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The Higgs as a Supersymmetric Nambu-Goldstone Boson

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Contents

Abstract

This thesis deals with the idea that the Higgs scalar doublet of the Standard Model is a Goldstone boson arising from a spontaneous breaking of an approximate global symmetry, driven by new physics effects, otherwise unobserved. This would regulate the behavior of the Higgs potential with the aim of addressing the Standard Model hierarchy problem, limiting the validity of the Standard Model as an effective theory to processes at energies below a cutoff around the TeV scale. After introducing this paradigm, also reviewing some literature, we describe few explicit models, minimal in some sense, built in the context of supersymmetric field theories. We show the details of the constructions, in particular its novel properties given by the presence of supersymmetry. We take into account the most important experimental constraints and we argue that some versions of these theories can be soon tested at collider experiments.

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Foreword

This PhD thesis contains part of the work done by the author in the period 2012-2014 which led to the articles:

- [1] F. Caracciolo, A. Parolini and M. Serone, "*UV Completions of Composite Higgs Models with Partial Compositeness*", JHEP **1302** (2013) 066 [arXiv:1211.7290 [hep-ph]].
- [2] D. Marzocca, A. Parolini and M. Serone, "*Supersymmetry with a pNGB Higgs and Partial Compositeness*", JHEP **1403** (2014) 099 [arXiv:1312.5664 [hep-ph]].
- [3] A. Parolini, "*Phenomenological aspects of supersymmetric composite Higgs models*", [arXiv:1405.4875 [hep-ph]].

Chapter 1

Introduction

The Standard Model (SM) of particle physics is a renormalizable quantum field theory established in the last fifty years aiming to describe interactions among observed particles. Despite its accuracy in describing many observed processes there are several experimental hints suggesting that it has to be extended to take into account for other phenomena. The recent observation of a neutral scalar boson with a mass of 126 GeV [4, 5][1] resembling the Higgs boson carried renewed vigor to the, otherwise unverified, hypothesis of the breaking of the electroweak (EW) symmetry through the simplest mechanism, an elementary scalar field charged under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ with a non vanishing vacuum expectation value (VEV).

Despite its simplicity the idea of a light elementary field is not appealing from a theoretical point of view because light scalars are unnatural. The SM is no exception: it classically possesses an approximate scale invariance symmetry broken by the mass term of the only scalar field present, the Higgs field. This mass term in turn sets the EW scale, the VEV v of its neutral component, which is experimentally constrained to be $v = \left(\sqrt{2}G_F\right)^{-1/2} \simeq 246$ GeV. Quantum fluctuations make the mass square (quadratically) sensitive to any cutoff of the theory and so a natural theory, in which this quantity is what is expected to be, either has a cutoff at the EW scale or is valid on its own at arbitrary high energies. There are a lot of experimental hints

[1] In the past few months the experimental collaborations provided a measurement with smaller uncertainties, of the order of the percent; the value of the measured mass is closer to 125 GeV rather than 126 GeV [6, 7]. Nevertheless we will continue to refer to 126 GeV because most of the work collected here has been carried on before this measurement: in any case the change is small and no conclusion is affected by this new result.

suggesting that the second cannot be true,[2] the strongest among them is not a hint but a fact: gravity exists and becomes non negligible at energies comparable with the Planck scale $M_{Pl} \simeq 10^{18}$ GeV. Therefore in this case enormous cancellations among unrelated quantum contributions to the Higgs mass occur leading to the observed value

$$\left(\frac{v}{M_{Pl}}\right)^2 \simeq 10^{-32} \ll 1\,, \tag{1.1}$$

and at the present time we do not have any reason why this should happen. We stress that independently of the new physics entering at M_{Pl} the observed value of the Fermi scale is extremely unnatural because under the SM RG evolution this solution is highly unstable: if naturalness is restored it has to be done at a low scale. Otherwise we are led to accept the unnaturalness of the world in which we live and explain the needed Fine Tuning (FT) as a consequence of some environmental selection: this calls for extensions of the SM, as certain constructions from string theory, with a (large) number of vacua. In each of this vacuum (a universe) v assumes a different value and we observe only a value of v which is compatible with the existence of observers: determine which values are allowed, and which SM parameters (either dimensionless or dimensionful) are allowed to vary from one universe to another is a hard task. Without strict assumptions about the detailed structure of the landscape of vacua and assuming that the values for the varying parameters are distributed with a reasonable probability an efficient mechanism to fix them is to find conditions related to the presence of observers which put bounds (or corners in the case of a multidimensional parameter space) on these values. For instance consider the case of a single parameter x occurring with a probability density $p(x)$ on the vacua: then

$$\frac{\mathrm{d}P}{\mathrm{d}x} = p(x) \;\Rightarrow\; \frac{\mathrm{d}P}{\mathrm{d}y} = p(e^y)e^y \quad \text{where} \quad y = \log x \tag{1.2}$$

and a flat distribution $p'(x) = 0$ implies that larger values of x are exponentially favored. If at the same time there exists a limiting value x_c such that

[2] An example is the measured value of the physical Higgs mass: 126 GeV fixes the quartic coupling at the weak scale such that it crosses zero at a scale 10^{11} GeV becoming negative. Strictly speaking this is not a problem because this running is subject to large uncertainties and this scale can be shifted to M_{Pl} within 2σ. Moreover for the central value the lifetime of the EW breaking vacuum is larger than the age of the Universe [8,9].

universes with $x > x_c$ do not lead to the formation and survival of observers we conclude that most likely we live in a universe with a measured value of x of the same order of magnitude x_c, saturating the upper bound. From this discussion it is already clear that this line of reasoning can be extended to "explain" the value of other SM parameters different from the Higgs VEV v: in fact it was originally proposed for the cosmological constant [10], with several refinements as [11, 12], and we can also imagine to generalize it. In general it is not simple to decide which parameters (and how and why) are anthropically chosen. It is worthwhile mentioning that within the context of multiverse theories anthropic selection is not the only option and there is the possibility that some dynamics is responsible for selecting specific values for parameters, see e.g. the discussion of near-criticality in [9]. Finally we remark that detecting, and therefore confirming, the existence of other universes seems very infeasible. We will not further discuss this possibility.

The other possibility[3] is a low energy cutoff close to the EW scale but not necessarily coinciding with it: taking into account a possible loop suppression, the SM as an effective theory should be valid up to the TeV scale and the resulting theory would be perfectly natural. This second attitude is not on firm grounds, we can collect a number of arguments pointing to the direction of a high energy cutoff: flavor measurements, precision tests, a possible gauge coupling unification, neutrino masses through a see-saw mechanism, inflation. Any physics Beyond the SM (BSM) should take them into account. The most urgent threat to natural physics at the TeV comes from experimental searches, mostly at the LHC. Many models studied in the literature predict the existence of new states, typically charged under the $SU(3)_c$ SM color gauge group: their presence is related to the dominant role of the top quark in the loops correcting the Higgs mass in the SM. They should be copiously produced through colored interactions and then decay leaving detectable signatures. Currently bounds for stop and gluino masses are around

[3] To be fair we mention a third way to resolve the dichotomy: if for some reason new physics, including gravity, does not introduce any mass shift proportional to a threshold scale a small value of the Higgs mass is natural in the sense that it is stable under RG evolution: this is because its beta function is proportional to the mass itself. Despite the fact that it is not known if this possibility can be realized it has lately received attention, under the names of finite naturalness, UV naturalness, natural tuning and similar names. Also, in the context of the minimal supersymmetric SM (MSSM) specific boundary conditions at some high scale, as M_{GUT}, allow for the existence of a focus point and a Higgs soft mass not scaling with the other soft terms is obtained without a large FT.

600 GeV and above 1 TeV respectively, while for Kaluza Klein (KK) gluon excitations the bound is around 2 TeV and finally 800 GeV for composite top partners with exotic 5/3 electric charge [13]: see the supersymmetry (SUSY) and Exotics webpage of ATLAS and SUSY and Exotica of CMS collaboration. These bounds are derived under simplifying assumptions and are not in general valid for any model of BSM physics: on the contrary there are many explicit models evading these bounds and preserving naturalness and they are an active research direction. Anyhow in this regard we well resume the general feeling reinterpreting a quote of E. Fermi: *where is everybody?*

SUSY surely offers one of the best motivated possibilities to solve the hierarchy problem in a natural way. Relating the number of bosonic and fermionic degrees of freedom in the UV allows for cancellations in the loop contributions. The Minimal Supersymmetric Standard Model is the simplest SUSY generalization of the SM: it introduces SUSY partners for every SM field and it contains one Higgs doublet more, necessary for anomaly cancellations. Furthermore it has many other welcome features, namely it improves gauge coupling unification and it provides a dark matter candidate. Nevertheless the Higgs boson mass is predicted to be too low: at tree-level it has to satisfy the bound

$$M_H^2 \leq m_Z^2 \cos^2 2\beta \tag{1.3}$$

where $\tan\beta$ measures the relative importance of the two Higgs doublet in participating to the EWSB mechanism, it is the ratio of the two VEVs. Loop corrections are important and necessarily depend on SUSY breaking parameters, they can increase the Higgs mass at the price of a larger amount of FT, which is around 1% or less in the MSSM [14]. At tree-level the largest source of FT is given by higgsino masses, governed by the so called μ term of the superpotential; at one-loop also stop soft parameters enter and finally, at two-loops, gluino masses are responsible, if too heavy, for a certain amount of FT. The situation gets improved for non minimal extensions, new (super)fields with new interactions can increase the Higgs mass. For instance the Next to Minimal Supersymmetric SM (NMSSM) is just the MSSM coupled to a chiral superfield, gauge singlet, with appropriate superpotential interactions.

Moreover there is a more profound reason why the MSSM is not enough, and it is linked to the SUSY breaking. The breaking has to be soft, namely only relevant operators are allowed, such that the UV nice behavior of the theory is not spoiled. But we need other fields spontaneously breaking SUSY and we have to make sure that there exist interactions[4] among this new sector and the MSSM fields, the visible sector, such that SUSY breaking is induced also for visible fields, namely SUSY particles are much heavier than their ordinary partners, satisfying at the same time other constraints for instance from flavor physics. The necessity of a separate sector responsible for SUSY breaking comes from the supertrace mass formula, which relates the sum of the squared masses of all the particles in the spectrum, properly weighted according to their spin, to the D terms computed in the vacuum: it cannot be satisfied with only the MSSM fields.

Finally we stress that SUSY can be broken either at tree-level or by non perturbative effects. This second option is favored because it dynamically generates a hierarchy among mass scales. A very generic expectation is a SUSY theory spontaneously broken, by non perturbative effects, in a metastable vacuum. A central role is played by the R symmetry, a bosonic U(1) whose generator does not commute with the supercharges. It forbids (Majorana) gaugino masses and therefore it has to be broken but, on the one hand, if it is spontaneously broken in the vacuum it implies the presence of a too light R axion, the associated Goldstone boson; on the other hand it can be shown that in a generic theory without an R symmetry SUSY is not broken [15]. However it can be that we start with a theory without an R symmetry and besides one or more SUSY preserving vacua there exists a metastable vacuum, a local minimum of the potential, breaking SUSY. In this vacuum there exists an approximate R symmetry which still allows for gaugino masses, but the full theory does not respect it. If its lifetime is long enough it provides a viable way out of the aforementioned tension.

Composite Higgs Models (CHM) are another possible extension of the SM of the second attitude mentioned before, aiming to solve the naturalness problem in a natural way, namely introducing new physics at the TeV scale. The general idea is that the Higgs is not an elementary scalar but instead a bound state of some new objects charged under a new, still unknown, strongly coupled gauge theory. This is similar to technicolor theories in which a condensate breaks the EW symmetry $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ to the $\mathrm{U}(1)_Q$: the similarity is in having a new gauge interaction responsible for this breaking; the main difference is that in CHM there exists a light scalar in the spectrum, with the right quantum numbers, whose VEV breaks the EW group. Several

[4]Gravity will eventually couple the two sectors. One might want, for several reasons, to avoid relying on it and having some other mechanisms of mediation.

different realizations of this idea have been studied and they can be classified: an up to date review, containing also several other interesting discussions, is given in [16]. In this thesis we concentrate on models of Higgs as a pseudo Nambu Goldstone bosons (pNGB): we do not discuss little Higgs or other realizations. A doubtless elegance of these models is in the dynamical origin of EWSB, whereas many others models, as for instance the MSSM, simply accept it as a matter of fact item.

As mentioned before any light scalar is unnatural because its mass is additively renormalized. Moreover scalar bosons are not protected by any symmetry and the massless limit does not restore any symmetry. This is not the case for Goldstone bosons of a spontaneously broken global symmetry: in this case the broken symmetry is non linearly realized and the associated Goldstones transform under shift. Therefore they do not have non derivative interactions and they are massless: in fact they are so constrained that their low energy Lagrangian, a non linear σ-model, is determined only by the structure of the coset associated to the breaking, regardless any other detail [17, 18]. Identifying the components of the Higgs doublet as Goldstone fields would therefore predict their mass to be exactly zero. Since this is not the case we want the global symmetry spontaneously broken to be only an approximate symmetry of the theory, namely we introduce some terms of explicit breaking: these terms drive a non trivial profile for the would be Goldstones. This radiatively induced potential is responsible for the EW breaking providing a non vanishing VEV for the Goldsones identified with the Higgs.

The typical situation is as follows: a gauge theory is responsible, through non perturbative effects, to the breaking of a global symmetry G to a proper subgroup H: the scale associated, denoted by Λ, is dynamically generated and exponentially suppressed with respect to the scale where the theory is defined perturbatively, for instance M_{Pl}. The scale $f < \Lambda$, the decay constant of the σ-model, regulates the interactions of the associated Nambu Goldstone bosons. The unbroken symmetry H has to contain the SM group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$: the broken generators have to be charged under the EW group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ in order to be identified with the Higgs doublet. This means that the symmetry G is explicitly broken by a partial gauging: moreover the coupling to SM fermions, in particular the top, is also explicitly G violating. This explicit breaking is mediated by a mechanism named Partial Compositeness (PC): SM fields acquire small components in terms of bound states of the new sector, in contrast to the Higgs which is fully composite. The resulting neat effect is a potential for the Goldsone bosons: it provides a VEV $v < f$ to the neutral component of the Higgs. The ratio

$$\xi = \frac{v^2}{f^2} \tag{1.4}$$

quantifies the separation of scales occurring among the Higgs and new physics. While ξ is generically expected to be $O(1)$ a certain separation is requested from phenomenological reasons: therefore ξ encodes a measure of the FT of the model. In the following we focus on the benchmark value

$$\xi = 0.1\,. \tag{1.5}$$

In the models which we are going to discuss the FT is estimated in a refined way and it is computed numerically using the definition of [19] with logarhitmic derivatives: the actual level of FT is found around $1 - 2\%$.

Given the negative results so far obtained for any BSM physics search, and insisting on naturalness, it is more than worthwhile to explore less minimal scenarios and to pursue new ideas. Therefore it has been natural to study supersymmetric realizations of pNGB CHM; the benefits from the union of the two frameworks well studied in separation, SUSY and composite Higgs, are manifold as we will explain in detail: SUSY provides tools to deal with strongly coupled theories, namely it allows us to build explicit models whose range of validity as effective field theories is not limited by non perturbatively generated energy scales[5], and it helps in controlling the little hierarchy problem generically affecting BSM theories; at the same time stops or other superpartners are not already excluded and the same is true for fermionic resonances, while they are within the reach of Run II at LHC in their less tuned versions.

Pure four dimensional UV completions of the CHM paradigm have been recently investigated also in [20–22], focusing on constructions without elementary scalars. On the other hand holographic descriptions, based on five dimensional spacetime, of the strongly coupled gauge theory have been exploited: we will comment on them in the next chapter. SUSY allows, namely thanks to Seiberg duality, a greater level of calculability, and therefore predictivity, than generic gauge theories, without enlarging the number of spatial dimensions.

[5]We often encounter Landau poles at high energies, therefore a UV cutoff must exist at some scale below M_{Pl}; the interesting part is that its appearance is not strictly related to the strong dynamics producing the pNGB Higgs.

The rest of the thesis is organized as follows: in chapter 2 we introduce CHM based on a pNGB Higgs, we discuss partial compositeness, the Higgs potential, the main features and some experimental constraints; from chapter 3 on we concentrate on original results and we turn to supersymmetric models, we again analyze the Higgs potential on general grounds and we sequently focus on one specific model in some sense minimal. The parameter space is inspected with the help of numerical scans. In chapter 4 we move to another model with a richer gauge structure, reviewing the most important bounds from negative direct detection of heavy states combined with a numerical scan. We review how to deal with the RG flow of UV soft SUSY breaking terms in presence of confining dynamics. Since the model sits in a metastable vacuum we address the issue of its decay and we show that a lifetime larger than the age of the universe is fully compatible with all the other requirments. We also discuss a mechanism, different than partial compositeness, to transmit EWSB to all SM fermions (we do it explicitly for quarks): we rely on dimension five operators in the initial superpotential. We check also that they satisfy bounds coming from flavor observables, mainly from mesons physics. Chapter 5 is devoted to the study, both analytical and numerical, of an excluded model in which the right top is fully composite: the discussion proceeds, with less details, in resemblance to the one carried on in the prior chapter. Finally we draw our conclusions and we close the thesis with few appendices. In appendix A we collect some one-loop beta functions coefficients on which we rely in the main text and we explicitly show how they enter the scale dependence of the Higgs potential. In appendix B we study how the non linearities of the Higgs field and new massive resonances modify the high energy behavior of the two to two scattering of EW bosons in the two models discussed in the text: in particular we show that perturbative unitarity of the amplitude of the process is recovered. We report our group matrices conventions in appendix C.

Chapter 2

Composite Higgs Models

This chapter is devoted to introducing the pNGB Higgs. We begin reviewing the Callan Coleman Wess Zumino (CCWZ) description of low energy Lagrangian for Goldstone bosons in a generic theory and we employ it together with some general parametrization of the strong sector breaking the symmetry. We describe the interactions with the SM and how they generate a potential for the Higgs, outlining its most relevant features. We also make contact with extra dimensional theories, also via the AdS/CFT correspondence. Finally we survey the experimental constraints on these models, both from direct searches and indirect measurements.

2.1 The CCWZ Construction

We briefly recall the CCWZ construction of the low energy Lagrangian for Goldstone bosons [17,18]. Given a global symmetry (Lie) group G broken to a subgroup H we distinguish the generators of the associated algebra among unbroken T^i and broken X^a. They satisfy the following

$$[T^i, T^j] = if^{ijk}T^k, \quad [T^i, X^a] = if^{iab}X^b, \quad [X^a, X^b] = if^{abc}X^c + if^{abi}T^i \,. \tag{2.1}$$

The structure constants f are antisymmetric in their indices. We work with orthogonal and unitary groups and a simple realizations for the generators is in terms of hermitian matrices. If the constants f^{abc} vanish the coset is said to be symmetric and its Lie algebra is invariant under a discrete $\mathbf{Z}_2$ symmetry under which T^i are even and X^a are odd; we call R this automorphism of the algebra of G:

$$R(T^i) = T^i, \quad R(X^a) = -X^a\,. \tag{2.2}$$

In a neighborhood of the identity any $g \in G$ can be decomposed as

$$g = e^{i\xi\cdot X} e^{iu\cdot T} \tag{2.3}$$

such that

$$\forall\, g_0 \in G \quad g_0 e^{i\xi\cdot X} = e^{i\xi'\cdot X} e^{iu'\cdot T} \tag{2.4}$$

where ξ' and u' are appropriate functions of ξ and g_0. With the definition $U = e^{i\xi\cdot X}$ this last can be expressed as

$$U \to g_0 U h^{-1}(\xi, g_0)\,. \tag{2.5}$$

We focus on the product

$$U^{-1}\partial_\mu U = iX^a D^a_\mu + iT^i E^i_\mu \tag{2.6}$$

where the decomposition along the generators is meaningful because the l.h.s. is an element of the algebra of G. Under a transformation $g_0 \in G$

$$U^{-1}\partial_\mu U \to h(U^{-1}\partial_\mu U)h^{-1} - (\partial_\mu h)h^{-1}\,. \tag{2.7}$$

From this we can read:

$$\left\{ \begin{array}{l} D_\mu = X^a D^a_\mu \to h D_\mu h^{-1} \\ E_\mu = T^i E^i_\mu \to h(E_\mu - i\partial_\mu)h^{-1} \end{array} \right. . \tag{2.8}$$

With these objects we can build a Lagrangian invariant under a linear and local H symmetry, where E_μ acts as a gauge field and we can build out of it covariant derivatives and a field strength. This Lagrangian is also symmetric under any transformation $g_0 \in G$, which acts non linearly. In an expansion in the number of derivatives the leading term is given by

$$\frac{1}{2} f^2 D^a_\mu D^{a\mu}\,, \text{ where } D^a_\mu = \partial_\mu \xi^a + \ldots \tag{2.9}$$

One can prove that under a transformation generated by broken X^a, ξ transforms with a shift; moreover ξ^a are identified with the canonically normalized Goldstone bosons, $\xi^a = f^{-1}\pi^a$.

For symmetric coset space the matrix U^2 turns out to be useful because it transforms linearly under G: it can be proven that

$$U^2 \to gU^2R(g)^{-1} \tag{2.10}$$

where R is the automorphism defined in eq.(2.2).

With this procedure of constructing invariant Lagrangians one can miss terms which are not invariant but shift by a total derivative under a symmetry transformation [23, 24]. These terms in the action can be written as

$$S_{WZW} = \int_{S^4} \beta \tag{2.11}$$

where S^4 is the (compactified) spacetime. If β shifts by a total derivative under a transformation symmetry the 5-form $\omega = \mathrm{d}\beta$ is left invariant. Because of the Stokes theorem we can also write

$$S_{WZW} = \int_{S^4} \beta = \int_{\mathcal{B}} \omega \tag{2.12}$$

such that $\partial\mathcal{B} = S^4$. Therefore the allowed invariants are related to $H^5(G/H, \mathbb{R})$, the fifth cohomology class of the coset G/H [25]. However for the only coset we are going to discuss, SO(5)/SO(4), this object vanishes and none of these terms can be added to the effective Lagrangian.

Finally the present discussion can be generalized to the case of a gauged subgroup of G: the simplest way is to gauge the entire group G and discard the non dynamical fields at the end, consistently replacing derivatives with covariant ones and adding kinetic terms for the field strengths. As expected the Goldstone bosons along the broken gauged directions are eaten.

One may be interested in including in the low energy Lagrangian also other fields, massless or massive. Their interactions are not completely fixed by symmetry considerations, nevertheless they can be included in a G invariant way if they transform in a proper way under the unbroken subgroup H, namely if they fullfill representations of H: it is sufficient to note that with the help of the matrix U we can "lift" them to representations of G; this fact is simple to understand recalling eq.(2.5). From a top down perspective, knowing a UV Lagrangian spontaneously breaking $G \to H$, the redefinition of all the fields through the matrix U makes the Goldstones disappear from the non derivative part of the Lagrangian. In our context this operation is useful to treat the explicitly G breaking terms.

An outstanding example of the use of the CCWZ formalism is the construction of the pion Lagrangian in QCD (with two quarks only, up and down): it presents analogies with the case of composite Higgs. Pions are the Goldstone bosons associated to the spontaneous breaking of the chiral symmetry $SU(2)_L \times SU(2)_R \to SU(2)_V$ triggered by non perturbative effects. This formalism allows to overtake the difficulties of the strongly interacting QCD and we correctly identify the low energy degrees of freedom, the pions. The Wess Zumino Witten term, described in eq.(2.11) and eq.(2.12), for the QCD is responsible for a term in the Lagrangian of the form

$$\mathcal{L} \supseteq -\frac{N_c e^2}{48\pi^2 f_\pi}\pi^0 \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \tag{2.13}$$

where $N_c = 3$ is the number of QCD colors and f_π is the pion decay constant. This describes an interaction vertex among the neutral pion and photons and it is related to the decay process $\pi^0 \to 2\gamma$. Moreover the chiral symmetry is not a true global symmetry of the QCD Lagrangian: it is explicitly broken by quark masses and by the gauging of the electromagnetic group $U(1)_Q$. These breaking generate a potential for the pions, otherwise forbidden. Indeed pions in nature are not massless particles. Quark masses

$$\mathcal{L} \supseteq m_u \bar{u} u + m_d \bar{d} d = \bar{q} M q \tag{2.14}$$

can be written in a way formally respecting the chiral symmetry, namely pretending that the matrix M transforms as a bidoublet, $M \to LMR^\dagger$ acting with an element of $SU(2)_L \times SU(2)_R$. Imposing formal chiral invariance, with the help of this spurious mass matrix, we can argue that the low energy Lagrangian will contain

$$\mathcal{L} \supseteq c f_\pi^3 \mathrm{Tr}[MU] \tag{2.15}$$

where U contains the three pions $\pi^\pm$ and π^0, $U = \exp\left(\frac{i}{f}\sigma^a \pi^a\right)$, $a = 1, 2, 3$ and $\mathrm{Tr}[\sigma^a \sigma^b] = \delta^{ab}$. Expanding for large f_π we obtain a mass term for the pions,

$$\mathcal{L} \supseteq -\frac{c}{4} f_\pi (m_u + m_d) \pi^a \pi^a \,. \tag{2.16}$$

A more detailed computation leads to the exact determination of the pion masses, $c = \frac{\langle \bar{q} q \rangle}{f_\pi^3}$. The presence of the $U(1)_Q$ further breaks the chiral symmetry and the electric charge is responsible for the mass difference among the charged pions and the neutral one.

2.2 Composite Higgs Models

The main ingredient of pNGB CHM is a strongly interacting sector which we consider at first in isolation from the SM: it contains the new physics showing up at the TeV scale. We frequently refer to it as composite sector, in contrast to an elementary sector which contains the SM fields, to stress that in the composite sector a new gauge interaction is at work, resulting in bound states; more in general, with the aim of a unified discussion, we will continue to denote as composite the sector responsible for the pNGB Higgs also in cases where the fields are not composite objects. Original models were formulated in terms of a conformal field theory (CFT) and the relevance of holographic techniques have been extensively stressed [26]. They are characterized by two mass scales: starting from a UV point, either free or interacting, the theory flows to an IR scale, hierarchically suppressed, where it becomes strongly coupled and an operator breaks a global symmetry G to a proper subgroup H. Before entering the details of the construction we note en passant the analogy with the chiral Lagrangian for pions described at the end of section 2.1: in that case the symmetry breaking pattern is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$ and we have three Goldstone bosons, the pions; we instead focus on cosets with four Goldstone and we identify them with the Higgs. In the case of QCD we have mass terms for quarks and the electromagnetic interactions explicitly breaking the chiral symmetry, while in our pNGB Higgs we introduce mass mixings, under the name of partial compositeness, and we gauge the EW group, again bringing an explicitly breaking: in turn it induces a potential for the Goldstones. The main difference is that in QCD the pion masses are positive, here we obtain a negative (squared) mass for the Higgs, since it has to trigger EWSB.

In the literature several models have been extensively explored, for instance $\mathrm{SU}(N)/\mathrm{SO}(N)$ and $\mathrm{SO}(N)/\mathrm{SO}(N-1)$. We focus on the simplest realization allowing for a custodial symmetry with $G = \mathrm{SU}(3)_c \times \mathrm{SO}(5) \times U(1)_X$ and $H = \mathrm{SU}(3)_c \times \mathrm{SO}(4) \times U(1)_X$: the interesting part of the breaking is in $\mathrm{SO}(5) \to \mathrm{SO}(4)$ and the other factors are spectators, nevertheless they are needed for a realistic model[1]. The four Goldstone bosons live on the coset $\mathrm{SO}(5)/\mathrm{SO}(4) \equiv S^4$ and they are identified with the Higgs doublet of the SM: therefore we realize the SM symmetries gauging a subgroup of

[1] Strictly speaking G and H can be larger, the key point is that they contain $\mathrm{SO}(5) \times U(1)_X$ and $\mathrm{SO}(4) \times U(1)_X$ respectively.

the global symmetry of the composite sector: we recall the isomorphism $\mathrm{SO}(4) \simeq \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and we identify the hypercharge as $T_R^3 + X$, where the X can be seen as proportional to the SM $B - L$; the choice for the color group is self evident. With this identification the real fourplet in the **4** of SO(4) has the right quantum numbers, it is a composite state of the new physics sector and it has a vanishing potential: it is a good candidate to play the role of the Higgs doublet. Moving to the realistic case with the SM coupled the total Lagrangian schematically is

$$\mathcal{L} = \mathcal{L}_{comp} + \mathcal{L}_{el} + \mathcal{L}_{mix} \,. \tag{2.17}$$

The first term in the Lagrangian is coming from the strongly interacting theory and it contains, among others, the kinetic term for the pNGB Higgs from eq.(2.9). The second term, elementary in contrast to the composite one, is the Lagrangian for the SM fields without the Higgs, namely the kinetic terms for fermions and gauge bosons. These fields are neutral under the new strong gauge group and therefore they do not participate in the formation of bound states. The third part is the mixing part and it has the form

$$\mathcal{L}_{mix} = g_a J^a_\mu A^{a\mu} + \cdots + \sum_r \epsilon_r \bar{\psi}_r O_r + h.c. \,. \tag{2.18}$$

The first part contains the currents of the composite sector associated to symmetries gauged by the SM, where the dots include possible terms with higher powers of the gauge fields; the second part is a sum of portal interactions for SM fermions, in principle $\psi_r \in \{q_L, u_R, d_R, l_L, e_R\}$, with composite operators O_r with suitable quantum numbers. The energy scaling of ϵ_r's are dictated by the anomalous dimensions $\gamma_r \simeq \mathrm{dim}(O_r) - \frac{5}{2}$: the scaling dimensions of O_r can be significantly different from the classical ones and therefore each of these mixings ϵ_r can be driven weak or strong in the IR. Because of them the SM fermions are partially composite where the degree of compositeness varies for different r, and for different families, and it regulates the strength of their coupling to the Higgs boson: the mass eigenstates coming from larger mixings have a more significant component in terms of composite states and they couple more to the Higgs, therefore they are heavier. It is clear that the more composite SM fermion is the top and we expect the associated anomalous dimensions γ's to be the smallest.

Partial compositeness, introduced in [27], is studied in a simplified version in [28]. It consists in treating O_r as vector-like (Dirac) massive fermions and declaring that ϵ_r are mass mixings: the relevant part of eq.(2.17) can be written as

$$\mathcal{L} = \bar{\psi}_r i\partial\!\!\!/ \psi_r + \bar{O}_r(i\partial\!\!\!/ - M_r)O_r + \epsilon_r\bar{\psi}_r O_r + h.c. \tag{2.19}$$

The mass eigenstates are given by

$$\begin{pmatrix} \cos\phi_r & \sin\phi_r \\ -\sin\phi_r & \cos\phi_r \end{pmatrix} \begin{pmatrix} \psi_r \\ P_r O_r \end{pmatrix} \tag{2.20}$$

where P_r projects onto the suitable chirality state and the mixing angle is $\tan\phi_r = \frac{\epsilon_r}{M_r}$; the mass eigenvalues are 0 and $(M_r^2 + \epsilon_r^2)^{1/2}$. We understand this approximation looking at the two point function $\langle O_r(q)\bar{O}_r(-q)\rangle$: in the large N limit of a gauge theory we expect it to be the sum of narrow resonances with increasing masses and the above simplification is nothing more than the truncation to the first resonance; alternatively we may think to rotate to a basis where only one state, approximatively the lightest, has a mixing with the elementary ψ_r. An analogous reasoning line applies to vector currents and heavy spin-1 resonances.

In the SUSY models we will discuss the operators responsible for fermionic partial compositeness are marginal in the UV, where the BSM is weakly coupled, and they flow to relevant deformations. They are mesonic, rather than baryonic, fermion states of the confining gauge theory: their presence is well understood thanks to SUSY and Seiberg duality.

$\mathcal{L}_{mix}$ is explicitly SO(5) breaking and therefore the degeneracy of the Higgs is lifted. The potential generated by gauge interactions does not trigger EWSB because it tends to align the vacuum in a preserving direction [29], therefore the contribution of the matter fields, mainly the top fermion, is crucial. We will elaborate more on this compensation of effects as one of the sources of the FT.

2.2.1 Spurionic Formalism for the Mixing Lagrangian

The mixing Lagrangian eq.(2.18) can be made formally SO(5) invariant enlarging the symmetry to $\mathrm{SO}(5) \times \mathrm{U}(1)_X \times \mathrm{SU}(2)_{0,L} \times \mathrm{U}(1)_{0,R} \times \mathrm{U}(1)_{0,X}$; the operators O_r transform in $\mathrm{SO}(5) \times \mathrm{U}(1)_X$ representations, the SM fields ψ_r are charged under the other $\mathrm{SU}(2)_{0,L} \times \mathrm{U}(1)_{0,R} \times \mathrm{U}(1)_{0,X}$ factors and the couplings ϵ_r's are promoted to spurions charged under both symmetries: their VEV break the symmetry to a diagonal combination.

$$\sum_r \bar{\psi}_r \cdot \epsilon_r \cdot O_r + h.c.\,. \tag{2.21}$$

Similarly promoting the SM gauge coupling g and g' to spurions allows to recover SO(5) invariance [30]. Alternatively, but completely equivalently, we can embed the elementary fields ψ_r in spurionic representations of SO(5) ξ_r whose extra components are set to zero. In this way again the invariance is formally recovered:

$$\sum_r \epsilon_r \; \xi_r O_r + h.c. \, . \tag{2.22}$$

ξ_r is chosen such that the product with O_r contains the singlet, therefore it depends on the details of the composite sector. Typically the most studied representations are the smallest ones which are **4**, **5**, **10** and **14** for SO(5) [31]. The neat examples of the following chapters are based exclusively on the **5**.

If we redefine all the fields in the composite sector as explained at the end of section 2.1 with the aim of singling out the Goldstones we remove the Higgs dependence from the non derivative part of the Lagrangian and the only places where it remains are the SO(5) breaking terms of $\mathcal{L}_{mix}$ eq.(2.18).

2.3 Extra Dimensional Models

In the following we will move to study the main predictions of CHM but before doing it we comment on extra dimensional theories, in particular on possible extensions of the SM defined on a five dimensional spacetime where one of the spatial dimensions is compactified and we explain their relation with CHM introduced in the previous sections. Extra dimensional models have provived many calculable examples in particular as holographic descriptions of the strong sector postulated in section 2.2. With the language offered by holography extra dimensional theories and strongly coupled CFT are two equivalent ways of describing the same physics.

The simplest example is $\mathbb{R}^{1,3} \times S^1$, although it turns out that the choice of the orbifold $\mathbb{R}^{1,3} \times \frac{S^1}{\mathbb{Z}_2}$ is much more convenient and interesting. The first consequence of orbifolding the fifth dimension is that proper boundary conditions allow to introduce a chiral fermion field, despite the fact the the spinorial representation of $SO(4,1)$ is not reducible, that is a fermion in five dimensions is defined with both its left and right chirality components.

There is a second advantage from compactifying the extra dimension on an orbifold, again coming from the imposition of adequate boundary conditions: in the case of gauge theory we can reduce the gauge group at low energy. In particular on the branes gauge invariance is associated to Neumann boundary conditions for the corresponding vector field and a five dimensional gauge theory can be reduced in two different ways at the orbifold fixed points.

The description of fields in extra dimensional theories and their reduction to four dimensions take great advantage from the Anti de Sitter/Conformal Field Theory (AdS/CFT) correspondence. This conjectured correspondence helps in making contact between extra dimensional models and CHM, it provides a description of a BSM CFT in terms of a gravitational dual very often more calculable than the original field theory. Therefore in the following we introduce and briefly discuss warped spaces[2], and the correspondence and finally we comment back on CHM models.

In a theory with n extra dimensions the four dimensional Planck scale M_{Pl} is related to the $4+n$ one M by

$$M_{Pl}^2 = M^{n+2} V_n \tag{2.23}$$

where V_n is the volume of the compact extra dimensions and typically $V_n \sim r_c^n$ for a compactification radius r_c. In [34] it has been considered a setup, universally known as RSI, of a five dimensional theory with the topology of $\mathbb{R}^{1,d-1} \times \frac{S^1}{\mathbb{Z}_2}$: at the fixed points of the orbifold two 3 branes are placed, supporting four dimensional field theories. On these branes an $\mathrm{SO}(3,1) \subseteq \mathrm{SO}(4,2)$ isometry group survives.

It has been shown that, with a proper choice of the vacuum energies defined in the bulk and on the branes, the resulting spacetime has the geometry of a slice of an Anti de Sitter (AdS) space with a curvature function of M and another mass scale $k < M$. The compactification radius r_c is a free parameter but sensible constructions require

$$M_{Pl} > M > k > \frac{1}{r_c}\,. \tag{2.24}$$

The condition $k < M$ ensures the stability of the AdS solution against quantum gravitational corrections. The metric can be explicitly written (choosing a mostly minus signature) as

$$\mathrm{d}s^2 = e^{-2kr_c|y|} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu - r_c^2 \mathrm{d}y^2 \tag{2.25}$$

[2]Notice that holographic models with a flat extra dimensions have been also studied, see e.g. [32, 33].

where y parametrizes the fifth dimension, $y \in [-\pi, \pi]$ and the 3 branes are located at $y = 0$ and $y = \pi$, the orbifold fixed points; we emphasize the consequences of the non factorizability of this geometry. Eq.(2.23) becomes

$$M_{Pl}^2 = \frac{M^3}{k}[1 - e^{-2kr_c\pi}] \simeq \frac{M^3}{k} \tag{2.26}$$

and therefore $M < M_{Pl}$. The exact value of r_c does not affect the four dimensional Planck scale but it has a remarkable consequence: it enters the theory living at $y = \pi$ through the factor $e^{-2kr_c\pi}$ from the restriction of the metric to the brane, after properly rescaling the fields of the bulk action. A mass term v is redshifted to

$$v' = e^{-kr_c\pi} v\,. \tag{2.27}$$

For a generic relevant operator of dimension α we expect the coupling λ to be sent to $\lambda' = e^{-kr_c\pi(4-\alpha)}\lambda$. In this way a value $kr_c \sim 10$ naturally generates a hierarchy of 15 orders of magnitude among a fundamental mass scale and the physical one: this is the crucial observation employed to resolve the SM hierarchy problem.

The correspondence AdS/CFT has been originally formulated in the context of superstring theory but it is believed to have a wider range of applicability. The claim is that a gravitational theory on a warped background is dual to a gauge theory living on a manifold with one dimension less, typically the boundary of the manifold of the gravitational theory, see [35] and also [36, 37]. Given an operator belonging to the CFT there exists a bulk field φ, i.e. living on the entire five dimensional manifold, with a defined boundary condition φ_0 such that the gravity solution is unique: from both languages we can extract, in principle, the same amount of information simply relating the effective action for this field and the generating functional of the correlators of $\mathcal{O}$ in the CFT,

$$e^{-\Gamma[\varphi_0]} = \langle e^{-\int \mathrm{d}^4x \varphi_0 \mathcal{O}} \rangle \tag{2.28}$$

(in Euclidean spacetime). The boundary field acts as a source for the operator $\mathcal{O}$ and it allows to compute n point functions in the strongly coupled regime, typically a large N limit in case of a gauge theory, as functional derivatives of the on shell action for the field ϕ_0.

For the holographic interpretation of the RSI scenario we follow [38, 39]. The AdS metric eq.(2.25) can be expressed as

$$\mathrm{d}s^2 = \frac{1}{(kz)^2}(g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2) \tag{2.29}$$

with the change of variable $kz = e^{kr_c y}$. The orbifold fixed points are at the positions $z = 1/k$ and $z = e^{kr_c\pi}/k$. The fifth coordinate z is related to the RG scale of the CFT, therefore we refer to the branes as UV and IR brane respectively. The presence of the UV brane acts as a cutoff for the field theory. Moreover at the scale M_{Pl} the CFT couples to gravity, in a way such that conformal symmetry is broken only by Planck suppressed operators. As it is clear from eq.(2.26), written as

$$M_{Pl} = \left(\frac{M}{k}\right)^3 \left(\frac{1}{z_{UV}^2} - \frac{1}{z_{IR}^2}\right). \tag{2.30}$$

the limit $z_{UV} \to 0$ implies a CFT valid up to arbitrary high energies and completely decoupled gravity, because it implies $M_{Pl} \to +\infty$. An AdS background modified by the presence of a single brane has been considered in [40], whose scenario is known as RSII. In [40] indeed it has been shown that this extra dimensional setup of AdS_5 ending to a brane supports a massless normalizable graviton and a continuum of KK states. Remarkably this also shows that, with the only purpose of building a theory consistent with Newton's law and general relativity, extra dimensions do not need to be compact.

The addition of the second IR brane introduces a new boundary condition at $z_{IR} = e^{kr_c\pi}/k$ and quantizes the spectrum of KK excitations. Moreover the IR brane signals a departure from AdS background at low energies, therefore in the field theory language we expect a breaking of conformality in the IR. The position of this brane is the VEV of a dynamical radion field which is viewed as the dilaton, the Goldstone boson of the broken dilatation symmetry. This VEV can be adjusted, namely the position of the brane can be fixed and the radius of the extra dimension is stabilized, with a mechanism proposed in [41]: integrating over an additional single scalar field forced to have a nontrivial profile along the fifth dimension provides a potential for the radion. In the spirit of AdS/CFT correspondence this field is dual to a slightly relevant operator which deforms the CFT and it grows in the IR where it causes the breaking of conformal symmetry, and the hierarchy between scales is generated through dimensional transmutation.

The holographic interpretation of the RSI can be extended also to gauge fields, including the breaking obtained imposing proper boundary conditions [42], and to fermions [43]. In particular while in the original RS models the SM fields where localized, namely confined to live, on the IR brane, it has been realized the convenience of having fields propagating in all the dimensions [44]. The bulk mass of a fermion is related to the strength of the mixing of the associated massless four dimensional fermion with an operator of the CFT. In the language of the KK reduction this comes from the localization of the zero mode in the fifth dimension controlled by its bulk mass. A fermion whose profile is localized close to the UV brane is less sensitive to what happens in the bulk, i.e. the mixing with the CFT is weaker, and therefore we refer to it as more elementary, in contrast to a fermion more localized toward the IR brane, more related to the strong dynamics. The four dimensional Yukawa couplings are determined by the overlap in the fifth dimension of the wavefunctions of the fermions (left and right) with the one of the Higgs scalar.

2.4 The Higgs Potential

Regardless on the possible UV completion we try to draw conclusions on the shape of the generated Higgs potential, not relying on the details of the CFT: we make model independent statements, assuming the coset SO(5)/SO(4) and partial compositeness. The one-loop Higgs potential can be obtained in a two steps procedure, following for instance [45]: all the new physics is integrated out and can be encoded in form factors for SM fields; then the one-loop action is computed in the background of the Higgs. Once the strong sector has been integrated out we are left with a Lagrangian (in momentum space) for the SM fields coupled to the Higgs, specifically the top and the EW gauge bosons:

$$\begin{aligned}\mathcal{L} &= \bar{t}_L \not{p}\Pi_{t_L} t_L + \bar{t}_R \not{p}\Pi_{t_R} t_R - (\bar{t}_L \Pi_{t_L t_R} t_R + h.c.) + \\ &+\frac{P_t^{\mu\nu}}{2}\left(2\Pi_{W^+W^-}W^+_\mu W^-_\nu + \Pi_{W^3W^3}W^3_\mu W^3_\nu + \Pi_{BB}B_\mu B_\nu + 2\Pi_{W^3B}W^3_\mu B_\nu\right)\end{aligned} \tag{2.31}$$

where $P_t^{\mu\nu} = \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$. We neglect SM fermions different from top because their contribution is negligible, but this formalism can be straightforwardly extended to account for them. The Higgs dependence, not manifest in eq.(2.31), is in the various form factors:

$$\Pi = \Pi(h)\,. \tag{2.32}$$

The computation of the effective action is best performed in the Landau gauge where the ghosts and the longitudinal components of the vectors decouple: the resulting potential does not depend on the choice of the gauge.

A standard one-loop computation leads to

$$V(h) = -2N_c \int \frac{d^4q}{(2\pi)^4} \log\left[q^2\Pi_{t_L}\Pi_{t_R} + |\Pi_{t_L t_R}|^2\right] + \frac{3}{2}\int \frac{d^4q}{(2\pi)^4}\left[2\log\Pi_{W^+W^-} + \log\left(\Pi_{W^3W^3}\Pi_{BB} - \Pi^2_{W^3B}\right)\right] \tag{2.33}$$

where every form factor Π is a function on $-q^2$, $\Pi(-q^2)$, and q is the Euclidean four momentum; $N_c = 3$ is the number of QCD colors. In order to simplify our notation considerably, we work in the unitary gauge and denote by h the Higgs field in this gauge.

Since the Higgs is a pNGB associated to an approximate spontaneous symmetry breaking, its VEV is effectively an angle. For this reason it is often convenient to describe its potential not in terms of the Higgs field h itself, but of its sine:

$$s_h \equiv \sin\frac{h}{f}\,, \tag{2.34}$$

where f is the Higgs decay constant. Following a standard notation we also define

$$\xi \equiv \langle s_h^2\rangle\,. \tag{2.35}$$

The electroweak scale is fixed to be $v^2 = f^2\xi \simeq (246\,\mathrm{GeV})^2$. We focus on small values of ξ and in explicit results we set it to the benchmark value $\xi = 0.1$. Due to the contribution of particles whose masses vanish for $s_h = 0$ (such as the top, W and Z), the one-loop Higgs potential contains non-analytic terms of the form $s_h^4 \log s_h$ that do not admit a Taylor expansion around $s_h = 0$. In the phenomenological regions of interest, these terms do not lead to new features and are qualitatively but *not* quantitatively negligible. However, they make an analytic study of the potential slightly more difficult.

For $s_h \ll 1$, the tree-level + one-loop potential $V = V^{(0)} + V^{(1)}$ admits an expansion of the form

$$V(s_h) = -\gamma s_h^2 + \beta s_h^4 + \delta s_h^4 \log s_h + \mathcal{O}(s_h^6) \tag{2.36}$$

where

$$\gamma = -\left.\frac{1}{2}\frac{\partial^2 V}{\partial s_h^2}\right|_{s_h=0}\,, \quad \delta = \left.\frac{s_h}{4!}\frac{\partial^5 V}{\partial s_h^5}\right|_{s_h=0}\,, \quad \beta = \left.\frac{1}{4!}\frac{\partial^4 (V - \delta s_h^4 \log s_h)}{\partial s_h^4}\right|_{s_h=0} \tag{2.37}$$

and higher order terms are entirely neglected. In a naive expansion around $s_h = 0$ the presence of δ would be detected by the appearance of a spurious IR divergence in the coefficient β. At first order in δ around $\delta = 0$ the non trivial minimum of the potential, assuming it exists, is found at

$$\langle s_h^2 \rangle \equiv \xi = \xi_0 \Big(1 - \frac{\delta}{4\beta}(1 + 2\log \xi_0) \Big) \,, \tag{2.38}$$

where

$$\xi_0 = \frac{\gamma}{2\beta} \tag{2.39}$$

is the leading order minimum for $\delta = 0$. The physical Higgs mass, computed as the second derivative of the potential at its minimum, is given by

$$M_H^2 = \frac{8\beta}{f^2}\xi_0(1 - \xi_0) + \frac{4\delta\xi_0}{f^2}\Big(1 - \frac{\xi_0}{2} + \xi_0 \log \xi_0\Big). \tag{2.40}$$

For $\xi_0 \ll 1$ we get

$$M_H^2 \simeq (M_H^0)^2 \Big(1 + \frac{\delta}{2\beta}\Big), \tag{2.41}$$

where

$$(M_H^0)^2 \simeq \frac{8\beta}{f^2}\xi_0 \,. \tag{2.42}$$

In the limit $\delta = 0$ we would recover

$$\xi = \frac{\gamma}{2\beta}\,, \quad M_H^2 = \frac{8\beta}{f^2}\xi(1 - \xi)\,. \tag{2.43}$$

In all the models we will consider, there are two distinct sectors that do not couple at tree-level at quadratic order: a “matter” sector, including the fields that mix with the top quark and a “gauge” sector, including the SM gauge fields and other fields, neutral under color. The matter and gauge sectors contribute separately to the one-loop Higgs potential:

$$V(s_h) = V_{gauge}(s_h) + V_{matter}(s_h) \tag{2.44}$$

and

$$\gamma = \gamma_{gauge} + \gamma_{matter}\,, \quad \beta = \beta_{gauge} + \beta_{matter}\,, \quad \delta = \delta_{gauge} + \delta_{matter}\,. \tag{2.45}$$

The explicit SO(5) symmetry breaking parameters are the SM gauge couplings g and g' in the gauge sector and the mixing parameters ϵ_r's given by eq.(2.18) in the matter sector. Since the latter are sizably larger than the former, for sensible values of the parameters $\beta_{matter} \gg \beta_{gauge}$.[3] At fixed ξ, then, the Higgs mass is essentially determined by the matter contribution (in the numerical study, however, we keep all the contributions to the one-loop potential). The gauge contribution should instead be retained in γ because the FT cancellations needed to get $\xi \ll 1$ might involve γ_{gauge}.

In the models we considered, the particles massless at $s_h = 0$ are always the top in the matter sector and the W and the Z gauge bosons in the gauge sector. Correspondingly, the explicit form of $\delta = \delta_{gauge} + \delta_{matter}$ is universal and given by

$$\delta_{matter} = -\frac{N_c}{8\pi^2}\lambda_{top}^4 f^4 \,, \quad \delta_{gauge} = \frac{3f^4(3g^4 + 2g^2g'^2 + g'^4)}{512\pi^2} \,, \tag{2.46}$$

with $M_{top} \equiv \lambda_{top} v$.

In the most general case both γ_{gauge} and γ_{matter} are quadratically sensitive to high energy scales, namely if the potential is expressed as in eq.(2.33) as an integral in momentum space

$$\gamma \sim \int q \, \mathrm{d}q \, \left(1 + O(q^{-2}) + \ldots\right) \,. \tag{2.47}$$

At the same time β_{gauge} and β_{matter} assume the following behavior

$$\beta \sim \int q \, \mathrm{d}q \, \left(q^{-2} + \ldots\right) \,, \tag{2.48}$$

revealing a logarithmic sensitivity to the cutoff of the theory. To improve calculability and relax this UV sensitivity, therefore keeping under control the fine tuning, generalized sum rules have been studied [45,46]: if the parameters of the theory satisfy these sum rules the divergent behaviors are canceled; they have been derived in close analogy with the Weinberg sum rules for QCD.

In section 3.2 we will comment on the UV sensitivity of the SUSY cases. SUSY introduces contributions to the Higgs potential of opposite spin with respect to the ones discussed here, but a generalization of the procedure to get eq.(2.33) is straightforward: matter fermions are properly included in chiral superfields, while vectors are part of vector multiplets; also additional

[3] A numerical analysis confirms this result and shows that typically β_{gauge} is at least one order of magnitude smaller than β_{matter}.

chiral superfields will contribute to the gauge part of the potential. We anticipate now that only gaugino soft masses can contribute quadratically to the (negative) Higgs square mass. In the language of eq.(2.47) and eq.(2.48) this result can be seen as the fact that for the matter contribution the most divergent terms, quadratically and logarithmically divergent in γ_{matter} and β_{matter} respectively, are zero because the parameters satisfy a certain condition, while the logarithmic part in γ_{matter} is canceled by the sum of the effect of bosonic and fermionic degrees of freedom: at the end the Higgs potential is UV finite. In the gauge sector SUSY cancellations are not exact and there is a leftover dependence on the cutoff of the theory, as we will see in greater detail in the next chapter.

2.4.1 Non-SUSY Higgs Mass Estimates

Before analyzing the Higgs potential in SUSY CHM, it might be useful to quickly review the situation in the purely non-SUSY bottom-up constructions. We focus in what follows on models where the composite fields are in the fundamental representation of SO(5). Higher representations lead to a multitude of other fields, they are more complicated to embed in a UV model and they worsen the problem of Landau poles. Moreover they might lead to dangerous tree-level Higgs mediated flavor changing neutral currents [47]. It should however be emphasized that they can be useful and can result in qualitatively different results, see e.g. [31] for a recent discussion of the Higgs mass estimate for CHM with composite fermions in the **14** of SO(5). Generically, the Higgs mass is not calculable in CHM, since both γ and β defined in eq.(2.37) are divergent and require a counterterm. The situation improves if a symmetry, such as collective breaking [30, 48], is advocated to protect these quantities, at least at one-loop level, or if one assumes that γ and β are dominated by the lightest set of resonances in the composite sector, saturating generalized Weinberg sum rules [45, 46]. As far as the Higgs mass is concerned, we see from eq.(2.43) that, at *fixed* ξ, it is enough to make β finite to be able to predict the Higgs mass.

In CHM with partial compositeness, the largest source of explicit breaking of the global symmetry comes from the mass term mixing the top with the composite sector. In first approximation, we can switch off all other sources of breaking, including the electroweak SM couplings g and g'. The estimate of the Higgs mass is then necessarily linked to the mechanism generating a mass for the top. Let us first consider the case in which both t_L and t_R are elementary and mix with their partners. In this case two mass mixing terms ϵ_L and ϵ_R are required to mix them with fermion states of the composite sector. The top mass goes like

$$M_{top} \sim \frac{\epsilon_L \epsilon_R}{M_f} s_h \,, \tag{2.49}$$

where by M_f we denote the mass (taken equal for simplicity) of the lightest fermion resonances in the composite sector that couple to t_R and t_L. In the limit in which ϵ are the only source of explicit symmetry breaking, a simple NDA estimate gives the form of the factors γ, β entering in the Higgs potential (2.36):[4]

$$\gamma \sim \frac{N_c}{16\pi^2}\epsilon^2 M_f^2 \,, \quad \beta \sim \frac{N_c}{16\pi^2}\epsilon^4 \,. \tag{2.50}$$

Plugging eqs.(2.50) and (2.49) in eq.(2.43) gives

$$M_H^2 \simeq \frac{N_c \epsilon^4}{2\pi^2 f^2}\xi \sim \frac{N_c}{2\pi^2} M_{top}^2 \frac{M_f^2}{f^2} \qquad (t_R \;\text{ elementary}) \,. \tag{2.51}$$

This estimate reveals a growth of the Higgs mass with the top partners mass scale. If one assumes that the composite sector is characterized by the single coupling constant g_ρ [49], we expect that $M_f \simeq g_\rho f$. Indirect bounds on the S parameter require $g_\rho f \gtrsim 2$ TeV. For values of $f \lesssim 1$ TeV this implies $g_\rho \gtrsim 2$. In many explicit models [45, 46, 50, 51] it has been shown that such a choice results in a too heavy Higgs. Indeed, a 126 GeV Higgs is attained only if one assumes that another mass scale characterizes the composite sector and one has relatively light fermion resonances in the composite sector, with $M_f < g_\rho f$. Although the splitting required between M_f and $g_\rho f$ is modest, it is not easy to argue how it might appear in genuinely strongly coupled non-SUSY theories.

Another possibility is having t_L elementary and t_R fully composite. This means that the right top is not present among the elementary fields and in the meanwhile in the composite sector there is one state, with the right quantum numbers, that we identify with t_R[5]. We can now have a direct mixing between t_L and t_R, in principle with no need of composite massive

[4] The estimate (2.50) changes when fields in higher representations are considered. For instance, $\beta \sim \frac{N_c M_f^2 \epsilon^2}{16\pi^2}$ when fields in the **14** are considered [31].

[5] Gauge anomalies have to cancel non trivially, that is non independently, among elementary and composite fermions.

resonances, that can all be taken heavy. Denoting by ϵ this mass mixing term, we get

$$M_{top} \simeq \epsilon s_h \,. \qquad (2.52)$$

Proceeding as before, we get

$$M_H^2 \simeq \frac{N_c \epsilon^4}{2\pi^2 f^2}\xi = \frac{N_c}{2\pi^2} M_{top}^2 \frac{M_{top}^2}{v^2} \qquad (t_R \ \text{composite})\,. \qquad (2.53)$$

We see that the Higgs mass is at leading order independent of the details of the composite sector and tends to be too light.[6] Of course, this is the case in the assumption that the top mixing term is the dominant source of explicit SO(5) breaking. One can always add extra breaking terms to raise the Higgs mass. Clearly, this is quite ad hoc, unless these terms are already present for other reasons. This happens in the concrete model with composite t_R introduced in [1], where anomaly cancellation and absence of massless non-SM states require adding exotic elementary states that necessarily introduce an extra source of explicit SO(5) breaking. We have explicitly verified in the model of [1] that the estimate (2.53) captures to a good accuracy the top contribution to the Higgs mass. This is still too light, despite the presence of additional sources of SO(5) breaking, that cannot be taken too large for consistency. We conclude that models with a composite t_R, at least those where the top sector plays a key role in the EWSB pattern, lead to a too small Higgs mass.

Let us now briefly mention on how ξ can be tuned to the desired value. There are essentially two ways to do that in a calculable manner: either $|\gamma_{matter}| \gg |\gamma_{gauge}|$, in which case the cancellation takes place mostly inside the matter sector, or $|\gamma_{matter}| \sim |\gamma_{gauge}|$, so that the gauge and matter contributions can be tuned against each other (see, for example, the discussion in Section 4 of [45]). Both options are generally possible, with the exception of minimal (i.e. where one ϵ is the only source of SO(5) violation in the matter sector) models with a fully composite t_R embedded in a fundamental of SO(5), where one can rely only on the second option.

[6] The problem of a too light Higgs when t_R is fully composite (when embedded in a **5** of SO(5)) was already pointed out in [45], where a formula like eq.(2.53) (see eq.(5.14)) was derived for a particular model.

2.5 Experimental probes of Composite Higgs Models

2.5.1 Direct Searches

In this section we discuss the experimental consequences of pNGB CHM. The presence of new, still unobserved, massive states characterizes many models of BSM physics: in the case at hand heavy resonances are introduced as partners for SM fields. The discussion on the Higgs potential in section 2.4 outlined that the most relevant ones are top partners because their presence, and their relative "lightness", is linked to top and Higgs masses. They are colored fermions transforming in representations of $SO(5) \times U(1)_X$ where now the X charge is not negligible: as we explained a common choice is to obtain the SM hypercharge as $Y = T^3_R + X$ where T^3_R is a generator of $SU(2)_R \subset SO(4) \subset SO(5)$. Therefore under SM gauge group we decompose $\mathbf{5}_{2/3} = (\mathbf{4}+\mathbf{1})_{2/3} = \mathbf{2}_{7/6} + \mathbf{2}_{1/6} + \mathbf{1}_{2/3}$. We thus expect massive vectorlike quarks and new fermions with exotic electric charge $Q = 5/3$. The relevance of searches for resonances has been stressed in [52] where simplified models have been introduced; dedicated studies of CMS collaboration [13], along the lines of [53], put a limit of 800 GeV for masses of $Q = 5/3$ colored fermions at 95% c.l. using 19.5 fb^{-1} collected at $\sqrt{s} = 8$ TeV at LHC. The strategy is to look for top partners pair produced through colored interactions each of them decaying to a W boson and a top which decays to another W and a bottom : they analyze events with two same sign leptons coming from leptonic decays of the two (same sign) W bosons. While this bound applies universally to $Q = 5/3$ top partners the single production case is model dependent: the cross section for the single production is expected to overcome the pair production for masses of the resonance roughly around the TeV. The process involves the vertex interaction among an EW vector boson, a quark and one of its partners, therefore the model dependence rely on the strenght of the various mixings ϵ_r.

For $Q = 2/3$ top partners CMS published the results of an inclusive search based on 19.5 fb^{-1} at 8 TeV [54]. They can decay to top and Higgs, top and Z boson or bottom and W boson and masses below 700 GeV are excluced: the exact value depends on the various branching ratios.

Higher dimensional SO(5) representations contain other exotic partners, like the **14** where a fermion with $Q = 8/3$ is present. The case is studied in [55], where events with same sign dileptons are analyzed and a bound is put at a mass 940 GeV. $Q = 8/3$ partners do not couple directly to SM fields at the renormalizable level due to their large electric charge: their decay is mediated by a vertex with two W bosons and a top (it can be seen as a two step decay through a $Q = 5/3$ fermion). In [55] they also explored the more realistic scenario with all the states in the **14** roughly at the same mass, obtaining a comparable bound.

While fermion resonances are an essential ingredient of partial compositeness, vector resonances are not strictly necessary, even though they have received a lot of attention, for instance in [56]. Partners of Z and W bosons with masses below 1.5 TeV are excluded [57, 58], [59] for limits not from the experimental collaborations. While experimental collaborations focused on benchmark models, in [60] has been considered the more realistic case of a heavy spin-1 resonance accompanied by fermionic resonances, specifically those typically arising from fundamental representation of SO(5).

On general grounds one could also expect the presence of gluon resonances, massive vectors in the adjoint of $\mathrm{SU}(3)_c$: this is particularly true in extra dimensional models where they are identified with KK excitations of the ordinary SM gluons, while in pure four dimensional theories they are not unavoidable. They affect the two point functions of other fermionic resonances and they enter the Higgs potential at two loops, nevertheless their effect can be not negligible as long as they do not completely decouple: as stressed in [61] masses above the experimental limit, set by CMS at 2.5 TeV [62], can contribute to lower the Higgs physical mass by a few percentage points.

2.5.2 Indirect Measurements

Higgs Couplings

Direct detection of a resonance would surely be a reliable sign of BSM physics and it would also helps in discriminating among several BSM hypothesis. Other indirect indications might come from deviations in the measure of the Higgs couplings: in this case different models may results in comparable predictions and further experiments would be needed. In CHM such deviations are intimately related to the pNGB nature of the Higgs: couplings to fermions and EW gauge bosons are dictated by symmetry considerations. For instance for the coset SO(5)/SO(4) the Lagrangian for gauge bosons contains

$$\frac{1}{2}\frac{f^2}{4}s_h^2[g'^2 B_\mu B_\nu - 2g'gB_\mu W_\nu^3 + g^2(W_\mu^1 W_\nu^1 + W_\mu^2 W_\nu^2 + W_\mu^3 W_\nu^3)]\eta^{\mu\nu} \tag{2.54}$$

coming from the covariant derivative of the Higgs field. The identification $v^2 = f^2\langle s_h^2\rangle = f^2\xi$ yields to the correct values for W and Z masses and ensures the relation $m_W = \cos\theta_W m_Z$. If we expand around the VEV $\langle h\rangle + h$ for small fluctuations we find

$$f^2 s_h^2 \simeq v^2 + 2v\sqrt{1-\xi}h + (1-2\xi)h^2\,, \tag{2.55}$$

whereas the SM is recovered in the limit $\xi \to 0$ at fixed v. Therefore the coupling VVh and $VVhh$ are modified by a factor $\sqrt{1-\xi}$ and $1-2\xi$ respectively. The couplings of the Higgs to the EW bosons are reduced compared to the SM case in a variety of cosets: an enhancement is obtained in non compact cosets, like SO(4,1)/SO(4) [63, 64].

The couplings to SM fermions depend on the SO(5) representations of the fermion partners. If we choose the fundamental **5** for both left and right the mass term is proportional to

$$f\sin\frac{2h}{f} \simeq 2v\sqrt{1-\xi} + 2h(1-2\xi) = 2v\sqrt{1-\xi}\left(1 + \frac{h}{v}\frac{(1-2\xi)}{\sqrt{1-\xi}}\right) \tag{2.56}$$

and therefore the Higgs coupling to fermions $h\bar{f}f$ with respect to the SM case is modified by the factor $(1-2\xi)/\sqrt{1-\xi}$.

Couplings to gluons and photons are induced by loops of charged and colored particles and are model dependent. The quickest way to compute them is through the low energy theorem [65, 66] which states that an amplitude involving a soft Higgs as external leg can be computed as a function of the same amplitude without the Higgs, namely

$$\lim_{p_h\to 0}\mathcal{A}(X+h) = h\frac{\partial}{\partial v}\mathcal{A}(X) \tag{2.57}$$

if M_H is much smaller than the masses in the loop. This is because inserting a Higgs in a propagator means the following replacement in the amplitude $\mathcal{A}(X)$:

$$\frac{1}{\not{q}-m} \to \frac{1}{\not{q}-m}h\frac{1}{\not{q}-m} = h\frac{\partial}{\partial m}\frac{1}{\not{q}-m}\,. \tag{2.58}$$

The Jacobian $\frac{\partial m}{\partial v}$ contains the Yukawa coupling of the vertex. In fact interactions with the Higgs may be generated in a Lagrangian, with the kinetic term consistently neglected in the soft limit, with the formal substitution

$$m \to m\left(1 + g\frac{h}{v}\right) \tag{2.59}$$

where g is a proper coefficient ($g = 1$ in the SM); with different values for g the theorem can be generalized to new physics models. As an example of the use of eq.(2.57) we compute the coupling of the Higgs to two gluons in the SM mediated by a top loop. We start with the term in the action

$$\mathcal{L} \supseteq -\frac{1}{4g_3^2}F^a_{\mu\nu}F^{\mu\nu a} \tag{2.60}$$

with g_3 the coupling of $\mathrm{SU}(3)_c$ such that $\beta_{\frac{1}{g_3^2}} = \frac{b}{8\pi^2}$. We recall that

$$\frac{\partial \frac{1}{g_3^2}}{\partial \log M_{top}} = -\frac{2}{3}\frac{1}{8\pi^2} \tag{2.61}$$

due to the changing of the beta function at energies crossing M_{top}. With the identity $\frac{\partial}{\partial v} = \frac{1}{v}\frac{\partial}{\partial \log M_{top}}$ and putting everything together we obtain

$$\mathcal{L} \supseteq \frac{g_3^2}{48\pi^2}F^a_{\mu\nu}F^{\mu\nu a}\frac{h}{v} \tag{2.62}$$

where we have reabsorbed the gauge coupling constant in the vector fields, $A^a_\mu \to g_3 A^a_\mu$; it agrees with the full one-loop computation up to terms of order $O(\frac{M_H^2}{M_{top}^2})$. Notice that the effect does not depend on the top mass, as long as $M_{top} \gg M_H$ because the coupling is proportional to the Yukawa of the top: in other words a massive field does not decouple because the heavier it is the more it interacts. The vertex with two gluons and n external soft Higgs legs is obtained iterating eq.(2.57) on eq.(2.62) and dividing for $n!$ to avoid multiple counting. Noticing that $\left(\frac{\partial}{\partial v}\right)^{n-1} v^{-1} = (-1)^{n-1}(n-1)!v^{-n}$ the resummation of all the vertices $h^n gg$ results in

$$\mathcal{L} \supseteq \frac{g_3^2}{48\pi^2}F^a_{\mu\nu}F^{\mu\nu a}\log\left(1 + \frac{h}{v}\right) . \tag{2.63}$$

More in general the result can be expressed in terms of $\log\det|M(h)|^2$ and the Higgs dependence is obtained performing the replacement eq.(2.59), where $M(h)$ is the mass matrix of the fields entering the loop. This holds not only for the SM but has wider applicability. In [67] the couplings of the Higgs to gluons has been studied extensively taking into account both the contribution from heavy resonances and the non linearities coming from the coset structure.

So far all the data extracted from the experimental collaborations at LHC are in good agreement with the expectations from the SM, no significant deviation for any of the probed couplings of the Higgs to bosons (massive vectors, photons and gluons) or fermions (bottom, tau) is found and the uncertainties shrank to $O(10\%)$: see [68–70] and the many references reported therein.

Electroweak Precision Observables

In discussing electroweak precision tests (EWPT) we follow the discussion of [71]. New physics effects are included in form factors of EW gauge bosons

$$\Pi_V(q^2) = \Pi_V(0) + \Pi'(0)q^2 + \frac{1}{2}\Pi''(0)(q^2)^2 + \dots \tag{2.64}$$

where $V \in \{W^+W^-, W^3W^3, BB, W^3B\}$ as in eq.(2.31). The vector bosons need not to be mass eigenstates but simply they are the gauge fields appearing in the covariant derivatives of the SM fields with gauge couplings g_0 and g_0': namely they are the elementary fields and in principle they can mix with heavy vector resonances. Moreover we decide to work with canonically normalized fields. For these reasons in the following formulae both the pairs g_0, g_0' and g, g' appear. While g_0 and g_0' are the couplings associated to the gauging the couplings g and g' are the observed ones, taking into account possible mixings. An example of the mixing will be given in eq.(4.16).

Stopping at the second order in the expansion five out of the twelve parameters are fixed by phenomenological considerations:

$$\Pi'_{W^+W^-}(0) = -\frac{g_0^2}{g^2}\,, \quad \Pi'_{BB}(0) = -\frac{g_0'^2}{g'^2}\,, \quad \Pi_{W^+W^-}(0) = g_0^2\frac{v^2}{2}\,. \tag{2.65}$$

The masslessness of the photon $A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W$ is guaranteed by the following

$$\Pi_{W^3W^3}(0) = -\frac{g g_0}{g' g_0'}\Pi_{W^3B}(0)\,, \quad \Pi_{BB}(0) = -\frac{g' g_0'}{g g_0}\Pi_{W^3B}(0)\,, \tag{2.66}$$

Consequently we are left with the following:

$$\begin{cases} \frac{1}{g^2}\widehat{S} & = & -\frac{1}{g_0 g_0'}\Pi'_{W^3B}(0) \\ \frac{2}{gg' M_W^2}X & = & -\frac{1}{g_0 g_0'}\Pi''_{W^3B}(0) \end{cases}, \quad \begin{cases} \frac{2}{g^2 M_W^2}W & = & -\frac{1}{g_0^2}\Pi''_{W^3W^3}(0) \\ \frac{2}{g'^2 M_W^2}Y & = & -\frac{1}{g_0'^2}\Pi''_{BB}(0) \end{cases},$$

$$\begin{cases} \frac{1}{g^2}M_W^2\widehat{T} & = & -\frac{1}{g_0^2}(\Pi_{W^3W^3}(0) - \Pi_{W^+W^-}(0)) \\ -\frac{1}{g^2}\widehat{U} & = & -\frac{1}{g_0^2}(\Pi'_{W^3W^3}(0) - \Pi'_{W^+W^-}(0)) \\ \frac{2}{g^2 M_W^2}V & = & -\frac{1}{g_0^2}(\Pi''_{W^3W^3}(0) - \Pi''_{W^+W^-}(0)) \end{cases}. \tag{2.67}$$

We have distinguished them into three classes: $\widehat{T}$, $\widehat{U}$ and V vanish in a theory preserving either custodial symmetry or $\mathrm{SU}(2)_L$. $\widehat{S}$ and X are not protected by the custodial symmetry while W and Y are not protected neither by the custodial nor by the $\mathrm{SU}(2)_L$ symmetry.

They are related to the S, T, U parameters [72, 73]:

$$S = \frac{4\sin^2\theta_W}{\alpha}\widehat{S}, \quad T = \frac{\widehat{T}}{\alpha}, \quad U = -\frac{4\sin^2\theta_W}{\alpha}\widehat{U}. \tag{2.68}$$

where we use $\alpha = \frac{e^2}{4\pi}$, $e = g/\sin\theta_W$. In an expansion around $q^2 = 0$ of the form factors Π_V we expect the coefficient of the n-th term to be suppressed with respect to the $(n-1)$-th one by a factor $(M_W/\Lambda)^2$, where Λ is the scale at which new physics enters into the form factors. Therefore we can restrict ourselves to the set of parameters $\widehat{S}, \widehat{T}, Y, W$. In pNGB CHM a suppression by a factor $(g/g_m)^2$ is reasonable: thus the most relevant parameters for the present discussion are $\widehat{S}$ and $\widehat{T}$.

We are interested in contributions to form factors, and hence to $\widehat{S}$ and $\widehat{T}$, different from the SM ones and possibly due to new physics. Let us start considering $\widehat{S}$. A first important contribution to the $\Pi_{W^3B}(q^2)$ from the strong sector can be interpreted, in the strongly interacting limit, as the exchange of heavy narrow spin-1 resonances: due to partial compositeness they mix at tree-level with the EW gauge bosons. For example a heavy vector boson ρ in the **6** of SO(4) can be treated with a simplified Lagrangian including its coupling constant g_m and its mass $g_m f$: the mixing with the SM bosons is proportional to gf. Its effect is

$$\Delta\widehat{S}_\rho = \frac{g^2}{g_m^2}\xi\,. \tag{2.69}$$

At the loop level the form factors get more contributions from the strong sector: in the narrow resonances approximation other states enter the loop. In particular fermionic partners of the SM fermions, depending on their SO(4) properties, can affect the result. In [74, 75] a one-loop computation has been performed taking into account the non linearities of the couplings. In the presence of a $\psi_{\mathbf{4}}$ and a $\psi_{\mathbf{1}}$ resonances respectively in the **4** and **1** of SO(4) $\subseteq$ SO(5) a non derivative coupling with the Higgs is induced by

$$\bar{\psi}_{\mathbf{4}}\, \not{D}\, \psi_{\mathbf{1}} + h.c. \tag{2.70}$$

where D_μ is defined in eq.(2.8). The induced $\Delta\widehat{S}_f$ has no definite sign and it can vanish for a particular choice of the coefficient of the above operator: this corresponds to points with enlarged symmetries.

A third important effect is related to the non linearities of the Higgs boson [76]: as we discussed they translate into deviations of the couplings of the Higgs to EW vector bosons with respect to the SM values, reobtainable in the limit $\xi \to 0$ at fixed v. In the SM, being a renormalizable theory, $\widehat{S}$ has to be finite because we cannot add counterterms for it; on the other hand the effective theory at $\xi \neq 0$ is no longer renormalizable, a UV completion is given by the addition of the strongly interacting sector. Therefore we expect $\widehat{S}$ to be sensitive to the cutoff scale, Λ. Notice that, conversely to what happened before, this contribution does not depend on the details of the UV theory, namely masses and couplings of heavy resonances, but it is fully understood in the effective theory: in this sense it is a IR contribution. The total one-loop computation with the vectors, the Goldstones and the neutral Higgs with modified couplings results in [76]

$$\Delta\widehat{S}_H = \frac{1}{6\pi}\frac{g^2}{16\pi}\xi \log\frac{\Lambda}{M_H}\,. \tag{2.71}$$

For what concern $\widehat{T}$ there is no tree-level contibution because the composite sector respects a custodial symmetry. At the loop level this symmetry is spoiled therefore we expect a non vanishing $\Delta\widehat{T}$: in particular the contribution from fermionic resonances can be estimated to be [30, 49]

$$\Delta\widehat{T}_{ferm} \sim \frac{N_c}{16\pi^2}\frac{\epsilon_r^4 f^2}{M_L^2}\xi\,. \tag{2.72}$$

Higgs non linearities produces an IR effect similar to the one in eq.(2.71)

$$\Delta\widehat{T}_H = -\frac{3g^2}{32\pi^2\cos^2\theta_W}\xi \log\frac{\Lambda}{M_H}\,. \tag{2.73}$$

The numerical values for $\widehat{S}$ and $\widehat{T}$, although in terms of the non hatted quantities S, T and U, can be found in [77,78].

As pointed out in [79,80] models with **5** or **10** representations for O_r are favoured with respect to the minimal models with fermions embedded in the spinorial **4**. While corrections to $\widehat{T}$ vanish at tree-level if the BSM physics respects a custodial symmetry, one can also include an additional symmetry P_{LR} exchanging $L \leftrightarrow R$, on top of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Since $P_{LR} \in \mathrm{O}(4)$ and $P_{LR} \notin \mathrm{SO}(4)$[7] it can be forced enlarging to the breaking $\mathrm{O}(5) \to \mathrm{O}(4)$ which is equivalent at the level of the algebra but does not allow for spinorial representations. At the same time the former representations automatically guarantee that no deviations occurs at tree-level in the coupling of the bottom left to the Z boson with respect to the SM value: the bound from [81,82] on deviations is about 10^{-3}. The couplings of top to Z and of top and bottom to W are instead modified but they are less constrained experimentally. Notice that the requirements studied in [79] to protect the coupling Zbb cannot be satisfied if one wants to embed in (different) SO(5) spurions the top and the bottom quarks: therefore with the most common choice, the one we adopt in the following, a tree-level correction appears proportional to the mixings ϵ_{b_L} of the bottom quark.

[7] P_{LR} acting on the fundamental **5** of O(5) can be written as $\mathrm{diag}(-1,-1,-1,1,1)$.

Chapter 3

Supersymmetric Composite Higgs Models

3.1 General Setup

We now proceed to generalize the pNGB Higgs paradigm in SUSY theories: our models consist of an elementary sector, containing SM fermions, gauge bosons and their supersymmetric partners, coupled to a composite sector where both the global symmetry G and SUSY are spontaneously broken. On top of this structure, in order to have sizable SM soft mass terms, we need to assume the existence of a further sector which is responsible for an additional source of SUSY breaking and its mediation to the other two sectors. We do not specify it and we parametrize its effects by adding soft terms in both the elementary and the composite sectors. Our key assumption is that the soft masses in the composite sector are G invariant. See fig.3.1 for a schematic representation. The main sources of explicit breaking of G are the couplings between the elementary and the composite sectors, namely the SM gauge couplings and the top mass mixing terms. We assume that partial compositeness in the matter sector is realized through a superpotential portal of the form

$$W \supset \epsilon\, \xi_{SM} N_{comp}\,, \tag{3.1}$$

the supersymmetric generalization of eq.(2.18). In eq.(3.1) N_{comp} are chiral fields in the composite sector and ξ_{SM} denote the SM matter chiral fields. No Higgs chiral fields are present in the elementary sector, since the Higgs arises from the composite sector. The term (3.1) is the only superpotential term involving SM matter fields. For concreteness, we consider only the minimal custodially invariant SO(5) $\rightarrow$ SO(4) symmetry breaking pattern with N_{comp} in the fundamental representation of SO(5). Like in non-SUSY CHM, the SM Yukawa couplings arise from the more fundamental proto-Yukawa couplings of the form (3.1).[1] We do not consider SM fermions but the top since they are not expected to play an important role in the EWSB mechanism. They can get a mass via partial compositeness through the portal (3.1), like the top quark, or by irrelevant deformations, for instance by adding quartic superpotential terms.

As we are going to show, the SUSY models we consider can be seen as the weakly coupled description of some IR phase of a strongly coupled theory, in which case the Higgs is really composite, or alternatively one can take them as linear UV completions, in which case no compositeness occurs. Depending on the different point of view, general considerations can be made. If we want to take our models as UV completions on their own, we might want to extend the range of validity of the theory up to high scales, ideally up to the GUT or Planck scale. In this setting, introducing gauge fields in addition to the SM gauge fields is disfavoured, because the multiplicity of the involved fields would typically imply that the associated gauge couplings are not UV free and develop a Landau pole at relatively low energies. Avoiding analogous Landau poles for certain Yukawa couplings in the superpotential implies that the "composite sector" should be as weakly coupled as possible. However, reproducing the correct top mass forces some coupling to be sizable; in our explicit example a Landau pole is reached at a scale around $10^2 f$. Viceversa, additional gauge fields are generally required if we assume that the linear models considered are an effective IR description of a more fundamental strongly coupled theory, like in ref. [1]. We might now assume that the theory becomes strongly coupled at relatively low scales, such as $\Lambda = 4\pi f$. We can actually determine the low energy non-SM Yukawa and gauge couplings by demanding that they all become strong around the same scale Λ. As we will see, light fermion top partners still appear in both cases.

In light of these two different perspectives, we will consider in greater detail two benchmark models, with and without vector resonances.

[1] In the field basis where we remove non-derivative interactions of the pNGB Higgs from the composite sector, the Higgs appears in eq.(3.1).

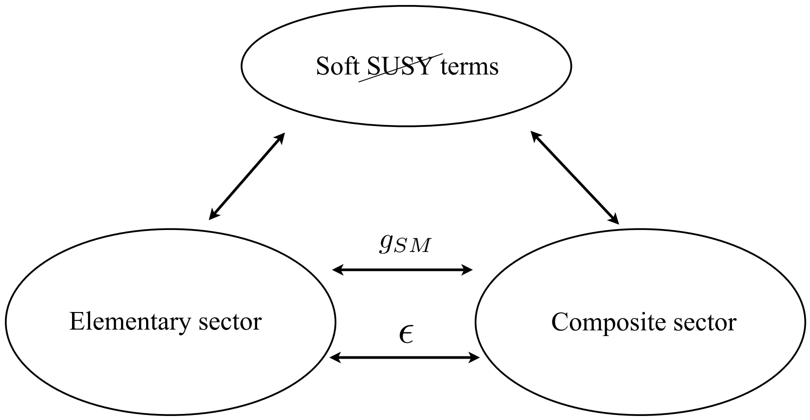

Figure 3.1: Schematic representation of the structure of our models.

3.2 General Features of the Higgs potential

We now move to realizations of pNGB CHM in concrete models which enjoy a rigid $\mathcal{N} = 1$ SUSY. First we consider more specifically the Higgs potential in SUSY models. No tree-level D-term contribution to the potential is present in our models, in contrast to many SUSY little Higgs constructions. We make comparisons with little Higgs models [83,84] because they are a realization of the idea of CHM different from the one we are pursuing and because they also have been considered in SUSY versions [85–89], therefore it is instructive to point out similarities and differences. The latter are based on global unitary symmetries, where one typically embeds the two MSSM Higgs doublets in two distinct multiplets of the underlying global symmetry group. Because of that, one generally ends up in having too large D-term contributions to the Higgs mass, whose cancellation usually requires some more model building effort. In our case, instead, the two Higgs doublets are embedded in a single chiral field $q_{\mathbf{4}}$ that is in the $\mathbf{4}$ of the unbroken SO(4) group. More precisely, the two Higgs doublets $H_{u,d}$ are embedded in $q_{\mathbf{4}}$ as follows:

$$q_{\mathbf{4}} = \frac{1}{\sqrt{2}}\Big(-i(H_u^{(u)} + H_d^{(d)}), H_u^{(u)} - H_d^{(d)}, i(H_u^{(d)} - H_d^{(u)}), H_d^{(u)} + H_u^{(d)}\Big)\,, \tag{3.2}$$

where the superscript denotes the up or down component of the doublet. Thanks to the underlying global symmetry, the H_u and H_d soft mass terms are equal, thus $v_u = v_d$ and $\tan\beta = 1$. The mass eigenstates are simply the real and imaginary components of $q_{\mathbf{4}}$. $\mathrm{Im}\, q_{\mathbf{4}}$ is identified with the heavy Higgs doublet, while $\mathrm{Re}\, q_{\mathbf{4}}$ is the light (SM) Higgs doublet. No D-term contribution affects the Higgs mass. In fact, at tree-level the SM Higgs is massless and its VEV is undetermined. Of course, the situation changes at one-loop level, because of the various sources of violation of the SO(5) global symmetry. The SM Higgs will still sit along the flat direction (i.e. $\tan\beta$ remains one at the quantum level), but quantum corrections will lift the flat direction, fix its VEV and give it a mass. As explained at the beginning of section 2.4, being the light Higgs doublet a pNGB, it is convenient to parametrize its potential in terms of the sine of the field, as in eq.(2.34). From now on, for simplicity, we denote the SM light Higgs doublet as the Higgs and denote by h the Higgs field in the unitary gauge, matching the notation with that introduced at the beginning of section 2.4.

In the Dimensional Reduction (DRED) scheme the one-loop Higgs potential $V^{(1)}$ is given by

$$\begin{aligned} V^{(1)}(s_h) &= \frac{1}{16\pi^2}\sum_n \frac{(-1)^{2s_n}}{4}(2s_n+1)m_n(s_h)^4\left(\log\frac{m_n^2(s_h)}{Q^2}-\frac{3}{2}\right) \\ &= \frac{1}{64\pi^2}\mathrm{STr}\left[M^4(s_h)\left(\log\frac{M^2(s_h)}{Q^2}-\frac{3}{2}\right)\right], \end{aligned} \tag{3.3}$$

where $m_n^2(s_h)$ are the Higgs-dependent mass squared eigenvalues for the scalars, fermions and gauge fields in the theory and we have denoted the sliding scale by Q. When the mass eigenvalues are not analytically available, we compute the $\log M^2$ term by using the following identity, valid for an arbitrary semi-positive definite matrix M, see e.g. [90]:

$$M^4\log M^2 = \lim_{\Lambda\to\infty}\left(\frac{1}{2}\Lambda^4 - \Lambda^2 M^2 + M^4\log\Lambda^2 - 2\int_0^\Lambda \frac{x^5 dx}{x^2+M^2}\right). \tag{3.4}$$

The RG-invariance of the scalar potential at one-loop level reads

$$\frac{\partial}{\partial\log Q}V^{(1)} + \beta_{\lambda_I}\frac{\partial}{\partial\lambda_I}V^{(0)} - \gamma_n\Phi_n\frac{\partial}{\partial\Phi_n}V^{(0)} = 0, \tag{3.5}$$

where the indices I and n run over all the masses and couplings (including soft terms) and all the scalar fields in the theory, respectively, and $V^{(0)}$

denotes the tree-level scalar potential with the addition of soft terms. By expanding eq. (3.3) for $s_h \ll 1$, we get the explicit form for γ and β defined in eq.(2.37). As we already pointed out in section 2.4, in first approximation we can switch off all SM gauge interactions and keep only the top mixing masses ϵ as explicit source of symmetry breaking. In this limit, only colored fermion and scalar fields contribute to the Higgs potential (3.3).

When all sources of SUSY breaking, denoted collectively by $\widetilde{m}$, are switched off, SUSY requires

$$\lim_{\widetilde{m}\to 0} V(h) = 0\,. \tag{3.6}$$

However, one has to be careful in properly taking the two limits $\widetilde{m} \to 0$, and $s_h \to 0$, since in general they do not commute. The cancellation (3.6) is only manifest when we first take the $\widetilde{m} \to 0$ limit. In practice, however, we only expand in s_h since the sources of SUSY breaking cannot be taken too small.

When the soft terms in the composite sector are SO(5) invariant and the SM gauge interactions are switched off the only SO(5) violating term is the superpotential term

$$W \supseteq \epsilon_L \xi_L O_{t_L} + \epsilon_R \xi_R O_{t_R}\,, \tag{3.7}$$

a superfield generalization of eq.(2.18). It contains the partial compositeness mass mixings among SM fermions and partners as well as an analogous term involving scalar fields, as dictated by SUSY. Also we point out that in the ungauged SM limit the β-functions β_{λ_I} and the anomalous dimensions γ_n appearing in eq.(3.5) are necessarily SO(5) invariant at one-loop level. As a consequence, neither the second nor the third term in eq. (3.5) can depend on s_h and hence the s_h-dependent one-loop potential $V^{(1)}$ is RG invariant and finite. In this case, in contrast to the MSSM, the electroweak scale ξ defined in eq.(2.38) is only logarithmically sensitive to the soft masses when these are taken parametrically large. A similar structure is found for the one-loop potential in SUSY little Higgs models, where a phenomenon named double protection is said to be at work [91,92]. The global symmetry breaking scale f is quadratically sensitive to the soft mass terms associated to the fields responsible for this breaking when these are taken parametrically large. In our models such fields are always in the gauge sector, where we provide a dynamical mechanism of SUSY breaking.

When the SM gauge interactions are switched on, β_{λ_I} and γ_n are no longer SO(5) invariant and can depend on s_h. Although holomorphy protects the superpotential from quantum corrections, the Kähler potential is renormalized and the gauging of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ explicitly breaks the SO(5) global symmetry. This implies that the physical, rather than holomorphic, couplings of the composite sector entering in the superpotential split into several components with different RG evolutions, depending on the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ quantum numbers of the involved fields. We discuss this aspect in more detail in appendix A.1. In what follows, we take the physical couplings to be all equal at the scale f. m_ψ introduced in eq.(3.12) and h, introduced in eq.(3.12) as well as in eq.(4.3),[2] are the couplings whose running is relevant for the Higgs potential.

Similarly, the RG flow induced by the SM gauge couplings gives rise to SO(5) violating contributions to the soft mass terms. In the models we will consider this dependence appears only at order s_h^2. It implies that the RG flow of the tree-level soft terms contributes to γ and induces a quadratic sensitivity to the wino and bino soft terms suppressed by a one-loop factor $\sim g^2/(16\pi^2)$. A "Higgs soft mass term" of the form $\frac{1}{2}\widetilde{m}_H^2 f^2 s_h^2$, even if absent at tree-level, is radiatively generated by the bino and wino masses $\widetilde{m}_g$. A radiatively stable assumption about the Higgs soft term $\widetilde{m}_H^2$ is to take it at the scale f of order

$$|\widetilde{m}_H^2| \sim \frac{g^2}{16\pi^2}\widetilde{m}_g^2 \,. \tag{3.8}$$

In this way, we can neglect its effect on the one-loop potential. Conversely this does not happen for the soft masses of squarks or other scalar fields. A term of the form (see [93] and references therein or [94])

$$g'^2 \mathrm{Tr}[Y_i \widetilde{m}_i^2] = g'^2 S \,, \tag{3.9}$$

present in the beta function of the Higgs soft mass, with i spanning over all relevant scalar fields identically vanishes at leading order. In fact soft masses from the composite sector are assumed to be SO(5) invariant, therefore they do not contribute to the SO(4) breaking soft Higgs mass. For what concerns soft masses for elementary scalar fields they do not enter the gauge contribution at one-loop level: this is a consequence of the fact that the hypercharge D term does not introduce quartic interactions among the Higgs and elementary fields, because for the models presented

$$\mathrm{Tr}\,\left[(U\langle\tilde{q}\rangle)^t T_R^3 (U\langle\tilde{q}\rangle)\right] \equiv 0 \,. \tag{3.10}$$

[2] Throughout the text we denote also the field associated to the physical Higgs particle with the same letter h. The difference should be clear from the context and no confusion should arise.

where the matrix U contains the pNGB Higgs, see eq.(3.17) and the relative discussion. This statement can be reprashed in the following equivalent way, thanks to the embedding of the Higgs in a **4** of SO(4) as given explicitly in eq.(3.2). The relation $\tan\beta = 1$ ensures that the physical Higgs is contained into the sum $H_u^{(d)} + H_d^{(u)}$: therefore the contribution proportional to S cancels out in the beta function of the soft mass, while of course it affects the mass of the other heavy doublet (although this effects does not enter the Higgs potential at one loop and we neglect it). We conclude that apart from the effect induced by EWinos a vanishing soft mass for the physical Higgs is a radiatively stable assumption at one-loop level.

3.3 Minimal Model without Vector Resonances

Given the features outlined in the previous sections we move to a specific example. This allows us to make real predictions, mainly about the spectrum of the theory. A simple supersymmetric pNGB Higgs Model with elementary t_L and t_R can be constructed using two colored chiral multiplets $N_{L,R}$ in the **5** of SO(5), two colored SO(5) singlet fields $S_{L,R}$, two color-neutral multiplets in the **5**, q and ψ, and a complete singlet Z. All these multiplets are necessary to have a linear realization of the global symmetry breaking SO(5) $\to$ SO(4) without unwanted massless charged states. The superpotential reads

$$W = \sum_{i=L,R} (\epsilon_i \xi_i^a N_i^a + \lambda_i S_i q^a N_i^a) + m_N N_L^a N_R^a + m_S S_L S_R + W_0(Z, q, \psi)\,, \quad (3.11)$$

where

$$W_0(Z, q, \psi) = hZ(q_a q_a - \mu^2) + m_\psi q_a \psi_a\,. \quad (3.12)$$

Notice that the gauge sector of the model can be seen as a supersymmetrization of the liner σ-model presented in [76] and in the Appendix G of [56].

We embed the elementary q_L and t_R into spurions ξ_L and ξ_R in the **5** of

	SO(5)	$SU(3)_c$	$U(1)_X$
N_L	$\mathbf{5}$	$\bar{\mathbf{3}}$	$-2/3$
N_R	$\mathbf{5}$	$\mathbf{3}$	$2/3$
S	$\mathbf{1}$	$\mathbf{3}$	$2/3$
S^c	$\mathbf{1}$	$\bar{\mathbf{3}}$	$-2/3$
q	$\mathbf{5}$	$\mathbf{1}$	0
ψ	$\mathbf{5}$	$\mathbf{1}$	0
Z	$\mathbf{1}$	$\mathbf{1}$	0

Table 3.1: Superfields of the model and their quantum numbers under the $SU(3)_c \times SO(5) \times U(1)_X \supseteq G_{SM}$ group.

SO(5) for minimality:[3]

$$\xi_L = \frac{1}{\sqrt{2}} \begin{pmatrix} b_L \\ -ib_L \\ t_L \\ it_L \\ 0 \end{pmatrix}, \qquad \xi_R = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t_R \end{pmatrix}. \tag{3.13}$$

The superpotential eq.(3.12) corresponds to an O'Raifeartaigh model of SUSY breaking. For $\mu^2 > m_\psi^2/(2h^2)$, this model has a SUSY breaking minimum with[4]

$$\langle q_a \rangle = \frac{f}{\sqrt{2}} \delta_a^5 \,, \tag{3.14}$$

where

$$f = \sqrt{2\mu^2 - \frac{m_\psi^2}{h^2}} \,. \tag{3.15}$$

The scalar VEV's of Z and ψ_a, undetermined at the tree-level, are stabilized at the origin by a one-loop potential. The symmetry breaking pattern is the minimal

$$SO(5) \times U(1)_X \to SO(4) \times U(1)_X \,, \tag{3.16}$$

[3] In order to keep the notation light, we omit in what follows the color properties of the fields, that should be clear from the context.

[4] With a common abuse of language, we denote with the same symbol a chiral superfield and its lowest scalar component, since it should be clear from the context the distinction among the two.

where $SU(2)_L \times U(1)_Y$ is embedded in $SO(4) \times U(1)_X$ in the standard fashion. The four NGB's $h^{\hat{a}}$ can be described by means of the σ-model matrix as

$$q_a = U_{ab}\tilde{q}_b = \exp\left(\frac{i\sqrt{2}}{f} h^{\hat{a}} T^{\hat{a}}\right)_{ab} \tilde{q}_b, \tag{3.17}$$

where $T^{\hat{a}}$ are the SO(5)/SO(4) broken generators defined as in the Appendix C and $\tilde{q}$ encodes the non-NGB degrees of freedom of q. In the unitary gauge we can take $h^{\hat{a}} = (0, 0, 0, h)$, and the matrix U simplifies to

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{1-s_h^2} & s_h \\ 0 & 0 & 0 & -s_h & \sqrt{1-s_h^2} \end{pmatrix}. \tag{3.18}$$

The effect of the SUSY breaking is not felt at tree-level by the colored fields $N_{L,R}$, $S_{L,R}$ mixing with the top. We add to the SUSY scalar potential the soft terms $V_{soft} = V_{soft}^E + V_{soft}^C$, transmitted from the external SUSY breaking sector, with

$$V_{soft}^E = \widetilde{m}_{tL}^2 |\xi_L|^2 + \widetilde{m}_{tR}^2 |\xi_R|^2 , \qquad V_{soft}^C = \sum_{\phi_i = N_{L,R}, S_{L,R}} \widetilde{m}_i^2 |\phi_i|^2 , \tag{3.19}$$

and soft masses for the elementary gauginos of the SM gauge group, $\widetilde{m}_{g,1,2,3}$. We neglect the smaller soft mass terms radiatively induced by W_0 and for simplicity we have not included B-terms. Let us analyze the tree-level mass spectrum of the model. We fix the mass parameter $m_S = 0$, since all the states remain massive in this limit,[5] and take $\lambda_L = \lambda_R = \lambda$, so that the composite superpotential enjoys a further $\mathbf{Z}_2$ symmetry (exchange of L and R fields), broken only by the mixing with SM fermions. We also assume all parameters to be real and positive. Before EWSB, the fermion mass spectrum in the matter sector is as follows. A linear combination of fermions, to be identified with the top, is clearly massless. The $SU(2)_L$ doublet with hypercharge 7/6 contained in $N_{L,R}$ does not mix with other fields and has a mass equal to $M_{Q_{7/6}} = m_N$. The doublet with hypercharge 1/6 mixes with

[5] We checked that, if taken non-zero, its contribution to the potential does not change qualitatively the conclusions of our analysis.

q_L and gets a mass $M_{Q_{1/6}} = \sqrt{m_N^2 + \epsilon_L^2}$. Two $SU(2)_L$ singlets get a mass square equal to $M^2_{S_\pm} = 1/2(m_N^2 + \epsilon_R^2 + \lambda^2 f^2 \pm \sqrt{(m_N^2 + \epsilon_R^2)^2 + 2m_N^2 f^2 \lambda^2})$. The scalar spectrum is analogous, with the addition of a shift given by the soft masses (3.19). After EWSB, the top mass is

$$M_{top} = \frac{\epsilon_L \epsilon_R f \lambda s_h \sqrt{1 - s_h^2}}{\sqrt{2}\sqrt{m_N^2 + \epsilon_L^2}\sqrt{2\epsilon_R^2 + f^2\lambda^2}} = \frac{\epsilon_L \epsilon_R f^2 \lambda^2 s_h \sqrt{1 - s_h^2}}{2\sqrt{2} M_{Q_{\frac{1}{6}}} M_{S_+} M_{S_-}} . \tag{3.20}$$

The gauge sector contains the SM vector superfields $w^{(0)}$ and $b^{(0)}$ and the chiral superfields q_a, ψ_a and Z. For simplicity, we neglect all soft mass terms in this sector, but the SM gaugino masses. Regarding the fermion spectrum, the SO(4) fourplets q_n and ψ_n ($n = 1, 2, 3, 4$) get a Dirac mass m_ψ. A linear combination of ψ_5 and Z, we call it p_5, gets a Dirac mass, together with q_5, $\sqrt{2(f^2h^2 + m_\psi^2)}$. The orthogonal combination of ψ_5 and Z (χ_5) is massless being the goldstino associated to the spontaneous breaking of SUSY. In the scalar sector, $\mathrm{Re}\, q_n$ are identified as the pNGB Higgs, while $\mathrm{Im}\, q_n$ get a mass $\sqrt{2} m_\psi$. These two are the mass eigenstates of the two Higgs doublets H_u, H_d introduced in section 3.2. The partners of ψ_n and p_5 will get the same mass as the fermions while the partner of the goldstino χ_5 is a pseudo-modulus, whose VEV is undetermined at the tree-level. This field is stabilized at the origin by a one-loop induced potential. Its detailed mass depends on the ratio $\mu^2 h^2/m_\psi^2$. In the region defined in the next subsection, its mass is of order $m_\psi h/(2\pi) \sim 50 \div 70$ GeV. The real and imaginary parts of q_5 have masses $\sqrt{2} f h$ and $\sqrt{2(f^2h^2 + m_\psi^2)}$, respectively.

Let us discuss the possible values of the parameters of the model. Demanding M_{top} to be around 150 GeV at the TeV scale gives a lower bound on the smallest possible value of the Yukawa coupling λ at the scale f, obtained by taking $\epsilon_{L,R} \to \infty$ in eq.(3.20):

$$\lambda_{min}(f) \gtrsim 1.2 . \tag{3.21}$$

An upper bound on λ is found by looking at its RG running. The relevant beta functions are reported in appendix A.2. For $h \ll 1$, the Yukawa coupling λ is UV free for $\lambda(f) \lesssim 0.9$ and develops a Landau pole for higher values. Demanding that the pole is at a scale greater than $4\pi f$ gives the upper bound:

$$\lambda_{max}(f) \lesssim 1.7 . \tag{3.22}$$

Putting all together, we see that the maximum scale for which the model is trustable and weakly coupled is obtained by taking $\lambda = \lambda_{min}$, in which case we get a Landau pole at around $300f$. This limiting value is never reached in realistic situations, but Landau poles as high as $100f$ can be obtained. The current bound on the top partner with 5/3 electric charge puts a direct lower bound on m_N [13]:[6]

$$M_{Q_{7/6}} = m_N \gtrsim 800 \text{ GeV} . \tag{3.23}$$

Demanding a value for λ as close as possible to the minimum value (3.21), the top mass (3.20) favours regions in parameter space where t_L and t_R strongly mix with the composite sector: $\epsilon_{L,R} \gg m_N$.

3.3.1 Higgs Mass and Fine Tuning Estimate

As we have already remarked, when V^C_{soft} is SO(5) invariant, the one-loop matter contribution to the Higgs potential is RG invariant and finite. Since the explicit form of β_{matter} is quite involved, there is not a simple analytic expression for the Higgs mass valid in all the parameter space. In particular an expansion for small values of $\epsilon_{L,R}$ is never a good approximation because, as explained, these mixings should be taken large.

The region of parameter space which realizes EWSB with $\xi = 0.1$ and gives $M_H = 126$ GeV is essentialy unique. In most of the parameter space γ_{gauge} and γ_{matter} are both positive and bigger than β_{matter}, and no tuning is possible to obtain the right value of ξ. The only region where $\gamma_{gauge} < 0$ is found for $\widetilde{m}_g, m_\psi \lesssim f$ where, however, the size of γ_{gauge} is smaller than the natural size of γ_{matter}, eq.(2.50). The bound (3.8) forces $\widetilde{m}_H^2$ to be negligibly small. From these considerations we see that γ_{matter} has to be tuned in order to become smaller than its natural value. The requirement of perturbativity up to $\Lambda = 100f$ fixes $\lambda(f) \simeq 1.3$. Regarding m_N, a lower bound is given by eq.(3.23) while an upper bound is given from the fact that, increasing m_N requires a higher value of ϵ_L in order to reproduce M_{top}, see eq.(3.20), and, as a consequence, γ_{matter} increases, which is the opposite of what it is necessary to get ξ. This forces $m_N \sim f$, near its lower bound. Reproducing M_{top} fixes

[6]This bound can be applied directly only if the lightest top partner is this one with $Q = 5/3$, in which case it decays in tW^+ with $BR \simeq 100\%$. For lower values of the BR the bound is weaker. We take a conservative approach and use the bound as a constraint on the mass of this particle.

also $\epsilon_L, \epsilon_R \gg f$. The Higgs mass is not sensitive to the stop soft masses $\widetilde{m}_{t_{L,R}}$ and its correct value is found for composite soft masses $\widetilde{m} \sim 3.5f$, taken all equal. Finally, in order to fix $\xi = 0.1$, $\widetilde{m}_{t_L}$ and $\widetilde{m}_{t_R}$ have to be tuned in the region $\widetilde{m}_{t_L} \gg \widetilde{m}_{t_R} \sim \widetilde{m}$.

An approximate analytic formula for M_H^2 in this region is obtained by expanding for $\lambda f \ll m_N, \widetilde{m} \ll \epsilon_{L,R}$, where $\widetilde{m}$ is a common universal soft mass term (the last one is a good approximation because M_H does not depend on the stop soft masses). In this limit we get

$$M_H^2 \simeq \frac{N_c}{2\pi^2} M_{top}^2 \frac{M_{top}^2}{v^2} \left(5 \log \left(\frac{\widetilde{m}^2}{\lambda_{top}^2 f^2} \right) + 4x \log \left(\frac{x}{1+x} \right) + \frac{1}{2} - 4 \log 2 \right), \quad (3.24)$$

where $x = \frac{\widetilde{m}^2}{m_N^2}$. It is immediate to see that for values of $\widetilde{m} \gtrsim m_N \sim f$,[7] a 126 GeV Higgs is reproduced.

Let us briefly discuss the fine tuning. We define it here as the ratio between the value of ξ we want to achieve and its natural value given by (2.39) in absence of cancellations. This is a crude definition, but it has the advantage to estimate the actual FT provided by cancellations rather than the sensitivity, without the need to worry about possible generic sensitivities. The electroweak scale is determined by eq.(2.39). As argued above most of the tuning arises within the matter sector. We can then neglect γ_{gauge} and determine the expected value of ξ by comparing γ_{matter} and β_{matter}. We get[8]

$$\gamma_{matter} \sim \frac{N_c}{8\pi^2} \lambda_{top}^2 f^2 \widetilde{m}^2 , \qquad \beta_{matter} \sim \frac{N_c}{8\pi^2} \lambda_{top}^4 f^4 . \quad (3.25)$$

The FT can then be written as[9]

$$\Delta \sim \frac{\widetilde{m}^2}{f^2} \frac{1}{\xi} , \quad (3.26)$$

and is always higher than the minimum value $1/\xi$. From eq.(3.24) we see that M_H grows with $\widetilde{m}$ in the region of interest and hence we expect a linear increase of Δ with the Higgs mass.

[7]We have numerically checked that the range of applicability of eq.(3.24) extends to the region with $m_N \sim f$.

[8]As explained below eq.(3.6), the limits $s_h \to 0$ and $\widetilde{m} \to 0$ do not commute. As a result, β_{matter} in eq.(3.25) does not vanish for $\widetilde{m} \to 0$.

[9]Another possible source of FT might arise from the origin of the scale f as the cancellation of the two terms in eq.(3.15). In the region of interest no significant cancellation occurs and we neglect this effect.

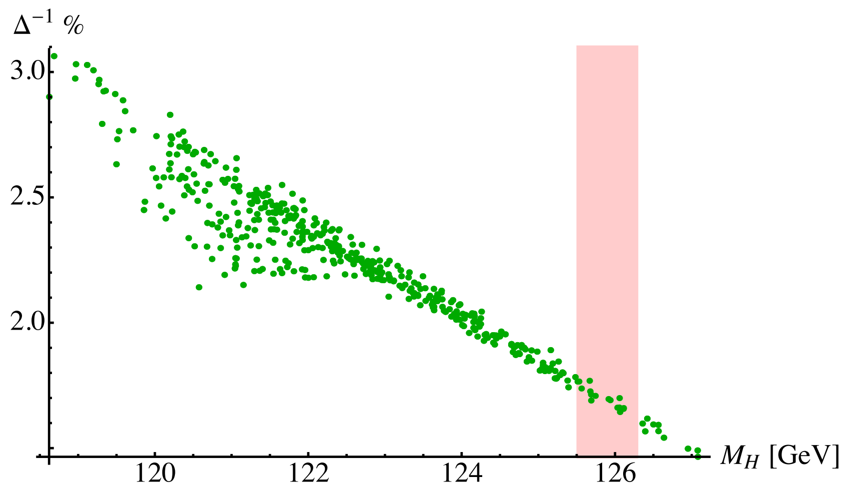

Figure 3.2: FT of the minimal model, (in %) as a function of the Higgs mass for $\xi \simeq 0.1$. In this scan we fixed $\lambda(f) = 1.29$ and $h(f) = 0.44$, so that both λ and h reach a Landau pole at the same scale $\Lambda \sim 100f$, $m_N = 1.2f$ and picked randomly $\epsilon_L \in [8.5f, 10f]$, $m_\psi \in [0.7f, 1.5f]$, $\widetilde{m}_g \in [0.5f, f]$, $\widetilde{m} \in [2.5f, 4.5f]$, $\widetilde{m}_{t_L} \in [4.5f, 6.5f]$, $\widetilde{m}_{t_R} \in [f, 3f]$ and $\widetilde{m}^2_H$ within the bound (3.8). We fixed M_{top} by solving for ϵ_R and then selected points with $\xi \simeq 0.1$. The pink strip represents the Higgs mass 1σ-interval as reported in ref. [95].

In order to check these considerations we performed a parameter scan in the restricted region described at the beginning of the section. We fixed the top mass by solving for ϵ_R and then obtained the minimum of the potential and the Higgs mass from the full one-loop expression of eq.(3.3). We report in fig.3.2 a plot of the FT computed using the standard definition of ref. [19] as a function of the Higgs mass. As can be seen, we obtain $\Delta^{-1} \sim 2\%$ for $M_H = 126$ GeV, in reasonable agreement with the rough estimate (3.26).

Let us now discuss the spectrum of new particles. In this region, the electroweak gauginos are relatively light, $\widetilde{m}_g \lesssim f \sim 800$ GeV and the two higgsino doublets (from ψ_n and q_n) have also a mass $m_\psi \sim 800$ GeV. The stops and their partners are heavy, above 2 TeV, while the fermion top partners are usually below the TeV, the lightest being the singlet with $Q = 2/3$ and a mass $M_{S_-} \simeq 660$ GeV.[10] The gluinos do not contribute to the Higgs

[10] The recent CMS analysis [54] rules out charge 2/3 top partners below ~ 700 GeV. A careful phenomenological analysis should be performed to check if the model with the

	SO(5)	$SO(4)_2$	$SU(3)_c$	$U(1)_X$
N_L	$\mathbf{5}$	$\mathbf{1}$	$\bar{\mathbf{3}}$	$-2/3$
N_R	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{3}$	$2/3$
X_L	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{3}$	$2/3$
X_R	$\mathbf{1}$	$\mathbf{4}$	$\bar{\mathbf{3}}$	$-2/3$
q	$\mathbf{5}$	$\mathbf{4}$	$\mathbf{1}$	0
Z	$\mathbf{1}\oplus\mathbf{14}$	$\mathbf{1}$	$\mathbf{1}$	0

Table 3.2: Chiral superfield content of the model with vector resonances.

potential at one-loop, therefore they can be taken heavy (above the experimental bounds) without increasing the fine tuning.

3.4 Road to Vector Resonances

A modification of the model is obtained introducing partial compositeness also for EW gauge bosons mixing them with heavy spin-1 resonances. In our linear realization this is achieved enlarging the pattern of symmetry breaking from SO(5) $\to$ SO(4) to SO(5) $\times$ $\mathrm{SO}(4)_2$ $\to$ $\mathrm{SO}(4)_D$: the gauging of the additional $\mathrm{SO}(4)_2$ provides the vector resonances and keeps the number of uneaten Goldstone modes to be identified with the components of the Higgs doublet equal to four. The field content is reported in table 3.2 while the superpotential is

$$W=\sum_{i=L,R}(\epsilon_i\xi_i^a N_i^a+\lambda_i X_i^n q_n^a N_i^a)+m_N N_L^a N_R^a+m_X X_L^n X_R^n+W_0(Z,q)\,, \quad (3.27)$$

where $N_{L,R}$ and $X_{L,R}$ are colored fields in the $(\mathbf{5},\mathbf{1})$ and $(\mathbf{1},\mathbf{4})$ of SO(5) $\times$ $\mathrm{SO}(4)_2$, respectively, and q is a color-singlet in the $(\mathbf{5},\mathbf{4})$ ($a=1,\ldots,5$, $n=1\ldots,4$). The spurions $\xi_{L,R}$ are taken as in eq.(3.13). The superpotential term W_0 reads

$$W_0=h\Big(q_a^n Z_{ab} q_b^n-\frac{f^2}{2}Z_{aa}\Big), \quad (3.28)$$

where Z is a field in the symmetric $\mathbf{14}\oplus\mathbf{1}$ of SO(5).

benchmark parameters taken is ruled out or not. Slightly decreasing ξ or the scale of the Landau pole are two possible solutions to increase the mass of M_{S_-} beyond 700 GeV.

The extra gauge sector is strongly coupled and it has a Landau pole at a low scale, around $10f$. Also, due to the augmented multiplicity of fields, the couplings λ and h enter the strong regime at the same scale. This case is discussed in [2] in a manner similar to the previous case. Nevertheless here we emphasize that it can be nicely seen as an effective field theory for a theory valid up to arbitrary high energy in the limit of ungauged SM. In fact the superfields, with the addition of a colored state Y, can be grouped as

$$q = \left(\frac{X_L,\ X_R}{q} \right), \quad M = \left(\begin{array}{c|c} Y & N_{L,R} \\ \hline N_{L,R}^t & Z \end{array} \right). \tag{3.29}$$

The colored fields are grouped embedding the $\mathrm{SU}(3)_c \times \mathrm{U}(1)_X$ in a larger SO(6) such that $\mathbf{6} = \mathbf{3}_{2/3} + \bar{\mathbf{3}}_{-2/3}$. The superfield q is written as a $(6+5)\times 4$ matrix while M is a $(6+5)\times(6+5)$ symmetric matrix. The transformation properties of the field Y are consequently deduced: it transforms as a $\mathbf{6}\times\mathbf{6}$. Also the couplings $\sqrt{2}\lambda_L$, $\sqrt{2}\lambda_R$ and h can be collected in a new coupling, which we call again h. The cubic terms of eq.(3.27) and eq.(3.28) can be written as

$$W = h\mathrm{Tr}[qMq]\,. \tag{3.30}$$

In the limit of decoupled SM the $\mathrm{SO}(6)\times\mathrm{SO}(5)$ group is a global symmetry, while the $\mathrm{SO}(4)_2$ is gauged. It can be viewed, through Seiberg duality, as the low energy description of a UV free supersymmetric QCD (SQCD) with a SO(11) gauge group and $N_f = 11$ flavors in the fundamental [1]: the gauge group $\mathrm{SO}(4)_2$ is emergent and we refer to it as magnetic, in contrast to SO(11) which we call electric. We recall the details of Seiberg duality focusing on orthogonal groups in the next section and we devote the next chapter to the discussion of this model from the UV completed point of view.

3.5 Seiberg Duality for Orthogonal Groups

In this section we briefly review Seiberg duality for orthogonal groups, [96,97]; similar considerations can be made for other Lie groups, although we focus on orthogonal for definiteness and because it is the case of our interest.

We first introduce $\mathcal{N}=1$ SQCD with $\mathrm{SO}(N)$ gauge group and N_f chiral superfields Q in the fundamental: they transform in the fundamental of a global $\mathrm{SU}(N_f)$. N and N_f play a crucial role in determining the dynamics of the theory and for different values we have qualitatively different situations,

<table>
<tr><td></td><td>$N-2$</td><td>$\frac{3}{2}(N-2)$</td><td>$3(N-2)$</td></tr>
<tr><td></td><td>free magnetic phase</td><td>interacting fixed point</td><td>free electric phase</td></tr>
<tr><td colspan="3">UV free</td><td>IR free</td></tr>
<tr><td colspan="2">confining</td><td colspan="2">non confining</td></tr>
</table>

Table 3.3: Diagram for SO(N) SQCD: the number of flavor N_f increases from left to right.

as we are going to recall. The coefficient of the beta function of the gauge coupling is $b_{el} = 3(N-2) - N_f$, therefore the theory is IR free for $N_f \geq 3(N-2)$; for $N_f < N-4$ gaugino condensation occurs, while the cases $N_f = N-4, N-3, N-2$ have a richer vacuum structure. For $0 < 3(N-2) - N_f \ll 1$ the theory is expected to have an interacting fixed point: in fact in this range of parameters the one-loop coefficient of the beta function for g is positive while the two-loop coefficient is negative, therefore it exists a value of g such that the beta function vanishes: it can be shown that $g \sim 1/N$. An elegant way to derive the two-loop result is to relate the physical gauge coupling to the holomorphic coupling which, being protected by SUSY, runs only at one-loop and whose beta function is exactly proportional to b_{el} [98]. It can be argued that the existence of an interacting fixed point extends within a conformal window defined by

$$\frac{3}{2}(N-2) < N_f < 3(N-2)\,. \tag{3.31}$$

The position of the lower bounds can be inferred inspecting the scaling dimension of the gauge invariant scalar operator $M = QQ$: because of an unitarity bound valid in any CFT the dimension of any scalar operator cannot be smaller than 1 [99]. At the same time the current associated to the R symmetry and the stress energy momentum tensor (related to the current associated to dilatations) sit in the same real vector superfield. In a superconformal field theory (SCFT) both currents are conserved and the associated charges are proportional, in particular[11]

$$2\dim M = 3R(M) \tag{3.32}$$

[11]This holds true for primary chiral operators: for a discussion see [37].

and the unitarity bound becomes $R(M) > 2/3$. Combining $R(M) = 2R(Q)$ and the expression of $R(Q)$ given in table 3.4 gives the bound

$$N_f > 3(N-2)/2\,. \tag{3.33}$$

An electric magnetic duality, named after Seiberg, can be formulated for SO(N) SQCD as long as

$$N-2 < N_f < 3(N-2)\,. \tag{3.34}$$

The dual theory is a supersymmetric gauge theory with magnetic quarks in the fundamental of an SO($N_f - N + 4$) gauge group and a neutral meson M: it has a non perturbative superpotential of the form

$$W_{mag} = hqMq\,. \tag{3.35}$$

Consistently the dual theory has an interacting fixed point in the same conformal window where both h and the gauge coupling are attracted to fixed values. Notably the magnetic theory is IR free for the values $N-2 < N_f \leq \frac{3}{2}(N-2)$. In this region the electric theory is strongly coupled in the IR and confining, and the magnetic dual can be seen as an effective theory for the IR dynamics, otherwise intractable. On the other hand the magnetic theory provides, in terms of weakly coupled degrees of freedom, a description valid up to a certain cutoff, above which it becomes inconsistent and it is UV completed by the electric theory; see for instance [100]. Finally for $N_f \leq N-2$ the magnetic dual is not defined and the original theory typically develops a non perturbative superpotential or a quantum moduli space of vacua: we do not discuss these cases. We summarize this behavior in table 3.3.

In table 3.4 we recall the superfield content of the electric and the magnetic theories with the gauged and global symmetry.

The running of the gauge coupling constant in a gauge theory produces a scale which is associated to non perturbative effects, as it is well known:

$$\Lambda = \mu \exp\left(-\frac{8\pi^2}{bg^2(\mu)}\right) \tag{3.36}$$

In our case of Seiberg duality the requirement that the two coupling constants, electric and magnetic, match (up to a phase) at a certain scale μ translates to the following

$$\Lambda_{el}^{b_{el}}\Lambda_{mag}^{b_{mag}} = (-1)^{N_f-N}\mu^{N_f} \tag{3.37}$$

	$\mathrm{SO}(N)_{el}$	$\mathrm{SU}(N_f)$	$\mathrm{U}(1)_R$
Q_I^N	$\mathbf{N}$	$\mathbf{N_f}$	$\frac{(N_f-N+2)}{N_f}$

	$\mathrm{SO}(N_f-N+4)_{mag}$	$\mathrm{SU}(N_f)$	$\mathrm{U}(1)_R$
$q^{I,n}$	$\mathbf{N_f-N+4}$	$\overline{\mathbf{N}}_\mathbf{f}$	$\frac{N-2}{N_f}$
M_{IJ}	$\mathbf{1}$	$\frac{\mathbf{1}}{\mathbf{2}}\mathbf{N_f(N_f+1)}$	$\frac{2(N_f-N+2)}{N_f}$

Table 3.4: Field content of the electric and magnetic theories.

The sign is fixed by other considerations, namely consistency of this relation and the duality. Eq.(3.37) tells us that $g_{el}^{-2}+g_{mag}^{-2}\sim const$ and in the conformal window at the IR fixed point the weaker one coupling is, the stronger the other.

In the range $N-2<N_f\leq\frac{3}{2}(N-2)$ the magnetic theory is IR free and it can be seen as an effective theory valid up to Λ_{mag}, while its UV completion, the corresponding electric theory, becomes non perturbative at and below a scale Λ_{el}. It is natural to choose the parameters of the theory such that there is only one scale

$$\Lambda=\Lambda_{el}=\Lambda_{mag}=\mu\,. \tag{3.38}$$

In the following we will assume this is the case and we will refer to the unique non perturbative scale of the theory Λ.

Several checks of this proposed duality have been verified: for instance they enjoy the same global symmetries, the 't Hooft anomalies match, and adding a mass term for quarks in a theory results in higgsing part of the gauge group of the dual theory and vice versa. The gauge invariant operators of the electric and magnetic theory are in one to one correspondence:

$$\begin{array}{ll} \spadesuit & Q_I^N Q_J^N \\ \heartsuit & \epsilon_{N_1\ldots N_N}Q_{I_1}^{N_1}\ldots Q_{I_N}^{N_N} \\ \diamondsuit & \epsilon_{N_1\ldots N_{N-4}W_1\ldots W_4}Q_{I_1}^{N_1}\ldots Q_{I_{N-4}}^{N_{N-4}}(W_{el}^{\alpha}W_{el,\alpha})^{W_1\ldots W_4}\,, \\ \clubsuit & \epsilon_{N_1\ldots N_{N-2}W_1W_2}Q_{I_1}^{N_1}\ldots Q_{I_{N-2}}^{N_{N-2}}W_{el,\alpha}^{W_1W_2} \end{array} \tag{3.39}$$

$$
\begin{array}{ll}
\spadesuit & M_{IJ} \\
\heartsuit & \epsilon_{I_1 \ldots I_N J_1 \ldots J_{N_f-N}} \epsilon_{n_1 \ldots n_{N_f-N} w_1 \ldots w_4} q^{J_1 n_1} \ldots q^{J_{N_f-N} n_{N_f-N}} \left(W^{\alpha}_{mag} W_{mag,\alpha}\right)^{w_1 \ldots w_4} \\
\diamondsuit & \epsilon_{I_1 \ldots I_{N-4} J_1 \ldots J_{N_f-N+4}} \epsilon_{n_1 \ldots n_{N_f-N+4}} q^{J_1 n_1} \ldots q^{J_{N_f-N+4} n_{N_f-N+4}} \\
\clubsuit & \epsilon_{I_1 \ldots I_{N-4} J_1 \ldots J_{N_f-N+2}} \epsilon_{n_1 \ldots n_{N_f-N+2} w_1 w_2} q^{J_1 n_1} \ldots q^{J_{N_f-N+2} n_{N_f-N+2}} W^{w_1 w_2}_{mag,\alpha}
\end{array}
. \quad (3.40)
$$

Notice that the dual quarks bilinear qq in the IR is fixed to zero by the F term equations implied by the superpotential eq.(3.35).

In the regime $N-2 < N_f \leq \frac{3}{2}(N-2)$ the SUSY gauge theory confines at a scale non perturbatively generated: below that scale we have a description in terms of an effective theory for vectors, scalars and fermions which are free in the IR but strongly coupled in the vicinity of the cutoff of the theory. Therefore we use this theory, gauging a subgroup of the global symmetries, as a candidate for the BSM sector, generically introduced in chapter 2, providing a pNGB composite Higgs. An interesting phenomenon occurs if a mass term is turned on for the quarks in the electrical theory: near the origin in moduli space there exists a metastable vacuum in which SUSY is spontaneously broken, to which we refer as the Intriligator Seiberg and Shih (ISS) vacuum [101]. The original computation has been possible only thanks to the Seiberg duality since the vacuum is found in terms of magnetic variables. Since the lifetime can be made parametrically large the vacuum is interesting also from a phenomenological point of view, as a SUSY breaking sector. We put ourselves in the ISS vacuum but we do not rely on this SUSY breaking to generate soft masses: we are concerned with the pattern of breaking of bosonic symmetries and, as already mentioned, we appeal to another unspecified SUSY breaking source.

Chapter 4

A UV Complete Model

4.1 The Basic Construction

As argued in subsection 3.4 the model there presented has a UV completion in terms of a confining SQCD theory described, at low energy, through Seiberg duality, recalled in section 3.5. Before discussing in detail the model we start this chapter with a more general approach, namely we do not completely fix the rank of the flavor group spontaneously broken: in fact the key points underlying our models are best illustrated in a set-up where we keep only the essential structure. We then converge to specific models. We focus on constructions where the Higgs is the NGB of an SO(5)/SO(4) coset, but the generalization to other cosets should be obvious.

Consider an $\mathcal{N}=1$ SUSY SO(N) gauge theory with $N_f = N$ flavors in the fundamental of SO(N), with superpotential

$$W_{el} = m_{ab} Q^a Q^b + \lambda_{IJK} Q^I Q^J \xi^K \,. \tag{4.1}$$

In the first term of eq.(4.1), we split the flavor index I in two sets $I=(i,a)$, $a=1,\ldots,5$, $i=6,\ldots,N$. The fields ξ^K are singlets under SO(N) and in general can be in some representation of the flavor group $H_f \subset G_f$ left unbroken by the Yukawa couplings λ_{IJK}. The ξ^K's are eventually identified as the visible chiral fields, such as the top fields. We take $\lambda_{IJK} \ll 1$, so that these couplings are marginally relevant, with no Landau poles, and can be considered as a small perturbation in the whole UV range of validity of the theory. We assume the presence of an external source of SUSY breaking, whose origin will not be specified, that produces soft terms for all the

SM gauginos and sfermions. For simplicity, we neglect for the moment the dynamics of the singlets ξ^K and the impact of the external source of SUSY breaking in the composite sector. We take the quark mass matrix proportional to the identity, $m_{ab} = m_Q \delta_{ab}$, to maximize the unbroken anomaly-free global group. For $\lambda_{IJK} = 0$, this is equal to

$$G_f = \mathrm{SO}(5) \times \mathrm{SU}(N-5)\,. \tag{4.2}$$

We take $|m_Q| = \epsilon^2 \Lambda \ll \Lambda$, where Λ is the dynamically generated scale of the theory.

For $N \leq 3(N-2)/2$, namely $N \geq 6$, the theory flows to an IR-free theory with superpotential [96, 97]

$$W_{mag} = h q_I M^{IJ} q_J - \mu^2 M_{aa} + \epsilon_{IJK} M^{IJ} \xi^K\,, \tag{4.3}$$

where

$$\epsilon_{IJK} = \lambda_{IJK} \Lambda, \qquad \mu^2 = -m_Q \Lambda = (\epsilon\Lambda)^2. \tag{4.4}$$

For simplicity, we identify the dynamically generated scales in the electric and magnetic theories,[1] whose precise relation is anyhow incalculable. The fields q_I are the dual magnetic quarks in the fundamental representation of the dual $\mathrm{SO}(N_f - N + 4)_m = \mathrm{SO}(4)_m$ magnetic gauge group, with coupling g_m, and $M^{IJ} = Q^I Q^J$ are neutral mesons, normalized to have canonical dimension one. The Kähler potential for the mesons M^{IJ} and the dual quarks q_I is taken as follows:

$$K = \mathrm{Tr}[M^\dagger M] + q_I^\dagger e^{V_{mag}} q_I\,, \tag{4.5}$$

where V_{mag} is the $\mathrm{SO}(4)_m$ vector superfield.

The original Yukawa couplings $\lambda_{IJK} Q^I Q^J \xi^K$ in the electric theory flow in the IR to a mixing mass term $\epsilon_{IJK} M^{IJ} \xi^K$ between elementary and composite fields, the SUSY version of the fermion mixing terms appearing in weakly coupled models with partial compositeness [28]. The quark mass term $m_Q Q^a Q^a$, introduced to break the flavor group from $\mathrm{SU}(N)$ down to $\mathrm{SO}(5) \times \mathrm{SU}(N-5)$, is also responsible for a spontaneous breaking of SUSY by the rank condition, as shown in [101]. Up to global $\mathrm{SO}(5) \times \mathrm{SO}(4)_m$ rotations, the non-SUSY, metastable, vacuum is at

$$\langle q^n_m \rangle = \frac{\mu}{\sqrt{h}}\, \delta^n_m\,, \tag{4.6}$$

[1] We refer to the UV and IR theories as electric and magnetic theories, respectively.

with all other fields vanishing. For simplicity, in the following we take μ and h to be real and positive. In eq.(4.6) we have decomposed the flavor index $a = (m, 5)$, $m, n = 1, 2, 3, 4$, and we have explicitly reported the gauge index n as well. When $\lambda_{IJK} = 0$, the vacuum (4.6) spontaneously breaks

$$\mathrm{SO}(4)_m \times \mathrm{SO}(5) \to \mathrm{SO}(4)_D\,, \tag{4.7}$$

where $\mathrm{SO}(4)_D$ is the diagonal subgroup of $\mathrm{SO}(4)_m \times \mathrm{SO}(4)$. The group $\mathrm{SU}(N-5)$ remains untouched, being q_a^n singlet under it. In the global limit $g_m \to 0$, this symmety breaking pattern results in 10 NGB's:

$$\mathrm{Re}\,(q_n^m - q_m^n) \quad : \quad \text{along the broken}\,\mathrm{SO}(4)_m \times \mathrm{SO}(4)\ \text{directions}\,, \tag{4.8}$$

$$\sqrt{2}\,\mathrm{Re}\,q_5^n \quad : \quad \text{along the broken}\,\mathrm{SO}(5)/\mathrm{SO}(4)_D\ \text{directions}\,. \tag{4.9}$$

For $g_m \neq 0$, the would-be NGB's (4.8) are eaten by the $\mathrm{SO}(4)_m$ magnetic gauge fields ρ_μ, that become massive, while the NGB's (4.9) remain massless and are identified with the 4 real components of the Higgs field.

The remaining spectrum of the magnetic theory around the vacuum (4.6) is easily obtained by noticing that all fields, but the magnetic quarks q_5^n and the mesons M_{5n}, do not feel at tree-level the SUSY breaking induced by the F-term of M_{55}:

$$F_{M_{55}} = -\mu^2. \tag{4.10}$$

The chiral multiplets $(q_n^m + q_m^n)/\sqrt{2}$ and M_{mn} combine and get a mass $2\sqrt{h}\mu$, as well as the multiplets M_{im} and q_i^m that form multiplets with mass $\sqrt{2}\sqrt{h}\mu$. The chiral multiplets $(q_n^m - q_m^n)/\sqrt{2}$ combine with the $\mathrm{SO}(4)_m$ vector multiplets to give vector multiplets with mass $\sqrt{\frac{2}{h}}g_m\mu$. As we have just seen, the NGB scalar components $\mathrm{Re}\,(q_n^m - q_m^n)$ are eaten by the gauge fields, while $\mathrm{Im}\,(q_n^m - q_m^n)$ get a mass by the $\mathrm{SO}(4)_m$ D-term potential. Similarly, the fermions $(\psi_{q_n^m} - \psi_{q_m^n})/\sqrt{2}$ become massive by mixing with the gauginos λ_{mn}. The chiral multiplets M_{ij} and M_{i5} remain massless.

The scalar field M_{55} is massless at tree-level and its VEV is undetermined (pseudo-modulus). This is stabilized at the origin by a one-loop induced Coleman-Weinberg potential, as we will shortly see. Its fermion partner is also massless, being the goldstino. Around $M_{55} = 0$, the fermions ψ_{q_5} and $\psi_{M_{5m}}$ mix and get a mass $\sqrt{2}\sqrt{h}\mu$, the scalars M_{5m} get the same mass. $\mathrm{Im}\,q_5^m$ get a mass $2\sqrt{h}\mu$, while $\mathrm{Re}\,q_5^m$ remain massless, the latter being indeed NGB's. The fate of M_{55} is determined by noticing that the superpotential of the $M_{55} - M_{5m} - q_5^m$ sector is

$$W_{mag} \supset -\mu^2 M_{55} + \sqrt{2} h \mu q_5^m M_{5m} + h (q_5^n)^2 M_{55} \,, \tag{4.11}$$

that is a sum of O'Raifeartaigh models. The associated one-loop potential is well-known (see e.g. appendices A.2 and A.3 of [101]). The pseudo-modulus M_{55} is stabilized at zero, and gets a one-loop mass

$$m^2_{M_{55}} = \frac{h^4 \mu^2}{\pi^2} f(h) \tag{4.12}$$

where $f(h) = \left[\frac{(1+h)^2}{h} \log \frac{1+h}{h} - \frac{(1-h)^2}{h} \log \frac{1-h}{h} - 2\right]$. The SM vector fields are introduced by gauging a subgroup of the flavor symmetry group

$$H_f \supseteq \mathrm{SU}(3)_c \times \mathrm{SU}(2)_{0,L} \times \mathrm{U}(1)_{0,Y} \tag{4.13}$$

that is left unbroken when we switch on the couplings ϵ_{IJK}. We embed $\mathrm{SU}(3)_c$ into $\mathrm{SU}(N-5)$ and $\mathrm{SU}(2)_{0,L} \times \mathrm{U}(1)_{0,Y}$ in $\mathrm{SO}(5) \times \mathrm{U}(1)_X$, where $\mathrm{U}(1)_X$ is a U(1) factor coming from $\mathrm{SU}(N-5)$ needed to correctly reproduce the SM fermion hypercharges. The details of the embedding are model-dependent and will be considered in the next sections. We identify $\mathrm{SU}(2)_{0,L}$ as the subgroup of $\mathrm{SO}(4) \cong \mathrm{SU}(2)_{0,L} \times \mathrm{SU}(2)_{0,R} \subset \mathrm{SO}(5)$. The hypercharge Y is given by $Y = T_{3R} + X$, where T_{3R} and X are the generators of the σ_3 direction $\mathrm{U}(1)_{0,R} \subset \mathrm{SU}(2)_{0,R}$ and of $\mathrm{U}(1)_X$, respectively. Denoting by A_μ^{aL} ($a = 1, 2, 3$), A_μ^{3R} and X_μ the $\mathrm{SU}(2)_{0,L} \times \mathrm{U}(1)_{0,R} \times \mathrm{U}(1)_X$ gauge fields and by g_0 (the same for $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_R$, for simplicity) and g_X their gauge couplings, we have (see appendix C for our group-theoretical conventions)

$$A_\mu^{aL} = W_\mu^a \,, \quad A_\mu^{3R} = c_X B_\mu \,, \quad X_\mu = s_X B_\mu \,, \tag{4.14}$$

where

$$c_X = \frac{g_X}{\sqrt{g_0^2 + g_X^2}} = \frac{g_0'}{g_0}, \quad s_X = \frac{g_0}{\sqrt{g_0^2 + g_X^2}}. \tag{4.15}$$

The $\mathrm{SU}(2)_{0,L} \times \mathrm{U}(1)_{0,Y}$ gauge fields W_μ^a and B_μ introduced in this way are not yet the actual SM gauge fields, because the flavor-color locking given by the VEV eq.(4.6) generates a mixing between the $\mathrm{SO}(4)_m \cong \mathrm{SU}(2)_{m,L} \times \mathrm{SU}(2)_{m,R}$ magnetic gauge fields and the elementary gauge fields. This explains the subscript 0 in $\mathrm{SU}(2)_{L,R}$ and $\mathrm{U}(1)_{Y,R}$ and in g and g' in eq.(4.15). The combination of fields along the diagonal $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset \mathrm{SO}(4)_D \times \mathrm{U}(1)_X$ group is finally identified with the SM vector fields. The SM gauge couplings g and g' are given by

$$\frac{1}{g^2} = \frac{1}{g_m^2} + \frac{1}{g_0^2}\,, \qquad \frac{1}{g'^2} = \frac{1}{g_m^2} + \frac{1}{g_0'^2}\,. \tag{4.16}$$

This mixing between elementary and composite gauge fields is analogous to the one advocated in bottom-up 4D constructions of composite Higgs models. The situation is simpler for the color group, since the gauge fields of $SU(3)_c$ are directly identified with the ordinary gluons of QCD; since the group $SU(N-5)$ in eq.(4.2) contains $SU(3)\times U(1)$, the minimal anomaly-free choices are SO(6) and SU(4)[2].

The set-up above is still unrealistic because of the presence of unwanted exotic massless states (M_{ij} and M_{i5}). There are various ways to address these points. We do that in the following, where we consider in greater detail the two models with SO(6) and SU(4), corresponding to $N_f = 5+6 = 11$ and $N_f = 5+4 = 9$ flavors, respectively.

4.2 A Semi-Composite t_R

With the general idea presented in the previous section 4.1, we build the explicit UV completion mentioned in section 3.4, namely we specialize to the case of a SUSY SO(11) gauge theory with $N_f = N = 11$ electric quarks. We also have two additional singlet fields, S_{ij} and S_{ia}, transforming as $(\mathbf{1}, \overline{\mathbf{20}}\oplus\mathbf{1})$ and $(\mathbf{5}, \overline{\mathbf{6}})$ of $SO(5)\times SU(6)$, respectively.[3] We add to the superpotential (4.1) the following terms:

$$\frac{1}{2}m_{1S}S_{ij}^2 + \lambda_1 Q^i Q^j S_{ij} + \frac{1}{2}m_{2S}S_{ia}^2 + \lambda_2 Q^i Q^a S_{ia}\,. \tag{4.17}$$

The mass terms in eq.(4.17) break the SU(6) global symmetry to SO(6). The total global symmetry of the model is then

$$G_f = SO(5)\times SO(6)\,. \tag{4.18}$$

[2] The algebras of SO(6) and SU(4) are isomorphic. We embed the $SU(3)_c$ such that the fundamental **3** is contained in the fundamental of SO(6) and SU(4), a **6** and a **4**.

[3] See [102] for a similar set-up in the context of models with direct gaugino mediation of SUSY breaking.

For $m_{1S,2S} > \Lambda$, the singlets S_{ij} and S_{ia} can be integrated out in the electric theory. We get[4]

$$W_{el}^{eff} = m_{ab}Q^aQ^b - \frac{\lambda_1^2}{2m_{1S}}(Q^iQ^j)^2 - \frac{\lambda_2^2}{2m_{2S}}(Q^iQ^a)^2 \,. \tag{4.19}$$

In the magnetic dual superpotential, the quartic deformations give rise to mass terms for the mesons M_{ij} and M_{i5}:

$$W_{mag} \supset -\frac{1}{2}m_1 M_{ij}^2 - \frac{1}{2}m_2 M_{ia}^2 \,, \tag{4.20}$$

where

$$m_i = \frac{\Lambda^2\lambda_i^2}{m_{iS}}, \quad i = 1, 2 \,. \tag{4.21}$$

The mass deformations do not affect the vacuum (4.6), but obviously change the mass spectrum given in section 4.1. The multiplets M_{ij} and M_{i5} are now massive, with masses given by m_1 and m_2, respectively, and the multiplets M_{im} and q_i^m form massive multiplets with squared masses $(m_2^2 + 16h\mu^2 \pm m_2\sqrt{m_2^2 + 32h\mu^2})/8$. We take the masses m_1 and m_2 as free parameters, although phenomenological considerations favour the values of m_2 for which the mesons M_{ia}, the ones that are going to mix with the elementary SM fields, have a mass around μ. We summarize in table 4.1 the gauge and flavor quantum numbers of the fields appearing in the electric and magnetic theories. We embed $SU(3)_c$ into $SO(6)$ and $SU(2)_{0,L} \times U(1)_{0,Y}$ in $SO(5) \times U(1)_X$, where $U(1)_X$ is a $U(1)$ factor coming from $SO(6)$ (see appendix C). We consider in what follows the top quark only, since this is the relevant field coupled to the EWSB sector. In terms of the UV theory, we might have Yukawa couplings of the top with the electric quarks, or mixing terms with the singlet fields. When the singlets are integrated out, we simply get a shift in the mixing of the top with the meson fields. So, without loss of generality, we can ignore mixing terms between the top and the singlets. The most general mixing term is then

$$\lambda_L(\xi_L)^{ia}Q_iQ_a + \lambda_R(\xi_R)^{ia}Q_iQ_a \,. \tag{4.22}$$

[4] Of course, we could have started directly by deforming the superpotential (4.1) with the irrelevant operators quartic in the quark fields appearing in eq.(4.19). However we want to emphasize how easy is to UV complete the above quartic terms. See [103] for studies of ISS theories deformed by irrelevant operators quartic in the quark fields.

	$\mathrm{SO}(11)_{el}$	SO(5)	SO(6)
Q_i^N	**11**	**1**	**6**
Q_a^N	**11**	**5**	**1**
S_{ij}	**1**	**1**	$\mathbf{20}\oplus\mathbf{1}$
S_{ia}	**1**	**5**	**6**

(a)

	$\mathrm{SO}(4)_m$	SO(5)	SO(6)
q_i^n	**4**	**1**	**6**
q_a^n	**4**	**5**	**1**
M_{ij}	**1**	**1**	$\mathbf{20}\oplus\mathbf{1}$
M_{ia}	**1**	**5**	**6**
M_{ab}	**1**	$\mathbf{14}\oplus\mathbf{1}$	**1**

(b)

Table 4.1: Quantum numbers under G_f and the strong gauge group of the matter fields appearing in the composite sector of model I: (a) UV electric and (b) IR magnetic theories.

We have written the mixing terms in a formal G_f invariant way in terms of the fields ξ_L and ξ_R. These are spurion superfields, whose only dynamical components are the SM doublet superfields $Q_L = (t_L, b_L)^t$ and the singlet t^c, whose θ-component is the conjugate of the right-handed top t_R. In order to write ξ_L and ξ_R in terms of q_L and t^c, we have to choose an embedding of $\mathrm{SU}(3) \subset \mathrm{SO}(6)$:

$$
(\xi_L)^{ia} = \begin{pmatrix} b^1 & -ib^1 & t^1 & it^1 & 0 \\ -ib^1 & -b^1 & -it^1 & t^1 & 0 \\ b^2 & -ib^2 & t^2 & it^2 & 0 \\ -ib^2 & -b^2 & -it^2 & t^2 & 0 \\ b^3 & -ib^3 & t^3 & it^3 & 0 \\ -ib^3 & -b^3 & -it^3 & t^3 & 0 \end{pmatrix}_{2/3} , \quad (\xi_R)^{ia} = \begin{pmatrix} 0 & 0 & 0 & 0 & (t^c)^1 \\ 0 & 0 & 0 & 0 & i(t^c)^1 \\ 0 & 0 & 0 & 0 & (t^c)^2 \\ 0 & 0 & 0 & 0 & i(t^c)^2 \\ 0 & 0 & 0 & 0 & (t^c)^3 \\ 0 & 0 & 0 & 0 & i(t^c)^3 \end{pmatrix}_{-2/3} , \tag{4.23}
$$

in terms of $\mathrm{SO}(6) \times \mathrm{SO}(5)$ multiplets, where the superscript in the fields denote the color $\mathrm{SU}(3)_c$ index. The subscript $\pm 2/3$ denotes the $\mathrm{U}(1)_X$ charge of the fermion. The terms in eq.(4.22) explicitly break the global group G_f of the composite sector and in the magnetic theory they flow to

$$
\epsilon_L (\xi_L)^{ia} M_{ia} + \epsilon_R (\xi_R)^{ia} M_{ia} \,. \tag{4.24}
$$

These are the mixings for partial compositeness of the top, eq.(2.18). We add soft terms in analogy to eq.(3.19): from a phenomenological point of view the only needed soft terms are masses for elementary sparticles and gauginos, because the composite states are all massive. We then add

$$
V_{soft} = \widetilde{m}_L^2 |t_L|^2 + \widetilde{m}_R^2 |t_R|^2 + \Big(\frac{1}{2}\widetilde{m}_{g,\alpha}\lambda_\alpha\lambda_\alpha + h.c.\Big), \tag{4.25}
$$

where λ_α are the SM gauginos and $\alpha = 1, 2, 3$ runs over the $\mathrm{U}(1)_{0,Y}$, $\mathrm{SU}(2)_{0,L}$ and $\mathrm{SU}(3)_c$ groups. In order to simplify the expressions below, we take the SM soft terms larger than μ. Due to the terms in eq.(4.24) and the interactions with the SM gauginos, the SUSY breaking is transmitted to the composite sector as well. More in detail, the Dirac fermions $\left(\lambda_{mn}, (\psi_{q_n^m} - \psi_{q_m^n})/\sqrt{2}\right)$ mix with the SM gauginos: as a result the former get splitted into two Majorana fermions with masses $\sqrt{\frac{2}{h}} g_m \mu \pm \delta\widetilde{m}_\lambda$. Expanding for heavy SM gauginos, we have

$$\delta\widetilde{m}_{\lambda,\alpha} \sim \frac{g_\alpha^2 \mu^2 \frac{2}{h}}{2\widetilde{m}_{g,\alpha}} \,. \tag{4.26}$$

Similarly, the scalar mesons and magnetic quarks that mix with the stops get soft terms of order

$$\widetilde{m}_s^2 \sim -|\epsilon_{L,R}|^2 \,, \tag{4.27}$$

that tend to decrease their SUSY mass value. The spectrum of the fields in the M_{i5} and in the M_{im}-q_i^m sectors is affected by the terms in eq.(4.24), while all the other sectors are unchanged. We see that a linear combination of fermions given by t_R and the appropriate components of $\psi_{M_{ia}}$ remains massless. This field is identified with the actual SM right-handed top. A similar argument applies to t_L. At this stage, the "goldstino" $\psi_{M_{55}}$ is still massless. In the case in which we also consider soft terms in the electric $\mathrm{SO}(N)$ theory, the mesons M_{ab} get a non-vanishing VEV and a mass for $\psi_{M_{55}}$ can be induced from higher dimensional operators in the Kähler potential. Independently of this effect, a linear combination of $\psi_{M_{55}}$ and the goldstino associated to the external SUSY breaking is eaten by the gravitino, while the orthogonal combination gets a mass at least of order of the gravitino mass (see [104] for an analysis of goldstini in presence of multiple sectors of SUSY breaking and specifically [105] for a set-up analogous to the one we are advocating here). We do not further discuss the mechanisms through which $\psi_{M_{55}}$ can get a mass.

More generally SUSY breaking is mediated to both the elementary sector and the composite one and without introducing further complications we assume it is mediated by the same mechanism, although we do not specify it: if the mediation scale M is lower than the Seiberg scale Λ the appropriate description is in terms of magnetic operators, while if the mediation occurs at higher scale we have to use electric operators. In both cases the RG evolution of the $\mathrm{SO}(11)_{el}$ SQCD is well captured by Seiberg duality only if the SUSY breaking parameters are small compared to Λ: the numerical analysis we performed showed us that this is always the case. The condition $M < \Lambda$, together with $F < M^2$ where F is the non vanishing SUSY breaking F term of the hidden sector results in unacceptably low soft masses because $\Lambda \simeq 10$ TeV, as can be seen from the discussion above, and from the results we illustrate in the following sections. Moreover we do not have a clear description of such soft terms in terms of UV operators. Therefore we exclude the case of a low scale mediation and we concentrate on the other: we add to the Lagrangian eq.(4.25) the following potential in terms of electric scalar fields

$$V_{soft,el} = \widetilde{m}^2_{1el} Q^{\dagger a} Q^a + \widetilde{m}^2_{2el} Q^{\dagger i} Q^i = \widetilde{m}^2_{1el} (Q^{\dagger a} Q^a + \omega\, Q^{\dagger i} Q^i)\,, \tag{4.28}$$

accompanied with $\mathrm{SO}(11)_{el}$ gaugino soft Majorana masses; the last equality defines $\omega = (\widetilde{m}_{2el}/\widetilde{m}_{1el})^2$. The deformation induced in the magnetic theory is

$$\begin{aligned} V_{soft} \;=\;& \widetilde{m}^2_{t_L} |\widetilde{t}_L|^2 + \widetilde{m}^2_{t_R} |\widetilde{t}_R|^2 + (\frac{1}{2}\widetilde{m}_{g,\alpha}\lambda_\alpha\lambda_\alpha + h.c.) + \\ & + \widetilde{m}^2_1 |M_{ia}|^2 + \widetilde{m}^2_2 |M_{ab}|^2 + \widetilde{m}^2_3 |q_i|^2 - \widetilde{m}^2_4 |q_a|^2 - \widetilde{m}^2_5 |M_{ij}|^2\,, \end{aligned} \tag{4.29}$$

In section 4.5 we discuss how to derive soft magnetic terms in eq.(4.29) from eq.(4.28). The result is

$$\begin{aligned} &\widetilde{m}^2_1 = \frac{1}{8}(3+2\omega)\widetilde{m}^2, \quad \widetilde{m}^2_2 = \frac{1}{8}(11-6\omega)\widetilde{m}^2, \quad \widetilde{m}^2_3 = \frac{5}{16}(1-2\omega)\widetilde{m}^2, \\ &-\widetilde{m}^2_4 = -\frac{1}{16}(11-6\omega)\widetilde{m}^2 \quad -\widetilde{m}^2_5 = -\frac{5}{8}(1-2\omega)\widetilde{m}^2 \end{aligned} \tag{4.30}$$

where $\widetilde{m} = \widetilde{m}_{1el}$. Similarly magnetic $\mathrm{SO}(4)_m$ gauginos acquire a Majorana mass of the form

$$\frac{1}{2} m_\lambda \lambda^A \lambda^A + h.c. \tag{4.31}$$

and in section 4.5 we derive m_λ starting from a mass for $\mathrm{SO}(11)_{el}$ gauginos, even though we do not use this expression and we let m_λ be a free parameter because $\mathrm{SO}(11)_{el}$ gaugino mass does not enter any other computation.

The condition $\omega < \frac{1}{2}$ ensures the positivity of all squared masses as defined above. This means we have some tachyonic direction: q^n_a, which are acquiring a vev in any case and M_{ij} which have a nonzero (and possibily "large") supersymmetric mass m_1. Because of this soft terms the vacuum eq.(4.6) gets modified to

$$\langle q^m_n \rangle = \frac{\tilde{\mu}}{\sqrt{h}} \delta^m_n, \quad \text{where} \quad \tilde{\mu} = \sqrt{\mu^2 + \frac{\widetilde{m}^2_4}{2h}}\,. \tag{4.32}$$

Finally in full analogy with eq.(3.17) and eq.(3.18) the matrix describing nonlinearly the Goldstone bosons is extracted from

$$q_b^n = \exp\Big(\frac{i\sqrt{2}}{f} h^{\hat{a}} T_{\hat{a}} + \frac{i}{\sqrt{2}f}\pi^a T_a\Big)_{bc} \widetilde{q}_c^m \, \exp\Big(\frac{i}{\sqrt{2}f}\pi^a T_a\Big)_{mn}, \tag{4.33}$$

where $f = \tilde{\mu}\sqrt{\frac{2}{h}}$ and the broken generators $T^{\hat{a}}$ are defined in appendix C. The broken generators T^a collect $T^a_{L,R}$. In the unitary gauge the π^a are eaten by the vector bosons; in the unitary gauge also for the EW breaking bosons the U matrix becomes exactly of the same form as in eq.(3.18).

4.3 Landau Poles

Similarly to what happens in models with direct gauge mediation of SUSY breaking, where the SM group is obtained by gauging a global subgroup of the hidden sector, one should worry about the possible presence of Landau poles in the SM couplings, the QCD coupling α_3 in particular, due to the proliferation of colored fields. Our model is no exception and Landau poles develop for the SM gauge couplings. In order to simplify the RG evolution, we conservatively take all the masses of the magnetic theory to be of order μ, SM superpartners included, with the exception of the mesons M_{ij}, whose mass m_1 is determined in terms of m_{1S} and Λ. We run from m_Z up to μ with the SM fields, from μ up to Λ with the degrees of freedom of the magnetic theory and above Λ with the degrees of freedom of the electric theory.

A one-loop computation shows that the $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)_{0,L}$ and $\mathrm{U}(1)_{0,Y}$ couplings develop Landau poles at the scales

$$\begin{aligned}
\Lambda_3^L &= m_{2S}\, \exp\Big(\frac{2\pi}{21\alpha_3(m_Z)}\Big)\Big(\frac{m_Z}{\mu}\Big)^{-\frac{1}{3}}\Big(\frac{\mu}{\Lambda}\Big)^{\frac{2}{7}}\Big(\frac{\Lambda}{m_{2S}}\Big)^{\frac{16}{21}}, \\
\Lambda_2^L &= m_{2S}\, \exp\Big(\frac{2\pi}{17\alpha_2(m_Z)}\Big)\Big(\frac{m_Z}{\mu}\Big)^{-\frac{19}{102}}\Big(\frac{\mu}{\Lambda}\Big)^{\frac{22}{17}}\Big(\frac{\Lambda}{m_{2S}}\Big)^{\frac{11}{17}}, \\
\Lambda_1^L &= m_{2S}\, \exp\Big(\frac{2\pi}{91\alpha_1(m_Z)}\Big)\Big(\frac{m_Z}{\mu}\Big)^{\frac{41}{546}}\Big(\frac{\mu}{\Lambda}\Big)^{\frac{336}{546}}\Big(\frac{\Lambda}{m_{2S}}\Big)^{\frac{215}{273}}.
\end{aligned} \tag{4.34}$$

We have taken $\lambda_{1,2} \sim 1$ in the superpotential (4.17), so that $m_{2S} \sim \Lambda/\epsilon$ is the highest scale in the electric theory, $\alpha_{1,2,3}(m_Z)$ are the $\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L \times \mathrm{SU}(3)_c$

SM couplings evaluated at the Z boson mass m_Z. In deriving eq.(4.34) we have matched the $\mathrm{SU}(2)\times\mathrm{U}(1)$ couplings at the scale μ, using eq.(4.16) with

$$\alpha_m(\mu) = \frac{2\pi}{5\log\left(\frac{\Lambda}{\mu}\right)}\,. \tag{4.35}$$

Notice that the scale of the poles does not depend on m_{1S}, since it cancels out in the contributions coming from S_{ij} and M_{ij}. Demanding for consistency that $\Lambda_i^L > m_{2S}$ constrains $\epsilon = \frac{\mu}{\Lambda}$ to be not too small. This is welcome from a phenomenological point of view, since a too small ϵ leads to a parametrically weakly coupled magnetic sector (see eq.(4.35)) and too light magnetic vector fields. On the other hand, ϵ cannot be too large for the stability of the vacuum, but values as high as 1/10 or so should be fine, given the estimate eq.(4.46): we address the issue of the lifetime of the vacuum in section 4.4. By taking natural choices for μ around the TeV scale, we see that all the Landau poles occur above m_{2S}, with $\mathrm{SU}(3)_c$ being the first coupling that blows up, entering the non-perturbative regime in the $10^2 - 10^3$ TeV range.

The Yukawa couplings $\lambda_{1,2}$ and $\lambda_{L,R}$ in the superpotential eq.(4.17) and eq.(4.22) might also develop Landau poles. A simple one-loop computation, in the limit in which the SM gauge couplings are switched off, shows that these poles appear at scales much higher than those defined in eq.(4.34). In a large part of the parameter space the Yukawa's actually flow to zero in the UV. This is even more so, when the SM gauge couplings are switched on, due to their growth in the UV.

4.4 Lifetime of the Metastable Vacuum

In presence of the meson mass terms eq.(4.20), in addition to the ISS vacuum eq.(4.6), other non-SUSY vacua can appear [103]. They can be dangerous if less energetic than the ISS vacuum, since the latter can decay through tunneling too quickly to them. These vacua do not appear in our model, since the superpotential does not include meson terms of the form M_{ab}^2. Other non-SUSY vacua, if present, can be found at $q_m^n \sim q_i^n \sim M_{ij} \sim M_{nm} \sim M_{in} \sim \mu$, $q_5^n = 0$, $M_{n5} = 0$, $M_{i5} = 0$, while M_{55} is still a flat direction. They do not lead to the desired pattern of symmetry breaking and they do not allow us to embed the SM in the flavor group. All these vacua have however exactly the same tree-level energy of the ISS vacuum and would be irrelevant for the tunneling rate.

Supersymmetric vacua[5] are expected when the mesons get a large VEV, in analogy with [101, 103]. The scalar potential has a local maximum at the origin in field space, with energy $V_{Max} = 5\mu^4$, while at the local minimum $V_{Min} = \mu^4$. We look for SUSY vacua in the region of large meson values, $|M_{ij}| \gg \mu$, $|M_{ab}| \gg \mu$. For simplicity, we take

$$\langle M_{ab}\rangle = X\,\delta_{ab}\,, \quad \langle M_{ij}\rangle = Y\,\delta_{ij}\,, \quad M_{ia} = 0\,. \tag{4.36}$$

For $|X|, |Y| \gg \mu$, the magnetic quarks are all massive and can be integrated out. Below this scale, we get a pure SUSY SO(N) Yang-Mills theory with a set of neutral mesons M. The ISS superpotential admits $N-2$ SUSY vacua, their existence is guaranteed by an explicit computation of the Witten index [106, 107]. The same result for the index does not hold here because the meson mass term modifies the behavior of the superpotential at large field values, therefore we should recompute the index on our own. Nevertheless we explicitly find, inspired by the original construction, SUSY vacua. The magnetic superpotential, below the induced quark masses, is

$$W = 2\Lambda^{-\frac{5}{2}}(\det M)^{\frac{1}{2}} - \mu^2 M_{aa} - \frac{1}{2}m_1 M_{ij}^2 - \frac{1}{2}m_2 M_{ia}^2\,, \tag{4.37}$$

where we neglect the elementary sector, that gives rise to subleading corrections. The first term is non perturbatively generated by gaugino condensation and its dependence on $\det M$ is obtained by scale matching. By imposing the vanishing of the F-term conditions, we find SUSY vacua at

$$\begin{aligned} X =& \Lambda^{\frac{5}{6}}\mu^{-\frac{1}{3}}m_1^{\frac{1}{2}} = \epsilon^{-\frac{1}{3}}\sqrt{\Lambda m_1} = \epsilon^{-\frac{5}{6}}\sqrt{\mu\, m_1}\,, \\ Y =& \Lambda^{\frac{5}{12}}\mu^{\frac{5}{6}}m_1^{-\frac{1}{4}} = \epsilon^{\frac{5}{6}}\Lambda\Big(\frac{\Lambda}{m_1}\Big)^{\frac{1}{4}} = \epsilon^{-\frac{5}{12}}\mu\Big(\frac{\mu}{m_1}\Big)^{\frac{1}{4}}\,, \end{aligned} \tag{4.38}$$

where

$$\epsilon = \frac{\mu}{\Lambda} \tag{4.39}$$

is a parametrically small number. The vacua eq.(4.38) can also be found directly in the electric theory. In the region where S_{ij} is non-vanishing, all the quarks Q are massive and the theory develops an Affleck-Dine-Seiberg superpotential of the form [108]

$$W_{np} = (N-2-N_f)\left(\frac{\Lambda^{3(N-2)-N_f}}{\det M}\right)^{\frac{1}{(N-2)-N_f}}\,, \tag{4.40}$$

[5] By supersymmetric vacua we mean those that are SUSY in the limit where we switch off the external source of SUSY breaking.

where now $M = M_{IJ} = Q_I Q_J$. It is straightforward to check that this term induces in fact the SUSY vacua eq.(4.38). The vacuum eq.(4.38) lies in the range of calculability of the magnetic theory if

$$\mu \ll |X|, |Y| \ll \Lambda\,. \tag{4.41}$$

The conditions eq.(4.41), together with the requirement that the mesons M_{ij} are not anomalously light, $m_1 \geq \mu$, determine the allowed range for m_1. Parametrizing

$$m_1 = \Lambda \epsilon^{\kappa}\,, \tag{4.42}$$

we get

$$\frac{2}{3} < \kappa \leq 1\,. \tag{4.43}$$

We now want to estimate the lifetime of the metastable vacuum, namely the decay rate per unit time and per unit volume to the true vacuum given by eq.(4.38). It is given by

$$\frac{\Gamma}{VT} \sim e^{-S_b} \tag{4.44}$$

where S_b is the bounce action, the euclidean action computed on the solution of the e.o.m with proper boundary conditions. These boundary conditions are such that there is one direction in field space interpolating among the false vacuum far in the past and exactly reaching at some finite time the "top of the hill", the maximum of the potential between the two minima, the metastable one and the truly stable one. Heuristically we can think to this transition happening in a localized region of space, whereas far from this "bubble" of true vacuum the field is still in the metastable vacuum. If the energy difference between the two vacua is small compared to the other energy scales the bubble is delimited by a "thin wall", thin compared to its radius [109]. In the present case the true vacuum has exactly zero energy, being SUSY, and therefore the difference is exactly the energy of the metastable minimum eq.(4.6): the thin wall approximation is not valid. In fact

$$V_{Max} \sim V_{Min} \sim V_{Max} - V_{Min} \sim \mu^4\,, \quad V_{SUSY} = 0\,. \tag{4.45}$$

As a very crude estimate of the lifetime of the metastable vacuum, we can parametrize the potential using the triangular approximation [110], neglecting the direction in field space along the Y direction, which is always closer to the ISS vacuum, given the bound (4.43).

The bounce action is parametrically given by [101, 109, 110]

$$S_b \sim \frac{|X|^4}{V_{Max}} \sim \epsilon^{-\frac{16}{3}+2\kappa} \gtrsim \epsilon^{-\frac{10}{3}} \,. \tag{4.46}$$

We conclude that for small ϵ the metastable vacuum is parametrically long-lived and a mild hierarchy between μ and Λ should be enough to get a vacuum with a lifetime longer than the age of the universe.

4.5 RG Flow of Soft Terms

In this section we explain, following [111, 112], how to understand the fate of UV soft terms in a SUSY gauge theory at strong coupling[6]: this is an essential ingredient for the model presented in chapter 5. For concreteness we focus here on SO(N) gauge theories with $N-2 < N_f \leq 3/2(N-2)$ flavors in the fundamental, admitting a Seiberg dual IR free description. This is the case of interest for us, but what follows has clearly a wider applicability. More specifically, we want to determine the form of the IR soft terms in the magnetic theory in terms of the electric ones. We first consider the case with no superpotential: $W_{el} = 0$. Soft terms can be seen as the θ-dependent terms of spurion superfields whose lowest components are the wave-function renormalization of the Kähler potential and the (holomorphic) gauge coupling constant. The Lagrangian renormalized at the scale E is

$$\mathcal{L}_{el} = \int d^4\theta \sum_{I=1}^{N_f} Z_I(E) Q_I^\dagger e^{V_{el}} Q_I + \Big(\int d^2\theta S(E) W_{el}^\alpha W_{el,\alpha} + h.c. \Big) \,, \tag{4.47}$$

where

$$\begin{aligned} Z_I(E) &= Z_I^0(E)\Big(1 - \theta^2 B_I(E) - \bar\theta^2 B_I^\dagger(E) - \theta^2\bar\theta^2(\widetilde m_I^2(E) - |B_I(E)|^2)\Big)\,, \\ S(E) &= \frac{1}{g^2(E)} - \frac{i\Theta}{8\pi^2} + \theta^2 \frac{\widetilde m_\lambda(E)}{g^2(E)} \end{aligned} \tag{4.48}$$

[6] An alternative derivation of the results of [111, 112] has been formulated [113]. We follow the original papers because we have found easier in this way to estimate the corrections coming from superpotential effects, although a reformulation in terms of the flow of conserved currents and of the would-be conserved R-symmetry should be possible.

are the spurion superfields that encode the B-terms B_I, non-holomorphic mass terms $\widetilde{m}_I^2$ and the gaugino mass $\widetilde{m}_\lambda$. When there is no superpotential, the B_I terms are irrelevant and can be set to zero. The Lagrangian in eq.(4.47) is invariant under a $\mathrm{U}(1)^{N_f}$ symmetry under which

$$Q_I \to e^{A_I} Q_I \,, \quad Z_I \to e^{-A_I - A_I^\dagger} Z_I \,, \quad S \to S - \sum_{I=1}^{N_f} \frac{t_I}{8\pi^2} A_I \,, \tag{4.49}$$

where A_I are constant chiral superfields and t_I are the Dynkin indices of the representations of the fields Q_I, $t_I = 1$ for $\mathrm{SO}(N)$ fundamentals. In terms of these spurions, one can construct the following RG invariant quantities:

$$\Lambda_S = E e^{-\frac{8\pi^2 S(E)}{b}} \,, \quad \hat{Z}_I = Z_I(E) e^{-\int^{R(E)} \frac{\gamma_I(E)}{\beta(R)} dR} \,. \tag{4.50}$$

In eq.(4.50), $b = 3(N-2) - N_f$ is the coefficient of the one-loop β-function $\beta(R)$, γ_I are the anomalous dimensions of the fields Q_I, and $R(E)$ is defined as $S(E)$ in eq.(4.48), but in terms of the physical, rather than holomorphic, gauge coupling constant. In terms of Λ_S and $\hat{Z}_I$, one can further construct a $\mathrm{U}(1)^{N_f}$ and RG invariant superfield:

$$I = \Lambda_S^\dagger \Big(\prod_{I=1}^{N_f} \hat{Z}_I^{\frac{2t_I}{b}} \Big) \Lambda_S \,. \tag{4.51}$$

In the far IR, the dynamics of the system is best described by the magnetic theory, whose degrees of freedom are the mesons $M_{IJ} = Q_I Q_J$, the dual magnetic quarks q_I and the $\mathrm{SO}(N_f - N + 4)$ magnetic vector fields V_m. We can use the RG invariants I and $\hat{Z}_I$ and dimensional analysis to write the lowest dimensional operators in the low-energy Lagrangian:

$$\begin{aligned} \mathcal{L}_{mag} = & \int d^4\theta \Big(c_{M_{IJ}} \frac{M_{IJ}^\dagger \hat{Z}_I \hat{Z}_J M_{IJ}}{I} + c_{q_I} q_I^\dagger e^{V_{mag}} \hat{Z}_I^{-1} (\prod_J \hat{Z}_J^{\frac{t_J}{b}}) q_I \Big) \\ & + \int d^2\theta \Big(S_m(E) W_m^\alpha W_{m,\alpha} + \frac{q_I M_{IJ} q_J}{\Lambda_S} \Big) + h.c. \,, \end{aligned} \tag{4.52}$$

where

$$S_m(E) = \frac{1}{g_m^2(E)} - \frac{i\Theta_m}{8\pi^2} + \theta^2 \frac{\widetilde{m}_{m,\lambda}(E)}{g_m^2(E)} \tag{4.53}$$

is the magnetic version of the spurion S defined in eq.(4.48). As shown in [111], these terms are the leading sources of soft terms provided that

$\widetilde{m}_I \ll \Lambda$, condition that will always be assumed. The last term in the second row in eq.(4.52) is the induced superpotential in the magnetic theory. Demanding the invariance of W fixes the $\mathrm{U}(1)^{N_f}$ charges of the dual quarks q_I to be $Q_I(q_J) = 1/b - \delta_{IJ}$. These, in turn, fix the $\hat{Z}$-dependence of the Kähler potential term of the magnetic quarks. The coefficients $c_{M_{IJ}}$ and c_{q_I} are real superfield spurions, the IR analogues of the wave function renormalization constants $Z_I(E)$. A relation between IR and UV soft terms is achieved by noticing that in the far UV (IR) the electric (magnetic) theory is free. This implies that for sufficiently high E, we can identify $\hat{Z}_I$ with Z_I, neglecting quantum corrections, and identify $m_I^2(E) \equiv \widetilde{m}_I^2$ with the physical UV electric soft terms. Similarly, in the far IR, we can neglect the θ^2 and θ^4 corrections induced by quantum corrections to $c_{M_{IJ}}$ and c_{q_I}. We can then compute the IR soft terms by working out the θ^2 and θ^4 terms in the Lagrangian (4.52). The physical non-holomorphic soft masses for the mesons and magnetic quarks are

$$\widetilde{m}^2_{M_{IJ}} = \widetilde{m}_I^2 + \widetilde{m}_J^2 - \frac{2}{b}\sum_{K=1}^{N_f} \widetilde{m}_K^2\,, \qquad \widetilde{m}^2_{q_I} = -\widetilde{m}_I^2 + \frac{1}{b}\sum_{K=1}^{N_f} \widetilde{m}_K^2\,. \tag{4.54}$$

As can be argued from eq.(4.54), positive definite UV soft terms always flow in the IR to tachyonic soft terms for some mesons and/or magnetic quarks [114]. Indeed, the following sum rule holds:

$$\sum_{I,J=1}^{N_f} \widetilde{m}^2_{M_{IJ}} + 2N_f \sum_{I=1}^{N_f} \widetilde{m}^2_{q_I} = 0\,. \tag{4.55}$$

In our derivation we have tacitly taken the dynamically generated scale in the magnetic theory to coincide with the electric one. This implies that the same Λ_S defined in eq.(4.50) should be expressed in magnetic variables, namely

$$\Lambda_S = E e^{-\frac{8\pi^2}{b} S(E)} = E e^{-\frac{8\pi^2}{b_m} S_m(E)}\,, \tag{4.56}$$

where $b_m = 3(N_f - N + 2) - N_f$. Identifying the θ^2 components of eq.(4.56), we get

$$\lim_{E\to 0} \frac{\widetilde{m}_{m,\lambda}(E)}{b_m g_m^2(E)} = \lim_{E\to\infty} \frac{\widetilde{m}_\lambda(E)}{b g^2(E)}\,. \tag{4.57}$$

Notice that the θ^2 term of Λ_S introduces B-terms coming from both the D- and F-components of the magnetic Lagrangian $\mathcal{L}_{mag}$ that precisely cancel each other. This is evident by noticing that the holomorphic rescaling

$$M_{IJ} \to \Lambda_S M_{IJ} \tag{4.58}$$

removes Λ_S from the leading order Lagrangian (4.52).

Let us now apply these considerations to our specific set-up. We assume that the electric soft terms do not break the G_f symmetry, so we effectively have two U(1) symmetries, respectively rotating the quarks Q_a and Q_i, and two different soft terms, $\widetilde{m}_1^2 Q^{\dagger a} Q^a + \widetilde{m}_2^2 Q^{\dagger i} Q^i$. We therefore use eq.(4.54) and we obtain the result quoted in eq.(4.30). Similarly in section 5.1, where we consider a model with $G_f = \mathrm{SO}(5) \times \mathrm{SU}(4)$ and fully composite right top with $b = 12$, with this formalism we immediately obtain the soft terms reported in eq.(5.7).

Let us now see the effect of having $W_{el} \neq 0$. For concreteness, consider the following two terms,

$$W_{el} = m Q^a Q^a + \frac{1}{2} \lambda Q_i Q_j S_{ij} \,. \tag{4.59}$$

We promote m and λ to chiral superfield spurions in the spirit of considering an external unspecified SUSY breaking mechanism:

$$m \to m(1 + \theta^2 B_m)\,, \quad \lambda \to \lambda(1 + \theta^2 A_\lambda)\,. \tag{4.60}$$

We can still set $B_I = 0$ in eq.(4.48), their effect being a redefinition of the B_m and A_λ terms in eq.(4.60). We can also reabsorb in B_m and A_λ the effect of the field redefinition (4.58) that would induce additional B-like terms proportional to the gaugino soft terms. The above $\mathrm{U}(1)^2$ symmetry is unbroken provided m and λ transform as follows:

$$m \to e^{-2A_1} m\,, \quad \lambda \to e^{-2A_2} \lambda\,, \tag{4.61}$$

with S_{ij} invariant. Two further $\mathrm{U}(1)^2$ and RG-invariants can be constructed starting from m and λ:

$$I_m = m^\dagger \hat{Z}_1^{-2} m\,, \qquad I_\lambda = \lambda^\dagger \hat{Z}_2^{-2} \lambda\,. \tag{4.62}$$

The leading order Kähler potential for the mesons and the magnetic quarks is still of the form (4.52), but now $c_{M_{IJ}}$ and c_{q_I} are unknown functions of

$I_\lambda/(16\pi^2)$ and of I_m/I.[7] These corrections are sub-leading provided that $m \ll \Lambda$ and the effective coupling λ/Z_2, at some UV scale E where the theory is perturbative, is smaller than 4π. Both conditions can be satisfied in our models. In first approximation we can then neglect the superpotential corrections to the RG flow of the soft terms. Of course, even when taking $W_{el} = 0$, the relations (4.54) and (4.57) are only valid in the strict UV and IR limits and with vanishing mixing and SM gauge couplings. We have not estimated the corrections coming from relaxing the above approximations, assuming they are sub-leading in eqs.(4.54) and (4.57). It would be interesting to perform a more careful analysis to check the validity of this assumption.

There is an important consequence in having a non-vanishing W_{el}. In the IR, the first term in eq.(4.59) becomes linear in the mesons M_{aa} and the B_m term induces a tadpole for these fields. This is nothing else that the deformation discussed in the next section, 4.6. The tadpole changes the vacuum structure of the model, as we will fully show in the next section. Extremizing the whole scalar potential, soft terms included, we will get eq.(4.67) and eq.(4.69), namely

$$\langle q^m_n \rangle = \frac{\tilde{\mu}}{\sqrt{h}} \delta^m_n \,, \quad \langle M_{55} \rangle = X_5 \,, \qquad \langle M_{mn} \rangle = X \delta_{mn} \,. \tag{4.63}$$

The presence of soft terms in the composite sector affects also the analysis of the vacuum decay pursued in section 4.4. We have checked the bound on the soft terms in the composite sector (eq.(4.29), with the addition of the B-terms in the composite sector) above which $\mathcal{L}_{\not{SUSY}}$ can no longer be taken as a perturbation of the SUSY scalar potential in the region of large meson VEV's, eq.(4.36). In particular, we have verified under what conditions the vacuum displacements from the SUSY values $\delta X/X$ and $\delta Y/Y$, where X and Y are defined in eq.(4.36), are much smaller than one. Comparable bounds arise from the soft terms $\widetilde{m}_2$, $\widetilde{m}_{m,\lambda}$ and B_{μ^2}. We get

$$|\widetilde{m}_2| \sim |\widetilde{m}_{m,\lambda}| \sim |B_{\mu^2}| \ll \epsilon^{\frac{4}{3}-\frac{\kappa}{2}} |\mu| \,, \tag{4.64}$$

where κ is defined in eq.(4.42). Given the bound (4.43) on the allowed values of κ, we see that the soft terms are constrained to be parametrically smaller

[7] These functions are not completely unrelated, since the combination of Kähler terms associated to conserved global currents should precisely match in the UV and IR theories [113]. We have not studied this flow in detail, since we anyway neglect the effects of such corrections.

than μ. We have numerically explored also the region of soft terms larger than eq.(4.64), resulting in shifts $\delta X \gtrsim X$, $\delta Y \gtrsim Y$. Although it is not possible to draw a definite conclusion from this numerical analysis, we believe that the bound (4.64) is quite conservative, since the would-be SUSY vacuum energy becomes greater than the one of the ISS-like vacuum (4.32), when the soft terms become comparable to (or larger than) μ. This is intuitively clear by noticing that the dominant source of energy coming from $\mathcal{L}_{\not{SUSY}}$ is the soft term $\widetilde{m}_2^2 X^2$ (being $|X| \gg |Y|$) and this is positive definite. If this is the case, the ISS-like vacuum would become absolutely stable, provided that other non-SUSY vacua with lower energy do not appear elsewhere in field space.

4.6 Deformation

In this section we study soft deformations including, besides scalar and gaugino masses as in eq.(4.29), A and B terms for the couplings in the superpotential eq.(4.3). We have just discussed the inclusion of SUSY breaking terms in the theory, in the previous section 4.5: the techniques employed to follow the soft masses [111, 112] cannot be used in the case of A, B terms because the former can be computed only in absence of any superpotential and thus they are expected to be valid up to perturbative corrections in couplings, while the latter are identically zero if the superpotential vanishes.

The most general soft terms may lead to unwanted tachyonic directions and we restrict to safe cases where they only modify the spectrum: this happens if they are not too large with respect to the holomorphic and soft masses. As we already argued the presence of B terms for the electric quarks induces a term of the form

$$\mathcal{L}_{soft} \supseteq -\mu^2 B_{\mu^2} \mathrm{Tr}\, M \tag{4.65}$$

which introduces a new qualitative feature, even for arbitrary small B_{μ^2}. In fact the scalar potential now includes

$$V \supseteq |2hM_{nn}q_m^n|^2 + |hq_m^n q_m^n - \mu^2|^2 + (\mu^2 B_{\mu^2} M_{aa} + h.c.) + \widetilde{m}_2^2 |M_{nn}|^2 - \widetilde{m}_4^2 |q_m^n|^2 \,. \tag{4.66}$$

The VEV of the magnetic quarks, eq.(4.32), becomes

$$\langle q_n^m \rangle = \frac{\tilde{\mu}}{\sqrt{h}} \delta_n^m \,, \quad \text{where} \quad \tilde{\mu} \simeq \sqrt{\mu^2 + \frac{\widetilde{m}_4^2}{2h}} - \frac{h\mu^4 (B_{\mu^2})^2}{\sqrt{\mu^2 + \frac{\widetilde{m}_4^2}{2h}} (4h\mu^2 + 2\widetilde{m}_4^2 + \widetilde{m}_2^2)^2} \tag{4.67}$$

expanding for small B_{μ^2}: the true value for $\tilde{\mu}$ is a solution of the equation cubic in $\tilde{\mu}^2$:

$$[\tilde{\mu}^2 - (\mu^2 + \frac{\widetilde{m}_4^2}{2h})](\widetilde{m}_2^2 + 4h\tilde{\mu}^2)^2 + 2h\mu^4 B_{\mu^2}^2 = 0 \,. \tag{4.68}$$

At the same time magnetic mesons also acquire a VEV

$$\langle M_{55} \rangle = X_5 \,, \qquad \langle M_{mn} \rangle = X \delta_{mn} \quad \text{where} \quad \begin{cases} X_5 & = & -\frac{\mu^2 B_{\mu^2}}{\widetilde{m}_2^2} \\ X & = & -\frac{\mu^2 B_{\mu^2}}{4h\tilde{\mu}^2 + \widetilde{m}_2^2} \end{cases} . \tag{4.69}$$

Contrary to the previous case the mesons M_{ab} have a nonzero VEV. Notice that the symmetry breaking is still of the form eq.(4.7), $SO(4)_m \times SO(5) \to SO(4)_D$. The spectrum of the theory gets modified but the only qualitative difference is about the Goldstone bosons: the four uneaten ones, identified with the Higgs, are now contained in the massless field combination

$$\cos\alpha \ \text{Re}\ q_5^n + \sin\alpha \ \text{Re}\ M_{5n} \,, \quad \text{where} \quad \sin\alpha = \frac{2(X - X_5)}{f} \,. \tag{4.70}$$

Their kinetic term comes from $|D_\mu q_a|^2$ and $|D_\mu M_{ab}|^2$; at the non linear level they are described by a σ-model through a matrix U as in eq.(3.18) with a new decay constant given by

$$f = \sqrt{\frac{2}{h}\left(\tilde{\mu}^2 + 2h(X - X_5)^2\right)} \,. \tag{4.71}$$

In the limit of vanishing B terms we have $X = X_5 = 0$ and $\cos\alpha = 1$. For numerical analysis we work in the regime of small B_{μ^2}, in particular we neglect its effects on the Higgs mass: this is consistent as long as the suppression between B_{μ^2} and other masses, both holomorphic and soft, is at least of the order of one-loop effects.

4.7 Quark Masses

The plain generalization of partial compositeness in our SUSY setup to all quarks and leptons does not work: it requires the presence of a large number of (super)partners, leading to tremendously large flavor symmetry of the composite sector and aggravating the problem of SM Landau poles. Therefore we abandon it for all the fermions but the top.

To induce all the SM Yukawas we need a further explicit breaking of the SO(5) global symmetry, proportional to two matrices λ_U^{AB} and λ_D^{AB} where $A, B = 1, 2, 3$ are family indices. The extension to leptons through another pair of matrices is straightforward. Deformations in the electric superpotential[8]

$$W_{el} \supseteq \frac{\lambda_U^{AB}}{\Lambda_L}\left(\xi_{L,U}^{ia}\right)^A\left(\xi_U^{ib}\right)^B Q_a Q_b + \frac{\lambda_D^{AB}}{\Lambda_L}\left(\xi_{L,D}^{ia}\right)^A\left(\xi_D^{ib}\right)^B Q_a Q_b \tag{4.72}$$

generate Yukawa terms if the dual mesons $M_{ab} \sim \frac{Q_a Q_b}{\Lambda}$ get a vev, eq.(4.69). $\xi_{L,U}$ and ξ_U are the spurionic embeddings of up type quarks in a fundamental of SO(5), as in eq.(3.13). $\xi_{L,D}$ and ξ_D are the spurions for down type quarks and can be defined in analogy to the up case but with a different X charge assignment, $X = -1/3$. The most general low-energy Lagrangian will contain

$$\mathcal{L} \supseteq \bar{q}_L^A \varepsilon u_R^B H^c + \bar{q}_L^A \lambda_u^{AB} u_R^B H^c + \bar{q_L}^A \lambda_d^{AB} d_R^B H + ... + h.c. \tag{4.73}$$

where[9]

$$\lambda_{u,d} = \lambda_{U,D}^{\dagger} \frac{\Lambda}{\Lambda_L} \sin\alpha \tag{4.74}$$

and the dots stand for higher dimensional operators: eq.(4.73) arises from the expansion in powers of f^{-1} of $M_{ab} = U_{ca}\langle M_{cd}\rangle U_{db}$ once we make explicit the Higgs dependence through the matrix U defined in eq.(3.18). H is the Higgs doublet

$$H = \left(H^{(+)}, H^{(0)}\right)^t = \frac{1}{\sqrt{2}}\left(ih^1 + h^2, -ih^3 + h^4\right)^t . \tag{4.75}$$

Without loss of generality we can go to the top basis in which $\varepsilon \sim \frac{\epsilon_L^A \epsilon_R^{*B}}{f^2}$ is different from zero only for $A = B = 3$: this is the term generated by the mixings in eq.(4.24). The second and the third terms in eq.(4.73) are the new operators responsible for the other quark masses. They have similarities with technicolor theories, where quark bilinears are coupled to an operator H

[8] The simplest way to induce these operators, as well as the the ones in eq.(4.19), is through the exchange of heavy chiral superfields, schematically $W = \lambda^{AB}\xi^A\Phi\xi^B + Q\Phi Q + \frac{\Lambda_L}{2}\Phi^2$ as already explained in the main text.

[9] The dagger is only for notational convenience.

arising from a strongly interacting theory and responsible for EW breaking. The main differences are first that in our case the Higgs is protected by a shift symmetry and second that this coupling is not the dominant source for the top mass.

To estimate the size of these masses we restrict for a moment on a single generation:

$$\mathcal{L} \sim \lambda_D \sin\alpha \frac{\Lambda}{\Lambda_L} v\, \bar{b}_R b_L + h.c. \tag{4.76}$$

where $v = 246$ GeV is the Higgs VEV and Λ is the crossover scale dynamically generated. If $\Lambda_L \sim 10\Lambda$ the correct value for the bottom mass can be reached with $\lambda_D = O(1)$ and $\sin\alpha \sim 0.1$. As discussed before a natural value for Λ is around 10 TeV while Λ_L can be chosen to be the scale of Landau poles for $SU(3)_c$, in the region 10^2 - 10^3 TeV. Other quarks require smaller couplings: we do not explain neither the hierarchy among SM masses nor the hierarchical structure of the CKM matrix, we assume them and we only distinguish between the top, partially composite, and other quarks, elementary. This different origin of the masses results in the splitting between the top and other quarks, making them naturally live at two different scales.

Finally we define $Y_d = \lambda_d$ and $Y_u = \lambda_u + \varepsilon$, the down- and up-type quark Yukawa matrices; they can be diagonalized with

$$Y_u \to Y_u^{diag} = V_u^\dagger Y_u U_u \,, \quad Y_d \to Y_d^{diag} = V_d^\dagger Y_d U_d \,. \tag{4.77}$$

If we perform the transformations V_d, U_u and U_d we go to the basis in which Y_d is diagonal, while $Y_u \to V_d^\dagger V_u Y_u^{diag}$ and we can define the CKM matrix $V_{CKM} = V_u^\dagger V_d$. Thus

$$\lambda_u \to V_d^\dagger \lambda_u U_u = V_{CKM}^\dagger Y_u^{diag} - V_d^\dagger \varepsilon U_u \,. \tag{4.78}$$

Since the matrix $V_d^\dagger \varepsilon U_u$ has arbitrary entries the second term signals a departure from minimal flavor violation, as it is depicted in [115]. In the next section we elaborate on it and on its consequences.

4.8 Flavor Analysis of Dimension Five Operators

A number of processes involving transitions in flavor space, $\Delta F = 0, 1, 2$, results in flavor and CP observables and they very often receive sizeable contributions from the presence of new physics, which in a broad class of CHM are mainly induced by the mixing of quarks with their partners. Since for the lightest generations the mixings are weaker generically we have a protection mechanism, known as RS-GIM [116] (see also [117]).

Despite the fact that the mixings are related to SM Yukawas, the resulting suppression might be not enough for a generic composite sector and additional flavor symmetries are frequently postulated: a recent review is provided in [118]. If light quarks are not partially composite, that is there are no mixings with bound states, these contributions are absent: this is the case for the interactions introduced in subsection 4.7.

On the other hand our model exhibits the same tensions of the MSSM, due to the presence of sparticles around the TeV scale: squark mass matrices cannot be completely anarchic. Solutions to regulate the contributions to flavor processes are either to assume a certain level of degeneracy or alignment among squarks masses or to rely on some hierarchy between the first two generations and the third one, without threatening the naturalness, ending up with a scenario close to effective SUSY, depicted for instance in [119] where a discussion on flavor processes is also present. Correlations among new physics contributions in different processes could help in the future to distinguish among these possibilities [120]. We derive our results with aligned squarks masses[10] and we allow for small misalignment treated in the mass insertion approximation.

We do not perform a full analysis of all existent bounds; we instead concentrate in what follows on the effects of the physics leading to the superpotential in eq.(4.72): at the scale Λ_L other operators are plausibly generated. Under the spurionic flavor group $\mathrm{U}(3)_q \times \mathrm{U}(3)_u \times \mathrm{U}(3)_d$ we assign the quantum numbers:

$$q_L \sim (3, 1, 1), \quad u_R^c \sim (1, \bar{3}, 1), \quad d_R^c \sim (1, 1, \bar{3}), \quad \lambda_u \sim (3, \bar{3}, 1), \quad \lambda_d \sim (3, 1, \bar{3})\,. \tag{4.79}$$

Compatibly with these charges, with gauge invariance and with the holomorphy of the superpotential we can write the following dimension five opera-

[10] Alignement is nicely realized in a variety of SUSY breaking mediation schemes: for instance in gauge mediation A-terms vanishes at the mediation scale.

tors[11,12]

$$W \supseteq \frac{a_1}{\Lambda_L}\left(u_R^c \lambda_U q_L\right)\left(d_R^c \lambda_D q_L\right) + \frac{a_2}{\Lambda_L}\left(u_R^c \lambda_U t^{\hat{A}} q_L\right)\left(d_R^c \lambda_D t^{\hat{A}} q_L\right) = \tag{4.80}$$
$$= \frac{Y^{ABCD}}{\Lambda_L}\left[a_1\left(u^c_{R,A} q_{L,B}\right)\left(d^c_{R,C} q_{L,D}\right) + a_2\left(u^c_{R,A} t^{\hat{A}} q_{L,B}\right)\left(d^c_{R,C} t^{\hat{A}} q_{L,D}\right)\right]$$

where $Y^{ABCD} = \lambda_U^{AB}\lambda_D^{CD}$. Dimension five operators in the MSSM are discussed in [121–123]: they result in, among other terms, contact interactions between two quarks and two squarks. We assign the couplings $\lambda_{u,d}$ to the vertices quark-squark-higgsino, neglecting deviations for tops, stops and left sbottoms. With higgsino exchange we draw one-loop diagrams contributing to four fermions interactions, experimentally constrained by $\Delta F = 2$ transitions in mesons. The resulting operator is

$$(\bar{d}_{R,Ck} d_{L,Dl})(\bar{d}_{L,Ei} d_{R,Fj}) \cdot \tag{4.81}$$
$$\cdot \frac{\lambda_u^{EA}\lambda_d^{BF}}{(4\pi)^2 \widetilde{m}\Lambda_L}\left\{\delta_{ij}\delta_{kl}\left[Y^{ABCD}(a_1 - \frac{a_2}{6}) - Y^{ADCB}\frac{a_2}{2}\right] - \begin{pmatrix} B \leftrightarrow D \\ j \leftrightarrow l \end{pmatrix}\right\},$$

where $\widetilde{m}$ is a common soft mass for the squarks and the higgsino in the loop. For the general case of different masses the loop integral results in the following function of squarks and higgsino masses, $\widetilde{m}_Q$, $\widetilde{m}_u$ and $\widetilde{m}_h$ respectively:

$$I(\widetilde{m}_h, \widetilde{m}_Q, \widetilde{m}_u) = -\widetilde{m}_h \frac{\widetilde{m}_h^2 \widetilde{m}_Q^2 \log\frac{\widetilde{m}_h^2}{\widetilde{m}_Q^2} + \widetilde{m}_Q^2 \widetilde{m}_u^2 \log\frac{\widetilde{m}_Q^2}{\widetilde{m}_u^2} + \widetilde{m}_u^2 \widetilde{m}_h^2 \log\frac{\widetilde{m}_u^2}{\widetilde{m}_h^2}}{(\widetilde{m}_h^2 - \widetilde{m}_Q^2)(\widetilde{m}_Q^2 - \widetilde{m}_u^2)(\widetilde{m}_u^2 - \widetilde{m}_h^2)} \tag{4.82}$$

such that

$$I(\widetilde{m}, \widetilde{m}, \widetilde{m}) = \frac{1}{2\widetilde{m}}\,, \tag{4.83}$$

altough we use eq.(4.81) and we do not need this exact expression. For operators of the form

$$\frac{c}{\Lambda_F^2}(\bar{d}_R d_L)(\bar{d}_L d_R) \tag{4.84}$$

the most stringent bounds come from kaons (the strongest is on the CP violating part). The Wilson coefficient computed from eq.(4.81) identically

[11] At high energies the flavor spurions are $\lambda_{U,D}$ and not $\lambda_{u,d}$, the difference being a factor $\frac{\Lambda_L}{\Lambda \sin\alpha} \sim 100$.

[12] $t^{\hat{A}}$ are the SU(3)$_c$ generators such that $t^{\hat{A}}_{ij} t^{\hat{A}}_{kl} = \frac{1}{2}(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl})$.

vanishes, even in the non MFV limit of eq.(4.78). Non aligned squark masses at $\varepsilon = 0$ results, in the mass insertion approximation, in (the hadronic matrix element with $j \leftrightarrow l$ is less significant by a factor of 3 [124, 125])

$$\begin{aligned} \frac{c}{\Lambda_F^2} &\simeq \frac{1}{(4\pi)^2 \widetilde{m}\Lambda_L} A^2\lambda^5 \left(\frac{\Lambda_L}{\Lambda \sin\alpha}\right)^4 y_d y_s y_t^2 \delta \\ &\Rightarrow \begin{cases} \Lambda_F = 1 \text{ TeV} \\ c \simeq 10^{-8}\delta \left(\frac{1\text{TeV}}{\widetilde{m}}\right)\left(\frac{100\text{TeV}}{\Lambda_L}\right) \end{cases} \end{aligned} \tag{4.85}$$

where for concreteness we fix $a_1 = a_2 = 1$; A and λ are the parameters appearing in the Wolfenstein parametrization of the CKM matrix and δ measures the relevant misalignement of squarks: it is the mixing of the first two families left handed squarks normalized with a common mass $\widetilde{m}^2$, $\delta = \frac{\left(\widetilde{m}_Q^2\right)_{1,2}}{\widetilde{m}^2}$. From [126] we easily read:

$$\text{Re}\, c < 6.9 \times 10^{-9}\,, \quad \text{Im}\, c < 2.6 \times 10^{-11} \quad \text{if} \quad \Lambda_F = 1 \text{ TeV}\,. \tag{4.86}$$

Bounds from box diagrams for different processes, with squarks and gluinos at 1 TeV, are stronger, they set for δ an upper bound around 10^{-2}, see [127] and references therein[13], resulting in $c \simeq 10^{-10}$, below the bound eq.(4.86) (the bound on the CP violating effect computed here is not fully satisfied, it needs $a_1 \simeq a_2 = O(10^{-1})$ or so). The numerical value of c is of the same order also with general ε.

Similarly the up-type quarks are involved in D mesons oscillations: in this case the calculation is performed in the up-type mass basis, that is

$$\lambda_u = Y_u^{diag} - V_u^\dagger \varepsilon U_u\,, \quad \lambda_d = V_{CKM} Y_d^{diag}\,. \tag{4.87}$$

The relevant operator has the same form as in eq.(4.81) with the exchange $u \leftrightarrow d$. In this case the coefficient is identically zero only if $\varepsilon = 0$, and for $\varepsilon = O(1)$ it is controlled by $\left(\frac{\Lambda_L}{\Lambda\sin\alpha}\right)^2 y_b^2 A\lambda^2$; its value can be recast as

$$\Lambda_F = 1 \text{ TeV}, \quad c \simeq 10^{-10}\left(\frac{1\text{ TeV}}{\widetilde{m}}\right)\left(\frac{100\text{ TeV}}{\Lambda_L}\right) \tag{4.88}$$

[13]Notice that in box diagrams down squarks run into the loop while loops with vertices from eq.(4.80) are sensitive to up squark mass insertions, constrained by box diagrams for $D - \bar{D}$ oscillation.

with $a_1 = a_2 = 1$, below the experimental constraints, $c < 10^{-8}$ [126]. Small squarks mass insertions do not change this numerical value[14].

Hence we can infer that the inclusion of eq.(4.80) does not reintroduce violations and does not hack the solution settled to avoid flavor problems.

4.9 Numerical Analysis

We show now the numerical results from an extensive scan in parameter space in the determination of the mass spectrum of the model, particularly the Higgs mass. We fix ϵ_R by requiring the correct top mass $M_{top}(1\text{ TeV}) \simeq 150$ GeV and then scan randomly for the other parameters searching for points with $\xi \simeq 0.1$. For any such point we then extract the Higgs mass from the exact potential and compute the full spectrum.

We find that the Higgs mass is distributed in the range 70 GeV $\lesssim M_H \lesssim$ 160 GeV, peaking between $100 - 140$ GeV. The measured value $M_H \simeq$ 126 GeV is therefore a typical value for this model. For each point of the scan we obtain the FT computing numerically the logarithmic derivative of the logarithm of the Higgs mass with respect to all the parameters of the model, and taking the maximum value [19]. The FT ranges between ~ 10 and ~ 300, the typical value being around 50, with no evident correlation with the value of the Higgs mass.

Let us now discuss some properties of the spectrum in the gauge sector and in the matter sector.

Gauge Sector

The mass of the spin-1 resonances is given by $m_\rho = g_m f$, up to corrections of order $\mathcal{O}(g_{SM}/g_m)$ due to mixing with the elementary $SU(2)_L \times U(1)_Y$ gauge bosons. Considerations of metastability and perturbativity fix $g_m(f) \simeq 2.5$, which means $m_\rho \simeq 1950$ GeV for $\xi = 0.1$. Such values are still above the experimental limits from direct searches at the LHC [57, 130], [59] for limits not from experimental collaborations, but are in tensions with indirect bounds from the S parameter.

The lightest uncolored scalar resonance has a mass bounded from above by the same value as the vector resonance. It is usually the complex neutral

[14] Bounds on down-type squark mass mixings are reported in [128, 129].

singlet M_{55} with a mass $\sim 1-1.4$ TeV. With less frequency it is the $\mathrm{Im}\, q_5^n$ or the lightest eigenstate of the symmetric part of q_m^n.

Among the spin-1/2 states, the lightest ones in our scan are usually the winos ($200-1200$ GeV), the doublets $\tilde{h}_{u,d}$ arising from q_5^n and M_{5n} (around 1 TeV), or the fields in the $(1, \mathbf{3})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ coming from the magnetic gauginos ρ and the fermions in the antisymmetric part of q_m^n (in particular, the ones $\tilde{\rho}_R^{\pm}$ with $Y = \pm 1$, $600-1300$ GeV, which do not mix with the elementary bino and the ones $\tilde{\rho}_R^3$ which do mix, $200-1200$ GeV).

The goldstino, contained in the superfield M_{55}, combines with the goldstino coming from the external SUSY breaking: a combination of the two will be eaten by the gravitino and the orthogonal will stay in the spectrum as a massive particle. The exact value for their masses depends on the F terms and we can have different mixed situations in collider experiments, leading to a cascades of decays from neutralino to pseudogoldstino in turn decaying to the true goldstino, resulting in multiphoton events (and missing transverse energy) [131].

Matter Sector

As in some non-SUSY CHM, the lightest colored fermion resonance is the exotic doublet with $Y = 7/6$: the singlet with $Y = 2/3$ coming from a mixture of the elementary t_R and M_{i5} is heavier, typically ~ 1 TeV, while the mass of the lightest fermion ranges up to 900 GeV.

In the case of colored scalars, the lightest one belongs almost always to the fields which mix with the elementary $\tilde{t}_L$ and $\tilde{t}_R$, respectively doublets with $Y = 7/6$ and singlets with $Y = 2/3$. Actually the whole bidoublet (a doublet with $Y = 7/6$ and a doublet with $Y = 1/6$) is almost degenerate in mass, the mass difference between the two doublets being $\lesssim 100$ GeV. The mass of the lightest scalar ranges from 600 GeV to 1 TeV. See fig. 4.1 for a scatter plot.

The gluino has a mass M_3 which does not enter the Higgs effective potential at one-loop, therefore it can be heavier than the current bounds without affecting the FT of the model: contrary to what happens in the MSSM the EW scale is only logarithmically sensitive to stops masses [2]. It is also worth mentioning the existence of the chiral superfield M_{ij} with supersymmetric mass m_1, singlet under the electroweak group and in the symmetric components of the $(3+\bar{3}) \times (3+\bar{3})$ representation of $\mathrm{SU}(3)_c$. Since it does not enter the Higgs potential at one-loop, its holomorphic mass m_1 is free in

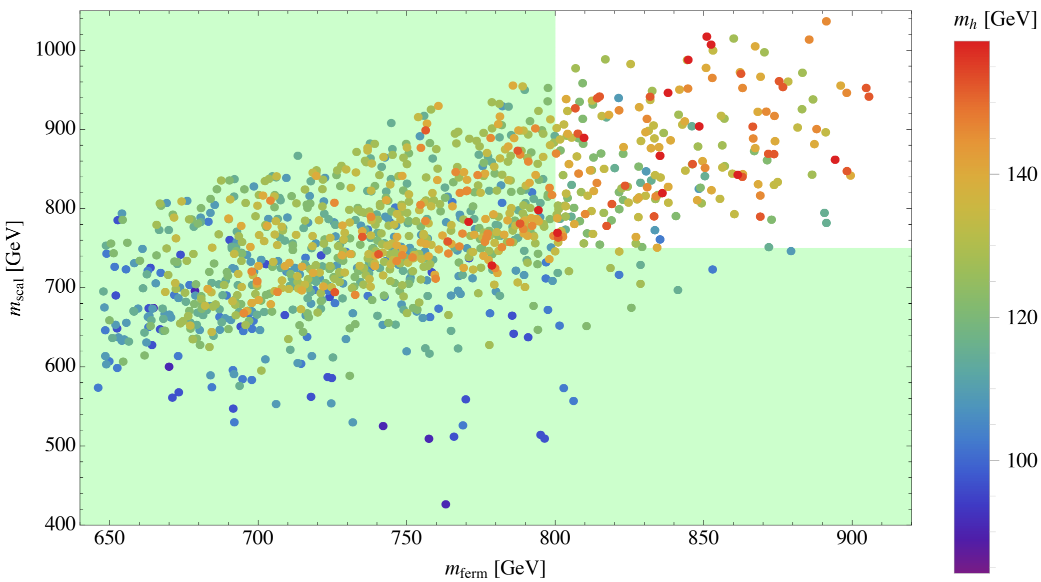

Figure 4.1: Masses of the lightest colored fermions and scalar resonances. In shaded green we superimposed the exclusions discussed in section 4.10. Colors represents Higgs mass as indicated aside.

our setup and for consistency we take it $m_1 < \Lambda \sim 10$ TeV. In the following we will assume that it is heavy enough and neglect its phenomenology.

4.10 Detection Bounds

Given the features outlined in the previous section direct searches should concentrate on colored states, in particular fermions and scalars with exotic electric charge 5/3. First we notice that there is a consistent R parity charges assignement. We have defined the gauge sector as the one which contributes to the one-loop Higgs potential via the SM electroweak gauge couplings, while the matter sector as the one which contributes through the mixings ϵ. This classification reflects also the R parity assignment for the superfields: it is the same as the one of the corresponding SM superfield with which the field mixes.

This implies, as usual, that scalar colored partners and EW fermion partners are pair produced and that the lightest among them is stable. Fermionic

	W, B, G	ρ_m	q_a^n	M_{ab}	q_L	t_R	M_{ia}	q_i^n	M_{ij}
R_P	+	+	+	+	−	−	−	−	+

Table 4.2: R_P assignment to the lowest component of the superfields.

top partners share the same R parity as elementary fields, because they mix with them; since they are a typical signature of CHM models [52] dedicated searches exist: a bound on their mass from events with a pair of same sign leptons has been put at 800 GeV by CMS [13]. For scalar particles there are not such searches but we can reinterpret the results for sbottoms pair produced and decaying into tops and charginos: events with two b-jets and isolated same sign leptons are considered by CMS in [132] and a bound is set at 450 GeV, well below the values found in the numerical scan, although the analysis is not based on the full dataset available but only on a portion corresponding to an integrated luminosity of 10.5 fb^{-1}.

In the model presented the scalar with $Q = 5/3$ is, up to EW effects, degenerate with a full bidoublet of SO(4), namely with other scalars with 1/3 and 2/3 electric charge. EW effects are Higgs VEV insertions, affecting the masses of these particles and removing this degeneracy inducing splitting of order 100 GeV. Thus we can convey as indirect bounds limits on the masses of the other components of the bidoublet: in particular the scalar with $Q = 2/3$ would behave similarly to a stop with decoupled gluinos. Bounds on stops decaying into top and neutralino or bottom and chargino in events with one isolated lepton are derived by CMS from the full 19.5 fb^{-1} dataset and stops are excluded with a mass approximatively below 650 GeV [133].

Also CMS collaboration provides a stronger bound, of 750 GeV [134], on pair produced stops decaying into tops and neutralino using razor variables.

Finally we stress that the simultaneous presence of fermions and scalars in the same mass range can strengthen the respective exclusion limits. Also in our setup multiple scalar stop partners appear and each of these can be produced and decay at the LHC thus heightening the number of expected events and consequently the exclusion bounds: this effect can be simply estimated as follow. We denote with $\sigma(M)$ the pair production cross section of one scalar top partner with mass M and with $M_{excl,n}$ the excluded mass in case of n identical scalars; if we assume a BR = 1 in top and chargino we

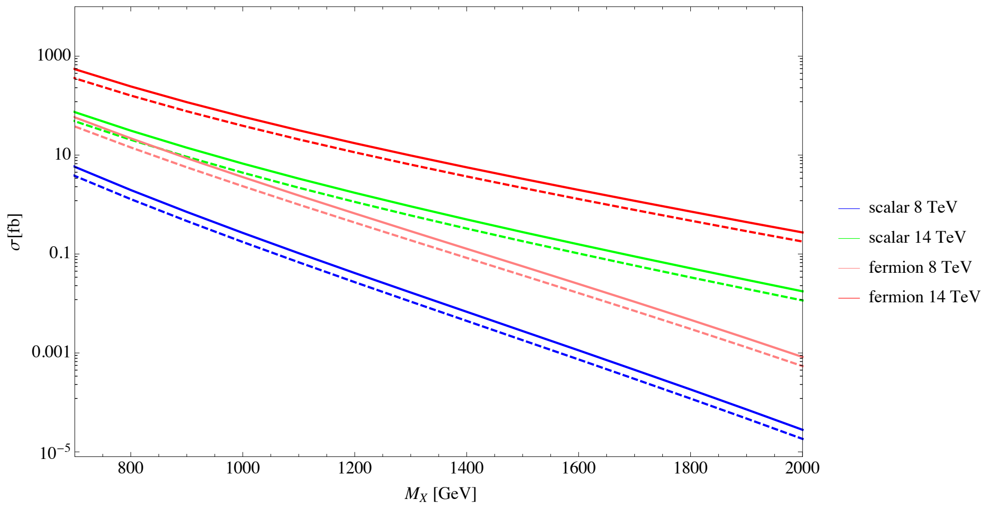

Figure 4.2: Pair production cross sections at LHC through QCD interactions. Dashed lines are the LO values, computed with MadGraph 5 [135], using CTEQ6L PDFs and the model produced with the package FeynRules [136]; solid lines are the NLO, using a common $K_{NLO} = 1.5$.

estimate

$$n\,\sigma(M_{excl,n}) = \sigma(M_{excl,1}) \quad \Rightarrow \quad M_{excl,n} = \sigma^{-1}(\frac{\sigma(M_{excl,1})}{n}) \tag{4.89}$$

assuming that the production cross section for n particles is just n times the case with a single scalar in the spectrum: we neglect decay chains and mutual interactions which deserve a dedicated study. We numerically have

$$\frac{M_{excl,n} - M_{excl,1}}{M_{excl,1}} \simeq 0.1 \quad \text{for} \quad n = 2, 3. \tag{4.90}$$

Therefore we claim the following: the exclusion limits for n particles in the spectrum with $n = 2, 3$ are $O(10\%)$ stronger then the limits obtained for a single particle.

Turning to non colored states the lightest particles are fermions with quantum numbers of EW gauginos or higgsinos. As recently summarized in [137] limits on charginos and neutralinos pair produced have been set by ATLAS [138] and CMS [139]: with all sleptons and sneutrinos decoupled they set limits at 350 GeV from events with three or more leptons in the

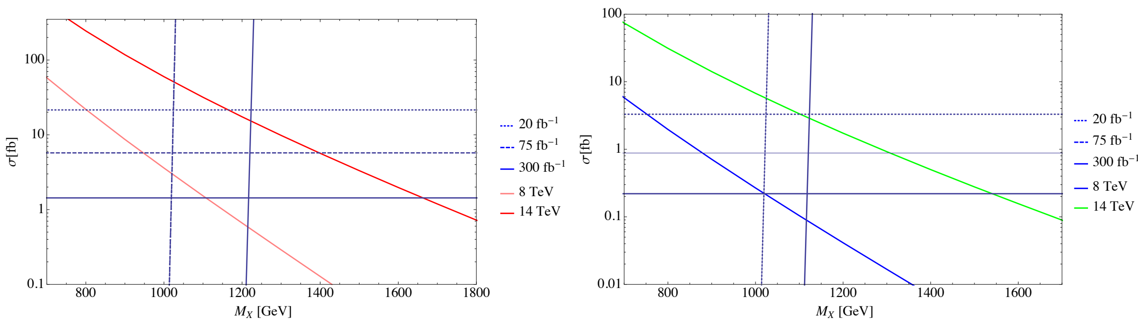

Figure 4.3: Expected exclusion bounds on masses of the lightest fermionic (left panel) and scalar (right panel) top partners from 75 (dashed line) and 300 (solid line) fb^{-1} at $\sqrt{s} = 14$ TeV. The dotted lines correspond to present bounds at 800 and 750 GeV discussed in the text.

final state. This analysis also allows CMS to put bounds on sbottoms and excludes at 95% CL masses below 570 GeV.

Projections for exclusion limits for scalar and fermionic top partners from LHC at a center of mass energy of 14 TeV can be obtained simply rescaling integrated luminosities[15]. Fig. 4.3 clearly shows that higher luminosities, and higher center of mass energies, data will probe the relevant part of the parameter space. We expect they will be able to exclude exotic 5/3 charge fermions up to 1400 (1650) GeV and scalars up to 1300 (1550) GeV with data corresponding to an integrated luminosity of 75 (300) fb^{-1}, assuming BRs = 1. For fermions we also expect single production to become more important than pair production at these energies.

We conclude this section noting that we can interpret already existing experimental searches to exclude portions of the parameters space of the main model described in this chapter, introduced in section 4.2: we expect future experiments, LHC at 14 TeV will play a preponderant role, to further probe it and constrain it to regions with higher level of FT.

[15] ATLAS released projections for future sensitivities in [140].

Chapter 5

A Fully Composite Top Right Case

As stressed in subsection 2.4.1 models of composite Higgs in which the right top entirely belongs to the BSM strongly interacting sector are generically disfavoured: the Higgs boson is predicted to be too light, not compatible with the measured value for its mass. Nevertheless we find instructive to present a simple realization of this idea, first introduced in [1], within the SUSY framework depicted in section 4.1: we do it in the following. Also the problem of SM Landau pole is less severe here than in the previous case, due to a more restricted number of partners. We allow for an additional elementary field, color and weak singlet with an exotic hypercharge: despite the further explicit SO(5) breaking induced by the pairing with the corresponding composite field no reasonable corner of the parameter space is found.

5.1 A Fully Composite t_R

We consider a SUSY SO(9) gauge theory with $N_f = 9$ electric quarks in the fundamental of the gauge group and an additional singlet S_{ij} in the $(\mathbf{1}, \mathbf{10})$ of the global $\mathrm{SO}(5)\times\mathrm{SU}(4)$. We add to the superpotential (4.1) the following term:

$$\lambda Q^i Q^j S_{ij} \,. \tag{5.1}$$

The terms (5.1) do not break any global symmetry. The total anomaly-free global symmetry of the model is

$$G_f = \mathrm{SO}(5) \times \mathrm{SU}(4) \,. \tag{5.2}$$

In full analogy with the previous case of section 4.2 the theory turns to strong coupling at a scale Λ, below which we rely on a magnetic description provided by Seiberg duality. In the magnetic theory eq.(5.1) turns into a mass term $\lambda\Lambda M^{ij}S_{ij}$. If we take $\lambda \sim O(1)$ around the scale Λ, the singlets S_{ij} and M^{ij} can be integrated out. At leading order in the heavy mass, this boils down to remove the chiral fields S_{ij} and M^{ij} from the Lagrangian. We summarize in table 5.1 the gauge and flavor quantum numbers of the fields appearing in the electric and magnetic theories.

The mass spectrum is the same as given in section 4.1, with the exception of the multiplet M^{ij} that has been decoupled together with the singlet S_{ij}. The multiplet M_{i5} is massless. We embed $\mathrm{SU}(3)_c \times \mathrm{U}(1)_X$ into SU(4) and $\mathrm{SU}(2)_{0,L}\times\mathrm{U}(1)_{0,Y}$ into $\mathrm{SO}(5)\times\mathrm{U}(1)_X$. The $\mathrm{U}(1)_X$ is identified as the diagonal SU(4) generator not contained in $\mathrm{SU}(3)_c$, properly normalized, such that $\mathbf{4} \to \mathbf{3}_{2/3} \oplus \mathbf{1}_{-2}$ under $\mathrm{SU}(3)_c \times \mathrm{U}(1)_X$. We identify t_R as the (conjugate) fermion component of $M_{\alpha 5}$, $\alpha = 6, 7, 8$. We also get an unwanted extra fermion, coming from M_{95}. Being an $\mathrm{SU}(2)_L$ singlet, $\psi_{M_{95}}$ corresponds to an exotic particle with hypercharge $Y = X = 2$. We can get rid of this particle by adding to the visible sector a conjugate chiral field ψ^c that mixes with M_{95}, in the same way as M_{ia} is going to mix with t_L. The field ψ^c is actually necessary for the consistency of the model, so that all gauge anomalies cancel. Consider for instance the cubic anomaly of the hypercharge, where once again $Y = T^3_R + X$ and $\mathrm{SU}(2)_R \subset \mathrm{SO}(4) \subset \mathrm{SO}(5)$: the contribution from the SM fermions without the right top is proportional to $A(\mathrm{SM} - t^c_R) = 3(2/3)^3$; from the composite sector we have $A(q^n_i) = 4[3(2/3)^3 + (-2)^3]$ and $A(M_{ia}) = 5[3(-2/3)^3 + 2^3]$. Therefore, to have $\mathrm{Tr}[Y^3] = 0$ we need to introduce a field with $Y = -2$. With this addition this and all the other anomalies vanish.

In the UV theory, the mixing terms are

$$\lambda_t \xi^{ia} Q_i Q_a + \lambda_\phi \phi^{ia} Q_i Q_a \,. \tag{5.3}$$

Like in the previous section, we have written the mixing terms in a formal G_f invariant way by means of the superfields ξ and ϕ. These are spurions, whose only dynamical components are the SM doublet q_L and the singlet ψ^c.

	$SO(9)_{el}$	SO(5)	SU(4)
Q_i^N	**9**	**1**	$\bar{\mathbf{4}}$
Q_a^N	**9**	**5**	**1**
S_{ij}	**1**	**1**	**10**

(a)

	$SO(4)_{mag}$	SO(5)	SU(4)
q_i^n	**4**	**1**	**4**
q_a^n	**4**	**5**	**1**
M_{ia}	**1**	**5**	$\bar{\mathbf{4}}$
M_{ab}	**1**	$\mathbf{14}\oplus\mathbf{1}$	**1**

(b)

Table 5.1: Quantum numbers under G_f and the strong gauge group of the matter fields appearing in the composite sector of model II: (a) UV electric and (b) IR magnetic theories.

More explicitly, we have

$$\xi^{\alpha a} = \frac{1}{\sqrt{2}}\begin{pmatrix} b_L \\ -ib_L \\ t_L \\ it_L \\ 0 \end{pmatrix}_{2/3}, \quad \xi^{9a} = 0\,, \quad \phi^{\alpha a} = 0\,, \quad \phi^{9a} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \psi^c \end{pmatrix}_{-2}, \tag{5.4}$$

where we have omitted the color index in q_L and ψ^c. In the magnetic theory the Yukawa's (5.3) become

$$\epsilon_t \xi^{ia} M_{ia} + \epsilon_\phi \phi^{ia} M_{ia}\,. \tag{5.5}$$

Thanks to the last term in eq.(5.5), the multiplets M_{95} and ψ^c combine and get a mass $\epsilon_\phi/\sqrt{2}$. The assumption of an external source of SUSY breaking affecting only the visible sector cannot work now, because t_R is a fully composite particle, and would result in an unacceptable light stop $\widetilde{t}_R$. We then also add SUSY breaking terms in the composite sector, by assuming that they respect the global symmetry G_f. In order to have a well-defined UV theory, we introduce positive definite scalar soft terms in the electric theory and analyze their RG flow towards the IR following [111]. The construction discussed in section 4.5, to whom we refer for all the details on how it is performed and the underlying approximations, here is crucial. Neglecting soft masses for the magnetic gauginos and B-terms, the non-SUSY IR Lagrangian reads

$$\begin{aligned} -\mathcal{L}_{\not{SUSY}} = {} & \widetilde{m}_L^2|\widetilde{t}_L|^2 + \widetilde{m}_\psi^2|\widetilde{\psi}|^2 + (\epsilon_L B_L(\xi_L)_{ia}M_{ia} + \frac{1}{2}\widetilde{m}_{g,\alpha}\lambda_\alpha\lambda_\alpha + h.c.) \\ & + \widetilde{m}_1^2|M_{ia}|^2 + \widetilde{m}_2^2|M_{ab}|^2 + \widetilde{m}_3^2|q_i|^2 - \widetilde{m}_4^2|q_a|^2\,, \end{aligned} \tag{5.6}$$

where

$$\begin{aligned}\widetilde{m}_1^2 &= \frac{1}{12}(4+2\omega)\widetilde{m}^2, \quad \widetilde{m}_2^2 = \frac{1}{12}(-8+14\omega)\widetilde{m}^2,\\ \widetilde{m}_3^2 &= \frac{1}{12}(-8+5\omega)\widetilde{m}^2, \quad \widetilde{m}_4^2 = \frac{1}{12}(-4+7\omega)\widetilde{m}^2\end{aligned} \tag{5.7}$$

are the soft mass terms for the scalars in the IR theory, determined in terms of the two SO(5)×SU(4) invariant soft terms in the electric theory, $\widetilde{m}_{1el}^2 Q^{\dagger a}Q^a + \widetilde{m}_{2el}^2 Q^{\dagger i}Q^i$, with $\widetilde{m}^2 \equiv \widetilde{m}_{1el}^2$ and

$$\omega = \frac{\widetilde{m}_{2el}^2}{\widetilde{m}_{1el}^2}\,. \tag{5.8}$$

As can be seen from eq.(5.7), there is no choice of ω for which all the magnetic soft terms are positive definite. If we take $\omega > 8/5$, the first three terms in the second row of eq.(5.6) are positive, while the last one is tachyonic. These tachyons are harmless, since the SUSY scalar potential contains quartic terms (both in the F and D-term part of the scalar potential) that stabilize them. Negative definite quadratic terms for the q_a are already present in the SUSY potential, resulting in fact in the vacuum eq.(4.6). The only effect of the Lagrangian (5.6), at the level of the vacuum, is to change the VEV as in eq.(4.32), namely

$$\langle q_n^m \rangle = \frac{\tilde{\mu}}{\sqrt{h}}\delta_n^m, \quad \text{where} \quad \tilde{\mu} = \sqrt{\mu^2 + \frac{\widetilde{m}_4^2}{2h}}\,. \tag{5.9}$$

The above treatment of soft terms as a perturbation of an underlying SUSY theory makes sense only for soft terms parametrically smaller than Λ. Notice that we cannot parametrically decouple the scalars in the composite sector, while keeping the fermions at the scale μ, by taking the soft terms $\widetilde{m}^2$ in the range $\mu \ll \widetilde{m} \ll \Lambda$. This is clear from eq.(5.9), since in this limit we decouple the whole massive spectrum in the composite sector. In order to keep the compositeness scale around the TeV scale and avoid too light scalars, we take the soft term mass scale around μ. In addition to that, we still have, like in the model in section 4.2, an "indirect" contribution to the composite soft masses coming from the mixing with the elementary sector, as given by eqs.(4.26) and (4.27). A linear combination of fermions given by t_L and the appropriate components of $\psi_{M_{im}}$ remains massless and is identified with the SM left-handed top. The "goldstino" $\psi_{M_{55}}$ is still massless in these approximations. See the considerations made in the last paragraph of section 4.2, that apply also here, for the possible mechanisms giving a mass to this particle.

5.2 Vacuum Decay

The non-supersymmetric vacuum we have found can be metastable and supersymmetric (in the sense explained in footnote 5) vacua might appear, due to non-perturbative effects in the magnetic theory. Contrary to the model in section 4.2, we have not found SUSY vacua in the regime of validity of the magnetic theory. The only SUSY vacua we found appear in the electric theory. Assuming $S_{ij} \neq 0$ with maximal rank, i.e. 4, all electric quarks are massive and the resulting theory develops the non-perturbative superpotential eq.(4.40) where now $N = N_f = 9$. Taking the ansatz eq.(4.36) for the gauge-invariant meson directions and $S_{ij} = S_0\delta_{ij}$, we get

$$
\begin{aligned}
F_X &= -5\Lambda^{-6} X^{\frac{3}{2}} Y^2 + m = 0\,, \\
F_Y &= -4\Lambda^{-6} X^{\frac{5}{2}} Y + \lambda S_0 = 0\,, \\
F_S &= \lambda Y = 0.
\end{aligned}
\tag{5.10}
$$

The only solution to eq.(5.10) is the runaway vacuum

$$
Y \to 0, \quad S \propto Y^{-\frac{7}{3}} \to \infty, \quad X \propto Y^{-\frac{4}{3}} \to \infty\,. \tag{5.11}
$$

We have found no other SUSY vacua at finite distance in the moduli space and we then conclude that the metastable vacuum eq.(4.6) is sufficiently long-lived, if not absolutely stable.

5.3 Landau Poles

Landau poles for the SM gauge couplings at relatively low energies are expected also in this model. Within the same approximations made in subsection 4.3, a one-loop computation shows that the $SU(3)_c$, $SU(2)_{0,L}$ and

$U(1)_{0,Y}$ couplings develop Landau poles at the scales

$$\begin{aligned}
\Lambda_3^L &= \Lambda \, \exp\left(\frac{\pi}{2\alpha_3(m_Z)}\right)\left(\frac{m_Z}{\mu}\right)^{-\frac{7}{4}}\left(\frac{\mu}{\Lambda}\right)^{\frac{1}{4}}, \\
\Lambda_2^L &= \Lambda \, \exp\left(\frac{2\pi}{9\alpha_2(m_Z)}\right)\left(\frac{m_Z}{\mu}\right)^{-\frac{19}{54}}\left(\frac{\mu}{\Lambda}\right)^{2}, \\
\Lambda_1^L &= \Lambda \, \exp\left(\frac{6\pi}{305\alpha_1(m_Z)}\right)\left(\frac{m_Z}{\mu}\right)^{\frac{41}{610}}\left(\frac{\mu}{\Lambda}\right)^{\frac{236}{305}},
\end{aligned} \tag{5.12}$$

where we have matched the $SU(2) \times U(1)$ couplings at the scale μ, using eq.(4.16) with

$$\alpha_m(\mu) = \frac{2\pi}{3\log\left(\frac{\Lambda}{\mu}\right)}\,. \tag{5.13}$$

The presence of less flavors and singlet fields here with respect to the previous case of $N = N_f = 11$ in section 4.2, allows for a significant improvement in the UV behavior of α_3, that now blows up at extremely high energies. However, the different embedding of $U(1)_X$ in the global group gives rise to several fields with hypercharge $|2|$ that significantly contribute to the running of α_1. As a result, the first coupling to explode is now α_1. For a sensible choice of parameters, e.g. μ around the TeV scale and $\epsilon \sim 1/10$, we see that Λ_1^L is about two orders of magnitude higher than Λ, around 10^3 TeV.

5.4 Numerical Analysis

The next step is the computation of the effective action for the Higgs field, performed in the unitary gauge. As we mentioned many times the Higgs is a NG boson of a spontaneous breaking of a global symmetry: the broken symmetry is not exact and it is explicitly broken by the SM EW gauge group and by the couplings ϵ_t and ϵ_ϕ. In the matter contribution to the Higgs potential we further distinguish between colored and non colored exotic fields. We then perform a numerical scan in the parameter space.

The value of ϵ_t is fixed by the top mass; the mixing ϵ_ϕ of the non colored field is in principle free. γ_{gauge}, $\gamma_{matter}^{(c)}$ and $\gamma_{matter}^{(nc)}$ are equally important and they cancel against each other: the size of these cancellations is a lower bound on the FT. For what concern the coefficient of the quartic term we have $\beta \sim \beta_{matter} \gg \beta_{gauge}$.

The Higgs turns out to be too light (~ 100 GeV) unless a sizable source of SO(5) breaking comes from the non colored sector, as in fact was expected. Since the Higgs mass square is proportional to the sum $\beta^{(c)}_{matter} + \beta^{(nc)}_{matter}$ in principle raising the non colored contribution controlled by ϵ_ϕ would be sufficient. At the same time large values for ϵ_ϕ are disfavored because generally $\gamma^{(nc)}_{matter} < 0$ and it tends to align the Higgs in a EW preserving vacuum. Due to this tension the model as it stands is excluded. We have chosen to report the results because, despite SUSY, the construction is quite minimal and we expect it to be representative for more general examples: it embodies a composite top right model with the addition of an extra massive singlet. The situation can be improved if we introduce more FT: due to the logarithmic dependence on soft masses we would need stops at a scale $O(100)$ TeV, definitely loosing the naturalness. We can also introduce more complication in the model or focus on SO(5) representations different from the fundamental, but we have not continued along this path.

Chapter 6

Conclusions and Outlook

In this thesis we collected the work of the author on SUSY CHM. We provided four dimensional examples of BSM sectors both weakly and strongly coupled. The Higgs is included as a pNGB living on the minimal coset studied in literature allowing for a custodial protection, SO(5)/SO(4). We introduced couplings to the top field through partial compositeness and we focused on top partners in the minimal representation of the global symmetry group, the fundamental **5**. The EW gauge fields may or may not be semi-composite, we investigated both cases.

Here we briefly summarize the main results:

- SUSY has offered tools (Seiberg duality) to deal with strongly coupled gauge theories. This allowed us to approach questions about UV completeness of the models: we considered explicit models of Higgs as pNGB, both as a truly elementary scalar and as a composite object. We reviewed how to constrain the most general form of soft SUSY breaking masses in presence of confining theories, with the help of global symmetries, treating them as spurion superfields: this allowed to keep a well defined UV completion. At the same time SUSY has soften the dependence of the Higgs potential on a low energy cutoff.

- We extensively analyzed, both analyticaly and numerically, the Higgs potential, expressed in terms of UV parameters. We neglected the impact of SM fermions different from top on the radiative potential, claiming they do not play a role in the EWSB. We have computed the FT of the models and shown that there exist regions in parameters space tuned at 1 - 2 % level, reproducing the correct top and Higgs masses.

- We proposed a mechanism different to partial compositeness to supply masses to other quarks, and to leptons, through irrelevant operators and we have considered the most stringent bounds from flavor processes. The introduction of dimension five operators generates subleading corrections to flavor observables therefore flavor violations can be addressed as in more conventional SUSY theories, as the MSSM: we explicitly focused on the case of aligned squark masses.

- We have shown which are the most important experimental bounds, mostly from direct searches of partners, on our models. The lightest colored particles in the BSM spectrum are generically colored partners of the top, both scalars and fermionic, and uncolored fermions with the quantum numbers of EWinos and higgsinos. We expect these predictions to be independent of the particular realizations we presented. Interestingly enough some versions of SUSY CHM can be soon accessible at Run II of LHC.

For other types of observables, like indirect measurements or deviations of (Higgs) couplings we refer to the many results and estimates present in the literature. LHC will soon let us know more about the detailed structure of the EWSB in nature: experimental collaborations have already achieved significant results constraining, from the available data, the simplest realizations of BSM physics, probing both the MSSM and its variations and models of composite Higgs, as well as extra dimensional and other exotic theories. We hope that future efforts will help in shedding light upon less minimal models, as the class proposed and discussed in this thesis.

Appendix A

Renormalization of the Higgs Potential

A.1 Renormalization of the Higgs Potential

Here we want to show explicitly the cancellations mentioned in the main text regarding the scale dependence of the effective potential of the Higgs, as imposed by eq.(3.5). We first focus on the case of the model with vector resonances, focusing on the superpotential

$$W_{mag} = h q_a M^{ab} q_b - \mu^2 M^{aa}\,. \tag{A.1}$$

The superpotential (A.1) is invariant under global SO(5) transformations. At tree-level, with a canonical Kähler potential, this leads to an SO(5) invariant (and hence Higgs independent) F-term scalar potential. At the radiative level, however, the Kähler potential is renormalized and the gauging of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ explicitly breaks the SO(5) global symmetry. This implies that although holomorphy protects the superpotential (A.1) from quantum corrections, the physical, rather than holomorphic, coupling h splits into several components with different RG evolutions, depending on the $\mathrm{SU}(2)_L \times U(1)_Y$ quantum numbers of the fields q_a and M^{ab}. For this reason, in order to better understand the one-loop behaviour of the scalar potential, it is convenient to directly start by the generalization of (A.1), where we distinguish all the couplings h with a different one-loop evolution. Recalling that q contains one singlet and two $\mathrm{SU}(2)_L$ doublets with $Y = \pm 1/2$ and M_{ab} contains three $\mathrm{SU}(2)_L$ triplets with $Y = -1, 0, 1$, two $\mathrm{SU}(2)_L$ doublets with

$Y = \pm 1/2$ and two singlets, we have:

$$W_{mag} = \sum_{i=1}^{5} h_i (q_a M^{ab} q_b)^{(i)} - \mu^2 M^{aa} \tag{A.2}$$

where i runs over all the five distinct possible combinations:

$$\begin{aligned} &(\mathbf{1}_0 \cdot \mathbf{1}_0 \cdot \mathbf{1}_0), \quad (\mathbf{1}_0 \cdot \mathbf{2}_{\pm 1/2} \cdot \mathbf{2}_{\mp 1/2}), \quad (\mathbf{2}_{\pm 1/2} \cdot \mathbf{3}_{\mp 1} \cdot \mathbf{2}_{\pm 1/2}), \\ &(\mathbf{2}_{\mp 1/2} \cdot \mathbf{3}_0 \cdot \mathbf{2}_{\pm 1/2}), \quad (\mathbf{2}_{\mp 1/2} \cdot \mathbf{1}'_0 \cdot \mathbf{2}_{\pm 1/2}) \,. \end{aligned} \tag{A.3}$$

We write the effective Higgs potential as

$$V(s_h) = V^{(0)}(s_h) + V^{(1)}(s_h) + \ldots . \tag{A.4}$$

The tree-level potential $V^{(0)}$ is given by

$$V^{(0)} = m_1^2 |q_5^n|^2 + m_2^2 |q_m^n|^2 + \sum_{i=1}^{5} |F_{ab}^{M(i)}|^2 . \tag{A.5}$$

Notice that the D-term scalar potential , as well as the F-term potential given by the dual quarks q manifestly vanish in the vacuum eq.(4.32), where h is now identified with h_5. The first two terms in eq.(A.5) are soft SUSY breaking terms. In the vacuum, modulo an irrelevant constant term, they give rise to a tree-level Higgs mass term of the form $m_H^2 s_h^2 \mu^2 / h_5$, where

$$m_H^2 = m_1^2 - m_2^2 \,. \tag{A.6}$$

The mass term (A.6) violates the SO(5) global symmetry of the composite sector, which is assumed to be exact in the limit of vanishing mixing terms ϵ_i and SM gauge couplings. We assume in the following that m_H at the scale f assumes a value of the same order of magnitude of the one expected to be given by radiative corrections, namely $g^2/(16\pi^2)$ in some mass unit. In this way, we are justified of neglecting its effect in the tree-level potential.

The RG-invariance of the scalar potential at one-loop level is governed by eq.(3.5), which reads

$$\frac{\partial}{\partial \log Q} V^{(1)} + \beta_{\lambda_I} \frac{\partial}{\partial \lambda_I} V^{(0)} - \gamma_n \Phi_n \frac{\partial}{\partial \Phi_n} V^{(0)} = 0, \tag{A.7}$$

It is easy to check that in the vacuum the last term in eq.(A.7) vanishes when we recover the SO(5) invariant limit with $h_i = h$. The relevant β-functions entering in the second term of eq.(A.7) are β_{h_i}, $\beta_{m_1^2}$ and $\beta_{m_2^2}$.[1] We have

$$\begin{aligned}
\beta_{m_1^2} - \beta_{m_2^2} &= \frac{(3g^2 M_2^2 + g'^2 M_1^2)}{8\pi^2}\,, \qquad \beta_{h_1} = h_1(2\gamma_S^q + \gamma_S^M)\,,, \\
\beta_{h_2} &= h_2(\gamma_{D_{1/2}}^q + \gamma_{D_{1/2}}^M + \gamma_S^q)\,, \qquad \beta_{h_3} = h_3(2\gamma_{D_{1/2}}^q + \gamma_{T_1}^M)\,, \\
\beta_{h_4} &= h_4(2\gamma_{D_{1/2}}^q + \gamma_{T_0}^M)\,, \qquad \beta_{h5} = h_5(2\gamma_{D_{1/2}}^q + \gamma_S^M)\,,
\end{aligned} \tag{A.8}$$

where

$$\begin{aligned}
\gamma_S^q &= \gamma_0^q\,, \qquad \gamma_S^M = \gamma_0^M \\
\gamma_{D_{1/2}}^q &= \gamma_0^q - \frac{1}{16\pi^2}\Big(\frac{3}{2}g^2 + \frac{1}{2}g'^2\Big)\,, \quad \gamma_{D_{1/2}}^M = \gamma_0^M - \frac{1}{16\pi^2}\Big(\frac{3}{2}g^2 + \frac{1}{2}g'^2\Big)\,, \\
\gamma_{T_1}^M &= \gamma_0^M - \frac{1}{16\pi^2}(4g^2 + 2g'^2)\,, \quad \gamma_{T_0}^M = \gamma_0^M - \frac{1}{16\pi^2}(4g^2)
\end{aligned} \tag{A.9}$$

are the anomalous dimensions of an $\mathrm{SU}(2)_L$ singlet with $Y = 0$, an $\mathrm{SU}(2)_L$ doublet with $|Y| = 1/2$, an $\mathrm{SU}(2)_L$ triplet with $|Y| = 1$ and an $\mathrm{SU}(2)_L$ triplet with $Y = 0$. The factors γ_0^q and γ_0^M are the SO(5) invariant contributions to the field anomalous dimensions, given by the couplings h_i themselves and by the SO(4) magnetic gauge interactions (for q only), that do not contribute to the Higgs-dependent scalar potential. They are reported in eq.(A.18).

After some algebra, the first term in eq.(3.5) reads

$$\begin{aligned}
\frac{\partial}{\partial \log Q} V^{(1)} &= -\frac{1}{16\pi^2}\frac{1}{2}\mathrm{STr}[M^4] = \\
&= \frac{1}{16\pi^2} f^2 s_h^2 \left(\mu^2(3g^2 + g'^2) - (3g^2 M_2^2 + g'^2 M_1^2)\right)\,.
\end{aligned} \tag{A.10}$$

It is straightforward to check that eq.(A.7) is indeed verified, when we take the couplings h_i all equal at the scale f:

$$h_1(f) = \ldots = h_5(f) = h(f). \tag{A.11}$$

The case of the model in section 3.3, without gauge resonances, is treated in full analogy. The relevant superpotential is given in eq.(3.12), namely

$$W_0(Z, q, \psi) = hZ(q_a q_a - \mu^2) + m_\psi q_a \psi_a\,. \tag{A.12}$$

[1] In principle, we also have β_{μ^2}, but it anyway cannot contribute to the one-loop Higgs-dependent potential, since M^{aa} is an $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ singlet.

The couplings whose running are affected by the SM gauging are m_ψ and h. The decomposition analogous to eq.(A.2) is a sum of two contributions,

$$W_0 = \sum_i \left[h_i Z (q_a q_a)^{(i)} + m_{\psi i} (q_a \psi_a)^{(i)} \right] - hZ\mu^2 \tag{A.13}$$

where i runs over the projections

$$(\mathbf{1}_0 \cdot \mathbf{1}_0)\,, \quad (\mathbf{2}_{\pm 1/2} \cdot \mathbf{2}_{\mp 1/2})\,. \tag{A.14}$$

Without reporting the full computation we just quote the results for the beta functions:

$$\beta_{h_2} - \beta_{h_1} = -\frac{3(3g^2 + g'^2)}{16\pi^2} h\,, \quad \beta_{m_{\psi 2}} - \beta_{m_{\psi 1}} = -\frac{3(3g^2 + g'^2)}{16\pi^2} m_\psi \tag{A.15}$$

where we have subtracted the SO(5) preserving part. The third tree-level coupling to be considered is the Higgs soft mass m_H, as defined in eq.(A.6). With these beta functions one can show that eq.(A.7) is satisfied.

In the next section we report the one-loop beta functions, computed with standard methods, of the couplings introduced for the models with partially composite top quark, namely the model without vector resonances introduced in section 3.3 and the version enlarged to include vector resonances, in the parametrization given in section 3.4.

A.2 SO(5) preserving anomalous dimensions and β functions

In appendix A.1 we just used some one-loop beta functions for fields appearing in the scalar potential of the main theories discussed. In particular since we were interested in the scale dependence of the Higgs potential we focused on the RG violating the SO(5) symmetry induced by SM gauging. Here we report the terms in the anomalous dimensions and in the beta functions which are SO(5) preserving. We quoted them in several points in the main text in discussions related to the behavior of certain couplings and the possible appearance of Landau poles: here we list them for reference.

The model with elementary t_R, without vector resonances, has the following anomalous dimensions for the fields appearing in the superpotential of eq.(3.11) and eq.(3.12):

$$\begin{aligned}
\gamma_{N_L} &= \frac{1}{16\pi^2}\Big(\lambda_L^2 - \frac{8}{3}g_3^2\Big)\,, \quad \gamma_{N_R} = \frac{1}{16\pi^2}\Big(\lambda_R^2 - \frac{8}{3}g_3^2\Big)\,,\\
\gamma_S &= \frac{1}{16\pi^2}\Big(5\lambda_L^2 - \frac{8}{3}g_3^2\Big)\,, \quad \gamma_{S^c} = \frac{1}{16\pi^2}\Big(5\lambda_R^2 - \frac{8}{3}g_3^2\Big)\,,\\
\gamma_q &= \frac{1}{16\pi^2}\Big(3\lambda_L^2 + 3\lambda_R^2 + 4h^2\Big)\,, \quad \gamma_Z = \frac{1}{16\pi^2}\Big(10h^2\Big)\,.
\end{aligned} \tag{A.16}$$

The β functions for the couplings appearing in the superpotential eq.(3.11) and eq.(3.12) are therefore given by:

$$\begin{aligned}
\beta_{\alpha_{\lambda L}} &= \frac{\alpha_{\lambda L}}{2\pi}\Big(9\alpha_{\lambda L} + 3\alpha_{\lambda R} + 4\alpha_h - \frac{16}{3}\alpha_3\Big)\,,\\
\beta_{\alpha_{\lambda R}} &= \frac{\alpha_{\lambda R}}{2\pi}\Big(9\alpha_{\lambda R} + 3\alpha_{\lambda L} + 4\alpha_h - \frac{16}{3}\alpha_3\Big)\,,\\
\beta_{\alpha_h} &= \frac{\alpha_\lambda}{2\pi}\Big(6\alpha_{\lambda L} + 6\alpha_{\lambda R} + 18\alpha_h\Big)\,,\\
\beta_{\alpha_3} &= \frac{\alpha_3^2}{2\pi}\Big(-3N_c + \frac{1}{2}N_{fund}\Big)\,,
\end{aligned} \tag{A.17}$$

where in the last line $N_c = 3$ is the number of colors and $N_{fund} = (3(1+1+2))_{SSM} + (2+10)_{extra} = 24$ is the number of fundamental representations of $SU(3)_c$ in the theory, divided into SM ones and the extra ones from the top-partner sector.

For what concerns the model with elementary t_R with also vector resonances, specified by the superpotential eq.(3.27) and eq.(3.28), the anomalous dimensions of the fields are given by:

$$\begin{aligned}
\gamma_{N_L} &= \frac{1}{16\pi^2}\Big(4\lambda_L^2\Big)\,, \quad \gamma_{N_R} = \frac{1}{16\pi^2}\Big(4\lambda_R^2\Big)\,,\\
\gamma_{X_L} &= \frac{1}{16\pi^2}\Big(5\lambda_L^2 - 3g_\rho^2\Big)\,, \quad \gamma_{X_R} = \frac{1}{16\pi^2}\Big(5\lambda_R^2 - 3g_\rho^2\Big)\,,\\
\gamma_q &= \frac{1}{16\pi^2}\Big(3\lambda_L^2 + 3\lambda_R^2 + 12h^2 - 3g_\rho^2\Big)\,, \quad \gamma_Z = \frac{1}{16\pi^2}\Big(8h^2\Big)\,.
\end{aligned} \tag{A.18}$$

From this we compute the β functions for the couplings in the superpotential eq.(3.27) and eq.(3.28):

$$\begin{aligned}
\beta_{\alpha_{\lambda L}} &= \frac{\alpha_{\lambda L}}{2\pi}\Big(12\alpha_{\lambda L} + 3\alpha_{\lambda R} + 12\alpha_h - 6\alpha_\rho\Big)\,, \\
\beta_{\alpha_{\lambda R}} &= \frac{\alpha_{\lambda R}}{2\pi}\Big(3\alpha_{\lambda R} + 12\alpha_{\lambda L} + 12\alpha_h - 6\alpha_\rho\Big)\,, \\
\beta_{\alpha_h} &= \frac{\alpha_\lambda}{2\pi}\Big(6\alpha_{\lambda L} + 6\alpha_{\lambda R} + 32\alpha_h - 6\alpha_\rho\Big)\,, \\
\beta_\rho &= \frac{\alpha_\rho}{2\pi}\Big(5\alpha_\rho\Big)\,.
\end{aligned} \tag{A.19}$$

In the anomalous dimensions and in the β functions we have neglected the contribution coming from the SM gauge couplings being surely subleading in the energy range of validity of eq.(A.18) and eq.(A.19), that is for energies roughly between 1 and 10 TeV.

Appendix B

Unitarization of WW Scattering

In theories where the Higgs is a pNGB the $2 \to 2$ scattering amplitudes between the longitudinal polarizations of the W and Z bosons, and the Higgs itself, for energies higher than the Higgs mass grow quadratically with the energy, $\mathcal{A}(E) \sim E^2/f^2$, violating perturbative unitarity at a scale $\Lambda \sim 4\pi f$. At this scale, or before, new degrees of freedom (in the form of either strong dynamics effects or new perturbative fields) must become important in the scattering to restore unitarity, namely to cancel the leading order dependence of the amplitude on the total energy. In the following we will see how the field content present in each of the two models described in section 3.3 and in chapter 4 is exactly what is needed to restore perturbative unitarity of WW scattering, as expected in genuine linear models.

By means of the equivalence theorem [141–144] the amplitude of the scattering involving longitudinal polarization of EW bosons are expressed in terms of the eaten Goldstone, up to corrections $O(\frac{M_V}{E})$, therefore we are led to study the UV properties of

$$\mathcal{A}(h^{\hat{a}}h^{\hat{b}} \to h^{\hat{c}}h^{\hat{d}})\,. \tag{B.1}$$

In the SO(5) $\to$ SO(4) coset, all $h^{\hat{a}}h^{\hat{b}}$ scattering amplitudes can be parametrized in terms of only two functions of the Mandelstam variables, $A(s,t,u)$ and $B(s,t,u)$ [56]:

$$\begin{aligned}\mathcal{A}(h^{\hat{a}}h^{\hat{b}} \to h^{\hat{c}}h^{\hat{d}}) \;=\;& A(s,t,u)\delta^{ab}\delta^{cd} + A(t,s,u)\delta^{ac}\delta^{bd} + A(u,t,s)\delta^{ad}\delta^{bc} + \\ &+ B(s,t,u)\epsilon^{abcd}.\end{aligned} \tag{B.2}$$

In our models, however, in the limit of zero SM gauging the gauge sector has a P_{LR} symmetry which fixes $B(s,t,u) = 0$. The NGB contribution to the scattering is universal and given by

$$A_{NGB}(s,t,u)=\frac{s}{f^2}. \tag{B.3}$$

The possible contributions to NGB scattering can be obtained simply by group theory and the fact that bosonic states must be symmetric under the exchange of identical particles:

$$h^{\hat a}h^{\hat b}\text{ scattering: }\mathbf{4}\otimes\mathbf{4}=(\mathbf{1};J=0)\oplus(\mathbf{6};J=1)\oplus(\mathbf{9};J=0)\ , \tag{B.4}$$

where J is the total spin.

B.1 Minimal Model $\mathbf{SO}(5)\to\mathbf{SO}(4)$

In the theory introduced in section 3.3 the only NGB present are the four components of the Higgs doublet, $h^{\hat a}$, and there are no gauge bosons other than the SM ones. The gauge sector of the model is a supersymmetrization of the linear σ-model presented in ref. [76] and in the Appendix G of ref. [56]. The only term in the Lagrangian relevant to WW scattering is the kinetic term of the real part of $q=(\phi+i\tilde{\phi})/\sqrt{2}$, which takes a VEV $\langle\phi\rangle=(0,0,0,0,f)$. Expliciting the NGB dependence as $\phi(x)=U(h^{\hat a}(x))\langle\phi\rangle\left(1+\frac{\eta(x)}{f}\right)$, where $\eta(x)$ is a real singlet scalar field with mass $M_\eta=\sqrt{2}hf$, we can write the Lagrangian as

$$\mathcal{L}_{kin}=\frac{1}{2}(\partial_\mu\eta)^2-\frac{1}{2}M_\eta^2\eta^2+\frac{f^2}{4}\mathrm{Tr}\left[d_\mu d^\mu\right]\left(1+\frac{\eta}{f}\right)^2\ , \tag{B.5}$$

where we defined the CCWZ structures [17], as in the general case of eq.(2.6), $d_\mu^{\hat a}T^{\hat a}+E_\mu^aT^a=iU^\dagger D_\mu U$ and $\nabla_\mu=\partial_\mu-iE_\mu$. The full NGB scattering amplitude in this theory can be written as

$$A(s,t,u)=\frac{s}{f^2}\left(1-\frac{s}{s-M_\eta^2}\right), \tag{B.6}$$

where the second term comes from the tree level exchange of the singlet field η. The total amplitude evidently recovers perturbative unitarity for $\sqrt{s}\gg M_\eta$.

B.2 Model with Vector Resonances

We now move to discuss unitarization in the model arising at low energy as Seiberg dual, see chapter 4 and in particular the gauge sector described in section 4.1 for the details on the Lagrangian. Now ten Goldstone bosons are present: six π^A in the adjoint representation of $\mathrm{SO}(4)_D$ and four $h^{\hat{a}}$ in the fundamental of $\mathrm{SO}(4)_D$. In the unitary gauge, where the Goldstone bosons in the adjoint are eaten by the ρ gauge bosons, the study of their scattering is shifted to the study of the $\rho\rho$ scattering. With the aim to connect our study with previous bottom-up studies of the effect of resonances in WW scattering in CHM and their phenomenology at the LHC [56], in the following we will concentrate only on the study of the scattering amplitudes among the four NGBs which form the Higgs doublet.

All contributions to NGB scattering, see eq.(B.4), come from the kinetic term of the fields in the multiplet which takes a VEV triggering the spontaneous symmetry breaking, in our case the real components of q_a^n:

$$\mathcal{L} = |D_\mu q_a^n|^2 = \left|iU^\dagger D_\mu q_a^n\right|^2 = |i\nabla_\mu \tilde{q} + d_\mu \tilde{q} - g_\mu \tilde{q}\rho_\mu|^2\,, \tag{B.7}$$

where we used eq.(4.33) to render explicit the NGB dependence. The fields $\tilde{q}_a^n$ transform under the unbroken group $\mathrm{SO}(4)_D \sim SU(2)_L \times SU(2)_R$ in the representations

$$\tilde{q}_a^n: \quad \mathbf{1} \oplus \mathbf{9} \oplus \mathbf{6} \oplus \mathbf{4} = (\mathbf{1},\mathbf{1}) \oplus (\mathbf{3},\mathbf{3}) \oplus (\,(\mathbf{1},\mathbf{3}) \oplus (\mathbf{3},\mathbf{1})\,) \oplus (\mathbf{2},\mathbf{2})\,. \tag{B.8}$$

Its decomposition in terms of component fields is

$$\tilde{q}_a^n(x) = \left(\frac{f}{\sqrt{2}} + \frac{\eta(x)}{2}\right)\delta_a^n + \Delta^{A_L B_R}(x)(2T^{A_L}T^{B_R})_a^n + \frac{\tilde{q}_\rho^A(x)}{\sqrt{2}}(T^A)_a^n + i\frac{\tilde{q}_5^n(x)}{\sqrt{2}}\delta_{a5}, \tag{B.9}$$

where the singlet η and the symmetric traceless Δ are complex, while the antisymmetric $\tilde{q}_\rho$ and the fundamental $\tilde{q}_5$ are real fields. From eq.(B.4) we see that the only states which can contribute to the scattering are the singlet (η), the gauge bosons (ρ) and the symmetric (Δ). Since we are interested only in the tree-level contribution to the scattering amplitude, we can study them separately.

Let us start with the singlet $\eta = (\eta_1 + i\eta_2)/\sqrt{2}$. Setting Δ and ρ_μ to zero in eq.(B.7) one can arrive easily to the Lagrangian[1]

$$\begin{aligned} \mathcal{L} &\supset |\partial_\mu \eta|^2 + \frac{1}{2}\mathrm{Tr}\left[d_\mu d^\mu\right] \left|\mu + \frac{\eta}{2}\right|^2 = \\ &= \frac{1}{2}\left((\partial_\mu \eta_1)^2 + (\partial_\mu \eta_2)^2\right) + \frac{f^2}{4}\mathrm{Tr}\left[d_\mu d^\mu\right]\left(1 + \frac{\eta_1}{f} + \frac{\eta_1^2+\eta_2^2}{4f^2}\right) . \end{aligned} \tag{B.10}$$

In the parametrization of ref. [56] it is easy to recognize $a_{\eta_1} = \frac{1}{2}$, $a_{\eta_2} = 0$ and $b_{\eta_1} = b_{\eta_2} = \frac{1}{4}$. From this we obtain the contribution of the η to the hh scattering amplitude:

$$A_\eta(s,t,u) = -\frac{1}{4}\frac{s}{f^2}\frac{s}{s - M_\eta^2}, \tag{B.11}$$

where $M_\eta = \sqrt{2}hf$. Setting to zero the scalars Δ and η we can obtain the contribution from the vector ρ_μ. The Lagrangian can be written as

$$\mathcal{L} \supset \frac{f^2}{4}\mathrm{Tr}\left[d_\mu d^\mu\right] + \frac{f^2}{2}\mathrm{Tr}\left[(g_\rho \rho_\mu - E_\mu)^2\right], \tag{B.12}$$

recognizing that, in the notation of ref. [56], $a_\rho = 1$. The contribution to the scattering amplitude which grows with the energy is

$$A_\rho(s,t,u) = -\frac{3}{2}\frac{s}{f^2}. \tag{B.13}$$

The scalar $\Delta = (\Delta_1 + i\Delta_2)/\sqrt{2}$ is a complex field in the $(\mathbf{3},\mathbf{3})$ of SO(4). Its Lagrangian can be written as

$$\mathcal{L} = \sum_{i=1,2}\left\{\frac{1}{2}\mathrm{Tr}[(\nabla_\mu \Delta_i)^2] - \frac{M_{\Delta_i}^2}{2}\mathrm{Tr}[\Delta_i^2] + a_{\Delta_i} f \,\mathrm{Tr}[\Delta d_\mu d^\mu] + \ldots\right\}, \tag{B.14}$$

where, in components, $\Delta_i = \Delta_i^{A_L B_R}(x)(2T^{A_L}T^{B_R})_a^b$ and where the dots represent terms not relevant for WW scattering. In our case $a_{\Delta_1} = 1$, $a_{\Delta_2} = 0$ and $M_{\Delta_1} = M_{\Delta_2} = \sqrt{2}hf$. The scattering amplitude is given by

$$A_\Delta(s,t,u) = \frac{(a_{\Delta_1}^2 + a_{\Delta_2}^2)}{4}\left(\frac{s}{f^2}\frac{s}{s-M_\Delta^2} - 2\frac{t}{f^2}\frac{t}{t-M_\Delta^2} - 2\frac{u}{f^2}\frac{u}{u-M_\Delta^2}\right). \tag{B.15}$$

[1]Here we also used that $\delta_c^n (T^A T^B)_{cd} \delta_d^n = \delta^{AB}$, $\delta_c^n (T^{\hat a} T^{\hat b})_{cd} \delta_d^n = \delta^{\hat a \hat b}/2$ and $\delta_c^n (T^A T^{\hat b})_{cd} \delta_d^n = 0$, where T^A and $T^{\hat a}$ are, respectively, the unbroken and broken generators of SO(5) $\to$ SO(4).

For energies larger than the masses of all these resonances we have

$$A_{tot}(s,t,u) = A_{NGB}(s,t,u) + A_{\eta}(s,t,u) + A_{\rho}(s,t,u) + A_{\Delta}(s,t,u) \simeq \text{const.} \tag{B.16}$$

We see that, as expected, the exchange of heavy resonances restores unitarity before the scattering amplitude becomes non perturbative.

Appendix C

SO(N)

In this appendix we report our conventions for $\mathfrak{so}(N)$ algebra, the same as [1].

$$\mathfrak{so}(N) = Span\{t^{ab},\, a,b = 1,\dots,N,\, a<b\} \quad \text{where} \quad t^{ab}_{ij} = \frac{i}{2}[\delta^a_i\delta^b_j - \delta^b_i\delta^a_j]\,. \tag{C.1}$$

The matrices t^{ab} satisfy

$$\begin{aligned} \mathrm{Tr}[t^{ab}t^{cd}] &= \frac{1}{2}(\delta^{ac}\delta^{bd} - \delta^{ac}\delta^{bd}) \\ [t^{ab}, t^{cd}] &= \frac{i}{2}(\delta^{bc}\delta^{ae}\delta^{df} - \delta^{bd}\delta^{ae}\delta^{cf} - \delta^{ac}\delta^{be}\delta^{df} + \delta^{ad}\delta^{be}\delta^{cf})t^{ef}\,. \end{aligned} \tag{C.2}$$

With $N = 5$ it is customary to define different generators:

$$\begin{aligned} &T^a_{L,R} = -\frac{1}{2}\epsilon^{abc}t^{bc} \mp t^{a4} \\ &T^1_L = t^{32} + t^{41}, \quad T^2_L = t^{13} + t^{42}, \quad T^3_L = t^{21} + t^{43}, \\ &T^1_R = t^{32} + t^{14}, \quad T^2_R = t^{13} + t^{24}, \quad T^3_R = t^{21} + t^{34}, \\ &T^{\hat{a}} = \sqrt{2}\,t^{a5}, \qquad \hat{a} = 1,2,3,4\,. \end{aligned}$$

It is straightforward to check that

$$\begin{aligned} \mathrm{Tr}[T^i_A, T^j_B] &= \delta^{ij}\delta_{AB}\,, \quad \mathrm{Tr}[T^{\hat{a}}T^{\hat{b}}] = \delta^{\hat{a}\hat{b}} \\ [T^i_A, T^j_B] &= i\epsilon^{ijk}T^k_A\delta_{AB} \\ [T^{\hat{a}}, T^{\hat{b}}] &= -it^{\hat{a}\hat{b}} = i\epsilon^{\hat{a}\hat{b}\hat{c}}\left(T^{\hat{c}}_L + T^{\hat{c}}_R\right) + i\delta^{\hat{b}4}(T^{\hat{a}}_L - T^{\hat{a}}_R) - i\delta^{\hat{a}4}(T^{\hat{b}}_L - T^{\hat{b}}_R) \end{aligned} \tag{C.3}$$

where $A, B = L, R$. In this basis, $T^{1,2,3}_L$ generate $\mathrm{SU}(2)_L$ and $T^{1,2,3}_R$ generate $\mathrm{SU}(2)_R$ of the $\mathrm{SO}(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ local isomorphism. The matrices

$T^{\hat{1},\hat{2},\hat{3},\hat{4}}$ generate the coset SO(5)/SO(4). The last line makes explicit that this coset is symmetric: the commutator among broken generators does not have components along broken generators.

The SO(6) generators are taken to be, for $N = 6$,

$$\begin{aligned} T^1 &= t^{32} + t^{14},\ T^2 = t^{31} + t^{42},\ T^3 = t^{12} + t^{43},\ T^4 = t^{16} + t^{52}, \\ T^5 &= t^{51} + t^{62},\ T^6 = t^{36} + t^{54},\ T^7 = t^{53} + t^{64},\ T^8 = \frac{1}{\sqrt{3}}(t^{12} + t^{34} + 2t^{65}), \\ T^9 &= t^{36} + t^{54},\ T^{10} = t^{14} + t^{23},\ T^{11} = t^{24} + t^{31},\ T^{12} = t^{16} + t^{25}, \\ T^{13} &= t^{36} + t^{45},\ T^{14} = t^{46} + t^{53},\ T^{15} = \sqrt{\frac{2}{3}}(t^{12} + t^{34} + t^{56}) \,. \end{aligned} \tag{C.4}$$

In this basis, $T^{1,\ldots,8}$ generate $\mathrm{SU}(3)_c$. The $\mathrm{U}(1)_X$ generator is given by $(4/\sqrt{6})\,T^{15}$, so that the fields ξ_L and ξ_R in section 4.2 have $\mathrm{U}(1)_X$ charges 2/3 and −2/3, respectively.

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