

When Vacuum Breaks: A Self-Consistency Test for Astrophysical Environments in Extreme Mass Ratio Inspirals

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Gravitational-wave signals are typically interpreted under the vacuum hypothesis, i.e., assuming negligible influence from the astrophysical environment. This assumption is expected to break down for low-frequency sources such as extreme mass ratio inspirals (EMRIs), which are prime targets for the Laser Interferometer Space Antenna (LISA) and are expected to form, at least in part, in dense environments such as active galactic nuclei or dark-matter spikes or cores. Modeling environmental effects parametrically is challenging due to the large uncertainties in their underlying physics. We propose a nonparametric test for environmental effects in EMRIs, based on assessing the self-consistency of vacuum parameter posteriors inferred from different portions of the signal. Our results demonstrate that this test can reveal statistically significant inconsistencies from vacuum signals—arising from, e.g., incomplete modeling, environmental effects, or deviations from general relativity—without introducing additional parameters or assumptions about the underlying physics.

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Introduction—With the latest release of gravitational-wave (GW) data by the LIGO-Virgo-Kagra (LVK) Collaboration, more than 200 GW events have been identified, with a few black hole (BH) binaries exceeding $\sim 100M_{\odot}$ [1,2]. The next decade of GW astronomy will potentially see more massive sources through space-based detectors such as the Laser Interferometer Space Antenna (LISA) [3,4]. One of the most interesting sources that LISA is expected to detect is extreme mass ratio inspirals (EMRIs), e.g., binaries of a stellar mass BH orbiting a supermassive one [5–7]. Because of their extreme mass ratio, these sources perform up to $\sim 10^5$ orbital cycles in the LISA band, thus requiring highly accurate waveforms to extract astrophysical parameters [8,9]. At least a fraction of EMRIs are expected to form in gas-rich astrophysical environments (e.g., active galactic nuclei—AGNs) [10]. EMRIs may also form in dense dark-matter environments (e.g., spikes in cold particle dark matter scenarios [11–14], or bosonic cores for ultralight dark matter [15,16]). These dense environments can leave a detectable imprint on their GW signals through physical effects such as migration

torques from accretion disks, gas or dark-matter accretion and dynamical friction, resonances, or even direct gravitational pulls from neighboring matter/objects [12,13,15–27]. Using vacuum EMRI waveform templates can thus potentially lead to incorrect inference of the source parameters [25].

Already in vacuum, EMRI waveforms have rich signal morphology and complexity across parameter space [5,6,28–31]. Conversely, the literature studying signatures of environmental effects in EMRIs typically makes simplifying assumptions about the astrophysical medium. For instance, migration torques, which are among the dominant effects, are typically studied in stationary, thin, and radiatively efficient accretion disks within Newtonian gravity [12,20,21,26,32]. With these approximations, disk-driven migration can be modeled as a power law correction to the vacuum GW flux [12,20,25]. Later studies confirmed that this power-law model remains robust even under the stochastic migration torques seen in simulations [27]. However, the physics underpinning accretion disks in AGNs is still largely uncertain when it comes, e.g., to disk geometry, density and angular momentum profile, viscosity, the role of turbulence and magnetic fields, etc. [12,20,25,33]. Even larger uncertainties characterize the interaction of EMRIs with dark matter, whose density profile and very nature are unknown, or the interaction with third bodies (due to their transient nature). Relativistic effects in the interaction between the source and the environment can also become important for EMRIs, and may lead to significant phase differences in the gravitational signals [34–38]. Finally, multiple environmental effects (e.g., migration

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and accretion in the case of disks) and even possible violations of general relativity (GR) may be simultaneously present, thus complicating the signal analysis and leading to degeneracies [25,39]. Due to these technical difficulties and physical uncertainties, a natural question is whether EMRI GW data can robustly reveal environmental effects, if the latter are (at least partially) mismodeled.

In this Letter, we show that one can indeed test for the presence of astrophysical environments, or even deviations from general relativity, using *only* vacuum EMRI waveform templates, i.e. assuming no specific model for effects that go beyond vacuum GR waveforms. Environmental effects are expected to be generally stronger during the early inspiral (i.e., at lower frequencies) [12] and are suppressed, in relative terms, closer to the plunge/merger. If that is the case, the last portion of the signal will be well described by a vacuum EMRI waveform, while inconsistencies/biases arising from neglecting environmental effects will build up from the early inspiral. By constructing data segments of increasing duration, we develop a self-consistency test that compares the vacuum EMRI parameter posteriors obtained from each segment. When the self-consistency test fails, our method indicates the presence of missing, unmodeled effects in the vacuum EMRI template. We apply this test to a fiducial EMRI signal containing an environmental effect (disk-driven migration), and quantify the significance of the inconsistency in the posteriors from different observation durations. We do so by computing the mismatch and relative systematic error between the estimated maximum likelihood points. We also estimate the probability that the inconsistency between the posteriors from different signal portions is due to the noise realization. In our fiducial system, we show that for observations of more than two years, the presence of an environmental effect can be robustly identified, irrespective of the noise realization. Throughout the Letter, we use M for primary mass, μ for secondary mass, and geometric units with $G = 1 = c$.

Waveforms and data analysis methods—We compute EMRI waveforms using the time-domain multivoice method [6,28] implemented in the Fast EMRI Waveform (few) package [30,31,40]. Specifically, these waveforms capture fully relativistic equatorial Kerr orbits evolving adiabatically within the self-force formalism [28,41]. To generate EMRI waveforms containing an environmental effect, we add suitable contributions to the adiabatic fluxes, which in turn modify the orbital evolution and thus the waveform. For simplicity, we implement contributions from disk-driven migration torques as computed for thin disks within Newtonian theory [12,20,25]. For circular prograde orbits, the angular momentum flux then takes the form $\dot{L} = \dot{L}_{\text{GW}} A (r/r^*)^n$, where $\dot{L}_{\text{GW}}$ is the Newtonian point-particle GW flux [42], $r^* = 10M$ is a characteristic separation, with A and n mapping to different accretion disk models [12,20,25]. As proof of principle of our

self-consistency test, we focus here on $n = 8$, corresponding to Type-I migration [43,44] in a Shakura-Sunayev α disk [45]. We choose $A = 1.92 \times 10^{-5}$, as in the analysis by [25], which is consistent, but not limited to, with an accretion Eddington ratio $f_{\text{Edd}} = 0.0005$ and a viscosity parameter $\alpha = 0.03$. In Supplemental Material [46], we also extend this analysis to other environmental effects (by varying n) and different source parameters, to further demonstrate the wider applicability of our self-consistency test [56].

We inject such an EMRI signal into the time delay interferometry (TDI) [57] channels of LISA using `fastlisaresponse` [58]. Using vacuum EMRI templates, we perform Bayesian parameter estimation of $\theta = \{\log_{10} M/M_{\odot}, \log_{10} \mu/M_{\odot}, a, r_0/M, D_L/\text{Gpc}, \cos \theta_S, \phi_S, \cos \theta_K, \phi_K, \Phi_{\phi_0}\}$. Note that a is the dimensionless spin of the primary, r_0/M and Φ_{ϕ_0} are the initial separation and orbital phase, D_L/Gpc is the luminosity distance, θ_S and ϕ_S are the azimuthal and longitudinal sky position in the solar system barycenter frame, and $\cos \theta_K, \phi_K$ are the primary spin orientation relative to the line of sight.

For the injection, we use vacuum EMRI parameters $\theta_{\text{inj}} = \{6.0, 1.69897, 0.9, 14.382, 3.16, 0.751, 0.236, 0.555, 0.628, 0.720\}$, which for a 3 yr observation [59] yields a signal-to-noise ratio (SNR) of 100 and plunges about 12 600 s before the end of the observation window. We note that this is reasonable SNR based on astrophysical population models [7,60–62]. In Supplemental Material [46], we also consider a different EMRI system with a lower SNR, to demonstrate how our test performs for weaker sources. We use uniform priors on all parameters except the luminosity distance, for which we use $p(D_L) \propto D_L^2$. Assuming stationary instrumental Gaussian noise, we write the joint log likelihood of the TDI channels (up to an additive constant) as [63]

$$\log \mathcal{L}(d|\theta) = -\frac{1}{2} \sum_i (d_i - h_i(\theta) | d_i - h_i(\theta))_i, \quad (1)$$

where d_i and $h_i(\theta)$ are the data and the waveform model, respectively, in TDI channel $i \in \{A, E\}$. The T channel is not included in our analysis, as it is an antisymmetric TDI combination and is thus mostly dominated by noise [63]. The matched filter inner product in channel i between time series $u_i(t)$ and $v_i(t)$ is defined as

$$(u_i | v_i)_i = 4 \text{Re} \int_0^\infty \frac{df}{S_n^i(f)} \tilde{u}_i(f)^* \tilde{v}_i(f), \quad (2)$$

where $S_n^i(f)$ is the one-sided noise power spectral density (PSD) of TDI channel i , with $\tilde{u}_i(f)$ and $\tilde{v}_i(f)$ the Fourier transforms of the respective time series. We obtain corresponding posterior samples using the parallel tempered Markov chain Monte Carlo sampler implemented in `eryn` [64] (see Supplemental Material [46] for more details).

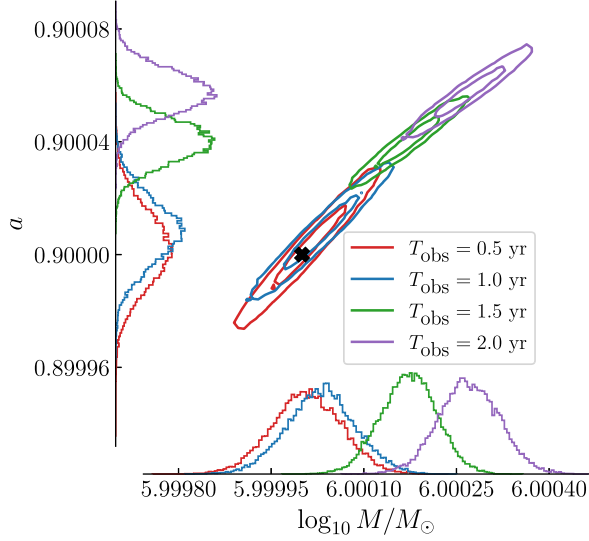


FIG. 1. Two-dimensional marginalized posteriors (with contours at 50% and 90% confidence level) for the primary mass M and spin a , for an injection with disk migration torques but with vacuum EMRI templates, for different observation durations. Significant inconsistencies in the posteriors reveal the neglected environmental effect. For clarity we indicate the true value of the parameters with a black cross.

We use the `scirv1` version of the LISA PSD with second generation TDI [65], to which we add the confusion noise from unresolved galactic binaries [66]. We also consider specific Gaussian noise realizations in the injected data, to assess their impact on the parameter estimation results.

Inconsistent inference across observation duration—To perform a self-consistency check of the vacuum EMRI

templates, we compare the parameter posteriors resulting from Bayesian inference, with different signal durations. Specifically, we consider signal segments with $T_{\text{obs}} \in \{0.5, 1, 1.5, 2, 2.5, 3\}$ yr, all ending at the same time (right after the plunge). Since most of the SNR is in the last part of the waveform, this ensures that we obtain informative posteriors for the vacuum EMRI parameters for all observation times. Given the injected EMRI signal with Type-I α -disk migration torques, we show in Fig. 1 the inferred one-dimensional and two-dimensional marginalized posteriors of the primary mass M and dimensionless spin a , for different T_{obs} . With increasing T_{obs} , the inferred posteriors become more and more inconsistent. In more detail, the posteriors obtained with the vacuum EMRI model for $T_{\text{obs}} = 2$ yr show minimal overlap with those for $T_{\text{obs}} = 0.5$ yr. This reveals the presence of a missing effect in the model—disk-driven migration in this example—that has accumulated significantly between $T_{\text{obs}} = 0.5$ and 2 yr of observation.

To contrast the role of instrumental noise with that of the environmental effect, we consider a vacuum EMRI injection with a Gaussian noise realization, and compare it with injections featuring disk-driven migration with and without such a noise realization. In Fig. 2, we compare the resulting one-dimensional marginalized posteriors for $\log_{10} M/M_{\odot}, a$ from these different injections (see Supplemental Material [46] for a full corner plot). When injecting a vacuum EMRI signal, we observe that the (green) posteriors are statistically self-consistent, with shifts induced by the noise realization that have no significant trend with observation duration. However, the (blue) posteriors resulting from the environment-affected

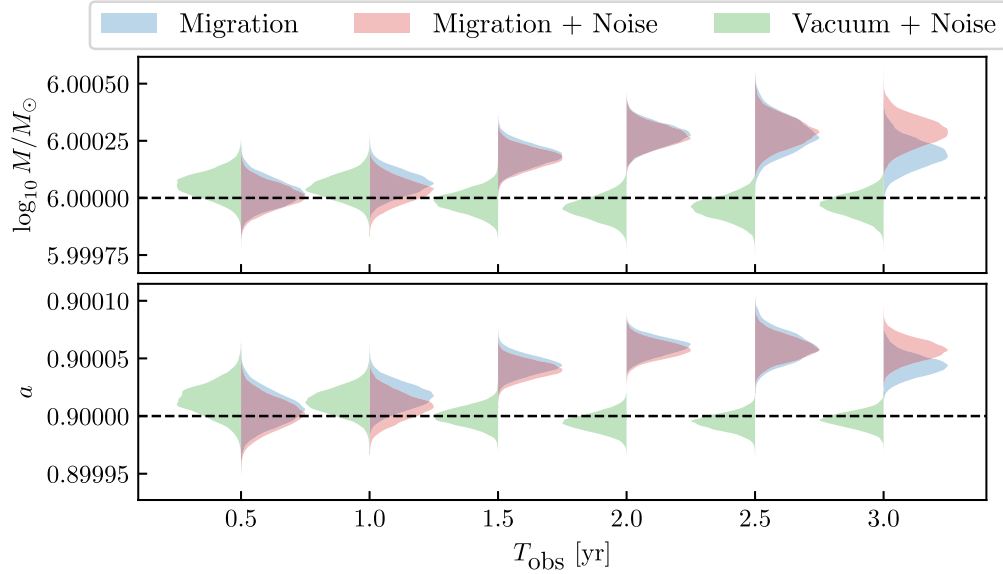


FIG. 2. One-dimensional marginalized posteriors of $\log_{10} M/M_{\odot}, a$ for different injections, namely: a vacuum EMRI with noise (green); a migration-affected EMRI with (red) and without (blue) noise. The inconsistency in the posteriors due to the environment can be distinguished from the effect of the noise realization.

EMRI injection without noise clearly show significant inconsistencies growing with the observation duration. Note that given the high SNR, the posteriors obtained from this noiseless injection are a good approximation for the ensemble average (over noise realizations) of the posteriors obtained with noisy injections [67,68]. Importantly, we also observe good agreement between these “average” posteriors and those obtained for an environment-affected EMRI with a specific noise realization (red). We find that for short observation durations $T_{\text{obs}} \leq 1$ yr, the inconsistency due to the missing environmental effect is comparable with the effect of the instrumental noise realization. For longer observation durations, the inconsistency is apparent and becomes distinguishable from the effects of the noise realization.

We will now quantify the statistical significance of the inference inconsistency, shown in Figs. 1 and 2, using both a *mismatch* criterion and a *relative systematic bias*, which we describe below. For a given waveform model (including the LISA response) $h(\theta)$, the match $\mathcal{M}(\theta_1, \theta_2)$ quantifies the distance between two points θ_1 and θ_2 in parameter space, with the mismatch given by $1 - \mathcal{M}(\theta_1, \theta_2)$. In a given channel i , the match is expressed as

$$\mathcal{M}(\theta_1, \theta_2) = \max_{\delta t, \delta \phi} \frac{(h_i(\theta_1) | h_i(\theta_2))_i}{\|h_i(\theta_1)\|_i \|h_i(\theta_2)\|_i}, \quad (3)$$

where $\|u_i\|_i^2 = (u_i | u_i)_i$ defines the norm squared with the inner products defined in Eq. (2). Following [69], the match is maximized over overall phase and time shifts, which are needed to align the waveforms. When computing the match across all channels, denoted as $\mathcal{M}_{\text{net}}$, we sum over each inner product as discussed in [70].

Using the maximum likelihood estimate (MLE) $\theta_{\text{MLE,ref}}$ obtained with $T_{\text{obs}} = 0.5$ yr as a reference, we compute the matches (using the vacuum EMRI waveform templates) to θ_{MLE} as obtained from other observation durations. We denote these matches by $\mathcal{M}_{\text{net}, T_{\text{obs}}} \equiv \mathcal{M}_{\text{net}}(\theta_{\text{MLE,ref}}, \theta_{\text{MLE}})$ for convenience. In each case, to compute the match, we generate the waveforms using an observation duration of $T_{\text{obs}} = 0.5$ yr, thus ensuring that their duration matches that of the reference. Based on [71–76], we consider the Bayesian inference to be significantly inconsistent when the mismatch satisfies

$$1 - \mathcal{M}_{\text{net}, T_{\text{obs}}} > \frac{\chi_{D, 90\%}^2}{2\rho_{\text{ref}}^2}, \quad (4)$$

with $\rho_{\text{ref}} = (\sum_i \|h_i(\theta_{\text{MLE,ref}})\|_i^2)^{1/2}$ being the optimal SNR [70] of the reference waveform, and $\chi_{D, 90\%}^2$ being the 90% quantile for the chi-square distribution with $D = \dim(\theta)$ degrees of freedom. For $D = 10$, we have $\rho_{\text{ref}} = 99$ and $\chi_{D, 90\%}^2 = 16$, resulting in threshold mismatch of 8.3×10^{-4} . We also assess the inconsistency in the

TABLE I. Mismatches and systematic biases relative to the reference MLE at $T_{\text{obs}} = 0.5$ yr. The SNR for 3 yr of observation is 100.

T_{obs} (yr)	$1 - \mathcal{M}_{\text{net}, T_{\text{obs}}}$	$\delta \log_{10} M / M_{\odot}$	δa
1.0	9.0×10^{-5}	0.10	0.10
1.5	4.8×10^{-4}	1.60	1.60
2.0	1.4×10^{-3}	2.58	2.60
2.5	1.6×10^{-3}	3.13	3.17
3.0	3.2×10^{-3}	3.10	3.14

MLEs by computing the relative systematic bias as

$$\delta\theta^\alpha \equiv \frac{|\theta_{\text{ML,ref}}^\alpha - \theta_{\text{ML}}^\alpha|}{\Sigma_{90\%}^\alpha}, \quad (5)$$

where α is the parameter index, and $\Sigma_{90\%}^\alpha$ is the parameter’s 90% credible interval for the reference case $T_{\text{obs}} = 0.5$ yr. With a large mismatch satisfying Eq. (4), one would typically expect significant biases in all parameters, with $\delta\theta^\alpha > 1$. However, $\delta\theta^\alpha > 1$ for a particular parameter does not imply that Eq. (4) is satisfied. For this reason, we primarily use Eq. (4) as the main quantitative metric to identify inconsistent inferences, and report the corresponding relative systematic biases for a few key intrinsic parameters such as $\log_{10} M$ and a .

In Table I, we show the mismatches with the reference MLE computed with Eq. (3), along with the relative systematic biases $\delta \log_{10} M / M_{\odot}$ and δa for each observation duration. We find that for $T_{\text{obs}} \leq 1.5$ yr, the mismatch is below the threshold [as given in Eq. (4)]. For a longer duration $T_{\text{obs}} = 2$ yr, the mismatch exceeds the threshold, indicating a significant inconsistency. For $T_{\text{obs}} \geq 2.5$ yr, the inconsistency is so significant that the probability that it is induced by noise is $\leq 5 \times 10^{-4}$, which corresponds to $\delta \log_{10} M / M_{\odot} > 3$ and $\delta a > 3$. Furthermore, in Supplemental Material [46] we verify that our results are robust to different noise realizations. Our quantitative analysis with Eqs. (4) and (5) therefore confirms the results show in Figs. 1 and 2.

Discussion—Detecting and discriminating environmental effects and/or deviations from GR in EMRI signals is an important challenge for LISA data analysis. While there are efforts underway aimed at accurately modeling specific environmental effects (due, e.g., to gas or dark matter), there is still a large uncertainty in their physics. Moreover, accurate models for environmental effects will necessarily involve several additional parameters, which may render their implementation in the LISA global fit [77–82] difficult in practice. In this Letter, we have therefore developed a self-consistency check solely based on vacuum GR EMRI waveforms. Our approach tests inconsistencies/biases in the posteriors as inferred from different portions of the signal, which can reveal missing physics in the

templates, due, e.g., to environmental effects and/or possible beyond-GR effects. For an injected EMRI signal affected by disk-driven migration, we showed that the parameter posteriors are significantly inconsistent, when inferred from different portions of the signal, for observations of more than two years.

Crucially, our model relies on the successful subtraction of louder sources in the LISA global fit. Indeed, residual power from imperfect subtraction of overlapping sources (e.g., Galactic binaries [56]) may be flagged by our test [46], we consider a slight mismodeling of the galactic noise, and find that it does not result in significant inconsistencies across observation times. We will explore a more rigorous treatment of this problem in future work. Here, we stress that our method can also be viewed as a consistency test for the “goodness” of the LISA global fit.

In the main text of this Letter, we have focused on disk-driven migration to showcase our self-consistency test. However, in Supplemental Material [46] we validate our results using two additional environmental effects (entering at -5.5 PN and -4 PN relative to the GW flux) and for a different set of source parameters. Additionally, we also consider an effect that contributes at 1PN order, in contrast to the negative-PN environmental effects, to demonstrate the broad applicability of the self-consistency test. An important data-analysis question concerns how to interpret the outcome of the self-consistency test. If the test fails, does this necessarily imply that the neglected physical effect would become measurable once it is explicitly modeled? Conversely, even if the test is successfully passed, could a neglected physical effect still be present but remain hidden through degeneracies with the vacuum EMRI parameters (a form of stealth bias [83,84])? Answering these questions requires a more systematic investigation of the performance of our self-consistency test across a wider range of environmental effects and over a broader region of the EMRI parameter space (for example including eccentricity). Such a study is beyond the proof-of-principle scope of this Letter and is left for future work.

We also emphasize that our approach identifies failures of the vacuum model but does not, by itself, identify which effect is missing or mitigate the parameter biases that arise. A common approach to addressing this is to introduce additional phenomenological parameters into the model, which can potentially mitigate the biases in vacuum EMRI parameters (for similar applications, see Refs. [85–92]), and also help identify missing physical effects. Finally, we note that our self-consistency test complements other methods such as the inspiral-merger-ringdown test [93,94] or signal reconstruction using unmodeled searches, which can also potentially be used to identify missing physical effects [95–97].

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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