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EXPLORING PRIMORDIAL MAGNETISM
VIA STRUCTURE FORMATION, GALAXIES,
AND INTERGALACTIC MEDIUM

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ABSTRACT This work examines Primordial Magnetic Fields (PMFs) and their effects on cosmological observables. PMFs can enhance the matter power spectrum on scales smaller than Megaparsec, because their presence before recombination induces additional baryon perturbations. This has various consequences; in this thesis we will present and discuss three directions: 1) PMF-related perturbations are different for baryons vs. dark matter, which leads to a higher baryon fraction in low-mass halos during early structure formation. 2) since PMFs are likely present in voids and the intergalactic medium (IGM), their disturbance on the neutral hydrogen fraction should have a limit based on Lyman- α forest observations. From this we obtain constraints on PMF parameters. 3) With newer JWST data on early galaxy formation, it is useful to explore how would PMFs contribute to high-redshift UV luminosity function, and what are the implications for reionization.

The thesis is based on our original contributions to the field of PMFs and cosmology. We have advanced the theoretical, numerical, and observational understanding of PMFs in two directions: evolution from initial conditions and observational constraints.

In our first study we have modified initial conditions of cosmological simulations so that the displacements due to Lorentz force are calculated directly from the magnetic field. Consequently, the non-linear evolution in cosmo-hydrodynamical simulations leads to low-mass galaxies with gas fraction several times larger than the cosmic average at high redshifts. This is a direct consequence of the initial conditions, where PMFs enhance perturbations of baryons, causing them to be larger than perturbations of dark matter. Some preliminary discussion is presented on how such a scenario could be tested by obtaining accurate estimates of the baryon fraction in high redshift galaxies. We also highlight the large theoretical uncertainty in the peak enhancement of the matter power spectrum due to PMFs, which was not considered in previous studies. Compatibility with other literature is shown through halo mass functions, and we show that they can be accurately reproduced using Sheth-Tormen formalism.

In the second study, we find constraints on PMFs from the Lyman- α forest. Compared to previous works focusing on Lyman- α constraints, we employ full cosmological hydrodynamic simulations. At the scales and redshifts probed by the data, the Lyman- α flux power spectrum is extremely sensitive to the extra power induced by PMFs in the linear matter power spectrum, at a scale that we parametrize with k_{peak} . We rely on a large set of flux models obtained by varying thermal and reionization histories and cosmological parameters. We find a hint of extra power that is well fitted by the primordial magnetic field model with $B \simeq 0.2$ nG, corresponding to $k_{\text{peak}} \simeq 50$ Mpc $^{-1}$. However, when applying additional modeling of the noise or disregarding the data from the lowest scales, we obtain a competitive 3σ C.L. upper limit on the strength of primordial magnetic fields: $B_{1\text{Mpc}} < 0.30$ nG, for nearly scale-invariant case $n_B = -2.9$. We argue that the constraints can be extended to a broader (B, n_B) plane.

Finally, we present our ongoing work in the direction that explores how PMFs impact galaxy luminosity function and reionization, with the new JWST data.

PUBLICATIONS

This thesis is based on the following publications

- [1] P. Ralegankar, M. Pavičević, and M. Viel,
PRIMORDIAL MAGNETIC FIELDS: CONSISTENT INITIAL CONDITIONS
AND IMPACT ON HIGH-Z STRUCTURES,
Journal of Cosmology and Astroparticle Physics, vol. 07, p. 027, 2024.
DOI: [10.1088/1475-7516/2024/07/027](https://doi.org/10.1088/1475-7516/2024/07/027). *arXiv*: [2402.14079](https://arxiv.org/abs/2402.14079) [[astro-ph.CO](#)].
- [2] M. Pavičević, V. Iršič, M. Viel, J. S. Bolton, M. G. Haehnelt, S. Martin-Alvarez,
E. Puchwein, P. Ralegankar,
CONSTRAINTS ON PRIMORDIAL MAGNETIC FIELDS FROM THE LYMAN- α FOREST,
Physical Review Letters, vol. 135, no. 7, p. 071 001, 2025.
DOI: [10.1103/77rd-vkpz](https://doi.org/10.1103/77rd-vkpz). *arXiv*: [2501.06299](https://arxiv.org/abs/2501.06299) [[astro-ph.CO](#)].
- (in preparation) M. Pavičević, A. D'Ambrosio, M. Viel, and V. Iršič.

Our research and results have been presented by the author at several visits, seminars and conferences; among them:

- Dec., 2025 | Cosmology seminar at the Central European Institute for Cosmology and Fundamental Physics (CEICO), Prague, Czech Rep. (correspondence with V. Danieli)
- Mar., 2025 | Seminar for the *LSS meeting* at the Institute for the Fundamental Physics of the Universe (IFPU), Trieste, Italy
- Feb., 2025 | Astrophysics & Cosmology seminar at Lyon Observatory, France
(correspondence with J. Rosdahl)
- Jul., 2024 | Short talk at *Cosmology at the Adriatic*, MEDILS, Split, Croatia
- May, 2024 | Contributed talk at the conference *Generation, evolution, and observation of cosmological magnetic fields*, Bernoulli Center–EPFL, Lausanne, Switzerland
- Feb., 2024 | Short talk at *Cosmology with CMB and LSS*, CNR/INAF, Bologna, Italy

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Introduction and Outline of the Thesis

Magnetism from the earliest stages of the universe, the ‘primordial’ epoch, has been motivated on many theoretical and observational grounds (§1.2.1). It is primarily because survival of Primordial Magnetic Fields (PMFs) from the post-Big Bang era into post-recombination cosmological stages would give them reputation of seeds for galactic magnetism and of Intergalactic Magnetic Fields (IGMFs). In the last two decades the primary focus was set precisely on those, since primordial nature would evolve them to be distributed on large scales and IGMF could explain certain features in the γ -ray spectra of blazar observations; in this scenario, a very small IGMF is capable of diverting the paths of charged particles, which can account for missing high-energy photons in the blazar spectra. One cannot repudiate that it is the seeding mechanism and the blazar observations, combined with theoretical arguments, which serve today as the two main pillars of possible PMF existence. To fully corroborate such hints of primordial magnetism, one must study how cosmic magnetic fields contribute to various environments in the universe. Our focus was set on pre-galactic structures (dark matter halos and baryon fraction), galaxies (their UV luminosity function) and the intergalactic medium (Lyman- α forest and reionization).

The seeds of our work were rooted for the first time when my colleagues and mentors, Matteo Viel and Pranjal Ralegankar, noticed that magnetism in the early universe affects the motions and statistics of baryonic particles to such an extent that—not only one must include these variations in the power spectrum of matter—one must specifically distinguish baryon and dark matter evolution, and carefully modify initial configurations of particles in cosmological simulations. Since I had been very curious about the theory of PMFs, and its cosmological and astrophysical implications, we joined our efforts in understanding whether, in the light of such new remarks, theoretical-numerical improvements could blossom into interesting predictions of pre-galactic physics.

Indeed, building on works of previous authors who studied signatures of PMFs on structure formation (§1.3.2), we found that by adding PMF-related perturbations to the initial conditions, cosmological evolution will lead to the formation of a higher number of low-mass dark matter halos at early redshifts ($z > 10$), and these halos will contain large baryon fractions in the form of gas (§2).

Such findings and their analysis are especially interesting in the current era of the James Webb Space Telescope (JWST) observations (§1.1.7), offering a window to rich physics of star formation and galaxy evolution, especially at very high redshifts. For this reason, at the later stages of our research, we were naturally compelled to explore the implications of the early universe presence of magnetism on the formation of stars,

galaxies, and their properties (§4). As expected, the birth of additional stars influences the process of reionization (§4.4).

*

At the same time, the parameters of PMFs cannot attain arbitrary values, since magnetic fields that are too strong would overly distort cosmological evolution and this would not fit many well-established experimental data points. Therefore—to avoid what astrophysicist L. Woltjer articulated in 1967 as:¹ *the larger one’s ignorance, the stronger the magnetic field*—a full multi-disciplinary process of elimination is developed by the community. Namely, constraining PMF parameters (§1.3.1) is currently the best guiding force to understand the importance (or redundancy) of PMFs;

Naturally, it was also our task to impose limits on PMFs. In this regard, to find permissible values of PMF parameters, we used the expertise of my mentor, along with essential insights of our collaborator Vid Iršič, on the Lyman- α (Ly- α) forest — that is, the absorption lines in the spectra of quasars whose light travels through the neutral hydrogen component of the intergalactic medium (§1.1.5.2). In this context, our strong constraints (§3) complement the already rich set of experimental and theoretical limits, and the Ly- α forest proved to be a very useful proxy for probing PMFs.

To update or maximize the constraining power, one must have access to observational data at smaller and smaller scales, much less than a Megaparsec. This is due to the fact that PMF parameters are related to a particular scale where magneto-gravitational effects are the strongest (§1.2.4.4). One could therefore say, in the context of constraints, *the smaller the scale of our ignorance, the weaker the primordial magnetic field*.

*

It is important to emphasize how crucial are numerical simulations (§1.1.3) in this field of research. Most of the advancements that our work has brought are a result of many cosmological simulations and post-processing. For this reason, in the present thesis, we attempt to provide certain details of numerical codes that are otherwise not completely transparent from our original publications.

*

¹In his review in the Remarks on the Galactic Magnetic Field [3], Woltjer concludes: “But the idea [...] that the magnetic fields might be of dominant dynamical importance, seems by now to be unlikely. The argument in the past has frequently been a process of elimination: one observed certain phenomena, and one investigated what part of the phenomena could be explained; then the unexplained part was taken to show the effects of the magnetic field. It is clear in this case that, the larger one’s ignorance, the stronger the magnetic field.”

In the next few paragraphs, we outline the contents of this thesis, with a small step-by-step guide how the reader can approach individual chapters and sections.

Outline of the thesis

This thesis is organized as follows.

In Chapter 1 we review the theoretical and observational background necessary for the study of primordial magnetic fields (PMFs) and their cosmological implications. We begin in §1.1 with an overview of modern cosmology and of the standard Λ CDM framework. In particular, we discuss the thermal history of the Universe, recombination and the Cosmic Microwave Background in §1.1.2, as well as cosmological perturbation theory and the linear matter power spectrum in §1.1.3. The process of structure formation is presented in §1.1.4, together with the halo mass function and baryon fraction. We further review the physics of the intergalactic medium in §1.1.5, including reionization and the Lyman- α forest, and discuss cosmological simulations and initial condition generation in §1.1.6. Finally, §1.1.7 introduces the current observational landscape in the era of the James Webb Space Telescope (JWST).

The second part of Chapter 1 focuses specifically on primordial magnetic fields. In §1.2.1 we summarize the main theoretical and observational motivations for PMFs, while §1.2.2 reviews several magnetogenesis scenarios proposed in the literature. The relevant magnetohydrodynamic equations governing PMF evolution in the early Universe are introduced in §1.2.3. Subsequently, in §1.2.4 we discuss how PMFs modify cosmological perturbations and the matter power spectrum, with particular emphasis on the damping scale and on the enhancement of small-scale power. Finally, §1.3 summarizes existing observational constraints on PMFs and reviews their expected signatures on cosmological observables such as the halo mass function and galaxy formation.

Chapter 2 presents the first original part of this thesis, devoted to the impact of PMFs on early structure formation. Building on previous studies of PMF-induced perturbations, we investigate the consequences of including distinct baryonic and dark matter perturbations directly in the initial conditions of cosmological simulations. In §2.1 we describe the numerical implementation adopted in our hydrodynamical simulations and analyze the resulting evolution of cosmic structures. The effects of PMFs on the halo mass function are discussed in §2.1.1, while §2.1.2 focuses on the enhanced baryon fraction observed in low-mass halos at high redshift. In §2.1.3 we explore possible implications for star formation and early galaxy evolution. The main conclusions of this analysis are summarized in §2.2.

In Chapter 3 we derive constraints on primordial magnetic fields from observations

of the Lyman- α forest. After introducing the observational data set in §3.2, we describe in §3.3 the PMF parametrization adopted in this work, with particular emphasis on the characteristic peak scale k_{peak} . We then present the suite of hydrodynamical simulations used to model the intergalactic medium in §3.3.1 and discuss the resulting one-dimensional flux power spectra and thermal histories in §3.3.2. By comparing simulations with observations, we obtain new constraints on PMF amplitudes and scales (§3.4), providing some of the strongest current limits on primordial intergalactic magnetic fields. We summarize, in §3.6 through a discussion on how to further improve such constraints, whether Lyman- α forest can be combined with other constraining methods, and what would new observational data reveal.

Chapter 4 investigates the influence of PMFs on high-redshift galaxy formation and reionization in the JWST era. In §4.2 we study the impact of PMFs on the ultraviolet luminosity function of galaxies and compare the theoretical predictions with recent JWST observations. We further discuss the resulting constraints on PMFs in §4.2.5 and comment on the implications of new observational data in §4.2.6. In §4.3 we compare our findings with other recent works addressing PMFs, dwarf galaxies, stellar mass density and magnetohydrodynamical simulations of reionization. Finally, §4.4 examines how PMFs may influence the reionization history of the Universe through their impact on early star formation and ionizing photon production.

The thesis concludes with a summary of the main results, we reflect on difficulties and possible improvements, and develop a discussion of future perspectives for probing primordial magnetic fields through cosmological observations, numerical simulations and upcoming high-redshift surveys.

*

Regarding the original contributions of the author, the thesis is laid out in a way that Chapter 1 is quite general and contains a survey of many works; some calculations, especially in §1.2.4, were part of our publication [1], but the author of this thesis was not the main author of those calculations, for this reason they belong in the introductory chapter.

The rest of the chapters contain components of theory, results, their interpretation, and discussion. In all of these, the author of the thesis was strongly involved or was initiating and developing the projects with other colleagues.

More specifically, throughout the PhD course, the author was mainly contributing with simulations, numerical results, fitting and statistics; beyond that, large amount of time was also spent with critical feedback on theoretical assumptions, checking validity of equations, learning and testing various codes, and finding relevant literature.

Chapter 1

Theoretical and Observational Background

Cosmology has rapidly evolved over the last three decades into a field of study based on precise measurements, owing to technological advancements. Since the study of the Universe incorporates many fields of physics—from tiny scales of quantum mechanics to large-scale gravitational dynamics—theoretical aspects of cosmology need to address all such contributions.

However, this comes with a great deal of compartmentalization of cosmological sub-fields, and consequently a plethora of theories emerges, with candidates to be tested by different observational means. On one hand, we currently find ourselves at a very generous period where the data from the CMB, reionization, and galaxy surveys provide an enormous playground for implementation of statistics and simulations, which results in strong restrictions on theoretical assumptions and related theoretical models. On the other hand, the faster we move, the louder our chains clink: we often need to revisit our basic assumptions, theoretical principles, and possible errors in data measurement, because for each new measurement, multiple theories are proposed to present their compatibility or to provide solutions for disagreement with other measurements. Indeed, both the theoretician and the observational cosmologist are masters and slaves to each other. Either of them must be careful to not confuse the brightness of their new chains with progress.

Fortunately, in the PMF research community, many are working with theory and experiment in parallel. In other words: let us not create new theories unless experiments ask for it, and let us collect observational data that can challenge existing theories. In our work, and in this thesis, we are inclined to such approach. Neither we propose new theories, nor we keep the existing ones unchallenged. This is how we tackle the question

of Primordial Magnetic Fields (PMFs); we simply ask two questions - what new is there to infer and learn from the existing theory of PMFs, and, with the use of observational data, can we eliminate some sections of the PMF parameter space?

To keep such an approach, we must first simplify the whole discourse of both cosmology and PMFs, and focus on things that have solid foundations. Unfortunately, we have to omit many interesting topics and theoretical models, that would otherwise deserve to be mentioned. In fairness, throughout this First Chapter, we will provide a selected bibliography and refer to specific publications, both pioneering or recent works, which our curious reader can consult for a deeper and broader understanding of relevant topics.

1.1 An overview of modern cosmology

Modern cosmology aims at describing the origin, evolution and large-scale structure of the Universe. The current cosmological paradigm is based on General Relativity (GR), which describes gravity as the manifestation of spacetime curvature induced by matter and energy. Under the assumption that the Universe is homogeneous and isotropic on sufficiently large scales, its dynamics can be described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, with the space-time interval

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right], \quad (1.1)$$

where $a(t)$ is the scale factor, K is the spatial curvature parameter, and $d\Omega^2$ is the line element on the unit sphere. The scale factor encodes the expansion history of the Universe, while the curvature parameter determines whether the geometry is open, flat or closed.

The dynamics of the scale factor are governed by the Einstein field equations,

$$G_{\mu\nu} = 8\pi G T_{\mu\nu},$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the stress-energy tensor describing the matter-energy content of the Universe. Assuming that the cosmic fluid, with density ρ and pressure p , can be approximated as a perfect fluid,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu},$$

one obtains the Friedmann equations,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2},$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p),$$

where $H(t)$ is the Hubble parameter, ρ is the energy density and p is the pressure of the cosmic fluid.

The conservation of the stress-energy tensor, $\nabla_\mu T^{\mu\nu} = 0$, yields the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0.$$

Introducing the equation of state parameter w through the relation $p = w\rho$, the continuity equation admits the solution

$$\rho(a) \propto a^{-3(1+w)}.$$

Different values of w characterize different components of the cosmic energy budget. Non-relativistic matter satisfies $w = 0$, implying $\rho_m \propto a^{-3}$, while radiation satisfies $w = 1/3$, leading to $\rho_r \propto a^{-4}$. A cosmological constant corresponds instead to $w = -1$, yielding a constant energy density.

An important observable in cosmology is the cosmological redshift. Due to the expansion of spacetime, the wavelength of photons stretches proportionally to the scale factor, leading to

$$1 + z = \frac{a_0}{a(t)},$$

where z is the redshift and $a_0 \equiv a(t_0)$ is the scale factor today (usually gauged to be $a_0 = 1$, corresponding to $z = 0$). Cosmological observations are therefore commonly expressed in terms of redshift z rather than cosmic time.

The homogeneous cosmological framework successfully explains several key observations, including the expansion of the Universe, the existence of the Cosmic Microwave Background (CMB), and the primordial abundances predicted by Big Bang Nucleosynthesis (BBN). However, the observed large-scale structures and anisotropies in the matter distribution require going beyond a perfectly homogeneous description. Tiny primordial fluctuations in the matter density field evolved gravitationally over cosmic time, eventually forming galaxies, galaxy clusters and the cosmic web observed today. The description

of these perturbations constitutes one of the central aspects of modern cosmology.

1.1.1 The standard Λ CDM model

The current standard model of cosmology is the Λ Cold Dark Matter (Λ CDM) model. It provides an excellent description of a broad range of cosmological observations, including the anisotropies of the CMB, baryon acoustic oscillations, Type Ia supernovae, gravitational lensing and the large-scale distribution of galaxies.

Within the Λ CDM framework, the energy content of the Universe is dominated by dark energy and dark matter, while ordinary baryonic matter constitutes only a small fraction of the total energy density. The expansion history of the Universe is governed by the Friedmann equation,

$$H^2(a) = H_0^2 (\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_K a^{-2} + \Omega_\Lambda),$$

where H_0 is the present-day Hubble parameter and the density parameters are defined as $\Omega_i = \rho_i/\rho_{\text{crit}}$, with the critical density $\rho_{\text{crit}} = 3H_0^2/8\pi G$.

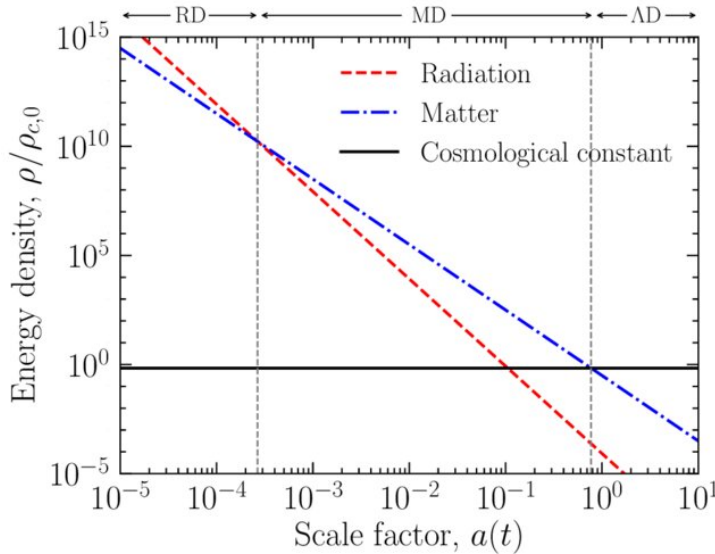


Figure 1.1: Evolution of the energy densities of radiation, matter and dark energy as a function of the scale factor in the Λ CDM model. Different cosmological epochs are characterized by the dominance of different components of the cosmic energy budget (RD - radiation domination, MD - matter domination). [Adapted from Ferreira & Roskill (2025)]

The different terms in the Friedmann equation correspond respectively to radiation, matter, spatial curvature and dark energy. Current observations indicate that the Universe is spatially very close to flat, implying $\Omega_r + \Omega_m + \Omega_\Lambda \simeq 1$.

Matter in the Λ CDM model is composed of both baryonic matter and cold dark matter. The latter is assumed to be non-relativistic during the epoch of structure formation, allowing density fluctuations to survive on small scales. Cold dark matter therefore plays a fundamental role in the hierarchical formation of cosmic structures.

Dark energy is commonly modeled as a cosmological constant Λ , characterized by an equation of state $w_\Lambda = -1$. The presence of a component with sufficiently negative pressure drives the late-time accelerated expansion of the Universe. Indeed, the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p),$$

shows that accelerated expansion occurs whenever $\rho + 3p < 0$.

As portrayed in Fig. 1.1, the thermal history of the Universe within the Λ CDM scenario consists of several important epochs. At very early times the Universe was radiation dominated and filled with a hot plasma of relativistic particles. As the Universe expanded and cooled, matter eventually became dominant at the epoch of matter-radiation equality. Later, recombination occurred at redshift $z \sim 1100$, when electrons and protons combined to form neutral hydrogen, allowing photons to decouple from matter. These photons are observed today as the CMB. At late times, dark energy became the dominant component of the cosmic energy density, initiating the present accelerated expansion phase. The transition between these cosmological epochs strongly influences both the evolution of perturbations and the formation of structures.

One of the cornerstones of the Λ CDM model is the existence of primordial density fluctuations, usually assumed to originate from quantum fluctuations generated during inflation. These perturbations are statistically characterized by the primordial power spectrum (see also Eq.(1.27)),

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*} \right)^{n_s-1},$$

where A_s is the scalar amplitude, n_s is the spectral index and k_* is a pivot scale. Observations indicate a nearly scale-invariant spectrum, consistent with the predictions of slow-roll inflation. The physics of inflation and its relationship to perturbations are briefly discussed in §1.1.3.2.

A summarized version of the evolution of the Universe is shown as a cosmic timeline in Fig. 1.2, and serves to accompany the following subsections on CMB, inflation, structure formation and reionization.

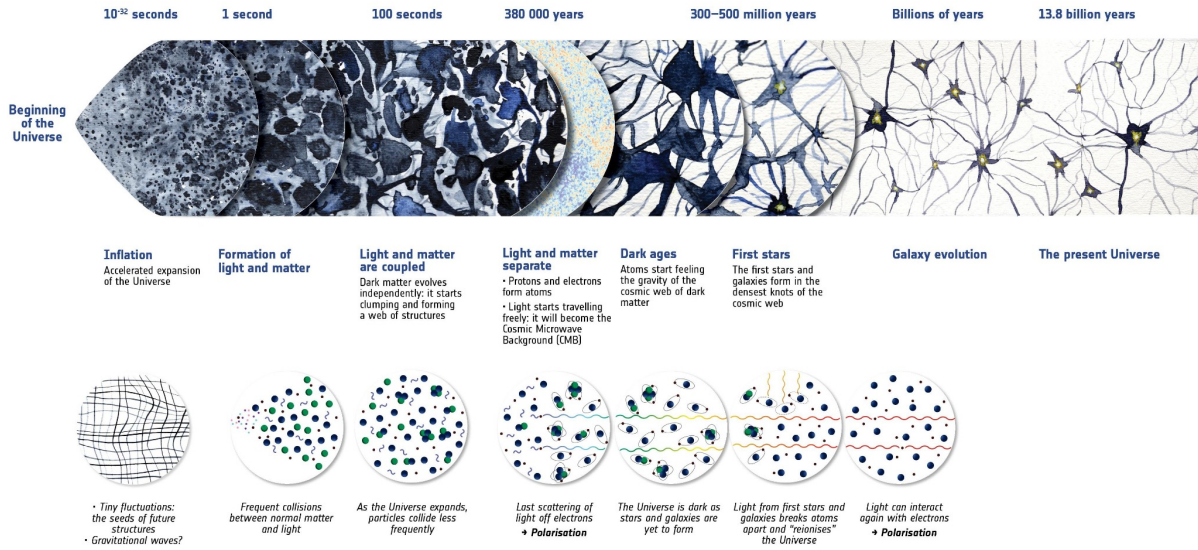


Figure 1.2: Timeline of the evolution of the Universe from its beginning to the present epoch. Important cosmological events include inflation, recombination, reionization, formation of galaxies and the formation of large-scale structures. [Credit: ESA/Planck]

Despite its remarkable observational success, several aspects of the Λ CDM model remain poorly understood. The physical nature of dark matter and dark energy is still unknown, while tensions between independent measurements of the Hubble constant may hint at possible deviations from the standard cosmological scenario.

1.1.2 Recombination and the Cosmic Microwave Background

One of the most important observational pillars of modern cosmology is the Cosmic Microwave Background (CMB), corresponding to the relic radiation emitted when the Universe became neutral approximately 380 000 years after the Big Bang. Prior to this epoch, the Universe consisted of a hot and dense plasma of photons, electrons and baryons tightly coupled through Thomson scattering. In this state, photons possessed a very short mean free path and the Universe was effectively opaque to radiation.

As the Universe expanded and cooled, the temperature eventually decreased below the ionization energy of hydrogen. Electrons and protons combined to form neutral hydrogen atoms in a process known as recombination. The reduction in the free electron density drastically suppressed Thomson scattering, allowing photons to decouple from matter and propagate freely through the Universe.

The ionization state of hydrogen in thermal equilibrium at temperature T is described by the Saha equation,

$$\frac{n_e n_p}{n_H} = \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-E_I/T},$$

where n_e , n_p and n_H are the number densities of electrons, protons and neutral hydrogen respectively, m_e is the electron mass and $E_I = 13.6 \text{ eV}$ is the ionization energy of hydrogen.

It is useful to express the Saha equation in terms of the free-electron fraction,

$$x_e \equiv \frac{n_e}{n_{\text{H,tot}}}, \quad (1.2)$$

where $n_{\text{H,tot}} = n_H + n_p$ denotes the total number density of hydrogen nuclei. Assuming charge neutrality, $n_e = n_p$, the Saha equation can then be rewritten as

$$\frac{x_e^2}{1 - x_e} = \frac{1}{n_{\text{H,tot}}} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-E_I/T}, \quad (1.3)$$

where natural units, $k_B = \hbar = c = 1$, have been adopted. The quantity x_e therefore describes the ionization state of the primordial hydrogen plasma: $x_e \simeq 1$ corresponds to an almost completely ionized medium, while $x_e \ll 1$ describes a predominantly neutral one.

Although the binding energy of hydrogen is $E_I = 13.6 \text{ eV}$, recombination does not occur when the temperature simply falls below this energy. The photon-to-baryon ratio in the early Universe is very large, $n_\gamma/n_b \sim 10^9$, and therefore even when the average photon energy is considerably smaller than E_I , the high-energy tail of the blackbody distribution still contains enough photons capable of ionizing hydrogen. As a consequence, hydrogen remains highly ionized until the temperature has fallen to $T \sim 0.3 \text{ eV}$, corresponding to a redshift of order $z \sim 1100$.

The Saha equation assumes thermodynamic equilibrium and consequently provides the equilibrium value of the ionization fraction. During the early stages of recombination this approximation is sufficiently accurate. As the Universe continues to expand, however, the interaction rates eventually become too slow to maintain equilibrium and the actual ionization fraction departs from the Saha prediction. A kinetic treatment of recombination is therefore required.

At the simplest level, the radiative recombination of an electron and a proton, $e^- + p \rightarrow \text{H} + \gamma$, occurs at a rate per unit volume

$$\Gamma_{\text{rec}} = \alpha(T) n_e n_p, \quad (1.4)$$

where $\alpha(T)$ is the temperature-dependent recombination coefficient. The quadratic dependence on the particle number densities is important: regions with a larger baryon density generally experience a larger recombination rate per unit volume.

The actual cosmological recombination process is more subtle than this simple reaction suggests. A direct recombination into the hydrogen ground state produces a photon with sufficient energy to ionize another neutral hydrogen atom, and therefore does not efficiently increase the net neutral fraction. Recombination proceeds predominantly through excited atomic states. In the effective three-level-atom treatment introduced by Peebles [4, 5], the evolution of the free-electron fraction can schematically be written as

$$\frac{dx_e}{dt} = -C [\alpha_B(T) n_{\text{H,tot}} x_e^2 - \beta_B(T) (1 - x_e)], \quad (1.5)$$

where α_B is the effective recombination coefficient to excited states, β_B is the corresponding photoionization coefficient, and C is the Peebles factor accounting for the probability that an atom reaching the first excited state ultimately reaches the ground state without being reionized.

Thus, recombination—as we adopt this nomenclature—is not an instantaneous transition or a single event, but rather a continuous departure from a highly ionized plasma towards a predominantly neutral Universe. In general, when we refer to the ‘recombination’ *as a process*, one must keep in mind that there is a period of time within which *three physical phenomena occur*: from the moment the first hydrogen atoms begin to form and free-electron abundance rapidly decreases, through decoupling of the photon fluid from the baryon plasma due to Thomson scattering becoming inefficient and the dramatic increase of the photon mean free path, until the last photons are scattered as they become free-streaming. The redshifts $z_{\text{rec.}} \gtrsim z_{\text{dec.}} \gtrsim z_*$ associated with these transitions are representing times of recombination, decoupling and the formation of last-scattering region.

Finally, recombination occurs approximately at redshift $z_{\text{rec}} \simeq 1100$, corresponding to a temperature $T_{\text{rec}} \sim 3000$ K. Due to cosmological redshift, the temperature of free-streaming photons in the expanding universe decreases proportionally to the inverse of the scale factor, $T_\gamma \propto 1/a(t)$.

The associated useful quantity is the sound horizon at the last-scattering time z_*

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)} dz, \quad (1.6)$$

where c_s is the sound speed in the photon-baryon fluid. The sound horizon quantifies maximum comoving distance that an acoustic wave could have traveled through the

photon–baryon plasma from the very early Universe up to photon last scattering.

*

Today, the CMB is observed as an almost perfect blackbody spectrum with temperature $T_0 = 2.725$ K. The blackbody spectrum is described by the Planck distribution,

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/(k_B T)} - 1} .$$

The remarkable agreement between the observed CMB spectrum and a perfect blackbody, demonstrated in Fig. 1.3, constitutes one of the strongest pieces of evidence supporting the Hot Big Bang model.

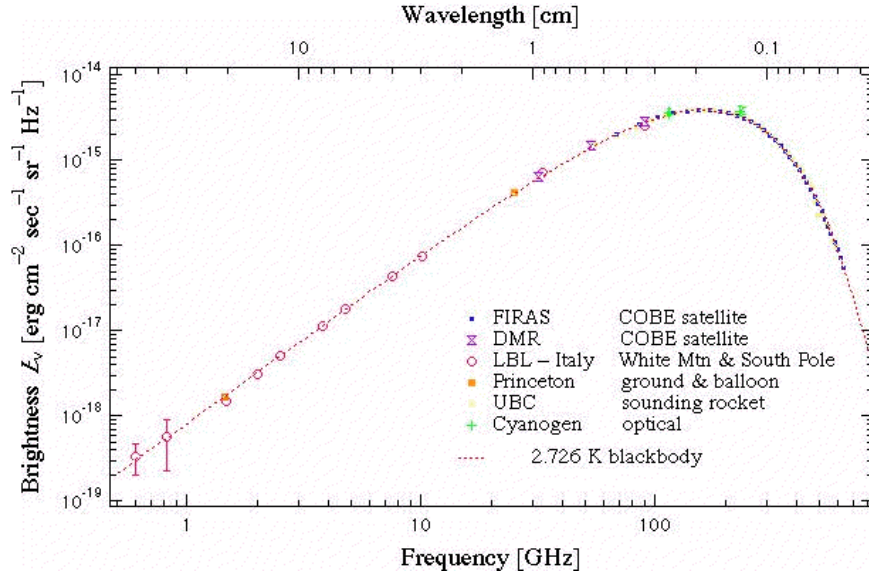


Figure 1.3: Spectrum of the CMB measured by COBE/FIRAS compared with the best-fitting blackbody spectrum. The extraordinary agreement with a thermal spectrum at $T_{\text{CMB}} \simeq 2.725$ K is manifest. Adapted from Fixsen+96 [6]; [Credit: NASA/COBE Science Team]

Although highly isotropic, the CMB exhibits temperature anisotropies at the level of $\frac{\Delta T}{T} \sim 10^{-5}$. These anisotropies trace the primordial density fluctuations (§1.1.3) that later evolved into the large-scale structures observed today. The temperature fluctuations are commonly expanded in spherical harmonics,

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell, m} a_{\ell m} Y_{\ell m}(\theta, \phi),$$

where $Y_{\ell m}$ are the spherical harmonic functions. The statistical properties of the anisotropies are encoded in the angular power spectrum,

$$C_\ell = \langle |a_{\ell m}|^2 \rangle.$$

The acoustic peaks observed in the CMB power spectrum originate from oscillations in the photon-baryon plasma prior to recombination. Their positions and amplitudes provide extremely precise constraints on cosmological parameters, including the Hubble constant, the matter density and the baryon fraction. The plot in Fig. 1.4 shows the rescaled CMB temperature angular power spectrum

$$D_\ell^{TT} = \frac{l(l+1)}{2\pi} C_\ell^{TT},$$

where TT stands for temperature-temperature correlation.

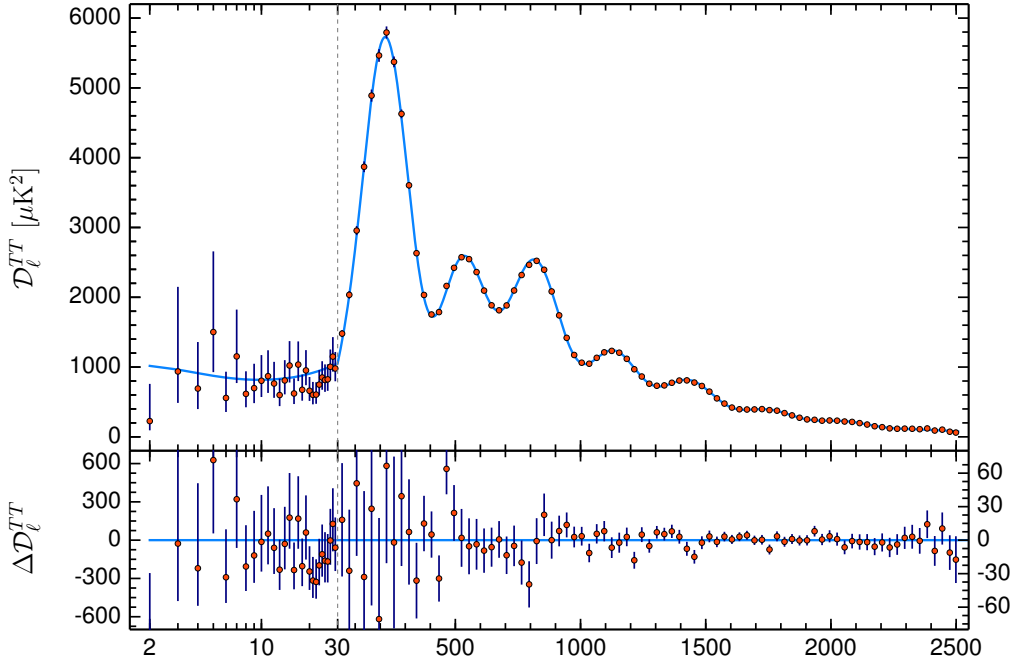


Figure 1.4: Angular power spectrum of the CMB temperature anisotropies measured by the Planck satellite. The acoustic peak structure encodes information about the cosmological parameters and the composition of the Universe. The solid curve represents the best-fitting base- Λ CDM model. Adapted from the Planck Collaboration [7].

Observations from experiments such as COBE, WMAP and Planck have measured the CMB anisotropies with extraordinary precision, establishing the Λ CDM model as the standard cosmological framework. In addition to temperature anisotropies, the CMB also exhibits polarization generated by Thomson scattering, providing further information about primordial perturbations and the epoch of reionization.

The CMB therefore represents a unique probe of the early Universe, linking primordial

physics to the large-scale structure observed at late times. As could be expected, numerous studies relate primordial magnetism and the CMB. Some of them will be mentioned in the motivations for studying PMFs (§1.2.1).

1.1.3 Cosmological perturbations

The FLRW geometry and the assumption of a homogeneous and isotropic matter distribution provide an accurate description of the Universe on sufficiently large scales, of order several hundred Megaparsec (Mpc) and above. On smaller scales, from the tens-of-Mpc scales of the cosmic web down to galaxy clusters ($\sim 1\text{--}10\text{Mpc}$) and individual galaxies, observations reveal increasingly pronounced anisotropies and inhomogeneities. Galaxies, clusters and the filamentary structure of the cosmic web originate from primordial fluctuations that evolved gravitationally over cosmic time. Cosmological perturbation theory describes these deviations from homogeneity and isotropy.

General relativity provides us with dialectics: *geometry tells matter how to move, and matter tells geometry how to bend*. In order to deviate from isotropy and homogeneity, we must deviate from the FLRW metric, whose spacetime element is in (1.1). The metric can be decomposed into a homogeneous background component and a small perturbation,

$$g_{\mu\nu}(\mathbf{x},t) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(\mathbf{x},t).$$

Similarly, the matter density can be written as $\rho(\mathbf{x},t) = \bar{\rho}(t) + \delta\rho(\mathbf{x},t)$, where $\bar{\rho}$ denotes the background density and $\delta\rho$ represents the perturbation. A particularly useful quantity is the density contrast,

$$\delta(\mathbf{x},t) = \frac{\delta\rho(\mathbf{x},t)}{\bar{\rho}(t)}.$$

Since primordial perturbations are initially very small, the Einstein equations can be linearized around the FLRW background. In Fourier space, *different perturbation modes evolve independently at linear order*. The Fourier transform of the density field is defined as

$$\delta(\mathbf{k},t) = \int d^3x \delta(\mathbf{x},t) e^{-i\mathbf{k}\cdot\mathbf{x}}. \quad (1.7)$$

In the Newtonian approximation and on sub-horizon scales, the evolution of matter perturbations is governed by the continuity, Euler and Poisson equations. Combining these equations leads to the standard growth equation,

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}_m\delta = 0. \quad (1.8)$$

During matter domination, the growing mode evolves approximately as

$$\delta(a) \propto a, \quad (1.9)$$

showing that perturbations grow linearly with the scale factor. An important quantity characterizing the growth of perturbations is the linear growth factor $D(a)$,

$$\delta(\mathbf{k}, a) = D(a)\delta(\mathbf{k}, a_i), \quad (1.10)$$

where a_i denotes the scale factor at a chosen initial epoch, i.e., initial conditions.

The evolution of perturbations strongly depends on the cosmological background. Radiation pressure suppresses the growth of matter perturbations before matter-radiation equality, while dark energy suppresses growth at late times by accelerating the expansion of the Universe.

As we have seen in the previous subsection, perturbations in the baryon-photon plasma before recombination generated acoustic oscillations, which are imprinted both in the anisotropies of the CMB and in the large-scale distribution of galaxies. The anisotropy seen in the CMB temperature map (Fig. 1.5) is directly linked to metric and matter perturbations.

Primordial perturbations are commonly assumed to be Gaussian random fields. This means that probability to find density contrast in the range $[\delta, \delta + d\delta]$ will be given by the distribution function [8]

$$\mathcal{P}[\delta] d\delta = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) d\delta, \quad (1.11)$$

where σ^2 is the variance of the density perturbation field. It can be evaluated through the correlation function as $\sigma^2 = \xi(0)$, where the correlation function is defined as $\xi(r) = \langle \delta(\mathbf{x}_1)\delta(\mathbf{x}_2) \rangle$, with $r = |\mathbf{x}_2 - \mathbf{x}_1|$.¹ In this case, their statistical properties are completely characterized by the two-point correlation function or equivalently by the power spectrum (Fourier transform of the correlation function). Deviations from Gaussianity may instead reveal important information about the inflationary epoch and possible non-linear processes in the early Universe.

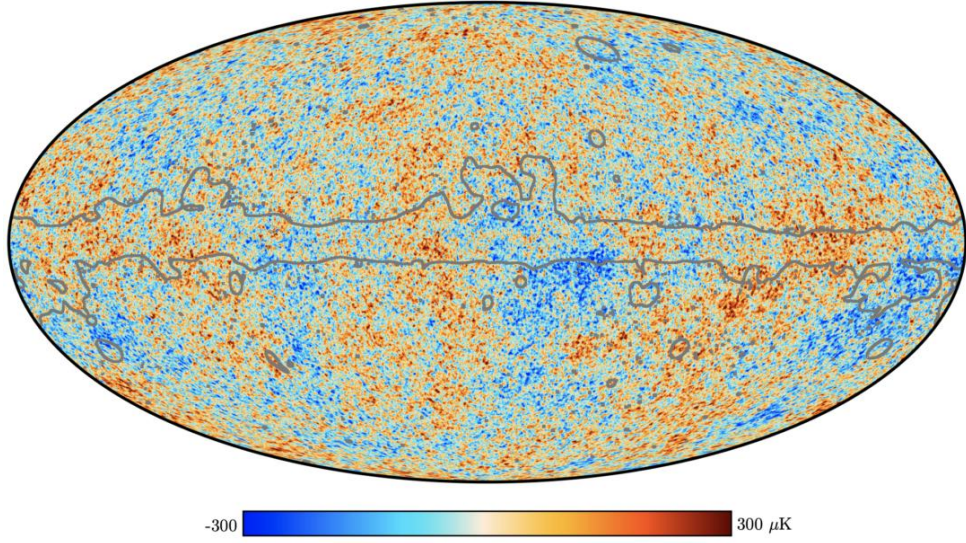


Figure 1.5: Temperature anisotropies of the Cosmic Microwave Background as measured by the Planck satellite. The observed temperature fluctuations encode the primordial density perturbations generated in the early Universe, which provided the seeds for the subsequent formation of cosmic structure. [Credit: ESA/Planck Collaboration [9]].

1.1.3.1 The linear matter power spectrum

The statistical properties of cosmological perturbations are commonly described through the matter power spectrum. Assuming statistical homogeneity and isotropy, the two-point correlation function in Fourier space can be written as

$$\langle \delta(\mathbf{k}) \delta^*(\mathbf{k}') \rangle = (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') P(k), \quad (1.12)$$

where δ_D is the Dirac delta function and $P(k)$ is the power spectrum, depending only on the modulus of the wavevector $k = |\mathbf{k}|$.

The dimensionless power spectrum is defined as

$$\Delta^2(k) = \frac{k^3}{2\pi^2} P(k), \quad (1.13)$$

and quantifies the contribution to the variance of the density field per logarithmic interval in wavenumber.

The late-time matter power spectrum differs from the primordial one because perturbations evolve differently depending on their scales and on the cosmic expansion history. The linear matter power spectrum can be related to the primordial spectrum P_{prim} as

$$P(k, z) = D^2(z) T^2(k) P_{\text{prim}}(k), \quad (1.14)$$

where $D(z)$ is the growth factor defined in Eq. (1.10), and $T(k)$ is the transfer function.

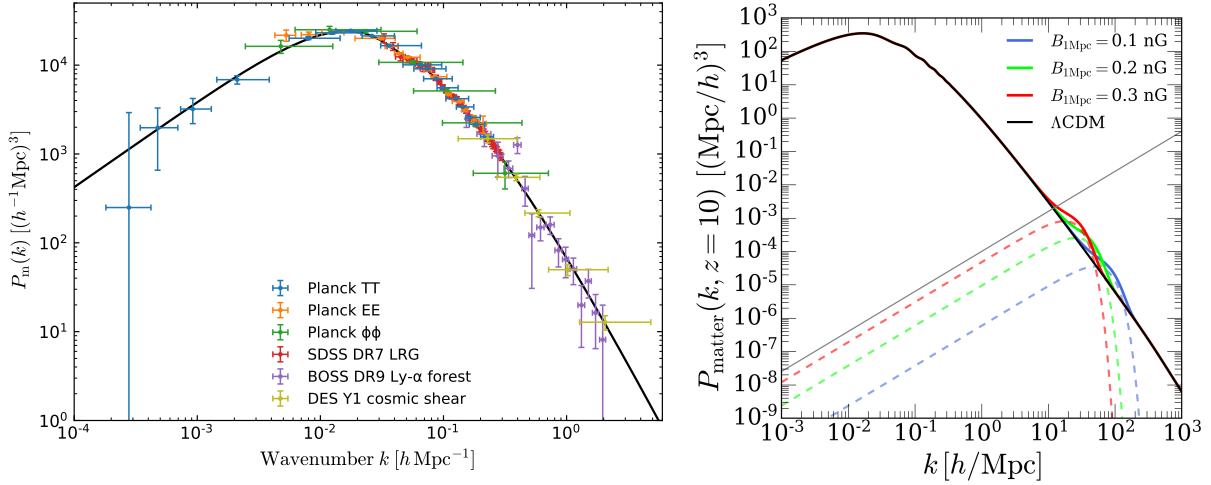


Figure 1.6: Linear matter power spectrum as a function of comoving wavenumber k . The shape of the spectrum reflects both the primordial fluctuations generated during inflation and their subsequent evolution through cosmic history. **Left:** from Planck Collaboration [10], redshift $z = 0$. **Right:** $P_m(k)$ generated using the code CLASS at redshift $z = 10$. In addition to the Λ CDM case, we show PMF-induced linear matter spectra $P_m^{\text{PMF}} \sim k^{2n_B+7}$ ($n_B = -2.9$) for three different models (dashed), and the sum $P_m^{\text{CDM}} + P_m^{\text{PMF}}$ (solid color curves). Details are in Section 1.2.

On large scales, perturbations remain in the linear regime and evolve proportionally to the growth factor. On smaller scales, non-linear gravitational collapse becomes important and linear theory eventually breaks down. This is visualized in Fig. 1.7.

Measurements of the matter power spectrum from galaxy surveys, weak lensing and the Lyman- α forest provide powerful constraints on cosmological parameters, neutrino masses and models of dark matter.

1.1.3.2 The physics of inflation

Since inflation was mentioned several times in previous sections, we now briefly remind the reader of the basics of inflationary physics and especially their relationship to primordial perturbations.

The inflationary paradigm was originally introduced to address a number of conceptual problems associated with the standard Hot Big Bang scenario [12–14]. Although the Hot Big Bang model successfully describes the thermal evolution of the Universe from very early times onwards, it requires rather special initial conditions in order to explain some of its observed properties. One of the most important examples is the *horizon problem*. Regions of the Cosmic Microwave Background separated by sufficiently large

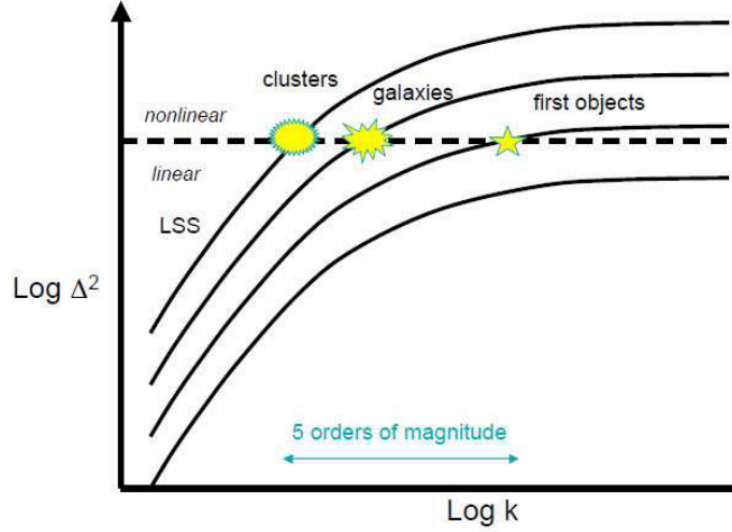


Figure 1.7: Evolution of the dimensionless matter power spectrum $\Delta^2(k)$ as a function of comoving wavenumber k , at four redshifts $z_1 < z_2 < z_3 < z_4$. As density perturbations grow, progressively larger scales enter the non-linear regime, conventionally characterized by $\Delta^2(k_{\text{nl}}) \simeq 1$, giving rise to collapsed objects and structures at different scales. Adapted from [11].

angular distances could not have been in causal contact in a purely radiation- and matter-dominated Universe, and yet their temperatures agree to a remarkable precision. Closely related is the *flatness problem*: the present-day value of the spatial curvature is very small, which in a decelerating Universe requires the curvature contribution to have been extraordinarily fine-tuned at early times. Finally, many extensions of particle physics predict stable relics, such as magnetic monopoles, whose expected abundance would be incompatible with observations.

Inflation provides a common solution to these problems by postulating a period of accelerated expansion, i.e., $\ddot{a} > 0$, during which the scale factor grows approximately exponentially. In particular, the comoving Hubble radius, $(aH)^{-1}$, decreases during inflation. Consequently, regions which today appear causally disconnected may originate from a much smaller region that was in causal contact before inflation. At the same time, any initial spatial curvature or unwanted relic abundance is strongly diluted by the rapid expansion. A sufficiently long period of inflation, typically of order 50–60 e-folds or more depending on the post-inflationary thermal history, can therefore account for the observed large-scale homogeneity and near-flatness of the Universe.

The simplest dynamical realization of inflation is obtained by introducing a scalar field

ϕ , commonly referred to as the *inflaton*, minimally coupled to gravity. Its dynamics can be described by the action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \quad (1.15)$$

where $M_{\text{Pl}} = (8\pi G)^{-1/2}$ is the reduced Planck mass and $V(\phi)$ denotes the inflaton potential. For a spatially homogeneous field $\phi = \phi(t)$, the corresponding energy density and pressure are

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi). \quad (1.16)$$

The Friedmann equations then become

$$H^2 = \frac{1}{3M_{\text{Pl}}^2} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right), \quad \dot{H} = -\frac{\dot{\phi}^2}{2M_{\text{Pl}}^2}, \quad (1.17)$$

while variation of Eq. (1.15) with respect to ϕ gives the Klein–Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0. \quad (1.18)$$

Accelerated expansion occurs when the potential energy dominates over the kinetic energy, $\dot{\phi}^2 \ll V(\phi)$, such that $p_\phi \simeq -\rho_\phi$. In the so-called *slow-roll regime* the acceleration of the scalar field is also small compared with the Hubble friction term, and Eqs. (1.17) and (1.18) reduce approximately to

$$H^2 \simeq \frac{V}{3M_{\text{Pl}}^2}, \quad 3H\dot{\phi} \simeq -V_{,\phi}. \quad (1.19)$$

The validity of this approximation is conventionally characterized by the potential slow-roll parameters

$$\epsilon_V \equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2, \quad \eta_V \equiv M_{\text{Pl}}^2 \frac{V_{,\phi\phi}}{V}, \quad (1.20)$$

which must satisfy $\epsilon_V, |\eta_V| \ll 1$ during inflation. The duration of the inflationary phase is conveniently described by the number of e-folds,

$$N \equiv \ln \left(\frac{a_f}{a_i} \right) = \int_{t_i}^{t_f} H dt \simeq \frac{1}{M_{\text{Pl}}^2} \int_{\phi_i}^{\phi_f} \frac{V}{V_{,\phi}} d\phi. \quad (1.21)$$

Besides solving the horizon and flatness problems, inflation provides a natural mechanism for generating the primordial perturbations from which all later cosmic structure develops. This represents one of its most important predictions. The inflaton is a quan-

tum field and therefore necessarily exhibits quantum fluctuations around its homogeneous background value,

$$\phi(\mathbf{x}, t) = \bar{\phi}(t) + \delta\phi(\mathbf{x}, t). \quad (1.22)$$

During inflation, physical wavelengths $\lambda_{\text{phys}} = a/k$ grow rapidly while the Hubble scale H^{-1} remains approximately constant. A Fourier mode initially satisfying $k \gg aH$ therefore eventually crosses the Hubble radius when $k = aH$.

On sub-Hubble scales the perturbations behave approximately as ordinary quantum vacuum fluctuations. As a mode is stretched to scales larger than the Hubble radius, however, its amplitude becomes approximately constant. In this way microscopic quantum fluctuations are converted into macroscopic classical perturbations.

For a canonical single-field model, the scalar fluctuations are most conveniently described in terms of the gauge-invariant comoving curvature perturbation $\mathcal{R}$. In the spatially flat gauge it is related to the inflaton fluctuation through

$$\mathcal{R} = \frac{H}{\dot{\phi}} \delta\phi. \quad (1.23)$$

The Fourier modes of the perturbations can be described by the Mukhanov–Sasaki variable $v = z\mathcal{R}$, where $z = a\dot{\phi}/H$. Its equation of motion is

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0, \quad (1.24)$$

where primes denote derivatives with respect to conformal time. Equation (1.24) explicitly illustrates the two regimes relevant for inflationary perturbations. For $k^2 \gg z''/z$, corresponding approximately to scales well inside the Hubble radius, the mode oscillates as a quantum vacuum fluctuation. Once $k^2 \ll z''/z$, the growing solution approaches a constant value and the curvature perturbation becomes frozen on super-Hubble scales for adiabatic single-field perturbations [15, 16].

The resulting primordial scalar power spectrum is conventionally defined by

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} |\mathcal{R}_k|^2. \quad (1.25)$$

For slow-roll inflation its amplitude, evaluated approximately at Hubble crossing $k = aH$, is

$$\mathcal{P}_{\mathcal{R}}(k) \simeq \frac{1}{8\pi^2 M_{\text{Pl}}^2} \frac{H^2}{\epsilon} \bigg|_{k=aH}, \quad (1.26)$$

where $\epsilon \equiv -\dot{H}/H^2$. Since the background quantities H and ϵ evolve only slowly during

inflation, fluctuations generated at neighbouring scales have similar amplitudes. Inflation therefore naturally predicts a nearly scale-invariant primordial spectrum, commonly parametrized as

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_\star} \right)^{n_s - 1}, \quad (1.27)$$

where A_s is the scalar amplitude at the pivot scale $k_\star$ and n_s denotes the scalar spectral index. For single-field slow-roll inflation,

$$n_s - 1 \simeq -6\epsilon_V + 2\eta_V, \quad (1.28)$$

so exact scale invariance, $n_s = 1$, is approached in the limit of an exactly de Sitter background.

After inflation ends, the energy stored in the inflaton is transferred to the particles of the Standard Model and possibly other degrees of freedom during reheating, initiating the conventional Hot Big Bang evolution. The primordial curvature perturbations generated during inflation then provide the initial conditions for the subsequent evolution of density fluctuations. Modes evolve differently depending on when they re-enter the Hubble radius and on whether the Universe is radiation or matter dominated. These effects are encoded in the transfer function $T(k)$, while their later growth is described by the linear growth factor $D(z)$. Consequently, the shape of the late-time matter power spectrum discussed in the previous sections contains both the nearly scale-invariant primordial spectrum produced during inflation and the scale-dependent processing of those perturbations through the subsequent thermal and gravitational evolution of the Universe. Inflation therefore provides the physical link between microscopic quantum fluctuations in the very early Universe and the macroscopic distribution of galaxies, clusters, and large-scale structure observed today.

For a self-contained description of inflation and related physics, the reader can consult textbooks [17, 18], while specifically cosmological perturbation theory is treated in detail in [16].

1.1.4 Structure formation

The large-scale structures observed in the present-day Universe originate from the gravitational amplification of tiny primordial fluctuations generated in the early Universe. In the standard cosmological paradigm, structures form hierarchically: small overdense regions collapse first and subsequently merge into progressively larger systems such as galaxies, groups and galaxy clusters.

The evolution of matter perturbations is governed by gravitational instability. Regions with density larger than the cosmic average generate deeper gravitational potentials, attracting surrounding matter and growing over time. In the linear regime, where $\delta \ll 1$, perturbations evolve independently and their growth can be described analytically. However, once the density contrast approaches unity, non-linear effects become important and structures collapse gravitationally.

A useful framework to study structure formation is provided by the spherical collapse model. Consider a spherically symmetric overdensity embedded in an expanding Universe. The evolution of the overdense region follows a modified Friedmann equation,

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{R^2}, \quad (1.29)$$

where $R(t)$ is the physical radius of the overdense region. Initially, the perturbation expands together with the background Universe. However, due to its excess density, the expansion slows down until the overdensity reaches a maximum radius, known as the turnaround radius. After this point, the structure collapses under its own gravity and eventually virializes.

The virial theorem implies $2K + U = 0$, where K and U denote the kinetic and potential energies respectively. Virialized objects formed through gravitational collapse are commonly referred to as dark matter halos. These halos provide the gravitational potential wells within which baryonic matter cools and forms galaxies.

An important quantity in structure formation is the critical overdensity for collapse. In the Einstein-de Sitter approximation, linear theory predicts a threshold value

$$\delta_c \simeq 1.686.$$

Regions whose linearly extrapolated density contrast exceeds this threshold are expected to collapse into bound structures.

The growth of structures depends strongly on the cosmological model. During radiation domination, perturbations in the matter component grow only logarithmically due to the dominant radiation pressure. After matter-radiation equality, matter perturbations grow approximately linearly with the scale factor. At late times, the accelerated expansion driven by dark energy suppresses the growth of structures by reducing the efficiency of gravitational collapse.

The distribution of structures observed today traces the underlying dark matter field. Numerical simulations show that matter is organized into a complex filamentary network known as the cosmic web, composed of filaments, sheets, clusters and voids. The sta-

tistical characterization of this network provides important constraints on cosmological parameters and on the nature of dark matter.

1.1.4.1 Halo mass function

One of the most important statistical quantities describing structure formation is the halo mass function (HMF), which characterizes the abundance of dark matter halos as a function of mass and redshift. The halo mass function provides a direct link between the primordial density field and the observed population of galaxies and galaxy clusters.

The earliest analytical description of the HMF was developed within the Press-Schechter formalism. In this framework, structures form from regions where the smoothed density field exceeds the critical threshold for collapse. The comoving number density of halos in the mass interval $[M, M + dM]$ is given by

$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}_m}{M^2} \frac{\delta_c}{\sigma(M)} \left| \frac{d \ln \sigma}{d \ln M} \right| \exp \left[-\frac{\delta_c^2}{2\sigma^2(M)} \right], \quad (1.30)$$

where $\bar{\rho}_m$ is the mean matter density and $\sigma(M)$ is the variance of the density field smoothed over the mass scale M . The variance is related to the linear matter power spectrum through

$$\sigma^2(M, z) \equiv \langle \delta^2(x, z; R(M)) \rangle = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P_m(k, z) \tilde{W}^2(kR), \quad (1.31)$$

where $\tilde{W}(kR)$ is the Fourier-transformed smoothing window function (e.g., top-hat) and R is related to the halo mass through

$$M = \frac{4\pi}{3} \bar{\rho}_m R^3. \quad (1.32)$$

The Press-Schechter formalism provides a simple analytical description of the abundance of collapsed dark matter haloes, but its predictions do not perfectly reproduce the halo abundances measured in cosmological N -body simulations. In particular, the assumption of spherical collapse is an idealization of the intrinsically anisotropic gravitational collapse of density perturbations. Sheth & Tormen [19] introduced a modification of the Press-Schechter multiplicity function, calibrated against the cosmological simulations, which provides a considerably improved description of the simulated halo population. The Sheth-Tormen (S-T) mass function may be written as

$$\frac{dn}{dM} = \frac{\bar{\rho}_m}{M^2} \left| \frac{d \ln \nu}{d \ln M} \right| A [1 + (a\nu)^{-p}] \sqrt{\frac{a\nu}{2\pi}} \exp\left(-\frac{a\nu}{2}\right), \quad \nu \equiv \left[\frac{\delta_c}{\sigma(M, z)} \right]^2, \quad (1.33)$$

where $\bar{\rho}_m$ is the mean comoving matter density, $\sigma(M, z)$ is the rms amplitude of the linearly evolved density field smoothed on the scale corresponding to mass M , and $\delta_c \simeq 1.686$ is the critical linear overdensity for collapse. The original Sheth–Tormen parameters are $a = 0.707$ and $p = 0.3$, while the normalization condition gives $A \simeq 0.322$.

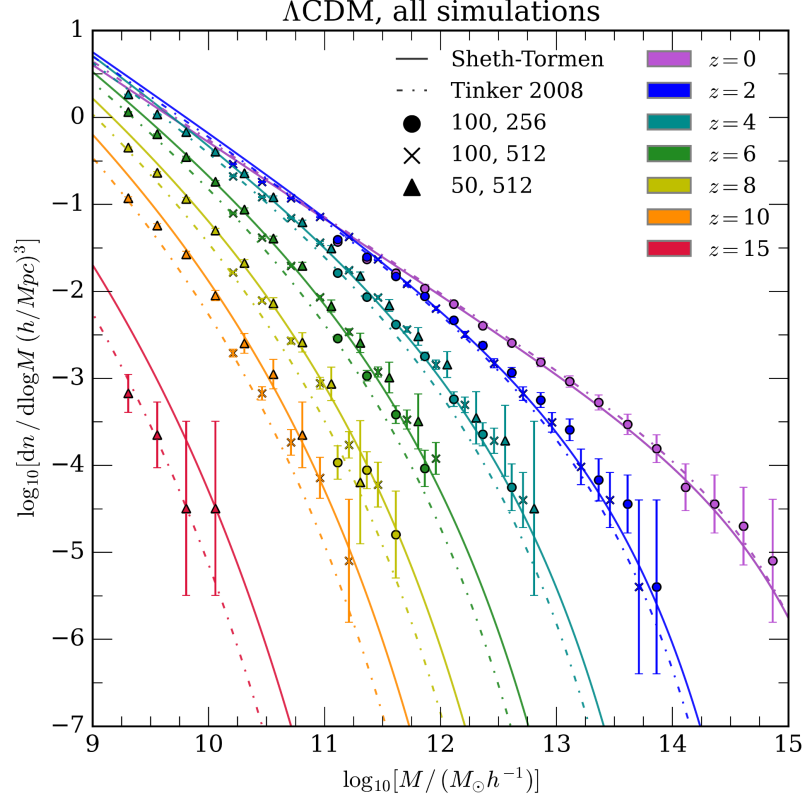


Figure 1.8: Sheth–Tormen halo mass function at different redshifts, compared with the HMF of Tinker+2008 [20]. The curves were generated with PYLIANS code; the data points represent halo counts from cosmological simulations with different particle numbers and volumes. The abundance of massive dark matter haloes decreases rapidly towards earlier cosmic times, reflecting the hierarchical nature of structure formation.

The modification of Sheth & Tormen changes both the low-mass behaviour and the exponential suppression of rare, massive haloes relative to the Press–Schechter prediction. Physically, this improvement can be understood by relaxing the assumption of perfectly spherical collapse. The subsequent work of Sheth, Mo & Tormen [21] showed that the collapse threshold depends on the ellipticity of the initial perturbation and on the surrounding tidal shear, leading to a mass-dependent collapse barrier. This ellipsoidal-collapse picture provides a physical motivation for the form of the Sheth–Tormen mass function and produces a better description of halo abundances in numerical simulations. Since $\sigma(M, z)$ decreases towards larger masses and towards higher redshifts, massive ob-

jects correspond to increasingly large values of ν and therefore lie in the exponentially suppressed tail of Eq. (1.33). Consequently, massive halos become progressively rarer at earlier cosmic times, as expected in the hierarchical picture of structure formation.

The halo mass function is extremely sensitive to cosmological parameters and to the amplitude of matter fluctuations. In particular, the abundance of massive clusters provides powerful constraints on σ_8 , Ω_m , neutrino masses and models of dark energy.

At high redshift, the halo mass function determines the abundance of the first collapsed objects and therefore plays a central role in galaxy formation and cosmic reionization studies. Observations from JWST are beginning to probe the high-redshift halo population, potentially challenging the predictions of the standard Λ CDM scenario.

1.1.4.2 Baryon fraction

Although dark matter dominates the total matter density of the Universe, baryons play a crucial role in the formation of luminous structures such as stars and galaxies. The cosmic baryon fraction is defined as

$$f_b = \frac{\Omega_b}{\Omega_m}, \quad (1.34)$$

where Ω_b and Ω_m denote the baryonic and total matter density parameters respectively.

Measurements of the CMB and Big Bang Nucleosynthesis indicate a universal baryon fraction of approximately

$$f_b \simeq 0.16. \quad (1.35)$$

However, the baryon content within halos does not necessarily coincide with the cosmic average. Several astrophysical processes can redistribute baryons and suppress their accumulation inside dark matter halos.

Gas cooling constitutes one of the fundamental mechanisms driving galaxy formation. The cooling time of the gas can be approximated as

$$t_{\text{cool}} = \frac{3nk_B T}{2\Lambda(T)n_e n_i},$$

where n is the particle number density, T is the gas temperature and $\Lambda(T)$ is the cooling function. Efficient cooling allows baryons to collapse toward the halo center and form stars.

Feedback processes can instead expel baryons from halos. Stellar winds, supernova explosions and active galactic nuclei (AGN) inject energy into the interstellar and circumgalactic media, heating the gas and reducing star formation efficiency. In Fig. 1.9 is

a schematic representation of such dynamics. These effects are particularly important in low-mass halos, where shallow gravitational potentials allow gas to escape more easily. The same physical picture is supported by simulation work showing that stellar feedback strongly regulates baryons in lower-mass halos, while AGN feedback becomes increasingly important toward group and cluster scales.

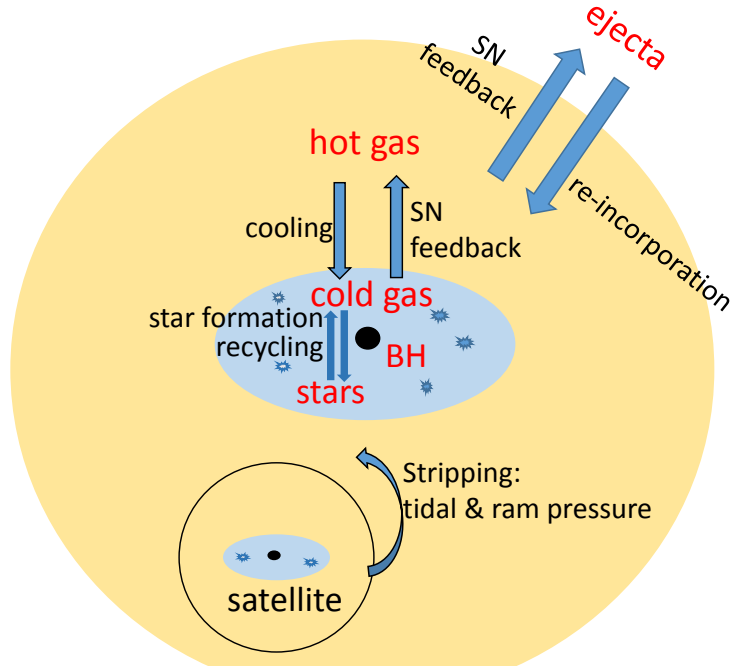


Figure 1.9: Schematic representation of the baryonic components and their cycling within a dark matter halo. Gas can cool from the hot halo into the central galaxy, where it forms stars and feeds the central black hole. Stellar and AGN feedback can return material to the hot phase or eject it beyond the halo, thereby modifying the baryon content relative to the cosmic baryon fraction. Adapted from Zhang et al. [22].

The baryon fraction also plays a crucial role in cosmological observations. Galaxy clusters are often assumed to provide approximately fair samples of the cosmic baryon content, allowing cosmological constraints through measurements of the cluster gas fraction.

At high redshift, baryonic physics strongly influences the formation of the first galaxies and the reionization history of the Universe. Understanding how baryons interact with dark matter therefore remains one of the major challenges in modern cosmology and galaxy formation theory.

1.1.5 Intergalactic medium

The intergalactic medium (IGM) is the diffuse baryonic component filling the space between galaxies and dark matter halos. Although galaxies contain the most luminous baryonic structures in the Universe, the majority of baryons reside in the IGM, particularly at high redshift. The physical state of the IGM is therefore closely connected to the thermal and ionization history of the Universe, as well as to the formation of cosmic structures.

Following recombination, the baryonic gas became predominantly neutral and remained weakly ionized until the appearance of the first astrophysical sources. During this period, often referred to as the “Dark Ages”, density fluctuations continued to grow under gravitational instability. Eventually, the formation of the first stars, galaxies and quasars produced ultraviolet radiation capable of ionizing the surrounding hydrogen gas, initiating the epoch of reionization.

The thermal evolution of the IGM is determined by the competition between several physical processes, including adiabatic cooling due to cosmic expansion, photoheating from ionizing radiation, Compton scattering and radiative cooling. The temperature evolution of the gas can be approximately described by

$$\frac{dT}{dt} = -2HT + \frac{2}{3k_B n}(\Gamma - \Lambda), \quad (1.36)$$

where T is the gas temperature, n is the particle number density, Γ represents heating processes and Λ cooling processes.

The ionization state of the IGM is commonly described through the ionized fraction x_e , whose evolution depends on the balance between ionization and recombination processes,

$$\frac{dx_e}{dt} = \Gamma_{\text{ion}}(1 - x_e) - \alpha_B n_e x_e^2, \quad (1.37)$$

where Γ_{ion} is the photoionization rate, α_B is the case-B recombination coefficient and n_e is the electron number density.

The IGM also traces the underlying dark matter distribution on sufficiently large scales. In low-density environments, baryons approximately follow the dark matter field, making the IGM an important cosmological probe. In particular, absorption features produced by neutral hydrogen along the line-of-sight to distant quasars provide a unique way to study the matter distribution at high redshift.

At low redshift, a significant fraction of baryons is believed to reside in the warm-hot intergalactic medium (WHIM), characterized by temperatures in the range $10^5 \text{ K} \lesssim T \lesssim 10^7 \text{ K}$. This gas is heated through gravitational collapse and shock heating associated with

structure formation.

The study of the IGM therefore provides crucial information about the thermal history of the Universe, the formation of the first luminous sources and the evolution of large-scale structures.

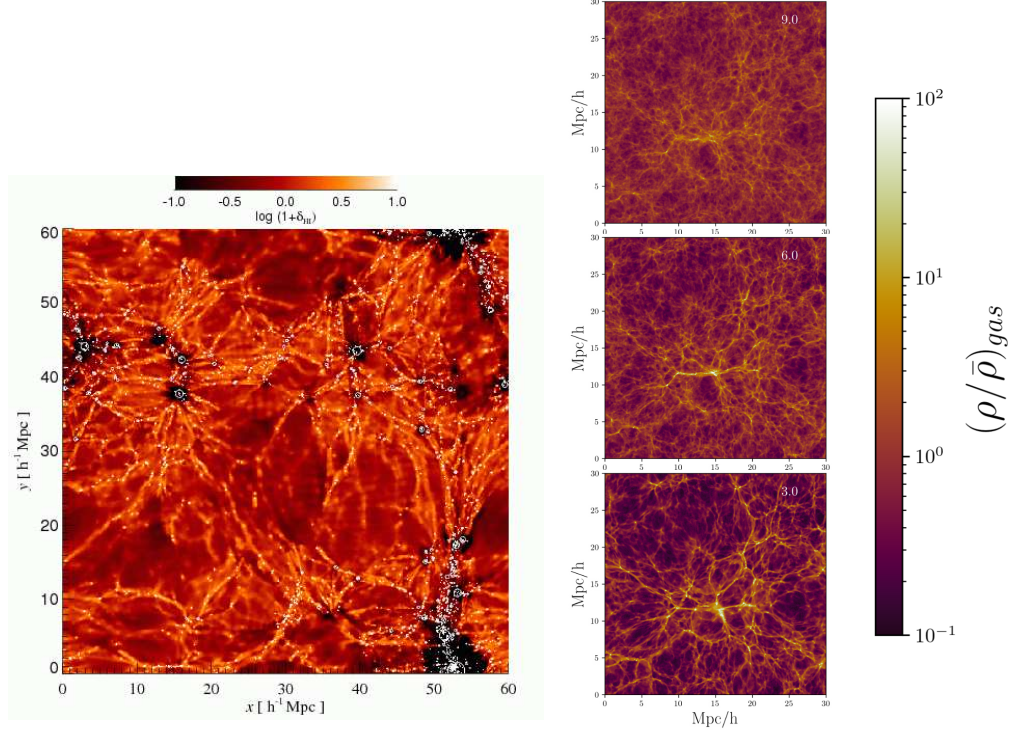


Figure 1.10: **Left:** Distribution of neutral hydrogen within the cosmic web in a cosmological hydrodynamical simulation. The diffuse intergalactic gas traces a network of sheets and filaments, while dark matter haloes, indicated by white symbols, preferentially occupy the denser regions. Adapted from Pierleoni, Branchini & Viel [23]. **Right:** Slices from our Λ CDM simulations, performed by GADGET-3 code, visualizing the total gas component, at redshifts $z = 9, 6$ and 3 .

1.1.5.1 Reionization

One of the most important transitions in the thermal history of the Universe is the epoch of reionization. After recombination, the baryonic gas became predominantly neutral, leaving the Universe filled with neutral hydrogen and helium. The formation of the first stars and galaxies subsequently produced ionizing ultraviolet photons capable of reionizing the surrounding medium.

Reionization is believed to occur approximately in the redshift range

$$6 \lesssim z \lesssim 15, \quad (1.38)$$

although the precise timing and duration of the process remain uncertain. The first

ionized regions formed around luminous sources and progressively expanded until they overlapped, eventually ionizing most of the hydrogen in the Universe.

The evolution of ionized regions can be approximately described through the growth of Strömgren spheres. The radius R_S of an ionized region surrounding a source emitting ionizing photons at a rate $\dot{N}_\gamma$ is determined by the balance between ionizations and recombinations,

$$\frac{4\pi}{3}R_S^3\alpha_B n_H^2 = \dot{N}_\gamma, \quad (1.39)$$

where n_H is the hydrogen number density.

An important quantity characterizing reionization is the Thomson optical depth,

$$\tau = \int_0^{z_{\text{re}}} c\sigma_T n_e(z) \frac{dt}{dz} dz, \quad (1.40)$$

where σ_T is the Thomson scattering cross section and $n_e(z)$ is the free electron density. Measurements of the CMB polarization provide constraints on the integrated optical depth and therefore on the timing of reionization.

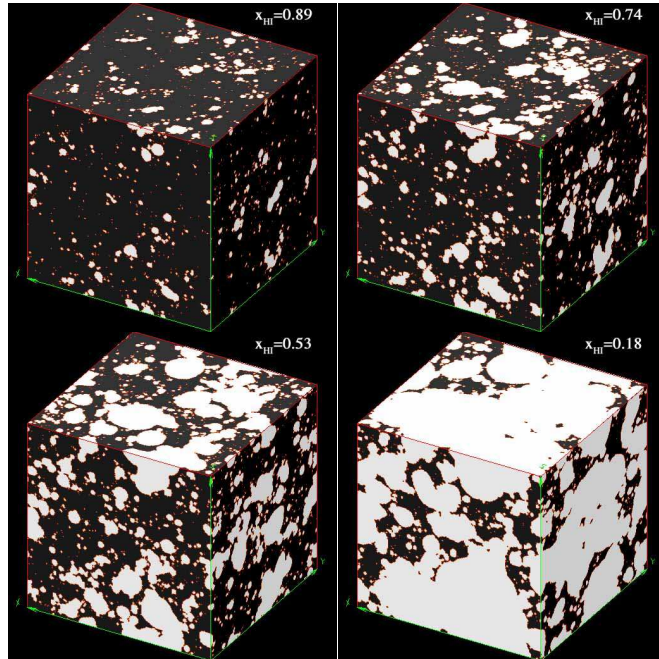


Figure 1.11: Evolution of the ionization field during the epoch of reionization. Ionized regions, shown in white, initially form as isolated bubbles around the first ionizing sources and progressively grow and merge as reionization proceeds. The panels correspond to mean neutral hydrogen fractions $\bar{x}_{\text{HI}} = 0.89, 0.74, 0.53$, and 0.18 , at redshifts $z = 10, 9, 8.25, 7.25$ respectively, illustrating the transition from an almost neutral intergalactic medium to a largely ionized one. Adapted from Mesinger & Furlanetto [24].

The ionization of the IGM also dramatically changes its thermal state. Photoionization heats the gas to temperatures of order $T \sim 10^4$ K, increasing the Jeans mass and suppressing gas accretion in low-mass halos. Reionization therefore affects both the thermal history of the IGM and the formation of dwarf galaxies.

Observationally, reionization can be probed through several methods, including CMB polarization, high-redshift galaxy surveys and quasar absorption spectra. In particular, the disappearance of the Gunn-Peterson trough in quasar spectra at $z \lesssim 6$ indicates that the IGM becomes highly ionized by this epoch (see next subsection 1.1.5.2 on the Lyman- α forest).

The study of reionization has gained renewed importance with the advent of JWST, which is now detecting galaxies at redshifts well within the epoch of reionization. These observations provide important constraints on the sources responsible for ionizing the early Universe.

1.1.5.2 Lyman- α forest

The Lyman- α forest consists of a large number of absorption lines observed in the spectra of distant quasars; an example is provided in Fig. 1.12 below. These absorption features trace continuous fluctuations in the diffuse, highly ionized IGM, with a small residual neutral hydrogen fraction, and constitute one of the most powerful probes of the high-redshift Universe.

Neutral hydrogen absorbs photons through the Lyman- α transition at a rest-frame wavelength $\lambda_\alpha = 1215.67$ Å. As photons emitted by a distant quasar travel toward the observer, they encounter neutral hydrogen regions at different redshifts. Each region absorbs photons at the local Lyman- α wavelength, producing a characteristic absorption feature in the observed spectrum.

The transmitted flux fraction is related to the optical depth through

$$F = e^{-\tau}, \quad (1.41)$$

where τ is the optical depth of neutral hydrogen. Under the fluctuating Gunn-Peterson approximation [25], the optical depth depends on the local baryon density approximately as

$$\tau \propto (1 + \delta_b)^2 T^{-0.7}, \quad (1.42)$$

where δ_b is the baryonic overdensity. The observed Ly α forest is understood through the relation (1.41), with $F(v) = \exp[-\tau(v)]$, where v denotes the coordinate along the

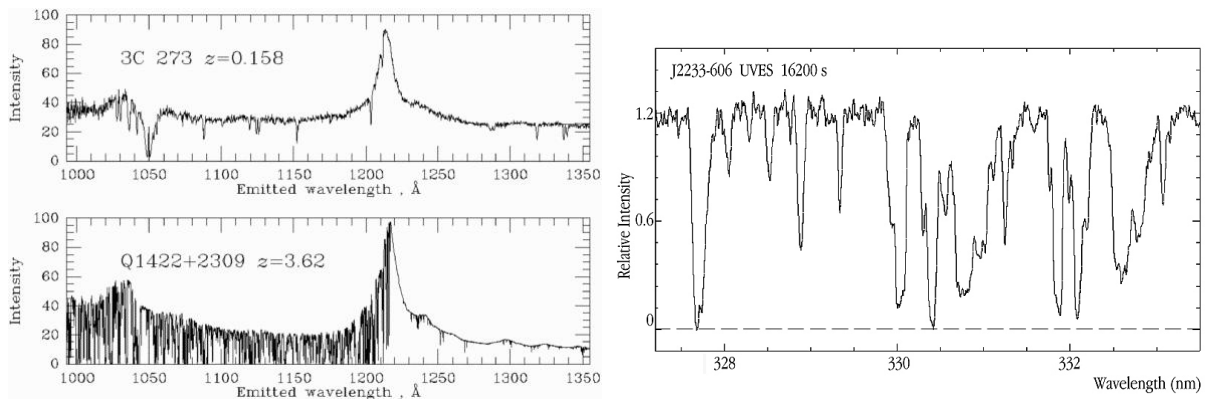


Figure 1.12: **Left:** Comparison of the rest-frame spectra of the low-redshift quasar 3C 273 ($z = 0.158$, upper panel) and the high-redshift quasar Q1422+2309 ($z = 3.62$, lower panel), illustrating the strong increase in the number of Ly- α absorption features toward high redshift. The 3C 273 spectrum is based on HST/FOS observations, while the Q1422+2309 spectrum was obtained with Keck/HIRES. Figure credit: W. C. Keel; Q1422+2309 data provided by M. Rauch. **Right:** Example of a quasar spectrum showing the Ly- α forest. The numerous absorption features blueward of the quasar Ly- α emission are produced predominantly by intervening neutral hydrogen in the intergalactic medium along the line-of-sight. [European Southern Observatory, UVES observations of quasar J2233–606 in the Hubble Deep Field South. Instrument: UVES on the VLT. Image credit: ESO.]

line-of-sight, commonly expressed in velocity units (km/s); this is explained in more detail within section §3.1. The optical depth is determined by the distribution of neutral hydrogen and is affected by the density and temperature of the gas, thermal broadening, and peculiar velocities. Consequently, *the transmitted flux constitutes a non-linear tracer of the underlying matter-density field*. Nevertheless, its statistical fluctuations retain information about the amplitude and shape of the matter power spectrum, which can be extracted by comparison with hydrodynamical simulations [26, 27].

In order to characterize the spatial fluctuations of the forest, the flux is normalized by its mean value at a given redshift. One therefore defines the transmitted-flux contrast as

$$\delta_F(v) = \frac{F(v)}{\langle F \rangle} - 1, \quad (1.43)$$

where $\langle F \rangle$ is the mean transmitted flux. The latter is often equivalently described by the effective optical depth,

$$\tau_{\text{eff}} = -\ln \langle F \rangle. \quad (1.44)$$

Removing the mean transmission isolates the fluctuations around the average absorption level. These fluctuations trace the inhomogeneous distribution of intergalactic gas and,

through it, the underlying matter distribution. The precise relation is not purely gravitational, however, since it also depends on the thermal and ionization state of the IGM [27].

Since a quasar spectrum samples the IGM along a single line-of-sight, the natural statistic is the one-dimensional flux power spectrum. The Fourier transform of the flux contrast may be written as

$$\tilde{\delta}_F(k) = \int dv \delta_F(v) e^{-ikv}, \quad (1.45)$$

and the one-dimensional flux power spectrum is defined through

$$\langle \tilde{\delta}_F(k) \tilde{\delta}_F^*(k') \rangle = 2\pi \delta_D(k - k') P_{F,1D}(k). \quad (1.46)$$

For a finite line-of-sight of length L , this corresponds, up to the adopted Fourier-transform convention, to

$$P_{F,1D}(k) = \frac{1}{L} \left| \tilde{\delta}_F(k) \right|^2. \quad (1.47)$$

In numerical analyses, this quantity is computed for a large number of synthetic lines-of-sight extracted from the simulation volume and is then averaged in bins of wavenumber.

The connection between the 3D matter and 1D flux power spectra is provided by the relation

$$P_{1D}(k_{\parallel}) = \int_0^{\infty} \frac{k_{\perp} dk_{\perp}}{2\pi} P_{3D}(k_{\parallel}, k_{\perp}), \quad (1.48)$$

where $k_{\parallel}$ refers to the line-of-sight in Fourier space, and $k_{\perp}$ are perpendicular directions.

It must be emphasized that the 1D statistic uses correlations of pixels along the same quasar spectrum, making it especially sensitive to smaller scales and to the transition between the linear and nonlinear regimes. In general, simulations compute the 1D power from thousands of lines-of-sight and average the resulting estimates.

The resulting $P_{F,1D}(k)$ quantifies the amplitude of transmitted flux fluctuations as a function of line-of-sight scale. Small values of k correspond to large-scale correlations, while large k probe progressively smaller-scale structure in the IGM. The flux power spectrum is therefore sensitive to the underlying matter power spectrum, but the mapping between the two is non-linear and depends on the thermal and ionization history of the IGM, pressure smoothing, thermal broadening, peculiar velocities, and the mean transmitted flux. For this reason, hydrodynamical simulations are required to predict $P_{F,1D}(k)$ for a given cosmological model [26–28].

The Lyman- α forest traces matter fluctuations on relatively small scales and at high redshift ($6 \geq z \geq 3$), making it an important probe complementary to galaxy surveys and the CMB. Measurements of the flux power spectrum constrain the matter power spectrum, neutrino masses, warm dark matter models and the thermal state of the IGM.

One of the major advantages of the Lyman- α forest is its sensitivity to scales that are difficult to probe through galaxy clustering. This makes it particularly valuable for studying the suppression of small-scale structures induced by massive neutrinos or non-cold dark matter scenarios.

The statistical properties of the Lyman- α forest are commonly analyzed through hydrodynamical simulations, which model the coupled evolution of dark matter, baryons and radiation fields. The comparison between simulations and observations provides stringent constraints on both cosmology and baryonic physics.

1.1.6 Cosmological simulations

Cosmological simulations constitute one of the main theoretical tools used to study the non-linear evolution of structures in the Universe. While linear perturbation theory provides an accurate description of the early stages of structure formation, it breaks down once density fluctuations become sufficiently large, $\delta \gtrsim 1$. In this regime, gravitational collapse, halo formation and, when baryons are included, gas dynamics, cooling and feedback processes must be followed numerically. Modern cosmological simulation codes are therefore designed to evolve the gravitational and hydrodynamical equations over a very large dynamical range and in an expanding cosmological background [29, 30].

In its simplest form, a cosmological simulation evolves a set of particles representing the dark matter distribution inside an expanding, usually periodic, volume. The particles move under their mutual gravitational interaction, while the background expansion is described by the scale factor $a(t)$. In comoving coordinates, the physical position is written as $\mathbf{r}(t) = a(t)\mathbf{x}(t)$, where $\mathbf{x}$ is the comoving coordinate. The equation of motion for a particle in an expanding Universe is

$$\ddot{\mathbf{x}} + 2H\dot{\mathbf{x}} = -\frac{1}{a^2}\nabla_{\mathbf{x}}\Phi, \quad (1.49)$$

where Φ is the peculiar gravitational potential. The latter is determined by the Poisson equation,

$$\nabla_{\mathbf{x}}^2\Phi = 4\pi G a^2 \bar{\rho}_{\text{m}} \delta, \quad (1.50)$$

with δ the matter density contrast. The term $2H\dot{\mathbf{x}}$ arises from the use of comoving coordinates in an expanding background and acts as a Hubble-drag term on the peculiar

motion. Efficient evaluation of the gravitational force in large cosmological particle distributions is commonly achieved through tree, particle-mesh, or hybrid TreePM methods, as implemented, for example, in the GADGET family of cosmological simulation codes [29, 31].

In dark-matter-only simulations, the matter distribution is represented by collisionless numerical particles. Each simulation particle does not correspond to an individual microscopic dark matter particle, but rather samples a macroscopic element of the collisionless phase-space distribution. The continuum evolution is described by the collisionless Boltzmann, or Vlasov, equation,

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{a^2 m} \cdot \nabla_{\mathbf{x}} f - m \nabla_{\mathbf{x}} \Phi \cdot \nabla_{\mathbf{p}} f = 0, \quad (1.51)$$

coupled to the Poisson equation. Here $f(\mathbf{x}, \mathbf{p}, t)$ is the phase-space distribution function. The cosmological N -body method provides a discrete numerical representation of this collisionless system and has become the standard approach for studying non-linear dark-matter clustering and halo formation [29].

In hydrodynamical simulations, baryons are included in addition to the collisionless components. Their evolution is governed by the equations of fluid dynamics in an expanding background. Besides gravity, the simulation must account for gas pressure and shocks and, depending on the physical problem, radiative cooling and heating, star formation, stellar feedback, chemical enrichment, black-hole growth and AGN feedback. Many of these processes occur on physical scales considerably smaller than the spatial and mass resolution attainable in cosmological volumes. Their effect on the resolved gas is therefore commonly incorporated through *sub-grid* or *sub-resolution* prescriptions. Examples include multiphase descriptions of star-forming gas and stellar feedback [32]; the choice and calibration of such prescriptions represent an important source of astrophysical uncertainty in simulations of galaxy formation.

The numerical methods used for cosmological hydrodynamics can be broadly divided according to how the gas is discretized. Smoothed Particle Hydrodynamics (SPH), introduced independently by Lucy [33] and Gingold & Monaghan [34], is a Lagrangian method in which the fluid is represented by particles and continuous quantities are reconstructed through smoothing kernels. SPH became widely used in cosmology because it couples naturally to particle-based gravitational solvers; GADGET-2, for example, combines an N -body TreePM solver with an SPH treatment of the gas [29]. Eulerian approaches instead solve the hydrodynamical equations on a mesh. Adaptive Mesh Refinement (AMR) methods refine the grid dynamically in regions where higher spatial resolution is required, allowing dense halos and galaxies to be resolved while retaining a much larger cosmologi-

cal volume. Well-known examples include RAMSES [30] and ENZO [35]. A further class is represented by moving-mesh methods such as AREPO [36], which solves the hydrodynamical equations on a moving unstructured Voronoi mesh and combines aspects of Eulerian finite-volume schemes with an approximately Lagrangian motion of the resolution elements.

A central difficulty in cosmological simulations is the enormous dynamical range involved. A representative cosmological volume may extend over tens or hundreds of comoving Megaparsecs, while many processes relevant for galaxy formation occur on kiloparsec, or even smaller, scales. One way to overcome this limitation is through *zoom-in* or resimulation techniques, in which a selected region or halo is represented with much higher mass and spatial resolution while the surrounding large-scale environment is retained at progressively lower resolution. Such techniques allow the object of interest to evolve within the correct cosmological tidal field without requiring the entire simulation volume to be evolved at the highest resolution; early applications of this approach to cosmological structure formation were developed, for example, by Katz & White [37].

Cosmological simulations are essential for connecting theoretical models to observations. They can be used to predict quantities such as the halo mass function, galaxy clustering and baryonic properties, as well as to generate synthetic observables that mimic real astronomical measurements. In the context particularly relevant for this thesis, hydrodynamical simulations are indispensable for modeling the Ly- α forest because the transmitted flux depends non-linearly on the density, peculiar velocity, temperature and ionization state of the intergalactic medium. Large suites such as the Sherwood simulations therefore evolve the cosmological matter distribution hydro-dynamically and subsequently extract synthetic Ly- α absorption spectra that can be compared directly with observed quasar spectra [38]. In this sense, numerical simulations provide the bridge between a specified cosmological model and the observables used to test it.

1.1.6.1 Initial conditions

The accuracy of a cosmological simulation depends crucially on the initial conditions (ICs). These specify the initial positions and velocities of the particles at a sufficiently high redshift, when density perturbations are still in the linear regime. The goal is to construct a particle realization whose statistical properties reproduce a chosen linear matter power spectrum $P(k)$.

The starting point is usually a Gaussian random field. In Fourier space, the density contrast is generated such that Equation (1.12) is satisfied. The Fourier amplitudes are drawn from a Gaussian distribution, while the phases are random. Statistical homogeneity

and isotropy require the power spectrum to depend only on the modulus $k = |\mathbf{k}|$.

The particles are initially placed either on a regular grid or on a glass distribution. A grid is simple and computationally convenient, but it introduces preferred directions; while a glass distribution is obtained by evolving particles under reversed gravity until they reach a nearly force-free configuration, reducing artificial anisotropies.

To transform the initial density field into particle positions and velocities, one commonly uses Lagrangian perturbation theory. The Eulerian comoving position $\mathbf{x}$ of a particle is related to its Lagrangian coordinate $\mathbf{q}$ by

$$\mathbf{x}(\mathbf{q}, a) = \mathbf{q} + \mathbf{d}(\mathbf{q}, a), \quad (1.52)$$

where $\mathbf{d}$ is the displacement field. In the Zel'dovich approximation, corresponding to first-order Lagrangian perturbation theory, the displacement field factorizes as

$$\mathbf{d}(\mathbf{q}, a) = D(a) \mathbf{d}^{(1)}(\mathbf{q}), \quad (1.53)$$

where $D(a)$ is the linear growth factor from Eq. (1.10). The displacement is related to a scalar potential ϕ through

$$\mathbf{d}^{(1)}(\mathbf{q}) = -\nabla_{\mathbf{q}} \phi(\mathbf{q}). \quad (1.54)$$

Mass conservation is expressed as

$$\delta(\mathbf{q}, a) = -\nabla_{\mathbf{q}} \cdot \mathbf{d}(\mathbf{q}, a), \quad (1.55)$$

and therefore the displacement potential satisfies (see Poisson Eq. (1.50))

$$\nabla_{\mathbf{q}}^2 \phi(\mathbf{q}) = \delta(\mathbf{q}). \quad (1.56)$$

In Fourier space, this relation becomes particularly simple, if we use Eq. (1.7); then the first-order displacement field is

$$\mathbf{d}^{(1)}(\mathbf{k}) = i \frac{\mathbf{k}}{k^2} \delta(\mathbf{k}). \quad (1.57)$$

The initial particle positions are then assigned according to Eqs. (1.52) & (1.53), with $D(a_{\text{init}})$, while the peculiar velocities are obtained from the time derivative of the displacement,

$$\mathbf{v} = a \dot{\mathbf{x}} = a \dot{D}(a) \mathbf{d}^{(1)}(\mathbf{q}). \quad (1.58)$$

It is common to express this in terms of the logarithmic growth rate $f(a) = \frac{d \ln D}{d \ln a}$, so that

$$\mathbf{v} = aH(a)f(a)D(a)\mathbf{d}^{(1)}(\mathbf{q}). \quad (1.59)$$

The Zel’dovich approximation captures the leading-order gravitational displacement of particles and is sufficient when the simulation is initialized at very high redshift. However, if the starting redshift is not sufficiently high, transients may appear. For this reason, many modern simulations use second-order Lagrangian perturbation theory (2LPT), in which the displacement field is written as

$$\mathbf{d} = D_1(a)\mathbf{d}^{(1)} + D_2(a)\mathbf{d}^{(2)}. \quad (1.60)$$

The second-order correction reduces numerical transients associated with the initialization procedure and provides a more accurate representation of the growing mode, particularly when simulations are initialized at moderate redshift. Simulations initialized using the Zel’dovich approximation contain larger decaying-mode transients, whereas second-order Lagrangian perturbation theory substantially suppresses these effects [39]. For sufficiently high starting redshifts the Zel’dovich approximation can nevertheless provide an adequate description, although the appropriate choice depends on the required accuracy and numerical resolution.

From a numerical point of view, the generation of cosmological ICs typically follows a sequence of steps. A realization of a Gaussian white-noise field is first generated, usually in Fourier space, with random phases fixed by a chosen random seed. The amplitudes are then modulated according to the desired linear matter power spectrum, or equivalently through the corresponding transfer function. The resulting density field is converted into a displacement field by solving the appropriate Poisson equations, conveniently performed in Fourier space. Finally, particles are displaced from their initially uniform Lagrangian positions and assigned peculiar velocities according to the chosen order of Lagrangian perturbation theory [40]. The equations describing this procedure are summarized in Appendix A, where additional details relevant for the implementation of primordial magnetic fields are discussed.

Several numerical packages have been developed; one commonly used code is N-GENIC [41], developed for the generation of cosmological ICs for GADGET simulations. In its standard implementation, a Gaussian realization of the prescribed power spectrum is generated in a periodic volume and the particles are displaced from a regular grid or glass-like particle load using the Zel’dovich approximation. The resulting particle positions and velocities can then be used directly as the starting configuration of an N -body or

hydrodynamical simulation. For simulations requiring non-uniform resolution, a widely used alternative is MUSIC (Multi-Scale Initial Conditions) [40]; it was designed particularly for multi-scale and zoom-in simulations.

The reliability of a cosmological simulation therefore depends not only on the numerical solver employed during the subsequent non-linear evolution, but also on the accuracy with which its ICs are constructed. The starting redshift, perturbative order, particle load, random realization and transfer functions can all influence the subsequent evolution. In simulations containing several matter components, the treatment of the individual transfer functions and relative perturbations of baryons and dark matter is particularly important. Errors introduced at the initial-condition stage can propagate into the evolved density field and consequently affect quantities such as the halo mass function, baryon distribution and galaxy population. This issue becomes especially relevant in the presence of additional sources of perturbations, such as primordial magnetic fields, and will be discussed in detail in Chapter 2 and Appendix A.

1.1.7 The JWST era

The launch and successful commissioning of the James Webb Space Telescope (JWST) have opened a new observational window onto the high-redshift Universe. Designed primarily for observations at near- and mid-infrared wavelengths, JWST combines a large collecting area with high angular resolution and unprecedented infrared sensitivity [42, 43]. These capabilities make it possible to probe the epoch of reionization and the formation of the first galaxies with a level of detail that was previously inaccessible. In particular, JWST observations are providing new constraints on the abundance, luminosities, stellar populations and physical properties of galaxies during the first few hundred million years of cosmic history [44].

One of the primary scientific goals of JWST is the detection and characterization of galaxies at very high redshift. Due to cosmological redshift, ultraviolet radiation emitted by young stellar populations in early galaxies is shifted into the near-infrared bands observed by JWST. Rest-frame ultraviolet emission therefore provides one of the principal means of identifying and characterizing star-forming galaxies during Cosmic Dawn and the epoch of reionization [44, 45].

Early JWST observations revealed candidate galaxies at redshifts $z \gtrsim 10$, corresponding to cosmic ages younger than approximately 500 Myr. Several particularly luminous candidates were identified at $z \sim 10\text{--}15$ in the first deep JWST fields [46–48]. These discoveries initially suggested a surprisingly large abundance of bright galaxies at very early times and motivated extensive discussion of whether star formation and galaxy assembly

proceeded more efficiently than expected in pre-JWST models. Subsequent measurements of the high-redshift galaxy population have continued to refine the abundance of these systems and the corresponding ultraviolet luminosity functions [49].

The abundance of galaxies is commonly characterized through the galaxy luminosity function. A widely used parametrization is the Schechter function,

$$\phi(L)dL = \phi_* \left(\frac{L}{L_*} \right)^\alpha \exp \left(-\frac{L}{L_*} \right) \frac{dL}{L_*}, \quad (1.61)$$

where L_* is the characteristic luminosity, ϕ_* is the normalization and α is the faint-end slope. Measurements of the ultraviolet luminosity function at high redshift provide important constraints on the abundance of star-forming galaxies, the efficiency with which baryons are converted into stars, and the connection between galaxies and their host dark-matter halos [44, 49]. The faint end of the luminosity function is particularly important for estimating the total ultraviolet luminosity density and, consequently, the contribution of galaxies to cosmic reionization.

The existence of luminous galaxies at very high redshift provides an important test of models of early galaxy formation. Within the Λ CDM framework, whose cosmological parameters are tightly constrained by observations such as those of the Planck satellite [7], the abundance of massive dark-matter halos decreases rapidly toward increasing redshift. Very luminous or massive galaxies are therefore expected to become progressively rarer at early cosmic times. The unexpectedly large number of bright galaxy candidates found in some of the first JWST analyses motivated investigations into enhanced star-formation efficiencies, altered feedback prescriptions, dust attenuation, stellar-population modeling, and uncertainties in the conversion between observed luminosity and halo mass [46, 48, 49]. At present, these observations are more appropriately regarded as a powerful test of high-redshift galaxy-formation physics than as evidence against the underlying Λ CDM cosmological model.

JWST is also providing important information on the epoch of reionization. The high-redshift galaxy population detected by JWST is expected to contribute substantially to the ionizing photon budget responsible for transforming the intergalactic medium from a predominantly neutral to an ionized state [44, 50]. The comoving production rate of ionizing photons escaping into the IGM can be written schematically as

$$\dot{n}_{\text{ion}} = f_{\text{esc}} \xi_{\text{ion}} \rho_{\text{UV}}, \quad (1.62)$$

where f_{esc} is the escape fraction of hydrogen-ionizing photons, ξ_{ion} is the ionizing photon production efficiency and ρ_{UV} is the ultraviolet luminosity density. The latter is

obtained by integrating the galaxy UV luminosity function down to an assumed limiting luminosity. Consequently, measurements of the abundance of faint high-redshift galaxies are directly relevant for reconstructing the reionization history [44, 50].

The study of reionization with JWST is complemented by spectroscopy capable of detecting rest-frame ultraviolet and optical emission lines from high-redshift galaxies. Such measurements can constrain star-formation rates, nebular conditions, metallicities and ionizing properties of the galaxy population, while $\text{Ly}\alpha$ emission additionally carries information about the ionization state of the surrounding intergalactic medium [44, 50]. In combination with measurements of the UV luminosity function, these observations make it possible to investigate both the sources responsible for reionization and the physical properties of the early galaxies producing the ionizing radiation.

The high-redshift observations performed by JWST also provide an opportunity to test cosmological models that modify the small-scale matter power spectrum. Scenarios such as warm dark matter or ultralight dark matter generally suppress the formation of low-mass halos relative to cold dark matter and can therefore modify the abundance of faint galaxies and the timing of reionization. Measurements of galaxy number densities at progressively higher redshifts and lower luminosities can consequently provide constraints on the abundance of small dark-matter halos and complement other probes of small-scale structure.



Figure 1.13: JWST’s First Deep Field, showing the galaxy cluster SMACS 0723 observed with NIRCcam. The near-infrared image contains thousands of galaxies, while the foreground cluster acts as a gravitational lens, magnifying and distorting the images of more distant background galaxies. [Credit: NASA, ESA, CSA, and STScI].

Although the Λ CDM model remains broadly consistent with the current observational picture, JWST is revealing a considerably richer population of galaxies at early times than was observationally accessible before its launch. Determining whether apparently unexpected galaxy abundances arise from uncertainties in star-formation efficiency, stellar populations, feedback and dust, or instead indicate modifications to the underlying cosmological model, requires increasingly precise measurements and self-consistent theoretical modeling. The combination of deep imaging, spectroscopy and complementary multi-wavelength observations will therefore play a central role in understanding galaxy formation, reionization and the small-scale structure of the Universe in the JWST era.

* * *

1.2 Primordial Magnetic Fields

In this section, we review several theoretical and observational aspects of PMFs.

After laying out some basic motivation for studying PMFs and emphasizing their testability (§1.2.1), we organize the subsections in a chronological order within the cosmological evolution.

In §1.2.2 some PMF-generation mechanisms are presented; importantly, the categorization of such magnetogenesis scenarios is later translated into one parameter, spectral index n_B , which determines the shape and relevant scales of the PMF power spectra.

Once the PMFs enter the early universe physics, it is necessary to carefully compute how they evolve with the plasma prior to the recombination. This is presented in §1.2.3.

The subsection §1.2.4 is one of the most crucial theoretical elements in this thesis; therein we review some well-established facts about PMF-related perturbations, and we elaborate on some additional features, especially in terms of the PMF-induced power spectrum of baryons.

1.2.1 Motivation: theory and observations

Magnetic fields are observed over an enormous range of astrophysical scales, from stars and galaxies to galaxy clusters and, possibly, the much more diffuse intergalactic medium. While magnetic fields of order μG are routinely measured in galaxies and clusters, their ultimate origin remains uncertain. In particular, the existence of coherent magnetic fields in large-scale structures raises two conceptually different possibilities: magnetic fields may have been generated relatively late by astrophysical processes associated with the first stars, galaxies and active galactic nuclei, or a seed magnetic field may already have

been present in the primordial Universe and subsequently evolved together with the cosmic plasma. This second possibility defines the broad class of *primordial magnetic field* (PMF) scenarios [51–53].

One of the early examples in which these two aspects were connected was the work of Wasserman [54]. Assuming the existence of a tangled cosmic magnetic field around recombination, Wasserman showed that the Lorentz force can source both compressional and rotational perturbations of the baryonic fluid. Once radiation pressure and drag become inefficient, these perturbations can grow gravitationally, while the magnetic field frozen into the plasma is carried into the collapsing structures. In this picture, primordial magnetism could therefore participate simultaneously in the formation of protogalaxies, the generation of their angular momentum and the origin of galactic magnetic fields. Although the original scenario is necessarily model dependent, its central observation remains important: a PMF is not merely a passive seed subsequently amplified during structure formation, but can itself act as a source of density and velocity perturbations. Modern calculations of magnetically-induced perturbations build on precisely this idea [55, 56].

From the theoretical point of view, the early Universe provides several possible epochs at which such fields could have been generated. Inflation is particularly attractive because the accelerated expansion can stretch fluctuations to cosmological scales. However, the conformal invariance of standard electromagnetism prevents significant amplification of electromagnetic vacuum fluctuations in a conformally flat background. Inflationary magnetogenesis therefore generally requires a mechanism that breaks conformal invariance, as in the classic models of Turner & Widrow [57] and Ratra [58]. Such scenarios can naturally generate correlations on very large scales and may produce approximately scale-invariant magnetic spectra. We also note that the first serious quantitative consideration of PMFs was introduced by Zel’dovich in 1970 [59].

An alternative class of mechanisms operates during causal processes after inflation, most notably cosmological phase transitions. For example, magnetic fields may be generated during the electroweak or QCD transitions through charge currents, gradients of fields, bubble collisions, or turbulent motions in the primordial plasma [60]. Because these mechanisms operate within the causal horizon, the resulting magnetic spectra are generically much bluer and initially concentrated on smaller characteristic scales than those produced during inflation. If the generated field carries magnetic helicity, subsequent magnetohydrodynamical evolution may transfer power towards larger scales through an inverse transfer or cascade [61]. Inflationary and phase-transition scenarios therefore predict qualitatively different combinations of field amplitude, correlation scale, spectral

index and helicity. These differences motivate the parametrization of PMFs introduced in the following subsection.

A particularly influential observational motivation emerged from very-high-energy γ -ray observations of distant blazars. TeV photons emitted by a blazar interact with the extragalactic background light and produce electron–positron pairs. These charged particles subsequently inverse-Compton scatter background photons, generating a secondary electromagnetic cascade which should contain a GeV component. Neronov & Vovk [62] argued that the absence of the expected GeV cascade emission in Fermi observations can be explained if the electron–positron pairs are deflected by an intergalactic magnetic field. They obtained the widely quoted lower limit

$$B_{\text{IGM}} \gtrsim 3 \times 10^{-16} \text{ G}, \quad (1.63)$$

for sufficiently large magnetic coherence lengths, with a coherence-length dependent limit for smaller scales. This result provided an important new motivation for cosmological magnetism: unlike the strong fields measured within galaxies, a weak magnetic field permeating low-density cosmic voids is much more difficult to produce and distribute through conventional late-time astrophysical activity. The interpretation of the missing cascade is not unique and depends on assumptions concerning source activity and the propagation of the pair plasma, but γ -ray observations remain one of the principal methods of probing intergalactic magnetic fields [51, 63].

The same basic principle extends to charged cosmic rays. Ultra-high-energy cosmic rays (UHECRs) propagating over extragalactic distances are deflected by the Lorentz force, so that their spectra and arrival directions contain information about the strength and geometry of the intervening magnetic field. Alves Batista & Saveliev [64] demonstrated, for example, that angular information in UHECR arrival distributions could in principle constrain not only the magnitude but also the sign of magnetic helicity. Together with γ rays and neutrinos, cosmic rays therefore turn cosmic magnetism into a multimessenger problem [63]. The difficulty is that these observables also depend on the unknown distribution and composition of the sources and on the Galactic magnetic field. Cosmological propagation simulations are consequently essential for separating these effects from those of the intergalactic field [65].

It is nevertheless essential to distinguish the existence of cosmic magnetic fields from evidence for their *primordial* origin. There are several well-motivated astrophysical mechanisms capable of producing seed fields after recombination. The Biermann battery, for example, generates magnetic fields whenever gradients of electron density and pressure are not parallel, and may operate efficiently in shocks, ionization fronts and collapsing

protogalactic gas. Other possibilities include magnetic-field injection by supernovae and active galactic nuclei. Gruzinov [66] has emphasized an even stronger version of this argument: the first gravitationally collapsing baryonic structures can spontaneously produce magnetic fields through plasma instabilities and subsequent turbulent evolution, without requiring any primordial seed field.

Once a seed field exists, turbulent dynamo processes can amplify it extremely efficiently. Small-scale and large-scale dynamos convert kinetic energy of turbulent motions into magnetic energy and can amplify initially weak fields until non-linear saturation is reached [67]. Consequently, the observation of μG fields in a present-day galaxy does not uniquely reveal whether the original seed was primordial or astrophysical: gravitational compression, turbulence, feedback and dynamo amplification can largely erase the memory of the initial field.

This degeneracy has been demonstrated directly in cosmological simulations. Garaldi, Pakmor & Springel [68] compared primordial seeding, the Biermann battery, magnetic fields generated around ionization fronts and supernova injection. Although these mechanisms produce very different magnetic configurations at early times, the magnetic fields inside sufficiently evolved galaxies become remarkably similar after amplification. Importantly, however, low-density environments retain substantially more information about the initial conditions.

This observation shifts the search for primordial magnetism from galaxies towards the intergalactic medium, cosmic filaments and voids. Cosmological simulations by Vazza and collaborators show that primordial scenarios typically produce a considerably more homogeneous and volume-filling magnetization of the cosmic web, whereas astrophysical scenarios driven by stars and AGN tend to concentrate magnetized material around collapsed structures [69]. It is precisely in filaments and voids, where dynamo amplification and baryonic feedback are less efficient, that the different magnetogenesis scenarios remain most distinguishable. Vazza et al. [70] evolved PMF configurations consistent with CMB constraints and showed that their signatures in cosmic filaments can in principle be discriminated by future radio observations. The problem of cosmic magnetogenesis is therefore not expected to be resolved by a single measurement, but by combining CMB, radio, γ -ray, cosmic-ray and large-scale-structure information.

Primordial magnetism can also affect observables that at first sight appear unrelated to cosmic magnetic fields. A notable example is cosmological recombination. A stochastic PMF produces small-scale baryon inhomogeneities: the Lorentz force redistributes baryonic matter, and because the recombination rate depends non-linearly on the local electron and proton densities, baryonic clumping can accelerate recombination. Jedamzik

& Pogosian [71] pointed out that this shifts photon decoupling to earlier times and reduces the sound horizon, allowing the CMB-inferred value of H_0 to move towards larger values and thereby potentially relieving the Hubble tension. More recent calculations incorporating detailed MHD evolution and recombination physics continue to find this possibility interesting [72], although its statistical significance remains dependent on the combination of cosmological datasets. Thus, the Hubble tension provides an unexpected independent motivation for studying small-scale primordial magnetism.

The CMB remains one of the cleanest probes of PMFs because a field present before recombination affects scalar, vector and tensor perturbations, the thermal and ionization history, and the polarization of the CMB through Faraday rotation. The work of Paoletti and collaborators has played an important role in developing and constraining these signatures. In particular, the LiteBIRD forecasts of Paoletti et al. [73] demonstrate how future high-precision CMB polarization measurements can combine several complementary PMF signatures. For a nearly scale-invariant spectrum with $n_B = -2.9$, the forecast from magnetically-induced perturbations reaches approximately

$$B_{1\text{ Mpc}} < 0.8 \text{ nG} \quad (95\% \text{ C.L.}), \quad (1.64)$$

while information from the thermal history, Faraday rotation and polarization non-Gaussianity provides independent constraints. The combination of these effects is particularly valuable because each depends differently on the PMF amplitude and spectral shape.

The observational prospects are therefore rapidly expanding beyond the CMB. At radio wavelengths, the Square Kilometre Array (SKA) will provide high-sensitivity measurements of synchrotron emission and dense grids of Faraday rotation measures. Cosmological MHD simulations indicate that the largest differences between primordial and astrophysical magnetogenesis scenarios should appear in low-density filaments rather than in the centres of galaxy clusters, making the cosmic web a particularly promising target [70, 74]. The combination of total-intensity and polarimetric measurements expected from the SKA should therefore probe a region of parameter space and a cosmic environment complementary to the CMB.

At the opposite end of cosmic history, observations of the first galaxies with the James Webb Space Telescope provide another route to primordial magnetism. PMF-induced density perturbations enhance the small-scale matter power spectrum and can increase the abundance of low-mass haloes at high redshift, thereby modifying star formation, the stellar mass density, the ultraviolet luminosity function and ultimately the history of reionization. High-redshift galaxy observations have consequently begun to be used

to constrain PMF parameters [75, 76]. Together with future 21-cm measurements, CMB polarization and radio observations of the cosmic web, JWST therefore extends the search for primordial magnetism to the epoch in which the first luminous structures formed.

The motivation for studying PMFs is thus twofold. On the theoretical side, they provide a possible relic of fundamental processes operating at energies and epochs inaccessible to laboratory experiments, while simultaneously acting as an additional source of cosmological perturbations. On the observational side, they connect phenomena ranging from CMB anisotropies and recombination to the propagation of γ rays and cosmic rays, the magnetization of the cosmic web, and the formation of the first galaxies. None of these observations alone presently establishes a primordial origin for cosmic magnetism. Their complementarity, however, makes PMFs a falsifiable hypothesis whose parameter space can increasingly be tested across a wide range of cosmological epochs and physical scales.

1.2.2 Magnetogenesis models

Having motivated the possible existence of primordial magnetic fields, we now briefly discuss physical mechanisms capable of generating them in the early Universe. A complete review of primordial magnetogenesis is beyond the scope of this work, and we shall instead emphasize the distinction between the two broad classes most relevant for the phenomenology considered below: *inflationary* magnetogenesis and *causal* magnetogenesis associated with processes occurring after inflation, in particular cosmological phase transitions. These mechanisms operate at very different epochs and therefore naturally predict different correlation scales and magnetic power spectra [57, 58, 60, 61, 77].

A useful starting point is the standard electromagnetic action in curved spacetime,

$$S_{\text{EM}} = -\frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}, \quad (1.65)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (1.66)$$

is the electromagnetic field-strength tensor. In four space-time dimensions this action is conformally invariant. Since the spatially flat FLRW metric is conformally related to Minkowski spacetime, the expansion during inflation does not, by itself, gravitationally amplify the vacuum fluctuations of a standard electromagnetic field. In the absence of additional interactions, the physical magnetic field simply redshifts as

$$B_{\text{phys}} \propto a^{-2}. \quad (1.67)$$

Consequently, inflationary magnetogenesis requires a breaking of the conformal invariance of electromagnetism [57, 58].

One commonly studied realization introduces a coupling between the inflaton and the electromagnetic sector, schematically replacing Eq. (1.65) by

$$S_{\text{EM}} = -\frac{1}{4} \int d^4x \sqrt{-g} I^2(\phi) F_{\mu\nu} F^{\mu\nu}, \quad (1.68)$$

where the coupling function $I(\phi)$ evolves while the inflaton field ϕ rolls down its potential. The time dependence of $I(\phi)$ breaks conformal invariance and allows electromagnetic quantum fluctuations to be amplified. Their wavelengths are subsequently stretched by the rapid inflationary expansion, in analogy with the generation of primordial curvature perturbations discussed in §1.1.3.2. A particularly important property of this mechanism is that inflation can establish correlations on scales that would otherwise have been causally disconnected. Inflationary magnetogenesis can therefore generate magnetic fields coherent on very large comoving scales and, depending on the specific coupling, can produce red or nearly scale-invariant magnetic spectra [57, 58].

The details are strongly model dependent. The same coupling responsible for amplifying the gauge field must remain compatible with the inflationary background, avoid excessive backreaction on the inflaton dynamics, and remain within a regime in which the effective electromagnetic theory is under control. We shall not attempt to select a particular inflationary model here. Instead, its phenomenological consequence for the present work will be encoded by the amplitude and spectral shape of the resulting stochastic magnetic field.

A conceptually different possibility is that magnetic fields were generated *after* inflation by causal processes in the primordial plasma. Cosmological phase transitions provide a natural environment for such mechanisms. In particular, the electroweak and QCD epochs may generate magnetic fields through charge separation, gradients of particle or Higgs fields, bubble-wall dynamics, shocks and turbulent motions [60, 78]. Vachaspati [60], for example, showed that spatial variations of the Higgs field during the electroweak transition can source electromagnetic fields.

Unlike inflationary mechanisms, causal magnetogenesis cannot initially correlate the field over distances larger than the causal horizon at the epoch of generation. Its characteristic correlation length therefore satisfies schematically

$$\lambda_{B,*} \lesssim H_*^{-1}, \quad (1.69)$$

where the subscript $*$ denotes the epoch of magnetogenesis. Since the comoving horizon

at an early phase transition is very small compared with cosmological scales today, causal mechanisms naturally generate fields whose initial power is concentrated on relatively small scales. Furthermore, causality together with the divergence-free character of the magnetic field strongly constrains its large-scale spectrum. For a statistically homogeneous and isotropic causal field, the lowest allowed large-scale spectral behaviour is the blue, or Batchelor-like, spectrum

$$P_B(k) \propto k^2, \quad k \ll k_*, \quad (1.70)$$

corresponding to a magnetic spectral index $n_B = 2$ [77].

The field produced at magnetogenesis does not, however, remain unchanged until recombination. Once embedded in the highly conducting primordial plasma, it evolves according to magnetohydrodynamics. In the ideal-MHD limit magnetic flux is approximately frozen into the plasma, leading to the scaling in Eq. (1.67), while turbulent interactions redistribute magnetic energy between scales. Dissipation suppresses power below a characteristic damping scale, whereas turbulent evolution can increase the magnetic correlation length. The details depend on the initial spectrum and, importantly, on whether the magnetic field possesses helicity [61]. These aspects of the subsequent evolution will be discussed further in the following subsection.

For the cosmological applications considered in this thesis, it is neither necessary nor practical to follow the microscopic magnetogenesis mechanism explicitly. Instead, its outcome can be represented statistically. We assume that the primordial magnetic field is a statistically homogeneous and isotropic stochastic field with vanishing mean,

$$\langle \mathbf{B}(\mathbf{x}) \rangle = 0, \quad (1.71)$$

and satisfying the absence of magnetic monopoles,

$$\nabla \cdot \mathbf{B} = 0. \quad (1.72)$$

For a non-helical field its Fourier-space two-point correlation function may then be written as

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{k}') \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \frac{P_B(k)}{2}, \quad (1.73)$$

where $P_B(k)$ denotes the magnetic-field power spectrum. The transverse projector in parentheses follows directly from $\nabla \cdot \mathbf{B} = 0$.

Over the range of scales relevant for cosmological observables, the primordial spectrum

is commonly approximated by a power law,

$$P_B(k) = A_B k^{n_B}, \quad (1.74)$$

with a suppression of power below the magnetic damping scale. In the phenomenological model adopted in this work we shall write this as

$$P_B(k) = A_B k^{n_B} \exp(-k^2 \lambda_D^2), \quad (1.75)$$

where $\lambda_D = k_D^{-1}$ characterizes the damping scale. The spectral index n_B is therefore the quantity that retains much of the information about the magnetogenesis scenario. A value

$$n_B \simeq -3 \quad (1.76)$$

corresponds to an approximately scale-invariant magnetic spectrum and is typically associated with inflationary magnetogenesis, whereas causal post-inflationary mechanisms naturally produce much bluer spectra, with $n_B = 2$ representing the canonical large-scale causal spectrum.

The origin of this terminology can be seen by considering the magnetic power per logarithmic interval in wavenumber. Up to normalization factors,

$$\Delta_B^2(k) \equiv \frac{k^3}{2\pi^2} P_B(k) \propto k^{n_B+3}. \quad (1.77)$$

Hence $n_B = -3$ gives equal magnetic power per logarithmic interval in k , while $n_B > -3$ corresponds to progressively more power on smaller scales. The spectral index thus provides a convenient phenomenological connection between the underlying magnetogenesis physics and the scales on which PMFs affect subsequent cosmological evolution.

The normalization A_B is not usually quoted directly. Instead, the field strength is characterized by smoothing the stochastic magnetic field on a fixed comoving length scale λ . For a Gaussian smoothing window we define

$$B_\lambda^2 \equiv \int \frac{d^3k}{(2\pi)^3} P_B(k) \exp(-k^2 \lambda^2). \quad (1.78)$$

For a pure power law and a smoothing scale sufficiently larger than the damping length, this gives

$$B_\lambda^2 = \frac{A_B}{4\pi^2} \lambda^{-(n_B+3)} \Gamma\left(\frac{n_B+3}{2}\right), \quad (1.79)$$

or equivalently

$$A_B = \frac{4\pi^2 B_\lambda^2 \lambda^{n_B+3}}{\Gamma[(n_B+3)/2]}. \quad (1.80)$$

Following the common convention, throughout this thesis we choose

$$\lambda = 1 \text{ Mpc}, \quad (1.81)$$

and therefore characterize the amplitude of the primordial field by $B_{1 \text{ Mpc}}$. Our basic stochastic PMF model is consequently specified, before introducing the damping scale, by the pair

$$\boxed{(B_{1 \text{ Mpc}}, n_B)}. \quad (1.82)$$

The former fixes the normalization of the magnetic fluctuations on a reference comoving scale, while the latter controls how magnetic power is distributed among different spatial scales.

More generally, a stochastic magnetic field may also contain a helical component. In this case the most general statistically isotropic two-point function contains an additional parity-odd contribution,

$$\langle B_i(\mathbf{k}) B_j^*(\mathbf{k}') \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') \frac{1}{2} \left[\left(\delta_{ij} - \hat{k}_i \hat{k}_j \right) P_B(k) + i \epsilon_{ijl} \hat{k}_l P_H(k) \right], \quad (1.83)$$

where $P_H(k)$ characterizes the helical component. Magnetic helicity is related to the quantity

$$\mathcal{H}_B = \int d^3x \mathbf{A} \cdot \mathbf{B}, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad (1.84)$$

and is approximately conserved in a highly conducting plasma. Helicity can substantially modify the subsequent MHD evolution of PMFs, in particular by facilitating the transfer of magnetic energy from smaller to larger scales [61]. It also leads to parity-odd observational signatures. Although helical primordial fields are therefore physically interesting, in this thesis we restrict our analysis to statistically isotropic **non-helical** PMFs, $P_H(k) = 0$, and leave the study of helicity for future work.

*

In summary, inflationary and causal magnetogenesis scenarios differ substantially in their microscopic physics, characteristic scales and expected spectral shapes. For the purposes of the present work, however, these differences are encoded phenomenologically in the stochastic magnetic power spectrum through the amplitude $B_{1 \text{ Mpc}}$, the spectral index n_B , and the damping scale k_D . This parametrization allows us to remain largely agnostic about the precise magnetogenesis mechanism while studying its subsequent cosmologi-

cal consequences. Once generated, the magnetic field evolves together with the highly conducting primordial plasma; we therefore turn next to the relevant magnetohydrodynamical description.

1.2.3 MHD in the early universe

1.2.3.1 Complete set of equations

We shall focus on scales much larger than $\sim \text{kpc}$, where we can safely assume that the conductivity of plasma is infinite.

Then the non-relativistic MHD equations for the baryon fluid are given by

$$\frac{\partial \vec{v}_b}{\partial t} + H\vec{v}_b + \alpha(\vec{v}_b - \vec{v}_\gamma) + \frac{(\vec{v}_b \cdot \nabla)\vec{v}_b}{a} + \frac{c_b^2}{a}\nabla\delta_b = \frac{(\nabla \times \vec{B}) \times \vec{B}}{4\pi a^5 \rho_b} - \frac{\nabla\phi}{a} \quad (1.85)$$

$$\frac{\partial\delta_b}{\partial t} + \frac{\nabla \cdot \vec{v}_b}{a} + \frac{\nabla \cdot (\delta_b \vec{v}_b)}{a} = 0, \quad (1.86)$$

where $\vec{v} = a d\vec{x}/dt$ is the physical velocity, $\delta = (\rho(x) - \bar{\rho})/\bar{\rho}$ is the fluid density perturbation, $\vec{v}_\gamma$ is determined by the dipole of photon temperature perturbation, ϕ is the metric perturbation, $\alpha = 4\rho_\gamma/(3\rho_b l_\gamma)$, l_γ is the photon mean free path, c_b^2 is the baryon sound speed, $B = a^2 B_{phys}$ is the comoving magnetic field with B_{phys} being the physical magnetic field, and H is the Hubble rate. Note that we work in natural units where speed of light, $c = 1$.

The baryon motion feeds back onto the magnetic field via the induction equation,

$$\frac{\partial B}{\partial t} = \frac{1}{a} \nabla \times (\vec{v}_b \times B). \quad (1.87)$$

When baryon velocity is small, $v_b \ll aH$, the RHS above can be ignored and we find that the comoving magnetic field is constant. Once $v_b > aH$, the baryons feedback onto the magnetic fields and damp them.

We shall primarily be interested on scales larger than the scale where magnetic fields are damped by v_b . Consequently, on these scales $v_b < aH$ and hence the non-linear terms in the baryon Euler equation can be ignored. For such v_b even δ_b remains linear because $\delta_b \sim v_b/[aH]$. Thus baryon perturbation equations are simplified to

$$\frac{\partial\theta_b}{\partial t} + H\theta_b + \alpha(\theta_b - \theta_\gamma) + \frac{c_b^2}{a}\nabla\delta_b = \frac{S_0}{a^2} - \frac{\nabla^2\phi}{a} \quad (1.88)$$

$$\frac{\partial\delta_b}{\partial t} + \frac{\theta_b}{a} = 0, \quad (1.89)$$

where $\theta = \nabla \cdot \vec{v}$ and

$$S_0 = \frac{\nabla \cdot [(\nabla \times \vec{B}) \times \vec{B}]}{4\pi a^3 \rho_b}. \quad (1.90)$$

Note that S_0 is a constant (in time) for a time-independent B . We can readily interpret the physical meaning of what is essentially the *divergence of the Lorentz force*, S_0 : if we rewrite Eq. (1.90), using $\nabla \cdot \vec{B} = 0$, as

$$S_0 = \frac{1}{a^3 \rho_b} \nabla \cdot \left[-\nabla \left(\frac{B^2}{8\pi} \right) + \frac{1}{4\pi} [(\vec{B} \cdot \nabla) \vec{B}] \right]. \quad (1.91)$$

The first term represents effects associated with gradients of the magnetic pressure, $p_B = B^2/(8\pi)$, while the second arises from magnetic tension and depends on the orientation and curvature of the magnetic-field lines. The magnetic-pressure force tends to move baryons away from regions of higher B^2 towards regions of lower B^2 . Spatial variations of this force can therefore produce compression (or rarefaction) of the baryonic fluid, both contributing to S_0 . The magnetic-tension term can also have a non-zero divergence and therefore contributes to the scalar, density-sourcing part of the Lorentz force. In addition, the Lorentz force can possess a rotational component, which instead sources vortical motions of the baryons. Thus, the source term S_0 , which can locally drive compression or rarefaction of the baryonic fluid, is not determined by the magnetic energy density B^2 alone, but also by the anisotropic magnetic stress encoded in $B_i B_j$.

However, it is important to think statistically; if we assume Gaussian distribution,¹ PMFs through S_0 do not create a non-zero mean density perturbation (e.g., more compression than rarefaction), but they *create additional spatial fluctuations*. In other words, we have $\langle S_0 \rangle = 0$, while $\langle S_0^2 \rangle \neq 0$, and therefore $\langle \delta_{b,\text{PMF}}^2 \rangle \neq 0$.

We note that the original derivation of S_0 was done by Wasserman [54].

1.2.4 PMF perturbations and the power spectrum

1.2.4.1 Effective description with linear perturbations

In this section, based on [1], we provide simplified equations that primarily control the evolution of density perturbations due to PMFs.

Together with the previous assumption of infinite conductivity, we wish to ignore baryon thermal pressure, which puts us at scales much larger than ~ 10 kpc. Furthermore,

¹See appendix on [1] for discussion of non-gaussianities. Briefly, we do find small non-gaussian features of S_0 that propagate into δ_b , but they can be neglected for observationally allowed values of PMFs.

we shall focus on times after recombination, in this limit the governing equations are

$$\frac{\partial \vec{v}_b}{\partial t} + H \vec{v}_b = \frac{1}{a^2} \frac{(\vec{\nabla} \times \vec{B}) \times \vec{B}}{4\pi a^3 \rho_b} - \frac{\vec{\nabla} \phi}{a}. \quad (1.92)$$

$$\frac{\partial \delta_b}{\partial t} + \frac{\theta_b}{a} = 0, \quad (1.93)$$

where $\theta = \nabla \cdot \vec{v}$, ϕ is the metric potential which is determined by

$$\nabla^2 \phi = \frac{1}{2M_{Pl}^2} a^2 (\rho_b \delta_b + \rho_{DM} \delta_{DM}). \quad (1.94)$$

The magnetic fields can be damped by the backreaction of baryons via the induction equation,

$$\frac{\partial B}{\partial t} = \frac{1}{a} \nabla \times (v_b \times B). \quad (1.95)$$

When baryon velocity is small, $v_b \ll aH$, the RHS above can be ignored and the comoving magnetic field is constant. Once $kv_b \sim \theta_b > aH$, the baryons feedback onto the magnetic fields and damp them.

One can naively estimate the damping scale by applying divergence $\nabla \cdot$ on the Eq. (1.92); this gives a relation $H\theta_b \sim S_0 \sim k^2 B_{\text{phys}}^2 / [4\pi a \rho_b]$. Thus, feedback occurs when $\theta_b > aH$, or equivalently

$$k^{-2} < \frac{B_{\text{phys}}^2}{4\pi \rho_b} (aH)^{-2}. \quad (1.96)$$

Applying the square root, we can promptly identify the Alfvén speed as $v_A = \sqrt{B_{\text{phys}}^2 / [4\pi \rho_b]}$ and $(aH)^{-1}$ as the conformal time. In conclusion, the magnetic fields are damped on scales smaller than distance covered with Alfvén speed in a Hubble time.

Note that if we consider B to be constant, then $v_A/[aH]$ is also constant in a matter dominated era. Thus as damping of magnetic field is not dynamic our assumption of constant B is self consistent. What this tells us is that we can treat B and hence S_0 as a constant in our perturbation equations!

To directly see how PMFs cause an increase in δ_b , we need to replace $\vec{v}_b$ using Eq. (1.92). We do so by first taking the time derivative of the continuity Eq. 1.93, then transforming $\frac{\partial \vec{v}_b}{\partial t}$ with Eq. (1.92) and $\nabla \cdot \vec{v}_b$ using Eq. (1.93) again. By writing the equation in terms of scale factor instead of time, we obtain

$$a^2 \frac{\partial^2 \delta_b}{\partial a^2} + a \frac{3}{2} \frac{\partial \delta_b}{\partial a} = -\frac{S_0}{a^3 H^2} + \frac{\nabla^2 \phi}{(aH)^2}, \quad (1.97)$$

Combining the above with the Poisson Eq. (1.94) yields,

$$a^2 \frac{\partial^2 \delta_b}{\partial a^2} + a \frac{3}{2} \frac{\partial \delta_b}{\partial a} - \frac{3}{2} \frac{\Omega_b}{\Omega_m} \delta_b = -\frac{S_0}{a^3 H^2} + \frac{3}{2} \frac{\Omega_{DM}}{\Omega_m} \delta_{DM}. \quad (1.98)$$

Above we have replaced metric potential in terms of density perturbations.

Now there are two sources that generate baryon density perturbations: the initial condition from inflationary curvature perturbations and the source term S_0 from PMFs. Since the evolution equation of δ_b is a linear ODE equation, we can simply write δ_b as a superposition of solution sourced by PMFs and curvature perturbations,

$$\delta_b = (\delta_b)_B + (\delta_b)_{\Lambda CDM}. \quad (1.99)$$

The solution of δ_b sourced just by initial conditions, $(\delta_b)_{\Lambda CDM}$, will be the same as in standard cosmology and hence we will not focus on those for now.

For the solution of $(\delta_b)_B$ the initial condition are taken to be $\delta_b = \delta_{DM} = 0$ so that their final values are those sourced just by S_0 . As initially, δ_b is very small, the RHS term dominates and causes an increase in δ_b . We can model this increase with

$$a^2 \frac{\partial^2 \delta_b}{\partial a^2} + a \frac{3}{2} \frac{\partial \delta_b}{\partial a} \approx -\frac{S_0}{a^3 H^2}. \quad (1.100)$$

The RHS is constant in matter domination. So δ_b must grow logarithmically as $\delta_b = 2 \frac{S_0}{a^3 H^2} \log(a/a_{rec})$. Now this logarithmic growth continues until gravity becomes more important. This roughly occurs when

$$\frac{3}{2} \frac{\Omega_b}{\Omega_m} \delta_b \sim \frac{S_0}{a^3 H^2} \quad (1.101)$$

$$\ln(a/a_{rec}) \sim \frac{\Omega_m}{3\Omega_b}. \quad (1.102)$$

So we can see that shortly after recombination, the value of δ_b becomes large enough for gravity to take over. After this point δ_b simply grows as $\propto a^m$, with m is less than 1 because we have ignored the growth of dark matter perturbations. The baryon perturbations sourced by S_0 would also source DM perturbations that fall into the gravitational potential of baryons. Eventually, when DM catches up with baryons we obtain $\delta_b = \delta_{DM} \propto a$.

Qualitatively, therefore, one can see that for a matter-dominated universe, where $a^3 H^2$ is constant, the magnetic fields can enhance baryon density perturbations if $\vec{B}$ remains constant. Let us dive into a quantitative analysis.

Note that the evolution of dark matter and baryon perturbations need to be solved together as they gravitationally influence each other. The evolution equation of δ_{DM} is

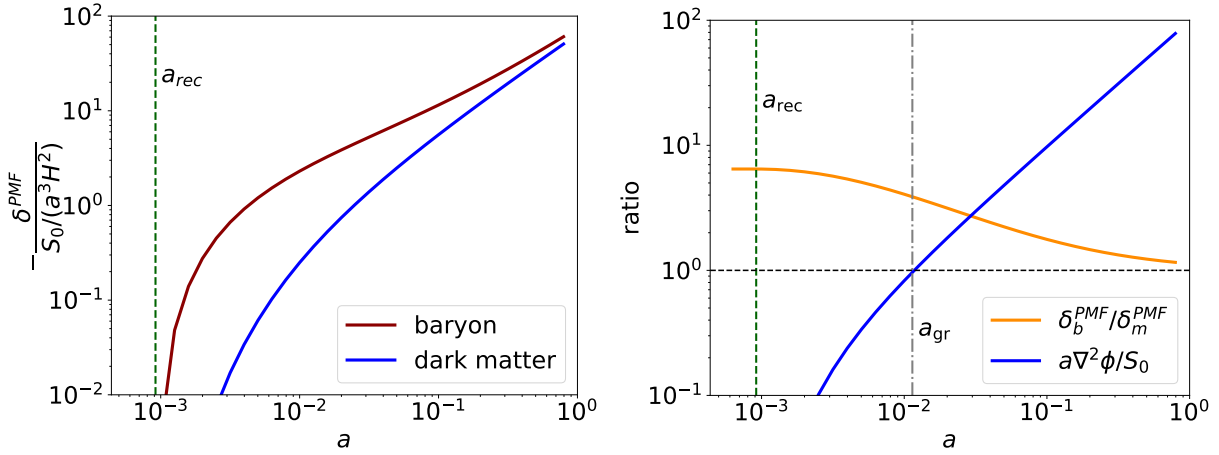


Figure 1.14: **Left:** Evolution of baryon (dark red) and dark matter (blue) density perturbations normalised with $S_0/[a^3 H^2]$, which parameterises the Lorentz force. **Right:** Blue line shows the ratio of gravitational force with the Lorentz force acting on baryons. Orange line represents the ratio of baryon fraction in an overdensity compared to the cosmic average Ω_b/Ω_m . Both the left and right panels are scale-independent. This is because all the scale dependence is sourced by the S_0 term and since we are plotting ratios the scale dependence is removed.

the same as Eq. (1.97) except the S_0 term is absent. Furthermore, by replacing ϕ using Poisson's equation (Eq. (1.94)) and neglecting dark energy density in the Hubble rate, we obtain

$$a^2 \frac{\partial^2 \delta_b}{\partial a^2} + a \frac{3}{2} \frac{\partial \delta_b}{\partial a} - \frac{3}{2} \frac{\Omega_b}{\Omega_m(1 + a_{\text{eq}}/a)} \delta_b = -\frac{S_0}{a^3 H^2} + \frac{3}{2} \frac{\Omega_{\text{DM}}}{\Omega_m(1 + a_{\text{eq}}/a)} \delta_{\text{DM}} \quad (1.103)$$

$$a^2 \frac{\partial^2 \delta_{\text{DM}}}{\partial a^2} + a \frac{3}{2} \frac{\partial \delta_{\text{DM}}}{\partial a} - \frac{3}{2} \frac{\Omega_{\text{DM}}}{\Omega_m(1 + a_{\text{eq}}/a)} \delta_{\text{DM}} = \frac{3}{2} \frac{\Omega_b}{\Omega_m(1 + a_{\text{eq}}/a)} \delta_b. \quad (1.104)$$

Here $\Omega_b = 0.048$ is the baryon energy density fraction today, $\Omega_{\text{DM}} = 0.261$ is the dark matter fraction, $\Omega_m = \Omega_b + \Omega_{\text{DM}}$, and $a_{\text{eq}} = 2.94 \times 10^{-4}$ is the matter-radiation equality [79].

The above system of equations can be solved once the initial conditions and the value of S_0 is specified. As the above equations are ordinary differential equations, their solution can simply be written as a linear combination of the homogeneous solution provided by the initial condition and the inhomogeneous solution sourced by S_0 , $\delta = \delta^{\text{CDM}} + \delta^{\text{PMF}}$. The solution of δ sourced just by initial conditions, δ^{CDM} , will be the same as in standard cosmology, and hence we will not focus on those solutions in this section.

The value of δ^{PMF} prior to recombination is suppressed due to the large photon drag experienced by baryons. Thus, we take the initial condition at $a = a_{\text{rec}}$ as $\delta_b^{\text{PMF}} = \delta_{\text{DM}}^{\text{PMF}} = 0$ and $v_b^{\text{PMF}} = v_{\text{DM}}^{\text{PMF}} = 0$.

After recombination, the evolution of δ^{PMF} is determined by the S_0 term. Specifically, we can write the post-recombination evolution of density perturbations as

$$\delta_b^{\text{PMF}} = -\xi_b(a) \frac{S_0}{a^3 H^2} \quad \delta_{\text{DM}}^{\text{PMF}} = -\xi_{\text{DM}}(a) \frac{S_0}{a^3 H^2}. \quad (1.105)$$

Here ξ_b and ξ_{DM} are dimensionless factors that contain the time-evolution of density perturbations and are independent of the strength of PMFs as well as the scales under consideration, as long as the scale is larger than λ_D .

In the left panel of Fig. 1.14 we show the evolution of ξ (see also [80]). One can see that S_0 causes growth in δ_b , which in turn gravitationally sources growth in δ_{DM} . For the most part of the evolution, δ_{DM} is lagging behind δ_b .

If these matter perturbations collapse into a halo, then they will naturally have a larger baryon fraction. One can estimate the baryon fraction in the halos at any redshift simply as,

$$f_b = \frac{\delta \rho_b}{\delta \rho_{\text{DM}} + \delta \rho_b} = \bar{f}_b \frac{\delta_b}{\bar{f}_{\text{DM}} \delta_{\text{DM}} + \bar{f}_b \delta_b} \equiv \bar{f}_b \frac{\delta_b}{\delta_m}, \quad (1.106)$$

where $\bar{f}_b = \Omega_b/\Omega_m \approx 0.154$ and $\bar{f}_{\text{DM}} = \Omega_{\text{DM}}/\Omega_m \approx 0.845$ are the cosmic averaged values of baryon and DM fraction [79], and δ_m is the matter density perturbation. In the right panel of Fig. 1.14, we show the evolution of $f_b/\bar{f}_b$ with scale factor. One can see that for halos collapsing at $z \sim 10$, one can expect a baryon fraction in halos about two times larger than the cosmic average. Note that this enhancement is independent of the halo mass. Later when we consider the influence of PMFs along with inflationary initial conditions, we see mass dependence in the enhancement of f_b .

There are two forces inducing growth in δ_b : gravity and the Lorentz force. In the left panel of Fig. 1.14, we show the evolution of the gravitational force $\nabla^2 \phi$ compared to the source term S_0 from PMFs. One can see that gravity quickly dominates over the Lorentz force by $10a_{\text{rec}}$.

To understand how δ_b evolves only under Lorentz force, let us focus on the initial evolution when δ_b and δ_{DM} are much smaller than $S_0/[a^3 H^2]$. In this limit, the terms with Ω in Eq. (1.103) can be ignored and one obtains a logarithmic growth for δ_b ,

$$\delta_b \approx 2 \frac{S_0}{a^3 H^2} \log(a/a_{\text{rec}}). \quad (1.107)$$

This logarithmic growth continues until self-gravity from baryons becomes important, i.e. when $\frac{3}{2} \frac{\Omega_b}{\Omega_m} \delta_b \sim \frac{S_0}{a^3 H^2}$. Substituting the above logarithmic growth of δ_b , the scale factor

when gravity takes over, a_{gr} , is then simply given by

$$a_{\text{gr}} \sim a_{\text{rec}} \exp\left(\frac{\Omega_{\text{m}}}{3\Omega_{\text{b}}}\right) \approx 10a_{\text{rec}}. \quad (1.108)$$

Thus, by redshift $z = 100$, the value of δ_{b} becomes large enough for gravity to take over. It is important to note that the time when gravity overcomes the Lorentz force is the same for all length scales (larger than λ_{D}) and is independent of the strength of PMFs.

1.2.4.2 The damping scale

The cutoff, or equivalently the magnetic damping scale, evolves as

$$a \frac{\partial}{\partial a} \left(\frac{1}{k_D^2} \right) = \frac{4V_A^2/3}{a^2 H(\alpha + H)}, \quad (1.109)$$

where $V_A^2 = \langle B^2 \rangle / (4\pi a^4 \rho_{\text{b}})$ is the Alfvén speed.

After recombination, $\alpha \ll H$ and $H \propto 1/a^3$. Moreover, $V_A^2 \propto 1/a$ for a constant $\langle B^2 \rangle$. Thus, k_D^{-1} roughly grows logarithmically post recombination. More accurately we have

$$k_D^{-1}(a) = k_D^{-1}(a_i) \left[1 + \frac{2(n_B + 5)}{3} \left(\frac{k_D^2 V_A^2}{a^3 H^2} \right)_{a=a_i} \ln(a/a_i) \right]^{1/(n_B+5)}, \quad (1.110)$$

where a_i is any $a > a_{\text{rec}}$. While this logarithmic growth of k_D is seen in simulations, in the literature that focus on the impact of magnetic fields on structure formation, this logarithmic growth of k_D is largely neglected. In such cases, $\langle B^2 \rangle$ remains constant after recombination and the damping scale is roughly calculated as V_A/aH , shown as gray dashed line.

Regardless of whether $\langle B^2 \rangle$ is constant or not, we can see that the damping scale of magnetic fields today is inversely related to the value of $\sqrt{\langle B^2 \rangle}$ today, B_0 ,

$$k_D = \kappa_{n_B} \text{Mpc}^{-1} (B_0/\text{nG})^{-1}. \quad (1.111)$$

1.2.4.3 PMFs and the matter power spectrum

To obtain the power spectrum of baryons and dark matter sourced by PMFs, one would first need to specify the spectrum for PMFs. In this study, we focus on non-helical PMFs

for simplicity, where

$$\langle B_i(k) B_j^*(k') \rangle = (2\pi)^3 \delta^3(k - k') \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \frac{P_B(k)}{2}. \quad (1.112)$$

Above $B_i(k)$ is the Fourier transform of $B_i(x)$, where we use the following convention for Fourier transforms: $A(k) = \int d^3x A(x) e^{ikx}$.

Furthermore, we shall consider P_B to be of the form

$$P_B(k) = A k^{n_B} e^{-k^2 \lambda_D^2}. \quad (1.113)$$

Above, the exponential term is added to model the damping from the back-reaction of baryons below the scale λ_D .² In this study, we focus on scenarios where the evolution of large-scale modes is insensitive to the behaviour on scales smaller than λ_D and hence the difference in modeling of damping is inconsequential. The amplitude A is determined by specifying the strength of PMFs averaged over $\lambda_{\text{Mpc}} = 1 \text{ Mpc}$,

$$B_{1\text{Mpc}}^2 \equiv \int \frac{d^3k}{(2\pi)^3} P_B(k) e^{-k^2 \lambda_{\text{Mpc}}^2} = \frac{A \lambda_{\text{Mpc}}^{-(3+n_B)}}{4\pi^2} \Gamma([n_B + 3]/2), \quad (1.114)$$

where to obtain the second relation we assumed $\lambda_{\text{Mpc}} \gg \lambda_D$ and $n_B > -3$ so that the integral is not sensitive to large and small scale cutoffs. Here Γ is the Gamma function. One can also calculate the total strength of PMFs, B^2 , by removing $e^{-k^2 \lambda_{\text{Mpc}}^2}$ from the above integral. The total PMF strength can then be related to $B_{1\text{Mpc}}$ using

$$B^2 = B_{1\text{Mpc}}^2 \left(\frac{\lambda_{\text{Mpc}}}{\lambda_D} \right)^{n_B+3}. \quad (1.115)$$

Having parameterised the spectrum of PMFs, we now come back to the evaluation of the baryon power spectrum induced by PMFs, P_b^{PMF} . From Eq. (1.105), one can see that P_b^{PMF} can be directly determined from the power spectrum of S_0 . Taking the Fourier transform of S_0 given in Eq. (1.90) and neglecting non-gaussianities in PMFs while taking the ensemble average $\langle \delta_b(k) \delta_b^*(k') \rangle$, we obtain (see also [55, 56])

$$P_b^{\text{PMF}}(k) = \xi_b^2(a) \frac{k^4}{8(4\pi a^3 \rho_b [a^3 H^2])^2} \int \frac{d^3q}{(2\pi)^3} \frac{P_B(q) P_B(k-q)}{(k-q)^2} \left[k^2 + 2q^2 + 4 \frac{(q \cdot k)^4}{k^4 q^2} - 4 \frac{(q \cdot k)^2}{k^2} - 4 \frac{(q \cdot k)^3}{k^2 q^2} + \frac{(q \cdot k)^2}{q^2} \right]. \quad (1.116)$$

²In reality, MHD turbulence would cause a power-law damping of PMFs below the scale λ_D [61], Trivedi:2018ejz

For $P_B(q) \propto q^{n_B}$ with $n_B > -3/2$, the above integral diverges at large q values and hence is sensitive to the small-scale cutoff, λ_D . Specifically, $P_b^{\text{PMF}} \propto k^4$ with the proportionality constant being sensitive to both n_B and λ_D . As the exact spectrum of PMFs near $k \sim \lambda_D^{-1}$ has large theoretical uncertainties, we do not focus on scenarios with $n_B > -3/2$ in this study.

For $-3 < n_B < -1.5$, the integral in Eq. (1.116) is largely sensitive to $q \sim k$ and hence is decoupled from both the small and large scale cut-offs. Considering $P_B = Ak^{n_B}$ in Eq. (1.116), replacing A using Eq. (1.114), integrating over q , and using present-day values of $\rho_{m0} \approx 1.15 \times 10^{-47} \text{GeV}^4$ in the Hubble rate and $\rho_{b0} \approx 1.64 \times 10^{-48} \text{GeV}^4$ we obtain

$$\Delta_b^{\text{PMF}}(k) \equiv \frac{k^3 P_b^{\text{PMF}}(k)}{2\pi^2} = 10^{-4} \xi_b^2(a) \left(\frac{k}{\text{Mpc}^{-1}} \right)^{2n_B+10} \left(\frac{B_{1\text{Mpc}}}{\text{nG}} \right)^4 G_{n_B} e^{-2k^2 \lambda_D^2}, \quad (1.117)$$

where G_{n_B} is a dimensionless number determined by

$$G_{n_B} = \int_0^\infty dx \int_{-1}^1 \frac{dy}{2} x^{n_B+2} (1+x^2-2xy)^{n_B/2-1} \frac{[1+2x^2+4y^4x^2-4y^2x^2-4y^3x+y^2]}{\Gamma^2([n_B+3]/2)}. \quad (1.118)$$

In Eq. (1.117) we added the exponential factor of $e^{-2k^2 \lambda_D^2}$ by hand to model the damping of baryon density perturbations near λ_D . The power spectrum of dark matter is of the same form with ξ_b replaced by ξ_{DM} .

The total baryon (or dark matter) power spectrum can be found by using the fact that the PMF-induced perturbations can be super-imposed on the Λ CDM perturbations (see also [81]),

$$\delta = \delta^{\text{PMF}} + \delta^{\text{CDM}}. \quad (1.119)$$

The above is valid as long as perturbations are small and in the linear limit.

Furthermore, one naturally expects δ^{PMF} to be uncorrelated with δ^{CDM} . This is because in typical inflationary magnetogenesis mechanisms, the PMFs are sourced by the vacuum fluctuations of the gauge field itself and not the inflaton [57, 58]. Consequently, PMFs are uncorrelated with inflationary curvature perturbations. The PMFs generated from first order phase transitions are typically generated from turbulent processes and are also expected to be uncorrelated with inflationary curvature perturbations. Thus, the total power spectrum can simply be written as,

$$P(k) = P^{\text{PMF}}(k) + P^{\text{CDM}}(k). \quad (1.120)$$

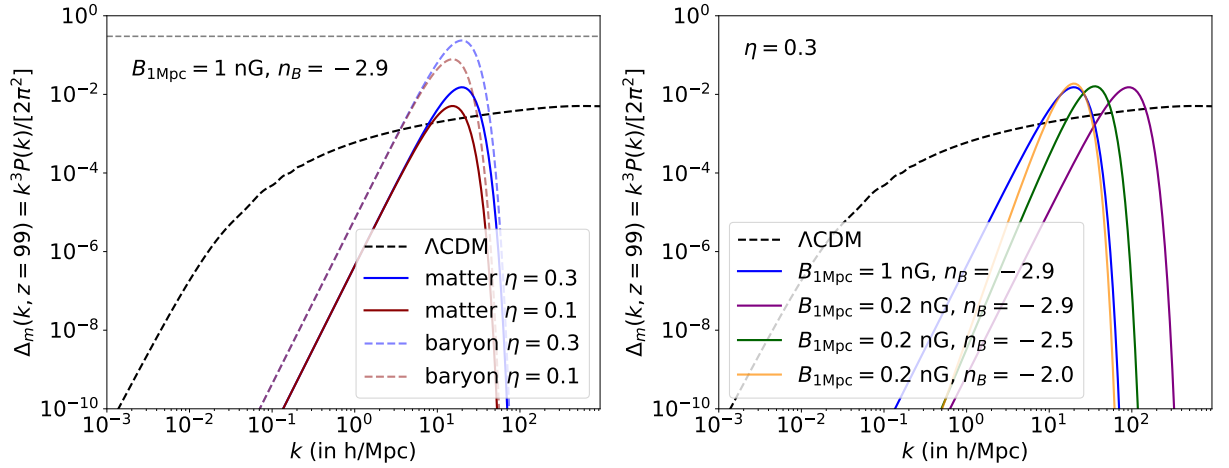


Figure 1.15: Comparison of dimensionless matter power spectra from PMFs (coloured solid lines) with that from Λ CDM cosmology (black dashed line). The coloured dashed lines in the left panel show baryon power spectra. The cutoff scale is determined by when the baryon power spectrum becomes equal to η . Increasing $B_{1\text{Mpc}}$ or n_B simply shifts the cutoff to a larger scale but keeps the maximum value of the power spectrum fixed.

The above equation is always true for dark matter but only for $k < \lambda_D^{-1}$ for baryons. For $k > \lambda_D^{-1}$, the baryon perturbations enter the non-linear regime and the total baryon power spectrum is expected to be suppressed due to turbulence.

For $n_B > -3$, which is required to keep PMFs insensitive to large-scale cutoff, we find the dimensionless baryon power spectrum sourced by PMFs is blue-tilted. In contrast, the dimensionless matter-power spectrum, Δ_m , in Λ CDM cosmology only grows logarithmically on small scales. Thus, PMFs can naturally enhance the small-scale matter power spectrum while keeping the large scales unchanged. In Fig. 1.15, we compare Δ_m obtained from PMFs with Δ_m from Λ CDM cosmology.

1.2.4.4 Details on the damping scale

Note that while the power-law amplitude of Δ_b in Eq. (1.117) is sensitive to the strength of PMFs, its peak value is insensitive to B . One can see this algebraically by setting $k = \lambda_D^{-1}$ in Eq. (1.117) and replacing $B_{1\text{Mpc}}$ in terms of B using Eq. (1.115). The resulting equation contains a factor of $(B/\lambda_D)^4$, which is a constant as λ_D is proportional to B , see Eq. (1.111). Parameterizing the proportionality constant with κ_{n_B} , such that

$$\lambda_D = 0.1 \kappa_{n_B} \text{Mpc} \left(\frac{B}{\text{nG}} \right), \quad (1.121)$$

we find

$$\Delta_b^{\text{PMF}}(k = \lambda_D^{-1}) = \xi_b^2(a) \kappa_{\text{nB}}^{-4} G_{\text{nB}} e^{-2}. \quad (1.122)$$

For $\kappa_{\text{nB}} \sim \mathcal{O}(1)$ and at $a = 0.01$ where $\xi_b \approx 2.5$, one finds that $\Delta_b(\lambda_D) \sim \mathcal{O}(1)$. The order one value of Δ_b is not a coincidence and is simply a result of the fact that λ_D is determined by the scales where PMFs induce non-linear fluctuations in baryon perturbations.

It is important to note that the maximum value of Δ_b is sensitive to κ_{nB}^{-4} . Thus, even a factor 3 change in κ_{nB} can introduce 2 orders of magnitude change in the maximum value of Δ_b . This large theoretical uncertainty in the maximum value of Δ_b has not been taken into account in previous literature. Hence, the earlier constraints on PMFs derived from PMFs' impact on the matter power spectrum could change substantially once the correct value of κ_{nB} is determined.

Determining the exact value of κ_{nB} is beyond the scope of this work as it requires detailed MHD simulations that include gravity. In this study, we set the value of κ_{nB} by requiring that the baryon power at the cut-off scale is equal to some order one number parameterized by η when gravity overcomes the Lorentz force,

$$\Delta_b^{\text{PMF}}(k = \lambda_D^{-1}, a = 0.01) = \eta. \quad (1.123)$$

Using $\xi_b(a = 0.01) \approx 2.5$, we obtain

$$\kappa_{\text{nB}} = \left(\frac{G_{\text{nB}}}{1.14\eta} \right)^{1/4}. \quad (1.124)$$

In the left panel of Fig. 1.15, we show both the baryon and matter power spectrum for two different values of η . The matter power spectrum is smaller than the baryon power spectrum because $\delta_{\text{DM}} \ll \delta_b$ when perturbations are sourced by PMFs.

In the right panel, we show the matter power spectrum for different values of B_{1Mpc} and n_B but keep η fixed to 0.3. One can see that the peak value of the matter power spectrum remains unchanged for different PMFs parameters but the scale at which PMFs enhance the power spectrum changes. In contrast, previous literature [56, 82–84] found the peak of the power spectrum to change with n_B because they neglected the n_B dependence in λ_D . For instance, when $\eta = 0.3$ in Eq. (1.124), we obtain: $\kappa_{-2.9} \approx 0.67$, $\kappa_{-2.5} \approx 1.07$, $\kappa_{-2} \approx 1.36$, and $\kappa_{-1.6} \approx 2.03$. In contrast, the naive λ_D employed by earlier studies assumed n_B independent cutoff scale with $\kappa = 0.7$ [82] or $\kappa = 0.35$ [84].

To take into account the uncertainty in the exact estimation of λ_D , in this study, we show the results for two values of η : $\eta = 0.3$ and $\eta = 0.1$. For a more detailed discussion

on choices of η and performed tests in the initial conditions, we recommend the appendix of [1] and a more recent study [85].

Relationship with the magnetic Jeans scale

The above description of gravity overcoming magnetic fields after $a > 10a_{\text{rec}}$ naively seems to be at odds with the main narrative in the literature where the dominance of gravity is expressed in terms of the magnetic Jeans scale. Specifically, the magnetic Jeans scale, λ_J , is evaluated by finding the scale where the magnetic pressure balances the gravitational collapse. Taking analogy with the thermal Jeans scale and replacing the sound speed with Alfvén speed, Ref. [55] finds the comoving magnetic Jeans scale to be

$$\lambda_J = \frac{v_A}{a} \sqrt{\frac{\pi}{G\rho_m}} = 2\pi \sqrt{\frac{2}{3}} \frac{v_A}{aH}. \quad (1.125)$$

In the last line, we used the fact that in a matter-dominated universe, $H^2 = 8\pi G\rho_m/3$.

The reasoning behind the magnetic Jeans scale is somewhat incomplete because it suggests that magnetic fields only act to oppose the growth of density perturbations. However, as we saw in the previous section, PMFs for the large part support the growth of density perturbations via the S_0 term. It is only when the baryon perturbations become large enough to back-react onto the magnetic fields, that magnetic fields suppress density perturbations by inducing turbulence.

Interestingly, the damping scale λ_D (Eq. (1.121), recall also Eq. (1.96)), which marks the scale where baryons back-react onto PMFs, is of the same form as λ_J above, if we use Alfvén speed $v_A \propto \sqrt{B^2}$. This similarity is surprising because gravity had no role in the derivation of λ_D . However, gravity does play an indirect role in the overall narrative behind λ_D once we consider the evolution of baryon perturbations on scales larger than λ_D . On these scales, the baryon perturbations sourced by S_0 are small and only grow logarithmically initially. It is gravity that ultimately escorts the perturbations to non-linear values. As the strength of gravity is much larger than the Lorentz force from PMFs by the time the perturbations become non-linear, the back-reaction from magnetic fields cannot inhibit the collapse of structures. Whereas perturbations on scales smaller than λ_D are suppressed by MHD turbulence before gravity overcomes the Lorentz force. Thus, λ_D effectively plays the same role as the magnetic Jeans scale.

1.3 Signatures of primordial magnetism on cosmological observables

The discussion of the previous sections shows that primordial magnetic fields can influence cosmological evolution through several conceptually different mechanisms. Before recombination, the magnetic energy density and anisotropic stress source scalar, vector and tensor perturbations and therefore modify the temperature and polarization anisotropies of the CMB. At the same time, the Lorentz force acts directly on the baryon–photon plasma and can generate small-scale baryonic inhomogeneities. After recombination, the same force sources additional matter perturbations, while dissipation of magnetic energy through ambipolar diffusion and decaying MHD turbulence alters the thermal and ionization history of the intergalactic medium. Finally, magnetic fields directly influence the propagation of charged particles and polarized electromagnetic radiation. The resulting observational signatures therefore span a remarkable range of epochs, from recombination to the formation of the first galaxies and the present-day cosmic web [51–53].

These different effects provide a correspondingly broad set of constraints on PMFs. An important point, however, is that the limits quoted in the literature are not always directly comparable. CMB and large-scale-structure studies commonly characterize a stochastic PMF by the smoothed amplitude $B_{1\text{Mpc}}$ and spectral index n_B , while studies of magnetic dissipation sometimes quote the total rms field $\sqrt{\langle B^2 \rangle}$ integrated up to the damping scale. Observations of extragalactic magnetic fields through Faraday rotation, γ rays or cosmic rays are instead often presented in the plane spanned by a field strength B and a coherence length λ_B . The conversion between these quantities depends on the assumed magnetic power spectrum and damping scale. Consequently, numerical values of limits should always be interpreted together with the PMF parametrization used to derive them.

1.3.1 Summary of constraints on PMFs

Cosmic Microwave Background. The CMB provides one of the most direct probes of magnetic fields present before recombination. The stress-energy tensor of a stochastic PMF sources additional scalar, vector and tensor perturbations, which contribute to both temperature and polarization anisotropies. Since the magnetic stress-energy tensor is quadratic in $\mathbf{B}$, PMFs also produce characteristic non-Gaussian signatures. In addition, the rotation of the polarization plane by Faraday rotation provides a frequency-dependent probe of the magnetic field along the line of sight.

A comprehensive analysis was performed by the Planck Collaboration [86]. For a non-

helical stochastic field, the CMB temperature and polarization power spectra constrain the smoothed field amplitude to

$$B_{1\text{ Mpc}} < 4.4\text{ nG} \quad (95\% \text{ C.L.}), \quad (1.126)$$

while for a nearly scale-invariant spectrum the limit becomes approximately

$$B_{1\text{ Mpc}} < 2.0\text{ nG}. \quad (1.127)$$

When the effect of magnetic energy dissipation on the ionization history is included, Planck obtained a stronger limit, $B_{1\text{ Mpc}} \lesssim 0.9\text{ nG}$ for a nearly scale-invariant spectrum. Constraints derived from magnetic non-Gaussianities are also at the few-nG level. By contrast, Faraday rotation of the CMB polarization was considerably less constraining with Planck, although future polarization experiments can substantially improve this sensitivity.

An additional and potentially much more sensitive CMB effect follows from the small-scale baryonic inhomogeneities induced by the Lorentz force. Since the local hydrogen recombination rate depends non-linearly on the baryon density, schematically

$$\Gamma_{\text{rec}} \propto n_e n_p, \quad (1.128)$$

a clumpy baryon distribution recombines differently from a homogeneous one with the same mean density. Jedamzik & Saveliev [87] investigated this process with compressible MHD simulations and found very strong limits on the *total* surviving magnetic field, obtaining approximately 47 pG for a scale-invariant spectrum and 8.9 pG for a Batchelor spectrum. These values should not be directly identified with $B_{1\text{ Mpc}}$, since the former correspond to the total magnetic energy integrated over the spectrum.

Related effects arise from the dissipation of PMF energy after recombination. Ambipolar diffusion and decaying MHD turbulence heat the gas, thereby changing the residual ionization fraction and subsequently the CMB anisotropies. Using Planck 2018 data, Paoletti et al. [88] obtained the marginalized constraint

$$\sqrt{\langle B^2 \rangle} < 0.69\text{ nG} \quad (95\% \text{ C.L.}), \quad (1.129)$$

when both dissipative effects are included.

An interesting extension of the recombination argument was proposed by Jedamzik & Pogosian [71]. PMF-induced baryonic clumping can accelerate recombination, shifting the visibility function towards higher redshift and decreasing the sound horizon at last

scattering (recall Eq. (1.6)). Since the angular acoustic scale measured by the CMB is extremely well constrained, a smaller sound horizon allows a larger CMB-inferred value of H_0 , potentially reducing the Hubble tension. More recent calculations using improved treatments of recombination and MHD evolution continue to find a moderate preference for a non-zero magnetic contribution for some combinations of cosmological data [72]. This should presently be regarded as an interesting possible indication rather than evidence for a cosmological magnetic field.

Future CMB polarization measurements will test these scenarios further. The Lite-BIRD forecasts of Paoletti et al. [73] predict, for example, a sensitivity

$$B_{1\text{Mpc}} < 0.8\text{ nG} \quad (n_B = -2.9, 95\% \text{ C.L.}) \quad (1.130)$$

from the gravitational effects of magnetically-induced perturbations, together with complementary information from the thermal history, Faraday rotation and magnetic non-Gaussianities.

Faraday rotation and the magnetization of the cosmic web. A conceptually different way of probing intergalactic magnetism is through the Faraday rotation of linearly polarized radio emission. For radiation propagating through an ionized magnetized plasma, the polarization angle changes according to

$$\Delta\chi = \text{RM} \lambda_{\text{obs}}^2, \quad (1.131)$$

where the rotation measure is proportional to

$$\text{RM} \propto \int \frac{n_e(z) B_{\parallel}(z)}{(1+z)^2} dl. \quad (1.132)$$

Here $B_{\parallel}$ denotes the magnetic-field component along the line of sight. Unlike the indirect effect of PMFs on the matter distribution, Faraday rotation therefore probes the magnetic field itself.

Rotation-measure catalogues of extragalactic radio sources have long provided upper limits on a volume-filling intergalactic magnetic field. Recent low-frequency observations with LOFAR have considerably increased their constraining power. Neronov, Vazza, Mtchedlidze & Carretti [89] combined the redshift dependence of residual rotation measures with cosmological MHD simulations and obtained a particularly strong limit for an inflationary, nearly scale-invariant field,

$$B_{1\text{Mpc}} \lesssim 70\text{ pG}, \quad (1.133)$$

at 90% confidence. The constraint becomes weaker and strongly correlation-scale dependent for bluer spectra. This result illustrates why the low-density cosmic web is particularly valuable: unlike fields inside galaxies and clusters, magnetic fields in filaments and voids have experienced less dynamo amplification and can preserve more direct information on their initial conditions.

Blazars and γ -ray cascades. Whereas the CMB and Faraday rotation primarily provide upper limits, very-high-energy γ -ray observations have motivated possible *lower* bounds on magnetic fields in cosmic voids. TeV photons from a distant blazar interact with the extragalactic background light, producing electron–positron pairs,

$$\gamma_{\text{TeV}} + \gamma_{\text{EBL}} \rightarrow e^+ + e^-. \quad (1.134)$$

The pairs subsequently inverse-Compton scatter background photons and are expected to generate a secondary GeV cascade. A sufficiently strong intergalactic magnetic field deflects the charged pairs, broadening and delaying the secondary emission and reducing the flux arriving along the original line of sight.

The classic analysis of Neronov & Vovk [62] interpreted the absence of the expected cascade in Fermi data as evidence for an intergalactic field and obtained, for sufficiently large coherence lengths, the approximate lower limit

$$B_{\text{IGM}} \gtrsim 3 \times 10^{-16} \text{ G}. \quad (1.135)$$

More recent analyses continue to use γ -ray cascades to probe fields in the approximate range 10^{-17} – 10^{-14} G, depending on the assumed coherence length, source lifetime and cascade model. These bounds do not by themselves prove a primordial origin: they constrain the magnetic field presently occupying low-density regions. Nevertheless, a truly volume-filling field in cosmic voids is particularly interesting for primordial magnetogenesis because it is more difficult to explain through magnetized outflows from galaxies and AGN. The interpretation also remains sensitive to source histories and to possible plasma processes affecting the pair beam, so the resulting lower limits should be regarded as model dependent.

Ultra-high-energy cosmic rays. Charged cosmic rays provide another direct probe of intergalactic magnetic fields. A cosmic ray of charge Ze and energy E is deflected by an angle which, for a turbulent field with coherence length λ_B and propagation distance

D , scales approximately as

$$\theta_{\text{CR}} \propto \frac{ZeB}{E} \sqrt{D\lambda_B}. \quad (1.136)$$

Consequently, the existence of localized UHECR excesses places an upper limit on the amount of magnetic deflection accumulated during propagation. For example, the interpretation of the Telescope Array hotspot associated with the Perseus–Pisces region yields a bound of order $B \lesssim 10^{-10}$ G for a coherent void field on very large scales [90]. Cosmic-ray arrival directions may also contain information about the geometry and even helicity of intergalactic fields [64]. In practice, these constraints are complicated by the uncertain nuclear composition of UHECRs, their unknown source distribution, and deflections within the Galactic magnetic field. They are therefore complementary to, rather than competitive with, the more direct cosmological limits.

Small-scale structure: dwarf galaxies and the Ly α forest. The enhancement of the small-scale matter power spectrum (derived in §1.2.4) provides another class of PMF constraints. The most sensitive objects are those whose characteristic mass or spatial scale lies close to the maximum of the magnetically-induced power spectrum. This makes low-mass dark matter haloes, dwarf galaxies, the Ly α forest and the first galaxies natural probes of PMFs.

Ultra-faint dwarf galaxies were used by Safarzadeh & Loeb [91] to constrain PMFs by requiring their host haloes to have formed with masses compatible with the observed systems. Depending on the assumed formation redshift, they obtained limits of approximately

$$B_0 \lesssim 0.3 - 0.5 \text{ nG}. \quad (1.137)$$

A more detailed numerical study was carried out by Sanati et al. [92], who performed cosmological dark-matter and chemo-hydrodynamical simulations with PMF-enhanced initial matter power spectra. Strong PMF models produced too many early low-mass structures, enhanced star formation and metallicity, overproduced luminous satellites and led to excessively early reionization. The analysis therefore excluded several combinations of $B_\lambda = 0.05\text{--}0.5$ nG and relatively blue magnetic spectra. More recent magneto-hydrodynamical zoom simulations confirm that dwarf galaxies remain highly sensitive to PMF-induced changes in the small-scale matter distribution [93].

A particularly direct probe of the small-scale matter distribution is provided by the Ly α forest, as we will see in Chapter 3. Since the one-dimensional flux power spectrum measures fluctuations in the neutral hydrogen distribution on comoving sub-Mpc scales, additional matter power sourced by PMFs can produce a measurable enhancement. In our study [2], full cosmological hydrodynamical simulations were used for the first time to

constrain PMFs directly with the Ly α flux power spectrum. A complete analysis yielded 3σ C.L. limit $k_{\text{peak}} > 30 \text{ Mpc}^{-1}$, which means

$$B_{1\text{Mpc}} < 0.30 \text{ nG} \quad (3\sigma, n_B = -2.9). \quad (1.138)$$

This places the Ly- α forest among the most sensitive probes of nearly scale-invariant PMFs on the small scales relevant for structure formation. As we argue in §3.4, these constraints can be extended to other spectral indices n_B , and with future implementation of consistent initial conditions, as in [1], the constraints could be considerably stronger, after more simulations and in-depth analysis of flux power spectra are done (see §3.5).

High-redshift galaxies, JWST and reionization. The same enhancement of small-scale structure increases the abundance of early dark matter haloes and can therefore modify the population of high-redshift galaxies. Measurements of the stellar mass density and ultraviolet luminosity function with JWST have consequently opened a new route for constraining PMFs. Using high-redshift stellar-mass density measurements, Zhang et al. [75] obtained joint 95% limits

$$B_{1\text{Mpc}} < 4.44 \text{ nG}, \quad n_B < -2.24, \quad (1.139)$$

with stronger amplitude limits when the spectral index is fixed. For example, their bound for $n_B = -2.9$ is $B_{1\text{Mpc}} < 2.21 \text{ nG}$.

More recent analyses combine the JWST UV luminosity function with the reionization history. Fairbairn et al. [76] found that sufficiently strong PMFs produce an excessive abundance of early galaxies and can generate a characteristic very-early phase of reionization. Including the CMB optical-depth constraint, they obtained

$$\sqrt{\langle B^2 \rangle} < 0.27 \text{ nG} \quad (n_B = -2), \quad \sqrt{\langle B^2 \rangle} < 0.18 \text{ nG} \quad (n_B = 2), \quad (1.140)$$

at 95% confidence. Again, these values correspond to a total rms field and should not be directly compared numerically with $B_{1\text{Mpc}}$ without specifying the spectrum and damping scale. Recent work has also emphasized that magnetic-energy losses through ambipolar diffusion and decaying turbulence should be followed consistently when deriving such high-redshift constraints [94]. Future measurements of the redshifted 21 cm signal will provide an additional probe of the same effects.

The limits obtained from structure formation deserve one additional qualification. They depend on the mapping between the PMF parameters $(B_{1\text{Mpc}}, n_B)$ and the matter power spectrum close to and below the magnetic damping scale. Recent dedicated MHD

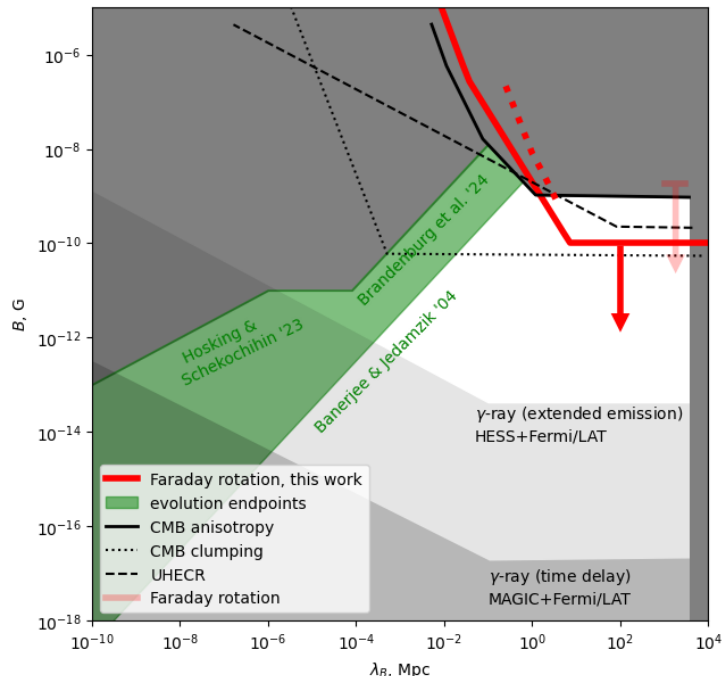


Figure 1.16: Summary of selected constraints in the $B - \lambda$ plane from [89].

simulations by Ralegankar, Garaldi & Viel [85] show that the non-linear PMF-induced matter spectrum can differ substantially from the sharp magnetic-Jeans cut-offs assumed in several earlier analytical studies. Some earlier limits from dwarf galaxies, reionization or reconstructed matter power spectra may consequently need to be reassessed with updated PMF transfer functions. This theoretical uncertainty is one reason why, in this thesis, particular attention is given to the characteristic peak scale of the magnetically-induced matter power spectrum rather than only to a nominal magnetic Jeans scale.

Taken together, present observations restrict nearly scale-invariant PMFs to amplitudes well below the few-nG level, while the precise constraint depends strongly on the spectral shape and on the physical quantity used to characterize the field. At the same time, γ -ray observations motivate lower limits on a possible volume-filling void field many orders of magnitude weaker than the cosmological upper limits. The remaining interval is therefore being probed from both directions. CMB polarization, Faraday rotation, $\text{Ly}\alpha$ forest measurements, high-redshift galaxies and future 21 cm observations provide particularly complementary approaches because they probe different epochs and different physical effects of primordial magnetism.

1.3.2 PMF imprints on the halo mass function

Sethi & Subramanian [83] provided for the first time a quantitative picture of the structure formation with PMFs included. They showed that, after recombination, the Lorentz force can generate additional baryonic perturbations and substantially increase the abundance

of early subgalactic objects. For nano-Gauss magnetic fields, structures could begin to collapse already at $z \gtrsim 15$, with the largest enhancement occurring around the characteristic scale set by the competition between magnetic forcing and magnetic pressure. Their analysis demonstrated that PMFs can therefore influence reionization not only through heating of the IGM but also indirectly, by changing the number of early collapsed objects.

The connection between the PMF-induced matter power spectrum and halo abundance can be understood from the variance of the density field smoothed on a mass scale M given by Eq. (1.31). The total linear matter power spectrum may be written as

$$P_m(k, z) = P_m^{\Lambda\text{CDM}}(k, z) + P_m^{\text{PMF}}(k, z), \quad (1.141)$$

provided the inflationary and magnetically-induced perturbations are uncorrelated, i.e. $\langle \delta^{\text{infl.}} \delta^{\text{PMF}} \rangle = 0$. Since P_m^{PMF} rises steeply towards small scales before reaching the damping/non-linear region, the value of $\sigma(M, z)$ can be greatly enhanced for low masses while remaining almost unchanged on cluster and larger scales.

From the perspective of halo abundance, where the HMF dependence on σ^2 shows (see Eqs. (1.30) & (1.33)) that even a modest change in $\sigma(M, z)$ can produce a large change in the abundance of rare objects. The effect is particularly strong at high redshift, when a low-mass halo that is common at the present epoch can still correspond to a relatively rare density peak.

This behaviour was studied in detail by Kahniashvili et al. [95]. They computed the PMF-induced matter power spectrum over a range of magnetic amplitudes and spectral indices and propagated it into the halo mass function. In the standard linear treatment, the magnetically-induced matter spectrum behaves approximately as

$$P_m^{\text{PMF}}(k) \propto \begin{cases} k^{2n_B+7}, & n_B < -3/2, \\ k^4, & n_B > -3/2, \end{cases} \quad (1.142)$$

before being suppressed close to the relevant damping scale. The additional small-scale power increases the abundance of low- and intermediate-mass haloes, while the high-mass end remains considerably less affected. Kahniashvili et al. considered the corresponding changes in cluster abundance and other large-scale-structure observables together with Ly α and CMB constraints.

The characteristic form of the modification is important. A PMF does not simply rescale the usual ΛCDM matter spectrum by a constant factor. Instead, it introduces a rapidly rising component at high k and a characteristic turnover associated with magnetic damping and non-linear backreaction. The halo abundance is consequently enhanced

most strongly around a corresponding range of masses. Both the position and amplitude of this enhancement depend on $B_{1\text{Mpc}}$, n_B and on the modeling of the damping scale. In particular, bluer magnetic spectra transfer progressively more magnetic power to small scales and can therefore alter substantially smaller haloes even when the field smoothed over 1 Mpc is extremely weak.

Early analytical calculations necessarily relied on simplifying assumptions concerning the magnetic Jeans or damping scale and on the use of Gaussian halo-mass-function prescriptions. This point is non-trivial because the Lorentz-force source is quadratic in the magnetic field, and hence PMF-induced density perturbations are not strictly Gaussian even if $\mathbf{B}$ itself is a Gaussian random field. Numerical simulations are therefore particularly useful for assessing whether the halo mass function can still be described by conventional fitting functions.

A first important numerical step in this direction was made by Sanati et al. [92], who performed both dark-matter-only and cosmological hydrodynamical simulations with magnetically-enhanced initial matter power spectra. Their simulations showed that PMF models can strongly increase the abundance of structures in the approximate mass range $10^7\text{--}10^9 M_\odot$, with corresponding consequences for the population of dwarf galaxies. These models demonstrated directly that an HMF enhancement induced at the level of the initial matter spectrum survives into the non-linear regime and can be translated into observable consequences for galaxy formation.

The non-linear evolution of the magnetically-induced matter spectrum was subsequently studied with dedicated gravity–MHD simulations by Ralegankar, Garaldi & Viel [85]. These simulations show that the behaviour close to the magnetic damping scale differs from the sharp suppression often assumed in earlier analytical models: the baryonic dimensionless power instead approaches a non-linear saturated regime. As a consequence, some previous prescriptions can overestimate the matter power close to the transition by orders of magnitude. Since the halo mass function is exponentially sensitive to $\sigma(M, z)$, this uncertainty propagates directly into predictions for low-mass halo abundances and into constraints based on dwarf galaxies or reionization.

Other cosmological MHD simulations have explored complementary aspects of the problem. For example, simulations of dwarf-galaxy formation [93] confirm that PMF-enhanced initial perturbations can accelerate halo collapse and star formation at $z \gtrsim 12$, while simulations of galaxy clusters by Mtchedlidze et al. [96] follow the subsequent amplification of inflationary and phase-transition magnetic seeds within collapsed objects. The latter are primarily studies of the *magnetization* of already forming clusters rather than calculations of a PMF-induced halo mass function, and should therefore be distinguished

from simulations in which the Lorentz force modifies the primordial density field itself.

The overall picture is therefore robust at a qualitative level: primordial magnetic fields source additional small-scale density fluctuations and can strongly increase the number of early low-mass haloes. Quantitatively, however, the predicted enhancement depends on the PMF spectral index, amplitude and damping prescription, and on the non-linear coupling between baryonic motions and the magnetic field. The halo mass function consequently provides both a useful summary of the effects of PMFs on structure formation and the natural bridge between the primordial power spectrum discussed above and the high-redshift galaxy observables investigated in the following chapters.

Chapter 2

Enhanced baryon fraction in low-mass, high-redshift halos

The chapter is based on our article [1]. We have moved the theoretical part to the previous sections §1.2.3 and §1.2.4 of this thesis. Similarly some motivations and references to other PMF studies are in sections above (§1.2.1 & §1.3). We believe this improves readability and emphasizes clearly our results. Comments and interpretations that do not appear in the original publication, but have emerged later, have been added to the footnotes.

2.1 Structure formation with PMFs in the initial conditions

In this study, we are primarily interested in obtaining the properties of dark matter halos at high redshifts ($z \sim 4 - 10$), while focusing on their masses, baryon fractions, and abundance of stars. To find these quantities and their relations, we resort to cosmological simulations. For this purpose, we use **P-Gadget-3**, an extended, non-public version of the publicly available **Gadget-2** code [29]. In the following, we will refer to it as ‘Gadget’.¹

Our implementation of the Gadget code carries out gravitational N-body collisionless dynamics for dark matter (DM) and smoothed particle hydrodynamics (SPH) for baryonic gas, including cooling of the gas and star formation. We emphasize that our simulations do not entail magneto-hydrodynamics (MHD) and, thus, cannot represent the realistic and complex nature of plasma physics. Some recent works used MHD simulations to study the evolution of PMFs through their impact on population of galaxies and galactic [97],

¹There is also a recent, updated version **Gadget-4**, that shares many features with our **P-Gadget-3** code. It is comprehensively presented in [31], whereby in Chapter 11 one can find a discussion on the development of the code and its public availability.

intergalactic [69] and extragalactic [98, 99] magnetic properties, also in comparison with astrophysical seeding mechanisms during galaxy formation [68]. However, as discussed in the previous section, on sufficiently large scales, the effects of the Lorentz force become sub-dominant to gravity after $a > 10 a_{\text{rec}}$, i.e., for redshift $z < 100$. Therefore, in the formation of structures on considered scales, we do not expect a significant difference in the evolution of density perturbations between our simulations and a full MHD picture.

The comoving box sizes (L_{box}) of the simulations are set such that the Nyquist frequency k_{Nyq} is about 1.5 times larger than the wavenumber corresponding to the damping scale, $k_{\text{D}} = 1/\lambda_{\text{D}}$. Each simulation box is filled with an equal number of gas and DM particles, $N = 512^3$ (with $N = N_{\text{DM}} = N_{\text{gas}}$).

The simulations are performed from initial conditions (ICs), provided at $z_{\text{in}} = 99$, and run down to $z_{\text{end}} = 0$ or 4 (see Table 2.1). For the Λ CDM cosmology, ICs were obtained by computing the Λ CDM power spectra of baryons and dark matter at z_{in} , using CLASS code [100], and inserting those in the N-GenIC software [41]. The ICs with PMFs (simulations A, D, E, and F) are evaluated by first generating magnetic fields on the lattice for a given PMF power spectrum. Then dark matter and baryon particles are displaced at each grid point according to the sum of Lorentz force, $L_{\text{B}} \propto (\vec{\nabla} \times \vec{B}) \times \vec{B}$, at that point and the displacement from original Λ CDM ICs. The resulting ICs are found to have a power spectrum smaller than the analytical estimate. The source of the mismatch is unclear and we leave its detailed investigation to future work. In this study, we amplify the displacement of particles along the Lorentz force direction to match with the analytical estimate. See Appendix A for a more detailed explanation of how the ICs are generated.

In Fig. 2.1 we show an example of a PMF configuration in the ICs of simulation A, and subsequent evolution of baryons and dark matter in the simulation, in comparison with the pure Λ CDM counterpart. The figure is a 2D projection of a thin slice (depth 0.2 Mpc/ h) of the whole simulation volume (with $L_{\text{box}} = 55$ Mpc/ h). In the top panel, the portrayed magnetic field vectors are in the middle of the slice and averaged over four mesh layers, while the magnitude of the local magnetic field is coloured in the background. The middle panel demonstrates the small-scale differences between the simulations when PMFs are present versus absent. In particular, one can observe more halos and stars in the PMF simulation at redshift 10. The small-scale differences are also prominent at redshift=0, where more bright spots are visible in the PMF case. In contrast, the large-scale structure is the same in both.

In Fig. 2.2, we show the matter power spectrum of the ICs generated by N-GENIC for different simulation parameter points described in Table 2.1. The numerical power spectrum for baryons starts overshooting the analytical estimate near the damping scale.

Table 2.1: PARAMETERS OF SIMULATIONS. From left to right, columns correspond to: the simulation name, PMF strength at 1 Mpc, spectral index of the PMF power spectrum, parameter setting the peak value of the baryon power spectrum, periodic box size, the final redshift of the simulation; the last column indicates whether the ICs are generated by the input of the enhanced power spectrum or by the displacement of particles with the Lorentz force. With and without isocurvature refers to whether the PMF-induced perturbations and the Λ CDM perturbations are uncorrelated or correlated, respectively. Below are listed values of some derived parameters: gravitational softening length, the Nyquist wavenumber, PMF damping wavenumber, and particle masses of DM and gas. At the bottom, we list the values of the total matter energy density, baryon energy density and the dimensionless Hubble parameter (with $H_0 = h \times 100 \frac{\text{km}}{\text{Mpc s}}$). All simulations start from redshift $z_{\text{in}} = 99$, contain $N = 512^3$ particles of both types, and have their Λ CDM counterpart.

Simulation	$B_{1\text{Mpc}}$ [nG]	n_B	η	L_{box} [Mpc/h]	z_{end}	particle displacement method with PMFs
A	1	-2.9	0.3	55	0	Lorentz force
B	1	-2.9	0.3	55	4	$P(k)$ without isocurvature
C	1	-2.9	0.3	55	4	$P(k)$ with isocurvature
D	1	-2.9	0.1	100	0	Lorentz force
E	0.2	-2.0	0.3	55	4	Lorentz force
F	0.5	-2.9	0.3	30	4	Lorentz force
Derived parameters						
Simulation	l_{soft} [kpc/h]	k_{Nyq} [h/Mpc]	$k_D = 1/\lambda_D$ [h/Mpc]	m_{DM} [$M_\odot/h$]	m_{gas} [$M_\odot/h$]	
{A,B,C}, E	4.30	29.25	{19.38}, 16.29	8.94×10^7	1.66×10^7	
D	7.81	16.08	14.93	5.37×10^8	9.97×10^7	
F	2.34	53.62	37.52	1.45×10^7	2.69×10^6	
Cosmological parameters (Planck 2015 data [101])						
$\Omega_m = 1 - \Omega_\Lambda$		Ω_b		h		
0.308		4.82×10^{-2}		0.678		

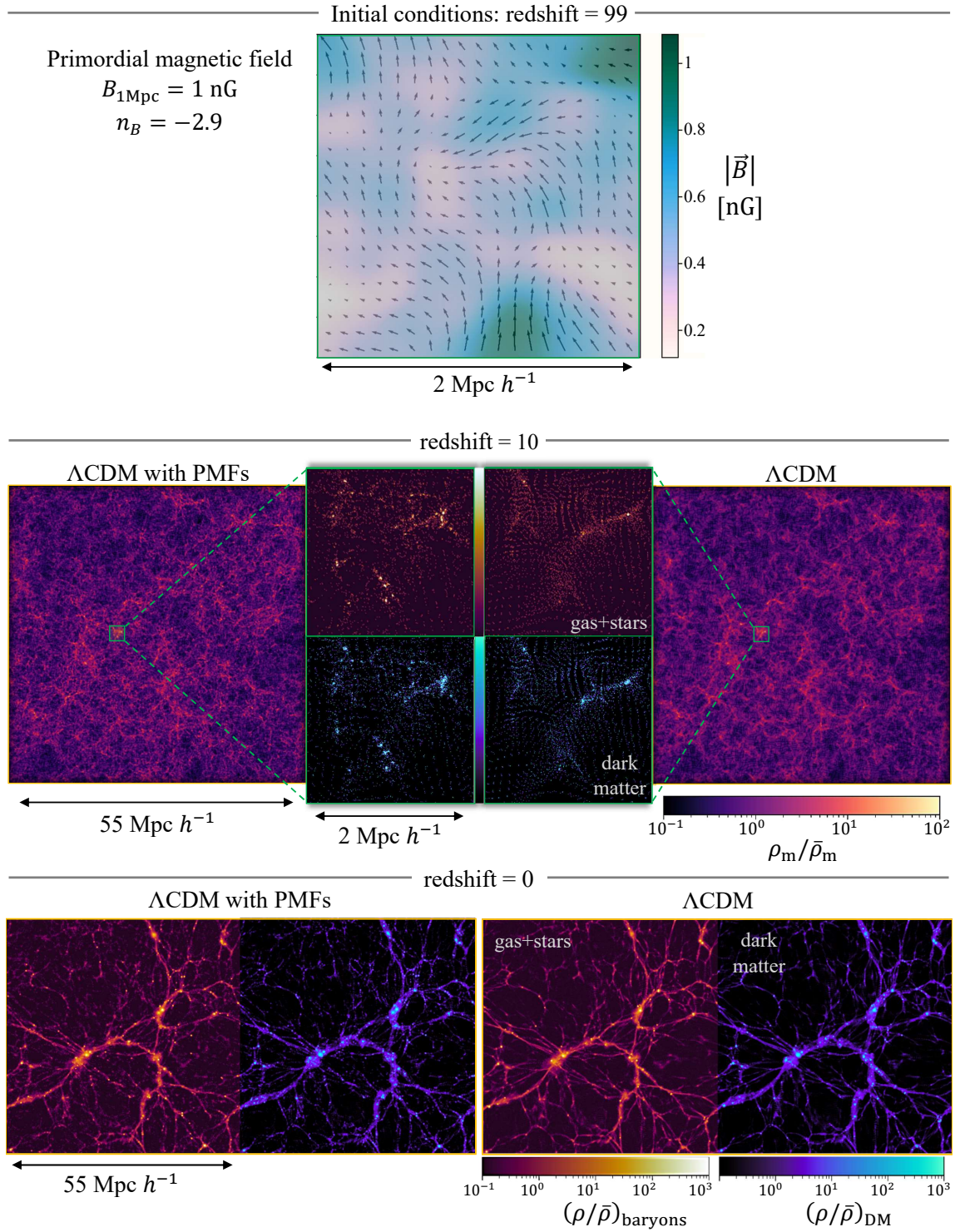


Figure 2.1: A thin slice of scale $0.2 \text{ Mpc}/h$ is cut out of the simulation volume and projected onto a 2D plane. **Top:** PMFs in the initial conditions. **Middle:** Zoom-in on a small region at redshift 10 (with same coordinates as in the Top panel), showing small-scale differences of overdensities $\rho/\bar{\rho}$. Color bars in the middle are linearly scaled between 0 and 100. **Bottom:** Overdensity maps of baryons and dark matter at zero redshift in the whole slice.

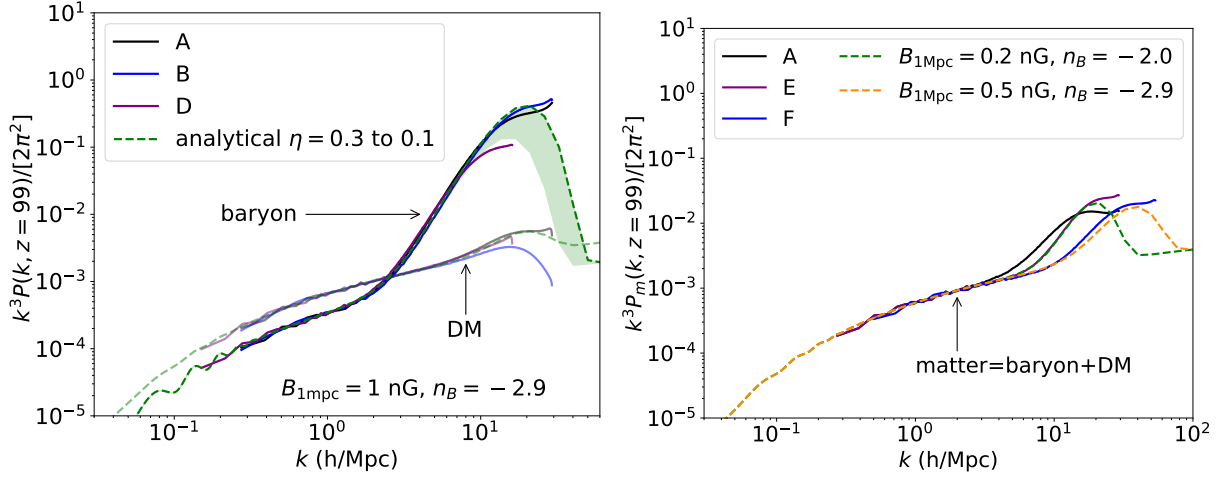


Figure 2.2: **Left:** The solid coloured lines show the dimensionless baryon power spectra of the initial conditions of different simulations. The slightly transparent lines are for dark matter. The power spectra for simulation C are completely identical to that of B and hence are not shown in the left panel. The dashed lines show the analytical power spectrum evaluated using Eq. (1.117). The green shaded region highlights the variation in the baryon power spectrum if η goes from 0.1 to 0.3. **Right:** Dimensionless total matter power spectrum for different simulation ICs. The dashed lines are analytical estimate for $\eta = 0.3$. These figures show that ICs generated for simulations match with analytical estimate on modes smaller than the Nyquist frequency.

This is because the perturbations start reaching large non-linear values near the damping scales. Consequently, the Zel’dovich approximation used in the generation of initial conditions becomes invalid near the damping scale. This is the reason the Nyquist scales of our simulations do not significantly exceed the damping scale (λ_D).

The ICs for simulation B were generated with the original N-GENIC algorithm, where particle positions are determined simply by the power spectrum and without generating magnetic fields on the lattice. To account for the influence of PMFs we simply input the PMF-enhanced power spectra for baryons and dark matter, $P(k) = P_{\text{CDM}}(k) + P_{\text{PMF}}(k)$. This algorithm implicitly enforces baryon and dark matter perturbations to be perfectly correlated with each other. This is because the original N-GENIC algorithm is built for ΛCDM cosmologies where baryon and dark matter perturbations are sourced from the same stochastic variable (inflationary curvature perturbation) and hence are correlated. However, PMF-induced perturbation is an independent stochastic variable uncorrelated with inflationary curvature perturbation. The interference between the two sources of perturbations generates an isocurvature between baryon and dark matter perturbations on small scales, i.e. $\delta_{\text{DM}}/\delta_b$ becomes a stochastic variable. This isocurvature is not captured in the initial conditions for simulation B.

For simulation C, we modify the N-GENIC code to capture the uncorrelation between

PMF-induced perturbations and inflationary perturbations, and hence the isocurvature. Specifically, the density fields are sourced by two independent Gaussian distributions: one originating from Λ CDM power spectrum and the other from PMF-induced power spectrum (Eq. (1.117)). The power spectra of dark matter and baryon for simulation C are identical to those of simulation B. However the total matter power spectrum of simulation B and C are different because of the presence of isocurvature between DM and baryon in simulation C.

Our primary method, where particles are displaced according to the Lorentz force, naturally takes into account that PMF-induced perturbations are uncorrelated with Λ CDM perturbations. Moreover, unlike simulation C, it also takes into account non-gaussianity in matter perturbations and the vortical motion of baryons.² However, we find that non-gaussianities and vortical motion play a negligible role in determining baryon fraction of halos.

Earlier studies [92, 102] that performed similar simulations generated initial conditions similar to our simulation B. However, unlike our simulation B, they implicitly assumed an equal enhancement to baryon and dark matter perturbations from PMFs, and thus could not capture the enhancement of baryon fraction of high redshift halos. While their initial conditions are less accurate than ours, their simulations include MHD [102] or galaxy chemical evolution [92]. In this study, as a first step, we choose to perform a dedicated set of hydrodynamic simulations to focus on the impact of improved initial conditions.

The following section is organized as follows. In §2.1.1 we compare the halo mass function obtained from our simulations with Λ CDM expectations. We show that the Sheth-Tormen analytic formula provides an excellent fit for the numerically obtained halo mass function. In §2.1.2, we show the baryon fraction of different halos as a function of their mass and redshift. We find that the presence of isocurvature widens the scatter in the baryon fraction of halos and that the baryon fraction is more enhanced for smaller values of n_B and larger values of λ_D . Finally, in §2.1.3, we show how the enhanced baryon fraction and matter power spectrum could boost star formation.

2.1.1 Halo mass function

The halo mass function (HMF) represents the abundance of dark matter halos in a small mass range $[M, M+dM]$, per unit volume. In this section, we show that the HMF obtained from simulations is in close agreement with that predicted from theory. Additionally, we discuss the variation of HMF as we change different PMF parameters.

²If PMFs are Gaussian distributed, then the matter perturbations would naturally have non-gaussianities because they are non-linearly related to PMFs, $\delta_b \propto \nabla \cdot [(\vec{\nabla} \times \vec{B}) \times \vec{B}]$.

Theoretical models can directly estimate the HMF from the linear matter power spectrum through the relation

$$\frac{dn_h}{dM}(M, z) = \frac{\bar{\rho}_m}{M} f(\sigma_R) \frac{d\sigma_R^{-1}}{dM}, \quad (2.1)$$

where the variance of the smoothed linear overdensity field of the total matter, σ_R^2 in Eq. (1.31), can be rewritten, using Eq. (1.13), in terms of the dimensionless power spectrum:

$$\sigma_R^2 = \int_0^\infty \Delta^2(k, z) \tilde{W}^2(kR) \frac{dk}{k}. \quad (2.2)$$

The shape of the differential mass function, f in Eq. (2.1), depends on a particular model of halo collapse or a fitting prescription. In this study, we employ the Sheth-Tormen model [19, 21], which is determined by the fitting formula of the form (recall (1.33))

$$f_{S-T}(\tilde{\nu}) = 2A \left[1 + \frac{1}{\tilde{\nu}^{2p}} \right] \frac{\tilde{\nu}}{\sqrt{2\pi}} e^{-\tilde{\nu}^2/2}, \quad \tilde{\nu} = \sqrt{a} \frac{\delta_c}{\sigma_R}, \quad (2.3)$$

and we will use the original parameters $A = 0.322$, $a = 0.707$, $p = 0.3$, and $\delta_c = 1.686$.³

In Fig. 2.3, we show the Sheth-Tormen HMF at different redshifts. We use the HMF libraries from the package `Pylians` [103] to obtain them. The input power spectra were in the Λ CDM case (dashed lines) obtained directly from `CLASS`, while in the case of PMF (solid lines) we added analytically computed enhancements, see Eq. (1.117). Since PMFs enhance matter power spectrum on small scales we see a greater abundance of low-mass halos compared to Λ CDM.

The markers in the plot show the binned HMF obtained from simulations. We identify halos using the Friends-of-Friends (FoF) algorithm [104], with a linking length parameter $b = 0.2$.⁴ We have not used spherical overdensity (SO) approach in our simulations. This

³Ref. [80] analytically finds the collapse threshold in PMF-only case (after turning off Λ CDM perturbations) to be redshift dependent. However, numerically we find that using the constant value of $\delta_c = 1.686$ always provides a good fit for halo abundance.

⁴Specifically, the FoF algorithm identifies a halo that has at least 32 dark matter particles. The gas particles are assigned to the halo by linking gas particles to the nearest dark matter particles in the halo. The mass of a halo is given by the sum of all components present in the halo, $M_{\text{halo}} = M_{\text{DM}} + M_{\text{b}}$, where $M_{\text{b}} = M_{\text{gas}} + M_{\text{*}}$ is the total baryon mass composed of gas and stars. After extracting masses of halos, we construct the binned HMF in the following way: we first put all halo masses $\{M\}$ in the log-space $\{\log_{10} M\}$, where we choose equally spaced bins of a fixed size $\Delta \log_{10}[M/(M_{\odot} h^{-1})] = 0.2$. We have tested other values between 0.1 and 0.3, and the results do not depend significantly on this particular choice. Number of bins at each redshift z is then determined by $[\log_{10}(M_{\text{max}}(z)/(32 \times m_{\text{DM}}))/0.2]$. With this, we find histogram values ΔN (number of halos) in each bin, which we then divide with the simulation box volume L_{box}^3 and with the bin width $\Delta \log M = 0.2$. In order to compare these values with theory, we multiply the LHS of (2.1) with mass M and $\ln(10)$ to get

$$\frac{dn_h}{d \log_{10} M} \simeq \frac{\Delta N}{L_{\text{box}}^3 \Delta \log M}. \quad (2.4)$$

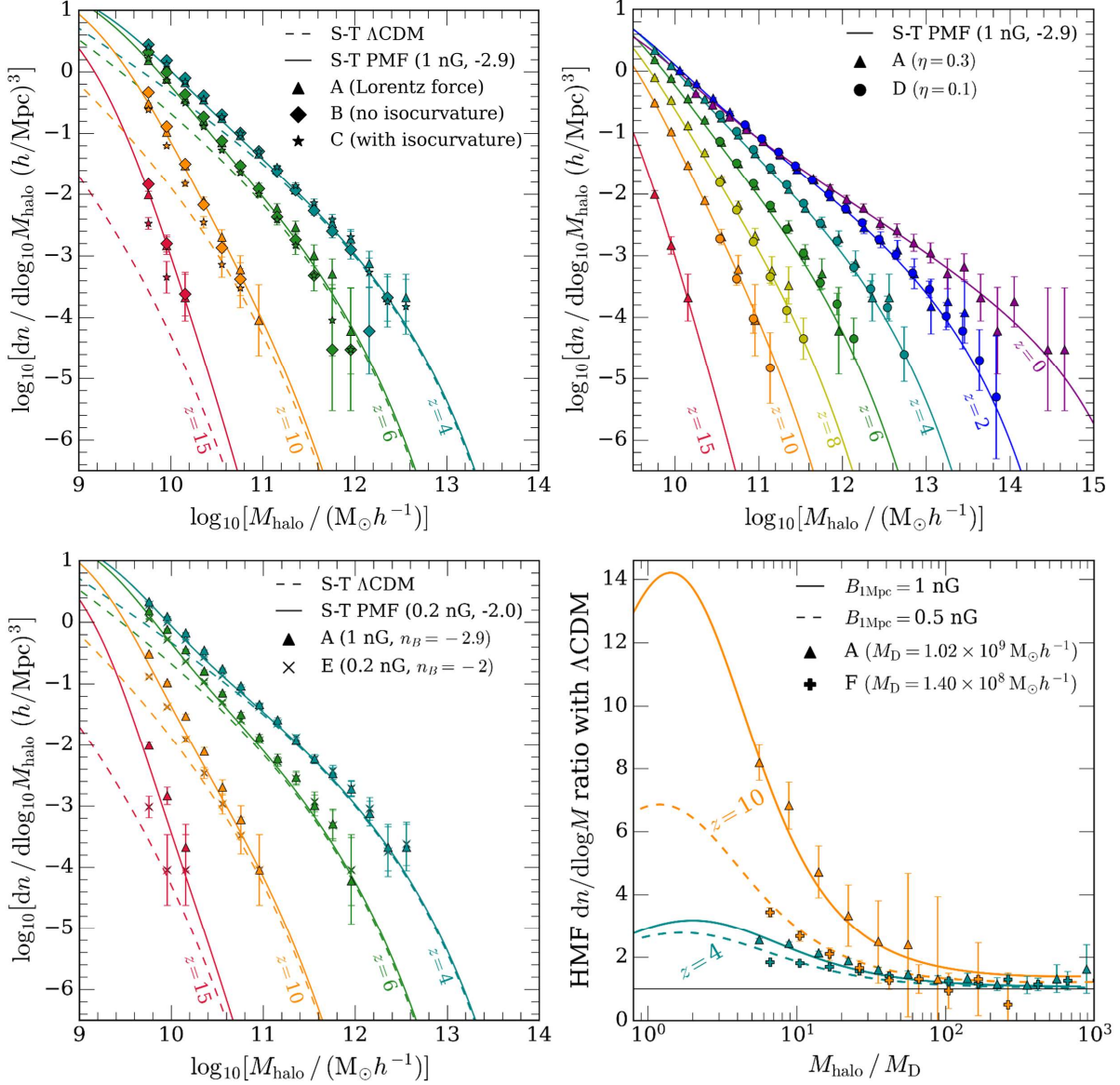


Figure 2.3: HALO MASS FUNCTION. Simulation points are shown at the centers of bins with a fixed width $\Delta \log M = 0.2$, count all FoF-identified halos with at least 32 DM particles, and are compared with the Sheth-Tormen theory at several redshifts z . Error bars are given by the Poisson shot noise of number of halos in each bin. The S-T lines (dashed for Λ CDM and solid for PMF in left panels) have different input power spectra, and their parameters are indicated in parentheses for the PMF case. **Top left:** Simulations A, B and C, with different particle displacement methods in the initial conditions. **Top right:** A and D, with different damping scales given by parameter η . **Bottom left:** A and E, with different PMF strengths and spectral indices, but similar power spectra. **Bottom right:** A and F, shown as ratios of HMFs $F_{\text{PMF}}/F_{\text{CDM}}$ shifted according to the damping mass M_D , with $F \equiv dn/d\log M$; the lines are ratios of S-T fits.

is because some previous studies (e.g., [107, 108]) found that FOF and SO give similar results, but FOF needs less (DM) particles in the low-mass end for determination of the HMF and is computationally cheaper.

In Fig. 2.3 we compare our simulations with the Sheth-Tormen theory and find an excellent agreement between the two. As the Sheth-Tormen lines are only a function of the linear matter power spectrum, the figures highlight that HMF is primarily sensitive to the linear matter power spectrum. Thus non-gaussianities in density distribution are unimportant (see appendix of [1]).

The first panel further supports the idea that HMF is primarily sensitive to the linear power spectra. Here, the different shaped markers come from simulations that have the same matter power spectrum but in one simulation (A) particles are displaced according to the Lorentz force while in the others (B and C) they are only displaced according to the input power spectrum.

The second panel shows how the HMF changes for different η values. It is apparent that changing η and the box size does not affect the HMF on scales larger than the damping scale, $1/k_D$. This figure highlights the numerical consistency of our simulations.

In the third panel, we vary both n_B and $B_{1\text{Mpc}}$ such that the total matter power spectrum for the two simulations is roughly the same (see Fig. 2.2). Again we see that the HMF for the two simulations are roughly the same as the total matter power spectra are similar.

Finally, the fourth panel shows the enhancement in HMF for different PMF strengths. The horizontal axis shows the halos mass relative to the damping mass, M_D , defined as

$$M_D = \frac{4\pi}{3}\rho_m(a_0) \times \left(\frac{2\pi}{k_D}\right)^3 \sim 7.4 \times 10^{12} \left(\frac{h/\text{Mpc}}{k_D}\right)^3 \frac{\Omega_m h^2}{0.14} \left(\frac{0.678}{h}\right)^2 M_\odot h^{-1}. \quad (2.5)$$

For all simulation points, halo masses are chosen to be at the center of each bin, which enables direct comparison of different runs. The mass of a halo is given by the sum of all components present in the halo, $M_{\text{halo}} = M_{\text{DM}} + M_b$, where $M_b = M_{\text{gas}} + M_*$ is the total baryon mass composed of gas and stars. This is in contrast to taking the mean halo mass in each bin, found, e.g., in [105, 106]. However, we evaluated that the difference is negligible. and, more importantly, in order to compare PMF results with ΛCDM runs, it is useful to have simulation bin points at the same positions in both cases.

The error bars in our HMF plots are related to the Poisson shot noise, for which the best estimate in each bin is $\sigma = 1/\sqrt{\Delta N}$. It is based on the assumption that halos are randomly sampled and form a Poisson realization of the underlying number density. Note that if ΔN has a Poisson shot noise error estimate $\sigma_{\Delta N} = \sqrt{\Delta N}$, then, based on the propagation of uncertainty, $F = \log_{10}(\Delta N/\text{const.})$ will have an error estimate given by

$$\sigma_F = \frac{dF}{d\Delta N} \sigma_{\Delta N} = \frac{1}{\Delta N \ln(10)} \sqrt{\Delta N} = \frac{1}{\sqrt{\Delta N} \ln(10)}.$$

However, we choose to show a larger estimate $\sigma = \ln(10)\sigma_F$, because there can be other factors influencing the error, such as binning, choice of M_{halo} representing the bin, lack of halos, etc. In addition, the same Poisson error estimate was considered in [106].

One can see that the shape of the enhancement is very similar and peaks at M_D mass. However, the amplitude of the peak decreases for smaller PMF strengths. The decrease is because smaller PMF strengths enhance the matter power spectrum on smaller scales (see Fig. 1.15). While the peak of the PMF-induced power spectrum is the same for different PMF strengths, the relative enhancement over Λ CDM power spectrum decreases because the Λ CDM power spectrum grows logarithmically on smaller scales.

For PMF strengths chosen in our simulations, $B_{1\text{Mpc}} \leq 1$ nG, there is a significant impact on the abundance of low mass halos, $M < 10^{11} M_\odot$. Consequently, the observed abundance of such halos can be used to put constraints on PMFs that are stronger than the $B_{1\text{Mpc}} \lesssim 1$ nG constraint derived from CMB and Faraday rotation measurements [86, 109]. Indeed Ref. [92] has used observations of dwarf galaxies to constrain $B_{1\text{Mpc}} < 0.5$ nG for scale invariant PMF. However, we expect these constraints to be modified after taking into account the self-consistent initial conditions we highlight in this study. We leave the detailed evaluation of modified constraints on PMFs from galaxy abundances to future work.

2.1.2 Baryon fraction

In the previous section we saw that at the level of halo mass function, the presence of PMFs was completely parameterised by the matter power spectrum. In this section, we show that the baryon fraction of halos can distinguish between scenarios that contain PMFs as opposed to a simple enhancement of the matter power spectrum.

The impact of PMFs on the baryon fraction of halos can be clearly seen in Fig. 2.4, where we show the scatter plot of all halos in our simulation as a function of halo mass and baryon fraction. The orange-to-purple shaded points are for the simulation with PMFs (simulation A) while the yellow-to-blue shaded points (outlined by a green line) are for Λ CDM-only simulation. Noticeably, at lower halo masses and higher redshifts, and when PMFs are present, the mean baryon fraction of halos sits precisely between the theoretical estimate $(\delta_b/\delta_m)_{\text{PMF}}$ and the mean Λ CDM value. Overall, in the PMF case, halo baryon fractions are larger than the cosmic mean $f_b^{\text{cosmic}} = \Omega_b/\Omega_m$, while those for Λ CDM are slightly lower (see also [110, 111]). In addition, the former is significantly changing with mass, merging with Λ CDM at the largest masses, reflecting the agreement on large scales, whereas the latter stays relatively constant for high redshifts and considered halo mass interval. The sharp downturn at lower masses can be ascribed mainly to resolution limits [110].

Note that using different approaches to assign gas particles to halo could introduce changes in the baryon fraction of halos. In this study, we only use the FoF algorithm

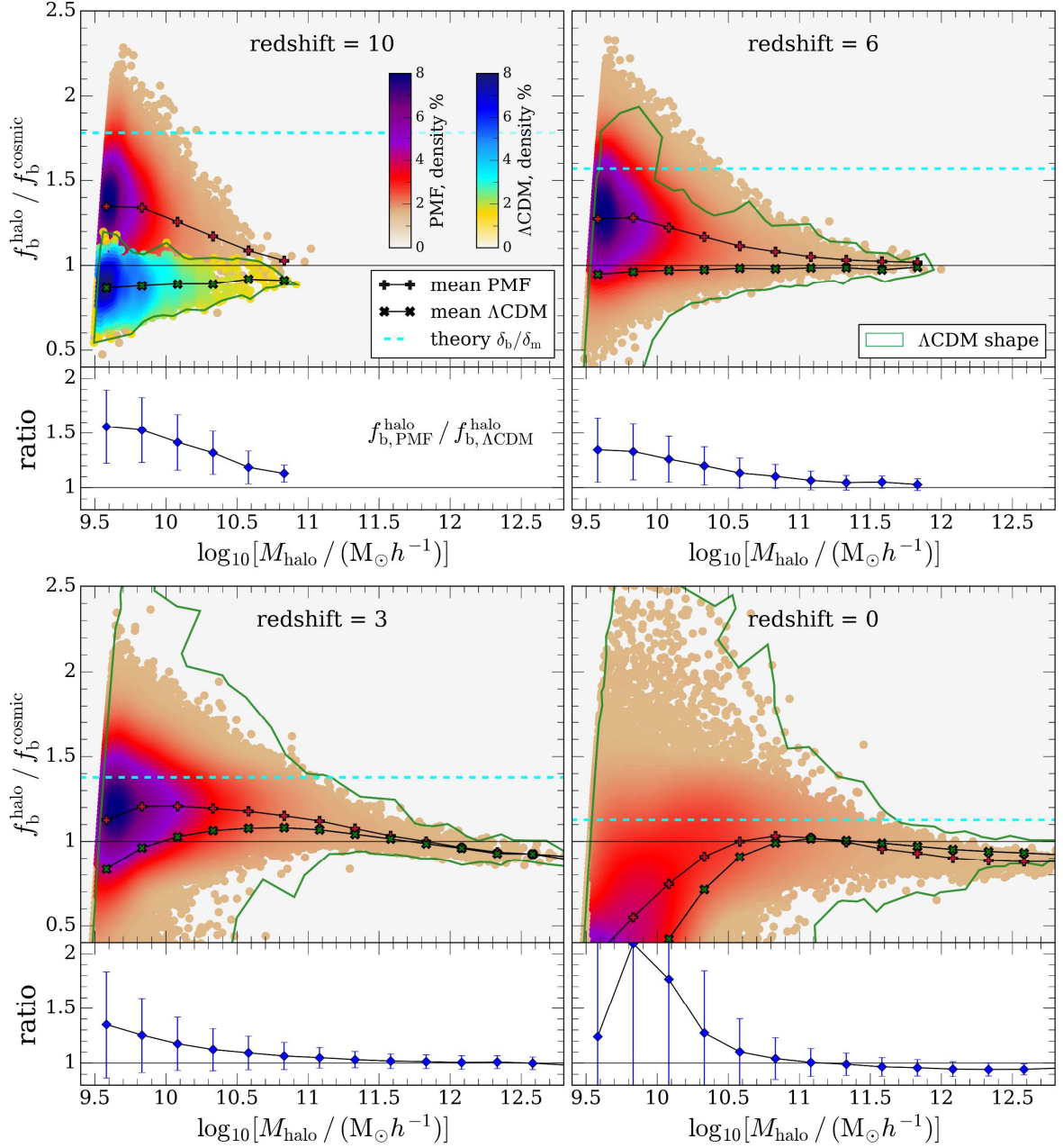


Figure 2.4: EVOLUTION OF THE BARYON FRACTION IN HALOS. For the simulation A, the scatter plot shows halo baryon fraction $f_b^{\text{halo}} = M_b/M_{\text{halo}}$ with respect to the cosmic mean value and comparison with ΛCDM -only evolution. Color coding represents the 2D histogram density. Dashed cyan line is a theoretical estimate given by Eq. (1.106) (see Fig. 1.14). Error bars in the ratio are computed from standard deviations in each bin. We only show the scatter plot for the ΛCDM simulation in the first plot. In other plots we simply show the boundary of the ΛCDM scatter plot as green line for better visualization.

and thus cannot quantify how sensitive the baryon fraction in a halo is to different algorithms. However, the relative enhancement of the baryon fraction in the Λ CDM+PMF case compared to the Λ CDM case should not change significantly.

Furthermore, before redshift ~ 6 , PMFs induce a wider scatter in the baryon fraction compared to the Λ CDM-only scenario. The large scatter is a result of interference between PMF-induced perturbations and those from inflationary initial conditions. This is more apparent when we rewrite the baryon fraction of a halo, given in Eq. (1.106), as

$$\frac{f_b}{\bar{f}_b} = \frac{\delta_b}{\delta_m} = \frac{\delta_b^{\text{PMF}} + \delta_b^{\text{CDM}}}{\delta_m^{\text{PMF}} + \delta_m^{\text{CDM}}}. \quad (2.6)$$

Since Λ CDM and PMF perturbations are uncorrelated and as baryon and dark matter perturbations are amplified differently by PMFs (see Fig. 1.15), there is an isocurvature component between baryons and dark matter, i.e. $\delta_b/\delta_{\text{DM}}$ is a stochastic variable. This in turn causes the baryon fraction of halos to also be a stochastic variable.

We have verified that the large scatter in the baryon fraction for the PMF scenario as seen in Fig. 2.5 is due to Λ CDM and PMF perturbations being uncorrelated. Specifically, in the top left panel, we show the mean and variance of the baryon fraction as a function of halo mass for simulations A, B and C. One can see that simulations A and C produce almost identical results, while simulation B has a narrower scatter (reflected also by a narrower 2σ band). The narrow scatter is because the PMF and Λ CDM perturbations are perfectly correlated and hence the baryon fraction in halos (Eq. (2.6)) is not a stochastic variable. Whereas simulation C takes into account the isocurvature between dark matter and baryons.

The isocurvature between dark matter and baryon is also the reason behind the mean of the baryon fraction scatter in Fig. 2.4 to be smaller than the analytical estimate (cyan dashed line) calculated in section 1.2.4. Specifically, we estimated the baryon fraction by only focusing on PMFs and neglecting contribution from inflationary initial conditions. This amounted to setting Λ CDM marked perturbations in Eq. (2.6) to zero. Once we include the contribution from Λ CDM marked perturbations we decrease the baryon fraction because the denominator in Eq. (2.6) increases more relative to the numerator.

The slope of the cutoff of baryon fraction scatter, as observed on the left side of top panels in Fig. 2.5, is a direct consequence of defining halos with at least 32 dark matter particles. More specifically, the halo baryon fraction from simulations reads

$$f_b^{\text{halo}} = \frac{M_b}{M_b + M_{\text{DM}}} = \left[1 + \frac{\Omega_{\text{DM}}}{\Omega_b} \frac{N_{\text{DM}}}{N_b} \right]^{-1}, \quad (2.7)$$

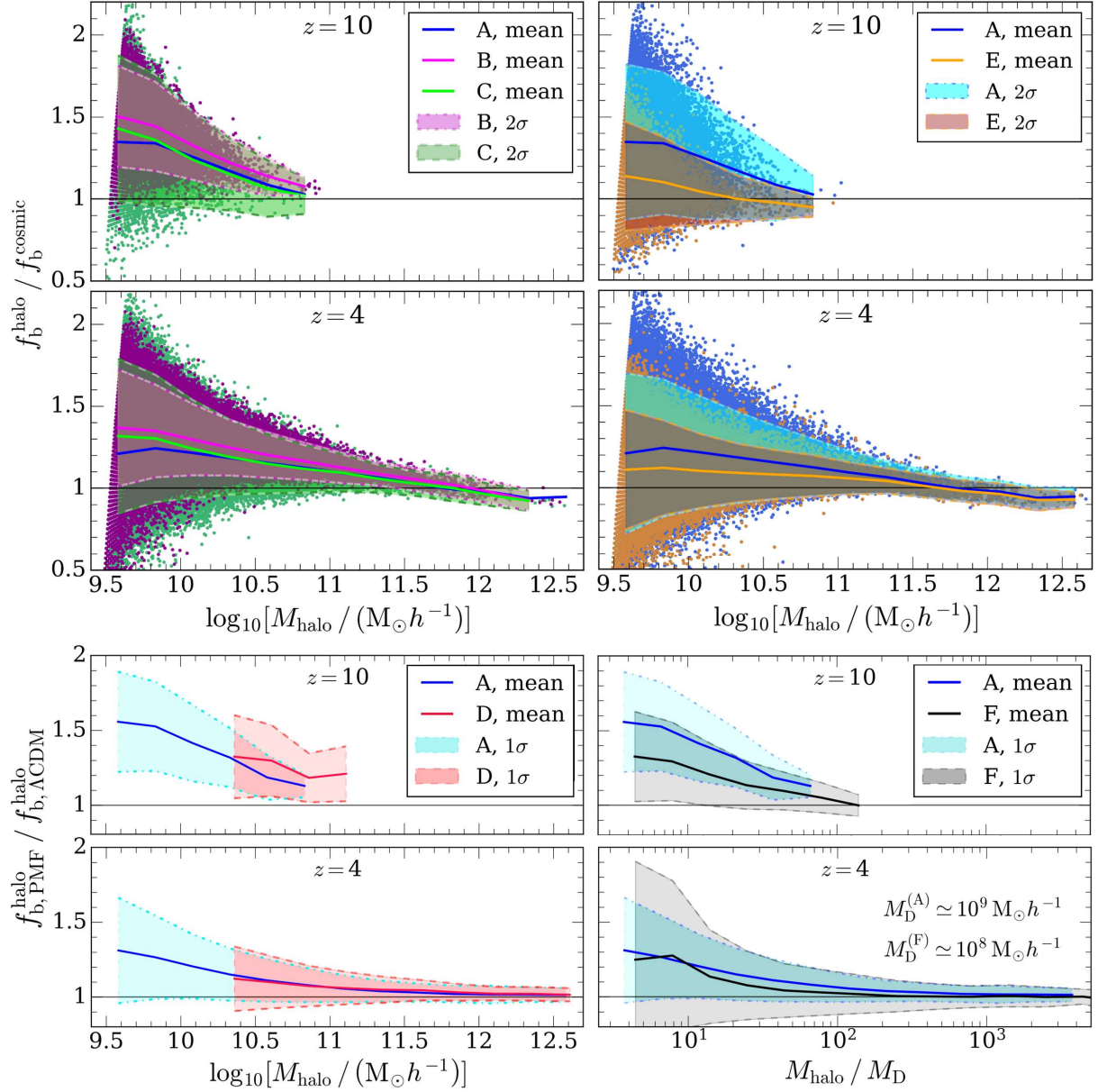


Figure 2.5: BARYON FRACTION IN HALOS - COMPARISON OF DIFFERENT RUNS. In the upper part are scatter plots of the halo baryon fraction in units of the cosmic mean value. In the lower part are ratios with Λ CDM simulations data, for corresponding box sizes. Means and error bands (1σ or 2σ) assume approximate Gaussian statistics in each bin. **Top left:** Simulations A, B and C, with scatter points and 2σ regions shown only for B and C. **Top right:** A (1 nG) and E (0.2 nG), with similar power spectra. **Bottom left:** A and D, with different damping scales quantified by $\eta = 0.1$ and $\eta = 0.3$, respectively. **Bottom right:** A and F (0.5 nG), with the x-axis shifted according to the damping mass M_D .

where we used the fact that each species' mass in the halo is given by $M_s = N_s m_s = N_s \Omega_s \rho_{\text{cr}} (L_{\text{box}}/N_{\text{mesh}})^3$, where N_s is the number of particles in the halo of species s . Thus, the initial slope of the baryon fraction scatter is given simply by having $N_{\text{DM}} = 32$ and adding baryons one by one, until it reaches a certain maximum value. Unsurprisingly, the theoretical formula for baryon fraction (1.106) can be written in the same way as (2.7), by interchanging $N_s \leftrightarrow \delta_s$.

Finally, it is worth noticing that at lower redshifts, the scatter in the baryon fraction for the PMF scenario overlaps more with the Λ CDM scenario. There are two reasons for this. First, as halos accrete more matter, their baryon fraction asymptotes to the cosmic average and hence the mean scatter of both Λ CDM and PMF scenario eventually asymptotes to the same values. Second, accretion and mergers of halos tend to increase scatter with time. For the PMF scenario, the halos are formed with a large scatter in the baryon fraction due to the presence of isocurvature. Hence, we believe the further enhancement of scatter by mergers/accretion is minimal. In contrast, in the Λ CDM scenario, the halos are initially formed with minimal scatter and accretion/mergers play a major role in the evolution of baryon scatter.

In Fig. 2.5 we also show how baryon fraction scatter changes as we change PMF parameters. Increasing n_{B} results in a reduction of baryon scatter (top right panel of Fig. 2.5). We postulate that this is because the range of scales where PMFs enhance the matter power spectrum over the Λ CDM one is reduced as n_{B} increases. Thus the baryon fraction quickly asymptotes to the Λ CDM value as halo mass increases.

As we change η from 0.3 to 0.1 (bottom left panel of Fig. 2.5), the baryon fraction scatter almost stays the same for large masses. This is expected as η only affects the location of the damping scale and thus properties of large massive halos should remain unchanged.

Finally, reduction in the strength of PMF, $B_{1\text{Mpc}}$, causes the baryon fraction enhancement to move to smaller masses because PMF-enhanced power spectra move to smaller scales (see Fig. 1.15). Thus, when we plot the enhancement of baryon fraction as a function of $M_{\text{halo}}/M_{\text{D}}$ in Fig. 2.5, we see a similar shape for both A and F simulation. However, simulation F has a slightly smaller enhancement in f_{b} because, on small scales, the logarithmic growth of Λ CDM power spectrum increases the contribution of Λ CDM f_{b} values in the halos.

Note that the bottom panels of Fig. 2.5 show the ratio of baryon fraction obtained in PMF simulation with the ones in Λ CDM simulations with the same box size. This approach allows us to mitigate potential systematic errors stemming from variations in box size when comparing simulations.

The baryon fraction shown in Figs. 2.4-2.5 could be suppressed by baryonic feedback processes (AGNs, winds, UV/X-ray background), which are not accurately modelled in our simulations. Specifically, our simulation has no feedback from AGNs and winds and we believe these effects can dramatically affect baryon fraction in halos [112, 113].

Our simulation does model UV background (UVB) based on Ref. [114]. The implemented UV background is consistent with observational data on QSO/galaxy luminosity functions, thermal history of the IGM, timing of reionization, and high-redshift Lyman- α forest data. However, this UVB model does not take into account explicitly the star formation rate from the simulation. Since star formation rate for the case with PMFs is expected to be much larger than the case for Λ CDM at high redshifts (see section 2.1.3), one could expect a much larger UVB in the case of PMFs. In Ref. [112], the primary impact of the star-formation UV feedback is to suppress the accretion of gas for $M_{\text{halo}} < 10^{10} M_{\odot}$. Since in the case of PMF-induced structure formation, halos are formed with large baryon fraction before star-formation, the local UVB-related suppression of accretion should have minimal impact on the final baryon fraction.

2.1.3 Analyzing impact on star formation

In the previous section, we saw that PMFs can significantly enhance the baryon fraction of high redshift halos. However, directly observing the baryon fraction of these halos poses significant challenges. Instead, a more promising observable is the stellar mass of halos. In this section, we show how PMFs might enhance the stellar mass of high redshift halos. Since our simulations do not incorporate MHD, the results of this section should not be taken as a final verdict on PMF's impact on star formation.

In our simulations, the star formation is based on the multi-phase model of Springel & Hernquist [32]. The model assumes that the gas is composed of a hot ($T \geq 10^5$ K) and a cold component ($T \leq 10^5$ K). Cold clouds ($T \simeq 10^3$ K) are formed due to thermal instability between the two gas phases. The clouds are allowed to grow due to the radiative cooling of the hot phase. Our simulations form stars within cold clouds, at locations where overdensity exceeds $\delta = 10^3$. A 10% fraction of formed stars is set to be short-lived and explode as supernovae, which heats the ambient hot gas and leads to the evaporation of cold clouds. Such a stellar feedback provides a self-regulatory star formation. This is the only feedback mechanism included in our simulations. The introduction of other feedback processes such as AGNs and galactic winds could further suppress star formation as they reduce the amount of gas available for star formation or eject the gas into halo outskirts.

From the perspective of PMF-induced structure formation, a relevant feature of the above prescription is that the mass fraction of cold clouds grows monotonically for very

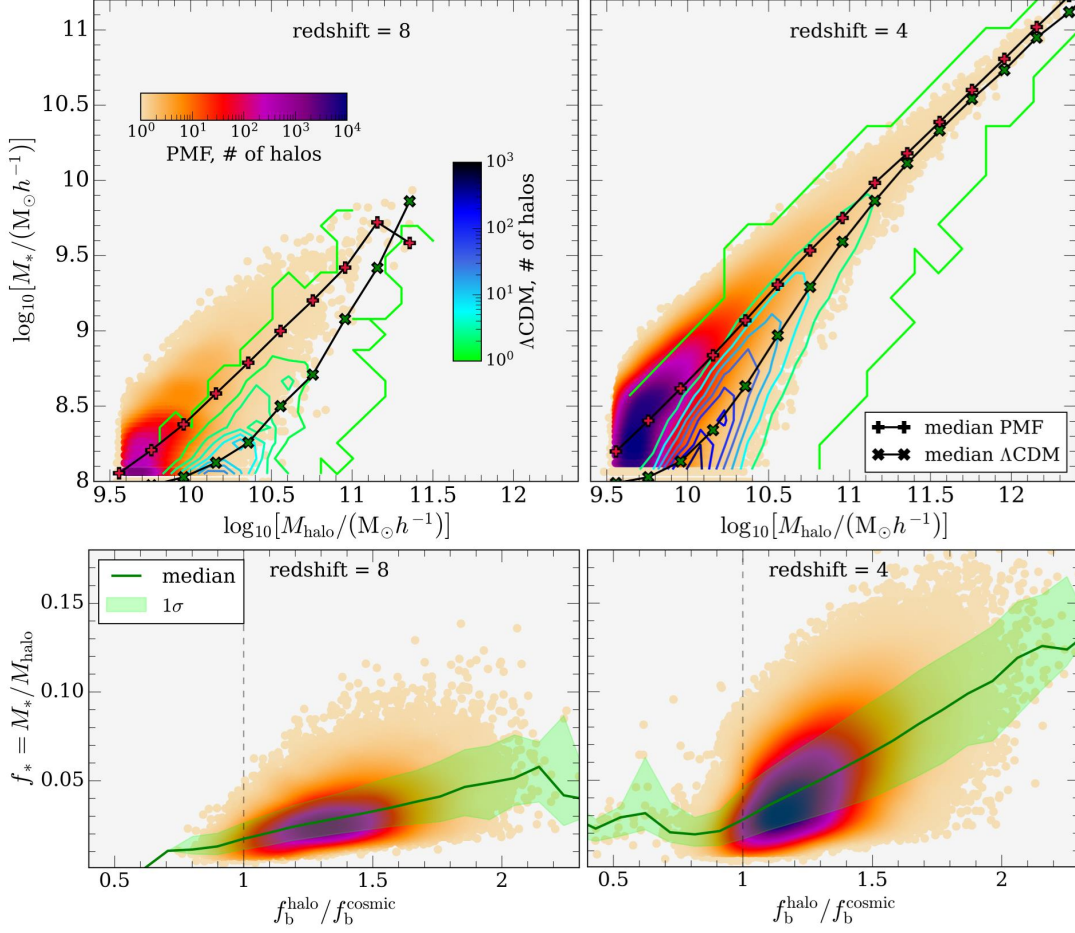


Figure 2.6: STELLAR MASS AND STAR FRACTION IN HALOS. For the simulation A, the scatter plots show halos which contain at least 1 star particle. **Top:** Stellar vs. halo mass in the log-log plane. Λ CDM is shown with colored contours. **Bottom:** Relationship between f_* and f_b^{halo} for the PMF simulation. The 1-sigma error band around the median is given by the 16/84-th percentiles in each f_b bin.

high baryonic overdensities. Since with PMFs, we can reach higher baryon overdensities in halos compared to Λ CDM, we expect a larger production of star-forming clouds and thus elevated star formation.

In the upper part of Fig. 2.6, we show a scatter plot of halos as a function of their stellar mass and halo mass. One can see that the stellar mass in halos is significantly enhanced compared to the Λ CDM simulation for $M_{\text{halo}} < 10^{11} M_\odot$. To estimate how much of the enhancement of stellar mass comes directly from higher baryon fraction, in the bottom panel of the Fig. 2.6 the scatter plot is shown as a function of $f_* \equiv M_*/M_{\text{halo}}$ and baryon fraction of halos. One can see that there is a direct correlation between the two, however, the correlation is weak as there is a significant scatter around the median. Additionally, we also find that the correlation between f_* and f_b increases with time. The

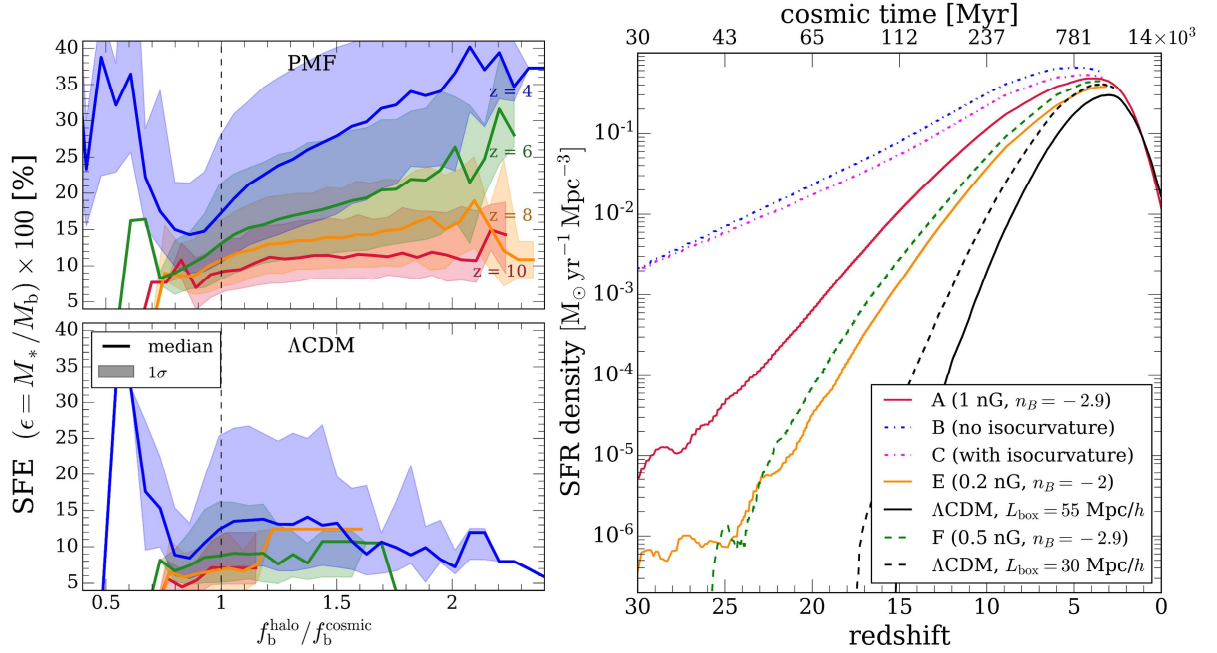


Figure 2.7: STAR FORMATION EFFICIENCY AND STAR FORMATION RATE. **Left:** Evolution of SFE, expressed as the efficiency of gas conversion into stars ϵ in relation to halo baryon fraction, for the simulation A (top) and Λ CDM (bottom). **Right:** Density of the total SFR (integrated over entire volume) for each simulation (see Table 2.1).

increase in correlation could be a consequence of larger time allowing conversion of more gas to stars, compared to the standard case. If this hypothesis is true, one would have an increasing value of the gas to star formation efficiency, $\epsilon \equiv M_*/M_b$, with time.

In the left panel of Fig. 2.7 we show the mean and 1σ scatter of halos as a function of ϵ and f_b for different redshifts. As expected, the median value of ϵ increases with time for each f_b bin. However, at larger f_b values one observes a larger growth in ϵ . This dependence of ϵ on f_b at lower redshifts suggests a non-trivial dependence of star formation on baryon fraction, which is absent in Λ CDM simulation. As Λ CDM simulations obtain large f_b values through accretion and mergers, whereas PMF simulations have an enhanced f_b because of the initial condition itself, the dependency of ϵ on f_b seen in PMF simulation likely corresponds to the historical evolution of f_b .

We also find PMFs to boost the star formation rate (SFR) density compared to Λ CDM as seen in the right panel of Fig. 2.7. The enhancement at high redshifts, $z > 10$, is primarily because halos collapse earlier due to the boosted power spectrum. Consequently, star formation begins much more in advance compared to Λ CDM. The suppression of SFR after $z < 6$, irrespective of the PMF strengths, is due to our simulations using a reionization model that is independent of star formation history [114].

Interestingly, the star formation rate in simulations B and C is much higher than the

rate in simulation A, even though all these simulations have similar halo mass functions and baryon fractions. This contrast arises from the absence of vortical motion in simulations B and C, where particles are not displaced according to the Lorentz force, as opposed to simulation A. Consequently, we hypothesize that the augmented angular momentum of halos induced by vortical motions acts to suppress star formation.

Note that our star formation prescription does not model feedback from winds or active galactic nuclei. For Λ CDM simulations, without such feedback mechanisms, it was found [32] that there is an overall overproduction of stars when compared to local disk galaxies. The main reason for such an overproduction is that radiative cooling is very efficient for high redshift and low-mass halos, with no mechanism for cold gas to escape from the gravitational potential of star-forming regions. In our PMF simulations, we should expect the same overproduction, which could, however, be even higher. This is because baryon overdensity enhancement is most pronounced for high redshifts and small halos. The inclusion of AGN/wind feedback might non-trivially change the behaviour of star formation rate and needs to be accounted for before connecting simulations with observations [115].

Finally, it is well-known that magnetic fields provide additional support against the collapse of molecular clouds into stars and play a role in the physics of the interstellar medium. Therefore, the evolution of PMFs, all the way to galaxy formation, should bring additional insights into our star formation results. It is worth mentioning that values of PMF strengths less than 1 nG were found to minimally impact the star formation histories [97]. However, this study assumes a uniform magnetic field configuration in the initial conditions. A stochastically distributed primordial magnetic field that is correlated with the displacement of particles in the initial conditions might change the result non-trivially.

2.2 Summary and Conclusions

In this work, we carefully delved into how primordial magnetic fields (PMFs) influence baryon and dark matter perturbations. We have pointed out three important aspects that have not been scrutinized thoroughly in previous research.

First, we found that PMFs can produce halos at high redshifts ($z > 4$) which have a baryon fraction several times larger than the cosmic average. This happens because PMFs directly boost baryon density perturbations, while dark matter perturbations only get a boost from the gravitational pull of baryon perturbations. As halos accrete surrounding matter, their baryon fraction eventually levels off to the cosmic average at lower redshifts.

Secondly, PMFs introduce isocurvature between baryon and dark matter perturba-

tions. This happens because the perturbations induced by PMFs are uncorrelated with the perturbations sourced by inflationary initial conditions in the standard cosmological model. The lack of correlation, combined with the asymmetric effect of PMFs on baryon and dark matter perturbations, leads to isocurvature. We show that this isocurvature increases the scatter in baryon fraction among high-redshift halos.

Lastly, we have shed light on the complexity of figuring out the magnetic damping scale (or magnetic Jeans scale). This scale sets the limit beyond which baryons' backreaction on magnetic fields suppresses the baryon power spectrum. Current estimates of the magnetic Jeans scale rely on simple calculations, which could overestimate the peak of the baryon power spectrum by orders of magnitude. To address this, we introduce a new variable, η , in order to parameterize the uncertainty in the magnetic damping scale and discuss our results for different η values.

To confirm the influence of PMFs on baryon fraction, we ran hydrodynamical simulations using `P-Gadget-3` code. It is important to note that our simulations did not include magnetohydrodynamics (MHD), and we only considered the effect of PMFs through the initial displacement field of particles. Since we ran the simulations after gravity overcame the Lorentz force from PMFs, the absence of MHD should have a minimal impact on the baryon fraction of halos in our simulations.

For our simulations, we selected PMF strengths permitted by CMB and Faraday rotation measurements, specifically $B_{1\text{Mpc}} \lesssim 1 \text{ nG}$. However, these parameter points may already be constrained by observations of Lyman-alpha, reionization optical depth, or galaxy abundances. Previous studies have derived constraints on PMFs from these measurements, but they did not account for several critical effects of PMFs on density perturbations that we have highlighted in this paper. We defer the task of estimating these modified constraints on PMFs to future research.

Through our simulations, we found that non-Gaussianities and vorticity of PMF-induced perturbations had negligible impact on halo mass function and baryon fraction of halos but still may play a non-trivial role in star formation. In particular, we ran two simulations: one where particles in the initial conditions were displaced along Lorentz force field lines, and a second where particles were displaced with a random Gaussian distribution determined by the power spectrum. Both simulations were identical at the level of matter power spectrum but the second simulation could not capture the non-gaussianity and vorticity present in the first one. However, the two simulations were found to provide identical halo mass function and baryon fraction statistics. The star formation rate was found to be suppressed in the simulation with Lorentz force-displacement and we hypothesize it could be due to the larger angular momentum of halos. Moreover, we

confirmed that the Sheth-Tormen formalism for calculating halo mass functions remains applicable even for the PMF-enhanced matter power spectrum.

Next, our simulations demonstrated that the baryon fraction of halos could be twice the cosmic average at redshifts of $z \sim 10$. This enhancement is most prominent for halos around the magnetic damping scale and levels off to the cosmic average for larger masses. Changing the strength of PMFs increases the magnetic damping scale, shifting the enhancement of the baryon fraction to larger halos. Altering the spectral slope of PMFs while keeping the damping scale fixed reduces the enhancement in the baryon fraction. This is because a larger spectral slope concentrates the impact on density perturbations to a smaller range of scales near the damping scale. Thus for halos on scales larger than the damping scales, i.e. scales where we trust the simulation results, we observe smaller enhancement in baryon fraction.

A higher baryon fraction as well as an enhanced small-scale power spectrum can boost star formation. Using the multi-phase model for star formation [32], we find that PMFs primarily enhance star formation in halos corresponding to the scales where PMFs enhance the matter power spectrum. The gas-to-star conversion efficiency, ϵ , was boosted on average by a factor of 2 to 3 compared to Λ CDM. Moreover, ϵ was found to develop a positive correlation with the baryon fraction with time, while such a correlation was absent in Λ CDM simulations.

Our results on star formation could be significantly modified after including more physical effects due to magnetic fields in our simulations. These include: 1) using MHD; 2) a star formation prescription that takes into account magnetic fields, 3) a self-consistent treatment of heating for the intergalactic medium from primordial magnetic field's dissipation; 4) appropriately calibrated galaxy formation models which include feedback from Supernovae winds and/or Active Galactic Nuclei.

In contrast, the baryon fraction of halos should be largely insensitive to the above-mentioned effects, except for the feedback from black holes and supernovas. However, these feedback processes typically reduce the baryon fraction in halos. The only processes that can enhance baryon fraction are non-linear interactions between halos and the existence of primordial magnetic fields. As in standard cosmology, the interaction between halos is not expected to produce $f_b > 1.5$ at high redshifts ($z > 8$), a detection of such enhanced baryon fraction values in high-redshift galaxies could provide a smoking-gun signal for the primordial nature of magnetic fields.

It is intriguing that recent observational campaigns have discovered: 1) a population of high-redshift and very luminous galaxies whose physical properties might be difficult to reconcile within the standard structure formation model (e.g. [116, 117]); 2) detection

of a very massive black hole at $z \sim 11$ which again might hint at an accelerated phase of star formation (and accretion from the surrounding medium) [118]; 3) a consistent picture of the rapid growth of black holes and galaxies at $z > 6$ from ALMA radio observations of quasar (QSO) environments in which star formation is found to be very high [119]; 4) bursty star-forming galaxies at $z = [4,6]$ and signatures of large galaxy-galaxy interactions [120]; 5) relatively large gas fractions in $z > 6$ QSO host galaxies [121]. All these observational results are likely to be impacted at some level by PMFs and could potentially return galaxies in better agreement with the data. Thus, it is clear that in the next few years, thanks to the combined efforts of JWST and ALMA, the baryon census in high redshift galaxies will become more complete and accurate providing a unique data set to test PMF models.

Chapter 3

Constraints on PMFs from the Lyman- α forest

In this chapter, we demonstrate that the Ly- α forest could be the ideal way to constrain cosmological PMFs. There are two reasons for this: firstly, it explores a vast variety of scales, and is distinctively capable of reaching very small scales by using the one-dimensional (1D) flux power spectrum; secondly, it probes the filamentary cosmic web in environments at around the mean density, far from galaxies, where the impact of magnetic fields generated by astrophysical sources should be minimal [122, 123]. Compared to previous works, in which the Lyman- α forest has been used to provide quantitative constraints on PMFs [84, 95, 124, 125], we perform a full extensive set of hydrodynamic simulations that follow the evolution of the perturbations at very non-linear scales. These simulations should provide a very accurate state-of-the-art description of the physical properties of the diffuse cosmic web and the main summary statistics that we will use: the 1D transmitted flux power.

The chapter is based on our article [2] and most of the sections follow the same structure as in the original publication. *Supplemental material* was integrated into the main body of the text. Two new sections were included to provide some details and additional analysis; in §3.1 we explain how are Ly- α quantities obtained from simulations, and in §3.5 we discuss and interpret some difficulties that emerge when separate transfer functions are used for gas and DM particles in the PMF cases. Comments and interpretations that do not appear in the original publication, but have emerged later—for instance through discussions during seminars—have been added to the footnotes.

3.1 Simulated Lyman- α forest

Let us begin by demonstrating how are quantities or statistics related to the Ly- α forest extracted from cosmological simulations.

The hydrodynamical simulations employed for Ly α forest studies evolve the gas distribution, temperature, peculiar velocity and ionization state of the intergalactic medium (IGM). These three-dimensional quantities are subsequently converted into synthetic one-dimensional absorption spectra that can be compared directly with quasar observations. In the P-GADGET-3 [29, 31] implementation used for the Sherwood [38] & Sherwood-Relics [126] simulation suite, this extraction can be performed *on-the-fly*, i.e. during the execution of the cosmological simulation rather than exclusively as a post-processing operation on saved snapshots. At selected redshifts, a set of sampling lines or *sightlines* hereafter, representing physically ‘lines of sight’ (l.o.s.), is drawn through the simulation volume and the relevant gas properties are interpolated from the smoothed-particle hydrodynamics (SPH) particle distribution onto regularly sampled sightlines.

The Sherwood implementation extracts $N_{\text{LOS}} = 5000$ lines of sight at intervals of $\Delta z = 0.1$, with each sightline sampled using $N_{\text{pix}} = 2048$ pixels [38]. The sightlines are taken parallel to the Cartesian axes of the periodic simulation volume. For a box of comoving length L_{box} , the spatial width of one pixel is therefore $\Delta x = L_{\text{box}}/N_{\text{pix}}$. It is useful to express the position along a sightline also in velocity units. The velocity extent associated with the Hubble expansion across the simulation box is

$$v_{\text{box}}(z) = \frac{H(z)L_{\text{box}}}{1+z}, \quad (3.1)$$

where the factor $(1+z)^{-1}$ converts the comoving box length into a proper distance. The corresponding velocity interval of one pixel is

$$\Delta v = \frac{H(z)L_{\text{box}}}{(1+z)N_{\text{pix}}}. \quad (3.2)$$

Since P-GADGET-3 uses SPH, the gas variables are carried by particles rather than being defined on a fixed Eulerian mesh. For a given l.o.s., the physical quantities at each pixel are therefore reconstructed by summing the contributions from gas particles whose smoothing kernels intersect the line of sight. Schematically, for a field A one may write

$$A_j \sim \sum_i A_i W_{ij}, \quad (3.3)$$

where i labels SPH particles, j labels pixels along the sightlines, and W_{ij} denotes the

appropriate SPH kernel weighting. This procedure follows the standard construction of synthetic Ly α spectra from SPH simulations, the details can be found in the appendix of Theuns et al. [127].

The neutral fraction is determined by the thermal and ionization state of the simulated gas, which in the standard optically thin simulations evolves in the presence of an imposed ultraviolet background.

The next step is the construction of the Ly- α optical depth – not on the fly. The absorption produced by a given element of neutral hydrogen is broadened thermally and displaced in velocity space by its peculiar motion. For thermal Doppler broadening of a hydrogen atom, the thermal velocity distribution is characterized by

$$b = \left(\frac{2k_{\text{B}}T}{m_{\text{H}}} \right)^{1/2}. \quad (3.4)$$

The total velocity of absorbing gas along the l.o.s. contains both the Hubble-flow contribution and the peculiar velocity,

$$v_{\text{tot}}(x) = \frac{H(z)x}{1+z} + v_{\text{pec}}(x), \quad (3.5)$$

where x is the comoving coordinate along the line of sight. The optical depth in an observed velocity pixel v may then be written schematically as

$$\tau_{\alpha}(v) = \sigma_{\alpha} \int dx, n_{\text{HI}}(x), \phi[v - v_{\text{tot}}(x); T(x)], \quad (3.6)$$

where n_{HI} is the neutral hydrogen number density, σ_{α} contains the Ly α atomic constants, and ϕ is the line profile. In practice the profile is evaluated using a Voigt-profile approximation; the Sherwood spectra adopt the analytic approximation developed by Tepper-Garcia [128]. Consequently, the optical depth at a given velocity pixel receives contributions from several neighbouring real-space elements. The τ_{α} field is therefore not simply proportional to the local neutral hydrogen density: thermal broadening redistributes absorption over neighbouring velocity pixels, while peculiar velocities shift absorbing structures from their real-space positions into redshift space.

Once the optical depth has been obtained, the transmitted flux follows directly from

$$F(v) = \exp[-\tau_{\alpha}(v)]. \quad (3.7)$$

Thus, the construction of Ly α forest spectra does not require radiative-transfer propagation of a quasar continuum through the simulation. Instead, the hydrodynamical and

ionization properties of the intervening IGM are used to calculate the absorption that would be imprinted on a hypothetical background source. The summary of extracted quantities for one l.o.s. is shown in the Fig. 3.1.

For the simulations considered in this work, the information written by the line-of-sight routine can be summarized schematically as

$$\{\rho, x_{\text{HI}}, T_{\text{HI}}, v_{\text{pec,HI}}\} \longrightarrow \tau_{\text{HI}} \longrightarrow F = e^{-\tau_{\text{HI}}}. \quad (3.8)$$

The binary line-of-sight files therefore contain both the reconstructed physical state of the gas and the resulting Ly α optical depth. An important advantage of the on-the-fly procedure is that a large number of one-dimensional absorption spectra can be stored without requiring an equally large number of complete three-dimensional simulation snapshots. The full particle distribution is available in memory while the simulation is running, allowing the SPH quantities to be projected directly onto the selected sightlines before the integration continues. Subsequent operations, such as rescaling the optical depth to reproduce an observed mean transmitted flux, are instead normally carried out in post-processing. One introduces a multiplicative factor A ,

$$\tau_{\alpha} \rightarrow A\tau_{\alpha}, \quad (3.9)$$

and determines A such that

$$\tau_{\text{eff}} = -\ln \langle e^{-A\tau_{\alpha}} \rangle \quad (3.10)$$

matches the desired effective optical depth, in our case gauged to observations of Boera et al. [129]. The resulting flux sightlines can then be used to construct observables such as the flux probability distribution function and the one-dimensional Ly α flux power spectrum.

Temperature-density relation An important theoretical ingredient in calculating optical depth from gas overdensity is the power-law relation between temperature T and baryon density ρ_b , based on the so-called Fluctuating Gunn Peterson Approximation [25], which provides analytical modeling of the IGM opacity by neglecting the effect of peculiar velocities and the thermal broadening

$$T = T_0(1 + \delta_b)^{\gamma-1}, \quad (3.11)$$

where T_0 denotes the temperature of the low-density IGM at the cosmic mean baryon density. The parameters T_0 and γ that appear in (3.11), enter into post-processing of our

simulations as described in §3.3.1.

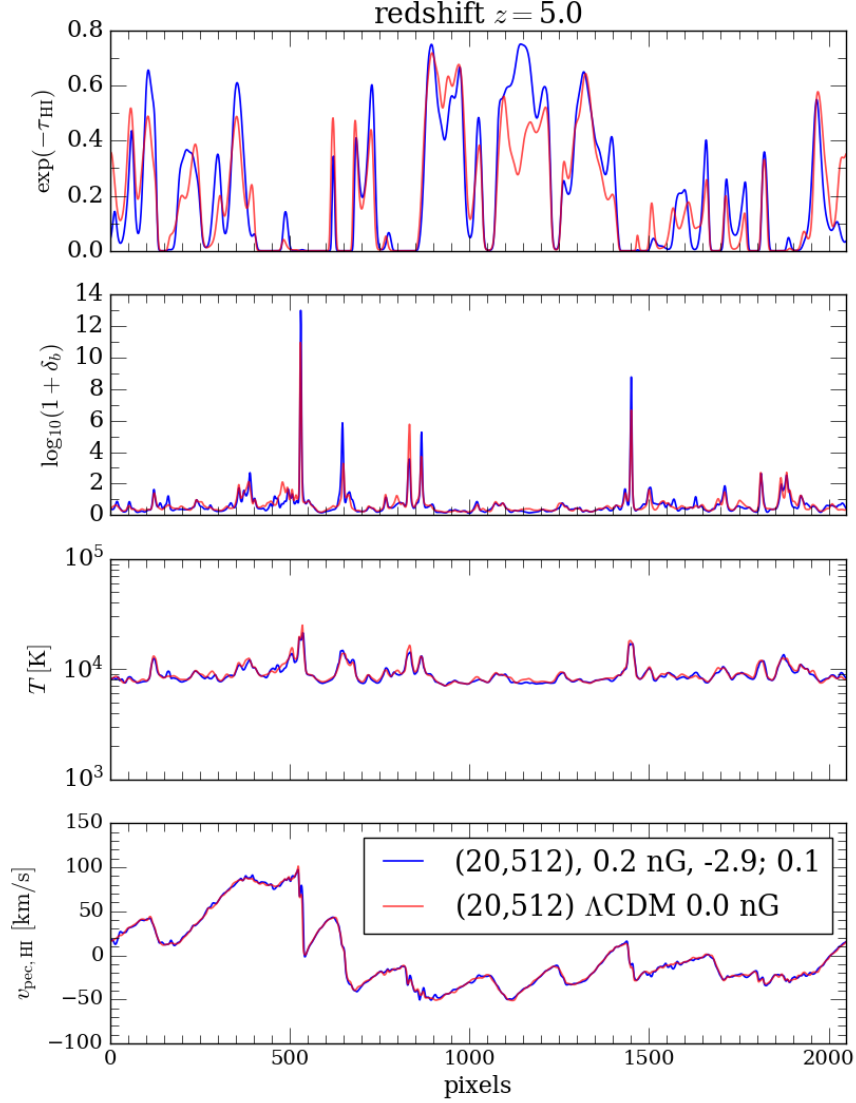


Figure 3.1: Extracted quantities along one line of sight from hydro-simulations. From top to bottom: transmitted flux, baryon density, temperature, peculiar velocity of neutral hydrogen. In this example we compare one PMF+ Λ CDM model $B = 0.2$ nG, $n_B = -2.9$ (blue), with the pure Λ CDM (red); the simulation has a box of 20 Mpc and $N_{\text{mesh}} = 512$.

The observational role of T_0 is that it relates to *broadening of absorption lines* in the Ly- α quasar spectra explained by Eqs. (3.4) & (3.6): thermal broadening depends mainly on the instantaneous gas temperature which determines the velocity variance of hydrogen, and optical depth depends on the width of the 1D velocity space. Larger T_0 means larger velocity variance and therefore optical depth is less narrowly determined. On the other hand, photoheating (e.g., from stars and AGNs) heats the gas and creates *pressure*

smoothing, preventing formation of small structures; this is a 3D effect that also depends on the whole thermal history, not only instantaneous gas temperature. Pedagogical details on how to qualitatively read quasar spectra are in the book [8].

Owing to the $T - \rho$ relation (3.11), one finds for the optical depth (see, e.g., [130])

$$\tau(z) \propto [1 + \delta_b(z)]^2 T^{-0.7}(z) = A(z) [1 + \delta_b(z)]^\beta, \quad (3.12)$$

with

$$\begin{aligned} A(z) = & 0.433 \left(\frac{1+z}{3.5} \right)^6 \left(\frac{\Omega_b h^2}{0.02} \right)^2 \left(\frac{T_0}{6000 \text{ K}} \right)^{-0.7} \\ & \times \left(\frac{h}{0.65} \right)^{-1} \left(\frac{H(z)/H_0}{3.68} \right)^{-1} \left(\frac{\Gamma_{\text{HI}}}{1.5 \times 10^{-12} \text{ s}^{-1}} \right)^{-1}, \end{aligned} \quad (3.13)$$

where $\beta = 2 - 0.7(\gamma - 1)$ and Γ_{HI} is the photoionization rate. The latter determines the rate at which neutral hydrogen atoms are ionized by the ultraviolet background, while the associated *photoheating* rate additionally depends on the excess energy of the absorbed photons above the hydrogen ionization threshold. Schematically, the heating rate per H,I atom may be written as $\epsilon_{\text{HI}} = \Gamma_{\text{HI}} \langle E_{\text{excess}} \rangle$, so that the corresponding heating rate per unit volume is $\mathcal{H}_{\text{HI}} = n_{\text{HI}} \epsilon_{\text{HI}} = n_{\text{HI}} \Gamma_{\text{HI}} \langle E_{\text{excess}} \rangle$; analogous contributions arise from helium He,I and He,II. The integrated thermal history used in Ly α forest analyses is then characterized by the cumulative heat injection

$$u_0(z_0) = \int_{z_0}^{z_{\text{max}}} \frac{\mathcal{H}(z)}{\bar{\rho}_b(z) H(z)(1+z)} dz, \quad \mathcal{H} = \sum_i n_i \epsilon_i, \quad (3.14)$$

where $\bar{\rho}_b$ is the mean baryon mass density and $i = \text{HI}, \text{HeI}, \text{HeII}$. Thus u_0 is formally a cumulative energy *per unit mass*; it is conventionally quoted in eV, m_p^{-1} and can therefore be interpreted approximately as the deposited energy per proton mass. Importantly, Γ_{HI} and u_0 should not be identified directly: in photoionization equilibrium $n_{\text{HI}} \Gamma_{\text{HI}} \simeq n_e n_{\text{HII}} \alpha_{\text{HII}}(T)$, so an increase in Γ_{HI} simultaneously lowers the neutral fraction. The thermal input therefore depends on both the ionizing intensity and spectral shape, in particular on the mean excess energy per photoionization, rather than on Γ_{HI} alone [129, 131].

The connection between u_0 and the timing of reionization arises because earlier-reionized gas is exposed to photoheating for a longer period and consequently has more time to develop pressure smoothing. In the spatially homogeneous simulations used by Nasir et al. and Boera et al., the gas becomes optically thin at a prescribed redshift z_{OT} , which effectively marks the onset/completion of the imposed rapid reionization transition;

the subsequent photoheating is then integrated down to the redshift of interest. For a fixed UV background and heating prescription, increasing z_{OT} therefore generally produces a larger $u_0(z)$, and hence stronger pressure smoothing at fixed observation redshift. There is, however, *no unique one-to-one relation* between u_0 and the redshift at which reionization ends: the same u_0 may result from an earlier but weaker/softer heating history or a later but stronger/harder one, and the result also depends on the duration and spatial inhomogeneity of reionization. Consequently, u_0 is best regarded as a measure of the *integrated thermal memory* of reionization rather than a direct measurement of z_{re} . A constraint on the end or timing of reionization can be obtained from u_0 only after specifying a family of reionization and UV-background models and computing the mapping $z_{\text{re}} \rightarrow u_0$ within those models [129, 131].

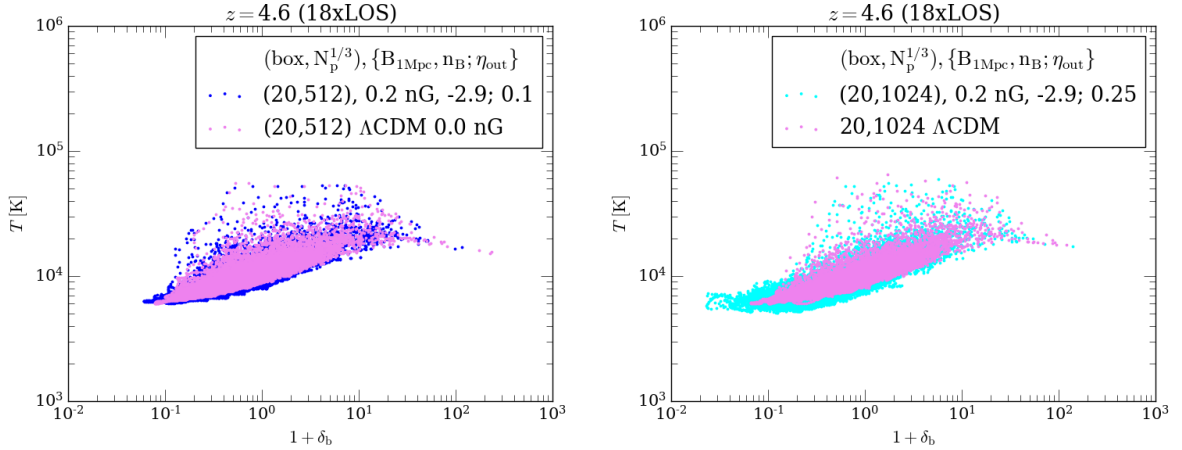


Figure 3.2: RELATION BETWEEN TEMPERATURE AND BARYON DENSITY. The scatter plots show distributions for 18 lines of sight. Comparison PMF+ Λ CDM (blue/cyan) vs. Λ CDM (pink) **Left:** $B = 0.2$ nG, $n_B = -2.9$ and $\eta = 0.1$; the simulation has $L_{\text{box}} = 10$ Mpc and $N_{\text{mesh}} = 512$. **Right:** B & n_B are the same, but $\eta = 0.25$; the simulation has the same resolution, but covers larger volume ($L_{\text{box}} = 20$ Mpc and $N_{\text{mesh}} = 1024$) – for this reason it picks up more low-density regions or voids, and therefore more gas with $1 + \delta_b \leq 10^{-1}$ is shown in the scatter plot for the PMF case.

A major advantage of hydrodynamical simulations over the fluctuating Gunn–Peterson approximation (FGPA) is that the thermal state of the intergalactic medium is computed self-consistently from the gas evolution rather than imposed through a simple power-law temperature–density relation. In a hydrodynamical simulation, instead, the temperature of each gas element follows from the full thermal history, including adiabatic expansion and compression, photoheating, radiative cooling, shocks, and the timing of reionization. Consequently, the simulated IGM can exhibit scatter around a single temperature–density relation and can naturally describe gas outside the regime in which the FGPA is accurate.

The resulting Ly α optical depth is therefore constructed from the actual simulated density, neutral fraction, temperature and peculiar velocity fields, with thermal broadening and redshift-space distortions included explicitly, providing a more realistic connection between the underlying matter distribution and the observable Ly α forest.

In Fig. 3.2 we show two examples extracted from our simulations, which demonstrate the relationship between temperature and gas density. In all cases the power law between T and δ_b is statistically well represented by the power-law relation (3.11) in the interval around mean density, for values around $T_0 \simeq 10^4\text{K}$ and $\gamma \simeq 1.5$ (fiducial model at $z = 5$).

3.2 Observational data

We use the 1D flux power provided by [129], estimated using 15 high signal-to-noise spectra observed by VLT/UVES [132] and Keck/HIRES [133]. The measurements span $z = 4.2 - 5.0$ in bins of $\Delta z = 0.4$. In each bin the flux power spectrum is measured in 15 k -bins equidistantly spaced in the range of $\log_{10} k / [\text{km/s}] = [-2.2, -0.7]$ with logarithmic spacing of $\Delta \log_{10} k / [\text{km/s}] = 0.1$. The spectrograph resolution is very high, with $R \sim 50,000$ (FWHM of ~ 6 km/s for HIRES and ~ 7 km/s for UVES). The effects of resolution uncertainty are small, even for the highest wavenumber power spectrum bin measured. Here, we use the measurements with instrumental resolution corrected, and propagate this correction through the covariance matrix. The measurements are also corrected for metal contamination. The typical noise estimated in the flux power spectrum is white noise, with an amplitude of $0.1 - 0.2$ km/s, comparable to the estimated level of the models at the highest wavenumbers. Characterizing the noise levels is important and its impact will be discussed in the Results section. The k -range of the data reaches $k \sim 0.2$ s/km, which would roughly correspond to 50 (physical) kpc at these redshifts. Thus, the data span both high redshifts and the smallest scales, making this an ideal tool to exploit new small scale physics beyond ΛCDM , including PMFs.

3.3 PMF models and the peak scale

The PMF-induced matter power spectrum is of the form (see Eq. 1.117)

$$\Delta_{\text{m, PMF}}^2 \propto \left(\frac{B_{1\text{ Mpc}}}{1\text{ nG}} \right)^4 \left(\frac{k}{1\text{ Mpc}^{-1}} \right)^{2n_B+10} e^{-2k^2\lambda_D^2}. \quad (3.15)$$

The total matter power spectrum is given by the sum $\Delta_{\text{PMF}} + \Delta_{\text{CDM}}$, assuming that inflationary and PMF-induced perturbations are uncorrelated. The wave number at which

the $\Delta_m^2(k)$ reaches its maximum is

$$k_{\text{peak}} = \left[0.1 \kappa(n_B) \left(\frac{B_{1\text{Mpc}}}{1\text{nG}} \right) \right]^{\frac{-2}{(n_B+5)}} \sqrt{\frac{n_B+5}{2}} \text{ Mpc}^{-1}, \quad (3.16)$$

where $\kappa(n_B)$ is an $\mathcal{O}(1)$ number that precisely specifies λ_D for given $B_{1\text{Mpc}}$ and n_B . The exact value of κ has been a subject of recent debate [85]. In this study, we use the matter power spectrum and κ derived in [1], where $\kappa(-2.9) = 0.88$ and $\kappa(-2.0) = 1.79$.¹ To make our final results flexible to a more accurate determination of κ in the future, we use the above ‘peak’ scale as the PMF parameter which enters our analysis. Another advantage is that different pairs of $\{B_{1\text{Mpc}}, n_B\}$ with the same k_{peak} value roughly have similar impact on the Lyman- α , as we show below.

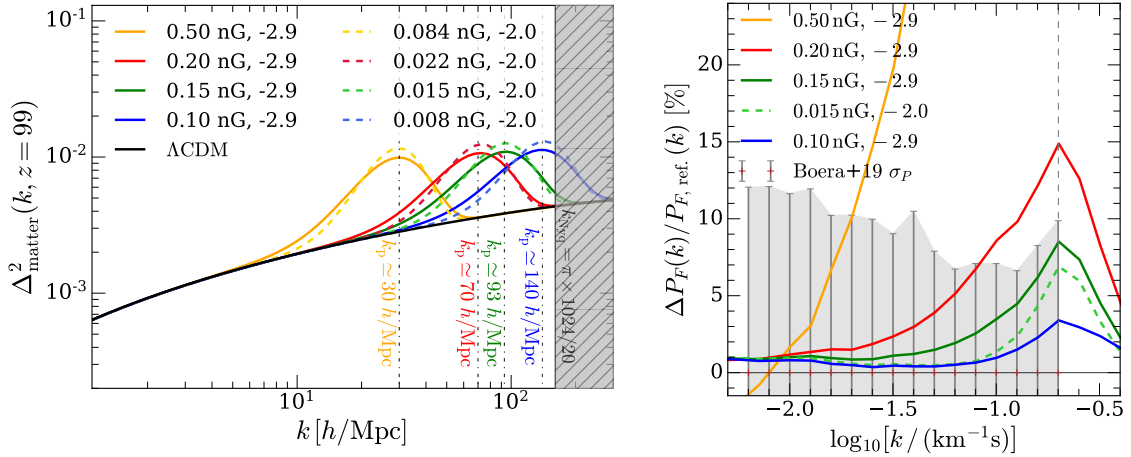


Figure 3.3: **Left:** Linear matter power spectra for PMF models with combinations of $\{B_{1\text{Mpc}}, n_B\}$ parameters for different peak scales k_{peak} . Only spectra with $n_B = -2.9$ (full colored lines) are used in all simulations. We also show the corresponding $n_B = -2$ spectra (dashed lines) to demonstrate their similarity. Shaded region represents scales beyond Nyquist. **Right:** Flux power spectra difference between PMF models and the reference ΛCDM model at $z = 4.6$, where $\Delta P_F = P_F^{\text{PMF}} - P_{F,\text{ref.}}$. The shaded region with error bars show the observational sensitivity, based on 1σ uncertainty of data points.

Here, we consider 4 different PMF models with $k_{\text{peak}} = 20.1, 48, 63.3, 92.9 \text{ Mpc}^{-1}$ which correspond to $B_{1\text{Mpc}} = 0.5, 0.2, 0.15, 0.1 \text{ nG}$ (with $n_B = -2.9$). The initial power spectrum is generated at $z = 99$, with the same transfer functions for baryons and dark matter

¹Ref. [1] only provides the expression for the baryon power spectrum in Eq. 2.23. We obtain matter power spectrum using

$$\Delta_{\text{m,PMF}}^2(k, z) = \left[\frac{\Omega_b}{\Omega_m} + \frac{\mathcal{G}_{\text{dm}}(z)}{\mathcal{G}_b(z)} \frac{\Omega_{\text{dm}}}{\Omega_m} \right]^2 \Delta_{\text{b,PMF}}^2(k, z), \quad (3.17)$$

where $\mathcal{G}_c(z)$ is redshift dependence of growth functions, $\delta_c^{\text{PMF}} \propto \mathcal{G}_c(z)S(x)$.

(but see [1] for another physically motivated approach when using two different transfer functions). In Figure 3.3, we show the power spectra. To demonstrate potential degeneracy between such models—i.e., whose enhancement of 3D power around k_{peak} could have similar effect in the increase of the 1D Lyman- α flux power—we test two models $\{0.15, -2.9\}$ and $\{0.015, -2.0\}$ with $k_{\text{peak}} \simeq 93h/\text{Mpc}$. It can be seen below in the Fig. 3.3 that, indeed, their 1D flux power differs by less than 5%, and is in agreement within the statistical 1σ error bars of the data.

3.3.1 A large set of hydrodynamic simulations

Capturing non-linear structure evolution of IGM structures and the physics of the gas which gives rise to Lyman- α absorption, such as Doppler broadening and thermal pressure smoothing, requires simulating several different thermal and reionization histories. We rely on a set of high-resolution cosmological hydrodynamical simulations from the Sherwood-Relics project, using a modified code P-GADGET-3, as described in section §3.1. This set is complemented by other 48 PMF simulations for the 4 different initial conditions described above. Basically, each PMF model is simulated for 12 different thermal and reionization histories which bracket observational constraints [134].

The (linear) box size is chosen to be $20 h^{-1}\text{Mpc}$ with 2×1024^3 dark matter and gas particles. We further correct for numerical convergence with both box size and resolution with a series of additional simulations described more extensively in [38, 134] and use a simple and computationally efficient star formation scheme [26]. For the CDM models, we assume a flat cosmology with $\Omega_{\Lambda} = 0.692$, $\Omega_{\text{m}} = 0.308$, $\Omega_{\text{b}} = 0.0482$, $h = 0.678$, and a primordial helium mass abundance of 0.24. The CDM grid will consist of 108 models which vary thermal history, amplitude and slope of linear matter power around the reference values of $\sigma_8 = 0.829$, $n_s = 0.961$, and PMF strength that defines the position of the peak on small scales of the linear matter power spectrum (Fig. 3.3).

For each of the simulations we probe the physical parameter space by generating in post-processing a set of $15 \times 10 \times 10$ models spanning mean flux values, temperature at the mean density, power-law index of the gas temperature-density relation. The thermal broadening is parameterized by $T_0(z)$ (K) the IGM temperature at mean density and $\gamma(z)$ (the power-law parameter describing the slope of the temperature-density relation) as independent parameters at each redshift. As the gas is heated during reionization, it hydrodynamically responds to the resulting increase in its temperature and pressure by expanding. The more heat is injected, the more the gas expands, erasing more small-scale structure. In our models we parametrised this effect with the cumulative heat injected per proton, $u_0(z)$ in eV/ m_{proton} . We also include a template for patchy reionization and

quantify the effect of a model dependent resolution correction.

Overall, a suite of 234,000 models for the 1D flux power spectrum for each of the 3 redshift bins probed by the data are fed as an input to our MCMC sampler [135].

3.3.2 1D flux power and thermal histories

In Figure 3.3 we show the PMF impact on the 1D flux power in terms of percentage difference with the reference Λ CDM (same thermal history). The extra power induced by PMFs is very prominent also at the level of 1D flux power and its magnitude and scale dependence can be visually compared with the statistical errors of the data.

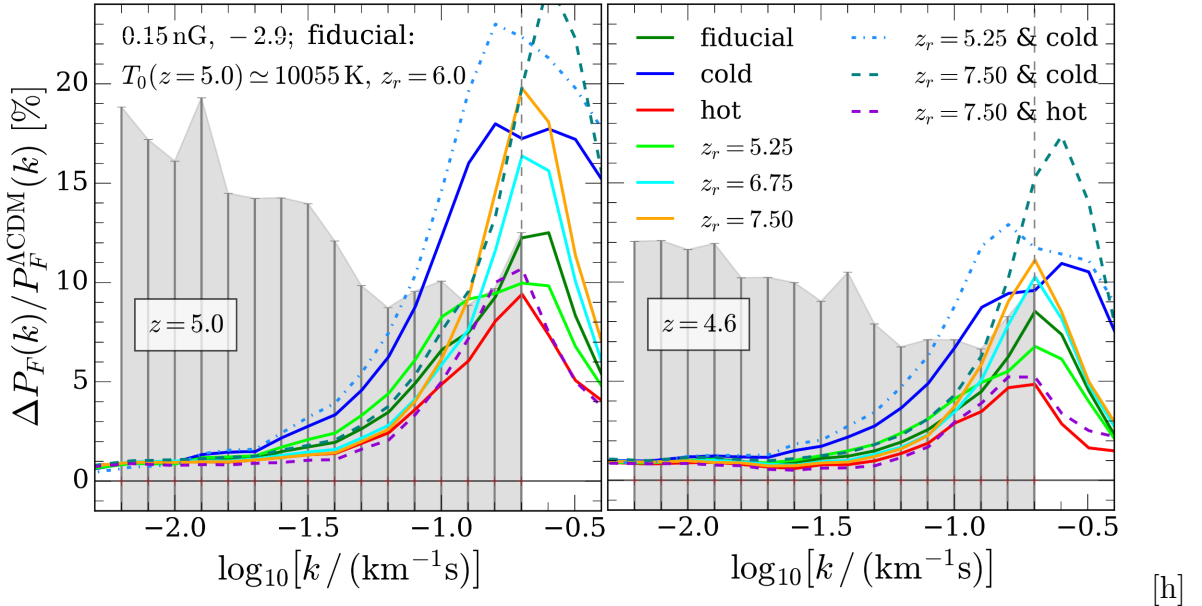


Figure 3.4: Flux power spectra difference between a PMF model, with $B_{1\text{Mpc}} = 0.15$ nG, $n_B = -2.9$, at two different redshifts $z = 5$ and $z = 4.6$, with varying T_0 and z_r . Here $\Delta P_F = P_F^{\text{PMF}} - P_F^{\text{CDM}}$, showing the impact of different thermal and reionization histories on the ratio of flux power spectra, in relation to observational sensitivity. Cold and hot thermal histories correspond to $T_0(z = 5.0) \simeq 6500$ K and $T_0(z = 5.0) \simeq 13560$ K, respectively. The redshift z_r denotes the end of reionization and it is a derived variable linked to u_0 .

In Figure 3.4 it is possible to appreciate the impact of the parameters describing the thermal and physical state of the gas on the 1D flux power by visually comparing a PMF model which is allowed by the data with $B = 0.15$ nG and the reference Λ CDM case without PMFs (for the same thermal history as the PMF model). In our analysis, z_r is the redshift of the end of the reionization and is a derived parameter, since the actual IGM pressure smoothing is fully described by u_0 . In this case z_r describes the 3D pressure

smoothing, while T_0 parameterizes the 1D thermal broadening. The reference parameters are $T_0(z = 4.6, 5) = 10060, 10000$ K, while for cold $T_0(z = 4.6, 5) = 6600, 6570$ K and for hot $T_0(z = 4.6, 5) = 13950, 13560$ K. The mapping between reionization redshift and pressure smoothing is such that, for our fiducial choice of the UV background, $z_r = 5.25, 6, 6.75, 7.50$ correspond to $u_0 = 6.6, 7.7, 9.6, 11.4$ (eV/ m_{proton}) at $z = 4.6$ and $u_0 = 4.7, 6.4, 8.1, 9.9$ (eV/ m_{proton}) at $z = 5.0$.

3.4 Results

We perform a full Monte Carlo Markov Chain (MCMC) likelihood analysis. In order to validate the robustness of our results we apply several different priors and data cuts. We use the following Gaussian priors on $\sigma_8 = 0.811 \pm 0.006$ and on the slope of the linear power spectrum at a pivot scale probed by the data $n_{\text{eff}} = -2.3089 \pm 0.005$, and we checked that this does not impact on our results.

The parameter which parameterizes PMFs is defined as k_*/k_{peak} with $k_* = 10 \text{ Mpc}^{-1}$, this is done in order to have the Λ CDM limit at zero value, when $k_{\text{peak}} \rightarrow \infty$ (similarly to what is done in warm dark matter analyses). We set a conservative flat prior on this parameter in the range $[0, 0.6]$ which allows for only a very small extrapolation from our PMF suite of simulations. For this choice the k_{peak} value in Mpc^{-1} would roughly correspond to the magnitude of the B field in nG.

A key prior is the one on the thermal history of the IGM. In this case, we have two different priors: the fiducial (ref) analysis assumes physical priors on the thermal history in the $u_0 - T_0$ plane as an envelope around our fiducial grid of simulations; the $u_0 - T_0$ priors case refers instead to two independent set of priors on T_0 and u_0 . The difference is that in this latter case the thermal priors have been informed by the measured T_0 evolution with redshift from other observational studies, rather than by our suite of hydrodynamic simulations.

Figure 3.5 shows the marginalized distribution k_*/k_{peak} for the reference case, alternative thermal priors, varying noise, or removal of the four largest value k -bins, leaving only data points at $k < 0.1 \text{ s/km}$. The reference case returns a tentative detection of a peak $k_*/k_{\text{peak}} = 0.188^{+0.094}_{-0.12}$ (2σ C.L.) with a $\chi^2_{\text{PMF}} = 28.63$ for 35 d.o.f. (p -value 0.76). Although the posterior is somewhat flat in the range $[0, 0.1]$, the Λ CDM limit (with no PMFs) is excluded at $\sim 3\sigma$. For comparison, the Λ CDM case gives $\chi^2_{\text{CDM}} = 40.8$ for 36 d.o.f. (p -value 0.27). Thus, the PMF analysis displays a significant $\Delta\chi^2 = -12.2$ improvement with only one extra free parameter. In Figure 3.6, we highlight that the constraints are especially driven by the smallest scales where PMFs clearly provide a better

match to the data. We also show, in order to appreciate the strong impact of PMFs on our observable, a $B = 0.5$ nG model which is extremely disfavoured by our analysis. When we include patchy reionization corrections and resolution corrections that depend on the thermal history we do not observe any appreciable shift in the 1D posterior for k_{peak} .

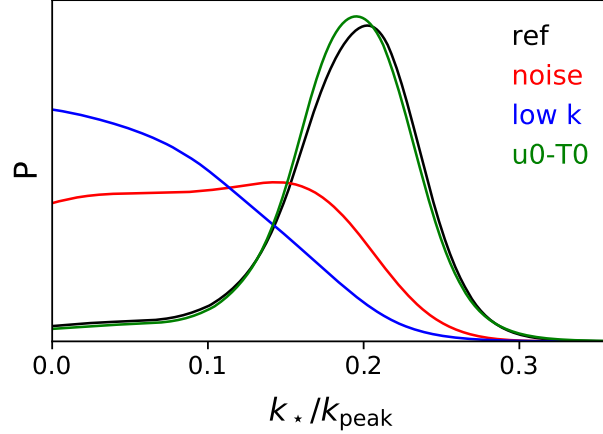


Figure 3.5: Marginalized 1D distribution for k_{peak} for 4 different PMF analyses: reference case (black), with a physical prior on thermal histories; a conservative treatment of the noise (red); without the small scale data (blue) and with separate priors on temperature and pressure smoothing (green).

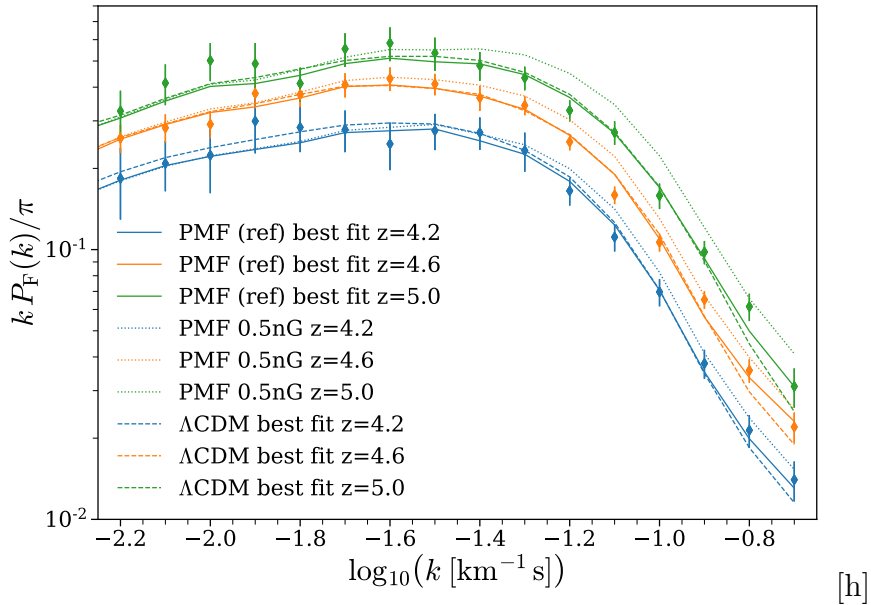


Figure 3.6: Bestfit PMF and Λ CDM models, a PMF model with $B = 0.5$ nG is also shown (all the other parameters being fixed at the best fit of the PMF ref. case).

In order to quantify the robustness of our findings, we also run two extra analyses, explicitly targeted to quantify the crucial role played by the power at small scales.

The first analysis consider a more conservative treatment of the noise, where three noise parameters are added to the theoretical model (one for each redshift bin), and the parameters' priors are assumed to be given by the approximate log-normal model of the noise distribution, which is measured from the data (see [134] for more details). These parameters describe a level of residual noise present in the data, and their marginalized values is larger than zero. In the reference analysis the noise level is instead added to the theory, without extra parameters. When noise is modelled in this way, the 'detection' becomes a robust upper limit $k_*/k_{\text{peak}} < 0.25$ (99.7% confidence level).

If we instead remove the smallest scale data, the bound $k_*/k_{\text{peak}} < 0.23$ (99.7% confidence level) is similar. It is worth stressing that even in this case, the derived limit is extremely competitive, especially when compared with other probes like the CMB upper limits, or complementary to lower limits from blazars. This implies that also the intermediate and large scales of 1D flux power are sensitive to the increase of power in the linear matter power spectrum, making evident the constraining power of the forest.

In all our analyses, we do not find any strong degeneracy between k_{peak} and other parameters, apart from relatively weak degeneracies with the parameters describing the noise and with the injected heat values $u_0(z)$, which is more prominent for $z = 4.6, 5$; see triangle plot below in Fig. 3.7.

These results demonstrate that an overall very conservative upper bound on the strength of PMFs at the Mpc scale that will bracket all our analyses could be safely placed as $B < 0.3$ nG, *since this value is excluded at 3σ also in the reference PMF analysis.*

In Figure 3.7 we show the full triangle plot for the Λ CDM, PMF (ref) and PMF with the conservative noise treatment. In all the three cases we impose a joint prior for $u_0 - T_0$ to follow the theoretical expectation. The shaded bands represent observational constraints obtained in other studies. There is broad agreement between the different analyses, with a tendency for PMF to have slightly higher u_0 values. In this case a higher value would imply a later end of reionization and more IGM pressure smoothing of that will be 'compensated' by the PMFs-induced extra power. The weak degeneracy between u_0 and k_{peak} can be noticed and it will be more prominent if a much wider prior is applied on thermal history (not shown). We find no degeneracies between the 3 extra parameters describing the noise and k_{peak} . As already found by [134], there is also a mild ($\sim 2\sigma$) preference for a larger effective optical depth, τ_{eff} (or equivalently, a ~ 20 per cent lower background photoionisation rate, [136]) for our best fit Λ CDM and PMF models at

$z = 4.2$ compared to the earlier analysis of [129]. The interplay between τ_{eff} values and PMF strength has also been investigated in [124].

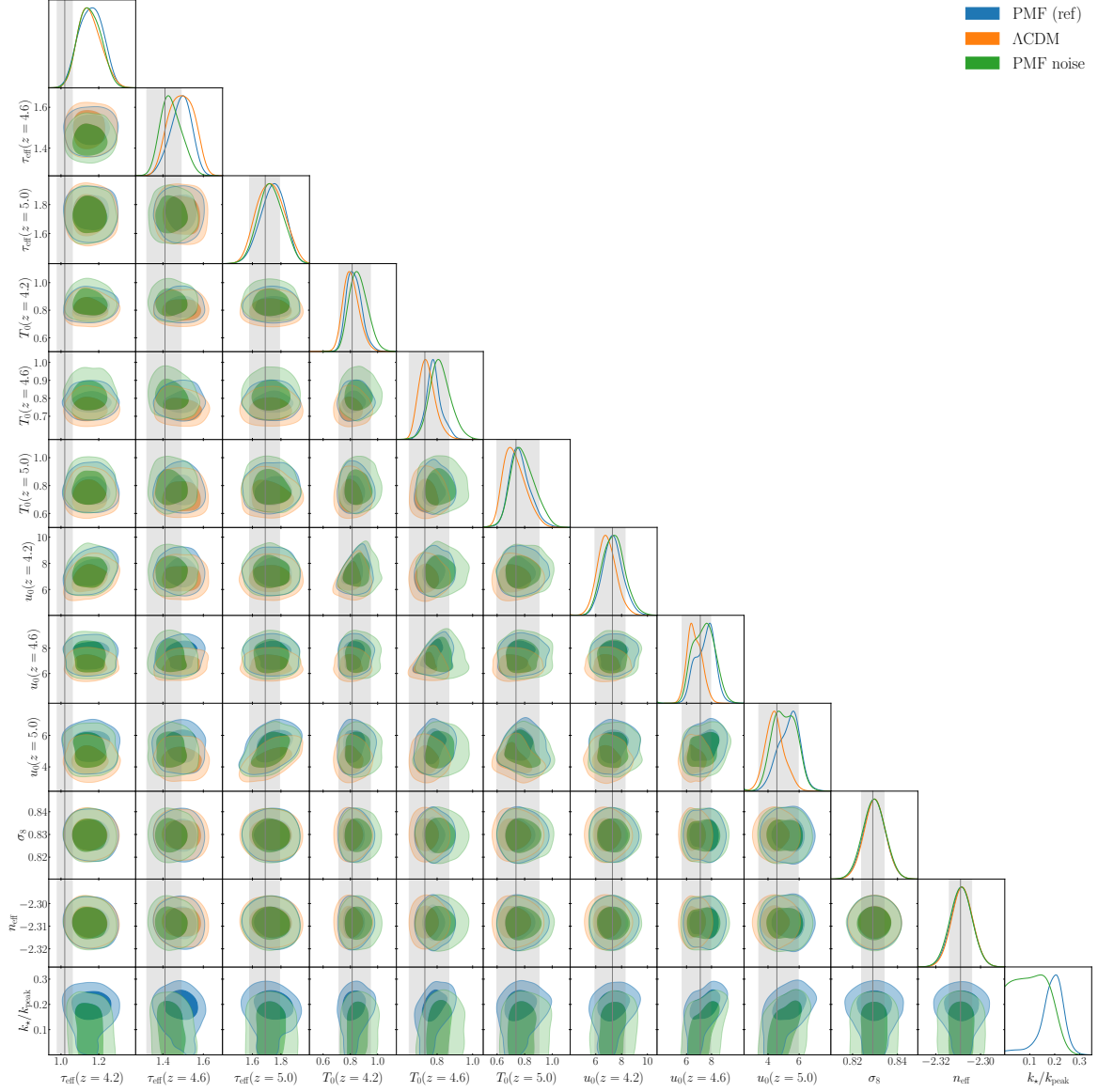


Figure 3.7: Triangle plot for the main analyses performed in this work. The Λ CDM case is shown in orange, reference PMF case in blue and PMF case with a conservative treatment of noise in the data in green (1σ and 2σ contours are shown). There is a trend for the PMF (ref) case to have higher slightly higher values of u_0 , which is the injected heat integrated from early times to $z = 4.2, 4.6, 5$. When compared to the Λ CDM case: this will result in larger pressure smoothing in the 1D flux power that will be compensated by the extra power induced by PMFs. There are no strong degeneracies present between k_{peak} and other astrophysical or cosmological parameters, apart from weak degeneracies with $u_0(z)$. The vertical shaded bands represent observation from independent studies [129].

It is possible to approximately extend our findings to cases for which $n_B \neq -2.9$. In Figure 3.8 we show how the bounds obtained for $n_B = -2.9$ can be extended to other n_B values. These considerations are only approximate since addressing this issue would require a much larger set of hydrodynamic simulations. It is however tempting to provide some semi-quantitative bounds that would apply to a wide range of PMF models. The first consideration that we make is that in this work we are providing a lower bound for a wavenumber k_{peak} which can be readily related to the pair $(B, n_B = -2.9)$ via Eq. 3.16. The second consideration is that the test with two potentially degenerate models in terms of 3D matter power spectrum i.e. $(B(\text{nG}), n_B) = (0.15, -2.9)$ and $(B(\text{nG}), n_B) = (0.015, -2)$, for which we perform the full hydro simulations,² returns 1D flux power that are consistent with each other within the 1σ statistical error bars of the data; see left panel of 3.8. We can thereby claim that our lower bounds on PMFs could be approximately extended to a wider set of PMF models with different n_B . In this case we will obtain the bounds shown in Figure 3.8.

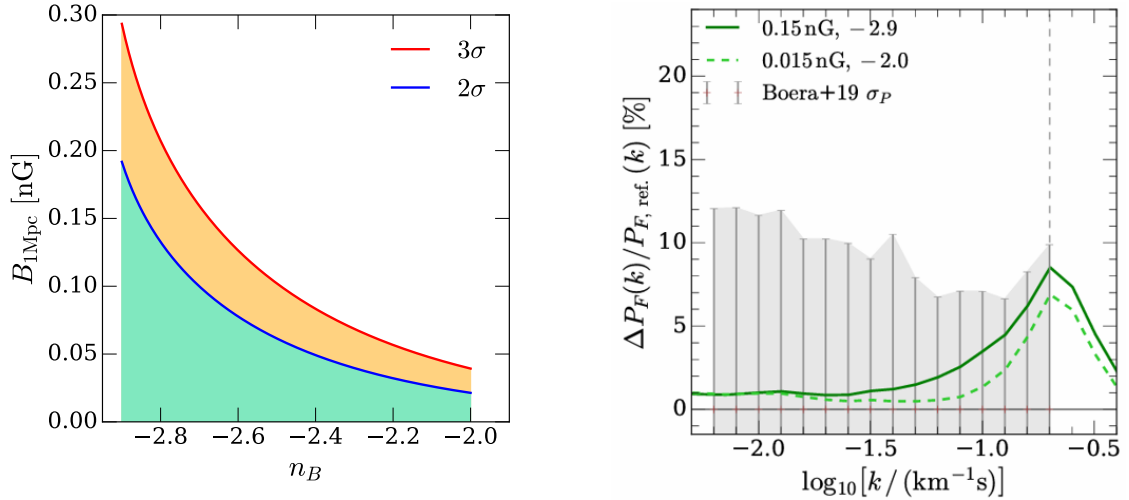


Figure 3.8: **Left:** Approximate limits from our reference analysis for PMFs models whose $\{B, n_B\}$ pairs combine into equal k_{peak} , the colored regions show allowed models within 2σ and 3σ C.L.. **Right:** A working example of two quasi-degenerate PMF models for which simulations and full analysis were employed. Looking at the plot on the left, both models lie under the 2σ curve.

²Another example is provided in the section §3.5, specifically Fig. 3.9, where quasi-degeneracy in terms of the flux power can be seen for pairs $P_1 := (0.1 \text{ nG}, -2.9)$ and $P_2 := (0.01 \text{ nG}, -2)$. This is because P_2 is similar to the model with $(0.008 \text{ nG}, -2)$ indicated in Fig. 3.3 as the model with equal k_{peak} as P_1 .

3.5 Separate transfer functions for baryons and dark matter: advantages and difficulties

*Hide nothing from the masses of our people. **Tell no lies.** [...] Mask no difficulties, mistakes, failures. **Claim no easy victories.***

— Amílcar Cabral (1965, directives for PAIGC, source: *Revolution in Guinea*, 1974)

It was briefly mentioned in the Section 3.3 that *the initial power spectrum is generated at $z = 99$, with the same transfer functions for baryons and dark matter.* The main reason for this choice is that one- $P(k)$ -for-all is a common, robust practice in Ly- α studies, and has been well tested; the second reason is that our constraints can be readily compared to the majority of constraints in the literature, which in their theory and codes use the common power spectrum for baryons and dark matter in the initial conditions; and the third reason emanates from the discussion that follows.

The original endeavor of our work [2] was to find PMF constraints from the Ly- α forest while using modified initial conditions from our preceding study [1], where PMF-induced spectra differ for gas and DM. However, we were unable to immediately interpret certain new features of the resulting flux power spectra.

To better explain this issue, in Fig. 3.9 we show redshift evolution of flux spectra of three PMF models (two of which were used in our constraints work, 0.1 nG and 0.2 nG), from $z = 6$ down to $z = 4$. One immediately notices a development of a local minimum, a ‘valley’ hereafter, from redshift $z \sim 5.4$ onward. It appears for all PMF models at the same $k \simeq 10^{-1}$ s/km. As a result, the flux power of PMF+ Λ CDM models ends up below the Λ CDM at small scales — instead of enhancement of power, we get suppression!

We ran several tests to explain this new feature in the flux power. First, we compared simulations with box-size and mesh $(L, N) = 20, 1024$ to simulations $(L, N) = 10, 512$ (equal resolution) & $(L, N) = 20, 512$ (lower resolution). In Fig. 3.10 we can see that equal (high) resolutions result in the same valley (red and yellow lines for $B = 0.01$ nG). Lower resolution, however, does not develop the valley (exemplified with blue and green lines, for $B = 0.2$ nG).

Next, we proceeded with a test in which we take the temperature and peculiar velocity of Λ CDM simulations and insert them into the computation of flux power for the PMF case. The result is in Fig. 3.11. One notices immediately that replacement of T does not remove the valley, while $v_{\text{pec.}}$ does the job. Nonetheless, we do not think that the reason is truly physical.

We believe the reason is a numerical artifact; there are two candidates for this: 1) the initial power spectra tend to develop a plateau if their peak is far from k_{Nyquist} (see [1]), 2) the grid on which the particles are initiated creates preferred directions.

Ad 1): After carefully comparing power spectra from initial conditions (both input and output) we deduce that is possible due to the plateau at values $\Delta_b^2 \simeq 1$ and the non-linear dependence of the 1D baryon power spectrum (related to flux spectrum) on the 3D spectrum, that valley appears into the flux power. To confirm this, we would need to perform more simulations where we follow the 3D and 1D power spectra in real time. On the other hand, ref. [85] argues that the plateau is inevitable even when MHD is employed, the turbulence does not suppress the power after the damping scale.

Ad 2) When we implement PMFs explicitly into ICs, we need to use one particle-grid $(I, J, K)_B$ for their computation-initialization. The other two particle grids are for gas and dark matter. This setup we call 3-grid. In this case, in N-GENIC the dark matter particles are shifted *before* the power spectra are applied to ICs, in a specific way

$$(I, J, K)_{\text{dm}} = (i, j, k)_{\text{gas}} + 0.5(d, d, 0),$$

where $d = \pi/k_{\text{Nyq}}$ is the interparticle distance. In contrast, for 2-grid (only gas and DM)

$$(I, J, K)_{\text{dm}} = (i, j, k)_{\text{gas}} + 0.5(d, d, d).$$

The results and plots we showed in this section are done on the 3-grid. Unfortunately, we do not have enough simulations with the 2-grid and separate transfer functions to compare them. It is possible that these shifts create artifacts in the velocity and therefore in the flux power — a study by Khan et al. [137] explores exactly this issue. They report a distortion which is close to 50% at $k \simeq 10^{-1}$ s/km, the k we quoted above. They also show that removing the valley is possible if v_{pec} is substituted from simulations which use identical transfer functions for gas and DM.

We conclude therefore, at least for now, that it is due to the shifts in the initialization particle-grid that the valley appears in the flux power.

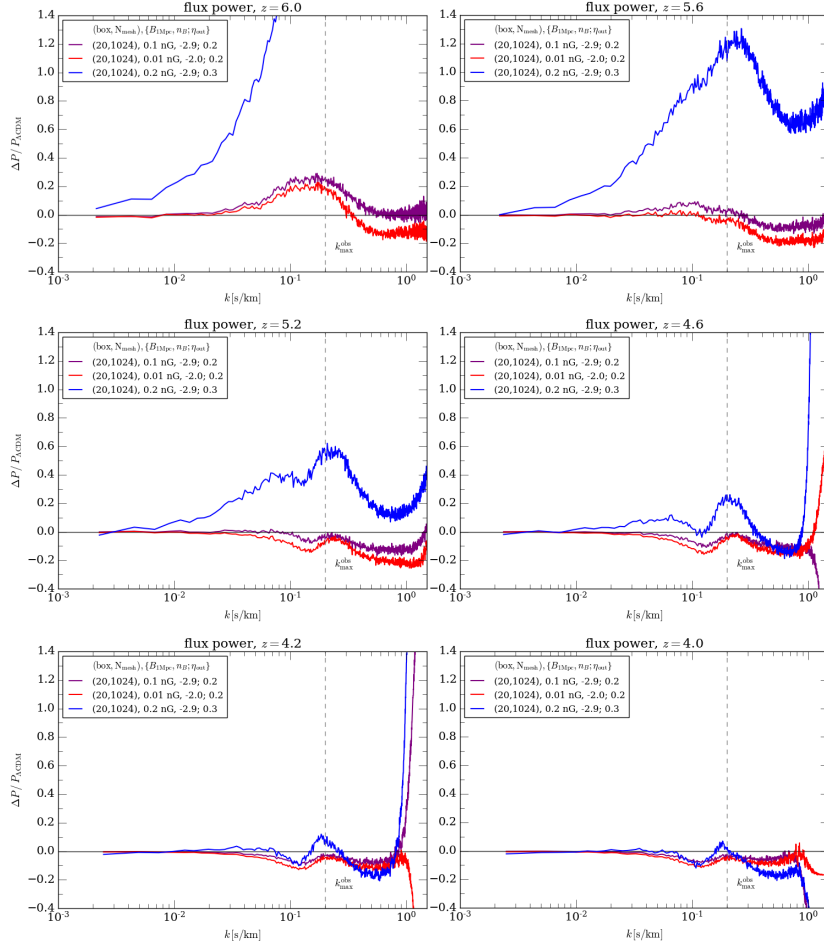


Figure 3.9: Redshift evolution of Ly- α flux power for 3 PMF models in the scenario where different transfer functions are applied to gas and dark matter in the initial conditions.

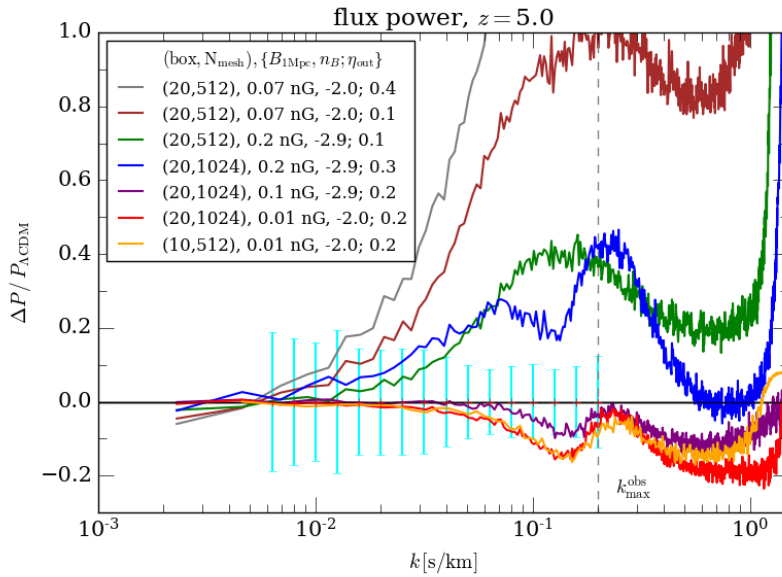
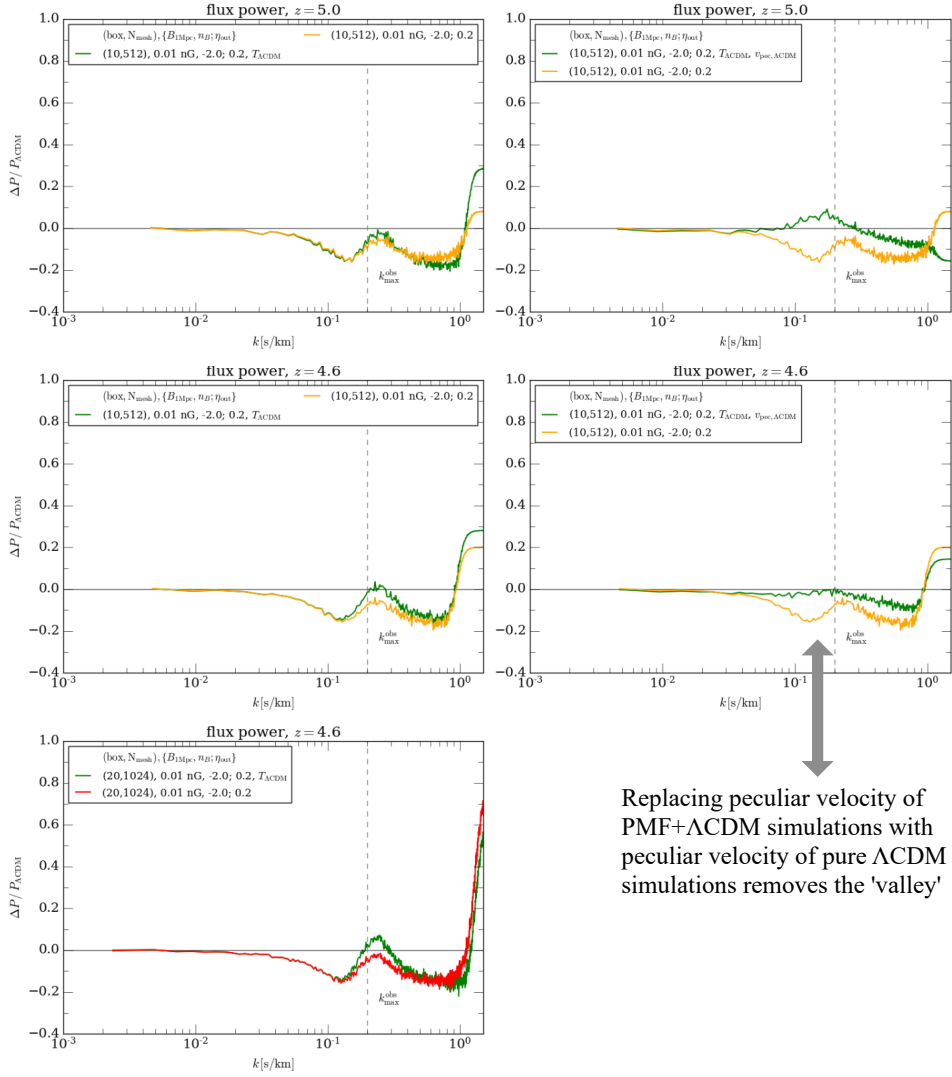


Figure 3.10: Ratios of flux power spectra (w.r.t. corresponding Λ CDM simulations). Cyan errorbars are from Boera+19 [129], and are placed on the x-axis to give a range of sensitivity.



Replacing peculiar velocity of
PMF+ ΛCDM simulations with
peculiar velocity of pure ΛCDM
simulations removes the 'valley'

Figure 3.11: Examples of tests performed to identify how the local minimum (valley) can be neutralized.

3.6 Summary and conclusions

We have performed an analysis of the 1D Lyman- α flux power with a suite of hydrodynamic simulation which incorporate the PMF-induced increase of power in the linear matter power spectrum. The data displays extra power when compared to the standard Λ CDM analysis without PFMs, which is well fitted by a PMF model $B = 0.2 \pm 0.05$ nG (1σ) and $n_B = -2.9$.³ The quoted value is rounded, see Table below. However, when a very conservative modeling of the noise level is applied or when removing the smallest scales data, this bound becomes an upper limit $B < 0.3$ nG (3σ). These results are stable also once different priors on thermal histories and cosmological parameters, patchy reionization, and resolution corrections are considered (see Figs. 3.4 & 3.7). We emphasize that the total bounds we choose to quote as the main result are quite safe, as they are compatible with all the posterior results. In total, we can summarize the limits as follows (for nearly scale-invariant case $n_B = -2.9$):

setup	$(10 \text{ Mpc}^{-1})/k_{\text{peak}}$	$k_{\text{peak}} [\text{Mpc}^{-1}]$	$B_{1\text{Mpc}} [\text{nG}]$
reference PMF	$0.188^{+0.094}_{-0.120} (2\sigma)$	$53^{+94}_{-18} (2\sigma)$	$0.18^{+0.10}_{-0.12} (2\sigma)$
additional noise modeling	$< 0.25 (3\sigma)$	$> 40.0 (3\sigma)$	$< 0.24 (3\sigma)$
removing last 4 k -bins from data	$< 0.23 (3\sigma)$	$> 43.5 (3\sigma)$	$< 0.22 (3\sigma)$

These limits improve the bounds obtained from a combination of CMB data with the inclusion of PMF impact on ionization history by a factor three [86] and by an order of magnitude the limits obtained from CMB data alone [138].

Earlier Ly α -based constraints were obtained by Pandey & Sethi [125] using the effective optical depth and a semi-analytical model of the forest. For $n_B \simeq -2.9$, they obtained $B_0 \lesssim 0.6\text{--}0.7$ nG at 5σ , numerically about a factor of two weaker than our conservative $B_{1\text{Mpc}} < 0.30$ nG (3σ) limit, although the confidence levels and modeling of the magnetic damping scale differ. Chongchitnan & Meiksin [124] similarly found that fields $B_1 \gtrsim 0.3$ nG for $n_B = -2.99$ can significantly modify the Ly α effective optical depth, but emphasized its strong degeneracy with the metagalactic photoionization rate and did not derive a formal likelihood bound on the PMF amplitude.

As we discussed in the results section, through the triangle plot (Fig. 3.7 we showed the values found for additional parameters in the model are consistent with measurements; nevertheless, we notice that one exception is mean flux (τ_{eff}), especially at $z = 4.2$. Although the offset is not excluded, it is worth remarking that the *models with PMF*

³It should be noted that in the Abstract of our paper [2], it says that $B = 0.2 \pm 0.05$ nG result corresponds to $k_{\text{peak}} \simeq 20 \text{ Mpc}^{-1}$. This is a mistake, which appears only in the Abstract, and we apologize for the error. The correct rough value is of course $k_{\text{peak}} \simeq 50 \text{ Mpc}^{-1}$, or $k_{\text{peak}} \simeq 70 h/\text{Mpc}$, as can be seen in Fig. 3.3.

prefer higher τ_{eff} . This is in agreement with insights from [124], who interpreted this as a need for a higher ionizing background. In future work, we will attempt to give an estimate for how high the ionizing background must be increased to bring the peak τ_{eff} at $z = 4.2$ to the measured value.

Additionally, a more accurate estimate of the damping scale as well as of the baryon and dark matter transfer functions could further affect our results [85]. In this work we neglect the additional heating of IGM that could be produced by ambipolar diffusion and/or MHD turbulent decay (see, for example, [83, 139–142]). This extra heating also impacts the ionization history, potentially influencing the Ly- α forest. The results of [102, 141] indicate that for our relevant values of PMFs, $B \sim 0.1 - 0.3$ nG, the change in temperature will be at most of the order ~ 100 K, leaving the flux power of the forest essentially unaffected.

Our measurement would have exciting consequences in terms of high-redshift galaxy formation and galactic feedback: the dark matter halo mass function will be enhanced below the mass of $10^9 M_{\odot}/h$, leading to earlier formation of galaxies. This would have a measurable impact on the interpretation of high-redshift galaxy luminosity functions as observed by the JWST. In order to discriminate between a bound and a detection, it will be extremely important to further characterize the smallest scales of the 1D flux power by collecting more data at high signal-to-noise and quantify the level of metal contamination. This effort is likely to be achieved easily and on a short time-scale by incrementing the number of high-redshift quasars and increasing the signal-to-noise level to reduce the noise power, using spectrographs like ESPRESSO⁴ [143].

⁴To entertain our reader after three chapters: since our research institute is located in Trieste, one of few Italian places where caffè espresso is called “nero” (*black* coffee) or one could order the famous “capo in B” (small cappuccino in a glass [Bicchiere]), we should consider naming future observations or simulation codes in honor of these beverages that keep us sharp, awake and unified. CAPO-in-B could certainly have usage in PMF research community, if B stands for the magnetic field amplitude.

Chapter 4

Impact of PMFs on the UV luminosity function and reionization

The analysis presented in this chapter should be regarded as work in progress. Our aim here is to establish the physical and numerical framework connecting PMF-induced small-scale power to the high-redshift UV luminosity function and reionization, and to identify the observables and astrophysical uncertainties that will be most relevant for a quantitative analysis. The next step is to turn the present qualitative comparison into a statistical inference of the PMF and galaxy-formation parameters. The same framework can subsequently be extended to include reionization observables, in particular measurements of the neutral hydrogen fraction and the Thomson optical depth. These developments form part of an ongoing study for which a new publication is currently being prepared.

4.1 Introduction

Primordial magnetic fields modify the formation of cosmic structures through the additional density perturbations generated by the Lorentz force acting on the baryonic component. As discussed in Chapters 1 and 2, the resulting matter power spectrum can exhibit a pronounced enhancement on sub-Mpc scales. This enhancement increases the abundance of low-mass dark matter haloes at high redshift and can therefore propagate to observables related to the first galaxies, their star-formation activity, and ultimately the reionization of the intergalactic medium.

The ultraviolet luminosity function (UV LF) provides a particularly direct connection between these scales. Rest-frame ultraviolet emission is dominated by short-lived massive stars and therefore approximately traces the recent star-formation rate of a galaxy. At the same time, the abundance of galaxies hosting these stars is controlled by the halo mass

function (HMF), which depends on the matter power spectrum through the mass variance $\sigma(M, z)$. A modification of small-scale power can consequently produce an observable modification of the UV LF.

This chain of dependencies makes high-redshift galaxy observations a natural probe of primordial magnetism:

$$P_m(k) \longrightarrow \sigma(M, z) \longrightarrow \frac{dn_h}{dM_h} \longrightarrow \dot{M}_h \longrightarrow \dot{M}_\star \longrightarrow L_{UV} \longrightarrow \Phi_{UV}. \quad (4.1)$$

Unlike the Ly α forest considered in Chapter 3, which probes the density and thermal state of diffuse intergalactic gas, the UV LF tests the same underlying PMF-induced enhancement through collapsed objects and star formation. These two observables are therefore highly complementary.

The situation has become particularly interesting with the advent of JWST. The measured abundance of luminous galaxies at $z \gtrsim 10$ is higher than predicted by several early galaxy-formation models, although increasingly flexible treatments of star formation, stellar populations and feedback can reproduce the observations within standard Λ CDM [144]. Consequently, the high-redshift UV LF should not by itself be interpreted as evidence for additional primordial power. Rather, it provides an opportunity to constrain any additional contribution, such as that induced by PMFs, once the astrophysical freedom is properly marginalized over.

In this chapter we first introduce the relation between the HMF and the UV LF and apply it to PMF-modified matter power spectra. We then compare the resulting luminosity functions with HST and JWST measurements and discuss the corresponding constraints on primordial magnetism. Particular attention is paid to theoretical uncertainties associated with star formation, baryonic accretion and the PMF-induced matter power spectrum. Finally, we consider the consequences for cosmic reionization and discuss how the UV LF can be combined with Ly α forest measurements, neutral-fraction constraints and future 21-cm observations.

4.2 UV Luminosity Function with PMFs

4.2.1 From halo abundance to UV luminosity

The UV luminosity function is defined as the comoving number density of galaxies per luminosity interval,

$$\Phi_{UV}(L_{UV}, z) \equiv \frac{dn_g}{dL_{UV}}, \quad (4.2)$$

or, more commonly, per interval in absolute AB magnitude,

$$\Phi_{\text{UV}}(M_{\text{UV}}, z) \equiv \frac{dn_g}{dM_{\text{UV}}}. \quad (4.3)$$

For the rest-frame specific luminosity L_ν , the absolute UV magnitude may be written as

$$M_{\text{UV}} = M_{\text{ref.}} - 2.5 \log_{10} \left(\frac{L_\nu}{\text{erg s}^{-1} \text{ Hz}^{-1}} \right). \quad (4.4)$$

At lower redshift the UV LF is often represented by a Schechter function,

$$\Phi(M_{\text{UV}}) = 0.4 \ln(10) \phi_\star 10^{-0.4(M_{\text{UV}} - M_\star)(\alpha+1)} \exp \left[-10^{-0.4(M_{\text{UV}} - M_\star)} \right], \quad (4.5)$$

where $\phi_\star$ is the characteristic number density, $M_\star$ defines the knee of the luminosity function, and α is the faint-end slope. At the highest redshifts a double-power-law description is also frequently adopted, since the observed bright end declines less rapidly than the exponential Schechter form.

Observations determine Φ_{UV} directly from galaxy samples. The connection to dark matter haloes is a theoretical step. In the simplest deterministic model, where a halo of mass M_h is assigned a unique luminosity, one obtains

$$\Phi_{\text{UV}}(M_{\text{UV}}, z) = f_{\text{duty}}(M_h, z) \frac{dn_h}{dM_h} \left| \frac{dM_h}{dM_{\text{UV}}} \right|, \quad (4.6)$$

where f_{duty} denotes the fraction of haloes hosting an actively star-forming galaxy.

More generally, scatter in the galaxy–halo relation can be included as

$$\Phi_{\text{UV}}(M_{\text{UV}}, z) = \int dM_h \frac{dn_h}{dM_h} P(M_{\text{UV}} | M_h, z), \quad (4.7)$$

where $P(M_{\text{UV}} | M_h, z)$ contains the stochasticity of star formation as well as, if desired, dust attenuation and observational scatter. Equation (4.6) follows in the limit in which this distribution becomes a delta function.

For the computation performed here we follow the semi-analytic framework of Refs. [145, 146]. The star-formation rate is related to the baryonic accretion rate through

$$\dot{M}_\star = f_\star(M_h, z) \dot{M}_b, \quad (4.8)$$

where $f_\star$ is the star-formation efficiency. In the standard approximation,

$$\dot{M}_b = \frac{\Omega_b}{\Omega_m} \dot{M}_h. \quad (4.9)$$

The intrinsic UV luminosity is then proportional to the recent star-formation rate,

$$L_{UV} \propto \dot{M}_\star \propto f_\star(M_h, z) \dot{M}_h. \quad (4.10)$$

The PMF contribution therefore affects the UV LF twice: primarily through the abundance of haloes, but also through their typical growth and accretion histories.

4.2.2 The PMF contribution

For uncorrelated adiabatic and magnetically-induced perturbations, the matter power spectrum can be schematically decomposed as

$$P_m(k, z) = P_{\Lambda\text{CDM}}(k, z) + P_{\text{PMF}}(k, z). \quad (4.11)$$

As described in subsection 1.2.4, P_{PMF} grows rapidly towards small scales before reaching a characteristic maximum associated with magnetic damping and non-linear backreaction. The corresponding increase of $\sigma(M, z)$ primarily affects low-mass haloes.

The effect on the UV LF is therefore qualitatively different from a simple overall change in its normalization. At fixed astrophysical parameters, PMFs preferentially increase the abundance of faint galaxies residing in low-mass haloes. The relative enhancement also becomes larger with increasing redshift because these haloes correspond to progressively rarer peaks in the underlying density field. Consequently, the most sensitive regime is expected at the combination of faint luminosities and high redshifts.

There is nevertheless an important degeneracy with galaxy astrophysics. A decrease in $f_\star$ at low halo masses, a lower duty cycle, stronger feedback, or changes in the scatter of the luminosity-halo relation can partly compensate the excess of low-mass haloes generated by PMFs. Conversely, redshift-dependent changes in stellar populations can enhance the UV luminosity at fixed halo mass without requiring any modification of the matter power spectrum. Robust constraints on PMFs therefore require the astrophysical parameters to be marginalized over rather than fixed to their low-redshift values.

4.2.3 Baryonic accretion and star-formation efficiency

An additional effect is particularly relevant for the PMF scenario considered in this thesis. Equation (4.9) assumes that the baryonic mass associated with a growing halo is always the universal fraction Ω_b/Ω_m . However, the simulations presented in previous chapters show that PMF-induced perturbations initially act more strongly on baryons than on dark matter, producing an enhanced baryon fraction in low-mass haloes at high redshift.

If (recall Fig. 1.9 to see how baryon fraction changes)

$$M_b = f_b(M_h, t) M_h, \quad (4.12)$$

then the baryonic accretion rate is instead

$$\dot{M}_b = f_b \dot{M}_h + M_h \left[\frac{\partial f_b}{\partial t} + \frac{\partial f_b}{\partial M_h} \dot{M}_h \right], \quad (4.13)$$

or, equivalently,

$$\frac{d \ln M_b}{dt} = \frac{\partial \ln f_b}{\partial t} + \left(1 + \frac{\partial \ln f_b}{\partial \ln M_h} \right) \frac{d \ln M_h}{dt}. \quad (4.14)$$

Thus PMFs may affect the predicted luminosity function not only by changing the number of haloes, but also by changing the amount of gas available to form stars in each halo.

In the present semi-analytic calculation we initially retain the standard relation in Eq. (4.9). This choice provides a useful controlled comparison in which only the modified matter power spectrum is propagated into the UV LF. It should not, however, be interpreted as the complete PMF prediction. A more self-consistent calculation would calibrate

$$f_b = f_b(M_h, z; B_{1\text{Mpc}}, n_B) \quad (4.15)$$

directly from hydrodynamical simulations and use Eq. (4.13) when computing the star-formation rate.

Whether the resulting increase in available gas translates linearly into additional UV emission is itself an astrophysical question. Magnetic pressure, cooling, feedback and the structure of the interstellar medium can alter the efficiency with which the additional gas forms stars. Therefore f_b and $f_\star$ should ultimately be calibrated separately rather than absorbing both effects into a single effective star-formation efficiency.

A second theoretical uncertainty concerns the normalization of the PMF-induced power close to its maximum. As discussed previously, the position of the peak and the magnitude of the enhancement depend on the modeling of magnetic damping and non-linear backreaction. It is therefore useful to consider η defined in Eq. (1.123), and test the

sensitivity of the UV LF to variations in η . We could work as well with the peak value $\Delta_b^2(k_{\text{peak}})$, connecting the strategy with our Ly- α study.

4.2.4 Qualitative results for the Galaxy Luminosity Function

For the computation of the UV LF we follow the semi-analytic procedure described above and in Appendix B. Figure 4.1 shows the predicted luminosity functions at several redshifts together with HST and JWST measurements. All PMF models shown (0.2, 0.5, 0.8 nG) are with $n_B = -2.9$.

The physical trend can be understood from Eq. (4.1). A PMF-induced increase of the variance $\sigma(M, z)$ produces additional low-mass haloes and therefore raises the predicted number density of galaxies in the corresponding luminosity range. The fractional effect becomes increasingly important at higher redshift, where these haloes lie further into the exponentially suppressed tail of the standard Λ CDM HMF.

In Fig. 4.1 we can see that PMF models with $B \geq 0.5$ nG could be easily excluded with data at redshifts $z \gtrsim 16$, while at lower redshifts the data would need to reach more fainter galaxies. The situation is interesting for the $B = 0.2$ nG model, which seems to be in a very good agreement with data for all the covered redshift intervals. Considering that in our Ly- α analysis in §3 the model with $B = 0.2$ nG was preferred over Λ CDM, or at very least, well within final constraints, it motivates us to think towards a joint UVLF+Ly- α +reionization analysis.

What can also be seen in Fig. 4.1 is that the semi-analytic galaxy LF of PMF models depart from Λ CDM at different magnitudes. This is because these models have linear power spectra that depart from Λ CDM at different scales ($k_{\text{dep.}}$) which translates into a threshold mass for each model, and therefore UV luminosity and magnitude as well.

As is apparent from appendix B or other studies, in the MCMC analysis we would have to work with 4 astrophysical parameters of the UV LF; among which we can emphasize the faint-end slope and star formation efficiency.

This behaviour also identifies the region of the UV LF with the greatest constraining power. Measurements of faint galaxies directly probe the low-mass haloes most affected by PMFs, whereas increasing the redshift amplifies the relative departure from Λ CDM. In practice these two advantages must be balanced against rapidly decreasing survey volume, spectroscopic completeness and increasing contamination at extreme redshift. For present data, pushing reliably to fainter magnitudes at $z \simeq 8$ –12 may therefore be at least as informative as relying on a small number of photometric candidates at $z \gtrsim 15$.

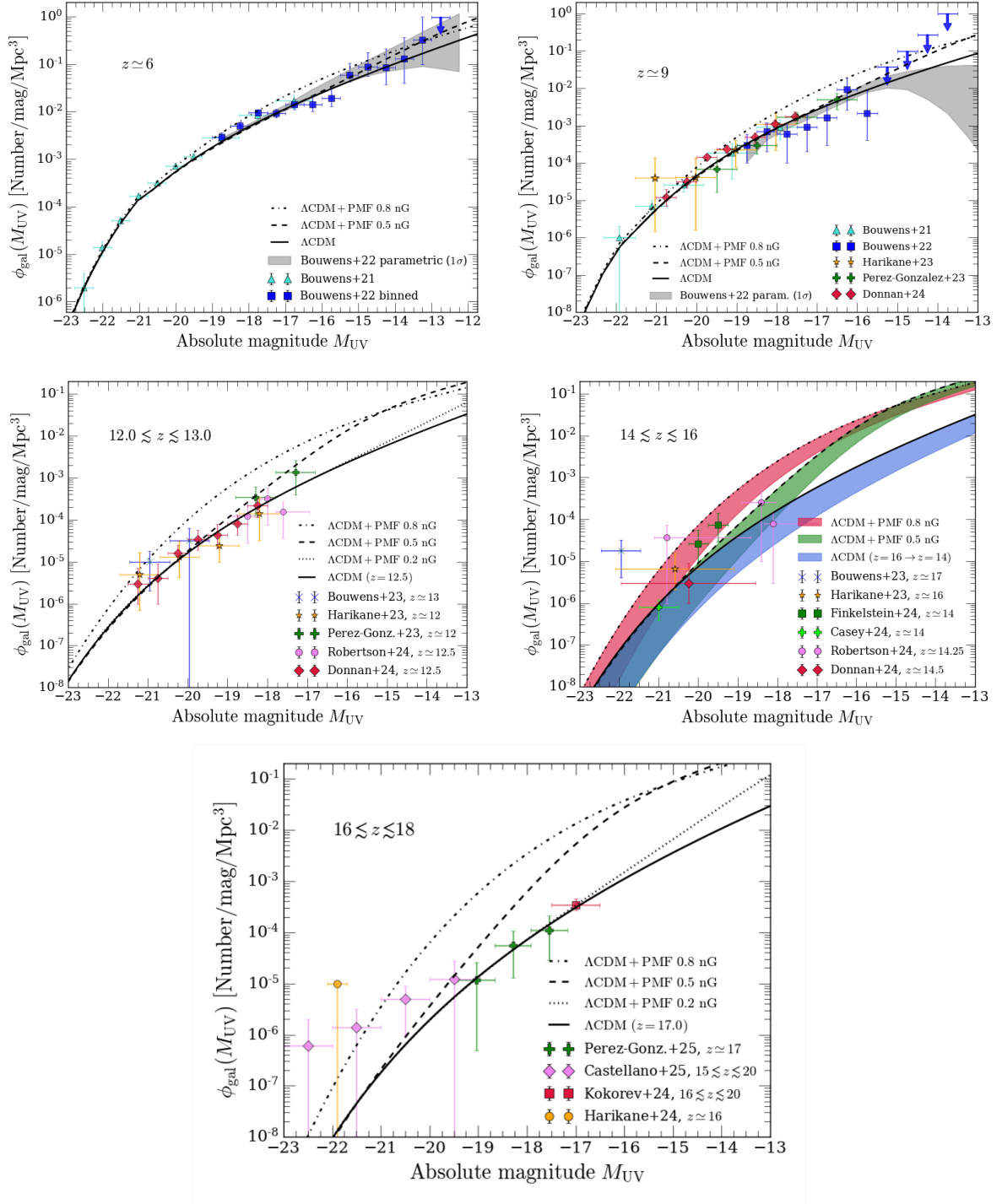


Figure 4.1: GALAXY LUMINOSITY FUNCTION WITH PMFs. At various redshifts, we compare the semi-analytic computation of the UV Luminosity Function with data points from *HST*: Bouwens+21 [147], Bouwens+22 [148], and *JWST*: Bouwens+23a, 23b [149, 150], Harikane+23 [151], Pérez-González+23 [152], Donnan+24 [153], Robertson+24 [154], Finkelstein+24 [155], Casey+24 [156], Harikane+24 [157], Kokorev+24 [158], Pérez-González+25 [159], Castellano+25 [160].

4.2.5 Constraints on PMFs from JWST

High-redshift galaxy measurements have only recently begun to be used as direct constraints on primordial magnetic fields. Zhang et al. [75] considered the cumulative stellar mass density at $6 < z < 10$. Their calculation follows the sequence

$$P_{\text{PMF}}(k) \rightarrow \text{HMF} \rightarrow \rho_*, \quad (4.16)$$

and uses the relation

$$M_* = f_* \frac{\Omega_b}{\Omega_m} M_h. \quad (4.17)$$

Allowing both the PMF amplitude and spectral index to vary, they obtained

$$B_{1\text{Mpc}} < 4.44 \text{ nG}, \quad n_B < -2.24 \quad (4.18)$$

at 95% confidence. For fixed $n_B = -2.90$ the corresponding constraint is $B_{1\text{Mpc}} < 2.21 \text{ nG}$. These results demonstrate the sensitivity of high-redshift galaxies to PMF-enhanced structure formation, although their assumption of a universal halo baryon fraction neglects the additional baryonic effect described in §4.2.3.

A more direct UV LF analysis has recently been performed by Fairbairn et al. [76]. They combine HST and JWST UV luminosity functions with a flexible star-formation prescription and a PMF matter power spectrum calibrated against MHD simulations. For representative spectral indices $n_B = -2$ and $n_B = 2$, they find the UV LF limits

$$\sqrt{\langle B^2 \rangle} < 0.87 \text{ nG} \quad (n_B = -2), \quad (4.19)$$

and

$$\sqrt{\langle B^2 \rangle} < 0.85 \text{ nG} \quad (n_B = 2), \quad (4.20)$$

at 95% credible level. Here $\sqrt{\langle B^2 \rangle}$ denotes the rms field integrated over the relevant magnetic spectrum and should not be identified directly with $B_{1\text{Mpc}}$ without specifying the damping scale.

The principal effect again appears at the faint end of the UV LF. Stronger PMFs increase the abundance of the lowest-mass haloes, and part of this effect can be compensated by changing the low-mass dependence of the SFR. The data nevertheless permit only a finite amount of such compensation. Importantly, the analysis finds no requirement for the PMF-induced excess: within a sufficiently flexible star-formation model, the current UV LF is well described by standard CDM.

This conclusion illustrates an important distinction between *explaining* an unexpect-

edly large galaxy abundance with new primordial power and *constraining* such power. The latter is more robust. Once astrophysical parameters are allowed to absorb reasonable changes in stellar populations and feedback, PMFs can be bounded by requiring that they do not produce more faint galaxies than observed.

An important refinement was introduced by Facchinetti et al. [94]. In addition to the enhancement of structure formation, they follow the loss of magnetic energy through ambipolar diffusion and decaying MHD turbulence from recombination through reionization. They write the comoving field evolution schematically as

$$\frac{B(z)}{(1+z)^2} = \chi_B(z) \frac{B(z_{\text{rec}})}{(1+z_{\text{rec}})^2}, \quad (4.21)$$

where $\chi_B = 1$ corresponds to pure flux freezing, whereas $\chi_B < 1$ describes the reduction of magnetic energy by dissipation.

This distinction is relevant when comparing constraints quoted at recombination with a present-day smoothed amplitude. In their notation,

$$\sigma_{B,0} = s_B \chi_B(0), \quad (4.22)$$

and the two quantities can differ appreciably once magnetic energy losses are included.

Facchinetti et al. implement the coupled evolution of PMFs, the IGM temperature and ionization fraction in `HyRec` and `exo21cmFAST`, and perform an MCMC analysis combining UV LF and reionization information. They find that consistently including PMF energy losses substantially relaxes constraints compared with treatments that assume the comoving magnetic amplitude remains constant. For the same n_B , their bounds are approximately an order of magnitude weaker than those obtained in the earlier 21-cm/reionization analysis of Cruz et al. The constraint at $n_B = -2.9$ is nevertheless of the same order as our independent Ly α forest result [2].

It is worth noting that the UV LF likelihood used by Facchinetti et al. is based on HST rather than JWST measurements. The authors explicitly avoid the JWST bright-end data because reproducing them would require extensions of the stellar model employed in `21cmFAST`. This highlights again that the limiting uncertainty is increasingly not the measurement of the UV LF itself, but the astrophysical mapping between halo mass and luminosity.

4.2.6 New observations

The observational situation at the highest redshifts continues to evolve rapidly. Increasing survey area and depth have significantly improved the UV LF at $z \simeq 9\text{--}15$, but the distinction between photometrically selected candidates and spectroscopically confirmed galaxies becomes essential at still higher redshift.

For example, the JADES analysis of Whitler et al. [161] obtains UV luminosity functions from several hundred candidates at $z \simeq 9\text{--}16$, reaching approximately $M_{\text{UV}} \simeq -17$. They find both a high normalization and a steep faint-end slope, while finding no candidates in their search at $z \simeq 16\text{--}22.5$.

By contrast, Pérez-González et al. [159] identify a small sample of F200W- and F277W-dropout sources whose photometric solutions place them in the range $16 < z < 25$. These objects should at present be described as *high-redshift galaxy candidates*. Their inferred UV luminosity functions at $z \simeq 17$ and $z \simeq 25$ are potentially extremely interesting for small-scale structure, but their use for precision PMF constraints should await stronger spectroscopic confirmation and a better quantification of the selection function and low-redshift contaminants.

From the point of view of primordial magnetism, the principal value of future JWST observations is therefore not simply reaching the largest possible redshift. Deep measurements of the faint-end luminosity function at redshifts where robust samples and spectroscopic calibration are available directly target the low-mass haloes in which the relative PMF effect is largest. Very high-redshift candidates provide an additional, high-leverage test, but should currently be treated separately rather than combined indiscriminately with the lower-redshift UV LF likelihood.

4.3 Relation to recent studies and modeling uncertainties

Several recent studies have used high-redshift structures to constrain additional small-scale power. Besides the PMF-specific analyses discussed above, these works are useful because they reveal the astrophysical degeneracies that any PMF analysis must confront.

Sanati et al. [92, 93] used cosmological simulations of dwarf-galaxy formation and showed that sufficiently strong PMF-induced perturbations produce earlier halo collapse and star formation, leading to excessive stellar populations and an altered reionization history. These studies demonstrate that the mapping between additional matter power and galaxy luminosity cannot always be described solely by modifying the HMF.

Zhang et al. [75] instead employ a semi-analytic halo-to-stellar-mass mapping and constrain PMFs using high-redshift stellar mass densities. Their treatment is conceptually close to the present calculation, although it assumes that the halo baryon fraction remains equal to the cosmic mean.

A useful non-PMF comparison is provided by Urrutia et al. [144], who fit HST and JWST UV luminosity functions in a wide class of dark-matter and enhanced-small-scale-power models. They find that the evolution of stellar populations and feedback can account for much of the high-redshift luminosity excess, and find no preference for an additional enhancement of the primordial matter spectrum. Their result emphasizes the need to marginalize over astrophysical freedom when deriving cosmological constraints from the UV LF.

4.4 Influence of PMFs on Reionization

The connection between the UV luminosity function and reionization is direct: the same young stellar populations that dominate the rest-frame UV continuum also produce photons above the hydrogen ionization threshold. Consequently, any PMF-induced change in the abundance or properties of early galaxies can alter the ionization history of the intergalactic medium.

A commonly used volume-averaged description follows the evolution of the ionized hydrogen fraction Q_{HII} ,

$$\frac{dQ_{\text{HII}}}{dt} = \frac{\dot{n}_{\text{ion}}}{\bar{n}_{\text{H}}} - \frac{Q_{\text{HII}}}{t_{\text{rec}}}, \quad (4.23)$$

where $\dot{n}_{\text{ion}}$ is the comoving rate density of ionizing photons entering the IGM and t_{rec} is the effective recombination time. The ionizing emissivity can be connected directly to the UV LF,

$$\dot{n}_{\text{ion}}(z) = f_{\text{esc}}(z) \int_{L_{\text{min}}}^{\infty} dL_{\text{UV}} \Phi_{\text{UV}}(L_{\text{UV}}, z) \xi_{\text{ion}}(L_{\text{UV}}, z) L_{\text{UV}}, \quad (4.24)$$

where ξ_{ion} is the ionizing-photon production efficiency and f_{esc} is the fraction of ionizing photons escaping from galaxies into the IGM.

The corresponding Thomson optical depth is

$$\tau_e = c\sigma_{\text{T}} \int_0^{z_{\text{max}}} dz \frac{n_e(z)}{(1+z)H(z)}, \quad (4.25)$$

providing an integral constraint on the entire ionization history.

Primordial magnetic fields can modify Eq. (4.23) through several distinct physical channels. The most direct effect in the models considered here is the increase in the

abundance of low-mass haloes and therefore of early star-forming galaxies. An enhanced baryon fraction can additionally modify their gas supply. Magnetic fields may change the internal structure of the ISM and hence the escape fraction of ionizing photons. Finally, ambipolar diffusion and decaying MHD turbulence transfer magnetic energy to the IGM, changing its temperature, ionization state and the magnetic field itself. A consistent treatment should in principle follow all of these processes simultaneously.

Radiation-MHD simulations

One of the most complete numerical studies of these effects is the SPHINX-MHD suite of Katz et al. [102]. SPHINX-MHD combines cosmological structure formation, ideal MHD and multi-frequency on-the-fly radiative transfer, allowing galaxy formation, the escape of ionizing radiation and inhomogeneous reionization to evolve self-consistently.

The simulations vary both the initial magnetic-field amplitude B_0 and the spectral index n_B . Katz et al. found that sufficiently strong or blue PMF models form low-mass dwarf galaxies too early and consequently produce an electron-scattering optical depth inconsistent with the CMB. Their approximate exclusion criterion is

$$n_B > -0.562 \log_{10} \left(\frac{B_0}{1 \text{ nG}} \right) - 3.35. \quad (4.26)$$

For models satisfying the optical-depth constraint, the change in the global reionization history and UV LF was found to be comparatively small.

SPHINX-MHD nevertheless reveals effects that are absent in a purely semi-analytic HMF calculation. Strong PMFs can increase the escape fraction of ionizing photons by up to approximately 25% and reduce galaxy effective radii by approximately 44%. Magnetic fields therefore modify not only the abundance of galaxies but potentially their observed sizes and the efficiency with which they ionize the IGM.

The broader SPHINX simulations also demonstrate that f_{esc} is not expected to be a universal constant. Rosdahl et al. [162] find substantial halo-to-halo and temporal variation, with the escape fraction linked to galaxy mass, metallicity and recent star formation. This represents an important uncertainty for any semi-analytic conversion from a UV LF into a reionization history.

Semi-numerical reionization and the 21-cm signal

Cruz et al. [163] studied the same basic enhancement of small-scale structure using a modified version of 21cmFAST. PMF-induced perturbations generate an earlier population

of low-mass haloes and consequently stronger stellar radiation backgrounds during Cosmic Dawn. This shifts both the timing of reionization and the global and fluctuating 21-cm signals.

Their analysis uses the CMB optical depth to exclude models that reionize too early and then forecasts the sensitivity of future 21-cm interferometers. The important result is that the 21-cm signal is sensitive to PMF amplitudes at approximately the pG level for favourable spectral indices, potentially improving substantially over constraints based only on existing reionization and CMB measurements.

The reason is that the 21-cm observable contains more information than the integrated optical depth. Schematically,

$$\delta T_{21} \propto x_{\text{HI}}(1 + \delta_{\text{b}}) \left(1 - \frac{T_{\text{CMB}}}{T_{\text{S}}} \right), \quad (4.27)$$

and is therefore simultaneously sensitive to the neutral fraction, baryonic fluctuations and the thermal state of the IGM. All three can be modified by PMFs.

Reionization constraints in the JWST era

The advent of deep JWST measurements has made it possible to calibrate the galaxy population responsible for reionization more directly. Fairbairn et al. [76] combine their UV LF model with measurements of the neutral hydrogen fraction and the CMB optical depth. They find that strong PMFs can produce a distinctive non-monotonic or “double reionization” history. The enhancement of very small haloes generates an early burst of ionizing photons, producing substantial ionization at $z \gtrsim 20$ before the later galaxy population completes ordinary reionization.

Such an early ionized component contributes strongly to the integrated optical depth even if it leaves relatively little direct imprint on the observed UV LF at lower redshift. The reionization constraint is therefore considerably stronger than their UV-LF-only result. Using the Planck optical-depth prior they obtain

$$\sqrt{\langle B^2 \rangle} < 0.27 \text{ nG}, \quad n_B = -2, \quad (4.28)$$

and

$$\sqrt{\langle B^2 \rangle} < 0.18 \text{ nG}, \quad n_B = 2, \quad (4.29)$$

at 95% credible level.

Facchinetti et al. [94] subsequently emphasized an important physical qualification. Magnetic fields lose energy through ambipolar diffusion and decaying turbulence; the field

amplitude relevant during galaxy formation need not therefore equal the value obtained by simply redshifting the recombination-era field as a^{-2} . When this decay is followed consistently, the PMF impact on later observables is reduced and the resulting reionization bounds become substantially weaker.

The comparison between these studies demonstrates that high-redshift constraints have reached a regime where modeling the *evolution* of the magnetic field itself is as important as specifying its initial power spectrum. Future analyses should therefore evolve the PMF amplitude, the matter perturbations and the ionization history as a coupled system.

4.4.1 Towards a joint PMF analysis with the Ly α forest

The complementary information contained in the Ly α forest provides a natural extension of the present analysis. The UV LF, reionization and Ly α forest respond to PMFs in related but physically different ways. The UV LF primarily probes the abundance and star-formation properties of collapsed haloes; reionization integrates their production of ionizing photons; and the Ly α forest directly probes the density, temperature and neutral-hydrogen distribution of the diffuse IGM.

Our Ly α analysis [2] already constrains the additional small-scale matter power using high-resolution hydrodynamical simulations while marginalizing over a broad range of thermal and reionization histories. This provides an opportunity to construct a single framework in which the same PMF model is required simultaneously to reproduce the abundance of high-redshift galaxies, the ionization history and the Ly α flux power spectrum.

Such a combination would be particularly useful for breaking degeneracies. For example, changing the star-formation efficiency can compensate a PMF enhancement in the UV LF but does not directly remove the corresponding extra matter fluctuations from the Ly α forest. Conversely, thermal pressure smoothing can suppress small-scale Ly α flux power but does not erase the additional number of collapsed haloes predicted by the HMF. The combination therefore has greater constraining power than either observable separately.

A convenient model-independent parametrization would use the position and amplitude of the PMF-induced matter-power enhancement,

$$\{k_{\text{peak}}, \mathcal{P}\}, \quad \mathcal{P} \equiv \Delta_{\text{b}}^2(k_{\text{peak}}),$$

while marginalizing over the four astrophysical parameters controlling the UV LF, and

over the parameters for reionization. Constraints could subsequently be translated into $(B_{1\text{Mpc}}, n_B)$ for a specified PMF damping model. In this way the analysis would remain less sensitive to theoretical uncertainties in the non-linear PMF damping scale.

A further improvement would be to replace independent nuisance parametrizations of the reionization history with physically linked histories. For each PMF model, the UV LF would determine an ionizing emissivity through Eq. (4.24); the resulting $Q_{\text{HII}}(z)$ would then determine a physically motivated reionization and thermal history for the Ly α simulations. Conversely, the Ly α constraints on pressure smoothing and the thermal state of the IGM would restrict the reionization histories allowed by the galaxy model. This would close the loop

$$\begin{aligned} \text{PMF} &\rightarrow P_m(k) \rightarrow \text{HMF} \rightarrow \Phi_{\text{UV}} \rightarrow Q_{\text{HII}}(z) \\ &\rightarrow T_{\text{IGM}}(z), u_0(z) \rightarrow P_{F,1D}(k, z). \end{aligned} \quad (4.30)$$

An especially useful new observational ingredient is the rapidly improving measurement of the neutral fraction during reionization. Kageura et al. [164] recently combined Ly α forest and damping-wing constraints over approximately $5 \lesssim z \lesssim 14$ to obtain an optical-depth constraint that does not use the large-scale CMB E -mode polarization likelihood. They find

$$\tau = 0.0552^{+0.0019}_{-0.0026} (\text{stat.})^{+0.0075}_{-0.0049} (\text{sys.}), \quad (4.31)$$

consistent with the standard CMB determination. Such information would provide a valuable cross-check of PMF models whose strongest reionization signature is an additional high-redshift contribution to τ .

Finally, future 21-cm data can add information that neither the UV LF nor the Ly α forest contains. In [94] they find that for $n_B = -2.5$ an amplitude as small as $s_B \simeq 10^{-1.8}$ nG can already produce a change comparable to the forecast 2σ sensitivity of HERA in their fiducial astrophysical model. A future combined analysis of galaxy counts, Ly α absorption and 21-cm fluctuations would therefore probe the PMF-induced small-scale power at three complementary stages of cosmic evolution.

4.4.2 Concluding remarks and outlook

The results presented in this chapter represent an intermediate stage rather than a finalized analysis. We have shown how the PMF-induced enhancement of the matter power spectrum propagates into the halo mass function and, through a semi-analytic model of baryonic accretion and star formation, into the UV luminosity function. The clearest qualitative signature is an enhancement of the abundance of faint galaxies, whose relative

importance increases towards higher redshift. From there we can see that the best data would be either at the very faint end ($-17 \lesssim M_{\text{UV}}$), or at very high redshift $z \gtrsim 14$.

However, a quantitative interpretation requires a simultaneous exploration of the astrophysical and PMF parameter space. In particular, we plan to perform an MCMC analysis in which the PMF parameters are varied jointly with the parameters describing star-formation efficiency and the halo-to-luminosity mapping. This will allow us to determine the preferred astrophysical parameters within Λ CDM and PMF cosmologies, identify possible degeneracies, obtain best-fitting models, and derive statistically meaningful limits in the $(B_{1\text{Mpc}}, n_B)$ plane or, alternatively, directly in terms of the position and amplitude of the PMF-induced peak in the matter power spectrum. The analysis will also be updated as larger and increasingly spectroscopically confirmed JWST samples become available, with particular attention to the faint end of the UV luminosity function, where the PMF signal is expected to be strongest.

A further step will be to connect the galaxy calculation self-consistently to reionization. For each model, the predicted UV luminosity function can be converted into an ionizing emissivity and hence into an ionization history, which can then be confronted with measurements of the neutral hydrogen fraction and the Thomson optical depth. Such information is especially valuable because PMFs can generate additional star formation at very high redshift and therefore contribute to the optical depth even when their effect on the directly observed galaxy population is modest. Ultimately, the same reionization and thermal histories could be propagated into the Ly α -forest simulations discussed in Chapter 3, enabling a joint analysis of the UV luminosity function, reionization and the Ly α flux power spectrum. Future 21-cm measurements would provide an additional and complementary probe of the neutral fraction, baryonic fluctuations and thermal history. The development of this combined framework, together with a full statistical analysis of current and forthcoming high-redshift data, constitutes the natural continuation of the work presented here and is the basis of a new publication currently in preparation.

Chapter 5

Final summary, reflections and future perspectives

Summary

This thesis has explored the cosmological consequences of primordial magnetic fields, with particular emphasis on their impact on small-scale structure formation and on observables associated with the high-redshift Universe. The main physical mechanism considered throughout the thesis is the generation of additional baryonic density perturbations related to the Lorentz force after recombination. These perturbations modify the small-scale matter power spectrum and subsequently affect the formation and baryonic content of dark matter halos, the distribution of neutral hydrogen probed by the Ly- α forest, and the abundance and properties of the first galaxies. The work presented here has therefore explored many regimes, between linear and non-linear theory, through simulations, and with different observational data.

We investigated how PMFs should be implemented in the initial conditions of cosmological simulations. By generating the magnetic field explicitly and calculating the corresponding particle displacements induced by the PMFs, it naturally accounted for the different response of baryons and dark matter. Our hydrodynamical simulations showed that the additional small-scale power increases the abundance of low-mass halos at high redshift, while the halo mass function can still be described accurately by the Sheth–Tormen formalism. More distinctively, low-mass halos can acquire baryon fractions substantially above the cosmic mean, reaching approximately twice the cosmic baryon fraction around $z \sim 10$ in the simulations considered here. Preliminary results further indicate enhanced early star formation, although this observable is considerably more sensitive to the adopted feedback and star-formation prescriptions and therefore requires a more complete physical treatment.

We turned these theoretical effects into an observational constraint using the high-redshift Ly- α forest. Because the Ly- α flux power spectrum probes the matter distribution on comoving sub-Mpc scales, it is particularly sensitive to the additional small-scale power generated by PMFs. We used a large suite of cosmological hydrodynamical simulations and varied the thermal and reionization histories of the intergalactic medium together with cosmological and observational nuisance parameters. The reference analysis shows a preference for additional small-scale power, with a central value $k_{\text{peak}} \simeq 50 \text{ Mpc}^{-1}$, corresponding to $B_{1\text{Mpc}} \simeq 0.20 \text{ nG}$ for $n_B = -2.9$. Since this preference is sensitive to the treatment of the smallest-scale data and of the observational noise, we adopt the more conservative result $B_{1\text{Mpc}} < 0.30 \text{ nG}$ at 3σ as the main constraint. This demonstrates that the Ly- α forest is a powerful probe of PMFs.

Finally, we extended the same physical picture to early galaxies and reionization in the JWST era. We constructed a semi-analytic connection between the PMF-modified matter power spectrum, halo mass function, baryonic accretion, star formation and the UV luminosity function. The characteristic PMF signature is an enhanced abundance of faint galaxies hosted by low-mass halos, with the relative effect becoming stronger toward higher redshift. This is work in progress; our next steps are performing an MCMC analysis, with resulting goodness of fit and constraints. We keep an eye on current galaxy observations that reveal strong degeneracies with star-formation efficiency, feedback, stellar populations and the mapping between halo mass and luminosity. The present analysis should therefore be regarded as a first step toward a self-consistent PMF study of the UV luminosity function and reionization.

Taken together, the results of this thesis show that primordial magnetism can leave a coherent sequence of signatures across apparently different observables. The most characteristic effects arise on small scales and at high redshift, where the distinction between baryons and dark matter becomes particularly important. At the same time, the work demonstrates that extracting robust constraints requires following not only the magnetic contribution to the matter power spectrum, but also its subsequent baryonic, thermal and astrophysical evolution.

We should also mention that some original contributions of the author to the publications [1] & [2] (e.g., calculations related to ambipolar diffusion) were not included in this thesis for the sake of emphasis of the main results and methods used. We have also attempted to provide a pedagogical review of numerical methods, constraints and recent observations that, in our view, can be helpful to students, or scientists from other departments, that enter into this field of research.

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Reflections

A recurring lesson throughout this thesis is that the same property which makes primordial magnetic fields potentially observable also makes them particularly difficult to model. Their strongest impact on structure formation occurs on small scales and at high redshift, precisely where the evolution becomes non-linear and where baryonic and astrophysical processes become increasingly important. Consequently, translating a primordial magnetic field specified by $(B_{1\text{Mpc}}, n_B)$ into an observable is not a single-step problem, but requires following several intermediate physical processes, each introducing its own uncertainty.

The first major difficulty concerns the magnetic damping scale and the shape of the matter power spectrum around its maximum. Much of the earlier literature relied on simplified analytical estimates of the magnetic Jeans or damping scale. However, the back-reaction of baryons and the non-linear evolution of the magnetic field can substantially modify both the position and amplitude of the peak. Since observables such as the halo mass function and the Ly α flux power spectrum are highly sensitive to this region, uncertainties in the damping prescription propagate directly into constraints on PMF amplitudes. In this thesis we partly addressed this problem by introducing a parameter describing the uncertainty in the peak scale, but a fully self-consistent MHD calculation would ultimately be preferable.

A second limitation follows from the treatment of baryons and dark matter. One of the main results of this work is precisely that PMFs do not initially affect these two components in the same way. This motivated our construction of separate initial perturbations and led to the prediction of enhanced baryon fractions in high-redshift halos. Nevertheless, our hydrodynamical simulations did not evolve the magnetic field dynamically with MHD. They also did not include several processes important for galaxy formation, such as galactic winds, AGN feedback and a radiation field coupled self-consistently to the simulated star-formation history. We therefore regard the baryon-fraction result as more robust than the quantitative prediction for star formation. In particular, the interesting indications that PMF-induced vorticity and angular momentum may suppress star formation deserve to be tested with dedicated MHD simulations rather than interpreted from the present simulations alone.

The Ly- α analysis presented a different set of difficulties. Its constraining power originates predominantly from the smallest measured scales, which are also the scales most affected by observational noise, spectral resolution, metal contamination and modeling uncertainties. The preference for additional small-scale power weakens when the smallest-scale data are removed or when the noise treatment is made more conservative, motivating

our choice to quote an upper limit rather than interpret the result as a detection. The mild preference of the PMF models for a modified mean flux, particularly at $z = 4.2$, also suggests that the interplay between PMFs and the ionizing background should be investigated more carefully. Finally, careful and well-tested implementation of separate baryon and dark matter transfer functions would provide another important improvement, that would lead us to much stronger constraints from the Ly- α forest.

These limitations do not invalidate the main conclusions of the thesis, but they indicate where their interpretation is most secure. The enhancement of small-scale structure and the distinct response of baryons and dark matter are robust physical consequences of PMF-induced perturbations, whereas predictions involving star formation, galaxy luminosities and reionization require progressively more astrophysical modeling. A central improvement would therefore be to replace the present sequence of partially independent models by a framework in which the magnetic field, baryonic perturbations, galaxy formation, thermal history and ionization state are evolved consistently.

Perspectives on the future

While much of the work is still in progress and some immediate improvements can be made for the already published work, we wish to present several other paths the author would consider in the future.

FILAMENTS From a structure formation perspective, we know that filaments form before halos and are between the linear and non-linear regime. In addition, the gas of the IGM prefers to follow the filamentary structure. Since PMF effects on baryons must be studied at very early redshifts to isolate them from complicated astrophysical processes, filaments present themselves as a natural environment. It would be interesting to analyze the statistics and topologies of filaments through simulations; for instance, do PMFs change the number of filaments? (we obtain more low-mass halos, perhaps this removes gas from filaments); what is the impact on the shape, temperature, and density profile of filaments? Similar questions and studies are performed, e.g., in [165] within the Λ CDM. Finally, Faraday rotation provides another promising means of probing magnetic fields in cosmic-web filaments. Since the rotation measure is proportional to the line-of-sight integral of the free-electron density and magnetic field, magnetized filaments can contribute to the residual rotation measures of background polarized radio sources. Cosmological MHD simulations indicate that the Faraday-rotation signal of filaments can retain significant sensitivity to the assumed primordial magnetic-field model [53, 166, 167]. It would be quite interesting to jointly simulate lines-of-sight for Ly- α at redshifts

$6 \geq z \geq 4$, that then propagate into simulated Rotation Measure data on the same lines-of-sights, at lower redshifts $z \leq 2.5$.

VORTICITY Something that was not discussed extensively in this thesis were vortical motions or related vector perturbations of baryons. In the linear regime of Λ CDM vorticity $\vec{\omega} = \nabla \times \vec{v}$ is simply diluted with the expansion of the universe ($\omega \sim a^{-1}$); on the other hand, with the presence of PMFs, vorticity can be sustained post-recombination through [52]

$$\frac{\partial \vec{\omega}}{\partial t} + H\vec{\omega} = \vec{\nabla} \times (\vec{j} \times \vec{B}).$$

where $\vec{j} = e(Zn_{\text{ion}}\vec{v}_{\text{ion}} - n_e\vec{v}_e)$ is a background quantity. It would be interesting to consider vorticity for several reasons: in our work we have not explored helical magnetic fields, and these would considerably contribute to vorticity; the star formation results, specifically star formation rate (SFR) density, that were discussed at the end of Chapter 2, were interpreted that having lower SFR for PMFs-in-ICs case (vs. only increase of power) could be due to larger angular momentum that vortical motions could induce. This can be and needs to be tested through simulations. Another connected issue is how to properly generate ICs in this case, and whether to use 2nd order Lagrangian Perturbation Theory codes. The last curiosity is a possible connection to filaments — a study of Wang et al. [168] claimed observational evidence for spin of filaments. They measured a systematic redshift difference between galaxies located on opposite sides of filament spines, consistent with coherent rotation around the filament axis.

MHD CODES Along with participation in workshops, schools and conferences, the author had extensive discussions with users and builders of codes that run magneto-hydrodynamics (AREPO [36], RAMSES [30]/SPHINX [102], ENZO [35], PENCIL-CODE [169]). It is natural to learn and use some of these codes for serious research on magnetism in cosmology and astrophysics. For each of these codes there are examples that interest us: AREPO allows for a simple translation from GADGET, and has been used to study PMFs [68] & [85]; RAMSES-based SPHINX-MHD codes are studying JWST and reionization [162, 170]; ENZO can be used for the Faraday rotation mentioned above; PENCIL-CODE is excellent to study early-universe physics, including PMFs and gravitational waves [171].

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These are only some examples of possible journeys in the PMF research community. Overall, we and many others are demonstrating that much is to be explored regarding primordial magnetism in the context of cosmology. The author is grateful for the opportunity to work in this humble, yet exciting field of science.

Appendix A

Initial Conditions with PMFs

In this appendix, we outline the algorithm behind our code that generates initial conditions for baryons and dark matter for hydrodynamical simulations. We have modified the publicly available code, N-GENIC, to produce initial conditions with PMFs.

Brief review of the N-GenIC algorithm

The N-GENIC code [41] takes in as input the density power spectrum of baryons and dark matter at a desired high redshift. Then as output, it produces the position of dark matter and baryon particles on a grid and also assigns each particle a velocity vector.

The N-GENIC code utilizes the Zeldovich approximation to obtain displacement fields from density perturbations. Specifically, in the linear limit, the continuity equation is of the form

$$\frac{d\delta}{dt} = -\frac{\nabla \cdot \vec{v}}{a}. \quad (\text{A.1})$$

Integrating the above yields,

$$\delta = -\nabla \cdot \vec{x}_{\text{dis}}. \quad (\text{A.2})$$

Here $\vec{x}_{\text{dis}}$ is the comoving displacement vector from original position. The original position should be such that $\delta = 0$, i.e. when particles are distributed uniformly on a grid.

Finding the displacement vector field, $\vec{x}_{\text{dis}}$, from density perturbations, δ , is simplified in Fourier space, where

$$\vec{x}_{\text{dis}}(k) = i \frac{\vec{k}}{k^2} \delta(k). \quad (\text{A.3})$$

In the above equation, a critical assumption has been made that particles are not displaced in directions perpendicular to $\vec{k}$, i.e. there is no vortical motion. This assumption is appropriate in Λ CDM cosmology as there are no sources of vortical motions.

Thus, N-GENIC first initializes the displacement vector field in Fourier space and then takes the inverse Fourier transform to obtain the displacements in real space. The velocity vector for each particle is obtained by simply multiplying their displacement vector with the Hubble rate, H .

N-GenIC with PMFs

We modified N-GENIC to produce initial particle positions from a given magnetic field power spectrum. We do so by first generating vector potential A on our grid, where $\vec{B} = \nabla \times \vec{A}$. Then we calculate $\vec{B}$ and displace particles according to the Lorentz force, $\vec{L}_B = (\nabla \times \vec{B}) \times \vec{B}$.

Initializing $\vec{B}$ fields

The power spectrum of $\vec{A}$ is related to that of magnetic fields as

$$\epsilon_{irs}\epsilon_{jab}k_rk_a\langle A_b(k)A_b(k')\rangle = (2\pi)^3\delta^3(k-k')\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\frac{P_B(k)}{2}. \quad (\text{A.4})$$

In the above form it is hard to understand the spectrum of A fields. Things are simplified if we change our coordinates, such that the z -axis is along $\vec{k}$. In this coordinate, we obtain

$$\langle A_2A_2\rangle = \langle A_1A_1\rangle = (2\pi)^3\delta^3(k-k')\frac{P_B(k)}{2k^2}, \quad \langle A_1A_2\rangle = 0. \quad (\text{A.5})$$

here the subscript 1 and 2 are simply two directions perpendicular to $\vec{k}$. The value of A_3 does not contribute towards the final value of the B field and hence we set it to zero.

At every grid point in the Fourier space, we first initialize the value of vector fields (A_1 and A_2) in the directions perpendicular to $\vec{k}$. Considering direction of $\vec{k}$ is of form

$$\hat{k} = (\sin\theta\sin\phi, \sin\theta\cos\phi, \cos\theta), \quad (\text{A.6})$$

and choosing one perpendicular direction to $\vec{k}$ to lie in $\hat{k} - \hat{z}$ plane and the other to lie in $x - y$ plane, we obtain

$$A_z = A_1\sin\theta \quad A_x = A_2\cos\phi - A_1\cos\theta\sin\phi \quad A_y = -A_2\sin\phi - A_1\cos\theta\cos\phi. \quad (\text{A.7})$$

The above procedure gives us $\vec{A}$ field in Fourier space. We then inverse Fourier transform $\vec{A}$ to obtain its values in real space. Subsequently, the magnetic fields are obtained by evaluating $\nabla \times \vec{A}$ in the real space. This procedure ensures that the magnetic fields are generated with minimal divergence.

Initializing particle positions

To obtain the displacement vector for baryons due to magnetic fields, we integrate the baryon Euler equation (Eq. (1.92)) and neglect the contribution from gravity to obtain

$$\vec{x}_{\text{dis}}^{\text{PMF}} \propto \frac{(\nabla \times \vec{B}) \times \vec{B}}{(4\pi\rho_b a^3) a^3 H^2}. \quad (\text{A.8})$$

Note that $\vec{x}_{\text{dis}}^{\text{PMF}}$ vector field has both irrotational and vortical (divergence-free) components in it, unlike the displacements generated in Λ CDM cosmology.

Considering the dimensionless proportionality constant in Eq. (A.8) to be ξ^{num} , the total displacement for a particle is simply given by adding the displacement from Λ CDM power spectrum with the displacement from PMFs,

$$\vec{x}_{\text{dis}} = [\vec{x}_{\text{dis}}]_{\text{CDM}} + \xi^{\text{num}} \times \frac{(\nabla \times \vec{B}) \times \vec{B}}{(4\pi\rho_b a^3) a^3 H^2}. \quad (\text{A.9})$$

The above prescription ensures that when perturbations are in the linear regime, the density perturbation field is given by

$$\delta = -\nabla \cdot \vec{x}_{\text{dis}} = \delta_{\text{CDM}} - \xi^{\text{num}} \frac{S_0}{a^3 H^2}, \quad (\text{A.10})$$

which matches what we expect from theory, see Eq. (1.105).

A.1 What does *consistent* ICs mean?

Let us write each Fourier mode of perturbations as

$$\delta(\vec{k}) = \mathcal{A}(\vec{k}) e^{i\phi}, \quad (\text{A.11})$$

where the amplitude $\mathcal{A}$ is drawn from a distribution determined by $P(k)$, while the phase is random. For example, using two random numbers r_1, r_2 uniformly distributed in $[0,1]$, one may write

$$\mathcal{A}(\vec{k}) \propto \sqrt{-P(k) \ln r_1}, \quad \phi = 2\pi r_2. \quad (\text{A.12})$$

A first, but naive, way of including PMFs would be simply to replace the standard matter power spectrum by

$$P(k) = P_{\Lambda\text{CDM}}(k) + P_{\text{PMF}}(k) \quad (\text{A.13})$$

and use this total spectrum as the input of an otherwise unchanged initial condition generator, i.e., inserting the total $P(k)$ under the square root of Eq. (A.12). This prescription reproduces the desired total two-point power spectrum, but it does not reproduce the physical origin of the PMF-induced perturbations. In particular, it treats the additional perturbations as another Gaussian scalar realization and does not distinguish between the response of baryons and dark matter.

A somewhat improved prescription is to generate the ΛCDM and PMF contributions separately,

$$\delta_{\Lambda\text{CDM}}(\vec{k}) = \mathcal{A}_{\Lambda\text{CDM}}(\vec{k})e^{i\phi}, \quad \delta_{\text{PMF}}(\vec{k}) = \mathcal{A}_{\text{PMF}}(\vec{k})e^{i\phi}, \quad (\text{A.14})$$

and subsequently add the two fields. However, if the same phase ϕ is used for both components, the PMF-induced and inflationary perturbations are artificially correlated. This is not the physical situation considered here: the stochastic PMF realization is independent of the primordial adiabatic perturbations, and the two contributions should therefore not in general share the same phases. Such a prescription would also miss the isocurvature component generated by the different response of baryons and dark matter.

The consistent procedure is instead to generate the primordial magnetic field itself and calculate the resulting particle displacements from the Lorentz force. It is important that the Lorentz force acts directly only on the baryonic component. Dark matter subsequently responds gravitationally to the magnetically induced baryonic perturbations. Consequently, the PMF-induced parts of the baryon and dark-matter density fields have different growth histories,

$$\delta_b = \delta_{b,\Lambda\text{CDM}} + \delta_{b,\text{PMF}}, \quad \delta_{\text{DM}} = \delta_{\text{DM},\Lambda\text{CDM}} + \delta_{\text{DM},\text{PMF}}, \quad (\text{A.15})$$

with, in general,

$$\delta_{b,\text{PMF}} \neq \delta_{\text{DM},\text{PMF}}. \quad (\text{A.16})$$

Thus the baryon and dark-matter initial conditions cannot be obtained consistently by assigning both species the same PMF-modified total matter power spectrum. Their distinct response to the magnetic source must be retained.

There are two further advantages of constructing the initial conditions directly from the magnetic-field realization. First, $\vec{L}_B$ is not in general a pure gradient, and therefore

the PMF-induced displacement contains both irrotational and vortical components. This information would be lost if the perturbations were generated only through Eq. (??). Second, even if the primordial magnetic field is itself a Gaussian random field, the Lorentz force is quadratic in $\vec{B}$. The resulting density and displacement fields are therefore intrinsically non-Gaussian. Generating a Gaussian density realization solely from $P_{\text{PMF}}(k)$ reproduces its two-point statistics but not these higher-order properties.

The three approaches can therefore be summarized schematically as

$$\begin{aligned}
\text{naive:} \quad & P_{\text{in}}(k) = P_{\Lambda\text{CDM}}(k) + P_{\text{PMF}}(k), \\
\text{power-spectrum approach:} \quad & \delta = \delta_{\Lambda\text{CDM}} + \delta_{\text{PMF}}, \\
\text{full approach:} \quad & \vec{B} \longrightarrow \vec{L}_B \longrightarrow \{\vec{x}_{\text{dis,b}}^{\text{PMF}}, \vec{x}_{\text{dis,DM}}^{\text{PMF}}\} \longrightarrow \{\delta_{b,\text{PMF}}, \delta_{\text{DM,PMF}}\}.
\end{aligned}
\tag{A.17}$$

The last prescription retains the physical origin, relative phases, baryon–dark-matter isocurvature, vorticity and non-Gaussianity of the PMF-induced perturbations, and is therefore the appropriate starting point for a self-consistent numerical treatment of structure formation with primordial magnetic fields.

Appendix B

Computation of the Luminosity Function

The Galaxy Luminosity Function $\phi_g(L)$ is defined as a number density of galaxies per luminosity interval $[L, L+dL]$, and can be related to the HMF dn_h/dM_h and UV luminosity [146]

$$\phi_g(L) \equiv \frac{dn_g}{dL} = \frac{dn_h}{dL_{UV}} = \frac{dn_h}{dM_h} \left(\frac{dL_{UV}}{dM_h} \right)^{-1}. \quad (\text{B.1})$$

We want to express ϕ_g as a function of the absolute magnitude

$$\mathcal{M}_{UV} = -2.5 \log_{10}(L_{UV}/\text{erg.s}^{-1}.\text{Hz}^{-1}) + \mathcal{M}_{\text{ref}}. \quad (\text{B.2})$$

implicitly through the halo mass M_h , i.e., $\phi_g = \phi_g(\mathcal{M}_{UV}(M_h))$. We will use the following expression for the GLF¹

$$\phi_g(\mathcal{M}_{UV}) = \frac{1}{2.5} \left[\frac{dn_h}{d \log_{10} M_h} \right]_{\text{S.-T.}} \left(\frac{d \log_{10} L_{UV}}{d \log_{10} M_h} \right)^{-1}, \quad (\text{B.3})$$

where S.-T. denotes the Sheth-Tormen HMF.

The UV luminosity is related to the star formation rate (SFR) as

$$\dot{M}_* = K_{UV} L_{UV}, \quad (\text{B.4})$$

¹It holds

$$\phi_g(\mathcal{M}_{UV}) \equiv \frac{dn_h}{d(-\mathcal{M}_{UV})} = \frac{dn_h}{d \log_{10} L_{UV}} \frac{d \log_{10} L_{UV}}{d(-\mathcal{M}_{UV})} = \frac{\ln(10)L_{UV}}{2.5} \phi_g(L_{UV}).$$

where $K_{\text{UV}} = 1.15 \times 10^{-28} \text{ M}_{\odot} \text{ yr}^{-1} / (\text{erg.s}^{-1} \cdot \text{Hz}^{-1})$ [145] (while ref. [172] uses $K_{\text{UV}} = 6.67 \times 10^{-29} \text{ M}_{\odot} \text{ yr}^{-1} / (\text{erg.s}^{-1} \cdot \text{Hz}^{-1})$). The SFR $\dot{M}_{*}$ is proportional to the gas accretion rate $\dot{M}_{\text{b}} = (\Omega_{\text{b}}/\Omega_{\text{m}})\dot{M}_{\text{h}}$, with the proportionality factor being set by the star formation efficiency (SFE) $f_{*}(M_{\text{h}}, z)$. Thus,

$$L_{\text{UV}} = L_{\text{UV}}(M_{\text{h}}, z) = K_{\text{UV}}^{-1} f_{*}(M_{\text{h}}, z) \frac{\Omega_{\text{b}}}{\Omega_{\text{m}}} \frac{dM_{\text{h}}}{dt}, \quad (\text{B.5})$$

where the halo mass accretion rate is given by

$$\frac{dM_{\text{h}}}{dt} = M_{\text{h}} \left(\frac{d \ln \sigma}{d \ln M_{\text{h}}} \right)^{-1} \frac{d \ln \sigma}{dt} \quad (\text{B.6})$$

The plan is straightforward:

1. compute $\sigma^2(M_{\text{h}}, z)$ from the linear power spectra,
2. find the accretion rate (B.6),
3. set $f_{*}(M_{\text{h}}, z)$ as in [145],
4. find $L_{\text{UV}} = L_{\text{UV}}(M_{\text{h}}, z)$ from (B.5), and set $\mathcal{M}_{\text{ref}}$.
5. compute the derivative $\left(\frac{d \log_{10} L_{\text{UV}}}{d \log_{10} M_{\text{h}}} \right)^{-1}$ and use Sheth-Tormen HMF in Eq. (B.3),
6. plot $\phi_{\text{g}}(\mathcal{M}_{\text{UV}}(M_{\text{h}}, z))$ with observational data points.

Ad 1.: For the computation of σ^2 we use COLIBRI, on a mass interval $M \in [10^7, 10^{14}] M_\odot/h$, with 400 points. The PMF-modified $P(k)$ input power spectra must be in units $(\text{Mpc}/h)^3$ to be compatible with COLIBRI.² In the following figure we can see how variance σ^2 varies between models. Between the three tested models $\{0.2, 0.5, 0.8\} \text{ nG}$, the peak of the HMF ratio changes approximately by a factor of 2 (at different masses, due to different peak scales k_{peak} of the 'bump' in the linear power spectra).

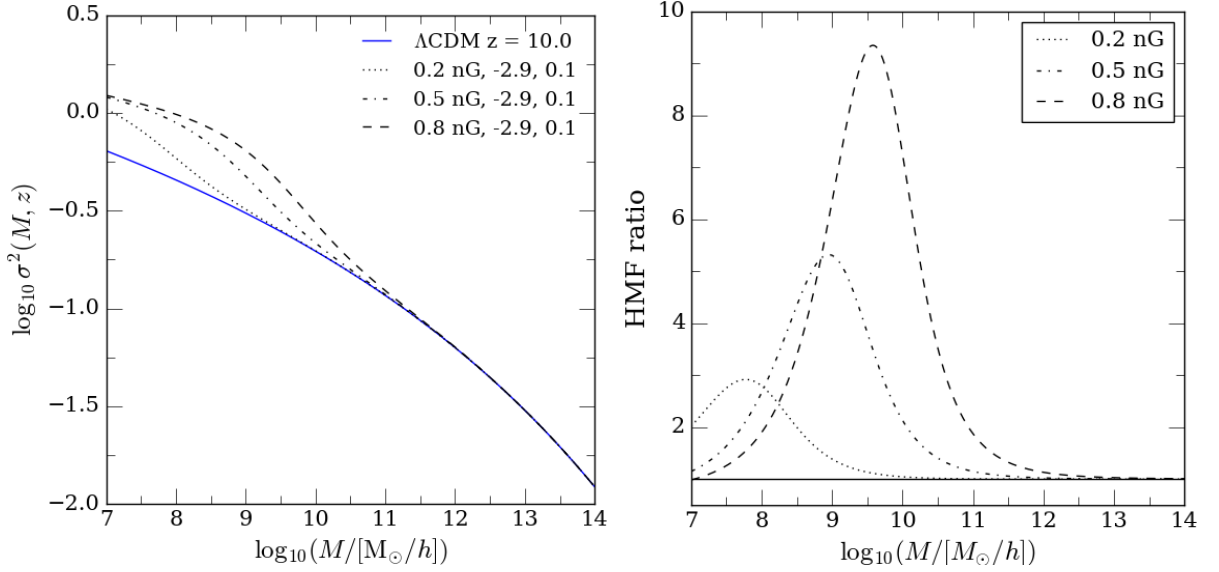


Figure B.1: **Left:** Mass variance at redshift $z = 10$. **Right:** Ratios of halo mass functions ($z = 10$), where the ratio is $(\text{PMF} + \Lambda\text{CDM})/\Lambda\text{CDM}$.

Apart from the HMF, this change from ΛCDM propagates also into mass accretion rate.

Ad 2.: Let us first express the time derivative into the redshift derivative³

$$\frac{d \ln \sigma}{dt} = -\frac{H_0}{2} [\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_{\text{rad}}(1+z)^4]^{1/2} (1+z) \frac{d \ln(\sigma^2)}{dz} \quad (\text{B.7})$$

We set $\Omega_\Lambda = 0.692$, $\Omega_m = 0.308$, $\Omega_{\text{rad}} = 5.4 \times 10^{-5}$, $H_0 = 67.8 \text{ km/s/Mpc} = 6.93 \times 10^{-11} \text{ yr}^{-1}$. The numerical derivative for $f := \sigma^2$ is $f'(z) = (f(z + \varepsilon) - f(z - \varepsilon))/2\varepsilon$, with

²COLIBRI documentation:

:k: array | Scales in h/Mpc ; Basically the same 'k' of the input.
:pk: 2D array of shape `((len(z), len(k)))` | Power spectrum in units of $(\text{Mpc}/h)^3$.
:param logM: log base 10 of the masses at which to compute the variance, in units of $M_\odot/h$.

³It follows from

$$\frac{d}{dt} = \left(\frac{dz}{da}\right) \left(\frac{da}{dt}\right) \frac{d}{dz} = \left(\frac{-1}{a^2}\right) (aH) \frac{d}{dz} = -\frac{H(z)}{a(z)} \frac{d}{dz} = -H(z)(1+z) \frac{d}{dz}$$

$\varepsilon \equiv \Delta z = 0.1$, and error $\mathcal{O}(\varepsilon^2)$. Similarly, the first derivative in (B.6) is computed with $\varepsilon \equiv \Delta \log M = 7/400 = 0.0175$.

The accretion rate is plotted in Fig.B.2 below. In the left panel, the solid lines are from our computation (within Λ CDM), while the dashed and dot-dashed lines are fits from Trac+15 [173] (used in [145] for comparison) and Correa+15 [174] (which [146] refer to as the compatible one), respectively. The latter two are modeled with the following equations

$$\left(\frac{dM_h}{dt}\right)_{\text{Trac+15}} = 0.21 \text{ M}_\odot \text{ yr}^{-1} \left(\frac{M_h}{10^8 \text{ M}_\odot}\right)^{1.06} \left(\frac{1+z}{7}\right)^{5/2}, \quad (\text{B.8})$$

(well applicable to a mass range $10^8 < M_h/\text{M}_\odot < 10^{13}$ and a redshift interval $6 \leq z \leq 10$)

$$\left(\frac{dM_h}{dt}\right)_{\text{Correa+15}} = 71.6 \text{ M}_\odot \text{ yr}^{-1} \left(\frac{M_h}{10^{12} \text{ M}_\odot}\right) f(M_0) ((1+z) - a) (\Omega_\Lambda + \Omega_m(1+z)^3)^{1/2}, \quad (\text{B.9})$$

where $f(M_0)$ and a are computed from σ^2 and growth factor, with $M_0 \equiv M_h(z=0)$. For mere visualization purpose, we approximate $f(M) = 0.1 \log_{10}(M) - 0.4$ (see Fig. 2 in [174]).

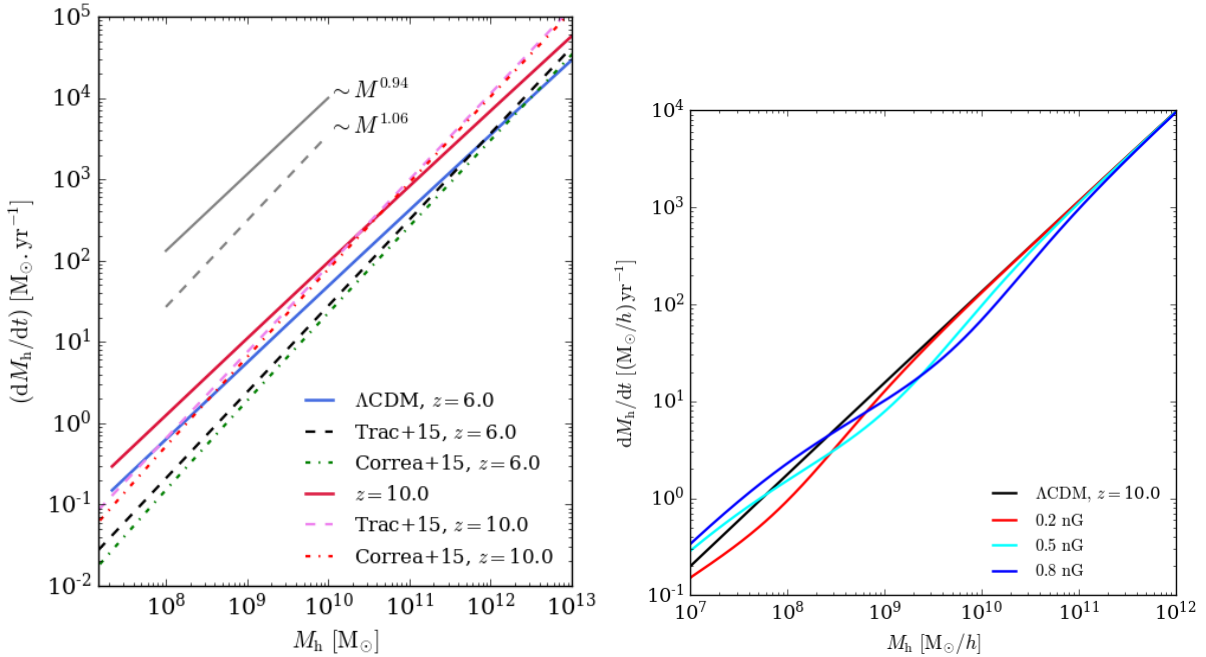


Figure B.2: MASS ACCRETION RATES. *left*: Comparison of accretion rates in Λ CDM models; solid lines are from our computation. *right*: Comparison with PMF models at $z = 10$.

Ad 3.: Following [145], we find the SFE $f_*(M_h, z)$ in an energy-driven feedback model

with virial shock (hereby referred to as 'energy-regulated'), using the following equations:⁴

$$f_*^{(\text{E})}(M_{\text{h}}, z) = \frac{f_{\text{shock}}(M_{\text{h}}, z)}{f_{*, \text{max}}^{-1} + \eta_{(\text{E})}(M_{\text{h}}, z)},$$

with

$$f_{\text{shock}}(M_{\text{h}}, z) = 0.47 \left(\frac{M_{\text{h}}}{10^{12} \text{ M}_{\odot}} \right)^{-1/4} \left(\frac{1+z}{4} \right)^{0.38}$$

(+ impose $f_{\text{shock}} \leq 1$),

$$\eta_{(\text{E})}(M_{\text{h}}, z) = 10 \epsilon_K \mathcal{O}(1) \left(\frac{M_{\text{h}}}{10^{11.5} \text{ M}_{\odot}} \right)^{-2/3} \left(\frac{9}{1+z} \right).$$

In the fiducial model, which we use, it is set to be $\epsilon_K = 0.1 = f_{*, \text{max}}$. We plot the above SFE in Fig. B.3, where we also include a redshift-independent model of SFE of Tacchella+18 [175] (who cite Moster et al. 2010, 2018)

$$f_*(M_{\text{h}}) = 2f_0 \left[\left(\frac{M_{\text{h}}}{M_{\text{c}}} \right)^{-\beta} + \left(\frac{M_{\text{h}}}{M_{\text{c}}} \right)^{\gamma} \right]^{-1}. \quad (\text{B.10})$$

We use this functional form, however the values we adopt are from a more recent study of Donnan+24 [153] who also use the above model, with $\{f_0, M_{\text{c}}, \beta, \gamma\} = \{0.16, 10^{11.7}, 0.9, 0.65\}$. The original values of Tachella+18 [175] are $\{f_0, M_{\text{c}}, \beta, \gamma\} = \{0.26, 7.1 \times 10^{10}, 1.09, 0.36\}$, found from the best-fit of the UV LF at $z = 4$.

Ad 4.: With the previous steps, we have all ingredients for obtaining $L_{\text{UV}} = L_{\text{UV}}(M_{\text{h}})$ from (B.5), which we translate to $\mathcal{M}_{\text{UV}} = \mathcal{M}_{\text{UV}}(M_{\text{h}})$ using (B.2). To gauge this relation, we need $\mathcal{M}_{\text{ref}}$. In ref. [145] (see their Fig. 2), the magnitude is gauged in such a way to have

$$\mathcal{M}_{\text{UV}}(M_{\text{h}} = 10^{10} \text{ M}_{\odot}, z = 8) = -17.$$

We then deduce the $\mathcal{M}_{\text{ref}}$ value from the $\mathcal{M}_{\text{UV}}$ vs. M_{h} plot in Figure B.4; the curve has to pass through the coordinates $(10^{10} \text{ M}_{\odot}, -17)$ at redshift $z = 8$. The resulting value is therefore $\mathcal{M}_{\text{ref}} = 52.45$.

⁴A simple way to satisfy the requirement $f_{\text{shock}} \leq 1$ for all masses and redshifts, is to insert a line in the computation code, which divides the computed values with their maximum: `fshock = fshock / max(fshock)`.

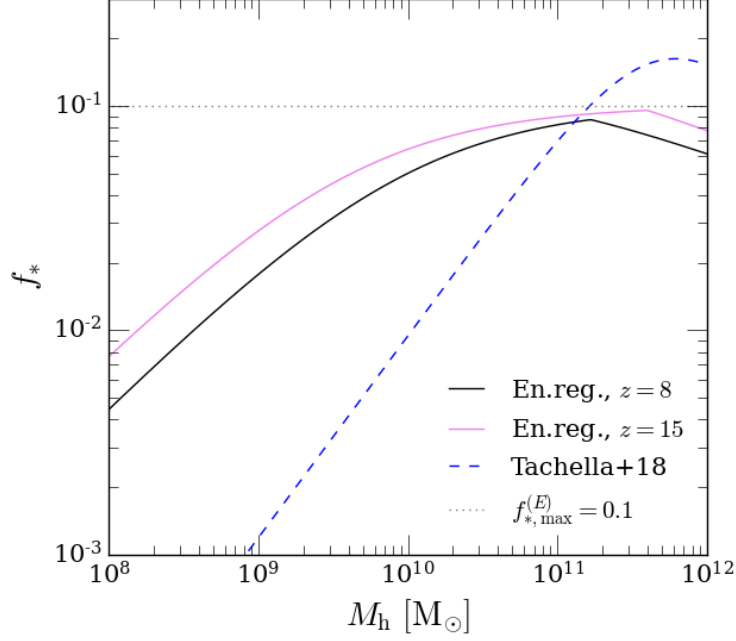


Figure B.3: STAR FORMATION EFFICIENCY MODELS. Solid lines are computed by following Furlanetto+17 [145], and compared with the z -independent model of Tacchella+18 [175] with values from Donnan+24 [153] (dashed).

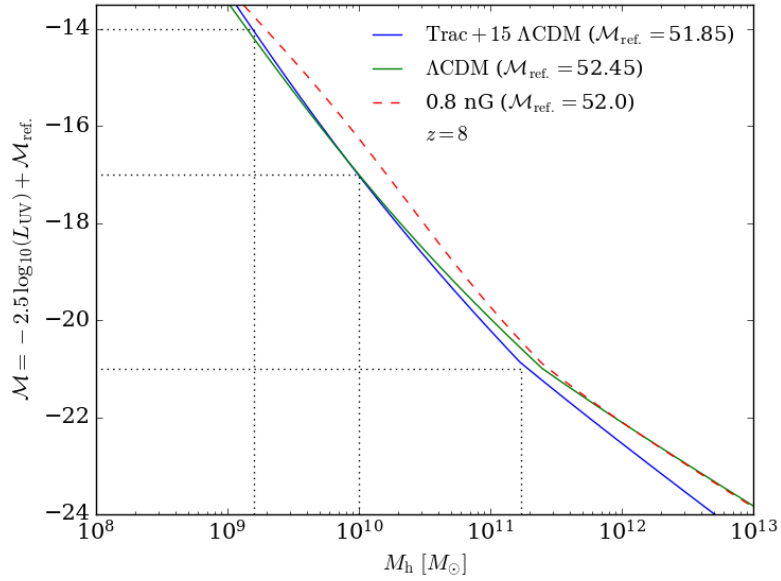


Figure B.4: ABSOLUTE MAGNITUDE VS. MASS.

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