

Bootstrapping the simplest deconfined quantum critical point

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We study the $N = 3$ case of the CP^{N-1} model, which is a field theory of N complex scalars in $3d$ coupled to an Abelian gauge field with $SU(N) \times U(1)$ global symmetry. Recent evidence suggests the $N = 2$ theory is not critical, which makes the $N = 3$ theory the simplest possibility of deconfined quantum criticality. We apply the conformal bootstrap to correlators of charge $q = 0, 1, 2$ scalar operators under the $U(1)$ symmetry, which gives us access also to $q = 3, 4$ operators. After imposing that only the lowest $q = 0, 1, 2$ scalar operators are relevant, we find that the bootstrap bounds are saturated by the large N prediction for $q = 1, 2, 3, 4$ scalar monopole operator scaling dimensions, which were shown earlier to be accurate even for small N , as well as a lattice prediction for the $q = 0$ non-monopole scalar operator. We also predict the scaling dimensions of the lowest spinning monopole operators, which we match to the large charge prediction for spinning operators. This suggests that the critical CP^2 model is described by this bootstrap bound.

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Introduction. Deconfined quantum critical points (DQCPs) describe the continuous phase transition between one phase that preserves a symmetry H but breaks another symmetry H' , and a second phase that preserves H' but breaks H [1,2]. The phase transition is described by a conformal field theory (CFT) with symmetry $H \times H'$. The simplest example of a DQCP is quantum electrodynamics in $2 + 1$ dimensions (QED3) with $N > 1$ matter fields (either fermions or bosons), where $H = U(1)$ and $H' = SU(N)$, and the gauge fields are emergent at the critical point (i.e., deconfined). At large N , it can be proven that this theory flows to a CFT in the IR, and observables can be computed in a $1/N$ expansion [3,4]. However, at finite N , the theory is

strongly coupled, which makes it challenging to compute observables or even determine if the theory flows to a CFT.

The simplest putative DQCP would have $N = 2$ bosons or fermions [5]. The conformal bootstrap was used in [6] to show that the $N = 2$ fermionic theory is likely not conformal [7], and instead the theory is believed to spontaneously break $SU(2) \times U(1) \rightarrow U(1)$ [13,14]. The bootstrap similarly rules out the possibility that the $N = 2$ bosonic theory is critical [6,15], i.e., that it is a CFT with one relevant $SU(2) \times U(1)$ singlet [16]. Instead, the theory is believed to either be a tricritical CFT (i.e. two relevant singlets) [18,19] or be weakly first order [20]. Since N must be even in the parity preserving fermionic case [21], this leaves the $N = 3$ bosonic theory as the simplest possible DQCP with one relevant singlet.

QED3 with N bosons, also known as the CP^{N-1} model [22], has been extensively studied with lattice simulations, but there is no agreement on the values of scaling dimensions, or even for what value of N the theory flows to a CFT. A recent simulation of the lattice Abelian-Higgs model suggested that the theory is conformal for

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$N = 10$ but not for $N = 4$ [23], which suggests it is also not conformal for $N = 3$ [24]. A previous simulation of the Néel-VBS transition in quantum $SU(N)$ antiferromagnets, using the JQ model, suggested that the theory is conformal for $N = 3$ and higher values [25,26].

In this work, we will show evidence that the CP^2 model flows to a critical point, and predict the scaling dimensions of low-lying operators. We will make use of the fact that the large N expansion of scaling dimensions Δ_q of the lowest dimension scalar operators with charge $q \in \mathbb{Z}$ under $U(1)$ [28,29], which are called monopole operators [30], has been shown to be extremely accurate for all values of N by comparison to lattice predictions for small values of N [27,29,31–33], and the duality to the critical $O(2)$ model for $N = 1$ [34–37].

We then apply the conformal bootstrap to $U(1)$ invariant CFTs in $3d$ whose only relevant operators are scalars with $q = 0, 1, 2$, as suggested for the CP^2 model from large N . This excludes the critical $O(2)$ model, which also has a relevant $q = 3$ operator [38]. The bootstrap rigorously bounds the space of allowed scaling dimensions of these operators, as well as the $q = 3, 4$ operators that appear in the operator product expansion (OPE) of the $q = 0, 1, 2$ operators. Physical theories often appear approximately [39] on the boundary of allowed regions, as has been shown for many other models [40]. In our case, by looking at the point on the boundary given by the large N value of Δ_1 and then minimizing the scaling dimension Δ_0 of the lowest singlet operator (which is not a monopole), we can read off the values of all the other scaling dimensions. As shown in Table I, our results match the large N estimates of the scalar monopole scaling dimensions Δ_q for $q = 2, 3, 4$, as well as the lattice estimate in [25] for Δ_0 .

We also make predictions for the lowest spin ℓ monopole operators with scaling dimension $\Delta_{q,\ell}$ for $\ell \leq 4$, reported in Table II. We can compare these values to the large charge q effective theory at small $\ell \neq 1$ [55,56], whose coefficients were fixed using the large N expansion for $\ell = 0$ in [57]. The extrapolation is summarized in Table III. We find that these values match our bootstrap predictions for

TABLE I. Comparison of scaling dimensions Δ_q of the lowest dimension scalar operators with $U(1)$ charge q in the CP^2 model, as determined from the bootstrap study here, the large N expansion for monopoles $q > 0$, and a lattice study for both monopoles and the lowest non-monopole singlet $q = 0$. The asterisk by Δ_1 for bootstrap means we put it in to determine the others, while the bootstrap errors come from extrapolation in the bootstrap truncation parameter Λ .

CP^2	Δ_1	Δ_2	Δ_3	Δ_4	Δ_0
Bootstrap	0.755*	1.841(1)	3.173(4)	4.65(9)	1.61(1)
Large N	0.755	1.81	3.10	4.59	...
Lattice [25]	0.71(4)	...	...	...	1.46(7)

TABLE II. Scaling dimensions $\Delta_{q,\ell}$ of the lowest dimension operators with $U(1)$ charge $q \in \mathbb{Z}$ and spin $\ell > 0$ as determined from the bootstrap study here, with errors estimates coming from the extrapolation in the bootstrap truncation parameter Λ . Note that odd spin $q = 4$ operators exist, but do not appear in the correlators we bootstrap here. The $\ell = 1, q = 0$ operator is the conserved $U(1)$ current which has exact dimension $\Delta = 2$.

Bootstrap	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$
$q = 1$	3.328(4)	3.21(1)	4.1(1)	5.42(3)
$q = 2$	4.83(3)	3.634(6)	5.64(9)	5.6(1)
$q = 3$	6.0(1)	4.893(4)	5.6(1)	6.82(2)
$q = 4$	...	6.1(2)	...	7.87(7)
$q = 0$	2	3.76(2)	4.328(2)	5.0(1)

$\ell \leq q$, which is presumably where this small ℓ expansion is accurate. Curiously, the exact same pattern is observed for the critical $O(2)$ model, where bootstrap results from [58] match the large q effective theory from [59] for $\ell \leq q$, as summarized in Supplemental Material [60,62].

The CP^{N-1} model. The Lagrangian of N complex scalar fields ϕ_i coupled to an Abelian gauge field A_μ in $3d$ is

$$\mathcal{L} = \sum_{i=1}^N [|(\partial_\mu - iA_\mu)\phi^i|^2 + m^2|\phi^i|^2] + u \left[\sum_{i=1}^N |\phi^i|^2 \right]^2 + \frac{F^2}{4e^2}, \quad (1)$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength. At large N , we can tune $m^2 = 0$ to get a critical theory in the IR [63]. One can then decouple the quartic term by introducing a Hubbard-Stratonovich field σ and replacing $u(\sum_{i=1}^N |\phi^i|^2)^2$ by $\sigma \sum_{i=1}^N |\phi^i|^2 - \sigma^2/4u$, such that the functional integral over σ reproduces (1). Since the scaling dimension of σ is $\Delta_0 = 2$ at large N , in the IR we set $e, u \rightarrow \infty$ to get the conformally invariant action

$$\mathcal{L} = \sum_{i=1}^N [|(\partial_\mu - iA_\mu)\phi^i|^2 + \sigma|\phi^i|^2]. \quad (2)$$

TABLE III. Scaling dimensions $\Delta_{q,\ell}$ of the lowest dimension scalar operators with $U(1)$ charge $q \in \mathbb{Z}$ and spin ℓ as computed from the large q effective theory for general $\ell \neq 1$, with coefficients fixed by large N for $\ell = 0$. The italicized values for $\ell > q$ are where this small ℓ expansion is observed to be less accurate.

Large charge	$\ell = 0$	$\ell = 2$	$\ell = 3$	$\ell = 4$
$q = 1$	0.750	2.482	<i>3.199</i>	<i>3.912</i>
$q = 2$	1.803	3.536	<i>4.253</i>	<i>4.966</i>
$q = 3$	3.093	4.825	5.543	6.256
$q = 4$	4.583	6.315	7.032	7.745

This is in the same universality class as the CP^{N-1} model, since shifting σ by one gives a nonlinear sigma model with CP^{N-1} target space [64]. The theory has a $SU(N)$ flavor symmetry that rotates ϕ_i , as well as a $U(1)$ topological symmetry whose current $\epsilon_{\mu\nu\rho}F^{\nu\rho}$ is conserved due to the Bianchi identity. Including discrete groups, the faithful global symmetry for $N > 2$ is $(SU(N)/\mathbb{Z}_N \times U(1)) \rtimes \mathbb{Z}_2^c$, where $\mathbb{Z}_2^c$ is charge conjugation and $\mathbb{Z}_N$ is the center of $SU(N)$.

We can construct local operators out of ϕ_i , A_μ , and σ that transform under $SU(N)$ but not $U(1)$. For instance, the unique relevant singlet scalar operator is σ , whose scaling dimension is $\Delta_0 = 2$ at $N \rightarrow \infty$. The scaling dimension of these operators can be computed at large N using Feynman diagrams to get $\Delta \sim N^0$ and the subleading correction [3,65–69], but this expansion is not very accurate for small N [70].

Monopole operators: We define monopole operators as local gauge invariant operators that transform under $U(1)$ with charge $q \in \mathbb{Z}$. These operators are not built from fields in the action, but instead are defined as inserting magnetic flux $q = \frac{1}{2\pi} \int F$ [73,74]. The lowest dimension monopoles are scalars and singlets under $SU(N)$.

Their scaling dimensions Δ_q are identified via the state-operator correspondence with the ground state energies on $S^2 \times \mathbb{R}$ with $2\pi q$ magnetic flux through S^2 , which can be computed at large N using a saddle point expansion [28,30] to get $\Delta_q \sim N$. This calculation was carried out to subleading order in [29,75], and the results were found to be extremely accurate even at small N . We list some values extrapolated to $N = 3$ in Table I.

Large q effective theory: As shown in [55,56], operators in any CFT with large charge q under a $U(1)$ global symmetry are described by an effective theory. This effective theory can be used to compute the scaling dimension $\Delta_{q,\ell}$ of the lowest dimension primary operator with charge q and spin $\ell \ll q^{\frac{1}{2}}$ as

$$\Delta_{q,\ell} = c_{\frac{3}{2}} q^{\frac{3}{2}} + c_{\frac{1}{2}} q^{\frac{1}{2}} - 0.0937 + \sqrt{\frac{\ell(\ell+1)}{2}} + O(q^{-\frac{1}{2}}), \quad (3)$$

except for $\ell = 1$, where the formula describes a conformal descendant [76]. Note that the q^0 terms are universal for any theory and any $\ell \ll q^{\frac{1}{2}}$. For the CP^{N-1} model, the lowest dimension monopoles are described by this effective theory. The large N results from [29], valid for $\ell = 0$ and finite q , were used in [57] to compute the theory-dependent coefficients $c_{\frac{3}{2}}$ and $c_{\frac{1}{2}}$ to subleading order in N (and verify the universal q^0 term), which for $N = 3$ gives

$$c_{\frac{3}{2}} = 0.4983, \quad c_{\frac{1}{2}} = 0.3449. \quad (4)$$

We show the values of $\Delta_{q,\ell}$ from this expansion in Table III, which for $\ell = 0$ match the finite q large N results in Table I to good accuracy.

Conformal bootstrap. We will apply the conformal bootstrap to just the $O(2) \cong U(1) \rtimes \mathbb{Z}_2^c$ sector. We follow the setup for $O(2)$ invariant systems in [38], which we review in more detail in Supplemental Material [62]. Four point functions of local scalar primary operators $\varphi_q(x)$ with charge q under a $U(1)$ global symmetry can be expanded in conformal blocks $g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(u,v)$ as

$$\begin{aligned} & \langle \varphi_{q_1}(x_1) \varphi_{q_2}(x_2) \varphi_{q_3}(x_3) \varphi_{q_4}(x_4) \rangle \\ &= \frac{x_{24}^{\Delta_{12}^-} x_{14}^{\Delta_{34}^-}}{x_{14}^{\Delta_{12}^-} x_{13}^{\Delta_{34}^-} x_{12}^{\Delta_{13}^+} x_{34}^{\Delta_{24}^+}} \sum_{\mathcal{O}} \lambda_{\varphi_{q_1} \varphi_{q_2} \mathcal{O}} \lambda_{\varphi_{q_3} \varphi_{q_4} \mathcal{O}} T_{\mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4}^{\mathbf{r}} \\ & \times g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(u,v), \end{aligned} \quad (5)$$

where $\Delta_{ij}^\pm \equiv \Delta_i \pm \Delta_j$, $x_{ij} = x_i - x_j$, and we define the conformal cross ratios as $u \equiv \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $v \equiv \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$. The primary operators $\mathcal{O}$ that appear in both OPEs $\varphi_{q_1} \times \varphi_{q_2}$ and $\varphi_{q_3} \times \varphi_{q_4}$ have scaling dimension Δ , have spin ℓ , and transform in an irreducible representation $\mathbf{r}$ of $O(2)$ that appears in both of the tensor products $\mathbf{r}_1 \otimes \mathbf{r}_2$ and $\mathbf{r}_3 \otimes \mathbf{r}_4$. The global symmetry tensor structures are denoted by $T_{\mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4}^{\mathbf{r}}$, whose explicit expression can be found in [38]. Finally, $\lambda_{\varphi_i \varphi_j \mathcal{O}}$ are the OPE coefficients. In the following, we denote by $\lambda_{ijk} \equiv \lambda_{\varphi_i \varphi_j \varphi_k}$ the OPE coefficients involving the “external” operators, whenever they are exchanged in the respective OPE. The lhs of (5) is invariant under swapping $\varphi_{q_1}(x_1)$ with $\varphi_{q_3}(x_3)$, but the block expansion on the rhs changes, giving the crossing equation:

$$\begin{aligned} 0 &= \sum_{\mathcal{O}} \lambda_{\varphi_{q_1} \varphi_{q_2} \mathcal{O}} \lambda_{\varphi_{q_3} \varphi_{q_4} \mathcal{O}} T_{\mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4}^{\mathbf{r}} v^{\frac{\Delta_{23}^+}{2}} g_{\Delta,\ell}^{\Delta_{12}^-, \Delta_{34}^-}(u,v) \\ & - \sum_{\mathcal{O}} \lambda_{\varphi_{q_3} \varphi_{q_2} \mathcal{O}} \lambda_{\varphi_{q_1} \varphi_{q_4} \mathcal{O}} T_{\mathbf{r}_3 \mathbf{r}_2 \mathbf{r}_1 \mathbf{r}_4}^{\mathbf{r}} u^{\frac{\Delta_{12}^+}{2}} g_{\Delta,\ell}^{\Delta_{32}^-, \Delta_{14}^-}(v,u). \end{aligned} \quad (6)$$

By projecting this equation into a basis for the tensor structures $T_{\mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_3 \mathbf{r}_4}^{\mathbf{r}}$, one can get a finite set of crossing equations, each a function of u, v . One can then truncate the infinite dimensional vector space of functions of u, v to the space of their derivatives at the crossing symmetric point up to a given order Λ [77]. By imposing the reality of the OPE coefficients that follows from unitarity, one can then use this finite set of crossing equations to compute bounds on the allowed scaling dimensions and OPE coefficients that appear in (5), and these bounds monotonically shrink as Λ is increased [79]. We use the Navigator algorithm to efficiently scan over the allowed region [80], and also Simpleboot [81].

We first consider correlators of φ_0 and φ_1 for $\Lambda = 19$, which yields 7 crossing equations as given in [41]. We assume that these are the only relevant scalar operators with their $U(1)$ charges, and scan over the OPE coefficient ratio $\lambda_{000}/\lambda_{110}$ to further shrink the bound. The allowed region in the space of (Δ_0, Δ_1) is shown in Fig. 1, which generalizes Fig. 2 of [41] by extending to larger values of Δ_1 [82]. Note that the critical $O(2)$ Wilson-Fisher model and the CP^{N-1} model for $N > 2$ (if critical) should appear in the allowed region, since they both have one relevant $q = 0, 1$ operator [83]. In particular, while the critical $O(2)$ theory appears in a small “island” as shown first in [41], the lattice and large N estimates for the CP^2 theory appear on the lower boundary of the “continent”, which motivates us to minimize Δ_0 to find this theory. Note that as Δ_1 gets bigger, the lower bound on Δ_0 goes to one, so with these assumptions the CP^{N-1} model for higher N cannot lie on this bound, as we expect $\Delta_0 \approx 2$ for large N . In Supplemental Material [62], we discuss results for

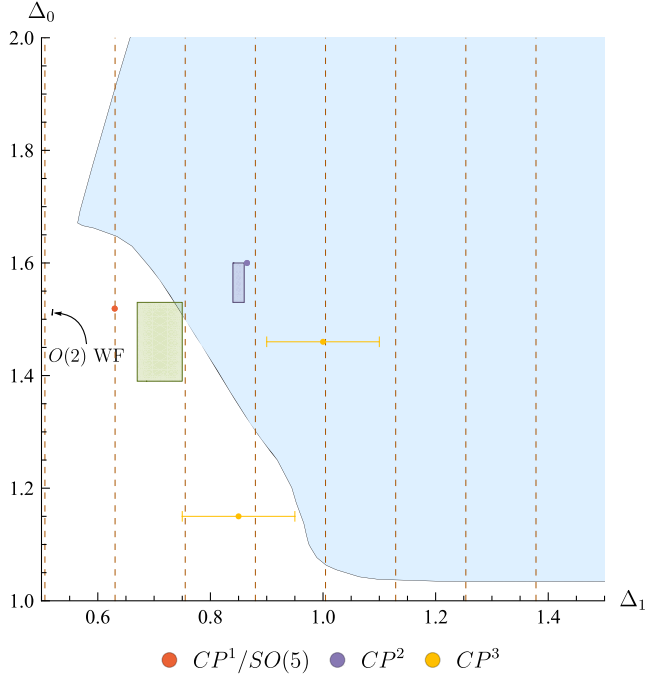


FIG. 1. Allowed region for (Δ_1, Δ_0) in 3d CFTs with $O(2)$ global symmetry and only one relevant $q = 0, 1$ operator, computed with $\Lambda = 19$. These assumptions allow for the critical $O(2)$ Wilson-Fisher model, contained in the tiny island indicated by the arrow. They also allow for the CP^{N-1} model for $N > 2$. We show lattice results for $N = 3$ [25], $N = 4$ [25,27], and $N = 5$ [84], where errors bars are shown where available, and otherwise are shown by colored dots. The dashed vertical lines correspond to the value for Δ_1 extrapolated from large N estimates [29] down to $N = 1, 2, \dots, 8$ from left to right. The red point denotes the scaling dimensions of the $SO(5)$ bootstrap [18] which is proposed to describe the tricritical $N = 2$ theory, and is ruled out by our assumption of just one relevant $U(1)$ singlet.

$N = 4, 5$, which show some agreement with the large N expansion for scalar monopole scaling dimensions.

We next consider correlators of φ_0, φ_1 , and φ_2 for $\Lambda = 19$, which yield 22 crossing equations as given in [38,85]. We now assume that these are the only relevant operators, by imposing that all $q = 3, 4$ scalar operators are irrelevant. This excludes the critical $O(2)$ model, which has a relevant $q = 3$ operator [38]. The output of the bootstrap is an allowed region in the 6-dimensional space $(\Delta_0, \Delta_1, \Delta_2, \lambda_{000}/\lambda_{110}, \lambda_{220}/\lambda_{110}, \lambda_{112}/\lambda_{110})$. We look at the point on the boundary of this region given by inputting the value $\Delta_1 = 0.755$, as given by large N , and minimizing Δ_0 as suggested by the previous plot. In Fig. 2, we look at a cross section of this allowed region with $\Delta_1 = 0.755$ in the space of (Δ_0, Δ_2) , which shows that minimizing Δ_0 corresponds to a corner of this allowed region.

We can then read off all CFT data that appear in the approximate solution to crossing at the boundary of this allowed region. We report the values of the lowest dimension $q = 0, 2, 3, 4$ scalar operators in Table I, and compare them to large N and lattice estimates. To obtain these values, we ran the bootstrap for $\Lambda = 19$ to $\Lambda = 31$ in steps of two, and then extrapolated to $\Lambda \rightarrow \infty$, so the error shown in our results comes from this extrapolation [see Supplemental Material [62] for more details]. We also report some of the lowest dimension operators with higher

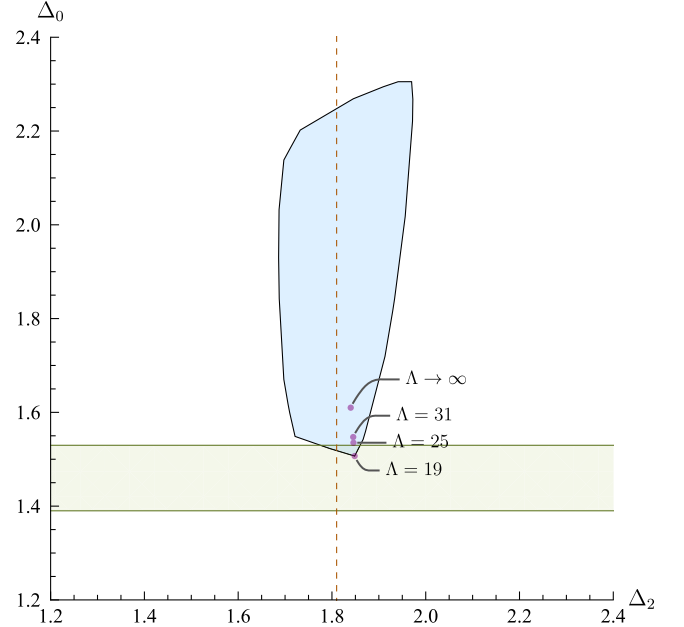


FIG. 2. Allowed region for (Δ_2, Δ_0) in 3d CFTs with $O(2)$ symmetry and only one relevant $q = 0, 1, 2$ operator, with $\Delta_1 = 0.755$, and computed with $\Lambda = 19$. The green band is the lattice result for Δ_0 from [25], while the dashed vertical line is the large N value of Δ_2 extrapolated to $N = 3$. The purple dots are the result of Δ_0 minimization for some values of Λ , together with the $\Lambda \rightarrow \infty$ extrapolation.

spin in Table II. We do not report higher twist operators, since these are less accurate.

Discussion. In this work, we bootstrapped the CP^2 model, which is the simplest DQCP. In particular, we showed that the point on the boundary of the allowed region of $U(1)$ CFTs in $3d$ given by inputting Δ_1 as computed from the large N expansion of the CP^{N-1} model extrapolated to $N = 3$ gives Δ_q for $q = 2, 3, 4$ that match their large N values, as well as Δ_0 that matches the lattice estimate from [25]. We also made predictions for $\Delta_{q,\ell}$ for $\ell > 0$, which match the large q effective theory as computed using the large N expansion in [57] for $\ell \leq q$.

It is striking that the large q expansion matches our bootstrap results for every $\ell \leq q$ that we computed, as strictly speaking this expansion should only be valid for $\ell \ll q^{\frac{1}{2}}$. A similar successful match was also observed for the critical $O(3)$ model in [86] for $\ell = 2$ and $q = 1, \dots, 6$. In Supplemental Material [62], we compare the bootstrap results from [38,58] to the large q expansion for the critical $O(2)$ from [59], where in this case lattice simulations for $\ell = 0$ data were used to fix $c_{\frac{3}{2}}$ and $c_{\frac{1}{2}}$ in (3). We find the exact same pattern as observed here for CP^2 . This suggests the general large q effective theory works for a larger regime of ℓ than one might expect [87].

Looking ahead, we would like to access the $SU(3)$ sector of the CP^2 model, perhaps by performing a mixed correlator study between adjoints of $SU(3)$ as well as the monopoles considered in this work. Unfortunately, we found that the simplest bootstrap setup, which only assumes one relevant $q = 0, 1, 2$ and one adjoint, leads to weak bounds in the $SU(3)$ sector, which are far from the lattice estimates of lowest dimension adjoint operator in [25]. This is because in this setup, we have found the mixing between the adjoint correlator and the correlators in this work to be very weak. Indeed, it was known from earlier work that the $SU(N)$ adjoint bootstrap yields weak bounds for small N [47,89]. It would be interesting to find well motivated physical assumptions that could overcome this problem.

It would also be nice to confirm our predictions for higher spin monopoles for general q by generalizing the large N and finite q calculation for lowest dimension monopoles in [28,29] to nonlowest monopoles, as all the lowest monopoles in the CP^{N-1} theory have zero spin. This would require one to compute the energy of excited states

on the cylinder. Such a calculation could also show evidence for the putative $SO(5)$ symmetry enhancement for the $N = 2$ theory [17–19].

We also expect that some of our predictions can be compared to lattice simulations in the near future. In particular, Δ_3 can be compared to the ω component of lattice simulations on the honeycomb lattice, while $\Delta_{2,2}$ could be compared to the ω component on the square lattice [90].

We would also like to generalize our study to larger N , even just in the $U(1)$ sector [91]. Note that the only specification of N into our $U(1)$ bootstrap was by inputting the value of Δ_1 as given from the large N expansion, so one could in principle bootstrap CP^{N-1} for larger N by inputting a larger value of Δ_1 . As noted above, though, the lower bound on Δ_0 goes to one as Δ_1 increases, so the CP^{N-1} model cannot lie on this lower bound in that regime. Preliminary investigation suggests that increasing the gap above Δ_0 above three, which was what we assumed in this work, might address this problem, but we have no independent way of determining how large that gap should be.

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Data availability. The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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