SISSA - SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI
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ISAS - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

# Symmetry Perception in Human Vision

## *Psychophysics and Modeling*

Thesis Submitted for the Degree of

*Doctor Philosophiae*

Candidate:
Ling Zhang

Supervisor:
Prof. Walter Gerbino

Academic Year 1991/92

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# Introduction

Human brain functions well in dealing with visual information. When we look around, information provided by the surroundings is processed in such a way that a useful and rich representation of the world becomes perceptually available. Among other things, this process leads to the extraction of patterns. One basic characteristic of pattern perception is the distinction between figure and ground. Furthermore, we can perceive object features as shape, color, depth, orientation, location, and movement. We are also able to memorize visual forms, to recognize them, and to retrieve them after long time intervals. The study of how visual information is processed, stored, or recalled leads to the understanding of the structure and the function of our visual system.

Mirror symmetry is an important property of natural and man-made objects. It can be perceived immediately and without effort. The perception of symmetry affects other visual processes such as the identification of a spatial reference frame, representation, memory, and recall. Mach (1886) was the first one who brought attention to the importance of symmetry in our understanding of the visual system. Since then, symmetry has been studied extensively and widely. Many phenomena have been observed. Hypotheses and models have been raised. Symmetry detection in random-dot textures is one way to study how patterns are processed and evaluated.

This thesis is based on the studying of information processing in symmetry detection. Three groups of experiments were performed. The first group of experiments regards the influence of a property of the axis, its explicitness, on symmetry detection. Results shows that symmetry is more salient and easier to detect when the axis is explicit; which suggests that the identification of axis location facilitates the process of symmetry evaluation. The second group of experiments regards the relative importance of two aspects of vertical bilateral symmetry, the horizontality of virtual lines and the verticality of the axis. To study the relative contribution of these two components, two types of skewed symmetry were compared, following the paradigm recently developed by Johan Wagemans (1992). Results show that skewed symmetry with horizontal virtual lines and an oblique axis is easier to detect than skewed symmetry with a vertical axis and oblique visual lines, suggesting that the superiority of vertical bilateral symmetry over other types of symmetry is more explained by the horizontality of virtual lines than by the verticality of the axis. Matching along the horizontal dimension is easier than along oblique dimensions. The role of visual information about the 3D orientation of the pattern was studied too, since skewed symmetry corresponds to the projection of normal symmetry on a slanted or tilted plane. Results show that such information can improve the detection of skewed symmetry, especially when provided simultaneously. The third group of experiments explores symmetry detection in three-color patterns, to clarify the role of grouping processes involved in symmetry detection and the specific contribution of point-by-point matching. Two kinds of grouping were modified and compared: the grouping of symmetrical paired dots between two halves and the grouping of dots within each half. Same- and opposite-contrast patterns were used. Results show that symmetry is easier to detect when paired dots have the same contrast than when they have opposite contrast. This suggest that point-by-point comparison is the basic process involved in symmetry detection. Both kinds of grouping, grouping between and within, facilitates symmetry detection. Grouping between the two halves is more useful than within each half.

The structure of the thesis is the following: Chapter 1 introduces theories of perceptual organization; Chapter 2 demonstrates phenomena observed in symmetry detection and presents some models accounting for the experimental results; Chapter 3, Chapter 4, and Chapter 5 describe details of Experiment Group 1, Experiment Group 2, and Experiment Group 3 respectively. Chapter 6 provides a model of symmetry detection with random-dot patterns, and presents the results of a computer simulation.

# Chapter 1

# Perceptual Organization

The overwhelming of information in our visual environment is proverbial. From retinal image to perceived patterns, visual information is analyzed and organized in various ways by our visual system. Parts of the retinal image are segregated and appear as figures on a background. Some elements of a pattern are grouped together to form a single object, and others to form different objects. Then, individual objects are discriminated from each other and identified. Perceptual organization is the essential process which precedes cognition and affects its contents and procedures. For this reason the factors which mediate perceptual organization have been studied extensively.

## 1.1 Figure-Ground Segregation

A fundamental process of visual perception is the extraction of information useful for separating the figure from the background. Certain patterns tend to be seen as the thing, while the rest is perceived as the background. In essence, we divide the world into two categories: into something we shift our attention to, and into remaining items that form the not-thing and become ignored (Julesz, 1971). In some instances, the figure-ground organization is bistable, as shown in Fig. 1.1, in which perception alternates between a white vase on a black background or two black profiles on a white background. Factors influencing figure-ground organization have been studied by psychologists following the Gestalt tradition (Koffka, 1935; Kanizsa, 1979). It is found that figure-ground organization is influenced, among other things, by stimulus factors like relative area, closure and symmetry.

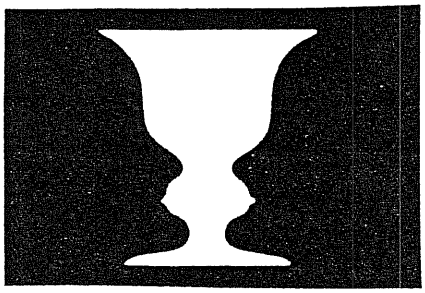


Fig. 1.1. Figure-Ground segregation

**Relative area** The smaller a region is the more it tends to be seen as figure; and conversely, the larger a region is the more it appears to be ground, as shown in Fig. 1.2.

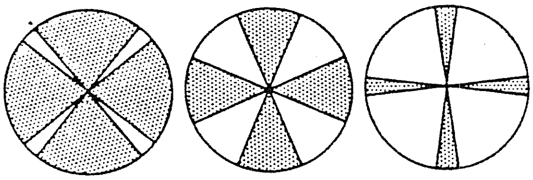


Fig. 1.2. Factor influences figure-ground organization: relative area.

**Closure** Areas with closed contours are more likely to be seen as figures than areas with open contours are, as shown in Fig. 1.3.

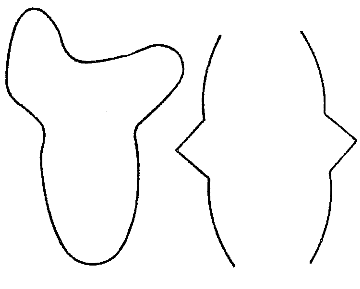


Fig. 1.3. Factor influences figure-ground organization: closure.

**Symmetry** The more symmetrical a region is the more likely it appear as the figure. The greater the amount of symmetry, the stronger is this tendency. As shown in Fig. 1.4.

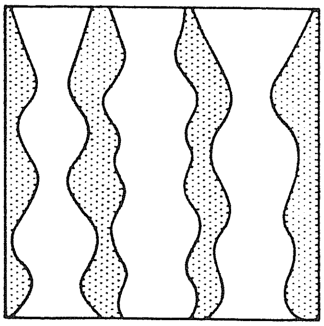


Fig. 1.4. Factor influences figure-ground organization: symmetry.

## 1.2 The Gestalt Principle of Organization

In principle, a pattern can be interpreted in many ways. Usually, only one interpretation is preferred. Even in the rather simple line drawing in Fig. 1.5, several different interpretations are possible, but the preferred one will be two squares.

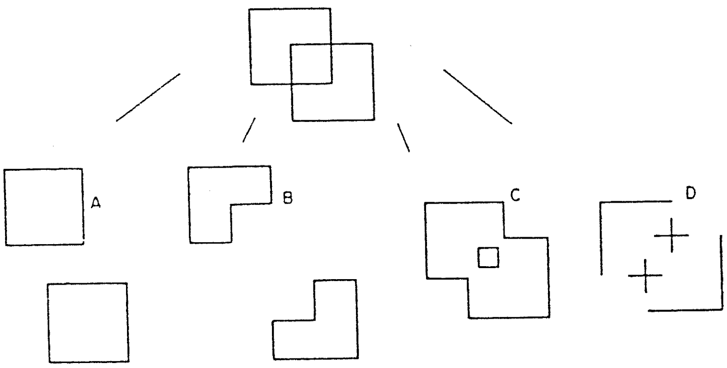



Fig. 1.5. Interpretations.

In order to be able to predict such preference, scientists in shape perception have searched for the underlying principles that govern the human interpretation of patterns and the nature of grouping.

Some scientists (e.g. von Helmholtz, 1867; Gregory, 1973), advocate the likelihood principle, which states that the preferred interpretation of a pattern is the one which reflects the most probable situation. Others, like the Gestaltists, advocate the minimum principle, which states that human pattern perception is guided by simplicity. For instance, grouping would occur according to proximity, similarity, good continuation, and common fate (Wertheimer, 1923; Kohler, 1920).

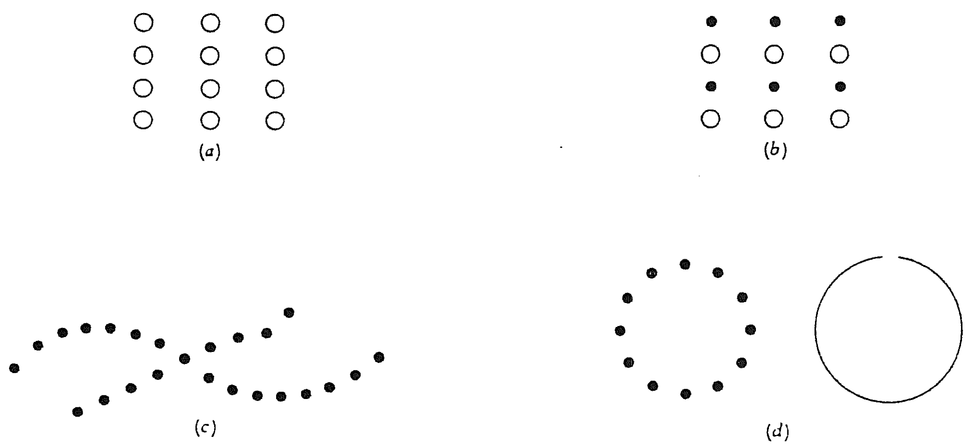



Fig. 1.6. The Gestalt principles of pattern organization: (a) proximity; (b) similarity; (c) continuation; (d) closure.

**Proximity** Spatial adjacent elements are grouped, as shown in Fig. 1.6 (a). The circles are grouped into three columns rather than four rows.

**Similarity** Elements of similar brightness, color, and shapes are grouped, as shown in Fig.1 .6 (b), black dots and white circles are grouped into rows.

**Continuation** Among many possible perceptual interpretations those that will minimize changes or interruptions in the contours of the constituents will be perceived as figures, as shown in Fig. 1.6 (c).

**Common fate** Elements which move or change together are seen as a unit or with a common fate.

**Closure** Elements are grouped according to closure, as shown in Fig. 1.6 (d), circle is perceived rather than individual dots.

Gestalt principles predict the likelihood with which certain elements are grouped into the same object and separated from other elements grouped into different objects. These principles have great heuristic values, but so far attempts of ordering them according to their relative strength have been unsuccessful; i.e., sometimes one law appears to be stronger, sometimes another law. Koffka (1935) suggested that all laws are specifications of a general minimum principle, the tendency towards Prägnanz; but his proposal does not clarify if the minimization must occur at a local level or at a global level.

## 1.3 Textural Segregation

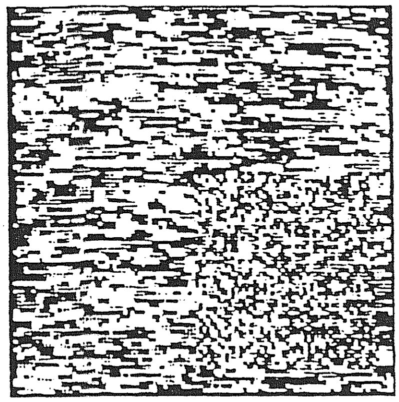


Fig. 1.7. An example of texture segregation.

A texture is an aggregate of elements in a random or semi-regular way. When textures are observed with little or no apparent effort, one can have the impression of either a unified texture or of several separate subtextures. This is demonstrated by Fig. 1.7. Julesz defined the effortless, spontaneous, and rapid performance of differentiating juxtaposed textures as visual texture segregation (Julesz, 1962). Study of texture segregation has attracted extensive attention, mainly focused on the factors that mediate the segregation and the nature of the segregation process itself. It also provides a way to study perceptual organization as well.

Julesz (1981), after a series of researches, proposed his texton theory, which suggests that texture segregation is based either on the difference in textons or on the difference in their first-order statistics (Julesz, 1981). Besides color, Julesz has identified three additional texton classes, which are: (1) elongated blobs of given orientation, width and aspect ratio; (2) terminators, and derived features like corners, closure and connectivity which can be described by differences in terminators; (3) line crossings.

Julesz's texton theory can explain many phenomena but it has been criticized by several authors (Treisman & Gormican, 1988; Gurney & Browse, 1985, 1989; Bergen & Adelson, 1988). The necessity of some textons seems controversial, and other features inducing similar effects as textons have been found.

Beck (1982, 1983) hypothesized that the retinal intensity array is transformed into texture elements following the rules of proximity, similarity, and good continuity through the linking operation. The segregation is based on the differences in distribution of local features, such as the slopes, sizes, and brightness, of texture elements and their parts. Beck proposed that the difference signals are proportional to the difference in activation of feature analyzers stimulated by texture elements belonging to neighboring spatial regions and decreased by shared features that stimulate common analyzers. In a sense, similarity can be regarded as noise which reduces the segregation.

Other determinants of textural segregation, different from textons and Beck's features, have been hypothesized. They include micropattern size (Gurney & Browse, 1989; Bergen & Adelson, 1988), and responses of circular symmetric center-surround operators or simple orientation selective filters (Bergen, 1986; Caelli, 1985; Marr & Hildreth, 1980; Fleet, Hallett & Jepson, 1985; Pollen & Ronner, 1983; Bergen & Julesz, 1983).

The convergence point of all theories is that texture segregation occurs automatically and in parallel, without attention. As other preattentive processes, it affects subsequent processing, for instance by switching on the attentive perception system, as suggested by Julesz.

## 1.4 Perceptual Organization and Attention

The process of selective attention enables organisms to respond selectively to individual objects in a cluttered visual scene. The study of attention by visual search task and other methods provides ways to understand the nature and the use of perceptual organization and the processes of the allocation of attention. Research in this area is influenced by two popular ideas. One is the spotlight illuminating or zoom lens magnifying metaphor (Broadbent, 1982; Eriksen & Eriksen, 1974; Eriksen & James, 1986; Posner, 1980; Tsal, 1983; Treisman & Gelade, 1980; Treisman, 1982), which claims that attention is directed to a particular region at any instant. Space has a particular role in visual attention as they predicted. The other is the grouping hypothesis (Duncan, 1984; Kahneman & Henik, 1981; Prinzmetal, 1981; Driver & Baylis, 1989, 1992), which claims that attention is directed to the perceptual groups formed according to Gestalt principles of organization. In such a case, spatial relation is not the unique factor mediating the direction of attention.

The spotlight hypothesis suggests that attention is focused on a contiguous region of the visual field for further processing. Eriksen & Eriksen (1974) provided their influential supports from their experiment on response competition. They asked subjects to make a two-choice responses to a letter appearing at a fixed position. Distracting letters associated with the alternative response slowed down the response only when they were close to the target. This distance effect has been interpreted in terms of spotlight metaphor: attention is focused on the known target location. Distracters close to this focus fall within the illuminated or magnified region. They will be processed more fully than will those that are far away. Thus close distracters produce more influences than do far distracters.

This idea of a spotlight is also implied in Treisman's feature-integration theory (Treisman & Gelade, 1980) which suggests that object identification depends on focal attention, directed serially to different locations to integrate the feature registered within the same spatial temporal spotlight into a unique percept. They assumed that the visual scene is initially coded along a number of separate dimensions, such as color, orientation, spatial frequency, brightness, direction of movement. Features of these dimensions are registered early, automatically, and in parallel across the visual field. Without prior knowledge, the separable features of objects are correctly combined only when focused attention is directed to each item in turn. If items are preattentively grouped, however, attention may be directed to groups rather than single items whenever no recombination of features within a group could generate an illusory target (Treisman & Gelade, 1980; Treisman, 1982).

The grouping hypothesis argues that attention might sometimes be assigned to non contiguous regions of the visual field. It suggests that attention is directed to perceptual groups determined by Gestalt principle of organization. As indicated by Driver and Baylis, in many cases the two hypotheses make the same prediction because proximity is a powerful grouping factor. The Eriksen distant effect could be explained either on the grouping account if one suppose that the near distracters are more strongly grouped with the target because of their proximity (Driver and Baylis,1989).

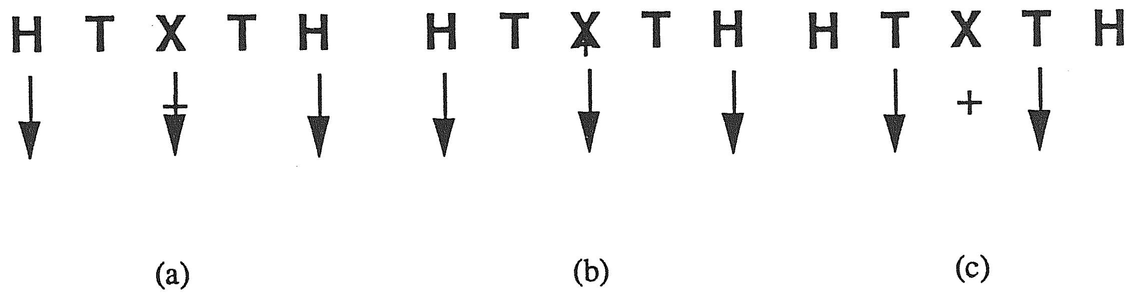



Fig. 1.8. Stimuli used in Drive and Baylis' experiments: (a), (b) the target and far distracters move together; (c) the near distracters move while the target and far distracters remain stationary.

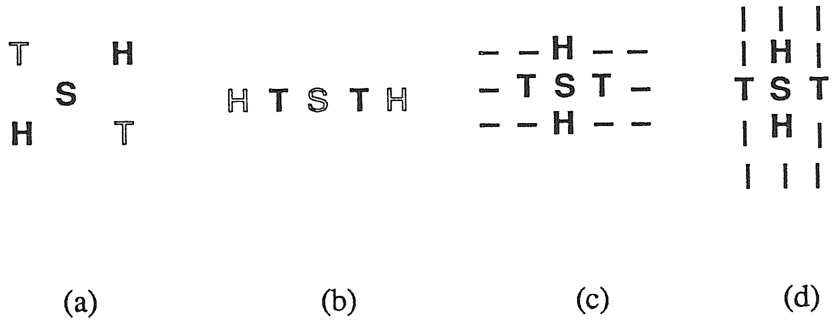



Fig. 1.9. Stimuli used in Drive and Baylis' experiments: (a), (b) the grouping of target and distracters by color against proximity; (c), (d) the grouping of target and distracters by good continuation.

To distinguish between the spotlight and the grouping hypothesis, Driver and Baylis designed two blocks of experiments. In the first block of experiments (Baylis & Drive, 1992), the grouping of target and distracters was set against their spatial proximity by means of common movement. Fig. 1.8 shows example of stimuli used in the experiments. Their results showed that distant distracters moved with a target could produce more interference than did distracters that were closer to the target but did not share its motion. In the second block of experiments (Baylis & Drive, 1992), the grouping of target and distracters was set against their spatial proximity by means of common color or good continuation respectively. Fig. 1.9 shows example of stimuli used in the experiments. Results demonstrated that distracters shared the target's color produced more interference than did those that did not (Experiment 1), and the effects of color could be sufficiently strong to override those of proximity under some circumstances (Experiment 2). Distracters grouped with the target by good continuation produced more influence than did those that were equally close but in a different good-continuation group (Experiment 5, Experiment 6). All these findings suggest that visual attention is directed to groups according to Gestalt principles of organization rather than to unparsed regions of visual field. Attention is not directed on the basis of position information alone, even when the position of the target is known in advance. Grouping by the common fate, color, or good continuation can override the grouping by the proximity in some circumstances.

## 1.5 Minimum principle theory

Hochberg and McAlister (1953) and Leeuwenberg (1969, 1971) suggest that perceptual interpretation process is guided by the minimum principle. The preferred interpretation of a pattern is reflected by the simplest description of that pattern. A pattern description can be seen as the formal counterpart of the way in which a pattern is represented by the perceptual system.

Many researches have been done to specify this minimum principle more exactly. Some scientists (Simon & Feigenbaum, 1964; Attneave, 1982; Biederman, 1987) regard simplicity as an internal aspect of the perceptual process. This idea of procedural simplicity implies that the preferred interpretation of a pattern corresponds to the pattern description generated from the most efficient procedure within some process model. Better in line with the Gestalt tradition, other scientists (e.g. Hochberg & McAlister, 1953) regard simplicity as being based on properties of a pattern itself. This starting point of phenomenal simplicity implies that the preferred interpretation of a pattern is thought to be reflected by the pattern description that expresses the largest amount of regularity in the pattern.

The transformational approach suggests, in different way, that the perceptual system reveals regularity in pattern by means of a fixed set of pattern transformations such as translation, rotation, and reflection. Regularity is specified as being constituted by certain arrangements of identical pattern parts. Garner (1970) takes the number of such invariant transformations, as allowed by a pattern, as a measure for the figure goodness of that pattern. Palmer (1983), on the other hand, describes pattern by means of a network in which pattern parts and their properties (revealed by invariant transformations) are stored and related to one another. Palmer suggests that the preferred interpretation is reflected by the "best" reference frame: each frame allows specific transformations, and the "best" frame is the one that reveals a maximum of symmetry in the pattern. Leyton (1986a,b) proposes a criterion for the internal structure of pattern descriptions which are formulated directly in terms of reference frames and invariant transformations. This criterion enables an explicit specification of perceptual differences between possible interpretations of a pattern. Leyton, by the way, defines a description as a mapping from a set of transformations onto a pattern, i.e. the pattern is not converted into some description but just has to be reconstructable from that set of transformation.

In the encoding approach, descriptions of a pattern are obtained on the basis of a symbolic representation of the pattern, in which identical symbols represent identical pattern parts. By means of coding rules, the symbolic representation is reduced into a code. The encoding approach starts with Hochberg & McAlister (1953), who proposed a simple method to measure the complexity of 2- and 3-dimensional patterns. Later, more intricate coding systems have been proposed. These systems are applied to the encoding of visual pattern (Vitz, 1966; Leeuwenberg, 1969, 1971; Restle, 1979).

**Leeuwenberg's Structural Information Theory** Leeuwenberg (1969) proposed a hypothesis that pattern descriptions are meant to express regularity, where regularity is specified as being constituted by certain arrangements of identical pattern parts and is thought to be revealed by the perceptual system. His coding system is based on coding rules which prescribe the combining of single identities, and codes express the allowed combinations. On the basis of such combinations, the human system is thought to extract information. Leeuwenberg embodied the minimum principle in the coding rules, and claimed that the preferred interpretation of a pattern is reflected by the simplest description of that pattern.

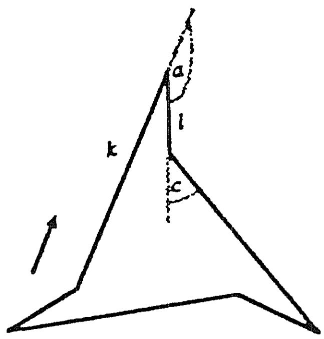



Fig. 1.10. Tracing the contour of the pattern, yields the symbol series '*kalckclckclc*'.

By means of Leeuwenberg's coding approach (Leeuwenberg, 1969, 1971; Van der Helm & Leeuwenberg 1989), pattern interpretations can be represented in a pattern code. The general procedure runs as follows. A pattern is first represented by a symbol series. Each symbol corresponds to an element of the pattern, For instance, in Fig. 1.10 the contour of the pattern consists of subsequent angles and line segments, each of which is labeled with a symbol, called pattern symbol, and angles or line segments of equal size are labeled with an identical symbol. This pattern is represented by the symbol series *kalckalckalc*. Next, the symbol series is encoded into pattern code.

The encoding consists of describing regularity in the symbol series which corresponds to regularity of the pattern. The symbol series is encoded as far as possible by applying onto it a number of coding rules. By means of these coding rules all redundant information is eliminated from the symbol series so that after this encoding process the pattern is represented by a code which only contains information about the essential pattern elements and their interrelationships.

**Coding rules** Leeuwenberg's structural information theory provides the concept of accessibility for the choice of appropriate coding rules. Regularity and hierarchy in a code of a pattern should correspond directly to regularity and hierarchy in the pattern. The criterion of accessibility is that a coding rule should be both holograph and transparent in order to be appropriate for the encoding of symbol series that represents a visual pattern. This enables a differentiation between coding rules. The coding rules that are in line with the concept of accessibility, not only account for an easy extraction of information from the pattern in order to construct a pattern code, but also account for an easy extraction, at higher cognitive levels, of a pattern information from the code. Based on the concept of accessibility there are three essential coding rules in Leeuwenberg's coding approach, each of which describes a specific class of regularity, namely iteration, symmetry, and alternation (ISA). The definitions of theses rules are as follow:

**Iteration rule:**

$$kk \ldots kk \rightarrow N * ( k )$$

it is applied to express the series contains successive identical symbols.

**symmetry rule:**

$$k_1 \, k_2 \ldots k_n \, pk_n \ldots k_2 \, k_1 \rightarrow S \, \langle \, ( \, k_1 \, ) \, ( \, k_2 \, ) \ldots ( \, k_n \, ) \, , ( \, p \, ) \, \rangle$$

it can be applied to express that a series contains pairs of identical symbols, nested around a so-called pivot.

**Alternation rules:**

$$kx_1\ kx_2\ \dots\ kx_n \rightarrow \langle\ k\ \rangle\ /\ \langle\ (\ x_1\ )\ (\ x_2\ )\ \dots\ (\ x_n\ )\ \rangle$$

$$x_1\ kx_2\ k\ \dots\ x_n\ k \rightarrow \langle\ (\ x_1\ )\ (\ x_2\ )\ \dots\ (\ x_n\ )\ \rangle\ /\ \langle\ k\ \rangle$$

it can be applied to express that a series contains successive subseries which either all begin or all end identically. According to these coding rules the pattern shown in Fig. 1.10 can be represented by the code 3*(*kalc*).

Three aspects should be noted in applying the coding rules. First, the symbols are considered to be variables standing for arbitrary subseries. This implies that the coding rules can be applied not just to express identity of single symbols but, in general, to express identity of subseries in a series. For instance, *ababab*-→3 * (*ab*). any subseries between parentheses in an ISA_form is called a chunk. Secondly, a code of a symbol series does not have to consist of just on ISA-form. In general, a code is a series consisting of single symbols and ISA-forms obtained by applying the coding rules to subseries of the symbol series. For instance: *akpkpfrstsrq* → *a* * 2(*kp*)*f*S<(*r*)(*s*),(*t*)>*q*. Thirdly, the subseries inside a chunk in an ISA-form can be encoded just like any symbol series. For instance: *bapabapa* → 2 * (*bapa*) → *a* * (*b*S<(*a*),(*p*)>). In such a case, the ISA-form is said to be hierarchically nested.

**Minimum Principle** Leeuwenberg argues that a code provides a description of regularity in that series. The ultimate meaning of the code is constituted by the fact that a code provides a mean to obtain a classification and an organization of the series which is considered to reflect an interpretation of the pattern that is represented by the symbol series.

Because patterns can be partitioned into different groups of elements, and because coding rules can be applied to these sets of elements in different ways (e.g. by choosing different starting points or by applying the coding rules in different orders), different codes, representing different pattern interpretations, can be arrived at. Leeuwenberg coding approach uses the quantity structural information to measure the preference of different pattern interpretations. Structural information load is defined as the number of pattern symbols in a code of a series plus the number of I-forms and S-forms in code. For instance, in Fig. 1.11, the structural information load for the proposed interpretation is 5. The minimum code is the code that contains a minimum amount of structural information load. The preferred interpretation of a pattern is reflected by a minimum code of a symbol series that represents the pattern. Referring to Fig. 1.10, it is clear that interpretation (a) has the minimum code and it is a fact that this interpretation is perceptually preferred.

Experimental validity of Leeuwenberg's structural information theory is controversial. There are evidences against this theory as well as in favor of it.

# Chapter 2

# Symmetry Detection

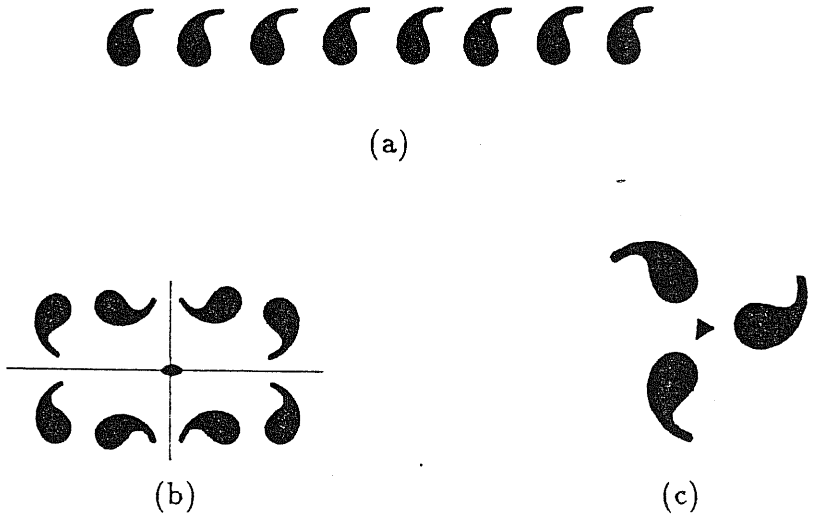



Fig. 2.1. Three basic forms of symmetry: (a) translation; (b) reflection; (c) rotation

In a general sense, symmetry is commonly used as a synonym for harmony of proportions, or balance. More particularly, the term refers to three basic transformations: (1) translation, in which the element moves up or down, left or right, or diagonally while keeping its orientation; (2) reflection, in which the element reflects as in a mirror; and (3) rotation, in which the element turns. Furthermore, these simple transformations can be combined to form complex regular patterns. The general form of symmetry, however, requires the invariance of a configuration of elements under a group of automorphic transformations (Weyl, 1952). In other words, symmetries are one of the simplest invariances of patterns that preserve their identities under certain specific transformations.

Symmetry is a salient property of objects. Many natural and man-made objects have at least one axis of symmetry. Our visual system can perceive it rapidly and efficiently. As discussed before, symmetry is one of the organization factors suggested by Gestalt psychologists. The presence of symmetry helps the detection of objects, making them to appear as figures segregated from the background. Many authors pointed out that symmetry is an important clue to the natural axes of the object. It helps to establish the object-centered coordinate frame (Marr & Nishihara, 1978; Barlow & Reeves, 1979). In addition, as Barlow and Reeves pointed out, symmetry reduces pattern complexity: any regularity such as symmetry is valuable for the very reason that it represents more of the image than an arbitrary or irregular feature. Symmetry reduces both the amount of information we must collect and the amount of information we must store. The perception of symmetry also affects other visual processes, such as encoding and representation (e.g., Garner & Sutliff, 1974; Enns, 1986; Freyd & Tversky, 1984), memory (Riley, 1963), recall (Royer, 1971, 1966; Clement & Varnadoe, 1967), and discrimination (Garner, 1962).

Mach (1886) was the first one to draw attention to the importance of symmetry for our understanding of visual processes and visual system. Since then, many researches have been done in various aspects of symmetry perception. More recently, works have been focused on how the information in symmetry displays is processed. Issues of the neurophysiology of symmetry processing (Corballis & Beale, 1976, Julesz, 1971) and of the processing models (Palmer & Hemenway, 1978; Jenkins, 1983; Pashler, 1990) have been raised. In this chapter we will present some observed effects on symmetry detection, variables mediating symmetry detection, and models or hypotheses suggested to account for the experimental findings. The discussion will be mainly on bilateral symmetry.

## 2.1 Effects on Symmetry Detection

**Efficiency of detection** Many reports demonstrate that symmetry can be detected efficiently in a wide range. We are capable to detect symmetry when the axis is vertical, horizontal, and in other orientations or when the axis is positioned left or right to the fixation point, though the performance is worse than that with vertical axis or with central axis (e.g., Barlow & Reeves, 1979). Barlow and Reeves (1979) also measured the discriminability between two populations of dots displays that contain mirror pairs and random dots in different proportion of symmetry. They found that the discriminability was a function of symmetry proportion. It was decreased gradually as the proportion of symmetry was decreased. They suggest that symmetry is represented in the brain as a graded rather than a discrete or all-or-nothing property. Overall they concluded that the mechanism of symmetry detection is efficient and versatile.

**Orientation Effects** It has been found that detection of symmetry depends on the orientation of the axis. This orientation effect was first observed by Mach (1886). He found that vertical symmetry is easier to detect than horizontal symmetry. Such preference for vertical symmetry is confirmed by many others and probably represents the most consistent finding in this area of research (Julesz, 1971; Goldmeier, 1972; Corbllis & Roldan, 1975; Palmer & Hemenway, 1978; Pashler, 1990). The order of horizontal and diagonal axes is debatable. Certain findings indicate that horizontal symmetry is more salient than diagonal symmetry, a superiority often explained as a kind of oblique effect (e.g., Palmer & Hemenway, 1978). Other findings indicate that diagonal symmetry is more salient than horizontal symmetry, in accordance with predictions derived from the hypothesis of mental rotation (e.g., Corballis & Roldan, 1975).

Mach (1886) thought that the special salience of vertical symmetry is due to the structural bilateral symmetry of the visual system and that perception of horizontal symmetry is accomplished by mentally rotating the figure or by an intellectual act. This means that shape might be mentally rotated to vertical before information about symmetry is extracted. This mental rotation hypothesis is supported by Corballis & Roldan's experiments. They tested symmetry detection in eight different orientations in 45 degree steps with dot patterns having a line indicating the location of the axis. They found that the decision time increased as the axis shifted from vertical. Further more, they found that tilting the head shifted the decision time function in the direction of head tilts, suggesting that retinal coordinates are more important than gravitational ones. They concluded that the retinal information is mapped on the phenomenal coordinate system. Information can be rotated mentally on the phenomenal coordinates to test the information against a template for detecting vertical symmetry embedded symmetrically in the brain. Attention to brain structure as a basis for the salience of vertical symmetry is also presented in Julesz's work (1971) which suggests that perception of symmetry requires a point-by-point comparison process based on neural anatomy that has a symmetrical organization around the center of fovea.

Rock and Leaman (1963) argued against a simple structural explanation for the salience of vertical symmetry, on the ground that the advantage of vertical symmetry is not a matter of retinal orientation; e.g., if the observer tilts his head through 45 degree, a figure with true vertical symmetry is still more salient that a figure with true horizontal symmetry, even though both figures are equally tilted on the retina. They suggested that the vertical orientation that is implicated in symmetry does not refer to a retinal-cortical direction but to a phenomenal direction. Symmetry about a vertical axis is widespread in the sphere of plant, animal life, and man-made objects. We become sensitized to vertical symmetry through learning or as a consequence of biological evolution. However, one could also interpret Rock and Leaman's data as further evidence for a process of mental rotation which normalizes the input before symmetry is perceived.

Recently, a two-stage model is favored by many researches. The model states that two processes are involved in symmetry detection: the selection of the axis by a global, rapid process and the evaluation of symmetry by a local, detailed point-by-point comparison process. There are biases in axis selection. This model will be discussed in detail later in this chapter.

**Redundancy in symmetry patterns** Redundancy reduction is involved in symmetry detection. Not all symmetry information available is utilized by the visual system. Barlow and Reeves (1979) have suggested that the organism reconstructs an image on the basis of minimal information. They pointed out that one advantage of symmetry is that it allows the image to be described economically. For instance, if one half of the an object is the mirror image of the other half, then one half need not to be described at all. The redundancy reduction in the perception of bilateral symmetry is achieved by ignoring the reflected half of the pattern. Jenkins (1982) provided data showing that information perceptually relevant to symmetry falls within a strip approximately 1.1 deg wide about the central axis, irrespective of the size of the pattern. Utilization of symmetry information is increasingly efficient from the outer boundary of 1.1 deg inwards, with a maximum at 0.3 deg. Outside the strip, symmetry information is found to be completely redundant. The importance of information near the axis is also reported by many others (Julesz, 1971; Barlow & Reeves, 1979; Bruce & Morgan, 1975). All these results are consistent with Julesz's proposal that redundancy reduction is achieved by giving greater weight to points close to the axis of symmetry.

**Multiple symmetries** The notion of mirror symmetry can be extended to cover patterns with more than one axis of symmetry. The salience of symmetry is increased when there are more than one axis. Experimental results obtained with polygons showed that quadruple symmetry is easier to detect than double symmetry, which in turn is easier than single symmetry (Palmer & Hemenway, 1978). Palmer & Hemenway thus suggested a variable bias in axis selection to explain the advantage of multiple symmetry. Another explanation of the salience of multiple symmetry is based on the invariance involved in the pattern. The more invariance the pattern has, the easier symmetry to be perceived.

## 2.2 Models of symmetry processing

Though the mechanisms of how symmetry is detected are still not clear, there are some models explaining how the information is processed in symmetry detection. Three of them are presented here.

**Palmer & Hemenway's two-stage model** The idea of two-stage processing is favored by many researches (Julesz, 1971; Bruce & Morgan, 1975). A rapid, preattentive process would be followed by a slow, scrutiny process. Palmer & Hemenway (1978) measured the reaction time to closed polygons with single, double, and quadruple axes of symmetry in different orientations. They found that response was faster to quadruple than to double symmetries, and faster to double than to single symmetries. Detection was fastest for vertical symmetry, next faster for horizontal symmetry, and slowest for diagonal symmetry. They interpreted their experimental results using the two-stage model. They suggested that in the first stage the observer selects a potential axis by a crude, but rapid analysis of symmetry performed in all orientations simultaneously. This process is biased toward the vertical axis and, to a less extent, the horizontal, rather than diagonal axes. The actual order of selection is assumed to be variable, thus it can explain the advantage of multiple symmetry. Second, the observer performs a detailed axis by explicitly comparing the two halves for mirror symmetry. Palmer & Hemenway did not indicated how the information is analyzed within the first stage and if there are interactions between the two stages.

**Jenkins' component processes model** Jenkins (1983) adopted the neurophysiological point of view that the mammalian visual system responds selectively to uniform orientation irrespective of the size of the elements that carry this stimulus property. He did a series of experiments on symmetry detection based on the detection of orientation uniformity and midpoint collinearity. Results showed that subjects were able to discriminate texture with uniformly oriented point-pairs from random dot texture, even midpoint collinearity was perturbed and subjects were more sensitive to horizontal orientation. On the other hand, in the task of detecting midpoint collinearity, subjects were more sensitive to symmetry with vertical axis. Furthermore, he found that the discrimination between the symmetrical and random textures was based on the presence or absence of a central coherent feature, once the midpoints were distributed randomly over a region larger than approximately 1.0 degree of visual angle, the visual system was unable to correlate them. This led him to suggest that a central fusion process is involved in symmetry detection. Jenkins' model suggested that there are three processes involved: first, a process that detects the orientation uniformity of the component point-pairs, irrespective of their size; second, a process that fuses the most salient point-pairs into a salient feature; and third, a process that determines whether this feature is symmetric. Further, Jenkins suggested that there are interactions among these processes but without specifying how the three processes are acting together. The preference for vertical symmetry comes out when all processes acting together.

**Pashler's grouping conjecture** Recent results showed that cueing subjects the orientation and the location of the axis does not eliminate the orientation effects (Pashler, 1990; Corballis & Roldan, 1975). In Corballis & Roldan's experiments, cueing has no improvements. In Pashler's experiments, cueing subjects the orientation of the axis provides a substantial and highly significant advantage in speed and accuracy. The difference in these two results might come from the difference of the stimuli they used, provided that Corballis & Roldan's stimuli contained a line representing the axis which was not present in Pashler's stimuli. In any case, in both experiments the orientation effects seem to depend on intrinsic factors involved in symmetry evaluation rather than on priming: they are not due to a bias in the selection of the axis, as suggested in the two-stage model. Pashler suggested that grouping principles already required for other visual functions could accomplish most of the work involved in detecting symmetry. He conjectured that symmetry is computed by grouping pairs of handedness-invariant shape descriptors from fine to coarse scale across the axis of symmetry. The shape descriptors are not orientation invariant. The computation of the handedness-invariant descriptors at various spatial scales is followed by a stage at which neighboring regions are grouped together when these descriptors match. The subjective perception of symmetry may simply be the conscious concomitant of this pattern of increasingly dense pairwise linkage across a common axis.

## 2.3 Skewed Symmetry

When a bilateral symmetrical pattern is on the plane other than frontal, bilateral symmetry is transformed to skewed symmetry, in which the orientation of the virtual lines (the lines connecting point pairs) is no longer orthogonal to the orientation of the axis (the line connecting the midpoints of virtual lines). Because of its connection with the recovery of three-dimensional spatial layout, skewed symmetry has attracted the attention of computer vision scientists (Kanade & Kender, 1983; Stevens, 1979).

Studies of orientation effects on symmetry detection focused on the influences of axis orientation. In fact two orientations contribute to the orientation effects on symmetry detection. One is the orientation of virtual lines, the other is the orientation of the axis. In the case of bilateral symmetry on the frontal plane, these two orientations are orthogonal, and therefore one implies the other. When shown on a plane different from the frontal one, a pattern with normal symmetry is transformed into a pattern with skewed symmetry, in which the axis is not orthogonal to the virtual lines. As indicated by Wagemans et al (1992), the independence of axis and virtual line orientations in skewed symmetry allows us to disentangle their effects on symmetry detection, and to study the component process involved in this task.

Wagemans et al (1992) have done a series of experiments measuring skewed symmetry detection in 12 different orientations, in 15 deg step (both clockwise and counterclockwise). They found that the detection of symmetry was largely disrupted in skewed symmetry. They suggested that skewing disrupted the preattentive stage of global axis selection. Referring to the orientation effects, they found that classically observed orientation effects were restricted to bilateral symmetry and that the orientation effects caused by the orientation of the virtual line was more pronounced and stable than the one caused by axis orientation. Symmetry was harder to detect when the orientation of the virtual lines deviated more from horizontal. The effect of the orientation of the axis was completely different at different levels of the skewing angle.

Wagemans et al suggested that an additional process is involved in skewed symmetry detection: the detection of the orientation in which the point-by-point comparisons have to be made, when following the axis-based route, or, when following the virtual lines-based route, the detection of the orientation of the axis that runs somewhere through them nonorthogonally. They suggested an invariance-based grouping mechanism postulating that a fixed association exists between local features and the relation specifying the orientation and distance of one local feature to another. They indicated that in bilateral symmetry the normal grouping processes that take place automatically leads to the detection of symmetry, because the possible relations confirm one another based on the Euclidean invariance in line length and orientations. In skewed symmetry, however, the Euclidean invariant relations between the element pairs disappeared as a consequence of the skewing. The remaining invariances such as parallelism and collinearity do not seem to be sufficient for the normal position-based grouping. This failure of the preattentive grouping that normally occurs as a first stage in symmetry detection forces the visual system to look explicitly for correspondences which are much harder to detect.

# Chapter 3

# Experiment 1: The Effect of Axis Explicitness on Symmetry Detection

The properties of the axis of symmetry, such as its orientation and location, have been found to affect symmetry detection. Furthermore, as mentioned before, the information distributed near the axis is dominant (Jenkins, 1982). It has also been noticed that the appearance of salient features around the axis facilitates symmetry detection. (Jenkins, 1983; Barlow & Reeves, 1979). Knowing only the orientation of the axis but not its location does not facilitate performance (Pashler, 1990). These results suggest that the identities of the axis and the information around the axis affect the anticipating of the axis position and thus affect the set up of local reference frames for further evaluation.

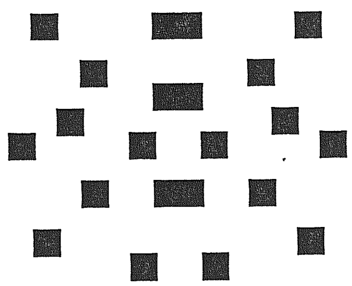


Fig. 3.1. A symmetrical pattern obtained by reflecting a homogeneous half-pattern.

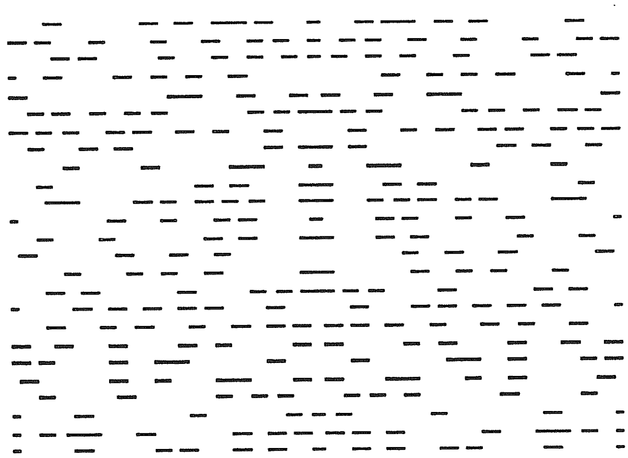
In the present experiments, we studied the effects of axis explicitness on bilateral symmetry detection. Random textures were used as stimuli. The orientation of the axis was always vertical and located on the center of the texture. We compared the sensitivities to symmetry in two conditions with different axis explicitness. The explicitness of the axis was manipulated by changing the statistic distribution of texture elements. In Condition 1, the elements were distributed homogeneously over the whole symmetrical pattern; thus the axis was not explicit. In Condition 2, the texture was obtained by reflecting a homogeneous half-pattern about the axis; thus the resulting texture was heterogeneous along the axis of symmetry, as shown in Fig. 3.1. In this condition, a new texture element, a rectangle, is formed along the axis. We call such elements emergent features. The emergent features could be size, shape, closure, etc. According to texture discrimination theory (Beck, 1982), the occurrence of such emergent features supports figural segregation, and therefore makes the axis explicit. In the present experiments, two kinds of emergent features had been studied: size (Experiment 1a) and shape (Experiment 1b). Random line textures (Experiment 1a) and random triangle textures (Experiment 1b) were used. Subjects were asked to discriminate symmetrical textures from noise textures. Results showed that subjects were more sensitive to symmetry with explicit axis.

## 3.1 Experiment 1a

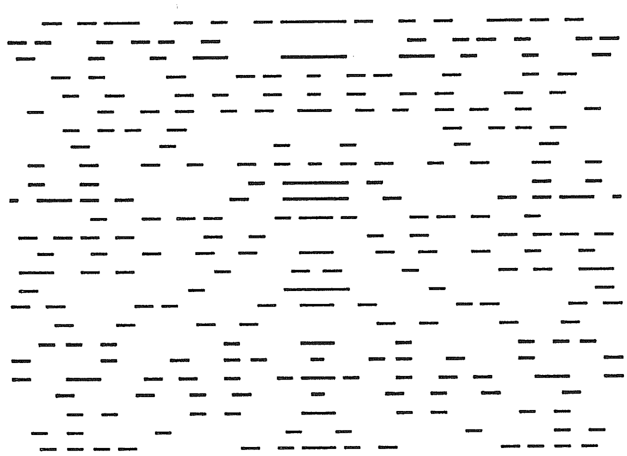
### Method

**Subjects** Sixteen subjects participated in this experiment. Most of them were college students. All of them were naive with respect to the aim of the experiment.

**Stimuli and Apparatus** The stimuli used in this experiment were random line textures. The textures in two conditions were generated by a computer plot program. They were 8.1 cm x 5.6 cm and were shown on the computer screen which is 24 cm x 16 cm. Subjects sat viewing the screen binocularly at a distance of about 80 cm. The visual angle was about 5 deg. Stimulus luminance was kept constant through all the experiments.



(a)



(b)

Fig. 3.2 Examples of symmetrical stimuli in Condition 1 (a) and Condition 2 (b) of Experiment 1a.

Each texture was composed of 25 rows of line segments. The basic texture elements were segments, either 2.5 mm or 5 mm long. Segments with a length of 10 mm emerged in Condition 2. Fig. 2 shows examples of symmetrical stimuli. The noise textures were obtained by shifting each row of symmetrical texture rightwards or leftwards randomly. The noise textures were not completely uncorrelated. All 5 mm segments in Condition 1 and all corresponding 10 mm segments in Condition 2 remained unshifted, in order to avoid the possibility that subjects may detect the position of these lines instead of detecting symmetry.

**Procedure** The experiment was composed of a training session and a test session. The training session contained a minimum number of 20 trials. Further training trials were presented until the subjects scored the critical level of 70% correct. The test session contained 80 trials: 20 symmetrical and 20 noise trials for each of the two conditions. The order of trials in the sequence was random.

To initiate a trial, the subject pressed the "START" key whenever ready. The stimulus texture was shown on the screen immediately and remained on for 500 ms. A fixation point, located on the axis, was on during the intertrial interval. Subjects were instructed to indicate whether the texture was symmetrical or not by pressing the key "YES" or "NO". Key "YES" and key "NO" were chosen by the subjects freely from the keyboard at the beginning of the experiment. Subjects were told to press the answer key as soon as they made the decision. The responses and the reaction times were registered by the computer.

## Results

Responses slower than 2 sec were discarded from the data analysis. Data were analyzed according to Signal Detection Theory. The sensitivity measure was zP'(A). Each subject contributed a pair of p(HIT) and p(F.A.). Only reaction times of correct responses were analyzed.

Two one-way ANOVAs were performed on zP'(A) and RT data respectively. The results showed that subjects were more sensitive to symmetry when axis was explicit. The mean zP'(A) was 1.217 in Condition 1 and 1.416 in Condition 2. The advantage was highly significant, $F(15,1)=4.985$, $P<0.04$. There was no difference in RTs, 1100 ms in Condition 1, 1095 ms in Condition 2, $F(15,1)=0.99$, $P>0.7$.

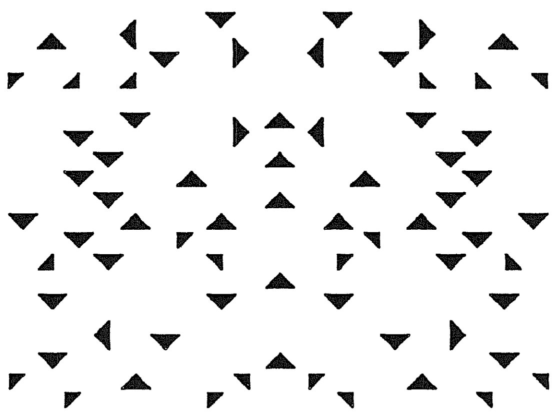
## 3.2 Experiment 1b

The logic and the procedure of Experiment 1b were identical to those of Experiment 1a.

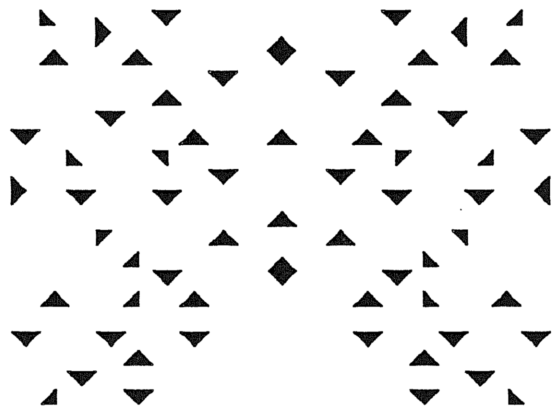
**Subjects** Sixteen subjects participated in this experiment. Two of them were involved in Experiment 1. All of them were naive with respect to the purpose of the experiment.

**Stimuli and Apparatus** The experiment was performed under the same condition as Experiment 1. Instead of using random line textures, random triangle textures were used as stimuli. The emergent feature studied in Experiment 1b was shape. The basic texture elements were right-angle isosceles triangles. In each half-pattern a configuration of two connected triangles was allowed. A new element shape, the diamond, appeared along the axis in Condition 2. As in Experiment 1a, the elements on the axis of symmetrical textures remained in noise textures to avoid the possibility that subjects may detect the occurrence of diamonds instead of symmetry. Furthermore, the noise textures had a symmetrical boundary. Fig. 3 shows examples of symmetrical stimuli of Experiment 1b.

**Procedure** The procedure of Experiment 1b was identical to the one of Experiment 1a, apart from the number of trials in test session, that was increased to 120 (30 symmetrical and 30 noise for each condition).



(a)



(b)

Fig. 3.3 Examples of symmetrical stimuli in Condition 1 (a) and Condition 2 (b) of Experiment 1b.

## Results

Responses slower than 2 sec were discarded from data analysis. Two one-way ANOVAs were performed on zP'(A) and RT data respectively. Subjects were more sensitive to symmetry when the axis was explicit. The mean zP'(A) was 1.298 in Condition 1 and 1.541 in Condition 2. The advantage is highly significant,

$F(15,1)=17.182$, $P<0.001$. There was no difference in RTs, 1025 ms in Condition 1, 1039 ms in Condition 2, $F(15,1)=0.46$, $P>0.5$.

## 3.3 Discussion

The results of Experiment 1a and Experiment 1b support the conclusion that symmetry is more detectable when the axis is more explicit. Emergent features like new size or shape enhance the visibility of the axis. Explicitness facilitates axis localization and thus affects symmetry detection. As Barlow and Reeves (1979) claimed: "it is certainly more difficult to assess symmetry when the position of its axis is unknown". Furthermore, Pashler (1990) found that the detection of symmetry is more efficient when subjects know the location of the axis in advance.

In my experiments, subjects knew that the axis was always vertical and central. The effect of axis explicitness was obtained in conditions in which preparation, prior knowledge, and anticipation were controlled. However, the information provided by the display is more important and may override the information got from priming. This suggestion can explain why in Corballis and Roldan's (1975) experiments cueing subjects the orientation of the axis didn't improve the performance. Because the display they used contained a line indicating the axis, cueing information thus became redundant. This also supports Pashler's (1990) suggestion that "subjects are unable to prepare mentally for a given orientation in a way that provides any benefit above and beyond whatever may be provided by the simultaneous presentation of the axis along with the pattern".

Previous researches also converge toward the conclusion that symmetry detection is a redundancy reduction process. Not all information is equally used in detecting symmetry. Information contained in the area around the axis is efficiently used (Julesz, 1971; Jenkins, 1979, 1983; Barlow & Reeves, 1979). The suggestion that symmetry detection is fundamentally a short-range process also predicts better performance when the axis is explicit. Any association or perturbation around the explicit axis is easier to detect.

# Chapter 4

# Experiment 2: Vertical Symmetry Preference and Processing of Environmental Information

Symmetry relative to a vertical axis is very salient and can be detected most accurately. Although this fact has been always connected to the salience of the vertical direction, both the verticality of the axis and the horizontality of the virtual lines might contribute to this advantage. In bilateral symmetry, these two factors are correlated, since the orientation of the axis and the orientation of the virtual lines are always orthogonal. As suggested by Wagemans et al (1992), the study of skewed symmetry enabled us to disentangle these two contributions. One of the purposes of the present experiments was to study the detection of symmetry in dot patterns perceived in a 3-D frame, in order to explore the sources of the advantage of vertical bilateral symmetry. Results indicated that the horizontality of the virtual line is more important than the verticality of the axis. There is an advantage of matching along the horizontal dimension.

Visual cues and prior knowledge of the axis were used in previous studies to explore the candidate component processes. One of the interesting findings is that cueing subjects the orientation of the axis does not eliminate the advantage for vertical (Corballis & Roldan, 1975; Pashler, 1990). This means that the advantage for vertical is due to intrinsic factors involved in computing symmetry itself rather than biases in preparing for a particular axis or computer symmetry in a specific sequence. The second purpose of the present experiments was to study how the visual environmental information influences symmetry detection. A 3-D outline frame providing schematic environmental information was presented either preceding or simultaneous with the dot pattern. Cueing facilitates symmetry detection but does not eliminate the orientation bias intrinsic to the matching process. A simultaneous frame was more useful than a preceding frame.

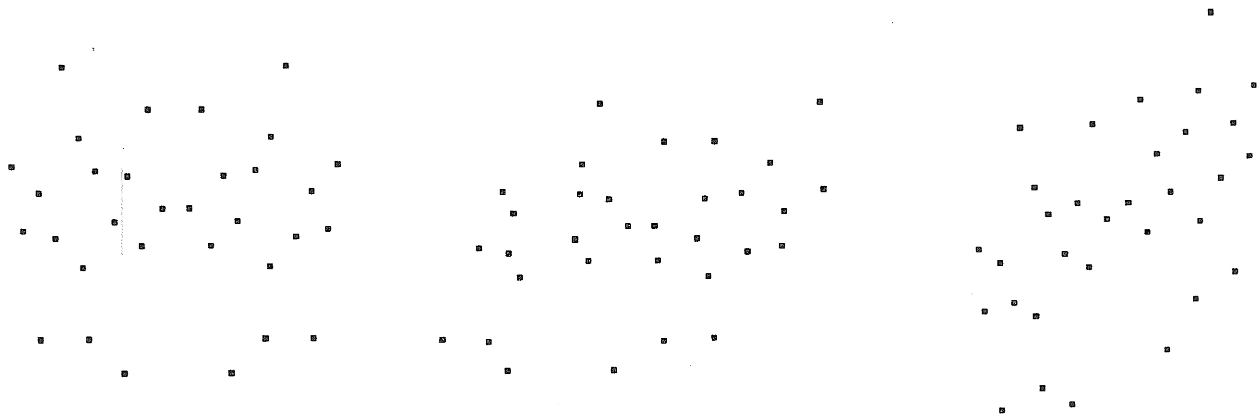
Random dot patterns were shown within frames simulating three different planes: frontal, slanted horizontal, and tilted vertical. The transformed symmetries were vertical bilateral symmetry, skewed symmetry with horizontal virtual lines and oblique axis, and skewed symmetry with oblique virtual lines and vertical axis. The angles between the axis and virtual line were the same in two skewed symmetries. Thus two skewed symmetries were in the same group of affine transformation. The environmental information was induced by presenting a corresponding 3-D frame. The frame provided the information of the orientation of the plane on which the dot patterns were shown. Two sides of the frame were parallel to symmetry axis and virtual lines respectively.

## 4.1 Experiment 2a

In this experiment, dot patterns were shown on one of the three planes: (1) frontal; (2) slanted horizontal; (3) tilted vertical. Subjects detected symmetry with (Group 2) or without (Group 1) the presence of the frames. The experiment was designed as a signal detection task. The signal was bilateral symmetry on the frontal plane or skewed symmetry on slanted or tilted plane. The dependent variables were the sensitivity zP'(A) and reaction time RT.

### Method

**Apparatus** The experiment was performed on MAC II computer. The stimuli were presented as black dots on the screen with grey background. The computer screen was covered by dark grey cardboard with a opening circle 16 cm in diameter. Subjects sat in a dim room viewing the stimuli binocularly at a distance of about 80 cm.



(a) (b) (c)

Fig. 4.1 (a) stimulus shown on frontal plane; (b) stimulus shown on a slanted horizontal plane; (c) stimulus shown on a tilted vertical plane.

**Stimuli Generation** Random dot patterns were generated by a computer program. Each pattern contained 16 pairs of dots which had uniform orientation. All dots were randomly distributed within a circle 4.8 cm in diameter. Half of the patterns were symmetrical and half of the patterns were random. The method used to generate the patterns was the following: The dot pattern was described as an array of dots. The location of each dot i was defined by two coordinates x(i) and y(i), generated by the computer's random number generator. The first 16 dots were generated independently under the constraint that the minimum distance between any pair of dots were greater than 2. Coordinates x(i) and y(i) ranged from 0 to 12, corresponding to 0 to 2.4 cm in real size, i=1, 2, ..., 16. x(i)=0 was avoided. For symmetrical patterns, the next 16 dots were obtained by reflecting the first 16 dots about the vertical axis, where x(i+16)=-x(i), y(i+16)=y(i). For the random patterns x(i+16) was a new number, ranging from -12 to 0, generated independently and y(i+16)=y(i). Three kinds of projected patterns used in the experiment were obtained under three transformations respectively. The three transformation matrices were:

$$T(1)=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\quad T(2)=\begin{pmatrix}1 & \cos\alpha\\ 0 & \sin\alpha\end{pmatrix},\quad T(3)=\begin{pmatrix}\sin\alpha & 0\\ \cos\alpha & 1\end{pmatrix}$$

where $\alpha$ is the angle between the axis and the virtual lines. In our experiments $\alpha=\pi/6$. Figure 4.1 shows examples of patterns used in each condition.

**Subjects** Two groups of twelve undergraduates at the University of Trieste participated in this experiment. All of them were naive with respect to the aim of the experiment. Corresponding frames were not shown to Group 1 subjects and shown to Group 2 subjects. The dot patterns were the same for both groups.

**Procedure** Before starting the experiment, each subject read written instructions. The instructions described the structure of symmetrical patterns in all three conditions. The relations of the frames to the projection planes were explained to Group 2 subjects. Examples of symmetrical and random patterns were presented. Subjects were asked to study these patterns with attention so as to be sure that they understood the structure of symmetry in each condition before proceeding with the experiment.

In order to eliminate learning effects, the experiment was composed of two sessions with the same blocks of trials in reversed order. In each session, there were 3 blocks of 60 trials each, 30 symmetrical and 30 random. The order of the trials in each block was random. The order of the blocks was different for each subject. A minimum number of 8 practice trials was given before each block. Further practice trials were presented until the subjects scored the critical level of 60% correct. The trials used in practice were shown on the same plane as those used in the block following.

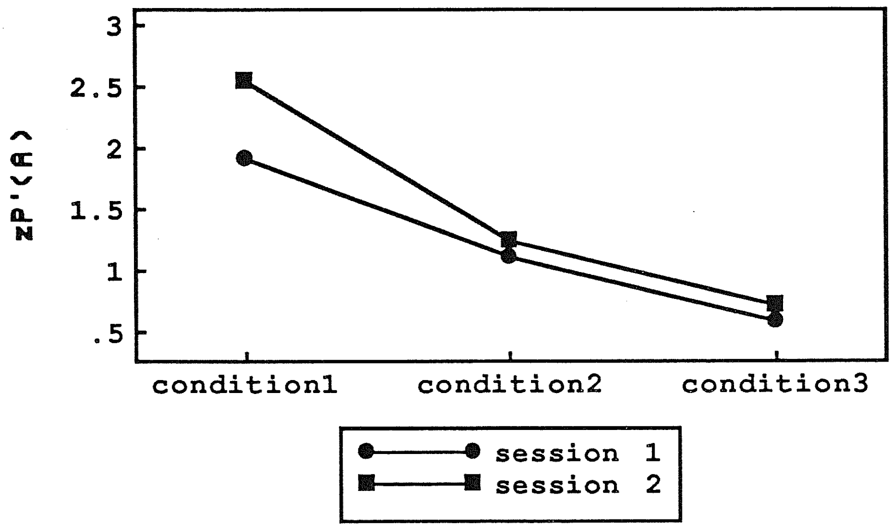
The procedure was the same for both groups of subjects before a trial began: a fixation point was displayed on the screen at the center of the axis for 1000ms. However, the procedure for the two groups was (differ) upon the cessation of the fixation point.

Group 1: the dot pattern was presented on the screen for 400 ms. Group 2: first a frame was presented and then after 400 ms, the dot-pattern was shown inside the frame for an additional 400 ms. Subjects gave their response by pressing a "YES" or "NO" key. A "YES" or "NO" key was freely chosen by the subjects at the beginning of the experiment. Instruction to subjects stressed both accuracy and reaction time. Responses and reaction times were registered by the computer.

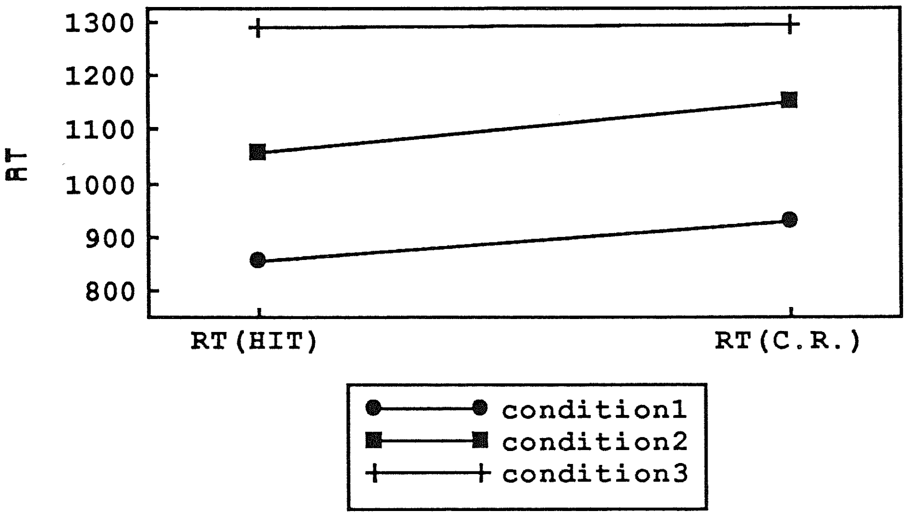
## Results and discussion

Data were analyzed according to Signal Detection Theory. The sensitivity measure was zP'(A). Each subject contributed with a pair of pHIT and pFA. Responses slower than 5000ms were discarded from the data analysis.

**Group 1 (dot-pattern without frames)** The means of sensitivity zP'(A), reaction of correct responses to symmetrical stimuli RT(HIT) and reaction of correct responses to noise stimuli RT(C.R.) in each condition are shown in Figure 4.2. Two-way ANOVA (learning x orientation) was performed to zP'(A). Generally, learning improved the performance 0.30 in average which was significant, $F(1,5)=17.148$, $P<0.009$. The sensitivities were 2.198 in condition 1 (frontal plane), 1.145 in condition 2 (slanted horizontal plane), and 0.631 in condition 3 (tilted vertical plane). The orientation effect was highly significant, $F(2,10)=47.547$, $P<0.0001$. The interaction between learning and orientation was marginal significant, $F(2,10)=4.002$, $P<0.053$. This arises from the fact that learning improved the performance in bilateral symmetry detection but did not influence skewed symmetry detection. The RTs of correct responses to symmetrical and noise stimuli were analyzed by a three-way ANOVA (learning x orientation x response). Learning had no effect on speed. The average RTs were 887ms in condition 1, 1098ms in condition 2, and 1287ms in condition 3. The orientation effect was significant, $F(2,10)=15.633$, P<0.0008. The average RTs of symmetrical stimuli were shorter than those of noise stimuli, 58ms in difference, F(1,5)=4.431, P<0.09. No interaction was found.




(a)




(b)

Fig.4.2 Means of sensitivities (a) and reaction times (b) of group 1 subjects in Experiment 1. The patterns were shown on frontal plane (condition 1), slanted horizontal plane (condition 2), and tilted vertical plane (condition 3).

There are three main results: (1) learning is useful only for the sensitivity of bilateral symmetry; (2) an advantage is conferred on bilateral symmetry both in sensitivity and in speed; (3) upon comparing the detection of two skewed symmetries, performance is better with regard to high sensitivity and speed when the patterns were shown on slanted horizontal plane (with horizontal virtual lines) than on tilted vertical plane.

**Group 2 (dot-pattern within frames)** The means of Group 2 subjects are shown in Figure 4.3. A two-way ANOVA (learning x orientation) was performed to zP'(A). The learning effect was significant, 0.420 in average difference F(1,5)=8.566, P<0.03; The sensitivities were 1.848 in condition 1 (frontal plane), 1.318 in condition 2 (slanted horizontal plane), and 0.999 in condition 3 (tilted vertical plane). The orientation effect was highly significant, F(2,10)=24.319, P<0001; There is no interaction between learning and orientation, F(2,10)=0.036, P>0.95.

The RTs of correct responses of symmetrical and noise stimuli were analyzed by a three-way ANOVA (learning x orientation x response). Learning had no effect on speed. The orientation effect was significant, 855 ms in condition 1, 936 ms in condition 2, 1086 ms in condition 3, F(2,10)=5.605, P<0.024. The average RTs of symmetrical stimuli were shorter than those of noise stimuli, 884 ms for RT(HIT), 1034 ms for RT(C.R.), F(1,5)= 6.992, P< 0.046. No interaction was found.

The main results obtained from Group 1 subjects was confirmed from Group 2 subjects. In addition, learning improved sensitivities to skewed symmetry as well.

To explore whether the cues on the plane influence performance, we compared the performance of Group 1 subjects with that of Group 2 subjects. Since the bilateral symmetry and skewed symmetry have large differences, the data of bilateral symmetry and skewed symmetry were analyzed separately. A two-way ANOVA with one between group factor (cueing) and one within group factor (learning) was performed to zP'(A) of bilateral symmetry. Cueing had no effect on bilateral symmetry, F(1,10)=2.383, P>0.15. A three-

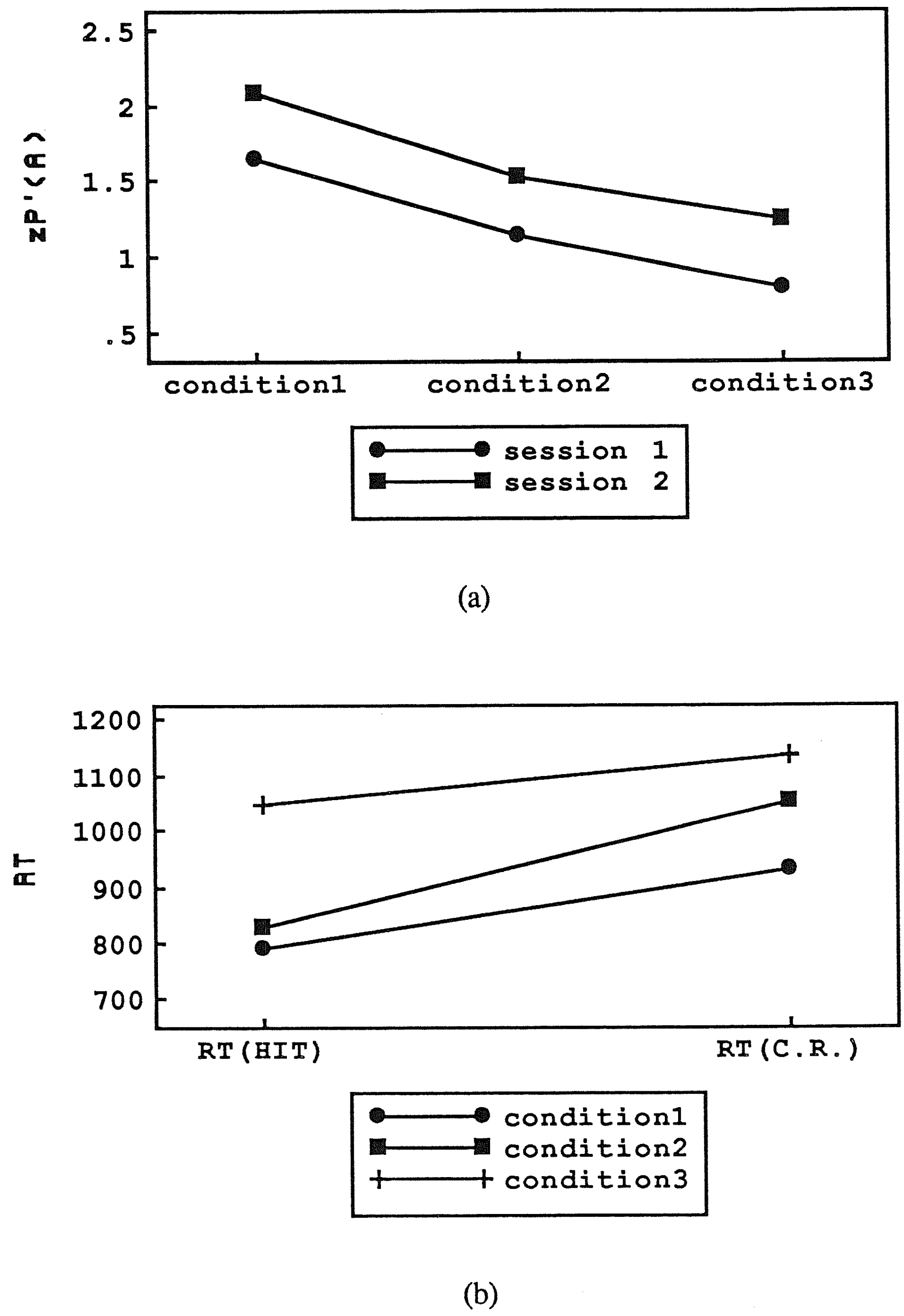



Fig. 4.3. Means of sensitivities (a) and reaction times (b) of Group 2 subjects in Experiment 1. The patterns were shown on frontal plane (condition 1), slanted horizontal plane (condition 2), and tilted vertical plane (condition 3).

way ANOVA with one between group factor (cueing) and two within group factors (learning x orientation) was performed to zP'(A) of skewed symmetries. Figure 4 shows the data. Cueing raised the sensitivity when subjects were trained, the sensitivity obtained from Group 2 was 0.416 higher than that obtained from group 1. This effect was reliable, $F(1,10)=19.067$, $P<0.001$. The orientation effects obtained from two groups were consistent. Subjects were more sensitive to skewed symmetry on slanted horizontal plane than on tilted vertical plane, 0.417 in average difference, $F(1,10)=22.531$, $P<0.0008$. A four way ANOVA with one between group factor (cueing) and three within group factors (learning, orientation, response) was performed to the RTs. Cues had no effects on RTs.

Results show that vertical symmetry is best detected when it is shown on frontal plane. Cues had no effects on vertical bilateral symmetry suggesting that in our experiment subjects took horizontal-vertical coordinates as default. Compare the sensitivities to two skewed symmetry, performances were better when the virtual line was horizontal than when the axis was vertical. This means the orientation of the virtual line is more important than the orientation of the axis as referring to the contributions to the advantage for vertical bilateral symmetry. Cues didn't eliminate such differences means that is due to intrinsic factors involved in symmetry evaluation rather than priming.

## 4.2 Experiment 2b

This experiment explores the temporal constraints of of the frame effect in skewed symmetry detection. The dot-patterns were shown on slanted horizontal plane or tilted vertical plane. The frame was presented before the dot-pattern in half of the trials, and simultaneously with the dot-pattern in another half. The independent variables were the orientation of the plane and the stimulus onset asynchrony (SOA). The dependent variables were sensitivity zP'(A) and reaction time RT.

## Method

The logic of Experiment 2 was similar to Experiment 1. The dot-patterns and two frames were generated in the same ways as those used in Experiment 1.

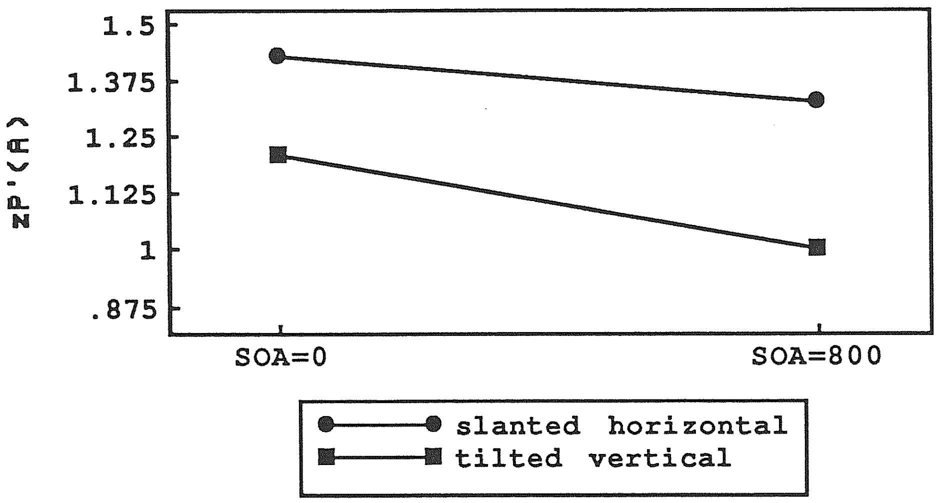
**Subjects** Ten undergraduate and graduate students participated in this experiment. All of them were naive with respect to the aim of the experiment.

**Procedure** A writing instruction similar to that in Experiment 1 was read by the subject before their experiments. The experiment comprised two test sessions. The projection plane was constant within each session but varied between two sessions. The order of the two sessions was balanced between subjects. In each session, the trials were divided into 4 blocks of 40 trials each, 20 random and 20 symmetrical. In half of the trials, SOA equals 800 ms: a frame was presented 800 ms before the dot-pattern. It lasted on for 400 ms then disappeared. 400 ms after its offset, the dot-pattern was on and it remained on the screen for 400ms. In another half of the trials, SOA equals 0: a frame was presented simultaneously with the dot-pattern. They remained on the screen for 400 ms. A fixation point on the center of the axis remained on the screen during the interval of intertrials. The time interval of the completion of one response and the presentation of the next stimulus was 2000 ms. Subjects gave their response by pressing a "YES" or a "NO" key. A "YES" or a "NO" key was chosen by the subjects freely at the beginning of the experiment. Subjects received feedback by a warning tone when a wrong response was made. Both the accuracy and reaction time were stressed.

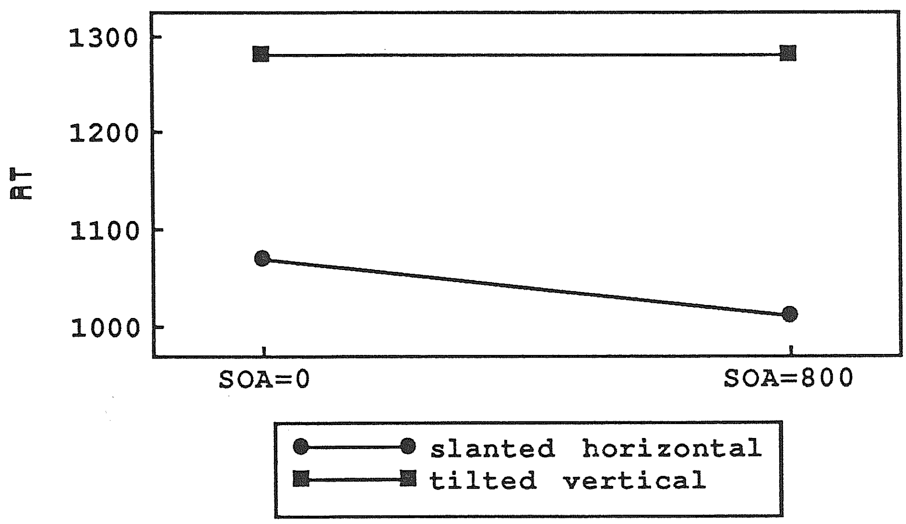
A minimum number of 8 practice trials for warming up was given before each block. Further practice trials were presented until the subjects scored the critical level of 60% correct. The trials used in practice were similar to those used in the block followed.

## Results and discussion

Responses slower than 5000 ms were discarded from the data analysis.




(a)




(b)

Fig.4.4 Means of sensitivities (a) and reaction times (b) in Experiment 2.

The means of sensitivity zP'(A) and reaction time of correct responses to symmetrical stimuli RT(HIT) and reaction time of correct responses to noise stimuli RT(C.R.) in each condition with two values of SOA are shown in Figure 5. Two-way ANOVA (orientation x SOA) was performed to zP'(A). Results show that subjects were more sensitive to symmetry in condition 1 (slanted horizontal image plane) than to symmetry in condition 2 (tilted vertical image plane). The difference in zP'(A) was 0.273, $F(1,9)=5.335$, $P<0.046$. Detection of symmetry was more efficient when SOA=0 than SOA=800 ms, 0.153 in average, $F(1,9)=10.755$, $P<0.01$. No interaction between orientation and SOA. A three way ANOVA (orientation x SOA x response) was performed to RT. Response was slower in condition 2 than in condition 1, 242 ms in average. The difference was reliable, $F(1,9)=15.628$, $P<0.0035$. The response was slower to noise stimuli, 100ms in average, $F(1,9)=14.133$, $P<0.005$. The SOA had no effect on speed. No interaction existed.

Experiment 2 confirmed the results of Experiment that the orientation effects are due to the processes of symmetry evaluation rather than preparing. The visual environmental information provided together with the stimulus benefits the performance most. This suggests that simultaneous visual information exceeds prior visual information.

## 4.3 General Discussion

These two experiments generated three main results: (1) Detection of vertical symmetry was best when it was shown on the frontal plane (bilateral symmetry); it was better when shown on the slanted horizontal plane (skewed symmetry with horizontal virtual lines) than on the tilted vertical plane (skewed symmetry with vertical axis). (2) Cueing subjects the orientation of the projection plane by presenting corresponding visual frame facilitated skewed symmetry detection but did not improve bilateral symmetry detection. (3) The appropriate condition of cueing was to present the cues simultaneously with the pattern.

The intrinsic differences between bilateral symmetry and skewed symmetry involves the differences in symmetry processing. One possibility is that view-centered coordinates are assigned to the pattern. Symmetry is evaluated depending on the assigned coordinates. Matching is easier when the axis and virtual lines are orthogonal. Cueing has no improvement to bilateral symmetry in our experiment, suggesting that a horizontal-vertical coordinates were taken as default in our experiments. An affine transformation is needed to set the coordinates in skewed symmetry detection. Cueing subjects the orientations decreased the differences between bilateral symmetry and skewed symmetry but did not affect the differences between two skewed symmetries. This indicates that the orientation effects are mainly due to intrinsic factors involved in symmetry evaluation rather than preparing for a specific coordinates specially when they are in the same affine transformation group.

The horizontality of the virtual line or the verticality of the axis remained unchanged when a vertical symmetrical pattern is shown on a slanted horizontal plane or on a tilted vertical plane respectively. Our results showed that subjects were more sensitive to symmetry when the pattern was perceived on a slanted plane than on a tilted plane. This suggests that the horizontality of the virtual lines is more important than the verticality of the axis, referring to the contributions to bilateral vertical symmetry preference. Matching along the horizontal dimension is easier than matching along oblique dimension though the axis is vertical.

The cueing effects observed in our experiments are consistent with Pashler's data, except that his experiment showed that cues improved vertical bilateral symmetry detection. Perhaps this exception is due to the different designs of the experiments. In Pashler's experiments, the orientation of the axis varied within each block whereas in our experiments it is constant within each block. Our subjects know the orientation of the plane, more concretely the orientation of the axis and the orientation of the virtual lines before the presence of stimuli. Thus cues facilitate processes other than preparation. In Pashler's Experiment 4 subjects were cued only the orientation but not the location. There was no effects on performance. Probably subjects can prepare a global reference frame from previous experience or other available cues. But this preparation is not sufficient. Detection of symmetry needs an real anticipation of a local reference coordinates relative to the position of the axis and the orientation of the virtual lines. Perhaps the setting of reference coordinates could occur only after the visual information (visual cues or pattern) has been perceived. Results showed that both advance cues and simultaneous cues raise the sensitivity to skewed symmetry. It supports Pashler's suggestion that "the effects of the cues might be automatic rather than voluntary; perhaps the perception of the cues themselves is sufficient to pull the frame of reference" (Pashler, 1990). Furthermore, from the fact that there is advantage for simultaneous cues and from the results of Corballis and Roldan that cues were useless when the patterns contain a line indicating the axis, suggest that the priorities are given to cues contained in the pattern itself and to those presented together with the pattern.

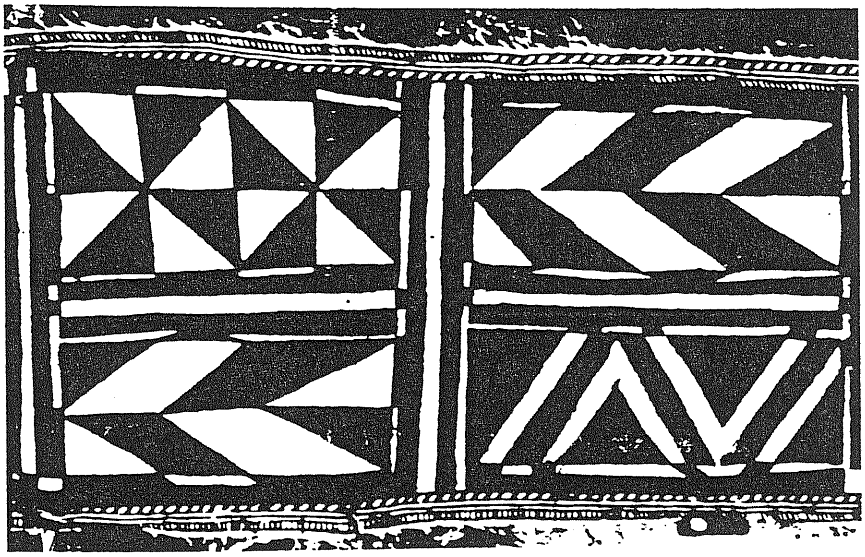
Another phenomenon has been widely noticed is that the perceiving of specific features or outlines which give a vivid impression of the axis is very important in symmetry detection (Barlow & Reeves, 1979; Jenkins, 1983). Our previous experiment on vertical symmetry detection with dot-patterns having different explicitness of axis, subjects were more sensitive to symmetry when the axis was more explicit. We conjecture that there are interactions between symmetry evaluation and axis assessing. On one hand, matching is more accurate and efficient about an explicit axis. On the other hand, when the paired points are matched, they grouped together and confirmed the axis. With the increasing of grouped points, the impression of axis is stronger and this in turn influences the matching process. In such a case, simultaneous cues facilitate not only the anticipation of the reference coordinates but also the evaluation of symmetry. This may be one of the reasons why simultaneous cues are better than prior cues.

# Chapter 5

# Experiment 3: Symmetry in Opposite-Contrast Dot Patterns

Symmetry is defined in terms of the balance of positional dimension. In the case of bilateral symmetry there is mirror correspondence between the elements of two halves divided by the axis of symmetry. Does the balance of other dimensions, such as color, influences the detection of symmetry in addition to positional balance? One aim of the experiment was to see if symmetry detection depends upon a contrast-insensitive mechanism.

Ornamental art provides many examples of two-color patterns in which mirror symmetry is associated with color reversal (Washburn & Crowe, 1988). Typically, black and white regions are reflected and reversed in color. In the resulting configuration, one half is the negative of the other in lightness and reflection of the other in spatial orientation (Figures 5.1 and 5.2). Symmetry is not so easily perceived in these patterns. One might speculate that such decorations are so ubiquitous because of the conflict between color dissimilarity and figural identity, which makes them pleasantly disordered. The same kind of unbalance is captured in their laboratory counterparts, the opposite-color random patterns of the type shown in Fig.5.3. However, two-color symmetrical patterns pose a problem. If figure-ground organization only supports the black-on-white or the white-on-black solution but never both at given time, these pattern should be considered functionally asymmetrical, despite their high degree of geometrical regularity. According to this interpretation, symmetry in two-color opposite-contrast patterns is perceptually weak not because of color dissimilarity per se, but because of its effect on figure-ground organization. To exclude the factor of figure-ground organization, three-color (black dots, white dots and middle grey background) patterns were used in the experiments. The patterns have unambiguous figure-ground organization.



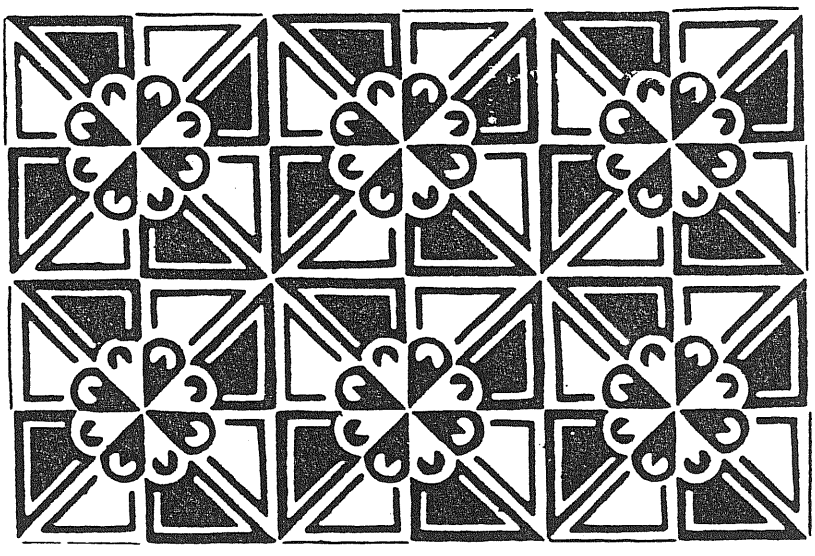


Fig. 5.1 Examples of two-color symmetrical pattern

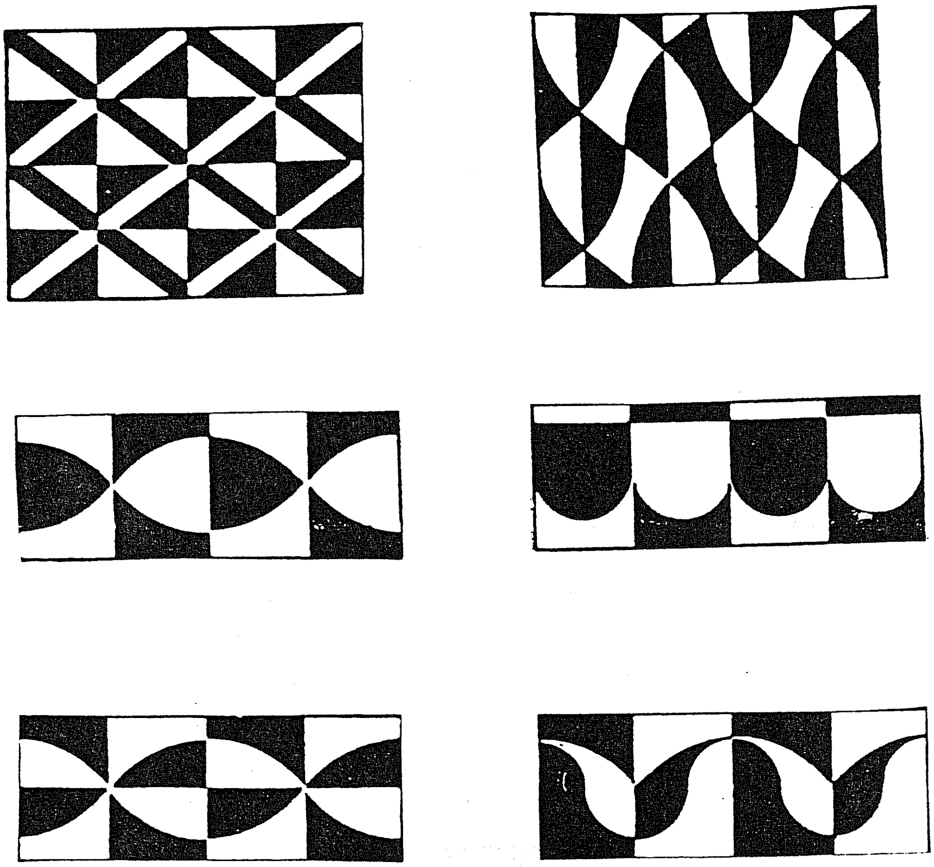


Fig. 5.2 Example of two-color symmetrical pattern

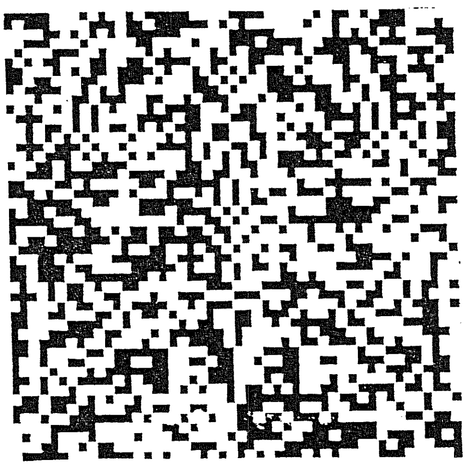


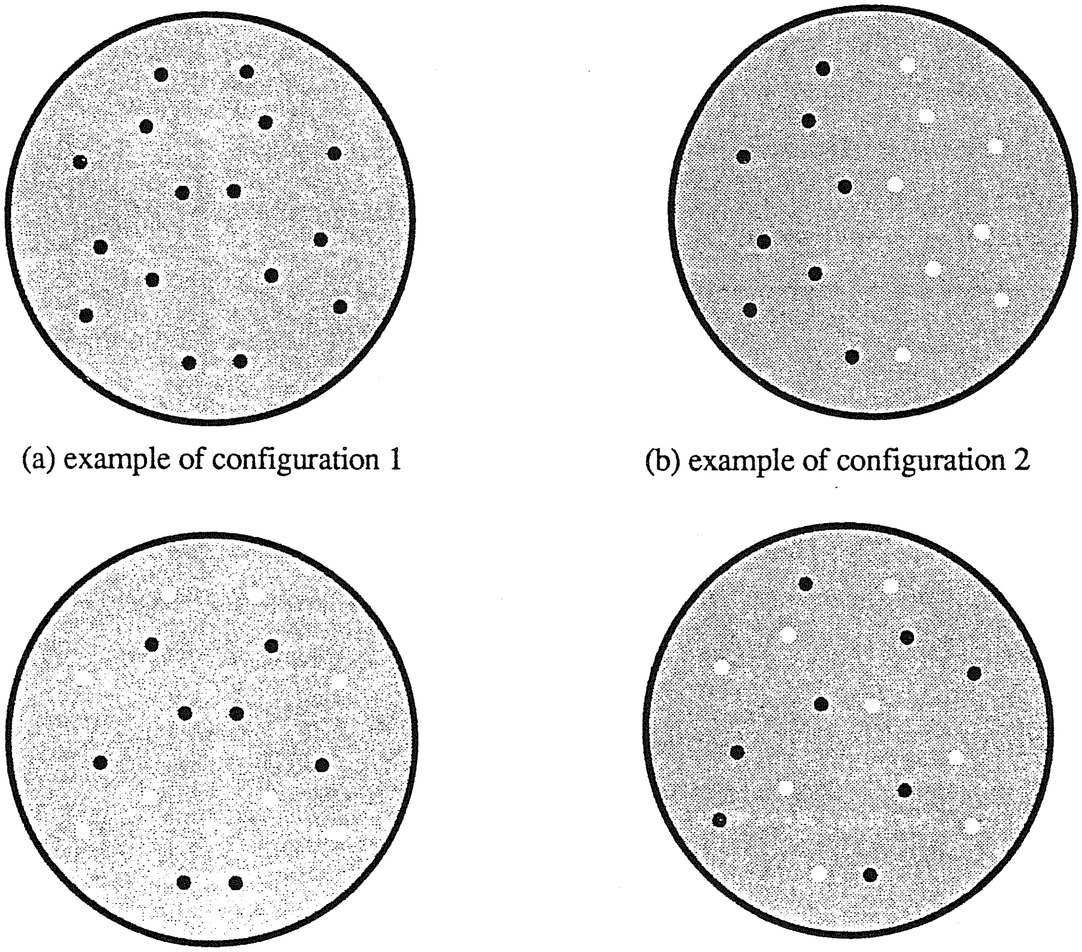
Fig. 5.3 Example of two-color random dot symmetrical pattern

Many features, such as point-by-point correspondence and identity of dot configuration, support the detection of bilateral symmetry in dot patterns. The use of three-color patterns also makes it possible to disentangle the relative contributions of two such features. In the extreme case of point-by-point correspondence, all paired dots have the same color. Symmetry could be evaluated by the point-by-point match or by the grouping of handedness-invariant pairs across the axis as conjectured by Pashler (1990). In the extreme case of identity of dot configurations, all paired dots have opposite color but all dots left to or right to the axis have the same color. The configuration of dots in the left side is the same as the configuration of the dots in the right side. In this case, symmetry could be evaluated by a holistic comparison, a mirror operation of the configuration of the dots.

Four contrast configurations obtained by manipulating two orthogonal factors: the dot-to-background contrast within each half pattern (same vs. opposite); and the dot-to-background contrast of corresponding dots across the axis (same vs. opposite), were used in the experiments. They were: (1) patterns composed of dots with the same contrast, either white or black; (2) patterns composed of dots with opposite contrast within each half, black on the left of the axis and white on the right, or vice verse; (3) patterns composed of dots with opposite contrast between two halves, half in white and half in black, paired with dots in same contrast; (4) patterns composed of dots in opposite contrast within and between halves, half in white and half in black, paired with dots in opposite contrast. The background was grey in all contrast configurations. Fig. 5.4 and Fig. 5.5 show examples of the patterns used. Thus, due to the similarity of dot color, the contribution of point-by-point correspondence is favored in configuration 1 and configuration 3, and the contribution of identity of dot configurations is favored in configuration 1 and 2. In configuration 4, the contributions of both two features are least.

Most theories imply that symmetry is evaluated by a matching process which occurs late in processing. If it is so, we could expect that symmetry detection works in a contrast-sensitive way. Since color is one of the grouping factors in early visual organization.

Previous experiments (Baylis & Drive, 1992) showed that under certain circumstances grouping by color similarity overrides grouping by proximity. The configurations of the dots will be different under different contrast configurations. Thus the detection of symmetry may varied according to different dots configurations.

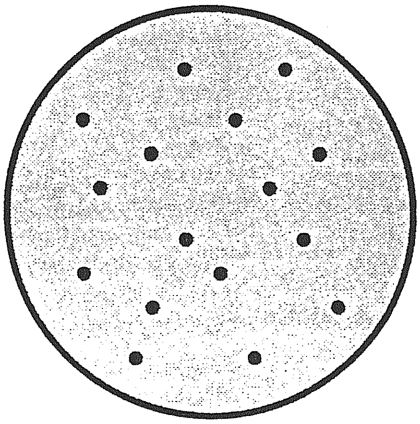


(a) example of configuration 1 (b) example of configuration 2

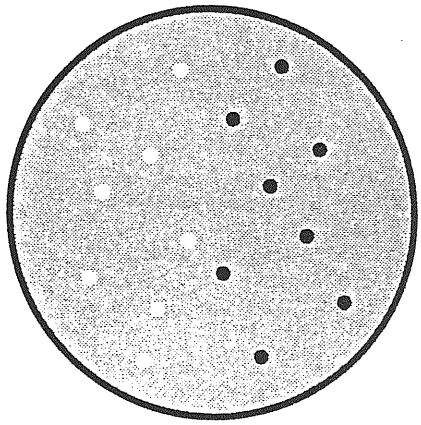
(c) example of configuration 3 (d) example of configuration 4

Fig.5.4 Examples of four configurations of symmetrical patterns.

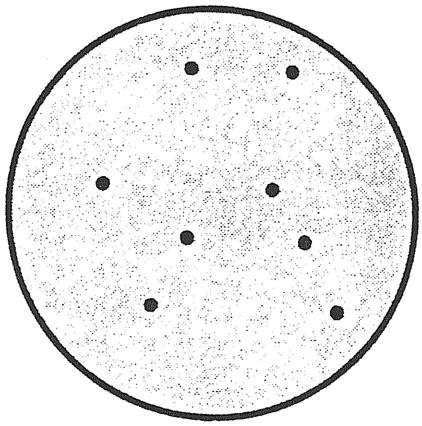
Furthermore three levels of numerosity, 8, 16 and 32, were used to see if the matching process is parallel. The salience of symmetry was defined as a function of the discrimination between symmetrical and noise patterns. Subjects were required to discriminate symmetrical patterns from equivalent noise patterns with the same numerosity and configuration. They were tested using either a random design (Group 1) or a block design (Group 2). Results allowed us to reject the hypothesis that symmetry detection depends upon a contrast-insensitive mechanism and provided evidence for a contrast-specific comparison process based on piont-by-point correspondence. In addition, when the dots had one of the two features, point-by-point correspondence or identical configurations, the performance was improved. Further more when both kinds of features were possible, the former the dominant one and it facilitated the detection most.



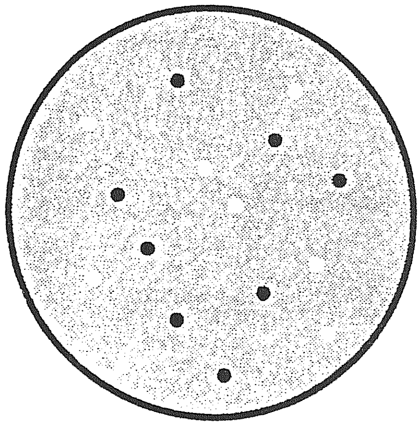
(a) example of configuration 1



(b) example of configuration 2



(c) example of configuration 3



(d) example of configuration 4

Fig.5.5 Examples of four configurations of noise patterns.

## 5.1 Experiment 3

### Method

**Subject** Two groups of six undergraduate students of psychology department in the University of Trieste participated the experiment. Group 1 subjects were tested using random design and Group 2 subjects were tested using block design. All of them were naive with the aim of the experiment.

**Stimuli Generation** The stimuli were generated by computer program in MAC II and stored in record files. The patterns were composed of random dots displayed on grey background. The dots were distributed homogeneously within a circle 8 cm in diameter. For symmetrical patterns, dots were generated in pairs. The orientation of each pair was vertical. The size of each pair and the location of the midpoint on the axis were generated by the computer's random generator. The constraint of minimum distance between two dots were varied according to the numerosity of the pattern. The color of each dot was controlled respectively according to each configuration condition. In configuration 3 and 4, the dots in white and black were mixed randomly. For the noise patterns, the orientation, size, and midpoint of each pair were random. Half of the dots were on the left and half were on the right. The constraint of minimum distance between two dots were the same as to symmetrical patterns with same numerosity.

**Procedure** Before starting the experiment, each subject read written instructions. The instructions described all kinds of configurations in detail. Examples of symmetrical and noise patterns in each configuration were presented. Subjects were stressed the positional symmetry irrespective of the color of the dots.

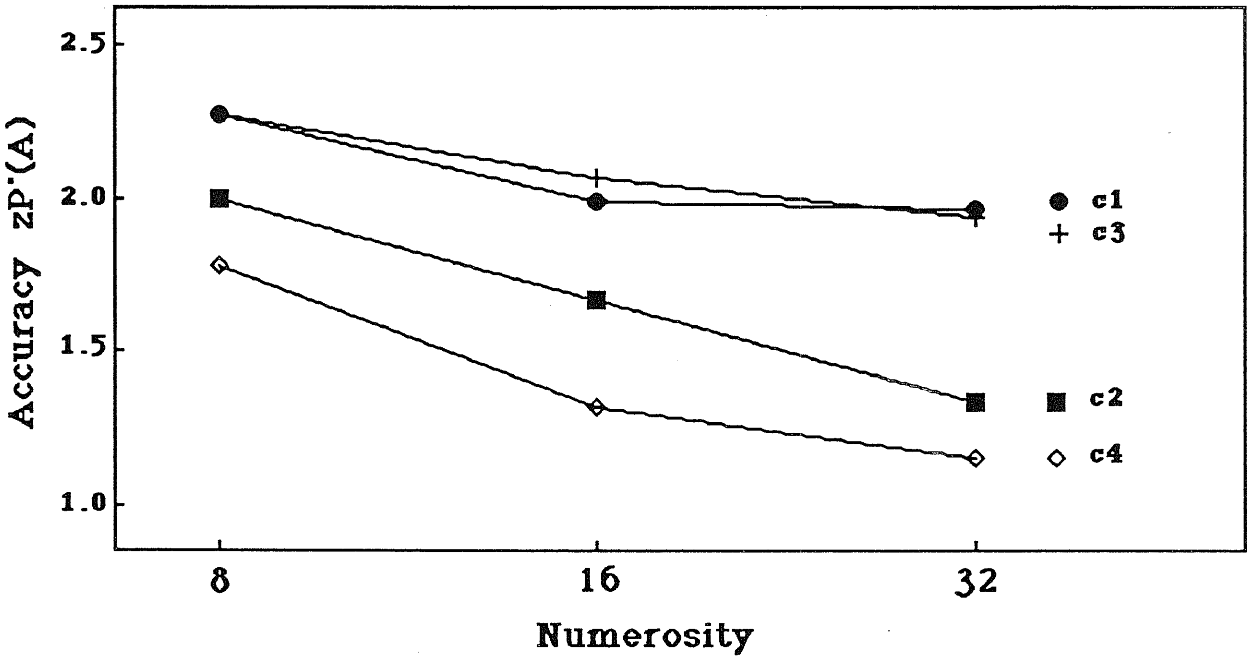
The experiment was performed on a Radius monochromatic screen controlled by a Mac II. The screen was covered by a dark grey cardboard with an opening circle. The experiment was composed of three sessions. The numerosity remained constant within each session and varied between sessions. Subjects did one session each time, so three sessions were finished in three different days. For Group 1 subjects, each session was divided into two blocks of 160 trials, 20 of each configuration either symmetrical or noise; the order of the patterns of different configurations, either symmetrical or noise, were mixed randomly. For Group 2 subjects, trials which have same configuration were blocked together; each session was divided into four blocks of 20 symmetrical patterns and 20 noise patterns in same configuration. The order of the blocks was balanced between subjects. For both two groups, the order of the numerosity was balanced between subjects.

A trial started with a fixation point located on the axis of symmetry. The fixation point lasted on the screen for 1000 ms. Immediately after its off set, the stimulus pattern was shown on the screen with an opening circle for 150 ms and then disappeared. Subjects were asked to determine if the stimulus was symmetrical or noise by pressing the answer key "YES" or "NO". Keys corresponding to "YES" and "NO" were chosen by the subjects freely at the beginning of the experiment. Subjects were stressed both the accuracy and speed. The responses and reaction times were registered by the computer.

## Results

The accuracy of symmetry, zP'(A) (a function of the discrimination between symmetrical and noise patterns), the error rate, and the reaction times of correct response of each configuration and numerosity were analyzed.

An ANOVA (design(2) x numerosity(3) x configuration(4) ) and an ANOVA (design(2) x numerosity(3) x contrast within(2) x contrast between(2) ) were performed to error rate, accuracy, and reaction times respectively.



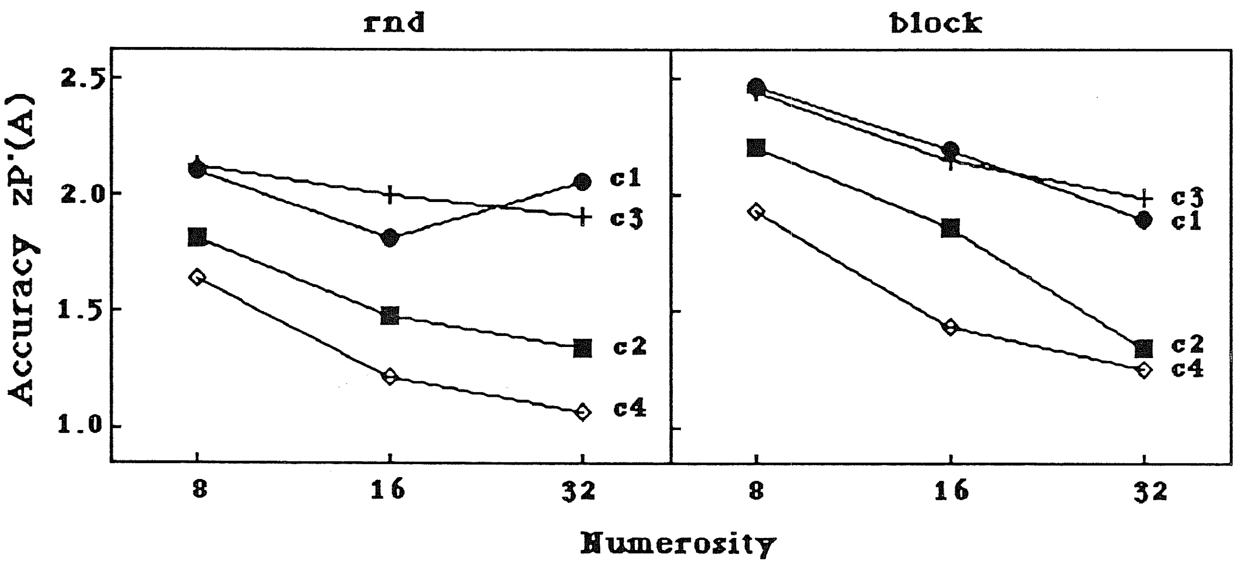




Fig. 5.6 Accuracy in condition 1 and 3 is better than in condition 2 and 4, for both designs.

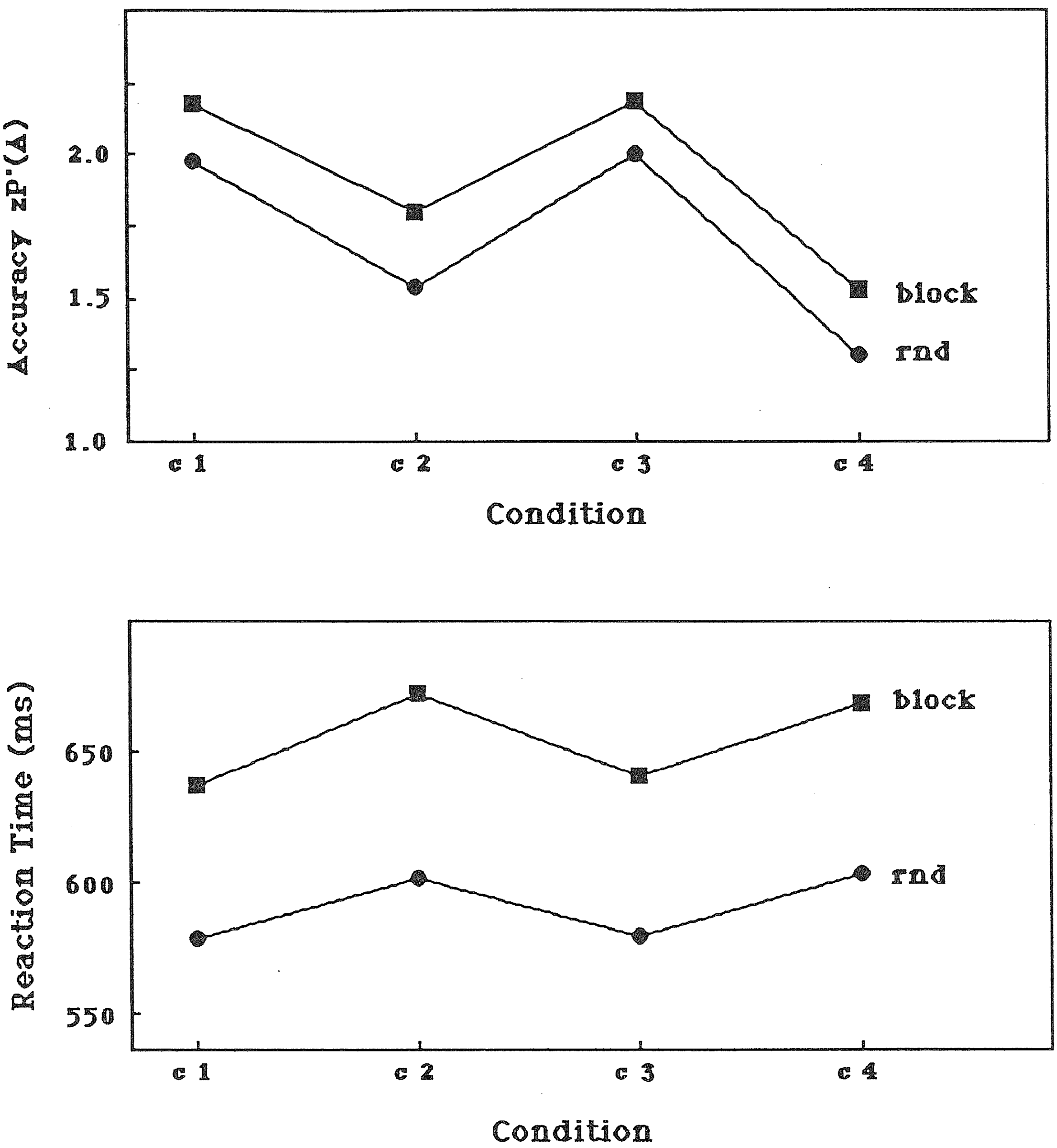



Fig. 5.7 Speed-accuracy tradeoff between the two designs

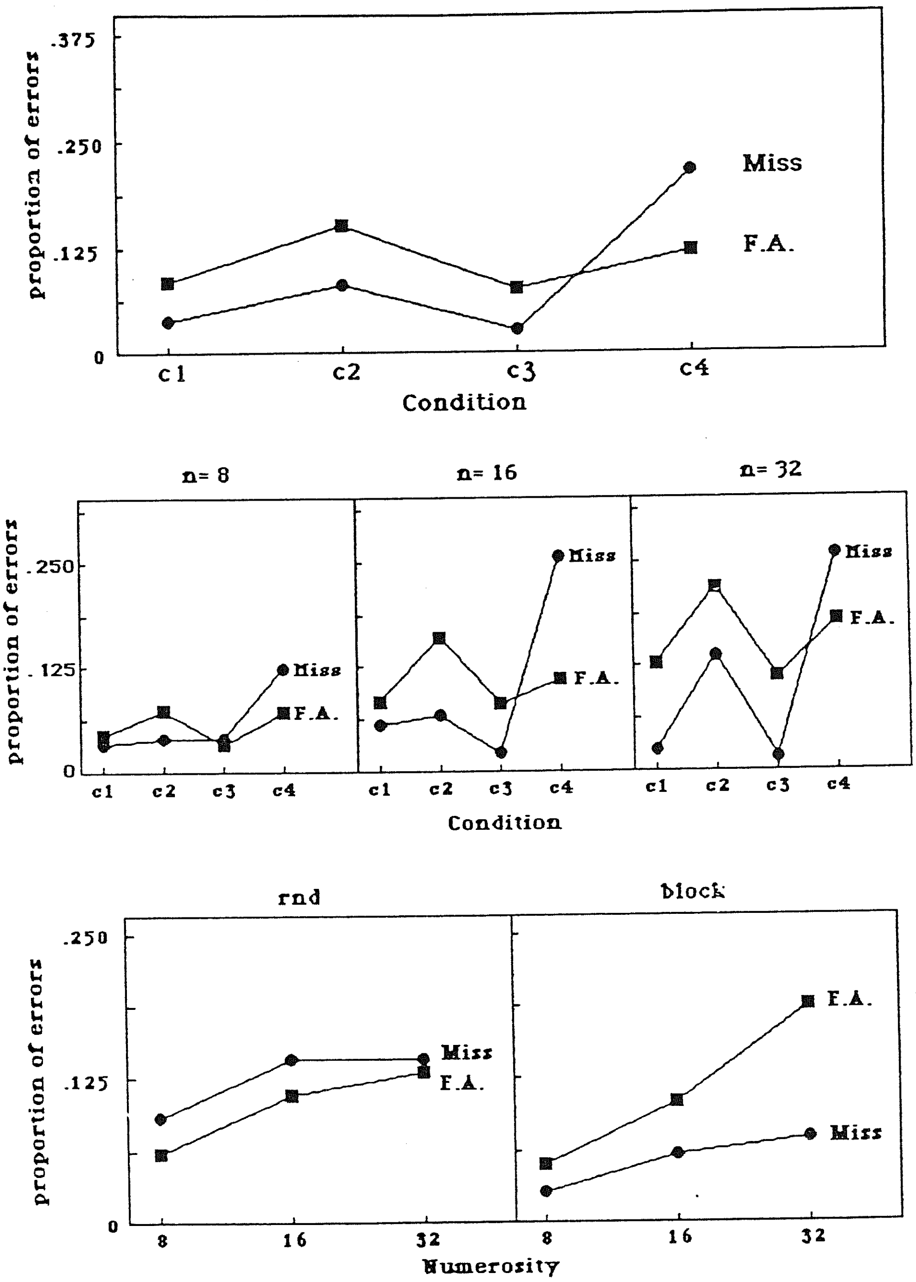



Fig. 5.8 error rates of in four conditions

As showed in Fig. 5.6, the accuracy of symmetry detection varied depending on the configurations. The performance was equally good in c1&c3 (2.070 & 2.087) and better than in c2 (1.661) which was in turn better than c4 (1.411). The effect on configuration was significant, $F(3, 30)=37.722$, $P<0.0001$.

Fig. 5.6 also shows the effect of point-by-point color correspondence and the effect of numerosity. zP'(A) was 2.079 in c1&c3 which had point-by-point correspondence and was 1.536 in c2&c4 which had not point-by-point correspondence. The difference was significant, $F((1,10)= 109.73$, $P<0.001$, for the overall comparison between c1&c3 vs c2&c4.

There was a main effect of numerosity: $F(2, 20)=12.29$, $P<0.001$. There was weak interaction between numerosity and point-by-point color correspondence, $F(2, 20)=2.565$, $P=0.102$, due to the fact that numerosity was less influential on c1&c3 ($F(2, 20)=2.969$, $P=0.074$) than on c2&c4 ($F(2, 20)=20.405$, $P<0.001$). But the influence of numerosity seems depend on the design of the experiment: in block design it also affected the performance of c1&c3.

Figure 5.7 shows that subjects were slower and more accurate in block design. However, this speed-accuracy tradeoff did not interact with basic experimental effects.

Furthermore, the detection in c4 was characterized by the high proportion of miss, as showed in Fig. 5.8. This could be explained by the noticing that grouping of elements on the basis of color masks the presence of a central axis of symmetry.

The results of reaction times analysis are coinsistance with that of accuracy.

## 5.3 Discussion

The experimental results shows that the performance of detection does not depend on the configuration of the pattern. Besides positional symmetry, the balance of color influences the detection of symmetry. Symmetry with paired dots in same color is more salient than that with paired dots in opposite color. This provides evidences that symmetry detection works in a contrast sensitive way suggesting that grouping of dots by color similarity occurs earlier than symmetry matching. Grouping on the basis of color similarity strongly affects symmetry detection also when the figure-ground organization is unambiguous.

The performance was best when the patterns were in configuration 1 (same contrast within each half and between two halves) and configuration 3 (opposite contrast within each half and same contrast between two halves), better in configuration 2 (same contrast within each half and opposite contrast between two halves), and worse in configuration 4 (opposite contrast within each half and opposite contrast between two halves). This suggests that the detection of symmetry is supported by either the feature of point-by-point correspondence or the feature of dot configurational identity and the former one is more efficient. The patterns of configuration 1 had both point-by-point correspondence and dot configurational identity. The equal performance of detection in configuration 1 and configuration 3 shows that point-by-point color correspondence is the dominant feature supporting symmetry detection. The large proportion of misses made in configuration 4 suggests that grouping of uncorrelated dots by color similarity overshadows spatial symmetry.

Symmetry can also be detected in configuration 4 suggests that the positional correspondence could be revealed by point-by-point comparisons overriding the color mismatch. However, this comparison is harder and takes more time.

The explicitness of the axis of symmetry might be another factor contributes to the experimental effects. From the results of Experiment Group 1, we know that symmetry is better detected when the axis is explicit. The occurrence of symmetrical features around the axis gives a vivid impression of the axis, and thus improves symmetry detections. Such features could appear in the patterns in configuration 1 and 3. In configuration 2, the segregation of two halves by the color difference makes the axis visible. In contrast, in configuration 4, the axis is less visible because of the lack of symmetrical features and left-right segregation. The grouping of unpaired dots masks the presence of a central axis of symmetry and makes symmetry less detectable.

# Chapter 6
# A Simulation of Mirror Symmetry Detection in Random-Dot Patterns

In previous chapters, we discussed many phenomena observed in symmetry detection and models accounting for experimental findings. Symmetry detection depends on the salience of symmetry which is found to be a function of axis identities and other properties of pattern and visual environment. Point-by-point comparison is a basic process in symmetry evaluation specially when the forming of dot-cluster is prevented. In present chapter, based mainly on Pashler's grouping conjecture and other experimental results, a computational model has been developed. The algorithm is based on the grouping of dot pairs described by symmetry descriptors and a pattern is mapped onto a potential surface of symmetry descriptors which represents salience. The pattern is analyzed by a set of parallel symmetry descriptors, each corresponding to a specific symmetry axis. The axis is specified by two variables: orientation and location. The same symmetry descriptor is stimulated by all pairs of dots which have a common axis, irrespective of their size (the length of the virtual line connecting the dots). Homologous pairs are grouped together in the sense that they raise the potential of a single descriptor. The sensitivity to symmetry depends on the difference between the potential surface of symmetrical pattern and noise pattern. Details will be presented later.

## 6.1 Algorithm

A simulation is composed of three steps: (1) pattern encoding; (2) symmetry evaluation; (3) decision making.

**Pattern encoding** A dot pattern is encoded as a matrix of 1 (dot) and 0 (background) elements. A horizontal-vertical reference frame is established and the original point is set at the center of the pattern. Two coordinates, $x_i$, and $y_i$, are assigned to each dot.

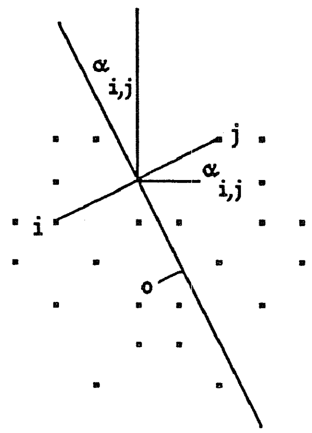



Fig. 6.1. An example of symmetrical pattern.

**Symmetry evaluation** Consider a pattern composed of N dots, as shown in Fig. 6.1. There are M= N(N-1)/2 pairs of dots. A set of descriptors {Dm} are stimulated by these pairs, where m=1,2,...,k, k≤M.

The algorithm used to compute all potential is as follows.

- Do from 1 to M.
- Pick up one dot pair (i,j)
- Calculate the orientation ($\alpha_{i,j}$) of the virtual line which connects the paired dots, and the location ($l_{i,j}$) of the axis. By definition, the orientation of virtual lines

refers to the horizontal. In the algorithm the same parameter defines the orientation of the axis, relative to the vertical, because in mirror symmetry the axis is orthogonal to the virtual line. Therefore, the value $\alpha_{i,j} = 0$ corresponds to a horizontal virtual line and vertical axis.

$$\alpha_{i,j} = \arctan\left(\frac{y_j - y_i}{x_j - x_i}\right)$$

$$l_{i,j} = \frac{\dfrac{y_i + y_j}{2} + \left(\dfrac{x_j - x_i}{y_j - y_i}\right)\left(\dfrac{x_i + x_j}{2}\right)}{\sqrt{\left(\dfrac{x_j - x_i}{y_j - y_i}\right)^2 + 1}}$$

- Stimulate the corresponding symmetry descriptor (e.g. descriptor $D_k$) by raising its potential

$$P(k) = P(k) + F_1(\xi_1)F_2(\xi_2)\ \dots F_n(\xi_n)$$

where P(k) is the potential of the descriptor $D_k$; $F_1, F_2, \dots, F_n$ are the modification functions; $\xi_1$, $\xi_2$, and $\xi_n$ are the variables which influence the salience of symmetry. In our simulations of mirror symmetries presented here, three factors, the orientation of the virtual line (orientation of the axis), the location of the axis, and the size of the dot pair are considered. So

$$P(k) = P(k) + F_1(\alpha_{i,j})F_2(l_{i,j})F_3(s_{i,j})C$$

where C is a constant and $s_{i,j}$ is the size of pair (i,j), defined as follows:

$$s_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \quad .$$

- Pick up a new pair (i,j).
- Loop.

Fig. 6.2 shows an example of the potential functions of descriptors corresponding to the symmetrical pattern shown in Fig. 6.1. The 3-D graph illustrates the distribution of final potentials (vertical) as a function of the two stimulus parameters used to describe the 2-D pattern (orientation and location of the axes of dot pairs).

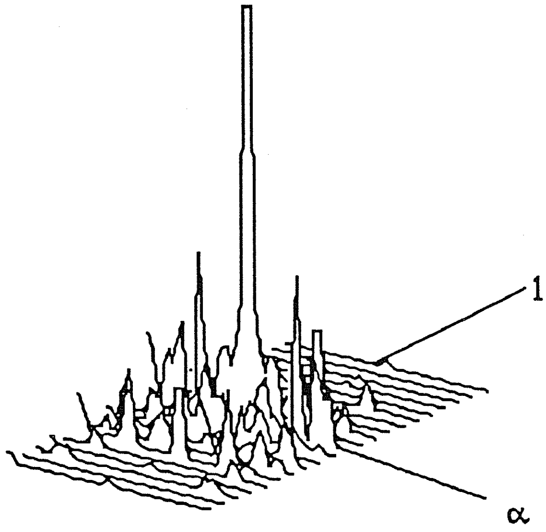



Fig. 6.2. Potential functions of symmetry descriptors.

**Decision making** Suppose there is a symmetry detector SD, which receives its input from the set of descriptors {Dm}, as shown in Fig. 6.3. Descriptors activate SD in a competitive way. This means that the activation SD is a function of the maximal value of

{P(m)}, Pmax. In the simulations reported here, a linear activation function was used: SD activation= aPmax, with a=1.

The parameter d* is used to describe the discrimination of symmetry from noise, where d* is defined as the ratio of the difference between the means of the activation distributions of symmetrical and noise patterns to the mean of the noise distribution. The range of d* is between 0 and N/2. The salience of symmetry is increased when d* is increased.

$$d^* = \frac{\text{Mean Activation (S)} - \text{Mean Activation (N)}}{\text{Mean Activation (N)}}$$

To simulate performance of a real observer, one should also choose a threshold value of activation corresponding to the shift from noise to symmetry responses. This is outside the scope of the present simulation, which is only concerned with stimulus sensitivity.

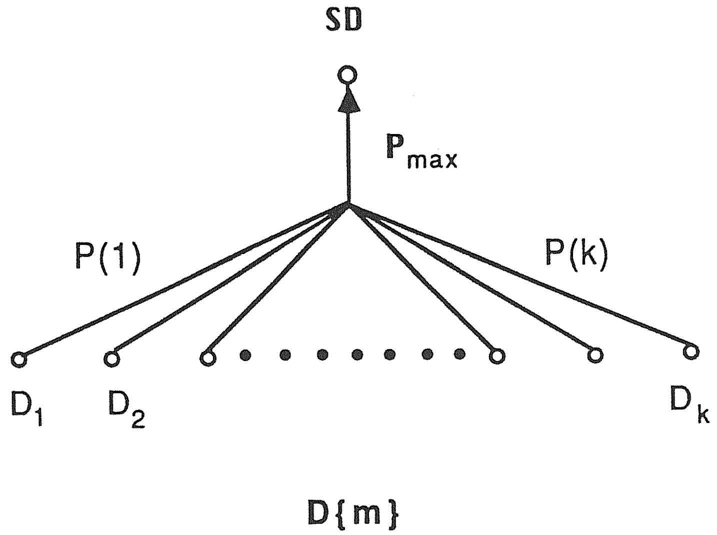



Fig. 6.3. A diagram of a symmetry detector.

## 6.2 Simulations

In the simulations presented here, dot patterns were encoded as 8x8 matrices. Most patterns were composed of 24 dots. For each kind of stimulus, there were 200 trials, corresponding to 100 symmetrical pattern and 100 noise patterns.

As regards the orientation effects, only the preference for vertical symmetry was considered, since it is supported by all researches. Two other facts were embodied in the model: (i) the detectability of symmetry is destroyed when the axis is displaced away from the center of the pattern; (ii) symmetry information is more relevant when corresponding elements are close to the axis. Therefore, the three modification functions were

$$F_1(\alpha_{i,j}) = \begin{cases} 1 & \text{if } |\alpha_{i,j}| \leq \frac{\pi}{10} \\ 0.9 & \text{otherwise} \end{cases}$$

$$F_2(l_{i,j}) = 1 - \frac{|l_{i,j}|}{3.5}$$

$$F_3(s_{i,j}) = \begin{cases} 1 & \text{if } s_{i,j} \leq 2 \\ \frac{18 - s_{i,j}}{16} & \text{otherwise} \end{cases}$$

### Simulation 1: orientation effects

Three blocks of 200 trials have been tested, with vertical, horizontal and diagonal axes, respectively. Fig. 6.4 shows the d* values as a function of axis orientation. There is an advantage for vertical axis. Since we didn't modify the preference for horizontal or diagonal, the d* of these two are almost the same.

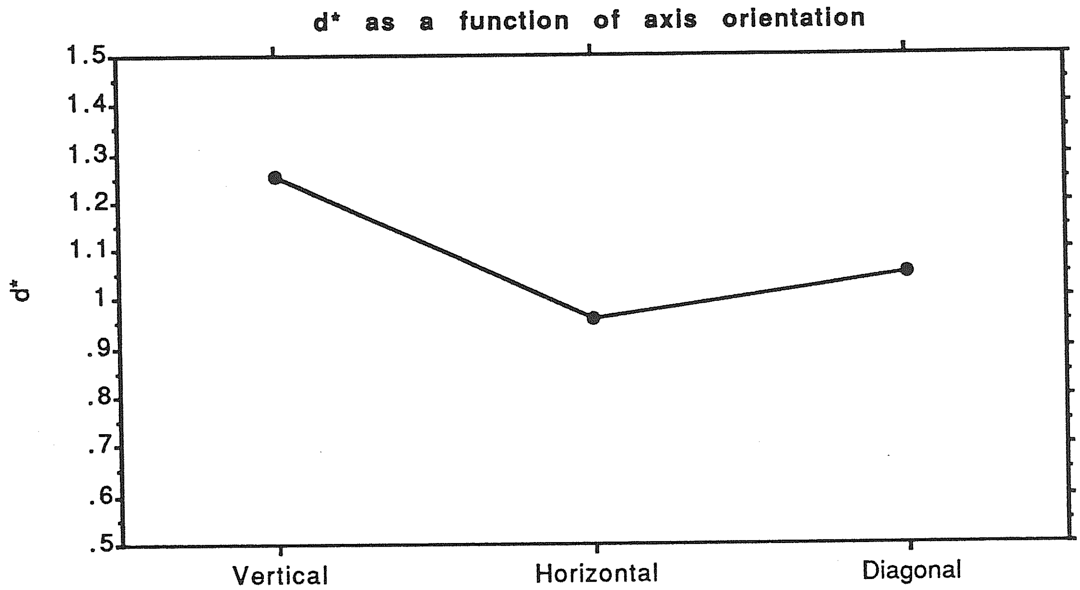



Fig. 6.4. Results of simulation 1.

## Simulation 2: double symmetry

Two blocks of double symmetry with vertical-horizontal axes and diagonal axes were tested. Results were shown in Fig.5. The activations of HV and DD were 1.224 and 1.142, slightly larger than single symmetry.

Among the 100 trials with HV symmetrical patterns, in 7 trials a horizontal axis were reported in the simulation. This can be interpreted as an effect of spatial constraints embodied in $F_3$ (proximity to the axis): despite the vertical bias embodied in $F_1$, in some cases the potential of smaller pairs across a horizontal axis can override the potential of larger pairs across a vertical axis. As expected, the simulation with two diagonal orientations had the same probability of generating the maximal activation. Other factors might influence the salience of multiple symmetry were not considered in this simulation.

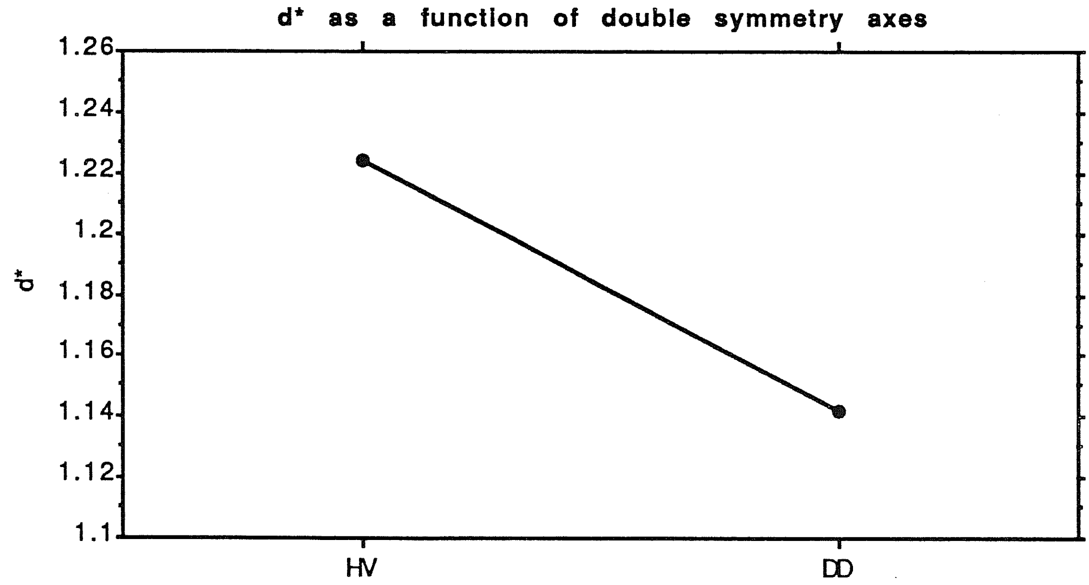



Fig. 6.5. Results of simulation 2.

In simulations 3, 4, and 5 only vertical symmetry was considered. In such cases, only dot pairs across the vertical axis were computed, while those across other axes were ignored. This is a way of approximating selective attention to symmetry about a specific axis in observers having prior information about it.

## Simulation 3: detection efficiency and symmetry proportion

Psychophysical studies found that symmetry detection does not occur in an all-or-none fashion. Symmetrical patterns are distinguishable from noise as a function of the proportion of corresponding elements. Six blocks of trials were tested, with symmetry proportion p equal to 1, 0.8, 0.6, 0.4, 0.2 and 0 respectively. All patterns contained 24 dots. To manipulate symmetry proportion in a pattern with a maximum of 12 symmetrical pairs, (1-p)x12 pairs were replaced by noise pairs.

Results are shown in Fig. 6.6. When 60% of symmetrical pairs (p= 0.4) were replaced by noise pairs, the activation distributions of symmetry and noise were almost the same. In this case symmetry cannot be detected efficiently. With larger symmetry proportion, the overlapping of symmetry and noise distributions is decreased. The efficiency of detection is increased accordingly. The results of this simulation fit experimental data by Barlow & Reeves (1979).

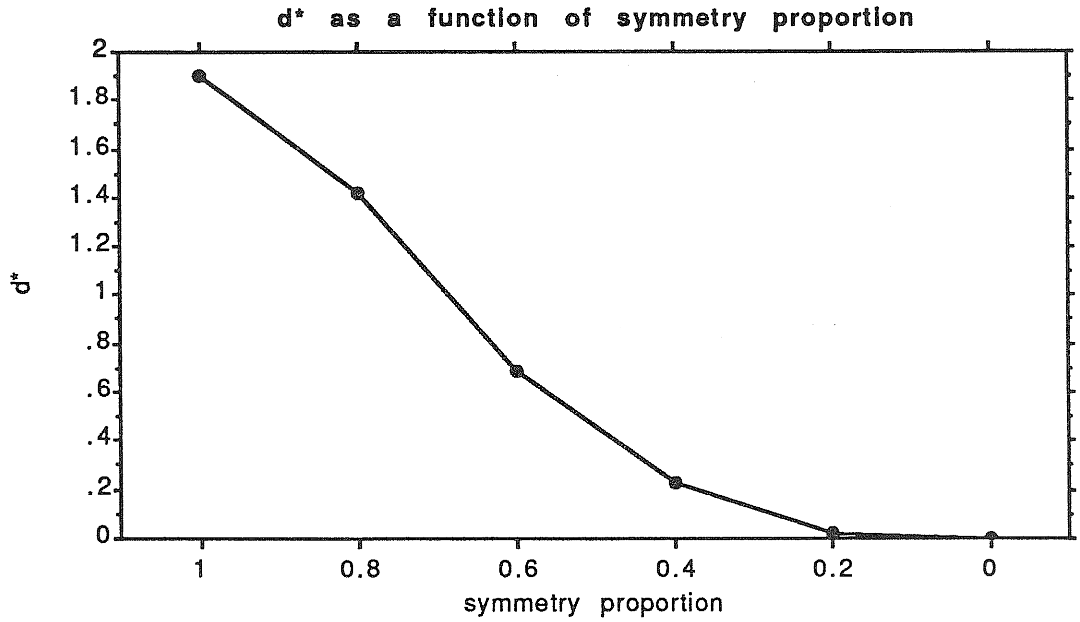



Fig. 6.6. Results of simulation 3.

## Simulation 4: redundancy of symmetry information

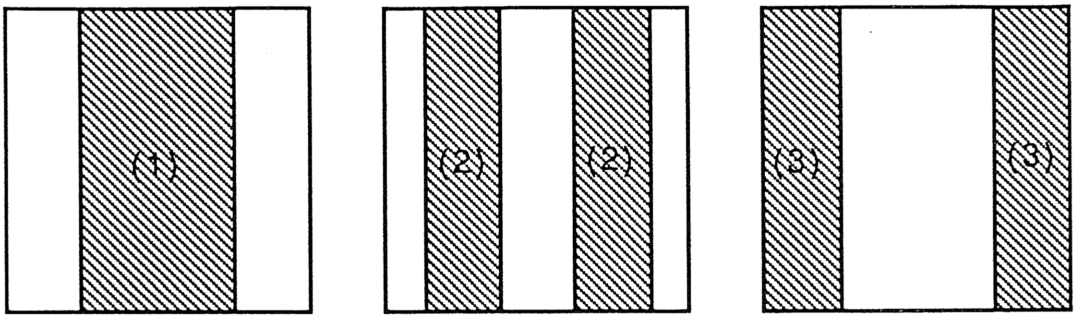



Fig. 6.7. Three noise regions.

In these tests, the pattern was divided as shown in Fig. 6.7. Only (1-p)x6 symmetrical pairs belonging to a certain region were replaced by noise pairs. Three regions have been chosen to test the redundancy distribution.

Results are shown in Fig. 6.8. Noise in region 1 largely influences the saliency of symmetry, whereas noise in region 3 affects the saliency of symmetry only slightly.

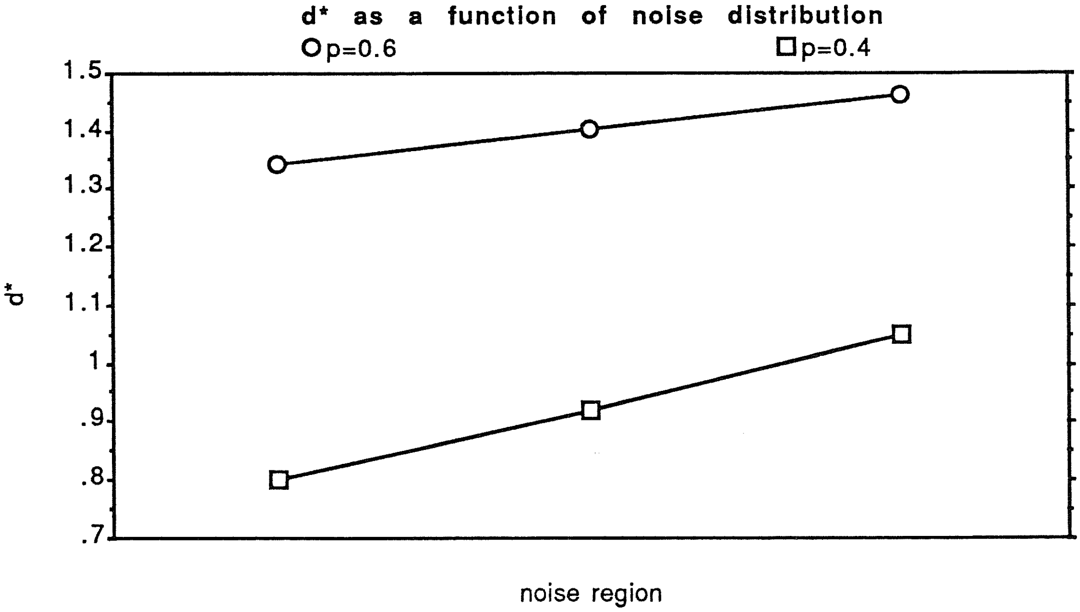



Fig. 6.8. Results of simulation 4.

## Simulation 5: d* and pattern numerosity

Six blocks of patterns were tested, with dot numerosity varying from 24 to 4, step 4. Results are shown in Fig. 6.9. The discrimination of symmetry from noise was decreased as the number of dots was increased. The influence of numerosity on detection can be interpreted as the number related grouping information processing. With lower number of dots, grouping of all symmetrical pairs can be performed within shorter time. On the other hand, the information needed to verify symmetry is less with less dots. We could expect the processing of grouping information could be parallel with finite capacity. If it is so, the shape of d* versus numerosity should be bell like.

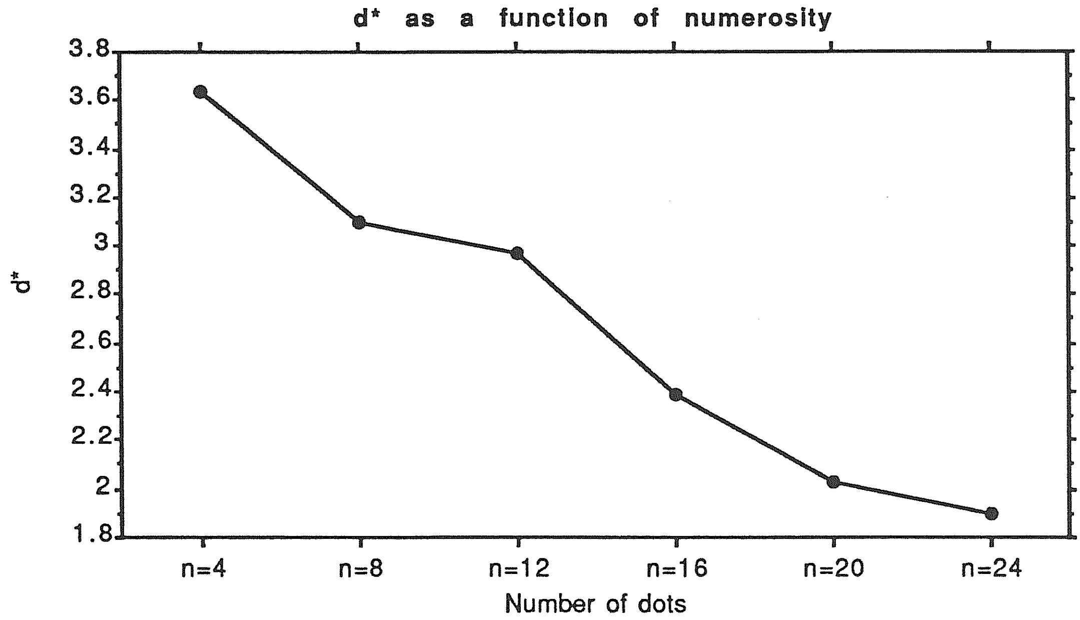



Fig. 6.9. Results of simulation 5.

## 6.3 Discussion

Symmetry detection in random dot patterns was simulated by using a computational model based on the grouping of homologous pairs. Three factors were assumed to act simultaneously: the preference for vertical symmetry; the weighted utilization of symmetry information referring to the distance of elements from the axis of symmetry; the decrease in detectability when the axis is located away from the center.

The simulation results fit the psychophysical experiment results on many aspects: orientation effects, symmetry information related detection efficiency, redundancy distribution, numerosity effects.

An interesting result emerged, which fits psychophysical data, is the preference for double symmetry. This may be due to the fact that two potentials of two descriptors are above the threshold and they are competitive. The distortions or mismatching along one axis reduce the potential of the corresponding descriptor but do not influence the potential of another descriptor. Thus the mean activation of the SD of multiple symmetry is larger than single symmetry. Our results of the simulation fit psychophysical results qualitatively. In order to explain the preference for multiple symmetry, Palmer & Hemenway (1978) assumed that the bias for axis selection is variable. The simulation of double symmetry has shown that in some cases horizontal or diagonal symmetry produces stronger activation than vertical symmetry. This is not due to the orientation bias but to other factors, like the sizes of symmetrical pairs present in certain patterns. Therefore, violations of vertical dominance in particular displays should be expected when all factors involved in symmetry evaluation are acting together.

The effect of multiple symmetry detection is not so strong as observed from the psychophysical experiments. There may exist other factors which enhance the salience of multiple symmetry and they are not considered in our simulation.

The present version of the computational model did not include other factors which affect human symmetry detection, such as second-order regularities (Wagemans, Van Horebeck, Van Gool, & Swinnen, 1991). However, by choosing the modification functions and the activation function appropriately, the model could be extended to fit a larger set of psychophysical results.

## 6.4 Simulation of skewed symmetry detection

Experiments on skewed symmetry detection found that our ability to detect symmetry is destroyed under skewed symmetry conditions. The orientation of the virtual line is more important than the orientation of the axis referring to the orientation effects. The salience of symmetry is also affected by the skewed angle $\beta$, $\beta$ is the angle between the normal axis of the virtual line and the axis of skewed symmetry. The detectability of symmetry decreased dramatically when $\beta$ deviated from 0. In the case of skewed symmetry detection, a specific descriptor corresponds to specific orientation of virtual line, orientation of axis, and the location of the axis. One pair of dots stimulates a series of descriptors which have the same orientation of virtual line but different axes. The amplitude depends on the orientation of the virtual line, the skewed angle, the size of the dot pair, and the location of the axis. We generalized the algorithm used in mirror symmetry simulation by adding one more modification function.

The algorithm used in skewed symmetry simulation is following

- Do
- Pick up one dot pair (i,j);
- Calculate the orientation of the virtual line $\alpha_{i,j}$;

$$\alpha_{i,j} = \arctan\left(\frac{y_j - y_i}{x_j - x_i}\right)$$

- Stimulate the corresponding symmetry descriptors, e.g. descriptor $D_k\{\beta)$ by raising its potential, $\beta$ is the skew angle which varies from $-\pi/2$ to $\pi/2$;

$$P_k(\beta) = P_k(\beta) + F_1(\alpha_{i,j})F_2(l_{i,j})F_3(s_{i,j})F_4(\beta)$$

where $P_k(\beta)$ is the potential of the descriptor $D_k(\beta)$, $s_{ij}$ is the size of pair (i,j), $l_{i,j}$ is the location of the skewed axis and C is constant.

$$l_{i,j} = \frac{\frac{y_i + y_j}{2} + \frac{x_i + x_j}{2}\,ctg(\alpha_{i,j} + \beta)}{\sqrt{ctg^2(\alpha_{i,j} + \beta) + 1}}$$

$$s_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$$

- Pick up a new pair (i,j);
- Loop

In our simulations of skewed symmetry, the four modification functions chosen were:

$$F_1(\alpha_{i,j}) = \begin{cases} 1 & \text{if } |\alpha_{i,j}| \le \frac{\pi}{10} \\ 0.9 & \text{otherwise} \end{cases}$$

$$F_2(l_{i,j}) = 1 - \frac{|l_{i,j}|}{3.5}$$

$$F_3(s_{i,j}) = \begin{cases} 1 & \text{if } s_{i,j} \le 2 \\ \dfrac{18 - s_{i,j}}{16} & \text{otherwise} \end{cases}$$

$$F_4(\beta) = \cos(\alpha_{i,j} - \beta)$$

Two blocks of 200 trials, 100 symmetric and 100 noise, have been tested. In block 1, the orientation of the virtual line is horizontal and the orientation of the axis is oblique. In block 2, the orientation of the vital line is oblique and the orientation of the axis is vertical. The skew angle is $\pi/6$ in both two blocks. Results are shown in Fig. 6.10.

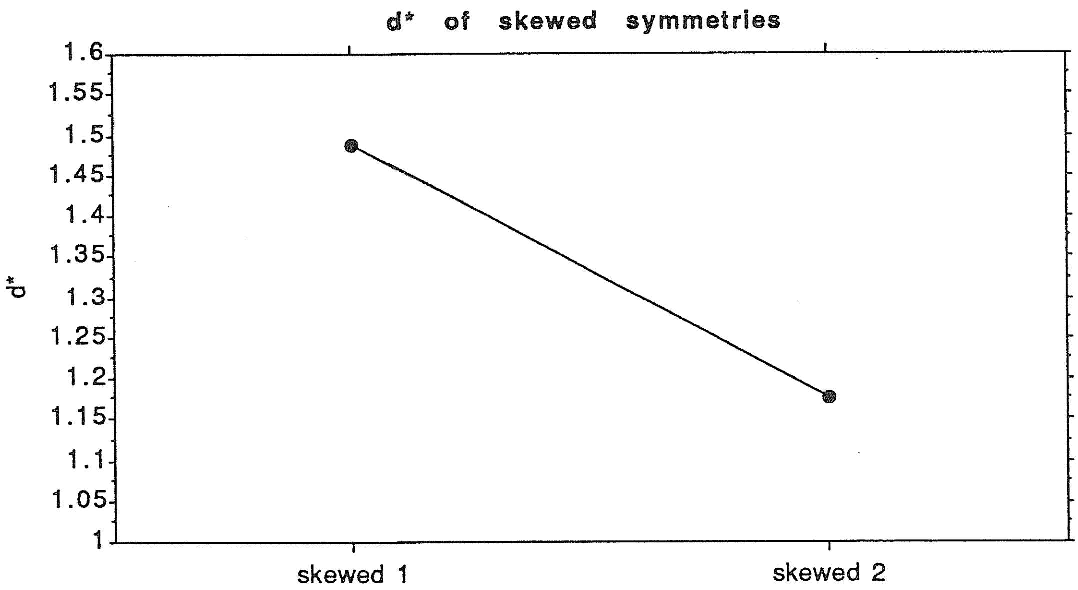



Fig. 6.10 the results of skewed symmetry detection

Compare with the results of mirror symmetry, d* is decreased from 1.900 of mirror symmetry to 1.489 of skewed symmetry 1 and 1.117 of skewed symmetry 2. Under the same condition of skew angle, skewed symmetry with horizontal virtual line, oblique axis is more salient than that with vertical axis, oblique virtual line. These are consistent with experimental results.

# References

1. Attneave, F. (1982). Pr?gnanz and soap-bubble systems: A theoretical exploration. In J. Beck (Ed.), *Organization and representation in perception,* 11-29, Hillsdale, NJ: Erlbaum.

2. Barlow, H.B. & Reeves, B.C. (1979). The versatility and absolute efficiency of detecting mirror symmetry in random dot displays. *Vision Res.* **19**, 783-793.

3. Baylis, G.C. & Driver, J. (1992). Visual parsing and response competition: The effect of group factors. *Perception & Psychophysics*, **51**, 145-162.

4. Beck, J. (1982). Textural segregation. In J. Beck (Ed.) *Organization and representation in perception,* 285-317, Hillsdale, NJ: Erlbaum.

5. Beck, J. (1983). A theory of textural segregation. In J. Beck, B. Hope, & A. Rosenfeld (Eds.) *Human and machine vision*, 1-38, New York Academic Press.

6. Bergen, J. (1986). Texture segregation and the psychophysics of texture. Paper presented at the center for visual science symposium, Rochester, NY.

7. Bergen, J. & Adelson, E. (1988). Early vision and texture perception. *Nature*, **333**, 363-364.

8. Bergen, J. & Julesz, B. (1983). Rapid discrimination of visual patterns. *IEEE Transactions on systems, man and cybernetics*, **13**,

9. Biederman, I. (1987). Recognition-by-components: A theory of human image understanding. *Psychological Review*, **94**, 115-147.

10. Broadbent, D.E. (1982). Task combination and selective intake of information, *Acta Psychologica*, **50**, 253-290.

11. Bruce, V.G. & Morgan, M.J. (1975). Vialations of symmetry and repetition in visual patterns. *Perception*, **4**, 239-249.

12. Caelli, T. (1985) Three processing characteristic of visual texture segregation. *Spatial Vision*, **1**,19-30.

13. Clement, D.E. & Varnadoe, K.W. (1967). Pattern uncertainty and the discrimination of visual patterns. *Perception & Psychophysics*, **2**, 427-431.

14. Corballis, M.C. & Beale, I.L. (1976). *The psychology of left and right.* Hillsdale, NJ: Erlbaum.

15. Corballis, M.C. & Roldan, C. (1975). Detection of symmetry as a function of angular orientation. *Journal of Experimental Psychology: Human Perception and Performance*, **1**, 221-230.

16. Driver, J. & Baylis, G.C. (1989). Movement and visual attention: the spotlight metaphor? breaks down. *Journal of Experimental Psychology: Human perception and Performance*, **15**, 448-456.

17. Duncan, J. (1984). Selective attention and the organization of visual information. *Journal of Experimental Psychology: General*, **113**, 501-517.

18. Enns, J. (1986). Seeing textures in context. *Perception & Psychophysics*, **39**, 143-147.

19. Eriksen, B.A. & Eriksen, C.W. (1974). Effect of noise-letters on identification of a target letter in a nonsearch task. *Perception & Psychophysics*, **16**, 143-149.

20. Eriksen, C.W. & St. James, J.D. (1986). Visual attention within and around the field of focal attention: A zoom lens model. *Perception & Psychophysics*, **40**, 225-240.

21. Fleet, D.J., Hallett, P.E., & Jepson, A.D. (1985). Spatiotemparal inseparablity in early visual processing. *Biological Cybernetics*, **52**, 153-164.

22. Freyd, J. & Tversky, B. (1984) Force of symmetry in form perception. American Journal of Psychology, 97, 109-126.

23. Garner, W.R. (1962). *Uncertainty and structure as psychological concepts.* New York: Wiley.

24. Garner, W.R. (1970). Good patterns have few attentives. *American Scientist,* **58**, 34-42.

25. Garner, W.R. & Sutliff, D. (1974). The effect of goodness on encoding time in visual pattern discrimination. *Perception & Psychophysics*, **16**, 426-430.

26. Goldmeier, E. (1972). Similarity in visually perceived forms. *Psychological Issues*, **8(1)**, Monograph 29 (Originally published in German, 1937).

27. Gregory, R.L. (1973). The unfounded eye. In R. Gregory & E. Combrich (Eds.), *Illusion in nature and art*, Duckworth, London.

28. Gurney, R. & Browes, R.A. (1985). Micropattern properties and presentation conditions influencing visual texture discrimination. *Perception & Psychophysics*, **38**, 141-171.

29. Gurney, R. & Browes, R.A. (1989). Asymmetries in visual texture discrimination. *Spatial Vision*, **4**, 31-44.

30. Hochberg, J.E. & McAlister, E. (1953). A qualitative approach to figural ``goodness". *Journal of Experimental Psychology*, **46**, 361-364.

31. Jenkins, B. (1982). Redundancy in the perception of bilateral symmetry in dot textures. *Perception & Psychophysics*, **32**, 171-177.

32. Jenkins, B. (1983). Component processes in the perception of bilaterally symmetric dot textures, *Perception & Psychophysics*, **34**, 433-440.

33. Julesz, B. (1962). Visual pattern discrimination. *IRE Transactions on Information Theory*, **8**, 34-43.

34. Julesz, B. (1971). *Foundations of cyclopean perception.* Chicago: University of Chicago Press.

35. Julesz, B. (1981). Textons, the elements of texture perception, and their interactions. *Nature*, **290**, 91-97.

36. Kahneman, D. & Henik, A. (1981) Perceptual organization and attention. In M. Kubovy & J.R. Pomerantz (Eds.), *Perceptual Organization.* Hillsdale, NJ: Erlbaum.

37. Kanade, T. & Kender, J.R. (1983). Mapping image properties into shape constraints: skewed symmetry, affine-transformable patterns, and the shape-from-texture paradigm. In J. Beck, B.Hope, & A. Rosenfeld (Eds.), *Human & Machine Vision*, 237-256.

38. Kanizsa, G. (1979). *Organization in Vision.* New York, Praeger.

39. Koffka, K. (1935). *Principles of Gestalt Psychology.* Harcourt, Brace & World, New York.

40. Kohler, W. (1929). *Gestalt Psychology*, New York: Liverright.

41. Leenwenberg, E.L.J. (1969). Quantitative specification of information in sequential patterns. *Psychological Review*, **76**, 216-220.

42. Leenwenberg, E.L.J. (1971). A perceptual coding language for visual and auditory patterns. *American Journal of Psychology*, **84**, 307-349.

43. Leyton, M. (1986a) A theory of information structure I: General principles. *Journal of Mathematical Psychology*, **30**, 103-160.

44. Leyton, M. (1986b) A theory of information structure II: A theory of perceptual organization. *Journal of Mathematical Psychology*, **30**, 257-305.

45. Mach, E. (1886) *The analysis of sensations and the relation of the physical to the psychical.* New York: Dover, 1959 (1st German ed. published 1886).

46. Marr, D. & Hildreth, E. (1980) Theory of edge detection. *Proceeding of the Royal Society, B*, **207**, 187-217.

47. Marr, D. & Nishihara, H.K. (1978). Representation and recognition of the spatial organization of three-dimensional shapes. *Preceedings of the Royal Society of Lodon,* **B**, **200**, 269-294.

48. Palmer, S.E. (1983). The psychology of perceptual organization: A transformational approach. In J.Beck, B.Hope, & A. Rosenfeld (Eds.), *Human and machine vision*, **1**, 269-339. New York: Academic Press.

49. Palmer, S.E. & Hemenway, K. (1978). Orientation and symmetry effects of multiple, rotational, and near symmetry. *Journal of Experimental Psychology: Human Perception and Performance*, **4**, 691-702.

50. Pashler, H. (1990). Coordinate frame for symmetry detection and object recognition. *Journal of Experimental Psychology: Human Perception and Performance*, **16**, 150-163.

51. Pollen, D.A. & Ronner, S.F. (1983). Visual cortical neurons as localized spatial frequency filters. *IEEE Transaction on Systems, Man, & Cybernetics*, **SMC-13**, 907-916.

52. Posner, M.I. (1980). Orienting of attention. *Quarterly Journal of Experiment Psychology*, **32**, 3-26.

53. Posner, M.I., Suyder, C.R., & Davidson, B.J. (1980). Attention and the detection of signals. *Journal of Experimental Psychology: General*, **109**, 160-174.

54. Prinzmetal, W. (1981). Principles of feature integration in visual perception. *Perception & Psychophysics*, **30**, 330-340.

55. Restle, F. (1979). Coding theory of the perception of motion configurations. *Psychological Review*, **86**, 1-24.

56. Riley, D.A. (1963). Memory for form. In L. Postman (Ed.), *Psychology in the making*. New York: Knopf.

57. Rock, I. & Leaman, R> (1963). An experimental analysis of visual symmetry. *Acta Psychologica*, **21**, 171-183.

58. Royer, F.L. (1966). Figural goodness and internal structure in perceptual discrimination. *Perception & Psychophysics*, **1**, 311-314.

59. Royer, F.L. (1971). Spatial orientational and figural information in free recall of visual figures. *Journal of Experimental Psychology*, **91**, 326-332.

60. Royer, F.L. (1981). Detection of symmetry. *Journal of Experimental Psychology: Human Perception and Performance*, **7**, 1186-1210.

61. Simon, H.A. & Feigenbaum, E.A. (1964). An information processing theory of some effects of similarity, familiarization, and meaningfulness in verbal learning. *Journal of Verbal Learning and Verbal Behavior*, **3**, 385-396.

62. Stevens, K.A. (1979). Surface perception by local analysis of textures and contour. *Tech. Rep.* No. **512**, Cambridge, MA:MIT Artificial Intelligence Laboratory.

63. Treisman, A. (1982). Perceptual grouping and attention in visual search for features and for objects. *Journal of Experimental Psychology: Human Perception and Performance*, **8**, 194-214.

64. Treisman, A. & Gelade, G. (1980). A feature-integration theory of attention. *Cognitive Psychology*, **12**, 97-136.

65. Treisman, A. & Gormican, S. (1988). Feature analysis in early vision: Evidence from search asymmetries. *Psychological Review*, **95**, 15-48.

66. Tsal, Y. (1983). Movements of attention across the visual field. *Journal of Experimental Psychology: Human Perception and Performance*, **9**, 523-530.

67. Van der Helm, P.A. & Leeuwenberg, E.L.J. (1989). Accesibility, a criterion for regularity and hierarchy in visual pattern codes.

68. Vitz, P.C. (1966). Preference for different amounts of visual complexity. *Behavioral Science*, **11**, 105-114.

69. Von Helmholtz, H. (1867). Treatise on Psysiological Opticas, Vol. 3. Translation (1962) from the 3rd German edition. Dover Publications, New York.

70. Wertheimer, M. (1923). Untersuchungen zur lehre von der gestalt II. *Psychologiche Forschung*, **4**, 301-350.

71. Weyl, H. (1952). *Symmetry*. Priceton, NJ: Priceton University Press

72. Wagemans, J., Van Gool, L. & d'Ydevalle, G. (1992). Orientational effects and component processes in symmetry detection. The Quarterly Journal of Experimental Psychology, 44A, 475-508.

73. Washburn, D. K. & Crowe D. W. (1988). Symmetries of Culture. Theory and Practice of Plane Pattern Analysis. Seattle: University of Washington Press.