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Topics in Exotic Fermion Phenomenology

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Contents

Preface

In the past decades the major task faced by particle theorists was that of explaining particles that where known to exists (as strange particles and the large number of baryonic resonances) and predicting other particles whose existence was fairly well established on the basis of indirect experimental and theoretical considerations (as the charm quark and the W and Z gauge bosons).

Nowadays, even if we still lack a completely satisfactory explanation for some important theoretical issues as fermion family replication, the origin of masses and of mass scales, etc... high energy experiments are in striking agreement with the predictions of the standard model, and all the known elementary particles are indeed accounted for.

However a few experimental cosmological and astrophysical observations exists that are challenging for theoretical physics and that could possibly be explained by invoking new particle physics models. Many efforts are in fact devoted in analysing the properties of the countless candidates (that most often are new unknown particles) for the dark matter of the galactic halos and for the missing mass of the universe, and several plausible explanation of the observed deficit of solar neutrinos also invoke non–standard properties of the known elementary neutral states.

Besides the search for explanations for these few unresolved issues, quite often the only motivation for studying the properties of new unknown particles is just that they have no reasons not to exist, and similarly non–standard properties of the known states are regarded as interesting as far as they do not conflict with the existing experimental results. Still any time we are able to rule out the existence of such new particles or to put tight constraints on non–standard parameters, we can claim we have learned something new.

The main topics discussed in this thesis concern exotics fermions, that are indeed particles whose existence lacks of any experimental evidence. Nevetheless, as I will show, some of the properties of these hypothetical particles can be effectively constrained. In addition, the presence of these new fermions is expected to induce deviations in the behaviour of the known fermions from what is expected in the frame of the standard model. These non–standard effects are analysed in some detail and constrained on the basis of the available experimental data.

The issues I will deal with in the different chapters of this thesis could seem to some extent unrelated one with the other, but there is a unique guideline throughout this research project (that is still far to be concluded) and it is worth to sketch it here.

In the summer 1989 A. De Rújula, S. Glashow and U. Sarid published an article in which they proposed a new and rather unconventional candidate to solve the dark matter problem. It was a *charged* massive particle (CHAMP) that was assumed to be absolutely stable and to survive annihilation in the early universe

[1]. Their work gave rise to a number of analysis pursuing a twofold goal, the first one was that of finding CHAMP's candidates among the various particles predicted by new physics models, the second one was that of ruling out this possibility. At that time my friend and collaborator Esteban Roulet had already achieved a good knowledge in the physics of the early universe and in the techniques for computing cosmological relic densities, while I had developed some experiences in working in the frame of E_6 models. The fundamental representation of E_6, to which fermions are assigned, contains 12 new degrees of freedom for each generation, among these we chose as our CHAMP candidate a colour triplet exotic quark with electric charge $Q = -1/3$ since, being a weak–singlet, this particle does not decay via gauge interactions and thus it had some chances to be absolutely stable. We started our analysis trying to figure out which could be the allowed window for the mass of our candidate, but eventually we had to recognize that, owing to the fact that our CHAMP's, besides being electrically charged were also strong interacting particles, there was no way to let enough of them survive primordial annihilation without conflicting with astrophysics or with the existing limits from searches for superheavy elements [2]. After an initial disappoint we realized that this seemingly negative result could be turned in the rather general claim that *on cosmological grounds, absolutely stable exotic quarks are not allowed to exist*, and this result was shown to hold beyond the frame of E_6 models.

A straightforward consequence of this result was that any model that predicts new strong interacting particles should also provide some mechanisms to let them decay. For our exotic weak singlet the most obvious way to insure its decay into light states was indeed that of allowing for a mixing with the ordinary light $Q = -1/3$ quarks. A mixing with exotic fermions induces non–universal modifications in the couplings of the ordinary mass eigenstate fermions to the gauge bosons. In the frame of E_6 models new neutral gauge bosons are also predicted to exist, and the fermion couplings to the (mass eigenstate) Z are also modified by a possible mixing between the standard Z and one (or two) additional Z' boson(s). This latter modification is universal and we were aware of several analysis where neutral current data were used to constrain the Z–Z' mixing angle, as well as the Z' mass. However in all these analysis the effects of fermion mixing had so far always been neglected.

The project of constraining E_6 models by simultaneously taking into account ordinary–exotic fermion mixings, the mixing between neutral gauge bosons and the effects of Z' exchange in off–resonance experiments, immediately shown itself to be a formidable one. In order to simultaneously bound the very large number of parameters (six mixing angles where needed to describe the mixings of the charged fermions and three parameters for the Z' effects, not to mention the rather complicated E_6 neutrino sector) the analysis had to be extended to include charged currents data as well.

A very general analysis of the limits on fermion mixings already existed, it was performed by Langacker and London in 1988 [3]. They had developed a general formalism to describe ordinary–exotic fermion mixings, and by using a very large set of experimental inputs they managed to constrain at the same time more than 20 mixing angles. Bounds on E_6 fermion mixings were derived as a particular case of the general analysis, however in doing so the effects of the new neutral gauge bosons and of a possible Z–Z' mixing were not taken into account.

A complete analysis of these models was (and is) still missing, and we realized that besides being an interesting research project, this could have been indeed a very good opportunity for acquiring a good knowledge in particle physics experimental procedures.

The analysis presented in [3] was carried out before the starting of the operation of the SLC and LEP machines, and it was clear that already the very first LEP results constituted at that moment a very important set of new experimental inputs. As a warming up exercise we decided to study how much the preliminary data from Z–peak experiments could improve the bounds on those mixings that were most poorly constrained. In spite of the fact that we included also Z–Z' mixing effects, the improvement in the constraints for the b and τ mixing angles that we could obtain with these new data was indeed quite a remarkable one [4].

In view of the fact that the analysis of the experimental data collected at LEP during the 1989 and 1990 runs was expected to be completed by all the four LEP collaborations by the end of the spring 1991, a few months in advance we started collecting and organizing all the available charged current and neutral current data. For this effort we asked our friend Daniele Tommasini to join us for working on the project. We decided to use the large amount of very precise data that we were collecting for a general analysis of fermion mixings, *i.e.* essentially for updating the original results of Langacker and London, leaving the particular case of E_6 models for a subsequent work. Our collaboration was rendered somewhat difficult by the fact that in the meanwhile Esteban left the SISSA and moved to Fermilab, but with no doubt this work has been of a great satisfaction for all of us. The area spanned by this research is a wide one, starting from theoretical topics as radiative corrections, neutrino physics and right handed currents reaching more experimental ones as cross section normalizations, detector acceptances and energy cuts, we had to learn and understand many different issues to deal properly with the experimental data. The large amount of CPU time required by the numerical analysis (a few *days* on the VAX 6410 of the SISSA) is probably unusual for particle physics theoreticians, and we had to develop a more than amateurish knowledge of computer programming, in order to optimize our procedures. Eventually the improvement that we obtained with respect to the existing limits was quite a remarkable one, for most of the parameters the bounds were strengthen by about one order of magnitude, and the fact that we could confirm an already existing indication that non–standard physics is possibly at work in the $\tau - \nu_\tau$ sector (a signal of a non–zero ν_τ mixing at 90 % c.l was found) added some thrill in the final phase of the interpretation of the results [5].

The organization of the work is the following: in the first chapter after a brief introduction to the techniques for computing cosmological relic densities, the mechanism leading to quark–antiquark annihilation is described in some detail. The present abundance of heavy exotic quarks is then computed as a function of their mass and the possibility of having a stable exotic quark is finally rejected on the light of the unsuccessful searches of anomalously heavy isotopes and for the astrophysical implications they would have.

PREFACE

A natural way to open decay channels for the exotic fermions is to allow for their mixing with the ordinary ones, provided they have the same quantum number assignments under the unbroken colour and electromagnetic gauge groups. The consequences of this mixing in the frame of E_6 models, where also new neutral gauge bosons are generally present, is considered in chapter two. After a short introduction to the formalism for describing Z–Z' mixing effects, the particular case of E_6 *charged* fermion mixing is analysed. It is shown that both these effects can induce similar modifications in the couplings of the ordinary fermions to the Z boson, so that they should be simultaneously taken into account when constraining these models. The Z partial decay widths into leptons and into b quarks determined at LEP are then used to illustrate how these mixings can be effectively bounded by measurements at the Z–peak.

The general formalism for describing fermion mixing with exotic states is presented in chapter three. In this chapter is a comprehensive analysis of the constraints on a general class of fermion mixings that can appear in many different extensions of the electroweak theory, such as models with mirror fermions or with new vector singlets and/or doublets (as the E_6 models) is carried out. In this analysis the known fermions are allowed to mix with new heavy particles with unconventional $SU(2)\times U(1)$ quantum numbers assignments (left-handed singlets or right-handed doublets) and limits on deviations of the lepton and quark weak–couplings from their standard values are obtained. As experimental constraints the new results on M_Z, Γ_Z, on the Z partial decay–widths and on the asymmetries measured at the Z resonance, as well as updated results on the W mass, on deep-inelastic ν–q and ν–e scattering and on atomic parity violation are used. Present constraints on lepton universality, unitarity of the quark mixing matrix and induced right-handed currents are also included. All these experimental inputs are briefly discussed. A global analysis of all the data leads to tight upper limits on the mixing factors $s^2 \equiv \sin^2\theta_{\rm mix}$. The interesting case when ν_τ mixes mainly with an ordinary heavy neutrino is discussed in more detail since in this situation a signal of non–zero mixing at 90 % c.l. is found. The analysis is performed both in the case when just one fermion is allowed to mix and in the more complicated situation when all the mixings are simultaneously present, thus allowing for accidental cancellations among different contributions.

Chapter 1

Can Exotic Quarks be Absolutely Stable ?

In this chapter we examine the possibility of the existence of new stable exotic quarks, as for example the $Q = -1/3$ charged quarks predicted by E_6 models. It is shown that their cosmological consequences combined with bounds from superheavy element searches and the requirement that heavy particles captured by neutron stars do not induce their collapse into a black hole exclude that possibility [2]. Thus in these models some mechanism must exist to allow the exotic quark decay.

1.1 Introduction

Several GUT extensions of the standard model enlarge considerably the particle content of the three standard generations, and some of them include heavy 'exotic' quarks among the new particles. An interesting example are the E_6 models [6] like those arising from ten dimensional superstring theories after compactification to the four physical dimensions [7]. In these models each generation of fermions is assigned to a **27**–dimensional (fundamental) representation that, together with the standard fifteen fermionic degrees of freedom, includes twelve additional new fields. The electromagnetic charge and colour quantum numbers of the new particles are univocally determined by the group structure. Besides the existence of new exotic charged and neutral leptons, each generation contains a new colour triplet and weak singlet quark that we will denote h of electric charge $Q = -1/3$. The mass of this quark is in principle arbitrary, but a lower bound on it comes from present accelerator experiments. Searches at LEP exclude the existence of new quarks lighter than about $M_Z/2$.

Since under the unbroken $SU(3) \times U(1)$ gauge group the heavy quarks transform with the same quantum numbers as the standard 'down type' quarks, a mixing among them is allowed. If this mixing is present, the h mass eigenstate should decay weakly into standard fermions. However, since the $SU(2) \times U_Y(1)$ quantum numbers are different, such a mixing induces deviations from the weak interactions of the down–type mass eigenstates predicted by the Standard Model (SM) and could also give rise to flavor changing neutral currents. Present experimental data restrict considerably the allowed values for the mixing angles, a quite general analysis, valid also beyond the assumption of E_6 fermion representations, yields the 90% confidence level limits $\sin\theta_{\rm mix} \lesssim 0.02, 0.04, 0.07$ [5] respectively for the first second and third generation $Q = -1/3$ quarks.

In this chapter we want to analyse the consequences of assuming the exotic quark to be stable (or nearly so, *i.e.* with lifetime larger than the age of the universe). For some superstring–inspired E_6 models, a stable exotic quark is a natural consequence of the particular structure of the superpotential. In fact in order to avoid low–energy baryon and lepton number violation, it is necessary to require that some potentially dangerous couplings vanish, and this is most easily done by introducing certain discrete symmetries. In turn, these symmetries often imply that the mixing between the exotic quarks and the ordinary down type quarks vanishes [8]. Although we have in mind the E_6 candidate, the analysis can be extended with minor modifications to other coloured particles (charge 2/3 quarks, sextet quarks, etc ...).

If the heavy quark is stable, it can have important cosmological consequences. Its present density can be computed by following the thermal evolution of the universe. At very early stages its abundance is determined by the thermal and chemical equilibrium. Subsequently, the cooling of the universe reduces the annihilation rate of the heavy quarks until, at the freeze out temperature, the chemical equilibrium can no longer be maintained. However, as we will show, the heavy particles still remain in thermal equilibrium. In the confinement transition the exotic quarks hadronize together with the ordinary quarks. At this stage the relevant annihilation cross section associated with the disappearance of the heavy quarks increases and can reach a typical hadronic size, so that a significant reduction of the relic density of heavy hadrons takes place after confinement. (Our results on the present abundance of heavy quarks differ from a previous analysis that overestimated the annihilation cross section after confinement [9].) The superheavy hadrons are subject subsequently to primordial and eventually stellar nucleosynthesis, where heavy nuclei are also produced. The stringent experimental bounds from searches of superheavy elements are a powerful test for the existence of these hypothetical stable exotic particles. We use also the bounds on very massive charged particles contributing to the cosmic dark matter that have been recently obtained from the study of their effects on the evolution of neutron stars [10]. These bounds, together with the cosmological requirements that the universe not be overclosed by these particles totally exclude the existence of a stable exotic quark.

In the following we assume that no asymmetry between particles and antiparticles is present. This assumption does not affect the generality of the conclusions since an asymmetry can only increase the abundance of superheavies, giving more strength to the bounds obtained.

1.2 Cosmological Relic Density

The equation that describes the evolution of the number density n of stable species is

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3\frac{\dot{R}}{R}\, n - \langle\sigma v\rangle(n^2 - n_{eq}^2) \tag{1.1}$$

where R is the scale factor of the universe, $\langle\sigma v\rangle$ is the thermally averaged annihilation cross section times the relative velocity and n_{eq} is the value of the density in chemical equilibrium. The integration of this equation yields, for the relic mass density at present time ρ_o [11],

$$\rho_o \simeq \frac{\sqrt{2\pi}}{M_{Pl}}\; s_o \;\left[\int_{x_{fo}}^{x_o} \frac{dx}{x^2}\; \sqrt{b}\; \langle \sigma v \rangle\right]^{-1} \tag{1.2}$$

where $s = \frac{2\pi^2}{45} g_{eff} T^3 \equiv (bT)^3$ is the entropy density, $g_{eff}(T)$ is the effective number of degrees of freedom at temperature T, $x \equiv m/bT$ with m the mass of the particle, and M_{Pl} is the Planck mass. The freeze out value x_{fo} is given by

$$x_{fo} \simeq \frac{1}{b}(ln\ B - \frac{1}{2} ln\ \frac{1}{b} ln\ B)\Big|_{fo} \tag{1.3}$$

where

$$B \equiv \Delta\ m \frac{M_{Pl}}{2\pi^2 b^2}\ \langle \sigma v \rangle \tag{1.4}$$

with the numerical factor Δ, of order one, depending on the criterion used to define the freeze out temperature. (A fit to the numerical integration of (1.1) leads to a preferred value $\Delta \simeq 1.5$.)

At temperatures above the confinement temperature $T_c \sim 200$ MeV, the relevant annihilation cross section of two heavy quarks h involves the channels $h\bar{h} \rightarrow gg\ ,\ q\bar{q}$, where g is a gluon and q an ordinary quark (we neglect the contribution to the annihilation cross section coming from electroweak channels). For a colour triplet quark in the non-relativistic limit ($T \ll m$) annihilating into N_f lighter flavors, we obtain

$$\langle \sigma v \rangle_{h\bar{h}} = \frac{\pi \alpha_s^2}{m^2} \left(\frac{2}{9} N_f + \frac{7}{27}\right) \tag{1.5}$$

In this expression, we take $\alpha_s(Q^2)$ renormalized to the scale $Q^2 \sim m^2$, since this is the relevant momentum transfer involved in the annihilation. Assuming only the standard physics at energies below m, we get for instance $\alpha_s((10\ \text{TeV})^2) \simeq \alpha_s(M_{W}^2)/2$. The appearance of new physics below the exotic quark mass could affect the value of α_s and could also open new channels for the annihilation. This is the case, for instance, if supersymmetry is present at the weak scale, since annihilations involving squarks and gluinos could contribute to (1.5).

When confinement occurs, due to the presence of a relatively large number of ordinary quarks, the heavy quarks h will hadronize mainly forming a system of 'superheavy kaons' ($h\bar{q}$ and $\bar{h}q$, with $q = u,\ d$) and, due to the baryon asymmetry, the $h\bar{q}$ mesons will finish as superheavy baryons hqq through annihilations with ordinary nucleons.

In a previous study of the survival of heavy quarks [9], the annihilation cross section below T_c was estimated to be equal to the ordinary nucleon-antinucleon cross section, $\sigma_{N\bar{N}} \sim 30$ mb$/v$. However, several reasons indicate that this is an overestimate. In fact, in (1.1) only the exclusive cross section that does not contain the two heavy quarks in the final state should be used. Since we are considering energies below Λ_{QCD}, the light quarks cannot be considered as spectators in the process of $h\bar{h}$ annihilation, which for instance could proceed through the hadronic process $\bar{h}q + hqq \to \bar{h}h + qqq$, with the formation of a $\bar{h}h$ bound state, which consequently decays into light particles. Although the associated cross section could be of hadronic size, since the Compton wavelength of TeV particles at MeV energies is $\lesssim$ fm, the total annihilation cross section cannot exceed the characteristic geometrical cross section associated to the range of the interactions ($\sim$ fm), *i.e.* $\sigma \lesssim 4\pi$ fm$^2 \sim 100$ mb. Instead, with the previously mentioned estimate of $\sigma \sim \sigma_{N\bar{N}}$ the very slow thermalized heavy hadrons (with $v = \sqrt{6T/m}$) would have a cross section much larger than one barn. The heavy hadrons, unlike the ordinary nucleons, enter the long wavelength regime where the annihilation cross section can grow beyond the geometric one only at temperatures ($<$ MeV) where the densities have been too diluted for the annihilation to be efficient. Furthermore, the exchange of ordinary mesons (π, ω, ...), that gives the main contribution to the total low energy annihilation of hadrons but does not affect the number of heavy quarks, should not be included in $\langle \sigma v \rangle$. Also, since the baryon asymmetry has washed out the ordinary antiquarks from the heavy hadrons, the vector exchange leads to repulsive interactions that further reduce the annihilation rate of $h\bar{h}$. In view of the previous discussion, we expect the relevant annihilation cross section to be much less than the corresponding geometrical cross section, and we will parametrize it as

$$\sigma \equiv f \cdot 100 \text{ mb} \tag{1.6}$$

with $f < 1$.

In figure 1 we show the resulting values of Ωh^2 for the upper value $f = 1$ and for $f = 0.1$. Here Ω is the present relic mass density of heavy quarks in units of the critical density, the Hubble constant h is given in units of 100 km/s Mpc and, from observations, $0.4 \leq$ h ≤ 1. Due to the large uncertainty in the estimation of f that is related with the non-perturbative effects involved, we have also plotted the other extreme case in which the annihilation rate after confinement is negligible. This

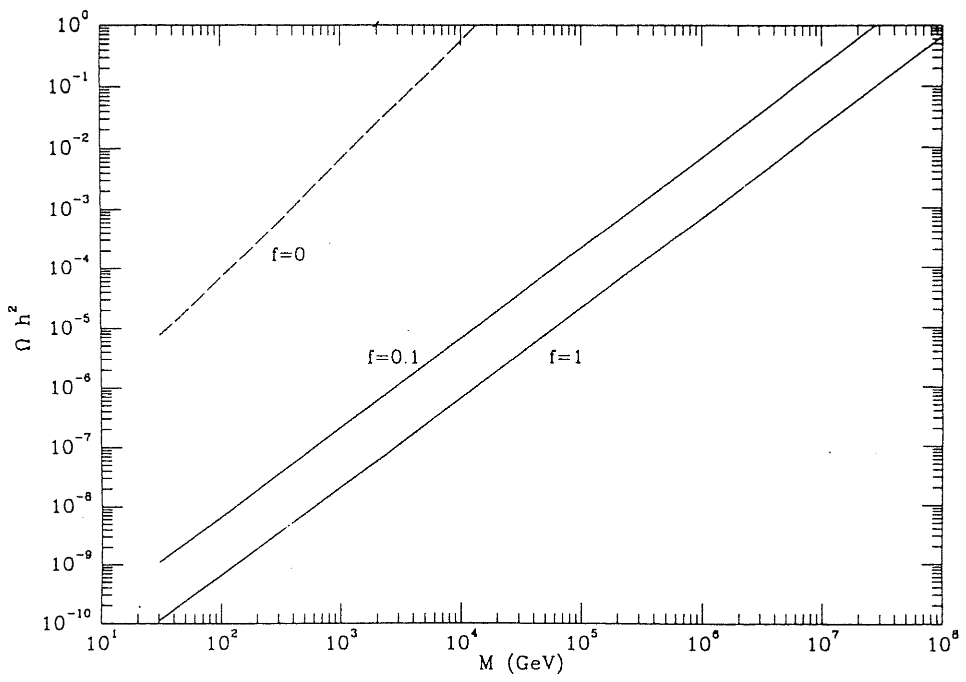

Fig. 1: Relic density of exotic stable quarks as a function of their mass for annihilation cross sections into light species after confinement of 100 mb ($f = 1$), 10 mb ($f = 0.1$) or negligible ($f = 0$).

would correspond to an annihilation process $\bar{h}q + hqq \rightarrow$ *light hadrons* proceeding essentially through a partonic–like cross section. In this case, Ω is determined with good accuracy (since $T_{fo} >> T_c$) by the free quark annihilation rate before hadronization. Quantitatively, it is safe to neglect annihilations below T_c as long as

$$\langle\sigma v\rangle_{T<T_c} \lesssim \frac{T_{fo}}{T_c}\langle\sigma v\rangle_{h\bar{h}} \sim \frac{m}{5\ \mathrm{GeV}}\langle\sigma v\rangle_{h\bar{h}} \tag{1.7}$$

where we have used a typical freeze out temperature $T_{fo} \sim m/30 - m/20$.

Figure 1 clearly implies that masses larger than $\sim 10^5$ TeV are cosmologically excluded, since they would yield $\Omega h^2 \gtrsim 1$, overclosing the universe. Moreover, the observed lifetime of the universe suggests $\Omega h^2 \lesssim 0.25$, making this constraint even stronger. For E_6 models, in the absence of any kind of mixing, each of the three flavours of exotic quarks contribute to Ω, and then the cosmological bounds should be applied to the sum of their contributions. This leads to an upper limit of $m \sim 3 \cdot 10^4$ TeV if the heavy quark masses are assumed to be similar.

If the heavy quark is not a triplet of colour (sextets of heavy quarks have been considered *e.g.* in [9]), or if new physics is present at energies below m, the annihilation cross section would differ (but in principle not drastically) from the case previously discussed, and the value of the relic density Ω, which is inversely proportional to it, will be correspondingly modified.

1.3 Is Thermal Equilibrium Maintained ?

In the previous computation we have assumed that the heavy particles remain in thermal equilibrium. An argument to justify this assumption goes as follows: thermal equilibrium is maintained if the amount ΔE of energy exchanged through collisions during an expansion time $\tau \simeq M_{Pl}/T^2$ is larger than the original energy

$E \sim T$ of the heavy particle, *i.e.*:

$$n\, v\, \sigma\, \tau\, \Delta E \gtrsim E \tag{1.8}$$

where n is the number density of the scatterers, v is their mean velocity, and σ their typical cross section. Before confinement, thermalization proceeds mainly through scattering off thermalized quarks and gluons through t–channel gluon exchange. Although the corresponding cross section has a Coulomb–like divergence associated with the exchange of soft gluons in the forward scattering, the relevant quantity for the thermalization is the energy transfer cross section

$$\sigma_{tr} = \int d\,\cos\theta\; \frac{d\,\sigma}{d\,\cos\theta}\,(1-\cos\theta) \tag{1.9}$$

where θ is the center of mass scattering angle. Since inside the quark-gluon plasma the color charges undergo an 'electric' screening with a typical length [12] m_{el}^{-1} with $m_{el}^2 \simeq (gT)^2(N+N_f/2)/3$ playing the role of an effective gluon mass (N=3 is the number of colors), after taking into account this effect we obtain a finite result:

$$\langle\sigma_{tr}\rangle \sim \alpha^2 \frac{m^2}{T^4}\, \ln(\frac{9T^2}{m_{el}^2})\ . \tag{1.10}$$

Since in this case the momentum transfer is $\Delta p \sim T$, $v \simeq 1$ and $n \sim T^3$, (1.8) is always satisfied.

After confinement, taking into account only the scattering off nucleons, we can derive from (1.8) an upper bound for the mass of a heavy hadron for it to be in thermal equilibrium: the baryonic asymmetry yields $n_N \sim 10^{-9}T^3$, while for non relativistic particles $v \sim \sqrt{T/m_N}$, and $\Delta E \sim T\sqrt{m_N/m}$, with m_N the mass of the nucleon. We then obtain:

$$m[\text{GeV}] \lesssim 10^{12} \left(\sigma\,[\text{mb}]\right)^2 \left(\frac{T}{\text{MeV}}\right)^3 \tag{1.11}$$

We see that for a typical hadronic elastic cross section, the assumption of thermalization is correct in the whole range of masses that we have considered. Moreover, for $T \sim m_\pi$ the large number of pions present will further contribute to the thermalization of the heavy hadrons, and for the charged ones also the scattering off photons and electrons will contribute, leading in general to a bound higher than (1.11).

1.4 Limits from Searches for Anomalous Elements

Now, in order to see what kind of superheavy elements we should expect to find at the present time, and where we should look for them, we will follow the evolution of the heavy quarks from the confinement transition until now.

From T_c and up to $T \sim 1$ MeV, the electroweak interactions among the different heavy hadrons (for instance $\bar{h}d + \nu \leftrightarrow \bar{h}u + e$) will determine, due to the mass difference of some MeV's between the up and down quarks, an excess of $\bar{h}u$ over $\bar{h}d$. Their abundances should have the ration

$$\frac{n_i}{n_j} = e^{-(m_i - m_j)/T} \tag{1.12}$$

and we expect their mass difference to be comparable to the ordinary K–meson or B–meson mass splittings.

For the heavy baryon, we will assume that the neutral isosinglet particle state $(hud)_{I=0}^{I_3=0}$ is lighter than the positively charged member of the isotriplet $(huu)_{I=1}^{I_3=1}$. This assumption is based on the same kind of analysis that explains qualitatively the mass relationship $m_{\Lambda^0} < m_{\Sigma^+}$: for s–wave baryons, the antisymmetry of the isosinglet $(ud)_A$ state in the internal isotopic–spin space forces the spins of the two light quarks to be antiparallel, and the energy of this configuration is lower with respect to the energy of the symmetric $(ud)_S$ triplet diquark state, that implies aligned spins [13].

At $T \sim 1$ MeV, electroweak interactions freeze out and primordial nucleosynthesis has begun. At these temperatures we expect that most of the heavy mesons will be positively charged $\bar{h}u$, while most of the heavy baryons should be $h(ud)_A$ neutral isosinglets. The surviving neutral $\bar{h}d$ mesons, if they do not bind to any nucleus, will weakly decay with a typical meanlife of a few seconds. If they bind to nuclei, their decay rate depends on the Coulomb barrier that they feel inside the positively charged nuclei, but in all cases they finish as charged elements. In contrast, due to the large mass splitting (O(100 MeV)), the few surviving isotriplet baryons should decay electroweakly into the neutral isosinglet even if they bind to nucleons. During nucleosynthesis, a fraction of these neutral baryons may bind to protons (and neutrons) giving rise to superheavy positively charged elements.

This fact has an important consequence, because when galaxies form, after the so called 'violent relaxation', all the atoms that have fallen into the potential well of the galaxy become ionized and, as shown in [1], the charged elements lighter than ~ 20 TeV fall into the disk together with the ordinary baryons. So for $m \lesssim 20$ TeV we expect most of the charged heavy elements to be found in the disk, while for larger masses they should originally remain mainly in the halos of galaxies. However, larger concentrations of superheavy elements in the disk are to be expected also for masses > 20 TeV, since these particles can be captured by the disk during the following evolution of the galaxy. It was recently suggested [14] that this could happen efficiently for particle masses up to 10^5 TeV. Clearly the heavy baryons that do not form charged nuclei during nucleosynthesis will for the most part remain in the halo, leading to an asymmetry between h and $\bar{h}$ concentrations in the disk. The heavy elements that had fallen into the disk will be subject to stellar nucleosynthesis forming also superheavy nuclei of large Z.

For $m \lesssim 20$ TeV the proportion of heavy hadrons H with respect to ordinary nucleons present in the stars (and in the earth) should be of the order of the ratio of their cosmological densities: $n_H/n_{bar} = (\Omega_H/m)/(\Omega_{bar}/m_p)$, with m_p the proton mass. The resulting concentrations are enormously large ($\gtrsim 10^{-9}$) and exceed by several orders of magnitude the existing experimental bounds. For instance, searches of superheavy water exclude concentrations of heavy hadrons with respect to ordinary hydrogen in water larger than $\sim 10^{-28}$ for masses $\lesssim 1$ TeV [15], larger than $\sim 10^{-24}$ for $m \leq 10$ TeV [16], and with less certainty $> 10^{-15}$ for larger masses [15]. Actually, in the earth, we expect an even larger concentration of elements containing H hadrons, since they did not evaporate away as most of the ordinary light elements did during the earth lifetime.

1.5 Astrophysical Constraints

For $m > 20$ TeV the heavy elements remain as dark matter or are captured in the disk, giving a contribution to the density of the galactic halo ρ_{halo} in the neighborhood of the disk of at least $\Omega_H \cdot \rho_{halo}$. However, the existence of large amounts of heavy CHArged Massive Particles (CHAMP's) [1] has been recently shown to be in contradiction with the observed long life of several neutron stars [10]. This is due to the fact that CHAMP's captured by the protostellar cloud should collapse into the interior of the stars forming a black hole that would destroy the star in a time scale $\sim$ yr. For the black hole to form it is necessary that the total mass of the captured heavy particles exceeds the Chandrasekhar mass, so that degeneracy does not prevent the gravitational collapse. Since the capture by the protostellar cloud depends on the electromagnetic cross section off hydrogen [10], although the bounds were deduced for leptonic CHAMP's, they also hold for the charged hadronic superheavies under consideration. In the range of masses 20 TeV $\lesssim m \lesssim 10^5$ TeV, which is less constrained by searches of superheavy elements, contributions to the halo densities larger than $10^{-7} - 10^{-8}$ are ruled out by this argument. (This should be compared with our prediction of more than

10^{-6} in this mass range.)

For E_6 models, since nearly all the $\bar{h}$'s form charged elements (*e.g.* $\bar{h}u$), while a sizeable fraction of the h's give rise to neutral states that are not efficiently captured by the protostellar cloud, we expect that a $\bar{h} - h$ asymmetry will be present inside the neutron star. This should be a general feature of models for which h and $\bar{h}$ belong to hadrons of different charge, or even both neutral, since in these cases we expect that they should bind differently with nuclei [9]. As a consequence, they should be captured by the protostellar cloud at different rates due to their 'chemical' difference, and since even a tiny asymmetry ($\lesssim 1\%$) between the concentrations of h and $\bar{h}$ would leave, after eventual $\bar{h} - h$ annihilation inside the star, enough superheavies to produce a black hole, the same conclusions deduced in [10] still hold in this case. In the case in which $\bar{h} - h$ were to form hadrons of equal charge, the large Coulomb barrier will prevent them from annihilating at the typical temperatures of neutron stars.

The previous analysis then leads to the conclusion that the existence of a stable exotic heavy quark can be safely ruled out.

We also note that in the presence of a particle–antiparticle cosmic asymmetry between the heavy quarks, all the bounds would be stronger: the relic abundance would clearly result larger, and hence the maximum cosmologically allowed mass would be smaller. For instance, an asymmetry $n_h - n_{\bar{h}}/n_\gamma \sim 10^{-10}$, which is comparable to the ordinary baryonic one, implies $m \lesssim 250$ GeV, and the corresponding larger density of relic superheavies would enhance the contradiction with the bounds previously discussed.

1.5 Astrophysical Constraints

The results obtained have been based on the very accurate determination of the concentration of heavy hadrons in water, and on the observation of long lived neutron stars, but it should be mentioned that several other experimental data (*e.g.* concentrations of heavy isotopes of different elements, bounds from satellite detectors, etc.) as well as other theoretical considerations (*e. g.* the possible influence of heavy hadrons on stellar evolution) also constrain the exotic quark mass. For instance, the stringent bounds on strongly interacting dark matter that come both from detector searches near the top of the atmosphere [17] as well as from underground experiments [18], will apply to the neutral superheavy baryons, restricting thus their possible contribution to the local density of the halo (the charged component is stopped before it can reach the detectors mentioned).

The conclusion is that a stable or very long lived quark would be present with too large a density to be compatible with the cosmological requirement of not overclosing the universe, with the bounds obtained from anomalous element searches and with some astrophysical implications. Hence, models with exotic quarks must include also a mechanism to allow for their decay. In the case of E_6 models this can be achieved by allowing for the presence of non–vanishing couplings of h with other (scalar) particles that could mediate their decay or induce, through a non zero vacuum expectation value, a sizeable mixing among exotic and ordinary quarks. The consequences of this mixing on the fermion neutral–current couplings as measured at the Z–peak will be the subject of the next chapter.

Chapter 2

Bounds on E_6 Ordinary–Exotic Fermion Mixing from LEP

In this chapter the problem of constraining E_6 models while allowing for the most general kind of effects that the presence of new fermions and new neutral gauge bosons can induce is addressed. It is shown that in these kind of extensions of the standard model the effects of a Z–Z' mixing (where the Z' boson corresponds to an additional effective U(1) factor) can produce similar effects to those due to fermion mixing. We conclude that to be fully consistent both these effects should be taken simultaneously into account.

A first attempt to set bounds on some of the mixing parameters in this general context was performed by using the very firsts LEP measurements of the partial decay widths of the Z boson [4]. A remarkable improvement with respect to the pre–LEP bounds on fermion mixings [3] was then achieved: about a factor 10 was obtained for the τ lepton and a factor 5 for the b quark mixing parameters.

The numerical analysis presented in this chapter updates the original results with the most recent LEP data on the leptonic [19] and b–quark [20] partial widths. Thanks to the increased statistics collected during one year of LEP run, the experimental errors on these quantities have been reduced by a factor between 2 and 4 with respect to the data originally used in [4].

It should be mentioned that an analysis similar to the present one, that leads to results that are in agreement with ours, has been also carried out quite recently in [21] where the author, while following closely the approach outlined in [4], obtains a factor 2 improvement with respect to the original bounds.

Finally it is worth to stress that a complete study of E_6 models in the frame of the general formalism that is sketched here is, to our knowledge, still missing, so that the spirit of the analysis that we are going to present here is indeed still topical.

2.1 Introduction

One of the striking results of the first period of running of the LEP-1 and SLC machines is that no new particles have been produced. The bounds on the masses of many new particles that are predicted by a large class of models (SUSY, Composites, GUTs,...) are already near the kinematic limit accessible with these two machines and thus it seems that the search for direct evidence of new physics must be delayed to the time when a larger centre–of–mass energy will be available.

On the other hand, the experimental data are in very good agreement with the Standard Model (SM) and, even if for the moment there is no evidence for deviations from the theoretical predictions, it is still possible that measurements at the Z–peak might reveal the existence of physics beyond the SM through indirect effects.

The increase in statistics, together with a better understanding of the systematical errors, will provide us with a set of high precision measurements that will be quite effective in the search for tiny new effects or, at least, for setting stricter bounds on the relevant parameters that are generally introduced in any extension of the electroweak theory.

For example, deviations from the SM predictions are expected if gauge groups larger than $\mathcal{G}_{\rm SM} = SU(2)_L \times U(1)_Y$ $[\times SU(3)_C]$ underlay the standard electroweak theory. In particular these deviations could be due to a mixing among the standard fermions and new exotic ones (that often occur in models with enlarged gauge groups), as well as to a mixing of the standard Z_0 with additional neutral vector bosons. Both these effects will modify the fermion couplings to the gauge bosons, and most of the quantities that are measurable at the Z–peak are particularly effective for detecting possible deviations from the standard neutral current couplings.

As far as the mixing among the gauge bosons is concerned, we will assume that only one new neutral Z_1 mixes appreciably with the Z_0, and then we are led to investigate the phenomenological consequences of an effective gauge group $\mathcal{G}_{\rm SM} \times U(1)'$. Since the direct product structure leaves the $U(1)'$ fermion quantum numbers, as well as the g' coupling constant completely arbitrary, a second assumption has to be made in order to obtain predictions: namely that our effective low energy gauge group originates from a *simple* group G_S, broken by some mechanism at a higher energy scale. Then, since $U(1)'$ belongs to the Cartan subalgebra of G_S, only a few choices for the quantum numbers of the particles present in the model will be allowed, and the possible range for the value of the coupling constant g' will also be constrained.

For the sake of definiteness we will carry out our investigation in the frame of a class of E_6 models. The consequences of the presence of a new Z_1 of E_6 origin on Z–resonance physics has been deeply investigated by many authors [22,23]. However, to consider the modifications of the Z-couplings due to a $Z_0 - Z_1$ mixing alone, is not totally consistent in these models, since similar effects can arise also from fermion mixing. In particular, since each fermion generation is assigned to a **27** representation of E_6, besides the 15 standard fields 12 additional 'exotic' particles per generation are predicted to exist. These are: a weak doublet of leptons (N, E^-) with its charged conjugate doublet (E^+, N^c) , a colour triplet weak singlet quark h of charge −1/3 together with h^c, and two neutral singlets ν^c and S. In general a mixing among particles that have the same quantum numbers under the unbroken $U(1)_Q \times SU(3)_C$ group will be allowed, modifying the standard couplings of the fermions.

Bounds on the mixing of a Z_1 of E_6 origin with the standard neutral boson in global analysis of the pre–LEP electroweak data have been derived in ref. [24,25]. The results of that analysis constrain the mixing to $\tan\Theta_{mix} \lesssim 0.22$ for a Z_1 almost decoupled from neutrinos, and to a much lower value ($\tan\Theta_{mix} \lesssim 0.05$) in the other cases. However, the analyses in [24,25] do not take into account the possibility of fermion mixing, so that a combined analysis of these two effects should turn out in slightly worse bounds than the ones quoted.

On the other hand, the implications of the fermion mixing alone in a very large variety of observables have been used to constrain the mixing angles between ordinary and exotic fermions in the frame of E_6 models [3], resulting in $s^2 \equiv \sin^2\theta \lesssim 0.030 - 0.050$ for the first generation and for ν_μ, $s^2_\mu < 0.055$ while the bounds for the fermions in the third generation and for the second generation quarks are much worse. For instance, for the $b - h_b$ mixing the bound is $s_b^2 < 0.43$ and for the $\tau - E_\tau$ mixing it is $s_\tau^2 < 0.22$ (all at 90% c.l.).

Although these mixings can in principle vanish, there are good reasons to believe that they are non–zero. In fact, it has recently been shown, using cosmological and astrophysical arguments, together with experimental bounds from heavy isotope searches, that new charged leptons [1] and new coloured particles [2] cannot be stable. Clearly, the mixing of the exotic particles with the ordinary ones provides a natural channel for their decay.

It is our purpose here to show that the present LEP results on partial widths of the Z boson already improve the previously mentioned bounds on s_b^2 by a factor of 6, taking into account also the possible effects of a $Z_0 - Z_1$ mixing, while for s_τ^2 the bound is improved almost by a factor 15 and this last result is essentially model independent.

In particular, it is important to constrain the b and τ mixings, not only because they are poorly bounded at present, but also because they are theoretically expected to be the largest ones since, if the masses arise from a seesaw mechanism, one has

$$\begin{aligned} \sin^2\theta &\simeq (m/M) && \text{linear} - \text{seesaw} \\ \sin^2\theta &\simeq (m/M)^2 && \text{quadratic} - \text{seesaw} \end{aligned} \tag{2.1}$$

where m and M are the light and heavy fermion masses respectively, and for a large class of models one typically expects that the mixings will fall within the range suggested in (2.1). This argument also leads us to expect tiny mixings in the first two generations ($\lesssim 10^{-3}$), since unsuccessful searches for exotic particles that couple to the Z-boson suggest $M > M_Z/2$. We will concentrate on the consequences of the mixing between the ordinary and exotic charged leptons and between h_b and b quarks. A general analysis of the effects of E_6 lepton and quark mixings in the charged and neutral sectors, as well as of the $Z_0 - Z_1$ mixing in several quantities measurable at LEP (partial widths and asymmetries) will appear elsewhere [26].

2.2 $Z - Z'$ Mixing: Formalism

The exceptional group E_6 [6] is one of the most interesting candidates as a unifying group, and the reason for this is at least twofold: first, it contains as subgroups the symmetry groups of the most popular grand unified and left-right symmetric theories, like *e.g.* $SO(10)$, $SU(5)$, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and second, it is the only phenomenologically acceptable group that can arise from ten–dimensional superstring theories after Calabi–Yau compactification down to the 4 physical dimensions [7]. A nice feature of the E_6 algebra is that the embedding of the colour and weak isospin subgroup $SU(2)_L \times SU(3)_C$ is unique, but clearly in going from rank 6 down to rank 3, three Cartan generators are left, and it follows that the identification of the hypercharge axis is not unique. Here we will consider the embedding of $\mathcal{G}_{SM}$ in E_6 through the maximal subalgebras chain:

$$\begin{array}{l} E_6 \longrightarrow U(1)_\psi \times SO(10) \\ \qquad\qquad\qquad \hookrightarrow U(1)_\chi \times SU(5) \\ \qquad\qquad\qquad\qquad\qquad \hookrightarrow \mathcal{G}_{SM} \end{array} \tag{2.2}$$

The most general form for $U(1)'$ compatible with (2.2) will then be a linear combination of the $U(1)_\psi$ and $U(1)_\chi$ generators that we will parametrize in terms of an angle α. Correspondingly, the couplings of the fermions to the Z' boson will depend on both the ψ and χ quantum numbers through the combination ($c_\alpha = \cos\alpha$, $s_\alpha = \sin\alpha$):

$$Q' = c_\alpha Q_\psi + s_\alpha Q_\chi. \tag{2.3}$$

For the left–handed fermions belonging to the **27** fundamental representation of E_6, the values of the Abelian Q_ψ and Q_χ charges are listed in Tab. I.

The multiplicative factors have been chosen in order to have the same normalization for the three Abelian axes: $\mathrm{Tr}\ Q_\psi^2 = \mathrm{Tr}\ Q_\chi^2 = \mathrm{Tr}\ (\frac{Y}{2})^2$, so that at the unification scale the same coupling constant g_Y is associated to both Y and Q' charges. Possible deviations that could arise at the 100 GeV scale, as a

Table I Quantum numbers for the left–handed fermions of the fundamental **27** representation of E_6. Abelian charges are normalized to the hypercharge axis according to: $\sum_{f=1}^{27}(Q^f)^2 = \sum_{f=1}^{27}(\frac{Y^f}{2})^2 = 5$.

	S_L	$\binom{E^+}{N_{E^+}}_L \quad h_L$	$\binom{N_{E^-}}{E^-}_L \quad h_L^c$	ν_L^c	$\binom{\nu}{e^-}_L \quad d_L^c$	$e_L^c \quad u_L^c \quad \binom{u}{d}_L$
$6\sqrt{\frac{2}{5}}Q_\psi$	4	−2			1	
$6\sqrt{\frac{2}{3}}Q_\chi$	0	2	−2	−5	3	−1

consequence of a different running of the couplings, can be taken into account by writing:

$$g' = \kappa \frac{e}{c_w} \tag{2.4}$$

where the SM relation $e = g_Y c_w$ has been used.

We will denote with Z_1 the $U'(1)$ vector boson gauge eigenstate that in general has a non–diagonal mass matrix with the standard Z_0. The mass eigenstates Z and Z' are related to Z_0 and Z_1 through an orthogonal transformation, parametrized in terms of a mixing angle Θ,

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} Z_0 \\ Z_1 \end{pmatrix}. \tag{2.5}$$

Then, the physical Z boson couples to fermions via the effective Lagrangian:

$$\mathcal{L}_{NC}(Z) = -\left(\frac{G_\mu \cos^2\Theta M_Z^2}{2\sqrt{2}}\right)^{1/2} \sum_f \bar{\psi}_f^o \gamma_\mu [\tilde{v}_f - \tilde{a}_f \gamma_5] \psi_f^o \cdot Z^\mu \qquad (2.6)$$

where the superscript "o" reminds us that for the moment we are considering unmixed fermions. In (2.6) the effective couplings $\tilde{v}_f$ and $\tilde{a}_f$ correspond to the SM couplings v_f and a_f shifted by a quantity proportional to the $Z_0 - Z_1$ mixing and dependent on the Q' fermion quantum numbers:

$$\tilde{v}_f = v_f + s_w \hat{t}_\Theta\ v'_f \qquad\qquad \tilde{a}_f = a_f + s_w \hat{t}_\Theta\ a'_f \qquad (2.7)$$

with

$$v_f = 2T_3^f - 4Q^f\ s_w^2 \ = \ 2T_3^f - Q^f(v+1) \qquad\qquad a_f = 2T_3^f \qquad (2.8)$$

$$v'_f = 2s_\alpha(Q_\chi^f - Q_\chi^{f^c}) \qquad\qquad a'_f = 4c_\alpha Q_\psi^f + 2s_\alpha(Q_\chi^f + Q_\chi^{f^c}) \qquad (2.9)$$

where T_3^f is the left-handed fermion weak isospin, Q^f the electric charge, $s_w^2 = \sin^2\vartheta_w$ with ϑ_w the weak mixing angle and $v = 4s_w^2 - 1$ is the charged lepton vector coupling. Since v is a small quantity that can be used as an expansion parameter for truncating expressions ($v \simeq -0.08$), it is useful to express all the fermions couplings as a function of v, as we have done in (2.8). In eq. (2.7), the ratio $g'/g_Y = \kappa$ has been absorbed by rescaling the mixing angle: $\hat{t}_\Theta = \kappa \cdot \tan\Theta$. On resonance, the shifts (2.9) of the standard couplings are by far the most important effects of the $Z_0 - Z_1$ mixing. Z' exchange and interference diagrams are in fact suppressed at least by a factor $\Gamma_Z \Gamma_{Z'}/M_Z M_{Z'}$ and can then be safely neglected.

We will discuss later the effects of the shift on the physical Z mass due to the mixing.

2.3 Charged Fermion Mixing in E_6 : Formalism

The next step is to allow for a mixing among the standard and exotic fermions that will further modify the couplings in eq. (2.7). In this section we will briefly develop only the formalism needed to describe the mixing between E_6 charged fermions. A more general formalism that describes also the mixing between neutral states and that applies to a more general class of models will be developed in the next chapter.

We note that since the electromagnetic (and colour) quantum numbers of the exotic quarks and leptons are the same as those of the ordinary ones (as it must be, since otherwise no mixing would be allowed), the electromagnetic current is unchanged. Moreover, table I shows that the $SU(2)_L$ transformation properties of the right–handed $Q = -1/3$ quarks and left–handed leptons also coincide with the ordinary ones, so that only the couplings of left–handed down–type quarks (weak isospin doublet) and right–handed ordinary leptons (weak singlets) will be modified by the mixing since their heavy partners are respectively singlets and doublets of weak isospin. Following [3] we introduce two vectors for the ordinary and exotic left– and right–handed weak eigenstates $\psi^o_{L(R)} = (\psi^o_{ord}, \psi^o_{ex})^T_{L(R)}$, and two other vectors for the light (i.e., standard) and heavy mass eigenstates $\psi_{L(R)} = (\psi_l, \psi_h)^T_{L(R)}$, where for example for the down-type light quarks $\psi_l = (d, s, b)^T$. The weak and mass eigenstates are related by unitary transformations

$$\psi^o_L = U_L \psi_L \; ; \qquad \psi^o_R = U_R \psi_R \tag{2.10}$$

with

$$U_{L(R)} = \begin{pmatrix} A & E \\ F & G \end{pmatrix}_{L(R)} \tag{2.11}$$

and from the unitarity of U

$$A^\dagger A + F^\dagger F = A^\dagger A + E^\dagger E = I \tag{2.12}$$

The 3×3 matrices E and F describe the mixing between the light and heavy states. The part of the weak neutral current that gets modified by the mixing can be written:

$$\begin{aligned} \frac{1}{4} J^\mu_Z &\sim \sum_f \bar{\psi}^o_f \left[t_3 I^f_L P_L + t_3 I^f_R P_R - Q s^2_w \, I \right] \psi^o_f \\ &= \sum_f \bar{\psi}_f \left[t_3 U^{f\dagger}_L I^f_L U^f_L P_L + t_3 U^{f\dagger}_R I^f_R U^f_R P_R - Q s^2_w \, I \right] \psi_f \end{aligned} \tag{2.13}$$

where the sum involves only the $Q = -1/3$ quarks $q = d, h$ and the charged leptons $\ell = e, E$, $P_{L(R)} = \frac{1}{2}(1 \mp \gamma_5)$ and

$$I^q_L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} , \qquad I^\ell_R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \tag{2.14}$$

while

$$I^d_R = 0 \qquad \text{and} \qquad I^\ell_L = I. \tag{2.15}$$

Using (2.13) and the unitarity relations (2.12) it is easy to see how the couplings of the light states d and ℓ are further modified with respect to eq. (2.7):

$$\begin{aligned} \tilde{v}_d \rightarrow \hat{v}_d &= \tilde{v}_d - 2t_3(F_L^{d\dagger} F_L^d) \qquad & \tilde{a}_d \rightarrow \hat{a}_d &= \tilde{a}_d - 2t_3(F_L^{d\dagger} F_L^d) \\ \tilde{v}_\ell \rightarrow \hat{v}_\ell &= \tilde{v}_\ell + 2t_3(F_R^{\ell\dagger} F_R^\ell) \qquad & \tilde{a}_\ell \rightarrow \hat{a}_\ell &= \tilde{a}_\ell - 2t_3(F_R^{\ell\dagger} F_R^\ell) \end{aligned} \tag{2.16}$$

The matrices $F^\dagger F$ are in principle 3×3 non–diagonal matrices that describe intergenerational mixing too. However, the off–diagonal terms that would induce flavour changing neutral currents at the tree level are severely constrained by experiments [3]. We will assume that these terms are negligibly small so that the light–heavy mixing occurs essentially between particles belonging to the same generation. We will then parametrize:

$$\mathrm{diag}(F^\dagger_{R(L)} F_{R(L)}) = \left((s_1^{R(L)})^2,\, (s_2^{R(L)})^2,\, (s_3^{R(L)})^2\right) \tag{2.17}$$

with $s_i^2 = \sin^2\theta_i$.

Clearly, in our procedure to define the effective coupling of the fermions to the Z-boson, second–order effects proportional to $\hat{t}_\Theta \cdot \sin^2\theta$ have been neglected. In the following we will consider as 'first–order terms' the following set of small parameters: $\hat{t}_\Theta$, $\sin^2\theta$ and v, and we will neglect terms involving products or higher powers of them.

2.4 Theoretical Expectations

Before beginning the analysis of the bounds that can be derived from the measurements of the partial widths $Z \rightarrow b\bar{b}$ and $Z \rightarrow \ell^+\ell^-$, we want to discuss an indirect effect of the fermion mixing that will enter as a theoretical uncertainty in any prediction for electroweak quantities. The set of electroweak parameters that is known with the best experimental accuracy is α, M_Z and G_μ (the Fermi constant measured in μ decay). In particular, G_μ is introduced to replace the W mass, whose experimental value is still affected by a large error. In order to do this, one uses the relation:

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha}{2M_W^2 \sin^2\theta_W(1-\Delta r)} \tag{2.18}$$

where Δr [27] is a radiative correction that (taking into account only the leading contributions) can be written as $\Delta r \simeq \Delta\alpha - (c_w^2/s_w^2)\Delta\rho$. Here, the effect of $\Delta\alpha$ ($\simeq 0.06$) is to renormalize the electromagnetic charge to the scale M_Z

$$\alpha(M_Z^2) = \frac{\alpha(0)}{(1-\Delta\alpha)} \tag{2.19}$$

while $\Delta\rho$ [28] contains potentially large corrections that in the SM are essentially due to the top–bottom mass splitting, but that in general can arise from a mass difference between the components of any additional isodoublet of fermion [28] or scalar [29] particles that is present in the model. In a theory that allows for a $Z_0 - Z_1$ mixing, it is possible to take into account this effect by replacing $M_Z^2 \to \rho_{mix}M_Z^2$ since ρ_{mix} enters any expression in the same way as a $\rho_o \neq 1$ generated by non-standard Higgses [22]. In such a theory, a possible (and useful) definition of the Weinberg angle is $c_w^2 = M_W^2/(\rho_{mix}M_Z^2)$. Allowing now also for a fermion mixing, eq. (2.18) will be modified into

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha(M_Z^2)}{2M_W^2(1-\frac{M_W^2}{\rho M_Z^2})}(1-\Delta s), \tag{2.20}$$

with

$$\rho \equiv \rho_{mix}(1+\Delta\rho),$$

and where

$$1-\Delta s \equiv c^L_{\nu_\mu} c^L_{\nu_e} \simeq 1-\frac{1}{2}({s^L_{\nu_\mu}}^2+{s^L_{\nu_e}}^2) \tag{2.21}$$

takes into account the effect of a possible neutrino mixing with additional exotic E_6 neutral fermions (see Table I) in μ–decay [3]. Two remarks are in order: first, both a $Z_0 - Z_1$ mixing and a heavy top produce positive deviations of ρ from the SM tree level value $\rho = 1$. Since the theoretical upper bound on m_t comes from measurements of the ρ parameter, in general allowing the top mass to vary in the range 90 GeV$< m_t <$200 GeV automatically takes into account the uncertainty related to a $Z_0 - Z_1$ mixing. This is not the case for the partial decay width into b–quarks, since this quantity receives an additional m_t–dependent contribution from the $Zb\bar{b}$ vertex correction that also involves the top mass and that almost cancels against the $\Delta\rho^{top}$ correction [30]. As a result, $\Gamma_{b\bar{b}}$ turns out to be nearly insensitive to the value of the top mass. Thus, in this particular case the uncertainty coming from $\Delta\rho_{mix} \equiv \rho_{mix} - 1$ must be separately taken into account. A second point that we need to discuss is the effect of ν_e and ν_μ mixing in the measured Fermi constant. We can include this effect simply by replacing $G_\mu \to G_\mu(1+\Delta s)$ in all the expressions, but in so doing Δs will induce an additional theoretical uncertainty. However, we expect this correction to be quite small since the mixings involved in μ–decay should be negligible ($\Delta s < 10^{-3}$ according to eq. (2.1)). In the forthcoming expressions we will keep trace of both these effects but, as we will see, they will not affect very much our numerical analysis since the overall error is largely dominated by the experimental uncertainty.

From the Lagrangian (2.6), after the replacements $\tilde{v} \to \hat{v}$ and $\tilde{a} \to \hat{a}$, we can write the tree–level expressions for the partial widths $Z \to f\bar{f}$ as:

$$\Gamma_{f\bar{f}} = \frac{\sqrt{2}\rho_{mix} G_\mu (1+\Delta s) M_Z^3}{48\pi} (\hat{v}_f^2 + \hat{a}_f^2) \tag{2.22}$$

Then, from the expression for the effective neutral couplings (2.7)–(2.9) and (2.16) we obtain for the partial decay width into b–quarks:

$$\Gamma_{b\bar{b}} = \Gamma_{b\bar{b}}^{SM} \left[1 + \frac{19}{13}(\Delta\rho_{mix} + \Delta S) - \frac{3}{13}\hat{t}_\Theta \left(\sqrt{10}\, c_\alpha - \sqrt{\frac{2}{3}}\, s_\alpha \right) - \frac{30}{13}(s_b^L)^2 \right] \tag{2.23}$$

with, at the tree level

$$\Gamma_{b\bar{b}}^{SM} = \frac{\sqrt{2} G_\mu M_Z^3}{48\pi} \frac{(13-4v)}{3} \tag{2.24}$$

Including the 1–loop electroweak and QCD corrections we have in the frame of the SM[1]

$$\Gamma_{b\bar{b}}^{SM} = 377 \cdot (1 \pm 0.008) \mathrm{MeV} \tag{2.25}$$

where the theoretical uncertainty corresponds to the variation of the top mass, Higgs mass and $\alpha_s(M_Z^2)$ in the ranges

$$90\,\mathrm{GeV} < m_t < 200\,\mathrm{GeV}; \qquad 45\,\mathrm{GeV} < M_H < 1\,\mathrm{TeV}; \qquad 0.11 < \alpha_s(M_Z^2) < 0.13 \tag{2.26}$$

[1] We thank W. Hollik for providing us with the Fortran program WIDTH that has been used to compute (2.25)

The lower bounds on m_t and M_H are from direct searches respectively at CDF [31] and LEP while the upper bounds are rather conservative theoretical estimations respectively from radiative effects and requirements of unitarity. The experimentally allowed range for $\alpha_s(M_Z^2)$ is determined from jet analysis [32].

According to our previous discussion we will neglect the effect of Δs in eq. (2.23). To estimate the uncertainty due to $\Delta\rho_{mix}$, we use again eq. (2.20) with the experimental value of the $W-Z$ mass ratio averaged over the UA2 and CDF experiments [33]: $M_W^2/M_Z^2 = 0.773 \pm 0.006$, and $M_Z = 91.175 \pm 0.021$ from LEP [19]. Neglecting again Δs, and subtracting the contribution of a 90 GeV top quark, we obtain at 90% c.l. $\Delta\rho_{mix} < 0.006$. We note that although the experimental value of ρ obtained in this way is slightly less precise than what could be obtained from low energy neutral to charged current ratio [24,25], this estimation is safer since it is insensitive to possible Z' exchange diagrams.

To estimate the uncertainty induced in $\Gamma_{b\bar{b}}$ by the $Z_0 - Z_1$ mixing we have evaluated the values of the corresponding term $\delta_\Theta^b = \frac{3}{13}\hat{t}_\Theta\left(\sqrt{10}\,c_\alpha - \sqrt{\frac{2}{3}}\,s_\alpha\right)$ in the range experimentally allowed for Θ as a function of α that is quoted in [24]. We obtain for this effect $-0.174 \le \delta_\Theta^b \le 0.013$. Although the bounds obtained in [24] ignored the effects of fermion mixing, we think that they should be reliable since they are derived from deep inelastic ν scattering off nucleons and from $e^+e^- \to \mu^+\mu^-$ data that involve only fermions for which mixing effects are expected to be small.

In conclusion, our numerical prediction for the partial decay width of the Z

boson into b quarks is the following:

$$\Gamma_{b\bar{b}} = 377 \cdot \left(1 \pm .008 \; {}^{+.009}_{-.0} \; {}^{+.013}_{-.174} - \frac{30}{13}(s_b^L)^2\right) \text{MeV} \tag{2.27}$$

where the first error comes from SM uncertainties, the second from ρ_{mix} and the third from $Z_0 - Z_1$ mixing. Since the effect of the $b - h_b$ mixing tends to decrease the decay rate, we have to compare the experimental data with the maximum allowed value of (2.27). Moreover, since also the effect of $Z_0 - Z_1$ mixing in this quantity turns out to be negative for almost all values of α, and the possible positive shift is bound to be quite small, the inclusion of this effect in our analysis does not change the bounds from what one would obtain assuming $\Theta = 0$. Clearly if any non–zero $Z_0 - Z_1$ mixing is experimentally found, this will generally result in a better bound on $(s_b^L)^2$.

As regards the partial width into τ and μ leptons, it looks convenient to get rid of the overall multiplicative coefficient in (2.22) by defining a quantity normalized with the electron width

$$R_\tau \equiv \frac{\Gamma_{\ell^+\ell^-}}{\Gamma_{e^+e^-}} = \frac{\hat{v}_\ell^2 + \hat{a}_\ell^2}{\hat{v}_e^2 + \hat{a}_e^2} \tag{2.28}$$

for which we obtain:

$$R_\ell = R_\ell^{SM} - 2(s_\ell^R)^2 + 2(s_e^R)^2 \tag{2.29}$$

and, neglecting the tiny effect of m_ℓ, $R_\ell^{SM} = 1$. Clearly the quantity R_ℓ is exactly one even in the presence of a $Z_0 - Z_1$ mixing, since the lepton couplings to the Z_1 boson also obey universality. In fact, the bound on the quantity $(s_\ell^R)^2 - (s_e^R)^2$ can be effectively thought of as a bound on any source of violation of universality, of which the mixing among fermions that we are considering now is probably one of the most obvious. In comparing (2.29) with the experimental data we will again neglect $(s_e^R)^2$, obtaining thus an upper limit on the $\tau - E_\tau$ and $\mu - E_\mu$ mixing.

2.5 Experimental Limits from LEP

We now discuss the experimental data. To obtain the partial width $Z \to b\bar{b}$ the decay mode $b \to \ell\nu X$ (with $\ell = e, \mu$) is most commonly used [20]. Then what is really measured is the quantity $\mathrm{Br}(b \to \ell)\, \Gamma_{b\bar{b}}/\Gamma_{had}$. Using the value $\mathrm{Br}(b \to \ell\nu X) = 0.119 \pm 0.006$ [34] for the branching–ratio of the b–quark into leptons, and $\Gamma_{had} = 1739 \pm 13\,\mathrm{MeV}$ for the value of the Z hadronic width averaged over the four LEP experiments [19] (for consistency only the determinations that do not assume lepton universality have been used) we finally obtain

$$\Gamma_{b\bar{b}} = 367 \pm 19 \text{ MeV} \tag{2.30}$$

In eq. (2.30) the uncertainty in the branching ratios for the decay of the b quark into e and μ leptons dominates the overall error. The average of the different determinations [19] of the flavor dependent leptonic $Z \to \tau^+\tau^-$ and $Z \to \mu^+\mu^-$ partial decay widths gives

$$\begin{aligned} \Gamma_{\tau^+\tau^-} &= 82.8 \pm 1.1 \text{ MeV}, \\ \Gamma_{\mu^+\mu^-} &= 83.4 \pm 0.9 \text{ MeV} \end{aligned} \tag{2.31}$$

while the weighted average yields for the partial width into electrons

$$\Gamma_{e^+e^-} = 83.2 \pm 0.6 \text{ MeV} \tag{2.32}$$

In the quoted averages (2.31) and (2.32) a 0.5% common systematic uncertainty arising from the luminosity measurements has been taken into account and in the case of $\Gamma_{e^+e^-}$ also the common error due to the procedure for subtracting the t–channel Bhabha scattering from the data has been included.

With these figures, the experimental value of our quantities *at the* 90 % *c.l.* are found to be

$$\begin{aligned} \Gamma_{b\bar{b}} &= 367\ (1 \pm 0.085)\ \text{MeV}; \\ R_\tau &= 0.995\ (1 \pm 0.025) \\ R_\mu &= 1.002\ (1 \pm 0.021) \end{aligned} \tag{2.33}$$

We note that in R_τ and R_μ the error is probably overestimated since we expect that the systematic uncertainty that originates from the measurement of the luminosity should cancel in this two ratios.

From (2.27), (2.29) and (2.33) we get the following bounds for the b–h_b, τ–E_τ and μ–E_μ mixing angles:

$$\begin{aligned} (s_b^L)^2 &\leq 0.060 \\ (s_\tau^R)^2 &\leq 0.015 \\ (s_\mu^R)^2 &\leq 0.010 \end{aligned} \tag{2.34}$$

It is interesting to note that the first two bounds in eq. (2.34) are already comparable with the maximum values for the mixings that can be derived from (2.1) given the present lower limits on the mass M of exotic particles.

In conclusion, in this chapter the consequences of a mixing among the ordinary fermions and new heavy exotic ones that are predicted to exist in E_6 models have been analysed. The attention has been focused on quantities relevant for experiments at the Z peak. Several effects that occur in this kind of theories, such as a Z_0–Z_1 mixing that will induce deviations from the SM couplings of the fermions and will also influence the value of the ρ parameter have been properly taken into account, and the theoretical expectations have been compared with the most recent LEP data, obtaining new and improved bounds on the mixing angles of the τ and μ leptons and of the b quark.

Chapter 3

A Comprehensive Analysis of the Constraints on Fermion Mixing

In this chapter we present a detailed analysis of the limits on deviations of the lepton and quark weak–couplings from their standard values, for a general class of models where the known fermions are allowed to mix with new heavy exotic particles with unconventional SU(2)$\times$U(1) quantum number assignments (left-handed singlets or right-handed doublets). The formalism that was introduced in the previous chapter in order to describe charged fermion mixing in the frame of E_6 models and that has been applied to Z–peak physics, is extended here (following ref. [3]) to include the mixing among different kinds of neutral particles and to the charged current sector. This formalism turns out to be powerful enough to describe, beyond the case of E_6 models, general extensions of the electroweak theory such as models with mirror fermions, additional generations, etc.

A detailed and up-to-date analysis of the available experimental data is also presented. The new results on M_Z, Γ_Z, on the Z partial decay–widths and on the asymmetries measured at the Z resonance, as well as updated results on the W mass, on deep-inelastic ν–q and ν–e scattering and on atomic parity violation are used to constrain the mixings. Present constraints on lepton universality, unitarity of the quark mixing matrix and induced right-handed currents are also included. A global analysis of all these data leads to upper limits that improve considerably the existing bounds on the mixing factors $s^2 \equiv \sin^2\theta_{\rm mix}$. When only one mixing angle is considered at a time, the limits for most of the mixing factors $(s^f)^2$ are below the 1 % level, with the exception of the mixings of u_R, d_R, c_R and $\nu_{\tau L}$ that are of the order of a few percent and those of s_R and b_R that are still poorly constrained to values $\lesssim 1/3$. Remarkably enough a signal of non–zero mixing at 90 % c.l. is found in the case that the τ neutrino mixes with an *ordinary* heavy neutral state. The general situation when all the mixings are simultaneously present and accidental cancellations among them are allowed to occur is also analysed, and it is shown that these cancellations can weaken the constraints by a factor between 2 and 5.

3.1 Introduction

In the last few years the ever increasing accumulation of precise electroweak experiments have been regularly employed to check the consistency of the standard model (SM), to determine $\sin^2\theta_W$ and to make predictions for the still unknown value of the top mass. Possible indirect signatures of physics beyond the SM, such as the effects of additional gauge bosons or of mixings of the standard fermions with exotic ones, as well as the contributions of non–decoupled physics to radiative corrections, have also been constrained by these measurements.

The first pre–LEP analyses [24,25,3] used the available information on gauge boson masses from colliders, neutral current (NC) data on ν scattering, parity violation, fermion asymmetries in e^+e^- annihilation below the Z resonance, and in some cases charged current (CC) constraints. By now the situation has improved considerably. A remarkable improvement has been achieved in the determination of the W boson mass from UA2 and CDF [33]. In the NC sector, there are new measurements on atomic parity violation in Cs [35] and new calculations of the atomic matrix elements involved [36], there are new results on $\nu_\mu e$ scattering [37,38,39] as well as new and updated analyses of the c and b asymmetries in γ–Z interference processes at PEP and PETRA [40,41,42]. In the CC sector, new constraints are available on the universality of the lepton couplings and on the unitarity of the quark mixing matrix, and the problem of the charm quark threshold [43], that affects the νq CC cross section used to normalize the deep-inelastic NC experiments, has been studied in more detail. The really new input, however, comes from the large set of accurate measurements carried out at the Z–peak at LEP and SLC. Besides M_Z, that is now very precisely known, the determination of the total and of the partial Z–widths and of the on–resonance forward–backward and τ polarization asymmetries has provided very precise informations about the fermion couplings to the Z.

Some of these data have been recently used to update the predictions on m_t [44] and to constrain extensions of the SM with extra U(1) gauge bosons [23], as well as technicolour models, strongly interacting Higgs–bosons and other kinds of heavy physics that could manifest itself through radiative corrections [45].

It is our purpose here to update the bounds on possible mixings between the known fermions and new exotic ones. There have been several earlier analyses of the limits on fermion mixings [46], and the first (pre-LEP) global analysis of this kind of new physics was done by Langacker and London [3]. Subsequently it was shown that the very first LEP data already improved some bounds significantly [4] (see also chapter 2) and, more recently, Langacker, Luo and Mann [47] have also discussed the sensitivity to some exotic mixings that will be attained with the forseeable precision of the ongoing or planned precision electroweak experiments.

The existence of new fermions with exotic weak couplings is a quite common feature in most of the extensions of the SM, being the 'superstring inspired' E_6 models well known and still popular examples [8] of these. A mixing between ordinary and exotic fermions is allowed whenever their $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{\mathrm{em}}$ quantum numbers are the same. If at the same time the new fermions have non canonical $SU(2)_L$ assignments, the couplings of the light states with both the W and the Z vector bosons will be modified, leading to deviations from the SM expectations. This is the kind of effects we aim to constrain by means of a careful analysis of the available experimental results.

The general formalism to describe fermion mixing that was introduced in [3] will be briefly surveyed in section 2. Section 3 is devoted to a brief discussion of the parameters needed to work out the numerical predictions. We then present in following sections 4 to 7 the theoretical expressions for the different observables that have been used to work out the constraints. A description of each measurement and a discussion of the experimental data are also given in these sections. Section 8 is devoted to comment on the results of the global analysis. The results are presented as 90 % c.l. upper limits on the ordinary–exotic mixing parameters, both in the case in which only one fermion is allowed to mix at a time and in the case were all mixings are simultaneously present so that accidental cancellations may occur. Finally, in section 9 I will summarize the main topics presented in the chapter and draw the conclusions.

3.2 Ordinary–Exotic Fermion Mixing: a General Formalism

The lack of observation of new particles in the last accelerator runs indicates that if possible new fermions exist, they will generally have large masses ($> 50-100$ GeV). Even if these particles cannot be directly produced with the experimental facilities available at present, it is still possible that their effects are indirectly detected as small deviations of the observed fermion couplings from the standard ones. In particular, this happens if the exotic fermions have non–canonical $SU(2)_L \times U(1)$ assignments and they mix with the ordinary ones. We consider a fermion to have canonical $SU(2)_L$ quantum numbers if it is a left–handed $SU(2)_L$ doublet or a right–handed $SU(2)_L$ singlet. These are called ordinary fermions while fermions with non–canonical quantum numbers are classified as exotics. Exotic fermions can appear in mirror models [48] in which generally whole mirror generations with

R-doublets and L-singlets are introduced, in models with vector doublets (singlets) where both left and right fermions have the same transformation properties under weak–isospin, or as singlet Weyl neutrinos. Fermions with exotic charges or colour assignments cannot mix with the known quarks and leptons and thus we will not consider them.

In order to describe the mixing between ordinary and exotic charged fermions, we introduce [3] two vectors for the left and right-handed ordinary and exotic weak eigenstates $\Psi^o_{L(R)} = (\Psi^o_O, \Psi^o_E)^T_{L(R)}$, and two different vectors for the light and heavy mass eigenstates $\Psi_{L(R)} = (\Psi_l, \Psi_h)^T_{L(R)}$. The weak and mass eigenstates are related by unitary transformations

$$\Psi^o_{L(R)} = U_{L(R)} \Psi_{L(R)}. \tag{3.1}$$

It is convenient to decompose the matrix U as

$$U_{L(R)} = \begin{pmatrix} A & E \\ F & G \end{pmatrix}_{L(R)}, \tag{3.2}$$

where A and F describe the overlap of the light eigenstates with the ordinary and exotic fermions respectively. Since we have included fermions of sequential families or canonical members of vector multiplets as ordinary states, the labels 'light' and 'heavy' should be taken as suggestive only. From the unitarity of U it follows that

$$A^\dagger A + F^\dagger F = AA^\dagger + EE^\dagger = I. \tag{3.3}$$

Hence, the matrix A describing the mixing with the ordinary fermions is non–unitary by small terms quadratic in the ordinary–exotic fermion mixings present in F.

The fermion current coupling to the Z is

$$\frac{1}{2}J_Z^\mu = \sum_f \bar{\Psi}_f^o \gamma_\mu (t_3^f I_L P_L + t_3^f I_R P_R - Q^f s_W^2 I)\Psi_f^o, \tag{3.4}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ are the L and R chiral projectors, $s_W^2 \equiv \sin^2\theta_W$, Q^f and t_3^f denote charge and third isospin component of f and $I_{L,R}$ project onto the subspaces of the ordinary and exotic weak doublets of Ψ^o, *i.e.*

$$I_L = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \quad , \quad I_R = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}. \tag{3.5}$$

Hence, (omitting the label l) the light fermions–Z vertex is given by the Lagrangian

$$\mathcal{L}_Z = -\left(\sqrt{2}G_F M_Z^2\right)^{1/2} \sum_f \bar{\Psi}_f \gamma_\mu (L^f P_L + R^f P_R)\Psi_f \; Z^\mu, \tag{3.6}$$

with

$$\begin{aligned} L^f &= t_3^f A_L^{f\dagger} A_L^f - Q^f s_W^2 \\ R^f &= t_3^f F_R^{f\dagger} F_R^f - Q^f s_W^2. \end{aligned} \tag{3.7}$$

Although the matrices $A^\dagger A$ and $F^\dagger F$ are in principle quite general, non–vanishing off diagonal terms would induce FCNC that are experimentally known to be very suppressed [3]. Hence, we will assume that different light mass eigenstates do not mix with the same exotic partner, in which case the absence of FCNC is automatically guaranteed. With this assumption one gets

$$(F_a^\dagger F_a)_{ij} = (s_a^i)^2 \; \delta_{ij} \; , \qquad a = L, R, \tag{3.8}$$

where $(s_a^i)^2 \equiv 1 - (c_a^i)^2 \equiv \sin^2\theta_a^i$, and $\theta_{L(R)}^i$ is the mixing angle between L(R) light and heavy partners.

Then the neutral–current couplings for the light fermions can be written as

$$\begin{aligned} \hat{\epsilon}_L(f_i) &\equiv (L^f)_{ii} = t_3^{f_i}(c_L^{f_i})^2 - Q^{f_i} s_W^2 \\ \hat{\epsilon}_R(f_i) &\equiv (R^f)_{ii} = t_3^{f_i}(s_R^{f_i})^2 - Q^{f_i} s_W^2, \end{aligned} \tag{3.9}$$

and we see that while the L–mixings reduce the strength of the isospin current, the presence of R–mixings induces a right–handed current. Clearly the electromagnetic current is left unchanged. The vector and axial–vector couplings in the presence of mixing are (omitting the generation index i)

$$\begin{aligned} v_f \equiv \hat{\epsilon}_L(f) + \hat{\epsilon}_R(f) &= t_3^f\left[(c_L^f)^2 + (s_R^f)^2\right] - 2Q^f s_W^2 \\ a_f \equiv \hat{\epsilon}_L(f) - \hat{\epsilon}_R(f) &= t_3^f\left[(c_L^f)^2 - (s_R^f)^2\right]. \end{aligned} \tag{3.10}$$

Henceforth v_f and a_f will always denote the true couplings of the light fermions including mixing terms.

The charged current between light states is

$$\frac{1}{2}J_W^\mu = \bar{\Psi}_u\gamma^\mu(V_L P_L + V_R P_R)\Psi_d. \tag{3.11}$$

The first 3 components of the vectors Ψ_u, Ψ_d represent the standard quarks, while the remaining $n-3$ 'light' fields correspond to possible extra sequential, or vector doublet, quarks. $V_L = A_L^{u\,\dagger} A_L^d$ and $V_R = F_R^{u\,\dagger} F_R^d$ generalize the SM Cabibbo-Kobayashi-Maskawa (CKM) quark mixing. In particular, the matrix V_L is non unitary due to the mixing with the exotic quarks, however it can be decomposed as

$$V_{L\,ij} = c_L^{u_i} c_L^{d_j} K_{L\,ij}, \tag{3.12}$$

where K_L is unitary [3]. For the induced right–handed currents, it is convenient to introduce the parameters

$$\kappa_{ij} \equiv \frac{V_{Rij}}{K_{L\,ij}}, \tag{3.13}$$

which are quadratic in the light–heavy mixings.

For the neutral fermions the situation is more complicated because in the presence of Majorana mass terms three kinds of neutral fields with different isospin assignments can mix at the same time, and also because due to the lack of experimental constraints the assumption on the absence of FCNC must be released. Besides the ordinary neutrinos that appear in L–doublets $(n_O^o,\ e_O^{o-})^T$, we can have exotic states that appear in the CP conjugates of $SU(2)$ R-doublets $(E_E^{o+} n_E^o)_L^T$, (these can mix with n_O^o through $\Delta L = \pm 2$ Majorana mass terms) and also exotic singlets n_{SL}^o can be present.

In analogy with the charged fermion case, we write the weak and mass eigenstates as

$$n_L^o = \begin{pmatrix} n_O^o \\ n_E^o \\ n_S^o \end{pmatrix}_L \quad , \quad n_L = \begin{pmatrix} n_l \\ n_h \end{pmatrix}_L . \tag{3.14}$$

These states are related through $n_L^o = U_L n_L$. The unitary matrix U can be decomposed as

$$U_L = \begin{pmatrix} A & E \\ F & G \\ H & J \end{pmatrix}_L , \tag{3.15}$$

with A, F, H describing the overlap of the light neutrinos with n_O^o, n_E^o and n_S^o respectively.

Note that we do not distinguish between left handed neutrinos and antineutrinos, they are all described by fields n_L and the right handed fields will be denoted as $n_R^c = C\bar{n}_L^T$. Clearly $n_R^{oc} = U_R n_R^c$ with $U_R = U_L^*$.

The LEP measurements of the number of light neutrino species implies that if neutrinos with large exotic n_E^o components exist, their masses must be heavier than $M_Z/2$. Light singlets, however, as in the case of Dirac neutrino masses, could be present and a mixing with exotic doublets would allow them to couple to the Z boson. For simplicity we will not consider this case, but our results are largely independent of this restriction. In conclusion we will assume the light neutrinos to be mainly ordinary states so that we will consider the elements of F and H as small light–heavy mixings.

We will chose the flavour basis such that the charged lepton flavour eigenstates coincide with the charged mass eigenstates up to light–heavy mixing effects. Hence, the charged current between light mass eigenstates reads

$$J_W^\mu = \bar{n}_L \gamma^\mu A_L^{\nu\dagger} c_L^e e_L + \bar{n}_R^c \gamma^\mu F_R^{\nu\dagger} s_R^e e_R. \tag{3.16}$$

The first term in this equation is the usual left–handed current with the overall strength reduced by the effect of light–heavy mixing, while the second term corresponds to an induced right–handed current that can produce neutrinos of the wrong helicity in weak decays. This term is present when both the light neutrino and charged lepton mix with the components of an exotic doublet.

It is convenient to write $A^{\nu\dagger} = K^\nu \mathcal{A}^{\nu\dagger}$, where the matrix K^ν is unitary and, being the leptonic analog of the CKM matrix, it is non–trivial if the light neutrinos have masses and ordinary mixings. The exotic mixings appear only in $\mathcal{A}$, which deviates from the identity only by terms of $O(s^2)$. In the charged current processes that we will consider, a sum has to be taken over the unobserved final neutrino mass eigenstates (the kinematical effects of ν masses are negligible) and thus the information in K^ν is lost. In weak decays, for example, the mixings induce a change in the decay rate with respect to the SM rate Γ_o that, to $O(s^2)$ and restricting ourself to the primary vertex, can be written as

$$\frac{1}{\Gamma_o}\sum_i \Gamma(e_a \to n_i) = (c_L^{e_a})^2 (A_L^\nu A_L^{\nu\dagger})_{aa} + O(s^4), \tag{3.17}$$

where $(A^\nu A^{\nu\dagger})_{aa} = (\mathcal{A}^\nu \mathcal{A}^{\nu\dagger})_{aa} \equiv (c_L^{\nu_a})^2$ accounts for the neutrino light-heavy mixing. As we see, the sum over the final undetected states allows us to take just one mixing angle per neutrino flavour to describe the exotic mixings, although in general the matrix $\mathcal{A}^{\nu\dagger}\mathcal{A}^\nu$ is not diagonal.

The weak neutral current for the light neutrino states is

$$J^\mu_{\nu Z} = \frac{1}{2}\bar{n}_L \gamma^\mu (A_L^{\nu\dagger} A_L^\nu - F_L^{\nu\dagger} F_L^\nu) n_L, \tag{3.18}$$

where $A_L^{\nu\dagger} A_L^\nu$ and $F_L^{\nu\dagger} F_L^\nu$ originate respectively from the ordinary n_O^o and exotic n_E^o neutrinos that have opposite isospin assignments.

Up to mixing effects in the target, and summing again over the undetected light n_i neutrinos, the scattering process $n_a \to n_i$ is modified with respect to the normal case as

$$\begin{aligned}\frac{1}{\sigma_o}\sum_i \sigma(n_a \to n_i) &= \frac{1}{(c_L^{\nu_a})^2}(A^\nu (A^{\nu\dagger} A^\nu - F^{\nu\dagger} F^\nu)^2 A^{\nu\dagger})_{aa} \\ &= 1 - 2\left(K^{\nu\dagger}(2F^{\nu\dagger}F^\nu + H^{\nu\dagger}H^\nu)K^\nu\right)_{aa} + O(s^4),\end{aligned} \tag{3.19}$$

where the factor $(c_L^{\nu_a})^2$ in the denominator comes from the normalization of the n_a produced in the weak decay of e_a. In (3.19) we have used the unitarity of U_L as well as the decomposition of the matrix A into the unitary K matrix, and we have neglected terms $O(s^4)$. Defining now $(K^\dagger F^\dagger F K)_{aa} \equiv \lambda_F^a (s_L^{\nu_a})^2$ and $(K^\dagger H^\dagger H K)_{aa} \equiv \lambda_H^a (s_L^{\nu_a})^2$ we finally get

$$\frac{1}{\sigma_o} \sum_i \sigma(n_a \to n_i) = 1 - \Lambda_a (s_L^{\nu_a})^2 + O(s^4). \tag{3.20}$$

Since the sum of the λ^a' s is constrained to be ≤ 1 from the unitarity of U_L in (3.15), the value of the effective parameter $\Lambda_a \equiv 4\lambda_F^a + 2\lambda_H^a$ must lie between 0 and 4, depending on the mixing involved. If the light states are mixed with ordinary states (that will be mainly heavy) then the couplings are not affected and $\Lambda_a = 0$. If only singlet states n_S^o mix with the known neutrinos then $\Lambda_a = 2$ while $\Lambda_a = 4$ describes mixings involving only exotic states n_E^o.

The decay rate of the Z boson into undetected neutrinos is proportional to the sum of the square of the neutrino neutral–current couplings. Using the same approximations as in the previous case we find

$$\mathrm{Tr}(A^{\nu\dagger} A^\nu - F^{\nu\dagger} F^\nu)^2 = 3 - \sum_a \Lambda_a (s_L^{\nu_a})^2 + O(s^4), \tag{3.21}$$

and we see that the effective parameter Λ_a could largely influence the reduction in the decay rate.

3.3 Overview on the Experimental Constraints

In this section we discuss the input parameters that we have used to work out the numerical predictions from the theoretical expressions, and we give a short overview of the general strategy that we have adopted. The measurements that have been used to constrain the fermion mixing angles will be discussed in detail in the forthcoming sections. In comparing the experimental results with the corresponding theoretical expressions some care is needed, since indirect effects of the mixings that depend on the particular experimental procedure used to extract the data could be present, and to match the precision reached by the 'last–generation' experiments, 1-loop effects should be taken into account in the evaluation of the theoretical expressions. We have followed the general attitude of including only the set of SM radiative corrections, which are nowadays completely known, neglecting the effect of mixings in them as well as the contribution of the additional states in the loops. We should stress however that the large number of exotic fermions present in the models under investigation could give rise to non negligible higher order effects [45] especially in the case of non-degenerate doublets [28]. QCD corrections have also been included in all the relevant cases when hadronic final states were involved.

Our set of fundamental input parameters consists of the QED coupling constant α measured at $q^2 = 0$, the mass of the Z boson M_Z and the Fermi coupling constant G_F. The numerical values of α and M_Z as extracted from experiments are not affected by the mixings. The position of the resonance–peak does not depend on the exact form of the fermion couplings with the Z (the shape and height of the peak, in contrast, are modified by the mixings) and the standard set of QED corrections needed to reconstruct the exact peak–position can be safely applied, since also the electromagnetic current is not modified.

Throughout this work we will fix the Z–mass at the value $M_Z = 91.175$ GeV [19] since the theoretical uncertainties induced by the present experimental error of ± 21 MeV are negligible.

In contrast with the previous two parameters, the Fermi coupling constant extracted from the measured life–time of the μ-lepton, $G_\mu = 1.16637(2) \times 10^{-5}\text{GeV}^{-2}$, is affected by fermion mixings. The relation between G_F and the effective μ-decay coupling constant (neglecting the $O(s^4)$ effect of induced right handed currents (RHC)) is

$$G_\mu = G_F c_L^{\nu_e} c_L^{\nu_\mu} c_L^e c_L^\mu. \tag{3.22}$$

Clearly, this indirect dependence on the light lepton mixing angles is propagated in all the expressions that contain G_F. This is the case for example for the W boson mass, for which no other explicit dependence on mixings appears.

In addition, also the value of the top-quark m_t and Higgs boson M_H masses must be specified, since they enter the expressions via loop corrections. The dependence on M_H is soft, and we keep its value fixed at 100 GeV. In contrast, varying the value of m_t can induce sizeable effects. We have chosen to fix the top mass at the value $m_t = 120$ GeV that corresponds approximately to the minimum of our χ^2 function when all the mixing parameters are set to zero.

Whenever some other experimental parameter enters our theoretical expressions, we have used those experimental determinations for which mixing effects are absent or negligible. This will be the case *e.g.* of the strong coupling constant $\alpha_s(M_Z^2)$, of the semileptonic branching ratio $Br(b \to \ell + X)$ and of the $B^0 - \bar{B}^0$ mixing parameter χ_B on which we will further comment in the following.

Experimental errors have been evaluated by adding statistical and systematic uncertainties in quadrature and correlations have been taken into account in all the relevant cases.

3.4 Indirect Constraint from the W Mass

The standard way of computing the value of the W mass is to compare the amplitude for W exchange at $q^2 \simeq 0$ in μ decay with the effective strength of the Fermi interaction. Radiative corrections are large and must be included [27]. The theoretical expression for the W mass reads

$$M_W^2 = \frac{\rho M_Z^2}{2} \left[1 + \sqrt{1 - \frac{G_\mu}{G_F} \frac{4\mathcal{A}}{\rho M_Z^2} \left(\frac{1}{1 - \Delta\alpha} + \Delta r^{rem} \right)} \right] , \tag{3.23}$$

where $\mathcal{A} = \pi\alpha/\sqrt{2}G_\mu$. The $1/(1-\Delta\alpha)$ term renormalizes the QED low energy coupling to the M_Z scale and resums to all orders the large logs contained in the photon vacuum polarization function. The leading top effects, quadratic in m_t, are included in the parameter $\rho \simeq 1 + 3G_\mu m_t^2/8\sqrt{2}\pi^2$ [28]. We have taken $\rho = 1$ at the tree level, that corresponds to the absence of non–doublet Higgs VEV's and of extra $U(1)$ gauge bosons with non–zero mixing with the standard Z. The non-leading top effects, Higgs and other small corrections are included in the Δr^{rem} term. We refer to [49] for a detailed discussion of all these corrections.

The expression for M_W in (3.23) is affected by the mixings only indirectly via the G_μ/G_F ratio. We note that increasing values of both the mixing angles and of the top mass tend to increase M_W. Since the same interdependence enters also the expression for the effective weak–mixing angle that defines the neutral–current couplings of the fermions, a sizeable anticorrelation between m_t and the light lepton mixings is to be expected, resulting into a stronger constrain for larger values of m_t.

Experimentally the value of the W mass, as measured by CDF, is $M_W = 79.91 \pm 0.39$ GeV [33]. The UA2 collaboration has measured the ratio of the W and Z masses, for which many systematic errors cancel, obtaining $M_W/M_Z = 0.8831 \pm 0.0055$ [33]. Using the LEP value for M_Z and averaging the two results yields

$$M_W = 80.13 \pm 0.31 \text{GeV}. \tag{3.24}$$

3.5 Charged Currents Constraints

In this section we present an updated review of the charged current experiments. The experimental data are presented in a form that is well suited to derive bounds on the fermion mixing parameters, and are organised in three subsections. Tests of lepton universality and on the unitarity of the CKM mixing matrix provide very stringent limits. Limits on the hadronic right handed currents are analysed in the third subsection, while the constraints on the leptonic right handed currents have not been included here since they are essentially unchanged with respect to a previous analysis [3].

i) Lepton universality

The ratios g_μ/g_e and g_τ/g_e of the leptonic couplings to the W boson, which in the SM are predicted to be unity (universality), are modified by fermion mixings according to

$$\left(\frac{g_i}{g_e}\right)^2 = \frac{(c_L^{l_i})^2(c_L^{\nu_i})^2 + (s_R^{l_i})^2(s_R^{\nu_i})^2}{(c_L^e)^2(c_L^{\nu_e})^2 + (s_R^e)^2(s_R^{\nu_e})^2} \simeq \frac{(c_L^{l_i})^2(c_L^{\nu_i})^2}{(c_L^e)^2(c_L^{\nu_e})^2}, \qquad i = \mu, \tau. \tag{3.25}$$

Experimentally these ratios can be determined by comparing two different leptonic decay processes. In table II we give the values of $(g_i/g_e)^2$ extracted from different experiments:

1) from the ratios of the partial cross sections

$$\frac{\sigma(p\bar{p} \to W)B(W \to l_i\nu_i)}{\sigma(p\bar{p} \to W)B(W \to e\nu_e)} \qquad i = \mu, \tau \tag{3.26}$$

as measured by UA1 and UA2 [50];

2) from the ratios of the τ and μ decay rates

$$\frac{\Gamma(\tau \to \mu\nu\bar{\nu})}{\Gamma(\tau \to e\nu\bar{\nu})}, \qquad \frac{\Gamma(\tau \to \mu\nu\bar{\nu})}{\Gamma(\mu \to e\nu\bar{\nu})}, \tag{3.27}$$

that can be evaluated using the experimental values of the τ branching fractions into e and μ and the measured τ–lifetime [51]. Combining the averages of [51] with

Table II Charged Current experimental constraints on lepton universality (g_i/g_e), unitarity of the quark mixing matrix V_{ij}, and induced hadronic RHC (κ_{ij}).

Quantity	Experimental value	Correlation	Processes
$(g_\mu/g_e)^2$	1.00 ± 0.20		$W \to l\nu$
$(g_\tau/g_e)^2$	1.00 ± 0.12		
$(g_\mu/g_e)^2$	1.016 ± 0.026	0.40	$\tau \to l\nu\bar{\nu}$ and
$(g_\tau/g_e)^2$	0.952 ± 0.031		$\mu \to e\nu\bar{\nu}$
$(g_\mu/g_e)^2$	1.014 ± 0.011		$\pi \to l\nu$
"	1.013 ± 0.046		$K \to l\nu$
$\sum_{i=1}^{3} \lvert V_{ui}\rvert^2$	0.9981 ± 0.0021		hadrons decays
$\sum_{i=1}^{3} \lvert V_{ci}\rvert^2$	1.08 ± 0.37		ν-d scatt. and D_{e3}
$\mathrm{Re}(\kappa_{ud})$	0 ± 0.0037		$K \to 3\pi, 2\pi$
$\mathrm{Re}(\kappa_{us})$	0 ± 0.0037		

two very recent measurement of the L3 and OPAL collaborations [52] we obtain $Br(\tau \to \mu\nu_\mu\nu_\tau) = 0.175 \pm 0.003$ and $Br(\tau \to e\nu_e\nu_\tau) = 0.178 \pm 0.003$ from which the values in table II are derived.

3) Finally, the ratios

$$\frac{\Gamma(\pi \to \mu\nu)}{\Gamma(\pi \to e\nu)}, \qquad \frac{\Gamma(K^+ \to \mu^+\nu)}{\Gamma(K^+ \to e^+\nu)}, \tag{3.28}$$

computed from the values reported in [51], have been also used for constraining universality of the μ and e couplings.

ii) CKM unitarity

Fermion mixings lead to violations of the 3-generation unitarity of the observable CKM matrix V_{ij}, $(i,j = 1,2,3)$, as is apparent from eq. (2.11–2.13). Thus, a measurement of the deviation from unity of the sum of the $|V_{ij}|^2$, for each matrix row, puts constraints on the mixings. V_{ud} and V_{us} are obtained by dividing by G_μ the measured vector coupling in β decay and in K_{e3} and hyperon decays, respectively. Hence [3],

$$V_{ui} = \frac{G_F}{G_\mu}(V_{L\,ui} + V_{R\,ui})c_L^e c_L^{\nu_e}, \qquad i = d, s. \tag{3.29}$$

The value of $|V_{ub}|$, obtained from the analysis of semileptonic B decays, is negligibly small for our purposes. Using the unitarity of the matrix K_L introduced in (3.12), and neglecting terms of $O(s^4)$ and $O(s^2 \sum_{i=4}^n |K_{L\,ui}|^2)$,

$$\sum_{i=1}^{3} |V_{ui}|^2 = \left(\frac{G_F}{G_\mu} c_L^e c_L^{\nu_e}\right)^2 \left\{ (c_L^u)^2 - \sum_{i=4}^{n} |K_{L\,ui}|^2 + \sum_{i=1}^{2} |V_{ui}|^2 \left[2\mathrm{Re}(\kappa_{ui}) - (s_L^{d_i})^2 \right] \right\} \tag{3.30}$$

where we have approximated $|K_{L\,ui}|^2$ with the experimental values $|V_{ui}|^2$ in the coefficients of the $O(s^2)$ terms.

V_{cd} is determined from the di-muon production rate of charm off valence d-quarks [53] while V_{cs} is extracted from D_{e3} decays [51]. Hence

$$|V_{cd}|^2 \simeq (c_L^c)^2 |K_{L\,cd}|^2, \qquad |V_{cs}|^2 \simeq |K_{L\,cs}|^2 [(c_L^c)^2 (c_L^s)^2 + 2\mathrm{Re}(\kappa_{cs})] \tag{3.31}$$

where, due to the comparatively large uncertainty affecting these measurements, those mixings that are more effectively constrained by other experiments have been neglected. Taking the sum, and neglecting also $|V_{cb}|^2$ and $O(s^4)$ terms, we obtain

$$\sum_{i=1}^{3} |V_{ci}|^2 \simeq (c_L^c)^2 - \sum_{i=4}^{n} |K_{Lci}|^2 + [2\mathrm{Re}(\kappa_{cs}) - (s_L^s)^2]|V_{cs}|^2. \qquad (3.32)$$

For the $|V_{ij}|$'s we use the values given in ref. [51], and the experimental constraints from unitarity are listed in table II. [2]

iii) Right handed currents

The RHC's induced by mixings with exotic quarks allow to constrain the κ_{ij} parameters of the hadronic sector.

Very stringent limits on the $\bar{u}_R\gamma^\mu d_R$ and $\bar{u}_R\gamma^\mu s_R$ RHC's were set [56] from the observation that the $K \to 3\pi$ amplitude and slope parameters are predicted, within an accuracy of $\sim$ 10%, by PCAC and by the measured $K \to 2\pi$ matrix elements. The resulting limits

$$|\mathrm{Re}\,\kappa_{ud}|, |\mathrm{Re}\,\kappa_{us}| < \frac{8 \times 10^{-4}}{|V_{ud}||V_{us}|} \simeq 0.0037 \qquad (3.33)$$

will be treated as 1σ experimental constraints [3] on $\mathrm{Re}(\kappa_{ud})$ and $\mathrm{Re}(\kappa_{us})$.

Limits on the $\bar{c}_R\gamma^\mu d_R$, $\bar{c}_R\gamma^\mu s_R$ RHC's were set by the CDHS collaboration [53] from the analysis of the y distribution in the di-muon charm production ($\nu d, \nu s \to \mu^- c$), from which the following 95% confidence level bound was obtained

$$\frac{|\kappa_{cd}|^2 + |\kappa_{cs}|^2 \left|\frac{V_{cs}}{V_{cd}}\right|^2 \frac{2S}{U+D}}{1 + \left|\frac{V_{cs}}{V_{cd}}\right|^2 \frac{2S}{U+D}} < 0.07. \qquad (3.34)$$

[2] Recent analyses [54] have reduced the error on V_{ud}. However, still unsettled theoretical issues concerning atomic corrections affect these conclusions [55], so we used the more conservative values of [51]. Similar considerations apply also for V_{cs}.

Here U, D, S denote the quark content of the isoscalar target [53], and $|V_{cd}|$ and $|V_{cs}|$ have been introduced to approximate $|K_{L\,cd}|$ and $|K_{L\,cs}|$. Using the experimental estimate [53] for $\left|\frac{V_{cs}}{V_{cd}}\right|^2 \frac{2S}{U+D}$ we can derive from this bound an experimental constraint on the parameters κ_{cd} and κ_{cs}.

The leptonic RHC's are limited by the measurements of the μ and τ Michel parameters, as well as by the electron polarization in β decays. The corresponding bounds on parameters such as $s_R^{e_a} s_R^{\nu_a}$ have not improved with respect to those obtained in [3], so we will not repeat them here.

3.6 Neutral Currents Constraints

Neutral–current experimental results are conveniently given as fits to the couplings appearing in the effective Lagrangians that describe the corresponding four–fermion processes [51]. The form of these effective Lagrangians relies only on the assumption of spin–one gauge boson exchange and of massless left–handed neutrinos, and thus the experimental values of the phenomenological parameters are essentially model independent.

We will treat separately the $\nu - q$, $\nu - e$ and the parity–violating $e - q$ sectors. For clarity we will only display the tree level expressions, but in our numerical computations the SM radiative corrections [57,58] have been included as well.

3.6 NEUTRAL CURRENTS CONSTRAINTS

i) Neutrino – quark sector

The effective Lagrangian for the neutral current interaction of the light neutrinos with quarks is

$$-\mathcal{L}^{\nu q} = \frac{G_\mu}{\sqrt{2}}\,\bar{\nu}\gamma_\mu(1-\gamma_5)\nu\ \left[\epsilon_L(q)\bar{q}\gamma_\mu(1-\gamma_5)q + \epsilon_R(q)\bar{q}\gamma_\mu(1+\gamma_5)q\right]. \qquad (3.35)$$

The values of the quark couplings $\epsilon_{L,R}(q)$ are extracted from deep–inelastic scattering experiments off isoscalar and proton targets, normalized to the charged current cross sections, i.e. from the ratios

$$R_\nu = \frac{\sigma(\nu_\mu N \to \nu X)}{\sigma(\nu_\mu N \to \mu^- X)} \quad , \quad R_{\bar{\nu}} = \frac{\sigma(\bar{\nu}_\mu N \to \bar{\nu} X)}{\sigma(\bar{\nu}_\mu N \to \mu^+ X)}. \qquad (3.36)$$

In comparing the experimental results with the theoretical expressions, the effect of the mixings in the normalization factors has to be taken into account as well [3], since the fermion charged–current couplings are modified according to (3.11) and, as discussed in the previous subsection, also the value of the CKM element V_{ud} obtained from β-decay experiments is affected.

Using now (3.6), (3.10) and (3.20) and taking the normalization effects properly into account, the values of the quark couplings as extracted from experiments correspond to

$$\epsilon_{L,R}(q) = \frac{1}{2}F_1(s^2,\kappa)(v_q \pm a_q), \qquad (3.37)$$

where

$$F_1(s^2,\kappa) = \frac{1-\Lambda_\mu(s_L^{\nu_\mu})^2/2}{1-(s_L^{\mu})^2-(s_L^{\nu_\mu})^2-\mathrm{Re}(\kappa_{ud})}. \qquad (3.38)$$

In this factor the numerator comes from the modified NC ν-couplings while the denominator comes from the experimental normalization.

The experimental values [51] are given in table III in terms of

$$g_a^2 \equiv \epsilon_a(u)^2 + \epsilon_a(d)^2 \quad , \quad \theta_a \equiv \tan^{-1}\left[\frac{\epsilon_a(u)}{\epsilon_a(d)}\right] \qquad a = L, R \tag{3.39}$$

that have negligible correlations.

ii) Neutrino – electron sector

The effective Lagrangian for the $\nu - e$ sector is

$$-\mathcal{L}^{\nu e} = \frac{G_\mu}{\sqrt{2}}\, \bar{\nu}\gamma_\mu(1-\gamma_5)\nu\; \bar{e}\gamma_\mu(g_V^e - g_A^e\gamma_5)e. \tag{3.40}$$

The electron vector and axial–vector couplings are extracted from $\nu_\mu - e$ scattering experiments that, as in the previous case, are normalized with the ν_μ-hadron charged–current cross sections. For high–energy neutrinos like those of CERN and FERMILAB, the CC deep–inelastic process leads to the same normalization factor as in the $\nu - q$ sector, so that the relation between the couplings extracted from experiments and the theoretical ones is

$$g_V^e = F_1\, v_e \quad , \quad g_A^e = F_1\, a_e. \tag{3.41}$$

For the low-energy neutrinos of BNL, the CC scattering is a quasi–elastic process so that the factor $F_1(s^2, \kappa)$ in eq. (3.41) is replaced by [3]

$$F_2(s^2) = \frac{1 - \Lambda_\mu (s_L^{\nu_\mu})^2/2}{1 - (s_L^\mu)^2 - (s_L^{\nu_\mu})^2}. \tag{3.42}$$

In table III we list the values of $g_{V,A}^e$ that we have used. The recent CHARM II [38] results, as well as the CHARM I [37] and BNL [39] data on both ν_μ and

Table III Neutral Current experimental constraints.

Deep-inelastic ν-q			
g_L^2	0.2977 ± 0.0042		
g_R^2	0.0317 ± 0.0034		
θ_L	2.50 ± 0.03		
θ_R	$4.59 {+0.44 \atop -0.27}$		
ν-e scattering		experiment	
g_V^e/g_A^e	0.047 ± 0.046	CHARM II	
g_V^e	-0.06 ± 0.07	CHARM I	
g_A^e	-0.57 ± 0.07	"	
g_V^e	-0.10 ± 0.05	BNL	
g_A^e	-0.50 ± 0.04	"	
e-q parity violation		correlation	
C_{1u}	-0.249 ± 0.066	-0.99	-0.95
C_{1d}	0.391 ± 0.059		0.95
$C_{2u} - \frac{1}{2}C_{2d}$	0.21 ± 0.37		

$\bar{\nu}_\mu$ scattering off electrons have all been included. In particular, CHARM II has measured g_V^e/g_A^e from the ratio between ν and $\bar{\nu}$ NC cross sections, leading to a clean measurement of v_e/a_e since the F_1 factor cancels.

iii) Electron – quark sector

iii) The measurements of parity violation effects in atoms and the polarized $e - D$ scattering experiments are sensitive to weak–electromagnetic interference effects and allow the determination of the $e - q$ parity violating couplings $C_{1,2}$. These parameters appear in the effective Lagrangian

$$-\mathcal{L}^{eq} = -\frac{G_\mu}{\sqrt{2}} \sum_i (C_{1i} \bar{e}\gamma_\mu \gamma^5 e \; \bar{q}^i \gamma^\mu q^i + C_{2i} \bar{e}\gamma_\mu e \; \bar{q}^i \gamma^\mu \gamma^5 q^i), \tag{3.43}$$

where $i = u, d$. Their relation with the theoretical couplings is

$$C_{1i} = 2\left(\frac{G_F}{G_\mu}\right) a_e v_i \quad , \quad C_{2i} = 2\left(\frac{G_F}{G_\mu}\right) v_e a_i . \tag{3.44}$$

For the determination of the coefficients C_1, parity violating transitions in Cs are quite effective since for heavy nuclei the vector couplings of the quarks are coherently enhanced, in addition since Cs has only a single electron outside a completely filled shell, rather clean theoretical calculations for the atomic effects are available [36]. The results are expressed in terms of the weak charge $Q_W = -2(C_{1u}(2Z + N) + C_{1d}(Z + 2N))$ whose value is [35] $Q_W({}^{133}_{55}Cs) = -71.04 \pm 1.58 \pm 0.88$ (the second error comes from atomic theory). The particular combination $C_{2u} - \frac{1}{2}C_{2d}$ has been also measured in the SLAC polarized $e - D$ scattering experiment [59]. The values of the parity–violating coefficients listed in table III have been derived from the quoted value of Q_W, and from the results given in table 1 of ref. [59].

3.7 Constraints from Measurements at the Z–peak

The recent experiments performed at the LEP and SLC Z–factories have provided us with a set of high precision measurements that are very sensitive to the values of the fermion couplings to the Z–boson.

Besides the accurate determination of the value of the Z–mass, that together with α and G_F completes the set of fundamental input parameters of the SM, also the total Z width and the partial decay widths into hadronic final states and into each of the three lepton flavours have been measured at LEP with very high precision. Less accurate results have been obtained also for the b and c partial widths.

The measurements of the different Γ_f's are sensitive to the particular combination of couplings $v_f^2 + a_f^2$, while the independent combination $v_f a_f/(v_f^2 + a_f^2)$ enters the expressions of the on–resonance forward–backward asymmetries $A_f^{\rm FB}$, that have been measured for $f = e$, μ, τ, c, b, and of the τ polarization asymmetry A_τ^{pol}. Already the first experimental results of LEP led to a drastic improvement of the bounds on the mixings of the heavy quarks and the τ–lepton [4], that were otherwise poorly constrained [3]. The present accuracy leads to a general improvement of the limits on all the mixing angles.

i) Z decay widths

All the four LEP collaborations have measured the total Z–width Γ_Z as well as the hadronic and the three flavour–dependent leptonic partial widths Γ_h,

Γ_e, Γ_μ and Γ_τ [19].

Due to the very high experimental accuracy, radiative corrections have to be carefully taken into account in all the theoretical expressions. At 1–loop, the partial decay width of the Z–boson to f–flavour fermions reads [49]

$$\Gamma_{Z\to f\bar{f}} = N_c^f \frac{M_Z}{12\pi}\sqrt{2}G_F M_Z^2 \rho_f (a_f^2 + v_f^2)(1+\delta_{QED}^f)(1+\delta_{QCD}^f), \qquad (3.45)$$

where $N_c^f = 3(1)$ for quarks (leptons). δ_{QCD}^f is the gluonic correction for hadronic final states ($\delta_{QCD}^f \simeq \alpha_s(M_Z^2)/\pi$ in leading order). For the strong coupling constant we have used the value $\alpha_s(M_Z^2) = 0.118 \pm 0.008$ determined from jet analysis in hadronic Z decays [32], and we have neglected the theoretical uncertainty related to the error on this parameter. δ_{QED}^f is a (small) additional photonic correction. Electroweak corrections appear in the ρ_f term as well as in the effective weak mixing angle that renormalizes the vector-coupling v_f :

$$\rho_f = \rho + \Delta\rho_f^{rem}, \qquad (3.46)$$

$$s_{eff}^2(f) = \frac{1}{2}\left[1 - \sqrt{1 - \frac{G_\mu}{G_F}\frac{4\mathcal{A}}{\rho M_Z^2}\left(\frac{1}{1-\Delta\alpha} + \Delta\bar{r}_f^{rem}\right)}\right]. \qquad (3.47)$$

In these equations the ρ–term is universal and includes the potentially large heavy–top effects. $\Delta\rho_f^{rem}$ and $\Delta\bar{r}_f^{rem}$ contain, among others, non universal flavour–dependent contributions arising from vertex form factors. These contributions are generally small, except in the case of b–quarks final states for which loops involving the top–quark appear in the correction to the Zbb vertex [30]. Finite mass effects not displayed in eq. (3.45) have been also taken into account. Analytical formulae for these corrections can be found in [49]. We note that, since G_μ/G_F enters the definition of the effective weak–mixing angle, all the LEP measurements contribute indirectly to bound the four mixings involved in the μ–decay.

The experimental values of the five width Γ_Z, Γ_h, Γ_ℓ ($\ell = e$, μ, τ) as measured by the four LEP collaborations are affected by common systematic errors. For the weighted averages listed in table IV we have assigned to Γ_Z a common systematic error of 5 MeV from point–to–point error in the LEP energy calibration, and to the partial widths a 0.5% error from luminosity [60]. Experimentally the widths are determined by fitting simultaneously the data for the reactions $e^+e^- \to hadrons$, e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$, and thus they are expected to have correlations that cannot be neglected, but that unfortunately are not always given. To overcome this inconvenience, we have adopted the following procedure. A second set of experimental quantities equivalent to $\{ \Gamma_Z, \Gamma_h, \Gamma_e, \Gamma_\mu, \Gamma_\tau \}$, but with much cleaner correlations, is provided by $\{ \Gamma_Z, \sigma_h^0, R_e, R_\mu, R_\tau \}$ where σ_h^0 is the peak hadronic cross section corrected for the effect of initial state radiation, and $R_l = \sigma_h^0/\sigma_l^0 = \Gamma_h/\Gamma_l$ ($l = e, \mu, \tau$). The correspondence between the two sets is given by

$$\sigma_f^0 = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}. \tag{3.48}$$

A remarkable property of this second set of quantities is that their systematic errors have in general different origins and that the correlation arising from their functional relation to the measured observables is negligible, with the exception of the one between Γ_Z and σ_h^0. In order to make a definite ansatz we have assumed an anticorrelation between these two quantities of –25%. The correlation matrix for the set of widths, shown in table IV, has been worked out via an iterative procedure by requiring that it reproduces this anticorrelation (together with vanishing small off–diagonal coefficients for the other entries) when we transform to the second set by means of eq. (3.48). We have explicitly checked that this procedure leads to quite acceptable results when confronted with the available correlations [19].

A direct measurement of the Z invisible width by single photon counting [61] $\Gamma_{\rm inv} = 500 \pm 76$ MeV has also been included in our analysis.

ii) Leptonic asymmetries.

Forward–backward asymmetries are defined as follows

$$A_f^{\rm FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}, \tag{3.49}$$

where σ_F (σ_B) is the cross section for events with the f- fermion scattered into the forward (backward) hemisphere with respect to the electron beam direction.

On resonance this gives

$$A_f^{\rm FB} = 3\,\frac{v_e a_e}{v_e^2 + a_e^2}\,\frac{v_f a_f}{v_f^2 + a_f^2}, \tag{3.50}$$

where again $v_{e,f}$ should be expressed in terms of the effective weak mixing–angle (3.47). For quarks, final state QCD corrections must also be included (see *e.g.*[62]). We have taken into account the bulk of the effects of QED initial–state radiation, that are known to yield large corrections, by convolving the $e^+e^- \to f\bar{f}$ differential cross sections with a suitable "radiator" kernel [63]. The convoluted complete s-dependent formulae [63] (instead of just eq. (3.50)) have then been used to fit the asymmetries.

Even if statistical errors are still large if compared with the uncertainties on the leptonic partial widths, the leptonic FB asymmetries constitute an additional important set of quantities for testing universality of the lepton couplings to the Z. In fact, while the Γ_ℓ's are mainly sensitive to the squared axial couplings, the asymmetries are sensitive to the ratios v_ℓ/a_ℓ. The combined measurement of these two sets of quantities allows for an independent determination of v_ℓ and a_ℓ, and turns out to be quite effective for constraining both the right and left mixing angles even in the "joint fits" where all the mixings are allowed to be present simultaneously. In table IV we give the values of the peak asymmetries averaged over the results of the LEP collaborations, but for the leptonic asymmetries we have also included in our analysis the data at ± 1 GeV around resonance in order to increase the statistics. Whenever available we have used the asymmetries determined from a maximum likelihood fit to the angular distribution $d\sigma_f/d\cos\theta \sim 1+\cos^2\theta+\frac{8}{3}A_f^{\mathrm{FB}}\cos\theta$ (where θ is the scattering angle), otherwise we have used the direct countings of the events and we have corrected for the relevant angular range of detection in the theoretical expressions.

The τ polarization asymmetry [64] has been also measured at LEP in τ pair production, using the distributions of its decay products [65]. At the Z resonance this quantity reads

$$A_\tau^{pol} = \frac{-2v_\tau a_\tau}{v_\tau^2 + a_\tau^2} \tag{3.51}$$

and it is very sensitive to the τ vector coupling to the Z, having the advantage with respect to the forward–backward asymmetry that it is not suppressed by the small electron vector coupling. Experimentally, the τ polarization is inferred from the slope of the energy distribution of the decay products under the assumption of pure V – A coupling with the $W^{\pm}$ bosons [64]. The presence of mixing–induced RHC could then in principle affect the quoted experimental results, but this effect is $O(s^4)$ and can thus be neglected. The weighted average of the ALEPH and OPAL results [65] is given in table IV.

iii) Heavy flavours

For the measurement of the width of the Z decay into b quarks, different methods have been used by different collaborations. ALEPH, L3 and OPAL at LEP and MARK II at SLC [20] used high momentum and high–p_T muons and/or electrons to tag the b quark, thus they measure the quantity $\Gamma_b/\Gamma_h\, Br(b \to \ell\nu X)$ for $\ell = e, \mu$. A value for the b–branching ratio into electrons and muons is then needed for the determination of Γ_b. This branching has been measured by the L3 collaboration [34] by analysing the ratio of the events where both b's decay semileptonically to the single lepton events. The L3 measurement can be combined with the PEP and PETRA determination of $Br(b \to \ell\nu X)$ (quoted in [34]) to obtain a value that is largely independent of assumptions on the b neutral–current couplings since the first result is, in first order, independent of Γ_b, and at the c.m. energies of the latter experiments weak effects contribute only a few percent to b–production. We have used the resulting value $Br(b \to \ell\nu X) = 0.119 \pm 0.006$ [34] for deriving Γ_b/Γ_h from the average of the first four measurements. The strategy adopted by DELPHI [20] in order to identify the b quarks is based on the fact that, due to the larger B–hadrons mass, a greater "sphericity" is expected for the

Table IV Results on Z–partial widths (in MeV) and on–resonance asymmetries. The values displayed for the leptonic asymmetries correspond to the peak–data and have been corrected only for angular acceptance. Also displayed are the values of the b and c axial–vector couplings extracted from the off–resonance $A^{\gamma Z}_{b,c}$ asymmetries (used only in the single fits) and the charm asymmetry measured through D^*–tagging (used in the joint fits).

Quantity	Experimental value	Correlation			
Γ_Z	2487 ± 10	0.52	0.52	0.29	0.25
Γ_h	1739 ± 13		−0.15	0.55	0.48
Γ_e	83.2 ± 0.6			−0.08	−0.07
Γ_μ	83.4 ± 0.9				0.26
Γ_τ	82.8 ± 1.1				
$A_e^{\mathrm{FB}}(peak)$	-0.019 ± 0.014				
$A_\mu^{\mathrm{FB}}(peak)$	0.0070 ± 0.0079				
$A_\tau^{\mathrm{FB}}(peak)$	0.099 ± 0.096				
A_τ^{pol}	-0.121 ± 0.040				
Γ_b	367 ± 19				
Γ_c	299 ± 45				
A_b^{FB}	0.123 ± 0.024				
A_c^{FB}	0.064 ± 0.049				
$a_b^{\gamma Z}$	-0.405 ± 0.095	(for single fits)			
$a_c^{\gamma Z}$	0.515 ± 0.085				
$A^{\gamma Z}_{c,D^*}$ (29 GeV)	-0.101 ± 0.027	(for joint fits)			
$A^{\gamma Z}_{c,D^*}$ (35 GeV)	-0.161 ± 0.034				

corresponding jets. This measurement gives a further direct determination of the Γ_b/Γ_h ratio.

From the overall average of these quantities and using the experimental value of Γ_h (table IV), we obtain $\Gamma_b = 363 \pm 19$ MeV, where the overall uncertainty is dominated by the error on the b–semileptonic branching ratio.

The Z partial decay width into charmed quarks has also been measured, but due to the greater difficulties in the identification of the primary c quarks the accuracy achieved is worse. ALEPH [66] uses high p and p_T electrons while OPAL uses muons [66]. Averaging these two measurements and using the value $Br(c \to \ell\nu X) = 0.096 \pm 0.006$ determined at PEP and PETRA and quoted by L3 in [34] a first value for Γ_c/Γ_h is obtained.

A second determination of the $c\bar{c}$ production rate has been performed by OPAL (last paper in [66]) with a different method. The Γ_c/Γ_h ratio is determined from the analysis of the reconstructed D^* momentum distribution produced in Z decays. The same ratio has been determined by the DELPHI collaboration [66] from the inclusive analysis of charged pions from $D^{*+} \to \pi^+ D^0$ decay. In combining these two measurements, the common systematic error coming from the $c \to D^*$ hadronization probability has been taken into account. The result of the average of the four measurement is given in table IV.

Three measurements of the b–quark forward–backward asymmetry have been reported [67]. In each case the b channel is selected using electronic and muonic b decays, with the requirement of high p and p_T for the final leptons, the consequent reduction in the statistics leads to rather large experimental errors. In addition the effect of $B^0 - \bar{B}^0$ mixing has to be taken into account. This effect tends to reduce the asymmetry since the neutral B^0 meson can transform into its charge conjugate before it decays. The relation between $A_b^{\rm FB}$ and the *observed* asymmetry is [68]

$$A_b^{\rm FB} = \frac{A_{obs}^{\rm FB}}{1 - 2\chi_B}, \tag{3.52}$$

where χ_B is a measure of the probability of a B^0 meson to oscillate into a $\bar{B}^0$ meson. Several measurements of the B–mixing parameter have been performed [69]. The method adopted, which is largely independent of the b–quark neutral couplings, is to count the ratio between like–sign to opposite–sign b–originated di–lepton events, since two leptons of the same charge are a signature that one B^0 meson has oscillated into its CP conjugate.

The result quoted in table IV for the b forward–backward asymmetry has been obtained by adjusting all the measurements to $\chi_B = 0$, and then correcting the average with $\chi_B = 0.146 \pm 0.016$ that was obtained by averaging the ALEPH, L3 and UA1 measurements [69]. [3]

The forward–backward asymmetry for charmed quarks has been measured only by the ALEPH collaboration [67] in a simultaneous fit with $A_b^{\rm FB}$. Their result is displayed in Tab. III.

We have included in our analysis also the FB b and c asymmetries $A_{b,c}^{\gamma Z}$,

[3] We have not included the ARGUS and CLEO results [70] since these observations stem from the analysis of $\Upsilon(4S)$ decays were only B_d mesons are produced, while at LEP the relative abundance of B_s to B_d mesons is estimated to be of 0.3-0.4.

measured in the γ–Z interference region at PEP and PETRA. These asymmetries are essentially determined by the product of the axial–vector couplings $a_e a_{b(c)}$, that is a different combination from what is measured on top of the resonance. High p and p_T leptons have been used for tagging both the heavy quarks [40,42], leading to non negligible correlations between the two asymmetries. For the c quark also the D^* tagging technique (largely independent of the b couplings) have been used [41,42]

In our individual fits we have used the PEP/PETRA averages for a_b and a_c quoted in ref. [40]. In the joint analysis it is no more consistent to include these results since each axial coupling is determined while keeping the others fixed at their SM value, so in this case we have restricted our set of data by including only the c FB asymmetry measured with the D^*–tagging technique [41,42].

The measurements of the leptonic asymmetries in weak–electromagnetic interference do not improve significantly the constraints, and have not been included in our fit.

3.8 Results and Discussion

To obtain the constraints on the mixing parameters s_i^2 we have confronted the theoretical expression X_α^{th} for each observable with the corresponding experimental result $X_\alpha^{exp} \pm \sigma_\alpha$ by constructing a χ^2 function

$$\chi^2 = \sum_{\alpha,\beta} \frac{(X_\alpha^{th} - X_\alpha^{exp})}{\sigma_\alpha} (C^{-1})_{\alpha\beta} \frac{(X_\beta^{th} - X_\beta^{exp})}{\sigma_\beta} \tag{3.53}$$

where the C represents the matrix of correlations.

Some care must be paid in the interpretation of the confidence levels from the χ^2 since the variables s_i^2 are bounded in [0,1]. For each parameter we then assume a probability distribution

$$P(s_i^2) = N_i \; e^{-\chi^2(s_i^2)/2} \tag{3.54}$$

with $N_i^{-1} = \int_0^1 \exp(-\chi^2(s_i^2)/2) \; ds_i^2$. For the joint fits, in which all mixing parameters are allowed to vary simultaneously, the χ^2 function in the expression for $P(s_i^2)$ is minimized with respect to all the remaining parameters for each value of s_i^2.

The 90% c.l. upper bounds $\bar{s}_i^2$ are computed by requiring

$$\int_0^{\bar{s}_i^2} P(s_i^2) \; ds_i^2 = 0.90 \tag{3.55}$$

under the additional condition $\chi^2(\bar{s}_i^2) > \chi^2(0)$ that, if not satisfied, would be a signature for non–zero mixing angles at 90% c.l..

Although there are more than 20 mixing parameters, the large number of observables allows to constrain all of them. The inclusion of the recent results from LEP, together with the updated NC and CC results, have considerably improved almost all the previous limits [3,4]. Our results for the 90 % c.l. bounds obtained in the individual and joint analyses are collected in table V.

For simplicity we have assumed $\Lambda_e = \Lambda_\mu = \Lambda_\tau$ (corresponding to ordinary-exotic mixings of the same kind for the three neutrinos) but these parameters could in principle differ. In the individual analysis, since only the bounds on the neutrino mixings may depend on the value of Λ, we just show the results for $\Lambda = 2$. Furthermore, since the electron and muon neutrino mixings are mainly constrained by CC measurements, they are largely independent of the value of Λ. In contrast, for the τ neutrino different values of Λ led to different bounds, since in this case the LEP measurement of Γ_Z gives an important constraint. The *upper* bounds for ν_τ are respectively $(s_L^{\nu_\tau})^2 < 0.098, 0.032, 0.015$ for $\Lambda_\tau = 0, 2, 4$ corresponding to neutrino mixings with heavy ordinary doublets in sequential or vector doublets, with heavy singlets and with exotic doublets respectively. For the joint bounds, we present all the results for $\Lambda = 0$, 2 and 4.

One possibility that we have not considered for simplicity is the presence of *light* neutrinos that are mainly singlets. These could appear for instance in models where the light neutrinos are Dirac particles. These light singlets could mix with exotic doublets and hence couple to the Z, giving rise to a new invisible decay channel. The additional parameters s_R^ν describing these mixings would then be constrained by the measurement of Γ_Z. This would affect mainly the bounds on the τ neutrino mixings in the joint analysis (for $\Lambda_\tau \neq 0$), while the other bounds would essentially remain unmodified.

In the last column of table V we list, for each mixing angle, the observables that are more important for establishing the constraints. Often a tight constraint set by some accurate measurements can be evaded in the joint analysis, since the deviations caused by the mixing under consideration may be canceled in these observables by adjusting other mixings to non–zero values. Other observables, for which the possibilities of cancellations are more restricted, can then become important in the determination of the joint bounds even if they were not decisive for the individual fits. In table V these observables are labeled with a (*). This happens *e.g.* for Γ_Z and Γ_{had}, that are crucial for the single bounds but that should be supplemented by other constraints in the joint analyses since they depend on several mixing parameters. Hence, the large number of measurements at our disposal plays a crucial rôle for setting the limits.

A look at table V makes apparent that the measurements of the Z partial and total widths contribute to the limits on all the exotic mixings. The bounds on the leptons and b–quark mixings receive further contributions from the on–resonance asymmetries, while PEP and PETRA off–resonance asymmetries help to strengthen the bounds for the c quark.

For the fermions of the first generation and for the μ_L and ν_L^μ leptons, both the 'low-energy' NC constraints (especially ν–q scattering and to a smaller extent the e–q sector) and the CC constraints on the unitarity of the CKM matrix are also important. These last quantities also bound the mixings $|V_{ui}|$ and $|V_{ci}|$ with sequential or vector doublets, as well as the parameters κ_{ij}, that are further constrained by direct searches of induced hadronic RHC's.

Due to the fact that the presence of the mixings modify the fermion couplings, the various determinations of the effective weak angle cannot be used as direct measurements. However since the theoretical expression for s_{eff} depends on the ratio G_F/G_μ, besides the direct constraints, the combination of all the LEP and NC experiments put also important indirect constraints to the mixings that appear in μ decay. These indirect bounds are quite effective for the electron and muon neutrino mixings, and are of some relevance also for $(s_L^\mu)^2$ in the joint analyses. These two indirect sources of constraints have been denoted respectively with s_{eff}^{LEP} and s_{eff}^{NC} in table V.

The W boson mass, that also constrains the ratio G_μ/G_F, is not very important in the individual analysis due to its present experimental error. However, it gains relevance in the joint fits since it does not allow for accidental cancellations between different mixings, as usually happens for LEP measurements.

For the left handed charged leptons and neutrinos, the constraints on lepton universality are also crucial. Some peculiarities arise in the $\tau - \nu_\tau$ sector, since to some extent the τ-decay measurements are better accounted for with non-universal CC lepton couplings (see e.g. ref. [71]). In particular, non–vanishing τ_L and/or ν_L^τ mixings weaken the $W\tau\nu_\tau$ coupling, allowing for a longer τ lifetime, as is favoured by experiments. The excellent agreement of the accurate LEP measurements with the SM predictions forces the overall probability distribution to be consistent with vanishing values for the τ and ν_τ mixings. However, if ν_τ mainly mixes with an ordinary sequential or vector doublet neutrino ($\Lambda_\tau \simeq 0$), the NC experiments are ineffective for constraining this mixing. In this case, both in the individual and in the joint analyses, we find that the value $s_L^{\nu_\tau} = 0$ falls out of the 90% confidence regions, that are respectively $0.0075 < s_L^{\nu_\tau} < 0.098$ and $0.0057 < s_L^{\nu_\tau} < 0.097$. However, within two standard deviations the data are consistent with zero mixing.

Another complication in the analysis is due to a peculiarity in the behaviour of the observables where the d_R-type quark mixings are involved. Indeed for $(s_R^q)^2 \simeq 0.3$ ($q = d,\ s,\ b$), the s^4 terms cancel against the quadratic ones inside both the combinations $v_q a_q$ and $v_q^2 + a_q^2$, that are the only combinations of couplings that can be measured at the Z–peak. Since the constraints on s_R^s and s_R^b are provided essentially by LEP experiments, the corresponding χ^2 distributions are characterized by two equivalent minima, one lying around vanishing value for the mixing and a second one near 0.3, and as a consequence the confidence intervals are split into two disjoint regions with the χ^2 function steeply rising between them. Actually, due to the low central value of the b-quark axial–vector coupling as extracted from the asymmetries in the γ–Z interference region [40], the value $(s_R^b)^2 \simeq 0.3$ is even in slightly better agreement with the data. For the bounds in table V we have conservatively integrated over both the regions, however a restriction to the interval consistent with zero mixing gives $(s_R^s)^2 \lesssim 0.09$ and $(s_R^b)^2 \lesssim 0.10$, *i.e.* about three times tighter limits.

We can add a final comment about the interplay between the mixings and the bounds on the top mass that are obtained by allowing this parameter free to vary. When all the mixing parameters are left free, a fit to m_t indicates that the preferred value is shifted downwards by about 25 GeV with respect to the case when all the mixings are kept fixed to zero. As we have already noted this is mainly due to the anticorrelation with the mixings that appear inside the ratio between the Fermi constant as measured in μ decay, and the true Fermi coupling constant: G_μ / G_F. As a consequence of the very large number of free parameters the error is larger, and eventually the upper bound on m_t is slightly relaxed. However, considering that also the loop effects of the new heavy fermions that are present in the models under consideration are expected to lower the upper bound on m_t (and in particular in the case of a large mass splitting inside the members of the same isodoublet) we can conclude that in these models a 'light' top is generally preferred.

Table V 90 % c.l. upper bounds on the ordinary-exotic fermion mixings for the individual fits, where only one parameter is allowed to vary, and for the joint fits where cancellations between different mixings can occur. The observables that mainly contribute to determine the numerical values of the bounds are listed in the last column (those labeled by an asterisk (*) are effective only in the joint analyses). s_{eff}^{LEP} and s_{eff}^{NC} refer to the effective weak mixing angle, from Z–peak and NC experiments, which contribute indirectly to constrain the mixings in G_F/G_μ.

	Individual	Joint			Source
		$\Lambda = 2$	$\Lambda = 0$	$\Lambda = 4$	
$(s_L^e)^2$	0.0047	0.015	0.0090	0.015	$\Gamma_e, M_W^*, A_\mu^{\mathrm{FB}*}, eq^*, g_e^*$
$(s_R^e)^2$	0.0062	0.010	0.0082	0.010	$\Gamma_e,\ A_e^{\mathrm{FB}},\ A_\mu^{\mathrm{FB}*},\ \nu e^*$
$(s_L^\mu)^2$	0.0017	0.0094	0.0090	0.011	$V_{ui}^2, \nu q, g_\mu, \Gamma_\mu, s_{eff}^{\mathrm{LEP}\,*}, M_W^*$
$(s_R^\mu)^2$	0.0086	0.014	0.014	0.013	$\Gamma_\mu,\ A_\mu^{\mathrm{FB}}$
$(s_L^\tau)^2$	0.011	0.017	0.015	0.017	$\Gamma_\tau,\ A_\tau^{\mathrm{FB}}, g_\tau,\ A_\tau^{pol*}$
$(s_R^\tau)^2$	0.011	0.012	0.014	0.012	$\Gamma_\tau,\ A_\tau^{pol}, A_\tau^{FB},\ g_\tau^*$
$(s_L^u)^2$	0.0045	0.019	0.015	0.019	$V_{ui}^2,\ \Gamma_h,\ \Gamma_Z, eq, \nu q$
$(s_R^u)^2$	0.018	0.024	0.025	0.024	$\nu q,\ \Gamma_h,\ \Gamma_Z,\ eq$
$(s_L^d)^2$	0.0046	0.019	0.016	0.019	$V_{ui}^2,\ \Gamma_h,\ \Gamma_Z,\ \nu q$
$(s_R^d)^2$	0.020	0.030	0.028	0.029	$eq,\ \Gamma_h,\ \Gamma_Z,\ \nu q$
$(s_L^s)^2$	0.011	0.038	0.039	0.041	$\Gamma_h,\ \Gamma_Z,\ V_{ui}^2$
$(s_R^s)^{2\,\dagger}$	0.36	0.67	0.63	0.74	$\Gamma_h,\ \Gamma_Z$
$(s_L^c)^2$	0.013	0.040	0.042	0.042	$\Gamma_h,\ \Gamma_Z,\ \Gamma_c^*,\ A_c^{\gamma Z*}$
$(s_R^c)^2$	0.029	0.097	0.10	0.099	$\Gamma_h, \Gamma_Z, A_c^{\gamma Z*}, \Gamma_c^*, A_c^{\mathrm{FB}*}$
$(s_L^b)^2$	0.011	0.070	0.072	0.069	$\Gamma_h,\ \Gamma_Z,\ \Gamma_b,\ A_b^{\mathrm{FB}*}$
$(s_R^b)^{2\,\dagger}$	0.33	0.39	0.40	0.39	$\Gamma_b,\ \Gamma_Z, \Gamma_h,\ A_b^{\gamma Z},\ A_b^{\mathrm{FB}*}$
$(s_L^{\nu_e})^2$	0.0097	0.015	0.016	0.014	$s_{eff}^{\mathrm{LEP}}, g_e, s_{eff}^{\mathrm{NC}}, M_W^*$
$(s_L^{\nu_\mu})^2$	0.0019	0.015	0.0087	0.011	$V_{ui}^2, g_\mu, \nu q, s_{eff}^{\mathrm{LEP}}, M_W^*$
$(s_L^{\nu_\tau})^{2\,\dagger}$	0.032	0.064	0.097	0.035	$\Gamma_Z,\ g_\tau$
$\sum_{i=4}^n K_{ui}^2$	0.0048	0.014	0.010	0.018	V_{ui}^2
$\lvert\kappa_{ud}\rvert$	0.0011	0.0059	0.0060	0.0058	V_{ui}^2, νq, RHC's, νe^*
$\lvert\kappa_{us}\rvert$	0.0054	0.0061	0.0061	0.0061	V_{ui}^2, RHC's
$\sum_{i=4}^n K_{ci}^2$	0.53	0.76	0.76	0.76	V_{ci}^2
$\lvert\kappa_{cd}\rvert$	0.31	0.31	0.31	0.31	RHC's
$\lvert\kappa_{cs}\rvert$	0.24	0.29	0.29	0.29	V_{ci}^2, RHC's

† For some peculiarities that occur for s_R^s and s_R^b, and for a discussion of the bounds on $s_L^{\nu_\tau}$, see text.

3.9 Concluding Remarks

As a summary, we have analysed the limits on the mixings of the known leptons and quarks with possible heavy fermions with exotic $SU(2)_L$ assignments. We have obtained significant constraints on a very large set of mixing parameters by performing a detailed global analysis of the available electroweak data.

In order to guarantee the experimentally observed absence of FCNC, we have assumed that each ordinary charged fermion mixes with a unique exotic state, in which case just one mixing angle per degree of freedom is enough to describe the effects of the mixing. For the neutrinos there is no experimental evidence of FCNC suppression, but since one has to sum over the flavor of the unobserved final ν states, again just one mixing angle per neutrino flavor allows to describe the mixings, with the addition of an effective parameter (Λ) that takes into account the type of exotic neutrinos involved.

In order to constrain the mixing angles we have analyzed the effects that they could induce in the couplings between the light fermions and the weak bosons. Very accurate measurements of the fermion weak–couplings are provided by several experiments, as for example tests on CC universality and on the unitarity of the CKM matrix, limits on induced right-handed currents, collider measurements of M_W, low-energy NC experiments (ν-scattering, atomic parity violation and polarized $e - D$ scattering) and in particular the huge amount of data obtained at LEP and SLC from experiments at the Z–resonance. The results of our analysis are collected in table V.

When just one mixing is constrained at a time, we obtain for most of the fermions the tight limits $s^2 \lesssim 0.002 \div 0.01$ at 90 % c.l.. For u_R, c_R and ν_τ the bounds are $s^2 < 0.03$, however if ν_τ mixes with an ordinary heavy neutrino the constraint is $s^2 \lesssim 0.1$ and a signal of non–zero mixing at 90 % c.l. is found. For s_R and b_R we find the much weaker bounds $s^2 \lesssim 0.35$. Allowing for accidental cancellations among different mixings the constraints are relaxed by a factor between 2 and 5.

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