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The Little-Higgs Mechanism in Electroweak and Flavor Physics

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Contents

Introduction

Almost forty years after its introduction [1], the standard model of elementary particles based on the $SU(3)_C \times SU(2)_W \times U(1)_W$ gauge symmetry with three families of leptons and quarks has been verified experimentally both at the level of its tree-level (classical) Lagrangian with the discovery at CERN [2] of the electrically charged and neutral gauge bosons and at that of its quantum corrections at LEP [3] and the Tevatron [4] up to energies of a few hundreds of GeVs. It provides a beautifully elegant picture of all known elementary particles and their interactions (except gravity) and the framework of our current understanding of cosmology, nuclear physics, chemistry and even biology.

Inspite of its many successes, the standard model may not be the ultimate theory and is probably incomplete and best considered as an effective theory only valid up to its characteristic energy scale. The main experimental evidences for physics beyond it are the smallness of neutrino masses (which points to heavy Majorana masses to make possible the see-saw mechanism [5]) and dark matter [6] (which requires the existence of new weakly interacting particles). In addition, there exist various theoretical reasons for extending the standard model—preeminent among them the hope of further unify the gauge interactions among themselves [7] and maybe even with gravity.

In recent years, the search for physics beyond the standard model has focused on what is arguably its least satisfactory part: the Higgs boson sector. This sector is necessary in order to trigger the spontaneous breaking of the electroweak symmetry and thus give mass to the gauge bosons as well as to the leptons and quarks.

Contrary to fermions and gauge bosons, the mass of the Higgs boson receives a one-loop quadratically divergent correction given by

$$\delta m^2_{H_{1-loop}} = \frac{3\Lambda^2}{32\pi^2}(\frac{3}{2}g^2 + g'^2 + 2\lambda - 4\lambda_t^2)\,,$$

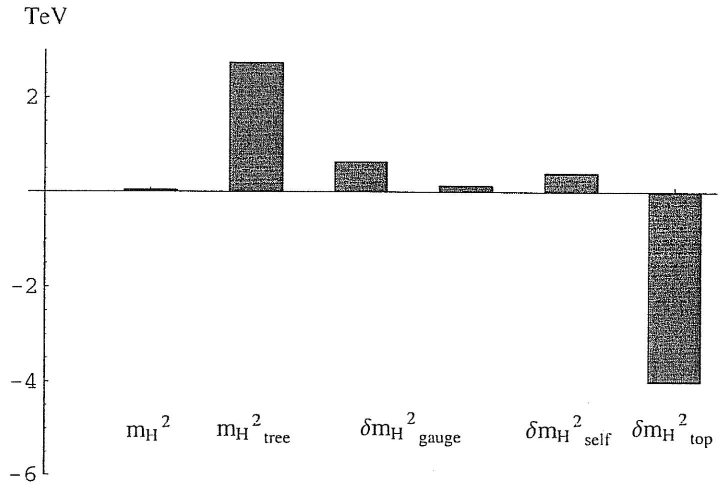
where g, g' are respectively the $SU(2)_W$ and $U(1)_W$ gauge couplings, λ the quartic self interaction coupling, λ_t the top Yukawa coupling and Λ the cut-off of the effective theory (the sub-leading contributions arising from all the other standard model fermions have been neglected). Substituting the gauge and Yukawa couplings with their numerical values and taking for λ a value that gives a Higgs boson mass at the central value of current electroweak data fits [8], that is $m_H = \sqrt{\lambda}v_w \sim 114$ GeV (where $v_w = 246$ GeV is the Higgs boson vacuum expectation value), we have that the Higgs boson mass is approximately given by

$$m_H^2 \simeq m^2_{H_{tree}} + \frac{\Lambda^2}{100}(0.64 + 0.13 + 0.4 - 4)\,.$$

This equation can be discussed in terms of the fine-tuning between the tree and one-loop contributions required in order to reproduce the desired value of m_H for a given cut-off. If we admit no fine-tuning at all, $m^2_{H_{tree}}$ and $\delta m^2_{H_{1-loop}}$ should be of the same order and Λ can be at most 700 GeV. If we want to accommodate the same mass for the Higgs boson with a cut-off $\Lambda \sim 1$ TeV, we have to tune $m^2_{H_{tree}}$ and $\delta m^2_{H_{1-loop}}$ at least within ten parts in one hundred. The fine-tuning becomes one part in one hundred if we push the standard model cut-off up to 10 TeV.

What amount of fine-tuning can be considered natural? 't Hooft's criteria for the naturalness of a dimensionful parameter [9] like the Higgs boson mass would require its value to be of the same order of the cut-off Λ. This means that we can adjust the relative cancellation between the tree level term and the one-loop corrections up to 10% before seriously running against this criteria.

Therefore, by only admitting a 10% fine-tuning, we reach the conclusion that we expect weakly coupled physics beyond the standard model around the TeV scale. New physics has to appear to cancel the quadratically divergent corrections to the Higgs boson mass and, at the same time, it has to be weakly coupled at the same scale in order not to affect the electroweak precision measurements (new strong dynamics may arise around 10 TeV [10]). This tension between the expected new physics scale, around the TeV, and the



The figure shows the size of the tree level and one-loop mass renormalization with a cut-off $\Lambda = 10$ TeV and compare them with the Higgs boson mass which is taken $m_H \simeq 113$. Radiative corrections are dominated by the top quark contribution.

scale indicated by the electroweak precision measurements, around 10 TeV, constitutes the so called *little hierarchy* problem.

The fine-tuning problem associated to the mass of the Higgs boson and therefore to the stabilization of the electroweak scale is not new. Accordingly to 't Hooft's criteria, a problem of hierarchy arises as soon as a scale larger than the electroweak scale is introduced, as it is the case of grand unified theories (GUTs) and Planck scale physics. In these examples, the scale is, respectively, 10^{16} GeV and 10^{19} GeV and therefore much bigger than in the case of the little hierarchy. Accordingly, it gives rise to an even more serious problem of hierarchy and fine-tuning.

Historically, technicolor models were the first attempts to solve this hierarchy problem [11]. In these models the standard model gauge group is extended to include a new $SU(N)$ strong interaction, Technicolor (TC), which breaks the electroweak gauge groups dynamically at a scale $\Lambda_{TC} \sim 1$ TeV. The Higgs boson is not an elementary particle but a composite one, made of techni-fermions, new strongly interacting fermions introduced by the model. TC models became extended technicolor models (ETC) to accomodate fermion masses. In these models there is an additional strong interaction which is spontaneously broken down to that of TC at a scale Λ_{ETC}. However, constraints on flavor changing neutral current processes impose Λ_{ETC} to be around 10^3 TeV, thus introducing a new hierarchy problem and therefore ruling out at least the simplest versions of these models. Even though earlier ETC and TC models do not satisfy the constraints arising from electroweak precision measurements, newly proposed TC models [12] have overcome the problems of the simpler versions to satisfy the electroweak constraints and predict a light composite Higgs mass.

The hierarchy problem may be considered also instrumental in the definition of the minimal supersymmetric extension of the standard model, the MSSM [13]. In supersymmetric theories each standard model field has a superpartner of opposite statistics and the quadratically divergent corrections to the Higgs boson mass arising from standard model fields are cancelled by those of their superpartners. If supersymmetry is softly broken around the TeV, the Higgs boson mass receives radiative corrections only of the order of the soft breaking terms and therefore fine-tuning is acceptable. The supersymmetric solution to the hierarchy problem has been enthusiastically accepted because of the other (many) good features of the MSSM, that is, it is a perturbative model, it is consistent with experimental data [14] and, last but not least, it indicates gauge coupling unification [15]. Even though supersymmetric models solve in the most natural way the hierarchy problem, recent electroweak data shows that even for them the light mass of the lightest Higgs boson can only be explained by admitting an amount of fine-tuning larger than originally expected [16].

Another recent proposal for solving the hierarchy problem between the electroweak and the Planck scale is low-scale gravity [17], an approach that circumvents the need for supersymmetry or TC. The hierarchy problem is solved by bringing the fundamental Planck scale down to the TeV scale, which becomes the only fundamental short-distance scale in nature. This is made possible by the presence of n large extra dimension. This kind of models are motivated by considerations inspired by string theory and by the observation that, even if the every day world looks four-dimensional, it is possible that at distances shorter than TeV^{-1} the Universe may best be described by more than four dimensions. In low-scale gravity models the large n extra dimensions are compactified, for example, on a circle of length $L_n = 2\pi r_n$, and the standard model fields live on a four-dimensional brane of the $(4+n)$ space, while gravity and perhaps other fields can propagate in the higher dimensions. The weakness of gravity at long distances is explained by the presence of the volume of the n large extra dimensions which enters the relationship between the four dimensional effective Planck scale, M_{pl}, and the $(4+n)$ dimensional one, M_*:

$$M_{pl}^2 \sim r_n^n M_*^{n+2} ,$$

where r_n may be interpreted as the size of the n^{th} large extra dimension.

In this thesis we concentrate on the little hierarchy problem and on a different approach to its solution. The basic idea consists in turning the Higgs boson into a pseudo-Goldstone boson (PGB) originating from a spontaneously broken global symmetry. Goldstone bosons (GBs) are massless scalar particles remaining after the spontaneous breaking of global symmetries. Their number is determined by the number of broken generators in the group algebra and they have no potential at all orders in perturbation theory and only couple derivatively to other fields. PGBs are spinless bosons that arise in theories in which scalar field interactions present accidental global symmetries larger than the gauge symmetries of the full Lagrangian. These global symmetries are then both spontaneously and explicitly broken: massless excitations arise after the spontaneous breaking but after the explicit breaking these would-be GBs acquire a potential and a mass proportional to the strength of the explicit breaking. The name of PGBs refers to their nature of would-be GBs: if the explicit breaking were to be turned off they would be real GBs. Historically they were first introduced by Weinberg [18]. Georgi and Pais [19] formulated a simple theorem that gives the conditions a scalar Lagrangian has to satisfy to assure the presence of PGBs.

Early attempts to use the idea of making the Higgs boson into a PGB go back to the eighties and were TC inspired composite Higgs models, denoted as *ultracolor (hypercolor) models* [20]. These models represent a sort of bridge between the standard model scalar sector description, with a fundamental Higgs boson, and ETC models. The Higgs boson becomes a composite state of ultrafermions bound by a strong new interaction, ultracolor or hypercolor. PGBs, among them the Higgs doublet, arise at a scale Λ_{UC} when ultrafermions condensate. Note that, in order to produce PGBs at the scale Λ_{UC}, the theory has to possess an accidental global symmetry larger than the ultracolor gauge group. Even if potentially interesting, ultracolor models and their extension, *extended ultracolor models*, were essentially ruled out by electroweak precision measurements.

More recently a new class of models that stabilize the electroweak scale by making the Higgs boson a PGB has been introduced. They can be relevant in the solution of the little hierarchy problem. These models are based on *deconstruction* [21, 22] and the physics of *theory space* [21, 23] and present no new strong interactions up to 10 TeV. In these models accidental global symmetries broken around 10 TeV protect the Higgs boson mass from receiving quadratically divergent contributions and the standard model quadratic divergences are canceled not by particle of opposite statistics as in supersymmetric theories but by particles of the same statistic. The name deconstruction is related to a higher, more than four, dimensional description that these models have and according to which the four-dimensional model can be thought as obtained by deconstructing the higher dimensions.

Contrary to extra-dimensional models in which starting from, for example, a five-dimensional theory the four-dimensional effective theory is obtained, in models based on deconstruction the fifth dimension may emerge dynamically at low energy. Suppose that at very high energies there is a four-dimensional theory with a gauge symmetry $SU(n)^N \times SU(m)^N$ with two kind of fermions, ψ_i and χ_i, transforming with respect to the gauge symmetry as bi-fundamental $(\bar{n}, m)$ and $(\bar{m}, n)$ respectively. At an intermediate scale the $SU(m)_i$ (or $SU(n)_i$) gauge groups become strongly interacting giving rise to a fermion condensate and non-linear sigma model fields that link the different $SU(n)_i$ (or $SU(m)_i$) gauge groups. Their fluctuations break the $SU(n)^N$ gauge group to the diagonal one and the gauge boson mass spectrum is identical to the Kaluza-Klein mass spectrum of a five-dimensional $SU(n)$ gauge symmetry with the fifth dimension compactified [21, 22]. In this way, the fifth dimension emerges dynamically: we start from a strongly interacting four-dimensional theory, we obtain an effective weakly interacting four-dimensional theory and discover that we would obtain the same effective theory if we started with a weakly interacting five-dimensional theory with a compactified fifth dimension.

Given the low energy four-dimensional description based on deconstruction, the model has a pictorial representation denominated *moose* diagram [24]. In general, a moose is given by N sites, representing N copies of a $SU(n)$ gauge group, by lines connecting the sites, representing the link fields, which may be bosonic or fermionic, and by faces, corresponding to plaquette operators. The space were the moose lives is called the theory space. Since in models based on deconstruction the extra-dimensions are not essential, the physics can be understood just in terms of its four-dimensional description. For this reason it is possible to construct a model directly from its moose in the theory space and completely forget about the five-dimensional correspondence [25].

The evolution of models based on deconstruction has been a class of models denoted as *little Higgs models.* These models present the same features of those based on deconstruction, that is, the Higgs boson is a PGB and its mass is protected by accidental global symmetries, but the correspondence between the description of the theory in four and in five dimensions that distinguishes deconstruction is lost. Moreover, most little Higgs models do not even have a description in terms of a moose in theory space. Many little Higgs models have been built in the past four years [26] but the best known is the *littlest Higgs model* [27].

Little Higgs models are appealing because the Higgs boson mass is protected from one-loop divergent contributions, that is quadratically divergent renormalizations to the Higgs mass arise only at the two-loop level. The standard model cut-off can thus be raised from few TeVs to more or less 10 TeV, thus solving the little hierarchy problem. The mechanism used to protect the Higgs boson mass is the so called *collective symmetry breaking*: the explicit breaking of the approximate global symmetries is arranged in such a way that more than one interaction term is always needed in order to break all symmetries. This mechanism can be implemented without doubling the standard model spectrum, such as in supersymmetric models, by simply adding a small number of new degrees of freedom. These considerations explain why there has been a lot of interest in the study of the phenomenology of different little Higgs models and in particular in the search of their possible experimental signatures at LHC [28]. Great attention has been devoted in analyzing the compatibility between the new processes induced in little Higgs models and the electroweak precision measurements [29, 30].

The first chapter of this thesis is mainly devoted to the littlest Higgs model. As a way of introduction, in the first section of the chapter we briefly discuss the main features of three models: a model based on deconstruction, a model based on a simple moose and a very simple little Higgs model. This discussion illustrates the link between models based on deconstruction and little Higgs models and provides an introduction to the concept of collective breaking.

In the remaining sections of the first chapter we turn to one specific model: the littlest Higgs model [27]. After describing the model in detail, we proceed by discussing how well it succeeds as a solution of the little hierarchy problem. We find that even though the model stabilizes the electroweak symmetry breaking, it requires a quite large physical mass for the Higgs boson, and hence partially fails in solving one of the problem it was initially built to solve, that is, the stabilization of a light Higgs boson mass.

The estimate of the required amount of fine-tuning in little Higgs models to satisfy the electroweak precision bounds, as done for instance in ref. [29, 30, 31], and the conclusion we reach at the end of the first chapter seem to suggest that little Higgs models are not better than the standard model from the point of view of the fine-tuning required in order to control the Higgs sector. Nevertheless the idea they are based on can be attractive in any non-supersymmetric context in which elementary scalar fields are needed together with scale differences between their masses and the theory cut-off. We thus turn to an example of the application of the little-Higgs mechanism beyond the Higgs sector proper.

A topical subject in which a scalar sector with hierarchical mass scales appears and the naturalness issue arises is flavor physics. The matter content of the standard model is given by three families of fermions. Each family is made of a $SU(2)_W$ left-handed doublet and two $SU(2)_W$ right-handed singlets of colored quarks, a $SU(2)_W$ left-handed lepton doublet and just one $SU(2)_W$ right-handed lepton singlet. In the standard model, neutrinos are massless and all the Yukawa couplings of the terms that give mass to the other fermions are free parameters, that is, the model does not explain neither their origin or their magnitude. Moreover, experimental data indicate that quark current and quark mass eigenvectors do not coincide, the three family bases being related by the Cabibbo-Kobayashi-Masaka (CKM) mixing matrix, which is characterized by three small mixing angles. Again, the standard model does not provide any explanation for the origin of this mixing matrix.

A similar structure is mirrored, as we now know, in the lepton sector. For a long time theoretical physicists have investigated the possibility that neutrinos were massive [32]. These speculations were supported by the fact that some GUTs naturally predict massive neutrinos [7]. The experimental discovery of neutrino oscillations [33] and the more and more accurate experimental data [34] have made the description of flavor physics even more challenging. Are the neutrino Majorana or Dirac particles? What is the origin of quark and lepton mixing? Why the quark mixing matrix presents three small angles, while the lepton one two almost maximal large angles and a small one? What is the origin of the hierarchy between the quark and charged lepton masses, the third family being always the heavier? Why neutrino masses do not present such a strong hierarchy?

Even though an adequate comprehension of flavor physics will probably be tightly linked to the structure of the still unknown fundamental theory of which the standard model is just the effective low-energy theory, it is worthwhile to study models in which flavor physics emerges from a framework in which the family structure is not included by hand and the various mass hierarchies (at least partially) explained and made stable.

The possibility that the hierarchy between the masses of the standard model fermions arises because of some global horizontal symmetry acting on the three generations of matter fields has been extensively discussed in literature [35, 36]. In most of these models, heavy scalars fields (referred to as *flavons*) carry the quantum number of this flavor symmetry, and are responsible for its breaking by acquiring non-vanishing vacuum expectation values. The hierarchy between fermion masses is then the result of the hierarchy between the vacuum expectation values and the cut-off scale of the theory.

In the second part of the thesis we discuss two models, the *little flavon* model and the *flhiggs* model in which a flavor symmetry is embedded in a little-Higgs inspired scenario.

The little flavon model presents a little-Higgs inspired scenario in which an $SU(2)_F \times U(1)_F$ gauge flavor symmetry is spontaneously (and completely) broken by the vacuum of the dynamically induced potential for two scalar doublets (the flavons) which are PGBs remaining after the spontaneous breaking of an approximate $SU(6)$ global symmetry. The vacuum expectation values of the flavons give rise to the texture in the fermion mass matrices. We show that with a small amount of fine-tuning of the parameters the experimental values of the fermion masses and mixing matrices are reproduced. Finally we discuss its phenomenology and show how flavor changing neutral current and lepto-quark compositeness set the most stringent bounds to the lowest possible value for the scale of the model.

The flhiggs model gives a unified picture of flavor and electroweak symmetry breaking at the TeV scale. The model is based on an $SU(3)_W \times U(1)_W$ extended electroweak gauge symmetry and a $U(1)_F$ global flavor symmetry incorporated in a $SU(10)$ approximate global symmetry. Flavor and Higgs bosons arise as pseudo-Goldstone modes of the spontaneous breaking of $SU(10)$ to $SO(10)$. In the model, explicit collective symmetry breaking terms yield stable vacuum expectation values of the electrically neutral components of the PGBs and their masses are protected at one-loop by the little-Higgs mechanism. The coupling to the fermions through a Yukawa Lagrangian invariant with respect to the $U(1)_F$ global flavor symmetry generates well-definite mass textures that, as in the little flavon model, correctly reproduce the mass hierarchies and mixings of quarks and leptons. We will comment on how the model is more constrained than usual little-Higgs models because of bounds on weak and flavor physics and argue that the main experimental signatures testable at the LHC are a rather large mass for the (lightest) Higgs boson and a characteristic spectrum of new bosons and fermions with masses around the TeV scale.

This thesis is based on original works published as

◇ F. Bazzocchi, M. Fabbrichesi, M. Piai, *The littlest Higgs is a cruiserweight,* hep-ph/0506175

◇ F. Bazzocchi, M. Fabbrichesi, *Flavor and electroweak symmetry breaking at the TeV scale*,
[hep-ph/0410107] Nuclear Physics B 715 (2005)372-412

◇ F. Bazzocchi, M. Fabbrichesi, *A Heavy Higgs Boson from Flavor and Electroweak Symmetry Breaking Unification,*
[hep-ph/0407358] Physical Review D Vol. 70 (2004) 115008

◇ F. Bazzocchi, *Phenomenology of the Little Flavon Model*,
[hep-ph/0401105], Physical Review D Vol. 70 (2004) 013002

◇ F. Bazzocchi, S. Bertolini, M. Fabbrichesi, M. Piai, *Fermion Masses and Mixing in the Little Flavon Model*,
[hep-ph/0306184] Physical Review D Vol. 69 (2004) 036002

◇ F. Bazzocchi, S. Bertolini, M. Fabbrichesi, M. Piai, *The Little Flavons*,
[hep-ph/0306184] Physical Review D Vol. 68 No. 9 (2003) 096007

Chapter 1

The littlest Higgs model

As already discussed in the introduction, there have been many, albeit not successful, attempts to stabilize the electroweak scale by making the Higgs boson a PGB. The first indication that the idea could be successful came about only recently when a new class of models based on deconstruction was proposed. The first of this kind of models was introduced by Arkani-Hamed, Cohen and Georgi [21]. In this model one-loop divergent quadratically contributions to the mass of the Higgs boson are canceled and the cancellation occurs between particles of the same statistics.

The first section of this chapter is devoted to a pedagogical overview of three simple models that make clear the link between models based on deconstruction and little Higgs ones. We describe the model of Arkani-Hamed *et al.*, then we introduce a model that could be thought as inspired by deconstruction and finally a simple version of a little Higgs model. Since the purpose of this first section is to lead the reader through this family of models in order to gain an understanding of the mechanisms used to build a little Higgs model, we will not enter into the details of each model and into the attempts to make them more realistic.

In the second part of the chapter we turn our attention to a particular example of little Higgs models: the littlest Higgs model [27]. We give a detailed description of it and study in detail its exact (one-loop) effective potential to determine the dependence of physical quantities, such as the electroweak vacuum expectation value v_W and the mass m_H

of the Higgs boson, on the fundamental parameters of the Lagrangian—masses, couplings of new states, the fundamental scale f of the model and coefficients of the quadratic divergent terms.

We show that it is possible to have the electroweak ground state and a relatively large cut-off $\Lambda = 4\pi f$ with f in the 2 TeV range without requiring unnaturally small coefficients for quadratically divergent quantities, and with only moderate cancellations between the contribution of different sectors to the effective potential of the Higgs. On the other hand, this cannot be achieved while at the same time keeping m_H close to its current lower bound of 114.4 GeV [8]. The natural expectation for m_H is $O(f)$, mainly because of large logarithmically divergent contributions to the effective potential of the top-quark sector. Even a fine-tuning at the level of $O(10^{-2})$ in the coefficients of the quadratic divergences is not enough to produce small physical Higgs masses, and the natural expectation is in the 800 GeV range for $f \sim 2$ TeV. We thus come to the conclusion that the littlest Higgs model is a solution of the little hierarchy problem—in the sense that it stabilizes the electroweak symmetry breaking scale to be a factor of 100 less than the cut-off of the theory—but only at the price of having a quite large mass for the Higgs boson.

In the last section of the chapter we consider a possible improved version of the littlest Higgs model [37] in which the top fermionic sector is completed to make its contribution to the Coleman-Weinberg (CW) one-loop potential finite [38]. We study this model and show that, even though (marginally) better than the littlest Higgs model, again the requirement of a Higgs boson mass around 200 GeV and f around 2 TeV leads to unreasonable values of the parameters and excessive fine-tuning.

1.1 From deconstruction to little Higgs models

1.1.1 Dimensional deconstruction

Every model based on deconstruction has its pictorial representation in a moose diagram. A moose is given by a number N of sites, which represent the gauge groups, by links between the sites, which can represent fermionic o non-linear sigma model link fields, and by faces, which represent plaquette operators. In this section we describe the main features of the model discussed by Arkani-Hamed *et al.* in ref. [21].

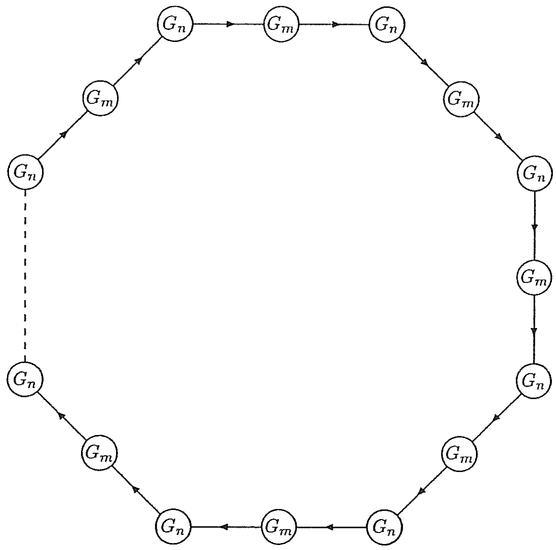


Figure 1.1: The moose diagram corresponding to the four dimensional high-energy description of the model based on a total gauge symmetry $G_n^N \times G_m^N$, with $G_n = SU(n)$ and $G_m = SU(m)$.

Consider a moose given by $2N$ sites where N copies of the two gauge groups $SU(n)$ and $SU(m)$ alternate and by $2N$ single directed lines that correspond to link Weyl fermionic fields, $\chi_{i,i}$ and $\psi_{i,i+1}$. When the fermions transform as the fundamental representation of the gauge group, their line is directed away from the corresponding site, when as the anti-fundamental of the gauge group, their line is directed toward it. For any choice of three subsequent gauge groups, $SU(m)_i \times SU(n)_i \times SU(m)_{i+1}$, the link fermions transform as

$$\begin{aligned} \chi_{i,i} &\rightarrow (m, \bar{n}, 1) \\ \psi_{i,i+1} &\rightarrow (1, n, \bar{m}), \end{aligned} \tag{1.1}$$

where $i = 1, \ldots, N$. We also impose the periodic boundary condition for which $i = 0$ is identified with $i = N + 1$.

The total gauge symmetry of the model is $SU(n)^N \times SU(m)^N$ and we assume for simplicity that all the $SU(n)$ gauge couplings g_{n_i} are equal, that is $g_{n_i} = g_n$ for $i = 1, \ldots, N$ and that also all the $SU(m)$ gauge couplings g_{m_i} are equal, that is $g_{m_i} = g_m$ for $i = 1, \ldots, N$. Suppose also that at a scale $\Lambda_{m(n)}$ the $SU(m)$ ($SU(n)$) become strongly interacting. At a very high-energy scale $\Lambda \gg \Lambda_{n,m}$ the theory is four dimensional and it is described by weakly interacting massless fermions and gauge bosons. Suppose now that $\Lambda_m > \Lambda_n$: when the energy approaches Λ_m the $SU(m)$ groups become strongly interacting and the fermions condensate in pairs

$$\langle \chi_{j,j}\psi_{j,j+1}\rangle \sim 4\pi f_m^3 U_{j,j+1} \quad j=1,\ldots,N\,, \tag{1.2}$$

with $f_m \sim \Lambda_m/4\pi$ and $U_{j,j+1}$ an $n \times n$ unitary matrix. The condensation produces a spectrum of *mesons* of masses close to $4\pi f_m$ and below the scale Λ_m the theory is described by an effective theory of N copies of the gauge group $SU(n)$ and N non-linear sigma model fields transforming as

$$U_{j,j+1} \quad \rightarrow \quad g_j U_{j,j+1} g_{j+1}^{-1}\,, \tag{1.3}$$

where g_j is the $SU(n)$ transformation associated to the gauge group $SU(n)_j$. The effective four-dimensional Lagrangian for the non-linear sigma model fields is given by

$$\mathcal{L} = \frac{1}{g_n^2}\,\mathrm{Tr}\,F_{\nu\mu}F^{\nu\mu} + f_m^2\Sigma_{j=1}^N \mathrm{Tr}\,[(D_\mu U_{j,j+1})^\dagger(D^\mu U_{j,j+1})]\,, \tag{1.4}$$

where $D_\mu U_{j,j+1} = \partial_\mu U_{j,j+1} - iA_\mu^j U_{j,j+1} + iU_{j,j+1}A_\mu^{j+1}$. The Lagrangian of eq. (1.4) may be interpreted as the effective four-dimensional Lagrangian of a five dimensional gauge theory with the fifth dimension compactified and latticized. According to this interpretation, the five dimensional lattice spacing a, the compactification radius R and the five-dimensional coupling g_5 are given and related by

$$a = \frac{1}{g_n f_m}\,, \quad R = Na\,, \quad \frac{1}{g_5^2} = \frac{1}{a g_n^2} = \frac{f_m}{g_n}\,. \tag{1.5}$$

The non-linear sigma model fields $U_{j,j+1}$ can be parametrized as

$$U_{j,j+1} = e^{\phi_j/f}\,, \tag{1.6}$$

where ϕ_j is given by $\phi_j^a(x)T^a$, with T^a the generators associated to the gauge group $SU(n)_j$ and $\phi_j^a(x)$ the corresponding fluctuations. The fluctuations ϕ_j^a of the non-linear sigma model fields break the $SU(n)^N$ gauge group to the diagonal $SU(n)$ giving mass to $N-1$ gauge boson multiplets (each multiplet composed by n^2-1 gauge bosons). The massive multiplets eat $N-1$ GBs, while the N^{th} multiplet ϕ corresponding to the linear combination $(\phi_1 + \phi_2 + \ldots + \phi_N)/\sqrt{N}$ gets a mass from gauge boson loop effects.

The fluctuations $\phi_j^a(x)$ can be interpreted as the GBs of a spontaneously broken global symmetry. In fact the condensed moose given by N copies of the $SU(n)$ gauge symmetry and N non-linear sigma model fields $U_{j,j+1}$ has a global symmetry $SU(n)^{2N}$ realized by

$$U_{j,j+1} \quad \rightarrow \quad L_j U_{j,j+1} R_{j+1}^{\dagger} \, . \tag{1.7}$$

This symmetry is spontaneously broken to the global symmetry $SU(n)^N$ realized by $L_j = R_j$. The spontaneous breaking $SU(n)^{2N} \rightarrow SU(n)^N$ produces $N(n^2-1)$ GBs that are the fluctuations of eq. (1.6).

Suppose now to turn off all the gauge couplings except that of the gauge group $SU(n)_i$. This gauge symmetry explicitly breaks the $SU(n)^{2N}$ global symmetry, but preserves an accidental approximate global symmetry $SU(n)_i^{2N-2}$ that protects all the N adjoints GBs. In fact if we have $SU(n)_{igauge} \times SU(n)_{i_{global}}^{2N-2}$, the spontaneous breaking of $SU(n)^{2N}$ in $SU(n)^N$ completely breaks the gauge symmetry $SU(n)_i$ and (n^2-1) among all the GBs arising in the spontaneous breaking $SU(n)^{2N} \rightarrow SU(n)^N$ are eaten by the massive gauge bosons. At the same time the global symmetry $SU(n)_i^{2N-2}$ is spontaneously broken to $SU(n)_i^{N-1}$ leaving exactly $(N-1)(n^2-1)$ GBs.

Let now turn on the gauge couplings of all the $SU(n)_i$: the gauge symmetry $SU(n)^N$ is broken to the diagonal $SU(n)$, $N-1$ GBs are eaten by the massive gauge bosons, but the linear combination ϕ becomes a PGB. Nevertheless, thanks to the accidental global symmetries $SU(n)_{i_{global}}^{2N-2}$, ϕ can acquire a mass from loop effects only if all the accidental global symmetries are broken, that is through a loop that involves all the N gauge bosons. The first divergent quadratically correction to its mass would be therefore proportional to $\Lambda_m^2 (g^2/16\pi^2)^N$ where $\Lambda_m = 4\pi f_m$ is the cut-off of the theory and f_m is the same of eqs. (1.2)–(1.4).

We can now rewrite eq. (1.6) just in term of the PGB ϕ

$$U_{j,j+1} = e^{\phi/f_m\sqrt{N}} \, , \tag{1.8}$$

and calculate the CW [38] potential for ϕ.

$$V(\phi) = \frac{3\Lambda_m^2}{32\pi^2}\mathrm{Tr}\, M^2(\phi) + \frac{3}{64\pi^2}\mathrm{Tr}\,(M^2(\phi))^2 \log\frac{M^2(\phi)}{\Lambda_m^2}\,, \tag{1.9}$$

where $M(\phi)$ is the gauge boson mass matrix in the presence of the background ϕ.

Let us for the moment neglect the dependence of the gauge boson masses from ϕ. In this approximation it is straightforward to check that the spectrum of the N $SU(n)$ gauge boson multiplets coincides with the spectrum of a five dimensional gauge theory $SU(n)$ with the fifth dimension latticized and compactified as we have already asserted after eq. (1.4). The symmetric $N \times N$ mass matrix M with $\phi = 0$ is easily obtained from eq. (1.4) and its entries are given by

$$\begin{aligned} M_{jj} &= 2g_n^2 f_m^2 \quad \text{for} \quad j = 1, \ldots, N \\ M_{j,j\pm 1} &= -g_n^2 f_m^2 \quad \text{for} \quad j = 1, \ldots, N \\ M_{j,i} &= 0 \quad \text{for} \quad i \neq j, j \pm 1\,, \end{aligned} \tag{1.10}$$

where we have imposed the boundary condition $j = N + 1 = 1$. The eigenvalues of M are given by [39]

$$M_k^2 = 4g_n^2 f_m^2 \sin^2\frac{\pi k}{N}\,, \tag{1.11}$$

with k an integer satisfying $0 < k \leq N$. For N very large and $k \ll N$ the masses become

$$M_k = g_n f_m \frac{2\pi k}{N} = \frac{2\pi k}{R}\,, \tag{1.12}$$

where for the last equality we have used eq. (1.5). The expressions obtained in eq. (1.12) for the gauge boson masses coincide with the masses of the Kaluza-Klein spectrum for a five-dimensional gauge boson compactified on a circle of radius R. The null eigenvalue of M obtained with $k = 0$ corresponds to the gauge boson multiplet of the four dimensional diagonal unbroken gauge group $SU(n)$. Since this unbroken gauge group is the diagonal one, the corresponding eigenvector of M is given by $(1, 1, \ldots\ldots, 1, 1)/\sqrt{N}$ and therefore the gauge coupling of the unbroken subgroup is $g^2 = g_n^2/N$.

Let us come back now to eq. (1.9). It is not possible to give a general expression for $V(\phi)$ therefore we have to specify which is the generic gauge group $SU(n)$. We take

$SU(n) = SU(2)$ and ϕ pointing in the σ_3 direction

$$\phi = \begin{pmatrix} \varphi & 0 \\ 0 & -\varphi \end{pmatrix} . \tag{1.13}$$

For $N > 2$, $M^2(\varphi)$ is an hermitian matrix with complex entries given by

$$\begin{aligned} M^2_{jj} &= 2g_n^2 f_m^2 \quad \text{for} \quad j = 1, \dots, N \\ M^2_{j,j\pm 1} &= -g_n^2 f_m^2 e^{\pm i\varphi/f\sqrt{N}} \quad \text{for} \quad j = 1, \dots, N \\ M^2_{j,i} &= 0 \quad \text{for} \quad i \neq j, j \pm 1 . \end{aligned} \tag{1.14}$$

From eq. (1.14) we see that $\mathrm{Tr}\, M^2(\varphi) = 2N\, g_n^2 f_m^2$ not depending on φ. As we expected the one-loop quadratically divergent term in eq. (1.9) does not contribute to any term of the potential of the PGB φ. If we now compute $(M^2(\varphi))^2$ from eq. (1.14), we discover that

$$M^4_{jj} = 6\, g_n^2 f_m^2 , \tag{1.15}$$

that is $\mathrm{Tr}\, (M^2(\varphi))^2$ does also not depend on φ and therefore even the logarithmically divergent term of eq. (1.9) does not give any contribution to the scalar potential. From eq. (1.14) the eigenvalues of eq. (1.11) in the presence of the background φ become

$$M_k^2 = 4g_n^2 f_m^2 \sin^2 \left(\frac{k\pi}{N} + \frac{\varphi}{f\sqrt{N}}\right) . \tag{1.16}$$

and by combining eq. (1.9) and eq. (1.16) it is possible to compute the complete one-loop CW ultraviolet (UV) finite potential for φ.

Note that the absence of the logarithmically divergent term is related to the condition $N > 2$. In fact, for $N = 2$, $M^2(\varphi)$ is real and it is given by

$$M^2_{N=2} = g_n^2 f_m^2 \begin{pmatrix} 2 & -2\cos 2\varphi/f\sqrt{N} \\ -2\cos 2\varphi/f\sqrt{N} & 2 \end{pmatrix} . \tag{1.17}$$

While there are no one-loop quadratically divergent contributions to the scalar potential, in this case $\mathrm{Tr}\, M^4(\varphi) = (8 + 8\cos^2 2\varphi/f\sqrt{N}) g_n^2 f_m^2$ and therefore one-loop logarithmically divergent contributions are present.

The most relevant ingredient given in this section is the introduction of the concept of collective symmetry breaking. The mass of the scalar boson φ receives suppressed divergent contributions thanks to the presence of N accidental global symmetries because only the collective breaking of all of them produces a mass term.

1.1.2 A minimal moose

In sec. (1.1.1) we have seen that the five-dimensional interpretation of a model based on a moose is possible if the number N of the sites is large. However it is possible to build a model inspired to deconstruction models, that is a model that has a moose description, even if N is small. It is clear that the minimal moose that can be build has $N = 2$. In this section we give a brief description of the model discussed by N. Arkani-Hamed *et al.* in ref. [40]. This further clarifies the concept of collective breaking introduced in sec. (1.1.1).

We consider a moose with only 2 sites: at one sites the gauge symmetry is $G_1 = SU(3)$ and at the other is $G_2 = SU(2) \times U(1)$. There are 4 link fields X_j with $j = 1 \dots 4$ transforming as bi-fundamental under $G_1 \times G_2$, where a fundamental of G_2 is given by $2_{1/6} + 1_{-1/3}$.

In absence of the gauge interactions the theory has an $SU(3)^8$ global symmetry realized by

$$X_j \to L_j \, X_j \, R_j^\dagger \, , \quad j = 1, \dots, 4 \, , \tag{1.18}$$

that is spontaneously broken to $SU(3)^4$ when the X_j acquire a vacuum expectation value (VEV) and they are represented in terms of non-linear sigma model fields $X_j = \exp i x_j / f$. The $SU(3)^4$ global symmetry is realized by the transformation of eq. (1.18) with the identification $L_j = R_j$ and we denote it as $SU(3)^4_{LR}$ for reasons that will soon be clear.

Let now consider the following Lagrangian

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_X \, , \tag{1.19}$$

given only by the kinetic and the plaquette terms. Let neglect for the moment the kinetic term. In the spontaneous breaking of $SU(3)^8$ into $SU(3)^4_{RL}$, 32 GBs arise, but in the presence of $\mathcal{L}_X$ given by the operators

$$\mathcal{L}_X = f^4 \, k \, \mathrm{Tr} \, (X_1 X_2^\dagger X_3 X_4^\dagger) + f^4 \, k' \, \mathrm{Tr} \, (X_1 X_4^\dagger X_3 X_2^\dagger) \, , \tag{1.20}$$

the GBs become PGBs since each of the two terms in eq. (1.20) breaks explicitly the global symmetry $SU(3)^8$. Nevertheless each of the two terms of eq. (1.20) preserves a different subgroup of $SU(3)^8$. The first term in eq. (1.20) proportional to k preserves a global symmetry $SU(3)_1^4$ realized by $R_1 = R_2$, $L_2 = L_3$, $R_3 = R_4$, $L_4 = L_1$. At the same time the term proportional to k' preserves another $SU(3)_2^4$ global symmetry realized by $R_1 = R_4$, $L_4 = L_3$, $R_3 = R_2$, $L_2 = L_1$. Notice that all the three global symmetry $SU(3)_{RL}^4$, $SU(3)_{1,2}^4$ are realized by different transformations. Both of the two global symmetries $SU(3)_1^4$ and $SU(3)_2^4$ are broken to a $SU(3)$ global symmetry by the spontaneous breaking of $SU(3)^8$ into $SU(3)_{RL}^4$, therefore of the 32 GBs arising when $SU(3)^8 \to SU(3)_{RL}^4$, 24 do not acquire a mass at tree level in the presence of the plaquette operators of eq. (1.20), while 8 become massive. The 8 PGBs that acquire a mass at tree level are given by the linear combination $x_1 - x_2 + x_3 - x_4$ and their mass is proportional to the explicit breaking of the global symmetry $SU(3)^8$, that is to $(k + k')f$.

16 PGBs among the 24 that do not acquire a mass at tree level, corresponding to the orthogonal combinations $x_1 - x_3$ and $x_2 - x_4$, acquire a quartic coupling, while the other 8 remain exact GBs even in the presence of the explicit breaking terms. The reason why this happens is that there is a global subgroup $SU(3)^2$ of the global groups $SU(3)_1^4$ and $SU(3)_2^4$ preserved by both the plaquette operators of eq. (1.20) and realized by $R_i = R$, for $i = 1, \ldots, 4$ and $L_i = L$, for $i = 1, \ldots, 4$. This subgroup is spontaneously broken to $SU(3)$ leaving 8 exact GBs, corresponding to the linear combination $x_1 + x_2 + x_3 + x_4$. If we now turn on the kinetic term $\mathcal{L}_G$ we see that the spontaneous breaking of $SU(3)^8$ into $SU(3)_{RL}^4$ breaks the gauge symmetry $G_1 \times G_2$ to the electroweak gauge group $SU(2)_W \times U(1)_W$. The 8 exact GBs are eaten by the massive gauge bosons, their masses being proportional to gf.

We are interested in the 16 PGB fields that do not receive a mass term but only a quartic coupling at tree level. It is quite simple to see how this happens if we parametrize the fields X_j as $\exp ix_j/f$ with the x_j fields given by

$$\begin{aligned} x_1 &= \frac{z}{4} + \frac{x}{\sqrt{2}} \\ x_2 &= \frac{z}{4} - \frac{x}{\sqrt{2}} \\ x_3 &= -\frac{z}{4} + \frac{y}{\sqrt{2}} \\ x_4 &= -\frac{z}{4} - \frac{y}{\sqrt{2}}, \end{aligned} \tag{1.21}$$

where we have not included the components eaten by the massive gauge bosons. Inserting eq. (1.21) in eq. (1.20) we finally obtain

$$\mathcal{L}_X = f^2 \frac{k}{2} \mathrm{Tr}\,(z + i\frac{[x,y]}{f} + \ldots)^2 + f^2 \frac{k'}{2} \mathrm{Tr}\,(z - i\frac{[x,y]}{f} + \ldots)^2 \tag{1.22}$$

from which we see that the z field acquires a mass given by $f^2(k+k')/2$. The main feature of eq. (1.22) is that if we put one of the two explicit breaking couplings, k and k', to zero, the tree level quartic coupling $\mathrm{Tr}\,([x,y]^2)$ is canceled by the effective quartic coupling obtained by integrating out the massive z field. On the contrary, if we leave both the explicit breaking couplings and integrate out the z field we obtain an effective quartic operator given by

$$\frac{kk'}{k+k'} \mathrm{Tr}\,([x,y]^2)\,, \tag{1.23}$$

which goes to zero if either k or k' is zero, as expected by the previous qualitative analysis. The mechanism described is an example of collective breaking since both of the explicit breaking terms are necessary in order to produce a potential for the fields which compose x and y. We can decompose x and y as

$$w = \begin{pmatrix} \varphi_w + \eta_w & h_w \\ h_w^\dagger & -2\eta_w \end{pmatrix}, \tag{1.24}$$

with $w = x, y$. $h_{x,y}$ transforms as a complex doublet $2_{1/2}$ with respect to the gauge group $SU(2) \times U(1)$, while $\varphi_{x,y}$ and $\eta_{x,y}$ are a real triplet 3_0 and a real singlet 1_0 respectively. Substituing eq. (1.24) into eq. (1.23) we obtain the quartic potential for the two doublets

$$\tilde{k}\, \mathrm{Tr}\,(h_x h_y^\dagger - h_y h_y^\dagger)^2 + \tilde{k}\,(h_x^\dagger h_y - h_y^\dagger h_x)^2\,, \tag{1.25}$$

with $\tilde{k} = kk'/(k+k')$ and where in eq. (1.25) we have neglected terms involving $\varphi_{x,y}$ and $\eta_{x,y}$. We can finally rewrite eq. (1.25) in terms of the two doublets $h_1 = h_x + ih_y$ and $h_2 = h_x - ih_y$

$$\tilde{k}\, \mathrm{Tr}\,(h_1 h_1^\dagger - h_2 h_2^\dagger)^2 + \tilde{k}\,(h_1^\dagger h_1 - h_2^\dagger h_2)^2\,, \tag{1.26}$$

obtaining a quartic potential similar to the MSSM one [41].

The two doublets h_1 and h_2 are the two *little Higgses*, so called because their masses and their quartic coupling are protected by the peculiar symmetry group structure even in the presence of explicit breaking terms of the spontaneously broken global symmetry from which they arise as GBs.

In the standard model the Higgs boson mass receives large one-loop quadratically divergent corrections both by the gauge bosons and the top quark. We have learned that it is possible to protect the mass of a scalar (doublet) if the couplings that may give rise to a mass term arise as explicit breaking terms of a global symmetry while preserving some subgroups of the global symmetry they explicitly violate. It is clear therefore that in order to protect the Higgs boson mass from the gauge bosons (top quark) one-loop divergent contributions, it is necessary that the gauge couplings (Yukawa coupling) be the explicit breaking couplings of a global symmetry opportunely chosen. In the next section we will see how this idea may be put into practice in a very simple model.

1.1.3 The little Higgs from a simple group

In the previous section we have seen how in a model based on a simple moose the collective symmetry breaking mechanism, realized as the interplay between the spontaneous and the explicit breaking of a global symmetry, may produce little Higgses. We now simplify even further the context in which little Higgses arise and, following the model of Kaplan *et al.* [42], show how it is possible to implement the same features in a simple extension of the standard model without referring to any moose description. In the following example we neglect the $U(1)_W$ hypercharge gauge symmetry in order to describe a very simple model and assume the standard model gauge symmetry to be only the $SU(2)_W$. The reader may find in ref. [42] how the same ideas are applied to the complete standard model gauge group $SU(2)_W \times U(1)_W$.

From sec. (1.1.2) we know that in order to produce little Higgses we need a gauge symmetry group G larger than the standard model electroweak $SU(2)_W$ gauge symmetry and an approximate global symmetry larger than the gauge group G. For these reasons, in the model we are going to discuss the $SU(2)_W$ standard model gauge group is enlarged to a $SU(3)$ gauge symmetry and 2 triplets, Φ_1 and Φ_2, of the gauge group $SU(3)$ are present transforming under two different global symmetries $SU(3)_{1,2}$. The total global symmetry is then $SU(3)_1 \times SU(3)_2$. We suppose that at a scale f around the TeV each of the 2 triplets acquires a VEV that breaks spontaneously the gauge symmetry $SU(3)$. We also suppose that the 2 VEVs are aligned so the $SU(3)$ gauge group is broken to $SU(2)_W$. If we turn off the gauge coupling, the VEVs of the scalars break the $[SU(3)]^2$ global symmetry into an $[SU(2)]^2$ global symmetry. For each scalar we have then 5 GBs and we parametrize the scalars as

$$\Phi_1 = e^{i\Theta_1/f} \begin{pmatrix} 0 \\ 0 \\ f+\rho_1 \end{pmatrix} \qquad \Phi_2 = e^{i\Theta_2/f} \begin{pmatrix} 0 \\ 0 \\ f+\rho_2 \end{pmatrix}, \tag{1.27}$$

where $\Theta_{1,2} = \Theta^a_{1,2}(x)\, T^a_{1,2}$ with $\Theta^a_{1,2}(x)$ the Goldstone modes and $T^a_{1,2}$ the broken generators of the two global group algebras. Let now turn on the gauge coupling: the presence of the gauge interaction breaks explicitly the $[SU(3)]^2$ global symmetry into the $SU(3)$ diagonal, leaving the $SU(3)$ axial broken. After the spontaneous breaking of $[SU(3)]^2$ in $[SU(2)]^2$ the $SU(3)$ gauge symmetry is broken to $SU(2)_W$. The 5 GBs corresponding to the diagonal $SU(3)$ global symmetry are eaten by the gauge bosons to become massive, while the 5 GBs corresponding to the axial broken $SU(3)$ become PGBs. Therefore eq. (1.27) can be rewritten in terms of the eaten GBs, Θ_{eaten} and of the PGBs, Θ,

$$\Phi_1 = e^{i\Theta_{eaten}/f} e^{i\Theta/f} \begin{pmatrix} 0 \\ 0 \\ f+\rho_1 \end{pmatrix} \qquad \Phi_2 = e^{i\Theta_{eaten}/f} e^{-i\Theta/f} \begin{pmatrix} 0 \\ 0 \\ f+\rho_2 \end{pmatrix}. \tag{1.28}$$

After having integrated out the two massive radial modes $\rho_{1,2}$ they can be parametrized as two non-linear sigma models

$$\Phi_1 = e^{i\Theta/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \qquad \Phi_2 = e^{-i\Theta/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}, \tag{1.29}$$

with Θ given by

$$\Theta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & \\ & & h \\ 0 & 0 & \\ & h^\dagger & & 0 \end{pmatrix} + \frac{\eta}{4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \tag{1.30}$$

The PGBs h and η are respectively a complex doublet, that we identify with the Higgs field, and a real singlet of $SU(2)_W$. Since the gauge interaction is the explicit breaking of the global symmetry we expect the PGB masses will be proportional to the gauge coupling. To analyze this, let turn now to the linear sigma model. The $SU(3)$ gauge invariant kinetic Lagrangian is given by

$$\mathcal{L}_K = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 = |(\partial_\mu + igA_\mu)\Phi_1|^2 + |(\partial_\mu + igA_\mu)\Phi_2|^2 \,, \tag{1.31}$$

where $A_\mu = A_\mu^a T^a$ with A_μ^a and T^a the $SU(3)$ gauge bosons and generators respectively. Each of the two terms of eq. (1.31) preserves the axial $SU(3)$ global symmetry: the 5 PGBs parametrized as in eq. (1.30) are the GBs of this global—explicitly broken— symmetry, therefore their masses can be generated only by an effective operator that violates it. If we compute the one-loop gauge bosons CW potential for the two scalars we obtain the following contributions

$$\Delta\mathcal{L} = \frac{g^2\Lambda^2}{16\pi^2}(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2) + \frac{g^4}{16\pi^2}|\Phi_2^\dagger\Phi_1|^2 \log(\Lambda^2/f^2)\,. \tag{1.32}$$

The first quadratically divergent operator of eq. (1.32) does not violate the axial global symmetry $SU(3)$ and therefore can not produce a quadratically divergent contribution to the mass of the Higgs and of the singlet η. The second logarithmic divergent operator on the contrary explicitly violates the axial global symmetry and produces a contribution to the Higgs mass proportional to $g^4 f^2/16\pi^2$. How should we interpret this result? The global symmetries structure of the model assures the absence of one-loop quadratically divergent contributions to the mass of the Higgs bosons. The one-loop logarithmic contributions to the Higgs mass is comparable to the expected two-loop quadratically divergente radiative corrections, that is $\mu_h^2 \sim g^4 f^2/16\pi^2 = \Lambda^2(g^2/16\pi^2)^2$ where Λ is the cut-off of the model. This implies that in order to have $\mu_h^2 \simeq 100$ GeV , the natural standard model cut-off of more or less few TeVs can be push up to 10 TeV. This discussion is purely qualitative since to have a realistic model we should produce dynamically the self quartic coupling for the Higgs boson and also we should include the fermions in such a way to protect the Higgs mass from the top one-loop quadratically divergent contribution. The lesson learned in this section suggests that in order to protect the Higgs mass, the Yukawa Lagrangian structure should contain terms that preserve the axial symmetry at tree level while the interplay between them has to give rise to operators that violates it at one-loop.

We are now ready to turn to the littlest Higgs model.

1.2 The littlest Higgs

In the previous section we gave a pedagogical description of the theoretical path leading from models based on deconstruction to the little Higgs models. We chose not to enter into the details of each model because it was not essential to understand the philosophy on which little Higgs models are based on and in particular the collective symmetry breaking mechanism that protects the Higgs mass by the one-loop quadratically divergent corrections.

In this section we analyze in detail the littlest Higgs model [27]. We will see that the model can be considered a serious candidate for an extention of the standard model: with a realistic Higgs boson potential and new gauge bosons, scalars and fermions which give rise to a rich phenomenology beyond the standard model.

We neglect all the implications related to the new particles introduced and to the new processes they are involved into and how they can affect the electroweak precision observable measurements (see the papers in ref. [29] for an extensive discussion of this point) and instead ask a very basic question: how natural is the littlest Higgs model? What is the amount of fine-tuning required by the little Higgs model in order to push the standard model cut-off from few TeVs to 10 TeV? Is it a real improvement with respect to the standard model? We will try to answer these questions in the last part of this chapter.

1.2.1 The model

Consider an approximate global symmetry $SU(5)$ at a scale Λ and a scalar fields Σ transforming as the adjoint with respect to this global symmetry. Suppose that the UV completion of the theory is such as the Σ acquires a VEV $\langle\Sigma_0\rangle$ that spontaneously breaks the $SU(5)$ global symmetry into $SO(5)$. The ten unbroken generators T^a of $SO(5)$ satisfy

$$T^a\Sigma_0 + \Sigma_0 T_a^T = 0\,, \tag{1.33}$$

while for the broken generators Θ^a holds

$$\Theta^a \Sigma_0 - \Sigma_0 \Theta_a^T = 0\,, \tag{1.34}$$

with Σ_0 given by

$$\Sigma_0 = \begin{pmatrix} & & I_{2x2} \\ & 1 & \\ I_{2x2} & & \end{pmatrix}, \tag{1.35}$$

where I_{2x2} is the two dimensional matrix identity.

The 14 GBs arising in the spontaneous breaking are parametrized as the components of a non-linear sigma model

$$\Sigma = e^{2i\Pi(x)/f}\Sigma_0 = e^{i\Pi(x)/f}\Sigma_0 e^{i\Pi(x)^T/f}\,, \tag{1.36}$$

with $\Pi(x) = \pi^a(x)\Theta^a$.

We gauge into $SU(5)$ two subgroups of it, G_1 and G_2, which are two copies of the gauge group $SU(2) \times U(1)$. The generators of G_1 and G_2 are embedded into $SU(5)$ as

$$\begin{aligned} Q_1^a &= \begin{pmatrix} \sigma^a/2 & \\ & \end{pmatrix} \qquad Y_1^a = \mathrm{diag}(-3,-3,2,2,2)/10 \\ Q_2^a &= \begin{pmatrix} & \\ & -\sigma^{a^*}/2 \end{pmatrix} \qquad Y_2^a = \mathrm{diag}(-2,-2,-2,3,3)/10\,. \end{aligned} \tag{1.37}$$

When $SU(5)$ is spontaneously broken into $SO(5)$ only the diagonal combination of the 2 gauge groups G_1 and G_2 survives, to be identified with the electroweak gauge group $SU(2)_W \times U(1)_W$. Four GBs arising in the spontaneous breaking $SU(5) \to SO(5)$ are eaten by the gauge bosons of the broken axial gauge group to become massive. On the other hand, the gauge interactions represent explicit breakings of the approximate global symmetry $SU(5)$, and therefore the other 10 degrees of freedom behave not as GBs, but as PGBs.

We can classify the remaining PGBs according to their transformation property with respect to the diagonal gauge group $SU(2)_W \times U(1)_W$, so we have a complex triplet

ϕ , 3_1, and a complex doublet φ, $2_{1/2}$. We can now express Π in terms of these two fields

$$\Pi = \begin{pmatrix} & \frac{\varphi^\dagger}{\sqrt{2}} & \phi^\dagger \\ \frac{\varphi}{\sqrt{2}} & & \frac{\varphi^*}{\sqrt{2}} \\ \phi & \frac{\varphi^T}{\sqrt{2}} & \end{pmatrix} . \tag{1.38}$$

Both the triplet ϕ and the doublet φ are PGBs arisen in the breaking of $SU(5)$ into $SO(5)$. Nevertheless there are 2 accidental global symmetry that further protect the doublet. In fact suppose to turn off one of the gauge couplings, for example g_1. Then there is an $SU(3)_1$ global symmetry living in the upper 3×3 block of $SU(5)$ with respect of which only the doublet transforms non linearly. If we now turn on g_1 and turn off g_2 we see that there is another $SU(3)_2$ global symmetry living in the lower 3×3 block and again only the doublet transforms non linearly. Only one gauge coupling is sufficient to break the global symmetry that protects the triplet ϕ while both are necessary to break all the global symmetries that protect the doublet φ. For this reason we expect that the leading terms of the scalar potential of the triplet will be proportional to $g_1^2 + g_2^2$, while that for the doublet to $g_1^2 g_2^2$. In order to produce an effective operator with a coefficient $g_{1,2}^2$ it is sufficient to make a simple gauge boson loop, and therefore the leading terms in the scalar potential of the triplet arise by one-loop divergent contributions. On the other hand, in order to obtain a coefficient proportional to $g_1^2 g_2^2$ we need at least one insertion or we are obliged to go up to two loops. As a consequence, the doublet potential does not have one-loop quadratically divergent contributions but only logarithmically divergent contributions, the first quadratically divergent contributions arising at two-loops.

1.2.2 The potential

The tree-level Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_t + \mathcal{L}_\psi \tag{1.39}$$

where $\mathcal{L}_K$ contains the kinetic terms for all the fields, $\mathcal{L}_t$ is the top and extra top-like quarks Yukawa Lagrangian, while $\mathcal{L}_\psi$ is the Yukawa Lagrangian for all the other standard model fermions, both quarks and leptons. What are the extra top-like quarks and why they have to be introduced it will slowly be clear. $\mathcal{L}_K$ is given by

$$\mathcal{L}_K = \frac{f^2}{8}\mathrm{Tr}\,(D_\mu\Sigma)(D^\mu\Sigma^*)\,, \tag{1.40}$$

f is the $SU(5)$ spontaneous breaking scale and

$$D_\mu = \partial_\mu - \sum_i \{ig_i W^a_{i\mu}(Q^a_i\Sigma + \Sigma Q^{aT}_i) + ig'_i B_i(Y_i\Sigma + \Sigma Y^T_i)\}\,, \tag{1.41}$$

where g_i and g'_i , $i = 1, 2$, are the gauge couplings of the two copies of $[SU(2) \times U(1)]_i$ gauge groups. The structure of the gauge interactions is such as to prevent the scalar potential of the doublet, that we identify with the Higgs boson, from receiving one-loop quadratically divergent contributions. In order to avoid the large quadratic divergence to the Higgs potential introduced by the top Yukawa coupling, it is necessary to have $\mathcal{L}_t$ with the same structure of the gauge interactions, that is given by the sum of two or more terms each of them preserves individually a global symmetry that protects the Higgs boson mass. In order to realize this, we add to the standard model third family, composed by the $SU(2)_W$ doublet $q_{3_L} = (t, b)_L$ and the two $SU(2)_W$ singlet u'^c_3 and d'^c_3, two colored Weyl fermions $\tilde{t}$ and $\tilde{t}^c$ of hypercharge 2/3 in such a way that $\chi = (b_3, t_3, \tilde{t})$ is a triplet of the global symmetry we previously identified with $SU(3)_1$. The top quark Yukawa Lagrangian is given by

$$\mathcal{L}_t = \sqrt{2}\lambda_1\, f\, \epsilon_{ijk}\epsilon_{xy}\,\chi_i\,\Sigma_{jx}\,\Sigma_{ky}u'^c_3 + \sqrt{2}\lambda_2\, f\,\tilde{t}\tilde{t}^c + h.c.\,, \tag{1.42}$$

where the term proportional to λ_1 preserves the approximate global symmetry $SU(3)_1$ and breaks $SU(3)_2$, while that one proportional to λ_2 preserves the approximate global symmetry $SU(3)_2$ and breaks $SU(3)_1$. Expanding eq. (1.42) to the first order in the Higgs field we have

$$\mathcal{L}_t = \sqrt{2}\lambda_1\,(q_3\varphi + f\tilde{t})u'^c_3 + \sqrt{2}\lambda_2 f\,\tilde{t}\tilde{t}^c + \mathcal{O}(\varphi^2)\,. \tag{1.43}$$

One combination of u'^c_3 and $\tilde{t}^c$ marries $\tilde{t}$ to become massive, its mass squared given by $f^2(2\lambda_1^2 + 2\lambda_2^2) + \mathcal{O}(\varphi^2)$; we identify the other combination with the standard model top quark so that the top Yukawa coupling is given by

$$\lambda_t = 2\frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}\,. \tag{1.44}$$

The Yukawa couplings present in $\mathcal{L}_\psi$ are small, so it is not necessary to add extra fermions to avoide one-loop divergent contributions arising from all the other standard model fermions. $\mathcal{L}_\psi$ is given by gauge invariant terms that break all the global symmetries protecting the Higgs boson potential and that once they are expanded to the first order in the Higgs field reproduce the standard fermion Yukawa couplings.

The effective potential of the Higgs boson and of the triplet is found by computing the CW potential [38] generated by the gauge boson and fermion loops. At the one-loop, it can be written as

$$\begin{aligned} V_1[c_i, g_i, g_i', \lambda_i; \Sigma] &= 3\,\frac{c_1\Lambda^2}{32\pi^2}\mathrm{Tr}\, M_B^2(\Sigma) - 12\,\frac{c_2\Lambda^2}{32\pi^2}\mathrm{Tr}\, M_F^2(\Sigma) \\ &\quad +3\,\frac{1}{64\pi^2}\mathrm{Tr}\, M_B^4(\Sigma)\log c_3 M_B^2(\Sigma)/\Lambda^2 \\ &\quad -12\,\frac{1}{64\pi^2}\mathrm{Tr}\, M_F^4(\Sigma)\log c_4 M_F^2(\Sigma)/\Lambda^2\,, \end{aligned} \tag{1.45}$$

where the factors 3 and 12 in front of the operators count the degrees of freedom of, respectively, bosons and colored fermions. The coefficients c_i are unknown constants, the values of which come (presumably) from the UV completion of the theory [37]. They are there because these terms are divergent and UV physics cannot be safely decoupled. Additional states may contribute to the relevant operators and their effect cannot be computed. From the effective theory point of view, these coefficients are arbitrary numbers to be determined. $c_{3,4}$ can be taken equal to 1 since they appear in the logarithmic contributions and their contribution cannot be crucial. The traces in eq. (3.15) over the effective (squared) masses are the one-loop quadratically divergent contribution of, respectively, bosonic and fermionic degrees of freedom:

$$\begin{aligned} \mathrm{Tr}\, M_B^2(\Sigma) &= \frac{f^2}{4}\Big(g_i^2\sum_a \mathrm{Tr}\,[(Q_i^a\Sigma)(Q_i^a\Sigma)^*] + g'_i\mathrm{Tr}\,[(Y_i\Sigma)(Y_i\Sigma)^*]\Big) \\ \mathrm{Tr}\, M_F^2(\Sigma) &= 2\,\lambda_1^2\, f^2\epsilon^{wx}\epsilon_{yz}\epsilon^{ijk}\epsilon_{kmn}\Sigma_{iw}\Sigma_{jx}\Sigma^{*my}\Sigma^{*nz} \end{aligned} \tag{1.46}$$

Expanding the quadratically divergent contributions up to the quadratic order in ϕ and to the quartic order in φ we have

$$\begin{aligned} &\tfrac{3}{4}[c_1(g_2^2+g'^2_2)+64c_2\lambda_1^2]\, f^2\,|\phi_{ij}+\tfrac{i}{2f}(\varphi_i\varphi_j+\varphi_j\varphi_i)|^2 \\ &\quad +\tfrac{3}{4}c_1(g_1^2+g'^2_1)\, f^2\,|\phi_{ij}-\tfrac{i}{2f}(\varphi_i\varphi_j+\varphi_j\varphi_i)|^2 \qquad , \end{aligned} \tag{1.47}$$

where ϕ_{ij}, $i = 1, 2$ are the components of ϕ written as

$$\phi = \begin{pmatrix} \phi^{++} & \phi^{+} \\ \phi^{+} & \phi^{0} \end{pmatrix},$$

and φ_i the $SU(2)_W$ components of φ.

In eq. (1.47) the first term preserves the first global symmetry $SU(3)_1$ while the second preserves the global $SU(3)_2$. If we neglect the second term and we expand the first we have, neglecting the constant factor,

$$f^2(\phi^* \phi + \frac{i}{2f}\phi^*_{ij}\varphi_i\varphi_j - \frac{i}{2f}\phi_{ij}\varphi^*_i\varphi^*_j + \frac{(\varphi^\dagger\varphi)^2}{4f^2}). \tag{1.48}$$

Eq. (1.48) presents the same feature of eq. (1.22): integrating out the triplet we produce an effective quartic coupling that cancels that one produced by the gauge interactions. This feature is not related to the truncation we did in the expansion, and it would be present at any order: in the absence of the explicit breaking of the $SU(3)_1$ global symmetry the Higgs is still a GB and no potential can be generated for it. We would come to the same conclusion if we had considered the second term of eq. (1.47). In the presence of both the explicit breaking terms a quartic term for the Higgs potential is produced and it is given by

$$\frac{3}{2}\frac{c_1(g_1^2 + g'^2_1)(c_1(g_2^2 + g'^2_2) + 64c_2\lambda_1^2)}{c_1(g_1^2 + g_2^2 + g'^2_1 + g'^2_2) + 64c_2\lambda_1^2}(\varphi^\dagger\varphi)^2 . \tag{1.49}$$

The triplet ϕ it is not protect by the accidental global symmetries $SU(3)_1$ and $SU(3)_2$ and receives a mass given by

$$m_\phi^2 = \frac{3f^2}{4}\left[c_1(g_1^2 + g_2^2 + g'^2_1 + g'^2_2) + 64c_2\lambda_1^2\right] . \tag{1.50}$$

The terms in eq. (1.45) proportional to logarithms of the cut-off give rise to the Higgs boson quadratic term but also contribute to the other terms in the potential. The latters are usually neglected [29, 31]. As it turns out, they are important and, as we shall show, crucial in determining the properties of the model. Their main contribution is to the quadratic terms of the potential which we write as

$$\mathcal{L}_{log} = \mu_h^2\varphi^\dagger\varphi + \mu_t^2\phi^\dagger\phi , \tag{1.51}$$

where we have neglected the sub-leading term $\mu_{th}\phi_{ij}\varphi_i^\dagger\varphi_j^\dagger + h.c.$.

Taking only the leading order of each term of eq. (1.51), we have

$$
\begin{aligned}
\frac{\mu_h^2}{f^2} &= -\frac{9}{256\pi^2} g_1^2 g_2^2 \log\frac{g_1^2+g_2^2}{64\pi^2} - \frac{3}{1280\pi^2} g'^2_1 g'^2_2 \log\frac{g'^2_1+g'^2_2}{320\pi^2} \\
&+ \frac{3}{\pi^2}\frac{\lambda_1^2\lambda_2^2}{2}(\lambda_1^2+\lambda_2^2)\log\frac{\lambda_1^2+\lambda_2^2}{8\pi^2} \\
\frac{\mu_t^2}{f^2} &= \frac{3}{128\pi^2}\left[\frac{1}{2}((g_1^2+g_2^2)^2 - 8g_1^2g_2^2)\log\frac{g_1^2+g_2^2}{64\pi^2} + \frac{1}{10}(g'^2_1 - g'^2_2)^2\log\frac{g'^2_1+g'^2_2}{320\pi^2}\right] \\
&+ \frac{24}{\pi^2}\frac{\lambda_1^2}{4}(\lambda_1^2+\lambda_2^2)\log\frac{\lambda_1^2+\lambda_2^2}{8\pi^2}\,. \qquad (1.52)
\end{aligned}
$$

We have written eq. (1.52) as function of the gauge couplings $g_{1,2}$, $g'_{1,2}$ and Yukawa couplings, λ_1 and λ_2, only for convenience and used $\Lambda = 4\pi f$. Usually [29, 31], the terms in the potential are reported as functions of the heavy gauge bosons and of the heavy top-like quark masses, which by eqs. (1.40)–(1.91) at the leading order are given by

$$
M^2_{W'} = \frac{1}{4}(g_1^2+g_2^2)f^2 \quad M^2_{B'} = \frac{1}{20}(g'^2_1+g'^2_2)f^2 \quad M^2_{\tilde{t}} = \frac{8\lambda_1^4}{4\lambda_1^2-\lambda_t^2}f^2\,. \qquad (1.53)
$$

Notice that in eq. (1.52) all the terms that enter in the Higgs boson mass have in front a coefficient given by the product of the two gauge or Yukawa explicit breaking terms of the global symmetry $SU(5)$ as we have anticipated at the end of sec. (1.2.1). On the contrary, the terms entering in μ_t^2 do not present the same feature since the triplet ϕ is not protected by the two accidental global symmetries $SU(3)_1$ and $SU(3)_2$.

1.2.3 How natural is the littlest Higgs model?

Now we would like to address the question of how much more natural is the littlest Higgs model with respect to the standard model. To do this we study the exact potential, rather than its truncation to terms quartic in the fields, and include all logarithmic terms—which are usually neglected in the most analysis [29, 31] or included only for the Higgs boson and not for the triplet [43]. These logarithmic terms are important and cannot be neglected; for the model to be successful they must be numerically small enough to give the Higgs boson the desired mass without further fine-tuning.

To proceed with our analysis we fix three combinations of gauge and Yukawa couplings to reproduce the standard model couplings g, g' and λ_t (that is, the mass m_t

of the top quark). The $g_{1,2}$ and ${g'}_{1,2}$ gauge couplings can be rewritten as functions of the $SU(2)_W$ and $U(1)_W$ electroweak g, g' gauge couplings and of two new parameters G, G' defined by

$$\begin{aligned} G^2 &= g_1^2 + g_2^2 \\ G'^2 &= {g'}_1^2 + {g'}_2^2 . \end{aligned} \tag{1.54}$$

Since

$$g^2 = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2} \qquad g'^2 = \frac{{g'}_1^2 {g'}_2^2}{{g'}_1^2 + {g'}_2^2}, \tag{1.55}$$

we have

$$g_{1,2}^2 = \frac{G^2}{2} \pm \frac{G}{2}\sqrt{G^2 - 4g^2}, \tag{1.56}$$

and similar expressions for the $U(1)_i$ ${g'}_i$ couplings. The standard model gauge couplings are given by $g = \sqrt{4\pi\alpha/\sin\theta_W}$ and $g' = g\tan\theta_W$ in terms of the fine structure constant α and the Weinberg angle θ_W.

In a similar way, by imposing that top-quark Yukawa coupling λ_t gives the experimental mass m_t, λ_2 can be expressed in terms of λ_t and λ_1. Renaming λ_1 x_L and using eq. (1.44) we have

$$\lambda_2 = \frac{x_L \lambda_t}{\sqrt{4x_L^2 - \lambda_t^2}}. \tag{1.57}$$

Together, eqs. (1.56)–(1.57) fix the range of the parameter G,G' and x_L. In fact by imposing the reality of g_i, ${g'}_i$ and λ_t we have

$$G \geq 2g(m_W) \qquad G' \geq 2g'(m_W) \qquad x_L \geq \frac{\lambda_t(m_W)}{2}. \tag{1.58}$$

The value $G = 2g$ corresponds to the maximally symmetrical case where $g_1 = g_2$ and the heavy bosons are decoupled from their lighter copies. The actual value is usually chosen so as to minimize the overall electroweak corrections [29, 31].

The littlest Higgs model is thus controlled by six parameters: 2 gauge and 1 Yukawa couplings, G, G' and x_L, the two coefficients c_1 and c_2, of the quadratically divergent terms, one for the bosonic and one for the fermionic loops and the symmetry breaking scale f. At the same time, we have six constraints given by the vanishing of the first derivatives in the doublet and triplet directions, the value v_W of the electroweak vacuum (that is, the value of the Higgs field in the minimum of the potential) and that of the triplet field, the mass m_H of the Higgs boson and of the triplet (the second derivatives of the potential at the minimum).

We therefore have an effective theory in which all parameters and coefficients are constrained and the model completely determined. We can study it as a function of the physically significant parameters v_W, m_H and f; in particular, what are the values of the coefficients c_1 and c_2? Are there any choices which allow for v_W at its physical value, the mass of the Higgs to be, say, around 115 GeV and f around 2 TeV, as suggested by the electroweak data? We will see in the next sections that the answer seems to be positive for the value of v_W (and in this respect the model is successful), it is negative for m_H, in the sense that there are no solutions, as we vary the gauge couplings and the coefficients, leading to m_H and f in the desired range. The main reason for this failure lays in the logarithmic contributions to the Higgs boson mass of eq. (1.52) which are $O(f)$ rather than $O(m_H)$ thus leading to a littlest Higgs with a mass around 800 GeV. Larger masses are also possible (and natural) but would lead the theory outside its perturbative definition.

At first, this negative result does not seem too troublesome since we know that the inclusion of the next-order (two-loop) corrections is crucial in the precise determination of m_H. They can be of various (and complicated) forms, and we indicate it by a generic operator of canonical dimension two:

$$V_2[c_5;\, \Sigma] = \frac{c_5 \Lambda^2}{(4\pi)^4} \mathcal{O}_{2-loop}(\Sigma)\,. \tag{1.59}$$

We are not going to compute these terms and just take c_5 to be the coefficient of a term of order $f^2/16\pi^2$, which controls the size of the two-loop quadratically divergent contributions.

What is surprising is that the size of the logarithmic corrections will turn out to be so large that we will be forced to introduce a proportionally large two-loop corrections thus rising some doubts on the entire perturbative expansion. Even after the two-loop corrections have been included, the possible choices in which the model gives m_H and f in the desired range lead to very unnatural values of the coefficients—at least one of the coefficients c_i must be unreasonably small—which defy the very purpose of introducing the model. In fact, as we already pointed out, these coefficients control the symmetry breaking operators, and if we were allowed to suppress them by fine-tuning we could have done it directly in the standard model without having to resort to the littlest Higgs model in the first place.

This problem seems to be more serious for the model than the amount of fine-tuning in the parameters imposed by electroweak precision measurements. Moreover, we will see that our analysis shows that the recent fit within the littlest Higgs model of the electroweak radiative corrections [31] falls in a region of the parameter space that is excluded by the requirement of having the ground state near zero, or alternatively of having $v_W \ll f$.

1.2.4 Approximate analysis

Before embarking in the analysis of the complete model, it is useful to examine qualitatively its main features. This will help in elucidating the numerical analysis in the next section. For simplicity, we ignore the $U(1)$ groups and therefore take ${g'}_{1,2} = 0$. We expand eq. (1.45) up to the fourth and second order in the doublet and triplet field components respectively that acquire a VEV responsible of the spontaneous breaking of the electroweak gauge group. The potential is then given by

$$V[h,t] = \mu_h^2 h^2 + \lambda_3\, hth + \lambda_4 h^4 + \lambda_\phi t^2\,, \tag{1.60}$$

where $h = \mathrm{Re}\,\varphi^0$ and $t = \mathrm{Im}\,\phi^0$ with φ^0 and ϕ^0 the neutral components of the doublet φ and the triplet ϕ respectively. If in eq. (1.60) we neglect the logarithmic contributions (except in μ_h^2), the coefficients λ_3, λ_4 and λ_ϕ coincide with that of eq. (1.47), eq. (1.49) and eq. (1.50), that we rewrite here as

$$\begin{aligned} \lambda_\phi/4 &= \lambda_4 = \frac{3}{16}(c_1 G^2 + 64 c_2 x_L^2) \\ \lambda_3 &= \frac{3}{4}(c_1 G^2 (s_g^2 - c_g^2) + 64 c_2 x_L^2)\,, \end{aligned} \tag{1.61}$$

where G and x_L have been defined in the previous section and

$$c_g = g_1/G \qquad s_g = g_2/G\,. \tag{1.62}$$

By imposing the conditions for the existence of a minimum in the potential,

$$\begin{aligned}\frac{\partial V[h,t]}{\partial h} &= 0\,,\\ \frac{\partial V[h,t]}{\partial t} &= 0\,,\end{aligned} \tag{1.63}$$

we find that the VEVs are given by

$$\begin{aligned}\langle t\rangle &= -\frac{\lambda_3}{2\lambda_\phi}\frac{\langle h\rangle^2}{f}\\ \langle h\rangle^2 &= -\frac{\mu_h^2}{\tilde{\lambda}}\,,\end{aligned} \tag{1.64}$$

where

$$\begin{aligned}\tilde{\lambda} &= 2\lambda_4 - \frac{\lambda_3^2}{2\lambda_\phi}\\ &= \frac{3}{2}\frac{c_1 G^2 c_g^2 (c_1 G^2 s_g^2 + 64 c_2 x_L^2)}{c_1 G^2 + 64 c_2 x_L^2}\,.\end{aligned} \tag{1.65}$$

Assuming $c_1 = c_2 = 1$, and hence no fine-tuning between UV and low energy sources of symmetry breaking, this reduces to:

$$\tilde{\lambda} = \frac{3}{2}\frac{G^2 c_g^2 (G^2 s_g^2 + 64 x_L^2)}{G^2 + 64 x_L^2}\,. \tag{1.66}$$

In order to make contact with electroweak physics we have to impose $\langle h\rangle = v_w/\sqrt{2}$, where v_W is 246 GeV. The mass of the physical Higgs boson and of the other physical scalar are therefore

$$\begin{aligned}m_h^2 = -2\mu_h^2 = \tilde{\lambda} v_w^2\,,\\ m_\phi^2 = \lambda_\phi f^2\,.\end{aligned} \tag{1.67}$$

Finally, let us give an estimate of μ_h^2, which as we have already pointed out is determined by the logarithmically divergent part of the CW potential plus finite terms. By combining eq. (1.52) with eq. (1.56) and eqs. (1.44)–(1.57) we have

$$\frac{\mu_h^2}{f^2} = -\frac{9}{256\pi^2} g^2 G^2 \log\frac{G^2}{64\pi^2} + \frac{3}{\pi^2}\frac{\lambda_t^2 x_L^4}{4x_L^2 - \lambda_t^2}\log\frac{4x_L^4}{8\pi^2(4x_L^2 - \lambda_t^2)}\,, \tag{1.68}$$

and using the constraints, on m_t and the gauge couplings,

$$\begin{aligned}x_L &> \lambda_t/2\\ G^2 &> 4g^2\,,\end{aligned} \tag{1.69}$$

we find that, for instance by taking $x_L \simeq 1$ and $G \simeq 2g$, we have

$$\mu_h^2 \simeq \left(0.01g^2G^2 - \frac{x_L^4}{4x_L^2 - 1}\right) f^2 \simeq -0.3f^2 . \tag{1.70}$$

With these, one gets for the Higgs mass:

$$m_h^2 \simeq 0.6f^2 . \tag{1.71}$$

If we want $f \simeq 2$ TeV, then

$$\tilde{\lambda} = -\frac{2\mu_h^2}{v_w} = \frac{0.6f^2}{v_w} \simeq 40 , \tag{1.72}$$

which is at the limit of validity of the perturbative expansion (the expansion parameter being roughly given by $\tilde{\lambda}/16\pi^2$). The mass of the triplet would be $m_\phi \simeq$ few TeV. This scenario would correspond to a cut-off of the theory $\Lambda \simeq 25$ TeV, which is what we wanted, but requires a mass for the physical Higgs $m_H \simeq 1$ TeV.

On the contrary, if we also demand that the Higgs boson mass be close to 115 GeV (from LEP lower bound) when $f \simeq 2$ TeV we must have $m_h^2/f^2 \simeq 3 \times 10^{-3}$. At the same time, the triplet must be a heavy state with $m_\phi \simeq f$. We therefore need

$$\begin{aligned} \mu_h^2 &\simeq 3 \times 10^{-3} f^2 \\ \tilde{\lambda} &\simeq 0.2 \\ \lambda_\phi &\simeq O(1) . \end{aligned} \tag{1.73}$$

The condition $\lambda_\phi \simeq O(1)$ yields

$$\frac{3}{4}(c_1 G^2 + 64 c_2 x_L^2) \simeq O(1) , \tag{1.74}$$

which implies that at least one of the two coefficients c_1 and c_2 must be of $O(1)$. On the other hand, the requirement (obtained by using (1.74) in (1.65) and by comparison with eq. (1.73))

$$\tilde{\lambda} \simeq \frac{3}{2} c_1 G^2 c_g^2 (c_1 G^2 s_g^2 + 64 c_2 x_L^2) \simeq 0.2 , \tag{1.75}$$

implies that at least one of the c_i coefficients must be fine-tuned to small values. Hence, the requirement of values for the Higgs mass close to the experimental bound would reintroduce the problem of fine-tuning that the little Higgs wanted to alleviate.

Finally, the ratio μ_h^2/f^2 is dominated by the top sector and is far from being of the desired order $O(10^{-3})$. We see by eq. (1.70) that the problem can be ameliorated only by allowing the coupling G to assume large values, and hence a very large fine-tuning between different sectors of the model (gauge and top loops) in order to cancel the top contribution to μ_h^2. Large values of G would also require even smaller values for the c_i coefficients, in order to adequately suppress $\tilde{\lambda}$.

These are all features that are confirmed by the more complete numerical analysis, to which we now turn, in which the logarithmically divergent contributions are properly taken into account. As discussed in the next section, the presence of the logarithmic contributions to the mass of the triplet will further constrain the region of the allowed coefficients.

1.2.5 Numerical analysis

All the electroweak precision data analysis and the fine-tuning estimates in the littlest Higgs model present in literature [29, 31] have been done expanding the CW potential up to the fourth and second order in the Higgs and triplet field respectively. In [43] the full potential is studied, but the logarithmic contributions for the triplet are neglected.

We study the full one-loop CW potential, with no approximations both in the Higgs and in the triplet field, in order to perform a detailed analysis of the parameter space. As done for the approximated potential of eq. (1.60), even the full one-loop CW potential is computed for the Higgs and triplet components that acquire a VEV responsible of the spontaneous breaking of the electroweak gauge group. Therefore, in the following, our field variables will be h and t as defined in sec. (1.2.4). As already discussed, the CW effective potential is controlled by six parameters and coefficients $(c_1, c_2, G, G', x_L, f)$ which are fixed by the six constraints provided by

- the existence (vanishing of first derivatives in the h and t directions),
- the value (to be v_W and v' for, respectively, the h and t fields) and
- the stability (m_H^2 and m_t^2 both larger than zero)

of the ground states in the Higgs and triplet directions. In addition, we can add a new coefficients c_5 (of the two-loop correction) in order to bring m_H closer to the desired value.

Since we study the potential numerically, we reverse the problem and instead of solving to find the values of these parameters and coefficients we generate possible sets of their values and check what m_H and f (as well as the corresponding quantities for the triplet field) are thus obtained.

We proceed in three steps by imposing the constraints which the potential must satisfy. As we shall see, these constraints greatly reduce the allowed values of the coefficients c_1, c_2 and c_5.

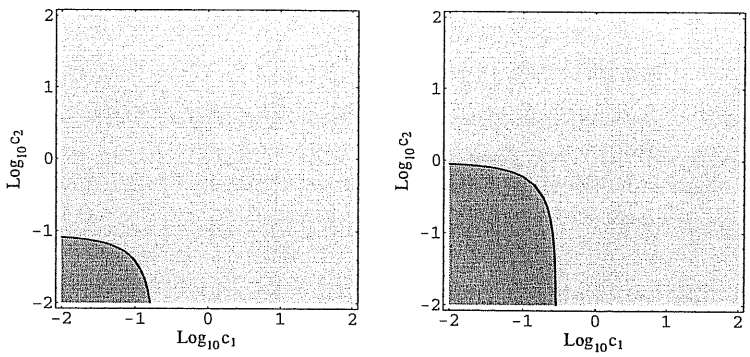


Figure 1.2: Possible values (on a logarithmic scale) of the coefficients c_1 and c_2. The two figures correspond to $G' = 0.72, x_L = 0.56$ and, respectively, two choices of $G = 3$ and $G = 8$. Each point in the light-gray region is a possible potential with a maximum at $h/f = \pi/2$, which means a possible minimum around $h = v_W/\sqrt{2}$. The darker region, where both c_i are small, corresponds to potentials with a minimum in $h/f = \pi/2$ which are not allowed.

1.2.5.1 First step: making v_W (and v') the ground state

A first constraint arises from the requirement of having the correct electroweak ground state for both the Higgs boson and the triplet fields. Here correct means for small values of the fields as opposed to larger values around $\pi f/2$. This is most easily implemented by studying the properties of the potential along one of its direction, for instance at large values of Higgs field h. The complete potential at one-loop $V_1[c_i, G, G', x_L, h/f, t/f, f]$ is a periodic function in h/f in the plane where the triplet $t = 0$. In order to have the ground state at the electroweak vacuum v_W around the origin, V_1 must be positive for $h/f = \pi/2$. This condition is sufficient to guarantee the existence of the correct ground state because the complete potential for the Higgs field h is given by (defining $V_1[0] = 0$)

$$V_1[h/f] = A \sin^2 h/f + B \sin^4 h/f + C \sin^6 h/f + D \sin^8 h/f \tag{1.76}$$

with A, B, C and D complicated functions of the coefficients and parameters and such as the first derivative of the potential with respect to h does not change sign between zero and $\pi/2$ when $V_1[h/f = \pi/2] < 0$. Another way to understand the same feature is that if $h/f = \pi/2$ is not positive, the mass squared of either h or t is negative and the electroweak ground state is unstable.

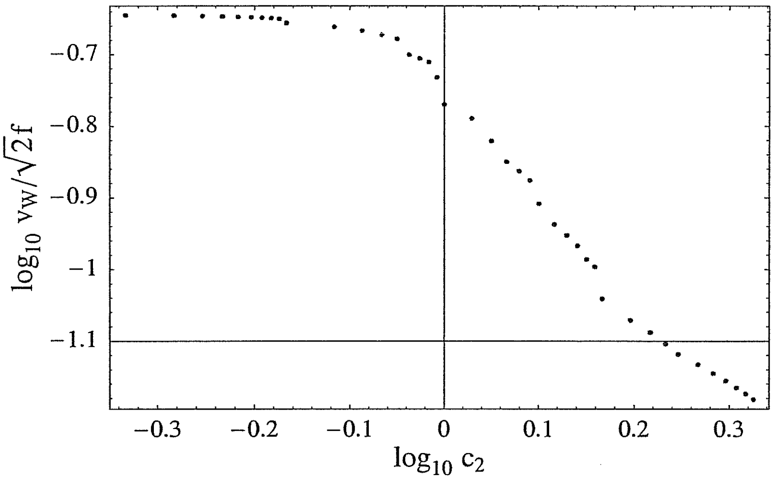


Figure 1.3: Dependence (on a logarithmic scale) of the minimum on one of the coefficients after having fixed the other ($c_1 = 1$) and all parameters ($G = 3$, $G' = 0.75$ and $x_L = 0.56$). The physical region, where $f \simeq 2$ TeV, corresponds to the line $\log v_W/\sqrt{2}f = -1.1$ (lower right hand corner in the figure).

This requirement makes possible to fix a region

$$V_1[c_i, G, G', x_L, h/f = \pi/2, t = 0, f] > 0 \tag{1.77}$$

of allowed values in the six-dimensional parameter space $(c_1, c_2, G, G', x_L, f)$ (no two-loop contributions are for the moment included and therefore there is no parameter c_5). The potential V_1 is given by

$$\begin{aligned} \frac{1}{f^4} V_1[c_i, G, G', x_L, h/f = \pi/2, t = 0] &= \frac{3}{16} c_1 \left(G^2 + G'^2\right) + 12 c_2 x_L^2 + \left(\alpha - \beta\, G^2\right) \log \frac{G^2}{64\pi^2} \\ &+ \left(\gamma - \delta\, G'^2 + \frac{15}{4096\pi^2} G'^4\right) \log \frac{G'^2}{64\pi^2} \end{aligned}$$

$$+ \quad \frac{3x_L^4}{4\pi^2(4x_L^2-1)}\left(1+4x_L^2\right)\log\frac{x_L^4}{2\pi^2(4x_L^2-1)}\,, \quad (1.78)$$

where $\alpha = 6.3 \times 10^{-4}, \beta = 1.8 \times 10^{-3}, \gamma = 2.0 \times 10^{-5}, \delta = 2.3 \times 10^{-4}$. The numerical coefficients in eq. (2.9) are obtained by giving their experimental values to the gauge and Yukawa couplings of the standard model.

Fig. 1.2 shows the values of c_1 and c_2 which satisfy the condition above for two choices of the gauge coupling G (the dependence on G' is weaker). A similar plot could be shown by varying x_L. In general, for given G, G', x_L, this condition forbids the configurations with both c_1 and c_2 of $O(10^{-2})$ and it is even more restrictive for larger values of the gauge coupling G (see plot on the right side of Fig. 1.2).

Therefore, the very requirement of having the electroweak vacuum as the ground state of the littlest Higgs model is far from obvious for arbitrary coefficients c_i. As we shall see, this is important for fits to the electroweak data.

We can plot this ground state as a function of one of these coefficients after the other one—and all the other parameters—have been fixed to some values. As shown in Fig. 1.3, in the physical region, where $f \simeq 2$ TeV—which corresponds to the line $\log v_W/\sqrt{2}f = -1.1$—we obtain the desired ratio $v_W/f \sim 1/10$ for $c_1 = 1$ and $c_2 \simeq 1.2$ and, therefore, with a natural choice of the coefficients. In addition, we would also like to find $v_W \ll f$ for a large range of values of these coefficients, that is, the logarithmic derivative should not be too large:

$$\frac{d\log(v_W/\sqrt{2}f)}{d\log c_i} < 10\,. \quad (1.79)$$

The result in Fig. 1.3 is a variation that is close to 1 for most values of c_2. In this respect, the model is therefore working well and it stabilizes the electroweak symmetry breaking scale to be a factor of one hundred less than the cut-off of the theory.

1.2.5.2 Second step: possible values of m_H and f in the one-loop CW potential

In the second step of our study—given the set of parameters (c_1, c_2, G, G', x_L) for which the scalar potential has the right behavior at large h and therefore $h = v_W/\sqrt{2}$ is its

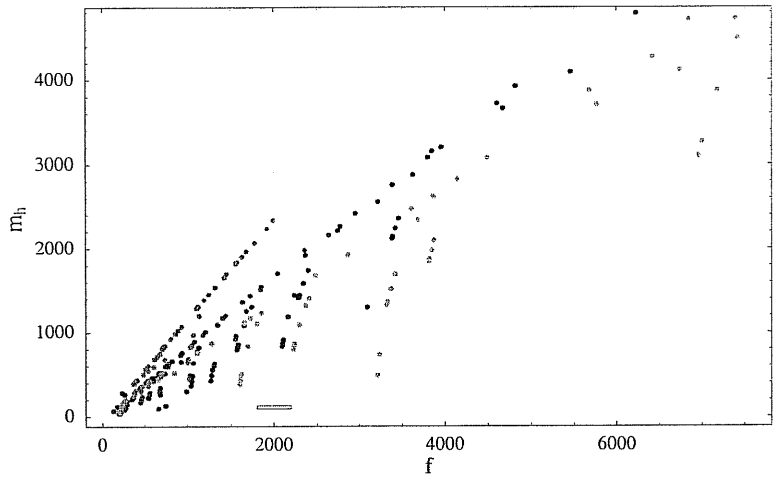


Figure 1.4: m_H vs. f for $c_{1,2}$ between 0.01 and 100 Each point represent a choice of c_1 and c_2. Four different values of G, G' and x_L ($G = 1.3, 3, 8, 10$, $G' = 0.72, 0.75, 2, 4$ and $x_L = 0.52, 0.56, 1.2$) are shown in different colors. No two-loop contribution is included. The little red box (rather squeezed by the axis scales) indicates the preferred values $f = 2000 \pm 200$ GeV and $m_H = 110 \pm 20$ GeV.

ground state—we look (see Fig. 1.4) at the possible values of m_H and f for a large range of parameters and coefficients. We take c_i between 0.01 and 100, and consider four different values of G, G' and x_L to show the dependence on the gauge and Yukawa parameters. Values of c_i not allowed (see Fig. 1.2) are automatically excluded.

No choice of values gives a light mass for the Higgs boson if f is larger than 1 TeV. Roughly speaking, the mass of the Higgs boson is a linear function of the scale f as we vary c_1 and c_2. The bigger the gauge coupling G (or the Yukawa x_L), the slower the raising of m_H with f. Notice, however, that by increasing the value of G we increase the difference in the values of the couplings g_1 and g_2 of the original gauge groups, and, for instance, at $G = 10$ we find $g_1 \simeq 10$ and $g_2 \simeq 0.65$. The same features are also shown in Figs. 1.5 and 1.6, where the ratio m_H/f is plotted against f for different choices of G and x_L. The natural values all lay on line at values of m_H of the same order as f and even stretching the parameters does not bring the ratio m_H/f near the desired values (for instance, 0.1 for $f \simeq 2$ TeV). The dependence on G' is instead rather weak.

Even for very small c_i's, the logarithmic contributions make m_H of the order of f so that if we want the mass of the Higgs boson to be small, we find that f is small as well.

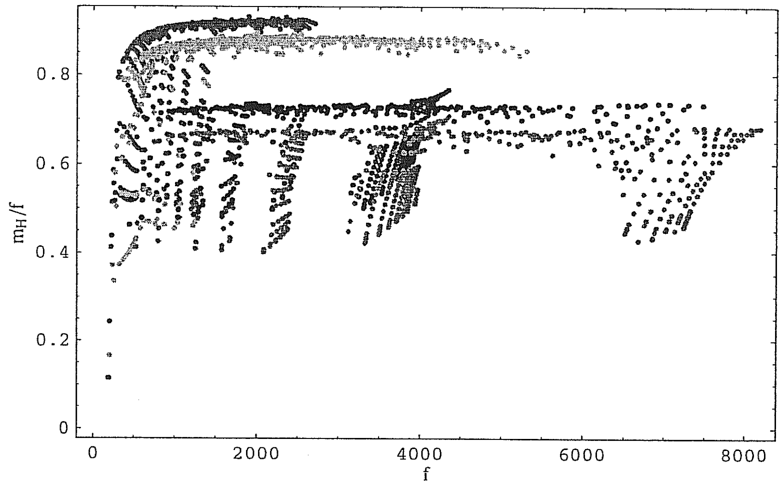


Figure 1.5: m_H/f vs. f for $c_{1,2}$ between 0.01 and 100 Each point represents a choice of c_1 and c_2 with c_1 increasing from the bottom to the top and c_2 from left to right. Holes in the dots distributions are an artifact of the numerical simulation mash. Four different values of $G = 1.3, 3, 8, 12$ (at fixed $x_L = 0.55$ and $G' = 0.72$) are shown in different colors with smaller values toward the bottom of the figure. No two-loop contribution is included. No choice of values of these coefficients gives $m_H \simeq 120$ GeV and f around 2 TeV.

Even though it is not surprising that m_H does not come out right—after all the (unknown and uncomputed) two-loop contributions have been usually introduced in the literature [29] to argue that the μ_h^2 term in the potential eq. (1.51) is essentially a free parameter to be adjusted in order to have the desired mass for the Higgs bosons—what is worrisome is that we find that the logarithmic terms are rather large and the coefficients of the two-loop corrections would have to be accordingly large to compensate them and fine-tuned to give a net mass one order of magnitude smaller.

1.2.5.3 Third step: including the two-loop term

We therefore proceed to the third and final step in our analysis and include a (quadratically divergent) two-loop contribution to the term quadratic in h in the scalar potential:

$$V'[c_5; h] = \frac{c_5 f^4}{16\pi^2} \left(\frac{h}{f}\right)^2 . \tag{1.80}$$

This is a somewhat *ad hoc* (and minimal) choice to simulate the actual two-loop computation which is vastly more complicated and the result of which would presumably be a series of

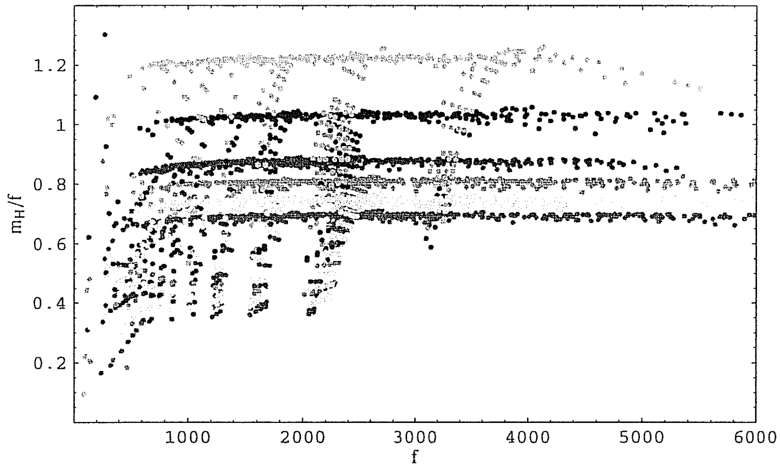


Figure 1.6: Same as Fig. 1.5. Four different values of $x_L = 0.55, 0.71, 0.91, 1.05, 1.55, 2.05$ (at fixed $G = 3$ and $G' = 0.72$) are shown in different colors with the largest value of x_L on top, smallest values corresponding to $x_L = 0.71$. No two-loop contribution is included. No choice of values of these coefficients gives $m_H \simeq 120$ GeV and f around 2 TeV.

operators similar to those we have included. Other terms proportional to t^2 or hth could be added (and if added would completely change the analysis) but they would correspond to two-loop corrections to already quadratically divergent one-loop terms and go against the very idea behind the little Higgs model.

Having added the two-loop term (1.80), it is possible to study the behavior of the potential

$$f^4 V_1[c_i, G, G', x_L, h, t, f] + V'[c_5, h]. \tag{1.81}$$

around the origin. For each choice of (G, G', x_L), by imposing the four constrains arising from the two first derivatives (to have a minimum and it to be at the correct value) and from the value of m_H and m_t, the three coefficients, c_1, c_2 and c_5, are fixed.

The study we performed shows that the new constraint of having a Higgs boson mass close to the current bound [8] drastically reduces the allowed region in the parameter space of $(c_1, c_2, G, G', x_L, f)$ and given G, G' and x_L the allowed regions are characterized by having either c_1 of $O(1)$ and c_2 of $O(10^{-2})$ or the opposite, as shown in Fig. 1.7. For each of these choices of coefficients c_1 and c_2, a value of c_5 must be chosen so as to obtain the desired mass m_H. This is only possible for rather large values of the coefficient c_5. If

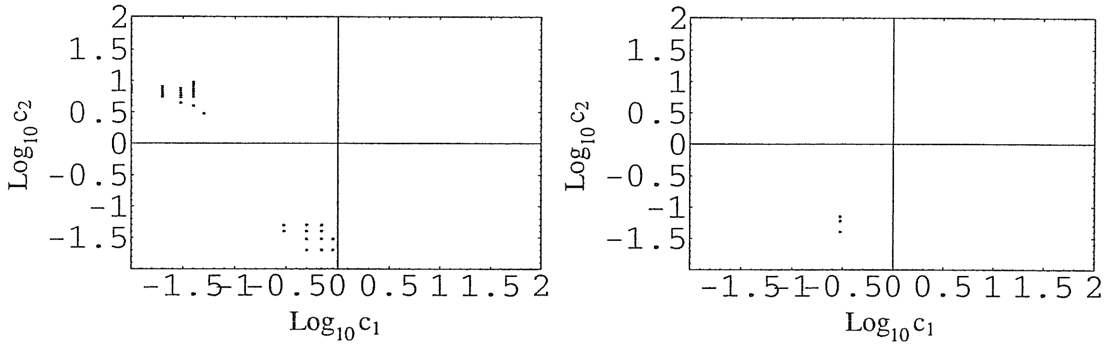


Figure 1.7: The allowed values of the coefficients c_1 and c_2 with the constrain on the Higgs boson mass (m_H between 110 and 200 GeV) enforced and the two-loop quadratic divergent term included. Only very few regions in the parameter space showed in Fig. 1.2 are still allowed. On the left side: $G = 3$ and $c_5 \simeq 50$, on the right side: $G = 8$ and $c_5 \simeq 35$.

we are willing to allow larger Higgs boson masses (that is, $m_H > 300$ GeV), c_5 will turn proportionally smaller but we will still have similar severe constraints on c_1 and c_2.

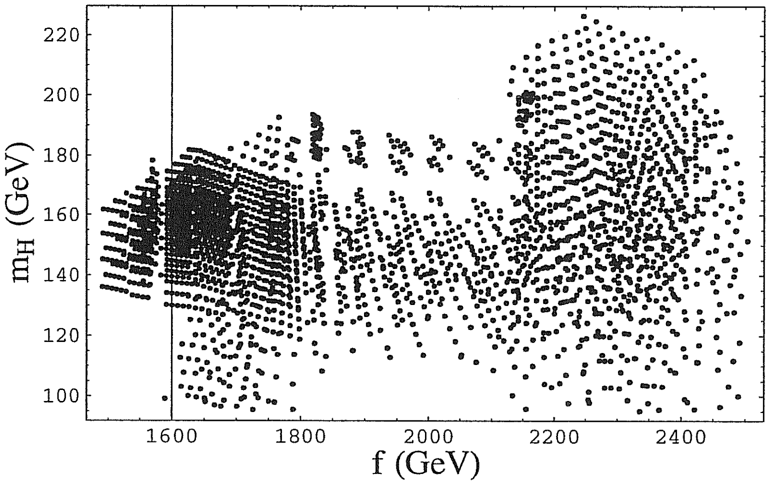


Figure 1.8: m_H vs. f. Each point corresponds to a choice of all coefficients and parameters in the range discussed in the text ($c_1 = O(10^{-2})$, $c_2 = O(1)$, $c_5 \simeq 50$) and varied in discrete steps; $G = 3$, $G' = 0.75$ and $x_L = 0.56$ are fixed.

1.2.6 Comments on the results

Fig. 1.8 shows the possible values of m_H and f close to the desired values for a range of the coefficients c_1, c_2 and c_5 in the allowed regions. These values are now possible but we pay a rather high price for it. The two main problems are that

- the natural case in which all the coefficients c_i are $O(1)$ seems to be ruled out. Values for f and m_H in the desired range are only obtained by taking c_1 of $O(10^{-2})$ and c_2 $O(1)$ or vice versa. A coefficient of order $O(10^{-2})$ clearly goes against the very rationale of introducing the littlest Higgs model in the first place because we have to make small by hand one of the symmetry breaking terms;

- the phenomenological two-loop term must have rather large coefficients (c_5 =45-55). This already anticipated feature reminds us of the importance of these terms in compensating the logarithmic contribution to the Higgs boson mass, which are therefore rather larger than one would wish and usually assume in the little-Higgs framework. Roughly speaking, these logarithmic terms are $O(f)$ whereas we expected them to be of $O(m_H)$. This is unfortunate since the naturalness of a scale f around 1–2 TeV is questionable once such a large two-loop term is included in order to bring m_H around its current bound. Moreover, given the size of our example of two-loop contribution, there is no way to argue that these two-loop contributions can be neglected in any other part of the potential and the entire approach at one-loop seems to break down. A similar conclusion was reached in a recent work were the fine-tuning of the littlest Higgs is discussed [44].

The analysis above shows that once the scale f is required to be larger than 1 TeV, after all coefficients have been fixed, the value of the Higgs boson mass—which is linked to that of the neutral component of the triplet—cannot be made as small as desired. In particular, it is not possible to have it close to the current experimental lower bound unless some the coefficients of the quadratically divergent terms are made unrealistically small while at the same time the two-loop correction is made rather large. The necessary smallness of some of the coefficients defeats the purpose of introducing the collective breaking mechanism to make the mass terms small and the littlest Higgs model stable against one-loop radiative corrections. Moreover, the mass of the Higgs boson itself comes out in a very unnatural way from the cancellation of terms one order of magnitude larger than its value.

This result seems to be a more serious problem for the model than that of the fine-tuning required in order to be consistent with electroweak precision measurements. The problem has been so far ignored in the literature because it has been assumed that it was always possible to add to the logarithmically divergent part of the potential the two-loop quadratically divergent contribution so as to obtain the desired Higgs boson mass. This is however only possible at the price of introducing an unreasonable large coefficient in this term and even then at the price of having at least one of the other two coefficients very small.

On the other hand, if we let instead the model to decide what value the mass of the Higgs boson should be, we find that it comes out close to the scale f and therefore, for f in the 1-2 TeV range, the Higgs is accordingly heavier than expected.

1.2.7 Comparison with other studies

There are many discussions in the literature about the littlest Higgs model and electroweak precision constraints [29, 31]. In many of these papers, however, the values of the coefficients of the divergent terms are assumed to be of $O(1)$ or, at most $O(0.1)$ and the logarithmic terms not included. Moreover in [29] only the parameters relevant to the effective operators in the gauge boson sector are discussed and the coefficients of the divergent terms are assumed of the desired order and not studied. The only reference in which the scalar potential is actually constrained is [31]. In order to show that our conclusions agree with what found in this reference, let us, following their notation, fix the coupling g and g' in terms of the fine-structure constant α and the Weinberg angle, v_W and v'—the VEV of the isospin triplet t—by means of the Fermi constant and reparametrize the top Yukawa couplings λ_1 and λ_2 in terms of

$$\begin{aligned} x_L &= \frac{\lambda_1^2}{\lambda_1^2+\lambda_2^2} \\ \frac{m_t}{v_W} &= \frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}\left[1+\frac{v^2}{2f^2}x_L(1+x_L)\right] \end{aligned} \tag{1.82}$$

we are thus left with a model that, after assigning a value to m_t and m_H, only depends on f, x_L, s and s' (as defined in ref. [31]) and the counterterms a and a' (which correspond to $3c_1/2$ and $6c_2$). These two can be found for each choice of the first four parameters by solving

$$\begin{aligned}
\frac{a}{2}\left[\frac{g^2}{s^2c^2}+\frac{g'^2}{s'^2c'^2}\right]+8a'\lambda_1^2 &= 2\frac{m_H^2}{v_W^2}\frac{1}{1-(4v'f/v_W^2)^2} \\
-\frac{a}{4}\left[\frac{g^2(c^2-s^2)}{s^2c^2}+\frac{g'^2(c'^2-s'^2)}{s'^2c'^2}\right]+4a'\lambda_1^2 &= 2\frac{m_H^2v'f}{v_W^4}\frac{1}{1-(4v'f/v_W^2)^2} \qquad (1.83)
\end{aligned}$$

We thus find that in order to have, for instance, $f = 2$ TeV while $m_H = 115$ GeV (and $v' = 3.54$ GeV, $x_L = 0.4$, $s = 0.22$ and $s' = 0.66$, as discussed in [31]) we must take the coefficients a and a' of order 1/100 (more precisely, $a = 0.036$ and $a' = 0.063$ in this case; small coefficients are found also for other allowed choices of f and v'), a choice that clearly defeats the very rationale for introducing the littlest Higgs model in the first place.

This result is consistent with our analysis as presented in the previous section in the case in which the logarithmic contributions are neglected and the two-loop terms included. However, as soon as the logarithmic contributions are not neglected (and we have shown that they cannot be neglected), the solution above does not exist because it would correspond to a negative value of the triplet mass and an unstable electroweak ground state. Going back to Fig. 1.2, the solutions studied in [31] is in the region ruled out where both coefficients c_i are very small.

1.3 A modified top sector

In the previous sections we have seen that the littlest Higgs model, given a cut-off $\Lambda = 4\pi f$ around 10 TeV, predicts a large Higgs mass around 500 GeV. Introducing a two-loop effective quadratic term allows to bring this mass to a value smaller than 200 GeV but the two-loop term coefficient must then be very large. Because the problem is largely due to the fermionic sector of the model, in this section we discuss a possible modification of the fermion content of the model, as proposed in [45], to see if it helps. We neglect in the following the triplet and focus on the Higgs doublet.

We consider next an improved version of the littlest Higgs model [37] in which the top fermionic sector is completed to make its contribution to the CW one-loop potential finite. We study this model and show that, even though (marginally) better than the littlest

Higgs model, again the requirement of a Higgs boson mass around 200 GeV and f around 2 TeV would lead to unreasonable values of the parameters and excessive fine-tuning.

1.3.1 The model

The Lagrangian of the model we consider differs from that of the littlest Higgs model only by the fermionic contributions $\mathcal{L}_\psi$. The fermionic content is given by an electroweak doublet $Q_L = (t_0, b_0)_L$, an electroweak singlet $t^c_{g_L}$ and by two colored $SU(5)$ quintuplets, X and $\bar{X}$. The Yukawa Lagrangian is given by an $SU(5)$ invariant term, and by two explicit breaking terms of the $SU(5)$ global symmetry, both of them preserves enough symmetry to prevent the Higgs to gain a mass. Only the loops contributions that involves both of them can produce a mass for the Higgs. The Yukawa Lagrangian is given by

$$\mathcal{L}_Y = \sqrt{2}\lambda_1 f\, \bar{X}\,\Sigma\, X + \sqrt{2}\lambda_2 f\,(a^c_{1_L} b_{0_L} + a^c_{2_L} t_{0_L}) + \sqrt{2}\lambda_3 f\, t^c_{g_L} t_{d_L}\,, \tag{1.84}$$

where

$$X = \begin{pmatrix} p_{1_L} \\ p_{2_L} \\ t_{d_L} \\ r_{1_L} \\ r_{2_L} \end{pmatrix} \qquad \bar{X} = \begin{pmatrix} a^c_{1_L} \\ a^c_{2_L} \\ t^c_{t_L} \\ b^c_{1_L} \\ b^c_{2_L} \end{pmatrix}. \tag{1.85}$$

By eq. (1.84) we obtain the fermion mass matrix given by

$$M^\dagger_{f_{RL}} = f \begin{pmatrix} -\sqrt{2}\lambda_1 \sin^2\frac{h}{f} & i\,\lambda_1 \sin\frac{2h}{f} & \sqrt{2}\,\lambda_2 & \sqrt{2}\,\lambda_1 \cos^2\frac{h}{f} \\ \sqrt{2}\,\lambda_1 \cos^2\frac{h}{f} & i\,\lambda_1 \sin\frac{2h}{f} & 0 & -\sqrt{2}\,\lambda_1 \sin^2\frac{h}{f} \\ 0 & \sqrt{2}\,\lambda_3 & 0 & 0 \\ i\,\lambda_1 \sin\frac{2h}{f} & i\,\sqrt{2}\,\lambda_1 \cos\frac{2h}{f} & 0 & i\,\lambda_1 \sin\frac{2h}{f} \end{pmatrix}, \tag{1.86}$$

where in eq. (1.86) we have put $t = 0$. By eq. (1.86) we see that

$$\begin{aligned} \mathrm{Tr}\, M^\dagger_{f_{RL}} M_{f_{RL}} &= 2(L_1^2 + L_2^2 + \lambda_1^2)\, f^2 \\ \mathrm{Tr}\,(M^\dagger_{f_{RL}} M_{f_{RL}})^2 &= 4(L_1^4 + L_2^4 + \lambda_1^4)\, f^4\,, \end{aligned} \tag{1.87}$$

where

$$\begin{aligned} L_1^2 &= \lambda_1^2 + \lambda_2^2 \\ L_2^2 &= \lambda_1^2 + \lambda_3^2 . \end{aligned} \quad (1.88)$$

Eq. (1.87) indicates that there are no one-loop fermionic divergent contributions to the mass of the Higgs. The only one-loop fermionic contributions are finite and therefore calculable. From now on we take $L_1 = L_2$.

One of the eigenvalues does not depend on h, and is given by:

$$m_3^2 = 2\lambda_1^2 f^2 . \quad (1.89)$$

The lightest mass is to be interpreted as that of the standard top quark, with mass approximated by

$$m_t^2 = \lambda_t^2 f^2 \sin^2 \frac{h}{f} + \left(-\lambda_t^2 + \frac{\lambda_t^4}{L_1^2}\right) \sin^4 \frac{h}{f} , \quad (1.90)$$

where

$$\lambda_t = 2 \frac{\lambda_1 \lambda_2 \lambda_3}{\sqrt{\lambda_1^2 + \lambda_2^2}\sqrt{\lambda_1^2 + \lambda_3^2}} . \quad (1.91)$$

It gives a negligible contribution to the effective potential. The two relevant eigenvalues can be expanded in powers of $\sin h/f$ obtaining:

$$\begin{aligned} m_1^2/f^2 &= 2L_1^2 + \sqrt{2} L_1 \lambda_t \sin\frac{h}{f} - \frac{\lambda_t^2}{2}\sin^2\frac{h}{f} - \left(\frac{L_1\lambda_t}{\sqrt{2}} - \frac{5\lambda_t^3}{8\sqrt{2}L_1}\right)\sin^3\frac{h}{f} \\ &+ \frac{1}{2}\left(\lambda_t^2 - \frac{\lambda_t^4}{L_1^2}\right)\sin^4\frac{h}{f} + O(\sin^5\frac{h}{f}) \\ m_2^2/f^2 &= 2L_1^2 - \sqrt{2} L_1 \lambda_t \sin\frac{h}{f} - \frac{\lambda_t^2}{2}\sin^2\frac{h}{f} + \left(\frac{L_1\lambda_t}{\sqrt{2}} - \frac{5\lambda_t^3}{8\sqrt{2}L_1}\right)\sin^3\frac{h}{f} \\ &+ \frac{1}{2}\left(\lambda_t^2 - \frac{\lambda_t^4}{L_1^2}\right)\sin^4\frac{h}{f} + O(\sin^5\frac{h}{f}) . \end{aligned} \quad (1.92)$$

The fermionic contribution to the potential for the Higgs field obtained when $L_1 = L_2$ (which is the most favorable case) is therefore

$$\frac{V_{tn}}{f^4} = -\frac{3\lambda_t^2 L_1^2}{4\pi^2}\sin^2\frac{h}{f} + \frac{\lambda_t^4}{16\pi^2}\left(-4 + \frac{12L_1^2}{\lambda_t^2} - 3\ln\frac{\lambda_t^2}{2L_1^2}\right)\sin^4\frac{h}{f} . \quad (1.93)$$

1.3.2 Approximate analysis

The bosonic sector of the model has not been modified, hence we can write (see eq. (1.60)) the (approximate) expressions:

$$\begin{aligned} \frac{\mu_h^2}{f^2} &= -\frac{9}{256\pi^2} g^2 G^2 \log\frac{G^2}{64\pi^2} - \frac{3\lambda_t^2 L_1^2}{4\pi^2}, \\ \lambda_4 &= \frac{3c_1 G^2}{16} + \frac{\lambda_t^2 L_1^2}{4\pi^2} + \frac{\lambda_t^4}{16\pi^2}\left(-4 + \frac{12L_1^2}{\lambda_t^2} - 3\ln\frac{\lambda_t^2}{2L_1^2}\right). \end{aligned} \tag{1.94}$$

Choosing $L_1 \sim \sqrt{2}$ (that is, a value close to the smallest possible after $\lambda_t = 1$), one obtains

$$\begin{aligned} \frac{\mu_h^2}{f^2} &\simeq 0.02 G^2 g^2 \log G^2 - 0.15, \\ \lambda_4 &\simeq \frac{3c_1 G^2}{16} + 0.2. \end{aligned} \tag{1.95}$$

The finite contributions to μ_h^2 in this variation of the model are comparable in size to the original logarithmically divergent ones. Further, the quartic coupling is now dominated by the gauge boson sector, since the top sector gives only a small contribution. From this, comparing with the original littlest Higgs model, we conclude that there is no substantial improvement: the cancellation of logarithmic divergences is not enough to reduce the large top contribution to the $\sin^2 h/f$ term in the potential. For this reason we leave out a more general numerical analysis of this modified model.

1.4 Conclusions

We started the chapter describing the origin of the little Higgs model, focusing on the collective symmetry breaking mechanism and underlying how appealing it was using this mechanism to protect the Higgs mass after having made the Higgs boson a PGB. Then we concluded pointing out that the most popular little Higgs model, the littlest Higgs, fails to predict a light Higgs mass, unless we accept a large amount of fine-tuning. In sec. (1.2.4) we have identified the large logarithmically divergent contributions as the reason of the difficulty to have a Higgs mass around the central value suggested by the electroweak data. Furthermore, in sec. (1.3) we have seen that a minimal extension of the fermion sector of the littlest Higgs model does not improve sufficiently the situation to be considered a real solution. At this point we are entitled to wonder if there is any possibility to further extend the littlest Higgs model has done in sec. (1.3) to reduce the finite term that are too large in eq. (1.95). Next, we could ask how much model dependent are the results we have obtained in sec. (1.2) and if it was possible that other little Higgs models could behave better than the littlest Higgs one.

In sec. (1.1.1) we have seen that in the model based on deconstruction the first quadratically divergent contributions to the PGB masses are proportional to $\Lambda_m^2(g^2/16\pi^2)^N$, where N is the number of the copies of the gauge group $SU(n)$. The hint is therefore to follow what done in sec. (1.3) and modify the top sector, and eventually the gauge one, by introducing more explicit breaking terms, each of them preserving an approximate global symmetries in order to further suppress the contributions to the Higgs mass. Note that if we want to modify the gauge sector and, for example, starting with three copies of the electroweak gauge group $SU(2)_W \times U(1)_W$ we should also enlarge the global symmetry $SU(5)$. At first sight this possibility seems encouraging, but Wacker [46] has noticed that there are always finite contributions to the little Higgs mass proportional to the mass of the heavy particles introduced by the model. It turns out that this finite contributions are of the same order of the two-loop quadratically divergent corrections, thus making useless the removing of higher order quadratic divergences, and the largest corrections to the Higgs boson mass is always of the order of a two-loop quadratic divergences. This means that even modifying, for example, the top sector in a manner similar to what done in sec. (1.3), we would eventually find finite contributions to the Higgs potential of the same order of those computed in sec. (1.3.2). It seems therefore that any hope of improving our conclusions lays in changing the group structure of the model.

The little Higgs models present in literature are based on a global symmetry H_1 spontaneously broken to H_2. Two subgroups of H_1, G_1 and G_2, are gauged and the gauging explicitly breaks the global symmetry H_1. At the same time the spontaneous breaking of H_1 into H_2 breaks the gauge group $G_1 \times G_2$ in the diagonal combination G, that has to coincide with the electroweak gauge group $SU(2)_W \times U(1)_W$. Among the PGBs arising in the spontaneous breaking an $SU(2)_W$ doublet with hypercharge 1/2 has to be present to be identified with the Higgs boson. The fermion sector is slightly modified and the final spectrum is enriched of a heavy Dirac top-like colored quark. Since the couplings to the standard model particles of the new gauge and scalar bosons and the new fermion introduced are model dependent, the electroweak precision observables are affected in a model dependent way and a detailed analysis is required to test the validity of every single little Higgs model. On the contrary, the low energy effective scalar potential for the Higgs boson has a general expression, being expressed in terms of the scale f, of the electroweak gauge and Yukawa coupling g, g' and λ_t and of the new gauge boson and fermion masses—the scalar contribution being neglected—which are proportional to the sum of the explicit breaking terms as in eqs. (1.65)–(1.68). The effective scalar potential for the Higgs boson has the standard form

$$\mu'^2_h h^2 + \tilde{\lambda}' h^4 \,, \tag{1.96}$$

and the quartic and quadratic couplings, $\tilde{\lambda}'$ and μ'^2_h respectively, satisfy the same conditions of eq. (1.73) we have found in sec. (1.2.4) for $\tilde{\lambda}$ and μ^2_h. The quartic coupling $\tilde{\lambda}'$ obtained in a generic little Higgs model will not depart from that given in sec. (1.2.4) except for numerical coefficients that we expect of the same order of that of eq. (1.65). At the same time the logarithmic contribution μ'^2_h will differ from that of eq. (1.68) in the numerical coefficients in front and inside each logarithmic term and in the number of the gauge boson logarithmic contributions, since it is possible that G_1 or G_2 is larger than the electroweak gauge group. The conclusion we reach is therefore the same as in sec. (1.2.4). To reduce the logarithmic contributions and bring μ'^2_h to the value indicated by eq. (1.73), we need large values of the explicit breaking terms, bounded by the constraints to reproduce the electroweak couplings, while the quartic coupling prefers values of the explicit breaking terms of $\mathcal{O}(1)$, larger values can not give a $\tilde{\lambda}'$ of $\mathcal{O}(10^{-1})$ as required by eq. (1.73) even with unreasonable small values of the UV coefficients.

These considerations support the conclusion that we do not expect we would find essentially different results if we had analyzed a different little Higgs model and the negative result we reached holds in general.

Chapter 2

The little flavon model

One of the most tantalizing clue for physics beyond the standard model that we know of comes from the Higgs sector and the closely related flavor structure. Data on particle masses and mixing angles present us with a wealth of information not too dissimilar to that once offered by Mendeleev's period table and seem to beg for a dynamical explanation of their regularities. These data are encoded in the standard model into the Yukawa lagrangian which gives mass to the fermions and shapes their mixing and mass hierarchies. This lagrangian is thus controlled by a (large) number of parameters that appear to be arbitrary insofar as their values are chosen by hand to match the experimental data; moreover, their values must be chosen in a precise manner and many of them vary across several orders of magnitude. The stability against radiative corrections of these patterns and hierarchies seems to require some amount of fine-tuning. While any fine-tuning of the parameters can always be seen as a mere coincidence (or, perhaps more speculatively, as anthropic selection at work), we take here the point of view that its explanation—like that for the little hierarchy—calls for new physics.

A first step in the direction of improving our understanding of the flavor structure of the standard model can be taken by re-organizing the parameters and considering the mass matrices of quarks and leptons not as 3×3 arbitrary matrices but as matrices having well-defined textures controlled by one or at most few parameters. In this picture, the mass matrices have entries that are powers of these few parameters modulated by dimensionless coefficients of order one—and thus requiring no further explanation. While this is not yet a dynamical model—it is really just *kinematics*—it helps in providing a framework in which to bring the dynamics eventually.

At least part of this dynamics comes from identifying the small parameters, the powers of which give rise to the textures, with the VEVs of some scalar fields with quantum numbers running across the horizontal family structure of fermions. In this picture—usually referred to as the Froggart-Nielsen mechanism [35]—the emerging textures are due to different charge assignments for the fermions, and therefore from the different powers of the small parameters, associated to the vacua of the scalar fields, making up the mass matrices arising from the Yukawa lagrangian.

The next, and crucial step consists in providing stability against radiative corrections for the patterns thus generated and therefore explaining away the apparent fine-tuning of parameters encountered. This problem can be rephrased in terms of the naturalness of the dimensionful parameters of the model that must be protected against corrections that tends to bring all of them to the highest mass scale in the problem, usually the cut-off of the effective field theory (all dimensionless parameters are then assumed of order 1 and therefore natural). Such a naturalness—in the 't Hooft's sense—is achieved by identifying the one or more symmetries that would be recovered in the limit of vanishing interactions and VEVs.

This problem of fine-tuning and overall stability is present in all models trying to describe the mass matrices of the fermions, and the textures by which they are characterized, in terms of the VEV v of one, or more, scalar fields, the flavons [35, 36]. In these models, the texture is written in terms of the ratio $\varepsilon = v/f$, where f is now the flavor symmetry breaking scale and ε is the small parameter of the texture typically of the order of the Cabibbo angle. These patterns tend however to be washed out by the quadratically divergent radiative corrections to the mass term μ^2 that make the VEV, for a generic quartic potential proportional to a parameter $\lambda \simeq 1$,

$$v^2 \simeq \mu^2 \simeq f^2 \tag{2.1}$$

and therefore $\varepsilon = 1$.

A small ε comes in a natural manner if the mass term is protected at the one-loop level and only logarithmically divergent so that

$$v^2 = \mu^2 \simeq \frac{\log(\Lambda^2/f^2)}{(4\pi)^2} f^2 \tag{2.2}$$

and we have $\varepsilon^2 \ll 1$ independently of the scale f.

In the first chapter we have seen how the mechanism of collective symmetry breaking is used in little Higgs models to stabilize against one-loop quadratic renormalization the Higgs potential. As a result quadratic renormalization of the Higgs boson masses arises only at the two-loop level and this make it possible to increase the standard model electroweak cut-off from few TeVs to few tens of TeVs.

The same idea can be attractive if applied to flavor physics. In this chapter we will see that it is possible to obtain dynamically a stable (non-supersymmetric) scalar potential by assuming that the flavons are PGBs originating from the breaking of an approximate global symmetry, spontaneously broken to a subgroup containing the flavor symmetry that acts on the standard model fermions. In this way, the field content of the flavon sector is determined.

The gauging of the flavor symmetry breaks explicitly the global symmetry and induces a potential for the flavons. The form of the potential as well the size of the scalar couplings is obtained by means of the CW potential [38] of the non-linear sigma model describing the PGB dynamics.

The potential induced by gauge interactions preserves the flavor symmetry we choose to be a $SU(2)_F \times U(1)_F$ flavor gauge symmetry, labeled by the index F in order to distinguish it from the electroweak group. The seed for the spontaneous breaking of the flavor symmetry is given by (gauge invariant) interactions of two doublet flavons with right-handed neutrinos. These interaction terms destabilize the symmetric vacuum and drive the complete breaking of the local flavor symmetry. As a consequence all flavor mediating gauge bosons become massive. At the same time, the one-loop stability of the flavon masses on the broken vacuum is preserved.

The general framework is similar to that discussed in ref. [47] and it actually uses the same high-energy global symmetry structure, albeit with a different pattern of gauge symmetry breaking. We have recalled this similarity in the naming of the *little flavons.*

2.1 The model

We choose as our basic flavor symmetry a gauged $U(2)_F \simeq SU(2)_F \times U(1)_F$. This choice is suggested by the approximate structure of the lepton sector: both neutrinos and charged-leptons can be classified in first approximation as two heavy flavor doublets—made by the τ and μ and the corresponding neutrinos—and two lighter singlets—the e and its neutrino.

To exploit the features of the little Higgs models, at least two copies of the flavor group should be embedded in a larger (approximate) global symmetry. The request that the flavon sector exhibits a vacuum structure that allows for the complete breaking of the final gauge flavor symmetry is satisfied minimally by two flavon doublets. The smallest group that satisfies these requirements is $SU(6)$, spontaneously broken to $Sp(6)$, which has been discussed as a little Higgs model in ref. [47].

In our model we assume that the electroweak and flavor symmetries are embedded in two independent collective-breaking frameworks with different cut-off scales Λ_H and Λ_F. We will not enter the details of the UV completion of the model, and only deal with the general structure of the effective theory below the non-linear sigma model scale $f = \Lambda_F/4\pi$, where all the physics of flavor takes place.

Consider then the spontaneous breaking of a global flavor symmetry $SU(6)$ down to $Sp(6)$. Fourteen of the generators of $SU(6)$ are broken giving 14 (real) GBs that can be written as a single field

$$\Sigma = \exp\left(i\Pi/f\right)\Sigma_0\,. \tag{2.3}$$

They represent fluctuations around the (anti-symmetric) VEV

$$\Sigma_0 \equiv \langle\Sigma\rangle = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}. \tag{2.4}$$

Within $SU(6)$ we can identify four subgroups as

$$SU(6) \supset [SU(2) \times U(1)]^2\,. \tag{2.5}$$

We choose to gauge these subgroups, in such a way as to explicitly break the global symmetry through the gauge couplings. Only the diagonal combination of these gauge groups survives the spontaneous breaking of the symmetry so that we have

$$[SU(2) \times U(1)]^2 \to SU(2) \times U(1)\,. \tag{2.6}$$

We will use the latter groups to classify our fermion and flavon states.

The generators of the two $SU(2)$ are given by the 6×6 matrices

$$Q_1^a = \frac{1}{2}\left(\begin{array}{cc|c} \sigma^a & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right) \tag{2.7}$$

and

$$Q_2^a = \frac{1}{2}\left(\begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & -\sigma^{a*} & 0 \\ 0 & 0 & 0 \end{array}\right), \tag{2.8}$$

where σ^a are the Pauli matrices; we choose the $U(1)$-charge matrices to be given by

$$\begin{aligned} Y_1 &= -\frac{1}{2\sqrt{15}}\,\mathrm{diag}\,(1,\ 1,\ -5,\ 1,\ 1, 1\,) \\ Y_2 &= -\frac{1}{2\sqrt{15}}\,\mathrm{diag}\,(1,\ 1,\ 1,\ 1,\ 1,\ -5\,)\ . \end{aligned} \tag{2.9}$$

Contrary to [47], our $U(1)$ charges belong to the generators of the group $SU(6)$. Notice that the gauged subgroup $[SU(2) \times U(1)]^2$ has rank 4: this means that one of the generators of the Cartan sub-algebra of $SU(6)$ (rank 5) is neither gauged nor explicitly broken. We identify this generator with

$$P = \mathrm{diag}\,[1,\ 1,\ 0,\ -1,\ -1, 0\,]\ , \tag{2.10}$$

in such a way that it commutes with the whole gauge group $[SU(2) \times U(1)]^2$, and it is orthogonal to all its generators. The generator P belongs to the algebra of $Sp(6)$, and generates a $U(1)_P$ exact global symmetry of the sigma model we are discussing. This symmetry is then explicitly broken by the couplings of flavons to fermions, as we shall discuss. We summarize the symmetry structure of the sigma model in Fig 2.1.

$$\begin{array}{ccc} SU(6) & \longrightarrow & Sp(6) \\ \big\downarrow & & \big\downarrow \\ [SU(2)\times U(1)]^2 \times U(1)_P & \longrightarrow & [SU(2)\times U(1)]\times U(1)_P \end{array}$$

Figure 2.1: Diagrammatic representation of the symmetry structure of the sigma model. Horizontal arrows indicate the spontaneous $SU(6) \to Sp(6)$ global symmetry breaking, vertical arrows the explicit breaking due to gauge interactions. A global $U(1)_P$ is preserved both by the spontaneous and the explicit breaking (induced by gauge interactions) while it is explicitly broken by the Yukawa sector of the model.

In the low-energy limit, there are two scalar bosons,

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \quad \text{and} \quad \phi_2 = \begin{pmatrix} \phi_2^0 \\ \phi_2^- \end{pmatrix}, \tag{2.11}$$

that are $SU(2)_F$-doublets with $U(1)_F$ charges respectively 1/2 and $-1/2$, and one $SU(2)_F$- and $U(1)_F$- singlet s. The remaining four bosons are eaten in the breaking of the (gauge) $[SU(2)\times U(1)]^2$ symmetries. Accordingly we find that in the low-energy limit we can write the PGB matrix as

$$\Pi = \begin{pmatrix} 0 & 0 & \phi_1^+ & 0 & s & \phi_2^0 \\ 0 & 0 & \phi_1^0 & -s & 0 & \phi_2^- \\ \phi_1^- & \phi_1^{0*} & 0 & -\phi_2^0 & -\phi_2^- & 0 \\ 0 & -s^* & -\phi_2^{0*} & 0 & 0 & \phi_1^- \\ s^* & 0 & -\phi_2^+ & 0 & 0 & \phi_1^{0*} \\ \phi_2^{0*} & \phi_2^+ & 0 & \phi_1^+ & \phi_1^0 & 0 \end{pmatrix}. \tag{2.12}$$

The singlet field becomes massive and has no expectation value in the vacuum configuration we will use; it is therefore effectively decoupled from the theory. The two doublets are our little flavons.

Under the action of $U(1)_P$ global transformations $U = \exp(i\,\alpha P)$, the doublets and singlet transform as:

$$\phi_{1,2} \longrightarrow e^{i\,\alpha}\,\phi_{1,2}\,, \quad s \longrightarrow e^{2\,i\,\alpha}\,s\,. \tag{2.13}$$

By construction, all GBs start out massless and with only derivative couplings. However, as anticipated, the gauge and flavon-fermion interactions explicitly break the symmetry and give rise to an effective potential for the PGBs, the form of which allows for the existence of a non-symmetric vacuum that completely breaks the residual flavor gauge symmetry.

2.1.1 The effective potential

The effective lagrangian of the PGBs below the symmetry-breaking scale is given by the kinetic term

$$-\frac{f^2}{4}\text{Tr}\,(D^\mu\Sigma)\,(D_\mu\Sigma)^*\,, \tag{2.14}$$

where the minus sign follows from the antisymmetric form of Σ. As already mentioned, we take undetermined the cut-off scale $\Lambda_F = 4\pi f$. The absolute value of this scale is immaterial to the generation of the lepton mass matrices that, as we shall see, only depend on the ratio between the VEVs, which are proportional to f, and f itself. In the last section of this chapter, we will see how it is possible to determine Λ_F studying the phenomenology and the flavor changing neutral current (FCNC) processes induced by the new particles present in the model.

The covariant derivative in (2.14) is given by

$$D_\mu\Sigma = \partial_\mu + ig_iA^a_{i_\mu}\left(Q^a_i\Sigma + \Sigma Q^{aT}_i\right) + ig'_iB_{i_\mu}\left(Y_i\Sigma + \Sigma Y^T_i\right) \tag{2.15}$$

where $A^a_{i_\mu}$ and B_{i_μ} are the gauge bosons of the $SU(2)_i$ and $U(1)_i$ gauge groups respectively and Q^a_i and Y_i their generators as given in (2.7), (2.8) and (2.9). Since the vacuum Σ_0 in eq. (2.26) breaks the symmetry $(SU(2)\times U(1))^2$ into the diagonal $SU(2)_F\times U(1)_F$, four combinations of the initial gauge bosons become massive; their masses are given by

$$M^2_{A'} = \frac{1}{2}\,(g_1^2+g_2^2)f^2 \quad \text{and} \quad M^2_{B'} = \frac{2}{5}\,(g'^2_1+g'^2_2)f^2\,. \tag{2.16}$$

The effective potential must break the $SU(2)_F\times U(1)_F$ remaining gauge symmetry and give mass to all surviving PGBs, little flavons included. At one-loop, the gauge interactions give rise to the CW potential given by the two terms

$$\frac{\Lambda_F^2}{16\pi^2}\text{Tr}\,[M^2(\Sigma)] + \frac{3}{64\pi^2}\text{Tr}\left[M^4(\Sigma)\left(\log\frac{M^2(\Sigma)}{\Lambda_F^2} + \text{const.}\right)\right]\,. \tag{2.17}$$

In agreement with the general framework of little Higgs models the quadratically divergent term gives mass only to the singlet field s. No mass is generated for the (doublet) little flavons. In addition, a trilinear coupling between the doublets ϕ_1 and ϕ_2 and s and a quartic term for the two doublets are generated:

$$\frac{\Lambda_F^2}{16\pi^2}\mathrm{Tr}\left[M^2(\Sigma)\right] = f^2\left(3\,g_1^2\left|s+\frac{i}{2f}\tilde{\phi_2}^\dagger\phi_1\right|^2 + 3\,g_2^2\left|s-\frac{i}{2f}\tilde{\phi_2}^\dagger\phi_1\right|^2\right), \tag{2.18}$$

where $\tilde{\phi}_{1,2} = i\sigma_2\phi_{1,2}^*$. From eq. (2.18) one obtains

$$m_s^2 = \frac{3}{2}(g_1^2+g_2^2)f^2\,, \tag{2.19}$$

and the quartic coupling

$$\lambda_4\,|\tilde{\phi_2}^\dagger\phi_1|^2\,. \tag{2.20}$$

After integrating out the heavy singlet s, one obtains for λ_4 the cut-off independent expression:

$$\lambda_4 = \frac{g_1^2 g_2^2}{g_1^2+g_2^2} \simeq O(g^2)\,, \tag{2.21}$$

which is the only term generated by the quadratic term in (2.17). This happens because of the mechanism of collective breaking for which the potential of the PGB doublets (little flavons) is generated by the interplay of both gauge interactions, thus breaking explicitly the global $SU(6)$ symmetry, while at the same time protecting the doublets from receiving a (quadratically divergent) mass at the one-loop level.

Mass terms as well as other effective quartic couplings for the little flavons arise from the logarithmically divergent term in eq. (2.17). One can verify that the one-loop potential induced by gauge interactions includes the following terms

$$\mu_1^2\phi_1^\dagger\phi_1 + \mu_2^2\phi_2^\dagger\phi_2 + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)\,. \tag{2.22}$$

The size of the mass terms and effective couplings are given by

$$\mu_i^2/f^2 \simeq \lambda_i \simeq c(\mu_i,\lambda_i)\frac{3g^4}{64\pi^2}\log\frac{M_V^2}{\Lambda_F^2} \lesssim 10^{-2}\,, \tag{2.23}$$

where c are numerical coefficients, related to the expansion of the Σ, and M_V is the mass of the massive gauge bosons (see, eq. (2.16)). The numerical estimate in eq. (2.23) takes into account that we take the (horizontal) gauge symmetry couplings to be of $O(1)$. This follows from requiring the flavor gauge bosons masses be between $10^{-1} f$ and f while the flavon masses be around $10^{-1} f$.

Since λ_4, induced by the leading divergent term in the CW potential, turns out to be a sizeable coupling, other relevant contributions to the effective potential may arise from integration of the doublet self-interaction in eq. (2.20) which contributes to the $\lambda_{1,2}$ terms with

$$\lambda_{1,2} \simeq \frac{\lambda_4^2}{64\pi^2} \log \frac{\Lambda_F^2}{M_\phi^2} \lesssim 10^{-2}, \tag{2.24}$$

which are in fact of the same order of those induced by the logarithmically divergent term in the gauge induced one-loop effective potential.

The one-loop flavon potential generated by gauge interactions in eqs. (2.20)–(2.22) has to be compared with the general potential for two $SU(2)$ doublets of opposite hypercharges that is given, up to four powers of the fields, by

$$\begin{aligned} V_4(\phi_1, \phi_2) &= \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 + (\mu_3^2 \tilde{\phi}_1^\dagger \phi_2 + H.c.) \\ &+ \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) \\ &+ \lambda_4 |\tilde{\phi}_1^\dagger \phi_2|^2 + \lambda_5 [(\tilde{\phi}_1{}^\dagger \phi_2)^2 + H.c.] \end{aligned} \tag{2.25}$$

Depending on the sign of the determinant of the mass matrix of the scalar fields and on relationships among the various couplings, the potential in eq. (2.25) can have different symmetry breaking minima [48]. In particular we are interested to the vacuum which completely breaks the $SU(2)_F \times U(1)_F$ gauge flavor symmetry. The residual exact $U(1)_P$ global symmetry, acting with opposite charge on ϕ_i and $\tilde{\phi}_i$ fields, $i = 1, 2$, forbids the generation of the μ_3 and λ_5 couplings. In the absence of μ_3^2 and λ_5 terms, the vacuum can be parametrized as

$$\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \tag{2.26}$$

with real VEVs. The complete breaking of the flavor symmetry allows us to avoid the presence in the physical spectrum of massless flavor gauge bosons and is necessary in order to generate the lepton mass matrices. This vacuum breaks also the $U(1)_P$ symmetry, however a linear combination of P and flavor isospin is still preserved. The corresponding global symmetry $U(1)_{P'}$, with

$$P' = \text{diag}\,[1,\,0,\,0,\,-1,\,0,\,0]\,, \tag{2.27}$$

is explicitly broken by the Yukawa sector.

The requirement that the potential is bounded from below gives the three conditions

$$\lambda_1 + \lambda_2 > 0, \quad 4\lambda_1\lambda_2 - \lambda_3^2 > 0, \quad \text{and} \quad \lambda_4 - |\lambda_5| > 0\,. \tag{2.28}$$

Assuming $\mu_{1,2}^2 < 0$ (and making use of $\mu_3^2 = \lambda_5 = 0$) the symmetry breaking vacuum in eq. (2.26) leads to the following flavon mass spectrum

$$\begin{aligned} m_{1,2}^2 &= m_{5,6}^2 = 0 \\ m_{3,4}^2 &= \frac{1}{2}\lambda_4\left(v_1^2 + v_2^2\right) \\ m_{7,8}^2 &= \lambda_1 v_1^2 + \lambda_2 v_2^2 \pm \sqrt{(\lambda_1 v_1^2 + \lambda_2 v_2^2)^2 - (4\lambda_1\lambda_2 - \lambda_3^2) v_1^2 v_2^2}. \end{aligned} \tag{2.29}$$

Positivity of the mass eigenvalues then requires $\lambda_4 > 0$ and $\lambda_1 v_1^2 + \lambda_2 v_2^2 > 0$.

The four massless degrees of freedom are eaten by the four gauge fields of the completely broken $SU(2)_F \times U(1)_F$ flavor symmetry which become massive at a scale determined by the VEVs size

$$v_1^2 = -\frac{2\lambda_2\mu_1^2 - \lambda_3\mu_2^2}{4\,\lambda_1\lambda_2 - \lambda_3^2} \quad v_2^2 = -\frac{2\lambda_1\mu_2^2 - \lambda_3\mu_1^2}{4\,\lambda_1\lambda_2 - \lambda_3^2}\,. \tag{2.30}$$

The condition μ_1^2, $\mu_2^2 < 0$ can be realized if there exist fermions coupled to the doublets that induce contributions to the scalar masses of opposite sign with respect to that induced by gauge interactions. This role is played in the model by heavy right-handed Majorana neutrinos, with mass $M \simeq f$. Radiative contributions to the flavon potential arising from global $SU(6)$ breaking couplings to Majorana right-handed neutrinos (as given in the next section) lead to scalar mass terms

$$\mu_{1,2}^2 \simeq -c_n^{(1,2)}\,\eta_n\,\frac{\Lambda_F^2}{16\pi^2} \simeq -c_n^{(1,2)}\,\eta_n\,f^2\,,. \tag{2.31}$$

where $c_n^{(1,2)}$ are coefficients of order unity. These quadratically-divergent corrections maintain the flavon mass scale below the f scale (and in the TeV regime) as long as $\eta_i \lesssim 10^{-2}$.

Thus, still avoiding a large fine-tuning of the couplings, no collective breaking mechanism is required for the lepton-induced renormalization (the only large couplings in the model are gauge and the Yukawa of the top quark).

Notice that, the contributions to the quartic couplings induced by the massive right-handed neutrinos are therefore given by

$$\lambda_{1,2,3} \simeq \frac{c_{nm}^{(1,2,3)} \eta_n \eta_m}{16\pi^2} \log \frac{\Lambda_F^2}{M^2} \lesssim 10^{-6} \tag{2.32}$$

and are subleading, with respect to those induced by gauge interactions.

From eq. (2.30) and eqs. (2.23)–(2.31) we obtain $v_1,\ v_2 = O(f)$, which in turn implies that besides the four flavor gauge bosons even two of the flavon states have masses of order f, while the remaining two scalars have masses of $O(10^{-1} f)$.

Assuming all of the above conditions satisfied (we will not be concerned with the detail of the UV completion of the theory) we now discuss the neutrino and charged lepton mass textures that arise by assigning non-trivial flavor transformation properties to the lepton families.

2.2 Textures generation

The spontaneous breaking of the global $SU(6) \to Sp(6)$ (approximate) symmetries leads to the breaking of the gauge $[SU(2) \times U(1)]^2$ to $SU(2)_F \times U(1)_F$. In this model, fermions of different families transform according to this $SU(2)_F \times U(1)_F$ gauge flavor symmetry. In the following, all Greek indices denote the flavor group while Latin indices refer to the electroweak group.

Textures in the mass matrices of fermions are generated by coupling the flavon fields to the fermions. The model does not explain the overall scales of the fermion masses, that have to be put in by hand; it explains the hierarchy among families that exists after that scale has been fixed.

The effective lagrangians are rather cumbersome because many different couplings are allowed by the flavor symmetry. The little flavon fields enter as components of the pseudo-Goldstone field Σ introduced in eq. (2.3) of the effective non-linear sigma model.

2.2.1 Generalities

After electroweak symmetry breaking, the effective lagrangian contains the following mass terms for fermions:

$$\mathcal{L}^{(m)} = -\bar{\psi}_R^{(i)} M^{(i)} \psi_L^{(i)} - \frac{1}{2}\chi^T C\, M^{(n)} \chi + H.c.\,, \tag{2.33}$$

where $\psi_{L,R}^{(i)}$ are chiral fields, $M^{(i)}$ are 3×3 matrices, $i = u, d, l$, $M^{(n)}$ is a 6×6 symmetric matrix, $\chi = (\nu_L, C\nu_R^*)^T$ and flavor indeces are understood. C is the charge conjugation matrix.

The neutrino mass matrix can be written in 3×3 block form as:

$$M^{(n)} = \begin{pmatrix} m_L & m_D^T \\ m_D & m_R \end{pmatrix}. \tag{2.34}$$

In the present case, $m_L = 0$ and the scale of m_R (whose generation does not involve neither electroweak nor flavor symmetry breaking at leading order) is of order f and therefore much larger than that of m_D. In the spirit of effective field theory, one can approximately block-diagonalize $M^{(n)}$, decouple three heavy states which are predominantly standard model singlets, and write the Majorana mass term for the light states as

$$\mathcal{L}^{(m)} = -\frac{1}{2}\nu_L^T C\, M^{(\nu)}\, \nu_L + H.c.\,, \tag{2.35}$$

where now the (symmetric) Majorana mass matrix for the light fields (with some abuse of notation, we identify the light fields with the left-handed components) is $M^{(\nu)} = -m_D^T\, m_R^{-1}\, m_D$.

All matrices are non-diagonal in flavor space. One can diagonalize them with appropriate bi-unitary trasformations,

$$\operatorname{diag} M^{(i)} = R^{(i)\dagger} M^{(i)} L^{(i)}\,, \tag{2.36}$$

$$\operatorname{diag} M^{(\nu)} = L^{(\nu)T} M^{(\nu)} L^{(\nu)}\,, \tag{2.37}$$

where $L^{(i)}$, $R^{(i)}$ ($i = u, d, e$) and $L^{(\nu)}$ are 3×3 matrices in flavor space. With these definitions one finds that the mixing matrices appearing in the charge-current interactions according to the standard notation are given by

$$V_{CKM} = L^{(u)\dagger} L^{(d)}\,, \tag{2.38}$$

$$V_{PMNS} = L^{(l)\dagger} L^{(\nu)}\,, \tag{2.39}$$

for quarks and leptons, respectively. We use the standard definitions of the mixing matrices, in which one writes the down-type quark (neutrino) flavor eigenstates d' (ν') in terms of the mass eigenstates d (ν)—in the basis in which up-type quarks (charged leptons) are diagonal— as

$$d' = V_{CKM}\, d, \tag{2.40}$$

$$\nu' = V_{PMNS}\, \nu. \tag{2.41}$$

The standard parameterization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in terms of three mixing angles θ_{12}, θ_{13} and θ_{23} and one phase δ reads:

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{2.42}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. An analogous expression is valid for the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [50], neglecting the flavor-diagonal Majorana phases.

2.2.2 Quarks

Quarks are characterized by small mixing angles. In this respect it is natural to consider them as singlets under non-abelian flavor symmetries. We take all standard model quarks—left-handed electroweak doublet components as well as right-handed electroweak singlets—to be singlets under $SU(2)_F$ while being charged under $U(1)_F$. Textures generated by abelian symmetries have been widely discussed in the literature (see for instance [49]). Here we embed this ansatz in the little flavon framework paying attention to the issue of the stability of the flavon potential, while avoiding the large hierarchies among the Yukawa couplings which are present in the standard model. A possible charge assignment is summarized in tab. (2.1).

Given the charges in tab. (2.1), we find for the up quarks the following effective Yukawa lagrangian

$$\begin{aligned} -\mathcal{L}_u &= \lambda_{31}\overline{t_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^3\tilde{H}^\dagger Q_{1L} + \lambda_{32}\overline{t_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^2\tilde{H}^\dagger Q_{2L} \\ &+ \overline{t_R}(\lambda_{33} + \lambda'_{33}\Sigma_{\alpha-1\,6}\Sigma_{3\,2+\alpha} + \lambda''_{33}\Sigma_{\alpha-1\,3}\Sigma_{6\,2+\alpha})\tilde{H}^\dagger Q_{3L} \end{aligned}$$

Table 2.1: Summary of the charges of quarks and flavon fields ($\alpha = 2,3$) under the horizontal flavor groups $SU(2)_F$ and $U(1)_F$. Q_{iL} stands for the electrweak left-handed doublets. q is an arbitrary charge that is not determined.

	$U(1)_F$	$SU(2)_F$
Q_{1L}	$q+3$	1
Q_{2L}	$q+2$	1
Q_{3L}	q	1
u_R	$q-3$	1
c_R	$q-1$	1
t_R	q	1
d_R	$q-4$	1
s_R	$q-2$	1
b_R	$q-2$	1
$\Sigma_{\alpha-1\,6} = (-i/f\ \phi_1 + ...)_{\alpha-1}$	$1/2$	2
$\Sigma_{\alpha-1\,3} = (+i/f\ \phi_2 + ...)_{\alpha-1}$	$-1/2$	2
$\Sigma_{3\,2+\alpha} = (-i/f\ \phi_1^* + ...)_{\alpha-1}$	$-1/2$	2^*
$\Sigma_{6\,2+\alpha} = (-i/f\ \phi_2^* + ...)_{\alpha-1}$	$1/2$	2^*

$$
\begin{aligned}
&+ \lambda_{21}\overline{c_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^4\tilde{H}^\dagger Q_{1L} + \lambda_{22}\overline{c_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^3\tilde{H}^\dagger Q_{2L} \\
&+ \lambda_{23}\overline{c_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})\tilde{H}^\dagger Q_{3L} \\
&+ \lambda_{11}\overline{u_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^6\tilde{H}^\dagger Q_{1L} + \lambda_{12}\overline{u_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^5\tilde{H}^\dagger Q_{2L} \\
&+ \lambda_{13}\overline{u_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^3\tilde{H}^\dagger Q_{3L} + H.c. \qquad (2.43)
\end{aligned}
$$

as well as

$$
\begin{aligned}
-\mathcal{L}_d \;=\; &\tilde{\lambda}_{31}\overline{b_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^5 H^\dagger Q_{1L} + \tilde{\lambda}_{32}\overline{b_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^4 H^\dagger Q_{2L} \\
&+ \tilde{\lambda}_{33}\overline{b_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^2 H^\dagger Q_{3L} \\
&+ \tilde{\lambda}_{21}\overline{s_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^5 H^\dagger Q_{1L} + \tilde{\lambda}_{22}\overline{s_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^4 H^\dagger Q_{2L} \\
&+ \tilde{\lambda}_{23}\overline{s_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^2 H^\dagger Q_{3L} \\
&+ \tilde{\lambda}_{11}\overline{d_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^7 H^\dagger Q_{1L} + \tilde{\lambda}_{12}\overline{d_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^6 H^\dagger Q_{2L} \\
&+ \tilde{\lambda}_{13}\overline{d_R}(\Sigma_{\alpha-1\,3}\Sigma_{3\,2+\alpha})^4 H^\dagger Q_{3L} + H.c. \qquad (2.44)
\end{aligned}
$$

for the down quarks.

Notice that even though the t quark has a large Yukawa coupling that could introduce a potentially destabilizing term in the flavon effective potential, the contribution to the flavon mass terms of t-quark loops induced by the couplings in eq. (2.43)

$$\mu_{1,2}^2 \simeq -\mathrm{Re}\,(\lambda_{33}^*(\lambda'_{33},\lambda''_{33}))\,\frac{\langle h_0\rangle^2}{f^2}\frac{\Lambda_F^2}{16\pi^2} \tag{2.45}$$

remains negligible compared to eq. (2.31).

2.2.3 Leptons

At variance with the quark sector, lepton mixings may differ by the presence of large angles, as a consequence of the neutrino oscillation data. The oscillation pattern together with the hierarchy structure in the charged lepton mass spectrum, suggests that the leptons of the second and third family may belong to flavor doublets. In the following we take the standard model electron doublet l_{eL} to be an $SU(2)_F$ singlet charged under $U(1)_F$, while the standard model doublets $l_{\mu,\tau L}$ to be members of a doublet in flavor space. Right-handed charged leptons are taken to follow a similar structure.

In order to have a see-saw-like mechanism [5], we introduce three right-handed neutrinos ν_R^i, $i = 1, 2, 3$, which are $SU(2)_F$ singlets. This choice allows us to take in the effective lagrangian right-handed neutrino mass entries at the scale $M \sim f$. Tab. (2.2) summarizes this possible charge assigment.

Table 2.2: Summary of the charges of all leptons under the horizontal flavor groups $SU(2)_F$ and $U(1)_F$.

	$U(1)_F$	$SU(2)_F$
l_{eL}	-2	1
e_R	2	1
$L_L = (l_\mu\,,\,l_\tau)_L$	1/2	2
$E_R = (\mu\,,\,\tau)_R$	1/2	2
ν_{1R}	1	1
ν_{2R}	-1	1
ν_{3R}	0	1

The neutrino lagrangian is obtained after integrating out the three right-handed neutrinos and, at the leading order in the right-handed neutrino mass and in number of Σ fields, is given by (see [51]):

$$-2\,\mathcal{L}_\nu = \frac{(\overline{l_{1L}^c}\,\tilde{H}^*)(\tilde{H}^\dagger\,l_{1L})}{M}\left[2\lambda_{1\nu}\lambda_{2\nu} + r\,\lambda_{3\nu}^2\right]\left[\Sigma_{\alpha-1\,6}\Sigma_{6\,2+\alpha}\right]^{-2Y_{1L}}$$

$$\begin{aligned}
&+ \frac{(\overline{l^c_{1L}}\ \tilde{H}^*)(\tilde{H}^\dagger\ l_{\alpha L}) + (\overline{l^c_{\alpha L}}\ \tilde{H}^*)(\tilde{H}^\dagger\ l_{1L})}{M} \lambda_{2\nu}(\lambda'_{1\nu}\ \epsilon_{\alpha\beta}\Sigma_{\beta-1\,6} + \lambda''_{1\nu}\ \Sigma_{6\,2+\alpha}) \\
&\quad [\Sigma_{\delta-1\,6}\Sigma_{6\,2+\delta}]^{-Y_{1L}+Y_{\nu_{2R}}} \qquad (2.46) \\
&+ \frac{(\overline{l^c_{\alpha L}}\ \tilde{H}^*)(\tilde{H}^\dagger\ l_{\beta L})}{2M_3}(i\sigma_2\sigma_\tau)_{\alpha\beta}\ (i\sigma_2\sigma_\tau)_{\delta\gamma}\left[(\lambda'_{3\nu})^2\ \Sigma_{\delta-1\,3}\Sigma_{\gamma-1\,3}\right. \\
&+ \left.\lambda'_{3\nu}\lambda''_{3\nu}\ (\epsilon_{\gamma\gamma'}\Sigma_{\delta-1\,3}\Sigma_{3\,2+\gamma'} + \delta \leftrightarrow \gamma) + (\lambda''_{3\nu})^2\ \epsilon_{\delta\delta'}\epsilon_{\gamma\gamma'}\Sigma_{3\,2+\delta'}\Sigma_{3\,2+\gamma'}\right] + H.c.\,,
\end{aligned}$$

where $r = M/M_3$, M and M_3 are the masses of the right-handed neutrinos, $\sigma_\tau/2$ are the generators of the $SU(2)_F$ gauge group ($\tau = 1, 2, 3$).

The lagrangian for the charged leptons is given by

$$\begin{aligned}
\mathcal{L}_e &= \overline{e_R}\ \left[\lambda_{1e}\ (\Sigma_{\alpha-1\,6}\Sigma_{6\,2+\alpha})^{(-Y_{1L}+Y_{1R})}\ (H^\dagger\ l_{1L})\right. \\
&\quad \left. + i\ (\lambda_{3e}\ \Sigma_{6\,2+\alpha} + \lambda_{2e}\ \epsilon_{\alpha\beta}\Sigma_{\beta-1\,6})(\Sigma_{\delta-1\,6}\Sigma_{6\,2+\delta})^{(Y_{1R}-1)}\ (H^\dagger\ l_{\alpha L})\right] \\
&+ \overline{E_{\alpha R}}\left[i\ (\lambda'_{1E}\ \Sigma_{6\,2+\alpha} + \lambda_{1E}\ \epsilon_{\alpha\beta}\Sigma_{\beta-1\,6})\ (\Sigma_{\delta-1\,6}\Sigma_{6\,2+\delta})^{-Y_{1L}}\right](H^\dagger\ l_{1L}) \\
&+ \overline{E_{\alpha R}}\left[\delta_{\alpha\beta}(-\lambda_{2E}\ + \lambda'_{2E}\ \Sigma_{\gamma-1\,6}\Sigma_{3\,2+\gamma} + \lambda''_{2E}\ \Sigma_{\gamma-1\,3}\Sigma_{6\,2+\gamma})\right. \qquad (2.47) \\
&\quad + \Big(\lambda_{3E}\ \Sigma_{\alpha-1\,6}\Sigma_{3\,2+\beta} + \lambda'_{3E}\ \epsilon_{\alpha\delta}\epsilon_{\beta\gamma}\ \Sigma_{6\,2+\delta}\Sigma_{\gamma-1\,3} + (3 \leftrightarrow 6)\Big) \\
&\quad \left. + \Big(\lambda_{4E}\ \Sigma_{\alpha-1\,6}\Sigma_{\gamma-1\,3}\epsilon_{\beta\gamma} + \lambda'_{4E}\ \epsilon_{\alpha\delta}\Sigma_{6,2+\delta}\Sigma_{3,2+\alpha} + (3 \leftrightarrow 6)\Big)\right](H^\dagger\ l_{\beta L})\ + H.c..
\end{aligned}$$

2.2.4 Leading order textures and masses

On the vacuum that completely breaks the the $SU(2)_F \times U(1)_F$ gauge symmtery, the little flavons acquire expectation values $v_{1,2} = \varepsilon_{1,2} f$, with $\varepsilon_{1,2} < 1$. By inspection of the Yukawa lagrangians introduced in the previous section, we can determine the fermion mass matrices.

Since all quarks are $SU(2)_F$ singlets the all entries of their mass matrices are proportional to powers of $k \equiv \varepsilon_1\varepsilon_2$. Due to the large number of possible higher-order terms, we only take for each entries the first non-vanishing term, and obtain

$$M^{(u)} = \langle h_0 \rangle \begin{pmatrix} \lambda_{11}k^6 & \lambda_{12}k^5 & \lambda_{13}k^3 \\ \lambda_{21}k^4 & \lambda_{22}k^3 & \lambda_{23}k \\ \lambda_{31}k^3 & \lambda_{32}k^2 & \lambda_{33} \end{pmatrix} \qquad (2.48)$$

and

$$M^{(d)} = \langle h_0 \rangle \, k^2 \begin{pmatrix} \tilde{\lambda}_{11} k^5 & \tilde{\lambda}_{12} k^4 & \tilde{\lambda}_{13} k^2 \\ \tilde{\lambda}_{21} k^3 & \tilde{\lambda}_{22} k^2 & \tilde{\lambda}_{23} \\ \tilde{\lambda}_{31} k^3 & \tilde{\lambda}_{32} k^2 & \tilde{\lambda}_{33} \end{pmatrix} . \tag{2.49}$$

The essential feature of the previous mass matrices is that the fundamental textures are determined by the vacuum structure alone—that is that obtained by taking all Yukawa couplings λ_{ij} and $\tilde{\lambda}_{ij}$ of $O(1)$. In fact, by computing the corresponding CKM matrix one finds in first approximation

$$V_{\text{CKM}} = \begin{pmatrix} 1 & O(k) & O(k^3) \\ O(k) & 1 & O(k^2) \\ O(k^3) & O(k^2) & 1 \end{pmatrix} , \tag{2.50}$$

that is roughly of the correct form and, moreover, suggests a value of $k \simeq \sin\theta_C \simeq 0.2$.

At the same time it is possible to extract from (2.48) and (2.49) approximated mass ratios:

$$\frac{m_u}{m_c} \simeq \frac{m_c}{m_t} \simeq \frac{m_d}{m_s} \simeq O(k^3) \quad \frac{m_s}{m_b} \simeq O(k^2) \tag{2.51}$$

which again roughly agree with the experimental values.

These results show that the quark masses and mixing angles can be reproduced by our textures. While a rough agreement is already obtained by taking alla Yukawa coupling to be equal, the precise agreement with the experimental data depends on the actual choice of the Yukawa couplings λ_{ij} and $\tilde{\lambda}_{ij}$. However, their values can be taken all of the same order, as we shall see in last section.

Notice that the textures used in this work do not satisfactorily address the flavor problem in a supersymmetric framework: the abelian nature of the flavor symmetry in the quark sector, and the large mixing angles in the right handed mixing matrices $R^{(i)}$ would in general induce large contributions to FCNC processes via diagram with gluino exchange. The diagonal entries of the squark mass matrices are not forbidden by the abelian symmetry, and in general one expects all of them to be determined only by the scale of supersymmetry breaking, up to $\mathcal{O}(1)$ coefficients. Once fermions are diagonalized, large off-diagonal entries are generated in the 3×3 right-handed down-type squark mass matrix, because of the large mixing angles in $R^{(d)}$ (this can be easily seen from the fact that second and third row of eq. (3.88) have entries of the same order). Phenomenologically, for generic choices of the diagonal elements of the squark mass matrices, this leads to contributions to $\Delta F = 2$ processes (K^0-$\bar{K}^0$ or B^0-$\bar{B}^0$ mixings and related CP violating observables) largely in excess of the experimental data [52]. This can be avoided allowing for a degeneracy of the diagonal entries themselves, albeit with a tuning at least at the percent level.

In the lepton sector, the VEVs of the little flavons gives us the left-handed neutrino and charged-lepton mass matrices (again, we only retain the first non-vanishing term for each entry):

$$M^{(\nu)} = \frac{\langle h_0 \rangle^2}{M} \begin{pmatrix} [r\ \lambda_{3\nu}^2 + 2\lambda_{1\nu}\lambda_{2\nu}]\,\varepsilon_1^4\varepsilon_2^4 & -\lambda_{2\nu}\lambda'_{1\nu}\varepsilon_1^2\varepsilon_2 & -\lambda_{2\nu}\lambda''_{1\nu}\varepsilon_1\varepsilon_2^2 \\ -\lambda_{2\nu}\lambda'_{1\nu}\varepsilon_1^2\varepsilon_2 & r\ \lambda'^2_{3\nu}\varepsilon_2^2 & r\ \lambda'_{3\nu}\lambda''_{3\nu}\varepsilon_1\varepsilon_2 \\ -\lambda_{2\nu}\lambda''_{1\nu}\varepsilon_1\varepsilon_2^2 & r\ \lambda'_{3\nu}\lambda''_{3\nu}\varepsilon_1\varepsilon_2 & r\ \lambda''^2_{3\nu}\varepsilon_1^2 \end{pmatrix}. \qquad (2.52)$$

The eigenvalues of this matrix are the masses of the three neutrinos. The scale M is just below or around f and therefore we are not implementing the usual see-saw mechanism that requires scales as large as 10^{13} TeV. At this level we can not say anything about the effective neutrinos Yukawa couplings, since we do not have yet fixed f. In order to reproduce the right neutrinos mass spectrum we should have $\lambda_\nu^2/f \simeq 10^{-10}/\text{TeV}$. This implies that the higher the scale f is the more realistic neutrino masses are obtained by less tuning the effective Yukawa couplings.

In the same approximation, the Dirac mass matrix for the charged leptons is given by

$$M^{(l)} = \langle h_0 \rangle \begin{pmatrix} \lambda_{1e}\,\varepsilon_1^4\varepsilon_2^4 & \lambda_{2e}\,\varepsilon_1^2\varepsilon_2 & \lambda_{3e}\,\varepsilon_1\varepsilon_2^2 \\ \lambda_{1E}\,\varepsilon_1^2\varepsilon_2^3 & \lambda_{2E} & (\lambda'_{14E} + \lambda'_{24E})\,\varepsilon_1\varepsilon_2 \\ \lambda'_{1E}\,\varepsilon_1^3\varepsilon_2^2 & -(\lambda_{14E} + \lambda_{24E})\,\varepsilon_1\varepsilon_2 & \lambda_{2E} \end{pmatrix}. \qquad (2.53)$$

In order to exhibit the main features of the underlaying textures, we study the limit

$$\varepsilon_1 \to 1 \quad \text{and} \quad \varepsilon_2 \to k\,, \qquad (2.54)$$

which is suggested by the additional constraint $\varepsilon_1\varepsilon_2 \simeq \sin\theta_C$, obtained from the study of the quark textures.

Notice that in ref. [51] a slightly different charged-lepton texture has been considered that accounts for maximal mixing in the limit $\varepsilon_1^2 \ll \varepsilon_2^2 \ll 1$ (or, equivalently $\varepsilon_2^2 \ll \varepsilon_1^2 \ll 1$).

In the limit (2.54), the matrices in eqs. (2.52)-(2.53) reduce—at the order $O(k^2)$, and up to overall factors—to

$$M^{(\nu)} = \begin{pmatrix} 0 & O(k) & O(k^2) \\ O(k) & O(k^2) & O(k) \\ O(k^2) & O(k) & 1 \end{pmatrix} \quad \text{and} \quad M^{(l)} = \begin{pmatrix} 0 & O(k) & O(k^2) \\ 0 & 1 & O(k) \\ O(k^2) & O(k) & 1 \end{pmatrix}, \tag{2.55}$$

where, as before, the 1 stands for $O(1)$ coefficients.

The eigenvalues of $M^{(l)}$ can be computed by diagonalizing $M^{(l)\dagger} M^{(l)}$. This product is—again for each entry to leading order in k:

$$M^{(l)\dagger} M^{(l)} = \begin{pmatrix} 0 & 0 & O(k^2) \\ 0 & 1 & O(k) \\ O(k^2) & O(k) & 1 \end{pmatrix}. \tag{2.56}$$

By inspection of the 2×2 sub-blocks, the matrix eq. (3.94) is diagonalized by three rotations with angles, respectively, $\theta_{23}^l \simeq \pi/4$ and $\theta_{12}^l \simeq \theta_{13}^l \ll 1$, leading to one maximal mixing angle and two minimal. On the other hand, the neutrino mass matrix in eq. (3.93) is diagonalized by three rotations with angles, rispectively, $\tan 2\theta_{12}^\nu \simeq 2/k$ and $\theta_{23}^\nu \simeq \theta_{13}^\nu \ll 1$ (the label 3 denotes the heaviest eigenstate). Therefore, the textures in the mass matrices in eqs. (2.52)–(2.53) give rise to a PMNS mixing matrix—that is the combination of the the two rotations above—in which θ_{23} is maximal, θ_{12} is large (up to maximal), while θ_{13} remains small.

The natural prediction when taking all coefficients $O(1)$ is then: a large atmospheric mixing angle θ_{23}, possibly maximal, another large solar mixing angle θ_{12}, and a small θ_{13} mixing angle; at the same time, the mass spectrum includes one light ($O(k^4)$) and two heavy states ($O(1)$) in the charged lepton sector (m_e, m_μ and m_τ respectively), two light states ($O(k^2)$) and one heavy ($O(1)$) in the neutrino sector, thus predicting a neutrino spectrum with normal hierarchy.

By flavor symmetry, one expects masses of the same order of magnitude for μ and τ. The ratio of the masses of τ and μ is given to $O(k)$ by:

$$R \equiv \frac{m_\mu}{m_\tau} \simeq \frac{\sqrt{\det\,(m^{(l)\,\dagger}\,m^{(l)})}}{\mathrm{Tr}\,(m^{(l)\,\dagger}\,m^{(l)})}\,, \tag{2.57}$$

where $m^{(l)}$ is the μ-τ sub-matrix of $M^{(l)}$. The experimental splitting can be explained only admitting a moderate amount of fine-tuning, of a factor 10, between the coefficients of the charge lepton mass matrix such as to make $R \simeq O(10^{-1})$. One can quantify the stability of this fine-tuning with the logarithmic derivatives $d^R_{Y_{ij}}$ of this ratio with respect to the corresponding Yukawa coefficients Y_{ij} [53]:

$$d^R_{Y_{ij}} \equiv \left|\frac{Y_{ij}}{R}\,\frac{\partial R}{\partial Y_{ij}}\right|\,. \tag{2.58}$$

Using the experimental value $R = m_\mu/m_\tau$, and the numerical solution given in sec. (2.3), we find $(i,j = 2,3)$ $d^R_{Y_{ij}} < 5$, where the largest value arises because of the leading order correlation between the diagonal Yukawas entries ($Y_{22} = Y_{33}$) in the charge lepton sector that doubles the sensitivity. In the absence of any fine-tuning one would expect values of $d^R_{Y_{ij}}$ at most around unity. Nevertheless, the tree level value of R is not destabilized by Yukawa radiative corrections, since they are very suppressed in the model.

The UV completion of the theory, in which all effective couplings should be computed from a restrict number of fundamental parameters, might explain possible correlations among the Yukawa couplings, together with the suppression of the the overall neutrino scale.

Finally, notice that even though the quark charges leave an undetermined factor q (see tab. (2.1)), gauge anomalies are present in the theory, as it can be easily seen by inspection considering the charges of the matter fields. They can be cancelled by adding appropriate Wess-Zumino terms [54].

2.3 Fitting the data

Let us first briefly review the experimental data and comment on the possible range of values we consider acceptable in reproducing these data within the model.

The CKM matrix is rather well known as are the masses of the quarks (see, e.g., the PDG [8]). We will estimate only ratios of masses which are renormalization group invariant, so that we only have to be careful in computing them at a common scale. Taking into account the uncertainties in the values of the quark masses, the mass ratio we would like the model to reproduce are given by

$$\frac{m_t}{m_c} = 248 \pm 70 \quad \frac{m_b}{m_s} = 40 \pm 10 \quad \frac{m_s}{m_d} = 430 \pm 300 \quad \frac{m_c}{m_u} = 325 \pm 200\,. \tag{2.59}$$

The CKM phase is determined [55] to be

$$\delta = 61.5^o \pm 7^o \quad (\sin 2\beta = 0.705^{+0.042}_{-0.032})\,. \tag{2.60}$$

Compelling evidences in favor of neutrino oscillations and, accordingly of non-vanishing neutrino masses has been collected in recent years from neutrino experiments [34]. Combined analysis of the experimental data show that the neutrino mass matrix is characterized by a hierarchy with two square mass differences (at 99.73% CL):

$$\begin{aligned} \Delta m^2_{\odot} &= (5.3 - 17) \times 10^{-5}\text{eV}^2 \\ |\Delta m^2_{\oplus}| &= (1.4 - 3.7) \times 10^{-3}\text{eV}^2\,, \end{aligned} \tag{2.61}$$

the former controlling solar neutrino oscillations [56] and the latter the atmospheric neutrino experiments [57]. In the context of three active neutrino oscillations, the mixing is described by the PMNS mixing matrix V_{PMNS} in eq. (2.39). Such a matrix is parameterized by three mixing angles, two of which (θ_{12} and θ_{23}) can be identified with the mixing angles determining solar [56] and atmospheric [57] oscillations, respectively (again, at 99.73% CL):

$$\begin{aligned} \tan^2\theta_{\odot} &= 0.23 - 0.69\,, \\ \sin^2 2\,\theta_{\oplus} &= 0.8 - 1.0\,. \end{aligned} \tag{2.62}$$

For the third angle, controlling the mixing ν_τ-ν_e, there are at present only upper limits, deduced by reactor neutrino experiments [58] (at 99.73% CL):

$$\sin^2\theta_{13} < 0.09\,. \tag{2.63}$$

Other observable quantities determined by the neutrino mass matrix have not been measured yet. These include: 1) the type of neutrino spectrum, with normal or inverted hierarchy (see for instance [59] for a definition), 2) the common mass scale, i.e. the actual value of the lowest mass eigenvalue m_1, 3) the (Dirac) phase δ^l responsible for CP violation in leptonic flavor changing processes, 4) the two Majorana flavor-diagonal CP-violating phases, 5) the sign of $\cos 2\theta_\oplus$. Several proposal appeared in the literature to measure all these quantities in the next generation neutrino experiments, together with the mixing angle θ [60]. Our model predicts a neutrino spectrum with normal hierarchy, with a very small mass for the lighter neutrinos $m_{1,2} \ll \sqrt{\Delta m^2_\oplus}$.

Finally, the values of the charged-lepton masses are given by $m_\tau \simeq 1777$ MeV, $m_\mu \simeq 106$ MeV and $m_e \simeq 0.51$ MeV, respectively. We therefore have

$$\frac{m_\tau}{m_\mu} \simeq 17\,, \quad \frac{m_\mu}{m_e} \simeq 207\,, \quad \frac{m_\tau}{m_e} \simeq 3484\,. \tag{2.64}$$

Tab. (2.3) summarizes all the experimental values and their uncertainties.

2.3.1 Masses and mixings

In order to show that the model reproduces in a natural manner all the experimental data we retain the first non-vanishing contribution to each entry in all mass matrices and then—having extracted an overall coefficient for each matrix according to eqs. (2.48)–(2.49) and eqs. (2.52)–(2.53)—treat the ratios of Yukawa couplings as a set of arbitrary parameters to be varied within a $O(1)$ range.

We keep the VEVs v_1 and v_2 fixed at the values obtained by taking $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.2$.

In practice, we generated for the quark matrices many sets of 18 complex Yukawa parameters whose moduli differ by at most a factor 10 and accepted those that reproduces the known masses and mixings. As an example, we found that the assignments

$$\begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} = \lambda_U \begin{bmatrix} 1.2+0.073i & 1.9+0.31i & -0.82+1.3i \\ -0.32-0.41i & -0.58+0.85i & -0.48-0.95i \\ 1.2+0.84i & -1.5+0.78i & 1.4+0.72i \end{bmatrix} \tag{2.65}$$

and

$$\begin{bmatrix} \tilde\lambda_{11} & \tilde\lambda_{12} & \tilde\lambda_{13} \\ \tilde\lambda_{21} & \tilde\lambda_{22} & \tilde\lambda_{23} \\ \tilde\lambda_{31} & \tilde\lambda_{32} & \tilde\lambda_{33} \end{bmatrix} = \lambda_D \begin{bmatrix} -0.55-1.5i & -0.76-0.42i & 0.55+1.2i \\ -1.3-0.83i & 0.32+1.2i & 0.58+0.67i \\ 0.75-1.0i & -1.4+0.17i & 0.09-1.6i \end{bmatrix} \tag{2.66}$$

with λ_U and λ_D of $O(1)$, give masses and mixing angles in excellent agreement with the experimental data. We have followed a similar procedure for the leptonic sector, generating random sets of 13 real parameters. Lacking experimental signature of CP violation in the leptonic sector, we have neglected, for the purpose of illustration, leptonic phases in the numerical exercise. Again, we obtain that for the representative choice

$$\begin{bmatrix} r\ \lambda_{3\nu}^2 + 2\lambda_{1\nu}\lambda_{2\nu} & -\lambda_{2\nu}\lambda'_{1\nu} & -\lambda_{2\nu}\lambda''_{1\nu} \\ -\lambda_{2\nu}\lambda'_{1\nu} & r\ \lambda'^2_{3\nu} & r\ \lambda'_{3\nu}\lambda''_{3\nu} \\ -\lambda_{2\nu}\lambda''_{1\nu} & r\ \lambda'_{3\nu}\lambda''_{3\nu} & r\ \lambda''^2_{3\nu} \end{bmatrix} = \lambda_\nu^2 \begin{bmatrix} 0.66 & -1.0 & 2.9 \\ -1.0 & 1.9 & 0.29 \\ 2.9 & 0.29 & -1.1 \end{bmatrix} \tag{2.67}$$

with $\lambda_\nu = O(10^{-4})$, and

$$\begin{bmatrix} \lambda_{1e} & \lambda_{2e} & \lambda_{3e} \\ \lambda_{1E} & \lambda_{2E} & \lambda'_{4E} \\ \lambda'_{1E} & -\lambda_{4E} & \lambda_{2E} \end{bmatrix} = \lambda_E \begin{bmatrix} 1.2 & 0.27 & 1.4 \\ -1.2 & 0.39 & 2.3 \\ 0.36 & 2.0 & 0.39 \end{bmatrix} \tag{2.68}$$

with $\lambda_E = O(10^{-2})$, the experimental values are well reproduced.

Tab. (2.3) summarizes the experimental data and compares them to the result of the above procedure. The agreement is quite impressive, keeping in mind that we have varied only the leading terms in the mass matrices. While the values of the overall constants (which are related to the scale of the heaviest state in the mass matrices) are not explained by the model, the hierarchy among the mass eigenvalues and the mixing angles are given in first approximation by the flavor symmetry and the flavor vacuum so that, within each sector, the Yukawa couplings remain in a natural range.

2.4 The phenomenology

In the previous sections we have seen that the model reproduces successfully fermion masses and mixing angles and that these predictions are independent of the scale f. On the other hand, the masses of the flavor bosons—both vector and scalar—arising in the breaking of the gauge symmetries depend on f. Since these masses enter in the processes mediated by the new particles, a computation of their effects allows us to constrain possible values of f.

Table 2.3: Experimental data vs. the result of our numerical analysis based on a rapresentative set of Yukawa couplings of order one (see text) and $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.2$. Uncertainties in the experimental inputs are explained in the main body.

	exp	numerical results
$\lvert V_{us}\rvert$	$0.219 - 0.226$	0.22
$\lvert V_{ub}\rvert$	$0.002 - 0.005$	0.0035
$\lvert V_{cb}\rvert$	$0.037 - 0.043$	0.040
$\lvert V_{td}\rvert$	$0.004 - 0.014$	0.0079
$\lvert V_{ts}\rvert$	$0.035 - 0.043$	0.039
δ	$61.5^o \pm 7^o$	61^o
$\sin 2\beta$	$0.705^{+0.042}_{-0.032}$	0.69
m_t/m_c	248 ± 70	219
m_c/m_u	325 ± 200	300
m_b/m_s	40 ± 10	45
m_s/m_d	430 ± 300	231
$\tan^2\theta_\odot$	$0.23 - 0.69$	0.32
$\sin^2 2\theta_\oplus$	$0.8 - 1.0$	1.0
$\sin^2\theta_{13}$	< 0.09	0.08
$\Delta m^2_\odot/\Delta m^2_\oplus$	$0.014 - 0.12$	0.043
m_τ/m_μ	17	15
m_μ/m_e	207	231
m_τ/m_e	3484	3465

We will now consider several processes and compare the predictions given by the little flavon model to their experimental bounds. We present the relevant processes that occur at tree level starting from those that give the less stringent bounds on f to that that give the most stringent. We will see that the latter bound comes from flavor changing neutral current in the K^0-$\bar{K}^0$ for which we have $\Lambda_F \geq 5 \times 10^4$ TeV. We also give an example of a one-loop process that gives a limit on f as stringent as that coming from K^0-$\bar{K}^0$.

2.4.1 Interactions

Before studying the processes induced by the new particles present in the spectrum of the little flavon model it is useful to repeat which are the new particles and their masses and which are the new interactions between standard model fermions and new bosons, both scalars and vectors.

The full effective lagrangian at the scale Λ_F is given by

$$\mathcal{L} = \mathcal{L}^{\Sigma}_{kin} + \mathcal{L}^{f}_{kin} + \mathcal{L}^{g}_{kin} + \mathcal{L}_Y \,, \tag{2.69}$$

where $\mathcal{L}^{\Sigma,f,g}_{kin}$ includes the kinetic terms for the PGBs, the fermions and the gauge bosons respectively and $\mathcal{L}_Y$ the Yukawa couplings. Explicitly we have (see eq. (2.14))

$$\begin{aligned}\mathcal{L}^{\Sigma}_{kin} &= -\frac{f^2}{4}\mathrm{Tr}\,(D^\mu\Sigma)\,(D_\mu\Sigma)^* \,,\\ \mathcal{L}^{f}_{kin} &= \bar{f}_{L,R}\gamma_\mu(\partial^\mu + ig_1 A^\mu_{1\,a}T^a + ig'_1 B^\mu_1)f_{L,R} \,,\end{aligned} \tag{2.70}$$

In sec. (2.1.1) we have seen that after the spontaneous breaking of global $SU(6)$ we are left with four massive gauge bosons, A'^μ_a $(a = 1,2,3)$ and B'^μ of masses (see eq. (2.16)

$$m^2_{A'} = \frac{1}{2}(g_1^2 + g_2^2)f^2 \quad \text{and} \quad m^2_{B'} = \frac{2}{5}(g'^2_1 + g'^2_2)f^2 \,, \tag{2.71}$$

and four massless gauge bosons, A^μ_a $(a = 1,2,3)$ and B^μ.

After the $SU(2)_F \times U(1)_F$ symmetry is broken we are left with one complex massive gauge boson, F^μ_3, 2 real massive gauge bosons, $F^\mu_{1,2}$, 2 real, $\varphi_{1,2}$, and one complex, φ_3, massive scalars, which are the flavon scalars. Their masses are given by

$$\begin{aligned}m^2_{F_3} &= \frac{1}{2}g^2(\epsilon_1^2 + \epsilon_2^2)f^2 \,,\\ m^2_{F_{1,2}} &= \frac{1}{2}(g^2 + g'^2)\epsilon^2_{1,2}f^2 \,,\\ m^2_{\varphi_{1,2}} &= [(\lambda_1\epsilon_1^2 + \lambda_2\epsilon_2^2 \pm \\ &\qquad \sqrt{(\lambda_1\epsilon_1^2 + \lambda_2\epsilon_2^2)^2 - (4\lambda_1\lambda_2 - \lambda_3^2)\epsilon_1^2\epsilon_2^2})f^2] \,,\\ m^2_{\varphi_3} &= \frac{1}{2}\lambda_4(\epsilon_1^2 + \epsilon_2^2)f^2 \,,\end{aligned} \tag{2.72}$$

where $g^2 = g_1^2g_2^2/(g_1^2 + g_2^2)$, $g'^2 = g'^2_1g'^2_2/(g'^2_1 + g'^2_2)$ are the effective gauge couplings, $\lambda_4 \simeq \mathcal{O}(1)$ and $\lambda_{1,2,3} \simeq \mathcal{O}(10^{-2})$ are the parameters of the potential as discussed in 2.1.1 and $\epsilon_{1,2}$ the ratios of the VEVs of ϕ_1 and ϕ_2 and the scale f. Unlike eqs. (2.16)–(2.29), where we have written the masses of the new gauge and scalars bosons in terms of the VEVs $v_{1,2}$ of the two flavor doublets, in eq. (2.72) we have given prominence to their dependence by the scale f and by $\epsilon_{1,2}$. This choice is motivated by the fact that in 2.2.4 we have seen that the fermion masses and mixing angles predicted by the model depend only by $\epsilon_{1,2}$.

Besides the numerical analysis in 2.3 indicates that $\epsilon_1\epsilon_2 \simeq 0.2$ if we want to fit the fermion masses and mixing angles. This allows us to reduce the number of degrees of freedom to perform the phenomenological analysis. Notice that the gauge bosons $F^{\mu}_{1,2}$ come from the mixing between A^{μ}_3 and B^{μ}, while F^{μ}_3 from the mixing between A^{μ}_1 and A^{μ}_2.

From the kinetic term in eq. (2.70) we have the following interactions between the gauge bosons and the fermions

$$y^f_{F_{L,R}}\left(\frac{\sqrt{g^2+g'^2}}{\sqrt{2}}\right)(\overline{f}_{L,R}\gamma_\mu f_{L,R})(F^{\mu}_1+F^{\mu}_2)\,, \tag{2.73}$$

if $f_{L,R}$ is a singlet of $SU(2)_F$ with flavor hypercharge $y^f_{F_{L,R}}$, and

$$\frac{g}{\sqrt{2}}\left[(\overline{\psi^1}_{L,R}\gamma_\mu\psi^2_{L,R})F^{\dagger\mu}_3+h.c.\right]+\frac{\sqrt{g^2+g'^2}}{\sqrt{2}}\left[(\overline{\psi^1}_{L,R}\gamma_\mu\psi^1_{L,R})F^{\mu}_2+(\overline{\psi^2}_{L,R}\gamma_\mu\psi^2_{L,R})F^{\mu}_1\right]\,, \tag{2.74}$$

if $\psi_{L,R}$ is a doublet of $SU(2)_F$ of flavor hypercharge 1/2 with components $\psi^1_{L,R}$ and $\psi^2_{L,R}$.

The interactions in eqs. (2.73)–(2.74) have been written in the flavor current basis for the fermions $f_{L,R}$ and $\psi_{L,R}$. In the next sections we will indicate as $e^{i=1,2,3}_{L,R}$ the charged lepton flavor current eigenstates, $e^{\alpha=1,2,3}_{L,R} = e_{L,R}, \mu_{L,R}, \tau_{L,R}$ the charged lepton mass eigenstates, $L^e_{i,\alpha}, R^e_{i,\alpha}$ the unitary matrices that diagonalize the non diagonal mass matrix M^{RL}_e through the bi-unitary transformation

$$R^{e\dagger}\,M^{RL}_e\,L^e = M^{RL^{diag}}_e\,. \tag{2.75}$$

The same conventions will be used for the quarks, where we have $u^i_{L,R}$, $d^i_{L,R}$, $u^{\alpha=1,2,3}_{L,R} = u_{L,R}, c_{L,R}, t_{L,R}$ and $d^{\alpha=1,2,3}_{L,R} = d_{L,R}, b_{L,R}, s_{L,R}$, $L^{u,d}_{i,\alpha}, R^{u,d}_{i,\alpha}$ and M^{RL}_u and M^{RL}_d. The non diagonal mass matrices are those of eqs. (2.48)–(2.49) and eqs. (2.52)–(2.53).

In the model all the standard model quarks are $SU(2)_F$ singlets charged under $U(1)_F$. Standard model leptons belonging to the first family, $l_{e\,L}$ and e_R, are $SU(2)_F$ singlets as well, while those of the second and third family, $l_{\mu,\tau\,L}$ and $(\mu,\tau)_R$, are members of a doublet in flavor space (see tabs. (2.1)–(2.2)). A consequence of this choice is that all lepton mass eigenstates interact with all the gauge bosons after $SU(2)_F\times U(1)_F$ is completely broken. For this reason it is useful to write down the interactions between flavor

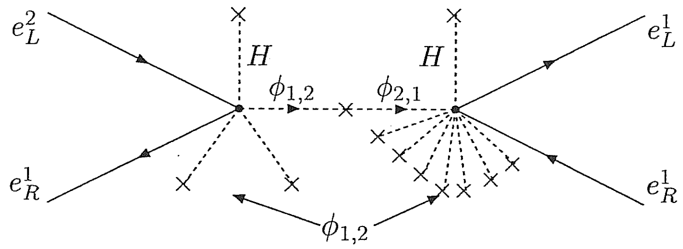


Figure 2.2: Flavon mediated contribution to the decay $\mu^- \to e^+ e^+ e^-$. The fields $\phi_{1,2}$ are the $SU(2)_F$ doublets.

gauge bosons and charged lepton mass eigenstates. The general interaction is given by

$$y^{e\alpha\beta}_{m_{L,R}} \left(\frac{\sqrt{g^2+g'^2}}{\sqrt{2}} \right) (\bar{e}^{\alpha}_{L,R}\gamma_\mu e^{\beta}_{L,R}) F^{\mu}_{m} \,, \tag{2.76}$$

where $m = 1,2,3$ and $y^{\alpha\beta}_{m_{L,R}}$ are given by

$$\begin{aligned} y^{e\alpha\beta}_{1_U} &= U^{e*}_{1\alpha} U^{e}_{1\beta} y^{e1}_{U} + U^{e*}_{3\alpha} U^{e}_{3\beta} \,, \\ y^{e\alpha\beta}_{2_U} &= U^{e*}_{1\alpha} U^{e}_{1\beta} y^{e1}_{U} + U^{e*}_{2\alpha} U^{e}_{2\beta} \,, \\ y^{e\alpha\beta}_{3_U} &= \sqrt{\frac{g^2}{g^2+g'^2}} U^{e*}_{2\alpha} U^{e}_{3\beta} \,, \end{aligned} \tag{2.77}$$

with $U^e = L^e$, R^e and y^{e1}_U the first family charged leptons flavor hypercharges (see tabs. (2.1)–(2.2)).

For completeness we report also the interaction between flavor gauge bosons and quark mass eigenstates. The interaction is given by

$$y^{q\alpha\beta}_{L,R} \left(\frac{\sqrt{g^2+g'^2}}{\sqrt{2}} \right) (\bar{q}^{\alpha}_{L,R}\gamma_\mu q^{\beta}_{L,R}) (F^{\mu}_{1} + F^{\mu}_{2}) \,, \tag{2.78}$$

where $y^{q\alpha\beta}_{L,R}$ are given by

$$y^{q\alpha\beta}_{U} = \sum_{i=1,2,3} U^{q*}_{i\alpha} U^{q}_{i\beta} y^{q\,i}_{U} \,, \tag{2.79}$$

with $U^q = L^q$, R^q and $y^{q\,i}_U$ the quarks flavor hypercharges (see again tabs. (2.1)–(2.2)).

2.4.2 Processes mediated by the flavons

All interactions between fermions and flavons come from the Yukawa lagrangian $\mathcal{L}_Y$ in eq. (2.33). These are the terms that give origin to the fermion mass matrices. After the breaking of $SU(2)_F \times U(1)_F$ it gives also the interactions we are interested here. Notice that in the following, for simplicity, we will indicate as *flavons* both the $SU(2)_F$ doublets, $\phi_{1,2}$, and the massive scalars, $\varphi_{1,2,3}$, arising after the breaking of the $SU(2)_F \times U(1)_F$ symmetry. Processes mediated by the flavons can occur at tree level and at one or more loops. Tree level processes concern direct interactions between fermions and only one flavon, for this reason couplings of this kind will follow the fermion mass matrices and all the flavor changing processes mediated by the flavons will be very suppressed since they will result be proportional to power of the ratio between the light fermion masses and the scale f.

In trying to constraint the flavon masses, let us first consider the lepton flavor violation (LFV) process $\mu \to 3e$. The limit on the branching ratio $\Gamma_{\mu^- \to e^+ e^+ e^-}$ is given as a function of the total branching ratio $\Gamma_{\mu \to \text{all}}$ [65]

$$\frac{\Gamma_{\mu \to 3e}}{\Gamma_{\mu \to \text{all}}} < 10^{-12}, \tag{2.80}$$

with

$$\Gamma_{\mu \to \text{all}} = \frac{m_\mu^5 G_F^2}{192\pi^3}. \tag{2.81}$$

In the model we have tree-level LFV processes mediated by the flavons which give rise to effective operators. They can be parametrized as

$$\begin{aligned}
&\frac{1}{\tilde{\Lambda}^2}\Big\{\eta_{LL}(\bar{e}(1-\gamma_5)\mu\,\bar{e}(1-\gamma_5)e) + \eta_{RR}(\bar{e}(1+\gamma_5)\mu\,\bar{e}(1+\gamma_5)e) + \\
&\eta_{LR}(\bar{e}(1-\gamma_5)\mu\,\bar{e}(1+\gamma_5)e) + \eta_{RL}(\bar{e}(1+\gamma_5)\mu\,\bar{e}(1-\gamma_5)e)\Big\}\,,
\end{aligned} \tag{2.82}$$

where $\tilde{\Lambda}$ is an effective scale given by

$$\frac{1}{\tilde{\Lambda}^2} = \frac{1}{4(4\lambda_1\lambda_2 - \lambda_3^2)(\epsilon_1\epsilon_2)^2 f^2} \tag{2.83}$$

and

$$\eta_{LL} = \left(\frac{R_{i1}^{e*}\, M_{e_{ij}}^{RL}\, L_{j2}^{e}}{f}\right)\left(\frac{R_{l1}^{*}\, M_{e_{lk}}^{RL}\, L_{k2}}{f}\right) F_{ijlk}(\lambda_1, \lambda_2, \lambda_3, \epsilon_1, \epsilon_2)\,, \tag{2.84}$$

with similar expressions for η_{RR}, η_{LR}, η_{RL}. Notice that the effective scale $\tilde{\Lambda}$ is obtained summing on the exchanges of the two lighter massive flavons, $\varphi_{1,2}$, that are the only ones which give rise to tree level processes. M_e^{RL} in eq. (2.84) is the non diagonal charged lepton mass matrix, L^e and R^e are defined in sec. (2.4.1), F_{ijlk} is a function of the potential parameters $\lambda_{i=1,2,3}$ discussed in sec. (2.1.1) and of $\epsilon_{1,2}$ that depends on the processes multiplicities in the current basis.

From eqs. (2.83)–(2.84) we can readily compute $\Gamma_{\mu\to e^+e^+e^-}$ that is given by

$$\Gamma^{flavoni}_{\mu\to 3e} = \frac{m_\mu^5}{6(16\pi)^3}\left(\frac{1}{(4\lambda_1\lambda_2-\lambda_3^2)(\epsilon_1\epsilon_2)^2 f^2}\right)^2 \left(|\eta_{LL}|^2+|\eta_{RR}|^2+|\eta_{LR}|^2+|\eta_{RL}|^2\right). \quad (2.85)$$

By imposing the experimental bound in eq. (2.80) and using eq. (2.81) we find

$$f > 200\,\mathrm{GeV}. \quad (2.86)$$

Such a rather weak bound is justified by the strong suppression of this process. This is best understood by going back to the current eigenstates. In this basis we have nine processes that sum to give $\mu\to 3e$ in the mass eigenstates. For simplicity we consider only one of them. The interaction terms that give rise to the tree level process are

$$\lambda_{2e}\bar{e}_R^1\,(h^{0*}e_L^2)\left(\frac{\phi_2^\dagger\phi_1}{f^2}\right)\frac{\phi_1^0}{f}+\lambda_{1e}\bar{e}_R^1\,(h^{0*}e_L^1)\left(\frac{\phi_2^\dagger\phi_1}{f^2}\right)^4+\lambda_1(\phi_1^\dagger\phi_1)^2+\lambda_2(\phi_2^\dagger\phi_2)^2+\lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)\,. \quad (2.87)$$

After the flavor and the electroweak spontaneous breaking the tree level effective coupling is (see fig. (2.2))

$$\frac{\lambda_{2e}\lambda_{1e}}{4(4\lambda_1\lambda_2-\lambda_3^2)(\epsilon_1\epsilon_2)^2 f^2}\epsilon_1(\epsilon_1\epsilon_2)^5\frac{\langle h^{0*}\rangle^2}{f^2}(\bar{e}(1-\gamma_5)\mu)(\bar{e}(1-\gamma_5)e)(16\lambda_1+8\lambda_2+12\lambda_3)\,, \quad (2.88)$$

where $(16\lambda_1+8\lambda_2+12\lambda_3)$ is the function indicated as F_{1211} in eq. (2.84). Processes mediated by the flavons are suppressed by powers of $\epsilon_{1,2}$ and by the ratio between the electroweak breaking scale and the flavor one.

Let us consider also a FCNC process mediated by the flavons in the quark sector. FCNC processes in the quark sector with $\Delta F = 2$ are responsible of meson-antimeson oscillations. Since meson mass eigenstates are a combination of mesons in the current basis, the splitting of the masses of the mass eigenstates is related to the possible FCNC processes. This statement is general and can be applied to K^0-$\bar{K}^0$ as well as B^0-$\bar{B}^0$ system. Nevertheless, the best experimental data are related to the splitting of Kaon masses [61]

$$\Delta m_{LS} = (3.46 \pm 0.01)\,\times\,10^{-12}\,\mathrm{MeV}, \quad (2.89)$$

and therefore we will consider only the processes with $\Delta S = 2$. Given an effective interaction $\mathcal{V} = C\mathcal{O}_{\Delta S=2}$, where C is a numerical coefficient and $\mathcal{O}_{\Delta S=2}$ the effective operator involving the quarks d and s, we have that

$$\Delta m_{LS} = 2C\,\frac{\mathrm{Re}\,\langle K^0|\mathcal{O}_{\Delta S=2}|\bar{K}^0\rangle}{2m_K}\,. \tag{2.90}$$

In order to estimate the contribution of the flavons to Δm_{LS} we have to consider all the possible effective operators with $\Delta S = 2$. We have three main operators that we parametrize as follows

$$\begin{aligned} -\frac{1}{\tilde{\Lambda}^2}\Big[&\rho_1\Big(\bar{d}(1+\gamma_5)s\,\bar{d}(1-\gamma_5)s\Big) + \rho_2\Big(\bar{d}(1-\gamma_5)s\,\bar{d}(1-\gamma_5)s\Big) \\ &+\rho_3\Big(\bar{d}(1+\gamma_5)s\,\bar{d}(1+\gamma_5)s\Big)\Big]\,, \end{aligned} \tag{2.91}$$

where

$$\frac{1}{\tilde{\Lambda}^2} = \frac{2(\lambda_1+\lambda_2+\lambda_3)}{4(4\lambda_1\lambda_2-\lambda_3^2)(\epsilon_1\epsilon_2)^2 f^2}. \tag{2.92}$$

The coefficients ρ_i are given by

$$\begin{aligned} \rho_1 &= \left(\frac{\Sigma_{ji}(R^{d*}_{j1}M^{RL}_{d_{ji}}N_{ji}L^{d}_{i2})}{f}\right)\left(\frac{\Sigma_{lk}(R^{d*}_{l2}M^{RL}_{d_{lk}}N_{lk}L^{d}_{k1})^*}{f}\right), \\ \rho_2 &= \left(\frac{\Sigma_{ji}(R^{d*}_{j1}M^{RL}_{d_{ji}}N_{ji}L^{d}_{i2})}{f}\right)^2, \\ \rho_3 &= \left(\frac{\Sigma_{lk}(R^{d*}_{l2}M^{RL}_{d_{lk}}N_{lk}L^{d}_{k1})^*}{f}\right)^2, \end{aligned} \tag{2.93}$$

where $M^{RL}_{d_{ji}}$ is the non diagonal quark mass matrices and N_{ji} is a multiplicity factor.

A comparison between eq. (2.89) and eq. (2.90) with eq. (2.91) indicates that we need f at least

$$f \simeq 10\,\mathrm{TeV}, \tag{2.94}$$

in order to satisfy the experimental bound.

However, as we shall see in the next sections, processes mediated by flavons are not the dominant ones and the limit obtained here must be increase. For this reason we will not further discuss this kind of processes, and concentrate on those mediated by the flavor gauge bosons.

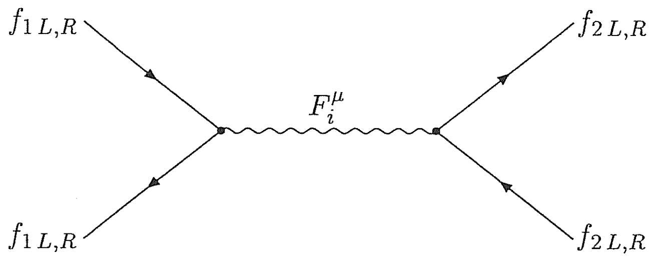


Figure 2.3: Processes of annihilation and production of $f\bar{f}$ mediated by gauge flavons.

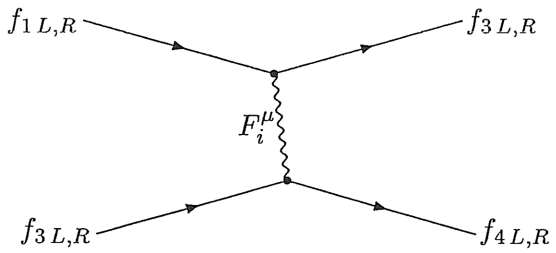


Figure 2.4: Parity violation processes mediated by gauge flavons.

2.4.2.1 Processes mediated by the gauge flavons

Processes mediated by the gauge bosons of the flavor groups are crucial in fixing the scale f. Most of the processes we discuss in the following arise from two classes of operators of the general form

$$\frac{1}{\bar{\Lambda}^2}(\bar{f}_1\Gamma^{V,A,S,P}f_1)(\bar{f}_2\Gamma_{V,A,S,P}f_2) \quad \text{and} \quad \frac{1}{\bar{\Lambda}^2}(\bar{f}_1\Gamma^{V,A,S,P}f_2)(\bar{f}_3\Gamma_{V,A,S,P}f_4)\,, \tag{2.95}$$

where in the second class of operators at least $f_2 \neq f_1$ (or $f_4 \neq f_3$). These operators arise from integrating out the gauge flavons. Notice that the longitudinal components of the gauge flavons propagators give contributions sub-leading with respect to that arising from the transverse components. The first class of operators in eq. (2.95) gives rise to processes of annihilation and production of fermion-antifermion couples and parity violation processes (see fig. (2.3)), while the latter to flavor changing processes (see fig. (2.4)). Tree level processes can give only the vectorial and the axial structure, while the scalar and pseudoscalar ones arise when we consider processes at least at one-loop. For this reason these structures are suppressed and we neglect them in the following.

2.4.2.2 $f\bar{f} \to f'\bar{f}'$ and parity violation

The number of four fermions operators belonging to the first class of eq. (2.95) which give rise to $f\bar{f} \to f'\bar{f}'$ and parity violation is very large, so we consider only those that contribute to the experimentally most constrained processes, that is, $e^+e^- \to e^+e^-$ and $e\,q_{L,R} \to e\,q_{R,L}$. They can be parametrized as

$$
\begin{aligned}
&-\frac{1}{f^2}\Big\{\eta^{ee}_{LL}\Big(\bar{e}_L\gamma_\mu e_L\,\bar{e}_L\gamma^\mu e_L\Big) + \eta^{ee}_{RR}\Big(\bar{e}_R\gamma_\mu e_R\,\bar{e}_R\gamma^\mu e_R\Big) + 2\eta^{ee}_{LR}\Big(\bar{e}_L\gamma_\mu e_L\,\bar{e}_R\gamma^\mu e_R\Big) + \\
&\eta^e_L\eta^u_L\Big(\bar{e}_L\gamma_\mu e_L\,\bar{u}_L\gamma^\mu u_L\Big) + \eta^e_R\eta^u_R\Big(\bar{e}_R\gamma_\mu e_R\,\bar{u}_R\gamma^\mu u_R\Big) + \eta^e_L\eta^u_R\Big(\bar{e}_L\gamma_\mu e_L\,\bar{u}_R\gamma^\mu u_R\Big) + \\
&\eta^e_R\eta^u_L\Big(\bar{e}_R\gamma_\mu e_R\,\bar{u}_L\gamma^\mu u_L\Big) + (u \to d)\Big]\Big\}\,. \qquad (2.96)
\end{aligned}
$$

The first line of eq. (2.96) has to be compared with the usual effective lagrangian of contact interactions [62]

$$
\begin{aligned}
&\tfrac{g^2}{\Lambda^2_{LL}}\Big(\pm\,\bar{e}_L\gamma_\mu e_L\,\bar{e}_L\gamma^\mu e_L\Big) + \tfrac{g^2}{\Lambda^2_{RR}}\Big(\pm\,\bar{e}_R\gamma_\mu e_R\,\bar{e}_R\gamma^\mu e_R\Big) + \\
&\tfrac{g^2}{\Lambda^2_{LR}}\Big(\pm\,\bar{e}_L\gamma_\mu e_L\,\bar{e}_R\gamma^\mu e_R\Big) + \tfrac{g^2}{\Lambda^2_{RL}}\Big(\pm\,\bar{e}_R\gamma_\mu e_R\,\bar{e}_L\gamma^\mu e_L\Big) \qquad (2.97)
\end{aligned}
$$

where the limits on Λ_{UU}, with $U = L, R$, are usually given imposing $g^2 = 4\pi$. From [63] we have

$$
\Lambda^+_{LL} = 8.3\,\mathrm{TeV} \quad \text{and} \quad \Lambda^-_{LL} = 10.1\,\mathrm{TeV}\,. \qquad (2.98)
$$

To compare these values with eq. (2.96), we write the η coefficients in terms of the model parameters as

$$
\begin{aligned}
\eta^{ee}_{LL} &= \frac{(y^{e11}_{1L})^2}{\epsilon_1^2} + \frac{(y^{e11}_{2L})^2}{\epsilon_2^2} + 2\frac{(y^{e11}_{3L})^2}{\epsilon_1^2+\epsilon_2^2}\,, \\
\eta^{ee}_{RR} &= \frac{(y^{e11}_{1R})^2}{\epsilon_1^2} + \frac{(y^{e11}_{2R})^2}{\epsilon_2^2} + 2\frac{(y^{e11}_{3R})^2}{\epsilon_1^2+\epsilon_2^2}\,, \\
\eta^{ee}_{LR} &= \frac{y^{e11}_{1L}y^{e11}_{1R}}{\epsilon_1^2} + \frac{y^{e11}_{2L}y^{e11}_{2R}}{\epsilon_2^2} + 2\frac{y^{e11}_{3L}y^{e11}_{3R}}{\epsilon_1^2+\epsilon_2^2}\,,
\end{aligned}
$$

where $y^{e\alpha\beta}_{mU}$ have been defined in eq. (2.77). A direct comparison imposes

$$
f \geq 36\,\mathrm{TeV}, \qquad (2.99)
$$

which is two order of magnitude bigger than the value we found in the previous section for LFV.

Let us turn now to parity violation processes. Parity violation is measured in term of the weak charge Q_W and the most recent experimental values give [62]

$$\Delta Q_w = 0.44 \pm 0.44\,. \tag{2.100}$$

From the contact parameters, ΔQ_w receives the contributions [62]

$$\Delta Q_w = (-11.4\mathrm{TeV}^2)(-\tilde{\eta}^{eu}_{LL} + \tilde{\eta}^{eu}_{RR} - \tilde{\eta}^{eu}_{LR} + \tilde{\eta}^{eu}_{RL}) + (-12.8\mathrm{TeV}^2)(-\tilde{\eta}^{ed}_{LL} + \tilde{\eta}^{ed}_{RR} - \tilde{\eta}^{ed}_{LR} + \tilde{\eta}^{ed}_{RL})\,, \tag{2.101}$$

where

$$\tilde{\eta}^{eq}_{AB} = \frac{4\pi}{\Lambda^{2\,eq}_{AB}} \eta^e_A \eta^q_B\,. \tag{2.102}$$

In eq. (2.96) $4\pi/\Lambda^{2\,eq}_{AB} = -1/f^2$ and the η coefficients are given by

$$\begin{aligned}
\eta^e_L &= \left(\frac{y^{e11}_{1_L}}{\epsilon_1^2} + \frac{y^{e11}_{2_L}}{\epsilon_2^2}\right)\,,\\
\eta^e_R &= \left(\frac{y^{e11}_{1_R}}{\epsilon_1^2} + \frac{y^{e11}_{2_R}}{\epsilon_2^2}\right)\,,\\
\eta^u_L &= y^{u11}_L\,,\\
\eta^u_R &= y^{u11}_R\,,\\
\eta^d_L &= y^{d11}_L\,,\\
\eta^d_R &= y^{d11}_R\,,
\end{aligned}$$

where $y^{e\alpha\beta}_{m_U}$ and $y^{q\alpha\beta}_U$ are given in eqs. (2.77)–(2.79). A direct comparison of eq. (2.100) with eq. (2.101) gives

$$f \geq 88\,\mathrm{TeV}. \tag{2.103}$$

2.4.2.3 Leptonic processes

The most stringent experimental limits for LFV processes comes from the processes $\mu \to 3e$ and $\mu \to e\gamma$, but in the little-flavon model only $\mu \to 3e$ is present at tree level. As already discussed in sec. (2.4.2) the limit on the branching ratio for muon decay LFV is given by $\Gamma_{\mu\to 3e}/\Gamma_{\mu\to \mathrm{all}} < 10^{-12}$.

As done in eq. (2.82) we parametrize the effective interactions as

$$-\frac{1}{f^2}\Big(g_{LL}(\bar{e}_L\gamma_\mu\mu_L\,\bar{e}_L\gamma^\mu e_L)+g_{RR}(\bar{e}_R\gamma_\mu\mu_R\,\bar{e}_R\gamma^\mu e_R)+g_{LR}(\bar{e}_L\gamma_\mu\mu_L\,\bar{e}_R\gamma^\mu e_R)+g_{RL}(\bar{e}_R\gamma_\mu\mu_R\,\bar{e}_L\gamma^\mu e_L)\Big)\,, \tag{2.104}$$

where

$$\begin{aligned}
g_{LL} &= \frac{y_{1_L}^{e12}y_{1_L}^{e11}}{\epsilon_1^2}+\frac{y_{2_L}^{e12}y_{2_L}^{e11}}{\epsilon_2^2}+2\frac{y_{3_L}^{e12}y_{3_L}^{e11}}{\epsilon_1^2+\epsilon_2^2}\,,\\
g_{RR} &= \frac{y_{1_R}^{e12}y_{1_R}^{e11}}{\epsilon_1^2}+\frac{y_{2_R}^{e12}y_{2_R}^{e11}}{\epsilon_2^2}+2\frac{y_{3_R}^{e12}y_{3_R}^{e11}}{\epsilon_1^2+\epsilon_2^2}\,,\\
g_{LR} &= \frac{y_{1_L}^{e12}y_{1_R}^{e11}}{\epsilon_1^2}+\frac{y_{2_L}^{e12}y_{2_R}^{e11}}{\epsilon_2^2}+2\frac{y_{3_L}^{e12}y_{3_R}^{e11}}{\epsilon_1^2+\epsilon_2^2}\,,\\
g_{RL} &= \frac{y_{1_R}^{e12}y_{1_L}^{e11}}{\epsilon_1^2}+\frac{y_{2_R}^{e12}y_{2_L}^{e11}}{\epsilon_2^2}+2\frac{y_{3_R}^{e12}y_{3_L}^{e11}}{\epsilon_1^2+\epsilon_2^2}\,.
\end{aligned}$$

The rate decay for this process is then given by

$$\Gamma_{\mu\to 3e}^{gauge}=\frac{m_\mu^5}{6(16\pi)^3f^4}(|g_{LL}|^2+|g_{RR}|^2+|g_{LR}|^2+|g_{RL}|^2) \tag{2.105}$$

and to satisfy the experimental bound we need

$$f > 580\,\text{TeV}, \tag{2.106}$$

which give us the stringent bound so far.

2.4.2.4 K^0-$\bar{K}^0$ mixing

As done in sec. (2.4.2) among all the FCNC processes with $\Delta F = 2$ in the quark sector that are responsible of meson-antimeson oscillations, we will consider only the processes with $\Delta S = 2$ since the best experimental data are related to the splitting of Kaon masses [61] (see eq. (2.89)).

Also this time, in order to estimate the contribution of the gauge flavons to Δm_{LS}, we have to consider all the possible effective operators with $\Delta S = 2$. To give a more complete analysis, we will take into account also the operators arising at one-loop level. Accordingly we have six main operators that we parametrize as follows

$$-\frac{1}{\Lambda_0^2}\Big[\eta_1\Big(\bar{d}\gamma_\mu(1-\gamma_5)s\,\bar{d}\gamma^\mu(1-\gamma_5)s\Big)\;+\;\eta_2\Big(\bar{d}\gamma_\mu(1+\gamma_5)s\,\bar{d}\gamma^\mu(1+\gamma_5)s\Big)\;+$$

$$+\eta_3\Big(\bar{d}\gamma_\mu(1-\gamma_5)s\,\bar{d}\gamma^\mu(1+\gamma_5)s\Big]-\frac{1}{\Lambda_1^2}\Big[\eta_4\Big(\bar{d}(1+\gamma_5)s\,\bar{d}(1-\gamma_5)s\Big)+$$
$$\eta_5\Big(\bar{d}(1-\gamma_5)s\,\bar{d}(1-\gamma_5)s\Big)+\eta_6\Big(\bar{d}(1+\gamma_5)s\,\bar{d}(1+\gamma_5)s\Big)\Big], \qquad (2.107)$$

where

$$\frac{1}{\Lambda_0^2}=\frac{1}{4f^2}\left(\frac{1}{\epsilon_1^2}+\frac{1}{\epsilon_2^2}\right) \quad \text{and} \quad \frac{1}{\Lambda_1^2}\simeq\frac{1}{(4\pi)^2}\frac{m_b^2}{4f^4}\left(\frac{1}{\epsilon_1^2}+\frac{1}{\epsilon_2^2}\right)^2, \qquad (2.108)$$

and

$$\begin{aligned}
\eta_1 &= (y_L^{d12})^2,\\
\eta_2 &= (y_R^{d12})^2,\\
\eta_3 &= y_L^{d12}y_R^{d12},\\
\eta_4 &= \frac{1}{m_b^2}\left(\Sigma_{ij}R_{j1}^{d*}y_R^{dj}M_{dji}^{RL}y_L^{di}L_{i2}^{d}\right)\left(\Sigma_{nm}R_{n2}^{d}y_R^{dn}M_{dnm}^{RL*}y_L^{dm}L_{m1}^{d*}\right),\\
\eta_5 &= \frac{1}{m_b^2}\left(\Sigma_{ij}R_{j1}^{d*}y_R^{dj}M_{dji}^{RL}y_L^{di}L_{i2}^{d}\right)^2,\\
\eta_6 &= \frac{1}{m_b^2}\left(\Sigma_{nm}R_{n2}^{d}y_R^{dn}M_{dnm}^{RL*}y_L^{dm}L_{m1}^{d*}\right)^2,
\end{aligned} \qquad (2.109)$$

where y_U^{d12} are defined in eq. (2.79). eq. (2.109) gives the relationships between the effective operators of eq. (2.107) and the model parameters and charges. The last three operators proportional to $\eta_{4,5,6}$ respectively arise from one-loop box-diagrams in which gauge flavon bosons are exchanged. These one-loop effects are not the dominant ones since they only require f to be ≥ 2 TeV to satisfy the experimental limit, as one can check comparing eq. (2.89) and eq. (2.90) with eq. (2.109). The first three operators come from tree level processes and a comparison to the experimental limits indicates that they impose at least

$$f\simeq 4\times 10^3\,\text{TeV}, \qquad (2.110)$$

to satisfy the bound in eq. (2.89). This result shifts the scale we have found in the lepton sector of more than an order of magnitude and definitively fixes the lowest scale for the breaking of the global symmetry that give rise to the little flavons.

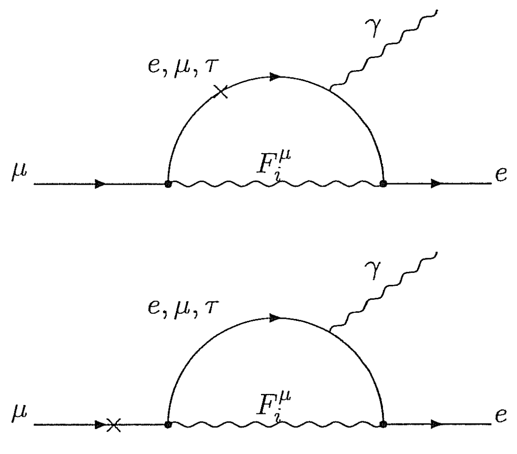


Figure 2.5: Gauge flavons mediated contribution to the decay $\mu \to e\gamma$

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2.4.3 Effects at one-loop

2.4.3.1 Rare processes

There are some rare decays that in the model occur only at one-loop, but give a bound on f which is comparable to the bound obtained from the analysis of the tree-level processes.

As an example let us consider the LFV process $\mu \to e\gamma$. For the $\mu \to e\gamma$ process we have the strong limit [64]

$$\frac{\Gamma_{\mu\to e\gamma}}{\Gamma_{\mu\to \text{all}}} < 1.2 \times 10^{-11} . \tag{2.111}$$

We can parametrize the interaction which gives rise to the decay as

$$\left(\bar{e}\, i\sigma_{\nu\mu}(1-\gamma_5)\,\mu\mathcal{M}^{LR} + \bar{e}\, i\sigma_{\nu\mu}(1+\gamma_5)\,\mu\mathcal{M}^{RL}\right)F^{\nu\mu} . \tag{2.112}$$

In the model we have two kind of diagrams that contribute to the process $\mu \to e\gamma$ (see fig. (2.5)). The second decay in fig. (2.5) is present also in the standard model—with the charged W bosons and massive neutrinos in the loop—and gives a contribution proportional to m_μ/m_F^2, where m_F^2 is the mass of the flavor gauge boson. On the contrary, the first is not present in the standard model and is possible because the flavor gauge bosons couple also to right handed fermions. It gives a contribution proportional to $m_\alpha/m_F^2 \log(m_\alpha^2/m_F^2)$

where m_α is the mass of the fermion circulating in the loop. For this reason the dominant contribution comes from the τ exchange. For this process we have

$$\mathcal{M}^{RL} = \frac{e}{2\pi^2}\frac{m_\tau}{f^2}\left(\log\frac{m_\tau^2}{f^2}\right)Y^{RL},$$

$$\mathcal{M}^{LR} = \frac{e}{2\pi^2}\frac{m_\tau}{f^2}\left(\log\frac{m_\tau^2}{f^2}\right)Y^{LR}, \tag{2.113}$$

where e is the electric charge and

$$Y^{RL} = \frac{y_{1L}^{e32}y_{1R}^{e13}}{\epsilon_1^2} + \frac{y_{2L}^{e32}y_{2R}^{e13}}{\epsilon_2^2} + 2\frac{y_{3L}^{e32}y_{3R}^{e13}}{\epsilon_1^2+\epsilon_2^2},$$

$$Y^{LR} = \frac{y_{1R}^{e32}y_{1L}^{e13}}{\epsilon_1^2} + \frac{y_{2R}^{e32}y_{2L}^{e13}}{\epsilon_2^2} + 2\frac{y_{3R}^{e32}y_{3L}^{e13}}{\epsilon_1^2+\epsilon_2^2}. \tag{2.114}$$

The rate decay for this process is then given by

$$\Gamma_{\mu\to e\gamma} = \frac{3\alpha}{8\pi^4}\frac{m_\mu^3 m_\tau^2}{f^4}\left(\log\frac{m_\tau^2}{f^2}\right)^2\left(Y^{RL^2}+Y^{LR^2}\right) \tag{2.115}$$

In order to satisfy the experimental bound of eq. (2.111) we need

$$f \simeq 4\times 10^3\,\text{TeV}. \tag{2.116}$$

which is of the same order of the value obtained in sec. (2.4.2.4).

The process corresponding to the LFV process $\mu\to e\gamma$ in the quark sector is the FCNC process $b\to s\gamma$. For the $b\to s\gamma$ process we have the limit [66]

$$\frac{\Gamma_{b\to s\gamma}}{\Gamma_{b\to \text{all}}} < (3.3\pm 0.4)\times 10^{-4}. \tag{2.117}$$

The standard model effective interaction that is responsible of this process is parametrized as [67]

$$\frac{-4G_F}{\sqrt{2}}V_{ts}^*V_{tb}\frac{em_b}{16\pi^2}C_7(m_W)\bar{s}_L\sigma_{\mu\nu}F^{\mu\nu}b_R, \tag{2.118}$$

where $C_7(m_W)$ is the Wilson coefficient and is a function of $m_t(m_W)$ and m_W as reported in [67]. In the following we will neglect of the renormalization effects and we will compare the effective interaction which gives rise to the decay $b\to s\gamma$ in our model with the one-loop electroweak operator of eq. (2.118). Analogously to what done for the process $\mu \to e\gamma$, we parametrize the interaction responsible of the decay $b \to s\gamma$ as

$$\left(\bar{s}\, i\sigma_{\nu\mu}(1-\gamma_5)\, b\mathcal{N}^{LR} + \bar{s}\, i\sigma_{\nu\mu}(1+\gamma_5)\, b\mathcal{N}^{RL}\right) F^{\nu\mu}\,. \tag{2.119}$$

All the considerations done for the process $\mu \to e\gamma$ may be applied in this context and for this reason the dominant contribution to the process $b \to s\gamma$ comes from the loops in which a quark b is exchanged. For the process we are considering we have

$$\begin{aligned} \mathcal{N}^{RL} &= \frac{e}{2\pi^2}\frac{m_b}{f^2}\left(\log\frac{m_b^2}{f^2}\right) X^{RL}\,, \\ \mathcal{N}^{LR} &= \frac{e}{2\pi^2}\frac{m_b}{f^2}\left(\log\frac{m_b^2}{f^2}\right) X^{LR}\,, \end{aligned} \tag{2.120}$$

where e is the electric charge and

$$\begin{aligned} X^{RL} &= y_L^{d33} y_R^{d23}\left(\frac{1}{\epsilon_1^2}+\frac{1}{\epsilon_2^2}\right), \\ X^{LR} &= y_R^{d33} y_L^{d23}\left(\frac{1}{\epsilon_1^2}+\frac{1}{\epsilon_2^2}\right). \end{aligned} \tag{2.121}$$

A comparison between eq. (2.118) and eq. (2.119) indicates that we need

$$f \simeq 31\,\text{TeV}, \tag{2.122}$$

in order to have the two contribution of the same order.

2.4.3.2 Muon anomalous magnetic moment

In sec. (2.4.3.1) we have seen how one-loop processes give a bound on f comparable to that one obtained by tree level processes. We may ask ourselves what is the limit on f we obtain if we consider the one-loop contribution to the anomalous magnetic moment of the muon. The uncertainty between the experimental data and the theoretical computation for the anomalous magnetic moment of the muon $a_\mu = (g_\mu - 2)/2$ is [68]

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27 \pm 14)\ \times 10^{-10}\,. \tag{2.123}$$

In order to obtain a limit for f from eq. (2.123) we have to consider the effective interaction that is proportional to a_μ. The interaction coincides to that of eq. (2.112) that is responsible of the rare decay $\mu \to e\gamma$ once we substitute the outgoing electron with an outgoing muon. Therefore it is parametrized as

$$\left(\bar{\mu}\, i\sigma_{\nu\mu}(1-\gamma_5)\,\mu\tilde{\mathcal{M}}^{LR} + \bar{\mu}\, i\sigma_{\nu\mu}(1+\gamma_5)\,\mu\tilde{\mathcal{M}}^{RL}\right)F^{\nu\mu}\,. \tag{2.124}$$

As in sec. (2.4.3.1) the dominant contribution comes from τ exchanging and for this reason we have

$$\begin{aligned}\tilde{\mathcal{M}}^{RL} &= \frac{e}{2\pi^2}\frac{m_\tau}{f^2}\left(\log\frac{m_\tau^2}{f^2}\right)\tilde{Y}^{RL}\,,\\ \tilde{\mathcal{M}}^{LR} &= \frac{e}{2\pi^2}\frac{m_\tau}{f^2}\left(\log\frac{m_\tau^2}{f^2}\right)\tilde{Y}^{LR}\,,\end{aligned} \tag{2.125}$$

where e is the electric charge and

$$\begin{aligned}\tilde{Y}^{RL} &= \frac{y_{1L}^{e32}y_{1R}^{e23}}{\epsilon_1^2} + \frac{y_{2L}^{e32}y_{2R}^{e23}}{\epsilon_2^2} + 2\frac{y_{3L}^{e32}y_{3R}^{e23}}{\epsilon_1^2+\epsilon_2^2}\,,\\ \tilde{Y}^{LR} &= \frac{y_{1R}^{e32}y_{1L}^{e23}}{\epsilon_1^2} + \frac{y_{2R}^{e32}y_{2L}^{e23}}{\epsilon_2^2} + 2\frac{y_{3R}^{e32}y_{3L}^{e23}}{\epsilon_1^2+\epsilon_2^2}\,.\end{aligned} \tag{2.126}$$

From eq. (2.124) we have

$$a_\mu = \frac{m_\mu(\tilde{\mathcal{M}}^{RL} + \tilde{\mathcal{M}}^{LR})}{e}\,. \tag{2.127}$$

A direct comparison between eq. (2.123) and eq. (2.127) gives

$$f \simeq 28\,\text{TeV}, \tag{2.128}$$

that does not change the previous results obtained in sec. (2.4.2.4).

2.5 Conclusions

The little flavon model discussed in this chapter provide an application of the mechanism used in the little Higgs models to stabilize the electroweak scale in the context of flavor physics.

We have seen that the masses and mixing matrices are reproduced successfully and are independent of the scale f at which the model lives, but a detailed analysis of the flavor changing processes induced by the new particles introduced by the little-flavon model shows that the scale $\Lambda_F = 4\pi f$ has to be around $\simeq 5 \times 10^4$ TeV.

This result has two consequences for the little flavon model. On the one hand, the determination of a bound on the scale f leads to a specific prediction for the scale for the see-saw mechanism, which in the model is used to give mass to the neutrinos and was left undetermined in sec. (2.2.4). The value found for f, about 10^4 TeV, allows to have the couplings of the Dirac neutrino mass term to be of the same order of the charged lepton ones, that is more or less $10^{-2} \div 10^{-3}$.

On the other hand, the scale of the model turns out to be quite high with respect to that of the electroweak symmetry breaking and therefore the Higgs mass is not protect against divergent contributions coming from loops of flavons and fermions. Since it would be nice to have both the electroweak and the flavor symmetry breaking scales stabilized, it is clear that some changes are needed in order to lower the flavor symmetry breaking scale. What are these changes and how it is possible to build a model *à la* little Higgs that unifies the electroweak and flavor symmetry breaking at a low scale is the subjects of the next chapter.

Chapter 3

The flhiggs model

In the previous chapter we analyzed in detail the little flavon model. We reached the conclusion that as viable as the model is, it leaves open the question of how the flavons and the weak Higgs field can be accomodated within an unified picture. Since the mass textures are but a modulation of the vacuum expectation value of the Higgs boson, the interplay between electroweak and flavor symmetries must lead us toward an unified picture of the two symmetries and their spontaneous breaking at closely related, if not the same, scales.

The energy scale of any horizontal flavor symmetry breaking is usually thought as well separated from that of the electroweak symmetry breaking mainly because of the constraints on FCNCs. The experimental bounds on flavor changing processes set rather stringent constraints on the value of the scale f at which flavor symmetry must be broken. In sec. (2.4.2) we have seen that in the case of the little flavon model, this scale turns out to be between 10^3 and 10^4 TeV. This bound comes from processes—like K^0-$\bar{K}^0$ mixing—mediated by the flavor gauge bosons. It puts the flavor scale several order of magnitude higher than that of electroweak physics and makes it difficult to think of them in a unified manner. Moreover, radiative corrections from the flavor to the electroweak sector become dangerously large and bring back into the picture some unwelcome fine-tuning. However, this result is heavily based on assuming the horizontal symmetry to be local and therefore having to include the effect of the corresponding gauge bosons. In the absence of these, the constraint can be relaxed and, depending on the specific model, the energy scale made closer and even the same as that of electroweak physics.

Interesting enough, contrary to those for the flavor gauge bosons, direct bounds on the effect of the scalar flavons are not very restrictive, giving, at least for some specific model, a scale f of the order of the TeV. This observation suggests to make the flavor symmetry into a global [1] (rather than local) symmetry and thus avoid the more stringent bounds on the gauge flavor bosons (that do not exist any longer) and bring the flavor symmetry breaking scale closer to that of electroweak physics. The unification of flavor and electroweak symmetry is thus made possible and an explicit example of it will be given in this chapter.

Needless to say, the above scenario—that is going to be realized in the model that follows—is still far from being a complete theory of flavor. In particular, it leaves open the question of the absolute value of the fermion masses, most notably the large difference between those of neutrinos and heavy quarks; this problem, and the much larger hierarchy implied, clearly requires a much deeper understanding of the dynamics in the UV and beyond the cut-off of the model. Nevertheless, the model we discuss does set the scene for a more profound dynamical understanding of the physics of flavor by creating a framework (with a special, and rather restrictive, choice of textures for the mass matrices of the fermions) and identifying the relevant symmetries and degrees of freedom at or around the TeV scale that make an unification between flavor and electroweak physics possible within a natural model. In doing so, it says something specific about physics in the range to be explored by LHC, giving a (lightest) Higgs boson mass in a well defined range and particles in addition to those of the standard model to be discovered.

3.1 The model

In order to have a single, unified model *à la* little Higgs describing the entire flavor structure as well as the electroweak symmetry breaking, the Higgs boson and the flavons must be the PGBs of the same spontaneously broken global symmetry. These PGBs—we

[1] This global symmetry, as well as those of the little Higgs model, must be thought as arising at same intermediate scale, well below that of string theory where all symmetries are necessarily local.

shall call them *flhiggs*—should transform under both flavor and electroweak symmetries. The symmetry breaking should leave the electroweak (or an extended symmetry, a subgroup of which is the electroweak) and the flavor symmetry unbroken. In a further step, the flavor and the electroweak symmetries break leaving, as in the standard model, the electric charge $U(1)_Q$ as the only unbroken symmetry.

To construct such a model, it is necessary first to identify the flavor and electroweak symmetry subgroups at the scale f of the spontaneous symmetry breaking of the global symmetry. The simplest choice would seem to be a product of the flavor symmetry G_F and the electroweak symmetry $[SU(2) \times U(1)]_W$, where for the flavor group G_F we can take, without loss of generality, $U(N)_F$. In this case the flhiggs bosons should transform, for example if we take $U(N)_F$ to be $U(2)_F \simeq SU(2)_F \times U(1)_F$, as doublets in the fundamental representations of the two $SU(2)$ groups, the electroweak and the $SU(2)_F$ in $U(2)_F$. However we have to reject this choice since with the scalar fields as doublets in both groups the scale of flavor breaking will necessarily coincide with that of the electroweak breaking with undesirable consequences for the phenomenology of the model.

This holds true for any choice of G_F. We are therefore necessarily lead to extend the electroweak symmetry and the minimal extention gives us $[U(N)]_F \times [SU(3) \times U(1)]_W$. In this case if the flhiggs bosons transform in the fundamental representations of both groups, the breaking of the flavor symmetry can happen at a scale different from the electroweak, that is, there is a limit in which the flavor symmetry is broken and its breaking induces the breaking of the $[SU(3) \times U(1)]_W$ electroweak symmetry to the standard $[SU(2) \times U(1)]_W$. This extention brings into the model an extra neutral gauge boson and exotic fermion states necessary to complete the weak doublets. These additional states give rise to new physics with crucial phenomenological consequences for the model. The mass and mixing of the extra gauge boson affect the neutral currents and impose rather severe bounds on the parameters of the model. Moreover, the masses and mixing of the exotic with standard fermions must be controlled by some additional symmetry that we take for simplicity to be an abelian $U(1)$. The exotic fermions, being charged under this abelian symmetry, only weakly couple to the standard fermions and acquire heavier masses.

3.1.1 What is the horizontal flavor symmetry?

The flavor symmetry could, in principle be abelian or nonabelian, that is a $U(2)$ since, for three generation at least, a $U(3)$ would introduce no differentiation. Let then consider the nonabelian $SU(2)$ case. [2] The flhiggs bosons arising as PGBs in a model of this kind are in the fundamental representations of both the flavor and the weak $SU(2)$ groups. Therefore, they transform as $(2,3)$ and $(2,\overline{3})$ under the flavor-electroweak symmetry.

In order to construct flavor-electroweak invariant Yukawa term we have to choose the representations for the standard fermions. The left-handed fermions have to transform as a 3 of $[SU(3)\times U(1)]_W$ since we want a doublet when $[SU(3)\times U(1)]_W$ is broken to $[SU(2)\times U(1)]_W$. As already noticed, we are obliged to introduce at least one exotic left-handed fields for each quark and lepton family. On the contrary, the right-handed ones could be singlets of $SU(3)$ or the third component of a triplet (anti-triplet) of weak $SU(3)$. Notice that in the latter case we would have to introduce other two exotic right-handed fermions for each quark and lepton family.

We still have to assign the representations with respect to the flavor group. We could have singlets, doublets or triplets. We reject the last case since it is impossible to reproduce the right hierarchies by this choice for either left-handed or right-handed fermions or both. If we use singlets and doublets we have, for instance, that two left-handed fermion families form a flavor doublet and the third is a singlet. In this case the choice for the left-handed fermion representations severely restricts that of the right-handeds while there is no mixing between the doublet and the singlet.

Consider for example the Yukawa term for the charged leptons and suppose that the second and the third family are in a flavor doublet, while the first family is a singlet with respect to the flavor symmetry. This assignment is motivated by the results obtained in [51, 73]. Each left-handed lepton family forms a triplet of $SU(3)_W$, so we have

$$L_L^e = \begin{pmatrix} \nu_L^e \\ e_L \\ \tilde{e}_L \end{pmatrix} = (1,3)_L \qquad E_L = \begin{pmatrix} \nu_L^i \\ e_L^i \\ \tilde{e}_L^i \end{pmatrix} = (2,3)_L\,. \tag{3.1}$$

[2] Which is the symmetry discussed in the little flavon model analyzed in the previous chapter and in [51, 73].

with $i = 1,2$, $e^1 = \mu$ and $e^2 = \tau$ and an exotic lepton for each family. In eq. (3.2) we have indicated in the brackets the fields representations with respect to flavor $SU(2)$ and weak $SU(3)$, respectively. There are only two possible choices for the representations of the right-handed charged leptons in order to have a Yukawa term involving L_L^e that give mass to the electron and these are

$$\begin{aligned} \overline{e}_R = (1,3) &\rightarrow (1,\overline{3})_R(2,3)_\phi(2,\overline{3})_\phi(1,3)_L \\ \overline{E}_R = (2,1) &\rightarrow (2,1)_R(2,\overline{3})_\phi(1,3)_L\,, \end{aligned} \tag{3.2}$$

and analogously for E_L with other two possibilities

$$\begin{aligned} \overline{e}^i_R = (1,1) &\rightarrow (1,1)_R(2,\overline{3})_\phi(2,3)_L \\ \overline{E}_R = (2,\overline{3}) &\rightarrow (2,\overline{3})_R(2,3)_\phi(2,\overline{3})_\phi(2,3)_L\,. \end{aligned} \tag{3.3}$$

In eq. (3.3) we have indicated the fields by the indices: ϕ stands for the PGBs, R and L for right-handed and left-handed leptons respectively. By comparing eq. (3.2) with eq. (3.3) we see that there is no choice for the right-handed fermions representations that permits mixing between the first family charged lepton and the other two. As it happens, we have the same problem in the neutrino sector, and this means that it is impossible to reproduce the experimental lepton mixing matrix since we cannot have an angle different from zero between the first and the second family. In conclusion, we are forced to use only flavor singlets. Notice that we would arrive at the same conclusion if we had started with a doublet composed by the first and the second family or if we had considered the quark sector.

The previous analysis shows that the introduction of a nonabelian flavor symmetry is not helpful since we are forced to use only flavor singlets as representations of the standard fermions if we want to reproduce the correct textures in the mixing matrices. Such a symmetry could in principle be useful if the PGBs content would be enlarged by making the flhiggs belong to different representations of $SU(3)$ and the flavor group, for instance, by having weak singlets in addition to doublets. Our aim is to build a model as simple as possible and therefore we try avoiding such an enlargement. This leads us to taking the abelian group $U(1)$ as our flavor symmetry.

3.1.2 Spontaneous symmetry breaking

Our discussion so far has lead us to identify the low-energy symmetry we expect to see realized in the model as $[SU(3) \times U(1)]_W \times U(1)_F$ plus the additional symmetry, which we take to be $U(1)_X$, that controls the exotic fermions.

Once chosen the symmetry at the lower scale, we have to identify the minimal global symmetry, the spontaneous breaking of which gives rise to the PGBs to be identified with flavons and Higgs boson. Since we need at least two copies of $SU(3) \times U(1)$, plus two copies of an extra $U(1)$ to control the masses of the exotic fermions, we end up with a group of rank 9, that we take to be $SU(10)$.

The $SU(10)$ global symmetry is spontaneously broken to $SO(10)$ at the scale f. This provides us with an effective theory with a cut-off at the scale $\Lambda = 4\pi f$. Fifty-four generators of $SU(10)$ are broken giving 54 real GBs we parametrize in a non-linear sigma model fashion as

$$\Sigma(x) = \exp\left[i\Pi(x)/f\right]\Sigma_0\,, \tag{3.4}$$

with $\Pi(x) = t^a\pi^a(x)$, where t^a are the broken generators of $SU(10)$, $\pi^a(x)$ the fluctuations around the vacuum Σ_0 given by

$$\Sigma_0 \equiv \langle\Sigma\rangle = \left(\begin{array}{cc|cc} 0 & I_{4\times4} & 0 & 0 \\ I_{4\times4} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right). \tag{3.5}$$

The vacuum state (3.5) can be rotated into its canonical form $I_{10\times10}$ by a change of basis. In this basis the breaking pattern is more evident but the sigma model dynamics more involved.

Within $SU(10)$ we identify seven subgroups

$$SU(10) \supset U(1)_F \times [SU(3) \times U(1)]^2_W \times [U(1)_X]^2\,, \tag{3.6}$$

where the $U(1)_F$ is the global flavor symmetry while the $[SU(3) \times U(1)]^2_W$ are two copies of an extended electroweak gauge symmetry, the need of which we discussed in the previous section. The groups $[U(1)_X]^2$ are two copies of an extra gauge symmetry we need in order

$$\begin{array}{ccc} SU(10) & \longrightarrow & SO(10) \\ \downarrow & & \downarrow \\ [SU(3) \times U(1)]_W^2 \times U(1)_F \times [U(1)_X]^2 & \longrightarrow & [SU(3) \times U(1)]_W \times U(1)_F \times U(1)_X \end{array}$$

Figure 3.1: Diagrammatic representation of the symmetry breaking structure of the sigma model. Horizontal arrows indicate the spontaneous $SU(10) \to SO(10)$ global symmetry breaking, vertical arrows the explicit breaking due to gauge interactions and plaquette terms (see the discussion in the text main body).

to separate standard fermions from the exotic fermions the model requires because of the enlarged $SU(3)_W$ symmetry that turns the weak doublets into triplets.

As discussed at the beginning of this chapter, we want the flavor symmetry proper to be global so as not to have in the theory flavor charged gauge bosons that would make impossible for flavor and weak symmetry breaking to be of the same order. On the other hand, all the other symmetries in addition to those of the standard model are local so as to reduce the number of GBs in the physical spectrum.

The generators of the five $U(1)$ are taken to be

$$\begin{aligned} Y_{F_1} &= \text{diag}\,(0,0,0,1,0,0,0,-1,0,0)/2 \\ Y_{W_1} &= \text{diag}\,(0,0,0,0,1,1,1,0,0,0)/\sqrt{6} \\ Y_{W_2} &= \text{diag}\,(1,1,1,0,0,0,0,0,0,0)/\sqrt{6}, \\ Y_{X_1} &= \text{diag}\,(0,0,0,0,0,0,0,0,1,0)/\sqrt{2} \\ Y_{X_2} &= \text{diag}\,(0,0,0,0,0,0,0,0,0,1)\sqrt{2}, \end{aligned} \tag{3.7}$$

while the generators of the two copies of $SU(3)_W$ can be identified with the corresponding generators Q_1^a and Q_2^a with $a = 1, ..., 8$ within $SU(10)$. Note that Y_{F_1} and $Y_{W_{1,2}}$ are generators of $SU(10)$, while $Y_{X_{1,2}}$ are not and their normalization is chosen for simple convenience.

The breaking of $SU(10)$ into $SO(10)$ also breaks the subgroups $[SU(3) \times U(1)]_W^2 \times [U(1)_X]^2$ and only a diagonal combination survives. On the contrary, the flavor symmetry

$U(1)_F$ survives the breaking and we eventually have that

$$U(1)_F \times [SU(3) \times U(1)]_W^2 \times [U(1)_X]^2 \to U(1)_F \times [SU(3) \times U(1)]_W \times U(1)_X \,. \tag{3.8}$$

The breaking in the gauge sector $[SU(3) \times U(1)]_W^2 \times [U(1)_X]^2 \to [SU(3) \times U(1)]_W \times U(1)_X$ leaves 10 gauge bosons massive after eating 10 of the 54 real GBs. The remaining 44 GBs can be labeled according to representations of the $U(1)_F \times [SU(3) \times U(1)]_W \times U(1)_X$ symmetry:

- 2 complex fields $\Phi_1\,[3_{(1,1/2,0)}]$ and $\Phi_2\,[3_{(1,-1/2,0)}]$, accounting for 12 degrees of freedom. They transform as triplets of $[SU(3)]_W$ and have the same $U(1)_W$ and opposite $U(1)_F$ charges. They are not charged under the exotic gauge symmetry $U(1)_X$.

- 2 complex fields $\Phi_3\,[3_{(1,0,1/2)}]$ and $\Phi_4\,[3_{(1,0,-1/2)}]$, accounting for other 12 degrees of freedom. They transform as triplets of $[SU(3)]_W$ and have the same $U(1)_W$ and opposite $U(1)_X$ charge. They are not charged under the flavor symmetry $U(1)_F$.

- a sextet of complex fields $z_{ij}\,[6_{(2,0,0)}]$, for 12 degrees of freedom,

- 4 complex fields $s\,[1_{(0,-1,0)}]$, $s_1\,[1_{(0,-1/2,1/2)}]$, $s_2\,[1_{(0,1/2,1/2)}]$, $s_3\,[1_{(0,0,-1)}]$, for for the remaining 8 degrees of freedom.

In the above notation, the representations with respect to the $SU(3)_W$ are indicated between square brackets and the indexes are the $U(1)$ charges: the first refers to the weak group, the second to the flavor group and the third to the exotic.

In terms of these representations, the field $\Pi(x)$ can then be written as

$$\Pi(x) = \left(\begin{array}{ccc|c|ccc|c|cc}
0 & 0 & 0 & & & & & & & \\
0 & 0 & 0 & \Phi_1/\sqrt{2} & & z_{ij} & & \Phi_2/\sqrt{2} & \Phi_3/\sqrt{2} & \Phi_4/\sqrt{2} \\
0 & 0 & 0 & & & & & & & \\
\hline
 & \Phi_1^*/\sqrt{2} & & 0 & & \Phi_2/\sqrt{2} & & s & s_1/2 & s_2/2 \\
\hline
 & & & & 0 & 0 & 0 & & & \\
 & z_{ij}^* & & \Phi_2^*/\sqrt{2} & 0 & 0 & 0 & \Phi_1^*/\sqrt{2} & \Phi_4^*/\sqrt{2} & \Phi_3^*/\sqrt{2} \\
 & & & & 0 & 0 & 0 & & & \\
\hline
 & \Phi_2^*/\sqrt{2} & & s^* & & \Phi_1/\sqrt{2} & & 0 & s_2^*/2 & s_1^*/2 \\
\hline
 & \Phi_3^*/\sqrt{2} & & s_1^*/2 & & \Phi_4/\sqrt{2} & & s_2/2 & 0 & s_3 \\
 & \Phi_4^*/\sqrt{2} & & s_2^*/2 & & \Phi_3/\sqrt{2} & & s_1/2 & s_3^* & 0
\end{array}\right), \tag{3.9}$$

where we have put zeros for the components that are going to be eaten by the gauge fields becoming massive. All these fields are still GBs with no potential; their potential arises after the explicit breaking of the symmetry to which we now turn.

3.1.3 Explicit collective symmetry breaking

The effective lagrangian of the PGBs must contain terms that explicitly break the $SU(10)$ global symmetry. These terms provide masses of the order of the scale f for the s, s_i and z_{ij} fields. However, each term separately preserve enough symmetry to keep the flhiggs fields Φ_i exact GBs. Only the simultaneous action of two or more of the terms (collective breaking) turns them into PGBs with a potential, even though there is still no mass term. Quadratic terms for the flhiggs will come from the coupling to right-handed neutrino, as we shall discuss presently.

The effective lagrangian is given by the kinetic term

$$\mathcal{L}_0 = \frac{f^2}{2}\mathrm{Tr}\,(D^\mu\Sigma)(D_\mu\Sigma)^*\,, \tag{3.10}$$

the covariant derivative of which couples the PGBs to the gauge fields:

$$D_\mu\Sigma = \partial_\mu + ig_i W^a_{i_\mu}(Q^a_i\Sigma + \Sigma Q^{a,T}_i) + ig'_i y_i B_{i_\mu}(Y_{W_i}\Sigma + \Sigma Y^T_{W_i}) + ik_i X_{i_\mu}(Y_{X_i}\Sigma + \Sigma Y^T_{X_i}) \quad i = 1, 2\,, \tag{3.11}$$

where $W^a_{i_\mu}$, B_{i_μ} and X_{i_μ} are the gauge bosons of the $SU(3)_{Wi}$, $U(1)_{Wi}$ and $U(1)_{Xi}$ respectively , Q^a_i, Y_{W_i} and Y_{X_i} their generators and y_i the $U(1)_{Wi}$ charges.

The lagrangian in eq. (3.11) gives mass to the z_{ij} and s_3 fields. On the other hand, each term of index i preserves a $SU(3)$ symmetry so that only when taken together they can give a contribution to the potential of the flhiggs fields.

At this point the fields s, s_1 and s_2 are still massless. They play no important role in the model but cannot remain massless. To give them a mass, we introduce plaquette terms—terms made out of components of the Σ field that preserve enough symmetry not to induce masses for the flhiggs fields.

As an example, one of these plaquette term can be written by looking at the Goldstone fields in the matrix eq. (3.9) after having rotate it by the vacuum Σ_0. We select the field s^* to which we want to give mass in the components (8, 8) and (4, 4) of the matrix $\Sigma(x)$. Both these choices leave a different $SU(9)$ symmetry acting on the remaining columns and rows that then prevents further terms to the potential of the fields that transform in the coset of $SU(10)/SU(9)$.

The other possible plaquette terms are given by choosing by the same token the two couples $(4,10)$ and $(8,9)$ and $(4,9)$ and $(8,10)$ components to give masses to the s_1 and s_2 respectively. Together they induce (harmless) terms and corrections into the coefficients of the flhiggs potential.

After adding the plaquette terms, we therefore have the effective lagrangian

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0 + a_1^2 f^2 \Sigma_{4,4}\Sigma^*_{4,4} + a_2^2 f^2 \Sigma_{8,8}\Sigma^*_{8,8} + a_3^2 f^2 \Sigma_{4,9}\Sigma^*_{4,9} \\ &+ a_4^2 f^2 \Sigma_{8,10}\Sigma^*_{8,10} + a_5^2 f^2 \Sigma_{4,10}\Sigma^*_{4,10} + a_6^2 f^2 \Sigma_{8,9}\Sigma^*_{8,9}\,, \end{aligned} \tag{3.12}$$

where a_i are coefficients of $\mathcal{O}(1)$. The relative signs of the plaquette terms are in principle arbitrary and presumably fixed by the UV completion of the theory. At this level we simply require $m^2_{s_i} > 0$.

In sec. (3.1.2) we said that in the breaking of $[SU(3)\times U(1)]^2_W \to [SU(3)\times U(1)]_W$ nine gauge bosons become massive. We now see that their masses are given by

$$M^2_{W'_a} = \frac{(g_1^2+g_2^2)}{2} f^2\,, \quad M^2_{B'} = \frac{(g'^2_1+g'^2_2)}{2} f^2\,, \quad M^2_{X'} = \frac{(k_1^2+k_2^2)}{2} f^2\,, \tag{3.13}$$

where $a = 1, \ldots, 8$.

These heavy gauge bosons—because of their mixing with those with lighter masses to be identified with the standard model gauge bosons—induce corrections on many observables that we know to be constrained by high-precision measurements, mainly coming from low-energy physics (like atomic parity violation and neutrino-hadron scattering). Their presence is the major constrain on the scale f and, accordingly, the naturalness of the model, as discussed for the littlest-Higgs model in [69]. We shall come back to them when we discuss these constrains in the flhiggs model in section 3.1.5.

The effective potential for the flhiggs fields is given by the tree-level contribution coming from the plaquettes and the one-loop CW effective potential arising from the gauge interactions:

$$\frac{\Lambda^2}{16\pi^2}\mathrm{Tr}\,[M^2(\Sigma)] + \frac{3}{64\pi^2}\mathrm{Tr}\left[M^4(\Sigma)\left(\log\frac{M^2(\Sigma)}{\Lambda^2} + \mathrm{const.}\right)\right], \tag{3.14}$$

where the second, logarithmic terms is very much suppressed and is not included in what follows.

The effective potential $O(f^{-2})$ is obtained by expanding the sigma-model field Σ and is given by

$$\begin{aligned}
\mathcal{V}_0[\Phi_i, z_{ij}, s, s_i] &= \frac{2}{3} g_1^2 f^2 |\frac{z_{ij}}{\sqrt{2}} - \frac{i}{2\sqrt{2}f}(\Phi_{1_i}\Phi_{2_j} + \Phi_{3_i}\Phi_{4_j})|^2 \\
&+ \frac{2}{3} g_2^2 f^2 |\frac{z_{ij}}{\sqrt{2}} + \frac{i}{2\sqrt{2}f}(\Phi_{1_i}\Phi_{2_j} + \Phi_{3_i}\Phi_{4_j})|^2 \\
&+ \frac{1}{3} g'^2_1 f^2 |\frac{z_{ij}}{\sqrt{2}} - \frac{i}{2\sqrt{2}f}(\Phi_{1_i}\Phi_{2_j} + \Phi_{3_i}\Phi_{4_j})|^2 \\
&+ \frac{1}{3} g'^2_2 f^2 |\frac{z_{ij}}{\sqrt{2}} + \frac{i}{2\sqrt{2}f}(\Phi_{1_i}\Phi_{2_j} + \Phi_{3_i}\Phi_{4_j})|^2 \\
&+ \frac{1}{2} k_1^2 f^2 |s_3 - \frac{i}{2f}(\frac{s_2 s_1^*}{2} + \Phi_3^\dagger\Phi_4)|^2 \\
&+ \frac{1}{2} k_2^2 f^2 |s_3 + \frac{i}{2f}(\frac{s_2 s_1^*}{2} + \Phi_3^\dagger\Phi_4)|^2 \\
&+ a_1^2 f^2 \, |s - \frac{i}{2f}(\frac{s_1 s_2}{2} + \Phi_1^\dagger\Phi_2)|^2 \\
&+ a_2^2 f^2 \, |s + \frac{i}{2f}(\frac{s_1 s_2}{2} + \Phi_1^\dagger\Phi_2)|^2 \\
&+ \frac{1}{4} a_3^2 f^2 \, |s_1 - \frac{i}{2f}(s s_2^* + s_2 s_3^* + \Phi_1^\dagger\Phi_3 + \Phi_4^\dagger\Phi_2)|^2
\end{aligned}$$

$$
\begin{aligned}
&+ \frac{1}{4}\, a_4^2\, f^2\, |s_1 + \frac{i}{2f}(s s_2^* + s_2 s_3^* + \Phi_1^\dagger \Phi_3 + \Phi_4^\dagger \Phi_2)|^2 \\
&+ \frac{1}{4}\, a_5^2\, f^2\, |s_2 - \frac{i}{2f}(s s_1^* + s_1 s_3 + \Phi_1^\dagger \Phi_4 + \Phi_3^\dagger \Phi_2)|^2 \\
&+ \frac{1}{4}\, a_6^2\, f^2\, |s_2 + \frac{i}{2f}(s s_1^* + s_1 s_3 + \Phi_1^\dagger \Phi_4 + \Phi_3^\dagger \Phi_2)|^2 \,. \qquad (3.15)
\end{aligned}
$$

From eq. (3.15) we see by inspection that the effective potential gives mass to the scalar fields s, s_1, s_2, s_3 and z, their masses given by

$$
\begin{aligned}
m_z^2 &= \frac{(2g_1^2 + 2g_2^2 + g'^2_1 + g'^2_2)}{6} f^2 \qquad m_{s_3}^2 = \frac{(k_1^2 + k_2^2)}{2} f^2 \\
m_s^2 &= (a_1^2 + a_2^2) f^2 \qquad m_{s_1}^2 = \frac{(a_3^2 + a_4^2)}{2} f^2 \qquad m_{s_2}^2 = \frac{(a_5^2 + a_6^2)}{2} f^2 \,, \qquad (3.16)
\end{aligned}
$$

respectively. The effect of these states must be included in the study of the low-energy observables together with that of the heavy gauge bosons.

After integrating out the massive states by means of their equations of motion, the potential of the four PGBs Φ_i, the flhiggs, is made of only quartic terms

$$
\begin{aligned}
\mathcal{V}_1[\Phi_i] &= \lambda_1(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_2(\Phi_3^\dagger\Phi_3)(\Phi_4^\dagger\Phi_4) + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_3^\dagger\Phi_3) + \lambda_4(\Phi_2^\dagger\Phi_2)(\Phi_4^\dagger\Phi_4) \\
&+ \lambda_5(\Phi_1^\dagger\Phi_1)(\Phi_4^\dagger\Phi_4) + \lambda_6(\Phi_2^\dagger\Phi_2)(\Phi_3^\dagger\Phi_3) \qquad (3.17) \\
&+ \xi_1|\Phi_1^\dagger\Phi_2|^2 + \xi_2|\Phi_3^\dagger\Phi_4|^2 + \xi_3(\Phi_1^\dagger\Phi_3)(\Phi_2^\dagger\Phi_4) + \xi_4(\Phi_1^\dagger\Phi_4)(\Phi_2^\dagger\Phi_3) \,,
\end{aligned}
$$

where the coefficients are given by

$$
\lambda_1 = \lambda_2 = \frac{(2g_1^2 + g'^2_1)(2g_2^2 + g'^2_2)}{2g_1^2 + g'^2_1 + 2g_2^2 + g'^2_2} \qquad \lambda_3 = \lambda_4 = \frac{a_3^2 a_4^2}{a_3^2 + a_4^2} \qquad \lambda_5 = \lambda_6 = \frac{a_5^2 a_6^2}{a_5^2 + a_6^2} \qquad (3.18)
$$

and

$$
\begin{aligned}
\xi_1 &= \frac{(2g_1^2 + g'^2_1)(2g_2^2 + g'^2_2)}{2g_1^2 + g'^2_1 + 2g_2^2 + g'^2_2} + \frac{a_1^2 a_2^2}{a_1^2 + a_2^2} \qquad \xi_2 = \frac{k_1^2 k_2^2}{k_1^2 + k_2^2} + \frac{a_5^2 a_6^2}{a_5^2 + a_6^2} \qquad (3.19) \\
\xi_3 &= \frac{(2g_1^2 + g'^2_1)(2g_2^2 + g'^2_2)}{2g_1^2 + g'^2_1 + 2g_2^2 + g'^2_2} + \frac{a_3^2 a_4^2}{a_3^2 + a_4^2} \qquad \xi_4 = \frac{(2g_1^2 + g'^2_1)(2g_2^2 + g'^2_2)}{2g_1^2 + g'^2_1 + 2g_2^2 + g'^2_2} + \frac{a_5^2 a_6^2}{a_5^2 + a_6^2} \,.
\end{aligned}
$$

The coefficients ξ_1, ξ_3 and ξ_4 differ only by the plaquette contributions. Notice that we can take them equal if we assume the plaquette coefficients to be equal as well.

Quadratic terms

$$
\mathcal{V}_2[\Phi_i] = \mu_1^2(\Phi_1^\dagger\Phi_1) + \mu_2^2(\Phi_2^\dagger\Phi_2) + \mu_3^2(\Phi_3^\dagger\Phi_3) + \mu_4^2(\Phi_4^\dagger\Phi_4) \qquad (3.20)
$$

that are necessary to induce vacuum expectation values for the flhiggs fields, and quartic terms of the type

$$\mathcal{V}_3[\Phi_i] = \chi_1(\Phi_1^\dagger\Phi_1)^2 + \chi_2(\Phi_2^\dagger\Phi_2)^2 + \chi_3(\Phi_3^\dagger\Phi_3)^2 + \chi_4(\Phi_4^\dagger\Phi_4)^2 \tag{3.21}$$

are not generated at one–loop in the bosonic sector discussed so far. In order to introduce them we couple the PGBs to right-handed neutrinos with masses at the scale f. This means that the flavor and electroweak symmetry breaking of the model is triggered by the right-handed neutrinos. This is done again along the lines of the little-Higgs collective symmetry breaking: to prevent quadratically divergent mass term for Φ_i—and thus render useless what done up to this point—the Yukawa lagrangian of the right-handed neutrinos sector is constructed by terms that taken separately leave invariant some subgroups of the approximate global symmetry $SU(10)$. In this way the flhiggs bosons receive a mass term only from diagrams in which all the approximate global symmetries of the Yukawa lagrangian are broken. Because of this collective breaking, the one-loop contributions to the flhiggs masses are only logarithmic divergent.

The right handed neutrino sector is given by sixteen 10-components multiplets

$$N_R^1 = \begin{pmatrix} 0_\alpha \\ \nu_R^1 \\ 0_\alpha \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad N_R^2 = \begin{pmatrix} 0_\alpha \\ 0 \\ 0 \\ \nu_R^2 \\ 0 \\ 0 \end{pmatrix} \quad N_R^3 = \begin{pmatrix} 0_\alpha \\ 0 \\ 0 \\ 0_\alpha \\ \nu_R^3 \\ 0 \end{pmatrix} \quad N_R^4 = \begin{pmatrix} 0_\alpha \\ 0 \\ 0 \\ 0 \\ 0_\alpha \\ \nu_R^4 \end{pmatrix}$$

$$N_R^5 = \begin{pmatrix} \nu_{R,\alpha}^5 \\ 0 \\ {\nu'}_{R,\alpha}^5 \\ \tilde{\nu}'^5_R \\ \hat{\nu}_R^5 \\ \hat{\nu}'^5_R \end{pmatrix} \quad N_R^6 = \begin{pmatrix} \nu_{R,\alpha}^6 \\ \tilde{\nu}_R^6 \\ {\nu'}_{R,\alpha}^6 \\ 0 \\ \hat{\nu}_R^6 \\ \hat{\nu}'^6_R \end{pmatrix} \quad N_R^7 = \begin{pmatrix} \nu_{R,\alpha}^7 \\ \tilde{\nu}_R^7 \\ {\nu'}_{R,\alpha}^7 \\ \tilde{\nu}'^7_R \\ 0 \\ \hat{\nu}'^7_R \end{pmatrix} \quad N_R^8 = \begin{pmatrix} \nu_{R,\alpha}^8 \\ \tilde{\nu}_R^8 \\ {\nu'}_{R,\alpha}^8 \\ \tilde{\nu}'^8_R \\ \hat{\nu}_R^8 \\ 0 \end{pmatrix}$$

$$N_R^9 = \begin{pmatrix} \nu_{R,\alpha}^9 \\ 0 \\ 0 \\ 0 \\ \hat{\nu}_R^9 \\ 0 \end{pmatrix} \quad N_R^{10} = \begin{pmatrix} \nu_{R,\alpha}^{10} \\ 0 \\ 0 \\ 0 \\ \hat{\nu}_R^{10} \\ 0 \end{pmatrix} \quad N_R^{11} = \begin{pmatrix} 0_\alpha \\ 0 \\ \nu'^{11}_{R,\alpha} \\ 0 \\ 0 \\ \hat{\nu}'^{11}_R \end{pmatrix} \quad N_R^{12} = \begin{pmatrix} 0_\alpha \\ 0 \\ \nu'^{12}_{R,\alpha} \\ 0 \\ 0 \\ \hat{\nu}'^{12}_R \end{pmatrix}$$

$$N_R^{13} = \begin{pmatrix} \nu_{R,\alpha}^{13} \\ \tilde{\nu}_R^{13} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad N_R^{14} = \begin{pmatrix} \nu_{R,\alpha}^{14} \\ \tilde{\nu}_R^{14} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad N_R^{15} = \begin{pmatrix} 0_\alpha \\ 0 \\ \nu'^{15}_{R,\alpha} \\ \tilde{\nu}'^{15}_R \\ 0 \\ 0 \end{pmatrix} \quad N_R^{16} = \begin{pmatrix} 0_\alpha \\ 0 \\ \nu'^{16}_{R,\alpha} \\ \tilde{\nu}'^{15}_R \\ 0 \\ 0 \end{pmatrix}, \quad (3.22)$$

where $\alpha = 1, 2, 3$ and $0_\alpha = (0, 0, 0)^T$.

Only the N_R^i with $a = 1, \cdots, 8$ couple directly to the fermions. The reason why we introduce so many fields is that we eventually want different, and independent, mass terms μ_i to be induced in the effective potential by the right-handed neutrino sector and also we do not want right-handed neutrinos massless.

The Yukawa lagrangian for the right-handed neutrinos can be written in a $SU(10)$-invariant manner as

$$\begin{aligned} \mathcal{L}_Y^{\nu_R} &= \eta_1 f(\overline{N_L^{1c}} \Sigma N_R^5) + \eta_2 f(\overline{N_L^{2c}} \Sigma N_R^6) + \eta_3 f(\overline{N_L^{3c}} \Sigma N_R^7) + \eta_4 f(\overline{N_L^{4c}} \Sigma N_R^8) \\ &+ \eta_5 f(\overline{N_L^{9c}} N_R^5) + \eta_6 f(\overline{N_L^{11c}} N_R^5) + \eta_7 f(\overline{N_L^{10c}} N_R^6) + \eta_8 f(\overline{N_L^{12c}} N_R^6) \\ &+ \eta_9 f(\overline{N_L^{13c}} N_R^7) + \eta_{10} f(\overline{N_L^{15c}} N_R^7) + \eta_{11} f(\overline{N_L^{14c}} N_R^8) + \eta_{12} f(\overline{N_L^{16c}} N_R^8) . \quad (3.23) \end{aligned}$$

In eq. (3.23) all the terms leave invariant different subgroups of $SU(10)$: the first four, four different $SU(9)$ global symmetries—easily identifiable by the zeros in N_{1-4}—the remaining eight different $SU(6)$—to be identified by the zeros in N_{9-12}.

Substituting in eq. (3.23) the expression $O(f^{-2})$ for Σ—as given in eq. (3.9)—and for the right-handed neutrino multiplets the expression given in eq. (3.22), we obtain the leading order lagrangian

$$
\begin{aligned}
\mathcal{L}_Y^{\nu_R} &= \eta_1 f \Big[\overline{\nu_L^{1c}}\tilde{\nu}'^5_R\Big(1-\frac{\Phi_1^\dagger\Phi_1}{2f^2}-\frac{\Phi_2^\dagger\Phi_2}{2f^2}-\frac{(2ss^*+s_1s_1^*+s_2s_2^*)}{2f^2}\Big) \\
&+ \frac{i}{f}\overline{\nu_L^{1c}}\Big(\Phi_2^T\nu_R^5+\Phi_1^\dagger\nu'^5_R+s_1\hat{\nu}'^5_R+s_2\hat{\nu}^5_R\Big)\Big] \\
&+ \eta_2 f \Big[\overline{\nu_L^{2c}}\tilde{\nu}^6_R\Big(1-\frac{\Phi_1^\dagger\Phi_1}{2f^2}-\frac{\Phi_2^\dagger\Phi_2}{2f^2}-\frac{(2ss^*+s_1s_1^*+s_2s_2^*)}{2f^2}\Big) \\
&+ \frac{i}{f}\overline{\nu_L^{2c}}\Big(\Phi_1^T\nu_R^6+\Phi_2^\dagger\nu'^6_R+s_2^*\hat{\nu}'^6_R+s_1^*\hat{\nu}^6_R\Big)\Big] \\
&+ \eta_3 f \Big[\overline{\nu_L^{3c}}\hat{\nu}'^7_R\Big(1-\frac{\Phi_3^\dagger\Phi_3}{2f^2}-\frac{\Phi_4^\dagger\Phi_4}{2f^2}-\frac{(2s_3s_3^*+s_1s_1^*+s_2s_2^*)}{2f^2}\Big) \\
&+ \frac{i}{f}\overline{\nu_L^{3c}}\Big(\Phi_4^T\nu_R^7+\Phi_3^\dagger\nu'^7_R+s_1^*\tilde{\nu}'^7_R+s_2\tilde{\nu}^7_R\Big)\Big] \qquad (3.24) \\
&+ \eta_4 f \Big[\overline{\nu_L^{4c}}\hat{\nu}^8_R\Big(1-\frac{\Phi_3^\dagger\Phi_3}{2f^2}-\frac{\Phi_4^\dagger\Phi_4}{2f^2}-\frac{(2s_3s_3^*+s_1s_1^*+s_2s_2^*)}{2f^2}\Big) \\
&+ \frac{i}{f}\overline{\nu_L^{4c}}\Big(\Phi_3^T\nu_R^8+\Phi_4^\dagger\nu'^8_R+s_2^*\tilde{\nu}'^8_R+s_1\tilde{\nu}^8_R\Big)\Big] \\
&+ \eta_5 f\Big[\overline{\hat{\nu}_L^{9c}}\hat{\nu}_R^5+\overline{\nu_L^{9c}}\nu_R^5\Big]+\eta_6 f\Big[\overline{\hat{\nu}'^{11c}_L}\hat{\nu}'^5_R+\overline{\nu'^{11c}_L}\nu'^5_R\Big]+\eta_7 f\Big[\overline{\hat{\nu}_L^{10c}}\hat{\nu}_R^6+\overline{\nu_L^{10c}}\nu_R^6\Big] \\
&+ \eta_8 f\Big[\overline{\hat{\nu}'^{10c}_L}\hat{\nu}'^6_R+\overline{\nu_L^{10c}}\nu'^6_R\Big]+\eta_9 f\Big[\overline{\tilde{\nu}_L^{13c}}\tilde{\nu}_R^7+\overline{\nu_L^{13c}}\nu_R^7\Big]+\eta_{10} f\Big[\overline{\tilde{\nu}'^{15c}_L}\tilde{\nu}'^7_R+\overline{\nu'^{15c}_L}\nu'^7_R\Big] \\
&+ \eta_{11} f\Big[\overline{\tilde{\nu}_L^{14c}}\tilde{\nu}_R^8+\overline{\nu_L^{14c}}\nu_R^8\Big]+\eta_{12} f\Big[\overline{\tilde{\nu}'^{16c}_L}\tilde{\nu}'^8_R+\overline{\nu'^{16c}_L}\nu'^8_R\Big]\,.
\end{aligned}
$$

From eq. (3.24) we see that, after integrating out the neutrinos, the divergent one-loop contributions to the PGBs masses in the effective potential $\mathcal{V}_2[\Phi_i]$ of eq. (3.20) are given by

$$
\begin{aligned}
\mu_1^2 &\simeq (\eta_1^2\eta_5^2+\eta_2^2\eta_7^2)\frac{f^2}{(4\pi)^2}\log\frac{\Lambda^2}{M_\eta^2} \\
\mu_2^2 &\simeq (\eta_1^2\eta_6^2+\eta_2^2\eta_7^2)\frac{f^2}{(4\pi)^2}\log\frac{\Lambda^2}{M_\eta^2} \\
\mu_3^2 &\simeq (\eta_3^2\eta_{10}^2+\eta_4^2\eta_{11}^2)\frac{f^2}{(4\pi)^2}\log\frac{\Lambda^2}{M_\eta^2} \\
\mu_4^2 &\simeq (\eta_3^2\eta_9^2+\eta_4^2\eta_{12}^2)\frac{f^2}{(4\pi)^2}\log\frac{\Lambda^2}{M_\eta^2}\,, \qquad (3.25)
\end{aligned}
$$

where in the logarithm of eq. (3.25) we have generically indicated the mass of right handed neutrinos with $M_\eta \simeq \eta f$. The one-loop quadratically divergent contributions are cancelled by the collective symmetry breaking and the masses are always proportional to two of the coefficients η.

From eq. (3.24), we can also estimate the one-loop divergent contributions to the quartic terms in the effective potential $\mathcal{V}_3[\Phi_i]$ ofeq. (3.21) coming from the right-handed neutrino sector. The coefficients χ_i turn out to be logarithmically divergent and proportional to four, not necessarily different, powers of η:

$$\chi_i \simeq \eta_{k,j}^4 \frac{f^2}{(4\pi)^2} \log \frac{\Lambda^2}{M_\eta^2} \,. \tag{3.26}$$

They only play a minor role in what follows.

3.1.4 Vacuum expectation value

The effective potential for the pseudo GBs is therefore made of the sum of eqs. (3.18), (3.20) and (3.21)

$$\mathcal{V}[\Phi_i] = \mathcal{V}_1 + \mathcal{V}_2 + \mathcal{V}_3 \,. \tag{3.27}$$

We want to find vacuum expectation values for the flhiggs fields Φ_i in this potential that breaks the symmetry $[SU(3) \times U(1)]_W \times U(1)_F \times U(1)_X$ down to the electric charge group $U(1)_Q$. Such a vacuum is, in general, given by the field configurations

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ v_W/2 \\ v_{F_1}/2 \end{pmatrix} \qquad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ v_W/2 \\ v_{F_2}/2 \end{pmatrix} \qquad \langle\Phi_3\rangle = \begin{pmatrix} 0 \\ 0 \\ v_{X_1}/2 \end{pmatrix} \qquad \langle\Phi_4\rangle = \begin{pmatrix} 0 \\ 0 \\ v_{X_2}/2 \end{pmatrix} . \tag{3.28}$$

The conditions to be satisfied, in order for eq. (3.28) to be a minimum, are the vanishing of the 24 first derivatives:

$$\left.\frac{\partial V[\Phi]}{\partial \Phi_i}\right|_{\Phi_i=\langle\Phi_i\rangle} . \tag{3.29}$$

Substituting the field configuration of eq. (3.28) in eq. (3.29), we have 16 equations satisfied and eight conditions that v_W, v_{F1}, v_{F2}, v_{X1}, v_{X2} and the parameters of the potential must satisfy. Among them we have the following two equations

$$\begin{aligned} \xi_4 v_{F1} + \xi_3 v_{F2} &= 0 \\ \xi_3 v_{F1} + \xi_4 v_{F2} &= 0 \,. \end{aligned} \tag{3.30}$$

We take the solution in which $\xi_4 = \xi_3$ and $v_{F2} = -v_{F1}$. This solution is quite natural if, as pointed out in sec. 3.1.3, we consider all plaquette terms to come with equal strengths. We also impose for simplicity that $v_{X1} = v_{X2} = v_X$ and $v_{F1} = v_{F2} = v_F$. The values of v_X and v_F need not be equal but we shall identify them to obtain a model with only two vacuum values and simpler expressions for them in terms of the parameters. On the other hand, we do want to keep v_F distinct from v_W because otherwise the increase in symmetry would lead to the presence of extra GBs and other undesirable phenomenological consequences for the model.

Under these assumptions, the field configuration of eq. (3.28) becomes

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ v_W/2 \\ v_F/2 \end{pmatrix} \qquad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ v_W/2 \\ -v_F/2 \end{pmatrix} \qquad \langle\Phi_3\rangle = \begin{pmatrix} 0 \\ 0 \\ v_F/2 \end{pmatrix} \qquad \langle\Phi_4\rangle = \begin{pmatrix} 0 \\ 0 \\ v_F/2 \end{pmatrix}, \tag{3.31}$$

that is the vacuum expectation value we are going to use in what follows.

At this point we are left with six independent conditions that reduce to four if

$$\xi_3 = \Big(1 - \frac{v_W^2}{v_F^2}\Big)\xi_1\,, \tag{3.32}$$

The four remaining equations yield the following expressions for the vacua as function of the coefficients of the effective potential:

$$\begin{aligned} v_W^2 &= \frac{(\lambda_3+\lambda_6-\lambda_4-\lambda_5)(\mu_1^2-\mu_2^2+\mu_3^2-\mu_4^2)+2(\chi_2-\chi_1)(\mu_4^2-\mu_3^2)+2(\chi_4-\chi_3)(\mu_1^2-\mu_2^2)}{4(\chi_1-\chi_2)(\chi_3-\chi_4)-(\lambda_4-\lambda_3)^2-(\lambda_5-\lambda_6)^2)} \\ v_F^2 &= \frac{(\lambda_3+\lambda_6-\lambda_4-\lambda_5)(\mu_1^2-\mu_2^2)+2(\chi_2-\chi_1)(\mu_3^2-\mu_4^2)}{4(\chi_1-\chi_2)(\chi_3-\chi_4)-(\lambda_4-\lambda_3)^2-(\lambda_5-\lambda_6)^2}\,, \end{aligned} \tag{3.33}$$

and also yield the following conditions on the μ_i^2 since we have reduced the number of degrees of freedom by imposing the previous equalities.

$$\begin{aligned} &- \frac{-\mu_2^2(2\xi_1-\xi_2-2\chi_3-\lambda_2-\lambda_6-\lambda_3)+\mu_3^2(\xi_1-2\chi_2-\lambda_1-\lambda_4-\lambda_6)}{(\lambda_1+\xi_1+2\chi_2)(2\xi_1-\xi_2-2\chi_3-\lambda_2-\lambda_6-\lambda_3)-(\lambda_6+\lambda_3)(\xi_1-2\chi_2-\lambda_1-\lambda_4-\lambda_6)} \\ &+ \frac{(-\mu_1^2+\mu_2^2)(-2\chi_3+2\chi_4+\lambda_4+\lambda_5-\lambda_6-\lambda_3)+(\mu_3^2-\mu_4^2)(-2\chi_1+2\chi_2+\lambda_4-\lambda_5+\lambda_6-\lambda_3)}{(\lambda_4+\lambda_5-\lambda_6-\lambda_3)(-2\chi_1+2\chi_2+\lambda_4-\lambda_5+\lambda_6-\lambda_3)+2(\chi_1-\chi_2)(-2\chi_3+2\chi_4+\lambda_4+\lambda_5-\lambda_6-\lambda_3)} = 0 \\ &- \frac{(-\mu_1^2+\mu_2^2)(\xi_1-2\chi_2-\lambda_1-\lambda_4-\lambda_6)+\mu_2^2(-2\chi_1+2\chi_2+\lambda_4-\lambda_5+\lambda_6-\lambda_3)}{(-\lambda_1-\xi_1-2\chi_2)(-2\chi_1+2\chi_2+\lambda_4-\lambda_5+\lambda_6-\lambda_3)+2(\chi_1-\chi_2)(\xi_1-2\chi_2-\lambda_1-\lambda_4-\lambda_6)} \\ &+ \frac{(-\mu_1^2+\mu_2^2)(-2\chi_3+2\chi_4+\lambda_4+\lambda_5-\lambda_6-\lambda_3)+(\mu_3^2-\mu_4^2)(-2\chi_1+2\chi_2+\lambda_4-\lambda_5+\lambda_6-\lambda_3))}{(\lambda_4+\lambda_5-\lambda_6-\lambda_3)(-2\chi_1+2\chi_2+\lambda_4-\lambda_5+\lambda_6-\lambda_3))+2(\chi_1-\chi_2)(-2\chi_3+2\chi_4+\lambda_4+\lambda_5-\lambda_6-\lambda_3)} = 0\,, \end{aligned} \tag{3.34}$$

Also notice that all the relationships discussed can only be approximate since the coupling of the scalar fields to the fermions introduces small corrections.

The vacuum in eq. (3.31) breaks the global symmetry $U(1)_F$ and there seem to be a GB in the spectrum. It can be removed by a mass term introduced by hand at an intermediate scale between v_F and f. However, as it is possible to see after fermion will be introduced in the model, this global symmetry is actually anomalous. This means that the would-be Goldstone is not part of the physical spectrum. [3] Also notice that similar anomalies in the gauge groups are automatically compensated by the GBs, as it always happens in spontaneously broken gauge theories [54]; they however reappear above the scale f and may help in the determination of the UV completion of the theory.

In order to give a back-of-the-envelope estimate of this solution—and to see that it satisfies the requirements outlined in the introduction—it is useful to make a few approximations. Let us for instance take

$$\xi_i \simeq \chi_i \simeq \lambda_1 = \lambda_2 = \lambda_3 = \chi \qquad \lambda_4 = \lambda_5 = \lambda_6 = \lambda \tag{3.35}$$

to reduce the number of coefficients. These approximations are rather natural and do not introduce any fine-tuning. Accordingly, the vacuum of the potential in eq. (3.27) can be given as

$$v_F^2 = \frac{\mu_1^2 - \mu_2^2}{\lambda - \chi} \quad \text{and} \quad v_W^2 = \frac{\mu_1^2 - \mu_2^2 + \mu_3^2 - \mu_4^2}{\lambda - \chi}\,. \tag{3.36}$$

In this simplified case, and by further taking $\lambda - \chi \simeq 1$, $\mu_1^2 - \mu_2^2 \simeq -\mu^2/2$ and $\mu_3^2 - \mu_4^2 \simeq 3\mu^2/8$ (and $\chi \simeq 1/2$, $\mu_3^2 \simeq 2\mu^2$ to satisfy eq. (3.33)) we obtain that

$$v_W^2 = -\mu^2/4 \quad \text{and} \quad v_F^2 = -\mu^2 \tag{3.37}$$

so that for the electroweak vacuum given by its experimental value $v_W = -\mu/2 = 246$ GeV, we find $v_F \simeq 500$ GeV. For $f \simeq 1$ TeV, the parameter $k \equiv v_F^2/f^2$ in the mass textures turns out to be small and of the order of the Cabibbo angle.

In section 3.3 we will come back to the vacuum solution in eq. (3.28) and study it for arbitrary parameters to show the range of masses allowed for the scalar particles as well

[3] Alternatively, one can think of the anomaly as an effective mass for the would-be GB that, like the η' of the $U(1)_A$ symmetry of chiral perturbation theory, becomes massive with a mass of the order of the symmetry breaking. In our case, this process would make the mass of the would-be GB heavier than those of the other flhiggs.

as for the other states of the model. Before that, we must study the gauge boson sector. As we are about to see, this sector is severely constrained and its consistency with precision electroweak data constrains the possible values of v_F and g' and therefore of f if we want to keep the texture parameter small enough.

3.1.5 Gauge bosons and currents

After symmetry breaking, the model is described at low-energy by a set of gauge and scalar bosons. We discuss first the gauge boson sector. Its structure is complicated by the mixing of the standard model gauge bosons to the new states we have introduced. Our general strategy is to impose that the charged currents of the model coincide with those of the standard model. This done, we are essentially left with the theory of the standard model with the addition of a massive neutral gauge boson Z' and we must check that its presence affects the ρ parameter, the Weinberg angle θ_W, the tree level coefficients of the neutral current and that the value of the mass of the Z' are all within the experimental bounds. In this way two of the free parameters of the model, namely v_F and g' are fixed.

At the scale f, the symmetry surviving the spontaneous breaking of $SU(10)$ into $SO(10)$ is $SU(3)_W \times U(1)_W \times U(1)_X \times U(1)_F$ and we can write the effective kinetic lagrangian for the four scalar triplets Φ_i as

$$L_K^\Phi = (D_\mu\Phi_1)^\dagger(D_\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D_\mu\Phi_2) + (D'_\mu\Phi_3)^\dagger(D'_\mu\Phi_3) + (D'_\mu\Phi_4)^\dagger(D'_\mu\Phi_4)\,, \tag{3.38}$$

where the covariant derivatives are given by

$$\begin{aligned} D_\mu &= \partial_\mu + igW_\mu^a t^a + ig' x_\Phi B_\mu \\ D'_\mu &= \partial_\mu + igW_\mu^a t^a + ig' x_\Phi B_\mu \pm ik\frac{1}{2}X_\mu\,, \end{aligned} \tag{3.39}$$

with, as before in eq. (3.11), W_μ^a the gauge bosons of the $SU(3)$ electroweak group, t^a its generators, B_μ the gauge boson of the $U(1)$ electroweak symmetry, X_μ that of the exotic $U(1)$ gauge symmetry and g, g' and k their coupling respectively, while x_Φ is the $U(1)$ extended electroweak charge of the triplets Φ_i. Since the $SU(3)_W \times U(1)_W \times U(1)_W$ gauge symmetry at the low scale is the diagonal combination surviving the spontaneous breaking of $SU(10)$ into $SO(10)$ their couplings are given, respectively, by

$$g^2 = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2}\,, \quad g'^2 = \frac{g'^2_1 g'^2_2}{g'^2_1 + g'^2_2} \quad \text{and} \quad k^2 = \frac{k_1^2 k_2^2}{k_1^2 + k_2^2}\,. \tag{3.40}$$

When the triplets acquire the vacuum expectation values given by eq. (3.28), we are left with nine massive and one massless gauge boson; this latter being the photon.

The eight massive gauge bosons can be written as 3 complex and 3 real gauge bosons. The lightest complex fields and the lightest real can be identified with the standard model weak gauge bosons W and Z. The remaining complex bosons are new massive charged gauge particles $\tilde{W}_{1,2}$. The masses of these complex gauge bosons are given by

$$m_W^2 = \frac{1}{4} g^2 v_W^2\,, \quad m_{\tilde{W}_1}^2 = \frac{1}{2} g^2 v_F^2 \quad \text{and} \quad m_{\tilde{W}_2}^2 = \frac{1}{2} g^2 (v_F^2 + \frac{v_W^2}{2})\,, \tag{3.41}$$

respectively. The charged W gauge bosons behave exactly like those of the standard model and can be directly identified with them. Contrary to the heavy gauge bosons in eq. (3.13), the gauge bosons $\tilde{W}_{1,2}$ do not mix with W and therefore do not induce additional effective operators in the low-energy theory. Similarly, the gauge boson of the exotic $U(1)$ gauge symmetry does not mix and acquires a mass given by

$$m_X^2 = \frac{1}{4} k^2 v_F^2\,. \tag{3.42}$$

The other three real gauge bosons, those associated to the diagonal generators of the $SU(3)$, W_μ^3 and W_μ^8, and the gauge boson of $U(1)_W$, B, do mix, and their mass matrix is given by

$$M_{WB}^2 = \begin{pmatrix} g^2 v_W^2/4 & -g^2 v_W^2/4\sqrt{3} & -g\tilde{g}' v_W^2/2 \\ -g^2 v_W^2/4\sqrt{3} & g^2(v_W^2/12 + 2v_F^2/3) & g\tilde{g}'(v_W^2 - 4v_F^2)/2\sqrt{3} \\ -g\tilde{g}' v_W^2/2 & g\tilde{g}'(v_W^2 - 4v_F^2)/2\sqrt{3} & \tilde{g}'^2(v_W^2 + 2v_F^2) \end{pmatrix}, \tag{3.43}$$

where in eq. (3.43) $\tilde{g}' = g' x_\Phi$. The 3×3 mixing arises because of the $SU(3)$ weak group we started with and leads to the most characteristic (and constrained) new physics in the model.

One eigenvalue of the matrix M_{WB}^2 in eq. (3.43) is zero and corresponds to the photon, the other two depend on the values v_F and $\tilde{g}'$, the lightest mass to be identified with that of the standard model Z, the heaviest with an extra gauge boson Z'.

The mixing between W_3, W_8 and B is delicate since it gives rise to electric and neutral currents for the standard fermions. We fix the value of v_F and $\tilde{g}'$ by imposing that the electric and neutral currents in our model coincide with those of the standard model. In order to analyze the neutral currents, consider the orthogonal matrix U_W that diagonalize M^2_{WB} according to

$$\text{diag}(0,\, M_Z^2,\, M_{Z'}^2) = U_W^T\, M^2_{WB}\, U_W\,. \tag{3.44}$$

Once g and v_W are fixed by their standard model values, the entries of the matrix U_W—three of which are independent variables—depend on the parameters $\tilde{g}'$ and v_F that we are going to determine by requiring consistency with the experimental data.

Consider now the interactions between a fermion triplet (antitriplet) of $SU(3)_W$, $Q_L\,(Q_L^*)$, of $U(1)_W$ charge x_L and two fermion singlets of $SU(3)_W$, $\psi_R^{1,2}$, of $U(1)_W$ charge $y_R^{1,2}$ respectively and a fermion singlet of $SU(3)_W$, $\tilde{\psi}_R$, of $U(1)_W$ charge $\tilde{y}_R$ and of $U(1)_X$ charge $-1/2$, with the electroweak gauge bosons, that is we neglect the exotic X-current. The first two components of the left-handed triplet (antitriplet) $Q_L\,(Q_L^*)$,ψ_L^1 and ψ_L^2 form a $SU(2)_W$ standard model doublet (antidoublet), and when $SU(3)_W \times U(1)_W \times U(1)_X$ is broken into $U(1)_Q$, $\psi^j = \psi_L^j + \psi_R^j$ has electric charge Q_{f_j}, with $j = 1, 2$. At the same time, the third component of the triplet (antitriplet) $Q_L\,(Q_L^*)$, $\tilde{\psi}_L$ and the exotic $SU(3)_W$ singlet $\tilde{\psi}_R$ give rise to an electric charged fermion $\tilde{\psi} = \tilde{\psi}_L + \tilde{\psi}_R$, with charged Q_{f_2}, where the index 2 refers tothe second component of the triplet (antitriplet) $Q_L\,(Q_L^*)$. Dividing the Standard model doublet (antidoublet) componets,

The kinetic lagrangian is given by

$$L_K^f = \overline{Q_L}\,\gamma \cdot D\, Q_L + \overline{\psi_R}\,\gamma \cdot D\, \psi_R + \overline{\tilde{\psi}_R}\,\gamma \cdot D\, \tilde{\psi}_R\,, \tag{3.45}$$

for a triplet and

$$L_K^f = \overline{Q^*}_L\,\gamma \cdot D^*\, Q_L^* + \overline{\psi_R}\,\gamma \cdot D\, \psi_R + \overline{\tilde{\psi}_R}\,\gamma \cdot D\, \tilde{\psi}_R\,, \tag{3.46}$$

for an antitriplet, with

$$D_\mu = \partial_\mu + igW_\mu^a t^a + ig' x^j_{L,R} B_\mu\,, \tag{3.47}$$

where in eq. (3.47) have been used the same notations as in eq. (3.39). Consider only the terms in eqs. (3.45)–(3.46) that give rise to the electromagnetic and the neutral current for all the fermions, that is

$$\begin{aligned}\mathcal{L}_K^f &= \overline{\psi_L^j}\gamma^\mu\left(\partial_\mu + igT_{3\,f_j}\,W^3_\mu + i\,g\,\frac{p}{2\sqrt{3}}\,W^8_\mu + i\,g'\,x_L B_\mu\right)\psi^j_L \\ &+ \overline{\psi_R^j}\gamma^\mu\left(\partial_\mu + i\,g'\,x^j_R B_\mu\right)\psi^j_R \\ &+ \overline{\tilde\psi_L}\,\gamma^\mu\left(\partial_\mu - i\,g\,\frac{p}{\sqrt{3}}\,W^8_\mu + i\,g'\,x_L B_\mu\right)\tilde\psi_L + \overline{\tilde\psi_R}\gamma^\mu\left(\partial_\mu + i\,g'\,\tilde x_R B_\mu\right)\tilde\psi_R\,, \end{aligned} \tag{3.48}$$

where we have explicited the standard model doublet (antidoublet) components $\psi_L^{1,2}$ and the exotic fermion $\tilde\psi_L$ and where p is equal to 1 or -1 for the left handed fermion coming from a triplet or an antitriplet respectively.

The gauge bosons W^3_μ, W^8_μ and B_μ mix through the U_W of eq. (3.44) giving the photon, A_μ the gauge boson Z_μ and an heavy Z-type gauge boson, ${Z'}_\mu$, in particular we have

$$\begin{pmatrix} W_3 \\ W_8 \\ B \end{pmatrix} = U_W \begin{pmatrix} A \\ Z \\ Z' \end{pmatrix}. \tag{3.49}$$

Substituting the expressions coming from eq. (3.49) in eq. (3.48) we can write the electric, the neutral and the extra neutral currents using the parametrization given in [8]

$$\begin{aligned}\mathcal{L}' &= -eQ_j\overline{\psi^j}\gamma^\mu\psi^j A_\mu - \frac{e}{2s_Wc_W}\left(1+\frac{\alpha T}{2}\right)\overline{\psi^j}\gamma^\mu(g^j_V - g^j_A\gamma^5)\psi^j\, Z_\mu \\ &- \frac{e}{2s_Wc_W}\,\overline{\psi^j}\gamma^\mu(\tilde h^j_V - \tilde h^j_A\gamma^5)\psi^j\, Z'_\mu \\ &- eQ_2\overline{\tilde\psi}\gamma^\mu\tilde\psi A_\mu - \frac{e}{2s_Wc_W}\,\overline{\tilde\psi}\gamma^\mu(g^3_V - g^3_A\gamma^5)\tilde\psi Z_\mu \\ &- \frac{e}{2s_Wc_W}\,\overline{\tilde\psi}\gamma^\mu(\tilde h^3_V - \tilde h^3_A\gamma^5)\tilde\psi Z'_\mu\,, \end{aligned} \tag{3.50}$$

where s_W and c_W are sine and cosine of the Weinberg angle θ_W, T is one of the oblique parameters,

$$g^j_{V,A} = g^{j\,SM}_{V,A} + \tilde g^j_{V,A}\,, \tag{3.51}$$

with

$$g^{j\,SM}_V = T_{3_j} - 2Q_js^2_* \quad\text{and}\quad g^{j\,SM}_A = T_{3_j}\,, \tag{3.52}$$

and $g^3_{V,A}$ and $\tilde h^3_{V,A}$ are related to the neutral currents of the exotic fermions $\tilde\psi$ that, as we shall show in section 3.2 below are only weakly coupled to the standard model states. The coefficients $\tilde{g}_{V,A}$ contain the deviation from the standard model, $\tilde{h}_{V,A}$ the strength of the coupling of Z' to the standard model fermions.

First of all, the entries of the orthogonal matrix U_W have to satisfy the following conditions in order to have in eq. (3.50) the correct electric charge current for the standard and exotic fermions:

$$\begin{aligned}
U_{W_{11}} &= s_W \\
\frac{U_{W_{21}}}{U_{W_{11}}} &= \frac{1}{\sqrt{3}} \\
\frac{1}{U_{W_{11}}}\Big(U_{W_{21}}\frac{p}{2\sqrt{3}} + x_L\frac{g'}{g}U_{W_{31}}\Big) &= x_L^{SM} \\
\frac{g'}{g}x_R^j\frac{U_{W_{31}}}{U_{W_{11}}} &= Q_j \\
\frac{g'}{g}\tilde{x}_R\frac{U_{W_{31}}}{U_{W_{11}}} &= Q_2\,,
\end{aligned} \tag{3.53}$$

where x_L and x_R^j are the extra $U(1)_W$ fermion charges, while x_L^{SM} is the $U(1)_W$ standard model charge of the electroweak doublet ψ_L. The conditions of eq. (3.53) together with

$$\frac{1}{U_{W_{11}}}\Big(U_{W_{21}}\frac{1}{\sqrt{12}} + \frac{\tilde{g}'}{g}U_{W_{31}}\Big) = 0\,, \tag{3.54}$$

that follows by inserting eq. (3.53) in eq. (3.38) and imposing zero electric charge for the second and the third components of the triplets Φ_i completely determines all the independent parameters of the matrix U_W.

Since the mass matrix of eq. (3.43) depends on $\tilde{g}' = g'x_\Phi$ the other three conditions give us the values of x_L/x_Φ , x_R^j/x_Φ and $\tilde{x}_R/x_\Phi$, that is the fermion charges in units of the triplet Φ_i charges, with the further constrain on $U_{W_{31}}$ of giving rational numbers for the charges.

By equating now the neutral currents, we obtain that

$$\begin{aligned}
(1+\frac{\alpha T}{2}) &= c_W\, U_{W_{12}}\Big(1 - \frac{U_{W_{32}}}{U_{W_{31}}}\frac{U_{W_{11}}}{U_{W_{12}}}\Big) \\
(1+\frac{\alpha T}{2})\, s_*^2 &= -c_W\frac{U_{W_{32}}U_{W_{11}}}{U_{W_{31}}} \\
(1+\frac{\alpha T}{2})\,\tilde{g}_V^j = (1+\frac{\alpha T}{2})\,\tilde{g}_A^j &= p\,c_W\frac{U_{W_{22}}}{2\sqrt{3}}\Big(1 - \frac{U_{W_{32}}}{U_{W_{31}}}\frac{U_{W_{21}}}{U_{W_{22}}}\Big) \\
\tilde{h}_V^j &= T_{3_j}U_{W_{13}}\Big(1 - \frac{U_{W_{33}}}{U_{W_{31}}}\frac{U_{W_{11}}}{U_{W_{13}}}\Big) + 2\frac{U_{W_{33}}}{U_{W_{31}}}U_{W_{11}}Q_j
\end{aligned} \tag{3.55}$$

$$
\begin{aligned}
&+ \quad p\,\frac{U_{W_{2\,3}}}{\sqrt{12}}\Big(1-\frac{U_{W_{3\,3}}}{U_{W_{3\,1}}}\frac{U_{W_{2\,1}}}{U_{W_{2\,3}}}\Big) \\
\tilde{h}^j_A \;&=\; T_{3_j} U_{W_{1\,3}}\Big(1-\frac{U_{W_{3\,3}}}{U_{W_{3\,1}}}\frac{U_{W_{1\,1}}}{U_{W_{1\,3}}}\Big)+p\,\frac{U_{W_{2\,3}}}{2\sqrt{3}}\Big(1-\frac{U_{W_{3\,3}}}{U_{W_{3\,1}}}\frac{U_{W_{2\,1}}}{U_{W_{2\,3}}}\Big)\,.
\end{aligned}
$$

A more complete analysis would require that also the corrections arising from the effective operators induced by the heavy gauge bosons in eq. (3.13) be included. Thy are important because they violate the $SU(2)$ custodial symmetry of the standard model. They affect the relationships in eq. (3.56) to $O(v_W^2/f^2)$ and, in the littlest Higgs model of ref. [27], force the scale f to be above 4 TeV [69]. As already mentioned, these constrains can be lessen by introducing an additional discrete symmetry [70]. Notice that the overall fit of these corrections against the experimental electroweak data can in principle be improved by the presence in the flhiggs model of the additional parameters in eq. (3.56) thus lowering the scale f with respect to that found in the case of the littlest Higgs model.

We said we are more interested in the consistency of the framework than in its detailed realization and therefore we neglect, in this context, these $O(v_W^2/f^2)$ corrections and consider eq. (3.56) as it stands.

The parameters and coefficients in eq. (3.56) are constrained by precision measuraments of neutral currents in low-energy observables like atomic parity violation in atoms and neutrino-hadron scattering. The mass of the Z' gauge boson is bounded by data on Drell-Yan production (with subsequent decay into charged leptons) in $p\bar{p}$ scattering to be larger than 690 GeV [71] and this constrain must be included as well. We require that deviation in the ρ-parameter

$$\rho = 1 + \alpha T \tag{3.56}$$

and the Weinberg angle be within 10^{-3} while the $\tilde{g}$ coefficients in eq. (3.56) be less than 10^{-2}. This choice put these deviations in the tree-level parameters in the ball park of standard-model radiative corrections.

The importance of these constrains resides in their fixing the values of the free parameters v_F and $\tilde{g}' = g' x_\Phi$. The bound on the ρ parameter essentially fixes the effective gauge coupling

$$\tilde{g}' \simeq 0.13\,. \tag{3.57}$$

For simplicity we take $x_\Phi = 1$ so we have $\tilde{g}' = g'$.

Once $\tilde{g}'$ has been fixed to this value, the bound on the mass of Z' requires

$$v_F \gtrsim 1260 \quad \text{GeV}\,. \tag{3.58}$$

We would like to have v_F as close to v_W as possible but the phenomenological constrains force it to a higher scale.

The rather large value we must take for v_F does imply unfortunately that some amount of fine-tuning in the parameters of the potential in eq. (3.15) is present. If we go back to our back-of-the-envelope estimate in section 3.1.4, we see that while there we had $v_f \simeq 2v_W$ with no fine-tuning (that is, the coefficient were chosen with a tuning of one out of four or 25%) on the values of the μ_i coefficients, we now must have $v_f \simeq 4v_W$ that can be obtained by taking, for instance, $\mu_1^2 - \mu_2^2 \simeq -\mu^2/2$ and $\mu_3^2 - \mu_4^2 \simeq 24\mu^2/50$ that means 1 out of 25, that is a fine-tuning of 4%. The actual fine-tuning in the model is however less than this because of the larger number of parameters involved and roughly of 10% or less.

3.1.6 The scalar sector and the lightest flhiggs boson

We now turn to the scalar sector of the model. The number of scalar bosons can readily be computed: the number of degrees of freedom of 4 complex triplets is 24. Of these 9 are eaten by the gauge fields, while 1—the would-be GB of the spontaneous breaking of the $U(1)_F$ global symmetry—is eliminated, after introducing the fermions in the model, by the anomaly. Therefore, the scalar sector contains $24 - 10 = 14$ massive fields. To describe them, we parametrize the Φ_i triplets with respect to these fifteen fields as follows

$$\begin{aligned}
\Phi_1 &= \begin{pmatrix} u^\rho_{11}\rho_1 \\ (v_W + u^\delta_{1i}\delta_i + u^\varphi_{1j}\varphi_j)/2 \\ (v_F + u^\delta_{2i}\delta_i + u^\varphi_{2j}\varphi_j)/2 \end{pmatrix} \\
\Phi_2 &= \begin{pmatrix} u^\rho_{21}\rho_1 \\ (v_W + u^\delta_{3i}\delta_i + u^\varphi_{3j}\varphi_j)/2 \\ (v_F + u^\delta_{4i}\delta_i + u^\varphi_{4j}\varphi_j)/2 \end{pmatrix}
\end{aligned}\,.$$

$$\Phi_3 = \begin{pmatrix} u^{\rho}_{32}\rho_2 \\ (u^{\delta}_{5i}\delta_i + u^{\varphi}_{5j}\varphi_j)/2 \\ (v_F + u^{\delta}_{6i}\delta_i + u^{\varphi}_{6j}\varphi_j)/2 \end{pmatrix}$$

$$\Phi_4 = \begin{pmatrix} u^{\rho}_{42}\rho_2 \\ (u^{\delta}_{7i}\delta_i + u^{\varphi}_{7j}\varphi_j)/2 \\ (v_F + u^{\delta}_{8i}\delta_i + u^{\varphi}_{8j}\varphi_j)/2 \end{pmatrix} . \tag{3.59}$$

with $i = 1,..,4$ and $j = 1,..,7$ and where $u^{\rho,\delta,\varphi}_{ij}$ are the entries of the unitary matrix which diagonalizes the mass matrix defined as

$$M^2_{\Phi_{ij}} = \left.\frac{\partial^2 V[\Phi]}{\partial\Phi_i\partial\Phi_j}\right|_{\Phi=\langle\Phi\rangle} , \tag{3.60}$$

and can be written in terms of the coefficients of the effective potential.

The scalar fields ρ_k, δ_i and φ_j are the Higgs-like components of the flhiggs fields and the most interesting experimental signature of the model.

The fields $\rho_{1,2}$ are electrically charged, their masses given respectively by by

$$m^2_{\rho_1} = 4\xi_1(v_F^2 - v_W^2) \quad \text{and} \quad m^2_{\rho_2} = (\xi_1 - \xi_2)v_F^2 - \xi_1 v_W^2 . \tag{3.61}$$

We shall call the lightest of the two $h^{\pm}$.

The masses of the neutral δ_i fields are given by

$$\begin{aligned} m^2_{\delta_1} &= 4\xi_1(v_F^2 - v_W^2) \\ m^2_{\delta_3} &= 2\xi_1 v_F^2 - \xi_1 v_W^2\left(1 - \frac{v_W^2}{v_F^2}\right) \\ m^2_{\delta_4} &= (\xi_1 - \xi_2)v_F^2 - \xi_1\frac{v_W^2}{v_F^2}v_W^2 . \end{aligned} \tag{3.62}$$

The missing δ_2 field, that in the diagonalization appears as a massless state, is the would-be GB eliminated by the anomaly.

The masses of the fields φ_j are obtained by diagonalization of the remaining sub-matrix. This sub-matrix written in terms of the vacua and the coefficients of the potential has a cumbersome form that is not particularly inspiring and that we do not include. We do not have an exact diagonalization for it but it must contain the lightest neutral scalar boson that we call h^0. This can be understood by thinking at one single flhiggs triplet for which the δ_i part would correspond to the imaginary component and the φ_j to the real part and therefore Higgs-like component.

We study the scalar sector spectrum numerically in section 3.3 to obtain an estimate of the allowed values for m_{h^0} and $m_{h^\pm}$ for arbitrary $O(1)$ coefficients in the potential and v_F fixed to the values determined in the previous section.

3.1.7 The flavorless limit

In the limit in which we factorize out the flavor part by taking $v_F = f$, the model has the littlest Higgs model of ref. [27] embedded inside. We can identify within the global symmetry $SU(10)$ a reduced symmetry $SU(5)$. The two flhiggs fields $\Phi_{3,4}$ decouple from this subsector that feels no $U(1)_X$ symmetry. In the notation of [27], the flhiggs fields $\Phi_{1,2}$ go into the Higgs boson h while the fields z_{ij} go into the weak triplet field ϕ. Clearly, all the Yukawa coupling of the next section become trivial and the fermion masses degenerate if we take the Yukawa coefficients to be all of $\mathcal{O}(1)$.

3.2 Introducing the fermions

According to the rules of the little-Higgs mechanism, the coupling of the fermions to the flhiggs must proceed by preserving enough symmetry not to give rise to 1-loop quadratic divergent contributions to their masses. This means that for every fermion with an Yukawa coupling of $O(1)$ we must introduce one (or even more) state to cancel the divergent diagram. This procedure brings in two more sets of fermions, one for the standard model fermions with large Yukawa couplings and one for the exotic states we introduced to complete the $SU(3)$ triplets.

Even though the introduction of new fermions seems to lead us to a structure of Baroque richness, notice that these states nicely fall into the fundamental representations of $SU(10)$ giving a natural structure to the Yukawa interactions in terms of the larger symmetry group that can be written in general, and by neglecting for the moment the flavor group, as

$$\mathcal{L}_Y \simeq \lambda_1 \mathcal{X} \bar{X} + \lambda_2 \bar{\mathcal{X}} \Sigma \mathcal{X} \tag{3.63}$$

where $\mathcal{X}$ is a decuplet of fermions in the fundamental representation of $SU(10)$.

The Yukawa lagrangians at the scale f is obtained by writing the $SU(3)_W \times U(1)_X \times U(1)_F$ invariant terms involving the four triplets Φ_1,Φ_2 ,Φ_3 and Φ_4, and the fermions. Standard model left-handed doublets are members of an $SU(3)_W$ triplet, the third component being an exotic fermion.

To help the reader in keeping track of the various terms, Tables 3.1–3.4 contain the representations and the charge assignments with respect to the exotic, the flavor and the electroweak groups of all fermions and flhiggs bosons, the latter having been determined solving eq. (3.53) and eq. (3.54).

Table 3.1: Representations and charges assignments for the flhiggs bosons.

	$U(1)_X$	$U(1)_F$	$SU(3)_W$	$U(1)_W$
Φ_1	0	1/2	3	1
Φ_2	0	−1/2	3	1
Φ_3	1/2	0	3	1
Φ_4	−1/2	0	3	1

3.2.1 Quarks

In order to avoid large quadratic corrections to the flhiggs masses induced by divergent one-loop contributions from the heaviest fermions present in the model, that is the top and the exotic quarks (and leptons) that complete the electroweak triplets, we introduce for each family a number of colored Weyl fremions, both triplets of the $SU(3)$ electroweak gauge group and singlets. Their charges are all summarized in tab. (3.2). The number of multiplets and singlets introduced is the smallest number that permit us to write a quark Yukawa lagangian composed by terms that singularly preserve enough symmetry in order to keep the four triplets Φ_1, Φ_2, Φ_3 and Φ_4 massless. In this way quadratic divergent contributions to the flhiggs masses arise only at two-loops.

The Yukawa lagrangian for the quarks is given by

$$\begin{aligned}\mathcal{L}_Y^q &= \lambda_{ab}^{u\,1}\, f\, \mathcal{U}_L^{a^c}\, \Sigma\, \mathcal{Q}_L^b \Big(\Sigma_{4,4}\Big)^{|y_Q^b+y_{\mathcal{U}}^a|} + \lambda_{ab}^{u\,2}\, f\, u''^{a^c}_L\, U_L^b \Big(\Sigma_{4,4}\Big)^{|1+y_Q^b-y_Q^a|} \\ &+ \tilde\lambda_{ab}^{u\,1}\, f\, \tilde{\mathcal{U}}_L^{a^c}\, \Sigma\, \mathcal{Q}_L^b \Big(\Sigma_{4,4}\Big)^{|y_{\tilde U}^b-y_{\tilde U}^a|} + \tilde\lambda_{ab}^{u\,2}\, f\, \tilde u''^{a^c}_L\, \tilde U_L^b \Big(\Sigma_{4,4}\Big)^{|y_{\tilde U}^b-y_{\tilde U}^a|} \\ &+ \eta_{ab}^u\, f\, \tilde Q_L^{a^c} \tilde Q_L^b \Big(\Sigma_{4,4}\Big)^{|y_Q^b-y_Q^a|} + \lambda_{ab}^d d_L^{a^c}\Big(\epsilon ijk Q_{L_i}^b \Sigma_{4,4+j}\Sigma_{8,4+i}\Big)\Big(\Sigma_{4,4}\Big)^{|y_Q^b+y_d^a|} \quad (3.64)\end{aligned}$$

with $a = 1,2,3$ and $d_L^{1,2,3^c} = d_L^c, s_L^c, b_L^c$. y_Q^a are the flavor charges of the Q_L^a and $\tilde Q_L^a$ triplets, now members of the $\mathcal{Q}^a$ multiplets defined in tab. (3.2), $y_{\mathcal{U}}^a$ that of the $\mathcal{U}^a$ multiplets and y_d^a the flavor charges of the Weyl fermions $d_L^{a^c}$. In eq. (3.64) the terms with the coefficients $\lambda_{ab}^{u\,1}$ preserve an $SU(8)$ subgroup of the approximate global symmetry $SU(10)$ while the terms with the coefficients $\lambda_{ab}^{u\,2}$ break it but preserve an $SU(9)$ subgroup of $SU(10)$. Analogously, the terms with coefficients $\tilde\lambda_{ab}^{u\,1}$ and $\tilde\lambda_{ab}^{u\,2}$ preserve different subgroups of $SU(10)$ making possible the protection of the flhiggs masses through the collective symmetry breaking mechanism.

The (4, 4) component of the sigma model field Σ that appears in eq. (3.64) is the only $SU(8)$ and $SU(9)$ singlet and therefore the only possible field we can include to balance the flavor charges to make eq. (3.64) invariant.

Expanding the Σ and keeping only the terms involving the Φ_i in eq. (3.64) yields

$$\begin{aligned}\mathcal{L}_Y^q &= \lambda_{ab}^{u\,1}\, f\,\Big[u'^{a^c}_L U_L^b\big(1-\frac{\Phi_1^\dagger\Phi_1}{2f^2}-\frac{\Phi_2^\dagger\Phi_2}{2f^2}\big)+u'^{a^c}_L\big(i\frac{\Phi_2^T}{f}Q_L^b+i\frac{\Phi_1^\dagger}{f}\tilde Q_L^b\big)\Big]\Big(\frac{\Phi_1^\dagger\Phi_2}{f^2}\Big)^{|y_Q^b+y_{\mathcal{U}}^a|} \\ &+ \lambda_{ab}^{u\,2}\, f\, u''^{a^c}_L U_L^b\Big(\frac{\Phi_1^\dagger\Phi_2}{f^2}\Big)^{|1+y_Q^b-y_Q^a|} \\ &+ \tilde\lambda_{ab}^{u\,1}\, f\,\Big[\tilde u'^{a^c}_L \tilde U_L^b\big(1-\frac{\Phi_3^\dagger\Phi_3}{2f^2}-\frac{\Phi_4^\dagger\Phi_4}{2f^2}\big)+\tilde u'^{a^c}_L\big(i\frac{\Phi_3^T}{f}Q_L^b+i\frac{\Phi_4^\dagger}{f}\tilde Q_L^b\big)\Big]\Big(\frac{\Phi_1^\dagger\Phi_2}{f^2}\Big)^{|y_{\tilde U}^b-y_{\tilde U}^a|} \\ &+ \tilde\lambda_{ab}^{u\,2}\, f\, \tilde u''^{a^c}_L \tilde U_L^b\Big(\frac{\Phi_1^\dagger\Phi_2}{f^2}\Big)^{|y_{\tilde U}^b-y_{\tilde U}^a|} \\ &+ \eta_{ab}^u\, f\, \tilde Q_L^{a^c}\tilde Q_L^b\Big(\frac{\Phi_1^\dagger\Phi_2}{f^2}\Big)^{|y_Q^b-y_Q^a|}+\lambda_{ab}^d d_L^{a^c}\Big(\epsilon ijk Q_{L_i}^b\frac{\Phi_1^*}{f}\frac{\Phi_2^*}{f}\Big)\Big(\frac{\Phi_1^\dagger\Phi_2}{f^2}\Big)^{|y_Q^b+y_d^a|}. \quad (3.65)\end{aligned}$$

After the symmetry breaking (3.31) that for convenience we rewrite here

$$\langle\Phi_1\rangle = \begin{pmatrix}0\\ v_W/2\\ v_F/2\end{pmatrix}\quad \langle\Phi_2\rangle = \begin{pmatrix}0\\ v_W/2\\ -v_F/2\end{pmatrix}\quad \langle\Phi_3\rangle = \begin{pmatrix}0\\ 0\\ v_F/2\end{pmatrix}\quad \langle\Phi_4\rangle = \begin{pmatrix}0\\ 0\\ v_F/2\end{pmatrix} \quad (3.66)$$

the Yukawa lagrangian eq. (3.65) at the leading order becomes

$$\begin{aligned}
\mathcal{L}_Y^q &= \lambda_{ab}^{u\,1}\, f\,\Big[u'^{ac}_L U_L^b + u'^{ac}_L (\frac{v_W}{f} u_L^b - \frac{v_F}{f}\tilde{u}_L^b + \frac{v_W}{f} n_L^b + \frac{v_F}{f}\tilde{n}_L^b)\Big](- k)^{|y_Q^b + y_u^a|} \\
&+ \lambda_{ab}^{u\,2}\, f\, u''^{ac}_L U_L^b (- k)^{|1 + y_Q^b - y_Q^a|} + \tilde{\lambda}_{ab}^{u\,1}\, f\,\Big[\tilde{u}'^{ac}_L \tilde{U}_L^b + \tilde{u}'^{ac}_L (\frac{v_F}{f}\tilde{u}_L^b + \frac{v_F}{f}\tilde{n}_L^b)\Big](- k)^{|y_{\tilde{U}}^b - y_{\tilde{U}}^a|} \\
&+ \tilde{\lambda}_{ab}^{u\,2}\, f\, \tilde{u}''^{ac}_L \tilde{U}_L^b (- k)^{|y_{\tilde{U}}^b - y_{\tilde{U}}^a|} + \eta_{ab}^u\, f\,(m_L^{ac} m_L^b + n_L^{ac} n_L^b + \tilde{n}_L^{ac}\tilde{n}_L^b)(- k)^{|y_Q^b - y_Q^a|} \\
&+ \lambda_{ab}^d d_L^{ac} d_L^b 2 v_W \sqrt{k} (- k)^{|y_Q^b + y_d^a|}\,, \qquad (3.67)
\end{aligned}$$

where $k = v_F^2/f^2$ is the parameter in terms of which we write the mass textures.

From the lagrangian in eq. (3.67), we can read off the mass matrices for the quarks (see also eq. (3.76) below). These matrices and their textures are discussed in section 3.2.4.

3.2.2 Collective breaking in the up-quark sector and decoupling of the exotic fermions

Let us now pause for a moment and show how the collective breaking mechanism works in preventing 1-loop quadratically divergent corrections to the flhiggs masses. Consider only the terms of the type "up" components of the third family

$$\begin{aligned}
\mathcal{L}_{top}^Y &= \lambda_{33}^{u\,1}\, f\,\Big[t'^c_L T_L + t'^c_L (\frac{v_W}{f} t_L - \frac{v_F}{f}\tilde{t}_L + \frac{v_W}{f} n_L^3 + \frac{v_F}{f}\tilde{n}_L^3)\Big](- k) + \lambda_{33}^{u\,2}\, f\, t''^c_L T_L (- k) \\
&+ \tilde{\lambda}_{33}^{u\,1}\, f\,\Big[\tilde{t}'^c_L \tilde{T}_L + \tilde{t}'^c_L (\frac{v_F}{f}\tilde{t}_L + \frac{v_F}{f}\tilde{n}_L^3)\Big] + \tilde{\lambda}_{33}^{u\,2}\, f\, \tilde{t}''^c_L \tilde{T}_L \\
&+ \eta_{ab}^u\, f\,(m_L^{3c} m_L^3 + n_L^{3c} n_L^3 + \tilde{n}_L^{3c}\tilde{n}_L^3)\,. \qquad (3.68)
\end{aligned}$$

We see that t'^c_L and t''^c_L mix into a heavy and light combination, the latter being the standard top quark t_L^c. Since the mixing is given by

$$\begin{aligned}
t_L^c &= \frac{\lambda_{33}^{u\,1}}{\sqrt{(\lambda_{33}^{u\,1})^2 + (\lambda_{33}^{u\,2})^2}} t''^c_L - \frac{\lambda_{33}^{u\,2}}{\sqrt{(\lambda_{33}^{u\,1})^2 + (\lambda_{33}^{u\,2})^2}} t'^c_L \\
\hat{t}_L^c &= \frac{\lambda_{33}^{u\,2}}{\sqrt{(\lambda_{33}^{u\,1})^2 + (\lambda_{33}^{u\,2})^2}} t''^c_L + \frac{\lambda_{33}^{u\,1}}{\sqrt{(\lambda_{33}^{u\,1})^2 + (\lambda_{33}^{u\,2})^2}} t'^c_L\,, \qquad (3.69)
\end{aligned}$$

the top Yukawa coupling is

$$\lambda_{33}^u = \frac{\lambda_{33}^{u\,1}\lambda_{33}^{u\,2}}{\sqrt{(\lambda_{33}^{u\,1})^2 + (\lambda_{33}^{u\,2})^2}}\,. \qquad (3.70)$$

Similarly, the exotic $\tilde{t'}^c_L$ and $\tilde{t''}^c_L$ mix into a heavy and a light combination, giving rise to the exotic quarks $\tilde{t}^c_L$ and $\tilde{\tilde{t}}^c_L$:

$$\begin{aligned}
\tilde{t}^c_L &= \frac{\tilde{\lambda}^{u\,1}_{33}}{\sqrt{(\tilde{\lambda}^{u\,1}_{33})^2 + (\tilde{\lambda}^{u\,2}_{33})^2}}\tilde{t''}^c_L - \frac{\tilde{\lambda}^{u\,2}_{33}}{\sqrt{(\tilde{\lambda}^{u\,1}_{33})^2 + (\tilde{\lambda}^{u\,2}_{33})^2}}\tilde{t'}^c_L \\
\tilde{\tilde{t}}^c_L &= \frac{\tilde{\lambda}^{u\,2}_{33}}{\sqrt{(\tilde{\lambda}^{u\,1}_{33})^2 + (\tilde{\lambda}^{u\,2}_{33})^2}}\tilde{t''}^c_L + \frac{\tilde{\lambda}^{u\,1}_{33}}{\sqrt{(\tilde{\lambda}^{u\,1}_{33})^2 + (\tilde{\lambda}^{u\,2}_{33})^2}}\tilde{t'}^c_L ,
\end{aligned} \tag{3.71}$$

and the exotic top Yukawa coupling is given by

$$\tilde{\lambda}^u_{33} = \frac{\tilde{\lambda}^{u\,1}_{33}\tilde{\lambda}^{u\,2}_{33}}{\sqrt{(\tilde{\lambda}^{u\,1}_{33})^2 + (\tilde{\lambda}^{u\,2}_{33})^2}} . \tag{3.72}$$

We can neglect the mixing between the top, the exotic quark and the components of the triplets $\tilde{Q}^3_L$ since they are much heavier thanks to the explicit mass term in eq. (3.69). Therefore, eq. (3.69) becomes

$$\begin{aligned}
\mathcal{L}^Y_{top} &= \lambda^u_{33}\, v_W\, t^c_L t_L(\,-\,k) - \lambda'^u_{33}\, v_F t^c_L \tilde{t}_L(\,-\,k) + \hat{m}\, \hat{t}^c_L T_L(\,-\,k) \\
&+ \tilde{\lambda}^u_{33}\, v_F\, \tilde{t}^c_L \tilde{t}_L + \tilde{\hat{m}}\, \tilde{\tilde{t}}^c_L \tilde{T}_L ,
\end{aligned} \tag{3.73}$$

where we have neglected the terms involving the exotic triplets $\tilde{Q}^3_L$. In eq. (3.73) there is mixing between the standard top t_L and the exotic one $\tilde{t}_L$ which is however very much suppressed, as we shall show shortly.

The same argument should in principle be applied to the first and second family. However in these cases we can neglect altogether the mixing between the $u'^{1,2c}_L$ and $u''^{1,2c}_L$ because it is strongly suppressed. Considering for example the the second family, we have

$$\begin{aligned}
c^c_L &= \frac{\lambda^{u\,1}_{22}(-k)^3}{\sqrt{(\lambda^{u\,1}_{22}(-k)^3)^2 + (\lambda^{u\,2}_{22})^2}} c''^c_L - \frac{\lambda^{u\,2}_{22}}{\sqrt{(\lambda^{u\,1}_{22}(-k)^3)^2 + (\lambda^{u\,2}_{22})^2}} c'^c_L \\
\hat{c}^c_L &= \frac{\lambda^{u\,2}_{22}(-k)^3}{\sqrt{(\lambda^{u\,1}_{22}(-k)^3)^2 + (\lambda^{u\,2}_{22})^2}} c''^c_L + \frac{\lambda^{u\,1}_{22}}{\sqrt{(\lambda^{u\,1}_{22}(-k)^3)^2 + (\lambda^{u\,2}_{22})^2}} c'^c_L ,
\end{aligned} \tag{3.74}$$

and from eq. (3.74) follows that

$$\begin{aligned}
c^c_L &\simeq c'^c_L \\
\hat{c}^c_L &\simeq c''^c_L .
\end{aligned} \tag{3.75}$$

In the exotic sector the situation follows what happens in the case of the top quark, and we define a light and a heavy exotic quark for both families, $\tilde{c}^c_L$ and $\tilde{\tilde{c}}^c_L$ for the second and $\tilde{u}^c_L$ and $\tilde{\tilde{u}}^c_L$ for the first one. At the end, the Yukawa lagrangian for the lightest quarks, both standard and exotic is given by

$$\begin{aligned}\mathcal{L}^q_Y &= \lambda^u_{ab}\, v_W\, u^{a^c}_L u^b_L(-k)^{|y^b_Q+y^a_{\bar{u}}|} - \lambda'^u_{ab}\, v_F\, u^{a^c}_L \tilde{u}^b_L(-k)^{|y^b_Q+y^a_{\bar{u}}|} \\ &+ \tilde{\lambda}^u_{ab}\, v_F\, \tilde{u}^{a^c}_L \tilde{u}^b_L(-k)^{|y^b_{\tilde{U}}-y^a_{\tilde{U}}|} + \lambda^d_{ab} d^{a^c}_L d^b_L 2 v_W \epsilon(-k)^{|y^b_Q+y^a_{\bar{d}}|}\,. \end{aligned} \tag{3.76}$$

To see that the mixing between the standard and the exotic fermions is negligible, consider the mass matrix at the leading order, that is by taking all the parameter λ equal to 1. We have the following 6×6 mass matrix

$$M^u_{RL} = \begin{pmatrix} v_W k^7 & v_W k^6 & v_W k^4 & v_F k^7 & v_F k^6 & v_F k^4 \\ v_W k^5 & v_W k^4 & v_W k^2 & v_F k^5 & v_F k^4 & v_F k^2 \\ v_W k^4 & v_W k^3 & v_W k & v_F k^4 & v_F k^3 & v_F k \\ 0 & 0 & 0 & v_F & v_F k & v_F k^3 \\ 0 & 0 & 0 & v_F k & v_F & v_F k^2 \\ 0 & 0 & 0 & v_F k^3 & v_F k^2 & v_F \end{pmatrix}. \tag{3.77}$$

To give an estimate of the mixing between standard and exotic fermions we have to consider $M^{d^\dagger}M^d$, that is

$$M^{u\dagger}_{RL}M^u_{RL} = \begin{pmatrix} v_W^2 k^8 & v_W^2 k^7 & v_W^2 k^5 & v_W v_F k^8 & v_W v_F k^7 & v_W v_F k^5 \\ v_W^2 k^7 & v_W^2 k^6 & v_W^2 k^4 & v_W v_F k^7 & v_W v_F k^6 & v_W v_F k^4 \\ v_W^2 k^5 & v_W^2 k^4 & v_W^2 k^2 & v_W v_F k^5 & v_W v_F k^4 & v_W v_F k^2 \\ v_W v_F k^8 & v_W v_F k^7 & v_W v_F k^5 & v_F^2 & 2v_F^2 k & 3v_F^3 \\ v_W v_F k^7 & v_W v_F k^6 & v_W v_F k^4 & 2v_F^2 & v_F^2 & 2v_F k^2 \\ v_W v_F k^5 & v_W v_F k^4 & v_W v_F k^2 & 3v_F^3 & 2v_F k^2 & v_F^2 \end{pmatrix} \tag{3.78}$$

Nine angles out of the 15 parametrizing the unitary matrix that diagonalize the mass matrix of eq. (3.78) contribute to the mixing between standard and exotic fermions. Let us call θ_{ij} with $i = 1,2,3$ and $j = 4,5,6$ one of these nine angles, we see that

$$\tan 2\theta_{ij} \simeq -2\, M_{ij}/M_{jj}\,, \tag{3.79}$$

that is

$$\theta_{ij} \simeq -k^{n_{ij}}\, v_W/v_F\,, \tag{3.80}$$

so the largest mixing angle between the standard and exotic fermions is $\theta_{36} \simeq -k^2\, v_W/v_F \simeq 10^{-2}$, while that with the first two standard families, are completely negligible and well beyond any current bound [72].

3.2.3 Leptons

While standard model quark doublets are put in $SU(3)$ electroweak antitriplets, standard model left-handed leptons are embedded in $SU(3)$ triplets. Since leptons are lighter than quarks we should worry only about the divergent quadratic one-loop corrections to the flhiggs masses coming from the exotic leptons. The lepton content of each family is given in table 3.3.

The right-handed neutrinos N_i with $i = 1, \cdots, 8$ couple to the left-handed triplets. In order to see their effect on the low-energy lagrangian, it is sufficient to consider a pair of them, for instance, ν_R^1 and $\tilde{\nu}_R^5$ since the equal coupling of the remaining three pairs only renormalizes the overall Yukawa coupling.

The Yukawa lagrangian for the leptons is given at the leading order for each term by

$$\begin{aligned}
\mathcal{L}_Y^l \;=\;& \frac{\eta_1 f}{2}(\overline{\nu_L^{1c}}\tilde{\nu}'^5_R + \overline{\tilde{\nu}'^{5c}_L}\nu_R^1) + \frac{\eta_2 f}{2}(\overline{\nu_L^{2c}}\tilde{\nu}_R^6 + \overline{\tilde{\nu}_L^{6c}}\nu_R^2) \\
+\;& \lambda_{1a}^\nu\, f\, \bar{\nu}_R^1\Big(\epsilon_{ijk}L_{L_i}^a\Sigma_{4,j}\Sigma_{8,k}\Big)\Big(\Sigma_{4,4}\Big)^{|y_L^a-1|} + \lambda_{5a}^\nu\, f\, \bar{\tilde{\nu}}'^5_R\Big(\epsilon_{ijk}L_{L_i}^a\Sigma_{4,j}\Sigma_{8,k}\Big)\Big(\Sigma_{4,4}\Big)^{|y_L^a+1|} \\
+\;& \lambda_{2a}^\nu\, f\, \bar{\nu}_R^2\Big(\epsilon_{ijk}L_{L_i}^a\Sigma_{4,j}\Sigma_{8,k}\Big)\Big(\Sigma_{4,4}\Big)^{|y_L|} + \lambda_{6a}^\nu\, f\, \bar{\tilde{\nu}}_R^6\Big(\epsilon_{ijk}L_{L_i}^a\Sigma_{4,j}\Sigma_{8,k}\Big)\Big(\Sigma_{4,4}\Big)^{|y_L^a|} \\
+\;& \lambda_{ab}^e\, f\, \mathcal{E}_L^{a^c}\,\Sigma\,\mathcal{L}_L^b\Big(\Sigma_{4,4}\Big)^{y_L^b-1/2+y_{\mathcal{E}}^a} \\
+\;& \tilde{\lambda}_{ab}^{e\,1}\, f\, \tilde{\mathcal{E}}_L^{a^c}\,\Sigma\,\mathcal{L}_L^b\Big(\Sigma_{4,4}\Big)^{y_L^b-y_L^a} + \tilde{\lambda}_{ab}^{e\,2}\, f\, \tilde{e''}^{a^c}_L\,\tilde{E}_L^b\Big(\Sigma_{4,4}\Big)^{|y_{\tilde{E}}^b-y_{\tilde{E}}^a|} + \eta_{ab}^e f \tilde{L}_L^{a^c}\,\tilde{L}_L^b\Big(\Sigma_{4,4}\Big)^{|y_{\tilde{L}}^b-y_{\tilde{L}}^a|}\,,
\end{aligned} \tag{3.81}$$

with $a = 1,2,3$, L_L^a defined in tab. (3.3), $\nu_R^{1,2,3} = \nu_R^e, \nu_R^\mu, \nu_R^\tau$, $e_L^{1,2,3^c} = e_L^c, \mu_L^c, \tau_L^c$, $\tilde{e}_L^{1,2,3^c} = \tilde{e}_L^c, \tilde{\mu}_L^c, \tilde{\tau}_L^c$. y_L^a are the flavor charges of the L_L^a and $\tilde{L}_L^a$ triplets members of the multiplets $\mathcal{L}_L^a$ defined in tabs. (3.3)–(3.4), $y_{\mathcal{E}}^a$ of the $\mathcal{E}_L^{a^c}$ multiplets , while the two right handed neutrinos, ν_R^1 and $\tilde{\nu}'^5_R$, flavor charges are $-1/2$ and $1/2$ respectively. In eq. (3.81) we have only two terms that preserve two different subgroups of the approximate global symmetry $SU(10)$, that is the terms with coefficients $\tilde{\lambda}^{e\,(1,2)}$. This permit us to protect the flhiggs $\Phi_{3,4}$ masses from the one-loop quadratic divergent contributions coming from the lepton triplets, since they couple to them with a large Yukawa coupling. As in eq. (3.64) for the quarks, the component $\Sigma_{4,4}$ is the only group singlet of the approximated global symmetries that can be introduced to make the lagrangian flavor invariant.

Like for the quarks, the exotic leptons $\tilde{e'}_L^{a^c}$ and $\tilde{e''}_L^{a^c}$ mix giving a light and a heavy exotic leptons, $\tilde{e}_L^{a^c}$ and $\hat{e}_L^{a^c}$. In terms of the standard leptons, of the light exotic leptons and of the Φ_i, eq. (3.81) becomes

$$\begin{aligned}
\mathcal{L}_Y^l &= \frac{\eta_1 f}{2}(\overline{\nu_L^{1c}}\tilde{\nu'}_R^5 + \overline{\tilde{\nu'}_L^{5c}}\nu_R^1) + \frac{\eta_2 f}{2}(\overline{\nu_L^{2c}}\tilde{\nu}_R^6 + \overline{\tilde{\nu}_L^{6c}}\nu_R^2) + \frac{\lambda_{1a}^\nu}{f}\bar{\nu}_R^1\Big(\epsilon_{ijk}L_{L_i}^a\Phi_j\Phi_k\Big)\Big(\frac{\Phi_2^\dagger\Phi_1}{f^2}\Big)^{|y_L^a-1|} \\
&+ \frac{\lambda_{5a}^\nu}{f}\bar{\nu}_R^5\Big(\epsilon_{ijk}L_{L_i}^a\Phi_j\Phi_k\Big)\Big(\frac{\Phi_2^\dagger\Phi_1}{f^2}\Big)^{|y_L^a+1|} + \frac{\lambda_{2a}^\nu}{f}\bar{\nu}_R^2\Big(\epsilon_{ijk}L_{L_i}^a\Phi_j\Phi_k\Big)\Big(\frac{\Phi_2^\dagger\Phi_1}{f^2}\Big)^{|y_L^a|} \\
&+ \frac{\lambda_{6a}^\nu}{f}\bar{\nu}_R^6\Big(\epsilon_{ijk}L_{L_i}^a\Phi_j\Phi_k\Big)\Big(\frac{\Phi_2^\dagger\Phi_1}{f^2}\Big)^{|y_L^a|} + \lambda_{ab}^e e_L^{a^c}\,\Phi_1^\dagger\,L_L^b\Big(\frac{\Phi_1^\dagger\Phi_2}{f^2}\Big)^{y_L^b-1/2+y_{\mathcal{E}}^a} \\
&+ \tilde{\lambda}_{ab}^e\tilde{e}_L^{a^c}\,\Phi_3^\dagger\,L_L^b\Big(\frac{\Phi_1^\dagger\Phi_2}{f^2}\Big)^{y_L^b-y_L^a} + H.c.
\end{aligned} \tag{3.82}$$

The neutrino sector in eq. (3.86) is given by four Majorana right-handed neutrinos (two copies of them, actually) and three left-handed neutrinos, these latter being the standard neutrinos. Right-handed neutrinos are heavy, since their masses is of the same order of the scale f, and we can integrate out them to obtain a Majorana mass matrix for the left-handed ones through the see-saw mechanism [5]. If we define the neutrino Dirac mass matrix, $M_{RL_{ia}}^D$ through

$$\bar{\nu}_{Ri}M_{RL_{ia}}^D\nu_L^a \simeq \frac{\lambda_{ia}^\nu}{f}\bar{\nu}_R^i\Big(\epsilon_{1jk}L_{L_1}^a\Phi_j\Phi_k\Big)\Big(\frac{\Phi_2^\dagger\Phi_1}{f^2}\Big)^{|y_L^a-y_{R_\nu}^i|} \tag{3.83}$$

and the right-handed Majorana mass matrix, $M_{RR_{ij}}$ by

$$\overline{\nu_L^{ic}}\,M_{RR_{ij}}\,\nu_{Rj} = \frac{\eta_1 f}{2}(\overline{\nu_L^{1c}}\tilde{\nu'}_R^5 + \overline{\tilde{\nu'}_L^{5c}}\nu_R^1) + \frac{\eta_2 f}{2}(\overline{\nu_L^{2c}}\tilde{\nu}_R^6 + \overline{\tilde{\nu}_L^{6c}}\nu_R^2)\,, \tag{3.84}$$

where we have defined $\nu_R^T = (\nu_R^1, \tilde{\nu'}_R^5, \nu_R^2, \tilde{\nu}_R^6)$ an $y_{R_\nu}^i$ the right-handed neutrinos flavor charges as reported in tab. (3.4), we have

$$M_{LL_{ab}} = M_{RL_{ia}}^{D^T}M_{RR_{ij}}^{-1}M_{RLjb}^D\,. \tag{3.85}$$

After the symmetry breakings in eq. (3.66) and after having integrating out the right-handed neutrinos , eq. (3.82) becomes

$$\begin{aligned} \mathcal{L}_Y^l &= \tilde{\lambda}^\nu_{ab} \frac{v_W^2}{f} \overline{\nu_L^{a^c}} k^{y_L^{l\,a} + y_L^{l\,b}} + \lambda^e_{ab} e_L^{a^c} \left(v_W e_L^b + v_F \tilde{e}_L^b \right) (-k)^{y_L^b - \frac{1}{2} - y_{\mathcal{E}}^a} \\ &+ \tilde{\lambda}^e_{ab} \tilde{e}_L^{a^c} \tilde{e}_L^b v_F (k)^{|y_L^b - y_{\mathcal{E}}^a|} + H.c. \,, \end{aligned} \tag{3.86}$$

where we can easily read the left-handed Majorana mass matrix of eq. (3.85) and where $O(\tilde{\lambda}^\nu_{ab}) = O([\lambda^\nu_{ia}]^2)$.

As for the quarks, the mixing between the standard charged leptons and the exotic one is negligible and in the discussion of the textures we will consider only the three standard lepton families.

The see-saw in eq. (3.85) is at low energy and therefore provides only a small part of the suppression of the neutrino Yukawa coefficient with respect to the others fermions. The problem of the absolute smallness of neutrino masses is left unsolved in the flhiggs model which only addresses the relative hierarchy in the fermion masses.

3.2.4 Fermion masses and mixing matrices

The fermion mass matrices are obtained from eq. (3.76) and eq. (3.86), respectively for quarks and leptons.

The quark mass matrices can be read off from eq. (3.76) by inserting the charges of all fermions according to Table 3.2. They are given by

$$M_u^{RL} = \lambda^u v_W k^2 \begin{pmatrix} \lambda^u_{11} k^6 & \lambda^u_{12} k^5 & \lambda^u_{13} k^3 \\ \lambda^u_{21} k^4 & \lambda^u_{22} k^3 & \lambda^u_{23} k \\ \lambda^u_{31} k^3 & \lambda^u_{32} k^2 & \lambda^u_{33} \end{pmatrix} \tag{3.87}$$

and

$$M_d^{RL} = \lambda^d v_W \sqrt{k} k^4 \begin{pmatrix} \lambda^d_{11} k^4 & \lambda^d_{12} k^3 & \lambda^d_{13} k \\ \lambda^d_{21} k^3 & \lambda^d_{22} k^2 & \lambda^d_{23} \\ \lambda^d_{31} k^3 & \lambda^d_{32} k^2 & \lambda^d_{33} . \end{pmatrix} , \tag{3.88}$$

where, we recall, the texture parameter is given by $k = v_F^2/f^2$. We have written the mass matrices by extracting an overall coefficient for each matrix according and then treating the ratios of Yukawa couplings as a set of arbitrary parameters to be varied within a $O(1)$ range

The essential feature of these mass matrices is that the fundamental textures are determined by the vacuum structure alone—that is that obtained by taking all Yukawa couplings $\lambda_{ij}^{u,d}$ of $O(1)$. In fact, by computing the corresponding CKM matrix one finds in first approximation

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & O(k) & O(k^3) \\ O(k) & 1 & O(k^2) \\ O(k^3) & O(k^2) & 1 \end{pmatrix}, \tag{3.89}$$

that is roughly of the correct form and, moreover, suggests a value of $k \simeq \sin\theta_C \simeq 0.2$, as anticipated.

At the same time it is possible to extract from (3.87) and (3.88) approximated mass ratios:

$$\frac{m_u}{m_c} \simeq \frac{m_c}{m_t} \simeq O(k^3) \quad \text{and} \quad \frac{m_d}{m_s} \simeq \frac{m_s}{m_b} \simeq O(k^2) \tag{3.90}$$

which again roughly agree with the experimental values.

These results show that the quark masses and mixing angles can be reproduced by our textures. While a rough agreement is already obtained by taking alla Yukawa coupling to be equal, the precise agreement with the experimental data depends on the actual choice of the Yukawa couplings $\lambda_{ij}^{u,d}$. Their values can be taken all of the same order, as we shall see in the appendix, and therefore the naturalness of the model is preserved.

Turning now to the leptons, eq. (3.86) yields the mass matrices

$$M_\nu^{LL} = (\lambda^\nu)^2 \frac{v_W^2}{\eta_1 f} \begin{pmatrix} \lambda_{11}^\nu k^2 & \lambda_{12}^\nu & \lambda_{13}^\nu k \\ \lambda_{12}^\nu & \lambda_{22}^\nu k^2 & \lambda_{23}^\nu k \\ \lambda_{13}^\nu k & \lambda_{23}^\nu k & \lambda_{33}^\nu \end{pmatrix}, \tag{3.91}$$

where λ^ν is an overall factor of the order of the yukawa coupling of the neutrino's Dirac mass matrix , and

$$M_e^{RL} = \lambda^e v_W \begin{pmatrix} \lambda_{11}^e k^3 & \lambda_{12}^e k^5 & \lambda_{13}^e k^4 \\ \lambda_{21}^e k & \lambda_{22}^e k & \lambda_{23}^e \\ \lambda_{31}^e k^2 & \lambda_{32}^e & \lambda_{33}^e k \end{pmatrix}, \tag{3.92}$$

where again we have extracted the overall factors and written the matrices in terms of the ratios of Yukawa couplings divided by the overall coefficients.

The matrices in eqs. (3.91)–(3.92) reduce—at the order $O(k)$, and up to overall factors—to

$$M^{(\nu)} = \begin{pmatrix} 0 & 1 & O(k) \\ 1 & 0 & O(k) \\ O(k) & O(k) & 1 \end{pmatrix} \quad \text{and} \quad M^{(l)} = \begin{pmatrix} 0 & 0 & 0 \\ O(k) & O(k) & 1 \\ 0 & 1 & O(k) \end{pmatrix}, \tag{3.93}$$

where, as before in the case of the quarks, the 1 stands for $O(1)$ coefficients.

The eigenvalues of $M^{(l)}$ can be computed by diagonalizing $M^{(l)\dagger} M^{(l)}$. This product is—again for each entry to leading order in k:

$$M^{(l)\dagger} M^{(l)} = \begin{pmatrix} 0 & 0 & O(k) \\ 0 & 1 & O(k) \\ O(k) & O(k) & 1 \end{pmatrix}. \tag{3.94}$$

By inspection of the 2×2 sub-blocks, the matrix eq. (3.94) is diagonalized by three rotations with angles, respectively, $\theta^l_{23} \simeq \pi/4$ and $\theta^l_{12} \simeq \theta^l_{13} \ll 1$, leading to one maximal mixing angle and two minimal. On the other hand, the neutrino mass matrix in eq. (3.93) is diagonalized by three rotations with angles, rispectively, $\tan 2\theta^\nu_{12} \simeq 2/k^2$ and $\theta^\nu_{23} \simeq \theta^\nu_{13} \ll 1$ (the label 3 denotes the heaviest eigenstate). Therefore, the textures in the mass matrices in eqs. (3.91)–(3.92) give rise to a PMNS mixing matrix [50]—that is the combination of the the two rotations above—in which θ_{23} is maximal, θ_{12} is large (up to maximal), while θ_{13} remains small.

The natural prediction when taking all coefficients $O(1)$ is then: a large atmospheric mixing angle θ_{23}, possibly maximal, another large solar mixing angle θ_{12}, and a small θ_{13} mixing angle; at the same time, the mass spectrum includes one light ($O(k^2)$) and two heavy states ($O(1)$) in the charged lepton sector (m_e, m_μ and m_τ respectively), two light states ($O(k^2)$) and one heavy ($O(1)$) in the neutrino sector, thus predicting a neutrino spectrum with normal hierarchy.

While the quark textures are the same of those discussed in the previous chapter and in ref. [51, 73], those for the leptons are slightly different because of the different flavor symmetry, an abelian $U(1)$ in the flhiggs model as opposed to the $SU(2)$ of the second chapter and of ref. [51, 73].

We have included in the appendix a numerical analysis in which all the experimental data for both quarks and leptons are reproduced by a random choice of the rescaled Yukawa coefficients $\lambda_{ij}^{u,d,e,\nu}$ of order 1. This analysis shows that we need the texture parameter to be $k = 0.14$ and therefore $f \simeq 3.4$ TeV for $v_F \simeq 1.3$ TeV.

3.3 Experimental signatures

The model contains many new particles. As explained, they are necessary in order to implement the collective symmetry breaking that solve the little hierarchy problem. Some live at the scale f, others at the lower scale v_F and all the way to reach the weak scale v_W below which the standard model particles live. In the low-energy range, these new states affect electroweak precision measurements and, as discussed in sec. 3.1.5, this essentially fixes the scale v_F of flavor symmetry breaking which cannot be lowered more than about the TeV. They also affect the overall fit of these precision data and can be included together with standard model radiative corrections.

The range of energies from v_F and f is going to be explored in the next few years by LHC. Let us here summarize these new particles predicted by the model and briefly discuss their main experimental signatures.

The most interesting experimental signature for LHC is in the scalar boson sector. The flhiggs model contains 12 scalar bosons, ten of which are neutral, two charged. For arbitrary coefficients of the potential we lack an analytic result for all their masses (see eqs. (3.61)–(3.62) for the analytically known part). Their values depend on v_F and g' and, after having fixed them, they are a function of the parameters of the potential. These parameters ξ_i, χ_i and λ_i can assume any value as long as they remain of order 1. To obtain an estimate of these masses, we vary the numerical value of the coefficients in the potential by a Gaussian distribution around the natural value 1 with a spread of 20% (that is $\sigma = 0.2$). This procedure gives us average values of these masses with a conservative error and we can consider the result the natural prediction of the model. The error is large enough to cover the uncertainty due to higher loop corrections.

For each solution we verify that all bounds on flavor changing neutral currents are satisfied. The most stringent of these is the potential contribution of the flhiggs fields to the K^0-$\bar{K}^0$ $\Delta S = 2$ amplitude. The presence of the flavor-charged flhiggs fields at such a low energy scale is possible because the relevant effective operators induced by their exchange are suppressed by powers of the fermion masses over f [51, 73, 74].

The lightest neutral scalar boson (what would be called the Higgs boson in the standard model) turns out to have a mass

$$m_{h^0} = 317 \pm 80 \quad \text{GeV}\,. \tag{3.95}$$

This is a rather heavy Higgs mass due to the value of $v_F \simeq 1$ TeV we were forced to take in order to satisfy the bounds on the Z' mass. It is still inside the stability bound for a cut-off of around a few TeVs. It is a value that only partially overlaps with the 95% CL of the overall fit of the electroweak precison data that gives $m_{h^0} < 237$ GeV [3] and gives the most characteristic prediction of the flhiggs model: a heavy Higgs boson (that is, with a mass larger than 200 GeV).

Notice that for a heavy Higgs boson like that we have found, and a cut-off f that we take around 3 TeV in order to generate the correct mass textures, we would have a little hierarchy problem with a fine-tuning of 1% that justifies the little-Higgs mechanism we have implemented in order to be solved.

Above the lightest, the other scalar boson masses are spread, the heaviest of them reaching above f. The lightest charged Higgs bosons has a mass $m_{h^\pm} = 560 \pm 192$ GeV.

Like all little-Higgs models, the presence of the heavy gauge bosons and the additional top-like quarks can be used as signatures in the experimental searches. In addition, the flhiggs model has also a number of exotic fermionic states of known masses and coupling. They couple only weakly with standard fermions, as explained in section 3.2. They can be used as further experimental signatures for the model.

Table 3.5 lists all the particles present in the flhiggs model ordered by the energy scale at which they live.

3.3.1 Estimating the residual fine-tuning

Even though the model was conceived to provide a framework for electroweak and flavor physics free of fine-tuning of the parameters, the requirement of having $v_F \gtrsim 1$ TeV together with that of having the texture parameter k of the order of the Cabibbo angle—and therefore $f \simeq 3$ TeV—reintroduce some amount of fine-tuning.

The bound on v_F implies relationships on the coefficients of the effective potential that, as already discussed, in turn give a fine-tuning of about 10%. We find the same amount of fine-tuning by considering the effect of having $f \simeq 3$ TeV and therefore of having the exotic quarks related to the top with masses of that order. They give a contribution to the (lightest) Higgs boson mass of the order of

$$-\frac{3f^2\lambda_t}{16\pi^2}\log\frac{\Lambda^2}{f^2} \qquad (3.96)$$

which, for $m_{h^0} \simeq 300$ GeV is a correction to be cancelled by the bare mass at the 10% level.

We conclude that while the flhiggs model has still a certain amount of fine-tuning in its parameters, this is substantially less than in the standard model with a light Higgs boson.

3.4 Numerical analysis of textures

In order to show that the mass textures we found reproduce in a natural manner all the experimental data we retain the first non-vanishing contribution to each entry in all mass matrices and then—having extracted an overall coefficient for each matrix according to eqs. (3.87)–(3.88) and eqs. (3.91)–(3.92)—treat the ratios of Yukawa couplings as a set of arbitrary parameters to be varied within a $O(1)$ range. The absolute value of f is immaterial to the textures that only depend on the ratio $k = v_F^2/f^2$. We keep the value of this texture parameter fixed and equal to $k = 0.14$. It corresponds in our fit to the values of $v_F = 1260$ GeV and $f = 3.4$ TeV.

In practice, we generated for the quark and lepton matrices many sets of Yukawa parameters whose moduli differ by at most a factor 10 and accepted those that reproduces the known masses and mixings.

For the leptonic sector, we generate random sets of 14 real parameters. Lacking experimental signature of CP violation in the leptonic sector, we have neglected, for the purpose of illustration, leptonic phases in the numerical exercise.

We obtain that for the representative choice

$$\begin{bmatrix} \lambda^{\nu}_{11} & \lambda^{\nu}_{12} & \lambda^{\nu}_{13} \\ \lambda^{\nu}_{12} & \lambda^{\nu}_{22} & \lambda^{\nu}_{23} \\ \lambda^{\nu}_{13} & \lambda^{\nu}_{23} & \lambda^{\nu}_{33} \end{bmatrix} = \begin{bmatrix} 1.6 & -2.9 & 1.0 \\ -2.9 & 0.55 & -0.40 \\ 1.0 & -0.40 & 2.9 \end{bmatrix} \tag{3.97}$$

with $\lambda_{\nu} = O(10^{-5})$ in eq. (3.91), and

$$\begin{bmatrix} \lambda^{e}_{11} & \lambda^{e}_{12} & \lambda^{e}_{13} \\ \lambda^{e}_{21} & \lambda^{e}_{22} & \lambda^{e}_{23} \\ \lambda^{e}_{31} & \lambda^{e}_{32} & \lambda^{e}_{33} \end{bmatrix} = \begin{bmatrix} -0.26 & -0.83 & 1.0 \\ -0.48 & -1.7 & -0.13 \\ -1.2 & 2.6 & -1.1 \end{bmatrix} \tag{3.98}$$

with $\lambda^{e} = O(10^{-2})$ in eq. (3.92), the experimental values are well reproduced.

We can see by inspection that there is a certain amount of tension between the request of a maximal mixing angle in the $(2,3)$ sector and the mass splitting between the μ and τ that forces an unnatural ratio of about 25 between the smallest and the largest of these ratios of Yukawa coefficients. This was already pointed out in the previous chapter and is a necessary feature of most textures discussed in the literature.

We proceed in a similar manner in the quark sector by generating this time 18 random complex parameters.

We obtain that for the representative choice

$$\begin{bmatrix} \lambda^{u}_{11} & \lambda^{u}_{12} & \lambda^{u}_{13} \\ \lambda^{u}_{21} & \lambda^{u}_{22} & \lambda^{u}_{23} \\ \lambda^{u}_{31} & \lambda^{u}_{32} & \lambda^{u}_{33} \end{bmatrix} = \begin{bmatrix} -1.1+1.3i & 0.37+0.37i & 0.36+0.42i \\ -0.22-1.6i & -0.39-1.2i & 1.0-0.56i \\ -0.16+1.2i & 0.39-1.1i & -1.3-0.22i \end{bmatrix} \tag{3.99}$$

with $\lambda^{u} = O(k^{-1})$ in eq. (3.87), and

$$\begin{bmatrix} \lambda^{d}_{11} & \lambda^{d}_{12} & \lambda^{d}_{13} \\ \lambda^{d}_{21} & \lambda^{d}_{22} & \lambda^{d}_{23} \\ \lambda^{d}_{31} & \lambda^{d}_{32} & \lambda^{d}_{33} \end{bmatrix} = \begin{bmatrix} -0.54+1.4i & -0.38+0.98i & -0.85-0.09i \\ 1.3-0.43i & -0.65+0.52i & 0.51-1.2i \\ -0.62-1.0i & 0.43+0.37i & -0.02-0.54i \end{bmatrix} \tag{3.100}$$

with $\lambda^{d} = O(k^{-1})$ in eq. (3.88), the experimental values are well reproduced.

Table 3.4 summarizes the experimental data and compares them to the result of the above procedure. The agreement is quite impressive. While the values of the overall constants (which are related to the scale of the heaviest state in the mass matrices) are not explained by the model, the hierarchy among the mass eigenvalues and the mixing angles are given in first approximation by the flavor symmetry and the flavor vacuum so that, within each mass matrix, the Yukawa couplings remain in a natural range.

3.5 Conclusions

In this chapter we have presented a unified picture of flavor and electroweak symmetry breaking based on a non-linear sigma model spontaneously broken at the TeV scale. Besides the flavor-electroweak unification, the main features of the model are the stabilization of the electroweak and flavor symmetry breaking scales, obtained by protecting at one-loop scalar VEVs and masses by the little-Higgs mechanism, and the success in reproducing the mass hierarchies and mixings of quarks and leptons.

The requirements to have both the simplest possible unification and the stabilization *á la* little-Higgs have forced both the choice for the global and gauge symmetry, and then indirectly the characteristic particle spectrum the model predicts. This implies that the number of new particles introduced by the model, as huge as it appears, must be considered as the minimal number for a model that aims to unify and stabilize the electrwoeak and flavor scale in a little-higgs inspired scenario. The new particles with masses at the TeV scale, new scalars, new gauge bosons and exotic fermions, even though only weakly coupled to the standard fermions, can be used as further experimental signatures at LHC, in addition to most important one of a rather large mass $m_{h^0} = 317 \pm 80$ GeV for the (lightest) Higgs boson. This large value is not ruled out by the electroweak precision measurements, since all the electroweak fits should be do again with the particle content of the flhiggs model.

The residual amount of fine-tuning arises in the parameters of the potential in eqs. (3.18)–(3.20) required by the split between v_F and v_W and the $O(3\lambda_t f^2/(8\pi^2)\log \Lambda^2/f^2)$ corrections to the Higgs mass coming from the top-like extra fermions loops. However the overall fine-tuning in the model with the mass $m_{h^0} = 317 \pm 80$ GeV for the Higgs boson and $f \simeq 3$ TeV is around 10% , substantially less than in the standard model with a light Higgs boson.

Table 3.2: Representations and charges assignments for the quarks. Different families run over the index i; they differ only for the flavor charges that are written as (q_1, q_2, q_3) for, respectively, the first, second and third family. $U(1)_W$ charges are determined by the data constrains (see text of main body).

		$U(1)_X$	$U(1)_F$	$SU(3)_W$	$U(1)_W$
$\mathcal{Q}_L^i = \begin{pmatrix} Q_L^i \\ 0 \\ \tilde{Q}_L^i \\ U_L^i \\ \tilde{U}_L^i \\ 0 \end{pmatrix}$	$Q_L^i = \begin{pmatrix} d_L \\ u_L \\ \tilde{u}_L \end{pmatrix}$	0	$(9/2, 7/2, 3/2)$	$\overline{3}$	1
	$\tilde{Q}_L^i = \begin{pmatrix} m_L^i \\ n_L^i \\ \tilde{n}_L^i \end{pmatrix}$	0	$(9/2, 7/2, 3/2)$	3	3
	U_L^i	0	$(4, 3, 1)$	1	2
	$\tilde{U}_L^i$	1/2	$(9/2, 7/2, 3/2)$	1	2
$\mathcal{U}_L^{ic} = \begin{pmatrix} 0_\alpha \\ u'^{ic}_L \\ 0_\alpha \\ 0 \\ 0 \\ 0 \end{pmatrix}$	u'^{ic}_L	0	$(3, 1, 0)$	1	-2
$\tilde{\mathcal{U}}_L^{ic} = \begin{pmatrix} 0_\alpha \\ 0 \\ 0_\alpha \\ 0 \\ 0 \\ \tilde{u}'^{ic}_L \end{pmatrix}$	$\tilde{u}'^{ic}_L$	$-1/2$	$(-9/2, -7/2, -3/2)$	1	-2
$\tilde{Q}_L^{ic}$	$\begin{pmatrix} x^{ci} \\ y^{ci} \\ \tilde{y}^{ci} \end{pmatrix}$	0	$(-9/2, -7/2, -3/2)$	$\overline{3}$	-3
	u''^{ic}_L	0	$(-3, -2, 0)$	1	-2
	$\tilde{u}''^{ic}_L$	$-1/2$	$(-9/2, -7/2, -3/2)$	1	-2
	d_L^{ic}	0	$(7/2, 5/2, 5/2)$	1	1

Table 3.3: Representations and charges assignments for the leptons. Different families run over the index i; they differ only for the flavor charges that are written as (q_1, q_2, q_3) for, respectively, the first, second and third family. $U(1)_W$ charges are determined by the data constrains (see text of main body).

		$U(1)_X$	$U(1)_F$	$SU(3)_W$	$U(1)_W$
$\mathcal{L}^i_L = \begin{pmatrix} \tilde{L}^i_L \\ 0 \\ L^i_L \\ 0 \\ \tilde{E}^i_L \\ 0 \end{pmatrix}$	$\tilde{L}^i_L = \begin{pmatrix} z^i_L \\ w^i_L \\ \tilde{w}^i_L \end{pmatrix}$	0	$(1,-1,0)$	$\overline{3}$	-2
	$L^i_L = \begin{pmatrix} \nu^i_L \\ e^i_L \\ \tilde{e}^i_L \end{pmatrix}$	0	$(1,-1,0)$	3	-4
	$\tilde{E}^i_L$	$-1/2$	$(1,-1,0)$	1	0
$\mathcal{E}^{ic}_L = \begin{pmatrix} 0 \\ e^{ic}_L \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	e'^{ic}_L	0	$(9/2, 1/2, 3/2)$	1	3
$\tilde{\mathcal{E}}^{ic}_L = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \tilde{e'}^c_L \end{pmatrix}$	$\tilde{e'}^{ic}_L$	1/2	$(-1,1,0)$	1	3
$\tilde{L}^{ic}_L$	$\begin{pmatrix} z^{c^i}_L \\ w^{c^i}_L \\ \tilde{w}^{c^i}_L \end{pmatrix}$	0	$(-1,1,0)$	3	4
	$\tilde{e''}^{ic}_L$	1/2	$(-1,1,0)$	1	3

Table 3.4: Representations and charges assignments for the two right-handed neutrinos.

	$U(1)_X$	$U(1)_F$	$SU(3)_W$	$U(1)_W$
ν^1_R	0	1	1	0
$\tilde{\nu'}^5_R$	0	-1	1	0
ν^2_R	0	0	1	0.
$\tilde{\nu}^6_R$	0	0	1	0

Table 3.5: Particles and energy spectrum of the model

energy scale	states
$f \simeq 3$ TeV	z_{ij}, s, $s_{1,2,3}$, W'_{1-8}, B', X', ν_R^{1-16}, $\tilde{Q}_f$, $\tilde{L}_f$
$v_F \simeq 1$ TeV	$\tilde{W}^{\pm}_{1,2}$, Z', X, $\tilde{q}_f$, $\tilde{l}_f$, $\hat{q}_f$, $\hat{l}_f$, $\hat{\tilde{q}}_f$, $\hat{\tilde{l}}_f$
between v_W and v_F	$\rho_1^{\pm}$, $\delta_{1,3,4}$, ϕ_{1-6}, $h^0(\phi_7)$, $h^{\pm}(\rho_2^{\pm})$
below $v_W = 246$ GeV	γ, $W^{\pm}$, Z, q_f, l_f

Table 3.6: Experimental data vs. the result of our numerical analysis based on a representative set of Yukawa couplings of order one (see text) and $k = 0.14$. Uncertainties in the experimental data are explained in the previous chapter.

	exp	numerical results
$\|V_{us}\|$	$0.219 - 0.226$	0.22
$\|V_{ub}\|$	$0.002 - 0.005$	0.003
$\|V_{cb}\|$	$0.037 - 0.043$	0.04
$\|V_{td}\|$	$0.004 - 0.014$	0.007
$\|V_{ts}\|$	$0.035 - 0.043$	0.04
δ	$61.5^o \pm 7^o$	53^o
$\sin 2\beta$	$0.705^{+0.042}_{-0.032}$	0.71
m_t/m_c	248 ± 70	222
m_c/m_u	325 ± 200	369
m_b/m_s	40 ± 10	40
m_s/m_d	23 ± 10	17
$\tan^2 \theta_\odot$	$0.23 - 0.69$	0.67
$\sin^2 2\theta_\oplus$	$0.8 - 1.0$	0.9
$\sin^2 \theta_{13}$	< 0.09	0.03
$\Delta m^2_\odot / \Delta m^2_\oplus$	$0.014 - 0.12$	0.06
m_τ/m_μ	17	17
m_μ/m_e	207	190

Conclusions

In this thesis we have discussed how the little-Higgs mechanism can be applied to stabilize the electroweak scale (the littlest Higgs model), the flavor scale (the little flavon model) and the two of them together (the flhiggs model). All the models achieve what they were designed to achieve but only up to a point.

The results obtained in discussing the little flavon model indicate that this mechanism may be used to stabilize the flavor scale and at the same time explains the fermion mass hierarchies and the quark and lepton mixings. However, the model is not completely satisfactory because the requirement of gauging the flavor symmetry forces the flavor scale to be around 10^4 TeV and re-introduces a problem of hierarchy.

The littlest Higgs and the flhiggs models share in the prediction of a large mass for the lightest scalar present in the two models—to be identified with the Higgs boson—if we require the spontaneous global symmetry breaking scale f to be around 2 TeV and accept at most a 10% fine-tuning. To be more precise, these predictions indicate a Higgs boson mass $m_H \sim 800$ GeV (see fig. (1.6)) in the littlest Higgs model and $m_H \sim 300$ GeV in the flhiggs model (see eq. (3.95)). These results are not in contrast with the value indicated by the electroweak data fits— that is, a Higgs boson mass $m_H = 114^{+69}_{-45}$ GeV [8]— because this result only holds within the standard model and all electroweak data fits should be computed again after taking into account the new particles and interactions introduced by the models. Nevertheless, as pointed out in ref. [30] in the littlest Higgs model a heavy Higgs boson with m_H around the TeV, may be compatible with the electroweak precision measurements only if the scale f of the model is rised up at 5 TeV. As a consequence, the residual amount of fine-tuning which is present in every little Higgs model turns out to be of the same order of the fine-tuning required within the standard model to stabilize a light Higgs boson mass with a cut-off of 10 TeV. This is somewhat disappointing.

The flhiggs model fares better than the littlest Higgs model. Even though the Higgs boson mass predicted is heavy with respect to the central value indicated by the electroweak precision measurements [8], it turns out to be of the same order of the electroweak scale and not around the TeV thus bypassing the electroweak constraints of ref. [30]. The model is (a lot) more complicated that the littlest Higgs but it achieves much more in providing an unified picture of flavor and weak physics. As the littlest Higgs model (and the little flavon as well), it predicts a distinctive spectrum of new particles and processes to be explored at the LHC.

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