

Supplemental Materials

Appendix A: Numerical bootstrap details

We remind the reader that representations of $O(2) \cong U(1) \rtimes \mathbb{Z}_2$ can be induced from the 1 dimensional complex representations of $U(1)$, labeled by $q \in \mathbb{Z}$. The $q = 0$ representation induces the trivial representation $\mathbf{0}^+$ (singlet) and the sign representation $\mathbf{0}^-$ (odd under $\mathbb{Z}_2$), both of which are 1 dimensional. The $q > 0$ representation induces a 2 dimensional representation $\mathbf{q}$, isomorphic to $q \oplus (-q)$ as $U(1)$ representation and where $\mathbb{Z}_2$ acts by swapping the two subspaces. Our bootstrap setup involves all 4-point functions of scalar primary operators in the $\mathbf{0}^+, \mathbf{1}, \mathbf{2}$ representations, which we denote by $\varphi_0, \varphi_1, \varphi_2$ respectively. Primary operators $\mathcal{O}_{\mathbf{r}, \Delta, \ell}$ exchanged in the conformal block decomposition are in representations of $O(2)$ determined by the tensor product decomposition of the representations of the possible pairs of external operators. These are

$$\begin{aligned} \mathbf{0}^+ \otimes \mathbf{0}^+ &= \mathbf{0}^+, & \mathbf{1} \otimes \mathbf{1} &= \mathbf{0}^+ \oplus \mathbf{0}^- \oplus \mathbf{2}, & \mathbf{2} \otimes \mathbf{2} &= \mathbf{0}^+ \oplus \mathbf{0}^- \oplus \mathbf{4}, \\ \mathbf{0}^+ \otimes \mathbf{1} &= \mathbf{1}, & \mathbf{0}^+ \otimes \mathbf{2} &= \mathbf{2}, & \mathbf{1} \otimes \mathbf{2} &= \mathbf{1} \oplus \mathbf{3}. \end{aligned} \quad (\text{A1})$$

Moreover, permutation symmetry of 3-point function implies that in the $\varphi_0 \times \varphi_0$, $\varphi_1 \times \varphi_1$ and $\varphi_2 \times \varphi_2$ OPEs the primary operators $\mathcal{O}_{\mathbf{0}^+, \Delta, \ell}$, $\mathcal{O}_{\mathbf{2}, \Delta, \ell}$ and $\mathcal{O}_{\mathbf{4}, \Delta, \ell}$ are exchanged with even ℓ , while $\mathcal{O}_{\mathbf{0}^-, \Delta, \ell}$ are exchanged with odd ℓ . Instead, in the $\varphi_0 \times \varphi_1$, $\varphi_0 \times \varphi_2$ and $\varphi_1 \times \varphi_2$ OPEs the primary operators $\mathcal{O}_{\mathbf{1}, \Delta, \ell}$, $\mathcal{O}_{\mathbf{2}, \Delta, \ell}$ and $\mathcal{O}_{\mathbf{3}, \Delta, \ell}$ are exchanged with any ℓ . The “external” operators appear themselves in the OPEs considered: φ_0 appears in $\varphi_0 \times \varphi_0$, $\varphi_1 \times \varphi_1$ and $\varphi_2 \times \varphi_2$ with OPE coefficients λ_{000} , λ_{110} and λ_{220} respectively; φ_1 appears in $\varphi_0 \times \varphi_1$ with coefficient $\lambda_{101} = \lambda_{110}$; φ_2 appears in $\varphi_1 \times \varphi_1$ and $\varphi_0 \times \varphi_2$ with coefficients λ_{112} and $\lambda_{202} = \lambda_{220}$ respectively.

When sitting at points that saturate bootstrap bounds, i.e. when we minimize Δ_0 at fixed Δ_1 , it’s possible to obtain approximate solutions to the set of crossing equations using the Extremal Functional Method (EFM) [1–3]. The EFM gives a non-rigorous estimate of the low lying spectrum and OPE coefficients of operators exchanged in OPEs involved in the crossing equations. With our bootstrap setup we can thus access the following operators: $\mathcal{O}_{\mathbf{0}^+, \Delta, \ell}$ and $\mathcal{O}_{\mathbf{4}, \Delta, \ell}$ with ℓ even; $\mathcal{O}_{\mathbf{0}^-, \Delta, \ell}$ with ℓ odd; $\mathcal{O}_{\mathbf{1}, \Delta, \ell}$, $\mathcal{O}_{\mathbf{2}, \Delta, \ell}$ and $\mathcal{O}_{\mathbf{3}, \Delta, \ell}$ with any ℓ . The estimates from EFM are subject to a dependence on the dimension of the space of derivative functionals used in the bootstrap bound (parametrized by Λ) and typically we can obtain a reliable estimate only for the lowest twist operators. See the Supplementary Material for the Λ -dependence of our bootstrap estimates of scaling dimensions.

The crossing equations we used are a subset of those already reported in [4], to which we refer to for conventions on the OPE coefficients.

The numerical bootstrap computations of this work were carried out using the `simpleboot` package [5]. The computation of the conformal blocks was done using `scalar_blocks`, while the semi-definite optimization was done using `SDPB` [6]. The feasibility bounds for scaling dimensions are obtained with a delaunay triangulation in scaling dimensions and by performing a scan over the OPE ratios of external operators using the cutting surface algorithm introduced in [7]. The constrained minimization of Δ_0 at fixed Δ_1 in the 5-dimensional space $(\Delta_0, \Delta_2, \lambda_{000}/\lambda_{110}, \lambda_{220}/\lambda_{110}, \lambda_{112}/\lambda_{110})$, reported in Tables III, V and VII, was carried out with the `Navigator` method, using a modified BFGS algorithm introduced in [8]. Both the cutting surface and the BFGS algorithm are implemented in `simpleboot`.

The parameters used in the numerical computation of the conformal blocks are reported in Table I for the various values of the bootstrap parameter Λ we explored. There, S_Λ denotes the following sets of spins included in the bootstrap constraints:

$$\begin{aligned} S_{19} &= \{0, \dots, 26\} \cup \{49, 52\}, \\ S_{23} &= \{0, \dots, 30\} \cup \{38, 49, 52\}, \\ S_{25} &= \{0, \dots, 30\} \cup \{38, 49, 52\}, \\ S_{27} &= \{0, \dots, 31\} \cup \{33, 34, 37, 38, 41, 42, 45, 46, 49, 50\}, \\ S_{29} &= \{0, \dots, 31\} \cup \{33, 34, 37, 38, 41, 42, 45, 46, 49, 50, 55, 56\}, \\ S_{31} &= \{0, \dots, 31\} \cup \{33, 34, 37, 38, 41, 42, 45, 46, 49, 50, 55, 56, 59, 60\}. \end{aligned} \quad (\text{A2})$$

The parameters relevant for the semi-definite optimization are reported in Table II, both for the feasibility bounds and the Navigator run. For the feasibility bounds, we assumed a bounding box on the OPE ratios for the cutting surface algorithm of $(\lambda_{000}/\lambda_{110}, \lambda_{220}/\lambda_{110}, \lambda_{112}/\lambda_{110}) \in [-10, 10] \times [-10, 10] \times [-10, 10]$.

For the extraction of the Δ_3 and Δ_4 scaling dimensions from the extremal functional, as well as for spinning operators, we used the `spectrum.py` package [9, 10].

Λ	spin-ranges	poles	order	precision
19	S_{19}	14	56	1024
21	S_{21}	18	72	1024
23	S_{23}	18	72	1024
25	S_{25}	18	72	1024
27	S_{27}	20	80	1024
29	S_{29}	22	88	1024
31	S_{31}	24	96	1024

TABLE I: Parameters of `scalar_blocks` used for the computation of the conformal blocks. See (A2) for the definition of the spin ranges.

Parameter	Feasibility	Navigator
<code>dualityGapThreshold</code>	10^{-20}	10^{-30}
<code>primalErrorThreshold</code>	10^{-60}	10^{-30}
<code>dualErrorThreshold</code>	10^{-60}	10^{-30}
<code>initialMatrixScalePrimal</code>	10^{20}	10^{20}
<code>initialMatrixScaleDual</code>	10^{20}	10^{20}
<code>maxComplementarity</code>	10^{100}	10^{100}
<code>maxIterations</code>	1000	1000
<code>precision</code>	1024	1024
<code>detectPrimalFeasibleJump</code>	true	false
<code>detectDualFeasibleJump</code>	true	false
<code>findPrimalFeasible</code>	false	false
<code>findDualFeasible</code>	false	false

TABLE II: Parameters of `SDPB` used for the semi-definite optimization.

Appendix B: Extended results for the CP^2 model

In this section we describe in more detail the results obtained for the CP^2 model, which corresponds to studying the crossing equations for $q = 0, 1, 2$ operators at $\Delta_1 = 0.755$, in accordance with the large N prediction. In the main text we showed the estimates for scaling dimension of the the lowest dimensional monopoles.

These estimates are obtained as follows. First we take the lowest dimension operator at multiple values of Λ , either as found using the EFM, or in the case of Δ_2 and the ratio of OPE coefficients, the values found through a rigorous minimization. Then we extrapolate our finite Λ results to infinite Λ assuming a linear fit in $1/\Lambda$. Linear convergence in $1/\Lambda$ has been previously observed in a wide range of observables (for sufficiently large Λ). Including a quadratic correction doesn't significantly improve the fit and does result in less physical behavior in the extrapolated regions. To illustrate the fitting procedure we show the fits used in extrapolation for the scalar monopole dimension in Figure 1. The data at finite values of Λ used for the extrapolations is also shown in Table III. The full data used for all the presented extrapolations can be found in the auxiliary file `QED3_data.nb`.

It should be noted that no rigorous errors can be assigned to the values extracted by the EFM. Instead we include a non-rigorous estimate of the error in the extrapolated value in the form of the variation of the extrapolated value when leaving up to 2 points out (this method of estimating the predictive value of a prediction based on finite set of points is known as *leave-p-out cross validation*, see for example Chapter 7.11 of [11]).

For some operators we interpret not the lowest but the second lowest operator detected in the EFM to be the true physical lowest operator. This is motivated because these spectra contain an extra operator very close to the unitary bound (and thus the assumed gap) with a significantly smaller OPE then the following operator. Moreover, this difference becomes larger as Λ increases. This spurious operator could be related to the *sharing effect* [3, 4]. Such an operator close to the unitary bound is also not expected to exist according to the large-charge expansion. The choice of fit and the alternative points that we could have chosen to fit are illustrated in Figure 2.

CP^2	$\Lambda = 19$	$\Lambda = 21$	$\Lambda = 23$	$\Lambda = 25$	$\Lambda = 27$	$\Lambda = 29$	$\Lambda = 31$	$\Lambda \rightarrow \infty$
Δ_0	1.507	1.519	1.528	1.535	1.540	1.544	1.547	1.608
Δ_2	1.848	1.847	1.847	1.846	1.846	1.845	1.845	1.841
Δ_3	3.209	3.204	3.203	3.198	3.199	3.196	3.195	3.173
Δ_4	5.130	5.137	5.050	5.026	5.012	4.975	4.952	4.653
$\lambda_{000}/\lambda_{110}$	0.182	0.200	0.213	0.225	0.234	0.241	0.246	0.344
$\lambda_{220}/\lambda_{110}$	1.766	1.775	1.782	1.786	1.789	1.792	1.794	1.836
$\lambda_{112}/\lambda_{110}$	1.695	1.698	1.700	1.701	1.703	1.703	1.704	1.717

TABLE III: Scaling dimensions and OPE ratios for the lowest dimensional scalars for $\Delta_1 = 0.755$ (corresponding to the CP^2 model) at various Λ values, together with the $\Lambda \rightarrow \infty$ fit. The values of Δ_0 are rigorous lower bounds. The values of Δ_2 and the OPE ratios correspond to the point on the boundary of the 5-dimensional allowed region ($\Delta_0, \Delta_2, \lambda_{000}/\lambda_{110}, \lambda_{220}/\lambda_{110}, \lambda_{112}/\lambda_{110}$) obtained by minimizing Δ_0 at fixed Δ_1 . The values of Δ_3 and Δ_4 are non-rigorous estimates from truncated crossing equations.

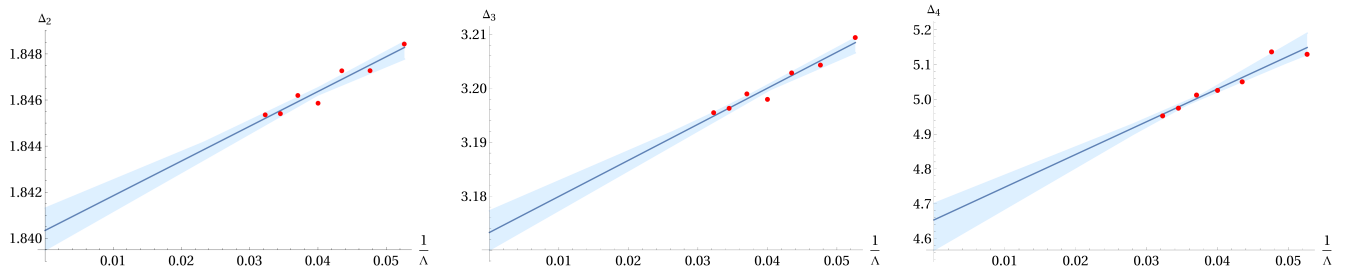


FIG. 1: Linear least-squares fits in $1/\Lambda$ of the dimension of the lowest scalar monopoles in the CP^2 model. In the case of Δ_4 we ignore a “fake” operator at the imposed gap of 3. These fits are used to obtain the extrapolations at $\Lambda \rightarrow \infty$ shown in the main text. A blue band indicate the range of fits that can be obtained by omitting up to 2 points. The red dots indicate the dimensions (obtained by EFM except for Δ_2 which is one of the variables in the search space).

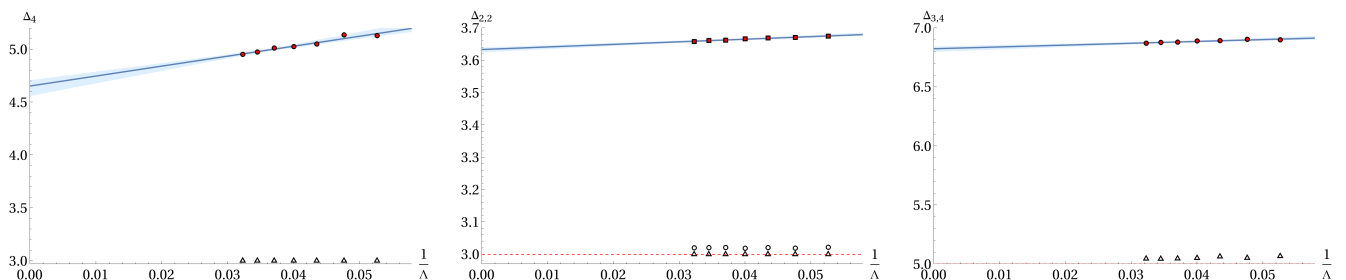


FIG. 2: Linear least-squares fits in $1/\Lambda$ of the suspected physical lowest dimensional operators in the CP^2 model. These fits are used to obtain the extrapolations at $\Lambda \rightarrow \infty$ shown in the main text. The operator dimensions obtained by EFM are plotted as black triangles (lowest dimensional operator), circles (next-lowest) and squares (next-to-next-to-lowest). The red dots indicate the dimensions used for extrapolations presented in the main text. Rather than using the lowest dimensional operators from the EFM as we did for the other sectors, in these case we treat operators at the imposed gaps or close to it as *spurious*. This is supported by the fact that the OPE coefficients of these operators near the unitarity bound are significantly smaller than the OPE coefficients of the fitted operator and are getting smaller still as Λ increases.

Appendix C: Results for the CP^3 and CP^4 models.

In this appendix we describe the results obtained for CP^3 and CP^4 models by analogous bootstrap methods, i.e. by setting Δ_1 to 0.88 and 1.005 (as predicted by the large N prediction). In Tables IV and VI we show the prediction of the lowest operator dimensions for scalars with charge q in the CP^3 and CP^4 models, respectively, and compare these against large N and lattice estimates. The bootstrap estimate is obtained by extrapolating our finite Λ results to infinite Λ according to the linear fits shown in Figure 3 and 4, as described in Appendix B. The data at finite values of Λ is included in Tables V and VII for CP^3 and CP^4 , respectively.

CP^3	Δ_1	Δ_2	Δ_3	Δ_4	Δ_0
Bootstrap	0.880*	2.12(1)	3.65(4)	5.8(2)	1.43(2)
Large N	0.880	2.12	3.64	5.40	—
Lattice [12]	0.85(1)	—	—	—	1.57(4)
Lattice [13]	0.865	—	—	—	1.60

TABLE IV: Comparison of scaling dimensions Δ_q of the lowest dimension scalar operators with $U(1)$ charge q in the CP^3 model, as determined from the bootstrap study here, the large N expansion for monopoles $q > 0$, and lattice studies. The asterisk by Δ_1 for bootstrap means we put it in to determine the others, while the bootstrap errors come from extrapolation in the bootstrap truncation parameter Λ .

CP^3	$\Lambda = 19$	$\Lambda = 21$	$\Lambda = 23$	$\Lambda = 25$	$\Lambda = 27$	$\Lambda = 29$	$\Lambda \rightarrow \infty$
Δ_0	1.303	1.317	1.329	1.336	1.342	1.347	1.432
Δ_2	2.161	2.157	2.151	2.151	2.148	2.149	2.123
Δ_3	3.766	3.753	3.737	3.735	3.727	3.728	3.647
Δ_4	6.734	6.669	6.542	6.510	6.436	6.435	5.795
$\lambda_{000}/\lambda_{110}$	0.096	0.103	0.108	0.112	0.115	0.118	0.161
$\lambda_{220}/\lambda_{110}$	1.636	1.645	1.650	1.656	1.658	1.662	1.711
$\lambda_{112}/\lambda_{110}$	1.633	1.635	1.643	1.644	1.647	1.648	1.681

TABLE V: Scaling dimensions and OPE ratios for the lowest dimensional scalars for $\Delta_1 = 0.88$ (corresponding to the CP^3 model) at various Λ values, together with the $\Lambda \rightarrow \infty$ fit. The values of Δ_0 are rigorous lower bounds. The values of Δ_2 and the OPE ratios correspond to the point on the boundary of the 5-dimensional allowed region ($\Delta_0, \Delta_2, \lambda_{000}/\lambda_{110}, \lambda_{220}/\lambda_{110}, \lambda_{112}/\lambda_{110}$) obtained by minimizing Δ_0 at fixed Δ_1 . The values of Δ_3 and Δ_4 are non-rigorous estimates from truncated crossing equations.

CP^4	Δ_1	Δ_2	Δ_3	Δ_4	Δ_0
Bootstrap	1.005*	2.58(2)	4.6(4)	3.5(4)	1.154(5)
Large N	1.005	2.43	4.18	6.21	–
Lattice rect. [14]	0.85(10)	–	–	–	1.15
Lattice honeyc. [14]	1.0(1)	–	–	–	1.46

TABLE VI: Comparison of scaling dimensions Δ_q of the lowest dimension scalar operators with $U(1)$ charge q in the CP^4 model, as determined from the bootstrap study here, the large N expansion for monopoles $q > 0$, and lattice studies (on rectangular and honeycomb lattices). The asterisk by Δ_1 for bootstrap means we put it in to determine the others, while the bootstrap errors come from extrapolation in the bootstrap truncation parameter Λ .

CP^4	$\Lambda = 19$	$\Lambda = 21$	$\Lambda = 23$	$\Lambda = 25$	$\Lambda = 27$	$\Lambda = 29$	$\Lambda = 31$	$\Lambda \rightarrow \infty$
Δ_0	1.063	1.071	1.080	1.086	1.091	1.094	1.098	1.154
Δ_2	2.534	2.552	2.539	2.547	2.545	2.554	2.553	2.576
Δ_3	–	3.331	–	4.575	4.558	4.588	4.569	4.589
Δ_4	–	4.129	10.348	3.787	3.746	3.758	3.731	3.534
$\lambda_{000}/\lambda_{110}$	–0.830	–0.918	–0.972	–1.051	–1.072	–1.131	–1.106	–1.615
$\lambda_{220}/\lambda_{110}$	1.567	1.561	1.560	1.557	1.563	1.568	1.569	1.571
$\lambda_{112}/\lambda_{110}$	2.892	2.979	3.003	3.065	3.058	3.099	3.065	3.396

TABLE VII: Scaling dimensions and OPE ratios for the lowest dimensional scalars for $\Delta_1 = 0.1005$ (corresponding to the CP^4 model) at various Λ values, together with the $\Lambda \rightarrow \infty$ fit. The values of Δ_0 are rigorous lower bounds. The values of Δ_2 and the OPE ratios correspond to the point on the boundary of the 5-dimensional allowed region ($\Delta_0, \Delta_2, \lambda_{000}/\lambda_{110}, \lambda_{220}/\lambda_{110}, \lambda_{112}/\lambda_{110}$) obtained by minimizing Δ_0 at fixed Δ_1 . The values of Δ_3 and Δ_4 are non-rigorous estimates from truncated crossing equations.

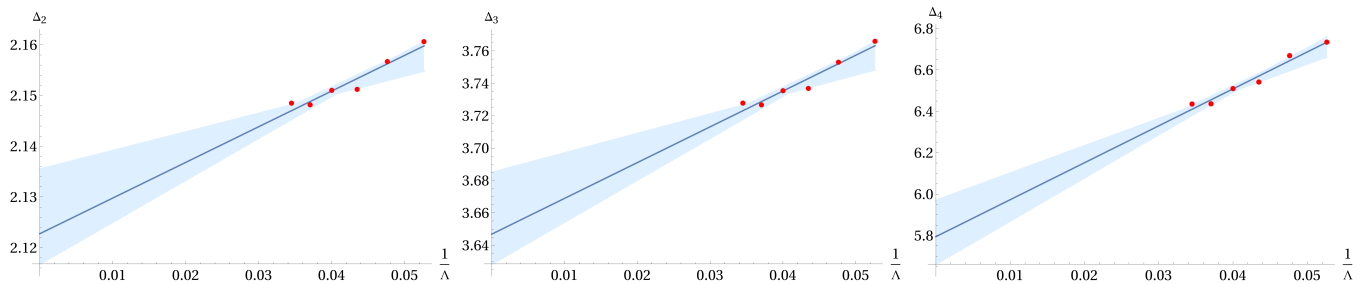


FIG. 3: Linear least-squares fits in $1/\Lambda$ of the dimension of the lowest scalar monopoles in the CP^3 model (excluding “fake” operators at the imposed gap, when present). These fits are used to obtain the extrapolations at infinite Λ shown in Table IV. To give an estimate of the stability of the fit and its predictive accuracy a blue band indicate the range of fits that can be obtained by omitting up to 2 points. The red dots indicate the dimensions (obtained by EFM except for Δ_2 which is one of the variables in the search space).

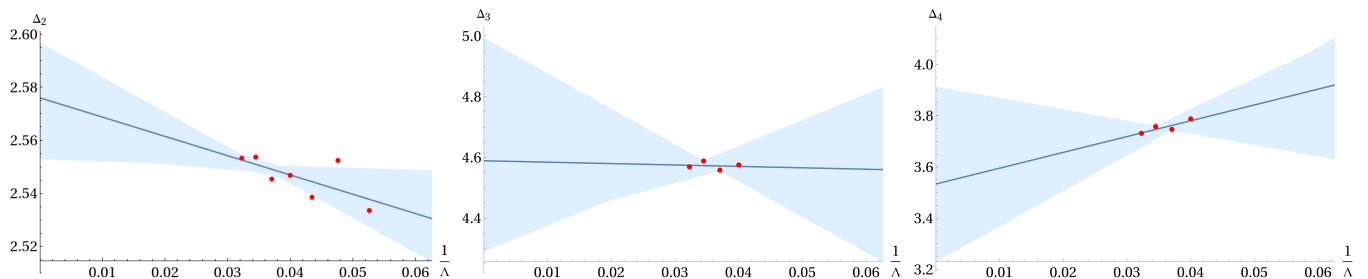


FIG. 4: Linear least-squares fits in $1/\Lambda$ of the dimension of the lowest scalar monopoles in the CP^4 model (excluding “fake” operators at the imposed gap, when present). These fits are used to obtain the extrapolations at infinite Λ shown in Table VI. To give an estimate of the stability of the fit and its predictive accuracy a blue band indicate the range of fits that can be obtained by omitting up to 2 points. The red dots indicate the dimensions (obtained by EFM except for Δ_2 which is one of the variables in the search space). Some figures contain less points since some operators are not detected at for all values of Λ .

Appendix D: Comparison of bootstrap to large q for critical $O(2)$

In [15], the scaling dimensions of scalar operators in the critical $O(2)$ model were computed for $q \leq 16$ using lattice Monte Carlo. These values were fit to the general large q expansion in the main text for $\ell = 0$ to get the coefficients

$$c_{\frac{3}{2}} = 0.3371, \quad c_{\frac{1}{2}} = 0.2658. \quad (D1)$$

We show the predictions from the large q expansion with these value of the coefficients for $q = 1, 2, 3, 4$ and $\ell = 0, 2, 3, 4$ in Table VIII. Note that just as for the CP^2 model, for some values of $\ell > q$ we observe violations of the unitarity bound, which suggests the expansion is not good for these small values of q .

The conformal bootstrap was used in [4] to compute scaling dimensions of operators in the critical $O(2)$, using the same set of correlators of $q = 0, 1, 2$ scalar operators considered in this paper, except with assumptions on the spectrum suitable to the critical $O(2)$ model. In Table IX, we summarize the data from this bootstrap study for $q = 1, 2, 3, 4$ and $\ell \leq 4$, where recall that odd spins do not appear for $q = 4$ in the correlators considered in this bootstrap. Some of this data is known to more digits of precision than shown here, but for the purpose of the comparison to large q , we find that the 4 digits is more than enough. As one can see from comparing Tables VIII and IX, the data from each method matches for $\ell \leq q$.

Note that [4] did not give data for $q = 3$ odd ℓ or $q = \ell = 1$, so we reran those values ourselves.

Large charge	$\ell = 0$	$\ell = 2$	$\ell = 3$	$\ell = 4$
$q = 1$	0.509	2.241	2.958	3.672
$q = 2$	1.236	2.968	3.685	4.399
$q = 3$	2.118	3.851	4.568	5.281
$q = 4$	3.135	4.867	5.584	6.297

TABLE VIII: Scaling dimensions of the lowest dimension scalar operators in the critical $O(2)$ model as computed from the large q effective theory for general $\ell \neq 1$ with coefficients fixed by Monte Carlo for $\ell = 0$. The red values for $\ell > q$ are where this small ℓ expansion is observed to be less accurate.

Bootstrap	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$
$q = 1$	0.519	2.950	3.650	4.615	5.700
$q = 2$	1.236	5.800	3.015	5.766	5.030
$q = 3$	2.106	2.078	3.884	4.582	5.852
$q = 4$	3.115	–	4.893	–	6.730

TABLE IX: Scaling dimensions of the lowest dimension scalar operators in the critical $O(2)$ model as computed from the conformal bootstrap.

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- [1] D. Poland and D. Simmons-Duffin, “Bounds on 4D Conformal and Superconformal Field Theories,” *JHEP* **05** (2011) 017, 1009.2087.
 - [2] S. El-Showk and M. F. Paulos, “Bootstrapping Conformal Field Theories with the Extremal Functional Method,” *Phys. Rev. Lett.* **111** (2013), no. 24 241601, 1211.2810.
 - [3] D. Simmons-Duffin, “The Lightcone Bootstrap and the Spectrum of the 3d Ising CFT,” *JHEP* **03** (2017) 086, 1612.08471.
 - [4] J. Liu, D. Meltzer, D. Poland, and D. Simmons-Duffin, “The Lorentzian inversion formula and the spectrum of the 3d $O(2)$ CFT,” *JHEP* **09** (2020) 115, 2007.07914. [Erratum: *JHEP* 01, 206 (2021)].
 - [5] N. Su. simpleboot: A mathematica framework for bootstrap calculations.
 - [6] W. Landry and D. Simmons-Duffin, “Scaling the semidefinite program solver SDPB,” 1909.09745.
 - [7] S. M. Chester, W. Landry, J. Liu, D. Poland, D. Simmons-Duffin, N. Su, and A. Vichi, “Carving out OPE space and precise $O(2)$ model critical exponents,” 1912.03324.

- [8] M. Reehorst, S. Rychkov, D. Simmons-Duffin, B. Sirois, N. Su, and B. van Rees, “Navigator Function for the Conformal Bootstrap,” *SciPost Phys.* **11** (2021) 072, 2104.09518.
- [9] Z. Komargodski and D. Simmons-Duffin, “The Random-Bond Ising Model in 2.01 and 3 Dimensions,” *J. Phys. A* **50** (2017), no. 15 154001, 1603.04444.
- [10] <https://gitlab.com/bootstrapcollaboration/spectrum-extraction>.
- [11] T. Hastie, R. Tibshirani, and J. Friedman, *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. Springer, New York, 2 ed., 2009.
- [12] J. Lou, A. W. Sandvik, and N. Kawashima, “Antiferromagnetic to valence-bond-solid transitions in two-dimensional SU(N) Heisenberg models with multispin interactions,” *Phys. Rev. B* **80** (Nov., 2009) 180414, 0908.0740.
- [13] K. Harada, T. Suzuki, T. Okubo, H. Matsuo, J. Lou, H. Watanabe, S. Todo, and N. Kawashima, “Possibility of deconfined criticality in SU(N) Heisenberg models at small N,” *Phys. Rev. B* **88** (Dec., 2013) 220408, 1307.0501.
- [14] M. S. Block, R. G. Melko, and R. K. Kaul, “Fate of CP^{N-1} Fixed Points with q Monopoles,” *Phys. Rev. Lett.* **111** (Sept., 2013) 137202, 1307.0519.
- [15] D. Banerjee, S. Chandrasekharan, and D. Orlando, “Conformal dimensions via large charge expansion,” *Phys. Rev. Lett.* **120** (2018), no. 6 061603, 1707.00711.