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Thesis submitted for the degree of Doctor Philosophiæ

BPS states and non-perturbative aspects of string theory

Candidate
J.F. Morales Morera

Supervisor
Prof. K.S. Narain

Accademic year 1997/98

SISSA - SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI
TRIESTE
Via Beirut 2-4

TRIESTE

SISSA ISAS
SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES
I-34014 Trieste ITALY - Via Beirut 4 - Tel. [+]39-40-37871 - Telex:460269 SISSA I - Fax: [+]39-40-3787528

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To Nadia, to my parents, this thesis.

Introduction

String theory is, at present, the only promising proposal for a consistent quantum theory of gravity (for a general review of the subject see for example [1],[2]). The fundamental objects in this construction are relativistic strings, with tension parameterized by $T = \frac{1}{2\pi\alpha'}$ and typical length $\sqrt{\alpha'}$, evolving in a D-dimensional spacetime. Elementary particles arise as the infinite tower of string vibrational modes with increasing spin and masses quantized in units of $1/\sqrt{\alpha'}$. The massless sector of this spectrum contains in particular, besides the standard gauge particles and matter, a spin two particle: the graviton, mediating the gravitational interactions. The correct strength of these interactions are reproduced by fixing $\sqrt{\alpha'} \sim 10^{-33} cm$. Massive string states will have therefore energies of the order of $(10^{-33} cm)^{-1} \sim 10^{19} GeV$ (far beyond our present experimental reach). In the low energy $\alpha' \to 0$ limit they can be integrated out leaving an effective supergravity theory in terms of the massless spectrum of physical states. The interactions are described by joining and splitting of strings during their evolution in the spacetime. This introduces an arbitrary dimensionless constant known as the string coupling constant g. A generic string scattering process is then given by a sum over all possible Riemann surfaces swept out by the string evolution weighted by $g^{-\chi}$, where χ is the Euler number of this surface. Unlike in standard quantum field theories, there is no well defined point in this Riemman surface where the string interaction occurs. This non-locality smoothes at least for closed strings the usual ultraviolet UV field theory divergences, opening a hope for a consistent quantization of gravity. For open strings different consistency conditions lead to the same conclusions. Indeed, consistent string theory constructions are believed to be UV finite at any order in the perturbation expansion. Another remarkable difference with standard quantum field theories is the dynamical nature of the string coupling constant, which can be reabsorved in the expectation value of a scalar field: the dilaton. This peculiarity makes clear that a pertubative expansion, which treats asymmetrically the dilaton from other scalars in the theory, is a rather unnatural limit in which many intrinsic facets of string theory can be at first sight hidden.

In the eighties, the interest in string theory was stimulated by the construction of the first phenomenologically interesting string models. Many chiral models in four dimensions with matter content very close to supersymmetric extensions of the standard electroweak theory were derived from the low energy effective actions arising in string compactifications. The observed world is clearly not supersymmetric. The understanding of the mechanisms of supersymmetry breaking stands as one of the most important open problem for the present string generation. For this goal, a deeper understanding of non-perturbative string phenomena is needed.

In the last few years, there has been a considerable improvement in the understanding of the non-perturbative structure of string theories. The key to these developments is the discovery of S-duality symmetries [3]-[15] which relate strong and weak coupling regimes of apparently very different string theories (for a general review see for example [16]). Our picture of "fundamentals" in string theory has, in this way, drastically changed and an increasing evidence for the existence of an underlying theory, denoted by M-theory [17, 3], has emerged. The non-perturbative techniques we have at present are still primitive to give a complete proof of any of these strong-weak duality conjectures. We can however reverse the arguments, and once enough evidence for the existence of a given duality relation is provided, take it as a fact, in order to learn about the strong coupling regime of the theory under study. The consistency of the final picture can be considered as a further support to the initial duality conjecture.

In this thesis we study string theories with sixteen supercharges. We will denote in the following by $\mathcal{N} = \frac{1}{4}$(number of supercharges), the number of four dimensional supersymmetries and by N the number supersymmetries in ten dimensions. The understanding of the non-perturbative physics of these string vacua is a first step toward the study of the phenomenologically most interesting (and more difficult) $\mathcal{N} = 1$ case. A common fact to all theories with $\mathcal{N} > 1$ supersymmetries is the existence of bounds in the mass spectra: the so called BPS bounds [24]. The spectrum of states saturating these bounds are called BPS states. They span short supermultiplets of the supersymmetry algebra and, for enough number of supersymmetries, they are stable under any deformation of the theory. This property allows us to follow them to regimes of strong coupling providing striking tests of the different duality conjectures. The first part of this thesis is devoted to the study of this important sector of the spectrum of states in string theories with 16 real supercharges. The models considered arise from toroidal compactifications of heterotic/type I string theories and as asymmetric orbifolds of IIB theory. We restrict our attention to nine and eight dimensional toroidal compactifications, where the relevant BPS states come, as we will see, from Kaluza-Klein and winding modes of fundamental strings or as excitations in a system describing a collection of D-strings. D-strings are one-dimensional BPS solitonic solutions of the low energy supergravity action that carry a charge with respect to the so called RR fields. They are special in the sense that they admit a very simple conformal description [18] as we will discuss below. We will see how ten dimensional type I/heterotic and IIB self- dualities implies the existence of an infinite tower of D-string bound states for lower dimensional compactifications. The electric spectra of these bound states will be studied in the effective gauge theories describing collections of nearby D-strings and will be shown to agree with the prediction of duality regarding the charges, masses and multiplicities of such BPS excitations [19]. In a second part we study string amplitudes which are essentially determined by the BPS spectrum of states. As a first example we consider the moduli dependence of D-instanton contributions to F^4 couplings in the eight dimensional low energy effective actions for the type II dual pair of string vacua. The results can be alternatively read from a perturbative computation in terms of worldsheet instantons of the dual fundamental string. The agreement between these two pictures through the IIB duality relations is shown [20]. Finally we discuss correlation functions which encode the leading order spin interactions for slowly moving D-branes [21], [22]. The amplitudes degenerate effectively at long(short) distances to a tree level (one-loop) spin effects in eleven dimensional supergravity (SYM). Once more, only contribution of BPS states are relevant and the truncation on each channel agrees as required by the M(atrix) model proposal for a non-perturbative description of M-theory. More general D-brane configurations are also discussed

The exposition is organized as follows: In a first chapter we briefly review some features of ten dimensional string dualities, BPS bounds and D-brane physics which will set the background and notations for the later discussions. In chapter 2 we study the spectrum of D-string bound states in the context of type I/heterotic and IIB self- dualities. Chapter 3 is devoted to the computation of "BPS saturated" threshold corrections in type IIB (4,0) string vacua (four refers to the number of four-dimensional supersymmetries). Finally in chapter 4 the SYM/SUGRA correspondence is discussed for a class of D-brane configurations describing spin effects and nontrivial fluxes/background data. Finally some conclusions and discussion are included.

Chapter 1

Low energy effective actions

1.1 Perturbative string theories

String theory is a quantum theory of relativistic strings. During its evolution, a string spans a two-dimensional surface in spacetime, known as the worldsheet. The quantization is implemented by the sum over all possible worldsheets Σ connecting the initial and final location of the string, weighted by the superstring action

$$S = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \sqrt{h} \left[\partial^\alpha X^\mu \partial_\alpha X_\mu - i\psi^\mu \rho^\alpha \partial_\alpha \psi_\mu - i\tilde{\psi}^\mu \rho^\alpha \partial_\alpha \tilde{\psi}_\mu \right], \tag{1.1}$$

For the bosonic string we simply omit the fermionic terms in (1.1). The X^μ's, $\mu = 0 \cdots D-1$, represent the spacetime coordinates and $\psi^\mu, \tilde{\psi}^\mu$ ten left and right moving Majorana-Weyl fermions respectively. The action (1.1) is invariant under the reparametrization of the worldsheet coordinates $\sigma_\alpha = \sigma, \tau$ and under the Weyl rescaling $h_{\alpha\beta} \to e^\gamma h_{\alpha\beta}$ of the worldsheet metric. In order to consistently fix these gauge invariances we should impose the cancellation of the two dimensional supercurrent and stress energy tensor associated to (1.1). Once this is done the bosonic part of this action reduces to the area of the string worldsheet. A careful study of these constraint equations at a quantum level reveals in general the existence of a conformal anomaly $T^\alpha_\alpha \neq 0$, with $T_{\alpha\beta}$ the stress energy tensor. The cancellation of this conformal anomaly is a strong constraint in the spacetime dimensions allowed for a sensible string theory construction, leaving only $D = 26$ and $D = 10$ for the bosonic and fermionic strings respectively. Further constraints are provided by modular invariance and tadpole cancellations for closed and open strings respectively. The modular invariance is just the statement that equivalent Riemman surfaces describing loops of closed strings, should give the same string answer. For example, for one-loop, when the string worldsheet is a torus, equivalent tori are described by complex structures $\tau, \tilde{\tau}$ related by an $SL(2,Z)$ transformation

$$\tilde{\tau} = \frac{a\tau + b}{c\tau + d} \tag{1.2}$$

with a, b, c, d integers satisfying $ad - bc = 1$. For open strings on the other hand the cancellation of tadpoles in the one-loop partition function, which ensures the vacuum stability, plays a similar constraining role. Tachyons in the spectrum of states are removed by demanding supersymmetry. We are left with five consistent possibilities, known as type IIA, type IIB, type I, $E_8 \times E_8$ heterotic and $Spin(32)/Z_2$ heterotic string theories. Let us briefly review the worldsheet content and massless spectrum of these theories

Type II strings: The worldsheet content of type IIA and IIB strings are built from ten scalar fields, ten left and ten right-moving Majorana-Weyl fermions with the same and opposite chiralities respectively in (1.1). All the fields are in the vector representation of the Lorenz $SO(1,9)$ group. On the compact σ direction, fermions can have either periodic R (Ramond) or antiperiodic NS (Neveu-Schwarz) boundary conditions. Once both sectors are included, modular invariance (1.2) requires a suitable projection (GSO) on the spectrum of states, leading to a sum over all possible boundary conditions for the fermions in both σ and τ directions. The projection is carried out independently for the left and right moving sectors, removing in each of them the tachionic NS-vacuum and one of the two spacetime $SO(1,9)$ Lorenz spinors realized by the Ramond ground states. On the massive tower of string excitations the projection leaves states transforming in tensor and spinor representations of the Lorenz group for the Neveu-Schwarz and Ramond sectors respectively. The resulting spectrum of physical states is invariant under a ten dimensional $N = 2$ supersymmetry which exchanges bosons, arising from NSNS and RR sectors, with fermions from the RNS and NSR ones. In particular, at the massless level, we have an $N = 2$ supergravity multiplet spanned by the NSNS metric G_{MN}, antisymmetric tensor B_{MN} and dilaton Φ, the RR antisymmetric forms, and their fermionic superpartners. The RR antisymmetric forms come from the expansion of a $SO(1,9)$ bispinor with the same and opposite chiralities depending of whether we are deal with type IIB or type IIA theories and give rise therefore to even and odd potential forms respectively. These massless contents determine completely the low energy effective action in the limit $\alpha' \to 0$, where as discussed above massive modes can be integrated out. The resulting field theories are known as IIA and IIB ten dimensional supergravities.

Heterotic string theories: Heterotic string theories are the heterosis of a left moving bosonic and a right moving fermionic string. Heterotic worldsheet contents can be realized by ten scalars and ten right moving Majorana-Weyl fermions, transforming both in the vector representation of the $SO(1,9)$ Lorenz group plus sixteen complex left moving Majorana-Weyl internal fermions. The worldsheet action can then be written as

$$S = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma\sqrt{h} \left[\partial^\alpha X^\mu \partial_\alpha X_\mu - i\psi^\mu \rho^\alpha \partial_\alpha \psi_\mu - i\tilde{\bar{\chi}}^A \rho^\alpha \partial_\alpha \tilde{\chi}_A\right], \tag{1.3}$$

with $A = 1\cdots 16$ running through the Cartan directions of the gauge group. The gauge groups arise from the Kac-Moody algebras represented by the internal fermions and are highly constrained by modular invariance. Indeed, only the $Spin(32)/Z_2$ and $E_8 \times E_8$ choice for the gauge group are allowed, corresponding to choose the boundary conditions for two groups of sixteen fermions with the same and opposite periodicity respectively. Supersymmetry arises from the left moving sector once one implement the GSO projection as before. In addition, an analog of this projection should be implemented for the internal fermions, ensuring the invariance under the modular group (1.2). The spectrum of physical states is now invariant under the $N = 1$ ten dimensional supersymmetry algebra with bosons and fermions arising now from the NS and R sector respectively. At the massless level in particular, we are left with two ten dimensional supermultiplets: the $N = 1$ supergravity multiplet which contains the metric G_{MN}, the antisymmetric tensor B_{MN}, the dilaton Φ and their fermionic superpartners; and the Yang-Mills supermutiplet containing the gauge vector and its fermionic superpartners. Again the low energy effective actions are determined by the number of supersymmetries and the choice for the gauge group. They are known as the $S0(32)$ and $E_8 \times E_8$ supergravities and corresponds to the only existing anomaly free ten dimensional super Yang-Mills theories coupled to gravity.

Type I theory: Finally we have the type I theory of open and closed unorientable strings. A novelty in the type I string perturbative expansion is the appearance of boundaries and non-orientable worldsheets. For example, at genus $\chi = -1$ a type I computation involve a sum over the disk and the projective plane contributions while at genus $\chi = 0$ we should include besides the torus; the Klein bottle, annulus and mobius strip contributions. Type I theory can be thought as the quotient of type IIB theory by the worldsheet parity operation Ω [23]. The projection keep only left-right symmetric states in the type IIB spectrum, while open strings appears as the twisted sectors of this operation. In particular, at the massless level the antisymmetric NSNS tensor, the zero and four RR form potentials are removed by this projection, leaving a content similar to the NSNS sector (common to all the previously discussed string theories), where now the antisymmetric tensor arises from the Ramond-Ramond sector. The ends of the open strings are taken to be free (Neumann boundary conditions) and carry a non-dynamical gauge index in some representation $\mathbf{R}$ of the gauge group. So in addition to the usual Fock space labels an open string state is described by a pair of indices (i, j), each one running in $\mathbf{R}$. Starting from $\mathbf{R}$ the fundamental representation of $U(N)$ two possible definitions for the Ω action on these Chan Paton indices are allowed. They lead to a surviving massless vector in the adjoint representation of the orthogonal $SO(N)$ or simplectic $USp(N)$ gauge groups. The $N = 1$ ten dimensional vector multiplet is completed by its fermionic superpartners coming from the R-sector. The tadpole cancellation requirement at one-loop leaves however the $SO(32)$ gauge group as the only consistent choice. At low energies the theory is described again by the $SO(32)$ ten dimensional supergravity, which as we will discuss below can be related to the one arising for the $SO(32)$ heterotic string by a simple change of variables. Although both theories share this low energy supergravity description, we should recall that they look as string theories very different and the statement of a strong-weak duality which relate then is, as we will see, far from straight.

1.2 Supersymmetry algebras and their central extensions

A common feature to all $N = 1, 2$ supergravities, describing low energy limits of the above discussed string theories, is the possibility of include "central" extensions in their associated superalgebras [24]. Aim of this section is to discuss how these extensions lead to bounds in the mass spectrum of physical states. States saturating BPS bounds provide short representations of the extended superalgebra and are then naively stable under any deformation of the theory. This property will be extensively exploited along the duality tests reported in this thesis. We will call these extensions "central" although they don't commute with Lorenz rotations. More rigorously, they are central extensions of the supertranslational algebra. Our discussion will follow mainly [25] and reference therein.

Let us start with the $N = 1$ ten dimensional superpoincare algebra. We can concentrate in the algebra of supertranslational invariance, since commmutators between Lorenz generators are determined as usual by the Lorenz indices. The supertranslational algebra is generated by translations P_M and a Majorana-Weyl supercharge Q_α. Translational invariance implies that P and Q commute, while for the $\{Q,Q\}$ anticommutator, we can write in general

$$\{Q_\alpha, Q_\beta\} = (\mathcal{P}\Gamma^M C)_{\alpha\beta} P_M + (\mathcal{P}\Gamma^M C)_{\alpha\beta} Z_M + (\mathcal{P}\Gamma^{MNPQR} C)_{\alpha\beta} Z^+_{MNPQR}. \quad (1.4)$$

where Γ^M are the 32×32 $SO(1,9)$ gamma matrices, $\mathcal{P} = \frac{1}{2}(1+\Gamma_{11})$ the chirality operator and C the charge conjugation matrix, which in the Majorana basis is identified with Γ_0. The Z_M's and Z^+_{MNPQR}'s commute with all the supertranslational generators, and are known as central extensions. They correspond to the existence of a string and a self-dual fivebrane solution of the free supergravity equation of motions (for a review and references see for example [26]). Indeed

$$Z_{M_1 \cdots M_p} = \mu_p \int_{\Sigma_p} dX_{M_1} \Lambda \cdots \Lambda \, dX_{M_p} \quad (1.5)$$

can be identified in general with the charge of a p-dimensional object (wrapped on a p-cycle Σ_p) under the corresponding p+1 form. The masses of each of these extended objects will be proportional to the volume of the cycle Σ_p. Once such a cycle exists in the ten dimensional space, an object carrying this topological charge provide a representation of the extended algebra (1.4). The simplest examples for finite energy configurations carrying these charges are given by the Kaluza-Klein and winding string modes. Suppose our space contain a compact X^9 direction of radius R. The unique-valuedness of the wave function e^{ipX} implies in this case the quantization of the Kaluza-Klein momentum P^9 in units of $1/R$. In addition one such compact direction exist we can wind it n times a string around it leading to a configuration with mass proportional to R. From the nine dimensional point of view we see that the Kaluza-Klein momentum P^9 and winding Z_9 are central extensions corresponding to a charge under the gauge fields $G_{\mu 9}$ and $B_{\mu 9}$ respectively. The Z^+_{MNPQR} in (1.4), on the other hand, represent the fivebrane winding charges of the known symmetric fivebrane solution of $N=1$ ten dimensional supergravity. The string and fivebrane solutions are electric-magnetic duals in ten dimensions [27]. In general a $(D-p-4)$-dimensional object carries a magnetic dual charge in D-dimensions respect to the p+1 form potential.

After these preliminaries we would like to study some particular representations of this $N=1$ algebra, which as we will see, behave more like the massless representations of the "standard" supersymmetry algebras. Let us first recall the structure of supersymmetry representations in the absence of central extensions $Z_M = Z^+_{MNPQR} = 0$. In this case we can distinguish two kind of representations: the massless $(2^{16/4} = 16)$-dimensional representation, and the massive $(2^{16/2} = 256)$-dimensional one. In order to see this, we write (1.4) in the rest frame $P_M = (M, 0, \cdots 0)$ for the massive case and in a frame such that $P_M = (E, \pm E, \cdots 0)$ otherwise. The algebra (1.4) reduces then to

$$\begin{aligned} \textbf{Massive} \quad \{Q_\alpha, Q_\beta\} &= \mathcal{P}_{\alpha\beta}\, M & (1.6) \\ \textbf{Massless} \quad \{Q_\alpha, Q_\beta\} &= \mathcal{P}(1 \pm \Gamma^{01})_{\alpha\beta}\, E & (1.7) \end{aligned}$$

In the massive case, (1.6) corresponds to a 16-dimensional Clifford algebra (recalling that 16 of the initial 32 components of the ten dimensional Majorana spinors are projected out by $\mathcal{P}$). A $2^{16/2} = 256$ dimensional representation is realized as usual acting with the 8 creation fermionic operators on a vacuum highest state. For the massless case (1.7) on the other hand, the further projection $(1 \pm \Gamma^{01})$ leave only 8 non trivial eigenvalues which give rise to a short $2^{8/2} = 16$ dimensional supermultiplet.

For a central extension the story is somehow similar. The positivity of the Q^2 operator in the LHS is, however, no longer ensured as before from $P^2 \geq 0$. Indeed, noticing that the matrices involved in (1.4) have only eigenvalues ± 1, and taken into account that we can always reverse the sign of $Z_{M_1 \cdots M_p}$ changing the orientation of the p-brane, we have a bound in the mass $M \geq |Z_{M_1 \cdots M_p}|$ for a p-brane solution oriented in the $M_1 \cdots M_p$ directions. For a massive state above this bound the non-degenerate matrix $\mathbf{1} + Z_{M_1 \cdots M_p} \Gamma^{0 M_1 \cdots M_p}$ implies that we are dealing again with a 16-dimensional Clifford algebra and therefore a "long" supermultiplet is realized. For a state saturating this bound this is not the case, and an algebra similar to (1.7) with E replaced by M and $(1 \pm \Gamma^{01})$ replaced by $(1 \pm \Gamma^{0 M_1 \cdots M_p})$ is found. Again, as in the massless case, we are left with just 4 creation-anhililation pairs spanning a $2^4 = 16$ short representation, now of the extended algebra (1.4). We notice that these extreme states break 8, out of the initial 16 supercharges, while the remaining ones are realized trivially. In general a state preserving some linear combination Q^- of the supercharges is called a *BPS state*

$$\sum_\alpha Q^-_\alpha |BPS state\rangle = 0, \qquad (1.8)$$

and the bound saturating by it, is called a *BPS bound*.

More complicate BPS configurations can be constructed by turning on various charges in (1.4) at the same time. Let us consider a compactification of the X^8, X^9 directions on a torus of radius R_8, R_9 respectively. In this case we can turn on, for example, a Kaluza-Klein momentum P_8 and a winding charge Z_9 in the 8 and 9 directions respectively. The matrices Γ^{08} and Γ^{09} anticommute and therefore the eigenvalues of $P_8\Gamma^{08}+Z_9\Gamma^{09}$ are given by $\pm\sqrt{P_8^2+Z_9^2}$. In order to get zero eigenvalues for the RHS matrix of (1.4) we should then saturate the bound $M=\sqrt{P_8^2+Z_9^2}$. We can notice that the mass of this state is lower than sum of the masses of the Kaluza-Klein and the winding constituents. When this is the case, we will call the state a *non threshold bound state.* Conversely if both winding and Kaluza-Klein modes are parallel the BPS bound is just $M=|P_9|+|Z_9|$ and we call it *a threshold bound state.* In both the cases the BPS states preserve, as before, 1/2 of the original supersymmetry and span 16-dimensional multiplets.

Finally let us discuss less supersymmetric BPS configurations. Let us consider for example a bound state of a string and a fivebrane wrapped on a fivedimensional torus oriented in the 12345 directions. In this case (as it will be always the case for parallel p-p+4, p-p+8, etc. brane configurations) the matrices Γ^{01} and Γ^{012345} commute and can be simultaneously diagonalized. The eigenvalues are just ±1 and then we have bounds in the mass given by $M\geq(|Z_1|+|Z_{12345}|)$. For states saturating this bound we will have 1/4 out of the initial 16 supercharges realized trivially. Acting with the remaining 12/2 creation operators we span a $2^6=64$ dimensional BPS supermultiplet. This is another example of a threshold bound state since the mass is equal to the sum of the BPS masses of the string and the symmetric fivebrane.

We can apply now a similar analysis to the $N=2$ ten dimensional superalgebras realized by the low energy effective actions for type II strings. The starting point, for the type IIA case, is now the supertranslational anticommutators:

type IIA:

$$\begin{aligned}\{Q_\alpha,Q_\beta\} &= (\Gamma^M C)_{\alpha\beta}P_M+(\Gamma_{11}C)_{\alpha\beta}Z+(\Gamma_{11}\Gamma^M C)_{\alpha\beta}Z_M+(\Gamma^{MN}C)_{\alpha\beta}Z_{MN}+\\ &\quad (\Gamma^{MNPQ}\Gamma_{11}C)_{\alpha\beta}Z_{MNPQ}+(\Gamma^{MNPQR}C)_{\alpha\beta}Z_{MNPQR}. \qquad (1.9)\end{aligned}$$

where Q_α combines in a non-chiral fermion the two Majorana-Weyl supersymmetries of opposite chiralities. The central extensions include now, besides the NSNS charges associated to the fundamental string and the fivebrane, a set of extended even-dimensional objects, which corresponds to p-brane solutions charged under the corresponding RR potentials. These solutions will be identified in the next section as the D0, D2 branes and their magnetic duals the D6, D4 branes respectively. Similarly for type IIB theory the supertranslational anticommutators are given by

type IIB:

$$\begin{aligned}\{Q^i_\alpha,Q^j_\beta\} &= \delta^{ij}(\mathcal{P}\Gamma^M C)_{\alpha\beta}P_M+(\mathcal{P}\Gamma^M C)_{\alpha\beta}\tilde{Z}^{ij}_M+\epsilon^{ij}(\mathcal{P}\Gamma^{MNP}C)_{\alpha\beta}Z_{MNP}\\ &\quad +\delta^{ij}(\mathcal{P}\Gamma^{MNPQR}C)_{\alpha\beta}Z^+_{MNPQR}+(\mathcal{P}\Gamma^{MNPQR}C)_{\alpha\beta}\tilde{Z}^{ij}_{MNPQR} \qquad (1.10)\end{aligned}$$

where now the RR charged brane solutions are odd-dimensional as should be in order to couple minimally to the even RR forms of type IIB. They correspond, as we will see, to the D1, D5 branes electric-magnetic pair and the self dual D3 brane.

Again we can detect bounds in the mass spectrum of states, and the existence of small representations of these extended algebras for states preserving some fraction of the initial supersymmetries, i.e. the BPS states.

At this point it is worth to stress a crucial fact. Although the similar definitions (1.5) for all these central extensions, the dilaton dependence of the masses M and charges Z's of these solutions are drastically different. For example, if we normalize to one the mass of the fundamental strings, their magnetic duals: the solitonic NS fivebranes in (1.4), (1.9) and (1.10), will carry masses going like $1/g^2$, g being the string coupling constant. The masses of the RR sources, on the other hand, can be read from the explicit p-brane supergravity solutions [26] to go like $1/g$. This dependence will become clear in the next section, where a very simple conformal description of these peculiar RR solitons will be given.

Finally we would like to say some words about the threshold bound states defined above. Many of the duality relations we will discuss in the following predict the existence of these bound states but a straight check of these predictions is rather tricky. The problem relies in the fact that there is no energy barrier which forbids the decomposition of such state in their constituents. The spectrum of states is then a continuous, in which the distinction of a one-particle from multiparticle state is non trivial. In ref. [28] an explicit and involved computation showed the existence of a unique bound state for $N = 2$ number of D0 branes in theories with 32 supercharges and no one for theories with less number of supersymmetries as required by string dualities. The arguments of [29] support the generalizations of these results for $N > 2$ zero branes. In the next chapter we present our contribution to this problem studying D-string bound states at threshold in the context of type I/heterotic and type IIB self- dualities.

1.3 D-branes and RR charges

There are wonderful reviews of the D-brane physics in the recent literature [30]. In this section we limit ourself to quote some basics results in the subject, which will be relevant in the future discussion of string dualities and D-brane spin dynamics. At the end of the previous section we have seen how p-extended solutions, charged under the Ramond-Ramond p+1-form potentials, are allowed by the supersymmetry algebra of type II supergravities. It is easy to see however, that no elementary state with this charge appear within the type II perturbation theory. Indeed the trilinear coupling

$$\langle closed|V_{RR}^{(p+1)}|closed\rangle \tag{1.11}$$

vanishes automatically on any closed Riemann surface since the RR vertex involves an odd number of left and right moving fermion emission vertices. The independent conservation of the left and right moving fermionic numbers is clearly a symmetry of (1.1). This is not the case if we allow boundaries in the string worldsheet. In this case only total fermionic number is conserved and the previous forbidding argument no longer holds. A boundary in the worldsheet of a string should be supplemented by either Neumann or Dirichlet boundary conditions

$$\begin{aligned} \partial_\sigma X^\mu &= 0 \quad \mathit{Neumann} \quad \mu = 0, \cdots p \\ \partial_\tau X^I &= 0 \quad \mathit{Dirichlet} \quad I = p+1, \cdots 9 \end{aligned} \tag{1.12}$$

for the open string coordinate X^μ. These boundary conditions defined a p-dimensional surface, we will call a *Dp brane*, where and open string can end. Alternatively, they can be thought as a point in which a closed string state, encoding the boundary condition data (1.12) [1], dissapears. They can therefore be represented by a suitable closed string operator creating this state from the vacuum. This formalism is called the boundary state formalism [31] and is particularly useful when we study string processes involving worldsheet boundaries, from the closed string point of view.

Our aim now is to illustrate how a Dp-brane, defined by (1.12) carries a charge under the Ramond-Ramond p+1 form potential. We will work in the G-S formulation of the type II strings which makes manifestly the supersymmetry of the theory. We follow closely the notation of [32]. Consider type II theory in the light-cone gauge $X^+ = x^+ + p^+\tau$. X^- is completely determined in terms of the transverse X^i's, in the $\mathbf{8}_\mathrm{v}$, and the left and right spinors S^a and $\tilde{S}^a$, in the $\mathbf{8}_\mathrm{s}$ of the $SO(8)$ transverse rotation group [2]. In this frame, the two light-cone directions $\pm = 0 \pm 9$ satisfy automatically Dirichlet boundary conditions while the transverse directions $i = 1, ..., 8$ can have either Neumann or Dirichlet boundary conditions. Since the time satisfies Dirichlet

[1] In terms of the closed string variables the boundary conditions can be written by simply change σ and τ in (1.12)

[2] In the following we display only the Type IIB expressions, for which the notation is somewhat friendlier; the Type IIA formulas can be easily obtained by switching dotted and undotted indices in the right-moving fermions.

boundary conditions, we are actually dealing with Euclidean branes; however we can identify the "time" with one of the transverse directions, say X^1. The usual metric is then recovered through a double analytic continuation $0 \to i1$, $1 \to i0$ in the final result. We can therefore include in our analysis only branes with $p = -1, ..., 7$. The coordinate transverse to the $\pm$ directions satisfy the Neumann-Dirichlet bosonic boundary conditions

$$(\partial X^I - M^I_J \bar{\partial} X^J)|B\rangle_p = 0 \tag{1.13}$$

with

$$M^I_J = \begin{pmatrix} -I_{p+1} & 0 \\ 0 & I_{7-p} \end{pmatrix}. \tag{1.14}$$

The boundary conditions on the fermions are then fixed by supersymmetry, that is, by the condition that the boundary state preserve some linear combination of the spacetime supercharges.

$$\begin{aligned} Q^a_+|B\rangle &= \frac{1}{\sqrt{2}}\left(Q^a + iM_{ab}\tilde{Q}^b\right)|B\rangle = 0 \\ Q^{\dot{a}}_+|B\rangle &= \frac{1}{\sqrt{2}}\left(Q^{\dot{a}} + iM_{\dot{a}\dot{b}}\tilde{Q}^{\dot{b}}\right)|B\rangle = 0 \end{aligned} \tag{1.15}$$

with

$$Q^a = \sqrt{2p^+}\oint d\sigma S^a \ , \ Q^{\dot{a}} = \frac{1}{\sqrt{p^+}}\gamma^i_{\dot{a}a}\oint d\sigma \partial X^i S^a$$

and similar expressions for the right moving supercharges. These conditions are solved by the boundary state

$$|B\rangle = \exp\sum_{n>0}\left(\frac{1}{n}M_{IJ}\alpha^I_{-n}\tilde{\alpha}^J_{-n} - iM_{ab}S^a_{-n}\tilde{S}^b_{-n}\right)|B_0\rangle \tag{1.16}$$

with M^I_J given by (1.14),

$$M_{ab} = (\gamma^1\gamma^2...\gamma^{p+1})_{ab} \ , \ M_{\dot{a}\dot{b}} = (\gamma^1\gamma^2...\gamma^{p+1})_{\dot{a}\dot{b}}. \tag{1.17}$$

and $|B_0\rangle$ being the zero mode part

$$|B_0\rangle = M_{IJ}|I\rangle|\tilde{J}\rangle - iM_{\dot{a}\dot{b}}|\dot{a}\rangle|\tilde{\dot{b}}\rangle \tag{1.18}$$

The coupling of the D-brane to a generic closed string state $|\Psi\rangle$ can be read from the corresponding one-point function on a disk with boundary conditions given by (1.12). In this language one point functions are simply given by the overlapping $\langle B|\Psi\rangle$. In particular, masses and RR charges can be read from the overlapping with the massless string states:

$$\begin{aligned} |\Psi_0^{NSNS}\rangle &= \xi_{mn}|m\rangle|\tilde{n}\rangle \ \Rightarrow \ \xi_{mn} \sim \phi\,\delta_{mn} + g_{mn} + b_{mn} \\ |\Psi_0^{RR}\rangle &= C_{\dot{a}\dot{b}}|\dot{a}\rangle|\tilde{\dot{b}}\rangle \ \Rightarrow \ C_{\dot{a}\dot{b}} \sim \sum_{k\ even}\frac{1}{k!}C^{i_1...i_k}_{(k)}\gamma^{i_1...i_k}_{\dot{a}\dot{b}} \end{aligned}$$

The NSNS insertion

$$\langle B_0|\Psi_0^{NSNS}\rangle = \xi_{ij}M_{ji}, \tag{1.19}$$

describes then the coupling of the Dp-brane to an specific combination of the dilaton ϕ and the diagonal graviton components $g_{11}...g_{p+1,p+1}$.

For the RR one point function we have on the other hand

$$\langle B_0|\Psi_0^{RR}\rangle = \sum_{k\ even} \frac{1}{k!} C_{(k)}^{i_1...i_k} \mathrm{Tr}_S[\gamma^{i_1...i_k} M] \tag{1.20}$$

The gamma-trace vanishes unless $k = p+1$, showing the RR charge of the Dp-brane under the $(p+1)$-form $C_{p+1}^{i_1 \cdots i_{p+1}}$. At this point we cannot compute the absolute value of the masses and charges given by (1.19) and (1.20) since the boundary state (1.18) is determined up to an overall constant. The relative coefficient between the absolute value of mass and charge is however fixed to one as required by the BPS condition (1.15). Moreover we can determine their dilaton dependence. Indeed since the vacuum expectation value of the dilaton is identified with the string loop parameter the masses and charges should go like $1/g$ ($\chi = -1$ for the disk). We conclude that a Dp-brane is a RR charged BPS state with mass of the order of $1/g$. They are the string realizations of the p-brane supergravity solutions carrying the RR charges in (1.9) and (1.10). Finally we should say that although we have discussed RR D-branes only for the type II strings, they appear also in the type I theory. Recalling the origin of this as an Ω projection of the type IIB theory which removes the NSNS antisymmetric tensor, the string and fivebrane charges in (1.4) are carried by the D1 and D5 branes. We will discuss in detail this case later on.

It is instructive to recover the same results about the masses and charges of D-solitons from the computation of the static force felt between two such objects. A consequence of the saturation of a BPS bound is a semmiclassical no-force condition. At large distances, two identical static D-branes interact through the exchanges of the massless fields in (1.18). The BPS no force condition implies then that the attractive forces mediated by the graviton and dilaton exchanges should be compensated by a repulsive RR interaction. We go now through this simple computation which illustrate the power of this boundary state techniques and set the basis for a more detailed study of the D-soliton dynamics later in this thesis.

The configuration space boundary state is given by

$$|B, \vec{x}\rangle = (2\pi\sqrt{\alpha'})^{4-p} \int \frac{d^{9-p}q}{(2\pi)^{9-p}} e^{i\vec{q}\cdot\vec{x}} |B\rangle \otimes |\vec{q}\rangle \tag{1.21}$$

where

$$\langle q|q'\rangle = V_{p+1}\,(2\pi)^{9-p}\delta^{(9-p)}(q-q')$$

and V_{p+1} is the space-time volume spanned by the p-brane.

With these normalizations, the static force between two parallel branes is given by the cylinder amplitude

$$\mathcal{A} = \frac{1}{16}\int_0^\infty dt\,\langle B,\vec{x}|e^{-2\pi t\alpha' p^+(P^- - i\partial/\partial x^+)}|B,\vec{y}\rangle \tag{1.22}$$

where

$$P^- = \frac{1}{2p^+}\left[(p^i)^2 + \frac{1}{\alpha'}\sum_{n=1}^{\infty}(\alpha^i_{-n}\alpha^i_n + \tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n + n\,S^a_{-n}S^a_n + n\,\tilde{S}^a_{-n}\tilde{S}^a_{-n})\right]$$

is the Hamiltonian in the light-cone gauge. The term $i\partial_+$ represents the subtraction of p^- (remember that in this gauge the effective Hamiltonian is $H - p^-$) and when applied to the boundary state, it reproduces simply the covariant p^2. The factor $1/16$ is needed to normalize correctly the D-brane charge; indeed, from eq.(1.22) we obtain

$$\mathcal{A} = \frac{1}{16}V_{p+1}\,(4\pi^2\alpha')^{4-p}\int_0^\infty dt\int\frac{d^{9-p}q}{(2\pi)^{9-p}}\,e^{i\vec{q}\cdot(\vec{x}-\vec{y})}\,e^{-\pi t\alpha' q^2}\,(8-8)\prod_{n=1}^{\infty}\frac{(1-e^{-2\pi tn})^8}{(1-e^{-2\pi tn})^8} \tag{1.23}$$

where the factor $(8-8)$ is due to the trace performed on the zero mode part of the boundary state, eq.(1.18). Performing the momenta and modulus integrations, one finds [18]

$$\mathcal{A} = 2\,V_{p+1}\,G_{9-p}(\vec{x}-\vec{y})\,(T_p^2 - \mu_p^2) \tag{1.24}$$

with $T_p = |\mu_p| = \sqrt{\pi}(4\pi^2\alpha')^{(3-p)/2}$ interpreted as the tension and charge density of a p-brane in units of the ten-dimensional Planck constant k^2 of Type II supergravity [18], and $G_d(\vec{x})$ the massless propagator of a scalar particle in d-dimensions

$$G_d(\vec{x}) = \frac{1}{4\pi^{d/2}}\frac{\Gamma(\frac{d-2}{2})}{|x|^{d-2}}.$$

We recognize in (1.23) the BPS no force condition as we have anticipated. We notice also that no α' corrections to this cylinder computation are present, since the contributions of massive modes cancel in (1.23) between boson and fermions. The exact result (1.24) (at this order in the string coupling constant) is therefore completely determined by the exchange of the massless graviton, dilaton and RR fields of type IIA supergravity. Alternatively we can think in the cylinder as a one-loop of an open string stretching between the two D-branes. In this picture (1.24) arises from the contribution of the lowest energy mode of the stretched string. This implies that we have an equivalent description of the D-soliton static force in terms of an effective theory of open strings.

Before going on, it is worthwhile to say some words about this dual picture, where a D-brane is seen as an end point of an open string rather than as a source of closed string states. This description is particularly helpful for nearby strings, regime in which the stretched open strings become light and constitute the relevant degrees of freedom. The effective theory describing the dynamic of N-nearby D-branes is defined by the quantization of the low energy modes of the strings stretching between them. We introduce, as in the case of type I open strings (which can be considered as strings ending on 32 D9 branes) Chan-Paton indices $i = 1, \cdots N$. The lowest energy bosonic modes are given then by the $\lambda_{ij}\partial_\tau X^\mu$ vectors and $\lambda_{ij}\partial_\sigma X^I$ scalars corresponding to the quantization of the longitudinal and transverse directions to the p-brane. The masses of these modes are proportional to the distances between the (i,j) branes, becoming massless when they are on top of each other. In particular, when M D-branes coincide we are left with a $U(M) \times U(1)^{N-M}$ enhancement of the initial $U(1)^N$ gauge group. Including the fermions we are left with a $U(M) \times U(1)^{N-M}$ gauge theory, which is just the reduction of $N = 1$ ten dimensional SYM theory to p+1 dimensions. In the case of unorientable strings, we should include, as before for type I theory, an Ω projection in the Chan-Paton indices leaving effective $O(N/2)$ gauge theories.

In the rest of this thesis we will alternate both pictures of D-solitons. The simplicity of these conformal descriptions will be extensively appreciated along the precision tests and D-brane dynamics analysis performed later.

1.4 String duality conjectures

In this section we will discuss the duality relations between ten dimensional string theories, from the point of view of their low energy effective actions. At the two-derivative level these actions are completely determined by their supersymmetry and the massless spectrum of the theory. From the string point of view they can be derived from the requirement of conformal invariance for the sigma model describing the motion of the string in the background of these massless fields. We will work in the string frame, where a factor $e^{-2\phi}$ appears in front of the Einstein kinetic term, recalling the σ-origin of this term from the propagator on the sphere ($\chi = -2$). If a duality transformation is a symmetry of a string theory should be a symmetry of the corresponding effective action. The study of the effective actions will then define the duality maps which will be promoted later to a symmetry of the full string theories. Evidences for these identifications, at the string level, will occupy the next chapters of this presentation.

1.4.1 Type I-heterotic duality

We start by describing the type I/heterotic duality conjecture [3]-[6]. The tree level (sphere) two derivative effective action of the $SO(32)$ heterotic string is given by

$$S^{\text{het}} = \int d^{10}x\sqrt{G}e^{-2\phi}\left[R + 4(\nabla\phi)^2 - \frac{1}{4}F^2 - \frac{1}{12}\hat{H}^2\right] . \tag{1.25}$$

with H the field strength of the antisymmetric tensor $B_{\mu\nu}$, $F_{\mu\nu}$ the $SO(32)$ field strength, $G_{\mu\nu}$ and ϕ the metric and dilaton respectively.

The duality map is defined by the change of variables

$$\phi_h = -\phi_I, \quad G^h_{\mu\nu} = e^{-\phi_I}G^I_{\mu\nu}, \quad A^h_\mu = A^I_\mu, \quad B^h_{\mu\nu} = B^I_{\mu\nu}. \tag{1.26}$$

Indeed, writing (1.25) in terms of the indexed $''I''$ variables we are left with

$$S^I = \int d^{10}x\sqrt{G}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2\right) - \frac{1}{4}e^{-\phi}F^2 - \frac{1}{12}\hat{H}^2\right], . \tag{1.27}$$

which is just the type I low energy effective action. We notice the no dilaton dependence of the H kinetic term since it comes from the RR sector. The dilaton and gauge kinetic terms are, on the other hand, weighted by $e^{-2\phi}$ and $e^{-\phi}$ since they comes from the sphere ($\chi = -2$) and the disk ($\chi = -1$) respectively.

The relations (1.26) suggest that the strong coupling regimes of the type I theory might be described by the weak coupling of the $SO(32)$ heterotic string theory and vice versa. This is the content of the type I/heterotic duality conjecture. If this is the case we can elucidate many of the non-perturbative properties of one theory from the knowledge of the perturbative regime of the other one. As we have discussed in the introduction a direct proof of this strong-weak duality is far from our present reach, but an increasing evidence for it, is collected in the recent literature. In this thesis we present some precision tests concerning the spectrum of physical states.

In general we don't expect a complete matching of both spectra, since a perturbative state on one side might becomes unstable and dissapears from the spectrum. This is not the case for states saturating a BPS bound as we have discussed previously. By a simple inspection of the $N = 1$ algebra (1.4) we see that the simplest finite energy BPS configurations we can constructed arise in a circle compactification to nine dimensions. In this case we can turn on Kaluza-Klein and winding modes of the elementary heterotic strings which should be mapped through (1.26) to Kaluza-Klein and RR charges under $B^I_{\mu\nu}$ respectively. In the next chapter we will make more precise this identification, providing a precision test to this string duality conjecture

1.4.2 IIB self-duality

As we described in the introduction the bosonic massless content of type IIB theory contains beside the universal NSNS sector, a zero, two and four RR form potentials. The RR four-form is self-dual, implying that the corresponding field strength is equal to its dual. It turns out that there is no simple covariant action describing this constraint, and we can write at most covariant equation of motions. The important point is that these field equations are invariant under the $SL(2,R)$ transformations [12]-[15]:

$$\lambda \to \frac{a\lambda + b}{c\lambda + d} \;, \quad \begin{pmatrix} B^N_{\mu\nu} \\ B^R_{\mu\nu} \end{pmatrix} \to \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \begin{pmatrix} B^N_{\mu\nu} \\ B^R_{\mu\nu} \end{pmatrix} \;, \tag{1.28}$$

where a, b, c, d are real with $ad - bc = 1$. These transformations mix the NSNS $B^N_{\mu\nu}$ and RR $B^R_{\mu\nu}$ antisymmetric tensors, acts fractionally on the complex scalar

$$\lambda = \chi + ie^{-\phi} \tag{1.29}$$

with ϕ, χ the dilaton and RR scalar respectively, and leave invariant the metric and RR four-form. It is easy to see that only a maximal $SL(2,Z)$ subgroup of these transformations can be a symmetry of the full string theory. The $B_{\mu\nu}$ charge carried by a string state must be quantized in units of the elementary string charge, which can be normalized to one. Acting with a $SL(2,R)$ on a state with integer $B_{\mu\nu}$ charge we are left in general with a forbidden fractional charge state. The matrices (1.28), with a, b, c, d restrict to be integers, are precisely the maximal set of transformations (up to a redefinition of $B_{\mu\nu}$) which leave an integer valued spectrum of charges. This $SL(2,Z)$ subgroup is precisely the conjectured self-duality symmetry of type IIB string theory [7].

An elementary string (denoted by (1,0)) carries an unit of charge under the NSNS antisymmetric tensor $B^N_{\mu\nu}$. Acting with the $SL(2,Z)$ transformation:

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{1.30}$$

we get a string state carrying an unit of charge respect to the RR antisymmetric tensor $B^R_{\mu\nu}$, which we will denote by (0,1). In a previous section we identify a physical state carrying this charge: the D-string. The complete $SL(2,Z)$ orbit produces then (p,q) strings with p(q) units of charge under the NSNS(RR) antisymmetric tensor. The content of $SL(2,Z)$ duality conjecture is that all those (p,q) strings provide equivalent descriptions of the full type IIB string theory. In the next chapter we will give some precision tests of this conjecture.

Before going on, it is useful to discuss some perturbative symmetries of the type IIB string theory which will be exploited in future discussions. As we mentioned, type IIB theory is invariant under the worldsheet parity transformation Ω. On the massless bosonic fields, Ω changes the sign of $B^N_{\mu\nu}, \chi$ and the four form $D_{\mu\nu\rho\sigma}$, leaving the other fields invariant. There is still an additional perturbative symmetry we denote by $(-)^{F_L}$. This operation changes the sign of all the states in the left moving Ramond sector. It is easy to see that both symmetry actions are related by the $SL(2,Z)$ transformation

$$S\,\Omega\,S^{-1} = (-)^{F_L} \tag{1.31}$$

with S given by (1.30). Type I theory is constructed from the orientifolding of type IIB with the Ω parity. The relation (1.31), combined with the type IIB S-duality, implies that a dual model can be constructed from the orbifolding of type IIB by $(-)^{F_L}$. This wil be the starting point for the construction of some dual pairs discussed in the next chapter.

1.4.3 The M(atrix) model conjecture

Type IIA supergravity is the dimensional reduction of $N=1$ supergravity in eleven dimensions [33]. The eleven dimensional theory is defined by the bosonic action [34]

$$L^{D=11} = \frac{1}{2\kappa^2}\left[R - \frac{1}{2\cdot 4!}G^2\right] + \frac{1}{2\kappa(144)^2} G\wedge G\wedge \hat{C} \tag{1.32}$$

where G is the field strength of the three-form potential $\hat{C}$. The fermionic action is determined from (1.32) by the $N=1$ eleven dimensional supersymmetry. The reduction of the eleven dimensional metric on a circle of radius $R=e^\sigma$

$$G_{\mu\nu} = \begin{pmatrix} g_{\mu\nu} + e^{2\sigma}A_\mu A_\nu & e^{2\sigma}A_\mu \\ e^{2\sigma}A_\mu & e^{2\sigma} \end{pmatrix}, . \tag{1.33}$$

give rises to the ten dimensional metric $g_{\mu\nu}$, gauge field A_μ, and dilaton $\phi \equiv \frac{3}{2}\sigma$ of the type IIA supergravity. The three-form $\hat{C}$ on the other hand gives rise to a three-form and a two-form in ten dimensions defined by

$$C_{\mu\nu\rho} = \hat{C}_{\mu\nu\rho} - \left(\hat{C}_{\nu\rho,11}A_\mu + \text{cyclic}\right) \quad , \quad B_{\mu\nu} = \hat{C}_{\mu\nu,11}\,. \tag{1.34}$$

In terms of these fields (after going to the string frame $g_{\mu\nu} \to e^{-\sigma} g_{\mu\nu}$), (1.32) reduces to the IIA supergravity bosonic action

$$\begin{aligned} \tilde{S}_{IIA} &= \frac{1}{2\kappa^2} \int d^{10}x \sqrt{g} \left[e^{-2\phi} \left(R + 4(\nabla\phi)^2 - \frac{1}{12} H^2 \right) - \frac{1}{2 \cdot 4!} \hat{G}^2 - \frac{1}{4} F^2 \right] + \\ & \frac{1}{2\kappa^2 (48)^2} \int B \wedge G \wedge G \end{aligned} \tag{1.35}$$

where H denotes the field strength of the antisymmetric tensor $B_{\mu\nu}$, G and F the ones corresponding to the RR A_μ and $C_{\mu\nu\rho}$ forms. We notice that no dilaton dependence for the kinematic RR terms appear, as expected.

The identification between the compactification radius and the string coupling constant $R = g^{\frac{2}{3}}$ involved in this reduction is remarkable. We can take it seriously and claim that type IIA theory at the strong coupling limit $g \to \infty$ becomes some eleven dimensional theory whose low energy regime is described by eleven dimensional supergravity [33], [3]. This conjectured theory is called M-theory. From its supergravity limit we know that M-theory should contain membrane and fivebrane states which couple to the eleven dimensional three-form $\hat{C}$ in (1.32) [35]. Upon compactification on a circle the membrane gives rise to a ten dimensional membrane or a string depending of whether we wrapped it or not around the eleventh direction. Using the identifications (1.33,1.34), it is possible to match the masses of these objects with the ones for the fundamental string and D2 brane of type IIA theory. Similarly the eleven dimensional fivebrane gives rise to a fivebrane and a fourbrane which are identified with the NS fivebrane and D4 brane present in type IIA theory. By further compactifying both theories, several evidences for this type IIA/M-theory duality conjecture has been accumulated in the recent literature, although not direct formulation of this theory has been formulated. Matrix theory pretends to be a parton definition of this mysterious eleven dimensional theory [36]. It is based in the observation that at the strong coupling limit $g \to \infty$, the D0 branes of type IIA theory become the lightest states of the theory and should determine the dynamics of it. The kinematical region that this proposal describes is the infinite momentum frame in the eleventh direction. Being the $U(1)$ type IIA photon identified with the $g_{\mu 11}$ component of the eleven dimensional metric, KK momentum modes along the eleventh direction maps to bound states in the D0 brane system. In particular a $P_{11} = \pm N/R$ KK momentum mode should be represented by a bound state of N D0 or anti D0 branes. Since in the infinite momentum frame all systems are constituted by partons with positive momentum we should deal only with D0 branes. Supergravity processes should then be mapped to correlations in the effective theory describing the interactions between clusters of D0 branes. The effective theory describing the interactions between N nearby D0 branes (N being the total number of D0 branes involved), was discussed at the end of section 1.3. This is given by the U(N) quantum mechanics

$$S = \int dt \frac{1}{2R} Tr(\dot{X^I}^2 + [X_I, X_J]^2 + \theta^T D\theta) \tag{1.36}$$

which is just the reduction of $N = 1$ ten dimensional SYM theory to 0+1 dimensions. The M(atrix) model conjecture then states that M-theory in the infinite momentum frame is exactly described by the $N \to \infty$ limit of (1.36). The first sign of a hope for such a conjecture is the existence of membrane-like solutions of the static equations of motion

$$[X^I, [X^J, X^I]] = 0 \tag{1.37}$$

arising from (1.36). Indeed the configuration

$$\begin{aligned} X^8 &= R_8 P \\ X^9 &= R_9 Q \end{aligned}$$

with

$$[Q, P] = 2\pi i \tag{1.38}$$

and all others $X^I = 0$ is a membrane-like solution oriented in the (89) direction. We notice that P and Q should be infinite by infinite matrices in order to satisfy (1.38) and have a non-vanishing trace, but this is precisely the $N \to \infty$ regime of validity of the conjecture.

A further evidence can be extracted from the study of matrix model interactions. We will present these results in a slightly different way from the original exposition in [36]. We will use the boundary state formalism described in the previous section. Our aim is to computed the tree level potential between two slowly moving D0 branes with impact parameter $\vec{b}$. Recall that we identify the "time" with X^1, the boundary state of a brane moving with velocity v^i is obtained from the static one (1.16) by applying the boost operator $\mathcal{O} = e^{iv_i J^{1i}}$ [37], with J^{1i} the generator of an $SO(8)$ rotation in the $(1i)$ plane. The potential between two D-branes moving with a relative velocity v can therefore be read from the insertion of the boost operator $\mathcal{O}$ in the static potential (1.22). As we have seen for the static case, this cylinder amplitude can be seen alternatively as a one-loop of the open string stretching between the branes. Once more, as in the static case, we will see that the leading order in the velocity expansion of this potential will collapse to a purely zero mode contribution, supporting equivalent descriptions in terms of the Yang-Mills (massless open string modes) or supergravity (massless closed string modes) degrees of freedom. Being interested only in the leading behavior in the velocity expansion of the D-brane potential we can consider the insertion of the boost operator $\mathcal{O}$ as a bunch of insertions of the vertex operator

$$V_B = v_i J^{i1} = v_i \oint_{\tau=0} d\sigma \left(X^{[1}\partial_\sigma X^{i]} + \frac{1}{2} S\gamma^{1i} S \right) \tag{1.39}$$

where we have used the boundary conditions (1.12), in order to write (1.39) in terms only of the left moving spinor S. The same operator (1.39) could also have been derived from that of photons in Type I theory with a constant field strength background after a T-duality transformation [38]. Before going on, it is important to point out that in computing leading orders of velocity-dependent potentials through correlation functions, we can actually directly extract potentials from the corresponding phase-shifts by simply dropping the overall time factor. This can be easily understood remembering that considering the velocity as insertions in the static cylinder amplitude instead of as a v-twisting in the boundary conditions, the extra bosonic zero mode integration will give simply the volume T of the time direction. The phase shift is define now by $\delta = VT$ with V a time independent potential.

Given these preliminaries, we can evaluate correlation functions involving n velocity insertions V_B's by compute

$$\mathcal{A}_n = \frac{1}{16}\int_0^\infty dt\, \langle B_p, \vec{x} = 0| e^{-2\pi t\alpha' p^+(P^- - i\partial/\partial x^+)} \frac{(V_B)^n}{n!} |B_p, \vec{y} = \vec{b}\rangle \tag{1.40}$$

where the $n!$ comes from the expansion of the boost operator. There is an evident analogy between eq.(1.40) and 1-loop amplitudes of massless states, in Type I string theory in the G-S formalism. In particular, the zero mode trace is vanishing unless all the eight zero modes S_0 are inserted [1], i.e.

$$\langle B_0 | R_0^N | B_0 \rangle = \mathrm{Tr}_V[R_0^N] - \mathrm{Tr}_S[R_0^N] = 0\ , \quad \text{for} \quad N < 4$$

where $R_0^{ij} \equiv \frac{1}{4} S_0 \gamma^{ij} S_0$ and the trace and matrix multiplication in both terms are over the vectorial and spinorial indices. Since the V_B (1.39) insertions provides at most two fermionic zero modes, a total of 4 velocity insertions is needed in order to get a non-zero result. The first non-vanishing trace is

$$\begin{aligned} t^{i_1\ldots i_8} &\equiv \mathrm{Tr}_{S_0}\, R_0^{i_1 i_2} R_0^{i_3 i_4} R_0^{i_5 i_6} R_0^{i_7 i_8} \\ &= -\frac{1}{2}\epsilon^{i_1\ldots i_8} - \frac{1}{2}\left[\delta^{i_1 i_4}\delta^{i_2 i_3}\delta^{i_5 i_8}\delta^{i_6 i_7} + \text{perm.}\right] \\ &\quad +\frac{1}{2}\left[\delta^{i_2 i_3}\delta^{i_4 i_5}\delta^{i_6 i_7}\delta^{i_8 i_1} + \text{perm.}\right] \end{aligned} \tag{1.41}$$

where "perm." means permutations of the pairs $(i_{2n-1}i_{2n})$ plus antisymmetrization within all the pairs.

The resulting potential then reads

$$V = \frac{\pi(2\pi)^{\frac{3}{2}}}{64} |v|^4 G_9(\vec{b}) \tag{1.42}$$

We notice that only the zero mode part of the vertex (1.39) gives a non trivial contribution to this result. Massive modes enter then only through the partition function and canceled out then between boson and fermions as in the static case. This ensures the result we have anticipated. On one side (1.42) defined the long range potential between two gravitons moving with a relative velocity v; equivalently we can read (1.42) from the one-loop effective action for the Yang-Mills quantum mechanics in terms of the massless open string modes. We can conclude that the one-loop matrix model effective action coming from (1.36), reproduces the long-range graviton-graviton potential.

In the last chapter we will see how spin-effects can be also correctly included in this frame, as far as more complicated supergravity configurations, providing further support to the matrix model conjecture for a non-perturbative description of M-theory.

Chapter 2

BPS spectra for string vacua with sixteen supercharges

In this chapter we study BPS spectra of physical states in string theories with sixteen real supercharges. The relevant supersymmetry algebra and its central extensions is given by (1.4). These extensions represent winding modes of string- and fivebrane-like objects, besides the standard Kaluza-Klein momenta. For toroidal compactifications to $D \geq 5$, fivebrane-like objects will be always infinitely heavy and finite energy BPS configurations can be constructed only from winding and Kaluza-Klein string excitations. However, string theories realize in different manners these central charges. The heterotic string, for example, contains both winding and Kaluza-Klein charges in its perturbative spectrum. This is not the case for type I theory, where the antisymmetric tensor arises from the RR sector, and therefore the winding charge is carried by a non-perturbative state: the D-string. These two theories are conjectured to be dual to each other under the duality map (1.26). These transformations map, in particular, the heterotic antisymmetric tensor $B^h_{\mu\nu}$ to the type I $B^I_{\mu\nu}$, exchanging winding of the fundamental heterotic string with D-string winding modes. If such a duality is true, both spectra of BPS states should match. More precisely, the complete spectrum of charges, masses and degeneracies of the N-winding sector of fundamental heterotic strings should be isomorphic to the spectrum of bound states of N type I D-strings each one wrapped once on a circle of radius $R_I = \frac{R_H}{\lambda_H}$. Similarly a strong-weak dual pair can be constructed starting from the self-dual type IIB theory and orbifolding it on one side by $(-)^{F_L}\sigma_V$ (σ_V an order two shift in the torus) and in other side by its S-image $\Omega\sigma_V$. Again the string-like central charges are carried in the former by fundamental string windings and by D-string windings in the later. Aim of this chapter is to show the precise match of the BPS spectra for these two dual pairs, providing a precision test of the type I/heterotic and type IIB self- duality conjectures.

The organization of the chapter is as follows: In the first two subsections, we study the infrared limit of the $O(N)$ gauge theory, describing N nearby type I D-strings. We show how this is described by an orbifold conformal field theory corresponding to N copies of the dual heterotic string, modded out by the permutation group S_N. For simplicity, we illustrate this arguments in the type I D-system. Identical lines of reasoning leads to the same conclusions for the second example: type IIB on $T^d/\Omega\sigma_V$. The subsection 3.1.3 is devoted to the computation of the elliptic genus. The computation is performed for a generic worldsheet content in order to include both examples. Finally, we identify the sector corresponding to real one-particle bound states and compare the results with the predictions of type I/heterotic and type IIB self- dualities.

2.1 Bound states of type I D-strings

We begin by recalling the form of the world volume action which describes the low lying modes of a system of N D-strings in the type I theory:

$$\begin{aligned} S &= \mathrm{Tr} \int d^2x \ -\frac{1}{4g^2}F^2 + (DX_I)^2 + g^2([X_I, X_J])^2 \\ &\quad + \Lambda \not{D} \Lambda + S \not{D} S + \sum_{A=1}^{16} \bar{\chi}^A \not{D} \chi^A + g\Lambda\Gamma^I[X_I, S] + \sum_{A=1}^{16} \bar{\chi}^A Y^A \chi^A. \qquad (2.1) \end{aligned}$$

The fields transform in various representations of the gauge group $O(N)$. X and S transform as second rank symmetric tensors, while Λ and χ transform in the adjoint and fundamental representations respectively. There is an $SO(8)_{\mathcal{R}}$, R symmetry group, under which X, S, Λ and χ transform as an $\mathbf{8}_V$ (this is the I label), an $\mathbf{8}_S$, an $\mathbf{8}_c$ and a singlet, respectively. The χ transforms under the $SO(32)$ in the vector representation with χ^A and $\bar{\chi}^A$ denoting the positive and negative weights. The Λ and χ are negative chiral (left-moving) world sheet fermions while the S are positive chiral (right-moving) fermions. Finally Y^A are the background holonomies (i.e. Wilson lines on the 9-branes) in the Cartan subalgebra of $SO(32)$. The Yang-Mills coupling g is related to the type I string coupling via $g^2 = \lambda_I/\alpha'$. The vev's of the X fields, appearing in the action above, measure the distances between the D-strings in units of $\sqrt{\alpha'\lambda_I}$. This fact will be important later, when we compare the spectrum with that of the heterotic theory.

Geometrically the fields appear in the following fashion [6, 39]. The above action arises as the Z_2 projection of the corresponding theory in the type II case. Recall that in the type II situation a system of N branes has a $U(N)$ symmetry. Write the hermitian matrices as a sum of real symmetric matrices and imaginary anti-symmetric matrices. The Z_2 projection, for type I D-strings, assigns to the world volume components of the gauge field the anti-symmetric matrices, that is, it projects out the real symmetric part and so reduces the gauge group to $O(N)$. On the other hand, the components of the gauge field in the transverse directions, X, have their imaginary part projected out and so are symmetric matrices transforming as second rank symmetric tensors under the $O(N)$. The diagonal components of the X give the positions for the N branes. The trace part represents the center of mass motion.

The χ carry the $SO(32)$ vector label, as they are the lowest modes of the strings which are stretched between the 9-branes and the D-strings.

We are interested in the counting of BPS bound states in the two dimensional system (2.1) with the σ-direction identified with the compact coordinate. A BPS bound state correspond to a vacuum in this gauge theory and therefore the information about multiplicities and masses of these states is encoded in the Witten index or more generally in the elliptic genus of (2.1).

Since the elliptic genus do not depend on the coupling constant, we can take the limit which is most convenient for our present purposes. We will consider the infra-red limit of the theory, as it has been conjectured in [40], that in this limit the theory flows to an $(8,0)$ orbifold superconformal field theory. This is in analogy with a similar conjecture for a system of type IIB D-strings [41]. In the following we give some support to this conjecture by, first, gauge fixing (2.1) and then performing a formal scaling which yields the orbifold theory directly.

2.1.1 Type II and the IR Limit

Before discussing the type I theory we make a digression on the type II theory that will prove useful later. Our aim here is to show that with a prudent choice of gauge one can simplify matters considerably. This prepares the way for taking the large coupling limit in a fashion that is, to a large extent, controllable. The starting point is then the $U(N)$ Yang-Mills theory defined by the dimensional reduction from $D = 10$ to $d+1 = 2$ dimensions. D dimensional vector labels are denoted by $M, N, \ldots$, those in $d+1$ dimensions are denoted by $\mu, \nu, \ldots$ and those in the remaining $D-d$ (reduced) dimensions by $I, J, \ldots$. To make contact with the type II D-string world volume theories one sets $D = 10$ and $d = 1$.

In any dimension the corresponding D dimensional Yang-Mills theory has a potential of the form

$$-\operatorname{tr}\frac{g^2}{4}[A_I, A_J]^2, \tag{2.2}$$

with flat directions along the Cartan subalgebra. Decompose the Lie- algebra, g, of the gauge group as $g = t \oplus k$, where t is the Cartan subalgebra and k its ortho-complement. It makes good sense, therefore, to perform a non-canonical split,

$$A_M = A^t_M + A^k_M, \tag{2.3}$$

where the superscripts indicate the part of the Lie-algebra that the fields live in. Before proceeding we need to gauge fix. Given the splitting of the algebra, it behaves us to choose the background field gauge

$$D^M(A^t)A^k_M = 0, \tag{2.4}$$

which preserves the maximal Torus gauge invariance. The ghosts come in as

$$\operatorname{tr}\overline{C}^k D_M(A^t)D^M(A)C^k + \operatorname{tr} g^2\overline{C}^k[\,[A^k_M, C^k]^t, A^{M\,k}]. \tag{2.5}$$

We choose a Feynman type gauge with a coefficient chosen to give the most straight-forward analysis, namely we add

$$\operatorname{tr} -\frac{1}{2}\left(D^M(A^t)A^k_M\right)^2 \tag{2.6}$$

to the action. With this choice the potential becomes

$$-\operatorname{tr}\frac{g^2}{2}[A^t_I, A^k_J]^2 + \ldots, \tag{2.7}$$

where the ellipses indicate higher order terms in A^k_I and which, directly, will be seen to be irrelevant.

We now perform the following sequence of scalings on the fields appearing in a $N = 1$ super Yang-Mills theory in D dimensions

$$A^k_M \to \frac{1}{g}A^k_M, \quad \psi^k \to \frac{1}{\sqrt{g}}\psi^k, \quad \overline{C}^k \to \frac{1}{g^2}\overline{C}^k. \tag{2.8}$$

On a torus T^d, with periodic boundary conditions on all the fields appearing, this scaling has unit Jacobian. We can now take the $g \to \infty$ limit. The action, in this limit reduces to:

$$\begin{aligned} S &= \operatorname{tr}\int d^{d+1}x - \frac{1}{4}F_{MN}(A^t)^2 + \psi^t \partial\!\!\!/ \psi^t - \frac{1}{2}[A^t_M, A^k_N]^2 \\ &\quad + \psi^k\Gamma^M[A^t_M, \psi^k] + [\overline{C}^k, A^t_M][C^k, A^t_M]. \end{aligned} \tag{2.9}$$

All the fields in the k part of the Lie-algebra can be integrated out and clearly give an overall contribution of unity to the path integral. Thus, we are left with a free, supersymmetric, system of Cartan valued fields. By invoking the Weyl symmetry that is left over, one finds that the target space of the theory is $(\mathbf{R}^{(D-d)r} \times T^{dr})/W$, where r is the rank of the group and W is the Weyl group. Specifying $D = 10$, $d = 1$, we are left with a 1+1 sigma model corresponding to N copies of a type IIB string moving in $(\mathbb{R}^8)^N/S_N$, with S_N the permutation group [41].

2.1.2 Type I and the IR Limit

The flat directions of the potential in this case require mutually commuting matrices once more. We denote those X's, with a slight abuse of notation, by X^t (for example one may choose these to be diagonal). A convenient way to proceed is to start with the (complexified) $SU(N)$ Lie algebra and to split it into a Cartan subalgebra t and into positive and negative roots, k_+ and k_-, respectively, that is, $k = k_+ \oplus k_-$. The Z_2 projection means that, in this basis, the world volume gauge fields are proportional to the anti-symmetric (imaginary part of k) generators, $m_- = k_+ - k_-$, while the X's are proportional to the symmetric generators, t and (real part of k) $m_+ = k_+ + k_-$. With these identifications the bosonic parts of the type I and type II theories coincide. We choose the same gauge fixing as in the type II theory, now restricted to the m_- directions,

$$\partial^\mu A_\mu^{m-} + g[X^t, X^{m+}] = 0 \tag{2.10}$$

and we scale the fields in a similar way, that is

$$\begin{aligned} A_\mu^{m-} \to \frac{1}{g} A_\mu^{m-}, \quad X_I^{m+} \to \frac{1}{g} X^{m+}, \quad \Lambda^{m-} \to \frac{1}{\sqrt{g}} \Lambda^{m-}, \\ S^{m+} \to \frac{1}{\sqrt{g}} S^{m+}, \quad \overline{C}^{m-} \to \frac{1}{g^2} \overline{C}^{m-}. \end{aligned} \tag{2.11}$$

The remaining fields X^t, S^t, C^{m-} and χ are unchanged. As before the Jacobian of these scalings is unity if we take periodic boundary conditions for the fermions S and Λ. There is no such requirement on the χ. Consequently the $g \to \infty$ limit may be safely taken.

The action now takes the form

$$\begin{aligned} S &= \operatorname{tr} \int d^d x - \frac{1}{2} \left|\partial_\mu X_I^t\right|^2 + S^t \not{\partial} S^t - \frac{1}{2}[X_I^t, A_\mu^{m-}]^2 - \frac{1}{2}[X_I^t, X_J^{m+}]^2 \\ &\quad + \Lambda^{m-} \Gamma^I [X_I^t, S^{m+}] + [\overline{C}^{m-}, X_I^t][C^{m-}, X_I^t] + \sum_{A=1}^{16} \bar{\chi}^A \not{D} \chi^A + \sum_{A=1}^{16} \bar{\chi}^A Y^A \chi^A. \end{aligned}$$

Formally, since the χ fields are chiral, only the right moving part of the gauge field is coupled to them and one can perform the integral over the left moving part of the gauge field which sets the left moving part to zero. Hence, on integrating out the massive modes, one would be left with a completely free theory of the massless modes X^t, S^t and χ. The determinant factors would then, at least formally, cancel between the fields of various statistics.

However, the above cancellation of the determinant factors is a bit quick. If correct, it would imply that even if we had started with an anomalous theory we would end up, in the limit, with a well defined superconformal field theory. For example, this would seem to be the case if we simply ignored the χ fields altogether. The point is that each fermionic determinant appearing is anomalous. These determinants, when defined in a vector gauge invariant way, involve extra quadratic terms in the gauge field. The presence of these would mean that the functional determinants would not cancel, since the gauge field contribution would not be $\mathrm{Det}(X^t)^2$. Happily, the condition that the theory be anomaly free means that the total sum of these extra pieces is zero and this is exactly what is required to make our formal argument above work.

On including the center of mass one gets N of the X's and S's, each transforming as a $\mathbf{8}_V$ and $\mathbf{8}_S$ of $SO(8)$ respectively and N χ's each transforming as a fundamental of $SO(32)$. The field content is like that of N copies of the heterotic string (1.3) in the light-cone gauge with an effective inverse tension

$$\alpha'_{\text{eff}} = \alpha' \lambda_I. \tag{2.12}$$

The condition (2.10) does not completely fix the gauge, there are still discrete transformations which leave the action invariant. There is the permutation group S_N which permutes the N copies of (X, S, χ) and which has the interpretation of permuting the N D-strings. There are also $O(N)$ transformations which leave invariant X and S but which act non-trivially on the χ's by reflection giving rise to a Z_2^N. The full orbifold group is therefore the semidirect product $S_N \times Z_2^N$.

2.1.3 Elliptic Genera and Symmetric Spaces

The arguments of the previous section implies that we can read the spectrum of N type I D-string bound states from the elliptic genus of the conformal theory describing N copies of heterotic strings moving on $(R^8)^N / S_N \times Z_2^N$. Similar arguments leads to the conclusion that the spectrum of D-string bound states for a $T^d/\Omega\sigma_V$ compactification of type IIB theory can be read from the elliptic genus of the conformal orbifold theory defined by N copies of the dual type IIB string on $T^d/(-)^{F_L}\sigma_V$, modded out by the Weyl symmetry group S_N. In this subsection we define the elliptic genera corresponding to these kind of orbifold conformal field theories, and work it out the generalities of their computations.

Given a two-dimensional CFT, the relevant elliptic genus for our present discussion, is the character-valued partition function in the right moving Ramond sector:

$$\chi(q) = Tr_{\mathcal{H}}(-)^{F_R} q^{L_0 - c/24} \bar{q}^{\bar{L}_0 - c/24} = Tr_{\mathcal{H}}(-)^{F_R} e^{-2\pi\tau_2 H + 2\pi i R \tau_1 P_\sigma}, \qquad (2.13)$$

where $q = e^{2\pi i \tau}$ and τ is the genus-one worldsheet modulus. R is a constant introduced for later convenience and H and P_σ are the Hamiltonian and momentum along the σ direction respectively. That (2.13) is an index can be seen easily from the fact that the right moving Ramond trace $Tr_R(-)^{F_R}$ get contributions only from ground states. Indeed, massive states come always in supersymmetric pairs which clearly cancel out in this trace. The elliptic genus (2.13) counts then effectively the number of BPS states (ground states of the right moving supersymmetric sector), or more precisely the difference between fermionic and bosonic ground states for the system described by $\mathcal{H}$.

In the following, we will be interested in computing this elliptic genera for two dimensional CFT's obtained by modding out the Hilbert space for N copies of a given world-sheet theory (usually a quotient Hilbert space $\mathcal{H}/Z_k$) by the permutation group S_N

$$\mathcal{H}_{N,Z_k} \equiv S_N(\mathcal{H}/Z_k) = \mathcal{H}^N/S_N \times Z_k^N. \qquad (2.14)$$

Z_k is an action on a Hilbert space $\mathcal{H}$, which preserves some number of supersymmetries, let us say $\mathcal{N}$. The relevant conformal field theories involved in our future discussions are more like Green-Schwarz strings, where the supersymmetry is realized through the zero modes for some periodic fermion in $\mathcal{H}/Z_k$, which we denote in the following by S. The Z_k action acts trivially on these modes.

Let us briefly review how the orbifold elliptic genus is computed [42]. The Hilbert space for a non-Abelian orbifold conformal field theory is built from the different twisted sectors labeled by the conjugacy classes of the orbifold group G. In each sector we project by the centralizer C_g in G, g being the twist element. The trace (2.13) in then given by

$$Z = \frac{1}{kN!} \sum_{[g]} \mathrm{h}\,\underset{\mathrm{g}}{\square} \qquad (2.15)$$

where [g]= $[(g,\epsilon)]$ denote the conjugacy class of a **g** group element in $G = S_N \times Z_k^N$, ($g \in S_N$, $\epsilon \in Z_k^N$). The elliptic genus for this CFT is zero, *viz.* $(2^{\mathcal{N}-1} - 2^{\mathcal{N}-1}) = 0$, due to the trace on the fermionic zero modes associated to the center of mass combination $S_0^1 + S_0^2 + \cdots S_0^N$. That this combination is invariant under the orbifold group is clear, since S_0^I are invariant under Z_k and get permutated under S_N. Indeed, these zero modes generate the $2^{\mathcal{N}-1}$ bosonic and $2^{\mathcal{N}-1}$ fermionic components of the short BPS supermultiplet, and being interested in computing the degeneracies of such supermultiplets only sectors with no additional fermionic zero modes will be relevant.

Let us start by identifying these sectors. In order to achieve this goal it is sufficient to consider the action of S_N, since as already stated Z_k^N does not act on the S fields. A general conjugacy class $[g]$ in S_N is characterized by partitions N_n of N satisfying $\sum nN_n = N$, where N_n denotes the multiplicity of the cyclic permutation (n) in the decomposition

$$[g] = (1)^{N_1}(2)^{N_2}\cdots(s)^{N_s}. \tag{2.16}$$

The centralizer for an element in this conjugacy class takes the form

$$C_g = \prod_{n=1}^{s} S_{N_n} \times Z_n^{N_n}, \tag{2.17}$$

We can now see that if $[g]$ involves cycles of different lengths, say $(n)^a$ and $(m)^b$ with $n \neq m$, then the corresponding twisted sector does not contribute to the elliptic genus. To see this, we note that there are now at least two sets of zero modes for S, which can be expressed, by a suitable ordering of indices, as $(S_1 + S_2 + \cdots S_{na})$ and $(S_{na+1} + \cdots S_{na+mb})$, where the two factors $(n)^a$ and $(m)^b$ act on the two sets of indices in the obvious way. These zero modes survive the group projection because the centralizer of g does not contain any element that mixes these two sets of indices with each other, thereby giving zero contribution to the elliptic genus. Thus we need only to consider those sectors with $[g] = (L)^M$ where $N = LM$.

The centralizer in the case where $[g] = (L)^M$ is $C_g = S_M \times Z_L^M$. From the boundary condition along σ it is clear that there are L combinations of S's that are periodic in σ. By suitable ordering, they can be expressed as $S^k = \sum_{i=Lk+1}^{L(k+1)} S_i$ for $k = 0, \ldots, M-1$. These zero modes have to be projected by the elements h in the centralizer C_g. In particular, when h is the generator of $Z_M \subset S_M \subset C_g$, it acts on the zero modes S^k by cyclic permutation. It is clear, therefore, that only the center of mass combination $\sum_{k=0}^{M-1} S^k$ is periodic along the t direction. Hence, this sector contributes to the elliptic genus. More generally any $h = (e,f) \in C_g = S_M \times Z_L^M$ will satisfy the above criteria provided $e = (M) \in S_M$ and f is some element of Z_L^M. The number of such elements h is $(M-1)! \times L^M$.

The full orbifold group G is specified by an element of S_N (discussed above) together with an element of Z_k^N. Let us consider a general element (g, ϵ). We denote by $e^{2\pi i \frac{t_\phi}{k}}$ the eigenvalue of a given field ϕ in $\mathcal{H}$ under the Z_k action. Now S_N acts as an automorphism in Z_k^N by permuting the various Z_k factors. We denote this action by $g(\epsilon)$. Then the semi-direct product is defined in the usual way: $(g, \epsilon).(g', \epsilon') = (gg', \epsilon g(\epsilon'))$. Twisted sectors will now be labeled by a conjugacy class in G. The relevant sectors, for the elliptic genus computation, as discussed above, are the conjugacy classes $[g]$ in S_N of the form $[g] = (L)^M$ with $N = LM$. One can easily verify that the various classes in G are labeled by $([g], \epsilon)$ with $\epsilon = \epsilon_1.\epsilon_2 \ldots \epsilon_M$ where each ϵ_i is in the quotient subgroup of Z_k^L by a Z_k subgroup. Combining this with the condition that we have found for h we may conclude that all the ϵ_i's must be equal (i.e. all ϵ_i's must have the same eigenvalue under Z_k^L) in order for such h to exist in the centralizer of $([g], \epsilon)$ in G. Different sectors are characterized then by a σ-twisted representative

$$\mathbf{g} = (g; \epsilon) = ((L)^M; \epsilon_1.\epsilon_2 \cdots \epsilon_M) \tag{2.18}$$

with $\epsilon_1 = \cdots \epsilon_M = (e^{2\pi i \frac{t\epsilon_g}{k}}, 1, 1, \cdots 1)$, $\epsilon_g = 0, 1, \cdots k-1$. A sector twisted by a group element (2.18) should be projected by the centralizer

$$\mathbf{h} = (h; \alpha) = (Z_M \times Z_L^M; \alpha_1.\alpha_2 \cdots \alpha_M)) \tag{2.19}$$

where $\alpha \in Z_k^N$ satisfies $\epsilon h(\epsilon) = \alpha g(\alpha)$. The number of independent such α's is k^M and therefore the order of the centralizer of $([g], \epsilon)$ is $M! L^M k^M$. The number of elements h in (2.19) that give rise to non-zero trace is $(M-1)! L^M k^M$, and therefore these are the relevant elements for the computation of elliptic genus. However, not all the h's of this form give different traces. Indeed, if h and h' are in the same conjugacy class in C_g, they will give the same trace. We can choose again a representative element $\mathbf{h}$. In a diagonal form, the actions (2.18),(2.19) for a given representative then read

$$\begin{aligned} \mathbf{g} &= e^{2\pi i (\frac{tt_g}{kL} + \frac{l}{L})} \\ \mathbf{h} &= e^{2\pi i (\frac{tt_h}{kLM} + \frac{ls}{LM} + \frac{r}{M})} \end{aligned} \tag{2.20}$$

with $l = 0, \cdots L-1$, $r = 0, \cdots M-1$ denoting the $N = M \cdot L$ copies of a generic field $\phi \in \mathcal{H}$ and $s = 0, \cdots L-1$ and $t_g, t_h = 0, \cdots k-1$ the orders of the Z_k elements in σ and τ directions respectively. It is easy to verify that the number of elements in the centralizer $\hat{C}_h$ in C_g, for a relevant h, is $kML = kN$. As a result, the number of elements in the conjugacy class of such h in C_g is $|C_g|/|\hat{C}_h| = (M-1)! L^{M-1} k^{M-1}$. The distinct conjugacy classes, that give non-zero traces, are labeled by the three integers s, t_g, t_h. Each of these classes appear with a prefactor, which is given by the number of elements in the class divided by the order of C_g, and is equal to $1/(kN)$.

We are now ready to compute the elliptic genus of the theory characterized by the Hilbert space (2.14) starting from the original ones for $\mathcal{H}$

$$\chi^{osc} \equiv \prod_{\phi} \chi \begin{bmatrix} \alpha_\phi \\ \beta_\phi \end{bmatrix} \tag{2.21}$$

where

$$\chi \begin{bmatrix} \alpha_\phi \\ \beta_\phi \end{bmatrix} = \prod_{n=1}^{\infty} (1 + e^{2\pi i \beta_\phi} q^{n-1/2-\alpha_\phi})^{\epsilon_\phi} \tag{2.22}$$

is the oscillator modes contributions for a generic, say right-moving, $\epsilon_\phi = -1(1)$ bosonic (fermionic) field ϕ with boundary condition data described by α_ϕ, β_ϕ. We will omit in the following the non trivial zero mode contributions which are included at the end of the computation.

We can now write the contribution to the elliptic genus of a given $\mathbf{g}, \mathbf{h}$ sector with eigenvalues given by (2.20) as

$$\chi_{L,M}^{s,t_g,t_h} \begin{bmatrix} \alpha_\phi \\ \beta_\phi \end{bmatrix} (q) = \prod_{r=0}^{M-1} \prod_{l=0}^{L-1} \prod_{n=0}^{\infty} (1 + e^{2\pi i \beta_\phi(l)} q^{n-1/2-\alpha_\phi(l)})^{\epsilon_\phi} \tag{2.23}$$

with

$$\begin{aligned} \alpha_\phi(l) &= \alpha_\phi + \frac{t t_g}{kL} + \frac{l}{L} \\ \beta_\phi(l) &= \beta_\phi + \frac{r}{M} + t\frac{t t_h}{kLM} + \frac{ls}{L}. \end{aligned}$$

Performing the products over r and l we are left with

$$\chi_{L,M}^{s,t_g,t_h} \begin{bmatrix} \alpha_\phi \\ \beta_\phi \end{bmatrix} (q) = \prod_{m=1}^{\infty} (1 + e^{2\pi i \beta} (q^{\frac{M}{L}} e^{2\pi i s \frac{M}{L}})^{n-1/2-\alpha})^{\epsilon_\phi} \tag{2.24}$$

in terms of the modified spin structure data

$$\begin{aligned} \alpha &= L(\alpha_\phi + \frac{1}{2}) + t\frac{t_g}{k} + \frac{1}{2} \\ \beta &= M(\beta_\phi + \frac{1}{2}) + s(\alpha_\phi + \frac{1}{2}) + t\frac{t_h + s t_g}{kL} + \frac{1}{2}. \end{aligned} \tag{2.25}$$

Finally the zero-mode contributions to the elliptic genus depend on the bosonic or fermionic nature of the state under consideration. For fermions one finds:

$$(2^{\mathcal{N}-1} - 2^{\mathcal{N}-1}) \sqrt{\prod_{j=1}^{M-1} (1 - e^{2\pi i j/M})^{2\mathcal{N}}} = (2^{\mathcal{N}-1} - 2^{\mathcal{N}-1}) M^{\mathcal{N}}. \tag{2.26}$$

For bosonic states there is a further distinction depending on the compactness of the bosonic coordinate. For d compact bosons one has

$$L^d \sum_{(p,\bar{p})\in\Gamma_{d,d}} (q^{\frac{M}{L}} e^{2\pi i \frac{s}{L}})^{p^2} (\bar{q}^{\frac{M}{L}} e^{2\pi i \frac{s}{L}})^{\bar{p}^2}. \tag{2.27}$$

For D non-compact bosons one gets

$$L^D \tau_2^{-D/2} \tag{2.28}$$

We can now collect the results for the two cases of interest:

Type I D-strings in S_1: The relevant worldsheet fields are given by $\mathcal{H} = X_I, S_a, \chi^A$ in (2.14). The Z_k operation corresponds to the Z_2 action $\chi^A \to -\chi^A$ ($t_\chi = 1, t_X = t_S = 0$) which leads to the four spin structures realizing the $SO(32)$ lattice. The original fields are all in the odd spin structure $\alpha_\phi = \beta_\phi = \frac{1}{2}$. The number of supersymmetries is $\mathcal{N} = 4$. Collecting then (2.24), (2.25) and (2.28) we are left with

$$\chi_N^I(q) = \frac{8-8}{\tau_2^4} \sum_{L,M} M^{-4} \frac{1}{2N} \sum_{s=0}^{L-1} \sum_{t_g,t_h=0,1} \frac{\vartheta\left[\begin{smallmatrix} t_g+\frac{1}{2} \\ \frac{t_h+st_g}{2L} \end{smallmatrix}\right] (q^{\frac{M}{L}} e^{2\pi i \frac{s}{L}})^8}{\eta^{24}(q^{\frac{M}{L}} e^{2\pi i \frac{s}{L}})} \tag{2.29}$$

where an overall factor $\frac{1}{N^8}$ have been included for later convenience.

Type IIB D-strings in $S_1/\Omega\sigma_V$: In this case the relevant worldsheet content is given by $\mathcal{H} = X_I, S_a, S_{\dot{a}}$, where the X_I's and S_a's are in the odd spin structure $\alpha_\phi = \beta_\phi = \frac{1}{2}$ while $S_{\dot{a}}$ represent an antiperiodic fermion $\alpha_\phi = 0, \beta_\phi = \frac{1}{2}$. We anticipate this result, which will be rigorously derived in section 2.3. There is no Z_k action in this case, we set then $t_g = t_h = 0$. The number of supersymmetries is $\mathcal{N} = 4$ as before. Collecting (2.24), (2.25) and (2.28) we are left now with

$$\chi_N^{\tilde{I}}(q) = \frac{8-8}{\tau_2^4} \sum_{L,M} M^{-4} \frac{1}{N} \sum_{s=0}^{L-1} \frac{\vartheta\left[\begin{smallmatrix} \frac{L}{2}+\frac{1}{2} \\ \frac{s}{2}+\frac{1}{2} \end{smallmatrix}\right] (q^{\frac{M}{L}} e^{2\pi i \frac{s}{L}})^4}{\eta^{12}(q^{\frac{M}{L}} e^{2\pi i \frac{s}{L}})} \tag{2.30}$$

where the same overall normalization $\frac{1}{N^8}$ have been included as before.

2.1.4 Longest string versus intermediate or short strings

The boundary conditions (2.18) represent a string starting in the first D-string, ending in the second one and so on, coming back to the first D-string after L steps. It can be thought then as M strings of length L. The longest string corresponds then to the twist element $[g] = (N)$, the shortest to the identity $[g] = (1)^N$ and intermediate strings to generic $[g] = (L)^M$ with $1 < M < N$. Even though the computation of the elliptic genus receives contributions from both the longest string sector and the intermediate or short string sectors, it is only the longest string sector that corresponds to a real threshold bound states of N D-strings. Consider for example the shortest string sector. It receives contributions only from the sector $\mathrm{tr}\, h(-1)^F$ where $[h] = (N)$. In this sector only the states having zero relative transverse momenta survive. In position space, therefore, the wave function of each of the N strings is constant along the relative separations. As a result such states are not normalizable. The same argument also applies to the intermediate strings. In this case there are M groups of strings of length L each and the wave function is constant as a function of the relative separations between these M groups. This state therefore represents a state of M strings, each of which is a threshold bound state of L strings. The analogue of a single particle state appears only in the longest string sector with $M = 1$ and $L = N$. This interpretation is also clear intuitively from the orbifold conformal field theory description, since the twisted states in this sector correspond to wave functions which are localized at the fixed point.

To see this more clearly, we can compactify one of the transverse directions, say X_8, on a circle of radius r and give the system a total momentum $1/r$ along this direction. Note that this is the minimal unit of quantized momentum. We will now show that only the longest string sector can carry this momentum.

The zero modes for X_8^i ($i = 1, \ldots, N$) for a general twisted sector of relevance labeled by (L, M) reads, after suitable ordering of indices, as:

$$X_8^i = a^i + \frac{kr}{M}\frac{2\pi t}{T} + \frac{\ell r}{L}\frac{\sigma}{R_I}, \tag{2.31}$$

where a^i satisfy

$$\begin{aligned} a^{i+L} &= a^i + \frac{2\pi k r}{M}, \\ a^{j+1} &= a^j + \frac{2\pi \ell r}{L}, \qquad j = 1, \ldots, L-1, \end{aligned} \tag{2.32}$$

and k and ℓ are arbitrary integers. Note that the integers k and ℓ are independent of i because only the center of mass X_8 is a zero mode under the combined actions of the twists along the t and σ directions. ℓ here denotes the winding number along X_8 direction. These zero modes contribute to the action as

$$\Delta S = \frac{\pi}{\alpha'_{\rm eff}}[\frac{N}{\tau_2}(\frac{kr}{M})^2 + N\tau_2(\frac{\ell r}{L})^2]. \tag{2.33}$$

The total momentum is $\frac{1}{2\pi i\alpha'_{\text{eff}}}\int d\sigma \sum_i \partial_t X_8^i = \frac{Nkr}{Mi\tau_2\alpha'_{\text{eff}}}$. We can now perform a Poisson resummation in order to go to the Hamiltonian formulation, with the result that the total momentum p along the X_8 direction is:

$$p = \frac{Mk}{r}, \tag{2.34}$$

with k some integer and the partition function is

$$\sum_{k,\ell} q^{\frac{p_L^2}{4\alpha'_{\text{eff}}N}} \bar{q}^{\frac{p_R^2}{4\alpha'_{\text{eff}}N}} \sqrt{\tau_2} Z(L, M), \tag{2.35}$$

where $p_L = M(\frac{\alpha'_{\text{eff}}k}{r} + \ell r)$ and $p_R = M(\frac{\alpha'_{\text{eff}}k}{r} - \ell r)$. This shows that the smallest unit of momentum, $p = 1/r$, gets contributions only from the $M = 1$ (i.e. the longest string). From the above partition function we see that this state carries an extra energy given by $\alpha'_{\text{eff}}p^2/2NR_I$ which, as we shall see below, is exactly what is expected from the dual side.

2.2 Type I/heterotic duality in D=9

We start by describing the electric spectrum of heterotic string states for compactifications to $D = 9$ on a circle of radius R_H with generic Wilson lines Y^a turned on. The spectrum of physical states is defined by the Mass formula and level matching condition

$$M_h^2 = \frac{1}{2}p_L^2 + \frac{2}{\alpha'}(N_L - 1) = \frac{1}{2}p_R^2 + \frac{2}{\alpha'}(N_R - c) \tag{2.36}$$

with $c = 0, 1/2$ the zero point energy for the NS and R sector respectively. The left and right moving momenta [43]

$$\begin{aligned} p_R &= \frac{k + YP + NY^2/2}{R_H} + \frac{NR_H}{\alpha'} \\ p_L &= \left(\frac{2}{\alpha'}(P + NY), \frac{k + YP + NY^2/2}{R_H} - \frac{NR_H}{\alpha'}\right) \end{aligned} \tag{2.37}$$

define a vector $p = (p_L, p_R)$ in the $\Gamma_{(1,17)}$ lattice of charges corresponding to the winding N, the Kaluza-Klein momentum

$$p_9 = \frac{1}{R_H}(k + Y.P + \frac{1}{2}Y^2 N) \tag{2.38}$$

and the $\Gamma_{(0,16)}$ vector P, associated to the $SO(32)$ charges.

For the spectrum of BPS states, we should in addition to set the right moving oscillator number to its ground state $N_R = c$. The physical condition (2.36) reduces then to

$$kN = \frac{1}{2}P^2 + N_L - 1 \tag{2.39}$$

The degeneracies $d(N_L)$ of these states can be found by expanding the left moving heterotic partition function in the absence of Wilson lines

$$Z^{het}(q) = \frac{1}{\eta(q)^{24}} \sum_{P \in \Gamma_{16}} q^{\frac{1}{2}P^2} = \sum_{N_L} d(N_L) q^{N_L - 1}, \tag{2.40}$$

since a continuous Wilson line cannot change these multiplicities. Furthermore, the mass m in the string frame is given by

$$M = \left| p_9 + \frac{NR_H}{\alpha'} \right|. \tag{2.41}$$

On the other hand, the Type I theory is related to the heterotic string theory through the relations (1.26). This imply that a heterotic state labeled by N, k and P is mapped to a type I state with Kaluza-Klein momentum p_9 and mass m, in the string frame, given by:

$$p_9 = \frac{1}{R_I}(k + Y.P + \frac{1}{2}Y^2 N) \tag{2.42}$$

$$M = \left| p_9 + \frac{NR_I}{\alpha' \lambda_I} \right|, \tag{2.43}$$

where R_I is the type I radius along the X^9 direction. Moreover the multiplicities should be given by the same $d(N_L)$'s as in (2.40).

As we have mentioned the information about charges, masses and multiplicities of bound states of N type I D-strings wrapped around the circle can be extracted from the elliptic genus of the $O(N)$ gauge theory (2.1). This index was computed in the previous section to be given by (2.29). The longest contribution ($L = N, M = 1$) can be written as

$$\chi_N^{typeI}(q) = \frac{(8-8)}{\tau_2^4} \frac{1}{N} \sum_{s=0}^{N-1} \frac{1}{\eta(e^{\frac{2\pi i s}{L}} q^{\frac{1}{N}})} \sum_{P \in \Gamma_{16}} e^{\frac{2\pi i s}{L}\frac{P^2}{2}} q^{\frac{1}{2N}(P+NY)^2}. \tag{2.44}$$

where the factor $(8-8)$ represents the 8 bosons and 8 fermions of the small $\mathcal{N} = 4$ vector multiplet. We can now compare this result with the predictions (2.40), (2.42) and (2.43) of type I/heterotic duality for the multiplicities, charges and masses of D-string bound states. The heterotic winding mode N is mapped to the D-string number. The Kaluza-Klein momentum p_9, on the other hand, is the longitudinal momentum P_σ of the D-string system along the σ direction. Given the fact that P_σ is the difference of the left and right Virasoro generators $L_0 - \bar{L}_0$, its charge is just given by the coefficient of $2\pi i R_I \tau_1$ in the partition function. From (2.44) and taking into account the projection implied by the sum over s, we conclude that

$$\begin{aligned} P_\sigma &= \frac{1}{R_I}(k + Y.P + \frac{1}{2}Y^2 N), \\ k &= \frac{1}{N}(\frac{1}{2}P^2 + N_R - 1) \in \mathbf{Z}, \end{aligned} \tag{2.45}$$

where $(N_R - 1)/N$ appears from the expansion of $\eta(q^{\frac{1}{N}})^{-24}$. Note that the multiplicity of these states is the same as the coefficient of q^{N_R-1} in the expansion of $\eta(q)^{-24}$. The value of k is bounded below by (-1) for $N = 1$ and by 0 for $N > 1$, while the value of P_σ is bounded by $-1/NR_I$. These two equations are exactly the ones appearing in (2.42) for the Kaluza-Klein momentum and the level matching condition (2.39) for the BPS states. It is also clear that the two multiplicities match, as both are given by the coefficient of q^{N_R-1} in the expansion of $\eta(q)^{-24}$.

Furthermore, the mass of the bound state is the original mass of N D-strings wrapped around the circle, plus the energy carried by the excitation, which is given by the coefficient of $T = R_I \tau_2$ in (2.44). Since the partition function depends only on q the latter is equal to P_σ. Thus the total energy is $\frac{NR_I}{\alpha' \lambda_I} + P_\sigma$. This is exactly the mass given in (2.43) predicted by the duality up to a sign. As mentioned earlier $P_\sigma \geq -1/NR_I$. Therefore, for $R_I^2 > \alpha' \lambda_I / N^2$ the quantity $\frac{NR_I}{\alpha' \lambda_I} + P_\sigma$ is positive definite and hence it coincides with the absolute value appearing in (2.43). However, for $R_I^2 < \alpha' \lambda_I / N^2$ this quantity is negative, for a suitable choice of the Wilson line Y, and the result would not make sense. But this is exactly the region in which the type I perturbation theory breaks down, as argued in [6].

Finally, we consider the situation discussed in the last section where a transverse direction is compactified on a circle of radius r_I and the system carries a momentum k/r_I. This does not alter the level matching condition and therefore the multiplicity of the state. Recalling that $\alpha'_{\text{eff}} = \alpha' \lambda_I$, we find that the extra energy is $k^2 \alpha' \lambda_I / 2NR_I r_I^2$. On the heterotic side the mass for a state with winding number N along the X_9 direction and carrying momentum k/r_H along the X_8 direction is given by

$$\frac{1}{\alpha'}\sqrt{N^2 R_H^2 + (\frac{k\alpha'}{r_H})^2}. \tag{2.46}$$

By using the duality relations (1.26) and expanding the square root to the leading order in λ_I we find that the extra mass is exactly $k^2 \alpha' \lambda_I / 2NR_I r_I^2$, in agreement with the prediction of duality.

2.3 Type IIB on $S_1/(-)^{F_L}\sigma_V$ vs. Type IIB on $T^d/\Omega\sigma_V$

We now consider a $(4,0)$ model, which is obtained by compactifying type IIB theory on the asymmetric orbifold $S_1/(-)^{F_L}\sigma_V$. F_L is the spacetime left-moving fermion number while σ_V is a shift of order two in the $\Gamma_{1,1}$ lattice of momenta. As a result of the orbifold projection, the left moving supercharges are projected out in the untwisted sector, while, due to the shift in the momentum lattice, no supercharge appears in the twisted sector. The orbifold projection removes all R-R states and therefore this model is effectively a type II vacuum without D-branes. The spectrum of BPS states is therefore completely perturbative much in the same way as in the previously consider heterotic case.

The BPS perturbative spectrum is defined as before by setting the right moving oscillators to their ground state $N_R = c_R$. We are left with the following contributions from the different orbifold sectors [44]:

$$Z_{+-} = \frac{(8_v - 8_s)}{\tau_2^4}\frac{\vartheta_2(q)^4}{\eta(q)^{12}} \sum_{P\in\Gamma_{1,1}} e^{2\pi i V\cdot P} q^{p^2/2}\bar{q}^{\bar{p}^2/2} \tag{2.47}$$

$$Z_{-+} = \frac{(8_v - 8_s)}{\tau_2^4}\frac{\vartheta_4(q)^4}{\eta(q)^{12}} \sum_{P\in\Gamma_{1,1}+V} q^{p^2/2}\bar{q}^{\bar{p}^2/2} \tag{2.48}$$

$$Z_{--} = -e^{\pi i V\bar{V}}\frac{(8_v - 8_s)}{\tau_2^4}\frac{\vartheta_3(q)^4}{\eta(q)^{12}} \sum_{P\in\Gamma_{1,1}+V} e^{2\pi i V\cdot P} q^{p^2/2}\bar{q}^{\bar{p}^2/2} \tag{2.49}$$

In the above expressions the first subscript $\pm$ refers to the twists in the σ direction whereas the second to the twist in the τ direction. The factor $8_v - 8_s$ comes from the right-moving fermionic zero modes and counts as before the 8 bosons and 8 fermions of the small $\mathcal{N}=4$ supermultiplet, V is the shift vector corresponding to σ_V. Finally $P = (p, \bar{p})$ are left- and right- moving momenta in the compact direction.

The only gauge fields in these models are those coming from the KK reduction of the ten dimensional metric G_{MN} and NS-NS antisymmetric tensor B^{NS}_{MN}, since, as discussed above, there are no gauge fields arising from the Ramond-Ramond sector. The corresponding charges belong to the (shifted) lattice appearing in eqs.(2.47) to (2.49). These are giving by the winding number N and KK momentum k/R corresponding to the 9-dimensional gauge fields $B^{NS}_{\mu 9}$ and $G_{\mu 9}$. We will take the "geometric" shift to be $V = (v, \bar{v}) = (1/2R, 1/2R)$. The level matching condition reduces then to

$$kN = N_L - c_L, \tag{2.50}$$

where k is integer (half-integer) in the untwisted (twisted) sector and c_L denotes, the left moving zero point energy due to the Z_2 twists. The multiplicity of these BPS states for a given k, N is defined by the coefficient of q^{N_L} in the expansion of eqs. (2.47-2.49), with N_L given by (2.50).

The conjectured dual pair for the type II orbifold under consideration is a type I theory *without open strings* [44], obtained by modding out type IIB theory with $\Omega\sigma_V$. The S-duality of ten-dimensional type IIB string theory, which exchanges the NS-NS and R-R antisymmetric tensors, maps, in particular winding modes of the fundamental string into D-string winding modes. We can conclude then that in the system of N "type I" D-strings, each one wrapped once around the circle, there should exist bound states carrying a given KK momentum k , with mass and degeneracy given by the above relations through the duality map (1.28). As before, in the context of type I - heterotic duality, these data about the bound states are encoded in the elliptic genus of the effective gauge theory describing the dynamics of N such nearby D-strings.

We come now to the determination of the effective gauge theory governing the D-string dynamics in the present situation. From eq (1.31) we see that S-duality maps the $(-)^{F_L}$ action to the world-sheet parity operation Ω. In $D = 10$ the two quotient theories are vastly different. On the one hand, projecting the type IIB theory by $(-)^{F_L}$ gives the type IIA theory since the twisted sector of the orbifold provides the extra (opposite-chirality) supercharges to restore (non-chiral) maximal supersymmetry. On the other hand, projecting the type IIB theory by Ω gives the type I theory as we have seen before.

Nevertheless accompanying the Z_2-projection by a shift in the compactification torus results in a dual pair in lower dimensions [45]. The relevant gauge theory describing the corresponding D-string system will be obtained by projecting the usual $U(N)$ gauge theory of type IIB D-strings [46] onto $\Omega\sigma_V$ invariant fields. The KK momentum of the fundamental string corresponds to the P_σ momentum in the D-string system, and therefore, σ_V, which is $+1$ or -1 depending on whether the KK momentum is even or odd, corresponds to periodicity (anti-periodicity) along the σ-direction on the D-string world-sheet. Recalling that in the action of Ω on the $U(N)$ Chan-Paton factors there is a relative sign between the transverse and longitudinal degrees of freedom, it is easy to obtain the field content resulting from the $\Omega\sigma_V$ projection. Let us denote by A_α, X_I, S_a and $S_{\dot{a}}$ the gauge field, the transverse scalars, the left- and right-moving fermions respectively, of the $U(N)$ type IIB D-string system. The indices $I, a, \dot{a}$ refer to the $\mathbf{8}_V$, $\mathbf{8}_s$ and $\mathbf{8}_c$ representations of the

$SO(8)_{\mathcal{R}}$ R-symmetry group as before. The result of the $\Omega\sigma_V$ projection is then:

$$\begin{aligned} &X_I^+, S_{\dot{a}}^+, S_a^-, A_\alpha^- \quad \text{for} \quad P_\sigma \quad \text{even} \\ &X_I^-, S_{\dot{a}}^-, S_a^+, A_\alpha^+ \quad \text{for} \quad P_\sigma \quad \text{odd} \end{aligned} \tag{2.51}$$

where + and − denote the symmetric and adjoint representations of $O(N)$ respectively.

Given these preliminary observations, one can, as before, compute the elliptic genus of this gauge theory in the infrared limit where the theory flows to a superconformal orbifold field theory. Indeed, the charged fields get massive and can be integrated out leaving a free field theory in terms of the diagonal fields X_I^t, $S_{\dot{a}}^t$ and S_a^t, $(t = 1 \cdots N)$ in (2.51). Finally, modding out by the remaining Weyl symmetry group, we are left with an orbifold theory with target space $(\mathbf{R}^8)^N/S_N$.

Let us analyze first the free $N = 1$ case. It is convenient to define new $\tilde{\sigma}_\alpha = 2\sigma_\alpha$ variables, in terms of which the field content (2.51) reduce to 8 $\tilde{\sigma}$-periodic bosons X_I and chiral fermions $S_{\dot{a}}$ and 8 $\tilde{\sigma}$-antiperiodic fermions S_a with all possible values of $P_{\tilde{\sigma}}$ momenta. The partition function (or elliptic genus, since we are in the odd spin structure for the S_a fields) is then given by

$$\frac{1}{\tau_2^4}(8_v - 8_s)\frac{\vartheta_4^4(\tilde{q})}{\eta^{12}(\tilde{q})} \sum_{P \in \Gamma_{d,d}} \tilde{q}^{p^2/2} \bar{\tilde{q}}^{\bar{p}^2/2} \tag{2.52}$$

where right-moving oscillators cancel out between the X_I and S_a supersymmetric fields, $(8_v - 8_s)$ represents, as before, the realization of the BPS supermultiplet and $\tilde{q} \equiv e^{\pi i \tau}$. The partition function (2.52) reproduces then the correct masses, charges and degeneracies coming from (2.48) (plus the $\tau \to \tau + 1$ amplitude (2.49) which implements the level matching condition (2.50) for $N = 1$), once we identify the radius of compactification of the dual theory $\tilde{R}$ with twice the radius R of the original one.

We now proceed to study the $N > 1$ case, which as was previously stated corresponds to N copies of the $N = 1$ field content modded out by the permutation group S_N. We can now use the results (2.30) for the elliptic genus of this symmetric space. The contribution of the relevant sector $(M = 1, L = N)$ is given by

$$\frac{1}{\tau_2^4}(8_v - 8_s)\frac{1}{N}\sum_{s=0}^{N-1} \frac{\vartheta\begin{bmatrix} \frac{N}{2}+\frac{1}{2} \\ \frac{s}{2}+\frac{1}{2} \end{bmatrix}^4 (\tilde{q}^{\frac{1}{N}}\omega^s)}{\eta^{12}(\tilde{q}^{\frac{1}{N}}\omega^s)} \sum_{P \in \Gamma_{d,d}} \omega^{s\frac{P^2}{2}} \tilde{q}^{p^2/2} \bar{\tilde{q}}^{\bar{p}^2/2} \tag{2.53}$$

with $\omega = e^{\frac{2\pi i}{N}}$. The sum over s projects on the modes

$$k = \frac{1}{N}\left(\frac{P^2}{2} + N_R\right) \in \mathbf{Z} \tag{2.54}$$

which is just the level matching condition (2.50). The multiplicities can be found upon expanding (2.53) in powers of $\tilde{q}^{\frac{1}{N}}$. In particular for N odd they come from the expansion of $\vartheta_4(\tilde{q}^{\frac{1}{N}})$ and $\vartheta_3(\tilde{q}^{\frac{1}{N}})$ reproducing the degeneracies arising from (2.48)+(2.49) once we apply the level matching condition (2.54). For N even, additional fermionic zero modes appears for the S_a field for even values of s, giving a vanishing contribution to the sum (2.30). In this case we are left with a sum over odd s of $\vartheta_2(\tilde{q}^{\frac{1}{N}}\omega^s)$, which reproduces the degeneracies, masses and charges coming from (2.47).

Chapter 3

Threshold corrections in type II string vacua

In the previous chapter we provided some evidences for the equivalences between various ten dimensional string theories by studying the BPS spectrum of physical states in lower dimensional compactifications. We proofed in particular the isomorphisms between the BPS spectra of states for the nine dimensional theories arising as compactifications of type I/heterotic string theories on a circle and for type IIB strings on $S_1/(-)^{F_L}\sigma_V$-$S_1/\Omega\sigma_V$. We would like now to apply these results in the study of the low energy effective actions describing these string vacua. In particular we study the moduli dependence for the special ("BPS saturated") $\mathcal{F}^4$ terms in the $D = 8$ dimensional low energy effective action for the conjecture pair of type II strings. We follow closely a sequence of works [47], where a similar analysis for the threshold corrections in the context of type I/heterotic duality has been performed.

The interest in the study of these terms relies on the fact that they are believed to receive only one-loop corrections for toroidal heterotic compactifications to $D > 4$ dimensions. Supersymmetry protects this term from higher loops perturbative corrections while the only identifiable source of non-perturbative corrections (the fivebrane instanton) is always infinitely heavy for compactifications to $D > 4$ dimensions. This seems to be also the case for the present type II model. The $(-)^{F_L}\sigma_V$ action removes the RR-fields leaving an effective type II theory without D-branes [52]. The only non-perturbative source we can think of is again the fivebrane instanton which cannot enter in the correction of a $D > 4$ dimensional effective action. On the "type I" side (type I without open strings or type IIB on $T^2/\Omega\sigma_V$ [44]) however, D-string instantons are expected to correct the effective actions for $D \leq 8$ dimensions. Indeed, using the conformal description of the infrared limit for the N D-instantons [1] system we will be able to compute these non-perturbative corrections, showing the agreement with the exact formula found in the dual computation.

3.1 Threshold corrections in type IIB on $T^2/(-)^{F_L}\sigma_V$

For simplicity we will restrict our attention to the $\mathcal{F}^4$ couplings in the low energy effective action. We will assume that, as it is the case for similar terms in toroidal heterotic compactifications to $D > 4$ dimensions, the moduli dependence for these terms in type IIB on $T^2/(-)^{F_L}\sigma_V$ receive only one-loop corrections. Let us recall how the arguments leading to this conclusion for the $SO(32)$ heterotic $\mathcal{F}^4$ gauge fields, work. The kind of term we will study are really closer to the $O(2,2)$ gauge fields $G_{\mu i}, B_{\mu i}$ arising from the KK reduction of the heterotic metric and antisymmetric tensor, but all these gauge fields are treated symmetrically by the T-duality group $O(2,18,\mathbb{Z}$ and therefore is reasonably to believe that the same arguments work in both cases. For $SO(32)$ gauge fields, the $\mathcal{F}^4$ terms (as it is also the case for the $\mathcal{R}^4$ and $\mathcal{F}^2\mathcal{R}^2$ terms) can be obtained by dimensional reduction of ten dimensional superinvariants, whose bosonic part reads [48]

$$\begin{aligned} I_1 &= t_8 tr\mathcal{F}^4 - \frac{1}{4}\epsilon_{10} B tr\mathcal{F}^4 \\ I_2 &= t_8 (tr\mathcal{F}^2)^2 - \frac{1}{4}\epsilon_{10} B (tr\mathcal{F}^2)^2 \end{aligned} \quad (3.1)$$

They are special because they contain CP-odd pieces related to the cancellation of gravitational and gauge anomalies in ten dimensions. Indeed in ten dimensions, the coefficients of these couplings are completely determined by supersymmetry and the anomaly cancelling mechanism. This is no longer true for lower dimensional compactifications, where supersymmetry relates, but not completely fixes, their dependence on the compactification moduli. The moduli dependence for the CP-odd pieces in (3.1) was studied in [49] for heterotic compactifications . As it was shown in that reference, they receive only one-loop perturbative corrections. Moreover non-perturbative corrections for compactifications to $D \geq 4$ dimensions are ruled out as we argued before. It is then plausible to assume that no higher corrections are present for their $\mathcal{F}^4$ superpartners.

[1] D-instantons in this context refer to instanton from the point of view of the eigth dimensional effective action. The D-recall the origin of these contributions from D-strings wrapped on the T^2 torus

A similar analysis for the type II model under study here, has not been done, but it looks more like the same. We will assume that this is the case, i.e. we assume that the one-loop formula we will obtain in this section for the moduli dependence of $\mathcal{F}^4$ terms in type IIB on $T^2/(-)^{F_L}\sigma_V$ is exact. The non-perturbative results for similar terms in the dual low energy effective action will support this assumption.

We will consider a compactification in a target space torus characterized by the complex moduli

$$\begin{aligned} T &= T_1 + iT_2 = \frac{1}{\alpha'}(B_{89}^N + i\sqrt{G}) \\ U &= U_1 + iU_2 = (G_{89} + i\sqrt{G})/G_{88} \ , \end{aligned} \tag{3.2}$$

where G_{ij} and B_{ij}^N are the σ-model metric and NSNS antisymmetric tensor.

We recall that projecting type IIB with $(-)^{F_L}\sigma_V$, we remove all RR massless fields. The only eight dimensional gauge bosons are then given by the $G_{\mu 8}, G_{\mu 9}$ and $B_{\mu 8}^N, B_{\mu 9}^N$ components of the metric and NSNS antisymmetric tensor. The corresponding vertex operators in the Green-Schwarz formalism can be written as

$$V_i^R = \int d^2z (G_{\mu i} - B_{\mu i})(\partial X^i - \frac{1}{4}p_\nu S\gamma^{i\nu}S)(\bar\partial X^\mu - \frac{1}{4}p_\rho \tilde S\gamma^{\mu\rho}\tilde S)e^{ipX} \tag{3.3}$$

$$V_i^L = \int d^2z (G_{\mu i} + B_{\mu i})(\partial X^\mu - \frac{1}{4}p_\nu S\gamma^{\mu\nu}S)(\bar\partial X^i - \frac{1}{4}p_\rho \tilde S\gamma^{i\rho}\tilde S)e^{ipX} \tag{3.4}$$

with $i = 8, 9$. The V_i^R are the vertices for the graviphotons, i.e. the gauge fields sitting in the supergravity multiplet. It is easy to see that they cannot contribute to one-loop corrections to an $\mathcal{F}^4$ term. Indeed, soaking the eight right moving zero modes S_0 in the torus with the fermionic piece $p_\nu S\gamma^{i\nu}S$ in (3.3) we bring already the correct power of momenta. At this order in the momenta, only the bosonic piece in the left moving part of the vertices can enter, but these leads always to total derivatives $\langle \bar\partial X^\mu(z_1)\bar\partial X^\mu(z_2)\rangle$ which cancel out after the z-integrations. Corrections to the $\mathcal{F}^4$ terms for the $G_{\mu i}$ and $B_{\mu i}$ gauge fields coincide then and are given by the correlations

$$\mathcal{A}_a = \langle (V_8^L)^a (V_9^L)^{4-a}\rangle = t_8 F_8^a F_9^{4-a} \langle \prod_{j=1}^{a} \int dz_j \bar\partial X^8(z_j) \prod_{k=a+1}^{4} \int dz_k \bar\partial X^9(z_k)\rangle \tag{3.5}$$

where t_8 is the tensor (1.41) arising from the right moving zero mode fermionic trace. It is convenient to define a generating function for all of these terms such that

$$\mathcal{A}_n = t_8 F_8^n F_9^{4-n} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} (\frac{U_2\tau_2}{\pi T_2})^4 \frac{d^n}{d\lambda_8^n}\frac{d^{4-n}}{d\lambda_9^{4-n}} Z(\lambda_i, \tau) \tag{3.6}$$

with F the fundamental domain for the T^2 torus, and $Z(\lambda_i,\tau)$ the partition function in terms of a perturbed Polyakov action with bosonic part

$$S_{\lambda_i} = \frac{2\pi}{\alpha'}\int d^2\sigma(\sqrt{g}G_{\mu\nu}g^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu + iB_{\mu\nu}\epsilon^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu + \sqrt{g}\frac{\alpha' T_2}{2U_2\tau_2}\lambda_i\bar{\partial}X^i). \quad (3.7)$$

and $\bar{\partial} = \frac{1}{\tau_2}(\partial_{\sigma_2} - \bar{\tau}\partial_{\sigma_1})$. This partition function involves a sum over all possible worldsheet instantons

$$\begin{pmatrix} X^8 \\ X^9 \end{pmatrix} = M\begin{pmatrix} \sigma^1 \\ \sigma^2 \end{pmatrix} \equiv \begin{pmatrix} m_1 & n_1 \\ m_2 & n_2 \end{pmatrix}\begin{pmatrix} \sigma^1 \\ \sigma^2 \end{pmatrix} \quad (3.8)$$

with worldsheet and target space coordinates σ_1, σ_2 and X^8, X^9 respectively taking values in the interval (0,1]. The entries m_1, n_1 are integer or half integers depending on the specific orbifold sector, while m_2, n_2 are always integers. We denote the three relevant sectors: n_1 half-integers, m_1 half-integers and both half integers by $\epsilon = +-, -+, --$ respectively. Clearly the untwisted sector $\epsilon = ++$ will not contribute since it has too many zero modes to be soaked by the four vertex insertions at this order in the momenta.

In the following we use the normalizations $\langle X^M X^N\rangle \sim G^{MN}$, with G^{MN} the ten dimensional metric defined by

$$G_{ij} = \frac{\alpha' T_2}{U_2}\begin{pmatrix} 1 & U_1 \\ U_1 & |U|^2 \end{pmatrix} \quad (3.9)$$

in the compact space and the flat metric $G_{\mu\nu} = \eta_{\mu\nu}$ in $\mathbb{R}^8$. For the worldsheet metric we choose

$$g^{\alpha\beta} = \frac{1}{\tau_2^2}\begin{pmatrix} |\tau|^2 & -\tau_1 \\ -\tau_1 & 1 \end{pmatrix}. \quad (3.10)$$

The generating function can then be written as

$$\int_F \frac{d^2\tau}{\tau_2^2} Z(\lambda_i,\tau) = \frac{V^8}{2^{10}\pi^4}\int_F \frac{d^2\tau}{\tau_2^2}\sum_\epsilon \Gamma^\epsilon_{2,2}(\lambda_i)\mathcal{A}_\epsilon \quad (3.11)$$

with

$$\Gamma^\epsilon_{2,2}(\lambda_i) = \frac{T_2}{\tau_2}\sum_{M_\epsilon} e^{2\pi i T \det M} e^{-\frac{\pi T_2}{\tau_2 U_2}\left|(1\ U)M\binom{\tau}{-1}\right|^2} e^{-\frac{\pi T_2}{\tau_2 U_2}(\lambda_8\ \lambda_9)M\binom{\tau}{-1}} \quad (3.12)$$

and $\mathcal{A}_\epsilon$ the anti-holomorphic BPS partition functions (2.47-2.49)

$$\mathcal{A}_{+-} \equiv \frac{1}{2}\frac{\vartheta_2(\bar{q})^4}{\eta(\bar{q})^{12}} \quad \mathcal{A}_{-+} \equiv \frac{1}{2}\frac{\vartheta_4(\bar{q})^4}{\eta(\bar{q})^{12}} \quad \mathcal{A}_{--} \equiv \frac{1}{2}\frac{\vartheta_3(\bar{q})^4}{\eta(\bar{q})^{12}} \quad (3.13)$$

Following Dixon, Kaplunovsky and Louis [50] we can express the sum over the M_ϵ matrices in (3.12) as a sum over $SL(2,Z)$ representative integrated in an unfolded domain. Notice that the complete generating function $Z(\lambda,\tau)$ has no definite modular transformation properties. This is not the case for the interesting term, the fourth λ-derivative of it appearing in (3.5) which is, indeed, modular invariant. In the following we keep in mind that we will consider at the end only this term, using (without point it out) modular manipulations which have sense only for the final result. In this sense the generating function (3.11) is invariant under the combined $SL(2,Z)$ actions

$$\tau \to \frac{a\tau+b}{c\tau+d}\,, \quad \text{and} \quad M_\epsilon \to M_\epsilon \begin{pmatrix} d & b \\ c & a \end{pmatrix} \tag{3.14}$$

Different ϵ-elements in the sum (3.12) get mixed in general by these transformations, but the sum is clearly invariant. An orbit is defined by the set of matrices M, which can be related by some $SL(2,Z)$ element V_i to a given representive M_0 through $M = M_0 V_i$. By a change of variables in the τ integration we can reduce the sum over matrices M in a given orbit to a single integration over an unfolded domain obtained as the union of V_i images of the fundamental domain through the modular transformations (3.14). These unfolded domain can be either the strip or the whole upper half plane depending on whether the matrix M is *degenerate* (det $M = 0$) or *non-degenerate* (det $M \neq 0$).

Let us consider first the degenerate case. Using the modular transformation properties of (3.12) and (3.13) we can write the contributions from the different orbifold sectors in (3.11) as

$$\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}(\Gamma^{+-}_{2,2}\mathcal{A}_{+-}(\tau) + \Gamma^{+-}_{2,2}\mathcal{A}_{+-}(\tau') + \Gamma^{+-}_{2,2}\mathcal{A}_{+-}(\tau'')) = \int_{\mathcal{F}_2} \frac{d^2\tau}{\tau_2^2}\Gamma^{+-}_{2,2}\mathcal{A}_{+-}(\tau) \tag{3.15}$$

with $\tau' = -\frac{1}{\tau}$ and $\tau'' = \tau' + 1$. The new fundamental domain F_2 is the quotient of the upper half plane by a Γ_2 subgroup of $SL(2,Z)$ transformations defined as the set of elements V which keep the form of an M_{+-} matrix ($m_1 \in \mathbb{Z}, n_1 \in \mathbb{Z}+\frac{1}{2}$). A generic element of Γ_2 can be written as

$$V = \begin{pmatrix} a & b \\ 2c & d \end{pmatrix}. \tag{3.16}$$

with $ad - 2bc = 1$. It is easy to see that acting with a V in this Γ_2 subgroup we can always bring a matrix M_{+-} with zero determinant to the form

$$M = \begin{pmatrix} 0 & j_1 - \frac{1}{2} \\ 0 & j_2 \end{pmatrix}. \tag{3.17}$$

The sum over M_{+-} matrices in (3.15) can be written then as a sum over the representatives (3.17) labeled by (j_1, j_2). The τ integration runs now over an unfolded domain defined by the union of all images of the F_2 fundamental domains under the Γ_2 actions (3.16). We should notice however that not all Γ_2 matrices define different M's. Indeed, a representative (3.17) is invariant under the action

$$V = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \quad \text{i.e. } \tau \to \tau + b. \tag{3.18}$$

The unfolded domain is then the upper half plane modded out by these transformations , i.e. the strip $|\tau_1| \leq \frac{1}{2}, \tau_2 > 0$. Substituing (3.17) in (3.15) we are left with

$$\int_{\text{strip}} \frac{d^2\tau}{\tau_2^2} \sum_{(j_1,j_2)\neq(0,0)} e^{-\frac{\pi T_2}{\tau_2 U_2}|j_1-\frac{1}{2}+j_2U|^2} e^{-\frac{\pi T_2(\lambda_1(j_1-\frac{1}{2})+\lambda_2 n_2)}{\tau_2 U_2}} \mathcal{A}_{+-}(\bar{\tau}). \tag{3.19}$$

The τ_1 integration picks up the $\bar{q}^0$ power in the expansion

$$\mathcal{A}_{+-}(\bar{\tau}) = \sum_{n=0}^{\infty} \mathcal{A}_{+-}^n \bar{q}^n. \tag{3.20}$$

which is just 2^3. After taking the λ_i derivatives and integrate in τ_2 we are left with the final result

$$\begin{aligned} &\langle (F_8)^a (F_9)^{4-a} \rangle_{deg} = \\ &\frac{V_8}{2^7\pi^6} t_8 F_8^a F_9^{4-a} T_2 \int_0^\infty \frac{d\tau_2}{\tau_2^6} \sum_{(j_1,j_2)\neq(0,0)} (j_1 - \frac{1}{2})^a j_2^{4-a} e^{-\frac{\pi T_2}{\tau_2 U_2}|j_1-\frac{1}{2}+j_2U|^2} \end{aligned} \tag{3.21}$$

Let us consider now the contributions from the non-degenerate orbits. In this case acting with an $SL(2,Z)$ transformation we can, at most, bring a matrix M to the form

$$M = \begin{pmatrix} m_1 & n_1 \\ 0 & n_2 \end{pmatrix} \tag{3.22}$$

where m_1, n_1 are in $\mathbb{Z}$ or $\mathbb{Z}+\frac{1}{2}$ as before depending on the orbifold sector. The $\tau \to \tau+b$ action (3.18) on this matrix shifts $n_1 \to n_1 + b\, m_1$. We can therefore bring always n_1 to a fundamental region $n_1 = 0(\frac{1}{2}), 1(\frac{3}{2}), \cdots, 0(\frac{1}{2}) + m_1 - 1$. The representatives for non-degenerate matrices M are then given by the matrices (3.22) with

$$\begin{aligned} \epsilon &= +- \quad m_1 \in \mathbb{Z};\ n_1 = \frac{1}{2}, \frac{3}{2}, \cdots, \frac{2m_1 - 1}{2} \\ \epsilon &= -+ \quad m_1 \in \mathbb{Z} + \frac{1}{2}; n_1 = 0, 1, \cdots m_1 - 1 \\ \epsilon &= +- \quad m_1 \in \mathbb{Z} + \frac{1}{2}; n_1 = \frac{1}{2}, \frac{3}{2}, \cdots, \frac{2m_1 - 1}{2} \end{aligned} \tag{3.23}$$

The unfolded domain is now the whole upper half complex plane since all $SL(2,Z)$ actions are allowed, i.e. distinct elements in a non-degenerate orbit are in one-to-one correspondence with the copies of the fundamental domain of τ in the upper half plane. We should notice however that different replicas of the fundamental domain in the upper half plane come from different orbifold sectors since $SL(2,Z)$ transformations which allow us to bring a given matrix M to a representative in (3.23) mixed the M_ϵ with different ϵ's Therefore only the unfolding of the sum over ϵ's in (3.11) have sense.

We can now perform the τ integrations. Expanding the antiholomorphic modular functions (3.13) as in (3.20) we are left with the integral

$$I_n = T_2 e^{2\pi i T m_1 n_2} \int_{C^+} \frac{d^2\tau}{\tau_2^2} e^{-\frac{\pi T_2}{\tau_2 U_2}|m_1\tau - n_1 - n_2 U|^2} e^{-\frac{\pi T_2}{\tau_2 U_2}[(m_1\tau - n_1)\lambda_8 - n_2\lambda_9]} e^{-2i\pi\bar{\tau} n} \qquad (3.24)$$

which after the τ integrations can be written as

$$\begin{aligned} I_n &= \frac{(U_2T_2)^{\frac{1}{2}}}{m_1} e^{-2\pi i T_1 m_1 n_2} e^{2\pi i n(\frac{n_1+U_1 n_2}{m_1})} e^{\pi i \lambda_1(\frac{n}{m_1} - \frac{T_2 m_1}{U_2})} \int_0^\infty \frac{d\tau_2}{\tau_2^{3/2}} e^{-\frac{\beta}{\tau_2} - \gamma\tau_2} \\ &= \frac{(U_2T_2)^{\frac{1}{2}}}{m_1} e^{-2\pi i T_1 m_1 n_2} e^{2\pi i n(\frac{n_1+U_1 n_2}{m_1})} e^{\pi i \lambda_1(\frac{n}{m_1} - \frac{T_2 m_1}{U_2})} \sqrt{\frac{\pi}{\beta}} e^{-2\sqrt{\beta\gamma}} \end{aligned} \qquad (3.25)$$

with

$$\begin{aligned} \beta &= \frac{\pi T_2}{U_2}(n_2 U_2^2 + \lambda_2 n_2 - \lambda_1 U_1 n_2 - \frac{\lambda_1^2}{4}) \\ \gamma &= \frac{\pi T_2}{U_2}(m_1 + \frac{n U_2}{T_2 m_1})^2. \end{aligned} \qquad (3.26)$$

In order to extract the $\mathcal{F}^4$ term we should still act with four λ-derivatives acting on (3.25) and set finally the sources λ's to zero. In the following we will restrict ourself to the leading behaviour in a $\frac{1}{T_2}$ expansion of this result. The interest in this particular expansion will become clear later. In this limit the four derivatives should hit one of the λ-linear terms in the exponential of (3.25) leaving the final expression

$$\begin{aligned} &\langle (F_8)^a (F_9)^{4-a} \rangle_{nondeg} = \\ &= \frac{V_8}{2^{10}\pi^6} t_8 F_8^a F_9^{4-a} \sum_\epsilon \sum_{m_1,n_1,n_2} \mathcal{A}_\epsilon^n e^{-2\pi i n(\frac{n_1+\bar{U}n_2}{m_1})} e^{-2\pi i m_1 n_2 T} \frac{d^a}{d\lambda_8^a} \frac{d^{4-a}}{d\lambda_9^{4-a}} e^{\frac{\pi T_2 m_1}{U_2^2}(\lambda_1 U - \lambda_2)} \\ &= \frac{V_8}{2^{10}\pi^6} t_8 F_8^a F_9^{4-a} \sum_\epsilon \sum_{m_1,n_1,n_2} \frac{m_1^4}{m_1|n_2|} \frac{U^a}{U_2^4} e^{-2\pi i T m_1 |n_2|} \mathcal{A}(\frac{n_1 + \bar{U} n_2}{m_1}) \end{aligned} \qquad (3.27)$$

where we have used the expansions (3.20) to recontruct the modular forms (3.13) evaluated now in an induced modulus $\frac{n_1+\bar{U}n_2}{m_1}$.

3.2 Threshold corrections in type I without open strings

We move on now to the study of $\mathcal{F}^4$ threshold corrections in the conjecture dual pair (type IIB on $T^2/\Omega\sigma_V$) of the previously studied type II orbifold model. If the one-loop formulas obtained for the moduli dependence of $\mathcal{F}^4$ terms in type IIB on $T^2/(-)^{F_L}\sigma_V$ are exact, as argued before, they should contain both the perturbative and non-perturbative corrections in the dual side. Aim of this section is to proof that this is the case. The results are in complete agreement with the predictions of the type IIB self-duality conjecture.

Let us begin by translating the exact results (3.21) and (3.27) in terms of the "type I" variables. The duality relations (1.28) implies the scaling of the lengths (in the σ model variables) as

$$L_F = \frac{L_{I'}}{\sqrt{\lambda_{I'}}} \tag{3.28}$$

where the subscripts "F, I"' denote the lengths in the orbifold $(T^2/(-)^{F_L}\sigma_V)$ and orientifold $(T^2/\Omega\sigma_V)$ compactifications of type IIB theory respectively. This implies in particular that the volume T_2 scale as $T_2^F = T_2^{I'}/\lambda_{I'}$ and therefore the expansion in $1/T_2$ of the exact result found in the previous section can be identified with the genus expansion in the "type I" side. Taking into account also the scaling of the gauge field $A_\mu^F = G_{\mu i}^F = G_{\mu i}^I/\lambda_I = A_\mu^I/\lambda$, we find that the relevant $\mathcal{F}^4$ terms in the eight dimensional effective action scales like

$$f(T_2^F, \lambda_F) \int d^8x \sqrt{G_F}\, F_F^4 = \frac{1}{\lambda_I^4} f(\frac{T_2^I}{\lambda_I}, \frac{1}{\lambda_I}) \int d^8x \sqrt{G_I}\, F_{I'}^4. \tag{3.29}$$

In the previous section we argue that the only non-trivial moduli dependence for these terms in the orbifold side are given by the one-loop (λ_F^0 order in the expansion of $f(T_2^F, \lambda_F)$) expressions (3.21) and (3.27). By plugging these results in (3.29) we can see that the contributions from degenerate orbits ($f(T_2^F) \sim (T_2^F)^{-4}$) correspond to one-loop effects (order λ_I^0) in the type I side; while the ones from non-degenerate matrices ($f(T_2^F) \sim e^{-2\pi T_2^F m_1 n_2}(1 + O(1/T_2^F))$ should arise as D-instanton corrections with instanton number $N = m_1 n_2$. We will now show how these corrections can be reproduced by a direct computation in the "type I" theory.

3.2.1 One loop threshold corrections

The one-loop effective actions for a "type I" theory without open strings involve a sum over the torus and Klein bottle contributions. We are interested in the corrections to $\mathcal{F}^4$ terms. There is as before four eight dimensional gauge fields $G_{\mu i}$ and $B_{\mu i}^{RR}$, but only the former ones couple to elementary string states (the Kaluza-Klein modes) in the spectrum. The relevant vertex operators are defined as before by

$$V_i = G_{\mu i}(\partial X^\mu - \frac{1}{4}p_\nu S\gamma^{\mu\nu}S)(\bar{\partial}X^i - \frac{1}{4}p_\rho\tilde{S}\gamma^{i\rho}\tilde{S})e^{ipX}, \tag{3.30}$$

since $\Omega\sigma_V$ is simply the worldsheet parity Ω when acting on a massless state. The sixteen fermionic zero modes in the torus, if soaked, would bring eigth power of momenta, and therefore the only relevant contributions comes from the Klein bottle. The Klein bottle partition function is defined by [51]

$$\begin{aligned}\mathcal{K} &= -\frac{1}{2}\int_0^\infty \frac{dt}{t}\text{Tr }\Omega e^{\pi i k_8}\, e^{-\pi t(k_N k_M G^{MN}+M^2)} \\ &= \frac{V_8}{2^{10}\pi^6}(8-8)\int_0^\infty \frac{dt}{t^6}\sum_{n_i} e^{-\frac{\pi}{t}n_i n_j G^{ij}}\end{aligned} \tag{3.31}$$

where the factor $(8-8)$ comes as before from the zero mode fermionic trace and the $\frac{1}{t^4}$ from the momentum integration in the uncompact directions. The second expression in (3.31) is given only by a sum over the classical configurations (Kaluza-Klein) since quantum bosonic and fermionic contributions cancel out by supersymmetry. The $e^{\pi i k_8}$ defines the action σ_V on a given state of Kaluza-Klein momentum k_8 running in the loop. We have performed a poisson ressumation on the integers k_8, k_9 expresing this projection as a $\frac{1}{2}$-shift in the Lagrangian mode n_8. The Kaluza-Klein momenta of states running in the loop is then labeled by the half-integer n_8 and the integer n_9. The metric G^{ij} is given by (3.9). Due to this shift in the momentum no massless closed string state flow in the transverse channel. This implies in particular that the O9 orientifold plane do not carry a RR charge and therefore there is no room for the introduction of D9 branes and as a result open string string sectors do not appear [52].

The insertion of four vertex operatos (3.30) in (3.31) will soak the eight left-right symmetric fermionic zero modes reproducing the right momentum structure $t_8 F_8^a F_9^{4-a}$. For the remaining part of the vertices only the zero mode bosonic pieces are relevant, leading to four Kaluza-Klein insertions in (3.31). Putting altogether we are left with the final expression

$$\langle (F_8)^a (F_9)^{4-a}\rangle_{one-loop} =$$

$$= \frac{V_8}{2^7\pi^6} t_8 F_8^a F_9^{4-a} T_2 \int_0^\infty \frac{dt}{t^6} \sum_{(j_1,j_2)\neq(0,0)} (j_1 - \frac{1}{2})^a j_2^{4-a} e^{-\frac{\pi T_2}{2U_2 t}|j_1-\frac{1}{2}+j_2 U|^2} \quad (3.32)$$

which reproduce precisely the contribution (3.21) from the degenerate orbits in the dual type IIB model on $T^2/(-)^{F_L}\sigma_V$.

3.2.2 D-Instanton contributions

Let us consider now non-perturbative corrections in the "type I" side. In eight dimensions the only identifiable source for these corrections in the present model are the contributions from D-instantons, arising from the wrapping of the D-string world-sheet on the target torus. Since the insertions of four gauge vertices can soak at most eight fermionic zero modes only the $\frac{1}{2}$-BPS D-instantons can contribute. We can use now the results of the previous chapter. There we study the partition function for the N-wrapped D-string excitations, by going to the infrared limit where the theory is described by an orbifold conformal theory. Indeed we can read directly the BPS N D-instanton partition function from (2.30) once the one-loop τ- parameter is identified with the complex structure of the target torus U. This summarizes the quantum contributions to the partition function in the D-instanton background. On top of this we should include the classical contribution arising from the N D-instanton action for this model. The bosonic part of this action coincides with the one for the N type IIB D-strings and can be written as

$$S_{\text{D-inst}} = \frac{2\pi}{\alpha'} \sum_{I=1}^{N} \int d^2\sigma (\sqrt{g} g^{\alpha\beta} \frac{1}{\lambda} G_{\mu\nu} + i B_{\mu\nu}\epsilon^{\alpha\beta}) D_\alpha X_I^\mu D_\beta X_I^\nu \quad (3.33)$$

where D_α represent the supersymmetric covariant derivatives, which in the complex basis can be written as

$$\begin{aligned} DX_I^\mu &= \partial X_I^\mu - \frac{1}{4} p_\nu S_I \gamma^{\mu\nu} S_I \\ \bar{D}X_I^\mu &= \bar{\partial} X_I^\mu - \frac{1}{4} p_\nu \tilde{S}_I \gamma^{\mu\nu} \tilde{S}_I \end{aligned}$$

and I label the N Cartan directions of the unbroken $U(1)^N$ gauge group. The X's are always in the static gauge $X_I^8 = \sigma_I^1, X_I^9 = \sigma_I^2$, where the τ parameter is identified with the target complex structure U. Plugging this in (3.33) for a trivial background in the remaining fields besides the G_{ij} and B_{ij} components, we are left simply with $S_{D-inst} = 2\pi N T$ with T the "dual" complex parameter

$$T_{I'} = T_1 + iT_2 = \frac{1}{\alpha'}(B_{89}^{RR} + i\frac{\sqrt{G}}{\lambda}) \quad (3.34)$$

In order to study $\mathcal{F}^4$ couplings with $\mathcal{F}$ one of the $G_{\mu i}$ components of the metric, we can turn on a background λ_i for this field and extract the coupling from the fourth derivative in the appropiate λ's. Notice that only the classical part of the D-instanton partition function will be modified by these insertions since quantum correlators are always given by total derivatives which drop out after the z-integrations. We can identify in (3.33) the relevant coupling as

$$S = 2\pi TN + \frac{N\pi}{\alpha' U_2}[(G_{\mu 9} + UG_{\mu 8})D_z X^\mu + (G_{\mu 9} - \bar{U}G_{\mu 8})D_{\bar{z}} X^\mu] + \cdots \qquad (3.35)$$

where $z = \sigma_1 + U\sigma_2$ is the complex worldsheet coordinate. Each $G_{\mu i}$ insertion should soak two right moving fermionic zero modes. These fermionic modes enter only in D_z. Therefore the fourth derivatives in λ hit always this term in (3.35) bringing a power of NU/U_2 for each $G_{\mu 8}$ insertion and a power of N/U_2 for the insertions of $G_{\mu 9}$. Collecting the different pieces:

- The classical contribution: $e^{2\pi i TN}$
- The quantum contributions from (2.30) omiting the (8-8) factor and replacing τ by U
- The fermionic zero mode trace: $t_8 F_8^n F_9^{4-n}$
- An NU/U_2 factor for each $G_{\mu 8}$ insertion and an N/U_2 for each $G_{\mu 9}$

we are left with the final result for the D-instanton contributions

$$\langle (F_8)^a (F_9)^{4-a} \rangle_{D-inst} =$$
$$= \frac{V_8}{2^{10}\pi^6} t_8 F_8^a F_9^{4-a} \frac{U^a}{U^4} \sum_{N,L,M} L^4 e^{2\pi i TN} \frac{1}{N} \sum_{s=0}^{L-1} \frac{\vartheta\left[\begin{smallmatrix} \frac{L}{2}+\frac{1}{2} \\ \frac{s}{2}+\frac{1}{2} \end{smallmatrix}\right](q^{\frac{M}{L}} e^{2\pi i \frac{s}{L}})^4}{\eta^{12}(q^{\frac{M}{L}} e^{2\pi i \frac{s}{L}})} \qquad (3.36)$$

which reproduces precisely the contributions from non-degenerate orbits (3.27) after trivial identifications.

Chapter 4

D-branes dynamics

4.1 Supersymmetry and higher spin BPS states

As we have seen, Dp-branes correspond to solitonic BPS saturated solutions of Type IIA(B) supergravity, which preserve one half of the supersymmetries. The remaining half is realized on a short-multiplet containing 256 p-brane configurations lying in the **44**+**84**+**128** representations of the little group $SO(9)$ for massive states. The various components of the short-multiplet are related by supersymmetry transformations generated by the 16 broken supercharges.

Different components of the supermultiplet spanned by these sources, are obtained by applying supersymmetry transformations to the scalar boundary state $|B\rangle$ through the operator

$$U = e^{\epsilon Q^-} = \sum_{m=0}^{16} \frac{1}{m!}(\epsilon Q^-)^m \tag{4.1}$$

We have used the $SO(9)$ notation $\epsilon = (\eta^a, \tilde{\eta}^{\dot{a}})$ and $Q^- = (Q^a_-, Q^{\dot{a}}_-)$. Different components of the supermultiplet, corresponding to the possible independent ϵ's, can be thought as the semiclassical multipole expansion of the source. In particular, terms in $U|B\rangle$ with an even (odd) number of Q^- describe couplings to bosonic (fermionic) closed string states Ψ_B (Ψ_F). We shall restrict ourselves to terms with an even number of supercharges, the relevant for the study of semiclassical D-brane dynamics in the eikonal approximation. For instance, the usual boundary (1.16) state corresponds to the universal part of the source, whereas applying two charges one obtains the part of the source due to angular momentum, and so on. As we are going to see in the following, the field theory counterpart of this source expansion is a power series in the transfered momentum, each momentum corresponding to the insertion of a pair of supercharges.

Among the different terms in expansion (4.1) we will always work out the ones with an equal number of dotted-undotted $SO(8)$ components $(\eta_a Q_-^a \tilde{\eta}_{\dot{a}} Q_-^{\dot{a}})^n$. All the other terms simply combine to reconstruct the covariant answer. We consider then boundary states of the form:

$$|B\rangle_{(n)} \equiv V_\eta^n |B\rangle \tag{4.2}$$

with

$$V_\eta = \eta_a Q^{-a} \tilde{\eta}_{\dot{a}} Q^{-\dot{a}} \tag{4.3}$$

The first interesting information we can extract from these higher spin boundary states is about their couplings to the massless bulk fields. This analysis for the D-instanton case was performed in the covariant NS-R formalism in ref.[53]. The formulae displayed in this section are "T-dual" versions of the ones reported in that reference.

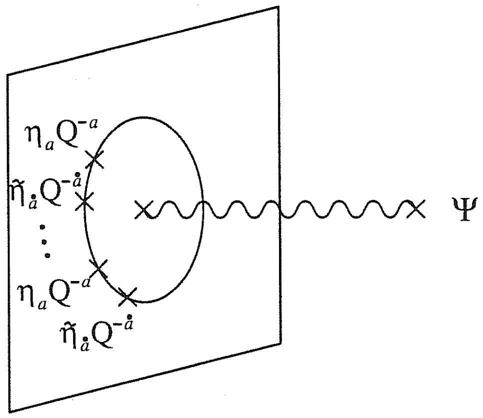

Fig. 2

In the following, we consider only terms with up to eight supercharges, $n = 0, ..., 4$, in eq.(4.2). This covers all the physical information relevant to our considerations. The one-point functions of the massless bosonic states of NSNS and RR sectors (in R-NS terminology) are obtained as before from the overlapping

$$\Psi_{(n)} \equiv \langle \Psi | B_0 \rangle_{(n)} \tag{4.4}$$

with $|\Psi\rangle$ given by the NSNS and RR massless states (1.19), (1.20) and $|B_0\rangle_{(n)}$ indicates the massless content of (4.2)

$$|B_0\rangle_{(n)} \equiv V_\eta^n |B_0\rangle = q_{j_1} \cdots q_{j_n} \omega_{i_1 \dots i_{2n}}^{j_1 \dots j_n}(\eta) R_0^{i_1 i_2} \dots R_0^{i_{2n-1} i_{2n}} |B_0\rangle \tag{4.5}$$

where

$$\omega^{j_1\dots j_n}_{i_1\dots i_{2n}}(\eta) \equiv \eta_{[a_1}\gamma^{j_1}\tilde{\eta}_{a_2}\cdots\eta_{a_{2n-1}}\gamma^{j_n}\tilde{\eta}_{a_{2n}]}\gamma^{i_1 i_2}_{a_1 a_2}\cdots\gamma^{i_{2n-1}2n}_{a_{2n-1}a_{2n}} \tag{4.6}$$

and have used the Fierz identity

$$S^a_0 S^b_0 = \frac{1}{2}\delta^{ab} + \frac{1}{4}\gamma^{ij}_{ab}R^{ij}_0,$$

the anticommutation relations of S_0^{-a} and the boundary conditions (1.12), in order to express (4.5) in terms only of the left moving $SO(8)$ generators R^{ij}_0.

In this way, we can use standard results for Type I theory. The R^{ij}_0 generators are represented in the $\mathbf{8_v}$ and $\mathbf{8_c}$ representations by

$$R^{mn}_0|i\rangle = (\delta^{ni}\delta^{mj} - \delta^{mi}\delta^{nj})|j\rangle$$
$$R^{mn}_0|\dot{a}\rangle = \frac{1}{2}\gamma^{mn}_{\dot{c}\dot{a}}|\dot{c}\rangle \tag{4.7}$$

After some simple algebra we can write (4.5) as

$$|B_0\rangle_{(n)} \equiv M^{ij}_{(n)}|i\rangle|\tilde{j}\rangle - iM^{(n)}_{\dot{a}\dot{b}}|\dot{a}\rangle|\tilde{\dot{b}}\rangle$$

with

$$M^{ij}_{(n)} = 2^n q_{j_1}\cdots q_{j_n}\omega^{j_1\dots j_n}_{ii_1i_1\dots i_{n-1}i_{n-1}i_n}(\eta)M^{i_n j}$$
$$M^{(n)}_{\dot{a}\dot{b}} = 2^{-n} q_{j_1}\cdots q_{j_n}\omega^{j_1\dots j_n}_{i_1\dots i_{2n}}(\eta)(\gamma^{i_1 i_2}\dots\gamma^{i_{2n-1}i_{2n}}M)_{\dot{a}\dot{b}} \tag{4.8}$$

The 1-point functions can then be written as (up to numerical and α' factors)

$$\Psi^{NSNS}_{(n)} = \xi_{ij}M^{ij}_{(n)} = \xi_{ij}q_{j_1}\cdots q_{j_n}\omega^{j_1\dots j_n}_{ii_1i_1\dots i_{n-1}i_{n-1}i_n}M^{i_n j} \tag{4.9}$$

$$\Psi^{RR}_{(n)} = \mathrm{Tr}_S[CM_{(n)}]$$
$$= \sum_{k\ even}\frac{1}{k!}C^{i_1\dots i_k}_{(k)}q_{j_1}\cdots q_{j_n}\omega^{j_1\dots j_n}_{k_1\dots k_{2n}}\mathrm{Tr}_S[\gamma^{i_1\dots i_k}\gamma^{k_1k_2}\dots\gamma^{k_{2n-1}k_{2n}}M] \tag{4.10}$$

Eqs.(4.9) and (4.10) contain all the non-minimal couplings of D-branes with the massless bosonic states of the corresponding supergravity theory. In particular, for even n the boundary state couples potentially to the NSNS components $\phi, g_{\mu\nu}$, g_{IJ} and $b_{\mu I}$ (μ,ν and I,J denoting Neumann and Dirichlet directions respectively), and to the remaining NSNS fields for odd n, as can be seen using the symmetry properties of $\omega^{j_1\dots j_n}_{i_1\dots i_{2n}}$. As a source of RR fields, we can see that non-minimal couplings arise from the non-vanishing gamma-traces in eq.(4.10), corresponding to forms with $k = p+1-2n, \dots, p+1+2n$. The specific form of these couplings depends on the polarization details encoded in ω. In particular the $n = 0$ case represent the spin independent part of the source content and is given by (1.19) and (1.20). The covariant form of these expresions can be written as

$$\Psi_{(0)}^{NSNS} = \xi_{\mu\nu} M^{\nu\mu} \ , \ \ \Psi_{(0)}^{RR} = \sum_{k\ even} \frac{1}{k!} C^{(k)}_{\mu_1 \dots \mu_k} \mathrm{Tr}_S[\Gamma^{\mu_1 \dots \mu_k} \mathcal{M}] \tag{4.11}$$

where Γ are $SO(1,9)$ gamma-matrices, $M^{\nu\mu}$ is the covariant extension of eq.(1.14), with diagonal entries only, -1 and +1 in Neumann and Dirichlet directions respectively, and $\mathcal{M} = \Gamma^0 \dots \Gamma^p$.

The first non-minimal NSNS coupling is given by

$$\Psi_{(1)}^{NSNS} = \xi_{ij} M_{jk} \omega^{ki} = \xi_{ij} M_{jk}\, q_l\, \eta \gamma^{kil} \tilde{\eta} \tag{4.12}$$

where we have used the fact that $q_j \xi_{ij} = q_i \xi_{ij} = 0$ and $q_k M_{kj} = q_j$ (there is a non-vanishing momentum transfer only along the Dirichlet directions). As anticipated, eq.(4.12) represents a non-minimal coupling of the brane with the antisymmetric tensor and graviton polarizations $b_{\mu\nu}$, b_{IJ} and $g_{\mu I}$. The covariant expression of eq.(4.12) is simply

$$\Psi_{(1)}^{NSNS} = \xi_{\mu\sigma} M^{\sigma}_{\nu} q_\rho \bar{\psi} \Gamma^{\mu\nu\rho} \psi \tag{4.13}$$

where ψ is the Majorana-Weyl fermionic parameter associated to the broken supersymmetry. In a chiral representation, it is simply $\psi = \binom{\epsilon}{0}$, where $\epsilon = (\eta^a, \tilde{\eta}^{\dot{a}})$. The corresponding RR coupling is

$$\Psi_{(1)}^{RR} = \sum_{k\ even} \frac{1}{k!} C_{(k)}^{i_1 \dots i_k} \mathrm{Tr}_S(\gamma^{i_1 \dots i_k} \gamma^{ij} M)\, q_l\, \eta \gamma^{ijl} \tilde{\eta} \tag{4.14}$$

where still the completely antisymmetric part in the fermion bilinear is the only non-vanishing contribution, since $q^{i_1} C_{(k)}^{i_1 \dots i_k} = 0$. The covariant form of eq.(4.14) is

$$\Psi_{(1)}^{RR} = \sum_{k\ even} \frac{1}{k!} C^{(k)}_{\mu_1 \dots \mu_k} \mathrm{Tr}_S(\Gamma^{\mu_1 \dots \mu_k} \Gamma_{\nu\rho} \mathcal{M}) q_\sigma \bar{\psi} \Gamma^{\nu\rho\sigma} \psi \tag{4.15}$$

The next coupling is $\Psi_{(2)}^{NSNS}$, that is

$$\Psi_{(2)}^{NSNS} = \xi_{ij}\, \omega^{ii_1 i_1 i_2} M^{i_2 j} = \xi_{ij}\, \omega_{a_1 \dots a_4} \gamma^{ii_1}_{a_1 a_2} \gamma^{i_1 i_2}_{a_3 a_4} M^{i_2 j} \tag{4.16}$$

After some algebra eq.(4.16) can be rewritten, neglecting q^2 contact terms which are irrelevant for our semiclassical analysis, as

$$\Psi_{(2)}^{NSNS} = \tilde{\xi}_{ij}\, q_m q_n \left(\eta \gamma^{ink} \tilde{\eta}\, \eta \gamma^{jmk} \tilde{\eta} - \eta \gamma^{im} \eta\, \tilde{\eta} \gamma^{jn} \tilde{\eta} \right) \tag{4.17}$$

where $\tilde{\xi}_{ij} \equiv \xi_{ik}M_{kj} + \xi_{jk}M_{ki}$. Notice that the combination of spinors in eq.(4.17) is the right one reproducing the covariant expression [53]

$$\Psi_{(2)}^{NSNS} = \tilde{\xi}_{\mu\nu}\, q_\alpha q_\beta\, \bar{\psi}\Gamma^{\mu\alpha\rho}\psi\, \bar{\psi}\Gamma^{\nu\beta}{}_{\rho}\psi \tag{4.18}$$

The RR coupling is

$$\Psi_{(2)}^{RR} = \sum_{k\ even} \frac{1}{k!} C_{(k)}^{i_1 \ldots i_k} \mathrm{Tr}_S(\gamma^{i_1 \ldots i_k}\gamma^{j_1 j_2}\gamma^{j_3 j_4} M)\omega^{j_1 \ldots j_4} \tag{4.19}$$

Using again the gauge condition $q^{i_1} C_{(k)}^{i_1 \ldots i_k} = 0$ and after some manipulations, similar to those so far performed, it is not difficult to put eq.(4.19) into the covariant form

$$\Psi_{(2)}^{RR} = \sum_{k\ even} \frac{1}{k!} C^{(k)}_{\mu_1 \ldots \mu_k} \mathrm{Tr}_S(\Gamma^{\mu_1 \ldots \mu_k}\Gamma_{\nu_1\nu_2}\Gamma_{\nu_3\nu_4}\mathcal{M})\, q_\alpha q_\beta (\bar{\psi}\Gamma^{\nu_1\nu_2\alpha}\psi\, \bar{\psi}\Gamma^{\nu_3\nu_4\beta}\psi) \tag{4.20}$$

Following the same procedure, it is possible to write down all the other terms.

4.2 Spin effects for the p-p brane system

In this section we compute the spin dependence of the brane potentials for two parallel slowly moving Dp-branes. The computation follows the lines of the one in section 2.4.3, where the universal potential (spin-independent) for two D0 branes have been worked it out. Spin effects can be included by the insertion of the supercharges vertices V_η (4.3) in universal potential (1.40). The cylinder correlation functions will then involve in general n velocity V_B's and m supercharge V_η's vertex insertions. The corresponding amplitudes are given by

$$\mathcal{A}_{n,m} = \frac{1}{16}\int_0^\infty dt\, \langle B_p, \vec{x} = 0| e^{-2\pi t\alpha' p^+(P^- - i\partial/\partial x^+)} \frac{(V_B)^n}{n!} (V_\eta)^m |B_p, \vec{y} = \vec{b}\rangle \tag{4.21}$$

Since V_η as V_B provide each at most two zero modes (4.21) will give a vanishing result untill $n + m = 4$ vertices are inserted. Moreover, for a fixed m, the leading order potential going like v^{4-m}, will display a similar decoupling of the massive mode contributions as was the case in (1.42). These amplitudes are therefore *scale invariant*, in the sense that their dependence on the distance $\vec{b}$ is exact, keeping the same functional form at any finite distance. In the following, expressions similar to eq.(4.21) will be denoted simply by $\mathcal{A}_n \equiv \langle V_B^n\, V_\eta^{4-n}\rangle$, in order to light the notation. We wrote in eq.(4.21) all the supercharges applied to the same boundary state; being fixed simply by a zero modes analysis, the computation will not depend on the choice of the boundary, whereas the physical interpretation as polarization effects will be different. The polarizations of the two D-branes are indeed given by the supersymmetric parameters $\eta_1^a, \eta_1^{\dot a}$ and $\eta_2^a, \eta_2^{\dot a}$ associated to the two boundaries, as shown in figure 3.

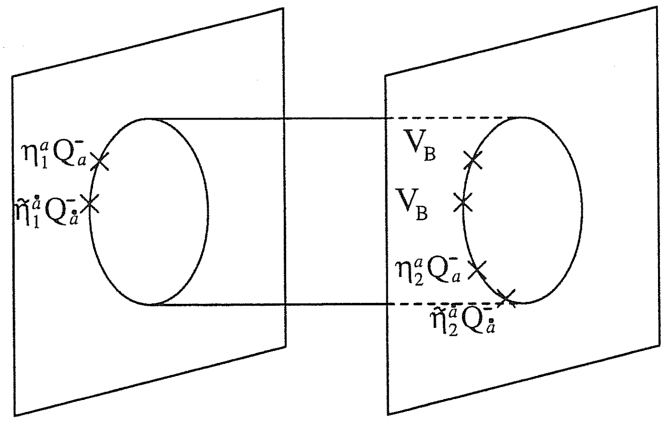

Fig. 3: example of a spin-dependent term at v^2 order, $m = n = 2$.

The $m = 0$ case is the one we have considered in 2.4.3 for D0 branes. For parallel Dp-branes this universal potential reads

$$\mathcal{A}_4 = \frac{V_{p+1}}{64} T_p^2 \, |v|^4 \, G_{9-p}(\vec{b}) \tag{4.22}$$

As before, possible contributions to a static force or to v^2-potentials are absent due to a compensation between the gravitational and dilatonic fields (attractive) and the RR A_{p+1} field, that for two Dp-branes is repulsive, of course. In this formalism, it is immediately clear that supersymmetry implies a contribution starting only like v^4.

The first spin effect is given by $\mathcal{A}_3 = \langle V_B^3 \, V_\eta \rangle$; going through the same steps and after some simple algebra, one obtains

$$\begin{aligned} \mathcal{A}_3 &= \frac{V_{p+1}}{8} |v|^2 \, (4\pi^2\alpha')^{4-p} \int_0^\infty dt \int \frac{d^{9-p}q}{(2\pi)^{9-p}} \, e^{i\vec{q}\cdot(\vec{x}-\vec{y})-2\pi t\alpha' q^2} \omega_{a_1 a_2}(\eta, q) \, \gamma^{1i}_{a_1 a_2} \, v^i \\ &= -i \frac{V_{p+1}}{16} T_p^2 \, |v|^2 v^i \, \tilde{\eta}\gamma^{1ij}\eta \, \partial_j G_{9-p}(\vec{b}) \end{aligned} \tag{4.23}$$

that represents a spin-orbit like coupling between branes. From eqs.(4.9) and (4.10) we can derive the NSNS and RR polarizations of the exchanged states, responsible of these non-minimal couplings. In order to perform the analytic continuation of eq.(4.23) to Minkowskian coordinates, it is convenient to write covariantly the term $\tilde{\eta}\gamma^{1ij}\eta$, whose $SO(1,9)$ expression is $\bar{\psi}\Gamma^{1\mu\nu}\psi$. Performing the rotation we obtain

$i\bar{\psi}\Gamma^{0\mu\nu}\psi$, and sending $v^i \to iv^i$ leads finally to

$$\mathcal{A}_3^{Mink.} = -\frac{V_{p+1}}{32} T_p^2 \, |v|^2 v_i \, \partial_j G_{9-p}(\vec{b}) \, J^{0ij} \tag{4.24}$$

where $i, j = 1, ..., 9$ and $J^{\mu\nu\rho} \equiv i\bar{\psi}\Gamma^{\mu\nu\rho}\psi$ The next spin effect is $\mathcal{A}_2 = \langle V_B^2 \, V_\eta^2 \rangle$; in this case we have to distinguish two possible configurations, depending on which boundary state we apply the supercharges:

$$\mathcal{A}_2^{(1)} = \langle V_B^2 \, V_\eta^2 \rangle; \quad \mathcal{A}_2^{(2)} = \langle V_{\eta_1} \, V_B^2 \, V_{\eta_2} \rangle$$

These two contributions can be written as

$$\mathcal{A}_2^{(1)} = \frac{V_{p+1}}{8} T_p^2 \, \omega^{ij}_{i_1 \ldots i_4}(\eta) \, t^{i_1 \ldots i_4 1k1l} \, v_k v_l \partial_i \partial_j G_{9-p}(\vec{b}) \tag{4.25}$$

$$\mathcal{A}_2^{(2)} = \frac{V_{p+1}}{8} T_p^2 \, \omega^{i}_{i_1 i_2}(\eta_1) \omega^{j}_{i_3 i_4}(\eta_2) \, t^{i_1 \ldots i_4 1k1l} \, v_k v_l \partial_i \partial_j G_{9-p}(\vec{b}) \tag{4.26}$$

Noting that

$$t^{i_1 \ldots i_4 1k1l} \, v_k v_l = v^2 \left(4\delta^{1i_2}\delta^{1i_4}\delta^{i_1 i_3} - \delta^{i_1 i_3}\delta^{i_2 i_4} \right) - 4 v^{i_1} v^{i_4} \delta^{i_2 i_3} \tag{4.27}$$

and working out the spinor algebra, it can be shown that eq.(4.25) reconstructs the covariant amplitude, that after analytic continuation, takes the following form:

$$\mathcal{A}_2^{(1)} = \frac{V_{p+1}}{192} T_p^2 \, v^2 (2J^{m0q} J^n{}_{0q} - J^{mpq} J^n{}_{pq} + 4J^{m\rho}{}_i J^n{}_{\rho j} \hat{v}^i \hat{v}^j) \partial_m \partial_n G_{9-p}(\vec{b}) \tag{4.28}$$

Latin letters $i, j, k, ...$ label $SO(9)$ indices running from 1 to 9, in contrast to $SO(1,9)$ indices, denoted with Greek letters. In the same way, one can reconstruct the explicit covariant form of eq.(4.26) and all the remaining spin effects that will follow. We do not report the explicit relations for all the cases, being quite lengthy, as well as the analytic continuation. The remaining spin effects are

$$\mathcal{A}_1^{(1)} = \langle V_B \, V_\eta^3 \rangle = \frac{V_{p+1}}{4} T_p^2 \, \omega^{ijk}_{i_1 \ldots i_6}(\eta) \, t^{i_1 \ldots i_6 1l} \, v_l \partial_i \partial_j \partial_k G_{9-p}(\vec{b}) \tag{4.29}$$

$$\mathcal{A}_1^{(2)} = \langle V_{\eta_1} \, V_B \, V_{\eta_2}^2 \rangle = \frac{V_{p+1}}{4} T_p^2 \, \omega^{i}_{i_1 i_2}(\eta_1) \omega^{jk}_{i_3 \ldots i_6}(\eta_2) \, t^{i_1 \ldots i_6 1l} \, v_l \partial_i \partial_j \partial_k G_{9-p}(\vec{b})$$

and the static force

$$\mathcal{A}_0^{(1)} = \langle V_\eta^4 \rangle = \frac{V_{p+1}}{4} T_p^2 \, \omega^{ijkl}_{i_1 \ldots i_8}(\eta) \, t^{i_1 \ldots i_8} \, \partial_i \partial_j \partial_k \partial_l G_{9-p}(\vec{b})$$

$$\mathcal{A}_0^{(2)} = \langle V_{\eta_1} \, V_{\eta_2}^3 \rangle = \frac{V_{p+1}}{4} T_p^2 \, \omega^{i}_{i_1 i_2}(\eta_1) \omega^{jkl}_{i_3 \ldots i_8}(\eta_2) \, t^{i_1 \ldots i_8} \, \partial_i \partial_j \partial_k \partial_l G_{9-p}(\vec{b})$$

$$\mathcal{A}_0^{(3)} = \langle V_{\eta_1}^2 \, V_{\eta_2}^2 \rangle = \frac{V_{p+1}}{4} T_p^2 \, \omega^{ij}_{i_1 \ldots i_4}(\eta_1) \omega^{kl}_{i_5 \ldots i_8}(\eta_2) \, t^{i_1 \ldots i_8} \, \partial_i \partial_j \partial_k \partial_l G_{9-p}(\vec{b}) \tag{4.30}$$

In all these cases the one-point functions considered in last section allows to see which are, in each configuration, the massless string excitations responsible of all these polarization effects.

4.3 More general brane configurations

We consider now more general D-brane configurations whose potentials display a similar SYM/SUGRA correspondence as the ones for the p-p system. From the supergravity point of view they represent non-trivial backgrounds, while in the SYM context they are described by fluxes in the worldvolume theory. We start considering spin potentials for parallel p-p+4 brane configurations. Like more general p-q systems with mixed Neumann-Dirichlet boundary conditions in four directions, these BPS configurations preserves 1/4 of the initial supersymmetries, rather than the 1/2 of the p-p system. This residual supersymmetry protects as before the leading order term in the velocity expansion from higher massive modes contributions; however, unlike in the p-p system, the reduced amount of supersymmetry allows now a non-trivial metric in the Dp moduli space. In particular the D0-D4 system, studied in [54], was proposed in [55] as a matrix description for the scattering of an eleven dimensional supergraviton off the background of a longitudinal fivebrane.

The leading spin dependence of the potential felt by slowly moving p-branes in the p+4 background is defined by a similar cylinder amplitude as (4.21) with one of the boundary state now representing a p+4 brane . The relevant zero mode traces are now of the form

$$\langle B_p|\mathcal{O}|B_{p+4}\rangle = \mathrm{Tr}_{S_0}[\mathcal{O}N] \equiv \mathrm{Tr}_V[\mathcal{O}N] - \mathrm{Tr}_S[\mathcal{O}N] \tag{4.31}$$

where $\mathcal{O}$ is a product of R_0^{ij} arising from the zero mode part of the V_B and V_η vertex insertions and

$$\begin{aligned} N^{ij} &\equiv (M_p^T M_{p+4})^{ij} = \begin{pmatrix} I_{p+1} & 0 & 0 \\ 0 & -I_4 & 0 \\ 0 & 0 & I_{3-p} \end{pmatrix} \\ N_{\dot a\dot b} &\equiv (M_p^T M_{p+4})_{\dot a\dot b} = (\gamma^{p+2}...\gamma^{p+5})_{\dot a\dot b} \end{aligned} \tag{4.32}$$

By simple inspection of eq.(4.31), using the matrices (4.32) and the representation of the operators (4.7), we get vanishing traces for $\mathcal{O} = 1, R_0^{ij}$. The first non trivial result is

$$\begin{aligned} t^{i_1...i_4} &\equiv \mathrm{Tr}_{S_0}\, R_0^{i_1i_2}R_0^{i_3i_4} \\ &= 2\,\epsilon^{i_1...i_4 p+2...p+5} \\ &\quad +2\left(\delta^{i_1p+2}\delta^{i_2p+3}\delta^{i_3p+4}\delta^{i_4p+5} + N^{i_2i_4}\delta^{i_1i_3} + \text{perm.}\right) \end{aligned} \tag{4.33}$$

where by "perm." we mean as before an antisymmetrization within each pair (i_1, i_2), (i_3, i_4) and symmetrization under the exchange of each of these pairs.

The relevant amplitudes describing leading spin-effects are then

$$\mathcal{A}_n = \frac{1}{16}\int_0^\infty dt\, \langle B_p, \vec{x}=0| e^{-2\pi t\alpha' p^+(P^- - i\partial/\partial x^+)} \frac{(V_B)^n}{n!}(V_\eta)^{2-n}|B_{p+4}, \vec{y}=\vec{b}\rangle \qquad (4.34)$$

where the total number of vertex insertions now is two providing the four zero modes required to get the first non-trivial result from eq.(4.33). The rest of the computation follow the lines of last section. We are left with the universal term

$$\mathcal{A}_2 = \frac{V_{p+1}}{4}\, T_p T_{p+4}\, |v|^2\, G_{5-p}(\vec{b}) \qquad (4.35)$$

and the leading spin potentials

$$\begin{aligned} \mathcal{A}_1 &= \frac{V_{p+1}}{4} T_p T_{p+4}\, \omega^i_{i_1 i_2}(\eta)\, t^{1j i_1 i_2}\, v_j\, \partial_i G_{5-p}(\vec{b}) \\ \mathcal{A}_0^{(1)} &= \frac{V_{p+1}}{4} T_p T_{p+4}\, \omega^{ij}_{i_1\ldots i_4}(\eta)\, t^{i_1\ldots i_4}\, \partial_i\partial_j G_{5-p}(\vec{b}) \\ \mathcal{A}_0^{(2)} &= \frac{V_{p+1}}{4} T_p T_{p+4}\, \omega^i_{i_1 i_2}(\eta_1)\omega^j_{i_3 i_4}(\eta_2)\, t^{i_1\ldots i_4}\, \partial_i\partial_j G_{5-p}(\vec{b}) \end{aligned} \qquad (4.36)$$

The appearance of T_{p+4} and G_{5-p} instead of the T_p and G_{9-p} for the p-p system is due to the lack of four Dirichlet-Dirichlet transfered momentum integrations.

We recall that eqs.(4.35) and (4.36) are exact to any order in the brane separation $\vec{b}$, supporting again a Super-Yang Mills description of the corresponding supergravity potentials. Of course this is again a peculiar property only of these leading order terms and of the supersymmetric p-p, p-p+4 configurations. Higher order terms or non supersymmetric brane configurations will involve contributions from the oscillator part of the vertices (1.39), (4.3) described by modular functions with non-trivial transformation properties which in general distinguish the large and short distance behaviors.

We should say however that this property is shared by an amount of other interesting brane systems. Indeed, there several examples [56]-[61] where a similar SYM/SUGRA correspondence of the leading D-brane interactions have been observed. These brane configurations fall in general into two main groups: supersymmetric brane configurations, which include besides the examples studied above, the p-p+8 systems, bound states between p-p+2, p-p+2-p+4, p-p+4, ... D-branes, and any S or T-dual combinations of these systems; and brane configurations which are supersymmetric only in certain limits of their moduli space.

Bound states can be considered in general as fluxes for the gauge field living on the boundary of the biggest D-brane, modifying therefore its boundary conditions. The corresponding light-cone boundary state for a generic condensate was constructed in

[32]. The cylinder amplitude defined by two of these boundary states, in the case that some of the supersymmetries are preserved (take for example two indentical p-p+2 bound states or the S-dual analog of two D-strings with equal electric fluxes turned on [62]) , will lead to similar vanishing traces as in eq.(4.31), unless $N/4$ velocity insertions soak the $N/2$ left zero modes, N being the number of supercharges left unbroken by the system. Again the spin-dependent dynamics can be studied by inserting supercharges on the cylinder, and once more an equivalent matrix-supergravity description for the slowly moving regime is guaranteed.

The second interesting class of configurations (and less straight) for which an analysis along the lines of this paper can be followed, is inspired from the brane systems studied in [58], which although not supersymmetric, become so in a given limit of the moduli space. In the analysis of these systems one can follow a strategy parallel to the one previously applied to the case of moving branes. In that case supersymmetry is broken for finite velocity v, but the existence of a supersymmetric limit $v = 0$, allow us to study leading orders by a simple analysis of the zero mode structure of amplitudes involving the insertion of vertex operators corresponding to the deformation (in that case v) from the supersymmetric point. Similarly, now we look at the neighborhood of a specific choice of flux for which some supersymmetry is restored. The fermionic part of the operators corresponding to deformations about this supersymmetric point coincide with the vertex (1.39), once we substitute the plane $(1i)$ defining the boost operation with the condensate euclidean plane (mn), and therefore the results can be read directly from the ones quoted above for the moving brane systems. We can illustrate this with the simple example of a Dp-brane, wrapped around a T^2 with a magnetic flux $f_1 = \frac{N}{2\pi R_m R_n}$ turned on. The boundary state for this specific condensate can be read from the more general one found in [32] to be defined by eqs.(1.16) and (1.18) through the matrices

$$
\begin{aligned}
M_{ij} &= \sqrt{(1+f^2)} \begin{pmatrix} -I_{p-1} & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & I_{7-p} \end{pmatrix} \\
M_{\dot a \dot b} &= (\gamma^{12} + f)\gamma^1 \cdots \gamma^{p+1}
\end{aligned}
\qquad (4.37)
$$

where $\cos\alpha \equiv -\frac{1-f^2}{1+f^2}$. Notice that eqs.(4.37) reduce, in the large f limit, to the matrices (1.14),(1.17) defining the D(p-2) brane, up to an overall f factor and the missing of two momentum modes corresponding to the Neumann-Dirichlet directions. As we discussed before, we can study the leading interactions of this bound state with a D(p-2) ordinary brane by simply perturbing the system by a small $c \equiv 1/f$ quantity from the supersymmetric $c = 0$ point. The spin independent potential is then defined by correlators involving insertions of $R_0^{i1}v^i$ and $R_0^{mn}\pi c$ in the D(p-2)-D(p-2) cylinder, and as before we have vanishing traces unless at least four of these insertions soak the eight zero modes, leaving

$$\langle B_{p-2}|B_p, c, v\rangle = \frac{1}{\pi c}\frac{V_{p-1}}{32}T_pT_{p-2}(v^4 + 2v^2(\pi c)^2 + (\pi c)^4)G_{7-p}(r). \tag{4.38}$$

The overall $1/c$ factor can be interpreted as the number of D(p-2) branes arising from the Dp-brane in the $c \to 0$ limit, the two missing powers in r represents the reduced transverse space to the system, and the relative coefficients are fixed by the kinematical tensor (1.41) [1]. Given, as before, by an exact string computation at the relevant order in the v, c expansion, this potential is valid at any transverse distance r and in particular admits equivalent Super Yang-Mills and supergravity descriptions. The p=2 case is the relevant one for the analysis performed in [58]. In that reference the authors study the graviton-membrane, static membrane-antimembrane and orthogonal moving membranes scattering. In each case the infinite boost ($N \to \infty$ or equivalently $c \to 0$) represents a point where the 16 supercharges are recovered (for $v = c = 0$). The leading orders in v, c are given by eq.(4.38), and the scale invariance of these terms is guaranteed by our previous analysis, and checked explicitly in that reference. The case of orthogonal membranes is particular in the sense that contains two line of deformations $c \equiv c_1 + c_2 = 0$ and $c' \equiv c_1 - c_2 = 0$ (this is the case studied in [59]), c_1, c_2 being defined by the fluxes in each membrane, along which half the supersymmetries of the D0-D0 system are preserved. Along these lines, the potential starts then with v^2 as for the previously discussed p-p+4 system. The leading scale invariant interactions are in general given by

$$\langle c_2, B_{p2}|B_{p1}, c_1, v\rangle = \frac{1}{\pi^2 c_1 c_2}\frac{1}{32}T_2T_2(v^2 + (\pi c)^2)(v^2 + (\pi c')^2)G_5(r). \tag{4.39}$$

The absence of the static c^4 and c'^4 terms reflects the surviving of half-supersymmetries along the aforementioned lines. In [59, 60] an exhaustive list of brane configurations was shown to present again agreement between the one-loop SYM and semi-classical supergravity descriptions of their potentials. Once more, homogenous polynomials of order four in the fluxes and velocities as in (4.38),(4.39) were found; an iteration of the analysis for the above discussed example provides a unified understanding of those results. We believe that this example can give a flavor of the generality of the analysis performed here, which extends to any supersymmetric (at least in a point

[1] A flip in the sign of the v^2 comes from the analytic continuation to the Minkowski plane.

of the moduli space) brane configuration and covers all (to our knowledge) one-loop scattering tests of a given matrix description. We should say that scale invariance is however stronger than what a correct matrix description of supergravity interactions really requires. In fact higher loop potentials will not display a simple decoupling of their massive modes as in the examples studied here and only a matching between the two open-closed massless truncated computations can be at most expected.

4.4 Field theory interpretation and D0-brane gyromagnetic ratio

In the present section we discuss the field theory interpretation of our results. We will show in particular that the knowledge of all the one-point functions of the massless fields of Type IIA/B supergravity allows to infer the complete and generic asymptotic form of the corresponding p-brane solution. Moreover, the spin-effects in scattering amplitudes that we have computed in section 4 and the supersymmetric cancellation of some of their leading orders proves to constitute an extremely efficient way to fix unambiguously the various coefficients entering the solution, and in particular the relative strength of the NSNS attraction and the RR repulsion (the fact that normalizations are better encoded in scattering amplitude than in one-point functions, especially through the vanishing of leading order, was already appreciated in Polchinski's computation of the Dp-brane charge [18]). As we will see, this approach yields a powerful technique to extract informations about a generic component of the p-brane multiplet. The analogous computation in supergravity would consist in performing supersymmetry transformations to the usual p-brane solution, to determine all the spinning superpartners; this requires looking up to eight variations, a program that, as can be appreciated from previous works [63, 64], is rather technical within the component fields formalism.

We will work it out the D0-brane case, for which the boundary state is defined through the matrices $M^0_0 = -1$, $M^i_j = \delta^i_j$ and $\mathcal{M} = \Gamma^0$; the other cases can be treated in the same way. Recall that in the NSNS sector, a generic field $\xi_{\mu\nu}$ is decomposed into trace, symmetric and antisymmetric parts ϕ, $h_{\mu\nu}$ and $b_{\mu\nu}$ as

$$\epsilon^{(\phi)}_{\mu\nu} = \frac{1}{4}(\eta_{\mu\nu} - q_\mu l_\nu - q_\nu l_\mu) \ , \quad \epsilon^{(h)}_{\mu\nu} = \xi_{(\mu\nu)} \ , \quad \epsilon^{(b)}_{\mu\nu} = \xi_{[\mu\nu]}$$

where l^μ is a vector satisfying $q \cdot l = 1$, $l^2 = 0$. Collecting the covariant one-point functions (4.11), (4.13), (4.15), (4.18) and (4.20), for up to four supercharge insertions, the NSNS and RR asymptotic fields for a generic component of the Dp-brane multiplet can be written as the multipole expansion in the coordinate space

$$\phi = \frac{3}{2}\kappa^2 M G_9(r) + \frac{1}{4}\kappa^2 C J^{mpq} J^n{}_{pq}\partial_m\partial_n G_9(r) + ...$$

$$\begin{cases} h_{00} = \kappa^2 M G_9(r) + \kappa^2 C J^{m0q} J^n{}_{0q}\partial_m\partial_n G_9(r) + ... \\ h_{ij} = \delta_{ij}\kappa^2 M G_9(r) + \kappa^2 C J^m{}_i{}^\rho J^n{}_{j\rho}\partial_m\partial_n G_9(r) + ... \\ h_{0i} = 2\kappa^2 A J_{0i}{}^m \partial_m G_9(r) + ... \end{cases}$$

$$\begin{cases} b_{ij} = \kappa^2 A J_{ij}{}^m \partial_m G_9(r) + ... \\ b_{0i} = 2\kappa^2 C J^m_{0q} J^n{}_i{}^q \partial_m\partial_n G_9(r) + ... \end{cases} \tag{4.40}$$

in the NSNS sector, and

$$\begin{cases} C_0 = 2\kappa^2 Q G_9(r) + \kappa^2 D J^{m\rho\tau} J^n{}_{\rho\tau}\partial_m\partial_n G_9(r) + ... \\ C_i = 2\kappa^2 B J_{0i}{}^m \partial_m G_9(r) + ... \end{cases}$$

$$\begin{cases} C_{0ij} = \kappa^2 B J_{ij}{}^m \partial_m G_9(r) + ... \\ C_{ijk} = 2\kappa^2 D J^m_{0[i} J^n{}_{jk]}\partial_m\partial_n G_9(r) + ... \end{cases} \tag{4.41}$$

in the RR sector. Dots stand for higher derivative terms associated to further supercharges insertions. We have restore the dependence on the ten dimensional Planck constant $||^2$. The constants M, Q, A, B, C, D describe the first three terms in the multipole expansion of the D0 brane solution and can be determined directly from the one-point function worked it out before carefully normalized. In order to avoid the technical complications of fixing absolute normalizations, we take a slightly different approach to determine these constants, or more precisely some relevant ratii of them.

Comparing eqs.(4.40) and (4.41) with the usual 0-brane solution [65] and the general result valid in D dimensions derived in [66], we conclude that M is the mass and Q the electric charge, whereas $2AJ_{0ij} = J_{ij}$ is the angular momentum and $BJ_{0ij} = \mu_{ij}$ the magnetic moment, so that the gyromagnetic ratio, defined by the relation $\mu^{ij} = (gQ)/(2M)J^{ij}$, is given by $g = (MB)/(QA)$. Also, the electric and gravitational dipole moments vanish, since they would correspond to one-derivative terms in C_0 and h_{00}, h_{ij} respectively. On the contrary, the presence of non-vanishing two-derivative terms in the gravitational and gauge fields, show the presence of quadrupole moments for D-particles. Analogously to the gyromagnetic ratio g, we can define the ratio of the gauge and gravitational quadrupole moments by $\tilde{g} = 4(MD)/(QC)$.

It is now straightforward to show how the semiclassical analysis of the phase-shift between two of these configurations can be used to determine in a simple way the value of the gyromagnetic ratio g and its quadrupole analogue $\tilde{g}$ associated to D0-branes. According to [67, 64], massive Kaluza-Klein states present a common value $g = 1$, contrarily to the usual and "natural" [68] value $g = 2$.shared by all the known elementary particles (neglecting radiative corrections, of course). This particular signature of Kaluza-Klein states can be useful to establish the 11-dimensional nature of D0-branes, implying $g = 1$. This consistency check has been recently performed in [64] considering D0-branes as extended extremal 0-brane solution of IIA supergravity. We present now an alternative and independent argument that relies on the "stringy" nature of D0-branes as points on which open strings can end; in particular, we show that $g = 1$ is the only possible value compatible with the cancellation of the linear term in velocity in the first spin effect, eq.(4.24). Similarly, we will show that our stringy analysis predicts for the quadrupole analog the value $\tilde{g} = 1$ from the cancellation of the static contribution to the second spin effect, eq.(4.25).

Consider first the scattering of a scalar 0-brane, taken as a probe, off a spinning 0-brane, acting as source. The effective action for the probe is (in the string frame)

$$\mathcal{S} = -M \int d\tau \, e^{-\phi} \sqrt{-g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu} - Q \int d\tau C_\mu \dot{X}^\mu \tag{4.42}$$

For a trajectory with constant velocity $v = \tanh \pi\epsilon$, we can choose $X^0(\tau) = \tau \cosh \pi\epsilon$, $X^i(\tau) = \tau \, \hat{v}^i \sinh \pi\epsilon$. Expanding for small velocities and weak fields ($\kappa \to 0$), one finds, dropping a constant term, $\mathcal{S} = \int d\tau \sum_{n\geq 0} v^n \mathcal{L}_n$ with

$$\begin{aligned}
&\mathcal{L}_0 = M\phi + \frac{1}{2} M h_{00} - QC_0 \\
&\mathcal{L}_1 = M h_{0i} \hat{v}^i - QC_i \hat{v}^i \,, \quad \mathcal{L}_2 = \frac{1}{2} M (h_{00} + h_{ij} \hat{v}^i \hat{v}^j) - \frac{1}{2} QC_0 \\
&\mathcal{L}_3 = M h_{0i} \hat{v}^i - \frac{1}{2} QC_i \hat{v}^i \,, \quad \mathcal{L}_4 = \frac{1}{2} M (h_{00} + h_{ij} \hat{v}^i \hat{v}^j) - \frac{3}{8} QC_0
\end{aligned} \tag{4.43}$$

We know from the amplitudes computed in section 4 that the leading non-vanishing contributions to the scattering amplitude behave like v^n / r^{7+n}, all lower orders in velocity cancelling by supersymmetry. Substituting the relevant asymptotic fields of the spinning 0-brane from eqs.(4.40) and (4.41), one then finds the following conditions:

$$\begin{aligned}
&\mathcal{L}_0|_G = 0 \;\Rightarrow\; M = Q \,, \quad \mathcal{L}_0|_{\partial^2 G} = 0 \;\Rightarrow\; MC = 4QD \\
&\mathcal{L}_1|_{\partial G} = 0 \;\Rightarrow\; MA = QB \\
&\mathcal{L}_2|_G = 0 \;\Rightarrow\; M = Q
\end{aligned} \tag{4.44}$$

Altogether, this yields

$$Q = M \,, \quad g = 1 \,, \quad \tilde{g} = 1 \tag{4.45}$$

The ratii $g = 1$ and $\tilde{g} = 1$ can be thought as the supersymmetric analog of the BPS saturation condition $Q = M$. We notice from (4.40) and (4.41) that they determine also the ratii of the strengths between the NSNS antisymmetric $b_{\mu\nu}$ and RR threeform $C_{\mu\nu\rho}$ multipole sources providing a complete spin description of the supergravity solution at this order in the multipole expansion.

Conclusions

In this thesis we studied some aspects of the non-perturbative structure of string theory in the context of different S- duality conjectures. We trace an important sector of the physical spectrum of states (the BPS spectrum), whose stability properties allow us to follow to regime of strong coupling. The "quasi" topological character of the quantities under study allow us always to go to some corners in the moduli space where the theory admits a rather tractable description of its strong coupling physics.

In the first part of the thesis we concentrated in the study of the complete (perturbative and non-perturbative) BPS spectrum of states for string vacua with sixteen supercharges ($\mathcal{N} = 4$ in four dimensions). The more familiar string vacua with this amount of supersymmetry arises from toroidal compactifications of heterotic and type I string theories as well as from $K3 \times T^d$ compactifications of type II strings. More sophisticate examples can be constructed from asymmetric orbifold and orientifolds of type IIB theory. These string vacua combines the perturbative $(-)^{F_L}$ and Ω symmetries of type IIB theory with a given shift in the $\Gamma_{d,d}$ lattice of momenta for a T^d compactification. The interest in these string vacua relies on the fact that, the $(-)^{F_L}$ and Ω actions being related by the S-duality of type IIB theory, modding out by these operations we yield to very simple dual pairs where our ideas of dualities can be tested.

In chapter two we study the BPS spectrum for a nine dimensional pair constructed in this way. The generalizations for an arbitrary toroidal compactification with $D > 4$ is straight. We have seen that the self duality conjecture of type IIB theory requires the existence of an infinite tower of D-string bound states with specific charges, masses and degeneracies. This is also the case for toroidal compactifications of type I and $SO(32)$ heterotic string. In this case winding of heterotic elementary strings are mapped to the D-string number in the type I side. In both cases the information about masses and multiplicities of these non-perturbative states are encoded in the elliptic genus of the relevant $O(N)$ effective gauge theory describing the excitations of the bound state system. We have computed this index using the conjectured description of the infrared limit of this two-dimensional gauge theory as an orbifold conformal fixed point. A subtlety in this computation is the existence of bound states at thresholds, which make difficult to distinguish the one particle states from multiparticle ones. Multiparticle states are characterized by non normalizable wave functions. We were able to distinguish those states between the different twisted sectors in the orbifold theory. The results for one-particle states identified as the "longest string sector" are in perfect agreement with the duality predictions.

The second part of the thesis is devoted to the non-perturbative study of certain higher derivative couplings in low energy effective actions for theories with sixteen and thirty-two supercharges. The considered examples although in very different contexts are quite close in spirit. In each case the considered string amplitudes receive contributions only from the BPS spectrum of states. They are extreme in this sense, and can be considered always as the leading order in a expansion of the effective action in terms of a perturbation from a given supersymmetric point.

We study first the threshold corrections to $\mathcal{F}^4$ terms in the low energy effective actions for the type II dual pair discussed before. The exact formula for these corrections can be extracted from a simple one-loop computation in type IIB theory compactified on $T^2/(-)^{F_L}$. The result is given in terms of an infinite double sum representing the winding and Kaluza-Klein BPS string excitations running in the loop (torus). On the other side we have both perturbative and non-perturbative corrections. Perturbative corrections are given by a single sum in terms of the Kaluza-Klein string states running into the loop (now represented by the Klein bottle). On top of this we have an infinite sum of N D-instantons corrections which combines with the perturbative contributions to reproduce the exact formula found in the dual theory. The results not only gives evidence for the equivalence of the low energy effective actions but once the duality is admitted can be used as a definition of precise rules for the D-instanton calculus [47].

In the last chapter we study different D-brane configurations which support equivalent SYM/SUGRA descriptions. We consider string amplitudes which can be considered as a description of the potentials between moving D-objects in a supergravity theory or as the one-loop effect for an open string moving in a corresponding background. We start always from a supersymmetric configurations and perturbe its from its supersymmetric point. Examples of these deformations from supersymmetry can be given by a relative velocity v, a magnetic worldvolume flux c or a spin characterized by a spinor η. We study several examples combining these deformations. In each case we show that the leading order in the v, c, η expansions collapsed to the contribution of a finite number of degrees of freedom which can be interpreted as coming from the exchange of the massless closed string states(supergravity) or as a loop of massless open (super Yang-Mill) string excitations. A simple analysis of the fermionic zero mode structure of the relevant string amplitudes show that they are given by homogeneous polynomials in v, c, η of order 4(2) for the p-p(p-p+4) D-brane systems. In this way we cover all (to us known) one-loop tests of the matrix model conjecture in an unified picture. An interesting application of these results results was the determination of the giromagnetic and quadrupole moments for D-particles. We have shown that they are completely determined by the supersymmetry and the BPS condition $Q = M$.

We would like finally to comment some unexplored directions in the present work. An interesting direction could be to extend the present analysis for less supersymmetric string vacua [2]. Some preliminary results concerning the matching of the BPS spectra of physical states for type II dual models with eight supercharges (type IIB on $(K3 \times S_1)/(-)^{F_L}\sigma_V - (K3 \times S_1)/\Omega\sigma_V$) are reported in [20]. A non-perturbative analysis of the threshold corrections in type I/heterotic $N = 2$ string vacua have been recently performed in [71] (for a perturbative study see for example [72]). Even with sixteen supercharges it would be interesting to study compactifications to four dimensional models. The main complication for an extension in this direction is the lack of a precise control of the NS fivebrane physics, but we hope that further investigations can improve this impasse.

[2]In this case the analysis can be more subtle since the arguments which lead to the stability of the BPS spectrum of states are no longer valid. Indeed, for some explicitly solved $\mathcal{N} = 2$ super Yang-Mills models [69] it has been found curves of marginal instability where a BPS state crossing it can decay [70].

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