

The impact of envelope binding energies on the merger rate density of binary compact objects

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ABSTRACT

The common envelope (CE) phase plays a key role in the formation of binary compact object systems. Its final outcome strongly depends on the envelope binding energy, but this quantity is often estimated using fitting formulas that are not fully consistent with the underlying stellar evolution models adopted in population-synthesis codes. Here, we investigate envelope binding energies across the most extensive stellar grid considered to date. Our stellar tracks, evolved with PARSEC v2.0, include hydrogen (H)-rich stars with metallicities ranging from $Z = 10^{-11}$ (Population III stars) to $Z = 0.03$, and initial masses between 2 and 2000 $M_{\odot}$, as well as pure-helium stars with masses from 0.36 to 350 $M_{\odot}$. We examine the sensitivity of the envelope binding energies to the selected core-envelope boundary definition and to different internal energy source contributions. For H-rich stars, we find that internal energy sources can alter the envelope binding energy by more than an order of magnitude, whereas the core boundary criteria play a secondary role. In contrast, for pure helium stars, the core-boundary criterion becomes the dominant factor. The envelope binding energies derived from different stellar tracks can show deviations of several orders of magnitude, with larger differences for more massive stars and higher metallicities. Finally, by implementing our new envelope binding energy prescriptions into the binary population synthesis code SEVN, we show that the predicted merger rate densities of compact binaries can differ by more than an order of magnitude compared to previous models. Our results highlight the importance of using envelope binding energies that are consistent with the underlying stellar evolution models and caution against extrapolating empirical fits beyond the considered parameter space.

Key words. gravitational waves – binaries: close – stars: black holes – stars: evolution

1. Introduction

The common envelope (CE) phase is one of the most prominent processes in binary stellar evolution, yet it is poorly understood. It denotes a rapid phase of evolution during which the two cores of the stars orbit within a shared envelope (Paczynski 1976; Webbink 1984; Iben & Livio 1993; Podsiadlowski 2001; Ivanova et al. 2013). The drag forces cause the orbit to shrink with exceptional efficiency, resulting in a merger of the two objects, or in a post-CE system significantly more compact than its original state.

The CE process is commonly invoked to explain several observed types of binary objects. Remarkable examples include cataclysmic variables (Meyer & Meyer-Hofmeister 1979),

type Ia supernovae (Iben & Tutukov 1984; Webbink 1984; Han & Podsiadlowski 2004), binary pulsars (van den Heuvel 1994; Vigna-Gómez et al. 2020), and close white dwarf binaries (Paczynski 1976; Han et al. 2002, 2003; Toonen & Nelemans 2013). Such systems typically require a phase of efficient spiral-in that reduces the orbital separation by several orders of magnitude (Röpke & De Marco 2023). Moreover, the CE phase holds a key role in the formation and merger of binary compact objects (Giacobbo & Mapelli 2018; Kruckow et al. 2018; Breivik et al. 2020; Broekgaarden et al. 2021; Bavera et al. 2021; Román-Garza et al. 2021; Mapelli et al. 2021; Belczynski et al. 2020; Chu et al. 2022; Iorio et al. 2023; Sgalletta et al. 2025; Chen et al. 2025).

The CE process poses a tough computational challenge due to the large ranges involved in both time and space (Ivanova et al. 2013; MacLeod et al. 2017; Fragos et al. 2019; Ivanova et al.

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2020). For this reason, hydrodynamical simulations still struggle to model the full CE evolution or fail to incorporate some physical ingredients, such as recombination energy, nuclear reactions that may take place within the dense accreted material, neutrino cooling, radiative and convective transport, and jet formation (see, e.g., Röpke & De Marco 2023 for a review). Moreover, there is still no consensus in the literature regarding the final outcomes of a CE event, with many hydrodynamical simulations failing to achieve complete ejection of the envelope (Taam & Sandquist 2000; Passy et al. 2012; Ricker & Taam 2012; Ohlmann et al. 2016; Iaconi et al. 2017; Moreno et al. 2022). Notably, Nandez et al. (2015) found that including the contribution from the recombination energy allows one to successfully unbind the envelope.

Binary population synthesis codes usually adopt simplified analytic approaches because of their easier implementation and computational efficiency. Usually these are parametric models based on general considerations of energy (Webbink 1984) and angular momentum (Nelemans et al. 2000; Nelemans & Tout 2005) conservation, although also new formalisms have recently been proposed (Trani et al. 2022; Hirai & Mandel 2022; Di Stefano et al. 2023). The (α, λ) -formalism (van den Heuvel et al. 1976; Webbink 1984) is one of the most commonly adopted approaches in population synthesis codes. This framework is based on an energy balance equation in which the two components at play are the binding energy of the envelope (parametrized by λ) and the orbital energy of the system. A second parameter, α , governs the efficiency of the energy transfer mechanism.

It has been clear for a long time that the (α, λ) -formalism fails to capture the physics of CE evolution; namely, all the sources and sinks of energy that are at play (Ivanova et al. 2013; Fragos et al. 2019). However, population-synthesis codes still use this formalism because of its intrinsic simplicity, and “hide” in α and/or λ all the uncertainties about missing physics. For instance, this is the reason why several authors adopt values of $\alpha > 1$ (Mapelli & Giacobbo 2018; Fragos et al. 2019; Hirai & Mandel 2022). Furthermore, α may not be constant across different systems, but there may be a discrepancy between low-mass and high-mass donors (Politano & Weiler 2007; De Marco et al. 2011; Wilson & Nordhaus 2022).

The parameter λ encapsulates the structural properties of a star’s envelope, and is intrinsically linked to its stellar mass, metallicity, and evolutionary phase. λ is typically derived from stellar evolution models. However, its determination can vary considerably due to differences among these models. Most population-synthesis codes adopt values of λ that are not calculated directly from the adopted evolutionary tracks. This introduces a methodological inconsistency between the adopted stellar evolution models and the envelope binding energy prescriptions. Additionally, there are still open questions regarding the correct evaluation of λ . The primary uncertainties arise from the criterion used to define the core-envelope boundary and from the contributing energy sources that should be considered in the evaluation of the binding energy (Ivanova et al. 2013; Wang et al. 2016; Kruckow et al. 2016). The uncertainties on α and λ , in turn, reflect on the demographics of binary compact objects, producing variations of orders of magnitude in the resulting merger rates (Belczynski et al. 2002; Neijssel et al. 2019; Broekgaarden et al. 2021, 2022; Iorio et al. 2023; Sgalletta et al. 2023; Boesky et al. 2024; Sgalletta et al. 2025).

Kruckow et al. (2016) and Klencki et al. (2021) conducted detailed studies of the envelope ejection conditions by massive stars. They showed that envelope binding energies are highly

sensitive to the adopted λ prescriptions, with different choices leading to large variations in the inferred values. Furthermore, while λ parametrizes the binding energy of the whole stellar envelope, the outcome of the CE phase may be more closely linked to the properties of the outer convective envelope, which is expected to dominate the initial dynamical inspiral, and therefore regulate the efficiency of envelope ejection (Klencki et al. 2021; Hirai & Mandel 2022; Picker et al. 2024).

Here, we present envelope binding energies for a wide set of stars, significantly extending the range of stellar masses and metallicities explored in previous studies. The set has been simulated with the PAdova and TRieste Stellar Evolution Code, PARSEC v2.0 (Bressan et al. 2012; Costa et al. 2019, 2021; Nguyen et al. 2022; Costa et al. 2025; Shepherd et al. 2025). Notably, we expanded this analysis to include pure-He stars, addressing previously unconstrained λ parameters. Moreover, we included the new sets of binding energies within the population synthesis code SEVN (Spera & Mapelli 2017; Spera et al. 2019; Mapelli et al. 2020; Iorio et al. 2023) and we employed the new formalism to test the cosmic merger rate densities for binary compact objects. In agreement with recent works, our results emphasize the importance of consistent envelope binding energies. Moreover, our results hint at a functional dependence of the α parameter.

In Sect. 2 we describe the methodology adopted in this work. In Sect. 3 we present the envelope binding energies, emphasizing the importance of convection. Furthermore, we describe their implementation in SEVN and show how they compare with existing models. We discuss our findings in Sect. 4 and draw our conclusions in Sect. 5.

2. Method

2.1. PARSEC

We used stellar tracks computed with the stellar evolution code PARSEC v2.0¹ (Bressan et al. 2012; Costa et al. 2019, 2021; Nguyen et al. 2022; Costa et al. 2025; Shepherd et al. 2025). We used two sets of tracks from Costa et al. (2025). In the first set, we started from the hydrogen main sequence (hereafter H tracks). In the second, we started from the helium main sequence, i.e., our tracks had the hydrogen fraction $X_H = 0$ since the beginning (hereafter, pure-He tracks). The H tracks have metallicity values of $Z = 10^{-11}, 10^{-6}, 0.0001, 0.001, 0.002, 0.004, 0.006, 0.008, 0.01, 0.014, 0.017, 0.02, \text{ and } 0.03$. The zero-age main-sequence (ZAMS) masses, M_{ZAMS} , range from 2.2 to 600 $M_\odot$ for $Z \geq 0.001$. M_{ZAMS} reaches up to 2000 $M_\odot$ for lower metallicities. The models assume overshooting on top of convective cores, allowing for the penetration of convective elements into stable regions under the “ballistic” scheme described by Bressan et al. (1981). Within this scheme the overshooting length – corresponding to the maximum distance traveled by the overshooting bubble across the unstable border – is roughly 0.25 pressure scale heights. Additionally, all the tracks have been evolved assuming no rotation. The tracks include, for each evolutionary timestep, detailed spatial grids of the main stellar properties. For the purposes of our work, we are interested in the radial distance (r), gravitational energy (E_g), internal energy (U), enthalpy (E_H), luminosity (L), and hydrogen and helium concentrations (X_H and X_{He} , respectively). For stars with $2 \leq M_{\text{ZAMS}}/M_\odot \leq 9$, we considered their evolution until the early asymptotic giant branch phase. Massive stars’ evolution

¹ <https://stev.oapd.inaf.it/PARSEC/index.html>

($M_{\text{ZAMS}} > 9 M_{\odot}$) was followed until either the start of core oxygen burning or the pair instability supernova regime. We refer to [Costa et al. \(2025\)](#) for further details on the stellar tracks.

For the pure-He tracks, the zero-age He main-sequence mass (M_{He}) ranges from 0.36 to 350 $M_{\odot}$ and metallicities: $Z = 10^{-6}$, 0.0001, 0.0002, 0.0005, 0.001, 0.002, 0.004, 0.006, 0.008, 0.01, 0.02, 0.03, and 0.05. We refer to [Costa et al. \(2025\)](#) for additional details on the tracks.

2.2. Envelope binding energy

The most general expression of the gravitational binding energy of the envelope is

$$E_G = - \int_{M_{\text{core}}}^M \frac{Gm}{r(m)} dm, \quad (1)$$

where M is the total stellar mass, M_{core} is the core mass, and m is the local mass coordinate. The quantity $r(m)$ is the radius from the center at mass coordinate m . The internal energy, U (including both thermal and recombination energies), may also contribute to lowering the binding energy of the envelope ([Dewi & Tauris 2000](#)); therefore, we can define E_B as

$$E_B = \int_{M_{\text{core}}}^M \left[-\frac{Gm}{r(m)} + U(m) \right] dm. \quad (2)$$

Finally, [Ivanova & Chaichenets \(2011\)](#) first suggested the pressure work term as an additional energy source, contributing to the overall binding energy of the envelope (E_H):

$$E_H = \int_{M_{\text{core}}}^M \left[-\frac{Gm}{r(m)} + U(m) + \frac{P(m)}{\rho(m)} \right] dm, \quad (3)$$

where the term $U + P/\rho$ is known as the specific enthalpy.

The boundary between the core and the envelope is a key quantity for the computation of these integrals. A number of criteria have been proposed (see, e.g., [Ivanova et al. 2013](#)). We defined the helium core of a star as the hydrogen-depleted region, where the fraction of hydrogen (X_H) is lower than a fixed threshold, $X_{H,0}$. The commonly adopted value in the literature is $X_{H,0} \sim 0.1$ ([Xu & Li 2010](#); [Ivanova 2011](#); [Klencki et al. 2021](#); [Marchant et al. 2021](#)). On the other hand, the PARSEC code defines the core as the region where $X_{H,0}$ reaches 10^{-3} by default. We kept $X_{H,0}$ as a free parameter and varied its value in our models in order to explore its effect. We evaluated E_G , E_B , and E_H through Eqs. (1), (2), and (3) at each timestep of the stellar evolution where $M_{\text{core}} \neq 0$. We repeated this procedure for every stellar track.

Whether or not a pure-He star can initiate a second unstable mass transfer episode is still unclear. According to the prescription by [Hurley et al. \(2002\)](#), mass transfer from pure-He stars becomes dynamically unstable if the donor-to-accretor mass ratio is larger than a critical mass ratio, $q_{\text{crit}} = 3$, for He main-sequence stars, and $q_{\text{crit}} \sim 1$ (or even lower) in more advanced evolutionary stages. Other works assume mass transfer from pure-He stars to always be dynamically stable ([Tauris et al. 2015, 2017](#); [Neijssel et al. 2019](#)). In particular, [Vigna-Gómez et al. \(2018\)](#) find that this condition is required in their models in order to explain the population of Galactic binary neutron stars. Nevertheless, [Sgalletta et al. \(2023\)](#) show that this assumption is not necessary, adopting SEVN and the PARSEC stellar tracks. We also studied the envelope binding energies for pure-He stellar tracks. In this case, E_G , E_B , and E_H were computed with Eqs. (1), (2), and (3), respectively. However, in this

case, the lower limit of integration was the carbon-oxygen (CO) core mass. The latter was defined, in turn, as the He-depleted region: $X_{\text{He}} < X_{\text{He},0}$, with $X_{\text{He},0}$ a concentration threshold similar to the one previously defined for hydrogen.

Here, we study how the binding energies vary for different parameters in our models. We test multiple core separation prescriptions, varying $X_{H,0} = 10^{-1}, 10^{-3}, 10^{-7}$ and $X_{\text{He},0} = 10^{-1}, 10^{-3}, 10^{-7}$. Moreover, we test how the different definitions E_G (Eq. 1), E_B (Eq. 2), and E_H (Eq. 3), impact the results.

2.3. Stellar Evolution for N-body code (SEVN)

SEVN² is a fast and versatile binary population synthesis code ([Spera & Mapelli 2017](#); [Spera et al. 2019](#); [Mapelli 2020](#); [Iorio et al. 2023](#)). It interpolates the main stellar properties (masses, radii, luminosities, etc.) on the fly from a precomputed set of stellar tracks, and accounts for binary-evolution processes through analytic and semi-analytic formulas.

For the interpolation of stellar properties, SEVN includes seven tables: evolutionary time, total stellar mass, He-core mass, CO-core mass, stellar radius, bolometric luminosity, and the stellar phase. Other properties can be included by adding further optional tables (see [Spera & Mapelli 2017](#); [Spera et al. 2019](#); [Iorio et al. 2023](#), for details). Here, we include the envelope binary energy (computed directly from PARSEC) as an optional table property in SEVN. These can be switched on as an alternative model for the CE treatment. The SEVN tables used in this work are publicly available in Zenodo³. Here below, we provide a description of the CE phase in SEVN and the major changes that our new treatment implies. We refer to [Iorio et al. \(2023\)](#) for a comprehensive description of the code.

2.3.1. ($\alpha\lambda$)–formalism

SEVN models the CE phase with the so-called ($\alpha\lambda$)–formalism ([van den Heuvel et al. 1976](#); [Webbink 1984](#); [Hurley et al. 2002](#)). This prescription is based on an energy budget criterion: the orbital energy lost due to the shrinkage of the orbit during CE is transferred to the envelope. This formalism is based on the assumption that the orbital energy constitutes the only energy reservoir able to unbind the envelope. The model depends on two parameters, α and λ . α represents the efficiency at which energy is transferred from the binary to the envelope. Therefore the binding energy of the envelope can be written as

$$E_{\text{bind}} = \alpha \Delta E_{\text{orb}} \quad (4)$$

$$= \alpha \left(-\frac{GM_1 M_2}{2a_{\text{ini}}} + \frac{GM_{\text{core},1} M_{\text{core},2}}{2a_{\text{fin}}} \right). \quad (5)$$

Here, M is the total mass of the star, M_{core} is the core mass of the star, and G is the gravitational constant. The subscripts 1 and 2, refer to the primary and secondary stars, respectively. a_{ini} and a_{fin} are the pre- and post-CE semimajor axis of the system, required in order to successfully eject the envelope. The value of a_{fin} sets the fate of the CE phase. In SEVN if neither of the stars fills its Roche lobe after the CE then the envelope is ejected; otherwise, the two stars coalesce. The second parameter, λ , enters in the definition of the binding energies:

$$E_{\text{bind}} = E_{\text{bind},1} + E_{\text{bind},2} = -G \left(\frac{M_1 M_{\text{env},1}}{\lambda_1 R_1} + \frac{M_2 M_{\text{env},2}}{\lambda_2 R_2} \right), \quad (6)$$

² In this work, we use the SEVN version V 2.16 (commit [8af02cc3](#)). SEVN is publicly available at the gitlab repository: <https://gitlab.com/sevncodes/sevn>

³ [10.5281/zenodo.105281](https://zenodo.org/record/105281)

where λ acts as a structural parameter, highly dependent on the stellar properties of the star at the time of CE. In the previous equations, which we took from [Hurley et al. \(2002\)](#), if the accreting star had not yet developed a core its contribution to the envelope binding energy (Eq. 6) was considered zero, and M_{core} was replaced by its total stellar mass, M , in Eq. (5).

The definition of α implies that its value should lie in the range between 0 and 1, where $\alpha = 1$ corresponds to the ideal scenario in which all the missing orbital energy is transferred to the envelope, with no dissipation. In principle, $\alpha > 1$ is still possible if additional energy reservoirs are considered, other than the orbital energy (e.g., recombination energy, outflows, and nuclear reactions, [Ivanova et al. 2013](#); [Giacobbo & Mapelli 2018](#); [Mapelli & Giacobbo 2018](#); [Fragos et al. 2019](#); [El-Badry et al. 2023](#)).

Currently, the default λ option in SEVN is based on the prescription by [Claeys et al. \(2014\)](#). A constant $\lambda = 0.5$ is used for pure-He stars. Additional λ formalisms can be chosen in SEVN, based on [Xu & Li \(2010\)](#), [Klencki et al. \(2021\)](#), or λ can be set to a constant value, as in [Spera et al. \(2019\)](#) with $\lambda = 0.1$ (see [Iorio et al. 2023](#), for more details).

2.3.2. Common envelope: New features

We used the envelope binding energies computed directly from the PARSEC stellar tracks, to evaluate

$$\lambda_i = -\frac{E_{\text{bind},i} R_i}{G M_i M_{\text{env},i}}, \quad (7)$$

where the subscript i refers to the primary (1) or secondary (2) star. We then used Eq. (6) to evaluate the total envelope binding energy in a CE phase. For a set $X_{\text{H},0}$, we computed the envelope binding energies (E_{bind} in the previous equation) through the integrals (1), (2), and (3). We stored λ as an optional table property in SEVN. In addition, different choices of the core–envelope boundary (i.e., different $X_{\text{H},0}$) affect the definition of the He and CO core masses. Therefore, for any choice of $X_{\text{H},0}$ and $X_{\text{He},0}$, we modified the corresponding tables accordingly. As a consequence, the time of core formation and of the end of the main sequence depends on the adopted $X_{\text{H},0}$, which in turn affects the evolutionary stage at which a star may enter a CE phase (see Sect. 3).

2.4. Initial conditions

In order to study the merger rates of binary compact objects, we evolved a population of binaries sampled according to the following initial conditions. We drew the primary star masses, $M_{\text{ZAMS},1}$, according to [Kroupa \(2001\)](#), in a range between 5 and $150 M_{\odot}$:

$$\mathcal{F}(M_1) \propto M_1^{-2.3}. \quad (8)$$

We sampled the mass ratios $q = M_{\text{ZAMS},2}/M_{\text{ZAMS},1}$ according to ([Sana et al. 2012](#)):

$$\mathcal{F}(q) \propto q^{-0.1}, \quad (9)$$

with $\max\left(\frac{2.2 M_{\odot}}{M_{\text{ZAMS},1}}, 0.1\right) \leq q \leq 1$. In this way we derived the secondary star mass ($M_{\text{ZAMS},2}$), within the range $[2.2, 150] M_{\odot}$. The orbital periods and eccentricities are distributed according to [Sana et al. \(2012\)](#):

$$\mathcal{F}(P_{\text{orb}}) \propto (\log P_{\text{orb}})^{-0.55}, \quad (10)$$

with $0.15 \leq \log(P_{\text{orb}}/\text{d}) \leq 5.5$, and

$$\mathcal{F}(e) \propto e^{-0.42}, \quad (11)$$

with $0 \leq e \leq 1 - (P/2 \text{ days})^{-2/3}$ ([Moe & Di Stefano 2017](#)). We sampled 2×10^6 binaries according to these initial conditions. We ran this set of binaries for 12 metallicities: $Z = 0.0002, 0.0004, 0.0008, 0.0012, 0.0016, 0.002, 0.004, 0.006, 0.008, 0.012, 0.016$, and 0.02 . The metallicities that are not included among the single stellar tracks are interpolated within SEVN, as explained in [Iorio et al. \(2023\)](#).

2.5. SEVN setup

We evolved the binaries drawn from the above initial conditions with SEVN following the fiducial setup as described by [Iorio et al. \(2023\)](#). We summarize here the main parameters. We adopted the *DelayedGauss* supernova model, which predicts black hole masses according to the delayed supernova model by [Fryer et al. \(2012\)](#), but draws the masses of neutron stars from a Gaussian with mean $1.33 M_{\odot}$ and standard deviation $0.09 M_{\odot}$ ([Özel et al. 2012, 2016](#)). We sampled the kicks following [Giacobbo & Mapelli \(2020\)](#). According to this model, the kicks are drawn from a Maxwellian distribution with $\sigma = 265 \text{ km s}^{-1}$ ([Hobbs et al. 2005](#)) and then rescaled by a factor of $\propto M_{\text{ej}}/M_{\text{rem}}$, with M_{ej} the ejected mass and M_{rem} the mass of the remnant. We considered the fiducial mass transfer stability criterion (QCRS in [Iorio et al. 2023](#)), which is the same as in [Hurley et al. \(2002\)](#), but donor stars in the main sequence or in the Hertzsprung gap always lead to stable mass transfer. We explored the parameters associated with the CE phase, by varying $\alpha = 0.5, 1, 3$, and 5 , and the λ prescriptions. We compared the models by [Claeys et al. \(2014\)](#) and by [Klencki et al. \(2021\)](#) with the λ parameters evaluated from the PARSEC stellar tracks, assuming $X_{\text{H},0} = X_{\text{He},0} = 10^{-3}$ and $E_{\text{bind}} = E_{\text{B}}$ (Eq. 2). We included α values > 1 , to bracket possible uncertainties related to physics that are not captured by the (α, λ) -formalism.

2.6. Merger rate density

We evaluated the merger rate density of binary compact objects with GALAXYRATE ([Santoliquido et al. 2022](#)). GALAXYRATE adopts data-driven observational scaling relations to generate a set of star-forming galaxies across cosmic time and populate them with binary compact objects. For this work, we used the fiducial setup described in [Sgalletta et al. \(2025\)](#). More details on the methodology adopted can be found in Appendix A.

3. Results

3.1. Binding energies

Figure 1 shows E_{G} (Eq. 1), E_{B} (Eq. 2), and E_{H} (Eq. 3) computed for different initial stellar masses and metallicities. For visualization purposes, we only show the evolution of massive stars at $Z = 0.01$ up to the point at which they become Wolf-Rayet stars. This is because, during this phase, stars begin to shrink (e.g., [Costa et al. 2025](#)), making them less likely to undergo Roche-lobe overflow. At this metallicity, stars with initial masses $\geq 150 M_{\odot}$ have already formed their cores during the Wolf-Rayet phase; thus, we omit their evolution from the figure⁴. Low-metallicity stars ($Z \leq 0.0001$) with $M_{\text{ZAMS}} \geq 100 M_{\odot}$ typically

⁴ The transition to Wolf-Rayet in a massive stellar model depends on the treatment of stellar winds and convection. Therefore, the limits discussed here may vary depending on different prescriptions.

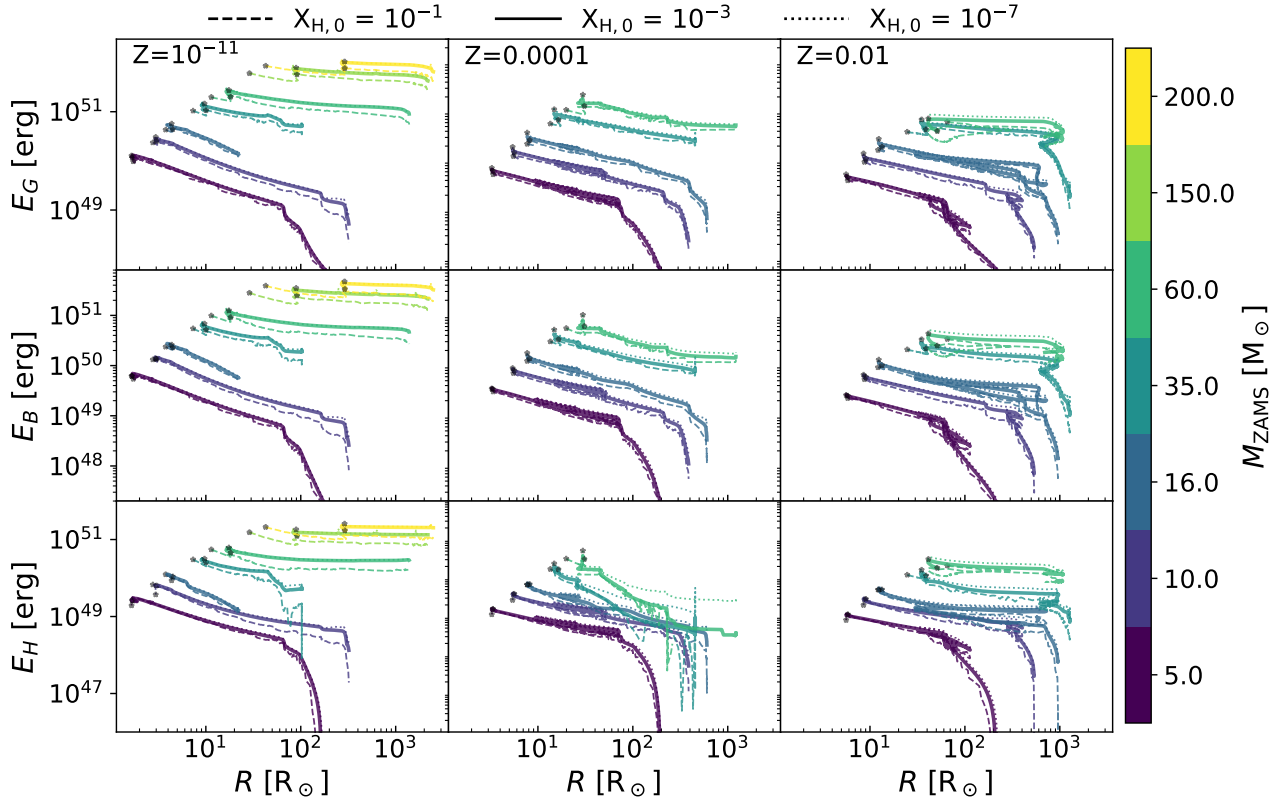


Fig. 1. Binding energy, E_H , evaluated from PARSEC as a function of the radius, R , for different stellar masses. The rows from top to bottom show E_G , E_B , and E_H , respectively. Different columns represent different metallicities: from left to right, $Z = 10^{-11}$, 0.0001, and 0.01. The black star markers indicate the formation of the He core. Different line styles show the results assuming different $X_{H,0}$ thresholds to define the core-envelope boundaries: $X_{H,0} = 10^{-1}$ dashed lines, $X_{H,0} = 10^{-3}$ solid lines, and $X_{H,0} = 10^{-7}$ dotted lines.

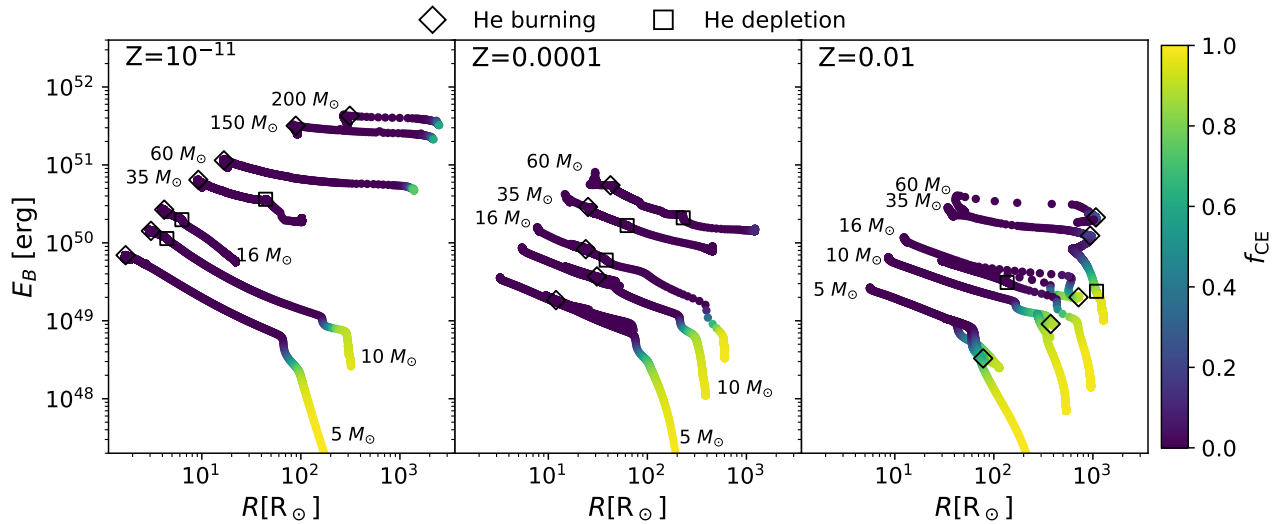


Fig. 2. Binding energy, E_B , evolution with radius for different stellar masses. The color code shows the mass fraction of the outer convective envelope, f_{CE} . Diamonds indicate the beginning of He burning; squares show the end of the core He burning phase. Different columns represent different metallicities: from left to right, $Z = 0.0001$, 0.001, and 0.01. Here, we assume that $X_{H,0} = 10^{-3}$.

reach their largest expansion at the end of core helium burning, after which they contract. While the final stellar radii depend on both metallicity and initial mass, they never exceed the maximum radius reached during helium burning (Costa et al. 2025). For this reason, we truncated the evolutionary tracks shown in the figures either at maximum expansion ($Z \leq 0.0001$) or when stars become Wolf-Rayet ($Z = 0.01$). The last point shown corresponds to a stellar age of $t = t_{\max}$. This evolutionary stage

is the most relevant for binary interactions, as the probability of initiating Roche-lobe overflow and a CE phase is maximized near maximum expansion. Finally, in the case of very massive stars ($M_{ZAMS} \geq 250 M_{\odot}$ at $Z = 10^{-11}$ and $M_{ZAMS} \geq 150 M_{\odot}$ at $Z = 0.0001$), radial expansion occurs before core formation. Any binary interaction would take place during the expansion phase. Therefore, from a binary population-synthesis perspective, the envelope binding energies of these stars are unlikely to

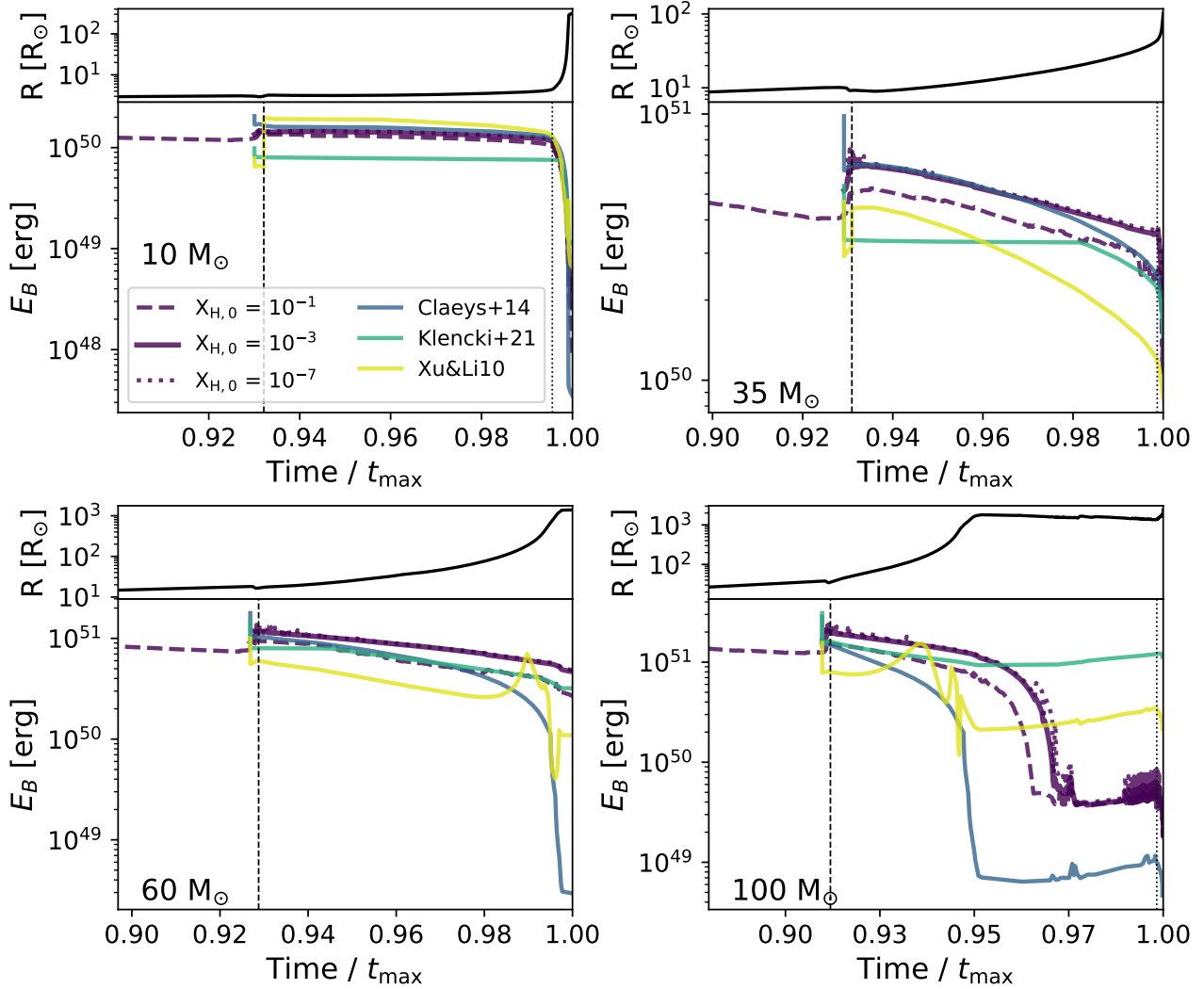


Fig. 3. Evolution of the envelope binding energy for a sample of stars with $M_{\text{ZAMS}} = 10 M_{\odot}$ (top left), $35 M_{\odot}$ (top right), $60 M_{\odot}$ (bottom left), and $100 M_{\odot}$ (bottom right). All the stars have metallicity $Z = 10^{-11}$. The x axis is the stellar age normalized to t_{max} , where t_{max} is defined as the time at which the star reaches its maximum radial expansion ($Z \leq 0.0001$) or becomes a Wolf-Rayet ($Z = 0.01$). The tracks are truncated at this point in the figures, since after maximum expansion stars typically contract and are unlikely to initiate further common-envelope interactions. The purple lines show the envelope binding energies, E_B , computed in this work, for different $X_{\text{H},0}$ thresholds (different linestyles): solid, $X_{\text{H},0} = 10^{-3}$; dashed, $X_{\text{H},0} = 10^{-1}$; and dotted, $X_{\text{H},0} = 10^{-7}$. The other colors show different envelope binding energies adapting different λ prescriptions to the PARSEC stellar tracks with SEVN: the solid blue line assuming Claeys et al. (2014), the solid green line Klencki et al. (2021), and the solid yellow line Xu & Li (2010). The lines start in different moments in time for the same star due to the different core formation conditions; however, for visualization purposes we do not show the full evolution of the $X_{\text{H},0} = 10^{-1}$ model, as it extends well before the other conditions but the star does not evolve much in the meanwhile. The upper panels of each plot show the corresponding evolution of the radii of the stars. The vertical dashed and dotted lines indicate the onset of core He burning and shell He burning, respectively.

play a major role in typical CE interactions. For clarity, we do not include these stars in the plots.

Generally, lower core masses exert a lower gravitational pull on the envelope. As a consequence, stars with lower M_{ZAMS} yield lower binding energies at equivalent radii. We find differences in the envelope binding energies within a factor of ~ 2 among the $X_{\text{H},0}$ models, for all the stars and metallicities. Massive stars at low metallicity ($M_{\text{ZAMS}} \geq 150 M_{\odot}$, $Z \leq 0.0001$) undergo a significant radial expansion during the early phases of core H burning. Thus, the definition of the helium core is particularly important for the description of mass transfer in such cases. For instance, if we define the helium core as the region where $X_{\text{H},0}$ falls below 0.1, the star already has a He core during its first Roche-lobe overflow, whereas if we use a stricter definition (e.g., $X_{\text{H},0} = 10^{-3}$) the star will still be core-less during mass transfer. On the other hand, the

model employing $X_{\text{H},0} = 10^{-1}$ displays higher fluctuations in the envelope binding energies, suggesting that the core is still growing because hydrogen burning is still ongoing.

Despite being important, the definition of the He core has a relatively minor impact compared to the definition of envelope binding energy, E_G , E_B , and E_H . Specifically, E_G consistently appears to be almost twice as large as E_B . The difference between E_H and E_G is even more pronounced, with E_H being about one order of magnitude lower than E_G . Nevertheless, E_H is affected by larger fluctuations (see, e.g., $M_{\text{ZAMS}} = 35 M_{\odot}$, $Z = 0.0001$). The reason for this is that the contribution from the enthalpy term progressively approaches the gravitational one as the stars reach the end of their lives. This leads to high fluctuations in E_H , as the two terms become similar. The effect is particularly evident for $X_{\text{H},0} = 10^{-1}$.

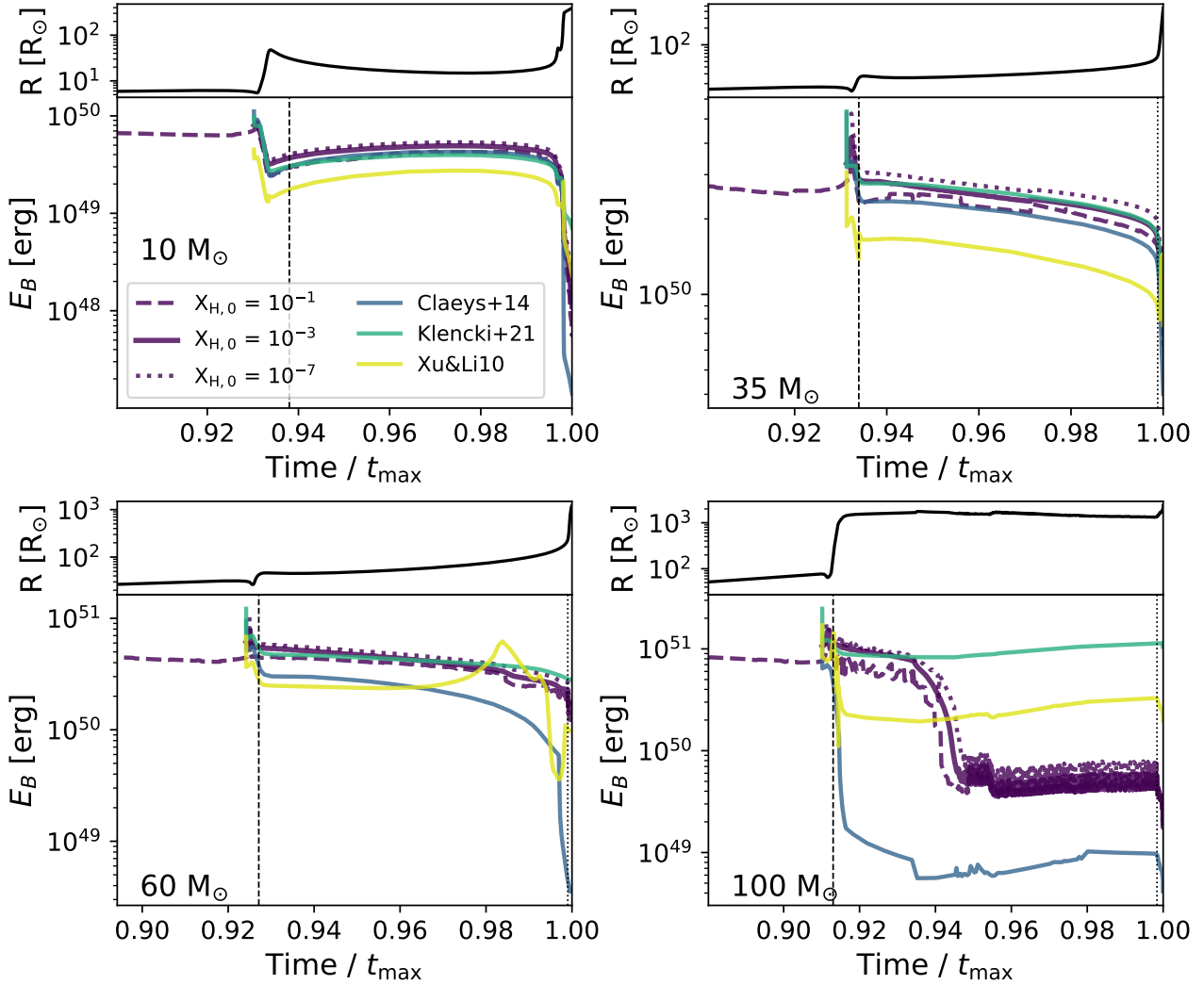


Fig. 4. Same as Fig. 3, but for metallicity $Z = 0.0001$.

Figure 2 shows E_B as a function of the radius, R , for several values of the ZAMS mass and metallicity. The color scale indicates the fraction of envelope contained within the outer convective zone:

$$f_{\text{CE}} = \frac{M_{\text{conv}}}{M_{\text{env}}}, \quad (12)$$

where M_{conv} is the mass of the outermost contiguous convective zone of the envelope. We can compare Fig. 2 with Fig. 1 in Klencki et al. (2021). Our models qualitatively agree with Klencki et al. (2021): we see a drop in the envelope binding energy as the envelope becomes mostly convective ($f_{\text{CE}} \geq 0.6$). As observed in Klencki et al. (2021), a deep convective layer does not necessarily mean lower binding energies. Indeed, the massive stars, $M_{\text{ZAMS}} \geq 60 M_{\odot}$ at $Z = 10^{-11}$ and $M_{\text{ZAMS}} = 150 M_{\odot}$ at $Z = 0.0001$, develop a deep convective envelope ($f_{\text{CE}} \sim 0.6$); however, they do not show a drop in E_B . In fact, these stars show a decrease of only $\sim 10\%$, compared to the $\sim 70\text{--}90\%$ decrease observed in less massive stars when they develop deep convective envelopes.

For massive stars ($M_{\text{ZAMS}} \gtrsim 60 M_{\odot}$), the point of core He depletion happens after the maximum radial expansion; therefore, it does not appear in the plot. At $Z = 0.01$, this stage occurs when the star has already become a Wolf-Rayet, while

at $Z \leq 0.0001$, helium depletion takes place after the star has reached its maximum expansion radius.

3.2. The importance of using consistent envelope binding energies in population synthesis

Figures 3, 4, and 5 show the envelope binding energy as a function of time, computed with different prescriptions. Among the models (different colors), we observe differences exceeding an order of magnitude. Such differences dominate over variations arising from distinct core-envelope boundary conditions independently of the metallicity and stellar mass.

The model proposed by Claeys et al. (2014), commonly employed in binary population synthesis codes, consistently underestimates envelope binding energies by more than an order of magnitude as stars approach the giant phase. This is particularly evident at higher metallicities, where the discrepancy can be as high as 3 orders of magnitude (e.g., $M_{\text{ZAMS}} = 60 M_{\odot}$). Our binding energies are instead in broad agreement with the model by Klencki et al. (2021), obtained with MESA. However, extrapolating the model by Klencki et al. (2021) outside of the original parameter space ($M_{\text{ZAMS}} \leq 80 M_{\odot}$, $Z \leq 0.0001$) leads to large discrepancies with our computed values. This is particularly evident for the star with $Z = 10^{-11}$ and $M_{\text{ZAMS}} = 100 M_{\odot}$. On the other hand, the prescription by Xu & Li (2010), based

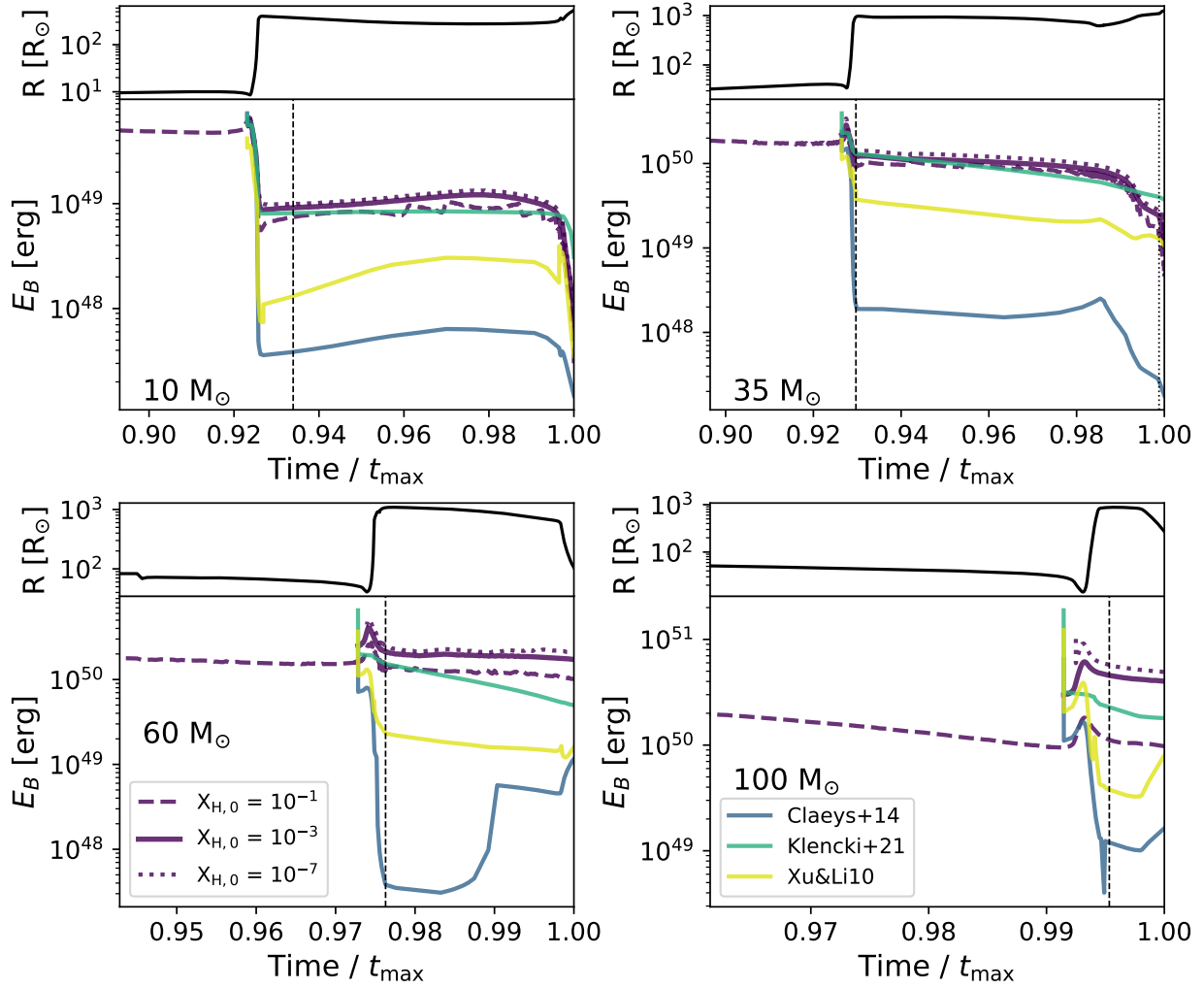


Fig. 5. Same as Fig. 3, but for metallicity $Z = 0.01$.

on stars with masses between 1 and $100 M_{\odot}$ at $Z = 0.001$ and 0.02 , significantly deviates from our model, even considering the same range of stars. These substantial divergences highlight the importance of using envelope binding energies that are consistent with the underlying stellar evolution tracks when modeling the CE phase in population synthesis simulations.

The adoption of different $X_{H,0}$ affects the core formation times, leading to an earlier formation (i.e., $X_{H,0} = 10^{-1}$) or a delay (i.e., $X_{H,0} = 10^{-7}$). This effect is likely negligible in binary evolution. In fact in most cases the stars do not undergo significant radial expansion during the period when the $X_{H,0} = 10^{-1}$ model already has formed an He core, while the $X_{H,0} = 10^{-3}$ one has not. We note that the envelope binding energies computed in Klencki et al. (2021) assume $X_{H,0} = 10^{-1}$, implying that these stars should form the core simultaneously with our dashed line. However, when this prescription is applied in SEVN with the PARSEC evolutionary tracks, the core forms instead according to the condition $X_{H,0} = 10^{-3}$. As a result, the starting point of the model from Klencki et al. (2021) coincides with the $X_{H,0} = 10^{-3}$ model, in the plot.

3.3. Pure-He stars

Figure 6 shows E_G , E_B , and E_H computed for pure-He tracks with different initial stellar masses and metallicities. For pure-He tracks we show the reference metallicities $Z = 0.0001$, 0.001 , and 0.01 .

Pure-He stars remain significantly more compact throughout their lifetimes compared to hydrogen-rich stars. As a result, their envelope binding energies also span a much narrower range. Moreover, for a given initial mass, the evolutionary tracks are similar across various metallicities. For a wide mass range ($M_{\text{He}} < 1 M_{\odot}$, $10 < M_{\text{He}} < 200 M_{\odot}$), these stars tend to shrink for most of their lives, reducing the likelihood of binary interactions.

Unlike H-rich stars, for which the contributions to the binding energy differ significantly depending on the considered energy contributions, pure-He stars show much smaller variations, typically no more than a factor of two between E_G and E_H . However, the choice of core-envelope boundary has a substantial impact, especially for high-mass stars ($M_{\text{He}} \geq 30 M_{\odot}$).

We also compare our new E_{bind} with two additional models. We show the prescription by Claeys et al. (2014), which assumes a constant $\lambda = 0.5$ for pure-He stars. This is the default value adopted in SEVN when using the models by Claeys et al. (2014) and Klencki et al. (2021). We also display the fit implemented in COMPAS (Riley et al. 2022):

$$\lambda = 0.3R_{\lambda}^{-0.8} \quad R_{\lambda} = \min(120, \max(0.25, R)), \quad (13)$$

where all quantities are in solar units. SEVN adopts this fit with the Xu & Li (2010) prescription (see Appendix A1.4 in Iorio et al. 2023 for more details).

The $\lambda = 0.5$ model deviates substantially from the PARSEC results. This is evident for low-mass stars ($M_{\text{He}} \leq 5 M_{\odot}$),

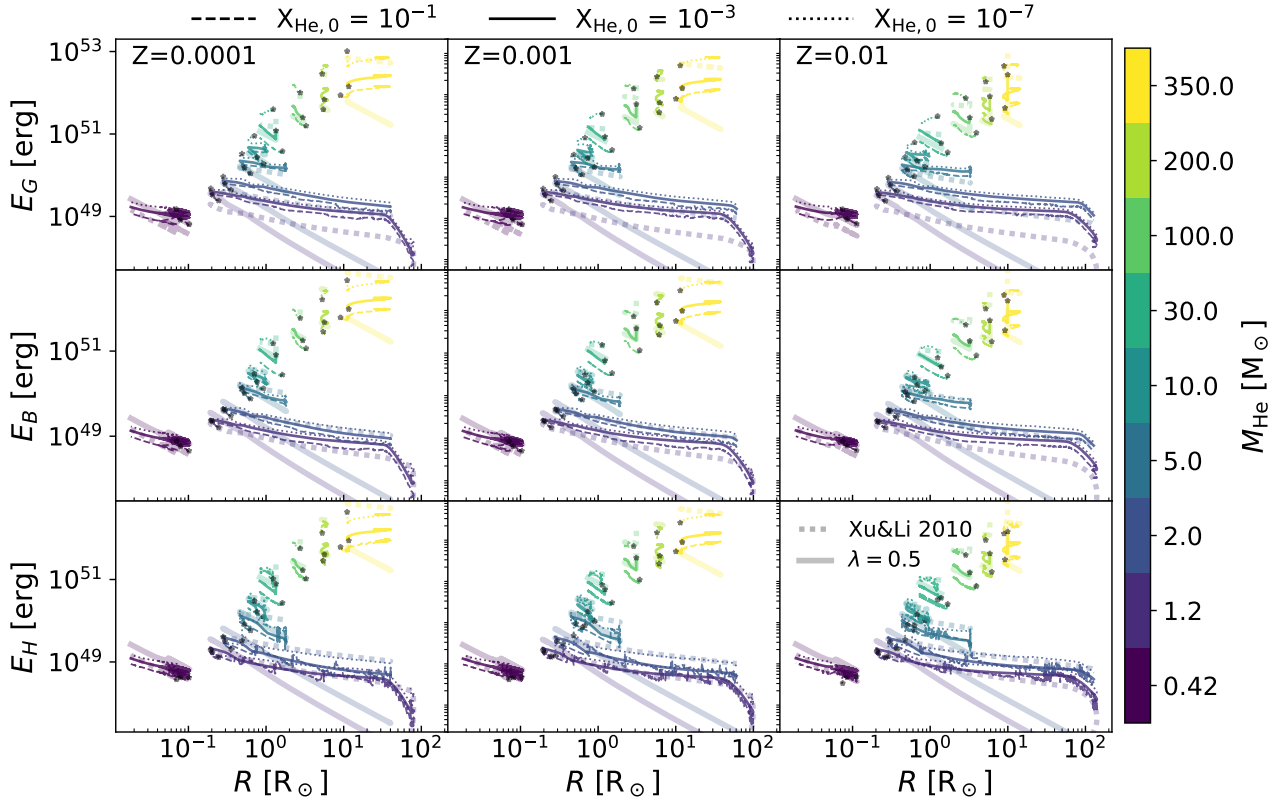


Fig. 6. Same as Fig. 1 but with pure-He tracks. The thick solid lines show the envelope binding energies with constant $\lambda = 0.5$, following [Claeys et al. \(2014\)](#). This is the default value adopted in SEVN for the prescriptions by [Claeys et al. \(2014\)](#) and [Klencki et al. \(2021\)](#). The thick dotted lines show the envelope binding energies as implemented in COMPAS. This model is used by SEVN with the fitting formulas by [Xu & Li \(2010\)](#).

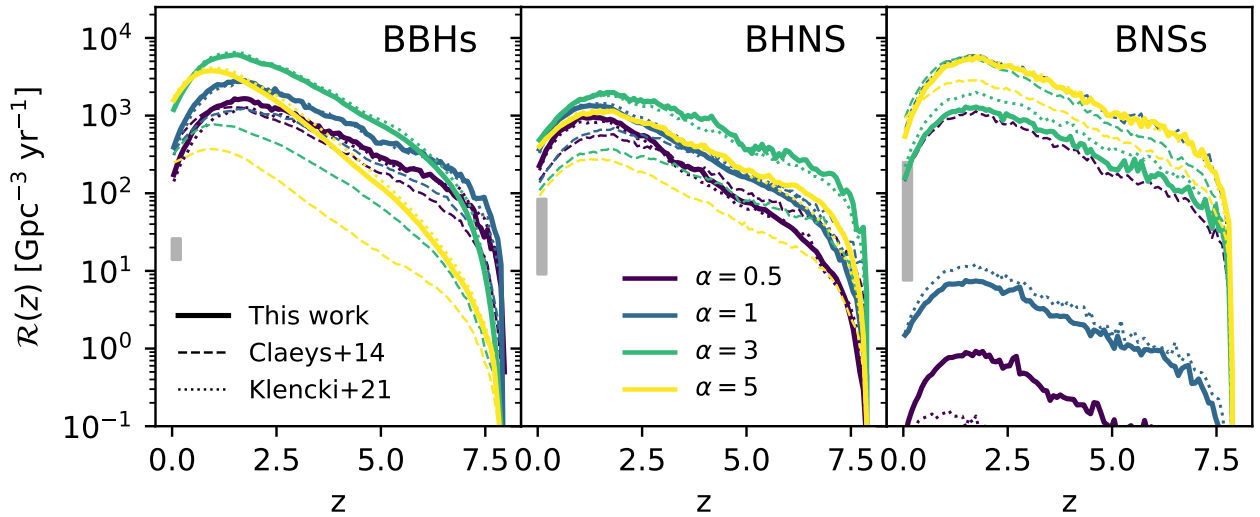


Fig. 7. Merger rate density of BBHs (left), BHNS (center), and BNSs (right) for different λ prescriptions and α parameters. The solid lines show the results adopting the λ self-consistent with the PARSEC tracks, derived from E_B and $X_{H,0} = X_{He,0} = 10^{-3}$. The dashed lines assume the λ prescription by [Claeys et al. \(2014\)](#). The dotted lines assume the prescription by [Klencki et al. \(2021\)](#). The different colors show the different α parameters: purple $\alpha = 0.5$, blue $\alpha = 1$, green $\alpha = 3$, and yellow $\alpha = 5$. The shaded gray areas show the merger rate densities inferred by LVK ([The LIGO Scientific Collaboration et al. 2025a](#)).

where discrepancies exceed an order of magnitude toward the end of stellar evolution. On the other hand, the envelope binding energies obtained with Eq. (13) are in much better agreement with the E_{bind} by PARSEC, especially when considering stars with $M_{\text{He}} \lesssim 30 M_{\odot}$ and the E_B model.

3.4. Merger rate density

We estimated the merger rate density of binary compact objects following the methodology outlined in Sect. 2.6. Figure 7 shows the resulting merger rate densities for binary black holes (BBHs,

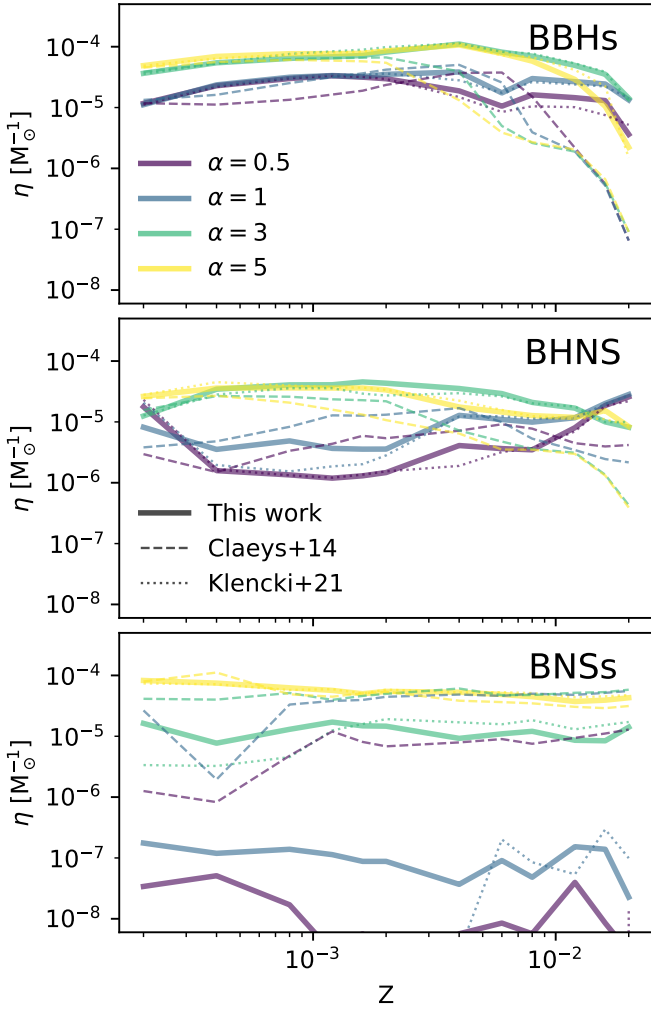


Fig. 8. Merger efficiency, η , for BBHs (top), BHNSs (middle), and BNSs (bottom) as a function of metallicity, Z . The solid lines represent the results assuming the PARSEC envelope binding energies (E_B) computed with $X_{\text{He},0} = X_{\text{He},0} = 10^{-3}$. The other line styles assume different λ prescriptions: dashed lines for Claeys et al. (2014) and dotted lines for Klencki et al. (2021). The different colors show results for different α parameters: $\alpha = 0.5$ (purple), 1 (blue), 3 (green), and 5 (yellow).

left), black hole–neutron stars (BHNSs, middle), and binary neutron stars (BNSs, right) with our new consistent envelope binding energies (solid lines). The merger rates’ differences between our new formalism and Claeys et al. (2014) are as high as a factor of two for many of the models. The effect of the new E_{bind} is to boost the merger rate densities of BBH and BHNS systems for $\alpha \gtrsim 1$. Conversely, the new values of E_{bind} quench the BNS merger rate densities, with a more apparent effect for $\alpha \leq 1$.

The merger rate densities agree well with the model by Klencki et al. (2021). This was expected, given the similarities between the two E_{bind} prescriptions.

Differences in the BBH merger rate densities are primarily driven by variations in the merger efficiency’s dependence on metallicity, as illustrated in Fig. 8. For $\alpha = 3, 5$ the envelope binding energies by Claeys et al. (2014) predict a drop in the BBH merger efficiency for $Z \sim 5 \times 10^{-3}$. In contrast, with the PARSEC envelope binding energies, the merger efficiency stays constant up to $Z \sim 10^{-2}$. The latter trend is very similar to the model by Klencki et al. (2021), where, as showed in Iorio et al. (2023), the drop occurs only for $Z \gtrsim 10^{-2}$. Since higher metal-

licities play a major role in the cosmic star formation history (Sgalletta et al. 2025), both the PARSEC and the Klencki et al. (2021) E_{bind} predict a boost in the BBH merger rate density. On the other hand, for $\alpha < 1$ the BBH merger efficiency is lower than the one obtained adopting the model by Claeys et al. (2014), considering metallicities $2 \times 10^{-3} \lesssim Z \lesssim 10^{-2}$. Similarly, the merger rate density of BNSs for $\alpha = 0.5, 1$ is significantly suppressed as a consequence of lower merger efficiencies.

The envelope binding energies by Claeys et al. (2014) are consistently lower than those predicted by PARSEC. Higher binding energies have two primary effects: making the shrinkage during CE more efficient (thus leading to lower post-CE separations) and increasing the occurrence of failed CE ejections. For BBH systems at high metallicity (i.e., $Z = 0.02$), the former effect dominates. While most BBHs remain wider in Claeys et al. (2014), the new envelope binding energies facilitate orbital shrinkage. This leads to a substantial population of systems being able to merge within a Hubble time even at $Z \sim 0.02$. Conversely, for BNSs and low values of α , failed CE ejections constitute the dominant effect.

4. Discussion

Sgalletta et al. (2025) pointed out that current binary population synthesis codes tend to predict higher BBH merger rate densities compared to inferred rates from the LIGO-Virgo-KAGRA collaboration (Abbott et al. 2023; The LIGO Scientific Collaboration et al. 2025b). The formalism presented here accentuates this discrepancy, pushing the BBH and BHNS merger rates well outside the 90% credible intervals if $\alpha > 1$. In order to relieve the tension, we might therefore favor models with $\alpha < 1$. However, the models with $\alpha < 1$ produce BNS merger rates below the 90% credible interval inferred from observations (The LIGO Scientific Collaboration et al. 2025a). A possible solution would be a varying α parameter, depending on the properties of the stars within the CE phase. Studies on double white dwarf systems have already highlighted the possibility of α being a function of the masses of the white dwarfs, as demonstrated by Ivanova et al. (2013) and Röpke & De Marco (2023). Similarly, Chruslinska et al. (2018) propose different CE ejection mechanisms to explain the rates of BNSs and BBHs. Our results support the scenario of a varying α parameter for massive progenitor stars. The recently developed two-stage formalism for CE treatment (Hirai & Mandel 2022) finds evidence of a broad range of α parameters, opposed to a universal value (Picker et al. 2024).

Nevertheless, several caveats remain. Our analysis is based purely on single stellar evolutionary tracks. A detailed response of stars to mass transfer and mass accretion could be crucial in understanding the final fate of these systems. Indeed, recent studies suggest that the CE channel might occur less frequently than is currently assumed in population synthesis calculations (e.g., Ge et al. 2020; Gallegos-Garcia et al. 2021; Marchant et al. 2021; van Son et al. 2022). An additional caveat concerns the fact that envelope binding energies are computed assuming envelopes bound to single stars in hydrostatic equilibrium, whereas during the CE phase the envelope becomes part of a highly dynamical binary system, and its initial binding energy may not be conserved. As a result, the envelope binding energy governing the interaction may differ from that inferred from the initial stellar configuration.

A final comment regards the role played by convection in affecting the fate of mass transfer events (Klencki et al. 2021; Hirai & Mandel 2022). Figure 9 is the same as Fig. 2, but the

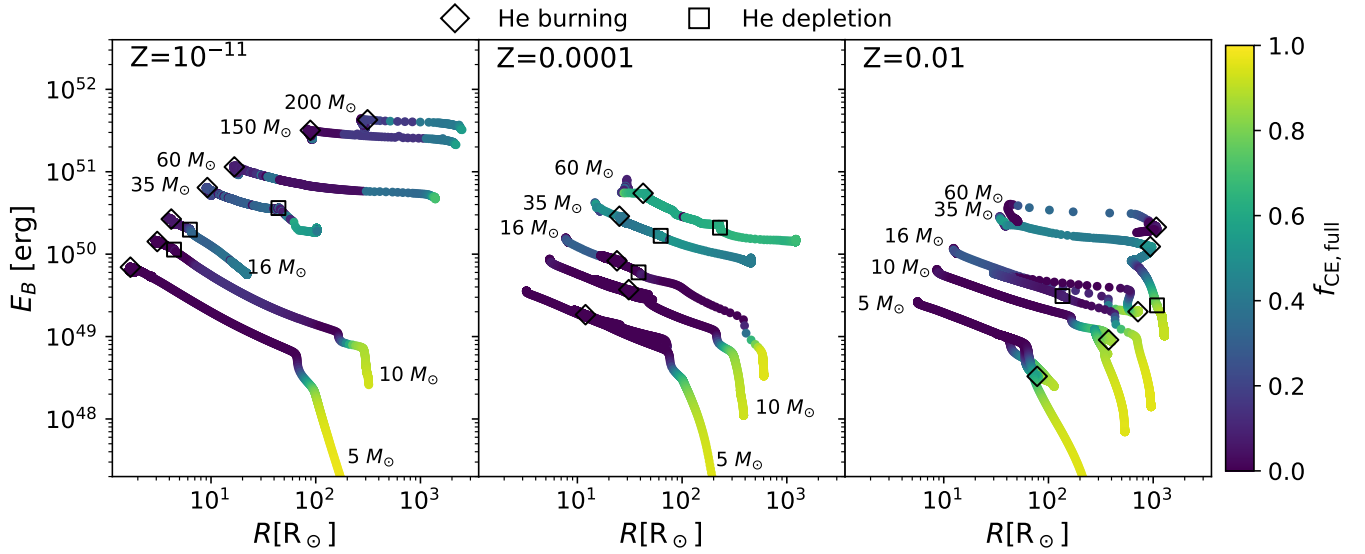


Fig. 9. Same as Fig. 2 but colored by $f_{\text{CE,full}}$, the fraction of the total convective envelope mass over the envelope mass.

colors show

$$f_{\text{CE,full}} = \frac{M_{\text{conv,full}}}{M_{\text{env}}}, \quad (14)$$

where $M_{\text{conv,full}}$ is the total mass of the convective envelope, accounting for both the outer convective zone and the inner convective zones within the stellar envelope. Compared to Fig. 2, we observe that stars (especially those with masses $\geq 30 M_{\odot}$) develop internal convective regions even before these zones reach the surface of the star. In these cases, the response of the star is thus unclear. There may be episodes in which the radiative layers are stripped away and the response of the star becomes the same as for a convective star. A model that considers only the outer convective layers fails to capture this complexity.

5. Summary

We have derived the envelope binding energies for a set of stars that is extensive in terms of both mass and metallicity. We have investigated different energy contributions to the envelope binding energy as well as alternative core-boundary criteria. Additionally, we studied how different prescriptions influence the predicted merger rate densities of compact object systems. We summarize our main findings below.

- In the case of H-rich stars, varying the $X_{\text{H},0}$ threshold affects the envelope binding energies by no more than a factor of ~ 2 in most cases. However, the choice of which energy sources to include (E_{G} , E_{B} , or E_{H}) has a more significant impact, with E_{H} being roughly an order of magnitude smaller than E_{G} . Conversely, for pure-He stars, the choice of $X_{\text{He},0}$ has a greater influence on the envelope binding energy than the selected energy contributions.
- The inconsistencies between various envelope binding energy prescriptions, each evaluated with different stellar evolution codes, emphasize the need to adopt binding energies that are consistent with the specific stellar models being used. Additionally, it is crucial to avoid extrapolating these prescriptions beyond the parameter space for which they were originally inferred.
- Differences in envelope binding energy models directly translate into differences in the predicted merger rate den-

sities, with discrepancies exceeding an order of magnitude. Compared to the prescription by Claeys et al. (2014), the higher PARSEC envelope binding energies cause a boost in the merger rate densities of BBHs and BHNS for $\alpha \geq 1$. Concurrently, they suppress the BNS merger rate density for $\alpha < 1$. This trend is in agreement with the model by Klencki et al. (2021), which also predicts similar envelope binding energies.

Data availability

The SEVN tables used in this work are publicly available in Zenodo at the link [10.5281/zenodo.18340609](https://doi.org/10.5281/zenodo.18340609).

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[//gitlab.com/Filippo.santoliquido/galaxy_rate_open](https://gitlab.com/Filippo.santoliquido/galaxy_rate_open). This research made use of NUMPY (Harris et al. 2020), SCIPLY (Virtanen et al. 2020), PANDAS (The pandas development Team 2024) and ASTROPY (Astropy Collaboration 2013, 2018, 2022). For the plots we used MATPLOTLIB (Hunter 2007).

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Appendix A: Our synthetic galaxy model

Here, we summarize the details of the model we used to calculate binary compact objects merger rate densities. We refer to the fiducial model described by Sgalletta et al. (2025) for additional details.

A.1. The galaxy stellar mass function

We simulate a comoving volume $V \sim (70 \text{ cMpc})^3$ and populate it with galaxy stellar masses M_* drawn from the galaxy stellar mass function derived by Chruslinska & Nelemans (2019). The fit takes the following form:

$$\phi(M_*, z) dM_* = \phi_N(z) e^{-M_*/M_{\text{cut}}(z)} \left(\frac{M_*}{M_{\text{cut}}(z)} \right)^{-\alpha_{\text{GSMF}}} dM_*, \quad (\text{A.1})$$

where $\phi_N(z)$ is a normalization factor and $M_{\text{cut}}(z)$ is the galaxy stellar mass at which the function transitions from a power law at low masses to an exponential decay at high masses. We sample the galaxy masses within the range $M_* \in [10^7, 10^{12}] M_\odot$. We generate in this way $N_{\text{gal}}(z)$ for an array of redshifts, from 0 to $z_{\text{max}} = 8$.

A.2. Galaxy mass – star formation rate

We assume the galaxy main sequence relation derived in Popesso et al. (2023) to determine the galaxies' star formation rates. We draw the galaxies' star formation rates from a double log-normal distribution (Sargent et al. 2012; Rodighiero et al. 2015; Schreiber et al. 2015):

$$\mathcal{P}(\log \text{SFR} | M_*, z) = A_{\text{MS}} \exp \left[-\frac{(\log \text{SFR} - \langle \log \text{SFR} \rangle_{\text{MS}})^2}{2\sigma_{\text{MS}}^2} \right] + A_{\text{SB}} \exp \left[-\frac{(\log \text{SFR} - \langle \log \text{SFR} \rangle_{\text{SB}})^2}{2\sigma_{\text{SB}}^2} \right], \quad (\text{A.2})$$

where $A_{\text{MS}} = 0.97$ and $A_{\text{SB}} = 0.243$ constants (Sargent et al. 2012). Here, the subscript MS (SB) stands for galaxy main sequence (starburst sequence). We assume the galaxy main sequence to follow the relation by Popesso et al. (2023):

$$\langle \log \text{SFR} (M_*, t) \rangle_{\text{MS}} = a_0 + a_1 t - \log \left[1 + \left(M_*/10^{a_2 + a_3 t} \right)^{-a_4} \right] \quad (\text{A.3})$$

with $a_0 = 2.693 \pm 0.012$, $a_1 = -0.186 \pm 0.009$, $a_2 = 10.85 \pm 0.05$, $a_3 = -0.0729 \pm 0.0024$ and $a_4 = 0.99 \pm 0.01$. On the other hand, the starburst sequence parameters are defined as in Sargent et al. (2012),

$$\langle \log \text{SFR} \rangle_{\text{SB}} = \langle \log \text{SFR} \rangle_{\text{MS}} + 0.59 \quad (\text{A.4})$$

We assume $\sigma_{\text{SB}} = 0.243$ dex.

A.3. Fundamental metallicity relation

Finally, we associate an average metallicity with each galaxy according to the fundamental metallicity relation by Andrews & Martini (2013):

$$12 + \log(\text{O}/\text{H}) = 0.43(\log M_* - 0.66 \log \text{SFR}) + 4.59. \quad (\text{A.5})$$

Within each galaxy, we distribute the metallicities following a log-normal function around this average value, with a scatter $\sigma_Z = 0.14$. We adopt the solar metallicity values from Caffau et al. (2011): $Z_\odot = 0.0153$ and $12 + \log(\text{O}/\text{H})_\odot = 8.76$ to convert the relation into absolute metallicity.

A.4. Merger rate density evaluation

We estimate the merger rate density as Santoliquido et al. (2022):

$$\mathcal{R}(z) = \frac{1}{V^3} \int_{z_{\text{max}}}^z \left[\int_{z_{\text{min}}}^{z_{\text{max}}} \mathcal{S}(z', Z) \mathcal{F}(z', z, Z) dZ \right] \frac{dt(z')}{dz'} dz', \quad (\text{A.6})$$

where V is the total simulated comoving volume. The star formation rate density $\mathcal{S}(z', Z)$ is given by:

$$\mathcal{S}(z', Z) = \sum_{i=1}^{N_{\text{gal}}(z')} \psi_i(z') p_i(Z|z') \quad (\text{A.7})$$

where we sum over all the galaxies $N_{\text{gal}}(z')$ formed at redshift z' . For the i -th galaxy, $\psi_i(z')$ is its star formation rate and $p_i(Z|z')$ its metallicity distribution. The term $\mathcal{F}(z', z, Z)$ is defined as:

$$\mathcal{F}(z', z, Z) = \frac{1}{M_{\text{sim}}} \frac{N(z', z, Z)}{dt} f_{\text{bin}} f_{\text{corr}}, \quad (\text{A.8})$$

where M_{sim} is the total simulated initial mass, $\frac{N(z', z, Z)}{dt}$ is the rate of binary compact objects that form at z' , with metallicity Z and merge at redshift z . The factor $f_{\text{corr}} = 0.251$ takes into account the incomplete sampling of the initial mass function (see Sect. 2.4), $f_{\text{bin}} = 0.5$ (Moe & Di Stefano 2017) represents the fraction of binaries.