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Electroweak Baryogenesis

in the Adiabatic Limit

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If we understand things, things are as they are,

if we do not understand things, things are as they are

Ringraziamenti

Avevamo fatto molto tardi quella notte ad Amburgo e ce ne stavamo tornando nella tana camminando silenziosi (la voglia di favellare me l'aveva tolta un Auf-lauf, una bomba ipercalorica di formaggio fuso e Diosasolocos'altro che, con un errore madornale, avevo ordinato a cena seguendo un becero consiglio).

Quand'ecco che il silenzio si spezza: "Vedete ragazzi, io sto cercando di tirare i fili... ....ed i fili, come e' noto, si spezzano".

Sguardi attoniti. Un attimo di smarrimento. Denis aveva parlato.

Impossibile, senza conoscerlo, apprezzare la saggezza di queste sentenze, di queste parole interrotte da interminabili pause ad effetto ed accompagnate da sguardi fissi nei tuoi occhi, quasi a chiedere un tacito assenso-consenso, tanto neanche la piu' ferrea delle volonta' potrebbe scalfire quell'uomo granitico.

Tutte tranne una....."Scon' ti', Denis, ma che cazzo dici?"

Sanguigno, diretto, e con una onesta' interiore senza pari, il Max non ha bisogno di mezze parole. La qual cosa va apprezzata e ti deve suonare quasi come una nota di merito poiche' si permette questo comportamento solo con gli amici, quelli veri.

Denis e Max sono stati per tutti questi anni di Dottorato i miei compagni di lavoro, di divertimento, di scazzatura, di impennate di felicita', di litigi.

Sono l'esempio vivente del fatto che, per lavorare bene con una persona, devi essere in sintonia con essa, ne devi condividere le gioie e le paure, senza avere vergogna di versare delle lacrime amare quando, montando su un taxi che ti porta all'Aereoporto, la saluti e lei ti guarda con occhi quasi disperati e pieni di solitudine.

Sono amici con cui condividere, oltre che discorsi sulle cosmocazzate, un letto ad una piazza e mezza a Parigi od un materassone sotto una mansarda triestina dopo essersi fatti fuori un Kg di risotto ai funghi e due dozzine di panzarotti alla marmellata preparati dalla mitica mamma Comelli. Amici con cui tentare, inutilmente, di entrare in un appartamento all'ultimo piano di uno stabile amburghese e starsene li', seduti sugli scalini, a domandarsi del perche' la chiave non entra nella toppa, fino alla balenante illuminazione che forse ci si era sbagliati di numero e che quella, forse, non era proprio la casa del Max.

Senza di loro questa Tesi non avrebbe potuto essere. Indubbiamente, i piu' diranno che cio' sarebbe stato molto meglio, io replico che pur di godere della loro compagnia di Tesi cosi ne scriverei anche mille.

Quella notte pioveva pure. Lungo tutto il tragitto fino alla tana, i due avevano continuato a discorrere, sotto i fumi della birra, di massimi sistemi e di fili da tirare. Io li seguivo in silenzio, le loro voci come sottofondo ai miei pensieri. Pensavo a Lucia. E' a lei che dedico non questa Tesi, che' non saprebbe che farsene, ma l'intera mia vita.

Introduction

The possibility of baryogenesis at the electroweak scale is a very popular but controversial topic. Despite the large number of related publications none of the key aspects of this subject can be considered to be on firm ground. First, it is well known that the requirement that the anomalous sphaleronic processes which violate baryon number B go out of equilibrium soon after the transition translates into a lower bound on the ratio between the vacuum expectation value of the Higgs field, $v(T)$, and the critical temperature of the transition, T_c, $v(T_c)/T_c \gtrsim 1$. In the Standard Model, improved perturbative evaluations of this ratio give a value which is badly less than unity for values of the mass of the Higgs scalar compatible with LEP results. On the other hand, the perturbative expansion cannot be trusted any more for values of the Higgs masses of the order of the $W^{\pm}$ boson mass or larger. Clearly, much work is still needed on this issue.

Another aspect of the problem is CP violation. This has been the subject of a recent debate in the literature about the need of further complex phases in the theory besides the one in the Cabibbo-Kobayashi-Maskawa mixing matrix. In models with more than one Higgs doublet, like the Minimal Supersymmetric Standard Model, further sources of CP violation can emerge naturally from the Higgs sector. In particular, the possibility of a spontaneous CP violation at finite temperature has been emphasized. This effect could give enough contribution for the baryogenesis and at the same time satisfy the upper bounds on CP violation coming from the electric dipole moment of the neutron.

Even assuming that the phase transition is strong and that CP violation is enough, we must face the other key issue, namely what is the mechanism responsible for the generation of baryons during the phase transition.

The most significant departure from thermodynamic equilibrium takes place at the passage of the walls of the expanding bubbles which convert the unbroken into the broken phase. According to the size and speed of the bubble walls, two different mechanisms are thought to be dominant. In the case of *thin* (width $\sim 1/T$) walls, typical of a very strong phase transition, the creation of baryons occurs via the asymmetric (in baryon number) reflection of quarks from the bubble wall, which biases the sphaleronic transitions in the region in front of the expanding bubble.

If the walls are *thick* (width $\sim (1-100)/T$) then the relevant mechanism takes place inside the bubble walls rather than in front of them. In this case we can make a distinction between fast processes (mediated by gauge, flavour diagonal, interactions and by top Yukawa interactions) and slow processes (mediated by Cabibbo suppressed gauge interactions and by light quarks Yukawa interactions). The former are able to follow adiabatically the changing of the Higgs VEV inside the bubble wall, while, in first approximation, the latter are frozen during the passage of the wall.

In the *adiabatic scenario*, as originally proposed by Cohen, Kaplan and Nelson (CKN), if CP violation, explicit or spontaneous, is present in the scalar sector then a space-time dependent phase for the Higgs VEVs is turned on inside the wall. The time derivative of this phase couples with the density of a quantum number non orthogonal to baryon number and then can be seen as an effective chemical potential, named *charge potential*, which has the effect of biasing the rates of the sphaleronic processes, creating an asymmetry proportional to $\dot{\theta}$, where θ is the phase of the VEVs.

This mechanism was shown to be successful in the framework of the Minimal Supersymmetric Standard Model where CP violation is spontaneous and occurs only at finite temperature.

This adiabatic scenario was reconsidered by different authors in different but related aspects.

First, Giudice and Shaposhnikov have shown the dramatic effect of non perturbative, chirality breaking, transitions induced by the so called QCD sphalerons. If these processes were active inside the bubble walls, then the equilibrium value for baryon number in the adiabatic approximation would be proportional to that for the conserved quantum number $B-L$ (L is the lepton number), up to mass effects suppressed by $\sim (M_{top}(T)/\pi T)^2$. Then, imposing the constraint $\langle B-L\rangle = 0$, where $\langle\cdots\rangle$ represents the thermal average, one would obtain zero baryon number (up to mass effects).

Dine and Thomas have considered the two Higgs doublets model in which the same doublet couples both to up and down quarks, the same model considered in the original work by Cohen Kaplan and Nelson. These authors have pointed out that $\dot{\theta}$ couples also to the Higgs density, so that the induced charge potential is for total hypercharge rather than for fermion hypercharge, as originally supposed by CKN. As long as effects proportional to the temperature dependent $v(T)$ are neglected, hypercharge is a exactly conserved quantum number and then, again imposing the constraint that all the conserved charges have zero thermal averages, no baryon asymmetry can be generated. So, one would find a $M_{top}(T)^2/T^2$ suppression factor.

Finally, Joyce, Prokopec and Turok (JPT) emphasized the very important point that the response of the plasma to the charge potential induced by $\dot{\vartheta}$ is not simply that of a system of fixed charges, because transport phenomena may play a crucial role. When a space time dependent charge potential is turned on at a certain point, hypercharged particles are displaced from the surrounding regions, so that even the thermal averages of conserved quantum numbers become locally non vanishing. As a consequence, the equilibrium properties of the system have to be reconsidered taking into account the local violation of the conserved quantum numbers.

Analyzing the adiabatic scenario through the linear response theory allows us to take transport effects into account. Assuming that a spacetime dependent charge potential is generated inside the bubble wall we can investigate its effect on the thermal averages of the various quantum numbers of the system. One finds that transport phenomena are really crucial, but in disagreement with JPT's early conclusion that as a consequence of the local violation of global quantum numbers there is no biasing of the sphaleronic processes. Actually, in the adiabatic approximation the local equilibrium configuration of the system is determined by the thermal averages of the charges conserved by all the fast interactions. The effect of transport phenomena is to induce space time dependent non zero values for these averages. We calculate these averages using linear response theory and then determine the local equilibrium configuration, showing that it corresponds to $\langle B + L \rangle \neq 0$.

The inclusion of transport phenomena also sheds a new light on the strong sphaleron effects and on the effect of a charge potential for total rather than fermionic hypercharge. The dramatic suppressions found by Giudice and Shaposhnikov and by Dine and Thomas respectively, are both a consequence of taking zero averages for conserved quantum numbers. Since these averages are no more locally zero we will find a non zero $\langle B + L \rangle \neq 0$, even in the case in which the charge potential is for total rather than for fermion hypercharge. In the case of QCD sphalerons we will find that the final result depends in a crucial way on the form of the charge potential which is considered.

Very recently, the original treatment by CKN which makes use of the charge potential has been criticized on very general grounds. Using a *general field theory approach* it was shown that fermionic currents in a CP violating and space-time dependent Higgs background arise only at one loop, so that a suppression factor $\mathcal{O}(h_t v/\pi T)^2$ with respect to *all* the previous computations was found. These results have been recently confirmed by Huet and Nelson with an independent semiclassical approach.

Contents

Chapter 1

Overture

1.1 Allegro

In the standard hot big bang model, relics from the early Universe can give us much information about microphysics. For instance, the abundances of the light elements, produced when the Universe was at a temperature of ~ 1 MeV, told us before the existence of LEP that there were at most four, and probably three, species of light neutrinos [1].

The most obvious of big bang relics are baryons, which make our own existence possible. Furthermore, the Universe seems to contain relatively few antibaryons. There is a clear evidence that at least the local cluster of galaxies is made of matter, and there is no plausible mechanism to separate matter from antimatter on such large scales. The observed abundance of baryons today implies that when the Universe was much hotter than a GeV the asymmetry between baryons and antibaryons have been about one part in 10^{10} [1].

The quantity we wish to explain is n_B/s–the ratio of the baryon density of the Universe to the entropy density–observed to be equal to [1]

$$B \equiv \frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s} = \simeq (0.8 \pm 0.2) \times 10^{-10}, \tag{1.1}$$

where $s = (2\,\pi^2\, g_*/45)T^3$, g_* being the effective number of relativistic degrees of freedom present at the temperature T, is the entropy density.

Approximately thirty years ago Sakharov [2] pointed out that this cosmological asymmetry could be a calculable result of particle interactions in the early Universe, teaching us several profound things about fundamental physics:

i) violation of the baryon number: this requirement is obvious since n_B/s is non-vanishing;

ii) C (charge conjugation symmetry) and CP (the product of charge conjugation and parity) are not exact symmetries: were C an exact symmetry, the probability of the process $i \rightarrow f$ would be equal to the one of the process $\bar{i} \rightarrow \bar{f}$. Since the baryon number of f is equal and opposite to that of $\bar{f}$, the net B would be vanishing. Furthermore, because of the CPT theorem, CP invariance is equivalent to T invariance (time reversal). The latter assures that the rate of the process $i(\mathbf{r}_i, \mathbf{p}_i, \mathbf{s}_i) \rightarrow f(\mathbf{r}_j, \mathbf{p}_j, \mathbf{s}_j)$ and that of the time-reversal process $f(\mathbf{r}_j, -\mathbf{p}_j, -\mathbf{s}_j) \rightarrow i(\mathbf{r}_i, -\mathbf{p}_i, -\mathbf{s}_i)$ are equal. Thus, even if it is possible to create a baryon asymmetry in a certain region of the phase space, when integrating over momenta and summing over spins, the net B would zero if CP were conserved;

iii) the Universe must have been out of equilibrium in order to produce a net baryon number aymmetry: the equilibrium thermodynamical distribution of a particle species is determined only by its energy E and by its chemical potential μ

$$n^{\rm EQ}(E,\mu) = \frac{1}{{\rm e}^{(E-\mu)/T} \pm 1}. \tag{1.2}$$

Since, according to the CPT theorem, the particle mass is equal to antiparticle mass and the chemical potential μ is vanishing at the thermodynamical equilibrium, we get

$$n_b = n_{\bar{b}} = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\, n^{\rm EQ}. \tag{1.3}$$

This tells us that $B = 0$ at equilibrium.

1.2 Allegro, ma non troppo

Sakharov himself suggested that *baryogenesis* took place immediately after the big bang, at a temperature not far below the Planck scale of 10^{19} GeV, when the Universe was expanding so rapidly that many processes were out of equilibrium.

The first models for baryogenesis satisfied the Sakharov's criteria in Grand Unified (GUT) theories [3], exploiting the fact that the GUT scale was not far from the Planck scale [4]. Thus baryon violation could exist without being in conflict with the proton lifetime, and departure from equilibrium could easily result due to the rapid expansion of the Universe during the GUT epoch. All such theories had to involve new sources of CP violation, as Kobayashi-Maskawa (CM) CP violation alone proved to be too small to explain the observed baryon asymmetry.

Two subsequently discovered effects threaten the viability of the GUT scale baryogenesis. The first is inflation, which serves to wash out GUT scale monopoles, but also washes out any baryon asymmetry in the Universe (BAU) produced prior to inflation. GUT scale baryogenesis to occur after inflation needs a high reheating temperature, which requires a strongly coupled inflaton, which in turn tends to give large perturbations inconsistent with structure formation. Moreover, in supersymmetric [5] GUTs, the gravitino (the supersymmetric partner of the graviton) decays may affect nucleosynthesis [6] unless the reheating temperature is below $\sim 10^9$ GeV, a temperature too cool to reinstate the baryon asymmetry through GUT processes.

A second difficulty is that in the standard model (SM) of electroweak interactions with gauge group $SU(2)_L \otimes U(1)_Y$ baryon number is known theoretically to be anomalous and not exactly conserved [7]. This anomalous baryon number violation acts on the left-handed fermionic doublets in processes like

$$t_L t_L b_L \tau_L \leftrightarrow 0, \quad t_L b_L b_L \nu_\tau \leftrightarrow 0.$$

Indeed, due to the presence of axial couplings, baryon and lepton number are not conserved. The baryon number current satisfies

$$\partial_\mu J^\mu_B = \frac{g^2\, N_f}{16\, \pi^2} \, \mathrm{tr}\left(F\tilde{F}\right), \tag{1.4}$$

where N_f is the number of flavours. Integrating Eq. (1.4) and discarding the surface terms of the baryonic current gives a change in the baryon number

$$\begin{aligned} \Delta B &= \frac{g^2\, N_f}{16\, \pi^2} \int d^4x \, \mathrm{tr}\left(F\tilde{F}\right) \\ &= N_f \int d^4x \, \partial_\mu K^\mu \end{aligned}$$

$$= N_f \oint d\sigma_\mu K^\mu, \tag{1.5}$$

where the last surface integral is taken over a large three-sphere S_3, of infinite radius, and the Cherns-Simons current is given by

$$K^\mu = \frac{g^2}{8\,\pi^2}\,\epsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\,\left[A_\nu\left(\partial_\alpha A_\beta - \frac{2}{3} i g A_\alpha A_\beta\right)\right]. \tag{1.6}$$

For finite-action gauge field configurations, $F = 0$ at spatial infinity and working in the static $A^0 = 0$ static gauge, in which the only non zero component of the Cherns-Simons current is K^0, we get

$$\Delta B = N_f\,\Delta N_{\mathrm{CS}}, \tag{1.7}$$

where the Chern-Simons number of the field configuration $\mathbf{A}(x)$ is defined as

$$N_{\mathrm{CS}} = \frac{i\,g^2}{24\,\pi^2}\int d^3\mathbf{x}\,\epsilon^{ijk}\,\mathrm{tr}\left(A^i\,A^j\,A^k\right). \tag{1.8}$$

If a generic element of the group $\mathcal{G}$ are parametrized as $u(x) = \exp\left[ig\alpha(x)\cdot\mathbf{t}\right]$, a generic transformation on the gauge fields is given by $A_\mu \to uA_\mu u^{-1} - i\partial_\mu u u^{-1}/g$.

Vacuum configurations $\mathbf{A}_{\mathrm{vac}} = -i\nabla u u^{-1}/g$ define a natural map $S_3 \to \mathcal{G}$ when restricted to $\alpha(\mathbf{x}) \to 0$ as $|\mathbf{x}| \to \infty$. With this restriction, points in space can be thought of as lying on a three-sphere, and the induced vacuum map is simply $\mathbf{x} \mapsto u(\mathbf{x}) \in \mathcal{G}$.

The Cherns-Simons number of these configurations is just the homotopy or winding number of this induced map. For semi-simple groups, such vacuum configurations can then be labelled by an integer and the true vacuum state is a linear superposition of the corresponding perturbative wave functionals, which each have support only over a definite winding number.

Note that, if $\mathcal{G} = U(1)$, any change in the Cherns-Simons number is vanishing, which is not the case for an abelian group $SU(2)$.

At low temperature baryon number violation only proceeds via an exponentially suppressed tunneling process, at a rate proportional to $\exp(-4\pi/\alpha_W)$, which is to say, it never happens. This low number can be understood in terms of the very large potential barrier, of height $\sim M_W/\alpha_W \sim 10$ TeV, separating the perturbative vacua of definite winding number [7].

At temperature above $\simeq 100$ GeV, however, electroweak baryon number violation is expected to proceed rapidly enough to equilibrate to zero any baryons produced by GUT baryogenesis [8, 9].

In the last decade there has been much work proving that this is the case [10]. However, anomalous electroweak processes conserve $B-L$, the difference between baryon and lepton numbers, and so a net $B-L$ generated at the GUT or other interactions will not be erased by electroweak interactions. *New and improved* GUT scale baryogenesis models must now posses and effective $B-L$ symmetry violated at high energies, and the baryogenesis must typically involve a scalar field that can store up $B-L$ during inflation, only realising it after inflation is over. Such a field can be the inflaton itself, or something like a squark or slepton field in supersymmetric (SUSY) models [11]. Unfortunately, the only new observable experimental consequences from such elaborate constructs are B and L violation, and even then the rates are very model dependent and can be adjusted to be out of reach. The additional CP violation cannot be observed directly. In fact, due to inflation, one can account for a baryon asymmetry within our horizon while having no net baryon number for the Universe as a whole, thus eliminating the need of CP violation.

1.3 Adagio

In 1985 Kuzmin, Rubakov and Shaposhnikov [10] suggested an elegant and simple solution to the baryogenesis problem. They argued that anomalous baryon number violation in the standard model of electroweak interactions is rapid at high temperatures and that the weak phase transition [12], if it is of first order with supercooling, provides a natural way for the Universe to get out of thermal equilibrium at weak scale temperatures.

Critical bubbles of the broken Higgs phase H eventually nucleate with a typical profile

$$H(r) = \frac{v(T)}{2}\left[1+\tanh\left(\frac{r}{L_w}\right)\right], \tag{1.9}$$

where r is the spatial coordinate, L_w is the bubble wall width and $v(T)$ is the vacuum expectation value (VEV) of the Higgs field at the critical temperature T_c. Bubbles expand until they fill the Universe; local departure from thermal equilibrium takes place in the vicinity of the expanding bubble walls. Since C and CP are known to be violated by the electroweak interactions, it is possible to satisfy all the Sakharov's baryogenesis conditions.

One bottleneck for electroweak baryogenesis (EWB) proves to be *i)* the need for departure from thermal equilibrium, which can only occur at the electroweak epoch only if there is a sufficiently strong first order phase transition; *ii)* the inadequate CP violation from the Kobayashi-Maskawa mechanism (as found in the original GUT baryogenesis models). Let us discuss these problems in more details.

1.3.1 A strong first order phase transition?

The standard picture of EWB assumes that $SU(2)_L \otimes U(1)_Y$ breaks via a first order phase transition, leading to bubble nucleation and a separation of phases. As said above, baryon number is expected to be badly violated at high temperatures. In ref. [9], it was shown that there exist static, unstable solutions to the field equations with one negative mode. These solutions are called *sphalerons* and represent saddle points of the potential energy functional in field space labelling the vacua with different baryon numbers.

This interpretation is further justified since it has a Cherns-Simons number half away between that of the successive perturbative vacua flanking the sphaleron.

It is quite probable that at very high temperatures, gauge field configurations can simply sail over the barrier separating two distinct vacua with different baryon number rather than tunnel through it, and then baryon number violation becomes unsuppressed. Indeed, at high temperatures, the system is well described by classical statistical mechanics.

In the broken phase around the electroweak scale the rate for barrier penetration is essentially the Boltzmann factor associated with forming a sphaleron and the rate per univ volume is

$$\Gamma_{\rm brok} \simeq \frac{M_W^7}{\alpha_W^3 T^3}\, {\rm e}^{-E_{\rm sp}/T}, \tag{1.10}$$

where $E_{\rm sp} \simeq (2\, M_W/\alpha_W)$ is the sphaleron energy (see Arnold and Mc Lerran in [10]).

In the symmetric phase, the situation is equivalent to a three dimensional field theory with no small dimensionless parameter. While no single classical configuration dominates the rate, a heuristic description in terms of instanton can be given. It is generally believed that the three dimensional field theory has a mass gap $\sim \alpha_W\, T$. Correspondingly, the correlation length of the high temperature theory (the so-called magnetic screening length) is $\xi = (\alpha_W\, T)^{-1}$. Consider now instantons at high temperatures. These will exist with typical size $(\alpha_W\, T)^{-1}$. This means that the number density of sphalerons per unit volume in a *sphaleron saturated* gas is around $(\alpha_W\, T)^3$. Since they decay with a rate $\sim \alpha_W\, T$, the rate per unit volume is estimated to be

$$\Gamma_{\rm sym} \simeq \kappa\, (\alpha_W T)^4, \tag{1.11}$$

where κ is a dimensionless constant evaluated by Ambjorn *et al.* in ref. [10] to be in the range $0.1 \lesssim \kappa \lesssim 1$).

In order to produce a baryon asymmetry, it is necessary that as the bubble of broken phase nucleates and expands, particle interactions with the bubble wall somehow produce a baryon excess in the region where B violation is rapid. However, this baryon asymmetry will be subsequently destroyed, unless B violation rate in the broken phase be extremely small. That is, the phase transition must be sufficiently strong so that sphalerons are heavy and play no role inside the bubbles. This suppression of the anomalous baryon number after the transition requires a large jump in the Higgs vacuum expectation value $v(T)$ during the transition [13]

$$\frac{v(T)}{T} \gtrsim 1. \tag{1.12}$$

The crucial question now is whether such a bound can be achieved in the SM.

Old common wisdom: andante: There has been extensive work on the nature of the electroweak phase transition (EWPT) [14], and there is general agreement that for the SM with one Higgs doublet, the transition is strongly first order in the limit $M_H \ll M_W$ while being second order in the opposite limit $M_H \gg M_W$. Perturbative calculations break down at $M_H \simeq M_W$. The maximum value for the Higgs mass where baryon asymmetry produced during the phase transition is not subsequently destroyed in the broken phase was thought to be about 45 GeV, which is experimentally excluded. It was also thought that extended Higgs sectors, such as the singlet majoron model [15] are still viable candidates for a sufficiently strong first order phase transition. A notable exception is the minimal supersymmetric standard model (MSSM), which is viable only if there is a relatively light squark [16].

New common wisdom: andante con brio: There have been recent advances in studies of the electroweak phase transition, expecially in lattice simulations of the electroweak phase transition (see [17] for a general discussion).

These simulations tend to suggest that nonperturbative effects are important for physically interesting Higgs masses, and that the phase transition tends to be more strongly first order than found in perturbation theory. For instance, one simulation found that for a Higgs mass of 80 GeV, $v(T)/T = 0.68$, as opposed to 0.3 from perturbation theory. Thus the upper bound of 45 GeV on the Higgs mass for EWB is probably too conservative, although we have not seen yet any definitive result replacing it.

1.3.2 Sufficient CP violation?

Old common wisdom: andante adagio: Since we see CP violation in the kaon system, it is of great interest to know whether the same CP violation is what gave rise to matter in the Universe.

A very rough (and optimistic) estimate of the amount of CP violation necessary to get $B \simeq 10^{-10}$ can be obtained as follows: since the baryon number violation rate in the symmetric phase is proportional to $\alpha_W^4 \simeq 10^{-6}$, if we indicate with δ_{CP} the suppression factor due to CP violation, we get

$$B \simeq \frac{\alpha_W^4 T^3}{s}\delta_{CP} \simeq 10^{-8}\delta_{CP}. \tag{1.13}$$

Neglecting all the suppression factors coming from the dynamics involved in EWB, we discover that

$$\delta_{CP} \gtrsim 10^{-3}. \tag{1.14}$$

Another naive estimate suggests that since CP violation would vanish in SM if any two quarks of the same charge had the same mass, the measure for CP violation should be the Jarlskog invariant

$$\begin{aligned} \frac{A_{CP}}{J} = & \left(M_t^2 - M_c^2\right)\left(M_c^2 - M_u^2\right)\left(M_u^2 - M_t^2\right) \\ & \left(M_b^2 - M_s^2\right)\left(M_s^2 - M_d^2\right)\left(M_d^2 - M_b^2\right), \end{aligned} \tag{1.15}$$

where J is twice the area of the unitarity triangle given, in the Kobayashi-Maskawa parametrization, by

$$J = \sin^2\theta_1 \, \sin\theta_2 \, \sin\theta_3 \, \cos\theta_1 \, \cos\theta_2 \, \cos\theta_3 \, \sin\delta. \tag{1.16}$$

The quantity A_{CP} has mass dimension equal to twelve. In the limit of high temperature, $T \gg M$, the only mass scale is the temperature T. Therefore, the dimensionless quantity δ_{CP} measuring the violation of CP is given by

$$\delta_{CP} \simeq \frac{A_{CP}}{T_c^{12}} \simeq 10^{-20}, \tag{1.17}$$

where $T_c \simeq 100$ GeV is the temperature at the onset of the electroweak phase transition. Obviously, these estimates are very rough and a definitive answer can only be given when studying particular models for EWB.

Contemporary common wisdom: veloce allegro: Recently Farrar and Shaposhnikov [18] challenged the above reasoning, which is admittedly naive. Their key point is the following: for quarks having three momentum $|\mathbf{p}| \simeq T \gg M_q$, the above estimate (1.17) is correct since light quarks are effectively degenerate in mass and the GIM suppression is operative; on the other side, this is no longer true when considering quarks having three momentum $|\mathbf{p}| \sim M_q$. Since the mass jump through the bubble wall is just M_q, quarks coming from the symmetric phase and with momentum $|\mathbf{p}| < M_q$ are reflected off from the wall, whereas the ones with momentum $|\mathbf{p}| \gtrsim M_q$ are partially reflected off and partially transmitted. In the reflection processes CP is violated because of the interference between the scattering phases and the CM phase and quarks and antiquarks acquire different probabilities of penetrating the bubble wall. In such a way a net baryon number flux from outside to inside the bubble wall is obtained.

If one considers momenta between the down quark mass M_d and the strange quark mass M_s (corresponding only to the fraction $(M_s - M_d)/T$ of the total number of quarks in thermal equilibrium), then all strange quarks are reflected off, whereas down quarks have a nonvanishing probability of being transmitted. In such a situation it is clear that the above naive estimate of considering the light quarks degenerate in mass is no longer trustable. Taking into account this fact, Farrar and Shaposhnikov got a suppression factor dependent upon the quark momenta: it is always of the order of (1.17) except in the interval $M_d < |\mathbf{p}| < M_s$, where it is much larger than (1.17). For some values of the velocity v_w and width L_w of the bubble wall (which must be extremely thin to make the reflection very efficient), such a mechanism seemed to generate a baryon asymmetry in the right amount.

Modern common wisdom: adagio lentissimo: The results of ref. [18] have been subsequently contradicted in refs. [19, 20]. The new effect, not taken into account by Farrar and Shaposhnikov and responsible for the final disagreement, is the loss of quantistic coherence of the quarks when scattering off the thermal gluons.

It is well known that fermionic dispersion relations are modified in a plasma [21]. Fermionic excitations appear, called quasi-particles, and acquire a thermal mass $\sim g_s T^2$. This was accounted for by Farrar and Shaposhnikov. However, quasi-particles during their propagation scatter with the thermal medium and this phenomenon induces a damping rate γ proportional to the imaginary part of the self-energy, which was not taken into account by Farrar and Shaposhnikov. The energy of quasi-particles is large compared to γ and it is then possible to speak about coherent excited states.

Quasi-particles have a lifetime $\sim (1/2\gamma)$ so that the quantum spatial coherence is lost after a distance of order of $(1/2\gamma)$. Since for quarks $\gamma \simeq (0.15\, g_s\, T)^{-1} \gg M_s$, the quark energy and momenta are not defined exactly, but have a spread of order of 2γ (analogously to resonance) much larger than the crucial range $(M_s - M_d)$ in which Farrar and Shaposhnikov mechanism is operative. In other words, the lifetime of the quantum packet is much shorter than the typical reflection time from the bubble wall

($\sim 1/M_s$): CP violation, which is based on coherence for (at least) a time ($\sim 1/M_s$), cannot be efficient.

Personally, we find the later work convincing, and believe that new CP violation must be added to any theory of baryogenesis. As mentioned above, typical CP violating angles have to be 10^{-3} or greater, which usually has experimental implications.

1.3.3 Transport

After having shortly discussed the issues of the strength of the electroweak phase transition and of CP violation, we are now ready to (try to) answer the fundamental question: how does the baryon asymmetry actually come from?

Assembling the ingredients is not enough to make a Universe, we still need a recipe. The recipe involves four distinct time scales governing the relevant interactions. These are the expansion time scale $\tau_{\rm U}$ given by the inverse of the Hubble parameter; thermalization scale $\tau_{\rm T}$, which characterizes how fast particles in the cosmic plasma equilibrate; the Higgs time scale $\tau_{\rm H}$, which governs the departure from thermal equilibrium as measured from a comoving observer while the expanding bubble wall passes through her; and the sphaleron time $\tau_{\rm sp} \simeq (\alpha_W^4\, T)^{-1}$ which governs the rate of baryon number violation in the symmetric phase.

Thermalization rate due to weak and strong interactions, defined for the different particles as the inverse of the mean free path $l_{\rm T}$, is much faster than $\tau_{\rm sp}^{-1}$. It is estimated [84] from strong or weak Coulomb scattering cross sections

$$\tau_{\rm T}^{-1} \simeq \begin{cases} 0.25\, T & q \\ 0.08\, T & W^{\pm},\, Z^0,\, l. \end{cases} \tag{1.18}$$

Yukawa interactions give rise to chirality changing processes ($q_L + g \leftrightarrow q_L + g$). For the top, the inverse rate can be estimated

$$\tau_{\rm T}^{\rm top} \simeq (\alpha_S\, h_t^2\, T)^{-1} \simeq \frac{30}{T}, \tag{1.19}$$

whereas for lighter quarks the above quantity must be rescaled by the factor $(h_q/h_t)^2$, h_q being the generic Yukawa coupling.

Thus, some rates correspond to fast interactions; some other standard processes are much slower, such as chemical equilibration between the first and the third family, or between the two chiralities of a light fermion.

The Higgs time scale is less easily determined (see, for instance, ref. [23] and references therein for a recent estimate of v_w in SM) and is quite model dependent, roughly given by

$$\tau_{\rm H}^{-1} = \frac{\dot{H}}{H} \simeq \frac{v_w}{L_w} \simeq (0.01 - 1)\, T, \tag{1.20}$$

where H indicates the generic Higgs field.

In general v_w depends on the bubble wall shape and, more in particular, on its width. Intuitively speaking, the motion of the bubble wall is determined by two factors, namely the pressure difference between inside and outside the bubble itself (which allows the expansion) and a friction force, proportional to v_w and due to the collisions of the plasma particles off the wall.

The equilibrium between these two forces should imply a steady state with a final velocity $v_w \sim 0.1$. However, if bubble walls are rather thick and larger than the typical particle free mean paths $\tau_{\rm T}$, thermodynamical conditions are established inside the wall and for the latter it is no longer possible to loose its energy by thermal dissipation (in other words, the friction force disappears and the total force acting on the wall is velocity independent). Under these conditions the bubble wall is accelerated until the approximation $\tau_{\rm T} \ll \tau_{\rm H}$ is no longer valid and the friction force is restablished.

Despite of these uncertainties, we see that baryon number violation is always out of equilibrium near the wall, since $\tau_{\rm sp} \gg \tau_{\rm H}$. However, other particle interactions may or may not be able to equilibrate near the bubble wall, depending on the relative size $\tau_{\rm H}$ and $\tau_{\rm T}$. This gives rise to two different regimes:

The adiabatic regime: $\tau_{\rm T}, \tau_{\rm T}^{\rm top} \ll \tau_{\rm H} \ll \tau_{\rm sp}$. Fast interactions mantain thermal equilibrium as the bubble wall passes by and the value of the Higgs field changes. This allows us to describe the plasma within the bubble wall in terms of equilibrium thermodynamics with quasistatic chemical potentials for quantities that equilibrate slowly compared to $\tau_{\rm H}$. In this adiabatic regime baryogenesis occurs within the bubble wall itself and the mechanism is called *spontaneous baryogenesis* (SB).

The nonadiabatic regime: $\tau_{\rm T}, \tau_{\rm T}^{\rm top}, \tau_{\rm sp} \ll \tau_{\rm U}$. The wall is thin compared to the free mean path, and particles reflect off the oncoming wall with calculable CP violating reflection coefficients. Baryogenesis occurs in an extended region preceding the phase boundary. The method for computing the flux of weak doublets reflecting from the bubble walls where anomalous baryon violation is occuring rapidly, as presented in refs. [25, 26, 27] for both the majoron model and the two Higgs model with a thin wall and top quark reflection. Transport properties were considered in [26], where a Monte Carlo calculation showed a significant *snowplot* effect: left-handed top quarks were pushed along in a region ranging from 20 to 100 thermal units in front of the wall. Such a model was shown to easily accomodate the desidered baryon asymmetry with a CP violating phase of $(10^{-2} - 10^{-3})$.

1.4 Overview

While the thin wall scenario has remained (approximately) unchanged, there has been a lot of work done recently for EWB with thick bubble walls.

The challenge of this Thesis is to provide a review of the recent progress in SB, which will turn out to be (for sure) not exhaustive.

SB was supposed to occur quasi-statically within the bubble wall and was first proposed by Cohen, Kaplan and Nelson (CKN) in ref. [28].

They considered the two Higgs doublet model in which up and down quarks couple to the same Higgs doublet. If CP violation, explicit or spontaneous, is present in the scalar sector then a space-time dependent phase for the Higgs VEVs is turned on inside the bubble wall propagating in the thermal bath during the electroweak phase transition.

In the adiabatic limit one can make a distinction between fast processes (mediated by gauge, flavour diagonal interactions and by top Yukawa interactions) and slow processes (mediated by Cabibbo suppressed gauge interactions and by light quarks Yukawa interactions). The former are able to follow adiabatically the changing of the Higgs VEV inside the bubble wall, while, in first approximation, the latter are frozen during the passage of the wall.

Making a rotation on the fermionic fields to remove the space-time dependent phase from the Yukawa interactions, the time derivative of this phase couples with the density of a quantum number non orthogonal to baryon number.

In the original paper [28] the rotation was taken to be proportional to the *fermionic hypercharge.*

The time derivative of the phase can be seen as an effective chemical potential, named *charge potential.* Each particle density acquires a nonvanishing value $n_i \sim Y_i^f \dot{\theta} T^2$, Y_i^f being the fermionic hypercharge of the species i, and the charge potential has the effect of biasing the rate of the sphaleronic processes, creating an asymmetry proportional to $\dot{\theta}$, where θ is the phase of the VEVs.

One of the basic assumptions of such a mechanism was that the conserved (by all the interactions) quantities, such as $B - L$ or the electric charge Q, had *vanishing thermal averages.*

This mechanism appeared to work, but only barely, in the original paper [28] because of the presence of different suppression factors. More in particular, CKN claimed that spontaneous baryogenesis could not work in the framework of MSSM in the case in which CP is broken spontaneously in the Higgs sector, see Chapter **2**.

This claim was shown to be incorrect by Comelli, Pietroni and Riotto (CPR) in ref. [29], where the authors made use of the fact that CP can be spontaneously broken at finite temperature evading any experimental limit coming from LEP at zero temperature [29, 30, 31]. Indeed, at high T there can be spontaneous CP violation such that the effective potential in the low-temperature phase has two nearly degenerate minima, with phases $\pm\pi$. Thus on roughly half the bubbles the CP violating phase will decrease from 0 to $-\pi$ in going from the unbroken to the broken phase, while in the other half it will increase from 0 to π. On each bubble the local baryon asymmetry production should be comparable to that of a maximal two Higgs doublet model. Then, it was argued that a tiny explicit CP violation, easily consistent with the limits on the electric dipole moment of the neutron (EDM), could produce a different in the surface tension on the two types of bubbles enough that the net BAU could be consistent with observations [29]. These results will be presented in Chapter **3**.

The adiabatic scenario was subsequently reconsidered by many authors in different, but related aspects, see Chapter 4.

Dine and Thomas [32] considered the same model discussed in the original work by CKN, the two Higgs doublets model in which the same doublet couples both to up and down quarks.

These authors pointed out that, when making an hypercharge rotation on fermionic fields to rotate away the space-time dependent phase from the Yukawa sector, $\dot{\theta}$ couples also to the Higgs density, so that the induced charge potential is for *total hypercharge rather than for fermion hypercharge.*

The key point was that total hypercharge is conserved by sphalerons: as long as effects proportional to the temperature dependent VEV $v(T)$ are neglected, hypercharge is a exactly conserved quantum number and then, imposing the constraint that all the conserved charges have zero thermal averages, no baryon asymmetry can be generated. A suppression factor of order of $M_t^2(T)/T^2$ in the particle densities was then expected with respect the original results given in [28], $B \sim 10^{-8}\,\Delta\theta$.

Giudice and Shaposhnikov showed the dramatic effect of non perturbative, chirality breaking, transitions induced by the so called QCD sphalerons [33]. If these processes were active inside the bubble walls, then the equilibrium value for baryon number in the adiabatic approximation would be proportional to that for the conserved quantum number $\langle B - L \rangle$ up to mass effects suppressed by $\sim M_t^2(T)/T^2$. Then, imposing the constraint $\langle B - L \rangle = 0$ we obtain zero baryon number (up to mass effects).

Finally, Joyce, Prokopec and Turok [34] emphasized the very important point that the response of the plasma to the charge potential induced by $\langle \dot{\theta} \rangle$ is not simply that of a system of fixed charges, because transport phenomena may play a crucial role. When a space-time dependent charge potential is turned on at a certain point, hypercharged particles are displaced from the surrounding regions, so that even the thermal averages of conserved quantum numbers become locally non vanishing. As a consequence, the equilibrium properties of the system have to be reconsidered taking into account the *local violation* of the conserved quantum numbers. The authors concluded that transport phenomena suppress the baryon asymmetry generated in the adiabatic limit.

In fact, it was shown by CPR [35] that transport phenomena are crucial and that, rather than being detrimental, they helped in alleviating the above mentioned problems.

CPR used the linear response theory in order to take properly transport effects into account and investigate the effects on the thermal averages of the various quantum numbers of the system in presence of a generic fermionic hypercharge potential.

They found that transport phenomena are really crucial, but were in disagreement with the conclusion of ref. [34] that, as a consequence of the local violation of global quantum numbers, there is no biasing of the sphaleronic processes. Actually, in the adiabatic approximation the local equilibrium configuration of the system is determined by the thermal averages of the charges conserved by all the fast interactions. The effect of transport phenomena is to induce space-time dependent non zero values for these averages. CPR calculated these averages using linear response theory and then determine the local equilibrium configuration, showing that it corresponds to a thermal average $\langle B + L \rangle \neq 0$.

The result in ref. [34] corresponded to freezing out any interaction inside the bubble wall, which is in contradiction with the adiabatic hypothesis.

Then CPR wrote down a rate equation in order to take into account the slowness of the sphaleron transitions and obtained an expression for the final baryon asymmetry explicitly containing the parameters describing the bubble wall, such as its velocity, v_w, its width L_w, and the width of the region in which the sphalerons are active.

Compared to previous estimates in which transport effects were not taken into account, they found an enhancement of nearly three orders of magnitude in the baryon asymmetry.

The inclusion of transport phenomena also shed a new light on the strong sphaleron effects and on the effect of a charge potential for *total rather than fermionic hypercharge*. The dramatic suppressions found by Giudice and Shaposhnikov and by Dine and Thomas respectively, were both a consequence of taking zero averages for conserved quantum numbers. In particular, the role of QCD sphalerons in cooperation with transport effects depends crucially on the particle species which enter into the charge potential.

Since thermal averages for conserved quantities are no more locally zero, CPR found a non zero $\langle B-L\rangle$ proportional to these averages, even in the case in which the charge potential is for total rather than for fermionic hypercharge. Thus, CPR showed that no suppression factor $M_t^2(T)/T^2$ is present when transport phenomena are taken into account.

We shall discuss transport phenomena in Chapter **5**.

Completely similar conclusions to those given in ref. [35] were obtained in ref. [36]. More in particular, CKN showed that diffusion allows doublet density produced within the bubble wall to venture out into the symmetric phase, where baryon number violation is fast and not suppressed by hypercharge violating quantities, eliminating the Dine-Thomas objection. It was also shown that particles are not in front of the wall enough for strong sphalerons to completely eliminate the left-handed doublet density, thus confirming the results obtained in [35].

However, a new and very general criticism was raised up very recently by CPR [37], see Chapter **6** (from this point of view, Chapter **6** should be read immediately after Chapter **2**).

All the previous criticisms (and ways out) to SB assumed as *a starting point* that the effect of a time-dependent CP violating phase in the Higgs sector was to induce a particle densities $n_i \sim \dot{\theta}\, T^2$.

In fact, in the original paper [28], making a redefinition of fermionic fields to get rid of the phase θ from the Yukawa sector, the effect of the CP violation on particle currents in the presence of CP violating Higgs background $H_i(x) = v_i(x) \exp\left[i\delta_i(x)\right]$ was calculated perturbating around the Higgs field configuration $\theta_i(x) = 0$, $v_i(x) \neq 0$, which, however, is *not* a solution of the equations of motions. In other words, the procedure adopted in the original treatment was equivalent to disantagle $\theta_i(x)$ from $v_i(x)$, whereas from the field equations one could see that $\partial_\mu \theta_i(x)$ vanishes as $v_i^2(x)$ for vanishing $v_i(x)$.

CPR calculated the particle currents induced in a bubble wall background at finite temperature in a model with CP violation in the Higgs sector. Using a *general field theory approach* they showed that fermionic currents arise only at one-loop, so that a suppression factor $\mathcal{O}(h_t\, v/\pi\, T)^2$ with respect to *all* previous computations was found. The contributions to the Higgs currents was also derived and their relevance for spontaneous baryogenesis mechanism discussed [37].

This suppression factor has nothing to do with the ones described in Chapter 4, the latter being based on particular hypothesis involving some physics (like the presence of QCD sphalerons) or some assumptions (like taking the thermal averages of conserved charges vanishing). On the contrary, the suppression factor depicted in Chapter 6 lies on very general theoretical grounds and is found when realized that the charge potential tool to describe particle currents in a space-time dependent and CP violating Higgs background suffers several problems.

Results of ref. [37] have been very recently confirmed and the same suppression factors obtained by Huet and Nelson [38], who computed the effects of CP violation on particle distributions inside the bubble walls, taking into account the effects of scattering from thermal particles.

That a typical suppression factor $\mathcal{O}(h_t v/\pi T)^2$ is really crucial can be understood when realizing that baryon number violation ceases to be effective for values of the Higgs VEV of order $v_{\rm co} \simeq \alpha_W/g$. A suppression factor of order 10^{-4} arises. The basic problem is that CP violating effects give rise to a doublet density in the region of the wall where the Higgs field is large, while baryon number violation occurs where the Higgs field is small.

In such a case, it might hard to reconcile the observed value of BAU with the mechanism of baryogenesis in the adiabatic limit, both in the case of spontaneous and explicit CP violation in the Higgs sector.

Nevertheless, CPR [37] suggested that, again, transport might help in eliminating this general drawback for spontaneous baryogenesis. This suggestion has been recently proven to be correct by Huet and Nelson [39] for the case of EWB in supersymmetric models: thermal scattering processes can convert CP violating charges into CP violating thermal particle distributions. The latter can diffuse into the symmetric phase, by CP even thermal processes, where sphalerons are active producing a baryon asymmetry of the same order of magnitude of that estimated in refs. [35, 36].

We are now ready to start: *adelante Pedro, ma con judicio!* [40].

Chapter 2

Spontaneous baryogenesis: the ancient days

The aim of this Chapter is to make the reader familiar with the mechanism of spontaneous baryogenesis as was originally described in ref. [28].

In the adiabatic limit it should be valid to treat the plasma in the bubble wall as being in quasistatic thermal equilibrium with a classical, time-dependent field. However, the plasma will not be in chemical equilibrium because some interactions, such as baryon number violation, are slower than $\tau_{\rm H}$. This deviation from chemical equilibrium may be treated introducing chemical potentials for the slowly varying quantities.

2.1 Toy model

To see how a time-dependent Higgs field can drive baryon number production, consider a toy model [24] with a conserved $U(1)_B$ baryon symmetry carried by fermions $\psi\,(B=1)$ and $\chi\,(B=-1)$, a scalar $\phi\,(B=2)$, and a Yukawa interaction $\bar{\psi}\chi\phi$.

During the phase transition ϕ takes the classical value

$$\phi(t) = v(t)\,\mathrm{e}^{i\theta(t)}, \tag{2.1}$$

leading to baryon production, even for $\dot{\phi}$ small compared to the fermion masses. To see this, redefine the fermion fields by the phase $\exp[iB\theta(t)/2]$. This removes the phase from the Yukawa interaction, but leads to a new interaction from the fermion kinetic term

$$\mathcal{L}_{\rm KE} \to \mathcal{L}_{\rm KE} + \frac{1}{2}\dot{\theta}\left(\bar{\psi}\gamma^0\psi - \bar{\chi}\gamma^0\chi\right) = \mathcal{L}_{\rm KE} + \frac{1}{2}\dot{\theta}n_B. \tag{2.2}$$

Thus $\dot{\theta}$ acts like a chemical potential for the baryon number. KCN used the term *charge potential* instead of chemical potential, since the effect is dynamical and does not arise from any constraint.

The interaction in Eq. (2.2) splits the energy levels of baryons versus antibaryons so that the free energy is minimized at non zero baryon number. The interaction violates C and CP and will spatially average to zero except in theories with explicit violation of these symmetries.

If B symmetry is violated in some other sector of the theory (*e.g.* through the electroweak anomaly, then Eq. (2.2) will cause B to try to equilibrate to the value $n_B = \mathcal{O}(\dot{\theta}T^2)$.

If the B violating interactions are rapid in comparison with $\ddot{\theta}/\dot{\theta}$, then this equilibrium value will be attained.

In the real case we wish to study, however, the rate of anomalous fluctuations is too slow to equilibrate because of the factor α_W^4. In this case, one simply integrates the master equation for weak scale baryogenesis over an appropriate time scale

$$\frac{dn_B}{dt} = -9\,\frac{\Gamma_{\rm sp}}{T}\,\frac{\partial {\rm F}}{\partial B} \equiv -9\,\frac{\Gamma_{\rm sp}}{T}\,\mu_B. \tag{2.3}$$

In Eq. (2.3) $\partial{\rm F}/\partial B$ is the derivative of the total free energy density ${\rm F}(B)$ calculated with all the conserved charges kept fixed, $\Gamma_{\rm sp}$ is the sphaleron rate per unit volume and μ_B is the baryon number chemical potential associated to the slowly varying B number.

To see how Eq. (2.3) is found (see [42] for a detailed analysis), imagine that the system is characterized by vanishing conserved charges, so that the free energy of the system is characterized, in presence of fermions, by a chemical potential μ_B such that ${\rm F} \sim \mu_B^2\,T^2$. As a consequence, transitions which increase the $B+L$ number are disfavoured with respect to the ones decreasing it.

The ratio between the transition rates with $\Delta N_{\rm CS} = +1$ and $\Delta N_{\rm CS} = -1$ is given by

$$\frac{\Gamma_+}{\Gamma_-} = e^{-\Delta f/T}, \tag{2.4}$$

where Δf is the difference of the free energies of the two vacua in presence of fermions. If we now indicate $\Gamma_+ \simeq \Gamma_- = \Gamma_{\rm sp}$, we get

$$\frac{dn_B}{dt} = 3\,(\Gamma_+ - \Gamma_-) = -3\,\frac{\Gamma_{\rm sp}}{T}\,\Delta f, \tag{2.5}$$

where the factor 3 comes from the fact that $\Delta B = 3$ in anomalous events. Recalling now that $\Delta f = 3\,\partial F/\partial B$, Eq. (2.3) is recovered.

What Eq. (2.3) tells us is that sphalerons cease to act only when $\Gamma_{\rm sp}$ is suppressed or when the system settles into its equilibrium state at which $\partial F/\partial B = 0$, *i.e.* the *total* free energy density of the system is at its equilibrium value.

It should be emphasized that it is not necessary for the charge potential to couple to the baryon number as in this toy model in order for having a nonvanishing value of $\dot{n}_B$. As long as the charge X is not orthogonal to B, an interaction coupling of the form $\dot{\theta}\,J^0_X$ will give rise to a non zero value of the chemical potential for the baryon number

$$\mu_B = \mathcal{N}\dot{\theta}, \tag{2.6}$$

where $\mathcal{N}$ is a calculable and model dependent constant.

After the bubble wall has passed a given point in space, the charge potential $\dot{\theta}$ returns to zero. If baryon number violation is still rapid compared to the cooling rate of the Universe, the baryon number produced will equilibrate at zero. However, the anomalous fluctuation rate $\Gamma_{\rm sp}$ goes rapidly to zero as the Higgs VEV turns on, so that the baryon number drops out of chemical equilibrium and remains to the present epoch.

Making the crude estimate that this *cutoff* value of the Higgs field occurs at[1]

$$v_{\rm co} \simeq \alpha_w/g \tag{2.7}$$

[1]The correct value of $v_{\rm co}$ is not currently known. The most optimistic consider it as large as $(14\,\alpha_W\,T/g)$ [41]. In this Thesis we shall support the pessimistic trend and use the value of Eq. (2.7).

we find that the final baryon number produced during the phase transition

$$n_B = -9\,\frac{\mathcal{N}}{T}\int_{-\infty}^{t_{\rm co}} dt\,\dot{\theta}\,\Gamma_{\rm sp}\,[\phi(t)] \simeq -9\,\frac{\mathcal{N}\,\Gamma_{\rm sp}}{T}\,\Delta\theta, \tag{2.8}$$

where we have estimated the integral by treating $\Gamma_{\rm sp}$ as a step function equal to its symmetric phase value (1.11) for $v(T) \lesssim v_{\rm co}$ and vanishing further inside the bubble wall.

The quantity $\Delta\theta$ is then the change in θ from the symmetric phase to the point in the bubble where baryon number violation effectively shuts off; it is a homogeneous function of the CP violating parameter δ_{CP} of the theory in question. Furthermore, $\Delta\theta$ is homogeneous in $v_{\rm co}$ and could be very small if $v_{\rm co}$ proves to be small [42].

Since $\Gamma_{\rm sp} \simeq (\alpha_W T)^4$, one finds

$$B \simeq -3\times 10^{-7}\left(\frac{\mathcal{N}}{0.1}\right)\left(\frac{100}{g_*}\right)\left(\frac{\Delta\theta}{\pi}\right), \tag{2.9}$$

to be compared with the observed value $B \simeq 10^{-10}$.

2.2 Recipe

Let us now list five steps that must be followed when analyzing EWB in the adiabatic limit:

1) Identify a model that has a first order phase transition with large time scale $\tau_{\rm H}$ and classical Higgs field evolution during the phase transition such that there is a CP violating, time-dependent phase in the Yukawa interaction.

2) Determine $\Delta\theta$ from the Higgs equation of motion in order to use Eq. (2.9) to compute the final baryon number.

3) Rotate the fermion fields of the theory to remove the time-dependent phase from the Yukawa interactions into a derivative coupling of the form $\dot{\theta}\,J_X^0$. The current X is ambiguous since the rotation is not unique–one can always rotate so that $X \to (X+Q)$, where Q is a classically conserved charge. It is simplest to choose X so that the rotation is anomaly free, so one needs not to treat couplings of the type $\theta(t)\tilde{F}F$.

4) Compute the dynamically generated value of μ_B. To do so one must introduce chemical potentials for each conserved or approximately conserved quantum number.

All the particle densities may be determined in terms of these chemical potentials and the charge potential $\dot{\theta}$; for each particle species i the density n_i is then

$$n_i = k_i \left[X_i \dot{\theta} + \sum_a \mu_a \, q_i^a \right] \frac{T^2}{6}, \tag{2.10}$$

where k_i is a statistical factor accounting for the effective degrees of freedom and differs for fermions and bosons, X_i is the X charge of the species i, and the sum is over all charges which are (approximately) conserved (including B). The values of the chemical potentials μ_a are fixed by requiring that the primordial plasma does not carry any quantum number. One can then determine μ_B and the factor $\mathcal{N}$.

5) Use Eq. (2.9) to compute the final baryon asymmetry. This formula is only expected to be valid in the adiabatic thick wall regime. If the equilibration time for particle distributions is long compared to $\tau_{\rm H}$, there may be additional suppression factors.

Before going on the reader should be alerted on two crucial issues: first, Eq. (2.9) is obtained assuming that thermal averages of conserved quantities, like $B-L$ and Q, are vanishing, see Chapter **4**. This is no longer true when transport phenomena are taken into account [35]; secondly Eq. (2.10) does not present any suppression factor recently discovered in ref. [37], see Chapter **6**.

Let us now apply these five dogma to a practical case.

2.3 The two Higgs doublet model

The two Higgs doublet model was first suggested by Mc Lerran as a model for EWB [44] and was analyzed by Turok and Zadrozny [45]. Here we strictly follow ref. [28].

The model is characterized by a Lagrangian

$$\mathcal{L} = \sum_{\rm fermions} \bar{\psi}_i \, \not{\mathcal{D}} \psi_i + \sum_1^2 |\mathcal{D}_\mu \phi_i|^2 - V(\phi_1, \phi_2) + \mathcal{L}_{\rm yuk} + \mathcal{L}_{\rm gauge}, \tag{2.11}$$

where the scalar potential is given by [46]

$$V(\phi_1, \phi_2) \;=\; \lambda_1 \left(\phi_1^\dagger \phi_1 - v_1^2\right)^2 + \lambda_2 \left(\phi_2^\dagger \phi_2 - v_2^2\right)^2$$

$$
\begin{aligned}
&+ \lambda_3 \left[\left(\phi_1^\dagger \phi_1 - v_1^2 \right) + \left(\phi_2^\dagger \phi_2 - v_2^2 \right) \right]^2 \\
&+ \lambda_4 \left[\left(\phi_1^\dagger \phi_1 \right) \left(\phi_2^\dagger \phi_2 \right) - \left(\phi_1^\dagger \phi_2 \right) \left(\phi_2^\dagger \phi_1 \right) \right] \\
&+ \lambda_5 \left[\mathrm{Re} \left(\phi_1^\dagger \phi_2 \right) - v_1 v_2 \cos \xi \right]^2 \\
&+ \lambda_6 \left[\mathrm{Im} \left(\phi_1^\dagger \phi_2 \right) - v_1 v_2 \sin \xi \right]^2 . \qquad (2.12)
\end{aligned}
$$

The Yukawa interactions in $\mathcal{L}_{\mathrm{yuk}}$ couple the ϕ_1 field to up quarks; down quarks and charged leptons may be coupled exclusively to either ϕ_1 or ϕ_2–either choice suppresses flavour changing neutral currents and is protected by a discrete symmetry which is softly broken.

For convenience CKN assumed that all fermion masses arise due to ϕ_1. In fact, the other option works equally well, since only the top quark Yukawa coupling proves to be relevant.

During the electroweak phase transition the two Higgs doublets will acquire VEVs. The neutral components of these doublet take the form

$$\phi_1^0 = v_1\, e^{-i\theta}, \quad \phi_2^0 = v_2\, e^{i\omega}. \qquad (2.13)$$

During the phase transition the fields evolve towards their zero temperature values

$$
\begin{aligned}
v_1(x) &\to v_1 \quad & v_2(x) &\to v_2 \\
\theta(x) &\to 0 \quad & \omega(x) &\to \xi.
\end{aligned} \qquad (2.14)
$$

Although $\theta(x) = 0$ in the vacuum, it will in general change by an amount $\Delta\theta$ during the phase transition, with the size and the sign determined by the CP violating angle ξ in the Higgs potential.

CKN then fixed the unitary gauge ensuring that θ is the physical pseudoscalar orthogonal to the Goldstone boson eaten by the Z^0. This gauge fixing eliminates the ω degree of freedom in Eq. (2.13) through the relation

$$\partial_\mu \omega = \left(\frac{v_1}{v_2} \right)^2 \partial_\mu \theta. \qquad (2.15)$$

During the phase transition, when v_1 and v_2 are function of the space-time, the Eq. (2.15) for ω is more complicated to integrate; luckily enough, since only the top Yukawa coupling proves to be important, no need of integration was needed.

In the model under consideration, the classical motion of the CP odd pseudoscalar field $\theta(x)$ gives rise to a charge potential. To see this, CKN removed $\theta(x)$ from the Yukawa couplings by performing a space-time dependent rotation on the fermion fields. Since they did not want to induce any coupling of $\theta(x)$ to gauge fields, they eliminated $\theta(x)$ by means of an anomaly free fermion rotation; thus they rotated the fermions by an amount proportional to their hypercharge.

CKN found that $\theta(x)$ had derivative couplings to twice the fermionic part of the hypercharge current $J^{\mu}_{Y_f}$

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\rm KE} + h_t \bar{q}_L \tilde{\phi}_1' t_R + h_b \bar{q}_L \phi_1' b_R \\ &+ \cdots + \text{h.c.} + 2\,\partial_\mu \theta \left[\frac{1}{6}\,\bar{q}_L \gamma^\mu q_L + \frac{2}{3}\,\bar{U}_R \gamma^\mu U_R - \frac{1}{3}\,\bar{D}_R \gamma^\mu D_R \right. \\ &- \left. \frac{1}{2}\,\bar{l}_L \gamma^\mu l_L - \bar{E}_R \gamma^\mu E_R \right], \end{aligned} \tag{2.16}$$

where there is an implicit sum over the three families in the current coupling to $\partial_\mu \theta$ and ϕ_1' is the scalar doublet ϕ_1 with the phase θ removed.

During the phase transition, CP violation in the Higgs sector will produce a non zero spatial average for the time derivative of θ.

The quantity $\langle \dot{\theta} \rangle$ is then a charge potential which splits particle-antiparticle energy levels inside the domain walls produced during the phase transition.

Such a charge potential produces a free energy that is minimized for *non zero baryon number.* Provided that there is baryon number violation during the epoch when $\langle \dot{\theta} \rangle \neq 0$, a baryon asymmetry will result. The source of baryon number violation is provided by the anomalous processes of the SM.

CKN first showed that the steady state value for the baryon number B was non zero and proportional to $\langle \dot{\theta} \rangle$.

Throughout the phase transition, the Universe was assumed to satisfy the constraints $\langle B - L \rangle = 0$ and $\langle Q \rangle = 0$: conserved charges were taken to be vanishing. To enforce these constraints, they introduced chemical potentials μ_{B-L} and μ_Q.

Assuming that all particles are lighter than the temperature and in thermal equilibrium with a non zero $\langle \dot{\theta} \rangle$, the net number (particle minus antiparticle) density for the ith particle type is given by

$$n_i = k_i \left(2\, q_i^{Y_f} \langle\dot{\theta}\rangle + q_i^{B-L}\mu_{B-L} + q_i^{Q}\mu_Q\right) \frac{T^2}{6}, \tag{2.17}$$

see Eq. (2.10).

Note that non zero $\langle\dot{\theta}\rangle$ contributes to fermion number densities proportional to fermionic hypercharge.

By applying the constraints $\langle B-L\rangle = \langle Q\rangle = 0$ and assuming sphalerons to be active, CKN solved for μ_{B-L} and μ_Q

$$\begin{aligned} \mu_{B-L} &= -16\,\frac{6+n}{111+13n}\,\langle\dot{\theta}\rangle \\ \mu_Q &= -\frac{66}{111+13n}\,\langle\dot{\theta}\rangle \\ n_B^{\rm EQ} = n_L^{\rm EQ} &= -12\,\frac{6+n}{111+3n}\,\frac{T^2}{6}\,\langle\dot{\theta}\rangle, \end{aligned} \tag{2.18}$$

where $n_B^{\rm EQ}$ and $n_L^{\rm EQ}$ are the baryon and the lepton number densities and n is the number of light charged scalars.

Although this calculation demonstrates the SP mechanism at the weak scale, it involves sever oversimplifying assumptions. In particular, in a given region of space a non zero charge potential is generated during an inverse time of order of $(1-100)$ GeV, and the rate for anomalous violation is too slow for baryon number to reach its equilibrium density given above.

It is possible that the time is long enough for weak interactions to remain in equilibrium, but among the right-handed fermions, only the top quark does have couplings strong enough to remain in thermal equilibrium with $\langle\dot{\theta}\rangle$.

To deal with a system in which some degree of freedom are in equilibrium while others are not, CKN needed to consider the rate equations for quantities that are slow to reach equilibrium.

In the two Higgs doublet model the only interactions which could possibly be in equilibrium inside the domain walls are: *i)* the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge boson exchange; *ii)* family diagonal $SU(2)_L$ gauge boson exchange; *iii)* top Yukawa interactions. Anomalous baryon number, Cabibbo suppressed charged current interactions and light fermion Yukawa interactions are presumed to be too slow to equilibrate inside the bubble wall. Thus CKN introduced chemical potentials for any particle which takes part to fast processes and then reduce the number of linearly independent chemical potentials by solving the corresponding system of equations, along the same lines of what done in refs. [43]. The abundances of any species in equilibrium can then be expressed in terms of the remaining linear independent chemical potentials, corresponding to the conserved charges of the system chosen by CKN to be $B-L$, Q and also B since the baryonic number changes very slowly inside the bubble walls and can be taken (approximately) conserved.

Thus, they fixed zero particle-antiparticle asymmetry for each of the right-handed fermions except the top and $\langle B-L\rangle = \langle Q\rangle = \langle B\rangle = 0$ to calculate $\partial \mathrm{F}/\partial B$, which is needed to solve Eq. (2.3) at the point where all slowly violated quantum numbers are approximately zero.

The result is

$$\frac{\partial \mathrm{F}}{\partial B} = -\frac{4}{3}\frac{6+n}{25+4n}\langle\dot{\theta}\rangle. \tag{2.19}$$

Since the temperature is constant during the phase transition, Eq. (2.3) can be easily integrated to yield

$$B = \frac{n_B}{s} \simeq -\frac{4}{3}\frac{6+n}{25+4n}\frac{\alpha_W^4\, T^3}{s}\,\Delta\theta \simeq 10^{-8}\,\Delta\theta. \tag{2.20}$$

$\Delta\theta$ can be estimated from the scalar potential to be

$$\Delta\theta \simeq \left[\xi - \tan^{-1}\left(\frac{\lambda_5}{\lambda_6}\tan\xi\right)\right]. \tag{2.21}$$

Therefore, if λ_5 and λ_6 are nondegenerate and of order one (so that Higgs fields are in thermal equilibrium) a CP violating phase $\xi \simeq 10^{-2}$ can explain the observed value of $\simeq 10^{-10}$.

CKN also commented about the fact that the final result for B is independent from h_t–even though baryon production must vanish in the limit of $h_t \to 0$. They claimed that h_t is not present because they assumed the top quark–Higgs scattering in thermal equilibrium, which is satisfied since the top quark is known to be heavy, and then the rate drops out of any calculation. They also claimed that CP violation occurs without loops in SB. The reason is that a CP violating phase θ does have a nonvanishing time derivative due to the equations of motions; in presence of this background field, tree level diagrams with different numbers of external θ quanta can interfere with each other and produce physical CP violating effects.

We shall see in Chapter **6** that this claim was incorrect. Loop suppression factors proportional to h_t^2 must be present in the final result for B *even if* top quark–Higgs scatterings are fast enough to be in equilibrium.

2.4 The supersymmetric legacy

In ref. [28] CKN shortly discussed also the applicability of what learned in the previous Section to the case of MSSM.

Indeed, supersymmetric standard model necessarily has two Higgs doublets and one might expect spontaneous baryogenesis to work quite naturally. However, CKN claimed that this is not the case. All CP violation may be removed from the Higgs potential and placed in interactions involving only superpartners of the ordinary particles. The phase ξ appearing in the potential (2.12) is absent in the MSSM because supersymmetry enforces the relation $\lambda_5 = \lambda_6 = \mathcal{O}(g^2)$, allowing ξ to be removed.

Provided that the squark and slepton fields do not acquire expectation values during the electroweak phase transition, the classical evolution of the scalar fields will be conserving. Since supersymmetry is softly broken, however, a finite splitting between λ_5 and λ_6 of order $(g^2/16\pi^2)$ will be generated at one-loop when heavy supersymmetric particles are integrated out of the theory, in particular from a box diagram with gauginos and Higgsinos running in the loop. This can feed a CP violation phase $\xi_{\rm SUSY}$ into the Higgs scalar sector, and result in CP violating evolution of the field θ.

From Eq. (2.21) CKN estimated the change in θ during the phase transition to be

$$\Delta\theta \simeq \frac{g^2}{16\,\pi^2}\,\xi_{\rm SUSY}. \tag{2.22}$$

The tiny limit on the neutron EDM requires that the new CP violating phases in SUSY models be smaller than $\sim 10^{-2}$, which, when combined with the loop suppression and the slow baryon number violating rate results in a net baryon number of about

$$B \simeq 10^{-10}\, \xi_{\rm SUSY} < 10^{-12}. \tag{2.23}$$

Thus, CKN concluded that the MSSM cannot be taken as a viable model for EWB in the adiabatic limit.

The aim of the next Chapter is to show that this conclusion was not correct: a space-time dependent phase for the Higgs VEVs in the MSSM can be generated at finite temperature by plasma effects, without being in conflict with any experimental bounds at zero temperature [29].

In fact this new effect is quite model independent and was shown to be working not only in the MSSM [29, 30], but also in the next-to-minimal supersymmetric standard model when a gauge singlet is added to the MSSM [31]: assuming CP conservation at zero temperature in the scalar sector, spontaneous CP violation can occur both inside the critical bubbles and the bubble walls (where SB occurs) thanks to the interaction of the Higgs scalar fields with the surrounding plasma, which helps in enhancing spontaneous violation of CP.

When the temperature cools down, the CP violating phases relax to $\pm\pi$ so that no spontaneous CP violation is present in the theory at zero temperature. This allows to avoid any stringent limit on CP violating phases of order of 10^{-2} coming from, *e.g.*, the present experimental bound on the neutron EDM and the possibility to have, during the electroweak phase transition CP violating phases much larger than 10^{-2}. The reader is referred to refs. [29, 30, 31] for more details about this new mechanism.

Chapter 3

Spontaneous baryogenesis in MSSM: the supersymmetric ancient days

It is rather intriguing that the three Sakharov's conditions for the generation of the baryon asymmetry in the early Universe might be fulfilled in the framework of MSSM.

In the MSSM the LEP lower bound on the lightest Higgs scalar is presently around 43 GeV [50].

The analysis which have been carried out by now [16] in the context of the one-loop approximation for the finite temperature effective potential indicate that the condition to have a sufficiently strong electroweak phase transition, $v(T)/T \gtrsim 1$, is indeed a strong constraint even in the MSSM, but it is not yet clear whether this limit rules out or not the possibility of generating the baryon asymmetry of the Universe at the EWPT [16].

It is likely that the actual upper bound on the lightest Higgs mass coming from Eq. (1.12) is not very far from the experimental lower bound the next generation of accelerators (LEP2, LHC) will eventually provide, making the EWB supersymmetric mechanism directly testable.

Moreover, if one considers also the dependence of $v(T)/T$ on the stop-sbottom masses and the ρ parameter, then a small region of parameter space is left, which will be explored by LEP2.

About the necessary amount of CP violation to generate the BAU in the right amount, the MSSM contains new sources of CP violation and for such a reason can provide a nice scenario for baryogenesis at the weak scale. In particular, the MSSM contains two extra explicit CP violating phases with respect to the SM.

The requirement that these phases provide the necessary amount of CP violation for the generation of the BAU, gives rise to additional very strong constraints on the parameter space of the model [51]. Indeed, the electric dipole moment of the neutron must be larger than 10^{-27} e $\cdot$ cm, while an improvement of the current experimental bound on it by one order of magnitude would constrain[1] the lightest chargino and the lightest neutralino to be lighter than 88 and 44 GeV, respectively [51].

However, in the MSSM another source of CP violation may emerge at finite temperature. As was shown in refs. [29, 30, 31], one-loop effects at a temperature $T = \mathcal{O}(100)$ GeV may induce a non zero relative phase δ between the VEV's of the two Higgs leading to spontaneous CP violation (SCPV). The phase δ varies with the VEV's and with the temperature and, as T goes to zero, it becomes trivial ($\delta = 0, \pm\pi$). Due to this fact, the *spontaneous* phase δ can assume all the values in the range $0 \leq \delta \leq \pi$ and, contrary to the *explicit* phases considered in ref. [51], does not receive any bound from current experimental constraints on the neutron EDM.

In this Chapter we will consider the implications of the finite temperature SCPV on baryogenesis. The spontaneous breakdown of CP can lead to different cosmological scenarios according to the nature of the EWPT.

We will investigate the relevance of the new source of CP violation for the SB scenario described in Chapter **2** and also discuss the case in which the EWPT is of the second order and the related problem of the formation of domain walls.

The presence of a space-time varying phase δ, can induce a shift in the energy levels between baryons and antibaryons and bias the anomalous, B violating, interactions, giving rise to a non vanishing baryon asymmetry. The presence of expanding bubbles in the thermal bath is essential to this mechanism, since the baryon asymmetry is generated inside the bubble walls where the phase δ changes its value and the B

[1]Strictly speaking, this is only true when transport phenomena are neglected, see Chapter **6**.

violating processes are active.

Since the phase δ can take values of order one, the baryon number creation mechanism by a single bubble turns out to be very efficient. For comparison, the explicit phases considered in [51] are constrained to be of order $(10^{-2} - 10^{-3})$ by the bounds on the neutron EDM, and the baryon number generated by a single bubble is estimated to be $(10^{-2} - 10^{-3})$ times the one generated if SCPV is present. However, if CP is broken *only* spontaneously, then the phase δ can take two opposite values, corresponding to two *exactly* degenerate vacua.

This degeneracy between the two vacua in $\pm\delta$ would induce an equal number of nucleated bubbles carrying phases with opposite signs, which in turn generate baryon asymmetries of opposite signs.

The BAU obtained averaging over the entire volume of the Universe would then be zero. We will find that the introduction of very small explicit phases, of order $(10^{-5} - 10^{-6})$, lifts the degeneracy, leading to a difference between the nucleation rates of the two kinds of bubbles, and possibly to a baryon asymmetry of the right order of magnitude. We wish to stress that the spontaneous phase δ and the explicit ones play very different roles in this scenario, the former being the real source of CP violation necessary to the primary production of baryons, and the latter lifting the degeneracy between otherwise equivalent vacua in order to achieve a global bias of the primary production.

From the phenomenological point of view, such very small phases give rise to negligible contributions to the neutron EDM. Moreover we shall prove that a light Higgs pseudoscalar is needed, and an upper bound on its mass will be obtained.

First, let us briefly discuss the additional possible sources of CP violation in MSSM, apart from the usual KM phase present in the SM.

As said before, it is useful to distinguish between *explicit* violation of CP, due to the present of complex parameters whose phases cannot be removed away by field redefinitions, and *spontaneous* CP violation, which is obtained when the vacuum expectation values of the Higgs scalar fields are complex even in the limit in which all the parameters of the theory are real.

3.1 Explicit violation of CP in MSSM

Let us consider the MSSM superpotential [5]

$$W^{\mathrm{MSSM}} = \mu \hat{H}_1 \hat{H}_2 + h^u \hat{H}_2 \hat{Q} \hat{u}^c + h^d \hat{H}_1 \hat{Q} \hat{D}^c + h^e \hat{H}_1 \hat{L} \hat{e}^c, \tag{3.1}$$

where we have omitted the generation indices for the superfields (indicated with hats). The Higgs sector contains two $SU(2)_L$ doublets

$$H_1 \equiv \begin{pmatrix} H_1^0 \\ H^- \end{pmatrix} \quad \text{and} \quad H_2 \equiv \begin{pmatrix} H^+ \\ H_2^0 \end{pmatrix}, \tag{3.2}$$

with hypercharge $-1/2$ and $1/2$, respectively.

The lepton Yukawa matrix h^e can be always taken real and diagonal, whereas h^u and h^d contain the KM phase.

The new contributions to the explicit violation of CP emerge from the operators which break softly supersymmetry [47]:

i) Trilinear scalar couplings:

$$\Gamma^u\ H_2 \tilde{Q} \tilde{u}^c + \Gamma^d\ H_1 \tilde{Q} \tilde{d}^c + \Gamma^e\ H_1 \tilde{L} \tilde{e}^c + \text{h.c.}, \tag{3.3}$$

where we have omitted the generation indices and the matrices $\Gamma^{(u,d,e)}$ are defined as

$$\Gamma^{(u,d,e)} \equiv m_{3/2}\, A^{(u,d,e)} \cdot h^{(u,d,e)}, \tag{3.4}$$

$m_{3/2}$ being the gravitino mass. Generally in supergravity models the matrices $A^{(u,d,e)}$ are assumed to be proportional to the identity matrix at the Planck scale

$$A^{(u,d,e)}(M_{\mathrm{Pl}}) \equiv A \cdot \mathbf{1}, \tag{3.5}$$

where the A parameter can be complex.

ii) Bilinear scalar coupling:

$$\mu\ B\ H_1 H_2 + \text{h.c.} \tag{3.6}$$

iii) Majorana gaugino masses:

$$\frac{1}{2}\left(M_1 \lambda_1 \lambda_1 + M_2 \lambda_2 \lambda_2 + M_3 \lambda_3 \lambda_3 + \text{h.c.}\right), \tag{3.7}$$

where the initial condition in Grand Unified models reads

$$M_1 = M_2 = M_3 = \tilde{m}. \tag{3.8}$$

iv) Scalar soft masses:

$$m^2_{ab} \tilde{z}^*_a \tilde{z}_b + \text{h.c.} \tag{3.9}$$

To avoid too large supersymmetric contributions to the $(K^0 - \bar{K}^0)$ mass difference, it is usually assumed that at the Planck scale

$$m^2_{ab} = m^2_{3/2} \delta_{ab}. \tag{3.10}$$

The new contributions to explicit violation of CP are given by the phases of the complex parameters A, B, and $\tilde{m}$, which break softly supersymmetry, and by the parameter μ in the superpotential. Two phases can be removed [48] by redefining the phase of the superfield $\hat{H}_2$ in such a way that the phase of μ is opposite to that of B, $\phi_\mu = -\phi_B$. The product μB in Eq. (3.6) is then real.

It is also possible to remove the phase of the gaugino mass $\tilde{m}$ by an R-symmetry transformation. The latter leaves all the other supersymmetric couplings invariant and only modifies the trilinear ones, which get multiplied by $\exp(-i\phi_{\tilde{m}})$ where $\phi_{\tilde{m}}$ is the phase of $\tilde{m}$.

The phases which are left are then

$$\phi_A = \arg(A\tilde{m}) \quad \text{and} \quad \phi_B = \arg(B\tilde{m}), \tag{3.11}$$

which are present in

$$A = |A| \; e^{i\phi_A}, \quad B = |B| \; e^{i\phi_B} \quad \text{and} \quad \mu = |\mu| \; e^{-i\phi_B}. \tag{3.12}$$

Note that the convention $\arg(\mu B) = 0$ leaves all the parameters of scalar potential real at the tree level.

The two new phases ϕ_A and ϕ_B do not have any effect on the CP violation in the $(K^0 - \bar{K}^0)$ system. On the other side, new important contributions to the neutron EDM d_n can be generated. The experimental upper bound on d_n is of order of

10^{-25} e · cm [49]. Assuming all the supersymmetric masses equal to $m_{3/2}$, the SUSY contribution to d_n is given by [48]

$$d_n^{\rm SUSY} \simeq \left(\frac{100\ \mathrm{GeV}}{m_{3/2}}\right)^2 \frac{\arg(BM_3^*) + \arg(\Gamma_{11}^u M_3^*)}{10^{-3}}\, 10^{-25}\ \mathrm{e}\cdot\mathrm{cm}. \tag{3.13}$$

Thus, for $m_{3/2} \simeq (10^{-1} - 1)$ TeV, we get

$$\phi_A,\ \phi_B \lesssim (10^{-3} - 10^{-2}). \tag{3.14}$$

3.2 Spontaneous breaking of CP

Let Φ_i be a generic scalar field, which transforms under CP as

$$CP\,\Phi_i\,CP^{-1} = \mathrm{e}^{i\phi_i}\,\Phi_i^*. \tag{3.15}$$

Assuming the the vacuum is invariant under CP, $CP|0\rangle = |0\rangle$, the phase δ_i of the field Φ_i and the phase ϕ_i are related through the equality

$$\langle 0|\Phi_i|\rangle 0 = v_i\,\mathrm{e}^{i\delta_i} = \langle 0|CP\,\Phi_i\,CP^{-1}|0\rangle = v_i\,\mathrm{e}^{i(\phi_i-\delta_i)}, \tag{3.16}$$

or

$$\delta_i = \frac{1}{2}\,\phi_i + n\pi, \tag{3.17}$$

where n is an integer number. If the above Equation is not satisfied, then $v_i = 0$.

If one requires that a generic potential $V(\Phi)$ be invariant under CP, different relations among the phases ϕ_i, and then on the δ_i through Eq. (3.17), are obtained. For instance, if the potential does have a trilinear term of the type $\lambda_{ijk}\Phi_i\Phi_j\Phi_k + \mathrm{h.c.}$, CP invariance is obtained if

$$\delta_i + \delta_j + \delta_k + \phi_{\lambda_{ijk}} = n\pi, \tag{3.18}$$

where $\phi_{\lambda_{ijk}}$ is the phase of the parameter λ_{ijk}.

If the phases δ_i, which are found by minimization of the scalar potential cannot satisfy relations similar to the one in Eq. (3.18), it will mean that CP is spontaneously broken.

Let us analyze a few examples to clarify the previous statements.

In the SM the scalar potential depends upon the modulus of the Higgs field and CP is conserved for any value of the Higgs VEV.

In the case of the two Higgs doublet model, we can consider the example of the operator

$$\lambda_5 \left(H_1 H_2\right)^2 + \text{h.c.} \tag{3.19}$$

Assuming λ_5 to be real, the operator is CP invariant if the phases of the the two Higgs fields, δ_1 and δ_2 satisfy the relation

$$\delta \equiv \delta_1 + \delta_2 = \frac{n\pi}{2}, \tag{3.20}$$

where we have used the definition

$$\langle H_1^0 \rangle = v_1\, \mathrm{e}^{i\delta_1} \quad \text{and} \langle H_2^0 \rangle = v_2\, \mathrm{e}^{i\delta_2}. \tag{3.21}$$

Thus, in absence of any other operator involving δ_1 and δ_2, a fully imaginary phase $\delta = i$ does not imply violation of CP. However, if the operator

$$m_2^2\, H_1 H_2 + \text{h.c.} \tag{3.22}$$

is present, among the values of δ satisfying Eq. (3.20), only those for which

$$\delta = n\pi \tag{3.23}$$

leave the theory CP invariant.

Let us now consider the most general two Higgs doublet model scalar potential

$$\begin{aligned} V &= m_1^2 \left|H_1\right|^2 + m_2^2 \left|H_2\right|^2 - \left(m_3^2 H_1 H_2 + \text{h.c.}\right) + \lambda_1 \left|H_1\right|^4 + \lambda_2 \left|H_2\right|^2 + \lambda_3 \left|H_1\right|^2 \left|H_2\right|^2 \\ &+ \lambda_4 \left|H_1 H_2\right|^2 + \left[\lambda_5 \left(H_1 H_2\right)^2 + \lambda_6 \left|H_1\right|^2 H_1 H_2 + \lambda_7 \left|H_2\right|^2 H_1 H_2 + \text{h.c.}\right]. \end{aligned} \tag{3.24}$$

The scalar potential for the MSSM is a particular case of the potential (3.24).

Let assume that all the parameters of the potential be real. Since both λ_5 and m_3^2 are present in V, from the discussion above we know that CP is spontaneously broken if $\delta = \delta_1 + \delta_2 \neq n\pi$. This condition is satisfied if

$$\lambda_5 > 0 \tag{3.25}$$

and

$$-1 < \Delta \equiv \frac{m_3^2 - \lambda_6\, v_1^2 - \lambda_7\, v_2^2}{4\,\lambda_5\, v_1\, v_2} < 1. \tag{3.26}$$

If it is the case, the relative phase δ is given by the relation

$$\cos\delta = \Delta. \tag{3.27}$$

Moreover, minimizing the scalar potential and taking into account the conditions necessary to avoid charge breaking through VEV's of H^+ and H^-, we have

$$\lambda_4 - 2\,\lambda_5 < 0. \tag{3.28}$$

Let us now focus on the supersymmetric standard model potential. At the tree level SUSY imposes the following relations for the couplings of the dimension four operators

$$\begin{aligned} \lambda_1 &= \lambda_2 = \frac{1}{8}\left(g_1^2 + g_2^2\right), \\ \lambda_3 &= \frac{1}{4}\left(g_2^2 - g_1^2\right), \\ \lambda_4 &= -\frac{1}{2}\,g_2^2, \\ \lambda_5 &= \lambda_6 = \lambda_7 = 0. \end{aligned} \tag{3.29}$$

We immediately see that SCPV cannot occur in MSSM *at the tree level.*

Due to the SUSY non-renormalization theorems [52], radiative corrections cannot change the tree level values of the couplings λ's when SUSY is an exact symmetry of the theory. The only radiative contributions able to modify the relations (3.29) and lead to SCPV must involve operators which softly break supersymmetry. These operators, of dimension two or three, can induce finite renormalizations of the λ parameters. Indeed, it was shown in ref. [53] that λ_5 can be renormalized at one loop and get a positive value when scalar quarks, neutralinos and scalar Higgses run in the loop.

However, the real problem is the condition (3.26). At one-loop the condition $|\Delta| < 1$ implies

$$\left|m_3^2 + \Delta m_3^2 - \Delta\lambda_6\, v_1^2 - \Delta\,\lambda_7\, v_2^2\right| < 4\,\Delta\lambda_5\, v_2\, v_2, \tag{3.30}$$

where the quantity $m_3^2 + \Delta m_3^2$ is related to the mass of the pseudoscalar[2] M_{A^0} by

$$M_{A^0}^2 \simeq 2 \left(m_3^2 + \Delta m_3^2\right) / \sin 2\beta, \tag{3.31}$$

where $\tan\beta = v_2/v_1$.

As a consequence, SCPV implies a very light pseudoscalar mass arising at one-loop, $M_{A^0} \lesssim 10$ GeV [54], already excluded by LEP [50]. This result is nothing else than the generalization of the Georgi-Pais theorem [55] stating that radiative corrections can induce SCPV only if the model contains a zero spin particle with vanishing mass at the tree level. In such a case, this particle identifies with the pseudoscalar A^0. SCPV in MSSM at zero temperature is then viable from the theoretical point of view, but experimentally excluded.

3.3 SCPV in the MSSM at finite temperature

In this Section we shall analyze the possibility of SCPV in the MSSM at finite temperature. We shall show that news contributions to the couplings $\lambda_{5,6,7}$ and m_3^2 can arise at finite temperature from radiative corrections. The effective potential can then have a CP violating minimum, with a nonvanishing phase δ. The latter, when the Universe cools down and relations (3.25) and (3.26) are no longer satisfied, goes to zero and the only sources for CP violation remain the KM phase and the $\bar{\theta}$ parameter of the QCD vacuum (apart from the possible explicit phases ϕ_A and ϕ_B). Furthermore, the mass of the pseudoscalar A^0, calculated at zero temperature, may be compatible with all the experimental bounds.

Differently from then phases ϕ_A and ϕ_B, which cannot be larger than $\sim (10^{-2} - 10^{-3})$ to avoid the limit on the neutron EDM, the *spontaneous* phase $\delta(T)$ may assume large values, $\delta(T) = \mathcal{O}(1)$, since at zero temperature it vanishes.

Radiative contributions to the parameters $\lambda_{5,6,7}$ and m_3^2 can be obtained either diagrammatically or calculating the one-loop effective potential. We shall follow this second option.

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[2]If CP is broken spontaneously, the mass eigenstates of the scalar sector are no longer eigenstates of CP. However, one of the eigenvalues of the mass matrix is given approximately by (3.31).

The one-loop contribution at finite temperature to the effective potential can be decomposed into the sum of a $T{=}0$ and a $T \neq 0$ term and reads in the 't Hooft-Landau gauge and in the $\overline{DR}$ scheme of renormalization [56]

$$\Delta V_{T=0} = \frac{1}{64\pi^2}\mathrm{Str}\left\{\mathcal{M}(\phi)^4\left(\ln\frac{\mathcal{M}(\phi)^2}{Q^2}-\frac{3}{2}\right)\right\}, \tag{3.32}$$

$$\Delta V_{T\neq 0} = \Delta V^{\rm bos}_{T\neq 0} + \Delta V^{\rm ferm}_{T\neq 0}, \tag{3.33}$$

where $\mathcal{M}^2(\phi)$, with $\phi \equiv (H_1^0, H_2^0)$, is the field dependent squared mass matrix; the supertrace

$$\mathrm{Str} = \sum_i (-1)^{2J_i}\, g_i \tag{3.34}$$

counts all possible particles with a ϕ-dependent mass, spin J_i and degrees of freedom g_i; Q is an arbitrary renormalization scale, and the Q^2-dependence is compensated by that of the renormalized parameters, so that the full effective potential is independent of Q^2 up to next-to-leading order[3]

Defining $a^2_{\rm b,(f)} \equiv \mathcal{M}^2_{\rm b,(f)}/T^2$, where $\mathcal{M}_{\rm b,(f)}$ is the bosonic (fermionic) mass matrix, the $T \neq 0$ contributions may be written as

$$\Delta V^{\rm bos}_{T\neq 0} = T^4\ \mathrm{Str}\left[\frac{1}{24}a_{\rm b}^2 - \frac{1}{12\pi^2}(a_{\rm b}^2)^{3/2} - \frac{1}{64\pi^2}a_{\rm b}^4\ln\frac{a_{\rm b}^2}{A_{\rm b}} - \pi^{3/2}\sum_{l=1}^{\infty}(-1)^l\,\frac{\zeta(2l+1)}{(l+1)!}\,\Gamma\left(l+\frac{1}{2}\right)\left(\frac{a_{\rm b}^2}{4\pi^2}\right)^{l+2}\right], \tag{3.35}$$

$$(a_{\rm b} < 2\pi)$$

$$\Delta V^{\rm ferm}_{T\neq 0} = T^4\ \mathrm{Str}\left[\frac{1}{48}a_{\rm f}^2 + \frac{1}{64\pi^2}a_{\rm f}^4\ln\frac{a_{\rm f}^2}{A_{\rm f}} + \frac{\pi^{3/2}}{8}\sum_{l=1}^{\infty}(-1)^l\,\frac{1-2^{-2l-1}}{(l+1)!}\,\zeta(2l+1)\,\Gamma\left(l+\frac{1}{2}\right)\left(\frac{a_{\rm f}^2}{\pi^2}\right)^{l+2}\right], \tag{3.36}$$

$$(a_{\rm f} < \pi)$$

[3] See ref. [57] for a complete analysis of two-loop corrections to the effective potential.

where $A_{\rm b} = 16A_{\rm f} = 16\pi^2 \exp(3/2 - 2\gamma_E)$, $\gamma_E = 0.5772$, ζ is the Riemann function.

Eqs. (3.35) and (3.36) give an exact representation of the complete one–loop effective potential at finite temperature [58] for $a_{\rm b} < 2\pi$ and $a_{\rm f} < \pi$, respectively.

As long as SUSY is exact, $\lambda_{5,6,7}$ are zero at any order in perturbation theory. So the only renormalization to these parameters may come from the soft SUSY breaking sector. In particular, we find that the dominant contributions are those coming from the gaugino mass terms, $M_{1,2}$, which enter the charginos and neutralinos mass matrices, and by the sfermion mass terms M_{LR} which appear in the stop mass matrices. In the following we will evaluate the one-loop renormalizations to $\lambda_{5,6,7}$ and m_3^2 at finite temperature, including in the effective potential the mass matrices of charginos, neutralinos and stops.

The stop mass matrix reads

$$\mathcal{M}_{\tilde{t}} = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{LR}^{*2} & M_{RR}^2 \end{pmatrix}, \tag{3.37}$$

where

$$\begin{aligned} M_{LL}^2 &= \tilde{M}_Q^2 + \Pi_{\tilde{q}} + h_t^2 \left|H_2^0\right|^2 \\ &+ \left(\frac{g_2^2}{12} - \frac{g_1^2}{4}\right)\left(\left|H_2^0\right|^2 - \left|H_1^0\right|^2\right), \\ M_{RR}^2 &= \tilde{M}_U^2 + \Pi_{\tilde{u}} + h_t^2 \left|H_2^0\right|^2 \\ &- \frac{g_1^2}{3}\left(\left|H_2^0\right|^2 - \left|H_1^0\right|^2\right), \\ M_{LR}^2 &= h_t\left(A_t\, H_2^0 + \mu^*\, H_1^{*0}\right), \end{aligned} \tag{3.38}$$

and

$$\Pi_{\tilde{q}} = \Pi_{\tilde{u}} \simeq \frac{4}{9}\, g_s^2\, T^2, \tag{3.39}$$

are the polarization squared masses of the stops in the limit of very heavy gluinos [16].

Taking $\tilde{M}_Q^2 = \tilde{M}_U^2$ the stop mass matrix takes the convenient form

$$a_t^2 = a_Q^2 \cdot \mathbf{1} + \tilde{a}_t^2 \tag{3.40}$$

where $a_Q^2 \equiv (\tilde{M}_Q^2 + \Pi_{\tilde{q}})/T^2$, $\mathbf{1}$ is the identity matrix, and $\tilde{a}_t^2$ is the field–dependent part of the mass matrix, *i.e.* $\tilde{a}_t^2 \to 0$ as the fields vanish. Summing up all the terms in Eq. (3.35) in $\tilde{a}_t^4$, $\tilde{a}_t^6$ and $\tilde{a}_t^8$, the following contributions are obtained from the stop [29, 30]

$$\Delta m_3^{(s)^2} = +3h_t^2 A_t T a_\mu e^{i\phi_\mu}\left[\frac{1}{8\pi a_Q} + \frac{1}{16\pi^2}\left(\ln\frac{Q^2}{A_b T^2} + \frac{3}{2}\right) + 4\mathcal{B}_4[a_Q^2]\right], \quad (3.41)$$

$$\Delta\lambda_5^{(s)} = -12h_t^4 \frac{A_t^2 a_\mu^2}{T^2} e^{2i\phi_\mu}\left[\mathcal{B}_8[a_Q^2] + \frac{1}{256\pi a_Q^5}\right], \quad (3.42)$$

$$\begin{aligned}\Delta\lambda_6^{(s)} &= -6h_t^2\frac{A_t a_\mu}{T}e^{i\phi_\mu}\left[\frac{3}{4}(g_2^2+g_1^2)\left(\mathcal{B}_6[a_Q^2] - \frac{1}{192\pi a_Q^3}\right)\right.\\ &+ \left.4h_t^2 a_\mu^2\left(\mathcal{B}_8[a_Q^2] + \frac{1}{512\pi a_Q^5}\right)\right], \end{aligned}\quad (3.43)$$

$$\begin{aligned}\Delta\lambda_7^{(s)} &= -6h_t^2\frac{A_t a_\mu}{T}e^{i\phi_\mu}\left[\left(6h_t^2 - \frac{3}{4}(g_2^2+g_1^2)\right)\left(\mathcal{B}_6[a_Q^2] - \frac{1}{192\pi a_Q^3}\right)\right.\\ &+ \left.4h_t^2\frac{A_t^2}{T^2}\left(\mathcal{B}_8[a_Q^2] + \frac{1}{512\pi a_Q^5}\right)\right], \end{aligned}\quad (3.44)$$

where we have defined $a_\mu \equiv |\mu|/T$ and

$$\mathcal{B}_{2n}[a_Q^2] \equiv \pi^{3/2}\sum_{l=\max[1,n-2]}^{\infty}(-1)^l\,\frac{\zeta(2l+1)}{(l+2)!}\,\Gamma\left(l+\frac{1}{2}\right)\binom{l+2}{n}\frac{(a_Q^2)^{l+2-n}}{(4\pi^2)^{l+2}} \quad (3.45)$$

$$n = 2,3,4\ldots \qquad a_Q < 2\pi.$$

The series in (3.45), having terms of alternate signs, may be easily evaluated numerically.

With the choice, $|\mu^2| = |M_1^2| = |M_2^2| = a_\mu^2 T^2$, the chargino and neutralino contribution are obtained in a completely analogous way and read (see also the Appendix of ref. [31] for a complete analysis)

$$\Delta m_3^{(c)^2} = +g_2^2\,T^2\,a_\mu^2\,e^{i\phi_\mu}\left[\frac{1}{8\pi^2}\left(\ln\frac{Q^2}{A_f T^2} + \frac{3}{2}\right) + 8\mathcal{F}_4[a_\mu^2]\right] \quad (3.46)$$

$$\Delta\lambda_5^{(c)} = 8\,g_2^4\,a_\mu^4\,e^{2i\phi_\mu}\mathcal{F}_8[a_\mu^2], \quad (3.47)$$

$$\Delta\lambda_6^{(c)} = \Delta\lambda_7^{(c)} = -\,4\,g_2^4\,a_\mu^2\,e^{i\phi_\mu}\left[3\mathcal{F}_6[a_\mu^2] + 4a_\mu^2\mathcal{F}_8[a_\mu^2]\right], \quad (3.48)$$

and

$$\Delta m_3^{(n)^2} = \frac{(g_2^2+g_1^2)}{2g_2^2}\Delta m_3^{(c)^2}, \qquad \Delta\lambda_i^{(n)} = \frac{(g_2^2+g_1^2)}{2g_2^2}\Delta\lambda_i^{(c)}, \quad i=5,6,7. \tag{3.49}$$

where

$$\mathcal{F}_{2n}[a_\mu^2] = \frac{\pi^{3/2}}{8}\sum_{l=\max[1,n-2]}^{\infty}(-1)^l\,\frac{1-2^{-2l-1}}{(l+2)!}\,\zeta(2l+1)\,\Gamma\left(l+\frac{1}{2}\right)\binom{l+2}{n}\frac{(|a_\mu|^2)^{l+2-n}}{(\pi^2)^{l+2}} \tag{3.50}$$

In ref. [29, 30], it is shown that the conditions for SCPV at finite temperature, *i.e.*

$$\Delta\lambda_5 > 0, \tag{3.51}$$

and

$$\left|\cos\delta(T) = \frac{\bar{m}_3^2 - \Delta\lambda_6 v_1^2(T) + \Delta\lambda_7 v_2^2(T)}{4\Delta\lambda_5 v_1^2(T) v_2^2(T)}\right| < 1 \tag{3.52}$$

where $\bar{m}_3^2 \equiv m_3^2 + \Delta m_3^{2(s)} + \Delta m_3^{2(n)} + \Delta m_3^{2(c)}$ and $\Delta\lambda_i \equiv \Delta\lambda_i^{(s)} + \Delta\lambda_i^{(c)} + \Delta\lambda_i^{(n)}$ $(i=5,6,7)$ can be satisfied, for temperatures around the electroweak scale, in a wide region of the parameter space compatible with the present experimental bound on the mass of the Higgs pseudoscalar. Typical numerical values for $\Delta\lambda_5$ are around 10^{-5}, whereas $\Delta\lambda_6, \Delta\lambda_7 \approx 10^{-4}$. This means that in order to have spontaneous CP violation, $\bar{m}_3^2$ must be of order $\Delta\lambda_6\, v_1^2(T) + \Delta\lambda_7\, v_2^2(T)$ within a 10 percent, see Eq. (3.52).

The fact that $\bar{m}_3^2$ must be small enough to have SCPV does not mean that we get necessarily a very light pseudoscalar, as in the case of zero temperature. Indeed, at $T=0$ it is the quantity $m_3^2 + \Delta m_3^2(T=0)$ which must be small, leading to $M_{A^0} \lesssim 10$ GeV. On the other side, at finite temperature, the quantity which must be small is $\bar{m}_3^2$: since

$$M_{A^0} = \frac{2}{\sin 2\beta}\left[\bar{m}_3^2 - \Delta m_3^2(T \neq 0)\right], \tag{3.53}$$

$\Delta m_3^2(T\neq 0)$ can be large enough for M_{A^0} to avoid the LEP limits.

One-loop contributions to $\lambda_{5,6,7}$ and m_3^2 from the Higgs scalars and the Goldstone bosons are always proportional to m_3^2 and can be small if m_3^2 is small (this does not create problems with the bounds on M_{A^0} since the latter receives large radiative contributions from stops, neutralinos and charginos). However, a negative contribution to λ_5 proportional to $-T/\bar{M}$, where $\bar{M}$ is the smallest eigenvalue of the Higgs mass matrix (with plasma effects taken into account) calculated in the origin, can arise reducing the region of the parameter space where SPCV violation occurs (this effect is analogous to what happens in the next-to-minimal supersymmetric standard model [31] where infrared divergences prevent to have SCPV inside the critical expanding bubbles unless the lightest of the stops has a mass close to its experimental lower bound of 45 GeV). However, in spite of these infrared effects, sufficiently large regions of the parameter space are left where CP can be spontaneously broken at finite temperature.

3.4 A time dependent phase

The spontaneous CP breaking can lead to different cosmological implications according to the nature of the EWPT.

If the latter is of the second order, all the quantities in Eq. (3.52) will evolve smoothly with the temperature, so that there will be a time interval during which the conditions (3.51) and (3.52) are satisfied, and $\cos\delta(T)$ varies from 1 to -1. As the temperature decreases below a critical value $T_{\rm rest}$, the T-dependent quantities in Eq. (3.52) become such that $|\cos\delta(T)| > 1$ and CP is restored.

During the time interval in which $|\cos\delta(T)| < 1$ domain walls separating positive and negative phases can form. Indeed, as long as the coefficients of the scalar potential are taken all real, the two vacua having $\delta(T) = \pm\text{arcos}(|\Delta(T)|)$, with $|\Delta(T)) < 1$, are completely *degenerate* from the energetic point of view.

The evolution time scale of the phases is dictated by the Hubble parameter and, as a consequence, inside each domain out of equilibrium conditions cannot be attained. However, if the degeneracy of two vacua is slightly lifted, domain walls move due to a pressure difference and out of equilibrium occurs inside the domain walls.

We shall analyze this possibility later one, showing that it cannot have relevant consequences as far as the baryogenesis is concerned.

On the other hand, if the EWPT is of the first order and proceeds by bubble nucleation, the temperature keeps constant until all the Universe is in the broken phase, then $\bar{m}_3^2$ and $\Delta\lambda_i$ are fixed during the phase transition, whereas v_1 and v_2 change their values at the critical temperature T_c from zero to $v_1(T_c)$ and $v_2(T_c)$ when a bubble wall passes through a fixed point. Any steady observer will then experience a change in the phase δ from 0 to $\pi(-\pi)$ in a time interval $\tau_H = v_w/L_w \simeq (1-100)/T$ (the time of the passage of the wall) if, as we assume, inside the bubble the modulus of the ratio in Eq. (3.52) is greater than one.

In the following Section we shall investigate the cosmological implications of the two different scenarios outlined above.

Let us only note that, as long as even a small explicit violation of CP is absent, bubbles with equal in modulus, but opposite sign of $\delta(T)$, $\delta(T) = \pm\mathrm{arcos}(|\Delta(T)|)$, are produced in equal number during a first order EWPT by tunneling since they are energetically equivalent. The consequences of this fact for baryogenesis will be discussed in the following. For the time being, let us focus on the fate of bubbles with a definite sign of $\delta(T)$.

3.5 Application of SCPV at finite temperature to spontaneous baryogenesis

As we have discussed in the previous Sections, the evolution of the effective potential as the temperature decreases may induce a non vanishing relative phase between the two Higgs VEV's. Such a new source of CP violation may be relevant for the generation of the baryon asymmetry of the Universe during the EWPT by means of the so called spontaneous baryogenesis scenario described in Chapter 2.

Let us recall that the main underlying idea of this mechanism is that a space-time varying phase δ may induce an interaction of the form

$$\mathcal{L}_{\mathrm{int}} \sim \partial_\mu\delta \; J^\mu. \tag{3.54}$$

If the current J^μ is not orthogonal to the baryonic one, then the interaction term (3.54) shifts the energy levels of baryons relative to antibaryons. The time derivative of the phase δ acts as an effective (charge) chemical potential for the baryon number. If an interaction like that in Eq. (3.54) is active when also baryon number violating interaction are in equilibrium, then the thermodynamic evolution of the system leads to a non zero baryon asymmetry.

An interaction of the type (3.54) emerges in the MSSM if one chooses the fermion basis in such a way that at any time and at any point in space the fermion masses are real. This implies that a space-time dependent rotation has to be done on the fermion fields in order to remove the phase δ from the Yukawa couplings

$$h_d\, v_1\, \mathrm{e}^{-i\delta_1}\, dd^c + h_t\, v_2\, \mathrm{e}^{-i\delta_2}\, uu^c + h_e\, v_1\, \mathrm{e}^{-i\delta_1}\, ee^c + \mathrm{h.c.} \tag{3.55}$$

at the price of introducing new interactions of the form (3.54) coming from the fermionic kinetic terms.

The minimization of the effective potential can only fix the combination $\delta = \delta_1 + \delta_2$, whereas the orthogonal combination corresponds to the Goldstone degree of freedom. A variation of δ is shared between δ_1 and δ_2 according to the variation

$$d\delta_1 = \frac{v_2^2(T)}{v_1^2(T) + v_2^2(T)} d\delta, \quad d\delta_2 = \frac{v_1^2(T)}{v_1^2(T) + v_2^2(T)} d\delta, \tag{3.56}$$

which is orthogonal to the unphysical Goldstone mode absorbed by the gauge fields.

The phases δ_1 and δ_2 can be removed from the Yukawa interactions (3.55) by a space-time dependent rotation of the fermionic fields

$$\begin{aligned} d &\to \mathrm{e}^{i\omega}\, d, \quad d^c \to \mathrm{e}^{i(\delta_1-\omega)}\, d^c, \\ u &\to \mathrm{e}^{i\omega}\, u, \quad u^c \to \mathrm{e}^{i(\delta_2-\omega)}\, u^c, \\ e &\to \mathrm{e}^{i\bar{\omega}}\, e, \quad e^c \to \mathrm{e}^{i(\delta_1-\bar{\omega})}\, e^c, \\ \nu &\to \mathrm{e}^{i\bar{\omega}}\, \nu, \end{aligned} \tag{3.57}$$

where $\omega(x)$ and $\bar{\omega}(x)$ are arbitrary phases.

We know that making a generic rotation on a Dirac field ψ

$$\psi \to \exp\left[i\,(a + b\gamma_5)\,\Theta\right]\, \psi, \tag{3.58}$$

where a and b are constants, generates the additional term in the Lagrangian

$$\Delta\mathcal{L} = \bar{\psi}\, m\, \left[\exp\left(2i\Theta b\gamma_5\right) - 1\right] \psi + \bar{\psi}\gamma^{\mu}\,(a + b\gamma_5)\,\psi\partial_{\mu}\Theta. \tag{3.59}$$

The axial rotation in the first piece will eventually cancel the complex phase induced in the Higgs potential. The second piece exhibits the conservation or otherwise of the associated current via the topological terms. There are two such contributions to the effective potential induced by the above rotation (3.57).

The first type is the anomaly term

$$\Delta\mathcal{L}_{\rm CS} = i\Theta\,(a-b)\,\frac{F_L\tilde{F}_L}{32\,\pi^2} - i\Theta\,(a+b)\,\frac{F_R\tilde{F}_R}{32\,\pi^2}, \tag{3.60}$$

where $F_{L(R)}$ is the gauge field coupling to $\psi_{L(R)}$. Such terms are temperature independent, as shown in ref. [59].

One of the two arbitrary phases ω and $\bar{\omega}$ can be eliminated by requiring that the rotation (3.57) does not introduce extra $SU(2)_L$ terms of the Chern-Simons type in the Lagrangian. This requires

$$\omega = -\frac{1}{3}\,\bar{\omega}. \tag{3.61}$$

In general, even is such a choice is made, there will be a second kind of contributions to the Chern-Simons like terms arising from a Taylor expansion in powers of momenta of non local one-loop triangle diagram at finite temperature [45, 83]

$$\begin{aligned} \Delta\mathcal{L}_{{\rm CS},T} &= ib\Theta\Lambda(T)\left(\frac{F_L\tilde{F}_L}{32\,\pi^2}+\frac{F_R\tilde{F}_R}{32\,\pi^2}\right), \\ \Lambda(T) &= \frac{14}{3}\,\zeta(3)\,\frac{m^2}{\pi^2\,T^2}\left(1-\frac{93\,\zeta(5)}{56\,\zeta(3)}\,\frac{m^2}{\pi^2\,T^2}+\cdots\right). \end{aligned} \tag{3.62}$$

The effect of these terms was studied in [83] where it was shown that their contributions to the baryon asymmetry is negligible with respect to the contribution coming from the charge potential[4] and we shall not consider them any longer in this Chapter.

Making the rotation (3.57) with the choice (3.61), the fermionic kinetic terms give rise to the derivative couplings

$$\mathcal{L}_{\rm int} = \partial_\mu\bar{\omega}\,J^\mu_{B-L} + \partial_\mu\delta_2\,J^\mu_{u^c} + \partial_\mu\delta_1\,J^\mu_{d^c} + \partial_\mu\delta_1\,J^\mu_{e^c}. \tag{3.63}$$

These currents correspond, respectively, to the $B-L$ vectorial current, to the antiquark up and down and to the antilepton currents.

[4]This statement was true when CPR wrote [29]. Nowadays, since CPR have shown that several suppression factors were missing in the original treatment of the charge potential mechanism, the importance of these terms should be reconsidered.

The first coupling is not suitable for SB since it introduces an energy splitting between the states with positive and negative $B-L$ and, since all the interactions of the SM conserve $B-L$, transitions between the two levels are forbidden. The other three couplings, however, involve charge numbers which are not conserved if the respective Yukawa interactions are active, so that they can play a role in generating the baryon asymmetry together with anomalous B violating processes processes.

Assuming for the time being that all the Yukawa interactions are in equilibrium, the number density of each species is given by

$$n_i = k_i \left(\dot{\theta}_i + q_i^{B-L}\,\mu_{B-L} + q_i^{Q}\,\mu_Q\right)\frac{T^2}{6}, \tag{3.64}$$

where θ_i represents the rotation phase of the i^{th} species in Eq. (3.57).

Imposing the constraints $\langle B-L\rangle = \langle Q\rangle = 0$, one gets

$$n_B^{\text{EQ}} = n_L^{\text{EQ}} = \frac{3}{137}\left(99\,\dot{\delta}_1 + 131\,\dot{\delta}_2\right)\frac{T^2}{6}. \tag{3.65}$$

However during the EWPT the simplifying assumptions leading to eq. (3.65) have to be revisited. In particular, the only right-handed fermion which might be in thermal equilibrium is the top (or, for large values of $\tan\beta$, also the right-handed bottom).

Moreover, since the typical time scale of the CP violation in this system is $(10^{-2}-1)\ \text{GeV}^{-1}$, the rate for anomalous baryon number violation is too slow to allow baryon number to reach its equilibrium density (3.65). For such reasons it seems more appropriate to make use of the master equation (2.3)

$$\dot{n}_B = -9\,\Gamma_{\text{sp}}\frac{\mu_B}{T}. \tag{3.66}$$

Introducing the chemical potentials μ_{B-L}, μ_Q and μ_B and setting the net densities of the light, right-handed particles to zero and $\langle B-L\rangle = \langle Q\rangle = \langle B\rangle = 0$, μ_B reads

$$\mu_B = \frac{72}{209}\,\dot{\delta}_1. \tag{3.67}$$

Eq. (3.66) will be integrated in Section **3.7** when we will estimate the baryon asymmetry produced under the hypothesis of a first order EWPT. Before turning to that scenario, which is the more promising from the point of view of baryogenesis, we want to make some comments on the cosmological implications of the spontaneous CP violation in the MSSM when the phase transition is of the second order.

3.6 Second order EWPT and domain walls

If we assume that the EWPT is of the second order, as expected for large values of the mass of the lightest Higgs, the spontaneous CP violation will lead to the formation of domain walls separating regions with opposite signs of the phases δ's, whose cosine is given by Eq. (3.26) [61].

Domain walls are an unavoidable prediction of models in which global discrete symmetries are spontaneously broken [62]. Since their energy density grows faster than that of the radiation, at a certain temperature they begin to dominate the evolution of the Universe, destroying its homogeneity and the isotropy of the relic radiation. However, if the discrete symmetry is broken only in a limited temperature interval, as for example in the scenario proposed in ref. [63], then the domain walls will decay as the symmetry is restored. If the temperature of symmetry restoration is higher than that at which the walls begin to dominate over radiation any dangerous consequence for the evolution of the Universe is avoided.

This is precisely what happens in the present case, since CP is restored at a temperature that is of the order of $1 - \lambda_5/\lambda_6$ times the temperature at which it is broken, *i.e.* $T_{\rm rest} = \mathcal{O}(100)$ GeV, whereas the temperature at which the walls begin to dominate is about $10^{-4}(v/100\ {\rm GeV})^{3/2}$ MeV, where we have assumed $v_1 \sim v_2 = v$.

Domain walls may also play a role in baryogenesis, as for instance in the left-right symmetric model considered in refs. [64, 65]. In principle, also in the model we are considering one could think to investigate on the possibility of implementing the spontaneous baryogenesis mechanism. Indeed, the phase changes from δ on one side of the wall to $-\delta$ on the other side, so that there is a net change of 2δ at any point when the wall passes by it.

The motion of the wall can be obtained if the degeneracy between the two vacua is slightly lifted, for example by introducing small *explicit* phases ϕ.

At the temperature at which the difference in energy density between the two vacua equals the energy density of the wall itself, which is of the order of $(\lambda_5 \phi M_{\rm Pl}^{1/2} v^{5/2})^{1/3} \sim$ 100 GeV for $\phi \sim 10^{-3}$, the walls start moving in the direction of the energetically disfavoured domain.

A necessary condition to make the spontaneous baryogenesis scenario work is however that the baryon number violating processes are active inside the walls and suppressed outside, otherwise the baryon asymmetry would be wiped out. Recalling the condition in Eq. (1.12), this could be achieved if for some temperature interval the ratio $v(T)/T$ were sensibly less than unity inside the wall and greater outside. Unfortunately this is not the case in this model, since the difference between the two VEV's is of the order of $(\lambda_6/g^2)^{1/2}\,v \sim 10^{-2}\,v$, consequently the B violating processes freeze out inside the wall and outside it almost contemporarily.

3.7 First Order EWPT and Spontaneous Baryogenesis

We now turn to the discussion of the consequences of the SCPV in the case in which the EWPT is of first order and proceeds via bubble nucleation. In our scenario the spontaneous baryogenesis mechanism is implemented through the change of the VEV's phase δ inside the bubble walls where the VEV's moduli change from 0 to $v_{1,2}(T_c)$.

If one approximates $\Gamma_{\rm sp}$ by a step function for $v \lesssim v_{\rm co}$, as explained in Section **2.1**, and takes into account that the temperature during the EWPT keeps constant, one can easily integrate the master Eq. (2.3) and find the baryon number density to entropy ratio induced by a *single* bubble

$$B \equiv \left(\frac{n_B}{s}\right)_{\rm local} \simeq 3\times 10^{-7}\left(\frac{\Delta\delta}{\pi}\right). \tag{3.68}$$

The local baryon number density to entropy ratio (3.68) generated by the passage of the wall would be erased if the anomalous interactions were still active in the true vacuum. If in the middle of the bubble $v(T)/T \gtrsim 1$, the sphaleron rate is safely suppressed.

This bound translates into an upper limit on the mass of the lightest higgs scalar in the MSSM. As we have discussed in the introduction to this Chapter, it is not clear whether this limit is or not compatible with LEP data [50].

In the following we assume that the first order phase transition is sufficiently strong so that the bound (1.12) is satisfied. Moreover, we will define the bubble wall as the region in which sphalerons are active, that is, in which $0 < v(T)/T \lesssim 1$.

Since the temperature keeps constant throughout the phase transition, the change of the phase δ at a given point is induced by the change of the VEV's $v_{1,2}(T)$ as the bubble wall passes through that point.

More precisely, one can easily infer from Eq. (3.26) that, as the VEV's change from $v_{\rm min}$ to $v_{\rm max}$, where

$$\begin{aligned} v_{\rm min}^2 &= \frac{\bar{m}_3^2}{\cos^2\beta}\,\frac{1}{\lambda_6+\lambda_7\tan^2\beta-4\lambda_5\tan\beta}, \\ v_{\rm max}^2 &= \frac{\bar{m}_3^2}{\cos^2\beta}\,\frac{1}{\lambda_6+\lambda_7\tan^2\beta+4\lambda_5\tan\beta}. \end{aligned} \tag{3.69}$$

The relative phase δ changes from 0 to $\pm\pi$ (we are assuming here that $\tan\beta$ keeps constant at any temperature) . If $v_{\rm max}/T \lesssim 1$, the variation of δ lies entirely inside the bubble wall, so that $\Delta\delta$ in Eq. (3.68) may be as large as π.

In refs. [29, 30] it is shown that the requirement $v_{\rm max}/T \lesssim 1$ is fulfilled in a phenomenologically acceptable region of the parameter space.

On the other hand, from Eq. (3.69) we read that this condition implies $0 < \bar{m}_3^2 \lesssim \lambda_{6,7}T^2$, which in turn induces a strong correlation between the parameters $m_{A^0}^2$ and $\tan\beta$ [29, 30].

In Fig. 1 we see that this requirement fixes the pseudoscalar mass with an accuracy of roughly 10 percent, which gives a measure of the *fine tuning* needed by this mechanism.

In Fig. 2 we have allowed the other free parameters of the model to vary in the range allowed by the condition (3.25) [29, 30], and we have obtained the absolute upper and lower bounds to the pseudoscalar mass. The dashed line represents the experimental lower bound [50], so that we can also see that values of $\tan\beta$ less than $\sim$7 are excluded.

Note that in the case of explicit breaking of CP like that considered in ref. [51] we have $\Delta\delta = \mathcal{O}(1)\delta_{CP}$ where δ_{CP} is bounded to values less than $10^{-2} - 10^{-3}$ by the experimental limits on the electric dipole moment of the neutron, which do not apply to the present case. Thus, we can read from Eq. (3.68) that the baryon production by the *single* bubble with the mechanism discussed in this Chapter is at least two or three orders of magnitude more efficient than in the case of an explicit CP violation.

As we have previously discussed, for any set of values of the temperature and the other parameters of the model, the effective potential has a double degeneracy in $\pm\delta$ as long as Eq. (3.26) is satisfied and there are no new complex phases other than the one in the Cabibbo-Kobayashi-Maskawa matrix.

Since we are assuming that the Higg's VEV inside the bubble is greater than $v_{\rm max}$, *i.e.* CP is restored, the vacuum in the broken phase is now unique, unlike the situation discussed in the previous Subsection.

On the other hand, inside the bubble walls, where the VEV is between $v_{\rm min}$ and $v_{\rm max}$, the condition (3.26)) is satisfied, and then the ambiguity between the two signs is present.

Stated in other words, the phase δ may follow two different paths as the VEV's change inside the walls, namely, from 0 to π or from 0 to $-\pi$. This results in two possible signs for $\dot{\delta}$, and then to the nucleation of two different kinds of bubbles, say *plus* and *minus* bubbles which, according to Eq. (3.68), create baryons of opposite signs.

Since there is no way to prefer one kind of bubble relative to the other one, the net baryon asymmetry, averaged over the entire Universe, will be zero.

In the following Section we show that allowing the soft SUSY breaking parameters and the μ parameter to take complex values, the effective potential takes on a $\sin\delta$ dependence, and then one sign of δ becomes energetically favourite with respect to the opposite one. In this way the two kinds of bubbles have slightly different surface tensions, and then free energies, so that their nucleation rates are no more equal. As a result, an abundance of one kind of bubbles relative to the other, and then a non zero average baryon asymmetry, is achieved. We will also find that the job may be done by explicit phases as little as $10^{-5} - 10^{-6}$, which give a negligible contribution to the neutron EDM.

3.8 Choosing the good bubbles

We recall that, defining

$$A = |A|\mathrm{e}^{i\phi_A}, \quad B = |B|\mathrm{e}^{i\phi_B} \tag{3.70}$$

where A and B are the trilinear and bilinear soft SUSY breaking parameters it is always possible to rotate the phase of the parameter μ taking

$$\mu = |\mu| e^{-i\phi_B}, \tag{3.71}$$

so that $m_3^2 = \mu B$ is real.

A and μ enter the one-loop corrections for $\bar{m}_3^2$ and $\lambda_{5,6,7}$ (see Section **3.3**), so these parameters take phases which, if $\phi_A, \phi_B \ll 1$, are given by

$$\begin{aligned} \phi_3 &= \frac{|\Delta m_3^{(s)}|}{|\bar{m}_3^2|}\phi_A - \frac{|\Delta m_3^{(s)}| + |\Delta m_3^{(n)}| + |\Delta m_3^{(c)}|}{|\bar{m}_3^2|}\phi_B, \\ \phi_i &= \frac{|\Delta\lambda_i^{(s)}|}{|\lambda_i|}\phi_A - \phi_B, \qquad i = 5,6,7, \end{aligned} \tag{3.72}$$

where we have defined

$$\bar{m}_3^2 = |\bar{m}_3^2|\, \mathrm{e}^{-i\phi_3}, \quad \lambda_5 = |\lambda_5|\, \mathrm{e}^{-2i\phi_5} \text{ and } \lambda_{6,7} = |\lambda_{6,7}|\, \mathrm{e}^{-i\phi_{6,7}}, \tag{3.73}$$

and approximated $\sin\phi_{A,B} \simeq \phi_{A,B}$.

The scalar potential now has also terms depending on $\sin\delta$, which are given by

$$\begin{aligned} \Delta V(\sin\delta) &\simeq 2\sin\delta \; v_1(T) \; v_2 T) \times \\ & \Big[|\bar{m}_3^2| \; (\phi_5 - \phi_3) - |\lambda_6| \; v_1^2(T) \; (\phi_5 - \phi_6) \\ &- |\lambda_7| \; v_2^2(T) \; (\phi_5 - \phi_7)\Big] \, . \end{aligned} \tag{3.74}$$

For fixed $v_{1,2}(T)$ there are two minima in δ. The potential (3.74) slightly removes the degeneracy between them, that is, it distinguishes between *plus* and *minus* bubbles.

We now come to an estimation of the abundance of one type of nucleated bubbles relative to the other one, due to the breaking of the degeneracy. Inside a comoving volume coincident with the horizon volume , the ratio between the number of plus and minus bubbles is

$$\frac{\mathrm{N}_+}{\mathrm{N}_-} = \mathrm{e}^{-\Delta \mathrm{F}/T}, \tag{3.75}$$

where ΔF is the difference between the free energies of the two kinds of critical bubbles.

The free energy of a bubble of radius R is given by the sum of the volume and surface energies[5]

$$\mathrm{F} = \frac{4\,\pi}{3}\,R^3\,V - 4\,\pi\,R^2\sigma, \tag{3.76}$$

where V is the density energy of the true vacuum and σ the surface energy density.

Reminding that the radius of a critical bubble, *i.e.* the minimum radius that a bubble must have in order to grow indefinitely, is given by $R_c = 2\sigma/V$, we can rewrite the exponential in Eq. (6.4) as

$$\frac{\Delta \mathrm{F}}{T} = \frac{\mathrm{F}(R_c)}{T}\left(\frac{3\Delta\sigma}{\sigma} - \frac{2\Delta V}{V}\right). \tag{3.77}$$

where $\Delta\sigma$ and ΔV are the differences in surface tension and volume energy density of the two kinds of bubbles.

The ratio F$/T$ during the bubble nucleation was evaluated in ref. [66] in the context of the Standard Model.

In that paper it was found that F$/T$ varies from $\sim$ 130 when the first bubble is nucleated into an horizon volume to $\sim$ 100 when all the Universe has been filled up by bubbles.

The above results are stable within a few percent as the Higgs mass is changed from 50 to 100 GeV [66] so we believe that these estimations may be valid for the MSSM as well.

Since we are supposing that inside both kinds of bubbles the condition (3.26) is not satisfied, the contribution to ΔF$/T$ coming from $\Delta V/V$ is zero.

In fact, in this case the minimization condition relative to the phase δ gives only one solution $\delta = \mathcal{O}(\phi_i)$ (i=3,5,6,7) and the vacuum inside the two kinds of bubbles is the same.

On the contrary, the surface tension contribution will be slightly different. A

[5]Rigously speaking, the expression for the free energy is only valid in the thin wall limit and it is disputable whether this limit is applicable to the MSSM case. However, since we are only interested in the relative difference of the free energies ΔF$/T$, and not the absolute value of ΔF, we are confident that the error be negligible.

convenient expression for σ is given by [66]

$$\sigma \approx \int_{v(R-\Delta R)}^{v(R+\Delta R)} \sqrt{2V(v)}\, dv, \tag{3.78}$$

where ΔR is the thickness of the wall.

Using the fact that inside the bubble wall $dv/dR = \sqrt{2V}$, σ can be approximated by $4\bar{V}\Delta R$, where $\bar{V}$ is the averaged value of V inside the bubble wall.

If, for instance, we take $\phi_A \ll \phi_B$ (this assumption is not restrictive) a straightforward calculation gives

$$\begin{aligned} \frac{\Delta \mathrm{F}}{T} &= \frac{\mathrm{F}}{T}\frac{\Delta \mathrm{F}}{\mathrm{F}} = 3\,\frac{\mathrm{F}}{T}\frac{\Delta\sigma}{\sigma}, \\ &\simeq 10\,\frac{\mathrm{F}}{T}\,\frac{m_{A^0}^2}{T_2^2}\,\frac{\cos^2\beta}{h_t^2}\,\phi_B, \end{aligned} \tag{3.79}$$

where $T_2 \sim 100$ GeV is the temperature at which the effective potential develops a flat direction in the origin, and h_t is the top Yukawa coupling.

The relative abundance of one kind of bubbles on the other one will give a global baryon number density to entropy ratio

$$B = \left(\frac{n_B}{s}\right)_{\rm local} \frac{\mathrm{N}_+ - \mathrm{N}_-}{\mathrm{N}_+ + \mathrm{N}_-} \simeq \left(\frac{n_B}{s}\right)_{\rm local} \frac{\Delta \mathrm{F}}{T}, \tag{3.80}$$

where $(n_B/s)_{\rm local}$ is given in Eq. (3.68).

Remembering that F/T is $\mathcal{O}(100)$, a value of B of order 10^{-10} can be achieved with a phase $\phi_B \simeq (10^{-5} - 10^{-6})$.

Such a small phase makes the supersymmetric contribution to the CP violation phenomenology unobservable. Moreover, this value is much smaller than that required in previous analysis on supersymmetric spontaneous baryogenesis which makes use of explicit phases as the only source of CP violation [51], whereas in our mechanism explicit phases are only needed to achieve an asymmetry in the number of *plus* and *minus* bubbles, while the dominant source of CP violation is the spontaneous one inside the bubble wall.

3.9 Finale

In the present Chapter we have investigated the role of spontaneous CP breaking at finite temperature in the MSSM [29, 30] for the generation of the baryon asymmetry at the electroweak scale.

We have shown that a space-time dependent relative phase between the two Higgs fields may act as a charge potential for the baryon number, along the lines of the SB mechanism.

This scenario requires a sufficiently strong first order EWPT, proceeding via bubble nucleation. This condition seems to be very critical in the MSSM, however a thorough analysis covering all the parameter space is still needed. In particular, in the case of a light pseudoscalar mass, which is relevant for SCPV, the two masses of the neutral scalar Higgses are not very different, and it is not possible to reduce the minimization of the scalar potential to a one-dimensional problem, as it is usually done [16].

A further source of uncertainty of the present results comes from the evaluation of the rate of B violating anomalous processes inside the bubble walls, where spontaneous baryogenesis is effective, and of the thickness of the walls themselves. It seems that in the MSSM, the walls are sufficiently thick to ensure equilibrium conditions for sphaleron-like transitions. We believe that these uncertainties might lead to an overestimation of the BAU by roughly one order of magnitude.

In general, the extra CP violation in the MSSM relative to the SM gives rise to *fine tuning* problems. Namely, the experimental bound on the neutron EDM, implies that the explicit phases must be smaller than $\sim 10^{-3}$, instead of their *natural* values $\phi \sim 1$. In the case of spontaneous CP violation *at zero temperature*, the fine tuning is even stronger: in order to have a *spontaneous* phase $\delta \sim 10^{-3}$ one must require that the ratio in Eq. (3.26) differs from unity by less than 10^{-6} [67].

In the scenario we have investigated the fine tuning is not so stringent. Indeed, we have found that in order to generate the observed amount of B, the *explicit* phases may be as small as 10^{-5}.

Numerical analysis indicate phases (at least) one order of magnitude too small are induced by the Yukawa couplings through the Renormalization Group Equations with initial real soft SUSY breaking parameters at the SUSY breaking scale [68]. Once again, one should ask for a new source of CP violation beyond the KM phase. Indeed, it has been recently argued [69] that in most grand unified supersymmetric theories, renormalization of the soft parameters between the Planck mass and the scale of grand unification will induce phases of order $(10^{-2} - 10^{-3})$ providing a new source of CP violation into the low energy effective theory.

We have also shown that, since the spontaneous phase δ disappears as T goes to zero, it does not receive any constraint from the neutron EDM.

On the other hand, as we have discussed in Section. **3.7**, the requirement that CP is violated inside the bubble walls at $T \sim 100$ GeV, constrains $\overline{m}_3^2$ to be less than $\sim \lambda_{6,7}\, T^2$. This condition is able to fix the mass of the pseudoscalar with an uncertainty of order of 10 GeV, as is shown in Fig. 1, so the fine tuning needed in the present scenario is only at the 10^{-1} level.

The upper bounds on the pseudoscalar mass shown in Fig. 2 are well below 100 GeV., so the next experimental results from LEP2 might be able to rule out the possibility of generating the BAU through spontaneous CP violation at the electroweak scale independently from the uncertainties coming from the knowledge of the EWPT. Of course, this conclusion might be taken *cum grano salis*: as it will become clear in the following, transport phenomena play a crucial role in the SB mechanism and all the results presented in this Chapter should be revisited. However, our feeling is that no dramatic quantitative changes can be expected when taking into account the new effects we are going to explain in next Chapter.

Chapter 4

The medieval era

In 1994 several authors reconsidered the spontaneous baryogenesis mechanism for the production of the baryon asymmetry, pointing out different disturbing drawbacks which might invalidate the whole picture.

Before launching into the specifics of these criticisms, let us remind the reader some crucial aspects of the adiabatic scenario to make the objections clear.

Given a fermion mass term of the form

$$\psi_i^T \, C \, M_{ij}(x_\mu) \, \psi_j + \text{h.c.}, \tag{4.1}$$

where we take all fermions left-handed and C is the charge conjugation matrix, we can make a space-time dependent unitarity change of basis on the fermions

$$\psi_i \to U_{ij}(x_\mu) \, \psi_j, \tag{4.2}$$

to make the fermion masses everywhere real, positive and diagonal; however the space-time dependence of U requires that we replace the kinetc energy terms in the Lagrangian by

$$\mathcal{L}_{\rm KE} \to \mathcal{L}_{\rm KE} + \bar{\psi}\gamma^\mu \left(U^\dagger \, i \, \partial_\mu \, U\right) \psi = \mathcal{L}_{\rm KE} + \left(U^\dagger \, i \, \partial_\mu \, U\right) \, J^\mu. \tag{4.3}$$

Note that, since U is a unitary matrix, $U^\dagger \, \partial_\mu \, U$ may be written

$$U^\dagger \, \partial_\mu \, U = i \, \partial_\mu \sum_a \alpha_a(x_\mu) \, \mathrm{t}_a, \tag{4.4}$$

where for N_f fermions the t's are generators of $U(N_f)$, and the functions $\alpha_a(x_\mu)$ are defined by the above Equation.

Thus the Lagrangian with the mass term (4.1) is equivalent to a Lagrangian with a real diagonal mass term, but also containing a term

$$-\sum_a \partial_\mu \alpha_a(x_\mu)\, \bar{\psi}\, \mathrm{t}_a \gamma^m u\, \psi. \tag{4.5}$$

If the transformation (4.2) has a gauge anomaly there will be also a modification of the Lagrangian

$$\sum_\beta \theta\, F_\beta \tilde{F}_\beta \rightarrow \sum_{a,\beta} \left(\theta + \frac{g_\beta^2}{16\,\pi^2}\, \alpha_a \,\mathrm{tr}\, \mathrm{t}_a\, \mathrm{t}_\beta^2 \right)\, F_\beta \tilde{F}_\beta, \tag{4.6}$$

where the t_β's are the gauge generators of the left-handed fermion representation and the F_β's are the gauge field strength.

In the original paper [28] CKN chose a transformation free from anomalies in order to get rid of the therm (4.6).

As already noted several times, the shift term in the kinetic term of the fermions in Eq. (4.3) was understood as a charge potential. What effect does this charge potential on a system? There are two possibilities:

- If there is a charge potential for an exactly conserved charge (*e.g.* electric charge or $B - L$) one can ignore it. Although it looks like the system could lower its free energy by producing a net charge, charge conservation imposes a zero charge constraint. This can be easily seen by integration by part the shift term in Eq. (4.3).
- Charge potentials for non conserved charges will lead to an asymmetry in the rates between processes which create and destroy the charges, until the system reaches its thermal equilibrium. For instance, if there is a small charge potential $\dot{\theta}_B$ for the baryon number density, for a system starting with no net quantum numbers, the constraint of zero net baryon number can be implemented by introducing a baryon chemical potential $\mu_B = -\dot{\theta}_B$. The chemical potential is just the force of constraint on the system, *i.e.* the derivative of the free energy density with respect to the baryon number. A net baryon asymmetry is then created through sphalerons by means of Eq. (2.3). In general, anomalous weak baryon number violation processes can be affected by a potential for any charge generator whose trace over left-handed fermion

weak doublets is non zero or by a non conserved charge current orthogonal to the baryonic current (in such a case, the charge currents introduces an energy splitting between the states with positive and negative charge numbers and baryon number is violated separately in each energy level; this baryon number violation can then be communicated to the other energy level since the charge current is not conserved).

In the original paper [28] the rotation to make the Yukawa couplings real was taken to be proportional to the *fermionic hypercharge* Y_f.

The key point here is that the fermionic hypercharge current $J^{\mu}_{Y_f}$ inside the bubble wall is violated only by the right-handed top-Higgs scatterings (the other right-handed fermions are out of equilibrium). From what said above, one should expect that the final result for the baryon asymmetry to be vanishing in the limit $h_t \to 0$ or $n = 0$, n being the number of light scalars.

The dependence upon h_t was not present in the original result [28], however this is certainly correct when assuming the right-handed top t_R interactions fast; had one written a Boltzmann like Equation for t_R, this dependence would have been recovered.

Nevertheless, as one can realize looking at Eq. (2.18), in their original work CKN did not found a vanishing baryon asymmetry for $n = 0$. The reason is they imposed the constraint $\langle Q \rangle = 0$, instead $\langle Y \rangle = 0$. As tacitly assumed by CKN in their original paper [28] (since there was no any evidence of suppression factors proportional to the Higgs VEV $v(T)$), baryons were thought to be efficiently produced while the Higgs VEV is small. As a consequence, it makes more sense to treat the system as being in the unbroken phase , where one can constrain $\langle Y \rangle = 0$ and then the nonabelian charges like Y_3 are automatically conserved. Alternatively one should constrain both $\langle Q \rangle$ and $\langle Y_3 \rangle$ to be zero. In the broken phase one should constrain $\langle Q \rangle = 0$.

Indeed, if we repeat the CKN calculation assuming Y, B and $B - L$ as the approximately conserved charges, the following system is obtained

$$\begin{aligned}
\langle Y \rangle &\propto 23\,\langle \dot{\theta} \rangle + (23 + 3n)\,\mu_Y - 2\,\mu_{B-L} + 4\,\mu_B = 0, \\
\langle B - L \rangle &\propto 8\,\langle \dot{\theta} \rangle + 8\,\mu_Y + 13\,\mu_{B-L} + 4\,\mu_B = 0, \\
\langle B \rangle &\propto \langle \dot{\theta} \rangle + \mu_Y + 2\,\mu_{B-L} + 2\,\mu_B = 0,
\end{aligned} \tag{4.7}$$

which, when solving in μ_B, gives the correct dependence $\dot{B} \propto \mu_B \propto \langle \dot{\theta} \rangle\, n$. Indeed, in Eq. (4.7) taking $n = 0$ is equivalent to reduce the number of unknowns from four $(\dot{\theta}, \mu_Y, \mu_{B-L}, \mu_B)$ to three $(\dot{\theta} + \mu_Y, \mu_{B-L}, \mu_B)$.

Thus, what one should take in mind is that in the original paper by CKN [28] and in all the following papers, the hypercharge fermionic current was considered and, consequently, the net baryon asymmetry was vanishing in the limit of decoupling the right-handed top quark t_R from the system.

Let us now analyze the three criticisms moved by several groups to the spontaneous baryogenesis scenario.

4.1 Why QCD sphalerons should not make baryons

In ref. [33] Giudice and Shaposhnikov (GS) considered the effect of non-perturbative chirality breaking transitions due to strong interactions on the spontaneous baryogenesis mechanism.

Indeed, it is well known that the axial vector current of QCD have a triangle anomaly, therefore one can expect axial charge violation due to topological transitions analogous to the sphaleronic transitions of the electroweak theory

$$\frac{dQ_5}{dt} = -\frac{12 \cdot 6}{T^3} \Gamma_{\text{QCD}}\, Q_5, \tag{4.8}$$

where Q_5 is the axial charge, the factor of 12 comes from the total number of quark chirality states, the factor of 6 from the relation between the asymmetry in the quark number density and the chemical potential $n_i \sim (\mu_i\, T^2/6)$.

The rate of these processes at hight temperature may be estimated [70] as

$$\Gamma_{\text{QCD}} = \frac{8}{3} \left(\frac{\alpha_S}{\alpha_W}\right) \Gamma_{\text{sp}} = \frac{8}{3}\, \kappa\, (\alpha_S\, T)^4\,, \tag{4.9}$$

where α_S is the strong fine structure leading to the characteristic time of order

$$\tau_{\text{QCD}} \simeq \frac{1}{192\, \kappa\, \alpha_S^4\, T}. \tag{4.10}$$

Since $\kappa = (0.1-1)$, we see that τ_{QCD} is comparable to the time of passage of the bubble wall, and might be even smaller. Hence GS concluded that strong QCD sphaleronic transitions must be taken into account.

The effect of QCD sphalerons may be represented by the operator

$$\Pi_{i=i}^{3}(u_L\, u_R^\dagger\, d_L\, d_R^\dagger)_i \tag{4.11}$$

where i is the generation index. Assuming that these processes are in equilibrium, we get the following chemical potentials equation

$$\sum_{i=1}^{3}(\mu_{u_L^i} - \mu_{u_R^i} + \mu_{d_L^i} - \mu_{d_R^i}) = 0. \tag{4.12}$$

Eq. (4.12) contains the chemical potentials for *all* the quarks, and imposes that the total *right-handed* baryon number is equal to the total *left-handed* one. In other words, including strong QCD sphalerons allow the generation of the right-handed bottom quark as well as first and second family quarks.

GS followed the original work by CKN [28] and considered a fermionic hypercharge potential. Assuming all the fermions in equilibrium inside the bubble wall (this assumption, however, is not crucial for their final conclusion), they wrote down the particle densities in terms of the time derivative of the phase $\langle\dot{\theta}\rangle$ and of the six chemical potentials associated to the conserved charges Y, Y_3 and A_i, $i = 1, ..., 4$ where

$$\begin{aligned}
A_1 &= \sum_{i=1}^{2} \bar{q}_L^i\gamma^\mu q_L^i + 2\,R - 2\,L_L, \\
A_2 &= \bar{t}_L\gamma^\mu t_L + \bar{t}_R\gamma^\mu t_R + R/2 - L_L, \\
A_3 &= \sum_{i=1}^{3} \bar{D}_R^i\gamma^\mu D_R^i - 3\,R/2, \\
A_4 &= \sum_{i=1}^{3} \bar{E}_R^i\gamma^\mu E_R^i, \\
R &= \sum_{i=1}^{2} \bar{U}_R^i\gamma^\mu U_R^i, \\
L_L &= \sum_{i=1}^{2} \bar{L}_L^i\gamma^\mu l_L^i.
\end{aligned} \tag{4.13}$$

Imposing

$$\langle A_i\rangle = \langle Y\rangle = \langle Y_3\rangle = 0, \tag{4.14}$$

the authors found

$$\langle B\rangle = \frac{2}{5}\,\langle B-L\rangle. \tag{4.15}$$

Thus, GS concluded that, assuming a vanishing $\langle B-L\rangle$ charge, the net baryon asymmetry would be vanishing in the presence of strong QCD sphalerons. The naive reason for such a behaviour is that QCD sphalerons deplete the number of left-handed quarks by converting them into right-handed quarks, making electroweak sphalerons unuseful since they only act on left-handed doublets.

However, GS pointed out that the exact proportionality between baryonic number and $\langle B-L\rangle$ was an artifact of the massless approximation.

Indeed, the most general relation between the asymmetry in quark number density and chemical potential reads

$$\begin{aligned} n_q - n_{\bar q} &= \int \frac{d^3\mathbf{k}}{(2\,\pi)^3}\,\left[n_q^{\rm EQ}(E,\mu_q) - n_q^{\rm EQ}(E,-\mu_q)\right] \\ &= \frac{\mu_q\,T^2}{6}\left(1-\frac{3\,M_q^2}{2\,\pi^2\,T^2}\right). \end{aligned} \tag{4.16}$$

When taking into account finite mass effects, GS found

$$B = -\frac{3\,M_t^2}{20\,\pi^2\,T^2}\,\rho_{Y_f} = -\frac{9\,n}{10(9+14\,n)\pi^2}\,M_t^2\,\langle\dot\theta\rangle, \tag{4.17}$$

where ρ_{Y_f} is the fermionic hypercharge density. Therefore, the presence of QCD sphalerons leads to a suppression factor $(126/185)\,(M_t/\pi\,T)^2$, where it was taken $n=2$.

To understand how much crucial this suppression factor could be, one has to realize that the sphaleron rate turns off rather fast, being the rate proportional to $\exp(-2\,M_W/T)$. As a consequence, sphaleron cease to be operative at a very small value of the Higgs VEV, $v_{\rm co} \simeq (\alpha_W\,T/g)$. This corresponds numerically to a suppression of about 10^{-4} for $h_t \simeq 1$ [33].

It was then concluded that QCD sphalerons wipes out the baryon asymmetry created by the electroweak sphalerons up to mass effect, reducing the original result of CKN [28] of roughly four orders of magnitude[1].

[1]Note, however, that the situation can be improved when taking into account that fermions do acquire a plasma mass $M_q^2(T) \simeq (g_S^2 T^2/6)$ when propagating through the thermal bath. This should reduce the suppression factor to about 10^{-2}.

Such a conclusion relies on the basic assumption that the $\langle B - L \rangle$ charge has a vanishing thermal average.

4.2 Why total hypercharge should not make baryons

In ref. [32] Dine and Thomas reconsidered the same two Higgs doublet model used by CKN in their original paper [28] to explain the spontaneous baryogenesis mechanism.

These authors pointed out that the final baryon asymmetry found by CKN was missing of a key ingredient, namely a suppression factor proportional to $(v(T)/T)^2$ which takes into account that the *total* hypercharge must be violated to bias the baryon violating anomalous interactions.

Indeed, if the small Higgs VEV limit is ignored, a chemical charge potential for the total hypercharge cannot bias the sphaleron processes, since the latter do not violate hypercharge. Moreover, scattering of top quarks from the Higgs particles in the plasma cannot help, even if the scatterings are rapid.

The problem is that the field redefinition by fermionic hypercharge, while removing phases from the fermion mass terms, induces phases in the Yukawa couplings of the fermions to the fluctuating part of the Higgs field. To avoid these, one has to write the Higgs field as

$$H_1 = (v_1 + H_1')\ e^{i\theta_1}, \tag{4.18}$$

where H_1' represents the (complex) fluctuating field. Neglecting v_1, the Lagrangian in terms of these fields contains the phase θ_1 in the coupling

$$\partial_\mu \theta_1\, J_Y^\mu, \tag{4.19}$$

where J_Y^μ represents the full hypercharge current, including the scalar parts. Thus, when making properly the redifinition of the fields of the theory, a total hypercharge potential and not a fermionic hypercharge potential is induced.

In the limit $v_1 \to 0$, J_Y^μ is conserved. Integrating by part Eq. (4.19) one realizes that the chemical hypercharge potential does not have any effect on spontaneous baryogenesis if hypercharge is not broken by a nonvanishing VEV of the Higgs field ϕ_1.

To explain this point in another way, we can consider the system of equations (4.7) valid in the unbroken phase, *i.e.* in the limit $v(T) = 0$. If one considers the total hypercharge potential, Higgs particle densities n_H receive a further contribution proportional to $Y_H \langle \dot{\theta} \rangle$, where Y_H is the hypercharge of the generic Higgs field H. Of course, this new contribution was missing in the original papers on SB since the fermionic hypercharge of the Higgs fields are zero. One immediately realizes that the system (4.7) leads to a vanishing final baryon asymmetry since the effective number of unknowns are reduced from four $(\dot{\theta}, \mu_Y, \mu_{B-L}, \mu_B)$ to three $(\dot{\theta} + \mu_Y, \mu_{B-L}, \mu_B)$.

Thus, in order to obtain an asymmetry in the adiabatic limit, terms involving $v(T)$ must be included. To quantify how much hypercharge is violated, Dine and Thomas considered the triangle fermion loop diagram at finite temperature which couples the hypercharge current to the background gauge fields

$$\begin{aligned} \partial_\mu J_Y^\mu &= a \, \frac{M_t^2}{T^2} \, \frac{F\tilde{F}}{32\,\pi^2}, \\ a &= \frac{14}{3\,\pi^2}\,\zeta(3). \end{aligned} \tag{4.20}$$

Eq. (4.20), taking into account Eq. (4.19), is then equivalent to the coupling

$$\mathcal{L}_{\theta_1} = a\,\theta_1 \, \frac{M_t^2}{T^2} \, \frac{F\tilde{F}}{32\,\pi^2}. \tag{4.21}$$

Integrating by parts, $\mathcal{L}_{\theta_1}$ can be written in terms of the Chern-Simons number

$$\mathcal{L}_{\theta_1} = a \left(\frac{M_t^2}{T^2} \right) \partial_0 \theta_1 \, N_{\rm CS}. \tag{4.22}$$

This gives, effectively, a chemical potential for the Chern-Simons number

$$\mu_{\rm CS} = a \left(\frac{M_t^2}{T^2} \right) \partial_0 \theta_1, \tag{4.23}$$

which biases anomalous baryon number violating interactions and gives rise to a final nonvanishing baryon asymmetry.

Taking $v_{\rm co} \simeq (\alpha_W\, T/g)$ and $h_t \sim 1$, it is then realized that a suppression factor of about 10^{-4} is present in the final expression for the baryon asymmetry with respect to the original result given in ref. [28] and subsequent papers[2].

Again we note that this conclusion is based on the crucial assumption that the thermal averages of the conserved charges are vanishing inside the bubble wall as one can infer from the above discussion about the system Eq. (4.7). Were this assumption false, the Dine-Thomas objection should be no longer valid.

4.3 Why diffusion should not make baryons

To see whether diffusion may be a significant effect, it is useful to consider the dimensionless quantity

$$\epsilon_D \equiv \frac{D}{L_w \, v_w}, \tag{4.24}$$

where D is the diffusion constant for quarks (classically equal to $l\,c/3$, l being the mean free path) ranging from $1/T$ to $5/T$. Evidently ϵ_D may be $\mathcal{O}(1)$, and hence one can expect significant redistributions of the particle densities due to transport.

Joyce, Prokopek and Turok (JPT) [34] were the first to emphasize the crucial role played by transport phenomena: the response of the plasma to any charge potential induced by $\langle\dot{\theta}\rangle$ is not simply that of a system of fixed charges because diffusion may play a crucial role.

When a space-time dependent charge potential is turned on at a certain point, hypercharged particles are displaced from the surrounding regions so that the thermal averages of the conserved charges acquire a nonvanishing *local* value. As a consequence, the equilibrium properties system should be reconsidered taking into account the *local violation* of the conserved quantum numbers.

As we have often emphasized, all the results obtained so far are based on the crucial assumption that the thermal averages of the conserved charges are constrained to be zero. But is it physical to impose such constraints? The region we are considering is surrounded by an effectively infinite bath of global charge which is pulled in by the

[2] Dine and Thomas also pointed out that this suppression might be even stronger in the two Higgs doublet model where $\Delta\theta_1$ is itself of order of $(v_{\rm co}/T)^2$ (times coupling constants), since in the absence of quartic couplings the Higgs potential is CP conserving. However, as seen in Chapter **3**, plasma effects can give rise to such quartic couplings and induce spontaneous CP violation at finite temperature with $\Delta\theta_1 \sim 1$.

applied or induced potentials. By imposing the constraints such as $\langle B-L\rangle = 0$ and $\langle B\rangle = 0$ *at any space-time point*, one is simply preventing some linear combination of particle species from moving in response to these potentials. This moving is the effect of screening: a local accumulation of charge drags in an exactly cancelling charge from the surrounding plasma[3].

To consider the rate at which the equilibrium distributions are established in the bubble wall, one can turn to the Boltzmann equation.

In the case of slow, thick walls, the particles passing the wall should be accurately described as a fluid (because the particle mean free path is much smaller than the wall thickness). Following the usual derivation of the linear perturbation equations for a relativistic fluid from the Boltzmann equation, JPT arrived to the following system

$$\begin{aligned} \dot{v}_i + \frac{1}{4}\,\delta' + \frac{3\,n_i}{4\,\rho_i}\left(Y_i\,\dot{\theta}' - Y_i\,E_x + \frac{6\,n_i}{D_i\,T^2}\,v_i\right) &= 0, \\ \dot{\delta}_i + \frac{4}{3}\,v_i' &= 0, \end{aligned} \tag{4.25}$$

where δ_i is the fractional energy density perturbation; v_i the velocity of the fluid describing the species i; n_i and ρ_i are the unperturbed number and energy density; E_x is the hypercharge electric field along the x axis and D_i is the diffusion constant for the species i, in terms of which the conductivity of the medium can be expressed. Finally primes denote spatial derivatives, dots time derivatives.

Now JPT looked for stationary solution to these equations describing the response to the potential $\dot{\theta}(x - v_w t)$ moving through the medium, where v_w is the wall velocity.

Using the Gauss' law and assuming all perturbations as functions of $x - v_w t$, they found

$$\left(\frac{1}{3} - v_w^2\right)\delta'' + \frac{n_i}{\rho_i}\left(Y_i\,\dot{\theta}'' - \frac{3}{4}\,Y_i\sum_j y_j\,n_j\,\delta_j + \frac{9\,n_i\,v_w}{2\,D\,T^2}\,\delta_i'\right) = 0, \tag{4.26}$$

where the term including the sum over all particles summarizes the effects of Debye screening and the diffusion constant D is assumed to be equal for all the quarks, $D \simeq (4\,\alpha_S^2\,T)^{-1} \sim 2\,T^{-1}$.

[3]A simple analogy is found in QED: when an electric charge is put into a neutral plasma, particles rearrange themselves moving around to screen the electric charge so that Q is globally, but not locally conserved.

The fluid equations tell us what is going on. The build up of particles in the charge potential $\dot{\theta}(x)$ describes a state of hydrostatic equilibrium, in which the force from the potential is balanced by the pressure of the particle fluid. As particles of a given species enter the wall, they slow down or speed up accordingly as their density decreases or increases. The time delay effect means that as the stationary state is being set up, deficits and excesses of particles and antiparticles are left behind it, but there is a little effect from them on or in front of the wall.

The question now is whether hydrostatic equilibrium is reached or not inside the bubble wall.

From Eq. (4.26) one can see that the *friction* term which opposes the state of hydrostatic equilibrium is dominant only when the wall thickness L_w satisfies the relation

$$L_w \gtrsim \frac{3\,D}{v_w}. \tag{4.27}$$

For slow walls, $v_w \lesssim 0.1$, this is typically larger than the perturbative estimate of typical wall thickness [23], $L_w \simeq (1-100)T^{-1}$. Thus, JPT concluded that hydrostatic equilibrium is effectively reached for slow bubble walls, precisely the regime in which spontaneous baryogenesis mechanism is argued to dominate.

Once checked that hydrostatic equilibrium is present inside the bubble wall, JPT considered its effect on the spontaneous baryogenesis scenario. For slow walls, the hydrostatic solution accurately follows the thermal distribution

$$f_i(x,p_x,t) = \frac{1}{e^{\beta(E_i+\mu_i)} \pm 1}, \quad E_i = \sqrt{p_x^2+m^2} + Y_i\,\theta(x). \tag{4.28}$$

This is an exact solution of the Boltzmann equation for any static potential $\dot{\theta}(x)$, in which all the collision terms are identically zero, provided the usual relations between the chemical potentials for the species involved in a reaction are satisfied.

Since from Eq. (4.26) one can infer that in the equilibrium distribution (4.28) many global conserved or approximately conserved quantum numbers are nonzero, JPT argued that all the chemical potentials μ_i in Eq. (4.28) are zero due to the absence of any constraints and that the density of the particle i is only proportional to its hypercharge, $n_i \sim Y_i\,\dot{\theta}$. Since anomalous baryon number violating processes do not change hypercharge, JPT concluded that diffusion is inimical to spontaneous baryogenesis.

In the next Chapter we want to show that, in fact, transport phenomena are not only friendly with SB, but can also shed a new bright light on the impacts of strong sphalerons and the use of the total hypercharge potential on SB.

Chapter 5

Linear response theory approach to SB: the Renaissance era

In this Chapter, we will reconsider the spontaneous baryogenesis mechanism as originally proposed by CKN focusing on the effects of transport phenomena. The contents of this Chapter are based on the work [35] by CPR.

We will analyze the adiabatic scenario using linear response theory [71, 72] in order to deal transport effects.

We shall assume that a space-time dependent charge potential for fermion hypercharge (the results of this Chapter are, however, insensitive to this choice and are confirmed even taking a total hypercharge potential) is generated inside the bubble wall, without discussing its origin, and investigate its effect on the thermal averages of the various quantum numbers of the system.

We shall find that transport phenomena are really crucial, but we disagree with JPT's early conclusion [34] described in the previous Chapter that as a consequence of the local violation of global quantum numbers there is no biasing of the sphaleronic processes. Actually, in the adiabatic approximation the local equilibrium configuration of the system is determined by the thermal averages of the charges conserved by all the fast interactions. The effect of transport phenomena is to induce space-time dependent non zero values for these averages. We shall calculate these averages using linear response theory and then determine the local equilibrium configuration, showing that it corresponds to $\langle B + L \rangle \neq 0$.

Then we will write down a rate equation in order to take into account the slowness of the sphaleron transitions and obtain an expression for the final baryon asymmetry explicitly containing the parameters describing the bubble wall, such as its velocity, v_w, its width L_w, and the width of the region in which the sphalerons are active.

The inclusion of transport phenomena also will shed a new light on the strong sphaleron effects and on the effect of a charge potential for total rather than fermionic hypercharge. The dramatic suppressions found by Giudice and Shaposhnikov and by Dine and Thomas respectively, are both a consequence of taking zero averages for conserved quantum numbers. Since these averages are no more locally zero we will find a non zero $\langle B + L \rangle \neq 0$, even in the case in which the charge potential is for total rather than for fermion hypercharge.

In the case of QCD sphalerons we shall find that the final result depends in a crucial way on the form of the charge potential which is considered. For example, if all the left-handed fermions plus the right-handed quarks contributed to the charge potential according to their hypercharge, then no bias of sphaleron processes would be obtained. In this case, we would find a non zero value for $B + L$ inside the bubble wall but no baryon asymmetry far from it inside the broken phase. On the other hand, if only right and left-handed top quarks participate to the charge potential, which seems the physical situation, then a final asymmetry is found and QCD sphalerons are harmless.

The plan of the Chapter is as follows: in Section **5.1** we will develop a chemical potential analysis for the equilibrium properties of the plasma inside the bubble wall in the adiabatic approximation, and we will write down the rate equation for the production of $\langle B + L \rangle$ due to sphaleron transitions; in Section **5.2** we will introduce our application of linear response analysis to the calculation of the variations of the thermal averages induced by $\dot{\theta}$. In particular we will show that the fundamental quantity to evaluate is the retarded two points Green's function for fermion currents. In Section **5.3** we will solve the rate equation, finding an expression for the final baryon asymmetry in terms of the various bubble wall parameters. In this context, we will also discuss the screening effects on the electric charge. In Section **5.4** we will discuss the role of QCD sphalerons in cooperation with transport phenomena, and finally we will discuss our results in Section **5.5**.

5.1 Local equilibrium inside the wall

For definiteness, we will work in the two Higgs doublet model in which one doublet couples to up quarks and the other one to down quarks. The phase transition is assumed to be strong enough so that sphaleron processes freeze out somewhere inside the bubble wall.

The relevant time scale for baryogenesis is given by the passage of the bubble wall, which takes a time $\tau_{\rm H} = L_w/v_w \simeq (1-100)/T$.

During this time the phase of the Higgs VEV's changes of an amount $\Delta\theta$. Thus, we can discriminate between fast processes, which have a rate $\gtrsim 1/\tau_{\rm H}$ and then can equilibrate adiabatically with $\dot{\theta}$, and slow interactions, which feel that θ is changing only when the bubble has already passed by and sphalerons are no more active.

Next, we introduce a chemical potential for any particle which takes part to fast processes, and then reduce the number of linearly independent chemical potentials by solving the corresponding system of equations, in a way completely analogous to that followed for example in refs. [43], the main difference here being that light quark Yukawa interactions and Cabibbo suppressed gauge interactions are out of equilibrium.

Finally, we can express the abundances of any particle in equilibrium in terms of the remaining linear independent chemical potentials, corresponding to the conserved charges of the system.

Since strong interactions are in equilibrium inside the bubble wall, and since the current coupled to $\dot{\theta}$ is color singlet, we can chose the same chemical potential for quarks of the same flavour but different color, and set to zero the chemical potential for gluons.

Moreover, since inside the bubble wall $SU(2)_L \otimes U(1)_Y$ is broken, the chemical potential for the neutral Higgs scalars vanishes[1].

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[1]This is true if chirality flip interactions, or processes like $Z \to Z^* H^0$, are sufficiently fast; since

The other fast processes, and the corresponding chemical potential equations are:
i) top Yukawa:

$$\begin{array}{ll} t_L + H_2^0 \leftrightarrow t_R + g & (\mu_{t_L} = \mu_{t_R}), \\ b_L + H^+ \leftrightarrow t_R + g & (\mu_{t_R} = \mu_{b_L} + \mu_{H^+}), \end{array} \tag{5.1}$$

ii) $SU(2)_L$ flavour diagonal:

$$\begin{array}{ll} e_L^i \leftrightarrow \nu_L^i + W^- & (\mu_{\nu_L^i} = \mu_{e_L^i} + \mu_{W^+}), \\ u_L^i \leftrightarrow d_L^i + W^+ & (\mu_{u_L^i} = \mu_{d_L^i} + \mu_{W^+}), \\ H_2^0 \leftrightarrow H^+ + W^- & (\mu_{H^+} = \mu_{W^+}), \\ H_1^0 \leftrightarrow H^- + W^+ & (\mu_{H^-} = -\mu_{W^+}), \end{array} \tag{5.2}$$

with $(i = 1, 2, 3)$.

Neutral current gauge interactions are also in equilibrium, so we have zero chemical potential for the photon and the Z boson.

Imposing the above constraints, we can reduce the number of independent chemical potentials to four

$$\mu_{W^+}, \quad \mu_{t_L}, \quad \mu_{u_L} \equiv \frac{1}{2}\sum_{i=1}^{2} \mu_{u_L^i}, \quad \text{and} \quad \mu_{e_L} \equiv \frac{1}{3}\sum_{i=1}^{3} \mu_{e_L^i}. \tag{5.3}$$

These quantities correspond to the four linearly independent conserved charges of the system.

Choosing the basis Q', $(B-L)'$, $(B+L)'$, and $BP' \equiv B_3' - 1/2(B_1' + B_2')$, where the primes indicate that only particles in equilibrium contribute to the various charges, and introducing the respective chemical potentials, we can go to the new basis using the relations

$$\left\{ \begin{array}{rcl} \mu_{Q'} &=& 3\,\mu_{t_L} + 2\,\mu_{u_L} - 3\,\mu_{e_L} + 11\,\mu_{W^+}, \\ \mu_{(B-L)'} &=& 3\,\mu_{t_L} + 4\,\mu_{u_L} - 6\,\mu_{e_L} - 6\,\mu_{W^+}, \\ \mu_{(B+L)'} &=& 3\,\mu_{t_L} + 4\,\mu_{u_L} + 6\,\mu_{e_L}, \\ \mu_{BP'} &=& 3\,\mu_{t_L} - 2\,\mu_{u_L}, \end{array} \right. \tag{5.4}$$

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the corresponding rates depend on the Higgs VEV, they will be suppressed by factors of $(v(T)/T)^2$ with respect for example to the rate for $ht_L \leftrightarrow t_R g$. This has led the authors of refs.[33, 34] to consider the system in the unbroken phase. Anyway, this choice does not lead to dramatic changes to the conclusion of this Chapter.

If sphaleron transitions were fast, then we could eliminate a further chemical potential through the constraint

$$2\sum_{i=1}^{3}\mu_{u^i_L}+\sum_{i=1}^{3}\mu_{d^i_L}+\sum_{i=1}^{3}\mu_{e^i_L}=0. \tag{5.5}$$

In this case, the value of $(B+L)'$ would be determined by that of the other three charges according to the relation

$$(B+L)'_{\rm EQ}=\frac{3}{80}Q'+\frac{7}{20}BP'-\frac{19}{40}(B-L)', \tag{5.6}$$

where we have indicated charge densities by the corresponding charge symbols.

We have neglected mass effects, which means that the excess of particle over antiparticle density is related to the chemical potentials according to the relation [43] $n_i-\bar{n}_i- = (a\, g_i\, \mu_i\, T^2/6)$, where $a=1$ for fermions and $a=2$ for bosons, and g_i is the number of spin and color degrees of freedom.

The above result should not come as a surprise, since we already know from ref. [43] that a non zero value for $B-L$ gives rise to a non zero $B+L$ at equilibrium. Stated in other words, sphaleron transitions erase the baryon asymmetry only if any conserved charge of the system has vanishing thermal average, otherwise the equilibrium point lies at $(B+L)_{\rm EQ}\neq 0$.

Actually, as pointed out several times, sphaleron rates are too small to allow $(B+L)'$ to reach its equilibrium value (5.6), $\tau_{\rm sp}\simeq(\alpha_W^4\,T)^{-1}\gg\tau_{\rm H}$, so equilibrium thermodynamics cannot be used to describe baryon number generation inside the bubble wall.

Therefore, we shall make use of the rate equation

$$\frac{d}{dt}(B+L)'_{\rm sp}=-\frac{\Gamma_{\rm sp}}{T}\frac{\partial{\rm F}}{\partial(B+L)'}. \tag{5.7}$$

We recall that F is the free energy of the system, and the derivative with respect to $(B+L)'$ must be taken keeping Q', $(B-L)'$, and BP', constant.

The meaning of Equation. (5.7) is straightforward. Sphaleron transitions (which change $(B+L)'$ but conserve Q', $(B-L)'$, and BP') will be turned on only if they allow the *total* free energy of the system to get closer to its minimum, *i.e.* equilibrium, value.

At high temperature ($\mu_i \ll T$) the free energy of the system is given by

$$\begin{aligned} \mathrm{F} = (T^2/12) \quad & \Big[3\,\mu_{e_L}^2 + 3\,\mu_{\nu_L}^2 + 6\,\mu_{u_L}^2 + 3\,\mu_{t_L}^2 + 3\,\mu_{t_R}^2 + 6\,\mu_{d_L}^2 + 3\,\mu_{b_L}^2 \\ & +6\,\mu_{W^+}^2 + 2\,\mu_{H^+}^2 + 2\,\mu_{H_1^0}^2 + 2\,\mu_{H_2^0}^2\Big]\,. \end{aligned} \tag{5.8}$$

Using (5.1), (5.2) and (5.4) to express the chemical potentials in terms of the four conserved charges in (5.4) we obtain the free energy as a function of the density of $(B+L)' = (\mu_{B+L}T^2/6)$,

$$\mathrm{F}\left[(B+L)'\right] = 0.46\,\frac{\left[(B+L)' - (B+L)'_{\mathrm{EQ}}\right]^2}{T^2} + \text{constant terms}, \tag{5.9}$$

where the constant terms depend on Q', $(B-L)'$, and BP' but not on $(B+L)'$, and $(B+L)'_{\mathrm{EQ}}$ is given by (5.6).

The total amount of $(B+L)'$ present in a certain point at a certain time is made up by two contributions: $(B+L)'_{\mathrm{sp}}$, generated by sphaleron transitions, and $(B+L)'_{\mathrm{TR}}$, which is not generated but is transported from nearby regions in response to the perturbation introduced by $\dot{\theta}$.

Thus, Eq. (5.7) now takes the form

$$\frac{d}{dt}(B+L)'_{\mathrm{sp}} = -0.92\,\frac{\Gamma_{\mathrm{sp}}}{T^3}\left[(B+L)'_{\mathrm{sp}} + (B+L)'_{\mathrm{TR}} - (B+L)'_{\mathrm{EQ}}\right], \tag{5.10}$$

with the initial condition $(B+L)'_{\mathrm{sp}} = 0$ before $\dot{\theta}$ is turned on.

Let us summarize our discussion up to this point. Consider an observer in the plasma reference frame during the phase transition. When a bubble wall passes by the observer, she (he) measures a space-time dependent charge potential for, say, fermionic hypercharge, which induces transport phenomena and then local asymmetries in particle numbers. The Q', $(B-L)'$, and BP' components of these asymmetries remain unaffected by fast interactions, while the other components are reprocessed as to obtain their equilibrium values. In the case of $(B+L)'$ the reprocessing is slow, so we have to use the rate equation in (5.10) to describe it.

The generation of $(B+L)'_{\mathrm{sp}}$ will go on until either Γ_{sp} goes to zero or the local equilibrium value is reached *i.e.* $(B+L)'_{\mathrm{sp}} + (B+L)'_{\mathrm{TR}} = (B+L)'_{\mathrm{EQ}}$.

After the passage of the wall, $(B+L)'_{\mathrm{EQ}}$ and $(B+L)'_{\mathrm{TR}}$ go rapidly to zero since $\dot{\theta}$ vanishes, and so the final asymmetry is given by the $(B+L)'_{\mathrm{sp}}$ generated until that time.

As we can see, the crucial question is now to calculate the induced values for Q', $(B-L)'$, BP', and $(B+L)'_{TR}$. We will do that in the next section by using linear response analysis [71, 72].

5.2 Linear response analysis

In this Section we want to discuss the effect on the thermal averages of the term induced in the Lagrangian when $\dot{\theta}$ is active, which we assume to have the form

$$\mathcal{L} \to \mathcal{L} + \dot{\theta} J^0_{Y_f}, \tag{5.11}$$

with

$$J^0_{Y_f} = \textstyle\sum_i' y^i_f J^0_i, \tag{5.12}$$

where $\sum_i'$ means that the sum extends on particles in equilibrium with $\dot{\theta}$ only[2].

The standard procedure is to consider $\dot{\theta}$ as an effective chemical potential, so that particle abundances at equilibrium are given by

$$n_i = q_i\,\mu_Q + (b-l)_i\,\mu_{B-L} + bp_i\,\mu_{BP} + y^i_f\,\dot{\theta}, \tag{5.13}$$

and then imposing

$$\langle Q' \rangle = \langle (B-L)' \rangle = \langle BP' \rangle = 0, \tag{5.14}$$

so that any particle abundance can be expressed in terms of $\dot{\theta}$ only.

The *key point* here is that taking transport phenomena into account, the above thermal averages are not zero, but depend on $\dot{\theta}$ themselves.

So we must first *calculate* their values and then use them to determine the chemical potentials.

Our starting point is the generating functional

$$Z[J_O;\dot{\theta}] = \int_{P(A)BC} \mathcal{D}\phi \exp\left\{ i \int_C d\tau \int_V d^3\mathbf{x} \left[\mathcal{L} + \dot{\theta} J^0_{Y_f} + J_O\, O + \text{sources} \right] \right\}, \tag{5.15}$$

where $\mathcal{D}\phi$ is the integration measure, O is the operator of which we want to calculate the thermal average and J_O the corresponding source. C is any path in the complex

[2] The reader should not be confused by the fact we are using an hypercharge potential. Qualitative results should be the same (apart numerical factor), had we used the total hypercharge potential.

τ plane connecting the point τ_{in} to $\tau_{out} = \tau_{in} - i\beta$ ($\beta = 1/T$) such that the imaginary part of τ is never increasing on the path [73]. P(A)BC means that periodic (antiperiodic) boundary conditions must be imposed on bosonic (fermionic) fields on the path.

The thermal average of the operator $O(\tau, \mathrm{x})$ in presence of $\dot{\theta}$ is obtained in the usual way

$$\langle O(t_x, \mathrm{x})\rangle_{\dot{\theta}\neq 0} = \frac{1}{i} \frac{\delta Z[J_O; \dot{\theta}]}{\delta J_O(t_x, \mathrm{x})}\bigg|_{J_O=0} . \tag{5.16}$$

where all the field sources are set to zero.

Now we make a functional expansion of (5.16) in $\dot{\theta}$ and truncate it at the linear term,

$$\begin{aligned}\langle O(t_x, \mathrm{x})\rangle_{\dot{\theta}\neq 0} &= \frac{1}{i}\frac{\delta}{\delta J_O(t_x, \mathrm{x})}\bigg\{ Z[J_O; \dot{\theta}=0] \\ &\quad + \int_C d\tau' \int_V d^3\mathrm{y}\, \dot{\theta}(\tau', \mathrm{y})\, \frac{\delta Z[J_O; \dot{\theta}]}{\delta \dot{\theta}(\tau', \mathrm{y})}\bigg|_{\dot{\theta}=0} + \dots \bigg\}\bigg|_{J_O=0} \\ &= \langle O(t_x, \mathrm{x})\rangle_{\dot{\theta}=0} + i\int_C d\tau' \int_V d^3\mathrm{y}\, \dot{\theta}(\tau', \mathrm{y})\, \langle T_C J^0_{Y_f}(\tau', \mathrm{y})\, O(t_x, \mathrm{x})\rangle_{\dot{\theta}=J_O=0},\end{aligned} \tag{5.17}$$

where T_C is the ordering along the path C.

Different choices for the *time* contour C lead to different formulations of thermal field theory, see Fig. 3.

One possibility is to take the vertical line connecting t_x to $t_x - i\beta$, so that $\tau' - t_x$ is pure imaginary on any point of the path.

This choice corresponds to the imaginary time formalism (ITF) of thermal field theory [73], and in this case we have to evaluate the euclidean two point thermal Green's function $\langle T J^0_{Y_f}(z_E), O(0)\rangle$ ($z_E^2 = -z_0^2 - |\mathrm{z}|^2$). This can be done in Matsubara formalism, where Feynman rules are straightforwardly obtained [72]. But, in this case, we would have to calculate $\dot{\theta}$ for complex times, whereas we are interested in its values at real times, during the passage of the wall.

So, in order to get a more direct physical interpretation of what we are calculating, we must turn to real time formalism. This corresponds to choose the path in Fig. 4, and then letting τ_{in} going to $-\infty$, and t_F to $+\infty$ [73].

Now, it is possible to see that the contributions to the integral coming from τ' on C_3 vanishes in the above limit [73], so we are left with the contributions from C_1 and C_2 only. The T_C ordering now allows us to rewrite the integral in (5.17) as

$$\begin{aligned} & i\int_{C_1\oplus C_2} dt_y \int_V d^3\mathbf{y}\, \dot\theta(t_y,\mathbf{y})\, \langle T_C J^0_{Y_f}(t_y,\mathbf{y})\, O(t_x,\mathbf{x})\rangle_{\dot\theta=J_O=0} \\ & = i\int_{-\infty}^{t_x} dt_y \int_V d^3\mathbf{y}\, \dot\theta(t_y,\mathbf{y})\, \langle \big[J^0_{Y_f}(t_y,\mathbf{y}), O(t_x,\mathbf{x})\big]\rangle_{\dot\theta=J_O=0}. \end{aligned} \tag{5.18}$$

Defining as usual the *retarded* Green's function

$$iD^R_{O,Y_f}(t_x,\mathbf{x};t_y,\mathbf{y}) \equiv \langle\big[O(t_x,\mathbf{x}), J^0_{Y_f}(t_y,\mathbf{y})\big]\rangle\Theta(t_x-t_y), \tag{5.19}$$

where $\Theta(x)$ is the step function, we arrive at the result

$$\langle O(t_x,\mathbf{x})\rangle_{\dot\theta\neq 0} = \langle O(t_x,\mathbf{x})\rangle_{\dot\theta=0} + \int_{-\infty}^{+\infty} dt_y \int_V d^3\mathbf{y}\, \dot\theta(t_y,\mathbf{y}) D^R_{O,Y_f}(t_x,\mathbf{x};t_y,\mathbf{y}). \tag{5.20}$$

The operators we are interested in are fermionic charge densities (Q', $(B-L)'$, $(B+L)'$, BP') of the form $Q'_A = \sum'_i q_i^A J_i^0$. Inserting it in (5.20), and using definition (5.12) we get

$$\langle Q'_A(t_x,\mathbf{x})\rangle_{\dot\theta\neq 0} = \textstyle\sum'_{ij} q_i^A\, y_f^i \displaystyle\int_{-\infty}^{+\infty} dt_y \int_V d^3\mathbf{y}\, \dot\theta(t_y,\mathbf{y})\, D^R_{ij}(t_x,\mathbf{x};t_y,\mathbf{y}), \tag{5.21}$$

where D^R_{ij} is the current-current retarded Green's functions for fermion i and j (i and j are flavour and color indices) and we used the fact that $\langle Q'_A(t_x,\mathbf{x})\rangle_{\dot\theta=0} = 0$.

Note that the Green's function has to be evaluated for $\dot\theta = 0$, *i.e.* we must use the unperturbed Lagrangian with the charge potential turned off and all the chemical potentials equal to zero in the partition function.

The problem of calculating the effect of the charge potential in (5.11) on the thermal averages for the particles in equilibrium is then reduced to the evaluation of the retarded Green's functions which enter in (5.21).

As we have discussed, the more natural framework for this calculation is real time thermal field theory, in which the physical sense of the various quantities is evident. Anyway, we have also seen that we can calculate the euclidean Green's function in the imaginary time formalism and then continue analytically to real times, thus obtaining the D^R_{ij}'s (this relation was established for the first time by Baym and Mermyn [74]). In energy-momentum space the analytical continuation is accomplished by the substitution

$$i\omega_n \longrightarrow \omega + i\varepsilon, \qquad \varepsilon \longrightarrow 0^+, \tag{5.22}$$

where $\omega_n = 2\,\pi\,n\,T$ are Matsubara frequencies and ω is the real energy.

5.3 Solution of the rate equation

Since the rate of the sphaleronic transition is suppressed by α_W^4, the asymmetry in $(B+L)'$ generated by the sphalerons, $(B+L)'_{\rm sp}$, is generally much smaller than both $(B+L)'_{\rm EQ}$ and $(B+L)'_{\rm TR}$ (we can check it *a posteriori*), so we can approximate the rate equation in (5.10) by

$$\frac{d}{dt}(B+L)'_{\rm sp} \simeq 0.92\,\frac{\Gamma_{\rm sp}}{T^3}\left[(B+L)'_{\rm EQ} - (B+L)'_{\rm TR}\right]. \tag{5.23}$$

Using the equations (5.6) and (5.21) we can determine $(B+L)'_{\rm LR} \equiv (B+L)'_{\rm EQ} - (B+L)'_{\rm TR}$ as

$$(B+L)'_{\rm LR}(t_x,{\rm x}) = \langle J^0_{(B+L)'}(t_x,{\rm x})\rangle = \Sigma'_{ij}c_{ij}\int_{-\infty}^{+\infty} dt_y \int_V d^3{\rm y}\;\dot\theta(t_y,{\rm y})\;D^R_{ij}(t_x,{\rm x};t_y,{\rm y}), \tag{5.24}$$

where

$$c_{ij} \equiv y_f^j\left(\frac{3}{80}q_i + \frac{7}{20}bp_i - \frac{19}{40}(b-l)_i - (b+l)_i\right).$$

Integrating eq. (5.23) in time from $-\infty$ to $+\infty$ we get the final density of $(B+L)'$,

$$\begin{aligned}\Delta(B+L)'_{\rm sp} &= \int_{-\infty}^{+\infty} dt_x \frac{d}{dt_x}(B+L)'_{\rm sp}(t_x,{\rm x}) \\ &= 0.92\,\frac{2\pi}{T^3}\int_{-\infty}^{+\infty} d\omega\,\tilde\Gamma_{\rm sp}(-\omega,{\rm x})\,\widetilde{(B+L)}'_{\rm LR}(\omega,{\rm x}),\end{aligned} \tag{5.25}$$

where $\tilde\Gamma_{\rm sp}(-\omega,{\rm x})$ and $\widetilde{(B+L)}'_{\rm LR}(\omega,{\rm x})$ are, respectively, the Fourier transformed of $\Gamma_{\rm sp}(t_x,{\rm x})$ and $(B+L)_{\rm LR}(t_x,{\rm x})$ with respect to time.

We recall that $\Gamma_{\rm sp}$ is $\kappa\,(\alpha_W T)^4$ in the unbroken phase and decreases exponentially fast as the Higgs VEV is turned on.

In order to solve eq. (5.23) analytically we approximate this behaviour by a step function. Moreover, we will consider a plane bubble wall moving along the z-axis with velocity v_w. So, our expression for $\Gamma_{\rm sp}$ will be

$$\Gamma_{\rm sp}(t_x, \mathrm{x}) \simeq \Gamma\,\Theta\left(t_x - t_1 - \frac{z_x}{v_w}\right)\,\Theta\left(t_2 - t_x + \frac{z_x}{v_w}\right), \tag{5.26}$$

with $t_1 \to -\infty$ and $\Gamma = \kappa\,(\alpha_W T)^4$.

Of course, more sophisticated approximations for $\Gamma_{\rm sp}$ may be used, at the price of solving eq. (5.23) numerically.

Analogously, we approximate $\dot\theta$ in such a way that it is constant in a region of width L_w inside the bubble wall, and is zero outside,

$$\dot\theta(t_y, \mathrm{y}) = \theta\,\Theta(z_y - v_w t_y)\,\Theta(v_w t_y - z_y + L_w), \tag{5.27}$$

where $\theta = v_w\,\Delta\theta/L_w$.

So, if we are at the point $\mathrm{x} = 0$, we observe an interaction of the form (5.11) turned on from $t = -L_w/v_w$ to $t = 0$, while the sphalerons are active till $t = t_2$, as we have shown in Fig. 5.

Putting all together we obtain

$$\Delta(B+L)'_{\rm sp} = 0.92\,\frac{(2\,\pi)^3}{T^3}\,\Gamma\,\theta\,{\textstyle\sum'_{ij}} c_{ij}$$

$$\int_{-\infty}^{+\infty} d\omega\,\frac{\mathrm{e}^{-i\omega t_2} - \mathrm{e}^{-i\omega t_1}}{\omega}\,\frac{1 - \mathrm{e}^{-i\omega L_w/v_w}}{\omega}\,\widetilde{D}^R_{ij}(p_x = p_y = 0, p_z = \frac{\omega}{v_w}, \omega). \tag{5.28}$$

Note the peculiar relationship between p_z and ω in the argument of $\widetilde{D}^R_{ij}$, due to the spacetime dependence of $\dot\theta(t_y, \mathrm{y})$, see (5.27).

A consideration of the general properties of retarded Green's functions [75] ensures that the imaginary part of $\widetilde{D}^R_{ij}$ is a even function of ω, while its imaginary part is odd. As a consequence the integral in (5.28) will always give a real result.

The lowest order contribution to $\widetilde{D}^R_{ij}$ comes from the fermion loop in Fig. 6, where the two crosses indicate the zero components of the fermion current.

We may evaluate the corresponding euclidean two point Green's function in the ITF and then continue analytically to real energies according to Eq. (5.22).

Moreover, since the relevant frequencies at which $\widetilde{D}^R_{ij}$ must be evaluated are such that $\omega \leq |\mathbf{p}| \leq v_w/L_w \simeq T/10^2$ we take only the leading terms in the high temperature expansion. These are given by [76]

$$\Pi_l(p_0 = 2n\pi T, \mathbf{p})\, \delta_{ij} = \frac{T^2}{3}\left[1 - \frac{ip_0}{2|\mathbf{p}|}\log\frac{ip_0 + |\mathbf{p}|}{ip_0 - |\mathbf{p}|}\right]\, \delta_{ij}, \tag{5.29}$$

where we have neglected fermion masses.

After continuing analytically to real energies and fixing the momenta as in (5.28) we get the lowest order contribution to $\widetilde{D}^R_{ij}$,

$$\widetilde{D}^{R\,0}_{ij}(p_x = p_y = 0, p_z = \frac{\omega}{v_w}, \omega) = \frac{T^2}{3}\left[1 - \frac{v_w}{2}\log\frac{1+v_w}{1-v_w} + i\,\frac{\pi}{2}\,v_w\,\mathrm{sign}(\omega)\right]\delta_{ij}\,. \tag{5.30}$$

When $v_w \to 1$ the above expression exhibits a collinear divergence, due to the fact that the fermions in the loops are massless in our approximation. This divergence disappears when plasma masses for fermions are taken into account. However, for our purposes, since $v_w \simeq 0.1$, the effects of plasma masses for fermions can be neglected [77].

Due to the constraint $p_z = \omega/v_w$ the real part of the correlation function in (5.30) does not depend on ω, while the imaginary part depends on its sign only. This implies that, in the high temperature limit, the response induced on the plasma through (5.30) has neither spatial nor temporal dispersion, *i.e.* inserting (5.30) in (5.21) gives rise to an induced thermal average for the charge Q_A which in any space-time point is proportional to the value of $\dot{\theta}$ in that point

$$\langle Q_A(t_x, \mathrm{x})\rangle^0 \propto \dot{\theta}(t_x, \mathrm{x}). \tag{5.31}$$

In particular also $(B+L)_{\mathrm{TR}}$ and $(B+L)_{\mathrm{EQ}}$ receive a contribution of this form and disappear as soon as $\dot{\theta}$ is turned off.

Inserting it into the rate equation we obtain from (5.28) the contribution to the asymmetry ($t_1 \to -\infty$)

$$\Delta(B+L)'^{\,0}_{\mathrm{sp}} = 0.92\,\frac{(2\,\pi)^3}{T^3}\,\Gamma\,\theta\,\textstyle\sum'_{ij} c_{ij} I^0(t_2)\delta_{ij}, \tag{5.32}$$

where

$$I^0(t_2) = \frac{2\,\pi}{3} T^2 \left(1 - \frac{v_w}{2} \log \frac{1+v_w}{1-v_w}\right) \times \begin{cases} 0 & (t_2 < -\frac{L_w}{v_w}), \\ \left(t_2 + \frac{L_w}{v_w}\right) & \left(-\frac{L_w}{v_w} < t_2 < 0\right), \\ \frac{L_w}{v_w} & (t_2 > 0). \end{cases} \quad (5.33)$$

We recall that t_2 is the time at which sphaleron transitions are turned off, while the charge potential induced by $\dot{\theta}$ is active for $-L_w/v_w < t < 0$.

So the asymmetry calculated in this approximation for D^R_{ij} grows linearly with t_2 until $t_2 = 0$ (for $t_2 < -L_w/v_w$ the asymmetry is obviously zero since there is no overlap between sphalerons and $\dot{\theta}$).

The result for $t_2 > 0$ is an artifact of our approximation $(B+L)'_{\rm sp} \ll (B+L)'_{\rm LR}$, which is no more appropriate in this case.

Actually, from (5.31) we know that when $\dot{\theta}$ goes to zero, as is the case for $t > 0$, $(B+L)'_{\rm EQ}$ and $(B+L)'_{\rm TR}$ vanish too, and the rate equation (5.10) becomes

$$\frac{d}{dt}(B+L)'_{\rm sp} = -0.92\,\frac{\Gamma_{\rm sp}}{T^3}\,\left[(B+L)_{\rm sp}\right] \qquad (t > 0), \qquad (5.34)$$

so that the asymmetry produced before decreases exponentially from $t = 0$ to $t = t_2$ with rate Γ.

However, due to the smallness of Γ, and to the fact that t_2 cannot be much larger than $\mathcal{O}(L_w/v_w)$, we can safely neglect this decreasing and take the result in (5.33).

Next, we consider the contribution to D^R_{ij} due to gauge bosons exchange. Since we are in the broken phase, and since the perturbation (5.11) induced by $\dot{\theta}$ is colorless, we will take into account only photons, which contribute through the graph in Fig. 7.

The blob in the middle represents the sum of all possible insertion of fermion loops.

In calculating the blob, we have to include not only the fermions which enter in Q', $(B-L)'$, and BP', *i.e.* the ones with fast flavour, or chirality, changing interactions, but we must instead take into account the contributions of all the charged fermions of the theory. This is because QED interactions are fast and then, for instance, pair production is in equilibrium also for right-handed light quarks.

The QED contribution of Fig. 7 gives

$$\widetilde{D}_{ij}^{R,\,\mathrm{QED}}(\omega,\mathbf{p}) = e^2\, q_i q_j\, \Pi_l^2(\omega,\mathbf{p})\, \frac{\mathcal{D}^{00}(\omega,\mathbf{p})}{1-\sum_k (e\, q_k)^2 \mathcal{D}^{00}\Pi_l}, \tag{5.35}$$

where eq_i is the electric charge of the fermion i. $\mathcal{D}^{00}(\omega,\mathbf{p})$ is the tree level $(0,0)$ component of the photon propagator in the Coulomb gauge

$$\mathcal{D}^{\mu\nu} = \frac{-1}{p^2} P_T^{\mu\nu} - \frac{1}{|\mathbf{p}|^2} u^\mu u^\nu\,, \tag{5.36}$$

where $p^2 = \omega^2 - |\mathbf{p}|^2$, $u_\mu = (1,\ 0,\ 0,\ 0)$ identifies the plasma reference frame, while

$$P_T^{00} = P_T^{0i} = P_T^{i0} = 0,$$

$$P_T^{ij} = \delta_{ij} - p^i p^j/|\mathbf{p}|^2.$$

As we have discussed, the sum in the denominator of (5.35) must be extended over all the charged quarks and leptons. Setting $p_z = \omega/v_w$ we get

$$\widetilde{D}_{ij}^{R,\,\mathrm{QED}}(\omega,\mathbf{p}) = -e^2\, q_i q_j\, v_w^2\, \frac{\Pi_l^2(\omega, p_z = \omega/v_w)}{\omega^2 + v_w^2\, \sum_k (eq_k)^2 \Pi_l(\omega, p_z = \omega/v_w)}. \tag{5.37}$$

Note that, unlike the *direct* contribution (5.30), the QED one is not flavour (or color) diagonal, so that even particles which do not enter into the expression for the charge potential (5.11) get a non zero thermal average depending on $\dot{\theta}$.

Moreover, this contribution has a genuine ω dependence.

Two points retarded Green's function are analytic in the upper half of the complex ω plane. $\widetilde{D}_{ij}^{R,\,\mathrm{QED}}(\omega, p_z = \omega/v_w)$ may then have poles of the form $\overline{\omega} = \omega_p - i\gamma_p$, with $\gamma_p > 0$. In order to determine them we have to solve the following equations

$$\begin{aligned} \omega_p^2 - \gamma_p^2 &= -C\, \mathrm{Re}\, \Pi_l(\overline{\omega}), \\ \omega_p &= \frac{C\, \mathrm{Im}\, \Pi_l(\overline{\omega})}{2\gamma_p}, \end{aligned} \tag{5.38}$$

where $C = v_w^2 \, \sum_k (e\, q_k)^2$.

Since the right hand side of the first of Eqs. (5.38) is negative, and $v_w \simeq 0.1 < 1$, we can approximate the solutions by

$$\begin{aligned} \overline{\omega}_{1,2} &= \pm\omega_p + i\gamma_p, \\ \omega_p &= \frac{\pi}{4}\, v_w\, \gamma_p, \\ \gamma_p &\simeq [C \, \mathrm{Re}\Pi_l(\overline{\omega})]^{1/2} \simeq v_w \frac{e\,T}{\sqrt{3}} \left[\sum_k (q_k)^2\right]^{1/2}, \end{aligned} \tag{5.39}$$

where $\gamma_p > 0$ as it should be.

Inserting (5.37) in Eq. (5.28) we obtain the QED contribution to the asymmetry

$$\Delta(B+L)_{\mathrm{sp}}^{\mathrm{QED}'} \simeq 0.92\, \frac{(2\,\pi)^3}{T^3}\, \Gamma\, \theta {\sum}'_{ij} c_{ij}\, I^{\mathrm{QED}}(t_2), \tag{5.40}$$

where, now,

$$I^{\mathrm{QED}}(t_2) = -\frac{2\,\pi}{3} T^2 \left(1 - \frac{v_w}{2} \log \frac{1+v_w}{1-v_w}\right) \frac{q_i q_j}{\sum_k (q_k)^2}$$

$$\times \begin{cases} 0 & (t_2 < -\frac{L_w}{v_w} \\ \left[t_2 + \frac{L_w}{v_w} + \cos\omega_p\left(t_2 + \frac{L_w}{v_w}\right) \frac{\mathrm{e}^{-\gamma_p(t_2 + L_w/v_w)}}{\gamma_p}\right] & (-\frac{L_w}{v_w} < t_2 < \\ \left[\frac{L_w}{v_w} - \frac{\mathrm{e}^{-\gamma_p t_2}}{\gamma_p}\left(\cos\omega_p t_2 - \mathrm{e}^{-\gamma_p L_w/v_w} \cos\omega_p\left(t_2 + \frac{L_w}{v_w}\right)\right)\right] & (t_2 > 0). \end{cases}$$

(5.41)

The same considerations about the case $t_2 > 0$ made after Eq. (5.33) apply also here.

We can notice that photon exchange gives two different types of contributions. The first one has the same behaviour of that in (5.33), *i.e.* a linearly increasing asymmetry from $t_2 = -L_w/v_w$ to $t_2 = 0$. On the other hand, the second term exhibits a well known feature of plasma physics, namely, plasma damped oscillations induced by an external perturbation [72]. The damping rate here is given by γ_p. In the case $-L_w/v_w < t_2 < 0$, we see that the oscillating term dominates over the linear one only for $t_2 \to -L_w/v_w$, and is rapidly damped as $t_2 \to 0$, since $\exp(-\gamma_p L_w/v_w) \simeq \exp(-T\, L_w) \simeq \exp(-40)$. When $t_2 > 0$ the amplitude of the oscillation is always suppressed at least by a factor $(10^{-1} - 10^{-2})$ with respect to the linear one, and then we can conclude that the effect of the oscillating term is negligible unless t_2 is very near to $-L_w/v_w$.

An interesting feature of our results (5.33) and (5.41) can be appreciated if we calculate, by means of eq. (5.21), the electric charge Q' induced by the phase $\dot{\theta}$, taking into account both the direct contribution (5.30) and the QED one (5.35) to $\widetilde{D}^R_{ij}$. It is easy to see that it is given by

$$\begin{aligned} \langle Q' \rangle \propto \; & \left[\textstyle\sum_i' y_i q_i \left(\sum_k q_k^2 - \sum_k' q_k^2 \right) \times \text{linear contribution} \right] \\ & + \text{damped contribution}, \end{aligned} \tag{5.42}$$

then, when *every* fermion is in equilibrium, the linear contribution to the thermal average of Q' vanishes, and we are left with the damped one.

The reason is that in this case Q' coincides with the total fermion electric charge, and this is perfectly screened as in the usual QED plasma. Since in the real situation not all the fermions are in equilibrium, the linear contribution to (5.42) does not cancel. Anyway, this considerations are valid for electric charge only, while the other interesting charges, $(B+L)'$, $(B-L)'$, and, BP', would have non zero linear contributions even if all the fermions were in equilibrium.

Putting all together, and neglecting the damped contribution, we find the final asymmetry in $(B+L)'$

$$\Delta(B+L)'_{\rm sp} = -0.92\, \frac{37}{240}\, (2\,\pi)^4\, \frac{\Gamma\, \theta}{T} \left(1 - \frac{v_w}{2} \log \frac{1+v_w}{1-v_w}\right) \left(t_2 + \frac{L_w}{v_w}\right), \tag{5.43}$$

where we have assumed that the sphalerons turn off when the phase is still active $(-L_w/v_w < t_2 < 0)$.

Recalling that $\theta = \Delta\theta v_w/L_w$, and assuming that sphalerons cease to be active after a time interval $t_2 + \Delta z/v_w = f\, L_w/v_w$ from the turning on of $\dot{\theta}$ we get

$$\Delta(B+L)'_{\rm sp} \simeq -2.3 \cdot 10^2\, \kappa\, T^3\, \alpha_W^4\, \Delta\theta\, f, \tag{5.44}$$

where we have taken the reasonable value $v_w \simeq 0.1$.

The above value is enhanced by nearly three orders of magnitude with respect to the original estimates by CKN [28] where transport phenomena were not taken into account.

The predicted baryon asymmetry of the Universe then comes out to be

$$B = \frac{n_B}{s} \simeq -10^{-6}\, \kappa\, \Delta\theta\, f. \tag{5.45}$$

κ is estimated in the range $(0.1 - 1)$ from numerical simulations [10].

Choosing $f \simeq (\alpha_W/g)$, the observed baryon asymmetryc $B \simeq 10^{-10}$, can then be reproduced for $\Delta\theta \simeq (10^{-2} - 10^{-3})$, values which can be obtained either by explicit CP violation or by spontaneous CP violation at finite temperature [30, 31] without entering in conflict with the experimental bounds on the electric dipole moment of the neutron.

The result (5.44) was obtained considering a charge potential of the form (5.11) where the sum extends on all the left-handed fermions plus the right-handed quark.

Considering the more physical situation in which only the top quarks (left- and right-handed) feel the effect of $\dot{\theta}$ the coefficient 37/240 in (5.43) should be changed into 9/32, thus leading to an enhancement of a factor 1.8.

5.4 The QCD sphalerons' legacy

As explained in Section 4.1 Giudice and Shaposhnikov analyzed the effect of these QCD sphalerons on the adiabatic baryogenesis scenario.

They showed that, as long as these transitions are in equilibrium and fermion masses are neglected, no baryon asymmetry can be generated. Thus, the final result will be suppressed by a factor $\sim (M_t(T)/\pi\, T)^2$. In this Section we will reconsider the issue taking transport phenomena into account.

The effect of QCD sphalerons may be represented by the operator

$$\Pi_{i=i}^{3}(u_L\, u_R^{\dagger}\, d_L\, d_R^{\dagger})_i, \tag{5.46}$$

where i is the generation index. Assuming that these processes are in equilibrium, we get the following chemical potentials equation

$$\sum_{i=1}^{3} (\mu_{u_L^i} - \mu_{u_R^i} + \mu_{d_L^i} - \mu_{d_R^i}) = 0. \tag{5.47}$$

Eq. (5.47) contains the chemical potentials for *all* the quarks, and imposes that the total *right-handed* baryon number is equal to the total *left-handed* one.

Using Eqs. (5.1) and (5.2) we can rewrite it as

$$4\,\mu_{u_L} + \mu_{t_L} - \mu_{b_R} - 2\,\mu_{d_R} - 2\,\mu_{u_R} - 3\,\mu_{W^+} = 0, \tag{5.48}$$

where

$$\mu_{u_{L,R}} \equiv \frac{1}{2}\sum_{i=1}^{2}\mu_{u_{L,R}^i}, \quad \mu_{d_R} \equiv \frac{1}{2}\sum_{i=1}^{2}\mu_{d_R^i}. \tag{5.49}$$

One of the three new chemical potentials, μ_{b_R}, μ_{d_R}, and μ_{u_R}, can be eliminated using Eq. (5.48), while the remaining two correspond to two more conserved charges that must be taken into account besides Q', BP', and $(B-L)'$ (now the primes mean that the summation has to be performed on right-handed quarks too, but not on right-handed leptons).

We can choose

$$X \equiv \sum_{i=1}^{3} d_R^i - \frac{3}{2}\sum_{i=1}^{2} u_R^i, \tag{5.50}$$

$$Y \equiv b_L + t_L + t_R + \frac{1}{2}\sum_{i=1}^{2} u_R^i - \sum_{j=1}^{3}(e_L^j + \nu_L^j), \tag{5.51}$$

corresponding respectively to A_3 and A_2 in the notation of ref. [33] and Section 4.1.

Following the usual procedure, we can now express the abundance of any particle number at equilibrium as a linear combination of Q', $(B-L)'$, BP', X, and Y. For $(B+L)'$ we obtain the result

$$(B+L)'_{\rm EQ} = -\frac{1}{5}\,(B-L)', \tag{5.52}$$

to be compared to Eq. (5.6), which we obtained considering QCD sphalerons out of equilibrium.

Thus, the equilibrium value for $(B+L)'$ depends only on the the density of $(B-L)'$, in agreement with what was obtained in ref. [33][3].

If transport phenomena were not present, as it was assumed in ref. [33], we could set $(B-L)'$ to zero and then conclude that QCD sphalerons allow no non vanishing $(B+L)'$ density, at equilibrium and in the massless approximation.

On the other hand, including transport effects, we can easily calculate the $(B-L)'$ density induced by $\dot{\theta}$ using Eq. (5.21), and then, through (5.52), the equilibrium value $(B+L)'_{\rm EQ}$, which, unlike in ref. [33], comes out to be non vanishing inside the bubble wall.

However this is not sufficient to conclude that we will have a non zero final baryon asymmetry when the bubble wall has passed by.

As we discussed in Sect. **5.2.**, the generation of baryons inside the bubble walls is described by Eq. (5.10), with the initial condition $(B+L)'_{\rm sp}=0$.

Then we must calculate $(B+L)'_{\rm TR}$, *i.e.* the contribution to $(B+L)'$ due to transport.

If all the particles in equilibrium participated to the charge potential then we would find

$$(B+L)'_{\rm TR}(t_x,\mathbf{x})=(B+L)'_{\rm EQ}(t_x,\mathbf{x}) \tag{5.53}$$

so that the system would always be on the minimum of the free energy inside the bubble wall and there would be no bias of the (electroweak) sphaleronic transitions.

As a consequence, $(B+L)_{\rm sp}$ would remain zero and no asymmetry would survive after the passage of the wall up to fermion mass effects, in agreement with what was find in ref. [33].

On the other hand, this is only an artifact of taking all the particles in equilibrium. Including only left-handed quarks and the right-handed top quark into the charge

[3]Incidentally, note that this is not a general property due to the insertion of QCD sphalerons into the set of processes in equilibrium, but is due to the fact that only top Yukawa interactions are fast. If, for instance, bottom quark Yukawa interactions were also fast, then we would find that $(B+L)'_{\rm EQ}$ is not simply proportional to $(B-L)'$, but depends also on the values of the other charges in equilibrium.

potential, Eq. (5.53) is no more satisfied and a non zero result for the final baryon asymmetry is recovered. In this case, the factor 37/240 in Eq. (5.43) should be changed to 25/72.

5.5 Discussion

In this Chapter we have analyzed the SB mechanism taking transport effect into account by using linear response theory.

We have assumed that inside the walls of the bubbles nucleated during the electroweak phase transition a space-time dependent charge potential for (partial) fermionic hypercharge is generated.

Our main interest here was to set a scheme for calculations in the the adiabatic scenario in the case in which such a charge potential is present, in order to determine the variations in the thermal averages induced by transport effects and the production of baryon number by sphalerons inside the bubble walls.

The physical limit we have considered is that of perfect transport, in which the abundances for particle species interacting via fast interactions assume their local equilibrium values.

These are not zero but driven to non vanishing values by transport phenomena. In particular, the local equilibrium configurations will correspond to non zero values for $(B+L)'$.

We have determined the local equilibrium configuration by means of a chemical potential analysis, calculating the values of the thermal averages for the conserved charges by using linear response theory.

We have considered only the dominant contribution to these averages, in particular, we have neglected any fermion mass effect and also the coupling of the Higgs field to the Chern-Simons number.

We find that, in contrast with previous claims [34], the presence of transport phenomena does not prevent baryon number generation inside the bubble walls. The main source of disagreement with JPT is the following. In their paper, JPT consider the rate equation in the form

$$\dot{B} = -\frac{\Gamma_{SP}}{T} (3\,\mu_{t_L} + 3\,\mu_{b_L} + \mu_{\tau_L} + \mu_{\nu_\tau}), \tag{5.54}$$

where the term on the right hand side has been obtained by considering the variation of the free energy of the system due to a sphaleron like transition involving only the third generation, *i.e.* due to the processes $t_L t_L b_L \tau_L \leftrightarrow 0$ and $t_L b_L b_L \nu_\tau \leftrightarrow 0$.

Then these authors find that the chemical potential of any particle is proportional to the value of its hypercharge, and so they argue that the right hand side vanishes as a consequence of the conservation of hypercharge (and of fermion hypercharge) in any sphaleronic transition.

The point is that, assuming local equilibrium of the fast interactions, the chemical potentials of the single particle species *are not* proportional to their hypercharge. In fact, since the single particle numbers are not conserved quantities of the system, their abundances are reprocessed by fast interactions as to obtain their local equilibrium values. On the other hand, imposing that the particle chemical potentials are proportional to hypercharge, would be equivalent to freeze out any interaction inside the bubble wall, both the fast and the slow ones, leaving transport phenomena as the only relevant process.

Also, if a charge potential for fermionic or total hypercharge is present, transport phenomena allow the generation of the baryon asymmetry even in the limit in which the Higgs VEV's go to zero, thus avoiding Dine and Thomas objection to spontaneous baryogenesis.

The reason is again that the thermal averages for Q', $(B-L)'$ and BP' are non vanishing and then a $(B+L)$ asymmetry can be generated even if the electroweak symmetry is unbroken.

This is strictly analogous to the well known result of the survival of a $B+L$ asymmetry when a $B-L$ density, eventually of GUT origin, is present [43].

Transport phenomena have been also studied by CKN in ref. [36] by means of kinetic equations. Their results are in perfect agreement with those described in this Chapter in the adiabatic limit of thick bubble walls[4].

[4]Note, however, that in ref. [36] the baryon number violating rate was used twice: in diffusion

In particular, the authors found that the Dine and Thomas criticsm is evaded since doublet densities produced within the bubble wall can venture out into the symmetric phase, where baryon number violation is effective. It was also shown that particles do not remain in front of the wall enough for the strong sphalerons to erase the left-handed doublet densities, thus making the QCD sphalerons harmless.

Let us make some comments before launching into the subject of the next Chapter.

We want to stress that no discussion about the origin of the charge potential has been given so far.

Throughout the whole Thesis we have been (tacitly) assuming that the effect of space-time dependent and CP violating phases (in the Higgs sector) on particle densities were described by a charge potential for the fermionic (or total) hypercharge. This assumption has been the starting point of all the the objections (and subsequent ways out) depicted in this Thesis.

However, since in the limit in which all the Yukawa couplings go to zero there is no communication between the Higgs and the fermion sectors, we expect that in this limit also the charge potential should go to zero.

In the traditional approach of CKN and in all the subsequent papers there was no trace of this behaviour.

Furthermore, in the limit in which the VEV's go to zero, also the charge potential should go to zero, since no complex phase can emerge from the Higgs sector in this case. Then, also this suppression, like the one due to vanishing h_t, should be made evident by an accurate discussion on the origin of the charge potential. As a consequence, our results for the baryon asymmetry, Eq. (5.44) should be multiplied by a further suppression factor roughly of order $(M_t(T)/\pi\, T)^2$, which has nothing to do with the presence of QCD sphalerons or with the Dine Thomas objection: we expect this suppression on very general theoretical grounds.

We reserve the discussion on this subject to the next Chapter.

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equations and to compute the final baryon number asymmetry through the Equation (2.8). They also used equilibrium number densities $n_i^{\rm EQ}$ proportional to $Y_f^i\,\dot{\theta}\,T^2$, which is not correct in the limit of fast processes since, when interactions are considered, this proportionality gets lost as we have seen in this Chapter. These conceptual errors have been corrected in ref. [39].

Chapter 6

Currents in a CP violating Higgs background and SB: the Modern era

Motivated by the opinion that describing by means of a charge potential the effects on particle currents of a space-time dependent and CP violating Higgs background presents some serious problems, in this Chapter we shall develop a field theoretic approach to the computation of particle currents on such space-time dependent and CP violating Higgs background. The contents of this Chapter are bases on two recent papers by CPR [37].

We shall consider the SM model with two Higgs doublets and CP violation in the scalar sector, and compute both fermionic and Higgs currents by means of an expansion in the background fields. We shall discuss the gauge dependence of the results and the renormalization of the current operators, showing that in the limit of local equilibrium, no extra renormalization conditions are needed in order to specify the system completely.

By means of this field theory approach we shall recover the suppression factor $(M_t(T)/\pi\, T)^2$ announced at the end of the previous Chapter.

Let us remind the reader again that this suppression factor has nothing to do with the ones depicted in Chapter 4 (the latter being derived from particular hypothesis involving the underlying physics, like the presence of QCD sphalerons, or some assumptions like $\langle B-L\rangle = 0$). From this point of view, this Chapter should have been read immediately after Chapter **2**).

The suppression factor we shall be discussing about is predicted by very general arguments and lies on very firmed theoretical grounds. Its presence signals that the charge potential tool is inadequate for a fully consistent approach to spontaneous baryogenesis.

In order to reliably compute the final baryon asymmetry, it is then necessary, as a first step, to determine the values of the currents induced by the background.

As said several times, in the approach of ref. [28] a rotation of the fermionic fields is done to make the Yukawa couplings real. As a consequence, a derivative coupling of the form

$$\mathcal{L}_{\rm int} \sim \partial_\mu \delta J^\mu, \tag{6.1}$$

where J^μ is the current corresponding to the rotation, is induced from the kinetic terms. We indicate here by $\delta(x)$ the space-time dependent relative phase between the two Higgs complex VEV's.

Nevertheless, using the interaction term in Eq. (6.1) as a starting point to compute the perturbations to the thermal averages presents some drawbacks.

Since the phase δ is communicated from the Higgs to the fermion sector through the Yukawa interactions, any perturbation in the fermion densities n_i should vanish in the limit of zero Yukawa couplings h_i.

Also, they should vanish in the limit of zero vacuum expectation value for the Higgs fields $H_i(x) = v_i(x)\exp[i\theta_i(x)]$ because no spontaneous CP violation is present in the Higgs sector in this limit.

Naively, one could then expect a suppression factor of order $(h_i^2 v_i^2(x)/T^2)$, where h_i is the relevant Yukawa coupling, for the perturbations in the fermionic particle number with respect to the original result. Since one is interested in regions of the bubble wall where sphalerons are still active, *i.e* for values at most $v_{\rm co}/T$ typically smaller than one, then the above mentioned suppression factor might be crucial for the scenario of SB.

The reason why these suppressions do not appear in the original treatment is that considering Eq. (6.1) as the only effect of the background is equivalent to perturbing around the Higgs field configuration $\delta(x) = 0$, $v_i(x) \neq 0$, which is not a solution of the field equations.

On the other hand, taking the field equations into account, one immediately sees that the expected suppression factors are recovered also in the original treatment, so that the result comes out to be rotation independent, as it should, also in this context. Indeed, by partially integrating Eq. (6.1), we obtain a perturbation term which is given by the hypercharge violating terms in the Lagrangian, coming from the fermionic mass terms and from the Higgs potential.

Also, from the field equations one can see that $\partial_\mu \delta(x)$ vanishes as $v_i^2(x)$ for vanishing $v_i(x)$. In other words, it is the perturbation (6.1) itself, if properly considered, to be vanishing in the limit of vanishing top Yukawa and Higgs couplings, or in the limit of vanishing values for the background Higgs fields.

In the limit in which some interaction rates (like for example those for the top Yukawa processes) are so large as to be in thermal equilibrium inside the bubble walls, the local equilibrium values for the corresponding particle numbers are of course *independent* on the reaction rates.

On the other hand, since their values are fixed by that of the perturbation term (6.1)), *they are still suppressed by the above mentioned factor of* $(h_t^2 v^2(x)/T^2)$. Thus, this suppression is *not* the one described at the end of Section **2.3**: it is always present, *even* for particles whose reactions are so fast that their equilibrium number densities do not depend upon the interaction rates.

In this Chapter we shall describe in more detail our approach to calculate the expectation values of a generic composite operator $\hat{O}(z)$ in a CP violating Higgs space-time dependent background.

For definiteness, we will work on the background of a bubble wall, however the formalism is quite general and can be easily extended to consider other interesting situations, like for example a non trivial background also for the gauge fields.

The method relies on a functional expansion in power series of the background $\Phi_i^c(x)$ where the coefficients of the expansion are the n-point 1PI Green's functions with one insertion of the operator $\hat{O}(z)$ computed in the *unbroken* phase.

We shall discuss the dependence of the expectation values from the gauge in which the background $\Phi_i^c(x)$ is expressed, showing explicitly that the expectation values of gauge invariant operators are gauge independent. Moreover, the problem of current renormalization will be addressed. In general, the computation of Green's functions with insertion of a composite operator requires the introduction of new counterterms besides those necessary for the renormalization of the basic Lagrangian.

This fact leads to the well known phenomenon of the mixing among the different renormalized currents of the theory. We shall discuss the mixing matrix for the renormalized currents present in the Standard Model model with two Higgs doublet and use the non renormalization properties for the conserved currents in order to reduce the number of independent counterterms.

Finally, we will discuss the limit of local equilibrium, showing that, in this case, the only expectation values that can be consistently computed are those for the conserved currents, so that no new renormalization condition is needed to specify the system in this limit.

The Chapter is organized as follows: in Section **6.1** our field theory method is described on general grounds and particular attention is given to the classical equations of motions and to the question of gauge invariance. In Section **6.2** the explicit calculations for the fermionic and Higgs currents in the specific model under consideration are given in details. Section **6.3** deals with the issue of current renormalization, whereas the role of the conserved currents in the thermodynamical limit is described in Section **6.4**. Section **6.5** presents our comments on the implications of the results of this Chapter for the SB mechanism. Finally, Section **6.6** briefly describes a new semiclassical approach due to Huet and Nelson, by which the same suppression factor proposed by CPR in [37] was found. Its application to EWB in supersymmetric models is shortly depicted in Section **6.7**.

6.1 The expansion

In this Section we will discuss our approach to the computation of the expectation value of an operator $\hat{O}(z)$ on a non zero background for the fields of the theory.

Our starting point is the finite temperature generating functional for the 1PI Green's functions with insertion of an operator $\hat{O}(z)$ (in the following $\hat{O}(z)$ will represent a particle current)

$$\Gamma\left[\Phi_i^c(x),\, \Delta(x)\right] = W\left[J_i(x),\, \Delta(x)\right] - \sum_j \int d^4x J_j(x)\Phi_j^c(x), \tag{6.2}$$

where $\Phi_i^c(x)$ are the classical fields of the theory and $J_i(x)$ the corresponding sources, while $\Delta(x)$ is the source for the operator $\hat{O}(x)$. Note that the Legendre transformation has been performed only on the fields and not on the operator.

The quantity we are interested in is the expectation value of the operator $\hat{O}(z)$ on the background given by the fields $\Phi_i^c(x)$, which we will specify later,

$$\langle \hat{O}(z)\rangle_{\Phi_i^c(x)} = \left.\frac{\delta\Gamma\left[\Phi_i^c,\, \Delta\right]}{\delta\Delta(z)}\right|_{\Delta=0} \equiv \mathcal{O}\left[\Phi_i^c(x)\right](z). \tag{6.3}$$

We can expand the functional $\mathcal{O}\left[\Phi_i^c(x)\right](z)$ in a power series of Φ_i^c

$$\mathcal{O}\left[\Phi_i^c(x)\right](z) = \sum_{n=0}^{\infty}\sum_{i_1,\ldots,i_n}\frac{1}{n!}\int d^4x_1\cdots d^4x_n \mathcal{O}^{(n)}_{i_1,\ldots,i_n}(x_1,\cdots,x_n;\, z)\Phi^c_{i_1}(x_1)\cdots\Phi^c_{i_n}(x_n), \tag{6.4}$$

where the coefficients of the expansion are the n-point 1PI Green's functions with one insertion of the operator $\hat{O}(z)$ computed in the *unbroken* phase

$$\mathcal{O}^{(n)}_{i_1,\ldots,i_n}(x_1,\cdots,x_n;\, z) = \frac{1}{i}\left.\frac{\delta^{n+1}\Gamma\left[\Phi_i^c,\, \Delta\right]}{\delta\Phi^c_{i_1}(x_1)\cdots\delta\Phi^c_{i_n}(x_n)\delta\Delta(z)}\right|_{\Phi_i^c=\Delta=0}. \tag{6.5}$$

Now we have to specify the background fields, $\Phi_i^c(x)$.

A priori, the relevant background is given by the solution of the full field equations,

$$\left.\frac{\delta\Gamma\left[\Phi_i^c,\, \Delta=0\right]}{\delta\Phi_i^c(x)}\right|_{\Phi_i^c=\bar{\Phi}_i^c} = 0 \tag{6.6}$$

with appropriate boundary conditions.

In practice, however, we have to consider some approximation of this complete, and unknown, solution. A possibility could be to consider the solutions of the classical equations of motion as the zero order approximation. However in many interesting cases the radiative corrections to the effective potential, either at zero or at finite temperature, are crucial in order to determine the shape of the potential and consequently of the background. A typical example is that of a first order phase transition induced by finite temperature corrections, in which the existence of bubble solutions is due to the $T \neq 0$ radiative corrections which induce a second minima in the effective potential.

Following refs. [78, 79], we will then consider the classical equations of motion in which the tree level effective potential is replaced by the radiatively corrected one.

In the following, we will consider the Standard Model with two Higgs doublets, H_1 and H_2. H_1 has $U(1)_Y$ charge $-1/2$, and couples to the down type right-handed fermions, whereas H_2 (charge $1/2$) couples to the up type ones.

Moreover, since only top and bottom quarks will be relevant in the following we will not consider the lighter quarks and leptons.

Neglecting all the Yukawa couplings but the one for the right-handed top, h_t, the classical Lagrangian is given by

$$\begin{aligned} \mathcal{L} \;=\; & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(\mathcal{D}_\mu H_1\right)^\dagger \mathcal{D}^\mu H_1 + \left(\mathcal{D}_\mu H_2\right)^\dagger \mathcal{D}^\mu H_2 \\ & + i\bar{t}_L\gamma_\mu \not{\mathcal{D}}^\mu t_L + i\bar{t}_R\gamma_\mu \not{\mathcal{D}}^\mu t_R + (t \to b) + h_t\left(H_2^0 \bar{t}_R t_L - H^+ \bar{t}_R b_L\right) - V(H_1, H_2), \end{aligned} \tag{6.7}$$

where the most general tree level scalar potential $V(H_1, H_2)$ is given in Eq. (3.24).

In Eq. (6.8) we have also omitted tha $SU(2)_L$ gauge part, which will not be relevant in the following.

Note that of the two phases of the Higgs fields, θ_1 and θ_2, defined by $H_i(x) = v_i(x)\exp[i\theta_i(x)]$, only the gauge invariant combination $\delta = \theta_1 + \theta_2$ appears in the scalar potential.

The orthogonal combination, $\chi = \theta_1 - \theta_2$, is the gauge phase, corresponding to the would be Goldstone boson after spontaneous symmetry breaking.

One can now write down the equations of motion coming from the Lagrangian in (6.7), with the tree level scalar potential $V(H_1, H_2)$ replaced by the (finite temperature) radiatively corrected one, $\overline{V}(H_1, H_2)$.

We assume that the parameters of the effective potential (and the temperature) are such that it has only one minimum in the charged directions, given by $H^+ = H^- = 0$, whereas there are two minima in the neutral Higgs directions. Then, a bubble solution exists, with $v_{1,2}(x)$ changing from a non zero value inside to zero outside, and $H^\pm = 0$ everywhere.

At zero order, we will also take the fermion fields and currents to be zero, so that the background is given by the solution of the following field equations:

$$\partial^2 v_{1,2} + \frac{\partial \overline{V}}{\partial v_{1,2}} - \frac{1}{4} v_{1,2} \left(\partial_\mu \delta \mp \mathcal{D}_\mu \chi\right)^2 = 0 \tag{6.8}$$

$$\partial_\mu F^{\mu\nu} = \frac{e}{4} \left[(v_1^2 - v_2^2)\partial^\nu \delta - (v_1^2 + v_2^2)\mathcal{D}^\nu \chi\right] \tag{6.9}$$

$$\frac{1}{4}\partial_\nu \left[(v_1^2 + v_2^2)\partial^\nu \delta - (v_1^2 - v_2^2)\mathcal{D}^\nu \chi\right] = -\frac{\partial \overline{V}}{\partial \delta}, \tag{6.10}$$

where the gauge invariant quantity $\mathcal{D}_\mu \chi$ is given by

$$\mathcal{D}_\mu \chi = \partial_\mu \chi + e A_\mu. \tag{6.11}$$

Assuming also that there are no electric or magnetic fields at zero order, *i.e.* $F_{\mu\nu} = 0$, and choosing the unitary gauge $A_\mu = 0$ (see below), we obtain from Eq. (6.9)

$$v_1^2 \partial_\mu \theta_1 = v_2^2 \partial_\mu \theta_2, \qquad (A_\mu = 0) \tag{6.12}$$

so that Eq. (6.10) now reads

$$\partial^\mu \left(v_1^2 \partial_\mu \theta_1\right) = -\frac{\partial \overline{V}}{\partial \delta}. \tag{6.13}$$

Inserting the solutions of the above equations into the expansion in Eq. (6.4) we will now be able to compute the various contributions to the expectation values of the currents as a loop expansion.

In the following Sections we will see that the Higgs currents get a contribution already at the tree level, whereas, consistently with our assumptions, the contributions to the fermionic currents, and to $\langle \partial_\mu F^{\mu\nu} \rangle$, arise only at one loop.

Before concluding this Section, we want to show explicitly that the expectation value for a gauge invariant operator, as obtained by the expansion in Eq. (6.4), is independent on the gauge in which the classical background has been computed.

In our discussion above, we have looked for a solution of the field equations with no electromagnetic fields, *i.e.* with $F_{\mu\nu} = 0$, so that it was possible to chose a gauge in which $A_\mu = 0$ everywhere (see Eqs. (6.12,6.13)). Choosing a different gauge gives rise to a different background, in which A_μ is different from zero, and we want to show that the result for the gauge invariant operator $\hat{O}(z)$ is the same as in the original background.

Following the usual procedure for the derivation of the Ward identities it is straightforward to obtain the relation

$$\frac{1}{\alpha}\partial^2\partial_\mu a^\mu + \partial_\mu \frac{\delta\Gamma}{\delta a_\mu} - i\frac{e}{2}\left(H_1^0 \frac{\delta\Gamma}{\delta H_1^0} - H_1^{0*}\frac{\delta\Gamma}{\delta H_1^{0*}} - H_2^0\frac{\delta\Gamma}{\delta H_2^0} + H_2^{0*}\frac{\delta\Gamma}{\delta H_2^{0*}}\right) = 0 \quad (6.14)$$

where α is the gauge parameter, a_μ, $H_{1,2}^0$, and ${H_{1,2}^0}^*$ are the classical gauge and neutral Higgs fields, and we have put the charged Higgses and the fermion classical fields to zero.

Differentiating the above expression with respect to the source $\Delta(x)$ keeping the background fields fixed, and then putting $\Delta(x) = 0$ we obtain the relevant identity containing the functional $\mathcal{O}$ of Eq. (6.3),

$$\partial_\mu \frac{\delta\mathcal{O}}{\delta a_\mu} - i\frac{e}{2}\left(H_1^0 \frac{\delta\mathcal{O}}{\delta H_1^0} - H_1^{0*}\frac{\delta\mathcal{O}}{\delta H_1^{0*}} - H_2^0\frac{\delta\mathcal{O}}{\delta H_2^0} + H_2^{0*}\frac{\delta\mathcal{O}}{\delta H_2^{0*}}\right) = 0. \quad (6.15)$$

The Green's functions entering the expansion (6.4) have to be computed in the 'unbroken phase' *i.e.* for vanishing background fields $\Phi_i^c = \{H_{1,2}^0, {H_{1,2}^0}^*, a_\mu\}$. Taking $\Phi_i^c = 0$ in Eq. (6.15), one gets

$$\partial_\mu \frac{\delta\mathcal{O}}{\delta a_\mu}\bigg|_{\Phi_i^c=0} = 0, \quad (6.16)$$

while, differentiating with respect to H_i^0,

$$-i\frac{e}{2}\frac{\delta\mathcal{O}}{\delta H_1^0(y)}\bigg|_{\Phi_i^c=0}\delta^{(4)}(x-y) + \partial_\mu \frac{\delta^2\mathcal{O}}{\delta a_\mu(x)\delta H_1^0(y)}\bigg|_{\Phi_i^c=0} = 0 \quad (6.17)$$

and similar identities are obtained differentiating with respect to the other fields.

Now, let's indicate with $\overline{\Phi}_i^c = \{\overline{H}^0_{1,2}, \overline{H}^{0\;*}_{1,2}, \overline{a}_\mu = 0\}$ the 'bubble' solution of the field equations discussed above, and with $\overline{\Phi}_i^{c\prime} = \{\overline{H}^{0\;\prime}_{1,2}, \overline{H}^{0\;\prime *}_{1,2}, \overline{a}'_\mu = -1/e\; \partial_\mu \theta\}$ the solution obtained from the former by a gauge transformation. Then, we consider the functional $\mathcal{O}$ on the background $\overline{\Phi}_i^{c\prime}$ and expand it according to Eq. (6.4). Since $\overline{a}'_\mu \neq 0$ in this case also Green functions involving gauge fields in the external legs will contribute to the expansion whereas, if we consider the background $\overline{\Phi}_i^c$, only the Green functions with neutral Higgses in the external legs contribute. However, by means of the above identities, one can easily show that there are cancellations among these new contributions, so that the result is the same as for $\overline{\Phi}_i^c$. Indeed, at the linear order in $\overline{a}'_\mu$ in the expansion we have

$$\int d^4x \left.\frac{\delta\mathcal{O}}{\delta a_\mu(x)}\right|_{\Phi_i^c=0} \overline{a}'_\mu(x) = \frac{1}{e}\int d^4x \partial_\mu \left.\frac{\delta\mathcal{O}}{\delta a_\mu(x)}\right|_{\Phi_i^c=0} \theta(x) = 0 \qquad (6.18)$$

where the last equality is due to Eq. (6.16). At the linear order in $\overline{H}_1^{0\prime}$ we have

$$\int d^4x \left.\frac{\delta\mathcal{O}}{\delta H_1^0(x)}\right|_{\Phi_i^c=0} \overline{H}_1^{0\prime}(x) = \int d^4x \left.\frac{\delta\mathcal{O}}{\delta H_1^0(x)}\right|_{\Phi_i^c=0} \overline{H}_1^0(x)(1-i\theta(x)/2) + \mathcal{O}(\theta^2). \quad (6.19)$$

The contribution $O(\theta(x))$ is canceled by the $\overline{H}_1^{0\prime}\overline{a}'_\mu$ term in the expansion by virtue of Eq. (6.17),

$$\int d^4x d^4y \left.\frac{\delta^2\mathcal{O}}{\delta a_\mu(x)\delta H_1^0(y)}\right|_{\Phi_i^c=0} \overline{H}_1^{0\prime}(y)\overline{a}'_\mu(x) =$$

$$\frac{1}{e}\int d^4x d^4y\; \partial_\mu \left.\frac{\delta^2\mathcal{O}}{\delta a_\mu(x)\delta H_1^0(y)}\right|_{\Phi_i^c=0} \overline{H}_1^0(y)\theta(y) + \mathcal{O}\left(\theta^2\right) = \qquad (6.20)$$

$$\frac{i}{2}\int d^4x \left.\frac{\delta\mathcal{O}}{\delta H_1^0(x)}\right|_{\Phi_i^c=0} \overline{H}_1^0(x)\theta(x) + \mathcal{O}\left(\theta^2\right).$$

The contribution of n-th order in θ in Eq. (6.19), is canceled by the terms coming from the $\overline{H}_1^{0\prime}\left(\overline{a}'_\mu\right)^i$ $(i = 0 \ldots n)$ orders in the expansion, due to the appropriate Ward identities obtained by differentiating Eq. (6.17) with respect to a_μ.

This result generalizes to the other terms of the expansion: all the contributions of the same order in the primed Higgs fields and of any order in a'_μ in the expansion for $\mathcal{O}[\overline{\Phi}_i^{c\prime}]$ sum up to the corresponding term of the same order in the non-primed Higgs fields in the expansion for $\mathcal{O}[\overline{\Phi}_i^c]$, so that

$$\mathcal{O}[\overline{\Phi}_i^{c\prime}] = \mathcal{O}[\overline{\Phi}_i^c]. \tag{6.21}$$

Since in practical applications one has to truncate the expansion to a finite order in the fields, it is clear from the above discussion that this can be done in a sensible way only if we consider the background $\overline{\Phi}_i^c$, where the classical gauge field is zero, *i.e.* in the unitary gauge.

6.2 Computation of the currents

We will now apply the method illustrated in the previous paragraph to the computation of $\langle J_i^\mu(z)\rangle_{\Phi_i^0(x)}$, the expectation value of the current operator $J_i^\mu(x)$ ($i = H_{1,2}^0, H^\pm, t_{L,R}, b_{L,R}$) on the background $\bar{\Phi}_{j,k}^c = \{H_1^0, H_1^{0*}, H_2^0, H_2^{0*}\}$, which, for definiteness, we assume to be the bubble solution to the field equation of motion discussed in the previous Section.

The computation of such currents is crucial for the spontaneous baryogenesis mechanism.

In the following, we will assume that there are no chemical potentials. Then, it is easy to realize that a necessary condition for the presence of non vanishing expectation values for the currents is a non trivial space-time dependence of the background.

Indeed, if $H_{1,2}^*(x)$ were constant then the only two vectors available for the construction of $\langle J_i^\mu(z)\rangle_{\Phi_j^c}$ would be z^μ and u^μ (the quadrivector which identifies the motion of the thermal reference).

By using translational invariance, we are left with $\langle J_i^\mu(z)\rangle_{\Phi_j^c} \sim \text{const} \times u^\mu$, but this is forbidden by CPT.

Also, by applying a charge conjugation transformation both to the operator and to the background, we see that

$$\langle J_i^\mu(z)\rangle_{\Phi_j^c(x)} = -\langle J_i^\mu(z)\rangle_{\Phi_j^c(x)^*}, \tag{6.22}$$

i.e. the expectation value for the current would vanish also if the background were real.

From these two considerations one can conclude that the expectation values we are looking for, will depend on *space-time derivatives* of a *complex background*, so that the two phases $\theta_1(x)$ and $\theta_2(x)$ discussed previously, (or $\delta(x)$ and $\chi(x)$), constrained by (6.12), will play a crucial role.

Computing the expectation value as a functional expansion on the background, as in Eq. (6.4), the problem is reduced to the calculation of 1PI Green's functions with one $J_i^\mu(x)$ insertion in the *unbroken* phase, as in Eq. (6.5).

We will work in the imaginary time formalism, so that our expressions for the Green's functions and for the currents are valid in the reference frame of the thermal bath, where the bubble wall is moving.

The zero order of the expansion in Eq. (6.5) vanishes since it is given by the 1PI part of $\langle J_i^\mu(z)\rangle_{\Phi_j^c=0}$ which is zero since translational invariance holds in the unbroken phase.

The linear order is also zero, as can be realized observing that in the unbroken phase the theory has still a *global* $U(1)$ invariance, whereas $\langle J_i^\mu(z)H_k^0(y)\rangle_{\Phi_j^c=0}$ is not a singlet under such symmetry.

The first non-vanishing contributions then come from the quadratic terms *i.e.*

$$\langle J_i^\mu(z)\rangle = \frac{1}{2}\sum_{j,k}\int d^4x\, d^4y\, \left.\frac{\delta^3\Gamma\left[\bar\Phi_j^c,\, \Delta_\mu^i\right]}{\delta\bar\Phi_j^c(x)\delta\bar\Phi_k^c(y)\delta\Delta_\mu^i(z)}\right|_{\bar\Phi_{j,k}^c=\Delta_\mu=0} \bar\Phi_j^c(x)\bar\Phi_k^c(y). \tag{6.23}$$

Using again the global $U(1)$, one can see that the only possible non-vanishing contributions to the above expression come from the 1PI part of $\langle J_i^\mu(z)\hat H_1^0(x)\hat H_2^0(y)\rangle_{\Phi_j^c=0}$, or $\langle J_i^\mu(z)\hat H_i^0(x)\hat H_i^0(y)^*\rangle_{\Phi_j^c=0}$ $(i=1,2)$. These Green's functions will now be computed as usual in perturbation theory, and the result inserted into the expression (6.23).

We start with the Higgs currents

$$J_{H_i}^\mu = i\left(H_i^\dagger \mathcal{D}^\mu H_i - \mathcal{D}^\mu H_i^\dagger H_i\right). \tag{6.24}$$

In the case of the neutral Higgses a contribution already appears at the tree level, see Fig. 8. Inserting this contribution into the expression (6.23) we get the classical current

$$\langle J^{\mu}_{H_i^0}(z)\rangle^{(1)} = -2\, v_i^2(z)\, \partial^{\mu}\theta_i(z). \tag{6.25}$$

Note that, as expected, the individual Higgs currents are zero if the phases are constant.

Moreover, from the equation of motion (6.12), we see that the total Higgs hypecharge current vanishes at the tree level.

At one-loop there is a contribution to the neutral and charged Higgs currents given by graphs like that in Fig. 9.

Since the computation has to be performed in the unbroken phase, we must use resummed propagators for the Higgs fields in order to deal with the infrared divergencies [80].

In the unbroken phase, the Higgs spectrum contains two complex electrically neutral fields and two charged ones. At the tree level, the squared masses of one of the neutral states and of one of the charged ones are negative, since the origin of field space becomes a minimum of the effective potential only after the inclusion of the finite temperature radiative corrections.

The resummation can be achieved by considering the propagators for the eigenstates of the thermal mass matrix, which has positive eigenvalues given by

$$M^2_{1,2}(T) = \frac{m_1^2(T) + m_2^2(T) \mp \sqrt{\left(m_1^2(T) - m_2^2(T)\right)^2 + 4\, m_3^4(T)}}{2}, \tag{6.26}$$

where the $m_i^2(T)$ are the thermally corrected mass parameters of the potential (3.24), $m_1^2(T) \simeq m_1^2 + 3g^2T^2/16$, $m_2^2(T) \simeq m_2^2 + h_t^2 T^2/4$, while $m_3^2(T)$ receives only logarithmic corrections in T, which were computed in ref. [30, 31].

Correspondingly, the neutral complex eigenstates are given by

$$\left\{ \begin{array}{lcrcl} h & = & \cos\alpha\, H_1^0 & + & \sin\alpha\, H_2^{0*} \\ & & & & \\ H & = & -\sin\alpha\, H_1^0 & + & \cos\alpha\, H_2^{0*} \end{array} \right. , \tag{6.27}$$

where

$$\sin 2\alpha = \frac{2m_3^2(T)}{M_1^2(T) - M_2^2(T)}. \tag{6.28}$$

Completely analogous formulae hold for the charged eigenstates.

The loop in Fig. 9 gives

$$\frac{\delta^3\Gamma}{\delta\Delta^1_\mu(z)\delta H^0_1(x)\delta H^0_2(y)} = -i\,\lambda_5\,\sin 2\alpha\;\delta^{(4)}(x-y)\int\frac{d^4q}{(2\pi)^4}e^{iq\cdot(z-x)}\mathrm{H}^\mu(q), \qquad (6.29)$$

where the function $\mathrm{H}^\mu(q)$ is the analytical continuation to Minkowski external momentum q of the finite temperature integral,

$$\mathrm{H}^\mu(\tilde{q}) = \sum\!\!\!\!\!\!\int dp\left\{\frac{p^\mu}{[p^2+M_1^2(T)]\;[(\tilde{q}-p)^2+M_2^2(T)]} - \Big((M_1^2(T)\leftrightarrow M_2^2(T)\Big)\right\}, \qquad (6.30)$$

where $\tilde{q}_4 = iq_0$, $\tilde{q}_i = q_i$, $p_0 = 2\pi n_p T$, $p^2 = p_0^2 + |\vec{p}|^2$, and

$$\sum\!\!\!\!\!\!\int dp \equiv \sum_{n_p}\int\frac{d^3\mathbf{p}}{(2\pi)^3}.$$

Note that the only quartic coupling contributing to the Higgs currents is λ_5, whereas there are no contributions from λ_6 and λ_7. Moreover, the contribution vanishes if there is no mixing $m_3^2(T)$ between the two Higgs gauge eigenstates (see Eq. (6.28)).

It is easy to check that Green function in (6.30) is ultraviolet finite, due to the minus sign between the two terms in the integrand. After some algebra, the integral in p can be casted into the form

$$\mathrm{H}^\mu(\tilde{q}) = \frac{i}{2}\tilde{q}^\mu\Big[(\tilde{q}^2+M_1^2(T))\sum\!\!\!\!\!\!\int dp\frac{1}{(p^2+M_2^2(T))((\tilde{q}-p)^2+M_1^2(T))((\tilde{q}+p)^2+M_1^2(T))} - \Big((M_1^2(T)\leftrightarrow M_2^2(T)\Big)\Big]. \qquad (6.31)$$

Now, the scale of the external momentum q is set by the bubble wall width, which, in the case of interest for us of thick bubble walls, is given by $L_w^{-1}\simeq(1-100)\,T$. Then, since we have $\tilde{q}^2 \ll M_{1,2}^2(T) \ll (2\pi T)^2$, we approximate the integral by neglecting the $\tilde{q}$ dependence in the integrand and by considering only the zero Matsubara mode for p_0. Inserting the result into the quadratic term of the expansion (6.4) we obtain

$$\langle J^\mu_{H_1^0}(z)\rangle^{(2)} = \langle J^\mu_{H_2^0}(z)\rangle^{(2)} \simeq \frac{\lambda_5}{8\pi}\frac{T\,m_3^2(T)}{(M_1(T)+M_2(T))^3}\,\partial^\mu\left[v_1(z)v_2(z)\sin\delta(z)\right], \qquad (6.32)$$

Each charged Higgs gets a contribution equal to that of the neutral Higgs belonging to the same doublet. As expected, also the Higgs currents vanish in the limit of vanishing $v_i(z)$.

The first contribution to the fermionic currents arises at one-loop and is given by graphs like that in Fig. 10.

Once inserted into the expansion (6.4), they give, in the case of the left-handed top,

$$\langle J^\mu_{t_L}(z)\rangle^{(3)} = i h_t^2 \int d^4x\, d^4y \,\mathrm{Im}\left(H_2^0(x) H_2^0(y)^*\right)\, \mathcal{G}^\mu(x,y,z), \tag{6.33}$$

where h_t is the top Yukawa coupling and

$$\mathcal{G}^\mu(x,y,z) = \int \frac{d^4l}{(2\pi)^4}\frac{d^4m}{(2\pi)^4} \mathrm{e}^{i\,l(x-z)}\mathrm{e}^{i\,m(y-z)}\,\mathrm{G}^\mu(m,l) \tag{6.34}$$

is the Green function corresponding to the diagram in Fig. 10, with

$$\mathrm{G}^\mu(\tilde m,\tilde l) = \sum\!\!\!\!\!\!\int dk \frac{\mathrm{Tr}\left[(\not k + \not{\tilde m})\,\not k(\not k - \not{\tilde l})\gamma^\mu\right]}{(k+\tilde m)^2 k^2 (k-\tilde l)^2}, \tag{6.35}$$

and the zero component of the fermionic loop momentum is $k^0 = (2n+1)\pi T$.

Contrary to the case of the scalar loop, this fermionic loop integral is now infrared finite, since the zero component of the loop momentum never vanishes. As a consequence, we will work with the tree level fermion propagators, without of resummation.

Again, in presence of a thick wall background we can perform a high temperature expansion.

This is achieved by approximating $\mathrm{G}^\mu(\tilde m,\tilde l) \simeq \mathrm{G}^\mu(\tilde m,0) + \mathrm{G}^\mu(0,\tilde l)$ (note that $\mathrm{G}^\mu(\tilde m,\tilde l) = -\mathrm{G}^\mu(\tilde l,\tilde m)$), and neglecting terms of order $\tilde l^2/T^2$ in the computation of $\mathrm{G}^\mu(0,\tilde l)$.

Now the integral has a logarithmic ultraviolet divergence, so renormalization is needed (see also the next section). In the $\overline{MS}$ scheme we obtain

$$\langle J^\mu_{t_L}(z)\rangle^{(3)} \simeq -\frac{h_t^2}{\pi^2}\, v_2^2(z)\partial^\mu\theta_2(z)\left(\log\left(\frac{\mu^2}{A_f\,T^2}\right)+\frac{3}{2}\right), \tag{6.36}$$

where $A_f = \pi^2\exp(3/2\; - 2\gamma_E) \simeq 13.944$.

Eq. (6.36) shows the expected dependence on h_t^2 and $v_2^2(z)$ which, in comparison to the original result given in ref. [28], gives a suppression factor $\mathcal{O}(h_t v_2/\pi\, T)^2$.

Note that Fig. 10 has the straightforward explanation: a left-handed particle leaves the point z and is scattered of the Higgs background at the point x where it becomes a right-handed particle. The latter is then reconverted into a left-handed fermion at the point y through another scattering off the Higgs bacground. Since these scatterings are CP violating, a net nonvanishing current arises.

A graph similar to that in Fig. 10 for the right-handed top quark leads to a contribution to $\langle J^{\mu}_{t_R}(z)\rangle$ given by $\langle J^{\mu}_{t_R}(z)\rangle^{(1)} = -\langle J^{\mu}_{t_L}(z)\rangle^{(1)}$. For the other fermion species, one finds analogous results, in which h_t is replaced by the appropriate Yukawa coupling, and $v_2(z)$ ($v_1(z)$) appears for the up (down) type fermions.

A contribution to $\langle J^{\mu}_{t_L}(z)\rangle$ proportional to $\mathrm{Im}(H_1^0 H_2^0) = v_1 v_2 \sin\delta$, and to h_t^2, appears at two-loops, given by the graph in Fig. 11.

The corresponding integral is

$$I(k) = \not{\int} dq \frac{1}{((q+k)^2 + M_1^2(T))(q^2 + M_2^2(T))} \mathrm{G}^{\mu}(-q-k, q) \tag{6.37}$$

where q is the bosonic loop variable and $\mathrm{G}^{\mu}(m,l)$ has been defined in (6.35).

Computing the fermionic integral in the approximation described above, we are left with a ultraviolet finite bosonic integration which receives important contributions only from the zero Matsubara mode. Inserting the result into the expansion we obtain, again in the high temperature approximation,

$$\langle J^{\mu}_{t_L}(z)\rangle^{(4)} \simeq -\frac{\lambda_5\, h_t^2}{64\pi^3} \frac{T\, m_3^2(T)}{(M_1(T) + M_2(T))^3} \left(\log\left(\frac{\mu^2}{A_f\, T^2}\right) + \frac{3}{2}\right) \partial^{\mu} \left[v_1(z) v_2(z) \sin\delta(z)\right]. \tag{6.38}$$

As for the one-loop result, the two-loop contribution with neutral Higgses in the internal lines for $\langle J^{\mu}_{t_R}(z)\rangle$ is opposite to the one for $\langle J^{\mu}_{t_L}(z)\rangle$.

This is no longer true when the graphs with charged Higgses in the internal lines are taken into account. In such a case, $\langle J^{\mu}_{t_R}(z)\rangle$ gets a a contribution equal to Eq. (6.38), whereas the result for $\langle J^{\mu}_{t_L}(z)\rangle$ is analogous but proportional to h_b^2 instead of h_t^2.

The charged Higgs loops give also rise to a non-vanishing left-handed bottom density opposite to Eq. (6.38).

When applying our calculations to the interesting case of spontaneous baryogenesis, one has to remember that the typical values of $v_i(x)/T$ are smaller than one, as required in regions of the bubble wall where sphalerons are still active. This fact enables us to stop the expansion series at the second order, being the remaining terms suppressed by powers of $v_i(x)/T$ with respect to it.

6.3 Current renormalization

In the previous Section we have seen that in order to calculate the finite temperature average of the particle currents, the expansion (6.4) can be used, whose coefficients are the 1PI Green's functions with the insertion of current operators J_i^μ, computed in the unbroken phase. With J_i^μ's we have indicated the current for the i-th particle, expressed in terms of the renormalized fields.

In general, the computation of Green's functions with the insertion of a composite operator requires the introduction of new counterterms besides those necessary for the renormalization of the basic lagrangian (see ref. [81] for a thorough discussion). For example, the one-loop computation of the Green function $\mathcal{G}^\mu(x,y,z) = \langle J^\mu_{t_L}(z) H_2^0(x) H_2^{0,\star}(y)\rangle$, defined in Eq. (6.34), gives a divergent part (in the $\overline{MS}$ renormalization scheme, and in the momentum space)

$$\tilde{\mathcal{G}}^{\mu,\mathrm{div}}(m,l) = +\frac{1}{8\,\pi^2} h_t^2 \frac{1}{\Gamma(3)}\frac{1}{\varepsilon}\left(m^\mu - l^\mu\right). \tag{6.39}$$

where m, l are the momenta associated to the external Higgs fields and $\varepsilon = 4 - n$. This divergence may be expressed as

$$\mathcal{G}^{\mu,\mathrm{div}}(x,y,z) = -\frac{1}{16\,\pi^2}\frac{1}{\varepsilon}\left\langle J^\mu_{H_2^0}(z) H_2^0(x) H_2^{0*}(y)\right\rangle \tag{6.40}$$

it i.e it is proportional to the *tree level* Green function involving the H_2^0 Higgs field current, corresponding to Fig. 8.

This is an example of a quite general phenomenon: a renormalized current $[J_i^\mu]$, can be defined as a linear combination of the currents J_j^μ

$$[J_i^\mu] = \sum_j (\delta_{ij} + \delta Z_{ij}) J_j^\mu \tag{6.41}$$

in such a way that all the possible Green's functions with $[J_i^\mu]$ insertions are finite. In the example discussed above, we have $\delta Z_{L2} = -\mu^2 h_t^2/(16\pi^2\,\varepsilon)$. The δZ_{ij} are in general new renormalization constants which are not contained in the counterterm lagrangian, so that new renormalization conditions are needed.

An important exception to this is the case in which J^μ is a conserved current for the renormalized lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{basic}} + \mathcal{L}_{\text{ct}}, \tag{6.42}$$

where $\mathcal{L}_{\text{basic}}$ is expressed in terms of renormalized quantities and $\mathcal{L}_{\text{ct}}$ is the counterterm Lagrangian. Let's consider here the simple case of a lagrangian containing only left and right-handed top quarks and the neutral Higgses, which will be enough for discussing the renormalization of the Green's functions relevant for this Chapter.

The conserved current under the group $U(1)$ is then

$$J^\mu + \delta J^\mu = q_L\left(1+\delta Z_L\right) J^\mu_L + q_R\left(1+\delta Z_R\right) J^\mu_R + q_1\left(1+\delta Z_1\right) J^\mu_{H^0_1} + q_2\left(1+\delta Z_2\right) J^\mu_{H^0_2}, \tag{6.43}$$

with $q_2 = q_R - q_L = -q_1 = 1/2$ and δJ^μ is the part of the current coming from $\mathcal{L}_{\text{ct}}$. The δZ_α ($\alpha = L,\ R,\ 1,\ 2$) are now the wave-function renormalization constants for the different fields, coming from $\mathcal{L}_{\text{ct}}$.

In this case, being the current conserved, no new counterterms besides those already present in (6.43) (and hence in $\mathcal{L}_{\text{ct}}$) are needed [81], *i.e.*

$$J^\mu + \delta J^\mu = \left[J^\mu\right] \equiv q_L\left[J^\mu_L\right] + q_R\left[J^\mu_R\right] + q_1\left[J^\mu_{H^0_1}\right] + q_2\left[J^\mu_{H^0_2}\right]. \tag{6.44}$$

Recalling this property of the conserved current it is possible to reduce the number of linearly independent renormalization constants δZ_{ij} (where now $i = L,\ R,\ 1,\ 2$) necessary in order to define the renormalized currents according to Eq. (6.41).

Indeed, from (6.44), recalling (6.41) and (6.43), the number of new linearly independent renormalization constants δZ_{ij}'s is reduced from 16 to 8.

The most general renormalization matrix is then given by

$$\begin{pmatrix} 1+\delta Z_{LL} & \delta Z_R - \delta Z_{RR} & \delta Z_{L1} & \delta Z_{L2} \\ \delta Z_L - \delta Z_{LL} & 1+\delta Z_{RR} & -\delta Z_{L1} & -\delta Z_{L2} \\ \delta Z_{1L} & \delta Z_{1R} & 1+\delta Z_{11} & \delta Z_{12} \\ \delta Z_{1L}+\delta Z_{LL}-\delta Z_L & \delta Z_{1R}-\delta Z_{RR}+\delta Z_R & \delta Z_1+\delta Z_{11}+\delta Z_{L1} & 1+\delta Z_2+\delta Z_{L2}+\delta Z_{12} \end{pmatrix} \tag{6.45}$$

We see then that, in general, eight extra renormalization conditions are required in order to define the set of renormalized currents $\{[J_i^\mu]\}$.

The only divergent graph encountered in Section **6.3**, is the fermionic one-loop (see Eq. (6.36)), so, for the present application, and to our order of computation, it is enough to compute the matrix (6.45) only to order $(g)^0 h_t^2$.

To this order, only one of the eight extra renormalization constants is non zero, δZ_{L2} and moreover we obtain, again from (6.44), that $\delta Z_{L2} = -\delta Z_2$ (+ higher order terms), so that no new renormalization condition is needed.

The mixing matrix is then given, at this order, by

$$\begin{pmatrix} [J_L^\mu] \\ [J_R^\mu] \\ [J_1^\mu] \\ [J_2^\mu] \end{pmatrix} = \begin{pmatrix} 1 & \delta Z_R & 0 & -\delta Z_2 \\ \delta Z_L & 1 & 0 & \delta Z_2 \\ 0 & 0 & 1 & 0 \\ -\delta Z_L & \delta Z_R & 0 & 1 \end{pmatrix} \begin{pmatrix} J_{t_L}^\mu \\ J_{t_R}^\mu \\ J_1^\mu \\ J_2^\mu \end{pmatrix}, \qquad (6.46)$$

where $\delta Z_{L2} = -\delta Z_2 = (-\mu^2 h_t^2 / 16\,\pi^2\,\varepsilon)$, $\delta Z_L = \delta Z_R = (-\mu^2 h_t^2/32\,\pi^2\,\varepsilon)$.

Working at higher order, we would need new renormalization conditions, defining the physical currents $[J_i^\mu]$ at a certain scale.

Due to the interactions, at a different scale the renormalized currents will be given by different mixings of the unrenormalized ones, so that, in general, the definition of 'pure' (*i.e.* non-mixed) currents is scale dependent.

As we have already stressed, this is not the case for the conserved currents, which do not suffer any mixing and do not require any new renormalization condition.

As we will discuss in the following section, the only relevant currents in the limit of local thermodynamical equilibrium are just the conserved ones, so that in this case, the state of the system is completely determined without the need of introducing extra renormalization conditions.

6.4 The limit of local equilibrium

Up to this point, we have computed the expectation values of the individual particle numbers starting from a partition function of the form

$$Z[J_i, \Delta_j] = \mathrm{Tr}\ \exp\left[-\beta\left(H + \int \Delta_j N_j\right)\right] \tag{6.47}$$

where H is the Hamiltonian of the system and we are now considering only the zero components ($N_j \equiv J_j^0$) of the currents considered in the previous Sections. We assume that all the chemical potentials are zero.

If we take generic values for the sources J_i and Δ_j the partition function (6.47) will not correspond to the one describing the limit of thermodynamical equilibrium, in which the individual particle numbers assume values such that the free energy of the system is minimized.

In the case under consideration of a space-time dependent Higgs field background, the limit of *local* equilibrium is achieved only if all the interaction rates are much higher that the inverse of the typical space-time scale of the variation of the background, *i.e.* the bubble wall width.

This condition is fulfilled in reality only by a few interactions (the gauge flavour conserving ones, the top Yukawa interactions, and possibly some Higgs-Higgs interactions), however it is anyway instructive to consider first this limit before discussing the more realistic situation in which some of the interactions are out of equilibrium inside the bubble wall.

The free energy of the system is obtained by performing the Legendre transformation of the functional defined in Eq. (6.2) with respect to the sources Δ_j of the density operators,

$$\begin{aligned} \mathrm{F}\left[\Phi_i^c, \mathcal{N}_j^c\right] &= W\left[J_i, \Delta_j\right] - \textstyle\sum_i \int d^4x J_j(x)\Phi_j^c(x) - \sum_j \int d^4x \Delta_j(x)\mathcal{N}_j^c(x) \\ &= \Gamma\left[\Phi_i^c, \Delta_j\right] - \sum_j \int d^4x \Delta_j(x)\mathcal{N}_j^c(x) \\ &= \langle H\rangle - TS. \end{aligned} \tag{6.48}$$

The last equality in the above equation is straightforwardly derived recalling that the entropy S is given by $S = -\partial W/\partial T$ and that $W = -T\log Z$ (we are taking the three dimensional volume $\Omega = 1$) .

The classical densities $\mathcal{N}^c_j(x)$ are defined as usual,

$$\mathcal{N}^c_j(x) = \left.\frac{\delta\Gamma[\overline{\Phi}^c_i, \Delta_j]}{\delta\Delta_j(x)}\right|_{\Delta_j=0}, \tag{6.49}$$

where the background is, as before (Eq. (6.6)), the solution of the field equations with appropriate boundary conditions, *i.e.* we have $J_i(x) = 0$ in Eq. (6.47).

Now, the local equilibrium situation on the background $\overline{\Phi}^c_i(x)$ is obtained once all the processes have driven the free energy to its minimum possible value. This condition corresponds to a system of M equations (where M is the number of processes in equilibrium) of the form

$$\delta^{(l)}\mathrm{F}\left[\Phi^c_i, \mathcal{N}^c_j\right] = \sum_j \nu^{(l)}_j \frac{\delta\mathrm{F}\left[\Phi^c_i, \mathcal{N}^c_j\right]}{\delta\mathcal{N}^c_j(x)} = -\sum_{j=1}^{n}\nu^{(l)}_j\Delta_j(x) = 0 \quad (l = 1, \ldots, M), \tag{6.50}$$

where $\nu^{(l)}_j$ is the multiplicity of the j-th particle in the l-th process (*i.e.* in the process $A + 2B \leftrightarrow C + D$ we have $\nu_A = -\nu_C = -\nu_D = 1$, and $\nu_B = 2$).

The system (6.50) is of course strictly analogous to the usual system of equations that one has to solve to determine the equilibrium chemical potentials. In this case, it restricts the set of the linearly independent sources to $\{\Delta'_a\}$ $(a = 1, \ldots, N)$, corresponding to the set of the N conserved charges N_a of the system. This means that the only composite operators that appear in the *equilibrium* partition function

$$Z_{eq}[J_i, \Delta'_a] = \mathrm{Tr}\,\exp\left[-\beta\left(H + \int \Delta'_a N_a\right)\right]\Big|_{\Delta'_a=0} \tag{6.51}$$

are those corresponding to the conserved charges,

$$\mathcal{N}^c_a(x) = \left.\frac{\delta\Gamma_{eq}[\overline{\Phi}^c_i, \Delta'_a]}{\delta\Delta'_a(x)}\right|_{\Delta'_a=0}, \tag{6.52}$$

where Γ_{eq} is obtained from Z_{eq} in the standard way.

After solving the system (6.50), the sources for the individual particle numbers, Δ_i, are expressed in terms of the new sources Δ'_a as

$$\Delta_i = \sum_a q^{(i)}_a\,\Delta'_a, \tag{6.53}$$

where $q_a^{(i)}$ is the a-charge value of the particle i. Then, the *equilibrium* values for the individual particle numbers are given by

$$\mathcal{N}_i^{eq}(x) = \sum_a \frac{\delta\Gamma_{eq}[\overline{\Phi}_i^c, \Delta_a']}{\delta\Delta_a'(x)}\bigg|_{\Delta_a'=0} \tilde{q}_a^{(i)} = \sum_a \mathcal{N}_a^c(x)\tilde{q}_a^{(i)}, \qquad (6.54)$$

where the $\tilde{q}_a^{(i)}$'s are found inverting the system (6.53).

Concerning renormalization we see that since now the only Green's functions that have to be computed are those involving the conserved charges, no new renormalization conditions for the composite operators are necessary when one is dealing with the local equilibrium case.

However in many interesting applications not all the processes have rates fast enough such that a complete local equilibrium situation can be attained.

In this case one can consider an adiabatic approximation, in which the slow processes are freezed out inside the bubble wall, while the fast ones are in equilibrium. As a consequence, on the bubble wall only the equations corresponding to such fast processess will contribute to the system (6.50) and one can repeat the above analysis with new effectively conserved charges now playing the role of the N_a's.

There is in this case a, mainly conceptual, problem, since these charges are now not conserved by the full theory, but only by the effective one obtained by sending to zero all the couplings contributing to the slow processes. Then they do in general mix one another under renormalization, as was discussed in the previous section, so that the interpretation in terms of physical currents will be scale dependent.

However, since the mixing is due only to the small couplings which are neglected in the effective theory, it will be in most of the practical applications a negligible effect.

6.5 Implications for the spontaneous baryogenesis mechanism

In this Chapter we have made use of a field theoretical approach based on a expansion in the background fields around the unbroken phase to compute in a consistent way the perturbations to the particle currents induced by a CP violating bubble wall background.

We have shown that the various contributions arise at the tree level, in the case of the Higgs currents, or as loop effects. In this way it has been possible to avoid the various ambiguities inherent to the traditional approach [28] based on the rotation of the fields, recovering all the expected suppression factors and including the Higgs fields.

Indeed, the proportionality between an individual particle density and its hypercharge in the original treatment was a direct consequence of the hypercharge rotation made to make the Yukawa couplings real.

Making a different rotation, the proportionality would of course drop. For instance, one could rotate the right-handed top and leave the left-handed one untouched, absorbing the anomaly by a proper rotation of the light fermion fields. In this case, in principle, the final value for the baryon asymmetry might come out to be dependent on the rotation that has been made.

In order to get a feeling of the implications of our results for the spontaneous baryogenesis mechanism, we can first consider an adiabatic approximation similar to that discussed in ref. [35].

Inside the bubble walls the $(B+L)$ violating sphaleron transitions are biased since a non zero local equilibrium value for $(B+L)$ is induced. $(B+L)_{\rm EQ}$ is a linear combination of the expectation values of all the currents conserved by the interactions in equilibrium inside the bubble wall.

Assuming that gauge flavor diagonal, top Yukawa and Higgs-Higgs interactions are in equilibrium, the conserved charges are Q', $(B-L)'$, $BP' = B_3 - 1/2\,(B_1+B_2)$ and $\tilde{Y}' = Y_H + Y_{t_L} + Y_{t_R} + Y_{b_L} + 1/3\; Y'_{\rm lep}$ where the prime means that only particles in equilibrium must be considered, and $Y'_{\rm lep}$ is the hypercharge of the leptons in equilibrium.

The values of these conserved currents are made up by the background perturbations computed above. Neglecting the contributions to the fermionic currents not proportional to h_t^2, it is straightforward to see that the contributions to $(B+L)_{\rm EQ}$ from the two Higgs currents cancel each other both at the tree level and at one-loop, and among the conserved charges only $\tilde{Y}'$ gets a non vanishing contribution at one-loop.

The two-loop diagrams where charged Higgses are exchanged give a further contribution to Q' and $\tilde{Y}'$.

The local equilibrium value $(B+L)_{\rm EQ}$ enters the rate equation of the form $\dot{B} \simeq -\Gamma_{\rm sp}(B+L)_{\rm EQ}/T^3$.

As a consequence, the force driving baryon number violation is just a one-loop effect.

Regarding the final value for the baryon asymmetry in the adiabatic approximation, the suppression terms that we have found would give rise to a suppression factor $\mathcal{O}(h_t^2\, v_{\rm co}^2/\pi^2\, T^2)$.

Taking $v_{\rm co}/T \simeq \alpha_W/g$ we see that typically a suppression $\mathcal{O}(10^{-4})$ arises.

In this case, it might be hard to reconcile the observed value for the baryon asymmetry with this mechanism for baryogenesis in the adiabatic limit, both in the case of spontaneous and of explicit CP violation in the Higgs sector.

In practice, however, the system is well far away from the idealized situation of perfect adiabacity.

Aa better description of the evolution of particle numbers can be achieved by employing a system of kinetic equations. For each particle number, the values expressed by (6.54) would then represent the infinite time limit of the corresponding rate equation,

$$\dot{\mathcal{N}}_i \sim \Gamma_{ij}(\mathcal{N}_j - \mathcal{N}_j^{\rm eq}). \tag{6.55}$$

As we have already stressed, the dependence of such equilibrium values on the coupling constants and on the value of the Higgs background should not be confused with the dependence of the rates Γ_{ij} on these quantities.

Also, the quantities we have computed here are just the perturbations induced by the CP violating background. They should be used as source terms for the departure from equilibrium in the equations describing dynamical processes, like gauge and Yukawa interactions, baryon number violation and particle diffusion.

Indeed, we know that particle diffusion may play an important role in the description of the spontaneous baryogenesis mechanism.

As a consequence of the perturbation induced by the bubble wall background, and of the different diffusion coefficients of different particle species, asymmetric densities are formed not only inside the bubble walls, as in the adiabatic approximation, but also in front of it, where the sphaleron transitions are not suppressed. These asymmetries are then transformed into a baryon asymmetry mainly in the region in front of the bubble wall, in a scenario similar to that occurring in the case of thin bubble walls.

In order to improve the adiabatic approximation a system of kinetic equations describing particle interactions and diffusion should then be solved, along the way of, *e.g.*, ref. [35, 36]. Indeed, it is now evident that transport processes, omitted in the original thick wall calculations, significantly enhance the baryon asymmetry produced during the EWPT. In fact, it has been suggested that the τ-lepton plays a leading role in baryogenesis due to its large diffusion constant [84]. However, due to the results found in this Chapter, we believe the τ-lepton is likely to be less important than the top quark for baryogenesis in the two Higgs doublet model, because the source of axial tau number is suppressed relative to the axial top source by a factor of $h_\tau^2/h_t^2 \sim 10^{-4}$.

The results of this Chapter have been recently confirmed by Huet and Nelson [38]. Although they adopted a semiclassical approach quite different from the one discussed in this Chapter, their results are in quantitative agreement with ours. In particular the same dependence on h_t and $v(x)$ is found. They also applied their formalism to the interesting case of EWB in supersymmetric models [39]. This will be the subject of the next two Sections.

Let us conclude this Section by saying that, due to its generality, we believe that the method illustrated in this Chapter can be easily applied to similar situations of interest in Cosmology, in which some of the fields acquire a space-time dependent classical value as for instance in inflationary models [82], or in presence of domain walls, as considered in in ref. [83] in the case of the Next to Minimal Supersymmetric Standard Model.

6.6 An alternative approach

An alternative method to calculate the effects of CP violation from extensions of the SM on the mechanism of EWB has been recently developed by Huet and Nelson [38].

This method reflects in a direct way the interplay between the coherent phenomenon of CP violation and the incoherent nature of the plasma physics, properly incorporating the decoherence effects which have been shown to have a major negative impact on the generation of a a CP violating observable in the SM [19, 20].

It is also valid for generic wall shape and size, reproducing the *thin wall* and the *thick wall* calculations with significant improvements over the original treatments.

In particular, when taking the thick wall limit, it predicts the correct behaviour with the mass background $\sim (M(T)/\pi\, T)^2$ in perfect agreement with the results described in the previous Sections and found in ref. [37].

Let us first consider a set of particles with (not necessarily diagonal) mass matrix $M(\mathbf{x})$ and moving, *in the rest frame of the wall*, with energy-momentum $(E, \mathbf{k})$.

At their last scattering point $\mathbf{x}_0$, these particles emerge from the thermal ensemble, propagate freely during a mean free time $\tau_{\rm T}$, then rescatter and return to the local thermal ensemble in the plane $\mathbf{x}_0 + \tau_{\rm T}\, v$, v being the velocity perpendicular to the wall, $|\mathbf{k}_\perp|/E$.

During the time $\tau_{\rm T}$, these particles evolve according to a set of Klein-Gordon, Dirac or Majorana equations coupled through the mass matrix $M(\mathbf{x})$. It is in the course of this evolution that CP violation affects the distribution of these particles.

To be specific, let us define $J_\pm$ the average current resulting from particles moving towards positive (negative) $\mathbf{x}$ between $\mathbf{x}_0$ and $\mathbf{x}_0 + \Delta$, $\Delta = \tau_{\rm T}\, v$.

The current J_+ receives contributions from either particles originating from the thermal ensemble at point $\mathbf{x}_0$, moving with positive velocity and being transmitted at $\mathbf{x}_0 + \Delta$, or from particles originating at $\mathbf{x}_0 + \Delta$, moving with velocity $-v$ and being reflected back towards $\mathbf{x}_0$ (a similar definition exists for J_-). $J_\pm$ are CP violating currents associated with each layer of thickness Δ moving along the wall. Once boosted in the plasma frame, these currents provide CP violating sources, which fuel EWB.

Let us work, for simplicity, with a Dirac mass $M(\mathrm{x}) = m(\mathrm{x})\,\mathrm{e}^{i\theta(\mathrm{x})}$ for a single fermion. It might be the top quark having its mass generated from the two Higgs doublet model, in which case τ_{T} is the free mean time for quark-gluon scatterings. As for the current, we follow [38] and choose the axial current[1].

The four-vectors currents $J_{\pm}$ take the form

$$\begin{aligned} J_{+}(\mathrm{x}_0) &= \int_{\tilde{v}>0} \frac{d^3\mathbf{k}}{(2\,\pi)^3} \langle [n(E,v) - n(E,-\tilde{v})]\; Q\,(\mathrm{x}_0, \mathbf{k}, \tau_{\mathrm{T}})\rangle_{\mathrm{x}_0}\; (1,0,0,\tilde{v}), \\ J_{-}(\mathrm{x}_0) &= \int_{\tilde{v}>0} \frac{d^3\mathbf{k}}{(2\,\pi)^3} \langle [n(E,v) - n(E,-\tilde{v})]\; Q\,(\mathrm{x}_0, \mathbf{k}, \tau_{\mathrm{T}})\rangle_{\mathrm{x}_0} (1,0,0,-v). \end{aligned} \quad (6.56)$$

In this expression, $v = |\mathbf{k}_{\perp}|/E$ is the velocity at the point x_0, $\tilde{v}$ is the velocity a distance Δ way, $\tilde{v}^2 = v^2 + \left[m^2(\mathrm{x}_0) - m^2(\mathrm{x}_0 + \Delta)\right]/E^2$, and $n(E,v)$ is the Fermi-Dirac distribution boosted to the rest frame of the wall,

$$n(E,v) = \frac{1}{\mathrm{e}^{\gamma_w\,E\,(1 - v\,v_w)} + 1}. \quad (6.57)$$

$Q\,(\mathrm{x}_0, \mathbf{k}, \tau_{\mathrm{T}})$ is the charge asymmetry which results along the axis x from the propagation of particles of momentum $\mathbf{k}$ in the interval $(\mathrm{x}_0, \mathrm{x}_0 + \Delta)$. In our specific example, Q is the chiral charge given by

$$Q\,(\mathrm{x}_0, \mathbf{k}, \tau_{\mathrm{T}}) = |T_L|^2 - |T_R|^2 - |T_{\bar{L}}|^2 + |T_{\bar{R}}|^2\,, \quad (6.58)$$

where T_L is the amplitude for the left-handed spinor to propagate over a distance Δ, $T_R = T_L(M \to M^{\dagger})$ and $T_{\bar{L}} = T_L(M \to M^*)$. Finally the brackets $\langle \cdots \rangle_{\mathrm{x}_0}$ average the location of the point x_0 within the given layer (parallel to the bubble wall) of thickness Δ as scattering occurs anywhere within the layer.

The standard thin wall and thick wall limits are obtained taking τ_{T}/L_w to ∞ to 0, respectively. In particular, in the thick wall situation, the currents $J_{\pm}$ yield, after a boost to the thermal frame, a locally defined space-time dependent source $S(\mathrm{z},t)$ which generalizes the local CP violating operator used in the original work [28].

As the wall crosses a small volume, it deposits into it the current density $(J_{+} + J_{-})$ every time interval τ_{T} so that the source per unit volume per unit time, located at

[1] In the two Higgs doublet model a combination of the axial top number and Higgs number diffuses efficiently into the symmetric phase and is approximately conserved by scatterings in the symmetric phase [36].

the point $\mathbf{z}$ fixed in the plasma, at a given time t is, to first order in v_w and τ_{T}/L_w,

$$S(\mathbf{z},t) = -\frac{\gamma_w\, v_w}{2\,\pi^2}\int_0^1 dv \int_{\mathbf{x}_0-\Delta/2}^{\mathbf{x}_0+\Delta/2} \frac{d|\mathbf{x}_\perp|}{\Delta} \int_{\gamma_w\, m(\mathbf{x})}^{\infty} dE E^3 \frac{dn}{dE}(2v)\, \frac{Q\,(\mathbf{x},\mathbf{k},\tau_{\mathrm{T}})}{\tau_{\mathrm{T}}}\Bigg|_{|\mathbf{x}_{\perp,0}|=\gamma_w(|\mathbf{z}_\perp|-v_w\,t)}, \tag{6.59}$$

where $\mathbf{x}_\perp$ represents the projection of $\mathbf{x}$ onto the direction perpendicular to the bubble wall.

In general $Q\,(\mathbf{x},\mathbf{k},\tau_{\mathrm{T}})$ is a charge asymmetry produced along the $\mathbf{x}$ axis by particles moving with momentum $\mathbf{k}$ between the planes $\mathbf{x}_0$ and $\mathbf{x}_0+\Delta$. Its calculation may require a different technique depending on the relative values of time scales involved and on the choice of the charge.

The physics of the generation of a CP violating observable is the physics of quantum interference. It is most easily dealt with by treating the mass $M(\mathbf{x})$ as a small perturbation, *i.e.* $|M(\mathbf{x})|/T < 1$.

Using the techniques developed in ref. [20], one has solve the associated Dirac equation with a space-time dependent mass $M(\mathbf{x})$

$$\begin{aligned} \left(-i\,\partial_{|\mathbf{x}|} - P_L\right)\,\chi_L(\mathbf{x}) &= -i\,\delta(\mathbf{x}-\mathbf{x}_0)\,\chi_L(\mathbf{x}_0) + M(\mathbf{x})\,\chi_R(\mathbf{x}), \\ \left(-i\,\partial_{|\mathbf{x}|} - P_R\right)\,\chi_R(\mathbf{x}) &= -M^\dagger(\mathbf{x})\,\chi_L(\mathbf{x}), \end{aligned} \tag{6.60}$$

where a δ-function source of left-handed particles has been located at $\mathbf{x}_0$ and $\chi_{L,R}$ are the two component left and right-handed spinors forming the Dirac spinor.

One finds, for the transmitted amplitude (up to an overall phase) along $\mathbf{x}_\perp$,

$$T_L(\mathbf{x}_0,\tau_{\mathrm{T}}) = 1 - \int_{\mathbf{x}_0}^{\mathbf{x}_0+\Delta} d|\mathbf{x}_{\perp,1}| \int_{\mathbf{x}_0}^{\mathbf{x}_1} d|\mathbf{x}_{\perp,2}|\ \mathrm{e}^{2i\omega(|\mathbf{x}_1|-|\mathbf{x}_2|)}\, M^\dagger(\mathbf{x}_2)\, M(\mathbf{x}_1) + \mathcal{O}(M/\omega), \tag{6.61}$$

where ω is the energy of the motion transverse to the wall. This expression has a straightforward explanation: the left-handed particle is scattered by the quark mass term M in the bubble at the point $\mathbf{x}_1$ becoming a right-handed particle which scatters again, via $M^\dagger$, leading to a contribution to the left-handed transmitted wave. A similar expansion can be written for the reflection amplitude R_L (depending linearly on $M^\dagger$).

Using the fact that CPT symmetry identifies the amplitude for a particle transmitted from the left with the amplitude of its CP conjugate to be transmitted from the right while unitarity relates trasmission and reflection amplitudes, one finds

$$\begin{aligned} Q\,(\mathrm{x}_0, \mathbf{k}, \tau_\mathrm{T}) &= 8 \int_{\mathrm{x}_0}^{\mathrm{x}_0+\Delta} d|\mathrm{x}_{\perp,1}| \int_{\mathrm{x}_0}^{\mathrm{x}_1} d|\mathrm{x}_{\perp,2}| \;\sin\left[2\,\omega\,(|\mathrm{x}_1| - |\mathrm{x}_2|)\right] \\ &\times \;\mathrm{Im}\left[M^\dagger(\mathrm{x}_2)\, M(\mathrm{x}_1)\right] + \mathcal{O}(M/\omega)^4. \end{aligned} \tag{6.62}$$

It is simplest to work out the case of the very thick wall, $L_w \gg \tau_\mathrm{T}$.

Using the derivative expansion $M(\mathrm{x}_i) = M(\mathrm{x}_0) + (|\mathrm{x}_i| - |\mathrm{x}_0|)\,\partial_{|\mathrm{x}|}\, M(\mathrm{x}_0)$, one obtains

$$\begin{aligned} Q\,(\mathrm{x}_0, \mathbf{k}, \tau_\mathrm{T}) &= \left[4\, f(\omega\,\Delta)/\omega^3\right]\; \mathrm{Im}\left[M^\dagger\, \partial_{|\mathrm{x}|}\, M\right]_{\mathrm{x}_0} \\ &= \left[4\, f(\omega\,\Delta)/\omega^3\right]\; m^2\; \partial_{|\mathrm{x}|}\; \theta|_{\mathrm{x}_0}\,, \end{aligned} \tag{6.63}$$

where

$$f(\xi) = \sin\xi\; (\sin\xi - \xi\;\cos\xi)\,. \tag{6.64}$$

Inserting this latter expression into formula (6.59) for the source $S(\mathbf{z}, t)$ yields

$$S(\mathbf{z}, t) = T\,\gamma_w\, m^2\; \partial_{|\mathbf{x}_\perp|}\;\theta|_{\mathrm{x}_0=\gamma_w(|\mathbf{z}_\perp| - v_w\, t)}\; \frac{2}{\pi^2}\, \mathcal{I}(\tau_\mathrm{T}, m, T) + \mathcal{O}\left(v_w^2, (m/T)^2, (\tau_\mathrm{T}/L_w)^2\right), \tag{6.65}$$

where

$$\mathcal{I}(\tau_\mathrm{T}, m, T) \simeq \int_{\mathcal{M}/T}^{\infty} dy\, \frac{\mathrm{e}^y}{(\mathrm{e}^y+1)^2} \int_0^{\tau_\mathrm{T} T\,\sqrt{y^2-(\mathcal{M}/T)^2}} \frac{dt}{t^2}\, f(t)\, \frac{2\,u^2}{u^4 + (t\,y/\tau_\mathrm{T}\,T)^2}, \tag{6.66}$$

where

$$u^2 = \sqrt{\frac{1}{4}\frac{\mathcal{M}^4}{T^4} + \frac{t^2\,y^2}{\tau_\mathrm{T}^2\,T^2}} - \frac{1}{2}\frac{\mathcal{M}^2}{T^2} \tag{6.67}$$

and

$$\mathcal{M}^2 = m^2 + M_T^2. \tag{6.68}$$

Thermal corrections, M_T, have been included in the mass dependence.

These kinematical corrections are required for the following reason. Constructive interference is maximal for particle whose transverse Compton wavelength $|\mathrm{k}_\perp|$ is of order of $v\,\tau_\mathrm{T}$. In such a regime there is a significant correlation between a scattering and the subsequent one.

In particular, the assumption that these particles propagate freely over the intermediate distance $v\,\tau_\mathrm{T}$ breaks down and their dispersion relation must be modified.

Typically $\tau_{\rm T} \geq T^{-1}$ so that the relevant momenta are $\leq T$. For these values of $|\mathbf{k}_\perp|$, the effect of scattering on particle propagation can be accounted for by substituting particles with quasiparticles, in which case $\tau_{\rm T}$ is replaced by $1/2\,\gamma$, where γ is the width of the quasiparticle.

Correspondingly, the dispersion relation is to be modified to incorporate self energy thermal corrections. In the particular case of quarks scattering off gluons, the width is $\gamma \simeq g_S^2\,T^2/3$ while the thermal mass corrections amount to the shift $E^2 \to E^2 + M_T^2$ with $M_T^2 \simeq g_S^2\,T^2/6$.

The form factor $\mathcal{I}$ vanishes as $\tau_{\rm T} \to 0$, it peaks at $\tau_{\rm T}\,T \sim 1$ and is well approximated by $\sim 1/\tau_{\rm T}$ in the range $\tau_{\rm T}\,T \gtrsim 5$.

The interpretation of this behaviour is straightforward. As explained earlier, constructive interference is maximal for particles whose Compton wavelength $|\mathbf{k}_\perp|$ is of order of $\tau_{\rm T}$, that explains the peak at $\tau_{\rm T}\,T \sim 1$. As $\tau_{\rm T}\,T$ increaes, fast oscillations along distinct paths tend to cancel against each other and the resulting asymmetry drops; as a matter of fact, in the extreme limit $\tau_{\rm T}\,T \gg 1$, the propagation is semiclassical and the asymmetry vanishes as it should. In the opposite limit, $\tau_{\rm T}\,T \to 0$, the asymmetry vanishes as the the quantum coherence required is washed away by rapid plasma interactions.

Let us apply the above calculation to the case of a top quark propagating in a thick bubble wall produced during the EWPT in the two Higgs doublet model. Here the free mean path is dominated by gluon scatterings $\tau_{\rm T} \simeq 3/(2\,g_S^2\,T) \sim (1-2)/T \ll L_w$ while $M_T \simeq T/2$. One finds

$$S(\mathbf{z},t) = \frac{2}{5\,\pi^2}\gamma_w v_w T m^2 \partial_{|\mathbf{x}_\perp|}\,\theta|_{|\mathbf{x}_{\perp,0}|=\gamma_w(|\mathbf{z}_\perp|-v_w\,t)} + \mathcal{O}\left(v_w^2, (m_t/T)^2, (\tau_{\rm T}/L_w)^2\right). \quad (6.69)$$

Let us now stress the complete similarity (apart from numerical factors and dimensions) between the above formula and Eq. (6.37) found in ref. [37], where CPR made use of tree level fermionic propagators.

Working with resummed fermionic propagators in [37] would have been equivalent to treat fermions as quasiparticles, whose dispersion relations are modified to incorporate both self energy thermal corrections and imaginary parts giving the width γ of the quasiparticles, which is strictly related to the scattering rate through the optic theorem.

Had CPR used resummed fermionic propagators in Fig. 10 one should have expected $\langle J^{\mu}_{t_L}(z)\rangle^{(3)}$ to be vanishing in the ideal limit of very large width, $\gamma_t \gg T$, as predicted in ref. [38]. This would happen because, in such an ideal limit, resummed fermionic propagators vanish as $1/\gamma_t$.

In the opposite ideal limit of very small width, $\gamma_t \ll T$, and then also *small* Yukawa h_t (since γ_t for the top receives also a contribution from the interaction of the top with the scalar H^0_2), again one should have expected $\langle J^{\mu}_{t_L}(z)\rangle^{(3)}$ to be vanishing as one can immediately realizes from Eq. (6.33).

We point out here that the dependence of $\langle J^{\mu}_{t_L}(z)\rangle^{(3)}$ from h_t, had CPR made use resummed propagators, would have been twofold: one in the part of the scattering rate, or partial top width γ_t, coming from the interaction between the top and the scalar H^0_2 and the second in the h_t factorized out in our expression (6.33). The latter, of course, remains even in the limit of very large γ_t or long times.

However, since in the realistic physical situation the major contribution to γ_t comes from the scattering off gluons and $\gamma_t \simeq T$, no significant suppression is expected and then making use of the tree level propagators is pretty reasonable. This feeling is well confirmed by the agreement between formulae (6.33) and (6.69).

A major advantage of the formulation described in this Section is that it is valid for all wall shapes and sizes and it easily applies to charges generated by flavour mixing through arbitrary mass matrices. In particular, it can be applied to cases, such as the supersymmetric model, for which there is no known semiclassical approximation.

6.7 EWB in MSSM: the contemporary days

Let us now discuss the application of the method discussed in the previous Section to the case of the MSSM [39]. As we shall see, taking into account all the previously neglected effects of transport and thermal scattering, a baryon asymmetry of the right order of magnitude can be explained provided the *explicit* CP violating phases are greater than $(10^{-3} - 10^{-5})$ and some superpartners are light enough to be relevant during the transition, which takes place at a temperature of about $(50 - 100)$ GeV.

Let us consider, for instance, the stop mass matrix (3.37) with

$$M_{LR}^2 \equiv a^2\, e^{i\alpha} = h_t \left(|A_t|\, e^{i\phi_A} v_2 + |\mu|\, e^{i\phi_B}\, v_1 \right), \tag{6.70}$$

the charge axial stop number is defined to be

$$Q_{\tilde{t}} = \text{Diag}\left(\frac{1}{2}, -\frac{1}{2}\right). \tag{6.71}$$

We proceed in computing current sources $J_\pm$ in the wall frame using Eqs. (6.56), which will then put in Eq. (6.59) to construct the source $S_{\tilde{t}}(\mathbf{x}, t)$.

Up to an overall phase and at leading order in $\mathcal{M}_{\tilde{t}}^2/|\mathbf{k}_\perp|^2$, one gets

$$T = \left[1 - \cdots - \int_0^\Delta dz_1 \int_0^{z_1} dz_2\, \frac{\mathcal{M}_{\tilde{t}}^2(z_2)}{2\,|\mathbf{k}_\perp|}\, \frac{\mathcal{M}_{\tilde{t}}^2(z_1)}{2\,|\mathbf{k}_\perp|}\, e^{2i|\mathbf{k}_\perp|(z_1 - z_2)}\right] + \mathcal{O}\left(\frac{\mathcal{M}_{\tilde{t}}}{2\,|\mathbf{k}_\perp|}\right)^3 \tag{6.72}$$

and

$$R = \cdots + \int_0^\Delta dz_1 \frac{\mathcal{M}_{\tilde{t}}^2(z_1)}{2\,|\mathbf{k}_\perp|}\, e^{-2i|\mathbf{k}_\perp|z_1} + \mathcal{O}\left(\frac{\mathcal{M}_{\tilde{t}}}{2\,|\mathbf{k}_\perp|}\right)^3, \tag{6.73}$$

where now T and R represent, respectively, the transmission and reflection probability of particles produced at $z = 0$ and evolving toward positive z (and analogous formulae are found for the transmission and reflection coefficients at the point $z = \Delta$).

Performing an expansion in powers of $\mathcal{M}_{\tilde{t}}/T$ and in first order in v_w one obtains

$$\begin{aligned}
(J_+ + J_-)^z &= 0, \\
(J_+ + J_-)^0 &= \gamma_w\, v_w\, \mathcal{I}_{\tilde{t}} \sum_{i=\tilde{M}_Q, \tilde{M}_U} \int \frac{d^3\mathbf{k}}{(2\,\pi)^3}\, \frac{g(|\mathbf{k}_\perp|\,\Delta)}{4\,|\mathbf{k}_\perp|^5}\, (2\,v_i) \\
&\times \frac{e^{E_i/T}}{\left(1 - e^{E_i/T}\right)^2}\, \frac{E_i}{T},
\end{aligned} \tag{6.74}$$

where $g(\xi)$ is defined as

$$g(\xi) = 1 - \cos 2\,\xi - \xi \sin 2\,\xi \tag{6.75}$$

and $\mathcal{I}_{\tilde{t}}$ is the CP invariant given by

$$\mathcal{I}_{\tilde{t}} = -\partial_z \alpha\, \frac{a^4}{T^5}. \tag{6.76}$$

After a few manipulations, one can derive the following expression for the stop axial source $S_{\tilde t}(\mathbf{x},t)$

$$S_{\tilde t}(\mathbf{x},t) = \gamma_w\, v_w\, N_c\, \frac{T^4}{4\,\pi^2}\;\mathcal{I}_{\tilde t}\;\left(\mathcal{S}_{\tilde t}^{\tilde M_Q} + \mathcal{S}_{\tilde t}^{\tilde M_U}\right) + \mathcal{O}\left((\mathcal{M}_{\tilde t}/T)^6, v_w^2, (\tau_{\rm T}/L_w)^2\right), \quad (6.77)$$

where $N_c = 3$ is the number of colors and the function $\mathcal{S}_{\tilde t}^{\tilde M}$ is defined as

$$\mathcal{S}_{\tilde t}^{\tilde M} = \frac{1}{4}\sqrt{\tau_{\rm T}\, T}\int_{\tilde M/T}^{\infty}\frac{dy}{\sqrt{y}}\frac{\mathrm{e}^y}{(1-\mathrm{e}^y)^2}\int_0^{\tau_{\rm T}\, T\left(y^2-\frac{\tilde M^2}{T^2}\right)/y} d\xi\;\frac{g(\xi)}{\xi^{5/2}}. \quad (6.78)$$

Note that $\mathcal{S}_{\tilde t}^{\tilde M}$ vanishes as $\tau_{\rm T}^3$ as incoherent plasma scatterings become overwhelming, *i.e.* as $\tau_{\rm T}\, T \to 0$, tracing the quantum nature of CP violation conflicting with the classical nature of the plasma physics. For larger coherence time $\mathcal{S}_{\tilde t}^{\tilde M}$ behaves approximately as

$$\mathcal{S}_{\tilde t}^{\tilde M} \simeq \frac{3}{40}\,(\tau_{\rm T}\,\tilde M)^{0.3}\log\tau_{\rm T}\;\frac{\mathrm{e}^{\tilde M/T}}{\left(1-\mathrm{e}^{\tilde M/T}\right)^2}. \quad (6.79)$$

The stop damping rate is dominated by strong interactions and was estimated in ref. [39] to be of order of $(2\,\alpha_S\, T) \simeq T/5$.

Another source of CP violation can derive from the chargino and neutralino mass matrices along the same lines of what briefly discussed here for the stops. We refer the interested reader to ref. [39] for the complete analysis.

Once computed the source per unit time and unit volume of CP violation, one can approximate the solution to the Boltzmann equations for particle distribution functions by writing down and solving a set of coupled differential equations for local particle densities including the source terms, transport and particle number changing reactions.

Only those particle species which partecipate in particle number changing transitions which are fast compared with the relevant time scales, but which carry some charge approximately conserved in the symmetric phase, can have a significant nonzero densities in the symmetric phase during the transition. Major simplifications of the diffusion equations take place when neglecting all the couplings except for gauge interactions and the top quark Yukawa coupling. Neglecting the weak sphalerons (in the first step) allows to forget about leptons in the differential equations and will turn out to be a good approximation when computing Higgs and quark densities. On the contrary, strong sphalerons will be included.

The particle densities one needs include $q \equiv (t_L + b_L)$, the right handed top quark $t \equiv t_R$, the Higgs particle $H \equiv (H^- + H_1^0 + H^+ + H_2^0)$ and their superpartners, $\tilde{q}$, $\tilde{t}$ and $\tilde{H}$. The individual particle numbers of these species can change through the top Yukawa interaction, the top quark mass, the Higgs self interactions, the anomalous weak interactions and the supergauge interactions.

Since it will turn out that EWB is only feasible if the superpartners of top and/or of the gauge and Higgs bosons are light, one may take the supergauge interactions to be in thermal equilibrium

$$\frac{q}{k_q} = \frac{\tilde{q}}{k_{\tilde{q}}}, \quad \frac{t}{k_t} = \frac{\tilde{t}}{k_{\tilde{t}}}, \quad \frac{H}{k_H} = \frac{\tilde{H}}{k_{\tilde{H}}}, \tag{6.80}$$

where k_i ($\simeq 2$ for bosons and $\simeq 1$ for fermions) are the degrees of freedom ($\simeq$ means that, for instance, $k = 2$ for bosons up to mass effects).

On can then describe describe the system by the densities $Q = q + \tilde{q}$, $T = t + \tilde{T}$ and $h = H + \tilde{H}$.

When including strong sphalerons (with a rate $\Gamma_{\rm QCD}$), right-handed bottom quark density will be produced, $B = b_R + \tilde{b}_R$, as well as first and second family quarks $Q_{(1,2)L}$, U_R, C_R, S_R and D_R. However, since strong sphalerons are the only processes which generate significant numbers of the first and second family quarks, and all the quarks have nearly the same diffusion constant, we can constrain these densities algebrically in terms of B to satisfy

$$Q_{1L} = Q_{2L} = -2\,U_R = -2\,D_R = -2\,S_R = -2\,C_R = -2\,B = 2\,(Q + T). \tag{6.81}$$

For simplicity, one can also assume all the squark partners of the light quarks degenerate and take

$$k_{Q_{1L}} = k_{Q_{2L}} = 2\,k_{S_R} = 2\,k_{D_R} = 2\,k_{U_R} = 2\,k_{C_R} = 2\,k_B. \tag{6.82}$$

Particle transport is treated by including a diffusion term and one can include scattering processes involving the top quark Yukawa coupling with rate Γ_y, axial top number violation at the rate Γ_m in the phase boundary and in the broken phase as well as Higgs violating processes with a rate Γ_h.

Taking all the quarks and squarks to have the same diffusion constant D_q and Higgs and Higgsinos to have diffusion constant D_h, one can write down the set of coupled diffusion equations

$$\begin{aligned}
\dot{Q} &= D_q \nabla^2 Q - \Gamma_y \left[Q/k_Q - h/k_h - T/k_T \right] - \Gamma_m \left[Q/k_Q - T/k_T \right] \\
&- 6\, \Gamma_{\rm QCD} \left[2\, Q/k_Q - T/k_T + 9\, \left(Q + T \right)/k_B \right] + S_{\tilde{t}}, \\
\dot{T} &= D_q \nabla^2 T - \Gamma_y \left[-Q/k_Q + h/k_h + T/k_T \right] - \Gamma_m \left[-Q/k_Q + T/k_T \right] \\
&+ 3\, \Gamma_{\rm QCD} \left[2\, Q/k_Q - T/k_T + 9\, \left(Q + T \right)/k_B \right] - S_{\tilde{t}}, \\
\dot{h} &= D_h \nabla^2 h - \Gamma_y \left[-Q/k_Q + h/k_h + T/k_T \right] - \Gamma_h\, h/k_h + S_{\tilde{h}},
\end{aligned} \tag{6.83}$$

where we have also included the CP violating source $S_{\tilde{h}}$ provided by neutralinos and charginos.

Several simplifications can be made. First, one can ignore the curvature of the bubble wall so that all the quantities are functions of $z \equiv |\mathbf{x} + \mathbf{v}_w\, t|$, the coordinate normal to the wall surface. Furthermore, assuming the rates Γ_y and $\Gamma_{\rm QCD}$ fast, one can write

$$Q/k_Q - h/k_h - T/k_T = \mathcal{O}\left(1/\Gamma_y\right), \quad 2\, Q/k_Q - T/k_T + 9\, \left(Q + T\right)/k_B = \mathcal{O}\left(1/\Gamma_{\rm QCD}\right). \tag{6.84}$$

Solving the above equations in function of h, the Higgs density satisfies

$$-\, v_w\, h' + \bar{D}\, h'' - \bar{\Gamma}\, h + \bar{S} = \mathcal{O}\left(1/\Gamma_y, 1/\Gamma_{\rm QCD}\right), \tag{6.85}$$

where $\bar{D}$ is an effective diffusion constant, $\bar{\Gamma}$ is an effective decay constant proportional to

$$\left(\Gamma_m + \Gamma_h\right) \simeq \frac{4}{21}\, \frac{M_W^2(T,z)}{g^2\, T}\, h_t^2\, \sin^2\beta + \frac{1}{35}\, \frac{M_W^2(T,z)}{g^2\, T}, \tag{6.86}$$

and $\bar{S}$ an effective source term proportional to $(S_{\tilde{t}} + S_{\tilde{h}})$. All of them easily readable from the above expressions.

Eq. (6.85) is easily solved approximating the source as a step function of the bubble wall width L_w

$$\bar{S} \;=\; S_*, \quad 0 < z < L_w,$$

$$\bar{S} = 0, \quad \text{otherwise}, \tag{6.87}$$

and similarly for the decay term

$$\begin{aligned} \bar{\Gamma} &= \Gamma_*, \quad z > 0, \\ \bar{\Gamma} &= 0, \quad \text{otherwise}, \end{aligned} \tag{6.88}$$

The effective diffusion constant is also spatially varying since the statistical factors k_i depend on spatially varying particle masses and since the weak interactions cross sections depend on the Higgs VEVs. However, making the approximation that $\bar{D}$ is a constant and with the boundary conditions $h(\pm\infty) = 0$, one easily finds that in the symmetric phase ($z < 0$)

$$h(z) = \mathcal{A}\, e^{(z\, v_w/\bar{D})}, \tag{6.89}$$

where

$$\mathcal{A} = \frac{4\, S_*\, \bar{D} \left(1 - e^{\left[\left(v_w + \sqrt{4\,\bar{D}\,\Gamma_* + v_w^2}\right)\, \bar{D}\right]}\right)}{\left(v_w + \sqrt{4\,\bar{D}\,\Gamma_* + v_w^2}\right)^2}. \tag{6.90}$$

From the form of Eq. (6.89) one can see that CP violating densities are nonzero for a time $t \simeq (\bar{D}/v_w^2)$ and so the assumption about which rates are fast which were used to derive Eq. (6.89) are valid provided $(\bar{D}\,\Gamma_{\rm QCD}/v_w^2)$, $(\bar{D}\,\Gamma_y/v_w^2) \gg 1$, $(\bar{D}\,\Gamma_{\rm sp}/v_w^2) \ll 1$ and the scattering processes due to Yukawa couplings other than top are slow.

Taking the Higgs diffusion constant D_h to be comparable to the diffusion constant for left-handed leptons, $\sim 110/T$ [27], and taking $D_q \simeq 6/T$, one finds that the effective diffusion constant is quite large

$$\bar{D} \simeq \frac{100}{T}, \tag{6.91}$$

indicating that the most of the transport of CP violating quantum numbers is done by weakly interacting particles, *i.e.* Higgs and Higgsinos. In the above expression we have assumed that all the supersymmetric particles are heavy compared with the temperature T except for neutralinos and charginos, so that

$$k_Q \simeq 6, \quad k_T \simeq 3, \quad k_B \simeq 3 \quad k_H \simeq 12. \tag{6.92}$$

Since Yukawa interactions readily convert Higgs number into axial top number, transport of axial top number is surprisingly efficient.

For the scattering rate due to the top quark Yukawa coupling one estimates $\Gamma_y \simeq (T/30)$ so that the assumption that this rate is fast is self consistent, $(\bar{D}\,\Gamma_y/v_w^2) \simeq (3/v_w^2) \gg 1$ (scattering due to Yukawa bottom quark can be safely neglected for natural values of $\tan\beta$).

As far as the anomalous fermion number violating rates are concerned, weak sphaleron rate may be safely taken to be slow provided

$$\frac{\kappa}{v_w^2} \lesssim 10^4, \tag{6.93}$$

and the strong sphaleron rate is fast if

$$\frac{\kappa}{v_w^2} \gtrsim 5. \tag{6.94}$$

We now turn to the weak sphaleron rate on, assuming it has a negligible effect on particle densities, *i.e.* Eq. (6.93) is valid, however it provides the only source for net baryon asymmetry. One can take n_B, the baryon number density, to be a function of z satisfying

$$D_q\, n_B'' - v_w\, n_B' - 3\, n_L\, \Gamma_{\rm sp} = 0, \tag{6.95}$$

where $\Gamma_{\rm sp}$ is the usual sphaleron step function rate (*i.e.* vanishing in the broken phase) and n_L is the total number density of left-handed weak doublet fermions. The above Equation has solution

$$n_B(z) = -3\,\frac{\Gamma_{\rm sp}}{v_w}\int_{-\infty}^{0} n_L(z)\,dz - 3\,\frac{\Gamma_{\rm sp}}{v_w}\int_0^z dz_1\, n_L(z_1)\left(1 - e^{v_w(z-z_1)/D_q}\right), \tag{6.96}$$

which is a constant fro $z > 0$ and vanishes as $z \to -\infty$. Thus, up to corrections of order of $(\bar{D}\,\Gamma_{\rm sp}/v_w^2)$, the baryon asymmetry inside the bubbles of broken phase is simply proportional to the integral of n_L in the symmetric phase.

If one neglects mass effects and take the limit $\Gamma_{\rm QCD} \to 0$, it is easy to show that $n_L = Q + Q_{1L} + Q_{2L} = 5Q + 4T = 0$ [33]. Thus, one needs to compute the $\mathcal{O}(1/\Gamma_{\rm QCD})$ corrections to particle densities. Assuming $\Gamma_y \gg \Gamma_{\rm QCD}$ ($\kappa \lesssim 7$) and taking

$$\begin{aligned} Q &= Q^0 + \delta_Q + \mathcal{O}(1/\Gamma_y), \\ T &= T^0 + \left(\frac{k_T}{k_Q}\right)\delta_Q + \mathcal{O}(1/\Gamma_{\rm QCD}), \end{aligned} \tag{6.97}$$

where Q^0 and T^0 are, respectively,the values of Q and T for large $\Gamma_{\rm QCD}$, one finds

$$n_L = \left(\frac{5\,k_Q + 4\,k_T}{k_Q}\right)\,\delta_Q + \left[\frac{9\,k_Q\,k_T - 8\,k_B\,k_T - 5\,k_B\,k_Q}{k_H\,(k_B + 9\,k_Q + 9\,k_T)}\right]\,h. \tag{6.98}$$

Assuming now the validity of Eq. (6.92), one finds

$$n_L = 7\,\delta_Q = -\frac{1}{56}\left(\frac{D_q\,h'' - v_w\,h'}{\Gamma_{\rm QCD}}\right), \tag{6.99}$$

so that the baryon asymmetry is proportional to $(\Gamma_{\rm sp}/\Gamma_{\rm QCD}) \simeq (3\,\alpha_W/8\,\alpha_S)$. Of course this proportionality disappears if higher order corrections in the masses or nondegenerate squark masses are considered.

With heavy squarks, the final baryon asymmetry will be given by

$$B \simeq -\left(\frac{3\,\mathcal{A}\,\Gamma_{\rm sp}}{56\,s\,\Gamma_{\rm QCD}}\right)\left(1 - \frac{D_q}{D_h}\right) \simeq -10^{-8}\,\gamma_w\,v_w\,\sin\phi_B\,\Delta\beta \tag{6.100}$$

where we have assumed $\phi_A \ll \phi_B$, $\Delta\beta$ is the total variation of β in the wall and light neutralinos with mass around 50 GeV and relatively heavy squarks with mass around 150 GeV have been considered.

EWB is then significant if

$$|\sin\phi_B\,\Delta\beta| \gtrsim 10^{-4}. \tag{6.101}$$

These conclusions is altered if, say, left-handed bottom squarks and left and right-handed top squark masses are rather light, but the other squark masses are heavy. Then the factor multiplying h in Eq. (6.98) does not vanish. Taking,

$$k_Q \simeq 18, \quad k_T \simeq 9, \quad k_B \simeq 3 \quad k_H \simeq 12, \tag{6.102}$$

we have

$$n_L = \frac{27}{82}\,H, \quad \bar{D} \simeq \frac{72}{T}, \tag{6.103}$$

and

$$B \simeq -\left(\frac{81\,\mathcal{A}\,\bar{D}\,\Gamma_{\rm sp}}{82\,s\,v_w^2}\right), \tag{6.104}$$

i.e. B is enhanced by a factor $\sim 18\,(\bar{D}\,\Gamma_{\rm QCD}/v_w^2)$ over the case with no light squarks. Taking the squarks with mass $\sim T \simeq 60$ GeV, one finds

$$B \simeq -4.5 \times 10^{-6}\,\frac{\kappa}{v_w}\,\sin\phi_B\,\Delta\beta, \tag{6.105}$$

where again it has been assumed that $\phi_A \ll \phi_B$. Thus, there is a significant contribution to B if

$$k \; |\sin\phi_B \, \Delta\beta| \gtrsim 4.5 \times 10^{-6}, \tag{6.106}$$

with $v_w \simeq 0.5$.

Note that in the limit of fast weak sphalerons or slow bubble walls, Eq. (6.93) is no longer valid and the final value of B will be insensitive to the sphaleron rate and determined by near equilibrium physics, along the lines of what found in ref. [35].

The attentive reader should be surprised by the fact that thermal scattering processes not only tend to interfere with baryogenesis by destroying the quantum coherence necessary for CP violation, but also can enhance the BAU. The ultimate reason for such a behaviour is that CP violating charges can be converted to CP violating thermal particles distributions inside the wall by incoherent thermal scattering processes. These CP violating thermal particle distributions diffuse into the symmetric phase, by CP even thermal processes, where they bias relatively rapid anomalous weak processes towards producing net baryon asymmetry.

Since axial top quark number is efficiently transported because the large top Yukawa coupling allows axial top number to convert to Higgs number, which is transported by weakly interacting Higgs particles, it is reasonable to believe that top quarks play a major role in EWB, certainly more important than τ's whose axial source is suppressed relative to the axial top source by a factor $h_\tau^2/h_t^2 \sim 10^{-4}$, as was first shown by CPR [37].

One of the main requirement for the mechanism to work is that the ratio of the Higgs VEVs is not fixed during the transition. The latter requirement implies that the effective theory during the transition has more than one light Higgs, which in turn means that at zero temperature the pseudoscalar and charged Higgs masses are not extremely heavy compared with the lightest Higgs mass. A light charged Higgs makes a potentially ruled out contribution to $b \to s\,\gamma$ [85] unless partially cancelled by a contribution from a loop containing light charginos and stops. Moreover, with small phases of order of $(10^{-4} - 10^{-5})$ the neutron EDM will be within two orders of magnitude of current experimental data.

It is then clear that sufficient baryon asymmetry can be produced with comfortably small explicit CP violating phases provided that either the stop squarks, the neutralinos or the charginos are light compared with the transition temperature. Moreover, a relatively light Higgs spectrum is needed.

This fact opens the exciting probability that the next generation of accelerators (LEP2, LHC) might say a definitive word about the possibility of electroweak baryogenesis in the framework of MSSM or, more in general, about the existence of supersymmetry.

References

[1] E.W. Kolb e M.S. Turner, *The Early Universe*, Addison Wesley (1990).

[2] A.D. Sakharov, Pis'ma Zh. Eksp. Teor. Fiz. **5** (1967) 32.

[3] P. Langacker, Phys. Rep. **72** (1981) 185.

[4] E.W. Kolb e M.S. Turner, Ann. Rev. Nuc. Part. Sci. **33** (1983) 645.

[5] H.P. Nilles, Phys. Rep. **110** (1984) 1; H.E. Haber e G.L. Kane, Phys. Rep. **117** (1985) 75; H.E. Haber, Lezioni TASI 1992, Preprint SCIPP-92-93.

[6] M.Yu. Khlopov e A.D. Linde, Phys. Lett. **B138** (1984) 265; J. Ellis, J.E. Kim, D.V. Nanopoulos, Phys. Lett. **B145** (1984) 181.

[7] G. 't Hooft, Phys. Rev. Lett. **37** (1976) 37; Phys. Rev. **D14** (1976) 3432.

[8] S. Dimopoulos e L. Susskind, Phys. Rev. **D18** (1978) 4500.

[9] N.S. Manton, Phys. Rev. **D28** (1983) 2019; R.F. Klinkhamer and N.S. Manton, Phys. Rev. **D30** (1984) 2212.

[10] V.A. Kuzmin, V.A. Rubakov, e M.E. Shaposhnikov, Phys. Lett. **B155** (1985) 36; P.Arnold e L. Mc Lerran, Phys. Rev. **D36** (1987) 581; Phys. Rev. **D37** (1988) 1020; S. Yu. Khlebnikov and M.E. Shaposhnikov, Nucl. Phys. **B308** (1988) 885; J. Ambjorn *et al.*, Phys. Lett. **B197** (1989) 49; J. Ambjorn *et al.*, Phys. Lett. **B244** (1990) 479; J. Ambjorn and J. Farakos, Phys. Lett. **B294** (1992) 248.

[11] I. Affleck, M. Dine, Nucl. Phys. **B249** (1985) 361; H. Murayama, Phys. Lett. **B322** (1994) 349; K. Olive, hep-ph 9404352.

[12] D.A. Kirzhnits e A.D. Linde, Ann. Phys. **101** (1976) 195; A.D. Linde, Rep. Progr. Phys. **42** (1979) 389.

[13] M. Shaposhnikov, Nucl. Phys. **B287** (1987) 757; Nucl. Phys. **B299** (1988) 797; Phys. Lett. **B277** (1992) 324, Erratum, Phys. Lett. **282** (1992) 483.

[14] For a nice review, see M. Quirós, *Field theories at finite temperature and phase transitions*, Helvetica Physica Acta, **67** (1994) 451; see also A. Hebecker, Ph.D. Thesis, DESY T-95-02 internal preprint.

[15] K. Enqvist, K. Kainulainen and I. Vilja, Nucl. Phys. **B403** (1992) 749.

[16] J.R. Espinosa, M. Quirós and F. Zwirner, Phys. Lett. **B307** (1993) 106; A. Brignole *et al.* Phys. Lett. **B324** (1994) 181.

[17] For a summary, see Z. Fodor, hep-lat 9603314.

[18] G. Farrar and M. Shaposhnikov, Phys. Rev. Lett. **70** (1993) 2833; *ibidem* **71** (1993) 210; Phys. Rev. **D** () da mettere;

[19] M.B. Gavela *et al.*, Nucl. Phys. **B430** (1994) 345; *ibidem* **B430** (1994) 382.

[20] P. Huet and E. Sather, Phys. Rev. **D51** (1995) 379.

[21] C. Quimbay and S. Vargas-Castillón, CERN-TH/95-72 preprint and references therein.

[22] M. Dine, R. Leigh, P. Huet, A. Linde, e D. Linde, Phys. Lett. **B283** (1992) 319; Phys. Rev. **D46** (1992) 550.

[23] G.D. Moore and T. Prokopec, hep-ph 9506475.

[24] A. Cohen, D. Kaplan, e A.Nelson, Ann. Rev. Nuc. Part. Sci. **43** (1993) 27.

[25] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Lett. **B245** (1990) 561; Nucl. Phys. **B349** (1991) 727.

[26] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Nucl. Phys. **B373** (1992) 453.

[27] J.M. Cline, Phys. Lett. **B338** (1994) 263; M. Joyce, T. Prokopec and N. Turok, hep-ph 9410281; J. Cline, McGill/95-34 preprint and references therein.

[28] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Lett. **B263** (1991) 86;

[29] D. Comelli, M. Pietroni and A. Riotto, Nucl. Phys. **B412** (1994) 441; see also Proceedings of the *Int. School on Cosmological Dark Matter*, Valencia, October 1993, World Scientific.

[30] D. Comelli, M. Pietroni, Phys. Lett. **306** (1993) 67; J.R. Espinosa *et al.*, Phys. Lett. **B319** (1993) 505.

[31] D. Comelli, M. Pietroni and A. Riotto, Phys. Rev. **D50** (1994) 7703; Phys. Lett. **B343** (1995) 207;

[32] M. Dine and S. Thomas, Phys. Lett. **328** (1994) 73.

[33] G. Giudice, M. Shaposhnikov, Phys. Lett. **B326** (1994) 118.

[34] M. Joyce, T. Prokopec and N. Turok, hep-ph 9401351.

[35] D. Comelli, M. Pietroni and A. Riotto, hep-ph 9406369, to be published in Astropart. Phys.

[36] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Lett. **B336** (1994) 41.

[37] D. Comelli, M. Pietroni and A. Riotto, DESY 95-070 preprint, to be published in Phys. Lett. **B**; DESY 95-109 preprint, hep-ph 9506278, submitted to Nucl. Phys. **B**.

[38] P. Huet and A.E. Nelson, preprint UW-PTH-95-03, hep-ph 9504427.

[39] P. Huet and A.E. Nelson, preprint UW-PTH-95-07.

[40] A. Manzoni, *I Promessi Sposi.*

[41] B. Liu, L. Mc Lerran and N. Turok, Phys. Rev. **D46** (1992) 2668.

[42] M. Dine *et al.*, Nucl. Phys. **B342** (1990) 341.

[43] J.A. Harvey and M.S. Turner, Phys. Rev. **D42** (1990) 3344; H. Dreiner and G.G. Ross, Nucl. Phys. **B410** (1993) 188.

[44] L. Mc Lerran, Phys. Rev. Lett. **62** (1989) 1075;

[45] N. Turok and J. Zadrozny, Phys. Rev. Lett. **65** (1990) 2331; Nucl. Phys. **358** (1991) 471.

[46] J.F. Gunion, H.E. Haber and S. Dawson, *The Higgs hunter's guide*, Addison Wesley (1990).

[47] R. Barbieri, S. Ferrara and C.A. Savoy, Phys. Lett. **B119** (1982) 343; H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. **B120** (1983) 346; E. Cremmer, P. Fayet and L. Girardello, Phys. Lett. **B122** (1983) 41; L. Ibãnez, Nucl. Phys. **B218** (1982) 514; L. Hall, J. Lykken and S. Weinberg, Phys. Rev. **D27** (1987) 2359; L. Alavarez Gaume and J. Polchinski, Nucl. Phys. **B221** (1983) 495.

[48] M. Dugan, B. Grinstein and L. Hall, Nucl. Phys. **B255** (1985) 413.

[49] K.F. Smith *et al.*, Phys. Lett. **B234** (1990) 191.

[50] M. Davier, LAL-91-48 preprint, *Rappourter talk* at the Conference *LP-HEP*, Geneva, July-August (1991); ALEPH Collaboration, Phys. Lett. **B265** (1991) 475.

[51] A.G. Cohen and A.E. Nelson, Phys. Lett. **B297** (1992) 111.

[52] M.T. Grisaru, M. Rocek and W. Siegel, Nucl. Phys. **B159** (1979) 429.

[53] N. Maekawa, Phys. Lett. **B282** (1992) 387.

[54] A. Pomarol, Phys. Lett. **B287** (1992) 331.

[55] H. Georgi and A. Pais, Phys. Rev. **D10** (1974) 1246.

[56] J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. **B257** (1991) 83.

[57] J.A. Casas, J.R. Espinosa, M. Quirós and A. Riotto, Nucl. Phys. **B436** (1995) 3.

[58] L. Dolan and r. Jackiw, Phys. Rev. **D9** (1974) 3320.

[59] M. Reuter and W. Dittricht, Phys. Rev. **D32** (1985) 513.

[60] S.A. Abel, W.N. Cottingham and I.B. Whittingam, Nucl. Phys. **B410** (1993) 173.

[61] Ya.B. Zel'dovich, I.Yu. Kobzarev and L.B. Okun, Zh. Eksp. Teor. Fiz. **67** (1974) 3 [Sov. Phys. JETP **40** (1975) 1].

[62] A. Vilenkin, Phys. Rep. **121** (1985) 263.

[63] V.A. Kuzmin, M.E. Shaposhnikov and I.I. Tkachev, Phys. Lett. **B105** (1981) 167.

[64] R. Holdom, Phys. Rev. **D28** (1983) 1419.

[65] H. Lew and A. Riotto, Phys. Lett. **B309** (1993) 258.

[66] G.W. Anderson and L.J. Hall, Phys. Rev. **D45** (1992) 2685.

[67] R. Garisto and G. Kane, hep-ph 9302282.

[68] F. Vissani and S. Bertolini, Phys. Lett. **B324** (1994) 164.

[69] S. Dimopoulous and L.J. Hall, Phys. Lett. **B344** (1995) 185.

[70] L. Mc Lerran, E. Mottola and M. Shaposhnikov, Phys. Rev. **D43** (1991) 2027.

[71] A.L. Fetter and J.D. Walecka, *Quantum theory of many-particle systems* (Mc Graw-Hill, 1971).

[72] J.I. Kapusta, *Finite temperature field theory*, Cambridge University Press, Cambridge, England, (1989).

[73] K.C. Chou, Z.B. Su, B.L. Hao, and L. Yu, Phys. Rep. **118** (1985) 1; N.P. Landsman and Ch.G. van Weert, Phys. Rep. **145**, (1987) 141 ; R. Baier and A. Niégawa, Bielfeld University preprint, BI-TP-93-39 (1993), hep-ph 9307362.

[74] G. Baym and N.D. Mermin, J. Math. Phys. **2**, 232 (1961).

[75] R.D. Pisarski, Nucl. Phys. **B309**, 476 (1988).

[76] R.D. Pisarski, Physica **A158**, 146 (1989).

[77] V.V. Lebedev and A.V. Smilga, Physica **A181**, 187 (1992).

[78] E.J. Weinberg, Phys. Rev. **D47** (1993) 4614.

[79] M. Gleiser, G.C. Marques and R.O. Ramos, Phys. Rev **D48** (1993) 1571.

[80] P. Arnold and O. Espinosa, Phys. Rev. **D47** (1993) 3546.

[81] J. Collins, *"Renormalization"*, Cambridge University Press (1984).

[82] See for instance A. Dolgov and K.Freeze, Phys. Rev. **D51**, (1995) 2693.

[83] S.A. Abel and P.L. White, OUTP-951P preprint , hep-ph/9505241.

[84] M. Joyce, T. Prokopec and N. Turok, hep-ph 9401352; J.M. Cline, K. Kainulainen and A.P. Vischer, hep-ph 9506284.

[85] A.J. Buras and S. Pokorski, Nucl. Phys. **B424** (1994) 374.

Figure Captions

Figure 1: The upper and lower limit on the pseudoscalar mass M_{A^0} as a function of $\tan\beta$ from the requirement $v_{\rm max}/T \lesssim 1$. Here $\tilde{M}_Q = \tilde{M}_U = 350$ GeV, $|\mu| = 200$ GeV, $|A_t| = 50$ GeV and $M_t = 130$ GeV.

Figure 2: The upper and lower limit on the pseudoscalar mass M_{A^0} (solid lines) when all the other parameters of the theory vary within their allowed ranges. The dashed line represents the experimental lower bound.

Figure 3: The path C corresponding to the imaginary time formalism of thermal field theory.

Figure 4: The path C corresponding to the real time formalism of thermal field theory. C_2 lies infinitesimally beneath the real axis.

Figure 5: Schematic representation of the behaviour of $\dot{\theta}$ and of the rate of the sphaleronic transitions at the point $\mathbf{x} = 0$.

Figure 6: Lowest order contribution to D^R_{ij}.

Figure 7: The contribution to D^R_{ij} due to photon exchange.

Figure 8: The tree level contribution to the neutral Higgs current.

Figure 9: The one-loop contribution to the neutral and charged Higgs currents.

Figure 10: The one-loop contribution to the left-handed top current.

Figure 11: The two-loop contribution to the left-handed top current. The scalar internal line can be either neutral or cherged Higgs fields.

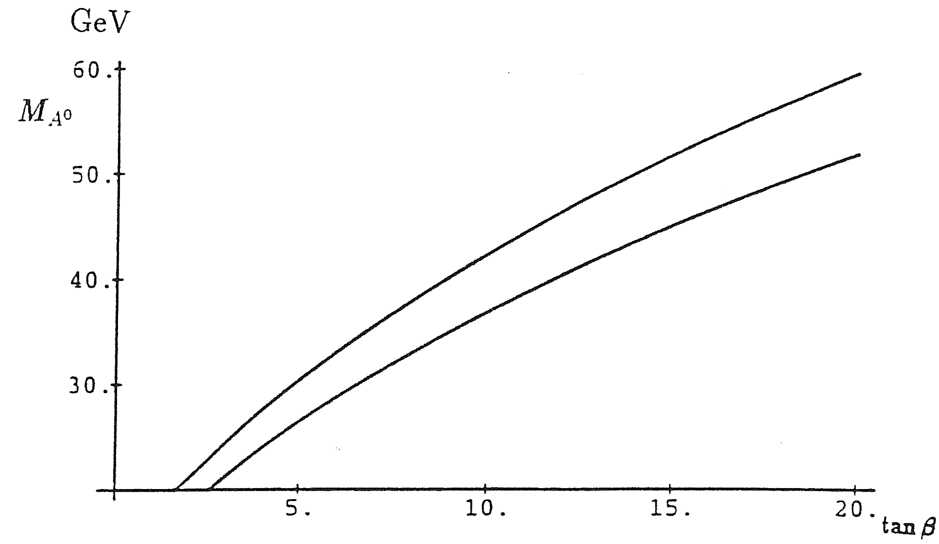


Figure 1

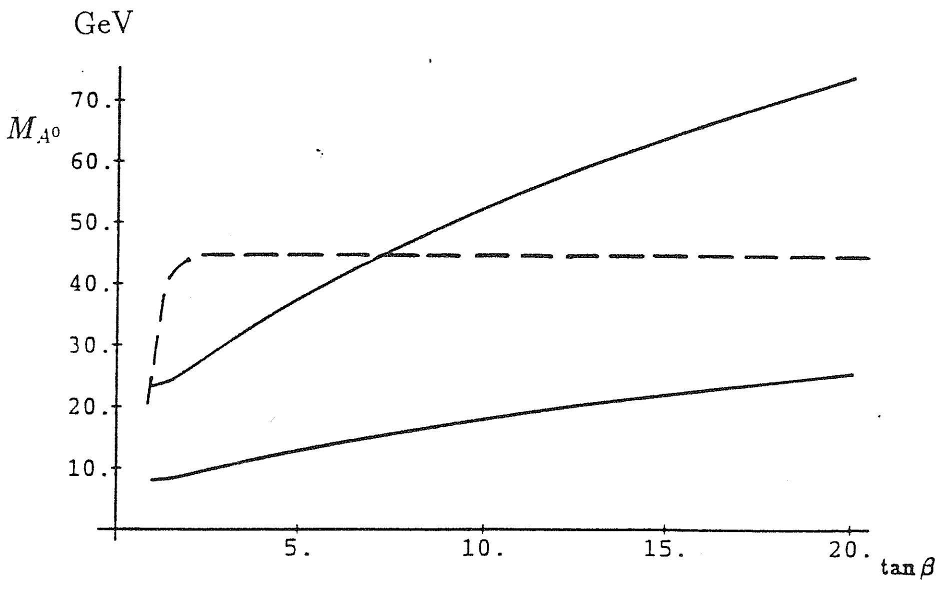


Figure 2

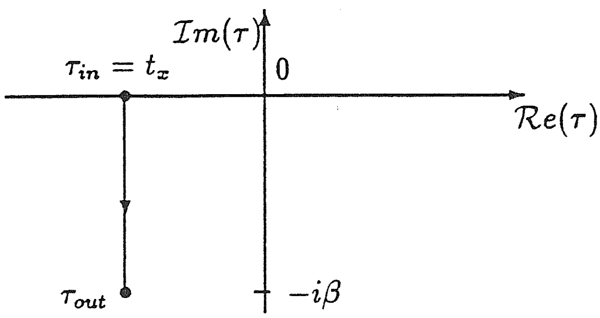
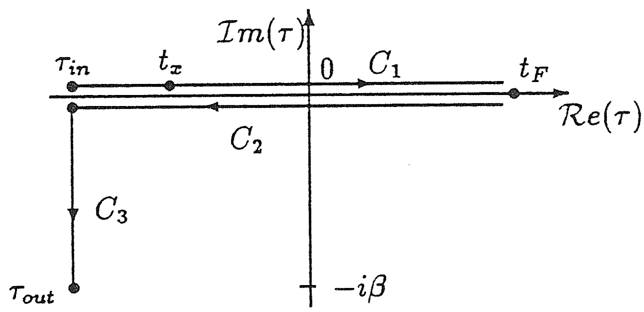


Figure 3



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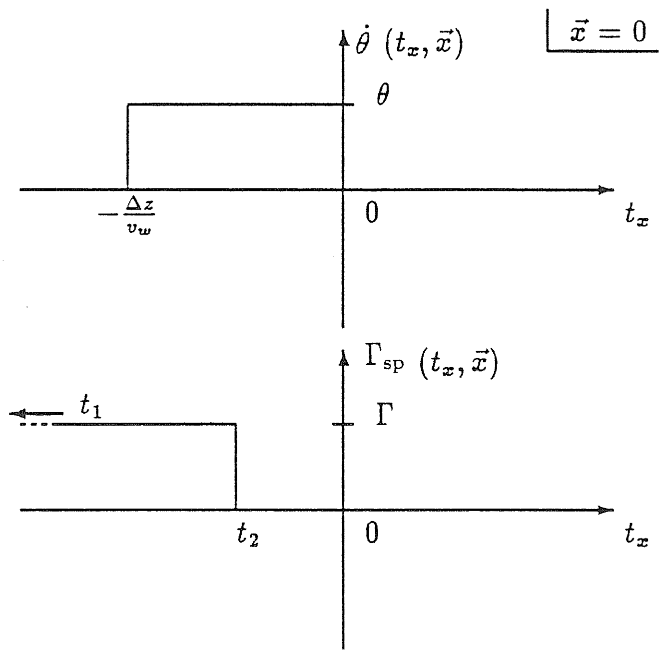
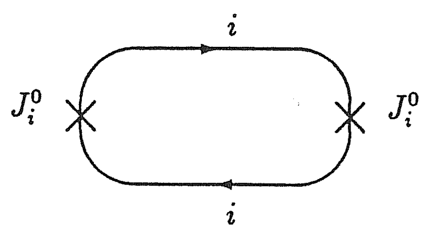


Figure 5



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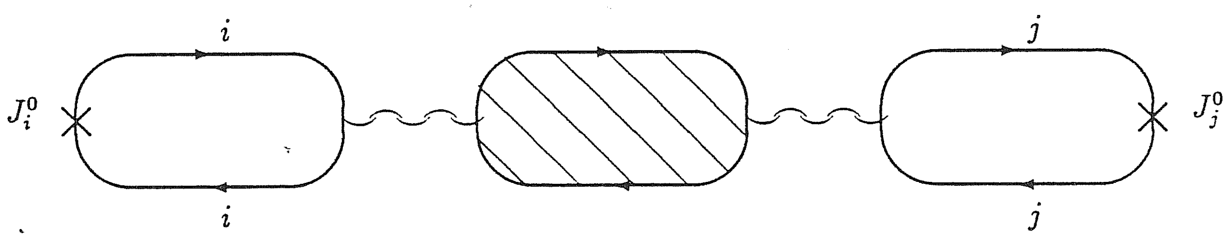

Figure 7

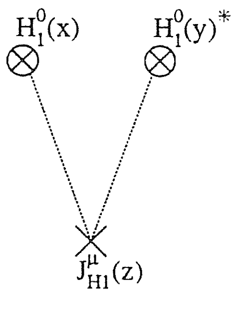

Figure 8

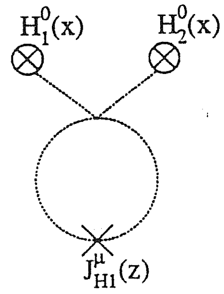

Figure 9

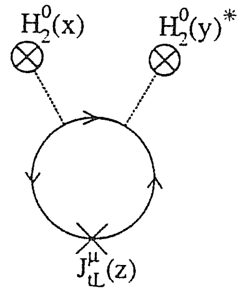

Figure 10

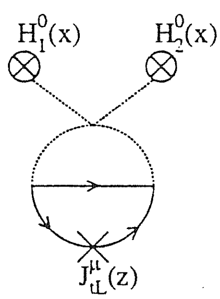

Figure 11