$

where $\hat{W}$ is the superpotential in the effective theory,

$$\hat{W}(\Phi') = W(\phi_0 + \Phi') \qquad (\Phi^G = 0). \quad (4.12)$$

Let us consider the different terms in eq. (4.11) in turn. The first term in the second line reproduces the contribution to the soft scalar masses in eq. (4.9). The second term gives rise to an additional contribution, only relevant to superfields that are gauge partners of the Goldstino superfield (and have the same quantum numbers under H as some of the generators of G)[1]. All in all, we have

$$\tilde{m}_{ij}^2 = 2g^2\left[(T_a^h)_{ij}\frac{F_0^\dagger T_a^h F_0}{M_{V_a}^2} + \frac{(T_a^h F_0)_i^*(T_a^h F_0)_j}{M_{V_a}^2}\right]. \quad (4.13)$$

[1]The latter contribution can be obtained in the context of the full theory by using the unitary gauge or in Wess-Zumino gauge from the F-term contribution to the scalar potential using eq. (4.24) below.

Note that the soft terms do not actually depend on the gauge coupling or on the normalization of the generators T, as $M^2_{V_a}$ is also proportional to g^2T^2. The second term in the first line of eq. (4.11) is a gauge-generated Yukawa interaction with coupling $\lambda = \mathcal{O}\left(|F_0|/M^2_V\right)$, usually absent in models of supersymmetry breaking. From a phenomenological point of view, such tiny Yukawa couplings might play a role in neutrino physics, where they could represent naturally small Dirac neutrino Yukawa couplings [68].

Finally, the first term in eq. (4.11), has to do with the existence of a hidden sector in the effective theory. In the phenomenological applications we have in mind, the light spectrum will contain the MSSM chiral superfields, as part of a light, "observable" sector. The latter will be charged under the residual gauge group $H \supseteq G_{\rm SM}$. On the other hand, the supersymmetry breaking superfields do not feel the residual gauge interactions. In the effective theory, therefore, the supersymmetry breaking sector is hidden from the observable sector from the point of view of gauge interactions. In order for the supersymmetry breaking sector to be hidden also from the point of view of superpotential interactions, it is sufficient to make sure that the first term in eq. (4.11) does not induce a direct coupling between the two sectors. To be more precise, we can write the chiral superfields of the effective theory, Φ', as

$$\Phi' = (Z, Q, \Phi^h). \tag{4.14}$$

The superfield Z is the only one getting an F-term vev, $\langle Z\rangle = |F_0|\theta^2$. Its fermion component is the Goldstino and therefore Z is a massless eigenstate of the mass matrix M^0 in eq. (4.4). The remaining mass eigenstates are divided in two groups, the heavy ones, Φ^h_i with masses $M^h_i \gg |F_0|$, and the light, or observable, ones Q_i, with masses $M^Q_i \lesssim |F_0|$. In order to hide supersymmetry breaking from the observable sector also from the point of view of superpotential interactions, we require that

$$\frac{\partial^2 \hat{W}}{\partial Z \partial Q_j}(Z, Q, \Phi^h = 0) = 0 \tag{4.15}$$

(at least for the renormalizable part of the superpotential).

We can then see supersymmetry breaking as arising in a hidden sector and then communicated from the to the observable sector by the diagrams on the right-hand side of Fig. 4.1. This can perhaps be considered as the simplest way to communicate supersymmetry breaking: through the tree level renormalizable exchange of a heavy gauge messenger. Since heavy gauge messengers at a scale not far from the Planck scale are automatically provided by grand-unified theories, this possibility is not only simple but also well motivated. The reason why it has not been pursued in the past is an apparent obstacle arising from the supertrace theorem that, as mentioned, can be easily evaded by providing heavy, supersymmetric masses to some of the superfields. Such mass terms can naturally arise in the context of grand-unified theories, as we will see.

We end this Section with some comments on integrating out heavy chiral superfields and on the corresponding possible tree level contributions to A-terms and soft scalar masses. The heavy vector superfields may not be the only fields living at the scale M_V, as chiral superfields could have mass terms of similar size or get it after gauge symmetry breaking. Such chiral fields should also be integrated out in order to get the effective theory below the scale M_V. In general, we want to integrate out all the heavy chiral superfields Φ^h. Since their masses M_i^h are assumed to be much larger than the supersymmetry breaking scale, it will still be possible to write the effective theory in a manifestly supersymmetric way. In order to integrate them out, let us write the superpotential as

$$\hat{W} = -|F_0|Z + \frac{M_i^Q}{2}Q_i^2 + \frac{M_i^h}{2}(\Phi_i^h)^2 + W_3(Z,Q,\Phi^h), \tag{4.16}$$

where W_3 is at least trilinear in its argument. The equations of motion $(\partial\hat{W})/(\partial\Phi_i^h) = 0$ give

$$\Phi_i^h = -\frac{1}{M_i^h}\frac{\partial W_3}{\partial\Phi_i^h}(Z,Q) + \mathcal{O}\left(\frac{1}{M_h^2}\right). \tag{4.17}$$

The effective superpotential for the light fields Z and Q is therefore

$$W_{\text{eff}}(Z,Q) = \hat{W}(Z,Q) - \frac{1}{2M_i^h}\sum_i\left(\frac{\partial W_3}{\partial\Phi_i^h}(Z,Q)\right)^2 + \mathcal{O}\left(\frac{1}{M_h^2}\right). \tag{4.18}$$

A contribution to the effective Kähler is also induced

$$\delta K_\Phi = \frac{1}{(M_i^h)^2}\sum_i\left|\frac{\partial W_3}{\partial\Phi_i^h}(Z,Q)\right|^2 + \mathcal{O}\left(\frac{1}{M_h^3}\right). \tag{4.19}$$

The effective contributions to the superpotential and to the Kähler in eqs. (4.18) and (4.19) may give rise to "chiral-mediated" tree-level A-terms and (negative) additional contributions to soft scalar masses respectively. The latter should be subleading with respect to the (positive) vector mediated contributions in eq. (4.13), at least in the case of the MSSM sfermions. Such tree level contributions could only arise in the presence of trilinear superpotential couplings in the form $ZQ\Phi^h$. In the following we will consider the case in which such a coupling is absent,

$$\frac{\partial^3\hat{W}}{\partial Z\partial Q\partial\Phi^h}(0) = 0, \tag{4.20}$$

so that the chiral-mediated tree tree level contributions also vanish. This is often the case, as illustrated by the model in [5].

4.2 One loop soft terms and gaugino masses

In this Section we consider the one loop contributions to soft masses, focusing mostly on gaugino masses and the enhancement factors compensating their loop suppression.

Gaugino masses do not arise at the tree level. They are however generated at the one-loop level, as in standard, "loop" gauge mediation models. The suppression of gaugino masses by a loop factor with respect to scalar masses represents a potential phenomenological problem. Given the present experimental limits on gaugino masses, a loop factor enhancement would make the sfermions heavier than $\mathcal{O}(10\,\text{TeV})$, beyond the reach of the LHC and heavy enough to introduce a serious fine-tuning problem, thus approaching the split supersymmetry regime [29]. However, it turns out that the loop hierarchy between gaugino and scalar soft masses is typically reduced or eliminated, as we will see in this Section.

We calculate gaugino masses in the full theory above M_V. There are two types of one loop diagrams contributing to gaugino masses, depending on whether the degrees of freedom running in the loop are components of the heavy vector superfields (including the Goldstone superfields), as in Fig. 4.2a, or physical chiral superfields, as in Fig. 4.2b. Correspondingly, we will distinguish a "vector" and a "chiral" contribution to the light gaugino masses,

$$M^g_{ab} = (M^g_{ab})_V + (M^g_{ab})_\Phi. \tag{4.21}$$

The latter may easily dominate on the former, as we will see. The source of supersymmetry breaking entering the diagrams of Fig. 4.2a and 2b is a tree level splitting among the components of the heavy vector and chiral superfields respectively. We now examine the two contributions in eq. (4.21) in turn and write the known results [69] in a form general enough to be suitable for the following discussion of their quantitative importance compared to the tree level scalar soft terms.

4.2.1 Vector contribution to gaugino masses

In the supersymmetric limit, the fields v^μ_a, λ_a, ψ^G_a, $\text{Re}(\phi^G_a)/\sqrt{2}$ form a massive vector multiplet with mass M_{V_a}. Once supersymmetry is broken, this spectrum is split by corrections to the fermion and scalar masses, which may also mix them with the components of the physical chiral superfields. Here, we are interested to the supersymmetry breaking fermion mass term in the form $-m_{ab}\psi^G_a\psi^G_b/2$, which is the source of the vector contribution to gaugino masses through the diagrams in Fig. 4.2a.

The mass term

$$m_{ab} = \frac{\partial^2 W}{\partial\Phi^G_a \partial\Phi^G_b}(\phi_0) \tag{4.22}$$

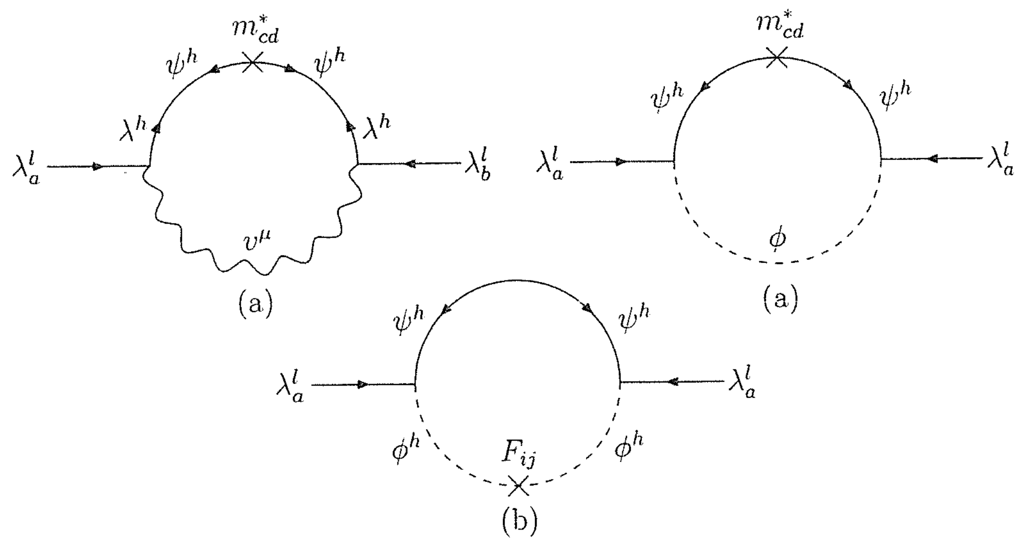


Figure 4.2: One loop contributions to light gaugino masses from the exchange of heavy vector (a) and chiral (b) degrees of freedom.

vanishes in the supersymmetric limit because of the gauge invariance of W. The situation is different in the presence of supersymmetry breaking, when the gauge invariance of W gives

$$m_{ab} = g^2 \frac{F_0^\dagger \{T^h_a, T^h_b\} \phi_0}{M_{V_a} M_{V_b}}. \tag{4.23}$$

Note also the more general expression for the mixed supersymmetry breaking terms

$$\frac{\partial^2 W}{\partial \Phi_i \partial \Phi^G_a}(\phi_0) = \sqrt{2} g \frac{F^\dagger_{0j} (T^h_a)_{ji}}{M_{V_a}}. \tag{4.24}$$

Before showing the expression for the gaugino masses induced by m_{ab}, let us remind that the heavy vector representation is in general reducible, under the unbroken gauge group H, to a set of irreducible components, each with a single value of the mass. Let us call $\hat{M}_{V_r}$ the value of the mass in the representation r and denote

$$g^2 \phi_0^\dagger \{T^h_a, T^h_b\} F_0 = m^*_{ab} \hat{M}^2_{V_r} \equiv \frac{\partial \hat{M}^2_{V_r}}{\partial Z} |F_0| \delta_{ab}, \tag{4.25}$$

if T^h_a, T^h_b belong to the representation r. In the limit $|F_0| \ll M^2_V$, the supersymmetry breaking source m_{ab} can be treated as a perturbation in the one loop computation of gaugino masses. At the leading order in m_{ab}, the diagram in

Fig. 4.2a generates a contribution to light gaugino masses given by

$$(M^g_{ab})_V = -2\frac{g^2}{(4\pi)^2}\sum_r S_{ab}(r)\frac{|F_0|}{\hat{M}^2_{V_r}}\frac{\partial \hat{M}^2_{V_r}}{\partial Z}, \tag{4.26}$$

where $S_{ab}(r) = \mathrm{Tr}(r(T^l_a)r(T^l_b))$ is the Dynkin index of the representation $r : T \to r(T)$ of the generator T. The above contribution to gaugino masses arises at the scale M_V where the heavy vectors live.

Let us now discuss the relevance of the above contribution to gaugino masses. First, let us note that in order for $(M^g_{ab})_V$ to be non-vanishing we need the following two conditions to be verified at the same time

$$\phi_0^\dagger\{T^h_a, T^h_b\}F_0 \neq 0 \text{ (for some } a, b), \qquad \phi_0^\dagger T^h_a F_0 = 0 \text{ (for all } a), \tag{4.27}$$

as it can be seen from eqs. (4.25) and (4.7). In particular, we need at least one irreducible (under the full group G) chiral superfield multiplet to get vev in both its scalar and F components. At the same time, we need

$$F_0^\dagger T^h_a F_0 \neq 0 \text{ (for some } a) \tag{4.28}$$

in order for the tree level contribution to scalar masses to be generated. The conditions in eqs. (4.27,4.28) may force the vector contribution to gaugino masses to vanish. On top of that, $(M^g_{ab})_V$ is always suppressed by a loop factor $g^2/(4\pi)^2$ compared to the typical scalar mass in eq. (4.13). If $(M^g_{ab})_V$ was the only contribution to gaugino masses, this would lead to an hierarchy between gaugino and scalar soft masses. Moreover, the present experimental lower limits on gaugino masses, $M_g \gtrsim 100\,\mathrm{GeV}$, would force the sfermion masses to be heavier than $\mathcal{O}(10\,\mathrm{TeV})$. However, as we will see in a moment, the chiral contribution to gaugino masses can be significantly larger than the vector contribution, thus reducing or even eliminating the loop suppression with respect to soft scalar masses. In this case, the vector contribution to gaugino masses typically ends up to be subdominant.

4.2.2 Chiral contribution to gaugino masses

The chiral contribution to gaugino masses arises from the one loop diagram in Fig. 4.2b, as in ordinary loop gauge mediation. The scalar and fermion components of the chiral superfields entering the loop are split by a supersymmetry breaking scalar mass term $-(F_{ij}\phi^h_i\phi^h_j + \text{h.c.})/2$. As a consequence of eqs. (4.15) and (4.20), the supersymmetry breaking couples directly only to the heavy chiral fields and F_{ij} can be treated as a perturbation in the calculation of gaugino masses. The mass term F_{ij} is then given by

$$F_{ij} = -\frac{\partial^3 \hat{W}}{\partial\Phi^h_i\partial\Phi^h_j\partial Z}(0)|F_0|, \tag{4.29}$$

which adds to the supersymmetric scalar mass term $-M_i^2|\phi_i'|^2$ (in the notation of eq. (4.5)).

The physical chiral superfield representation under the unbroken gauge group H is in general reducible to a set of irreducible components, each with its own mass $\hat{M}_r$. Let us denote

$$\frac{\partial^3\hat{W}}{\partial\Phi_i^h\partial\Phi_j^h\partial Z}(0)|F_0| = -F_{ij} \equiv \frac{\partial\hat{M}_r^h}{\partial Z}|F_0|\delta_{ij}. \tag{4.30}$$

At the leading order in F_{ij}, the diagram in Fig. 4.2b generates a contribution to light gaugino masses given by

$$(M_{ab}^g)_\Phi = \frac{g^2}{(4\pi)^2}\sum_r S_{ab}(r)\frac{|F_0|}{\hat{M}_r}\frac{\partial\hat{M}_r}{\partial Z}. \tag{4.31}$$

Each of the contributions in the sum in the RHS of eq. (4.31) arises at at the scale $\hat{M}_r$ at which the corresponding chiral superfield lives. If the heavy chiral superfields split in two conjugated representations $\Phi^h = \Psi + \overline{\Psi}$ with a mass term in the form $-\overline{\Psi}\mu\Psi$, eq. (4.31) still holds with $M \to \mu$ and a factor 2 multiplying the RHS.

Let us now discuss the size of the typical chiral contribution to gaugino masses M_g and compare it with the typical size of the tree level scalar soft masses $\tilde{m}^2$ in eq. (4.13). Let us consider for simplicity the case in which the scalar masses are due to the exchange of a single heavy vector and the irreducible (under H) components of the physical chiral superfields have definite charges Q_r under the corresponding generators. As for the dynamics giving rise to gaugino masses, let us assume that there are no bare mass terms in the superpotential, i.e. $(\partial^2W)/(\partial\Phi_i\partial\Phi_j)(\phi=0)=0$. Then both $\hat{M}_r = \lambda_{rs}\phi_{0s}$ and $(\partial\hat{M}_r)/(\partial\phi_{0s}) = \lambda_{rs}$ arise from the same trilinear term in $W(\Phi)$. Under the above assumptions, we have

$$\tilde{m}^2 = \frac{\sum_r(Q_r/Q)|F_{0r}|^2}{\sum_r(Q_r/Q)^2|\phi_{0r}|^2} \qquad M_g = \frac{g^2}{(4\pi)^2}\sum_r S(r)\frac{\sum_s\lambda_{rs}F_{0s}}{\sum_s\lambda_{rs}\phi_{0s}}, \tag{4.32}$$

where Q is the charge of the scalar acquiring the mass $\tilde{m}$. While the loop factor $g^2/(4\pi)^2$ suppresses M_g compared to $\tilde{m}$ by a $\mathcal{O}(100)$ factor, the expressions in eqs. (4.32) may give rise to several enhancements of $\tilde{m}/M_g$ reducing or even eliminating the loop hierarchy:

- In the context of grand unified theories the heavy vectors contributing to the soft scalar masses are either one (as in the case of the minimal possibility SO(10), see Section 4.3) or a few, unless the unified group is very large. On the other hand, the gaugino masses may always get contribution from several chiral messengers.

- Sfermion and gaugino masses depend on different group factors. Sfermions can get a mild suppression if $Q_r/Q > 1$. This is indeed what turns out to happen in simple models, as we will see in Section 4.3.
- The heavy vector masses whose exchange generates $\tilde{m}$ collect all the vevs breaking the corresponding charge Q. The scalar mass $\tilde{m}$ is therefore suppressed by all such vevs. On the other hand, gaugino masses are only suppressed by the vevs related to supersymmetry breaking by superpotential interactions λ_{rs}. Unless some of then have $Q = 0$, the vevs suppressing gaugino masses will be a subset of the vevs suppressing scalar masses, thus leading to an enhancement of gaugino masses. In the presence of an hierarchy between the vevs related to supersymmetry breaking and some of the other, Q-breaking vevs, this enhancement can be quite large.
- Different couplings λ_{rs} can appear in the numerator and denominator of the expression $(\sum_s \lambda_{rs} F_{0s})/(\sum_s \lambda_{rs}\phi_{0s})$. This is likely to be the case as a consequence of the relation $\sum_s Q_s(F^*_{0s}\phi_{0s}) = F_0^\dagger T^h \phi_0 = 0$, which can be satisfied without the need of cancellations only in the case in which the fields charged under Q do not have vevs in both the F and scalar components. If the couplings appearing in the numerator and the denominator are hierarchical, gaugino masses can be sizably enhanced (or, in this case, further suppressed).

The study of simple models shows that indeed the enhancement factors above can naturally arise (see Section 4.3.1 below and [5]). In particular, the first two factors reduce the hierarchy $\tilde{m}/M_g$ by factors 3 and $\sqrt{5}$ respectively. The third factor gives at least a factor $\sqrt{2}$ enhancement. The $\mathcal{O}(100)$ hierarchy arising from the loop factor is thus reduced by a factor 10, which is enough to bring the sfermions within the reach of the LHC. Such a milder hierarchy may even be necessary in the light of the bounds on the Higgs mass, which require the stops not to be too light. Still, the residual $\mathcal{O}(10)$ hierarchy can then be easily fully eliminated by the remaining two factors in the above list.

4.2.3 Other one-loop contributions to soft masses

Besides gaugino masses, which can be seen to arise from one loop corrections to the gauge kinetic function, a number of soft terms can be generated or get a contribution from the one-loop corrections to the Kähler. The latter can be computed by using the general results in [70], which give

$$\delta_{\text{1-loop}}K = -\frac{1}{32\pi^2}\left[\mathrm{Tr}\left[M_\Phi^\dagger M_\Phi\left(\log\frac{M_\Phi^\dagger M_\Phi}{\Lambda^2} - 1\right)\right] - 2\,\mathrm{Tr}\left[M_V^2\left(\log\frac{M_V^2}{\Lambda^2} - 1\right)\right]\right], \tag{4.33}$$

where

$$(M_\Phi)_{ij} = \frac{\partial^2 W}{\partial\Phi_i \partial\Phi_j}(\Phi), \qquad (M_V^2)_{ab} = \frac{\partial^2 K}{\partial V_a \partial V_b}(\Phi, V=0) \tag{4.34}$$

are functions of the chiral superfields, K is our canonical Kähler $K = \Phi^\dagger e^{2gV}\Phi$ and the indexes run on the heavy vector and chiral superfields. As in the case of gaugino masses, the soft terms might get a contribution from both heavy vector and chiral superfields running in the loop.

As the contribution to one loop soft terms are highly model dependent, we just remind and collect their general expression in terms of $\delta_{\text{1-loop}}K$. Let us expand $\delta_{\text{1-loop}}K$ in terms of powers of Q and Z around ϕ_0. The relevant terms are

$$\delta_{\text{1-loop}}K = \left(\alpha_{ij}^{(1)} Z Q_i^\dagger Q_j + \frac{\beta_{ij}^{(1)}}{2} Z^\dagger Q_i Q_j + \text{h.c.}\right) + \alpha_{ij}^{(2)} Z^\dagger Z Q_i{}^\dagger Q_j + \left(\frac{\beta_{ij}^{(2)}}{2} Z^\dagger Z Q_i Q_j + \text{h.c.}\right) + \ldots, \tag{4.35}$$

where $\alpha^{(1)}$, $\alpha^{(2)}$, are hermitian, $\beta^{(1)}$, $\beta^{(2)}$ symmetric and all are dimensionful. We have omitted $Z^\dagger Q_i$ terms, which are well-known to destabilize the hierarchy [71]. Their absence can be ensured for example by requiring that there are no light chiral fields with the same quantum numbers as Z.

The first term $\alpha^{(1)}$ gives rise to the following "A-terms"

$$\mathcal{L}^{A}_{\text{1-loop}} = -A_{ij} q_i \frac{\partial \hat{W}}{\partial Q_j}(q), \quad \text{with} \quad A_{ij} = |F_0|\alpha^{(1)} \tag{4.36}$$

(and to a two loop contribution to scalar soft masses), where q is the scalar component of Q. The second term $\beta^{(1)}$ generates a contribution to the "μ-term" in the superpotential

$$W^{\mu}_{\text{1-loop}} = \frac{\mu_{ij}}{2} Q_i Q_j, \quad \text{with} \quad \mu_{ij} = |F_0|\beta^{(1)}, \tag{4.37}$$

and the fourth term $\beta^{(2)}$ a contribution to the "B_μ-term"

$$\mathcal{L}^{B_\mu}_{\text{1-loop}} = -\frac{(B_\mu)_{ij}}{2} q_i q_j, \quad \text{with} \quad (B_\mu)_{ij} = -|F_0|^2\beta^{(2)}. \tag{4.38}$$

A more comprehensive discussion of the μ-term and the μ problem can be found in Section 4.4.

Finally, $\alpha^{(2)}$ gives 1-loop contributions to soft scalar masses

$$\delta\tilde{m}^2_{ij} = -|F_0|^2 \alpha^{(2)}_{ij} \tag{4.39}$$

that add to the tree level contributions in eq. (4.13).

Additional one-loop contributions to soft scalar masses can come from an induced 1-loop Fayet-Iliopoulos term [72] associated for example to the heavy H-singlet generators, in particular to those involved in the mediation of supersymmetry breaking at the tree level. Such terms vanish if the heavy chiral mass matrix and the matrix of their couplings to the spurion Z are diagonal in the same basis (in which case the condition in eq. (4.20) is also automatically satisfied) or if the latter matrix of couplings is hermitian in one basis in which the mass matrix is diagonal [73].

This completes the list of the soft terms arising at one loop. Two loop corrections to the scalar soft masses can also arise, of course, as in standard loop gauge mediation, and be sizable in the presence of an enhancement of one-loop gaugino masses [5].

4.3 Guidelines for model building

We now consider the possibility to obtain a phenomenologically viable model from the general formalism discussed so far. We will see that clear model building guidelines emerge from this analysis, leading, in economical schemes, to peculiar predictions for the pattern of MSSM sfermion masses. In particular, we will identify the assumptions underlying such predictions.

In a phenomenologically viable model, the unbroken gauge group H should contain the SM group, $G_{\text{SM}} \subseteq H$, and the light superfield content should contain the MSSM spectrum, $(q_i, u^c_i, d^c_i, l_i, e^c_i) \subseteq Q$, in standard notations, where $i = 1, 2, 3$ is the family index. We assume that the full gauge group G is a simple, grand-unified group, motivated by the well known successful predictions of the SM fermion gauge quantum numbers, of the strong coupling in the MSSM, and of the unification scale in the phenomenologically allowed region. The candidates for the unified group G in a four-dimensional theory are SU(N), $N \geq 5$, SO($4n$+2), $n \geq 2$, and the exceptional group E_6 [74]. In the following we will focus on the smallest (or unique) representatives of each class, SU(5), SO(10), and E_6.

We want the MSSM sfermions to get a positive, $\mathcal{O}$(TeV) mass through tree level gauge mediation. The general form of such mass terms is given in eq. (4.13). The latter contains two contributions, corresponding to the two diagrams on the right-hande side in Fig. 4.1. In order for the second contribution to play a role for sfermion masses, the corresponding chiral superfields should live in the same unified multiplet as the supersymmetry breaking source Z. This will not be the case in the models we consider (as a consequence, for example, of a matter parity telling the supersymmetry breaking multiplet from the matter ones). On the other hand, the second contribution might contribute to the Higgs masses, if some of the gauge generators have the same quantum numbers (which is not the case in

SO(10)	16			$\overline{16}$			10		45				54		
SU(5)	1	10	$\bar{5}$	1	$\overline{10}$	5	5	$\bar{5}$	1	10	$\overline{10}$	24	24	15	$\overline{15}$
X	5	1	-3	-5	-1	3	-2	2	0	-4	4	0	0	-4	4

Table 4.1: Quantum numbers of the non-trivial SO(10) representations with dimension $d < 120$ under the SO(10) generator X.

SO(10), the unified group we will consider in greater detail).

The MSSM sfermions then get their tree level soft masses from the first term in eq. (4.13) only. In order for $F_0^\dagger T_a^h F_0$ to be non-vanishing, the heavy generator T_a^h must be a SM singlet, since F_0 is. We therefore need a group G with rank 5 at least. This means that SU(5) cannot give rise to tree level gauge mediation, while SO(10) and E_6 are in principle suitable.

Let us first consider the "minimal" option, SO(10), which has also the well known virtue to be able to accommodate a whole MSSM family in a single irreducible spinorial representation. We will make a few considerations on the E_6 option at the end of this Section. In SO(10) there is exactly one (up to a sign) orthonormalized heavy SM-singlet generator, $T_h = 1/\sqrt{40}X$, where $X = 5(B-L) - 4Y$ is the SU(5) invariant SO(10) generator. The quantum numbers of the SO(10) representations with dimension $d < 120$ under X are given in Table 4.1. The values of the X quantum numbers are crucial because the soft terms turn out to be proportional to those charges. From eq. (4.13) we obtain in fact

$$\tilde{m}_f^2 = \frac{X_f(F_0^\dagger X F_0)}{\phi_0^\dagger X^2 \phi_0} \quad \text{at the scale} \quad M_V = \frac{g^2}{20}\phi_0^\dagger X^2 \phi_0, \tag{4.40}$$

where X_f is the X-charge of the sfermion $\tilde{f}$ and M_V is the mass if the vector superfield associated to the generator X (note that the gauge coupling and the normalization of the generator T_h cancel in eq. (4.40)). In order to predict the pattern of the tree level sfermion masses, we then just need to specify the embedding of the three MSSM families into SO(10), which we will do through their SU(5) embedding into three light $\bar{5}_i^l + 10_i^l$, $i = 1, 2, 3$.

We use two constraints to determine the embedding of the $\bar{5}_i^l + 10_i^l$ into SO(10) representations. The first one is related to quite a nice feature eq. (4.40): the soft terms turn out to be family-universal, thus neatly solving the supersymmetric flavor problem. Provided, of course, that the three families of each of the MSSM matter multiplets are embedded in the same type of SO(10) representation, which we will assume in order to ensure that family-universality indeed holds. On top of that, we want the MSSM sfermion soft masses in eq. (4.40) to be positive in order to avoid spontaneous symmetry breaking of color, electric charge, or lepton number at the scale $\tilde{m}$. Clearly, the standard embedding of a whole family into a 16 of SO(10) would not work, as it would lead to negative masses for the sfermions in either the $\bar{5}$ or the 10 of SU(5). This is in turn related to the tracelessness of the SO(10) generators, and in particular of X. As a consequence, whatever is the SO(10) representation in which we choose to embed a given MSSM matter multiplet with positive soft mass, that representation will necessarily contain extra fields with negative soft masses. This apparent obstacle can be easily overcome by splitting the SO(10) representation containing the MSSM multiplet through SO(10) breaking, in such a way that the extra fields with negative soft masses acquire a large supersymmetric mass term. The negative soft mass will then represent a negligible supersymmetry breaking correction to that large (positive) mass. It turns out that such a splitting is actually expected to arise, as will see, a fact that reinforces the logical consistency of tree level gauge mediation.

We are now ready to discuss the embeddings of the three $\bar{5}^l_i$ and 10^l_i of SU(5) containing the light MSSM families in SO(10). As $\phi_0^\dagger X^2 \phi_0$ is positive, the possible choices depend on the sign of $F_0^\dagger X F_0$. We limit ourselves to the SO(10) representations with $d < 120$, as in Table 4.1. There are then only two possibilities:

- $F_0^\dagger X F_0 > 0$. In this case we need to embed the $\bar{5}^l_i$'s and 10^l_i's into SO(10) representations containing $\bar{5}$ and 10 of SU(5) with positive charges under X. From Table 4.1 we see that the only possibility is to use three $16_i = (1^{16}_i, 10^{16}_i, \bar{5}^{16}_i)$ and three $10_i = (5^{10}_i, \bar{5}^{10}_i)$, $i = 1, 2, 3$, where we have explicitly indicated the SU(5) decomposition, and to embed the 10^l_i's into the 16_i's, $10^l_i \equiv 10^{16}_i$, and the $\bar{5}^l_i$'s into the 10_i's, $\bar{5}^l_i \equiv \bar{5}^{10}_i$. The spare components $\bar{5}^{16}_i$, 5^{10}_i get negative soft masses and need to acquire a large supersymmetric mass term.

- $F_0^\dagger X F_0 < 0$. In this case we need the $\bar{5}^l_i$'s and 10^l_i's to have negative charges under X. The only possibility is then to use three 16_i's as before and three $45_i = (1^{45}_i, 10^{45}_i, \overline{10}^{45}_i, 24^{45}_i)$, $i = 1, 2, 3$, with $\bar{5}^l_i \equiv \bar{5}^{16}_i$ and $10^l_i \equiv 10^{45}_i$. The spare components 10^{16}_i, $\overline{10}^{45}_i$, get negative or vanishing soft masses and need to acquire a large supersymmetric mass term.

In both cases the chiral content of the theory is still given by three 16 of SO(10). We have implicitly neglected the possibility of mixed embeddings in which, for example, the $\bar{5}_i$'s of SU(5) are a superposition of the $\bar{5}_i$'s in the 10_i's and 16_i's of SO(10). While this possibility is in principle not excluded, it would in general introduce a dependence of the sfermion soft masses on mixing parameters that are in general flavor violating, thus possibly spoiling the flavor universality result.

The two possibilities above give rise to two definite predictions for the patter of sfermion soft masses at the scale M_V:

$$(\tilde{m}_l^2)_{ij} = (\tilde{m}_{d^c}^2)_{ij} = m_{\bar{5}}^2\delta_{ij}, \quad (\tilde{m}_q^2)_{ij} = (\tilde{m}_{u^c}^2)_{ij} = (\tilde{m}_{d^c}^2)_{ij} = m_{10}^2\delta_{ij}, \quad \text{with}$$
$$m_{\bar{5}}^2 = 2m_{10}^2 \quad \text{if} \quad F_0^\dagger X F_0 > 0 \tag{4.41}$$
$$m_{\bar{5}}^2 = \frac{3}{4}m_{10}^2 \quad \text{if} \quad F_0^\dagger X F_0 < 0.$$

To summarize, the latter predictions are based on the following hypotheses: "minimal" unified gauge group SO(10), embedding of the MSSM families in the SO(10) representations with dimension $d < 120$ not containing the Goldstino, and absence of mixed embeddings to automatically preserve flavor-universality. The predictions on the ratios $m_{\bar{5}}/m_{10}$ in eq. (4.3) are peculiar enough to make a possible experimental test at the LHC a strong hint for tree level gauge mediation.

As for the source of supersymmetry breaking, $\langle Z\rangle = |F_0|\theta^2$, we need Z to have a non-vanishing charge under X. If we limit ourselves again to representations with $d < 120$, the only possibility is that Z has a component in the "right-handed neutrino" direction of a 16 or a $\overline{16}$. With the sign conventions we adopted, a component in a 16 gives a positive contribution to $F_0^\dagger X F_0$, while a component in a $\overline{16}$ gives a negative contribution.

We now want to show that the two embeddings of the light MSSM families described above can be obtained in a natural way. We have to show that it is possible to split the SO(10) representations in which the MSSM fields are embedded in such a way that the extra fields (with negative soft masses) get a heavy supersymmetric mass term from SO(10) breaking. It will turn out that the SO(10) breaking vevs of a $16+\overline{16}$, essential to break SO(10) to the SM (unless representations with $d \geq 126$ are used to reduce the rank) just provide the needed splitting. The fact that such vevs make heavy precisely the components of the SO(10) representations that get a negative soft supersymmetry breaking mass reinforces the logical consistency of this framework. In the following, we first discuss the $16_i + 10_i$ embedding in a general, top-bottom perspective, obtaining a generalization of the model in [5], and discuss the conditions for a pure (non mixed) embedding. We then discuss the possibility of a $16_i + 45_i$ embedding.

4.3.1 The embedding into $16_i + 10_i$, $i = 1, 2, 3$

Let us consider the embedding associated to the case $F_0^\dagger X F_0 > 0$. We assume the existence of a matter parity symmetry that tells matter superfields from Higgs superfields. Let 16, $\overline{16}$ be the SO(10) multiplets breaking SO(10) to SU(5) (we can always choose the basis in the space of the 16 ($\overline{16}$) representations in which a single 16 ($\overline{16}$) gets a vev in its scalar component). The most general renormalizable superpotential involving 16, $\overline{16}$, 16_i, 10_i, $i = 1, 2, 3$, and invariant under a matter parity under which the SO(10) Higgs fields 16, $\overline{16}$ are even and the the matter fields are odd is

$$W = h_{ij} 16_i 10_j 16 + \frac{\mu_{ij}}{2} 10_i 10_j + W_{\text{vev}}, \tag{4.42}$$

where W_{vev} takes care of providing a vev to the 16, $\overline{16}$ in the SM-singlet direction and does not depend on the matter fields (but can involve additional even fields[2]). The term $h_{ij}16_i10_j16$ is just what needed to split the SU(5) components of the $16_i = (1^{16}_i, 10^{16}_i, \overline{5}^{16}_i)$ and of the $10_i = (5^{10}_i, \overline{5}^{10}_i)$ and make heavy the unwanted components $\overline{5}^{16}_i$ and 5^{10}_j. Once 16 acquires a vev V in its singlet neutrino component, in fact, a mass term is generated for those components,

$$M_{ij} \overline{5}^{16}_i 5^{10}_j, \qquad M_{ij} = h_{ij} V. \tag{4.43}$$

The singlet neutrinos 1^{16}_i remain light at the renormalizable level but can get a mass at the non-renormalizable level through the operator $(\overline{16}16_i)(\overline{16}16_j)/\Lambda$.

It is remarkable that the components acquiring a large mass are precisely those that get a negative soft mass term. On the other hand, this is only true in the limit in which the μ_{ij} mass term in eq. (4.42) can be neglected. In the presence of a non negligible μ_{ij}, in fact, the full mass term would be

$$(\overline{5}^{16}_i M_{ij} + \overline{5}^{10}_i \mu_{ij}) 5^{10}_j, \tag{4.44}$$

which would give rise to a mixed embedding of the light $\overline{5}^l_i$'s in the 16_i's and 10_i's. In order to abide to our assumptions, which exclude the possibility of mixed embeddings, such a μ_{ij} term should be absent. This can be easily forced by means of an appropriate symmetry. Let us however relax for a moment that assumption in order to quantify the deviation from universality associated to a small, but non-negligible μ_{ij}. The MSSM sfermions in the $\overline{5}$ of SU(5) receive in this case two contributions to their soft mass, a positive one associated to the components in the 10_i's, proportional to $X(\overline{5}^{10}) = 2$, and a negative one associated to the components in the 16_i's, proportional to $X(\overline{5}^{16}) = -3$. The soft mass matrix for the light sfermions in the $\overline{5}$ of SU(5) can be easily calculated in the limit in which the μ_{ij} mass term can be treated as a perturbation. In this limit, the light MSSM fields in the $\overline{5}$ of SU(5) are in fact

$$\overline{5}^l_i \approx \overline{5}^{10}_i - (\mu M^{-1})^*_{ij} \overline{5}^{16}_j \tag{4.45}$$

and their soft scalar mass matrix at the scale M_V is

$$(\tilde{m}^2_{\overline{5}})_{ij} \approx \frac{2}{5} \tilde{m}^2 \left(\delta_{ij} - \frac{5}{2} \left(\mu^* M^{*-1} M^{T-1} \mu^T \right)_{ij} \right), \tag{4.46}$$

[2]The simplest possibility is $W_{\text{vev}} = X(\overline{16}16 - V^2)$, where X is an SO(10) singlet.

where $\tilde{m}^2$ is defined below. The mixed embedding induced by the mass term μ_{ij} leads to flavor-violating soft-terms. Setting $\mu_{ij} = 0$ allows to preserve the flavor blindness of the soft terms and to satisfy the FCNC constraints without the need of assumptions on the structure of the flavor matrices h_{ij} and μ_{ij}. We therefore assume that μ_{ij} is vanishing or negligible. We then have $\bar{5}^l_i = \bar{5}^{10}_i$, $10^l_i = 10^{16}_i$, with the extra components $\bar{5}^{16}_i$ and 5^{10}_i obtaining a large supersymmetric mass term $M_{ij}\bar{5}^{16}_i 5^{10}_i$, as desired. The soft masses for the light sfermions are

$$(\tilde{m}^2_l)_{ij} = (\tilde{m}^2_{d^c})_{ij} = \frac{2}{5}\tilde{m}^2\delta_{ij}, \quad (\tilde{m}^2_q)_{ij} = (\tilde{m}^2_{u^c})_{ij} = (\tilde{m}^2_{d^c})_{ij} = \frac{1}{5}\tilde{m}^2\delta_{ij}, \tag{4.47a}$$

$$\text{with} \quad \tilde{m}^2 = 5\frac{(F_0^\dagger X F_0)}{\phi_0^\dagger X^2 \phi_0} > 0, \tag{4.47b}$$

as anticipated in eq. (4.3). The reason for the factor $5 = X(1^{16})$ will become clear in a moment.

We now need to identify the embedding of the MSSM Higgs superfields and obtain the MSSM superpotential for them, in particular the MSSM Yukawa interactions. It is useful to discuss the Yukawa interaction in SU(5) language. The up quark Yukawa interactions arise from the SU(5) operator

$$\frac{\lambda^{(1)}_{ij}}{2} 10^l_i 10^l_j 5_H, \tag{4.48}$$

where 5_H contains the MSSM up Higgs. As $10^l_i = 10^{16}_i$, the operator in eq. (4.48) can arise at the renormalizable level from a SO(10) invariant operator only if 5_H has a component into a 10_H of SO(10), $10_H = (5^{10}_H, \bar{5}^{10}_H)$, with

$$5^{10}_H = \cos\theta_u 5_H + \dots, \quad 0 \leq \theta_u \leq \pi/2, \tag{4.49}$$

where $\cos^2\theta_u$ measures the size of the 5_H component from 10 representations of SO(10) (a basis in the space of the 10 representations can always be chosen such that 5_H is contained in a single one, the 10_H). The operator in eq. (4.48) will then originate as

$$\frac{y^H_{ij}}{2} 16_i 16_j 10_H = \frac{\lambda^{(1)}_{ij}}{2} 10^l_i 10^l_j 5_H + \dots, \quad \text{with} \quad \lambda^{(1)}_{ij} = \cos\theta_u y^H_{ij}. \tag{4.50}$$

The down quark and charged lepton Yukawa interactions arise at the renormalizable level[3] from the SU(5) operator

$$\lambda^{(2)}_{ij} 10^l_i \bar{5}^l_j \bar{5}_H, \tag{4.51}$$

[3]SU(5)-invariant renormalizable Yukawa interactions lead to wrong mass relations for the two lighter families of down quarks and charged leptons. This may indicate that the light family Yukawas arise at the non-renormalizable level, as also suggested by their smallness. We ignore this issue in the following and only consider the renormalizable part of the superpotential.

where $\overline{5}_H$ contains the MSSM down Higgs. As $10^l_i = 10^{16}_i$ and $\overline{5}^l_i = \overline{5}^{10}_i$, the operator in eq. (4.51) can arise at the renormalizable level from a SO(10) invariant operator only if $\overline{5}_H$ has a component into a 16_H of SO(10), $16_H = (1^{16}_H, 10^{16}_H, \overline{5}^{16}_H)$, with

$$\overline{5}^{16}_H = \sin\theta_d 5_H + \ldots, \quad 0 \leq \theta_d \leq \pi/2, \tag{4.52}$$

where $\sin^2\theta_d$ measures the size of the $\overline{5}_H$ component from 16 representations of SO(10). The operator in eq. (4.51) will then originate as

$$h^H_{ij} 16_i 10_j 16_H = \lambda^{(2)}_{ij} 10^l_i \overline{5}^l_j \overline{5}_H + \ldots, \quad \text{with} \quad \lambda^{(2)}_{ij} = \sin\theta_d h^H_{ij}. \tag{4.53}$$

It is tempting (and economical) to identify the 16_H with 16, the field whose vev breaks SO(10) to SU(5), in which case $h^H = h$ and the mass of the heavy extra components $\overline{5}^{16}_i$ and 5^{10}_i in eq. (4.43) turns out to be proportional to the corresponding light fermion masses (up to non-renormalizable corrections needed to fix the light fermion mass ratios)[4].

Having introduced the MSSM Higgs fields, let us now discuss their soft mass terms. To summarize the previous discussion, with our $d < 120$ representation content, the up (down) Higgs superfield h_u (h_d) can be embedded in either 10's or 16's ($\overline{16}$'s) of SO(10), in both cases through the embedding into a 5_H ($\overline{5}_H$) of SU(5). We have denoted by $\cos^2\theta_u$ ($\cos^2\theta_d$) the overall size of the h_u (h_d) component in the 10's. The overall size of the component in the 16's ($\overline{16}$'s) is then measured by $\sin^2\theta_u$ ($\sin^2\theta_d$). Correspondingly, the Higgs soft masses get two contributions from the first term in eq. (4.40) proportional to two different X charges:

$$m^2_{h_u} = \frac{-2c^2_u + 3s^2_u}{5}\tilde{m}^2, \quad m^2_{h_d} = \frac{2c^2_d - 3s^2_d}{5}\tilde{m}^2, \quad \text{so that} \tag{4.54a}$$

$$-\frac{2}{5}\tilde{m}^2 \leq m^2_{h_u} \leq \frac{3}{5}\tilde{m}^2, \quad -\frac{3}{5}\tilde{m}^2 \leq m^2_{h_d} \leq \frac{2}{5}\tilde{m}^2. \tag{4.54b}$$

Let us now consider gaugino masses. A general discussion of all possible contributions to gaugino masses in the embedding we are considering and in the presence of an arbitrary number of SO(10) representation with $d < 120$ would be too involved. We then consider a few examples meant to generalize the case considered in [5] and to illustrate the general properties discussed in Section 4.2.

Let us begin by illustrating in more detail the structure of supersymmetry breaking. With the representation content of Table 4.1, supersymmetry breaking can be associated to the F-term vevs of superfields in 16, $\overline{16}$, 45, 54 representations (the ones containing SM singlets). However, only the 16, $\overline{16}$, whose singlets have non-vanishing X-charges, can contribute to tree level soft masses. Let us call 16^H_α,

[4] This property can give rise to a predictive model of leptogenesis in the context of type-II see-saw models [55, 56].

$\overline{16}^H_\alpha$ the matter parity even superfields in the 16 and $\overline{16}$ representations of SO(10). In a generic basis, we can parametrize the vevs of their singlet components as

$$\langle 1^{16_H}_\alpha \rangle = V_\alpha + F_\alpha \theta^2 \qquad \langle 1^{\overline{16}_H}_\alpha \rangle = \overline{V}_\alpha + \overline{F}_\alpha \theta^2. \tag{4.55}$$

The D-term condition for the X generator requires

$$\sum_\alpha |V_\alpha|^2 \approx \sum_\alpha |\overline{V}_\alpha|^2, \tag{4.56}$$

while gauge invariance gives

$$\sum_\alpha V^*_\alpha F_\alpha = \sum_\alpha \overline{V}^*_\alpha \overline{F}_\alpha. \tag{4.57}$$

Sfermion masses are proportional to

$$\tilde{m}^2 = \frac{\sum_\alpha (|F_\alpha|^2 - |\overline{F}_\alpha|^2)}{\sum_\alpha (|V_\alpha|^2 + |\overline{V}_\alpha|^2)} \tag{4.58}$$

(due to the factor 5 in the definition of $\tilde{m}^2$), where $\sum_\alpha |F_\alpha|^2 > \sum_\alpha |\overline{F}_\alpha|^2$ by definition in the case we are considering. Note that $\tilde{m}^2$ is suppressed by *all* vevs contributing to X breaking.

Let us now comment on the vector contribution to gaugino masses. Let us assume to begin with that the $\overline{16}$'s do not break supersymmetry. Without loss of generality we can then assume that supersymmetry breaking is only associated to $16' \equiv 16^H_1$. The gauge invariance condition then gives $V_1 = 0$, i.e. a vev for both the F-term and scalar components is not allowed. Since the F-term and scalar components belong to different irreducible representations, no vector contribution to gaugino masses is generated by the 16's. A vector contribution can still be generated by the F-term vev of a 45, for example, for which the gauge invariance condition does now prevent a vev in both the scalar and F-term component. Or, it can be generated by the F-terms of the 16's if some of the $\overline{16}$ also breaks supersymmetry and cancels the contribution of the 16 to eq. (4.57).

Let us next consider the chiral contribution to gaugino masses. The massive components $\overline{5}^{16}_i$ and 5^{10}_j of the matter superfields will act as chiral messengers if they are coupled to supersymmetry breaking. Let us then consider as before the case in which the $\overline{16}$'s do not break supersymmetry, supersymmetry breaking is provided by the F-term vev F of the singlet component of the $16'$ and is felt by the chiral messengers through the $h'_{ij} 16_i 10_j 16'$ interaction. Let $16 \equiv 16^H_2$ be the field whose vev gives mass to the $\overline{5}^{16}_i$, 5^{10}_j through the $h_{ij} 16_i 10_j 16$ interaction, as in eq. (4.42). And let us assume that additional 16^H_α's and $\overline{16}^H_\alpha$'s get vevs in their scalar components. The chiral messengers $\overline{5}_i^{16}$, 5_j^{10} have therefore a supersymmetric mass $M_{ij} = h_{ij}V$ and their scalar components get a supersymmetry breaking term mass term $F_{ij} = h'_{ij}F$. The induced one loop chiral contribution to gaugino masses is then

$$M_g = \frac{g^2}{(4\pi)^2}\,\mathrm{Tr}(h'h^{-1})\frac{F}{V}. \tag{4.59}$$

The tree level soft mass of the stop (belonging to the 10 of SU(5)) is

$$\tilde{m}_t^2 = \frac{1}{5}\frac{|F|^2}{|V|^2 + \sum_\alpha |V_\alpha|^2 + |\overline{V}|^2 + \sum_\alpha |\overline{V}_\alpha|^2}. \tag{4.60}$$

We can then compare stop and gaugino masses (before radiative corrections). Their ratio is particularly interesting, as the gaugino mass M_g is at present bounded to be heavier than about 100 GeV, while $\tilde{m}_t$ enters the radiative corrections to the Higgs mass. Therefore, the ratio $\tilde{m}_t/M_g$ should not be too large in order not to increase the fine-tuning and not to push the stops and the other sfermions out of the LHC reach. From the previous equations we find

$$\frac{M_g}{\tilde{m}_t} = \frac{3\sqrt{5k}}{(4\pi)^2}\lambda, \quad \lambda = \frac{g^2\,\mathrm{Tr}(h'h^{-1})}{3}, \quad k = \frac{|V|^2 + \sum_\alpha |V_\alpha|^2 + |\overline{V}|^2 + \sum_\alpha |\overline{V}_\alpha|^2}{|V|^2} \geq 2. \tag{4.61}$$

Eq. (4.61) illustrates all the enhancement factors discussed in Section 4.2 that can compensate the loop suppression of gaugino masses. The factor 3 corresponds to the number of chiral messenger families ($\mathrm{Tr}(h'h^{-1}) = 3$ for $h = h'$) contributing to gaugino masses, to be compared to the single vector messenger generating sfermion masses at the tree level. The factor $\sqrt{5}$ comes from the ratio of charges $X(1^{16})/X(10^{16}) = 5$ suppressing the stop mass in eq. (4.32). The factor $k \geq 2$ is the ratio of the vev suppressing gaugino masses (the one related to supersymmetry breaking though superpotential interactions, $|V|^2$), and the combination of vevs suppressing sfermion masses (all of them). Note that in the presence of hierarchies of vevs, the factor k can be large. Finally λ represents a combination of couplings that can further enhance (or suppress, in this case) gaugino masses. All in all, we see that the loop factor separating $\tilde{m}_t$ and M_g is partially compensated by a combination of numerical factors: $(4\pi)^2 \sim 100$ (leading to $\tilde{m}_t \gtrsim 10\,\mathrm{TeV}$ for $\lambda = 1$) becomes at least $(4\pi)^2/(3\sqrt{10}) \sim 10$ (leading to $\tilde{m}_t \gtrsim 1\,\mathrm{TeV}$ for $\lambda = 1$). A largish value of the factors k or λ can then further reduce the hierarchy and even make $M_g \sim \tilde{m}_t$, if needed.

4.3.2 The embedding into $16_i + 45_i$, $i = 1, 2, 3$

Let us now consider the second type of embedding identified above, corresponding to $F_0^\dagger X F_0 < 0$. The most general renormalizable superpotential involving 16, $\overline{16}$

and 16_i, 45_i, $i = 1, 2, 3$ and invariant under matter parity is

$$W = h_{ij} 16_i 45_j \overline{16} + \frac{\mu_{ij}}{2} 45_i 45_j + W_{\text{vev}}. \tag{4.62}$$

The term $h_{ij} 16_i 45_j \overline{16}$ is just what needed to split the SU(5) components of the $16_i = (1_i^{16}, 10_i^{16}, \overline{5}_i^{16})$ and of the $45_i = (1_i^{45}, 10_i^{45}, \overline{10}_i^{45}, 24_i^{45})$ and make heavy the unwanted components 10_i^{16} and $\overline{10}_j^{45}$. Once 16 acquires a vev V, in fact, a mass term is generated for those components,

$$M_{ij} 10_i^{16} \overline{10}_j^{45}, \qquad M_{ij} = h_{ij} V. \tag{4.63}$$

It is remarkable that also in this case the components acquiring a large mass are precisely those that get a negative soft mass term. On the other hand, this is only true in the limit in which the μ_{ij} mass term in eq. (4.62) can be neglected. In order to abide to our pure embedding assumption, we will neglect such a term. Let us note, however, that such a term should arise at some level in order to make the 24_i^{45}'s components heavy. Note that the 24_i's do not affect gauge coupling unification at one loop and can therefore be considerably lighter than the GUT scale, consistently with the required smallness of μ_{ij}. The soft masses for the light sfermions are now

$$(\tilde{m}_l^2)_{ij} = (\tilde{m}_{d^c}^2)_{ij} = \frac{3}{5} \tilde{m}^2 \delta_{ij}, \quad (\tilde{m}_q^2)_{ij} = (\tilde{m}_{u^c}^2)_{ij} = (\tilde{m}_{d^c}^2)_{ij} = \frac{4}{5} \tilde{m}^2 \delta_{ij}, \tag{4.64}$$

$$\text{with} \quad \tilde{m}^2 = -5 \frac{(F_0^\dagger X F_0)}{\phi_0^\dagger X^2 \phi_0} > 0. \tag{4.65}$$

Unfortunately, the embedding we are discussing cannot be implemented with renormalizable interactions and $d < 120$ representations only. The problem is obtaining the Yukawa interactions. Let us consider the up quark Yukawas, arising as we saw from the SU(5) operator in eq. (4.48). Given its size, we expect at least the top Yukawa coupling to arise at the renormalizable level. As in the present case $10_i^l = 10_i^{45}$, the operator in eq. (4.48) can arise at the renormalizable level from a SO(10) invariant operator only if 5_H has a component in a SO(10) representation coupling to $45_i 45_j$. And the lowest dimensional possibility containing the 5 of SU(5) is the 210. For this reason, we do not pursue this possibility further here, although models with large representations are not a priori excluded.

4.3.3 E_6

We close this Section with a few considerations about the possibility to identify the unified group with E_6. Such a possibility looks particularly appealing in the light of what above. We have seen in fact that the most straightforward possibility to realize tree level gauge mediation in SO(10) requires the matter superfield content to include three $16_i + 10_i$, $i = 1, 2, 3$. This is precisely what E_6 predicts. The fundamental of E_6, in fact, a representation of dimension 27, decomposes as

$$27 = 16 + 10 + 1 \quad \text{under SO(10)}. \tag{4.66}$$

The matter content needed by the $16_i + 10_i$ embedding can therefore be provided in the context of E_6 by three matter 27_i, $i = 1, 2, 3$, and the 16_H and 10_H needed to accommodate the Higgs fields can also be provided by a single Higgs 27_H. All Yukawas can then in principle follow from the single E_6 interaction

$$\lambda_{ij} 27_i 27_j 27_H. \tag{4.67}$$

We postpone the analysis of this promising possibility to further study.

4.4 Some solutions to the μ-problem

In this Section, we discuss a few approaches to the μ-problem in the context of tree level gauge mediation. Let us remind what the μ-problem is. Any supersymmetric extension of the SM must contain two Higgs doublet chiral superfields $\hat{h}_u$, $\hat{h}_d$, with hypercharges $\pm 1/2$, within the light spectrum Q. Moreover, the lagrangian must contain a mass term for their Higgsino (fermion) components, $\mu \tilde{h}_u \tilde{h}_d$. It should also contain a corresponding term for the scalar components, $B_\mu h_u h_d$, where B_μ is a dimension two parameter. The Higgsino mass μ is constrained to be in the window $100\,\text{GeV} \lesssim \mu \lesssim \text{TeV}$ by the present bounds on chargino masses and by naturalness considerations. This coincides with the window for the supersymmetry breaking scale in the observable sector, $100\,\text{GeV} \lesssim \tilde{m} \lesssim \text{TeV}$. It is then tempting try to establish a connection between these two a priori independent scales, in such a way that $\mu \to 0$ when $\tilde{m} \to 0$, thus making the coincidence of the two scales not accidental. This is the μ-problem. In the absence of such a connection, there would be no reason why μ should not be of the order of a much larger, supersymmetry conserving scale such as the GUT or the Planck scale. Or, if a symmetry or some other independent principle suppressed μ, there would be no reason why μ should not be much smaller.

Fermion mass terms such as $\mu \tilde{h}_u \tilde{h}_d$ belong to the list of possible soft supersymmetry breaking mass terms [75]. The reason why they are usually omitted from the MSSM effective soft supersymmetry breaking lagrangian is that they can be always reabsorbed in the superpotential (through appropriate additions to the scalar soft lagrangian). Moreover, most models of supersymmetry breaking, including the ones we are considering, do not generate such supersymmetry breaking fermion mass terms. We can then assume that the Higgsino mass term arises from a corresponding term in the superpotential. The problem is then to relate the coefficient of that (supersymmetric) superpotential term, $\mu\, \hat{h}_u \hat{h}_d$, to the supersymmetry breaking scale in the observable sector, which in our case is given by $\tilde{m} \sim |F_0|/M_V$. We discuss in the following three possible connections. One is peculiar of tree level gauge mediation, the other two have been considered in other contexts, but have specific implementations in tree level gauge mediation. We classify them according to the dimension D of the SO(10) operator from which the μ term arises. Note that we are not addressing the origin of the smallness of $\tilde{m}$ and μ compared to the Plank scale, just their connection. The three options we consider are:

$D = 3$: μ comes from the operator $\mu\, \hat{h}_u \hat{h}_d \subset W$. It is the supersymmetry breaking scale to be derived from μ, and not viceversa: $F_0 \sim \mu M$, where $M = \mathcal{O}(M_V)$, and $\tilde{m} \sim F_0/M \sim \mu$.

$D = 4$: μ comes from the operator $\lambda S \hat{h}_u \hat{h}_d \subset W$. The light SM singlet S gets a vev from a potential whose only scale is $\tilde{m}$, so that $\mu \sim \lambda\langle S\rangle \sim \tilde{m}$.

$D = 5$: μ comes from the operator $a(Z^\dagger/M)\hat{h}_u \hat{h}_d \subset K$, so that $\mu = aF_0/M$.

Let us discuss each of those possibilities in turn.

4.4.1 $D = 3$

Such a possibility was anticipated in [5], where however no concrete implementation was given. Let us consider the $16_i + 10_i$ embedding. As discussed in Section 4.3.1, $\hat{h}_u$ is a superposition of the "up Higgs-type" components in the $\overline{16}$'s and 10's (with $R_P = 1$) in the model. Analogously, $\hat{h}_d$ will be a superposition of the "down Higgs-type" components in the ($R_P = 1$) 16's and 10's. The only possible $D = 3$ origin of the μ-term in the context of the full SO(10) theory are then $\mathcal{O}$(TeV) mass terms for the above $\overline{16}$'s, 16's, and 10's. As said, we do not address the origin of such a small parameter in the superpotential, as we do not address here the smallness of the supersymmetry breaking scale. The latter can for example be explained by a dynamical mechanism. We want however to relate such mass parameters, in particular the coefficient of a $\overline{16}16$ mass term, to the supersymmetry breaking scale. This is actually pretty easy, as the tree level gauge mediation embedding we are considering provides all the necessary ingredients and the result arises from their simple combination. We have seen in fact that the model needs a 16, $\overline{16}$ pair to get a vev in the SM singlet direction of the scalar component, in order to break SO(10) to the SM. Moreover, we have seen that an independent $16'$, $\overline{16}'$ pair is required to break supersymmetry trough the F-term vev of the SM singlet component in the $16'$. The simplest way to achieve such a pattern is through a superpotential like

$$W_1 = \lambda_1 Z(\overline{16}16 - M^2) + m16'\overline{16} + \lambda_2 X16\overline{16}', \tag{4.68}$$

where X, Z are SO(10) singlets and $M \sim M_{\text{GUT}}$. This is a generalization of an example in [54]. Finally, we have just reminded that the light Higgses may have a component in 16, $16'$, $\overline{16}$, $\overline{16}'$. Let α' be the coefficient of the h_d component in the $16'$ and α the coefficient of the h_u component in the $\overline{16}$. Then a μ parameter is generated in the form

$$\mu = \alpha' \alpha\, m \tag{4.69}$$

from the $m16'\overline{16}$ term in eq. (4.68). The parameter m is therefore required to be in the window $100\,\text{GeV}/(\alpha'\alpha) \lesssim m \lesssim \text{TeV}/(\alpha'\alpha)$. In the limit $\mu = 0$, supersymmetry is unbroken and 16, $\overline{16}$ acquire a vev that can be rotated in the SM singlet component $\langle 1^{16} \rangle = \langle 1^{\overline{16}} \rangle = M$. A non-vanishing μ, on the other hand, triggers supersymmetry breaking and induces an F-term vev for the singlet component of the $16'$, $\langle 1^{16'} \rangle = F\theta^2$, with $F = mM$. We therefore have

$$\tilde{m} \sim \frac{F}{M} = m = \frac{\mu}{\alpha'\alpha}, \tag{4.70}$$

providing the desired connection between μ and the supersymmetry breaking scale. Tree level gauge mediation plays a crucial role not only in providing the ingredients (and no need to stir) but also because it is the very SO(10) structure providing the heavy vector messengers to relate in a single irreducible representation (the 16') supersymmetry breaking (the F-term vev of its SM singlet component) and the down Higgs entering the μ-term (the lepton doublet-type component of the 16'). In the Appendix 6.3 we provide an existence proof of a (perturbative) superpotential that i) implements the mechanism above, thus breaking supersymmetry and SO(10) to SU(5), ii) further breaks SU(5) to the SM, iii) makes all the fields that are not part of the MSSM spectrum heavy, in particular achieves doublet-triplet splitting.

4.4.2 $D = 4$

This is an implementation of the NMSSM solution of the μ-problem (see e.g. [76] and references therein). As we will see, the implementation of such a solution in the context of tree level gauge mediation avoids some of the problems met in ordinary gauge mediation.

In order to implement the NMSSM solution of the μ-problem, an explicit term $\mu\, \hat{h}_u \hat{h}_d$ should be forbidden, for example by a symmetry; the light fields Q should include a SM singlet S, coupling to the Higgses through the superpotential interaction $\lambda \hat{S}\hat{h}_u\hat{h}_d$; and S should develop a non-zero vev. A μ parameter will then be generated, $\mu = \lambda\langle S\rangle$. In the absence of terms linear or quadratic in $\hat{S}$ in the superpotential, the scale of a vev for S can only be provided by the supersymmetry breaking terms in the soft lagrangian, $\langle S\rangle \sim \tilde{m}$, in which case $\mu = \lambda\langle S\rangle \sim \lambda\tilde{m}$, as desired.

In order to generate a non-zero vev for S, one would like to have a negative soft mass for S at the weak scale, along with a stabilization mechanism for large values of the fields. In ordinary gauge mediation this is not easy to achieve. While the stabilization can be simply provided by a S^3 term in W, as in the NMSSM (or by a quartic term in Z' extensions of the MSSM [77]), the soft mass term of S vanishes at the messenger scale because S is typically a complete gauge singlet. A non-vanishing negative mass term is generated by the RGE running but it is typically too small. Another problem is that the Higgs spectrum can turn out to be non-viable [78]. A sizable soft mass can still be generated by coupling S to additional heavy fields. Such possibilities can be implemented in our setup by promoting S to an SO(10) singlet and coupling it to the Higgses through a $S\,\overline{16}\,16$ or a $S\,10\,10$ coupling to the SO(10) representations containing (a component of) the Higgs fields.

Tree level gauge mediation offers a different avenue. A sizable, negative soft mass term for S can in fact be generated by embedding S in a $\overline{16}$ of SO(10) (this is the only choice within the fields in Table 4.1). On the other hand, the stabilization of the potential for S is not straightforward. A sizable S^3 term is not expected to arise, as it should involve a SO(10) operator with three $\overline{16}$. However, the S^3 term can be replaced by a term involving a second light singlet N,

$$W = \lambda\hat{S}\hat{h}_u\hat{h}_d + \kappa\hat{S}^2\hat{N}. \tag{4.71}$$

The latter can come from a $\overline{16}^2 126$ coupling, if N is in the 126 singlet, or from a $\overline{16}^2 16_1 16_2/\Lambda$ coupling, where N is the 16_1 singlet and 16_2 gets a vev.

The scalar potential for $V(h_u, h_d, S, N)$ can be written as

$$V = V_{\text{MSSM}} + |\kappa S^2|^2 + m_S^2|S|^2 + |\lambda h_u h_d + 2\kappa SN|^2 + M_N^2|N|^2, \tag{4.72}$$

where V_{MSSM} is the MSSM scalar potential with $\mu \to \lambda S$, $m_S^2 = -\tilde{m}^2$, and $m_N^2 = 2\tilde{m}^2$ or $\tilde{m}^2$ depending on whether N comes from a 126 or a 16. We have neglected the A-terms, which play a role in explicitly breaking R-symmetries that could lead to massless states. The potential above has a minimum with a sizable $\langle S\rangle$, and a μ parameter whose size is controlled by λ.

4.4.3 $D = 5$

Finally, let us consider the possibility to generate the μ parameter through a $D = 5$ correction to the Kähler in the form $a(Z^\dagger/M)\hat{h}_u\hat{h}_d$, as in the Giudice Masiero mechanism [79]. The F-term vev $|F_0|$ of Z would give in this case $\mu = a|F_0|/M$.

We show first that the operator above cannot arise at the tree level from integrating out heavy vector or chiral superfields. The corrections to the Kähler obtained by integrating out heavy vector superfields are given in eqs. (6.19). All terms are at least of second order in $1/M_V$ and no trilinear term is present. Moreover, no sizable trilinear term can be obtained through the vev of Φ', as by definition the scalar components of Φ' do not get a vev (and an F-term vev would give an additional F_0/M_V suppression). A similar conclusion can be obtained for the corrections one obtains by integrating out chiral superfields Φ_i^h with mass $M \gg \sqrt{|F_0|}$. We have seen in Section 4.1 that the equations of motion allow to express Φ_i^h in terms of the light fields as in eq. (4.17). Since W_3 contains terms at least trilinear in the fields, the expression for Φ_i^h is at least quadratic in the light fields. When plugging eq. (4.17) in the canonical Kähler for Φ_i^h one gets again terms that contain at least four light fields, with none of them getting a vev in the scalar component. Therefore, no operator $Z^\dagger\hat{h}_u\hat{h}_d$ can be generated at the tree level by integrating out heavy fields.

Let us now consider the possibility that the $D = 5$ operator above is obtained at the one loop level. This possibility raises two issues. First, μ would be suppressed compared to, say, the stop mass $\tilde{m}_t$ by a loop factor $\mathcal{O}(10^{-2})$. As for the case of gaugino masses vs sfermion masses, such a large hierarchy would lead to sfermions beyond the reach of the LHC and a significant fine-tuning. However, as we will see, this problem can be overcome in the same way as for the gaugino masses. We will see in fact in an explicit model that μ and $M_{1/2}$ get a similar enhancement factor. The second issue is the well known μ-B_μ problem. B_μ is a dimension two parameter generated, as μ, at the one loop level. Therefore, we expect an order of magnitude separation between $\sqrt{B_\mu}$ and μ: $\sqrt{B_\mu}/\mu \sim 4\pi$. This is however tolerable in a scheme in which $\tilde{m}_t \sim \sqrt{B_\mu} \sim 4\pi\mu \sim 4\pi M_{1/2}$, with $\tilde{m}_t \sim \sqrt{B_\mu} \sim$ TeV and $\mu \sim M_{1/2} \sim 100\,\text{GeV}$. The explicit model will show that the above pattern can be achieved in the large $\tan\beta$ regime. In turn, the large $\tan\beta$ regime raises a new issue. The minimization of the MSSM potential shows in fact that large $\tan\beta$ corresponds to small $B_\mu/(m^2_{\tilde{h}_u} + m^2_{\tilde{h}_d} + 2|\mu|^2)$, while in the situation we want to reproduce, $\tilde{m}_t \sim \sqrt{B_\mu}$, we expect $B_\mu/(m^2_{\tilde{h}_u} + m^2_{\tilde{h}_d} + 2|\mu|^2) \sim 1$. In order to make $\tan\beta$ large we therefore need to cancel the contribution to B_μ we get at one loop with an additional contribution, at least in the specific example we consider. Such a cancellation may not be required in different implementations of the one-loop $D = 5$ origin of the μ parameter. That is why we believe it is worth illustrating the example below despite the cancellation that needs to be invoked.

Let us consider as before a model involving the following $R_P = 1$ fields: 16, $\overline{16}$, $16'$, $\overline{16}'$, 10, with $\langle 1^{16}\rangle = \langle 1^{\overline{16}}\rangle = M$, $\langle 1^{16'}\rangle = F\theta^2$, $\langle 1^{\overline{16}'}\rangle = 0$. Let us denote the coefficients of the h_u and h_d components in the above SO(10) representations as follows: $16 \supset s_d\alpha_d h_d$, $16' \supset s_d\alpha'_d h_d$, $10 \supset c_d h_d$, $\overline{16} \supset s_u\alpha_u h_u$, $\overline{16}' \supset s_u\alpha'_u h_u$, $10 \supset c_u h_u$, where $|\alpha_d|^2 + |\alpha'_d|^2 = 1$, $|\alpha_u|^2 + |\alpha'_u|^2 = 1$, $c_d = \cos\theta_d$, $s_d = \sin\theta_d$, etc. The notation is in agreement with the definition of θ_u, θ_d in Section 4.3.1. The μ and B_μ parameters, as the gaugino masses, get a vector and a chiral one-loop contribution, see eqs. (4.33,4.35,4.37,4.38). The vector contribution turns out to be

$$|(\mu)_V| = \frac{3}{2}\frac{g^2}{(4\pi)^2}s_u s_d|\alpha'_d\alpha_u|\left|\frac{F}{M}\right| \tag{4.73a}$$

$$(B_\mu)_V = \frac{3}{4}\frac{g^2}{(4\pi)^2}s_u s_d|\alpha'_d\alpha_u|\left|\frac{F}{M}\right|^2 . \tag{4.73b}$$

As in the case of gaugino masses, the vector contribution to μ is suppressed with respect to the sfermion masses by a full loop factor. We therefore need a larger chiral contribution in order to reduce the hierarchy between μ and $\tilde{m}_t$. Let us then consider the one-loop chiral contribution associated to the superpotential

$$h_{ij}16_i 10_j 16 + h'_{ij}16_i 10_j 16'. \tag{4.74}$$

That is easily found to be vanishing because of a PQ symmetry of the superpotential. Such a PQ symmetry can however be broken by adding a term

$$\frac{M^1_{ij}}{2}1^{16}_i 1^{16}_j \tag{4.75}$$

to the above superpotential, coming for example from the non-renormalizable SO(10) operator $(\alpha_{ij}/\Lambda)(\overline{16}16_i)(\overline{16}16_j)$ after $\overline{16}$ gets its vev (note that $\Lambda \gg M$ would give $M^1_{ij} \ll M$). En passant, the singlet mass term in eq. (4.75) is nothing but the right-handed neutrino Majorana mass term entering the see-saw formula for light neutrino masses. Note however that no light neutrino mass is generated here, as the light lepton doublets do not have Yukawa interactions with the "right-handed neutrinos", 1^{16}_i. Once the PQ symmetry is broken by the mass term in eq. (4.75), the μ and B_μ parameters get a chiral one-loop contribution given by

$$|(\mu)_\Phi| = \frac{\lambda_t\lambda_b}{(4\pi)^2}f\left(\frac{\sqrt{(M^1M^{1*})_{33}}}{|h_{33}M|}\right)\frac{|M^1_{33}|}{\sqrt{(M^1M^{1*})_{33}}}\left|\frac{h'_{33}F}{h_{33}M}\right| \tag{4.76a}$$

$$(B_\mu)_\Phi = \frac{\lambda_t\lambda_b}{(4\pi)^2}g\left(\frac{\sqrt{(M^1M^{1*})_{33}}}{|h_{33}M|}\right)\frac{|M^1_{33}|}{\sqrt{(M^1M^{1*})_{33}}}\left|\frac{h'_{33}F}{h_{33}M}\right|^2, \tag{4.76b}$$

where λ_t, λ_b are the top and bottom Yukawa couplings respectively and the functions f, g are given by

$$f(x) = \frac{1 - x^2 + x^2 \log x^2}{(x^2-1)^2} x, \qquad g(x) = \frac{x^4 - 2x^2 \log x^2 - 1}{(x^2-1)^3} x. \tag{4.77}$$

We have assumed the Yukawa couplings h_{ij}, h'_{ij} to be hierarchical in the basis in which the down Yukawa matrix is diagonal.

We can see from eq. (4.76) that the one loop chiral contribution to μ is comparable to the corresponding contribution to $M_{1/2}$ if i) $\lambda_b \sim 1$, which corresponds to the large $\tan\beta$ regime (remember that the bottom mass is given by $m_b = \lambda_b \cos\beta v$, where $v = 174\,\text{GeV}$); ii) $|h'_{33}/h_3| \gtrsim |h'_{ii}/h_i|$, $i = 1, 2$; iii) $|M_{33}| \gtrsim |M_{3i}|$; iv) $|h_{33}M| \sim |M_{33}|$. If the above conditions are satisfied, $\mu \sim M_{1/2}$ and both parameters can easily be enhanced, as explained in Section 4.2.2, for example because $|h'_{33}/h_{33}| \gg 1$. The only non-trivial condition is the large $\tan\beta$ one. Remember in fact that $\tan\beta$ is determined by B_μ through the minimization of the MSSM potential, which gives

$$\sin 2\beta = \left. \frac{2B_\mu}{m^2_{h_u} + m^2_{h_d} + 2|\mu|^2} \right|_{M_Z} . \tag{4.78}$$

Therefore large $\tan\beta$, i.e. small $\sin 2\beta$, requires a small B_μ. This is in contrast with the situation we want to reproduce, $\tilde{m}_t \sim \sqrt{B_\mu}$. The RGE evolution of B_μ from the scale at which it is generated ($|h_{33}M|$) down to the electroweak scale can reduce the value of B_μ but not enough to make it as small as we need. A significant RGE contribution would in fact require $M_{1/2} \gtrsim \tilde{m}_t$, in contrast with the $\tilde{m}_t \sim 4\pi M_{1/2}$ we are trying to reproduce. We are then forced to invoke a cancellation between the one-loop contribution to B_μ in eq. (4.76b) and an additional contribution. For example, a tree level contribution to B_μ can be obtained as in Appendix B or in [54].

4.5 Conclusions

In this chapter we have considered what may be regarded as one of the simplest ways to communicate supersymmetry breaking from a hidden to the observable sector, through the tree level, renormalizable exchange of superheavy gauge (GUT) messengers, and we have studied the general properties of such a tree level gauge mediation (TGM) scheme.

We have first of all obtained the general structure of the tree-level soft terms arising from a supersymmetry breaking source that is part of a non-trivial gauge (GUT) multiplet. This is most conveniently done in the effective theory in which the heavy vector superfields associated to the broken generators are integrated out at the tree level (en passant, in Appendix A, we summarized the procedure to integrate out vector superfields and addressed a few minor issues, such as the generalization to the non-abelian case and the role of gauge invariance in a consistent supersymmetric generalization of the expansion in the number of derivatives). The scalar soft terms then obtain the two contributions in eq. (4.13), corresponding to the two diagrams on the right-hand side of Fig. 4.1. Only the first contribution is relevant for scalars that are not in the same gauge multiplet as the scalar partner of the Goldstino (or have not the same SM quantum numbers as some of the GUT generators). Because of the tracelessness condition, such a contribution gives both positive and negative soft masses. This potential phenomenological problem, which has long been considered as an obstacle to tree level supersymmetry breaking, is automatically solved in the models we consider because the fields getting a $\mathcal{O}\,(\text{TeV})$ negative soft mass also get an $\mathcal{O}\,(M_{\text{GUT}})$ positive, supersymmetric mass.

Gaugino masses do not arise at the tree level, but can be generated at the one-loop level, as in ordinary gauge mediation. They receive two contributions, from loops involving heavy vector or chiral superfields. The loop factor suppression of gaugino compared to sfermion masses must be at least partially compensated if the sfermions are to be within the LHC reach and the split-supersymmetry regime is to be avoided. We calculated in full generality the vector and chiral contributions to gaugino masses corresponding to the diagrams in Fig. 4.2. We have seen that the vector contribution is always suppressed by a full loop factor and is typically subdominant (often vanishing). On the other hand, the chiral contribution is typically larger. We listed four potential enhancement factors that can (do) compensate, at least partially, the loop suppression: a larger number of (chiral) messengers contributing to gaugino masses than (vector) messengers contributing to sfermion masses; group theoretical factors that in practice turn out to enhance gaugino masses; the fact that sfermion masses are suppressed by all the vevs with non-vanishing gauge coupling to the vector messengers, while gaugino masses are suppressed only by the vevs that are related to supersymmetry breaking through superpotential interactions; ratios of Yukawa couplings appearing in the expression for the gaugino masses. In minimal models the first two factors partially compensate the $\mathcal{O}\,(10^{-2})$ loop factor, reducing it to the level of a tolerable (and possibly necessary) one order of magnitude hierarchy between gauginos and sfermions. The last two factors are more model-dependent but can give rise to larger enhancements.

The general analysis of the TGM scheme allowed us to define the guidelines to obtain phenomenologically viable models from the general formalism and to identify the assumptions underlying the peculiar predictions one obtains. Clear model building guidelines emerge, identifying SO(10) and E_6 as the "minimal" grand-unified groups, while SU(5) is found not to have the necessary structure (rank ≥ 5) to realize the TGM scheme. The SO(10) possibility turns out to be quite appealing. It turns out in fact that the SO(10) breaking vevs of a $16+\overline{16}$, important to break SO(10) to the SM, typically make heavy precisely the components of the SO(10) representations that need to be made heavy because of their negative soft supersymmetry breaking masses. This reinforces the logical consistency of the TGM framework.

In SO(10), the tree level sfermion soft masses turn out to be proportional to their charges under the SU(5)-invariant SO(10) generator X. We find two possible embeddings of the MSSM superfields into SO(10) representations, depending on whether $F_0^\dagger X F_0$ is positive or negative. In the first case, $F_0^\dagger X F_0 > 0$, the three MSSM families are embedded in three 16_i and three 10_i, $i = 1, 2, 3$. The quark doublets, the up quark singlets, and the lepton singlets, unified in 10's of SU(5), are embedded in the 16_i's, while the lepton doublet and down quark singlets, unified in $\bar{5}$'s of SU(5), are embedded in the 10_i's. They all get positive soft masses. The spare components in the 16_i's and 10_i's get superheavy, positive, supersymmetric mass terms (and TeV scale negative soft masses). In the second case, $F_0^\dagger X F_0 < 0$, the three MSSM families are embedded in three 16_i and three 45_i, $i = 1, 2, 3$. The MSSM fields in 10's of SU(5) are embedded in the 45_i's, while the ones unified in $\bar{5}$'s of SU(5), are embedded in the 16_i's. As before, they all get positive soft masses. In both cases the chiral content of the theory is still given by three 16 of SO(10). An important property of the TGM soft terms is that they turn out to be family universal, thus solving the supersymmetric flavor problem. This property only depends on the hypothesis that the three MSSM families are embedded in the same SO(10) representations. Mixed embeddings, in which the MSSM fields are superpositions of fields in inequivalent SO(10) representations, are also possible, but can spoil the flavor universality property. Each of the two possible flavor-universal embeddings leads to specific and peculiar predictions for the soft masses at the GUT scale: $m^2_{\bar{5}} = 2m^2_{10}$ in the $F_0^\dagger X F_0 > 0$ case and $m^2_{\bar{5}} = (3/4)m^2_{10}$ in the $F_0^\dagger X F_0 < 0$ case, where $m^2_{\bar{5}}$ and m^2_{10} are common and family-independent soft masses for the fields in the $\bar{5}$ and 10 of SU(5) respectively. The latter predictions are only based on i) the use of the "minimal" unified gauge group SO(10), ii) the embedding of the MSSM families in the SO(10) representations with dimension $d < 120$ not containing the Goldstino, and iii) the absence of mixed embeddings to automatically preserve flavor-universality. The predictions on the ratios $m_{\bar{5}}/m_{10}$ in eq. (4.3) are determined by group theory factors and are peculiar enough to make a possible experimental test at the LHC a strong hint for tree level gauge mediation. The embedding into three $16_i + 10_i$'s has the advantage that the large top Yukawa coupling can be accounted for by a renormalizable superpotential interaction involving only low-dimensional ($d \leq 16$) representations for the chiral superfields. In the $16_i + 45_i$ case, a $d = 210$ representation of SO(10) must be used to reproduce the top Yukawa coupling at the renormalizable level.

The E_6 option is also quite appealing, as the matter superfield content of the $16_i + 10_i$ embedding is precisely the one obtained from three fundamentals 27_i of E_6. The latter decompose in fact as $27_i = 16_i + 10_i + 1_i$ under SO(10). We have postponed the investigation of this promising possibility to further study.

Finally, we have illustrated three possible approaches to the μ-problem in TGM, which we classify according to the dimension D of the SO(10) operator from which the μ-term arises. The $D = 3$ option provides a new approach to the μ-problem, peculiar of TGM. The idea is that the supersymmetry breaking scale turns out to coincide with the μ scale because supersymmetry is triggered by the same $D = 3$ SO(10) operator from which the μ-term arises. We have provided an explicit realization of such a possibility in Appendix B. The superpotential shown there also achieves supersymmetry breaking, SO(10) breaking to the SM, and ensures that only the MSSM fields survive below the breaking scale (in particular it provides doublet-triplet splitting). While it is not meant to be simple or realistic, that superpotential represents a useful existence proof. The $D = 4$ option is nothing but the NMSSM solution of the μ problem, in which the μ-term is obtained from the vev of a SM singlet superfield stabilized at the supersymmetry breaking scale. We pointed out that the above singlet can easily get a sizable, predictable, negative soft mass term in TGM. This makes giving a vev to the singlet easier than in ordinary gauge mediation (where its soft mass usually vanishes before RGE running), provided that the singlet potential can be made stable. The $D = 5$ option is nothing but the Giudice-Masiero mechanism realized at the loop level, as in gauge mediation. The consequent loop hierarchy between the μ-term and the sfermion masses can be reduced exactly as for the gaugino masses. We provided an explicit example, which however needs an extra contribution to the B_μ parameter in order to give rise to the necessary large $\tan\beta$.

Chapter 5

Conclusions and outlook

In this thesis work we discussed two main arguments, both related to the sector of the MSSM where SUSY is broken softly: the hierarchical soft terms scenario and Tree Level gauge Mediation.

In Chapter 2, we studied the framework of hierarchical soft terms, in which the first two generations of squarks and sleptons are heavier than the rest of the supersymmetric spectrum. This scheme gives distinctive predictions for the pattern of flavor violations, which we compared to the case of nearly degenerate squarks. Experiments in flavor physics have started to probe the most interesting parameter region, especially in $b \leftrightarrow s$ transitions, where hierarchical soft terms can predict a phase of B_s mixing much larger than in the Standard Model. One of the main results of this analysis is that the correlations between observables in $\Delta F = 1$ and $\Delta F = 2$ processes can be very sensitive to the SUSY spectrum. In the work [4], we only investigate the case of the gluino contribution in the approximation of having one flavor violating insertion $\hat{\delta}$ at time and we considered just the $B \to X_s\gamma$ process among all the $\Delta F = 1$ observables. A complete analysis of the hierarchical scenario would be particular interesting, also in light of the recent experimental results regarding B physics [22].

In Chapter 3, we proposed a new scheme in which supersymmetry breaking is communicated to the MSSM sfermions by GUT gauge interactions at the tree level. The (positive) contribution of MSSM fields to $\text{Str}(\mathcal{M}^2)$ is automatically compensated by a (negative) contribution from heavy fields. Sfermion masses are flavour universal, thus solving the supersymmetric flavour problem. In the simplest SO(10) embedding, the ratio of different sfermion masses is predicted and differs from mSugra and other schemes, thus making this framework testable at the LHC. Gaugino masses are generated at the loop level but enhanced by model dependent factors. In Chapter 4 we studied the general structure of TGM, in particular the general form of the tree level sfermion masses and the one loop gaugino masses. We discussed also several possibilities to solve the μ-problem in this new context.

There are several issues that can be studied beyond the results in [5, 6].

A study of TGM in an E_6 unified theory is actually in progress [81]. The interesting aspects to study in this extension are: the possibility to unify all the matter superfields in one irreducible representation of E_6, a possible contribution to the neutrino masses from SUSY breaking, highlight some aspects of TLG in the case of a larger group respect to the $SO(10)$ model.

Apart from this extension, it would be interesting to study in detail the phenomenology of TGM in a general context and in the specific $SO(10)$ model in particular. Several aspects need to be clarified: the effects of chiral messenger thresholds, the possibility that other fields beyond the MSSM remain light from the underlying GUT theory, the cosmological implications and the LHC phenomenology.

Chapter 6

Appendix

6.1 Perturbative diagonalization for hierarchical soft terms

In this Appendix we compute the fermion-sfermion mixing matrix $\mathcal{W}$ in the limit of hierarchical soft terms. We also discuss the conditions under which the heavy-squark contribution can be neglected in the amplitude of eq. (2.27) and the natural size of the flavor-violating parameters $\hat{\delta}$.

In a general basis in which the quark mass matrix is not necessarily diagonal, $\mathcal{W}$ is a combination of the matrices that diagonalize the quark and squark mass matrices M and $\mathcal{M}^2$ respectively,

$$\mathcal{W} = \begin{pmatrix} U_L & 0 \\ 0 & U_R \end{pmatrix} \mathcal{W}', \quad U_R M U_L^\dagger = \text{diagonal}, \quad \mathcal{W}'^\dagger \mathcal{M}^2 \mathcal{W}' = \text{diagonal}. \tag{6.1}$$

Because the relevant amplitudes will turn out to be dominated by loops with only third-generation squark exchange, we are justified to neglect chiral-violating entries in the squark mass matrix involving first or second generation indices. Under this assumption and working at leading order in an expansion in inverse powers of the heavy-squark mass scale, we obtain

$$\mathcal{W}' = \begin{pmatrix} \tilde{U}_L & \hat{\delta}^{LL} \cos\theta & 0 & -\hat{\delta}^{LL} \sin\theta e^{i\phi} \\ -\hat{\delta}^{LL\dagger} \tilde{U}_L & \cos\theta & 0 & -\sin\theta e^{i\phi} \\ 0 & \hat{\delta}^{RR} \sin\theta e^{-i\phi} & \tilde{U}_R & \hat{\delta}^{RR} \cos\theta \\ 0 & \sin\theta e^{-i\phi} & -\hat{\delta}^{RR\dagger} \tilde{U}_R & \cos\theta \end{pmatrix}, \tag{6.2}$$

where we have omitted the generation indices of the first two generations. The 2×2 unitary matrices $\tilde{U}_{L,R}$ diagonalize the 2×2 blocks of the heavy states in the squark mass matrix (which we call $\mathcal{M}^2_{\tilde{h}L}$ and $\mathcal{M}^2_{\tilde{h}R}$) according to

$$\tilde{U}_L^\dagger \mathcal{M}^2_{\tilde{h}L} \tilde{U}_L = \text{diagonal}, \quad \tilde{U}_R^\dagger \mathcal{M}^2_{\tilde{h}R} \tilde{U}_R = \text{diagonal}. \tag{6.3}$$

The two-component vectors $\hat{\delta}_{i3}^{LL,RR}$ $(i = 1, 2)$ are given by

$$\hat{\delta}_{i3}^{LL} \equiv -\sum_{j=1}^{2} \left(\mathcal{M}_{hL}^{-2}\right)_{ij} \mathcal{M}_{Lj,3}^{2}, \quad \hat{\delta}_{i3}^{RR} \equiv -\sum_{j=1}^{2} \left(\mathcal{M}_{hR}^{-2}\right)_{ij} \mathcal{M}_{Rj,3}^{2}. \tag{6.4}$$

It is easy to verify that this definition coincides with eq. (2.28), at the leading order in the expansion and neglecting quark rotation effects. Finally, θ and ϕ are the parameters determining the diagonalization of the light-squark sector and are defined by

$$\tan 2\theta \equiv \frac{2\left|\mathcal{M}_{L3,R3}^{2}\right|}{\mathcal{M}_{L3,L3}^{2} - \mathcal{M}_{R3,R3}^{2}}, \quad e^{i\phi} \equiv \frac{\mathcal{M}_{L3,R3}^{2}}{\left|\mathcal{M}_{L3,R3}^{2}\right|}. \tag{6.5}$$

The result presented in the text in eq. (2.29) can now be easily derived by replacing eq. (6.2) into eq. (2.27). Moreover, we can use eq. (6.2) to compare the contributions to flavor-violating amplitudes from heavy and light squarks . For instance, the flavor transition between the first and second generations in the down-left sector, obtained from eq. (2.27), is given by

$$f\left(\frac{\mathcal{M}_{D}^{2}}{M^2}\right)_{d_L s_L} = \frac{\tilde{m}_h^2}{M^2}\Delta_h f^{(1)}\left(\frac{\tilde{m}_h^2}{M^2}\right) + \hat{\delta}_{13}^{LL}\hat{\delta}_{23}^{LL*} f\left(\frac{\tilde{m}_\ell^2}{M^2}\right). \tag{6.6}$$

Here, for simplicity, we have neglected quark rotations and we have considered near degeneracy among the heavy squark states (with a common mass $\tilde{m}_h$) and among the light squark states (with a common mass $\tilde{m}_\ell$). We have defined $\Delta_h \equiv (\mathcal{M}_{hL}^2)_{12}/\tilde{m}_h^2$ to parametrize the mass insertion in the heavy sector. Using the property that, for large x, $f(x) \sim 1/x$ (and therefore $f^{(1)}(x) \sim 1/x^2$), we obtain that the second term in eq. (6.6) dominates over the first one when

$$\hat{\delta}^{LL} \gtrsim \Delta_h^{1/2} \frac{\tilde{m}_\ell}{\tilde{m}_h}. \tag{6.7}$$

Analogous considerations hold for $\hat{\delta}^{RR}$. When the condition in eq. (6.7) is satisfied, we are allowed to neglect the heavy-squark contribution in the loop diagram.

To establish if the condition is satisfied we have to discuss what is the natural range of values for $\hat{\delta}^{LL}$. A lower limit on $\hat{\delta}^{LL}$ is obtained from eq. (6.4) with the requirement that any chiral-conserving entry of $\mathcal{M}_D^2$ is at least of size $\tilde{m}_\ell^2$,

$$\hat{\delta}^{LL} \gtrsim \frac{\tilde{m}_\ell^2}{\tilde{m}_h^2}. \tag{6.8}$$

An upper limit on $\hat{\delta}^{LL}$ is derived by observing that the light left squark receives a contribution from the heavy sector to its mass square equal to

$$-\hat{\delta}^{LL\dagger}\mathcal{M}_{hL}^2\hat{\delta}^{LL}\cos^2\theta - \hat{\delta}^{RR\dagger}\mathcal{M}_{hR}^2\hat{\delta}^{RR}\sin^2\theta \sim \mathcal{O}(\hat{\delta}^{LL2}\tilde{m}_h^2). \tag{6.9}$$

Thus, barring special cancellations, the hierarchical separation between the light and heavy sectors is maintained only if

$$\hat{\delta}^{LL} \lesssim \frac{\tilde{m}_\ell}{\tilde{m}_h}. \tag{6.10}$$

The natural range for $\hat{\delta}^{LL}$ (or $\hat{\delta}^{RR}$) is defined by eq. (6.8) and eq. (6.10). In the absence of any GIM suppression in the heavy sector (*i.e.* when $\Delta_h \approx 1$), the natural values of $\hat{\delta}^{LL}$ are nearly inconsistent with the condition in eq. (6.7). However, as discussed in the text, the constraint from ϵ_K require that $\Delta_h < 10^{-2}\tilde{m}_h/(3\,\mathrm{TeV})$. In presence of a mechanism justifying the smallness of Δ_h (like, for instance, an approximate U(2) symmetry), the condition in eq. (6.7) can be satisfied.

When the ratio $\tilde{m}_h/\tilde{m}_\ell$ becomes very large, the quark rotation angles in $U_{L,R}$ can dominate over those of $\mathcal{W}'$ in eq. (6.1). In this case, eq. (6.7) is automatically satisfied, and the assumption of neglecting heavy squarks in the loop diagram is perfectly justified. Assuming that the CKM matrix $V = U_L^u U_L^{d\dagger}$ is dominated by the rotation in the down sector, we obtain

$$\hat{\delta}_{db}^{LL} \approx V_{td}^*, \qquad \hat{\delta}_{sb}^{LL} \approx V_{ts}^*. \tag{6.11}$$

Thus, excluding unexpected cancellations, $\hat{\delta}^{LL}$ cannot be smaller than the maximum between $\tilde{m}_\ell^2/\tilde{m}_h^2$ and what given in eq. (6.11). Although we cannot directly relate U_R to CKM angles, we expect that the result in eq. (6.11) will hold approximately for $\hat{\delta}^{RR}$ too if, for instance, the quark mass matrix is nearly symmetric.

6.2 Integrating out vector superfields

In this Appendix, after a few general comments, we write the effective theory one obtains by integrating heavy vector superfields at the tree level and in unitary gauge in a generic, non-abelian, $N = 1$ globally supersymmetric theory with renormalizable Kähler K and gauge-kinetic function (the superpotential W is allowed to be non-renormalizable). The general prescription has been studied in [65, 66, 67]. In particular, it has been shown in [65] that the usual expansion in the number of derivatives n_∂ can be made consistent with supersymmetry by generalizing n_∂ to the parameter

$$n = n_\partial + \frac{1}{2}n_\psi + n_F, \tag{6.12}$$

where $n_\psi/2$ is the number of fermion bilinears and n_F the number of auxiliary fields from chiral superfields. With such a definition, a chiral superfield Φ has $n = 0$ and $d\theta$ integrations and supercovariant derivatives have $n = 1/2$. Such an expansion makes sense when supersymmetry breaking takes place at a scale much smaller than the heavy superfield mass M and in particular when the F-terms and fermion bilinears from heavy superfields being integrated out are much smaller than M.

In the presence of vector superfields one should further assume that the D-terms and gaugino bilinears are small and should generalize eq. (6.12) to account for the number n_λ of gauginos and the number n_D of vector auxiliary fields. We claim that the correct generalization is

$$n = n_\partial + \frac{1}{2}n_\psi + n_F + \frac{3}{2}n_\lambda + 2n_D, \tag{6.13}$$

according to which a vector superfield V has $n = 0$. Note that the double weight of D-terms compared to F-terms is consistent with eq. (4.6). With such a definition, the initial lagrangian has $n = 2$, except for the gauge kinetic term, which has $n = 4$. Chiral and vector superfields can then be integrated out at the tree level by using the supersymmetric equations of motion

$$\frac{\partial W}{\partial \Phi} = 0 \quad \text{and} \quad \frac{\partial K}{\partial V} = 0 \tag{6.14}$$

up to terms with $n \geq 3$ when integrating out chiral superfields and $n \geq 4$ when integrating vector superfields (with the missing terms originating from the gauge-kinetic term having $n \geq 6$).

From a physical point of view, we are interested not only in the expansion in n but also, and especially, in the expansion in the power m of $1/M$. It is therefore important then to remark that using eqs. (6.14) amounts to neglecting terms with $m \geq 3$ when integrating chiral superfields and $m \geq 6$ when integrating out vector superfields.

We are now ready to present out results on the effective theory obtained integrating out the heavy vector superfields in a generic supersymmetric gauge theory as above. We are interested in operators with dimension up to 6 ($m \leq 2$) in the effective theory. We can then use the equation $\partial K/\partial V$. Neglecting higher orders in m, the latter equation can be rewritten as

$$V_a^h (M_V^2)_{ab} = -\frac{1}{2}\frac{\partial K_2}{\partial V_b^h}(\Phi', V^l), \tag{6.15}$$

where Φ' is defined in eq. (4.3), $K_2(\Phi', V) = \Phi'^\dagger e^{2gV}\Phi'$, the indices run over the broken generators, and M_V^2 is a function of the light vector superfields:

$$\begin{aligned}
(M_V^2)_{ab} &= \frac{1}{2}\frac{\partial^2}{\partial V_a^h \partial V_b^h}\left(\phi_0^\dagger e^{2gV}\phi_0\right)\Big|_{V^h=0} = (M_{V0}^2)_{ab} + (M_{V2}^2)_{ab} \\
(M_{V0}^2)_{ab} &= g^2 \phi_0^* \{T_a^h, T_b^h\}\phi_0 \\
(M_{V2}^2)_{ab} &= \frac{g^4}{3}\phi_0^* T_a^h V^l V^l T_b^h \phi_0 + (a \leftrightarrow b).
\end{aligned} \tag{6.16}$$

In order to solve eq. (6.15) for V_a^h, we need to invert the field-dependent matrix M_V^2. In the Wess-Zumino gauge for the light vector superfields, we get

$$(M_V^2)^{-1}_{ab} = (M_{V0}^2)^{-1}_{ab} - (M_{V0}^2)^{-1}_{ac}(M_{V2}^2)_{cd}(M_{V0}^2)^{-1}_{db}. \tag{6.17}$$

The effective contribution to the Kähler potential is

$$K_{\text{eff}} = -(M_V^2)_{ab}V_a^h V_b^h = K_0 + K_1 + K_2, \tag{6.18}$$

where

$$\delta K^0_{\text{eff}} = -g^2(M_{V0}^2)^{-1}_{ab}(\Phi'^\dagger T_a^h \Phi')(\Phi'^\dagger T_b^h \Phi') \tag{6.19a}$$

$$\delta K^1_{\text{eff}} = -2g^3(M_{V0}^2)^{-1}_{ab}(\Phi'^\dagger T_a^h \Phi')(\Phi'^\dagger \{V^l, T_b^h\} \Phi') \tag{6.19b}$$

$$\begin{aligned}\delta K^2_{\text{eff}} = &-\frac{4}{3}g^4(M_{V0}^2)^{-1}_{ab}(\Phi'^\dagger T_a^h \Phi')\Phi'^\dagger(T_b^h V^l V^l + V^l T_b^h V^l + V^l V^l T_b^h)\Phi' \\ &- g^4(M_{V0}^2)^{-1}_{ab}\left((\Phi'^\dagger\{T_a^h, V^l\}\Phi')(\Phi'^\dagger\{T_b^h, V^l\}\Phi') + \frac{1}{3}(\Phi'^\dagger[T_a^h, V^l]\Phi')(\Phi'^\dagger[T_b^h, V^l]\Phi')\right).\end{aligned} \tag{6.19c}$$

In recovering eq. (6.19c) we have used the identity

$$f_{\alpha ab}(M_{V0}^2)_{bc} = -f_{\alpha cb}(M_{V0}^2)_{ba}, \tag{6.20}$$

where f_{abc} are the structure constants of the gauge group, the latin indices refer to broken generators and the greek one refers to an unbroken one.

We are interested to soft supersymmetry breaking terms arising from eqs. (6.19) when some of the auxiliary fields get a vev. The relevant terms should contain up to two F-terms and one D-term (eq. (4.6)). The only relevant terms are therefore those in (6.19a).

6.3 An explicit example

In this Appendix we provide an example of a superpotential achieving supersymmetry breaking, SO(10) breaking to the SM, ensuring that below the scale of this breaking only the MSSM fields survive (in particular providing doublet-triplet splitting) and solving of the μ-problem. We do not aim at being simple or realistic, we just aim at providing an existence proof. We include in this example representations with dimension $d > 120$. It would be interesting to obtain a dynamical supersymmetry breaking, in particular the F-term vev of a 16 of SO(10).

SO(10) will broken to the SM at a scale $M \sim M_{\text{GUT}}$. Below this scale only the MSSM fields survive, in particular the Higgs triplets are made heavy via a generalization of the Dimopoulos-Wilczek mechanism [80, 56]. The μ-term is present in the theory in the form of a $D = 3$ operator present at the GUT scale and triggers supersymmetry breaking. B_μ is generated at the tree-level and turns out to be of the same order as the sfermion masses.

6.3.1 The superpotential

The superpotential we use is

$$W = W_Y + W_1 + W_2 + W_3 + W_4, \tag{6.21}$$

where

$$\begin{aligned}
W_Y &= y_{ij} 16_i 16_j 10 + h_{ij} 16_i 10_j 16 + h'_{ij} 16_i 10_j 16' \\
W_1 &= \lambda_1 Z(\overline{16}16 - M^2) + m\,\overline{16}16' + \lambda_2 X\,\overline{16}'16 \\
W_2 &= \overline{16}''(\lambda_3\, 45 + \lambda_4\, U)16 + \overline{16}(\lambda_5\, 45 + \lambda_6\, U')16'' + M_{45}\, 45\, 45 + \lambda_7\, 54\, 45\, 45 + M_{54}\, 54\, 54 \\
W_3 &= \lambda_8\, 16 16' 120 + \lambda_9\, \overline{16}\,\overline{16}'\, 120 + M_{120}\, 120\, 120 \\
W_4 &= \lambda_{10}\, 10'\, 45\, 10 + \lambda_{11}\, \overline{16}\,\overline{16}''10 + M_{10} 10' 10' + \lambda_{12}\, \overline{16}\,\overline{16}' 10 + \lambda_{13}\, 16 16'' 10 + \lambda_{14}\, Z\, 10\, 10.
\end{aligned} \tag{6.22}$$

Here we denote the fields according to their SO(10) representation, except the SO(10) singlet fields Z, X, U, U'. The mass parameter m is of the order of the TeV scale (we do not discuss the origin of such a small parameter here), while all other mass parameters are near the GUT scale

$$\text{TeV} \sim m \ll M \sim M_{45} \sim M_{54} \sim M_{10} \sim M_{120} \sim M_{\text{GUT}}.$$

Let us discuss the role of the different contributions to the superpotential and anticipate the vacuum structure and the spectrum. W_1 is responsible for supersymmetry breaking and the breaking of SO(10) to SU(5): as we are going to show below, this part of the superpotential generates $\mathcal{O}\,(M_{\text{GUT}})$ vevs for the scalar components of 16 and $\overline{16}$ along the SU(5) singlet direction $\langle S \rangle \sim M + \mathcal{O}(m^2/M)$ and $\langle \overline{S} \rangle \sim M + \mathcal{O}(m^2/M)$ and a supersymmetry breaking vev for the F-term component of $16'$ along the SU(5) singlet direction $\langle F_{S'} \rangle \sim mM$. It also provides small supersymmetry breaking vevs for the F-term component of X $\langle F_X \rangle \sim m^2$ and for the D-term of the vector superfield corresponding to the $U(1)_X$ generator of SO(10) $\langle D_X \rangle \sim M(\langle S \rangle - \langle \overline{S} \rangle) \sim m^2$. This D-term vev will generate sfermion masses along the lines of Section 4.3.1. This superpotential appears in eq. (4.68) and is a generalization of an example in [54].

W_Y contains the MSSM Yukawa couplings and provides supersymmetry breaking masses for heavy chiral superfields that will generate gaugino masses at 1-loop as in ordinary gauge mediation. The MSSM matter is embedded in both the 16_i and the 10_i, as explained in Section 4.3.1. The MSSM Higgs fields are linear combinations of different fields and have components in different representations,

$$h_u \subset 10, \overline{16} \qquad h_d \subset 10, 16, 16', 120.$$

Therefore the first term in W_Y contains the up-type Yukawas, while the second and third terms provide down-type and charged lepton Yukawas. The second term also gives a large mass to the additional fields $5_i^{10} \subset 10_i$ and $\overline{5}_i^{16} \subset 16_i$. The latter are also the only fields that couple to the F-term vev in the $16'$ and act as one-loop messengers of supersymmetry breaking. While this gives a subleading contribution to sfermion masses, it is the only source of gaugino masses in this model.

The role of W_2 is the breaking of SU(5) to the standard model gauge group. It provides a large vev for the 45 along the $B-L$ direction $\langle 45_{B-L}\rangle \sim M$ as needed for the Dimopoulos-Wilzcek mechanism. Also U, U' and the SM singlet in the 54 take large vevs. W_3 merely gives large masses to components in the $16'$ and $\overline{16}'$. Note that since the 120 does not contain SU(5) singlets, the neutrino component in the $16'$ stays massless as it should, being the dominant component of the Goldstino superfield. W_4 takes care of the Higgs sector: it keeps the MSSM Higgs doublets light and gives a large mass to the corresponding triplets. Its last term provides the B_μ term because Z gets a small supersymmetry breaking vev and both H_u and H_d have components in the 10. The μ term is contained in W_1, because H_d has a component also in the $16'$.

6.3.2 The vacuum structure

We are interested in a vacuum that does not break the SM gauge group. Thus only that part of the superpotential which involve SU(5) singlets is relevant for the determination of the ground state. We denote the singlets in $(16, \overline{16}, 16', \overline{16}', 16'', \overline{16}'')$ by $(S, \overline{S}, S', \overline{S}', S'', \overline{S}'')$ (which is different than the notation used in the main text) and the singlets in the 45, 54 by B, T, V, where B, T are the properly normalized fields corresponding to the $B-L$ and T_{3R} generators in SO(10). The relevant part of the superpotential is

$$\begin{aligned} W = {} & \lambda_1 Z(\overline{S}S - M^2) + m\,\overline{S}S' + \lambda_2 X\,\overline{S}'S \\ & + \overline{S}''\left(-\frac{\lambda_3}{2}T + \frac{\lambda_3}{2}\sqrt{\frac{3}{2}}B + \lambda_4 U\right) S + \overline{S}\left(-\frac{\lambda_5}{2}T + \frac{\lambda_5}{2}\sqrt{\frac{3}{2}}B + \lambda_6 U\right) S'' \\ & + M_{45}(B^2+T^2) + M_{54}V^2 + \lambda_7 V\left(\frac{1}{2}\sqrt{\frac{3}{5}}T^2 - \frac{1}{\sqrt{15}}B^2\right). \end{aligned} \tag{6.23}$$

The F-term and D_X-term equations show that SUSY is broken ($F_{S'} \neq 0$) and that all vevs are determined except V, B, T, for which there exist three solutions, all yielding $F_T = F_V = F_B = 0$. This tree-level degeneracy is lifted by one-loop corrections which select the solution with $T = 0, B \neq 0, V \neq 0$. One can check that the vevs are given by

$$\begin{gathered}
S' = \overline{S}' = S'' = \overline{S}'' = X = Z = T = 0 \\
S = M - \frac{m^2}{4M}\left(\frac{1}{\lambda_1^2} - \frac{1}{50g^2}\right) \qquad \overline{S} = M - \frac{m^2}{4M}\left(\frac{1}{\lambda_1^2} + \frac{1}{50g^2}\right) \\
U = -\frac{3\sqrt{5}}{2}\frac{\lambda_5}{\lambda_6\lambda_7}\sqrt{M_{45}M_{54}} \qquad U' = -\frac{3\sqrt{5}}{2}\frac{\lambda_3}{\lambda_4\lambda_7}\sqrt{M_{45}M_{54}} \\
V = \frac{\sqrt{15}M_{45}}{\lambda_7} \qquad B = \frac{\sqrt{30}}{\lambda_7}\sqrt{M_{45}M_{54}} \\
F_{S'} = -mM \qquad F_Z = \frac{m^2}{2\lambda_1} \qquad D_X = -\frac{m^2}{10g}.
\end{gathered} \tag{6.24}$$

6.3.3 Spectrum and soft terms

In order to identify the light (with respect to M_{GUT}) we can set $m = 0$ and consider the supersymmetric limit. Most fields are at the GUT scale, with the light ones being the MSSM ones, the Goldstino superfield S', and the right-handed neutrinos in the 16_i, which can easily be made heavy through a non-renormalizable superpotential operator $(\overline{16}16_i)(\overline{16}16_j)$. The MSSM matter fields are embedded in the 10_i^{16} and in the $\overline{5}_i^{10}$, as desired. The Higgs doublets are embedded into the $\overline{16}, 10$ and $10, 120, 16', 16$ according to

$$\begin{gathered}
h_u = \frac{1}{N_u}\left(\overline{L}_{\overline{16}} + 3\sqrt{5}\frac{\lambda_5}{\lambda_7\lambda_{13}}\frac{\sqrt{M_{45}M_{54}}}{M}\overline{L}_{10}\right) \\
h_d = \frac{1}{N_d}\left(L_{10} - \frac{\lambda_{12}}{\lambda_9}L_{120} + 2\frac{\lambda_{12}}{\lambda_8\lambda_9}\frac{M_{120}}{M}L_{16'} + \frac{1}{3\sqrt{5}}\frac{\lambda_7\lambda_{11}}{\lambda_3}\frac{M}{\sqrt{M_{45}M_{54}}}L_{16}\right)
\end{gathered} \tag{6.25}$$

with normalization factors N_u and N_d, where L_x, $\overline{L}_x$ denote the SM component with the quantum numbers of h_d, $\overline{h}_d$ in the SO(10) representation x.

After switching on m the soft supersymmetry breaking terms and μ-term are generated. The μ-term is already present in the high energy Lagrangian and is of order m, the vev of D_X generates sfermion and Higgs masses of order m^2 and the vev of F_X gives rise to a B_μ term of order m^2. The heavy fields 5_i^{10} and $\overline{5}_i^{16}$ act as messengers of SUSY breaking to the gauginos who get masses of order $m^2/(16\pi^2)$. The Goldstino will be mainly the fermion in S' but gets also small contributions from the gaugino corresponding to the $U(1)_X$ generator and the fermion in Z. The corresponding scalar will get a mass of order m^2.

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