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To cite this article: Pasquale Calabrese *J. Stat. Mech.* (2026) 034002

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The quantum Mpemba effect in closed systems: from theory to experiment

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Received 5 February 2026
Accepted for publication 18 February 2026
Published 13 March 2026



Online at stacks.iop.org/JSTAT/2026/034002
<https://doi.org/10.1088/1742-5468/ae4bb6>

Abstract. The Mpemba effect refers to a counterintuitive phenomenon whereby a system initially prepared further from equilibrium may relax faster than one prepared closer to equilibrium. While extensively studied in classical nonequilibrium physics, its extension to isolated quantum systems only started in the last few years. In this contribution we review recent progress on the quantum Mpemba effect in closed many-body systems, emphasizing the role of reduced density matrix, entanglement and symmetry restoration. We discuss why and how the entanglement asymmetry provides a natural and experimentally accessible framework to characterize Mpemba-like behavior in unitary quantum evolution.

Keywords: entanglement in extended quantum systems, quantum quenches, cold atoms

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1. Introduction

Nonequilibrium relaxation is a central theme in statistical physics. Even seemingly elementary questions, such as how fast a system approaches equilibrium, may lead to highly nontrivial and sometimes counterintuitive answers. A paradigmatic example is the Mpemba effect, named after Erasto B. Mpemba, who reported that, under certain conditions, hot water may freeze faster than cold water [1]. Remarkably, qualitative accounts of this phenomenon can already be traced back to ancient Greek writings, including Aristotle [2], and it is famously said to have been discussed informally during a dinner at Los Alamos [3]. Despite decades of experimental and theoretical investigation, the microscopic mechanisms underlying the Mpemba effect remain subtle, highly system-dependent, and far from universally understood.

From a modern viewpoint, the Mpemba effect highlights the limitations of describing nonequilibrium states through a single thermodynamic parameter, such as temperature. Indeed, if a time-dependent state can be assigned a well-defined temperature at each instant, cooling is expected to proceed incrementally, passing through all intermediate temperatures. Within such a quasi-equilibrium picture, a colder system should always reach a low-temperature equilibrium state faster than a hotter one. The Mpemba effect directly contradicts this intuition and signals a complete breakdown of equilibrium-based reasoning. Rather than being characterized by temperature alone, relaxation is governed by the full structure of the nonequilibrium state and by its projection onto the relevant relaxation modes. Several mechanisms leading to Mpemba-like behavior have been identified in classical systems, including the presence of multiple relaxation timescales, nontrivial energy landscapes, and strong coupling to the environment. A particularly illuminating example is provided by the experiment of Kumar and Bechhoefer

[4], where the Mpemba effect was observed for a single Markovian particle coupled to an effective thermal bath. In this context, the phenomenon admits a clear interpretation within the theory of Markovian dynamics [5], where the effect originates from the overlap of initial conditions with slow relaxation modes.

The extension of these ideas to quantum systems is especially intriguing. A natural first approach is to consider temperature quenches in open quantum systems, a direction that has been extensively explored theoretically (see the comprehensive reviews [6, 7]). Very recently, the first experimental observations of the Mpemba effect in an open quantum system have been reported using a single trapped-ion qubit [8, 9].

In this study, however, we focus on a conceptually different setting of closed quantum many-body systems. These systems evolve unitarily and never relax to thermal equilibrium at the level of the full density matrix. Thermalization, when it occurs, emerges only locally, through the relaxation of subsystems and of local observables. This fundamental difference raises a natural and nontrivial question: can a genuine quantum analog of the Mpemba effect exist in closed systems, and if so, how should it be properly defined and diagnosed?

2. Symmetry restoration and entanglement asymmetry

We focus on a global quantum quench protocol [10] in which a many-body quantum system is initialized in a *pure, far-from-equilibrium state* $|\psi_0\rangle$ and subsequently evolves under unitary dynamics:

$$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle. \quad (2.1)$$

Although the global state remains pure at all times and therefore never relaxes to a stationary density matrix, large quantum systems may exhibit local relaxation. In particular, the reduced density matrix of a finite subsystem A ,

$$\rho_A(t) = \text{Tr}_{\bar{A}}[|\psi(t)\rangle\langle\psi(t)|], \quad (2.2)$$

can approach at long times a stationary form $\rho_{A,\text{se}}$, corresponding to the restriction of an effective statistical ensemble ρ_{se} to the subsystem. The nature of this emergent ensemble depends on the dynamical properties of the system. For ergodic systems satisfying the eigenstate thermalization hypothesis, ρ_{se} is a thermal Gibbs ensemble [11–13], whereas for integrable systems it is described by a generalized Gibbs ensemble that accounts for the full set of conserved quantities [14, 15]. The von Neumann entropy of the reduced density matrix, $S_A = -\text{Tr}\rho_A \log \rho_A$, coincides with the entanglement entropy between the subsystem A and its complement. A particularly striking consequence of local relaxation is that, since $\rho_A(t)$ converges to $\rho_{A,\text{se}}$ and the entropy of the stationary ensemble is extensive, the thermodynamic entropy of ρ_{se} can be identified with the entanglement generated dynamically within subsystems during the unitary evolution [16]. This correspondence between thermodynamic entropy and entanglement growth has also been directly tested in a pioneering cold-atom experiment [17].

Our goal is to investigate the quantum Mpemba effect in situations where the initial state explicitly breaks a symmetry that is preserved by the Hamiltonian governing the time evolution but is nevertheless restored *locally* as the system relaxes. Throughout this study we restrict our attention to 1D systems. In this setting, the restoration of symmetry admits a natural interpretation in light of the Mermin–Wagner theorem, which forbids the spontaneous breaking of symmetries at finite temperature in 1D. After a global quench, the finite energy density of the initial state plays the role of an effective temperature for local observables, and therefore generically leads to symmetry restoration in the local steady state, with only a few exceptions associated with special conservation laws [18].

Within this framework, the quantum Mpemba effect manifests itself when initial states that break the symmetry more strongly end up restoring it more rapidly at the level of subsystems. To make this statement precise, one needs an experimentally accessible quantity that characterizes the degree of symmetry breaking in an arbitrary subsystem. This role is played by the *entanglement asymmetry* [19]. We briefly recall its definition. Let Q denote a conserved local charge generating, for simplicity, a $U(1)$ symmetry. Locality implies that for any bipartition one can write $Q = Q_A + Q_{\bar{A}}$ (a paradigmatic example being the longitudinal magnetization $\sum_j \sigma_j^z/2$ in a spin chain). If the reduced density matrix ρ_A is symmetric, it commutes with the local charge, $[\rho_A, Q_A] = 0$, and therefore admits a block-diagonal decomposition into sectors labeled by the eigenvalues of Q_A . This decomposition underlies the notion of symmetry-resolved entanglement [20]. In contrast, if the state breaks the symmetry, ρ_A has off-diagonal matrix elements between different charge sectors, which provide a direct measure of the degree of symmetry breaking. Starting from ρ_A , one can define its symmetrized counterpart by projecting onto the charge sectors,

$$\rho_{A,Q} = \sum_q \Pi_q \rho_A \Pi_q, \quad (2.3)$$

where Π_q denotes the projector onto the subspace with charge q in the subsystem A . The entanglement asymmetry is then defined as the difference between the von Neumann entropies:

$$\Delta S_A = S(\rho_{A,Q}) - S(\rho_A). \quad (2.4)$$

This quantity is nonnegative and vanishes if and only if $\rho_{A,Q} = \rho_A$. In this sense, it provides a faithful measure of symmetry breaking at the level of reduced density matrices, a property that can also be established within the framework of quantum resource theory [21–23].

In addition to the von Neumann case, one can introduce Rényi versions of the entanglement asymmetry:

$$\Delta S_A^{(n)} = \frac{1}{1-n} [\log \text{Tr} \rho_{A,Q}^n - \log \text{Tr} \rho_A^n]. \quad (2.5)$$

These quantities were originally introduced as technical tools to facilitate analytical and numerical calculations [19] but were later recognized as particularly well suited for

experimental measurements. Like their von Neumann counterpart, the Rényi entanglement asymmetries are positive definite and vanish if and only if the reduced density matrix is symmetric.

The time dependence of the entanglement asymmetry therefore provides a direct probe of the rate at which symmetry is restored at the level of subsystems, and hence of the possible emergence of a quantum Mpemba effect. In the following section we exploit this idea explicitly. We first present theoretical results for a simple 1D spin-chain, and subsequently discuss their experimental implementation and observation.

3. Quantum Mpemba effect

3.1. Quantum Mpemba effect in a simple spin-chain

We begin by considering a fully polarized ferromagnetic state in a 1D spin-chain, in which all spins are aligned along a direction forming an angle θ with the z axis. This state can be written as:

$$|\psi(\theta)\rangle = \exp\left(i\frac{\theta}{2}\sum_j\sigma_j^x\right)|\uparrow\uparrow\cdots\rangle. \quad (3.1)$$

We focus on the symmetry under rotations about the z axis, generated by the charge $Q = \frac{1}{2}\sum_j\sigma_j^z$. For $\theta=0$, the state is invariant under this symmetry, and the entanglement asymmetry vanishes identically. In contrast, for $\theta\neq 0$ the symmetry is explicitly broken and the asymmetry acquires a finite value. For a subsystem of length ℓ , the Rényi entanglement asymmetry can be computed exactly by elementary combinatorial arguments, yielding the following [19]:

$$\Delta S_A^{(n)} = \frac{1}{1-n}\log\left[\cos^{2n\ell}\left(\frac{\theta}{2}\right)\sum_{p=0}^{\ell}\binom{\ell}{p}^n\tan^{2np}\left(\frac{\theta}{2}\right)\right]. \quad (3.2)$$

As a function of the tilting angle θ , this expression exhibits a bell-shaped profile. For instance, in the von Neumann limit $n\rightarrow 1$, the asymmetry vanishes at $\theta=0$ and $\theta=\pi$, while reaching a maximum at $\theta=\pi/2$. This behavior is fully consistent with the interpretation of $\Delta S_A^{(n)}$ as a measure of symmetry breaking.

We now turn to the unitary time evolution of the state $|\psi(\theta)\rangle$ under a Hamiltonian that preserves the charge Q . A particularly convenient choice is the XX spin chain,

$$H = -\frac{1}{4}\sum_{j=-\infty}^{\infty}(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y), \quad (3.3)$$

which can be mapped onto free fermions and therefore allows for an analytic treatment. For technical reasons, the initial state must be chosen as the symmetric superposition $|\psi_0\rangle = (|\psi(\theta)\rangle + |\psi(-\theta)\rangle)/\sqrt{2}$ [19].

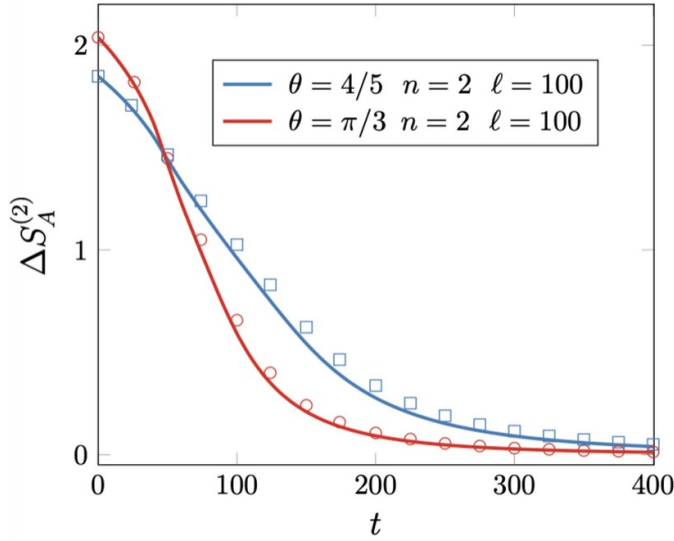


Figure 1. Time evolution of the entanglement asymmetry under the Hamiltonian (3.3), starting from two ferromagnetic initial states with different tilting angles θ , for a subsystem A of ℓ contiguous sites. Symbols denote exact numerical results, while solid lines correspond to the analytical large- ℓ asymptotics. Crossing of the two curves provides a clear signature of the quantum Mpemba effect.

Although the exact expression for the time-dependent asymmetry $\Delta S_A^{(n)}(t)$ is lengthy and not particularly transparent [19], its physical content is most clearly illustrated by the numerical and analytic results shown in figure 1, where the evolution is displayed for two different initial tilting angles. Several key features emerge. First, at long times, the asymmetry decays to zero, indicating local restoration of the symmetry. Second, the relaxation time depends strongly on the initial angle. For integer n , the long-time behavior ($t, \ell \rightarrow \infty$ with t/ℓ fixed) can be obtained analytically, yielding the following [19]:

$$\Delta S_A^{(n)}(t) \propto \left(1 + 8 \frac{\cos^2 \theta}{\sin^4 \theta}\right) \frac{\ell^4}{t^3}. \tag{3.4}$$

This result shows that larger subsystems require longer times to restore the symmetry and that the symmetry is never fully recovered in the limit of an infinite subsystem. More importantly, for our purposes, both figure 1 and equation (3.4) demonstrate unambiguously that states with stronger initial symmetry breaking restore the symmetry more rapidly. In other words, the farther the system is initially from equilibrium, the faster it relaxes. This is precisely the quantum Mpemba effect, whose most striking manifestation is the crossing of the entanglement asymmetry curves corresponding to different initial tilting angles, as can be seen in figure 1. The entanglement asymmetry is not the only observable capable of detecting the quantum Mpemba effect in closed systems; other diagnostics have been proposed and discussed, for instance, in [25, 47, 56].

A natural question at this stage is how robust these features are, and whether they rely crucially on the free-fermion structure of the XX model. To address this issue, we have performed extensive exact diagonalization studies starting from the same class of initial states $|\psi(\theta)\rangle$, but evolving them under a variety of Hamiltonians that preserve the charge Q , including both integrable and chaotic models, as well as systems with disorder (see also [24]). In all cases, we find clear and robust numerical evidence for the occurrence of the quantum Mpemba effect [19].

3.2. Quantum Mpemba effect in a trapped ion quantum simulator

Ultimately, physics is an experimental discipline, and any genuinely new nonequilibrium phenomenon calls for direct experimental validation. For isolated quantum systems, such a verification of the quantum Mpemba effect was achieved shortly after its theoretical proposal in a trapped-ion quantum simulator [25]. These platforms allow for the coherent control, manipulation and measurement of relatively large arrays of ions, which effectively realize spin-1/2 degrees of freedom. For the purpose of observing the quantum Mpemba effect, it is sufficient to consider a chain of 12 ions. The system is initialized in a product state with all spins polarized along the z direction, after which the spins are collectively rotated by an angle θ away from this axis. This rotation explicitly breaks the $U(1)$ symmetry associated with rotations about the z axis. The subsequent unitary evolution is governed by the native trapped-ion Hamiltonian, which features long-range spin–spin interactions and preserves the same $U(1)$ symmetry. In this sense, the dynamics is analogous to that generated by the XX Hamiltonian in equation (3.3), but with spatially extended couplings J_{ij} , decaying as a power law of the separation $|i - j|$. The tilting angle θ therefore controls the degree of symmetry breaking encoded in the initial state.

The Rényi-2 entanglement asymmetry can be experimentally accessed using randomized measurements and classical shadow techniques (see e.g. the review [26]). In the experiment, attention is focused on a subsystem A consisting of four ions, which may be chosen either contiguous or noncontiguous. The quality of state preparation and the reliability of the measurement protocol are independently verified by comparing the experimentally measured initial asymmetry with its theoretical prediction, as shown in the inset of figure 2. By preparing initial states with different values of the tilting angle θ , the experiment reveals a clear crossing in the time evolution of the entanglement asymmetry, displayed in the main panel of figure 2. This crossing provides direct experimental evidence of the quantum Mpemba effect; states that break the symmetry more strongly are observed to restore it more rapidly. Remarkably, this behavior persists under realistic experimental conditions, including long-range interactions, global dephasing, and decoherence.

The robustness of the phenomenon is further tested by introducing controlled disorder into the system. While a pronounced Mpemba effect remains visible at weak disorder, no crossing is observed within the accessible time window when the disorder strength is increased, suggesting a suppression of the effect in strongly disordered regimes.

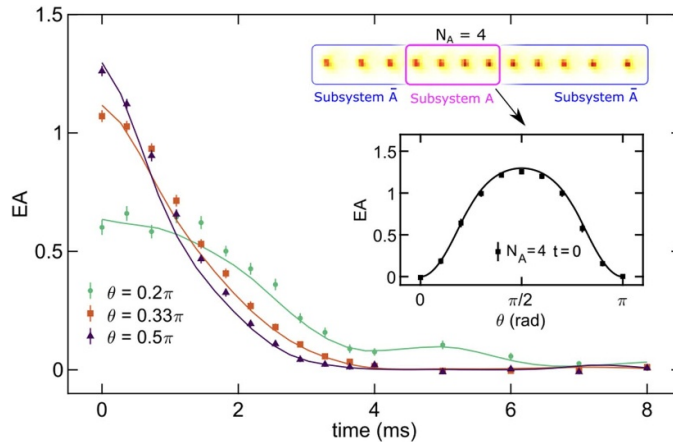


Figure 2. Entanglement asymmetry in a trapped-ion quantum simulator. Relaxation dynamics is monitored through the restoration of the Rényi-2 entanglement asymmetry in a four-ion subsystem A . Inset shows the initial values, in excellent agreement with the theoretical prediction of equation (3.2) (solid lines). In the main panel the continuous lines are the theory while the symbols are experimental measurements. They display a clear crossing of the curves, signaling the quantum Mpemba effect. Reprinted figure with permission from [25], Copyright (2024) by the American Physical Society.

Entanglement asymmetry, however, is not the only observable capable of revealing Mpemba-like behavior. In the same experimental quench protocol, the Frobenius distance between the time-dependent reduced density matrix of a subsystem and its expected stationary counterpart was also measured. This distance exhibits the same qualitative features observed for the entanglement asymmetry. Beyond confirming the quantum Mpemba effect, this analysis provides an independent probe of local thermalization in a nonintegrable quantum system. Furthermore, it can also be used in the absence of symmetry breaking [27].

We note that Mpemba-like behavior detected through the entanglement asymmetry has also been observed experimentally on a superconducting quantum processor [28].

3.3. Further examples of the quantum Mpemba effect

Over the past few years, a rapidly growing body of work has reported additional manifestations of the Mpemba effect across a wide range of classical and quantum systems. Here, we provide a brief overview of some of the most relevant developments.

A significant fraction of the recent progress concerns integrable quantum models, both free and interacting [29–32]. In this context, particular attention has been devoted to identifying general conditions under which Mpemba-like behavior can occur. Notably, a concrete criterion for the emergence of the Mpemba effect has been proposed in [33] and will be discussed in the next subsection. Related studies have also explored the role of finite symmetry groups [34, 35].

Beyond 1D settings, the Mpemba effect has been investigated in 2D fermionic [36] and bosonic systems [37], as well as in models with long-range interactions [38].

Extensions to open and dissipative systems have been analyzed from the perspective of symmetry and asymmetry [39]. Further studies include free-fermionic mixed states [40], periodically driven (Floquet) quantum systems [41], and quantum simple exclusion processes [42]. Field-theoretical approaches provide a more idealized yet analytically controlled framework to investigate the dynamics of the asymmetry, and have yielded valuable insights into the Mpemba-like relaxation [43–46]. It has also been pointed out that the Mpemba effect can be diagnosed through the restoration of time-translation invariance, a perspective that leads to a simplified criterion for its occurrence [47].

Important conceptual advances have been obtained in the study of the dynamics of asymmetric initial states under $U(1)$ -symmetric random quantum circuits [48, 49]. These works demonstrate that, when starting from tilted ferromagnetic states, Mpemba-like behavior appears generically, whereas the same phenomenon is absent for tilted antiferromagnetic initial states. A semi-quantitative explanation for this striking dependence on the choice of initial state has been proposed, highlighting the crucial role played by the structure of the initial symmetry breaking. The importance of the initial-state properties is further emphasized in [50]. Within the broader framework of random unitary dynamics, several related phenomena have been identified. These include the Mpemba effect associated with the restoration of translational symmetry [51, 52], the dynamics of the entanglement asymmetry in nonlocal random circuits [53, 54], and the behavior of charged dual-unitary circuits [55]. Complementary insights have also emerged from the perspective of quantum resource theory, which provides a unified framework to quantify the asymmetry and its dynamical evolution [56–58].

Finally, the interplay between measurements and symmetry has recently attracted considerable attention. It has been shown that the presence of continuous monitoring can qualitatively modify the relaxation dynamics of many-body quantum systems. In particular, monitored systems governed by effective non-Hermitian dynamics have been demonstrated to exhibit a quantum Mpemba effect [59, 60].

3.4. Microscopic origin of quantum Mpemba effect in integrable models

A central issue, already anticipated in the discussion above, is whether one can formulate a microscopic criterion that allows one to predict the appearance of a Mpemba effect by comparing two distinct initial states. At present, a complete and rigorous answer to this question is available only for a restricted but important class of systems, namely 1D integrable quantum many-body models. In this setting, both the underlying physical mechanism and a set of necessary and sufficient conditions for the emergence of Mpemba-like behavior have been identified and characterized in detail in [33].

Integrable systems form a highly special subclass of quantum models, distinguished by the presence of infinitely many conserved quantities. These conservation laws strongly constrain the dynamics, leading to relaxation behavior that differs qualitatively from that of generic chaotic systems. At the same time, integrability renders these models amenable to detailed analytical treatment. One of the most important consequences of integrability is the existence of long-lived elementary excitations, or quasiparticles,

which provide an effective description of both equilibrium properties and far-from-equilibrium dynamics. In particular, they offer a natural microscopic language to understand the quantum Mpemba effect.

Following a global quantum quench, the initial state can be expressed as a superposition of eigenstates of the postquench Hamiltonian. In integrable models, these eigenstates are composed of many quasiparticle excitations so that the quench effectively acts as a source that populates quasiparticle modes throughout the system. As time evolves, pairs of quasiparticles are produced uniformly across space and then propagate ballistically in opposite directions, each pair characterized by a definite velocity. Quasiparticles emitted at different spatial locations are statistically independent, whereas the two members of a pair generated at the same point are quantum mechanically entangled. This physical picture, known as the *quasiparticle picture* [61–63], has proven extremely successful in explaining the growth of entanglement entropy and the spreading of correlations following a quench in integrable systems. When the initial state breaks a symmetry, the entangled quasiparticle pairs carry correlations that violate that symmetry. Within this framework, the entanglement asymmetry of a subsystem A quantifies the contribution of those pairs whose two constituents both lie inside A and therefore actively break the symmetry there. The crucial observation is that once one member of an entangled pair crosses the boundary of the subsystem, that pair no longer contributes to symmetry breaking within A , and the entanglement asymmetry decreases accordingly. In an infinite system, as time progresses and the quasiparticles separate over arbitrarily large distances, the number of complete pairs contained within any finite subsystem vanishes. As a result, the symmetry is locally restored and the asymmetry relaxes to zero. This mechanism provides a transparent microscopic explanation for both symmetry restoration and the decay of entanglement asymmetry in integrable systems.

Within the same picture, the origin of the quantum Mpemba effect becomes clear. Consider two different initial states that break the symmetry to different extents. The more asymmetric state contains a larger weight of quasiparticle pairs that contribute strongly to symmetry breaking. However, these quasiparticles are characterized by a distribution of velocities. If, in the more asymmetric initial state, the dominant symmetry-breaking quasiparticles are also the fastest ones, they will leave the subsystem more rapidly, causing the asymmetry to decay sooner than in a less asymmetric state. In this situation, the system that starts farther from equilibrium appears to relax faster, realizing a quantum Mpemba effect. This reasoning underlies the results of [33], where precise and general conditions for the occurrence of the quantum Mpemba effect in integrable systems are derived. Remarkably, these conditions depend on only two pieces of information, the occupation densities of the quasiparticle modes in the initial state and their corresponding group velocities under the postquench Hamiltonian. Both quantities can, in principle, be computed using Bethe ansatz techniques for a broad class of integrable quantum models.

4. Conclusion

The Mpemba effect stands as a striking example of how nonequilibrium dynamics can defy naive intuition, in both classical and quantum settings. In the quantum domain, where relaxation must be understood in terms of reduced states and local observables, the phenomenon acquires new layers of conceptual richness. In this context, the entanglement asymmetry, designed to quantify symmetry breaking at the level of subsystems, has emerged as an especially powerful diagnostic. It has proven capable of capturing the quantum Mpemba effect in a clear and robust manner, providing a unified language for theoretical analysis and experimental observation.

Beyond its role as a probe of Mpemba physics, the entanglement asymmetry has found important applications in other areas of quantum many-body physics. A particularly intriguing example concerns Haar-random states and their connection to questions in black hole physics. Recent works have shown that, for a random pure state, the entanglement asymmetry of a subsystem vanishes identically as long as the subsystem occupies less than half of the total system, while it exhibits an abrupt transition to a logarithmic scaling once the subsystem exceeds this threshold [64–66]. This sharp behavior has a direct interpretational consequence; in the context of black hole evaporation, it implies that access to at least half of the emitted radiation is required in order to determine whether the original black hole state respected a given symmetry. In this sense, entanglement asymmetry encodes a precise notion of when global symmetry information becomes locally accessible.

Despite recent progress, several fundamental questions remain. Foremost among them is the problem of identifying a general microscopic mechanism underlying the quantum Mpemba effect in isolated chaotic systems. While integrable models admit a transparent explanation in terms of quasiparticle dynamics, no analogous framework is currently available for generic nonintegrable systems, where relaxation is governed by scrambling and many-body chaos rather than stable excitations. Finally, a broader and still unresolved issue concerns the possibility of a unifying theoretical framework that encompasses both classical and quantum manifestations of the Mpemba effect [56]. Understanding whether these phenomena share a common structural origin, or whether their similarity is merely phenomenological, remains an important challenge for future research.

Acknowledgments

I thank the many colleagues who, over recent years, have contributed to our collective efforts to understand the quantum Mpemba effect. In particular, I owe a special debt of gratitude to Filiberto Ares and Sara Murciano, with whom this line of research originated. Their insight, dedication and collaboration were essential from the very beginning, and this study would not have come into being without them. I also thank Vincenzo Alba, Bruno Bertini, Rainer Blatt, Konstantinos Chalas, Andrea De Luca, Florent Ferro, Alessandro Foligno, Johannes Franke, Lata Kh. Joshi, Manoj Joshi, Israel

Klich, Katja Klobas, Florian Kranzl, Lorenzo Piroli, Aniket Rath, Christian Roos, Colin Rylands, Xhek Turkeshi, Benoit Vermersch, Eric Vernier, Shion Yamashika and Peter Zoller. I acknowledge support from ERC under the Advanced Grant No. 101199196 (MOSE).

References

- [1] Mpemba E B and Osborne D G 1969 Cool? *Phys. Educ.* **4** 172
- [2] Aristotle 1923 *Meteorologica* (Clarendon)
- [3] Groves L R 1962 *Now It Can Be Told: The Story of the Manhattan Project* (Harper & Row)
- [4] Kumar A and Bechhoefer J 2020 Exponentially faster cooling in a colloidal system *Nature* **584** 64
- [5] Lu Z and Raz O 2017 Nonequilibrium thermodynamics of the Markovian Mpemba effect and its inverse *PNAS* **114** 5083
- [6] Ares F, Calabrese P and Murciano S 2025 The quantum Mpemba effects *Nat. Rev. Phys.* **7** 451
- [7] Teza G, Bechhoefer J, Lasanta A, Raz O and Vucelja M 2026 Speedups in nonequilibrium thermal relaxation: Mpemba and related effects *Phys. Rep.* **1164** 1
- [8] Shapira S A, Shapira Y, Markov J, Teza G, Akerman N, Raz O and Ozeri R 2024 Inverse Mpemba effect demonstrated on a single trapped ion qubit *Phys. Rev. Lett.* **133** 010403
- [9] Zhang J *et al* 2025 Observation of quantum strong Mpemba effect *Nat. Commun.* **16** 301
- [10] Calabrese P, Essler F H L and Mussardo G 2016 Introduction to quantum integrability in out of equilibrium systems *J. Stat. Mech.* **2016** 064001
- [11] Deutsch J M 1991 Quantum statistical mechanics in a closed system *Phys. Rev. A* **43** 2046
- [12] Srednicki M 1994 Chaos and quantum thermalization *Phys. Rev. E* **50** 888
- [13] Rigol M, Dunjko V and Olshanii M 2008 Thermalization and its mechanism for generic isolated quantum systems *Nature* **452** 854
- [14] Rigol M, Dunjko V, Yurovsky V and Olshanii M 2007 Relaxation in a completely integrable many-body quantum system: an Ab initio study of the dynamics of the highly excited states of 1D lattice hard-core bosons *Phys. Rev. Lett.* **98** 050405
- [15] Vidmar L and Rigol M 2016 Generalized Gibbs ensemble in integrable lattice models *J. Stat. Mech.* **2016** 064007
- [16] Calabrese P 2020 Entanglement spreading in non-equilibrium integrable systems *SciPost Phys. Lect. Notes* **20**
- [17] Kaufman A M, Tai M E, Lukin A, Rispoli M, Schittko R, Preiss P M and Greiner M 2016 Quantum thermalization through entanglement in an isolated many-body system *Science* **353** 794
- [18] Ares F, Murciano S, Vernier E and Calabrese P 2023 Lack of symmetry restoration after a quantum quench: an entanglement asymmetry study *SciPost Phys.* **15** 089
- [19] Ares F, Murciano S and Calabrese P 2023 Entanglement asymmetry as a probe of symmetry breaking *Nat. Commun.* **14** 2036
- [20] Castro-Alvaredo O A and Santamaria-Sanz L 2025 Symmetry resolved measures in quantum field theory: a short review *Mod. Phys. Lett. B* **39** 2430002
- [21] Vaccaro J A, Anselmi F, Wiseman H M and Jacobs K 2008 Tradeoff between extractable mechanical work, accessible entanglement and ability to act as a reference system, under arbitrary superselection rules *Phys. Rev. A* **77** 032114
- [22] Gour G, Marvian I and Spekkens R W 2009 Measuring the quality of a quantum reference frame: the relative entropy of frameness *Phys. Rev. A* **80** 012307
- [23] Chitambar E and Gour G 2019 Quantum resource theories *Rev. Mod. Phys.* **91** 025001
- [24] Liu S, Zhang H-K, Yin S, Zhang S-X and Yao H 2025 Quantum Mpemba effects in many-body localization systems *Sci. Bull.* **70** 3991
- [25] Joshi L K *et al* 2024 Observing the quantum Mpemba effect in quantum simulations *Phys. Rev. Lett.* **133** 010402
- [26] Elben A, Flammia S T, Huang H-Y, Kueng R, Preskill J, Vermersch B and Zoller P 2023 The randomized measurement toolbox *Nat. Rev. Phys.* **5** 9
- [27] Bhore T, Su L, Martin I, Clerk A A and Papic Z 2025 Quantum Mpemba effect without global symmetries *Phys. Rev. B* **112** L121109
- [28] Xu Y *et al*, Observation and modulation of the quantum Mpemba effect on a superconducting quantum processor (arXiv:2508.07707)

- [29] Murciano S, Ares F, Klich I and Calabrese P 2024 Entanglement asymmetry and quantum Mpemba effect in the XY spin chain *J. Stat. Mech.* **2024** 013103
- [30] Klobas K 2024 Non-equilibrium dynamics of symmetry-resolved entanglement and entanglement asymmetry: exact asymptotics in Rule 54 *J. Phys. A* **57** 505001
- [31] Rylands C, Vernier E and Calabrese P 2024 Dynamical symmetry restoration in the Heisenberg spin chain *J. Stat. Mech.* **2024** 123102
- [32] Chalas K, Ares F, Rylands C and Calabrese P 2024 Multiple crossing during dynamical symmetry restoration and implications for the quantum Mpemba effect *J. Stat. Mech.* **2024** 103101
- [33] Rylands C, Klobas K, Ares F, Calabrese P, Murciano S and Bertini B 2024 Microscopic origin of the quantum Mpemba effect in integrable systems *Phys. Rev. Lett.* **133** 010401
- [34] Ferro F, Ares F and Calabrese P 2024 Non-equilibrium entanglement asymmetry for discrete groups: the example of the XY spin chain *J. Stat. Mech.* **023101** **2024**
- [35] Hara R, Endo S and Yamashika S Dynamics of entanglement asymmetry for space-inversion symmetry of free fermions on honeycomb lattices (arXiv:2511.14114)
- [36] Yamashika S, Ares F and Calabrese P 2024 Entanglement asymmetry and quantum Mpemba effect in two-dimensional free-fermion systems *Phys. Rev. B* **110** 085126
- [37] Yamashika S, Calabrese P and Ares F 2025 Quenching from superfluid to free bosons in two dimensions: entanglement, symmetries and quantum Mpemba effect *Phys. Rev. A* **111** 043304
- [38] Yamashika S and Ares F The quantum Mpemba effect in long-range spin systems (arXiv:2507.06636)
- [39] Caceffo F, Murciano S and Alba V 2024 Entangled multiplets, asymmetry and quantum Mpemba effect in dissipative systems *J. Stat. Mech.* **2024** 063103
- [40] Ares F, Vitale V and Murciano S 2025 The quantum Mpemba effect in free-fermionic mixed states *Phys. Rev. B* **111** 104312
- [41] Banerjee T, Das S and Sengupta K Entanglement asymmetry in periodically driven quantum systems (arXiv:2412.03654)
- [42] Russotto A, Ares F, Calabrese P and Alba V Dynamics of entanglement fluctuations and quantum Mpemba effect in the $\nu=1$ QSSEP model (arXiv:2510.25519)
- [43] Chen M and Chen H-H 2024 Rényi entanglement asymmetry in (1+1)-dimensional conformal field theories *Phys. Rev. D* **109** 065009
- [44] Benini F, Godet V and Singh A H 2025 Entanglement asymmetry in conformal field theory and holography *Prog. Theor. Exp. Phys.* **6** 063B05
- [45] Fujimura H and Shimamori S Entanglement asymmetry and quantum Mpemba effect for non-Abelian global symmetry (arXiv:2509.05597)
- [46] Benini F, Calabrese P, Fossati M, Singh A H and Venuti M Entanglement asymmetry for higher and noninvertible symmetries (arXiv:2509.16311)
- [47] Ares F, Rylands C and Calabrese P 2025 A simpler probe of the quantum Mpemba effect in closed systems *J. Phys. A* **58** 445302
- [48] Liu S, Zhang H-K, Yin S and Zhang S-X 2024 Symmetry restoration and quantum Mpemba effect in symmetric random circuits *Phys. Rev. Lett.* **133** 140405
- [49] Turkeshi X, Calabrese P and De Luca A 2025 Quantum Mpemba effect in random circuits *Phys. Rev. Lett.* **135** 040403
- [50] Yu Y-H, Jin T-R, Zhang L, Xu K and Fan H 2025 Tuning the quantum Mpemba effect in an isolated system by initial-state engineering *Phys. Rev. B* **112** 094315
- [51] Klobas K, Rylands C and Bertini B 2025 Translation symmetry restoration under random unitary dynamics *Phys. Rev. B* **111** L140304
- [52] Gibbins M, Gammon-Smith A and Bertini B 2025 Translation symmetry restoration in integrable systems: the noninteracting case *Phys. Rev. B* **112** L180307
- [53] Ares F, Murciano S, Calabrese P and Pirolì L 2025 Entanglement asymmetry dynamics in random quantum circuits *Phys. Rev. Res.* **7** 033135
- [54] Li H-Z, Lee C H, Liu S, Zhang S-X and Zhong J-X, Quantum Mpemba effect in long-ranged U(1)-symmetric random circuits (arXiv:2512.06775)
- [55] Foligno A, Calabrese P and Bertini B 2025 Non-equilibrium dynamics of charged dual-unitary circuits *PRX Quantum* **6** 010324
- [56] Summer A, Moroder M, Bettmann L P, Turkeshi X, Marvian I and Goold J A resource theoretical unification of Mpemba effects: classical and quantum (arXiv:2507.16976)
- [57] Aditya S, Summer A, Sierant P and Turkeshi X Mpemba effects in quantum complexity (arXiv:2509.22176)

- [58] Moroder M, Culhane O, Zawadzki K and Goold J 2024 Thermodynamics of the quantum Mpemba effect *Phys. Rev. Lett.* **133** 140404
- [59] Di Giulio G, Turkeshi X and Murciano S 2025 Measurement-induced symmetry restoration and quantum Mpemba effect *Entropy* **27** 407
- [60] Ganguly K and Agarwalla B K Measurement induced faster symmetry restoration in quantum trajectories (arXiv:2601.18458)
- [61] Calabrese P and Cardy J 2005 Evolution of entanglement entropy in one-dimensional systems *J. Stat. Mech.* **2005** 04010
- [62] Alba V and Calabrese P 2017 Entanglement and thermodynamics after a quantum quench in integrable systems *PNAS* **114** 7947
- [63] Alba V and Calabrese P 2018 Entanglement dynamics after quantum quenches in generic integrable systems *SciPost Phys.* **4** 017
- [64] Ares F, Murciano S, Piroli L and Calabrese P 2024 An entanglement asymmetry study of black hole radiation *Phys. Rev. D* **110** L061901
- [65] Russotto A, Ares F and Calabrese P 2025 Non-Abelian entanglement asymmetry in random states *J. High Energy Phys.* **JHEP06(2025)149**
- [66] Yang J-N, Joshi L K, Ares F, Han Y, Zhang P and Calabrese P Probing entanglement and symmetries in random states using a superconducting quantum processor (arXiv:2601.22224)