ISAS - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

Particle physics and Cosmology confront the Cosmological constant

Thesis submitted for the degree of

"Doctor Philosophiae"

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Preface

This thesis deals with some of the most recent developments on the cosmological constant problem in particle physics and cosmology.

The fact that a relevant cosmological term appears to be incredibly small compared to any 'natural' particle physics scale has puzzled both cosmologists and particle theorists for decades. Today, that there seems to be a mounting evidence for a non–vanishing cosmological constant, the puzzle is even more severe. Two coincidences await explanation: the smallness of Λ – why it is so small, some 120 orders of magnitude less than the Planck scale, but still not zero – and the time coincidence – why the vacuum contribution to the cosmological energy density is of the same order of the matter one *today*.

A conclusive proposal for setting the cosmological constant (almost) to zero has not yet come out. In the last couple of years, then, the attitude has become more phenomenological. Following the belief that the cosmological constant will ultimately be set to zero by some yet unknown high energy mechanism, most of the recent efforts have focussed on studying the cosmology of rolling scalar fields (also referred to as 'Quintessence'). These fields could easily provide the tiny vacuum energy density in excess from zero required by cosmology, if they happen to be displaced from the minimum of the potential or in the case that the minimum is not zero. Moreover, for some classes of potentials, an attraction mechanism has been shown to work, thus providing a natural framework in which to weaken the fine–tuning in the initial conditions. Whether particle physics can provide any realistic model of Quintessence is still an open question.

In this thesis we will focus on some basic issues connected with the attempt to build a particle physics phenomenology of scalar fields with inverse power potentials, following the results discussed in refs. [112, 122, 134, 135]. In Chapter 3 a multi–field model for Quintessence is discussed in the framework of Supersymmetric QCD. This model turns out to be a natural particle physics candidate for 'Quintessence' and the multi–scalar dynamics is shown to enrich the phenomenology with respect to the one–field case. In Chapter 4 the problems implied by the interaction of the quintessence scalar with the fields of the Standard Model of particle physics are discussed and a possible way out is proposed. In Chapter 5, then, the paradigm of 'Quintessential Inflation' is introduced, in which the scalar field driving inflation, the inflaton, and the quintessence scalar are identified. It is shown that in this framework the exit from inflation can uniquely set the initial conditions for the subsequent quintessential rolling.

The first two chapters are introductory and aim at giving a quick overview of some work of the past and of the motivations for considering the cosmological constant problem. These chapters are meant to be illustrative rather then comprehensive. A more in depth historical introduction and a complete list of references can be found in earlier reviews [33, 29, 116, 137, 163] on the subject. In Chapter 1 we recall some basic ideas about the role of the cosmological constant in particle physics and cosmology. The aim is to convince the reader that after more than 70 years we still have some good reason to bother about the cosmological constant. In Chapter 2, then, the idea of Quintessence is introduced and recent results about cosmological rolling scalar fields are discussed. Here we set the framework for the results that will be presented in the following chapters.

Finally, in Chapter 6, we present our conclusions.

We will use natural units in which $c = \hbar = 1$ and $G^{-1/2} = M_P \simeq 10^{19}$ GeV. The metric tensor has signature $(+ - - -)$.

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Chapter 1

Introduction

The cosmological constant problem is one of the deepest and longest-standing unresolved issues at the interface between cosmology and particle physics. If on one hand particle physics is expected to 'compute' its value or eventually find some symmetry that could make it vanish, on the other cosmology is the best experimental ground on which to test the theory. Unfortunately, since the cosmological constant was first introduced by Einstein in 1917, no compelling explanation from fundamental physics has been proposed so far. The smallness of Λ, as implied by the cosmological constraints, still remains a mistery[1].

The question whether the cosmological constant is identically zero or plays a relevant role in cosmology has not yet been definitively settled, but there seems to be mounting evidence of a non-negligible vacuum energy presently contributing to the total energy density of our universe. Surprisingly, as it was noticed long ago by Zeldovich [167, 168], what is a 'large' cosmological constant from the cosmologist's point of view (say a vacuum energy density contributing as much as the matter one) appears instead unimmaginably small in the particle physicist perspective (some 120 orders of magnitude smaller than the 'natural' scale). The attemp to reconcile this paradox is what goes under the name of 'the cosmological constant problem'.

But what is the cosmological constant? When we speak about the 'vacuum energy density' ρ_V to be added to the matter and radiation densities, ρ_M and ρ_R, to make up the whole of the energy content of the universe, we mean the sum of

[1]For recent reviews on the cosmological constant problem see refs. [33, 29, 116, 137, 163]. For a non–technical introduction see also [1]).

two contributions of very different nature.

On one hand there is the constant Λ term originally added by Einstein to his equations of the general theory of relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu} . \quad (1.1)$$

If $\Lambda > 0$ these equations admit a static universe as a solution, which was Einstein's original motivation for introducing this term[2]. A positive Λ in eq. (1.1) acts like a 'repulsive' force that counterbalances the attraction of gravity.

In addition to this, however, another contribution should be considered. From the particle physics point of view, the vacuum is not just empty space and so the contribution of the so-called 'zero-point energy' should be taken into account. In other words, this means that the energy–momentum tensor in general does not vanish in the quantum state of zero particle excitations:

$$\langle 0|T_{\mu\nu}|0\rangle = \epsilon_0 \, g_{\mu\nu} \neq 0 . \quad (1.2)$$

The fact that $\langle 0|T_{\mu\nu}|0\rangle$ is proportional to the metric tensor $g_{\mu\nu}$ comes from the constraint that the vacuum state is Lorentz invariant. The value of the ground state energy density $\epsilon_0 \equiv \langle 0|\rho|0\rangle$ has no physical effect in quantum field theory and can be arbitrarily renormalized to zero. The problem arises when we face gravity. The zero-point of the energy does matter! In fact, the energy of the vacuum gravitates as well and we are not free anymore to add or subtract constants.

When speaking of the cosmological constant, then, we will be concerned with the sum of these two effects:

$$\Lambda_{eff} = \Lambda + 8\pi G\epsilon_0 . \quad (1.3)$$

If we define ρ as the total energy density of the universe, it will be composed of three contributions

$$\rho = \rho_M + \rho_R + \rho_V \quad (1.4)$$

where ρ_V is given by

$$\rho_V \equiv \frac{\Lambda_{eff}}{8\pi G} = \frac{\Lambda}{8\pi G} + \epsilon_0 . \quad (1.5)$$

[2]We should remember that when in 1917 Einstein first introduced the cosmological constant he wasn't aware of the expansion of the universe which Hubble was going to discover only much later, in 1929. Despite that, Friedmann's solutions of the Einstein equations predicted a non–static universe already in the early 20's.

Now, how can we explain the fact that, as we will see in what follows, ρ_V is vanishingly small compared to any particle physics scale? In other words, why the two terms in eq. (1.5) almost exactly cancel? This is the cosmological constant problem.

1.1 The zero–point energy in quantum physics

In order to understand the problems connected with the energy of the vacuum in quantum physics, let us start with a simple example: the computation of the one–dimensional harmonic oscillator energy spectrum in quantum mechanics.

Consider a particle of mass m with position coordinate q and momentum p. And suppose that it is subject to a restoring force $F = -m\omega^2 q$ proportional to the distance from the origin. The corresponding one–dimensional Hamiltonian is

$$H = \frac{1}{2m}\left(p^2 + m^2\omega^2 q^2\right) , \tag{1.6}$$

and the position and momentum variables are connected by the commutation relations

$$[q,\, p] = i \tag{1.7}$$

where we have set $\hbar = 1$.

One can then introduce the creation and annihilation operators $a^\dagger$ and a

$$a^\dagger = \frac{1}{\sqrt{2}}(q - ip) \quad , \qquad a = \frac{1}{\sqrt{2}}(q + ip) \tag{1.8}$$

which are Hermitean conjugates of each other and satisfy

$$\left[a,\, a^\dagger\right] = 1 . \tag{1.9}$$

The Hamiltonian can then be rewritten as

$$H = \omega\left(N + \frac{1}{2}\right) \tag{1.10}$$

where we have defined the 'number' operator $N \equiv a^\dagger a$. This operator can be easily shown to have a spectrum of eigenvalues given by the non–negative integers $n \geq 0$, from which we can compute the zero point energy of the theory. The spectrum of the Hamiltonian is then

$$H_n = \omega\left(n + \frac{1}{2}\right) \quad \text{with} \quad n \geq 0 . \tag{1.11}$$

Since the vacuum is defined as the state of minimum energy of the system, we find that it gives a non–vanishing contribution

$$\langle 0|H|0\rangle \equiv H_0 = \frac{1}{2}\omega \neq 0 \ . \tag{1.12}$$

The situation does not change if we go to quantum field theory. In the simple case of the canonical quantization of a real scalar field $\phi(\mathbf{x}, t)$ of mass m obeying the field equation

$$(\Box + m^2)\phi = 0 \tag{1.13}$$

we obtain a similar result. The scalar field $\phi(\mathbf{x}, t)$ and its conjugate momentum $\pi(\mathbf{x}, t) \equiv \delta L/\delta\dot{\phi}$, being L the lagrangian of the system, are operators which satisfy the commutation relations

$$[\phi(\mathbf{x}, t),\ \phi(\mathbf{y}, t)] = i\ \delta^3(\mathbf{x} - \mathbf{y}) \tag{1.14}$$

and the solution of eq. (1.13) can be expressed as an integral over the modes $\mathbf{k}$

$$\phi(\mathbf{x}, t) = \int \frac{d\mathbf{k}}{[(2\pi)^3 2\omega_{\mathbf{k}}]^{1/2}} \left[a_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_{\mathbf{k}} t)} + a^\dagger_{\mathbf{k}} e^{-i(\mathbf{k}\cdot\mathbf{x} - \omega_{\mathbf{k}} t)}\right] \tag{1.15}$$

where we have defined $\omega_{\mathbf{k}} = \sqrt{m^2 + \mathbf{k}^2}$ as the energy of the mode $\mathbf{k}$.

The coefficients of the expansion $a_{\mathbf{k}}$ and $a^\dagger_{\mathbf{k}}$ are the operators which destroy or create a particle excitation of momentum $\mathbf{k}$, respectively, and obey the commutation relations

$$\left[a_{\mathbf{k}},\ a^\dagger_{\mathbf{k}'}\right] = \delta^3(\mathbf{k} - \mathbf{k}') \tag{1.16}$$

which can be derived from eq. (1.14). The Hamiltonian of the theory is then

$$H = \frac{1}{2}\int d\mathbf{k}\ \omega_{\mathbf{k}} \left[a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + a_{\mathbf{k}} a^\dagger_{\mathbf{k}}\right] = \int d\mathbf{k}\ \omega_{\mathbf{k}} \left[N_{\mathbf{k}} + \frac{1}{2}\right] \tag{1.17}$$

with the operator $N_{\mathbf{k}} = a^\dagger_{\mathbf{k}} a_{\mathbf{k}}$ counting the number of excitations of momentum $\mathbf{k}$ in a given state.

If we compute the expectation value of the Hamiltonian in the vacuum we find an infinite answer since we are now summing over all the possible energy levels of the system

$$\langle 0|H|0\rangle = \frac{1}{2}\int d\mathbf{k}\ \omega_{\mathbf{k}} \ . \tag{1.18}$$

This problem is usually overcome by introducing an *ad hoc* renormalization procedure named 'normal ordering' (indicated by the symbol ::), which estabilishes that the creation and annihilation operators should be ordered in such a way that the creation one is always to the left. The normal–ordered vev of the Hamiltonian gives

$$\langle 0|\, :H:\, |0\rangle = 0 \ . \tag{1.19}$$

By means of the normal ordering procedure, the zero–point energy of quantum field theory can then be safely renormalized to zero.

However we should be warned that while in quantum field theory the choice of the absolute normalization of the energy has no physical meaning, in cosmology instead it is crucial. Since all the energy contributions have a gravitational effect, we are not allowed anymore to throw away the energy of the vacuum but we should more carefully take it into account. Of course, in the integral (1.18) it does not have physical meaning to integrate all the modes up to infinity, but we should rather introduce an ultraviolet cutoff k_{max} in momentum space following the belief that the theory will not hold anymore beyond a certain wavelength.

W. Pauli was probably the first to wonder about the cosmological consequences of the zero point energy of quantized fields, as reported in [149]. But we will start with the first published attempt which was due to Zeldovich [167] in 1967. He tried to estimate the vacuum energy integrating out the shortest wavelengths

$$\langle 0|H|0\rangle \;=\; \frac{1}{2}\int_0^{k_{max}} d\mathbf{k}\ \omega_{\mathbf{k}} \ . \tag{1.20}$$

Defining $k \equiv |\mathbf{k}|$ and approximating $\omega_{\mathbf{k}} \simeq k$, we find for the vacuum energy density

$$\epsilon_0 \;\simeq\; \frac{1}{(2\pi)^3}\int_0^{k_{max}} k \cdot 4\pi k^2\, dk \;=\; \frac{k_{max}^4}{8\pi^2} \ . \tag{1.21}$$

When Zeldovich gave his estimate, he used the mass of the proton as the cutoff energy and obtained an energy density $\epsilon_0 \sim (1\ \mathrm{GeV})^4$. If we believe that general relativity is a good approximation up to the Planck scale we should instead take that as a cutoff and so obtain $\epsilon_0 \sim (10^{19}\ \mathrm{GeV})^4$ or, being more conservative and using the electroweak scale as the limiting one, $\epsilon_0 \sim (100\ \mathrm{GeV})^4$. As we will see below, these energy scales – which seem so far apart from each other – are indeed all much larger than any cosmologically allowed ρ_V.

It is worth noticing that, even if he chose a much lower *cutoff* than the Planck scale, Zeldovich already realized that the energy scale for ϵ_0 compatible with cosmology had to be 12 orders of magnitude smaller than what he got on dimensional grounds. That's why he attempted a first 'theory' of the cosmological constant, making the hypothesis that the two terms in Λ_{eff}, as defined in (1.3), exactly cancel and that the residual cosmological constant was due to higher order effects like the gravitational interaction between the particles [167, 168]. Also in this case, however, he was still 8 orders of magnitude above the cosmologically allowed energy, which is ~ 0.003 eV.

Another interesting exercise is the attempt to fit to cosmology the parameters of the Higgs mechanism for symmetry breaking of the electroweak theory [163]. If this is the vacuum in which we live, it should be straightforward to compute the vacuum energy density from the parameters of the standard model.

The potential for the Higgs field is

$$V(\phi) \;=\; V_0 \;-\; \mu^2\phi^\dagger\phi \;+\; \lambda(\phi^\dagger\phi)^2 \tag{1.22}$$

with V_0 an arbitrary constant of mass dimension 4.

The minimum of the potential (1.22) is in $\phi^2 = v^2 \equiv \mu^2/2\lambda$ and has a value

$$V_{min} \;=\; V(\phi^2 = v^2) \;=\; V_0 \;-\; \frac{\mu^4}{4\lambda} \;\equiv\; \epsilon_0 \;. \tag{1.23}$$

We can set $V_0 = 0$ in order to have the unstable maximum of the potential at $\phi = 0$ normalized to zero. As a consequence, we would find that ϵ_0 is negative definite

$$\epsilon_0 \;=\; -\lambda\, v^4 \;\simeq\; -\lambda\; (300 \text{ GeV})^4 \tag{1.24}$$

which is by far too large even if we fine tune λ to be very small. Another possibility is that the two terms in eq. (1.23) almost exactly cancel. We can then simply require that $V_0 \simeq \mu^4/4\lambda$, but also in this case we do not improve the fine–tuning issue. Instead of requiring the coupling constant λ to be as small as $\sim 10^{-56}$, we should explain a cancellation to the 56th digit.

Before switching to cosmology, a final remark is to be done. When computing the vacuum energy in quantum field theory, all the particle species are to be taken into account. In particular we have to sum over both bosons and fermions. A result analogous of that of eq. (1.18) for bosons can be obtained for fermions and the same expression is found but with a minus sign, due to the fact that fermions obey anticommutation and not commutation rules.

Since in supersymmetric theories fermion and boson partners are characterised by having equal masses [170], in this context we would easily find the total energy density of the vacuum to be identically zero. This is a consequence of the algebra obeyed by the supersymmetry generators

$$\left\{Q_\alpha,\, Q^\dagger_\beta\right\} \;=\; (\sigma_\mu)_{\alpha\beta}\, P^\mu \tag{1.25}$$

where α and β are two–component spinor indices, σ_i are the Pauli matrices, $\sigma_0 = 1$ and P^μ is the 4–momentum operator. The condition for supersymmetry to be unbroken is that the vacuum state $|0\rangle$ satisfies

$$Q_\alpha|0\rangle \;=\; Q^\dagger_\alpha|0\rangle \;=\; 0\ . \tag{1.26}$$

From the last two equations it is easily shown that the vacuum has zero energy–momentum

$$\langle 0|P^\mu|0\rangle \;=\; 0\ . \tag{1.27}$$

Quantum effects do not change this conclusion, since supersymmetry also ensures the cancellation of boson and fermion loops.

This result seems very encouraging: we could set the cosmological constant equal to zero invoking supersymmetry. Unfortunately in the real world in which we live, supersymmetry is broken and so the minimum of the energy is not zero anymore.

1.2 Consequences of the cosmological constant

Consider now a homogeneous and isotropic universe, with line element given by the Robertson–Walker metric

$$ds^2 \;=\; dt^2 \;-\; R(t)^2\left\{\frac{dr^2}{1-kr^2} + r^2 d\vartheta^2 + r^2\sin^2\vartheta d\varphi\right\} \tag{1.28}$$

where (r, ϑ, φ) are comoving spatial coordinates, t is the proper time and $R(t)$ is the scale factor of the universe which measures its size. The constant k can take values 0 or ± 1 and is related to the spatial curvature of the universe. For a spatially flat universe $k = 0$.

The Einstein equations then read

$$\frac{\ddot{R}}{R} \;=\; -\frac{4\pi G}{3}\sum_i(\rho_i + 3p_i) \tag{1.29}$$

$$\frac{\dot{R}^2}{R^2} \;=\; \frac{8\pi G}{3}\sum_i \rho_i \;-\; \frac{k}{R^2} \tag{1.30}$$

where the contribution of the different forms of energy in the universe, with equation of state $p = w_i \rho$, has been taken into account. We can also define the Hubble 'constant' H, which measures the expansion rate, and the decelleration parameter q, which says if the universe is accelerating or decellerating

$$H \equiv \frac{\dot{R}}{R} \tag{1.31}$$

$$q \equiv -\frac{\ddot{R}R}{\dot{R}^2} = -\frac{\ddot{R}}{RH^2} \ . \tag{1.32}$$

With the subscript 0 we will indicate the present value of these quantities.

Equation (1.29) can be used to express q_0 as a function of the present fraction densities in radiation, matter and vacuum energy,

$$\Omega_R = \frac{\rho_R}{\rho_c} \ , \qquad \Omega_M = \frac{\rho_M}{\rho_c} \quad \text{and} \quad \Omega_V = \frac{\rho_V}{\rho_c} \ , \tag{1.33}$$

with respect to the critical energy density

$$\rho_c \equiv \frac{3H_0^2}{8\pi G} \simeq 8h^2 \times 10^{-47} \ \text{GeV}^4 \quad , \qquad 0.6 \lesssim h \lesssim 0.8 \ , \tag{1.34}$$

and get

$$q_0 = \frac{1}{2} \sum \Omega_i \ (1 + w_i) \ . \tag{1.35}$$

We have introduced the adimensional constant h which accounts for the uncertainty in the measure of the Hubble constant

$$H_0 = 100h \ \text{km sec}^{-1} \ \text{Mpc}^{-1} \ . \tag{1.36}$$

Since we know that today the universe is very nearly critical, that means that ρ_0 is within an order of magnitude of ρ_c, any cosmologically relevant vacuum energy contribution should be of the same order. From this we see that the scale $\rho_V \simeq 10^{-47}$ GeV is the one which would have an effect on present day cosmology. And this is clearly by far too smaller than the particle physics predictions discussed in the previous Section.

From eq. (1.35) we note that in order to have an accelerated expansion, $q_0 < 0$, the dominating energy component should have a negative equation of state $w < -1/3$. This cannot happen if radiation ($w_R = 1/3$) or matter ($w_M = 0$) are dominating, but it is possible in the case of vacuum energy ($w_V = -1$). In particular, during the present epoch in which the radiation contribution is completely negligible, it is sufficient to have $\Omega_V > \Omega_M/2$ in order for the universe to accelerate, since in this case

$$q_0 = \frac{\Omega_M}{2} - \Omega_V \ . \tag{1.37}$$

The net effect of a non–vanishing vacuum energy is then to make the universe expand faster and faster until it approaches an exponential regime in which the scale factor grows like $R(t) \sim \exp\sqrt{\Lambda_{eff}/3}\, t$.

The Friedmann equation (1.30) can be rewritten as

$$1 - \sum \Omega_i = 1 - \Omega = \frac{k}{H^2 R^2} \tag{1.38}$$

and casts the evolution of Ω for all times . The product H^2R^2 is always decreasing with time: it goes like $\propto R/R_0 = (1+z)^{-1}$ during matter domination and like $\propto (R_{EQ}/R_0)(R/R_{EQ})^2 \simeq 10^4(1+z)^{-2}$ during radiation dominaton, where we have defined the redshift z

$$z \equiv \frac{R_0}{R(t)} - 1 \tag{1.39}$$

and R_{EQ} is the scale factor of the universe at matter–radiation equivalence.

We see that Ω tends to run away from the critical value $\Omega = 1$. Since we know that today Ω is certainly within an order of magnitude of one, the universe should have been *very* nearly critical in the past. For example, at nucleosynthesis ($t \sim$ 1sec) it should have been $|\Omega - 1| < \mathcal{O}(10^{-16})$, and at the Planck time ($t \sim 10^{-43}$sec) it should have been $|\Omega - 1| < \mathcal{O}(10^{-60})$. This fine–tuning can be avoided only if the universe is exactly critical. Only in this case, indeed, Ω would remain fixed to one. This problem, the so called 'flatness problem' of standard cosmology, is one of the main motivations for introducing the paradigm of inflation in the early universe. The exponential expansion driven by the energy density of the inflaton is a very natural scheme in which to obtain a critical energy density. It is because of this fine–tuning problem, and *not* as a consequence of inflation, that we will then set $\Omega = 1$ in the following. From eq. (1.38) we also see that a critical universe is spatially flat, since $\Omega = 1$ implies $k = 0$.

We can now study the evolution of the universe with $\Omega = \Omega_M + \Omega_V = 1$. Eq. (1.30) can be rewritten in a more convenient form as a function of Ω_M and Ω_V:

$$\frac{1}{H_0^2 R_0^2}\left(\frac{dR}{dt}\right)^2 = \Omega_M \left(\frac{R}{R_0}\right)^{-1} + \Omega_V \left(\frac{R}{R_0}\right)^2 \ . \tag{1.40}$$

We then see that if a positive cosmological term is present, no matter how small, it will always grow with respect to the matter density and thus eventually always dominate.

The first consequence of this fact is that, with $\Omega_V \neq 0$, 'geometry is not destiny' anymore, in the words of L.M. Krauss and M.S. Turner [94]. If $\Omega_V = 0$, it is straightforward to see from eqs. (1.29)–(1.30) that a positive curvature geometry ($k = 1$) corresponds to a recollapsing universe (the Hubble parameter can change sign), while a negative curvature ($k = -1$) or flat ($k = 0$) geometry correspond to an ever expanding universe (the Hubble parameter is always positive and tends to zero only in the flat case). If a cosmological constant is present, instead, this one–to–one correspondence is lost. Since the ratio of the matter to vacuum energy density scales like $\propto R^{-3}$, the cosmological constant could eventually dominate even if we start with a very low Ω_V. Imagine that we had at present Ω_M as large as 1.1 (corresponding to a closed geometry), with a vacuum component of just $\Omega_V = 10^{-3}$. This last contribution, absolutely undetectable at the present time, would be enough to speed up the universe in such a way to prevent it from recollapsing [94]. In the case of a critical universe, as we have seen from eqns. (1.37) and (1.40), if a non-vanishing cosmological constant is present, it would constantly grow with respect to the matter component and eventually dominate, whatever its initial value, speeding up the expansion in contrast to the 'traditional' slowing down of critical models. An inflationary phase would start as soon as $\Omega_V > \Omega_M/2$. On the same footing, it can be checked that if the cosmological constant is negative, *i.e.* $\Omega_V < 0$, then the universe will certainly recollapse, even if it is open, *i.e.* $\Omega < 1$.

We thus conclude that the fate of the universe is determined by the ultimate equation of state rather than geometry. In this perspective, we will be able to make definite predictions about the future destiny of the universe only when we will be able to compute, from high energy principles, the exact matter/energy content of the universe in all its components. More speculative discussions about the future of civilization in an ever–expanding universe can be found in refs. [63, 95, 145].

Another effect of the cosmological constant is on the expansion age of the universe. When $\Omega_M + \Omega_V = 1$ the age of the universe t_0 as a function of the vacuum fraction density Ω_V is given by the following expression [88]

$$t_0 = \frac{2}{3H_0} \Omega_V^{-1/2} \ln \left[\frac{1+\Omega_V^{1/2}}{(1-\Omega_V)^{1/2}} \right] . \quad (1.41)$$

If $\Omega_V = 0$ this gives the well known result $t_0 = 2/3H_0$. But, if a vacuum energy component is present, this has the effect of lengthening the expansion age of the universe. As an example $\Omega_V = 2\Omega_M = 2/3$ would give a universe older by a factor ~ 1.4, with respect to the $\Omega_V = 0$ case. Taking $H_0 = 70$ km sec^{-1} Mpc^{-1}, we find that $t_0 \simeq 9$ Gyr for $\Omega = \Omega_M = 1$ but $t_0 \simeq 13.5$ Gyr for $\Omega_M = 0.3$ and $\Omega_V = 0.7$.

At present, the best way to infer the presence of a non–vanishing cosmological constant is by measuring deviations from linearity of the Hubble's law. If a distant object has an absolute luminosity $\mathcal{L}$, we can define its 'luminosity distance' d_L in terms of the measured flux $\mathcal{F}$ [88]

$$d_L^2 \equiv \frac{\mathcal{L}}{4\pi\mathcal{F}} = R^2(t_0) r_1^2 (1+z)^2 . \quad (1.42)$$

where r_1 is the comoving coordinate at which the source emits light (that will be detected at time t_0 at the comoving coordinate $r = 0$).

The Hubble's law gives a relation between the luminosity distance and redshift of observed objects [88]

$$d_L = H_0 z + \frac{1}{2} H_0 (1 - q_0) z^2 + \dots . \quad (1.43)$$

This relation is linear for small redshifts, but might deviate from linearity at high redshifts if the decelleration parameter $q_0 \neq 1$. Observations of the most distance objects are thus of crucial importance for determining the sign of q_0 and as a consequence the energy balance between matter and vacuum in recent times. By plotting in the Hubble diagram the luminosity distance *vs* redshift of distant objects it is then possible to discriminate among different cosmological models. This is what is beeing done studying SuperNovae explosions of type Ia, which are supposed to be 'standard candles'.

1.3 Anthropic considerations

Several authors have also used anthropic considerations in the attempt to explain the smallness of the cosmological constant. The idea is that, even if we cannot account for the value of Λ_{eff} from particle physics arguments, we could perhaps say something requiring that it is not in contrast with the possibility of intelligent life to develop in the universe.

There exist several formulations of the anthropic principle (see for example [13, 27, 72]), but the basic idea underlying this principle is that we should take the fact that intelligent life has emerged in the universe as an additional constraint to cosmological model building.

For example, with respect to the problem at hand, applying the antropic principle means trying to evaluate the range of values for the cosmological constant which are compatible with life to emerge.

The first to use this approach with respect to the cosmological constant problem was S. Weinberg in 1987 [162] (see also refs. [163, 164]). He has found that the most stringent constraint coming from the anthropic principle is the requirement that sufficiently large gravitationally bound systems can form in the universe. In order for this to happen we need the vacuum energy to have been subdominant until redshift $z \gtrsim 4$, the time at which gravitational condensation started in the universe. Otherwise, the accelerated expansions would have prevented bound systems to form. This translates to an upper bound

$$\rho_V \lesssim 100\rho_M \ . \tag{1.44}$$

The anthropic principle, then, does not forbid a cosmological term. The question now becomes: if ρ_V could be as large as $100\rho_M$ from anthropic consideration, why is it not so?

The last limit applies to a positive cosmological term. If instead a negative term $\Lambda_{eff} < 0$ is present in the Einstein equations, the universe would recollapse in a time T [13]

$$T = \pi\sqrt{8\pi G|\rho_V|} \ . \tag{1.45}$$

We should then require that the universe lives long enough for the appearence of life, say $T \gtrsim 0.5H_0^{-1}$. So we get the stringent bound

$$|\rho_V| \lesssim \rho_c \tag{1.46}$$

for the case $\rho_V < 0$.

Recent refinements of these kinds of computations can be found in refs. [49, 110]. A similar point of view is that of Vilenkin and al. [159, 64, 65] who studied the cosmological constant problem in the context of quantum cosmology. In this approach the probability for the universe to have the observed values of the physical constants is evaluated in the space of all possible universes.

In conclusion, we have seen that the cosmological constant problem is still an open issue both in particle physics and cosmology. As we will see in the next chapter it has become even more important in recent times when the data seem to point in the direction of a cosmological constant dominated universe.

In this thesis we will mainly focus on the proposal that the candidate for the vacuum energy of the universe is a scalar field rolling down an inverse power potential. However, other possibilities have also been discussed recently. See for example refs. [109, 118, 136, 138, 148]. For earlier literature on the subject, see the references in the existing reviews [33, 29, 116, 137, 163].

An alternative interesting approach to the cosmological constant problem was discussed by R. Brandenberger. In ref. [21] he proposes a relaxation mechanism for the cosmological constant arising from the back–reaction effect in the evolution of the cosmological perturbations.

Chapter 2

Quintessence: a dynamical Λ

According to the editors of *Science* magazine [140], the breakthrough of the year 1998 was the discovery of the acceleration of the Universe. This discovery is a consequence of the study of a bunch of high–redshift Supernovae of type Ia (SNe Ia) by two independent teams, the High–z Supernova Search Team [71] and the Supernova Cosmology Project [141]. The observation of the distant supernovae allowed to measure deviations from linearity of the magnitude–redshift relation of the Hubble diagram. As we have already discussed in Chapter 1, the study of the most distant objects gives direct measurment of the decelleration parameter q_0 and so gives a crucial information about the relative content in matter and cosmological constant of the present Universe.

Nevertheless, we should be warned that the SNe Ia data are still under scrutiny and, although they are the only direct evidence for the present acceleration of the universe, they probably are not the most compelling. Indirect evidence coming from the clustered matter distribution and the Cosmic Microwave Background (CMB) data seems to point in the very same direction. It is thus not premature to seriously confront ourselves with the possibility of a cosmological constant–dominated universe.

2.1 Recent data

Supernovae Ia. The idea underlying the recent claim of an accelerating universe is the belief that Supernovae of type Ia (see for example [107]) are 'standard candles'. That means that their intrinsic luminosity does not depend on their redshift (i.e. on their age) and so they can be used as a reliable cosmological distance ladder. The apparent luminosity of SN Ia should then just depend on their redshift and on the expansion velocity of the universe while their light is travelling towards our telescopes. If they appear dimmer than expected, this could be explained with an increasing expansion velocity of the universe, which in mathematical terms translates to having $q_0 < 0$.

This is the essence of the results presented last year by the two independent teams studying high–z Supernovae. The Supernova Cosmology Project [123, 124, 125] examined a sample of 42 Sn Ia at redshifts between 0.18 and 0.83 and gave a joint probability distribution for the cosmological parameters which can be approximated by the relation $0.8\Omega_M - 0.6\Omega_V \simeq -0.2 \pm 0.1$. The High–z Supernova Search Team [131, 56, 57, 62, 99] studied different samples which add to a total of 25 Sn Ia at redshifts between 0.16 and 0.97 and, plotting Ω_M vs. Ω_V, concluded $\Omega_M = 0.24 \pm 0.10$ with the assumption $\Omega_0 = 1$. The results of the two teams are in good accordance and both point in the direction of a cosmological constant dominated universe. We should remember that both rely on the assumption of a flat universe ($\Omega_0 = 1$), but this seems widely justified for two different reasons. On one hand we have to face the fine–tuning problem connected with the cosmological evolution of $\Omega(t)$, as already mentioned in Chapter 1, and so setting $\Omega = 1$ seems the best option from the theoretical point of view. On the other we have mounting evidence from the existing CMB anisotropy data (see below) for a critical universe. And clustered matter constraints limit the density of matter to relatively low values. All this data, put together, give independent constraints on Ω_M and Ω_V and are consistent with the SN Ia results.

It should be remembered that the assumption of the SN Ia being standard candels is presently under investigation and evolutionary effects may well be sources of systematic errors in the determinations of the cosmological parameters [132, 133, 47, 48]. There also have been claims that the progressive dimming of the Supernovae explosions could be due to absorption of their light by the interstellar dust on the way to us [5, 142, 151]. The hypotheisis that the observed effects are due to the fact that we live in a non-homogeneous universe has also been put forward [31]. For what concerns the direct measurment of the acceleration of the universe by SN Ia, we should then await confirmation from the forthcoming data and a better understanding of the Supernovae evolution.

The age problem. The problem of observing objects which appear to be older than the whole universe has puzzled the astronomers for some time (see for example [91]). If the universe is decellerating, its age should be less than the Hubble time H_0^{-1}. In particular, in a flat matter dominated cosmology we have $t_0 = 2/3H_0 = 9.7$ Gyr $(65/h)$. This fact is in contrast with the measured age of the oldest objects observed. The globular cluster are in fact estimated to be old as much as $t_{gc} = 11.5 \pm 1.3$ Gyr. The contradiction got even worse in recent times when the measure of the Hubble constant started to converge to relatively high values [59] giving $57 < H_0 < 85$ km/sec/Mpc at 95% confidence level. A low density universe would somehow ease the situation, giving $t_0 < 12.5$ Gyr $(65/h)$ for $0.2 < \Omega_M < 1$.

The most elegant way out of the puzzle seems [91] the introduction of a cosmological constant whoose net effect is to change the relation t_0–H_0 and so to allow for an older universe with the same value of H_0. For example a flat accelerating universe with $\Omega_V < 0.8$ should have an age $t_0 < 16.6$ Gyr $(65/h)$. This could easily reconcile the measured globular clusters age [91].

The baryon problem. Another striking evidence of a low mass universe comes from the counting of baryons in clusters of galaxies (see [11, 92] and references therein). Since clusters are the largest bound systems observed in the universe, they are supposed to be 'fair samples' of the average distribution of the different matter components. We can then reasonably suppose that the ratio of the baryonic to total matter in a cluster is the same as in the whole universe. Observation of X-ray emissions, together with theoretical models of clusters, gives a lower bound $\Omega_b/\Omega_M = R \gtrsim 0.05h^{-3/2}$ to the ratio of baryonic to total matter densities. If $\Omega_M = \Omega = 1$, then $R = \Omega_b$. The problem arises since nucleosynthesis gives instead an upper bound on the total density of baryons in the universe: $\Omega_b h^2 \lesssim 0.02$. The two bounds are clearly inconsistent and could only be reconciled if $\Omega_M < 1$. This can happen either in a open universe, $\Omega_M = \Omega < 1$, or in a flat accelerating universe, $\Omega_M + \Omega_V = \Omega = 1$.

Cluster dynamics. The study of rich clusters of galaxies (see refs. [11, 26, 30]) via their dispersion velocitiy, temperature of the hot intercluster gas and gravitational lensing distorsions, is a very powerful tool for estimating the energy density of matter in the universe. If we suppose that the universe is not biased in such a way that most of the dark matter is distributed on the largest scales, well beyond the clusters, then the measured mass density of the clusters can be taken as a reliable indication of the average mass density of the whole universe. All the methods mentioned above point in the direction of a low mass universe, with $\Omega_M \simeq 0.2 \div 0.3$. Again this result is consistent both with a low–mass open universe and with a cosmological constant dominated flat universe. This is due to the fact that the vacuum energy does not cluster and so it does not come into play in the dynamical evolution of galaxies and clusters of galaxies.

CMB anisotropy. The study of Cosmic Microwave Background anisotropies will have a tremendous impulse in the next millennium with the launch of two new satellites, the American Microwave Anisotropy Probe (MAP) [113] and the European Planck surveyor [129]. Neverthless, still with the presently avaliable data, it is possible do draw some conclusions about the geometry of the universe (see for example [17, 46, 50, 105, 106, 166]). The number of multiple l corresponding to the first doppler peak in the CMB anisotropy spectrum is in fact inversely proportional to the total energy density of the universe, $l \simeq 200/\sqrt{\Omega}$. From the identification of the position of the first peak is then possible to discriminate between open or flat universes. The present data, coming from balloons or ground based experiments, are still very preliminary but they seem to coverge towards $\Omega \simeq 1$. A flat universe seems then to be favored with respect to a open, low–mass one.

All the above constraints, taken together, are consistent with a flat cosmological model with a vacuum energy contributing as much as twice the matter energy density [12, 98, 92, 154, 155]. It should be stressed that even without the SN Ia data, which awaits confirmation, the case for a Λ–dominated universe is strong (see for example refs. [93, 117]). As we have seen in this Section, a non–vanishing cosmological constant would solve many problems at the same time. Only one issue, in the dark matter context, would still remain open. Even with a large cosmological constant, we would still need a large fraction of the matter density to be non–baryonic. The hunt for WIMPs (Weakly Interacting Massive Particles) as the favorite candidtes for the cold dark matter in the universe remains a challange for particle physics and cosmology also in the presence of a cosmological constant.

2.2 The idea of Q

The approach that has recently received the largest attention differs somehow from the past. The basic assumption is that the cosmological constant Λ_{eff}

as defined in eq. (1.3) exactly vanishes. The cosmological constant problem, then, is not solved but it is assumed that, with a better understanding of the physics beyond the Standard model, a mechanism which could set $\Lambda_{eff} = 0$ will eventually be found. Some other kind of energy is then needed in order to provide the observed acceleration of the universe. We need something which could contribute nearly as much as the critical energy density and which has a negative equation of state.

In the following we will name 'Quintessence' any dynamical and slowly evolving form of energy with negative pressure which results to be smoothly distributed in the universe [24, 61, 153]. Its equation of state is defined as the ratio of the pressure to the energy density, $w_Q = p_Q/\rho_Q$.

By definition, quintessence cosmology can be distinguished from the cosmological constant case. Not only because of the equation of state that now can be $\neq -1$ but also because of its past history. In fact, while the energy density corresponding to the cosmological constant does not evolve in time, in the Quintessence case we might have a much richer dynamics.

The easiest way to model Quintessence is with a scalar field ϕ rolling down a potential $V(\phi)$. In this case the scalar contribution to the cosmological energy density will be

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) \tag{2.1}$$

and the corresponding equation of state is

$$w_\phi = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \ . \tag{2.2}$$

It can be easily verified that the scalar field, depending on its dynamics, can have an equation of state ranging from 1 to -1 and so, from this point of view, it is a suitable candidate for Quintessence. A very appealing feature of scalar field cosmology is that, for some classes of potentials it admits attractor solutions in phase space. That means that for a very wide range of initial conditions, the scalar energy density will converge to a well defined attractor behaviour. This property allows to study scalar field cosmology with a high degree of generality and opens the hope to weaken some of the many fine–tuning issues connected with the cosmological constant problem.

A cosmological scalar field is not the only candidate for Quintessence. There are other components which could give a negative but $\neq -1$ equation of state.

For example a network of frustrated topological defects (such as strings or domain walls) would exibit a negative equation of state as well [16, 44, 79, 143, 158]. In particular they are carachterized by $w = -n/3$, with n beeing the dimension of the defect. We thus obtain $w = -1/3$ for strings and $w = -2/3$ for domain walls.

Scalar fields minimally coupled to gravity have also been considered as candidates for quintessence [7, 9, 15, 38, 45, 73, 127, 156]. We will briefly comment on some consequences of this possibility in Chapter 4.

More 'exotic' possibilities have also been proposed for explaining the Sn Ia results. For example, it has been shown that time variability of the fine structure constant α [14, 96] or of the gravitational constant G [8] could account for the Sn Ia data. R.R. Caldwell, instead, has discussed [25] the possibility that the SN Ia data could be accounted for by a 'phantom' component with equation of state $w < -1$.

Quintessence and dynamical Λ cosmological models have recently also been confronted with the issue of structure formation in the universe. Calculation of the evolution of perturbations in such models and constraints coming from the observed large scale structure of the universe seem to be in good agreement with the hypothesis that most of the dark energy has negative equation of state [10, 32, 37, 121, 126, 157].

At the same time, a great effort is beeing spent in the attempt to constrain the equation of state of the unknown dark component with present and future data [51, 74, 115, 160]. This could potentially resolve the degeneracy with the pure cosmological constant case [75] and also give some information about the scalar potential of Quintessence [76, 146]. Much other work on the observational side is on the way [171].

2.3 Scalar fields and attractors

In this section we will summarize some important results about rolling scalar fields in cosmology (see [119, 130, 165] and more recently [36, 54, 55, 100, 147, 169]). In particular we will focus on two types of potentials[1]: the exponentials, $V \sim e^{-\phi}$, and the inverse powers, $V \sim \phi^{-p} \ , \quad p > 0$. These potentials show an

[1]The cosmology of a scalar field with potential $V(\phi) = V_0(\cosh \lambda\phi - 1)^p$ is discussed by the authors of ref. [139]. In ref. [6] a potential which goes like $V(\phi) = V_p(\phi)\exp(-\lambda\phi)$, where V_p is a polinomial in ϕ, was proposed as a model for quintessence.

attractor behaviour in the evolution of the scalar field, with solutions which have respectively been denominated 'scaling' and 'tracking' attractors in literature.

If we consider the cosmological evolution of a scalar field ϕ with potential $V(\phi)$ in the approximation that its energy density is subdominant with respect to the background, *i.e.* $\rho_\phi \ll \rho_B$, (where the subscript B stands for radiation or matter), we have to solve the equations:

$$\begin{aligned} \ddot{\phi} + 3H\,\dot{\phi} &= -\frac{dV}{d\phi} , \\ H^2 &= \frac{8\pi G}{3}\left[\rho_B + \frac{\dot{\phi}^2}{2} + V(\phi)\right] , \end{aligned} \tag{2.3}$$

together with the assumption that the scalar field has a perfect fluid equation of state $w_\phi = p_\phi/\rho_\phi$.

Scaling solutions. In the case of an exponential potential,

$$V(\phi) = V_0 \, \exp(-\lambda\phi) , \tag{2.4}$$

the solution $\phi \sim \ln t$ is, under very general conditions [36, 54, 55, 100, 165], a 'scaling' attractor in phase space characterized by

$$\frac{\rho_\phi}{\rho_B + \rho_\phi} = \frac{m}{\lambda^2} , \tag{2.5}$$

where $m = 3, 4$ in the case of MD or RD respectively. Note that the ratio of the scalar to background energy density is independent on the scale V_0 in the potential. If $\lambda^2 > m$ this is the unique late–time attractor in phase space. This could potentially solve the so called 'cosmic coincidence' problem, providing a dynamical explanation for the order of magnitude equality between matter and scalar field energy today. Unfortunately, the equation of state for this attractor turns out to be

$$w_\phi = w_B , \tag{2.6}$$

which cannot explain the acceleration of the universe neither during RD ($w_R = 1/3$) nor during MD ($w_M = 0$). Moreover, Big Bang nucleosynthesis constrains the field energy density [36, 54, 55, 100] to values much smaller than the required $\Omega_\phi \sim 2/3$.

Traking solutions. If instead an inverse power-law potential is considered,

$$V(\phi) = M^{4+p}\,\phi^{-p} , \quad \text{with } p > 0 \tag{2.7}$$

the attractor solution is [100, 147, 169]

$$\phi \sim t^{1-n/m} \ , \qquad \text{with} \ \ n = 3(w_\phi + 1) \ , \ \ m = 3(w_B + 1) \ ; \tag{2.8}$$

and the equation of state turns out to be

$$w_\phi \ = \ \frac{w_B \, p - 2}{p + 2} , \tag{2.9}$$

which is always negative during MD ($w_B = w_M = 0$). This solutions maintains the condition

$$V'' \ = \ \frac{9}{2} \, (1 - w_\phi^2) \ \frac{p+1}{p} \ H^2 \tag{2.10}$$

and this implies that the scalar ϕ must be of order M_P today if ρ_ϕ is beginning to dominate, since $V'' \simeq \rho_\phi/\phi^2$ and $H^2 \simeq \rho_\phi/M_P^2$. Then we obtain that the ratio of the energies is not constant but scales as

$$\rho_\phi/\rho_B \ \sim \ R^{m-n} \tag{2.11}$$

thus growing during the cosmological evolution, since $n < m$. We see that ρ_ϕ could have been safely small during nucleosynthesis and have grown lately up to the phenomenologically interesting values. These solutions are then good candidates for Quintessence and have been denominated 'trackers' in the literature [100, 147, 169].

It should be noted that the inverse power-law potential does not improve the cosmic coincidence problem with respect to the cosmological constant case. Indeed, the scale M has to be fixed from the requirement that the scalar energy density today is exactly what is needed. This corresponds to choosing the desired tracker path. An important difference exists in this case though. The initial conditions for the physical variable ρ_ϕ can vary between the present critical energy density ρ_c^0 and the background energy density ρ_B at the time of beginning [147] (this range can span many tens of orders of magnitude, depending on the initial time), and will anyway end on the tracker path *before* the present epoch, due to the presence of an attractor in phase space [147, 169]. On the contrary, in the cosmological constant case, the physical variable ρ_Λ is fixed once for all at the beginning. This allows us to say that in the quintessence case the fine-tuning issue, even if still far from solved, is at least weakened.

Chapter 3

A particle physics model of Quintessence

As we have seen in the previous chapters, a great effort has been recently devoted to study the cosmology of Quintessence models and to find ways to constrain them with present and future data. On the other hand, the investigation of quintessence models from the particle physics point of view is just in a preliminary stage and a realistic model is still missing[1]. There are two classes of problems: the construction of a field theory model with the required scalar potential and the interaction of the quintessence field with the standard model (SM) fields. As first pointed out by Binétruy [20], scalar inverse power potentials appear in supersymmetrcic QCD (SQCD) theories [3, 4, 150] with N_c colors and $N_f < N_c$ flavors (for a pedagogical introduction see [128]). The second problem, instead, seems the thoughest [28] (see also [19]). Indeed the quintessence scalar today has typically a mass of order $H_0 \sim 10^{-33}$eV, then in general it would mediate long range interactions of gravitational strength, which are phenomenologically unacceptable.

In this and the next chapter we will consider in more detail these problems in the framework of SQCD (SQCD), following the treatment of ref. [112, 134]. In particular we note that these models have N_f independent scalar directions in the vacuum manifold. In ref. [20] all of them were given the same initial conditions, so that the dynamics reduced effectively to that of a single scalar field with an inverse power law potential. On the other hand, in a cosmological setting there is no a priori justification for this assumption, and the fields will in general start

[1]Some different proposals can be found in refs. [18, 20, 22, 39, 60, 80, 81].

from different initial conditions. The dynamics is then truly N_f-dimensional and it is relevant to know what is the late time behavior of the system, whether there are other attractors besides the single-scalar one discussed in ref. [20], and under what conditions is the latter reached by the system. Indeed, we will see that starting with the same initial total energy but different initial conditions for the N_f fields may prevent them to reach the attractor, so that SQCD cannot be considered as a simple one-scalar model for quintessence in these regions of initial conditions phase space [112, 134].

3.1 SUSY QCD

As already noted by Binètruy [20], supersymmetric QCD theories with N_c colors and $N_f < N_c$ flavors [3, 4, 150] may give an explicit realization of a model for quintessence with an inverse power law scalar potential. The remarkable feature of these theories is that the superpotential is exactly known non-perturbatively. Moreover, in the range of field values that will be relevant for our purposes (see below) quantum corrections to the Kähler potential are under control. As a consequence, we can study the scalar potential and the field equations of motion of the full quantum theory, without limiting ourselves to the classical approximation.

The matter content of the theory is given by the chiral superfields Q_i and $\overline{Q}_i$ $(i = 1 \ldots N_f)$ transforming according to the N_c and $\overline{N}_c$ representations of $SU(N_c)$, respectively. In the following, the same symbols will be used for the superfields Q_i, $\overline{Q}_i$, and their scalar components.

Supersymmetry and anomaly-free global symmetries constrain the superpotential to the unique *exact* form

$$W = (N_c - N_f) \left(\frac{\Lambda^{(3N_c - N_f)}}{\det T} \right)^{\frac{1}{N_c - N_f}} \tag{3.1}$$

where the gauge-invariant matrix superfield $T_{ij} = Q_i \cdot \overline{Q}_j$ appears. Λ is the only mass scale of the theory. It is the supersymmetric analogue of Λ_{QCD}, the renormalization group invariant scale at which the gauge coupling of $SU(N_c)$ becomes non-perturbative. As long as scalar field values $Q_i, \overline{Q}_i \gg \Lambda$ are considered, the theory is in the weak coupling regime and the canonical form for the Kähler potential may be assumed. The scalar and fermion matter fields have then canonical kinetic terms, and the scalar potential is given by

$$V(Q_i, \overline{Q}_i) = \sum_{i=1}^{N_f} \left(|F_{Q_i}|^2 + |F_{\overline{Q}_i}|^2 \right) + \frac{1}{2} D^a D^a \tag{3.2}$$

where $F_{Q_i} = \partial W / \partial Q_i$, $F_{\overline{Q}_i} = \partial W / \partial \overline{Q}_i$, and

$$D^a = Q_i^\dagger t^a Q_i - \overline{Q}_i t^a \overline{Q}_i^\dagger \ . \tag{3.3}$$

The relevant dynamics of the field expectation values takes place along directions in field space in which the above D-term vanish, *i.e.* the perturbatively flat directions $\langle Q_{i\alpha} \rangle = \langle \overline{Q}_{i\alpha}^\dagger \rangle$, where $\alpha = 1 \cdots N_c$ is the gauge index. At the non-perturbative level these directions get a non vanishing potential from the F-terms in (3.2), which are zero at any order in perturbation theory. Gauge and flavor rotations can be used to diagonalize the $\langle Q_{i\alpha} \rangle$ and put them in the form

$$\langle Q_{i\alpha} \rangle = \langle \overline{Q}_{i\alpha}^\dagger \rangle = \begin{array}{ll} q_i \delta_{i\alpha} & 1 \leq \alpha \leq N_f \\ 0 & N_f \leq \alpha \leq N_c \end{array} . \tag{3.4}$$

Along these directions, the scalar potential is given by

$$\begin{aligned} v(q_i) &\equiv \langle V(Q_i, \overline{Q}_i) \rangle = 2 \, \frac{\Lambda^{2a}}{\prod_{i=1}^{N_f} |q_i|^{4d}} \left(\sum_{j=1}^{N_f} \frac{1}{|q_j|^2} \right) , \\ a &= \frac{3N_c - N_f}{N_c - N_f} , \qquad d = \frac{1}{N_c - N_f} . \end{aligned}$$

In the following, we will be interested in the cosmological evolution of the N_f expectation values q_i, given by

$$\langle \ddot{Q}_i + 3H\dot{Q}_i + \frac{\partial V}{\partial Q_i^\dagger} \rangle = 0 \ \ , \ i = 1, ..., N_f \ . \tag{3.5}$$

In ref. [20] the same initial conditions for all the N_f VEV's and their time derivatives were chosen. With this very peculiar choice the evolution of the system may be described by a single VEV q (which we take real) with equation of motion

$$\ddot{q} + 3H\dot{q} - g \frac{\Lambda^{2a}}{q^{2g+1}} = 0 \ , \qquad g = \frac{N_c + N_f}{N_c - N_f} \ , \tag{3.6}$$

thus reproducing exactly the case of a single scalar field Φ in the potential $V = \Lambda^{4+2g} \Phi^{-2g}/2$ considered in refs. [100, 119, 130, 147]. We instead will consider the more general case in which different initial conditions are assigned to different VEV's [112, 134], and the system is described by N_f coupled differential equations. Taking for illustration the case $N_f = 2$, we will have to solve the equations

$$\ddot{q}_1 + 3H\dot{q}_1 - d \cdot q_1 \frac{\Lambda^{2a}}{(q_1 q_2)^{2dN_c}} \left[2 + N_c \frac{q_2^2}{q_1^2}\right] = 0 , \tag{3.7}$$

$$\ddot{q}_2 + 3H\dot{q}_2 - d \cdot q_2 \frac{\Lambda^{2a}}{(q_1 q_2)^{2dN_c}} \left[2 + N_c \frac{q_1^2}{q_2^2}\right] = 0 , \tag{3.8}$$

with $H^2 = 8\pi/3M_P^2\ (\rho_m + \rho_r + \rho_Q)$, where M_P is the Planck mass, $\rho_{m(r)}$ is the matter (radiation) energy density, and $\rho_Q = 2(\dot{q}_1^2 + \dot{q}_2^2) + v(q_1, q_2)$ is the total field energy.

3.2 The tracker solution

In analogy with the one-scalar case, we look for power-law solutions of the form

$$q_{tr,i} = C_i \cdot t^{p_i} , \qquad i = 1, \cdots, N_f . \tag{3.9}$$

It is straightforward to verify that – when $\rho_Q \ll \rho_B$ – the only solution of this type is given by

$$p_i \equiv p = \frac{1-r}{2} , \qquad C_i \equiv C = \left[X^{1-r}\ \Lambda^{2(3-r)}\right]^{1/4} , \qquad i = 1, \cdots, N_f , \tag{3.10}$$

with

$$X \equiv \frac{4\ m\ \ (1+r)}{(1-r)^2\ [12 - m(1+r)]} , \tag{3.11}$$

where we have defined $r \equiv N_f/N_c\ (= 1/N_c, \ldots, 1 - 1/N_c)$. This solution is characterized by an equation of state

$$w_Q = \frac{1+r}{2} w_B - \frac{1-r}{2} . \tag{3.12}$$

Eq. (3.12) can be derived as usual from energy conservation, $d(R^3 \rho_Q) = -3\ R^2 p_Q$.

Following the same methods employed in ref. [100] one can show [112] that the above solution is the unique stable attractor in the space of solutions of eqs. (3.8). Then, even if the q_i's start with different initial conditions, there is a region in field configuration space such that the system evolves towards the equal fields solutions (3.9), and the late-time behavior is indistinguishable from the case considered in ref. [20].

The field energy density grows with respect to the matter energy density as

$$\frac{\rho_Q}{\rho_m} \sim R^{\frac{3(1+r)}{2}} , \tag{3.13}$$

where a is the scale factor of the universe. The scalar field energy will then eventually dominate and the approximations leading to the scaling solution (3.9) will drop, so that a numerical treatment of the field equations is mandatory in order to describe the phenomenologically relevant late-time behavior.

The scale Λ can be fixed requiring that the scalar fields are starting to dominate the energy density of the universe today and that both have already reached the tracking behavior. The two conditions are realized if

$$v(q_0) \simeq \rho_c^0 , \qquad v''(q_0) \simeq H_0^2 , \tag{3.14}$$

where $\rho_c^0 = 3M_P^2 H_0^2/8\pi$ and q_0 are the present critical density and scalar fields VEV respectively. Eqs. (3.14) imply

$$\frac{\Lambda}{M_P} \simeq \left[\frac{3}{4\pi}\frac{(1+r)(3+r)}{(1-r)^2}\frac{1}{rN_c}\right]^{\frac{1+r}{2(3-r)}} \left(\frac{1}{2rN_c}\frac{\rho_c^0}{M_P^4}\right)^{\frac{1-r}{2(3-r)}} , \tag{3.15}$$

$$\frac{q_0^2}{M_P^2} \simeq \frac{3}{4\pi}\frac{(1+r)(3+r)}{(1-r)^2}\frac{1}{rN_c} . \tag{3.16}$$

Depending on the values for N_f and N_c, Λ and q_0/Λ assume widely different values. Λ takes its lowest possible values in the $N_c \to \infty$ (N_f fixed) limit, where it equals $4 \cdot 10^{-2}(h^2/N_f^2)^{1/6}$ GeV (we have used $\rho_c^0/M_P^4 = (2.5 \cdot 10^{-31} h^{1/2})^4$). For fixed N_c, instead, Λ increases as N_f goes from 1 to its maximum allowed value, $N_f = 1 - N_c$. For $N_c \gtrsim 20$ and N_f close to N_c, the scale Λ exceeds M_P.

The accuracy of the determination of Λ given in (3.15) depends on the present error on the measurements of H_0, *i.e.,* typically,

$$\frac{\delta\Lambda}{\Lambda} = \frac{1-r}{3-r}\frac{\delta H_0}{H_0} \lesssim 0.1 . \tag{3.17}$$

In deriving the scalar potential (3.2) and the field equations (3.8) we have assumed that the system is in the weakly coupled regime, so that the canonical form for the Kähler potential may be considered as a good approximation. This condition is satisfied as long as the fields' VEVs are much larger than the non-perturbative scale Λ. From eqs. (3.15) and (3.16), one can compute the ratio between the VEVs today and Λ, and see that it is greater than unity for any N_f as long as $N_c \lesssim 20$.

On the other hand, if we want to follow the cosmological evolution of the fields starting from an earlier cosmological epoch, we must impose the stronger condition that the q_i have been much larger than Λ throughout the time interval of interest. Taking the tracker solution (3.9) as a reference, at the initial redshift z_{in} we have,

$$\frac{q_{tr}^{in}}{\Lambda} = \frac{q_{tr,0}}{\Lambda}\frac{q_{tr}^{in}}{q_{tr,0}} = \frac{q_{tr,0}}{\Lambda} \left\{ \begin{array}{ll} z_{in}^{r-1}\, z_{eq}^{(1-r)/4} & \text{if} \quad z_{in} > z_{eq} \\ z_{in}^{3(r-1)/4} & \text{if} \quad z_{in} \leq z_{eq} \end{array} \right. , \tag{3.18}$$

where z_{eq} is the redshift at matter-radiation equivalence. For a given r, the condition $q_{tr}^{in} \gg \Lambda$ gives an upper bound on z_{in}. Taking for instance $N_c \to \infty$ (N_f fixed) we get $z_{in} \ll 10^{21}(N_f h)^{-1/3}$. In the numerical computations, we will consider initial conditions such that the weak coupling regime is always realized.

3.3 The two-fields dynamics

In this section we illustrate the general results of the previous sections for the particular case $N_f = 2$, $N_c = 6$.

In Fig.3.1 the energy densities *vs.* redshift are given. We have chosen the same initial conditions for the two VEVs, in order to effectively reproduce the one-scalar case of eq. (3.6), already studied in refs. [100, 119, 130, 147]. No interaction with other fields of the type discussed in the previous section has been considered.

We see that, depending on the initial energy density of the scalar fields, the tracker solution may (long dashed line) or may not be reached before the present epoch. The latter case happens either when initial scalar field energy is larger than ρ_B (dash-dotted line) or when it is smaller then the critical energy density today. Thus, all together, there are around 35 orders of magnitude in ρ_Q^{in} at redshift $z+1 = 10^{10}$ for which the tracker solution is reached before today. Cleary, the more we go backwards in time, the larger is the allowed initial conditions range. Correspondingly, the initial fields' values may range between the lower bound $q_{in}^2/M_P^2 \simeq [(1+z_{eq})/(1+z_{in})^4]^{1/g}$, where g is defined in eq. (3.6), and the upper bound given by the right hand side of eq. (3.18). In Fig.3.2 we plot the scalar field equation of state, w_Q, for the corresponding solutions of Fig.3.1.

Next, we explore to which extent the two-field system that we are considering behaves as a one scalar model with inverse power-law potential. In fig. 3.3 we plot solutions with the same initial energy density but different ratios between the initial values of the two scalar fields. Given an initial energy density such that – for $q_1^{in}/q_2^{in} = 1$ – the tracker is joined before today, there is always a limiting value for the fields' difference above which the attractor is not reached.

In conclusion, we have taken in full consideration the multi-scalar nature of the model, allowing for different initial conditions for the N_f independent scalar VEVs and studying the coupled system of N_f equations of motion. We have found that, starting with the same initial scalar energy density, but different fields' values, the tracking behavior becomes more difficult to reach the larger the difference among the initial conditions for the fields. Thus, an approximate flavor symmetry of the initial conditions is needed in order that SQCD may act as an effective quintessence model.

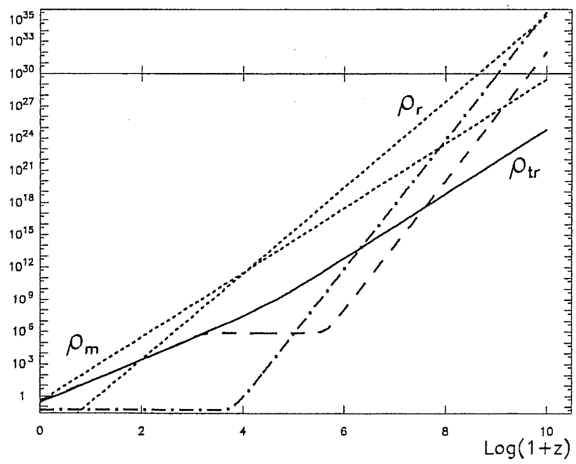

Figure 3.1: *The evolution of the energy densities ρ of different cosmological components is given as a funcion of red-shift. All the energy densities are normalized to the present critical energy density ρ^0_{cr}. Radiation and matter energy densities are represented by the short-dashed lines, whereas the solid line is the energy density of the tracker solution discussed in Section 3. The long-dashed line is the evolution of the scalar field energy density for a solution that reaches the tracker before the present epoch; while the dash-dotted line represents the evolution for a solution that overshoots the tracker to such an extent that it has not yet had enough time to re-join the attractor.*

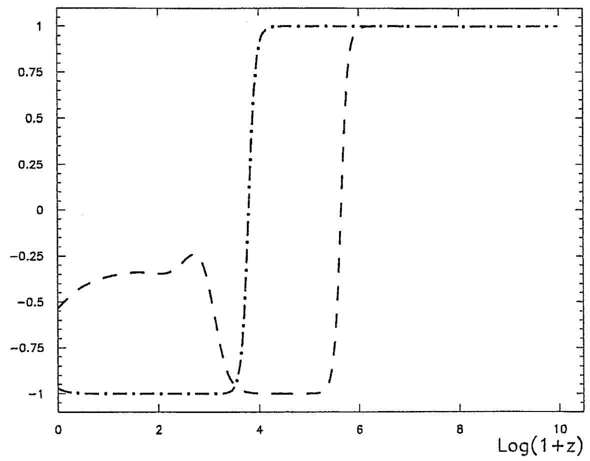

Figure 3.2: *The cosmological evolution of the equation of state $w_Q \equiv p_Q/\rho_Q$ for the scalar field Q is plotted as a function of red-shift. The two cases correspond to the energy densities of the two examples in Fig.1. Note that, in the long-dashed curve case, the attractor value of $-1/3$ for the equation of state (corresponding to $N_c = 6$) is joined well before the present epoch and only very recently abandoned, when the scalar field starts to become the dominating component of our universe and as a consequence its equation of state is rapidly driven towards -1.*

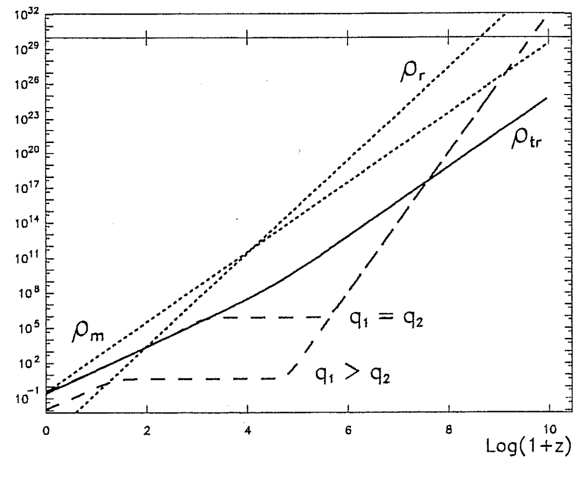

Figure 3.3: *The effect of taking different initial conditions for the fields, at the same initial total field energy. Starting with $q_1^{in}/q_2^{in} = 10^{15}$ the tracker behavior is not reached today. For comparison we plot the solution for $q_1^{in}/q_2^{in} = 1$.*

Chapter 4

Interaction with the visible sector

The second aspect that we will consider [112], in the attempt to build a particle physics motivated model for Quintessence, is the interaction of the scalars of the Supersymmetric $SU(N_c)$ model discusssed in the previous chapter with the Standard model (SM) fields. The quintessence fields are usually assumed to be singlets under the SM gauge group and to interact with the rest of the world only gravitationally, *i.e.* via non renormalizable operators suppressed by inverse powers of the Planck mass. This is however not enough. In order to prevent long-range interactions of gravitational strength it is necessary to assume that the – a priori unknown – couplings between the quintessence fields and the SM sector are strongly suppressed today. We do not solve this problem, but point out that if a *least coupling principle* of the type proposed by Damour and Polyakov [40] for the superstring dilaton were operative, quintessence models could be reconciled with the experimental constraints on the weak equivalence principle and on time variation of the SM coupling constants [112]. At the same time, during RD it would be quite likely to have SUSY breaking and mass generation for the quintessence fields, with masses proportional to H, by the same mechanism discussed by Dine, Randall, and Thomas in [43]. If present, these time-dependent SUSY breaking masses would prevent the fields from taking large values, thus driving the system towards the tracker solution [112].

4.1 The problem

In the treatment of the previous Chapter, the superfields Q_i and $\overline{Q}_i$ have been taken as singlets under the SM gauge group. Therefore, they may interact with the visible sector only gravitationally, *i.e.* via non-renormalizable operators suppressed by inverse powers of the Planck mass, of the form

$$\int d^4\theta\ K^j(\phi_j^\dagger, \phi_j)\ \left[\beta_n^{ji}\frac{\left(Q_i^\dagger Q_i\right)^n}{M_P^{2n}} + \overline{\beta}_n^{ji}\frac{\left(\overline{Q}_i\overline{Q}_i^\dagger\right)^n}{M_P^{2n}}\right]\ , \tag{4.1}$$

where ϕ_j represents a generic standard model superfield. From (3.15) we know that today the VEV's q_i are typically $O(M_P)$, so there is no reason to limit ourselves to the contributions of lowest order in $|Q|^2/M_P^2$. Rather, we have to consider the full (unknown) functions

$$\beta^{ji}\left[\frac{Q_i^\dagger Q_i}{M_P^2}\right]\ \equiv\ \sum_{n=0}^{\infty}\beta_n^{ji}\frac{\left(Q_i^\dagger Q_i\right)^n}{M_P^{2n}}\ , \tag{4.2}$$

and the analogous $\overline{\beta}$'s for the $\overline{Q}_i$'s. Moreover, the requirement that the scalar fields are on the tracking solution today, eqs. (3.14) implies that their mass is of order $\sim H_0^2 \sim 10^{-33}$ eV.

The exchange of very light fields gives rise to long-range forces which are constrained by tests on the equivalence principle, whereas the time dependence of the VEV's induces a time variation of the SM coupling constants [40, 28]. These kind of considerations sets stringent bounds on the first derivatives of the β^{ji}'s and $\overline{\beta}^{ji}$'s *today,*

$$\alpha^{ji}\ \equiv\ \left.\frac{d\log\beta^{ji}\left[x_i^2\right]}{dx_i}\right|_{x_i=x_i^0}\ ,\qquad\qquad \overline{\alpha}^{ji}\ \equiv\ \left.\frac{d\log\overline{\beta}^{ji}\left[x_i^2\right]}{d\,x_i}\right|_{x_i=x_i^0}\ , \tag{4.3}$$

where $x_i \equiv q_i/M_P$. To give an example, the best bound on the time variation of the fine structure constant comes from the Oklo natural reactor. It implies that $|\dot{\alpha}/\alpha| < 10^{-15}\ \mathrm{yr}^{-1}$ [41], leading to the following constraint on the coupling with the kinetic terms of the electromagnetic vector superfield V,

$$\alpha^{Vi},\ \overline{\alpha}^{Vi}\ \lesssim\ 10^{-6}\,\frac{H_0}{\langle\dot{q}_i\rangle}\,M_P\,, \tag{4.4}$$

where $\langle\dot{q}_i\rangle$ is the average rate of change of q_i in the past 2×10^9yr.

Similar –although generally less stringent– bounds can be analogously obtained for the coupling with the other standard model superfields [40]. Therefore, in order to be phenomenologically viable, any quintessence model should postulate that all the unknown couplings β^{ji}'s and $\overline{\beta}^{ji}$'s have a common minimum close to the actual value of the q_i's[1].

4.2 A possible way out

The simplest way to realize this condition would be via the *least coupling principle* introduced by Damour and Polyakov for the massless superstring dilaton in ref. [42], where a universal coupling between the dilaton and the SM fields was postulated. In the present context, we will invoke a similar principle, by postulating that $\beta^{ji} = \beta$ and $\overline{\beta}^{ji} = \overline{\beta}$ for any SM field ϕ_j and any flavor i. For simplicity, we will further assume $\beta = \overline{\beta}$.

The decoupling from the visible sector implied by bounds like (4.4) does not necessarily mean that the interactions between the quintessence sector and the visible one have always been phenomenologically irrelevant. Indeed, during radiation domination the VEVs q_i were typically $\ll M_P$ and then very far from the postulated minimum of the β's. For such values of the q_i's the β's can be approximated as

$$\beta\left[\frac{Q^\dagger Q}{M_P^2}\right] = \beta_0 + \beta_1 \frac{Q^\dagger Q}{M_P^2} + \dots \tag{4.5}$$

where the constants β_0 and β_1 are not directly constrained by (4.4). The coupling between the (4.5) and the SM kinetic terms, as in (4.1), induces a SUSY breaking mass term for the scalars of the form [43]

$$\Delta L \sim H^2 \beta_1 \sum_i \left(|Q_i|^2 + \left|\overline{Q}_i\right|^2\right) , \tag{4.6}$$

where we have used the fact that $\langle \sum_j \int d^4\theta K^j(\phi_j^\dagger, \phi_j) \rangle \sim \rho_R$ during radiation domination.

If present, this term would have a very interesting impact on the cosmological evolution of the fields. First of all one should notice that, unlike the usual mass terms with time-independent masses considered in [90], a mass $m^2 \sim H^2$ does not modify the time-dependence of the tracking solution, which is still the power-law given in eq. (3.9). Thus, the fine-tuning problems induced by the requirement

[1]An alternative way to suppress long-range interactions, based on an approximate global symmetry, was proposed in ref. [28].

that a soft-supersymmetry breaking mass does not spoil the tracking solutions [90] are not present here.

Secondly, since the Q and $\overline{Q}$ fields do not form an isolated system any more, the equation of state of the quintessence fields is not linked to the power-law dependence of the tracking solution. Taking into account the interaction with the SM fields, represented by H^2, we find the new equation of state during radiation domination ($w_B = 1/3$)

$$w'_Q = w_Q - 4\beta_1 \frac{1+r}{9(1-r)+6\beta_1} \tag{4.7}$$

where w_Q was given in eq. (3.12) and $r = N_f/N_c$.

From a phenomenological point of view, the most relevant effect of the presence of mass terms like (4.6) during radiation domination resides in the fact that they rise the scalar potential at large fields values, inducing a (time-dependent) minimum. In absence of such terms, if the fields are initially very far from the origin, they are not able to catch up with the tracking behavior before the present epoch, and ρ_Q always remains much smaller than ρ_B. These are the well-known 'undershoot' solutions considered in ref. [147]. Instead, when large enough masses (4.6) are present, the fields are quickly driven towards the time-dependent minimum and the would-be undershoot solutions reach the tracking behavior in time.

The same happens for the would-be 'overshoot' solutions, [147], in which the fields are initially very close to the origin, with an energy density much larger than the tracker one, and are subsequently pushed to very large values, from where they will not be able to reach the tracking solution before the present epoch. Introducing mass terms like (4.6) prevents the fields to go to very large values, and keeps them closer to the traking solution.

In other words, the already large region in initial condition phase space leading to late-time tracking behavior, will be enlarged to the full phase space.

The effect of the interaction with other fields is shown in Fig. 4.1. Here, we have included the mass term (4.6) during radiation domination with $\beta_1 = 0.3$ and we have followed the same procedure as for Fig. 3.1, looking for solutions which do not reach the tracker before today. As we see, the acceptable range of initial energy densities –and fields' values– for the solutions reaching the tracker is now enormously enhanced since, as we discussed previously, the fields are now prevented from taking too large values. The same conclusion holds even if different initial conditions for the two fields are allowed, for the same reason.

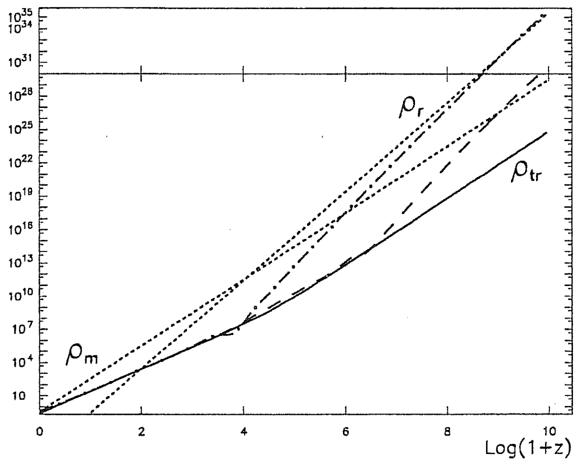

Figure 4.1: *The effect of the interaction with other fields, to be compared with Fig. 3.1. Adding a term like eq. (4.6) with* $\beta_1 = 0.3$ *the would-be overshooting solution (dash-dotted line) reaches the tracker in time.*

In conclusion, we have sketched a possible way out [112] to the common problem of all quintessence models considered so far, that is the presence of long-range interactions of gravitational strength mediated by the ultra-light scalar fields [28]. Our solution is inspired by the Damour-Polyakov relaxation mechanism for the superstring dilaton [42]. Basically, we postulate that all the couplings of the SQCD quark superfields with the SM ones are given by a unique function, which has a minimum close to today's values of the scalar fields' VEVs. Since all the deviations from Einstein gravity are parametrized by the slope of these couplings today, this could make the model phenomenologically safe with respect to limits on the weak equivalence principle and on the time dependence of the SM coupling constants. At the same time, during radiation domination the coupling with SM fields may have induced a SUSY-breaking -time dependent- mass to the scalar fields, with the effect of enhancing the initial configuration space leading to a late time tracking behavior.

An alternative solution to the problem of long range interactions of the quintessence scalar was proposed in ref. [15] in the context of scalar–tensor theories of gravity. Such theories represent a natural framework in which massless scalars may appear in the gravitational sector of the theory and so they have been considered as candidates for quintessence by several authors [7, 9, 38, 45, 73, 127, 156].

In particular, in ref. [15] a model in which a scalar field with potential $V \sim \phi^{-\alpha}$ is minimally coupled to gravity via a ϕ–dependent coupling $\omega(\phi)$ was proposed. The authors of ref. [15] find that in this case two attraction mechanisms are operative at the same time: one towards the tracker solution and the other towards general relativity. This last fact makes the scalar field phenomenologically safe today: while it can account for the accelerated expansion of the universe, at the same time it does not lead to violations of general relativity. Since in scalar–tensor theories of gravity the coupling of matter with gravity is purely metric, the equivalence principle and the constancy of all non–gravitational couplings are automatically preserved.

Chapter 5

Initial conditions for Quintessence

In Chapters 1 and 2 we have reviewd the motivations for the growing interest in cosmological models with $\Omega_M \sim 1/3$ and $\Omega_V \sim 2/3$. The phenomenological implications of 'tracking ' scalar fields has then been extensively discussed in Chapters 3 and 4.

As we have already explained, a good feature of these 'quintessence' models is that for a very wide range of the initial conditions the scalar field will reach the tracking attractor before the present epoch. This fact, together with the negative equation of state, makes the trackers feasible candidates for explaining the cosmological observation of a presently accelerating universe. Nevertheless, it should be stressed that in principle we do not have any mechanism to prevent ρ_ϕ^{in} from beeing outside the desired interval. In this respect, an early universe mechanism which could uniquely fix it at the end of inflation is needed [122, 135]. In other words, if we find a way to naturally set ρ_ϕ^{in} in the range of values which allows for late time–tracking, we will be assured that the 'quintessence' field is a good candidate for the unknown component which presently accelerates the universe.

5.1 Quintessential Inflation

A promising way to address the problem of initial conditions for quintessence is the paradigm of 'quintessential inflation' [120, 122], also refereed to as the 'non oscillatory' [53] scheme. The basic idea is to study an inflaton potential $V(\phi)$

which, as it is typical in quintessence, goes to zero at infinity [120, 122, 53, 58, 144, 77]. In this way it is possible to obtain a late time quintessential behaviour from the same scalar that in the early universe drives inflation. The hope is that the end of inflation could uniquely fix the initial conditions for the subsequent evolution of the scalar componet of the universe.

In ref. [120], a model with a potential which goes like $\sim \phi^4$ for $\phi < 0$ and like $\sim \phi^{-4}$ for $\phi > 0$ is studied in detail, and the authors use gravitational particle production for providing the entropy in the cosmological matter fields after the end of inflation. Although the shape of the potential for $\phi > 0$ is the same studied by Zatlev et al. in [169], they fail to show the 'tracking' behaviour of the scalar field at late times because the initial conditions for the scalar energy density after inflation lay out of the phase space region that leads to joining the attractor before the present epoch. Anyway they succeed in matching the present cosmological data because the cosmological constant-like behaviour ($w_\phi = -1$) that they find for the scalar field is also also a viable option. The reason why the scalar does not reach the tracker is the fact that its energy density at the end of inflation is so low that it did not jet have enough time to move towards the attractor.

The model in [120] suffers from some problems with respect to the reheating mechanism that are extensively discussed in ref. [53]. In particular the authors of [53] propose to use the 'instant preheating' [52] mechanism instead of gravitational particle production for the post-inflationary reheating phase. The main foucus of that work, though, is not on the initial conditions for quintessence.

We will instead address the issue of the initial conditions for quintessence in the context of the 'quintessential inflation' paradigm [122]. The aim is two–fold. On one hand we will discuss under which conditions an inflaton potential can leave a residual vacuum energy on its tail, as already discussed in [120, 53]. On the other we will show that in some specific models it is possible to have a late time tracking (*i.e.* a well defined constrained behaviour of the scalar and negative but $\neq -1$ equation of state) of the residual inflaton energy density.

After briefly recalling the constraints which inflation and quintessence tracking models should separately meet, in the next Section we will discuss to what extent they are compatible. We will then go on giving two specific examples, one in the context of first order inflation an the other in the hybrid case. In the first case we show that the 'escape point' from the tunneling naturally lies within the range which will produce a late time tracking. In the second, we re–examine the model proposed in [82, 83] where a particle physics motivated potential was studied in order to produce inflation. We find that in that model, a late time quintessential behaviour is already built in and discuss under which conditions it meets the observational constraints.

Before starting the construction of any unified model of inflation and quintessence, it is necessary to establish the constraints to which it should be subject. This is not a trivial task, since both inflation and quintessence model building require very precise characteristics in order to be successful and we must check if these separate needs are compatible with each other.

Regarding *inflation*, there are four main points to be taken into account (see for example [87, 101, 108]):

1. If we want the universe to be accelerating, the equation of state of the inflaton field ϕ

$$w_\phi = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \tag{5.1}$$

must satisfy the inequality $w_\phi < -1/3$. This can be achieved if $\dot{\phi}^2 < V(\phi)$.

2. If inflation is to solve the flatness and horizon problems, a sufficient number of e-foldings should take place. This means that the ratio of the final to the initial value of the scale factor a must satisfy $R_f/R_i = \exp N$ with $N \gtrsim 50$.

3. The fact that the amplitude of scalar perturbations in the cosmic microwave background, as measured by COBE in 1992, is of order $\sim 10^{-5}$ constrains the normalization of the inflaton potential.

4. We must ensure that at the end of inflation sufficient reheating takes place. This is needed in order to produce the observed particle species in the universe. At the same time, one has also to check that gravitinos are not overproduced. This puts on the reheating temperature an upper limit which depends on the mass of the gravitinos and which is typically around $10^9 \div 10^{12}$ GeV [1].

For what concerns *quintessence*, the following requirements should be fulfilled (see the discussion of Chapter 2):

1. In order for the scalar field modeling of the cosmological constant to be sufficiently general, we require that the post-inflationary shape of the potential

[1]This bound refers to thermal production. However, recently attention has been paid to gravitinos production during preheating, suggesting that this mechanism could overcome the thermal one [66, 78]. Since this depends on the precise form of the superpotential of the whole supergravity theory, we will not deal with it in the present work.

has the form[2]

$$V(\phi) \sim \frac{\Lambda^{4+q}}{\phi^q}, \qquad q > 0 . \tag{5.2}$$

In this way we are guaranteed that for a very wide range of initial conditions (indeed between the present critical energy density and the background energy density at the beginning, $\rho_c^0 \leq \rho_\phi^{in} \leq \rho_{\rm B}^{in}$) the scalar field will be rapidly driven to a well-known "tracking" attractor behavior $\rho_{\rm TR}$:

$$\frac{\rho_{\rm TR}}{\rho_{\rm B}} \propto R^{\frac{6(w_B+1)}{q+2}} , \tag{5.3}$$

where $w_B = 0, 1/3$ during MD and RD respectively. The attractor is characterized by an equation of state that during MD is always negative: $w_{\rm TR} = (q\, w_{\rm B} - 2)/(q+2)$. There are two main qualitative ways through which this can be achieved (for more details see [100, 147, 169]). If the initial conditions for ϕ are such that $\rho_c^0 \leq \rho_\phi^{in} \leq \rho_{\rm TR}^{in}$ (*undershoot* case), then it will remain "frozen" until $\rho_\phi \sim \rho_{\rm TR}$ and then start to scale following eq. (5.3). If, instead, initially $\rho_{\rm TR}^{in} \leq \rho_\phi^{in} \leq \rho_{\rm B}^{in}$ (*overshoot* case) then ϕ will pass through a phase of kinetic energy domination before remaining frozen at $\rho_\phi < \rho_{\rm TR}$ and eventually join the attractor.

2. Secondly, we want the field ϕ to be already on track today and its present energy density to correspond to what observations report, i.e. $\Omega_\phi \simeq 2/3$. These two conditions translate to

$$V''(\phi) \simeq H^2 \qquad \text{and} \qquad V(\phi) \simeq \rho_c^0 , \tag{5.4}$$

which together imply for the quintessence field $\phi \simeq M_P$ today. Moreover, eq. (5.4) provides a normalization for the mass scale Λ in the potential (5.2), giving

$$\Lambda \simeq \left(\rho_c^0\, M_P^q\right)^{\frac{1}{4+q}} \simeq 10^{-\frac{123}{4+q}}\, M_P . \tag{5.5}$$

This corresponds to choosing the desired tracker path to which the scalar will be attracted to.

While it is straightforward to find potentials with the required early and late-time behavior, the subtle issue resides in successfully matching the exit conditions for the scalar field after inflation with the range of initial conditions allowed for the trackers. For example, the naive guess of using the potential $V = \Lambda^{4+q}\phi^{-q}$

[2]This is the only class of potentials that admits an analytic "tracking" attractor solution. However, more general cases have been studied in ref. [147].

for quintessential inflation is easily shown not to work. This is due to the fact that, with this potential, the slow-roll conditions imply $\phi \gtrsim M_P$ *during inflation*, while the request that the quintessence field is presently dominating the universe translates to $\phi \simeq M_P$ *today*.

The first scenario that we will discuss [122] is first order inflation. In this context, if the potential $V(\phi)$ does not have an absolute minimum but goes to zero as ϕ runs to infinity, the exit conditions of the inflaton from the tunneling would set the starting point for the subsequent quintessential evolution of the same field ϕ. If instead hybrid inflation is considered [122], it is again possible to construct models in which the same field plays the role of the inflaton and of the late time dynamical cosmological constant. In this case, the critical value ϕ_c that makes inflation stop will determine the initial condition for the subsequent quintessential rolling.

5.2 Constructing workable models

First-order quintessential inflation

In the original proposal of inflation [68], the scalar inflaton field ϕ which leads the expansion "sits" on a metastable minimum of its potential $V(\phi)$ during the whole process. Inflation eventually ends when bubbles of true vacuum nucleate through the barrier and subsequently expand and collide reheating the universe. A measure of the efficiency of the nucleation is given by the ratio

$$\varepsilon = \frac{\Gamma}{H^4} \tag{5.6}$$

between the tunneling rate Γ and the Hubble constant H.

As it was soon noticed [69, 70], models where ε is constant in time cannot work, because one needs both (i) $\varepsilon \ll 1$ during inflation in order for the expansion to last enough to solve the flatness and the horizon problems and (ii) $\varepsilon \gtrsim 1$ to have an efficient nucleation. This puzzle is known as the "graceful exit problem". Many proposals have been suggested to solve this problem (see [86] for a review), based on the possibility of changing either H [97] or Γ [2, 102] with time. This is commonly achieved by the use of an "auxiliary" scalar field ψ, which is also employed to fit the amplitude of scalar perturbations in the cosmic microwave background measured by COBE[3]. Without entering the details of this procedure, we will fix $\varepsilon = 1$ in the toy model below, *assuming* that also in our case some auxiliary field (or some other mechanism) can be invoked in this regard[4].

In the model that we propose, the scalar field ϕ has a potential (see Fig. 5.1 below)

$$V(\phi) = \frac{\Lambda^{\alpha+6}}{\phi^\alpha \left[(\phi - v)^2 + \beta^2\right]} \quad , \quad \text{with } \frac{\beta}{v} \ll 1 \; , \tag{5.7}$$

where Λ, β and v are constants of mass dimension one. Eq. (5.7) has a barrier at $\phi \sim v$, after a metastable minimum in $\phi \sim v\alpha/(\alpha+2) \equiv \phi_m$, while it behaves like $\sim \Lambda^{\alpha+6}/\phi^{\alpha+2}$ for $\phi \gg v$.

The parameter Λ is constrained by quintessence (see eq. (5.4)-(5.5)):

$$\Lambda \simeq 10^{-\frac{123}{\alpha+6}} M_P \simeq 10^{\frac{19\,\alpha-9}{\alpha+6}} \text{ GeV} \; . \tag{5.8}$$

In this way we ensure that the residual vacuum energy after inflation does not overclose the universe and that at the same time it is not presently negligibly small.

Inflation, instead, requires that most of the energy density $V(\phi_m)$ which dominates the accelerated expansion is transferred, after the end of inflation, into a thermal bath of temperature $T_{rh} \equiv 10^{9+\gamma}$ GeV and this fixes the scale v in the potential[5]

$$v \simeq (\alpha+2)\left(\frac{10^{-83-4\,\gamma}}{140\,\alpha^\alpha}\right)^{\frac{1}{\alpha+2}} M_P \simeq (\alpha+2)\left(\frac{10^{19\alpha-45-4\gamma}}{140\;\alpha^\alpha}\right)^{\frac{1}{\alpha+2}} \text{GeV} \; . \tag{5.9}$$

The ratio β/v is fixed below by the condition $\varepsilon = 1$, so that the only free parameters of the model are the exponent α and the reheating temperature parametrized by γ.

Although the potential (5.7) does not have two minima (being the lower one at infinity), the problem can be analytically approached in the so called "thin wall limit" [34] as in the case in which the two minima are present. This limit applies

[3]If this second field ψ is slowly rolling down its own potential $V(\psi)$, the amplitude of the density fluctuations is given by [114] $10^{-5} \simeq \delta\rho/\rho \simeq \frac{3H^3}{dV(\psi)/d\psi} \simeq 3(8\pi/3M_P^2)^{3/2}\,\frac{V(\phi)^{3/2}}{dV(\psi)/d\psi}$.

[4]Since a late time quintessential behaviour obvoiusly cannot affect the exit from inflation, the solutions proposed so far (see for example [97, 2, 102]) may be assumed to work also in the present case.

[5]For the degrees of freedom of the Standard Model $V(\phi_m) \simeq \rho_{rad} \simeq 35\,T_{rh}^4$.

when the barrier is much higher than the difference between the two minima. This is our case, since

$$\frac{V(v)}{V(\phi_m)} = \mathrm{O}\left(\frac{v}{\beta}\right)^2 \gg 1 \,. \tag{5.10}$$

In order to get the decay rate Γ (see [34] for details) one has to integrate the equation of motion associated to the potential (5.7) and select the solution $\phi(x)$ which minimizes the Euclidean action S_E of the system. This solution is O(4) symmetric (in the whole Euclidean space) and approaches $\phi = \phi_m$ at $x \equiv |\mathbf{x}| = \infty$. The value $\phi(x=0) \equiv \phi_{\mathrm{es}}$ is called the "escape point" and corresponds to the point at which the field ϕ tunnels out and starts rolling under its classical equation of motion.

If the thin-wall limit holds, the solution is $\phi \simeq \phi_{\mathrm{es}}$ for an interval $0 < x < R$, and then $\phi \simeq \phi_m$ for $x > R$. We physically interpret it as a bubble with radius R of (nearly) true vacuum within separated by a thin wall from the false vacuum without. Continued to Minkowski space, the bubble appears to expand with a speed which asintotically approaches the speed of light. The universe can then be reheated by the particle production that occurs during the subsequent phase of collision of the bubbles recovering from the tunneling. The dynamics of this process in the present model is exactly analogous to the one which occurs in the usual case (when the minimum of $V(\phi)$ is at a finite value of ϕ) and is extensively discussed in ref. [89, 111, 152, 161].

Following [34], the Euclidean action and the initial radius of the bubbles are given by

$$\begin{aligned} S_E &\sim -\frac{27\,\pi^2}{2} \cdot \frac{S_1^4}{V(\phi_m)^3} \\ R &\simeq 3\,S_1/V(\phi_m) \;, \end{aligned} \tag{5.11}$$

where

$$S_1 = \int_{\phi_m}^{\infty} d\,\phi\,[2\,(V(\phi) - V(\phi_m))]^{1/2} \;. \tag{5.12}$$

In our case S_1 can be calculated analytically for any value of α in the potential (5.7) without any approximation, but a more readable and accurate enough estimate is given by

$$S_1 \simeq 2\int_{\phi_m}^{v} d\,\phi\,\sqrt{2}\left[\frac{\Lambda^{\alpha+6}}{v^\alpha}\,\frac{1}{(\phi-v)^2+\beta^2}\right]^{1/2} = \frac{2\,\sqrt{2}\,\Lambda^{(\alpha+6)/2}}{v^{\alpha/2}}\,\ln\left[\frac{4}{\alpha+2}\,\frac{v}{\beta}\right] \,. \tag{5.13}$$

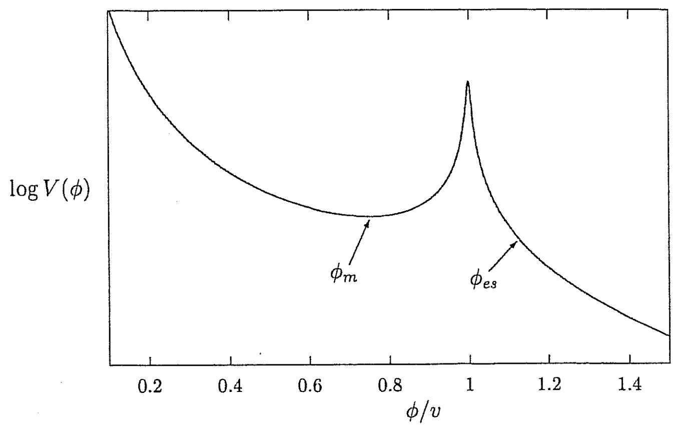

Figure 5.1: *The bump in the potential of eq. 5.7, shown here with parameters* $\alpha = 6$ *and* $\beta/v = 0.005$*, allows for an early stage of inflation while the inflaton field* ϕ *sits in the relative minimum at* ϕ_m*. After* ϕ *has tunneled out at* ϕ_{es}*, the quintessential phase starts with the scalar rolling down the slope* $\sim \phi^{-\alpha-2}$ *until today.*

The tunneling rate of ϕ is

$$\Gamma = A\, e^{-S_E} \tag{5.14}$$

where A is a parameter with mass dimension 4 of order $V(\phi_m)$.

From this equation, the condition

$$\varepsilon = \frac{\Gamma}{H^4} = \left(\frac{3}{8\pi}\right)^2 \frac{M_P^4}{V(\phi_m)}\, e^{-S_E} = 1 \tag{5.15}$$

is obtained for $S_E \simeq 84 - 9\,\gamma$, that is if the ratio β/v satisfies

$$\ln\left[\frac{4}{\alpha+2}\,\frac{v}{\beta}\right] = \left(\frac{84 - 9\,\gamma}{5.5 \cdot 10^5}\, 140^{\frac{\alpha+6}{\alpha+2}}\right)^{1/4} \left(\frac{\alpha+2}{\alpha^{\frac{\alpha}{\alpha+2}}}\right)^{\alpha/2} \left(10^{\alpha\,(\gamma-10)+63+6\,\gamma}\right)^{1/(\alpha+2)}. \tag{5.16}$$

Moreover, with this condition we also have

$$R = 0.36\ (84 - 9\,\gamma)^{1/4}\ 10^{-9-\gamma}\ \text{GeV}^{-1} \tag{5.17}$$

as the analytical estimate for the initial radius of the bubbles[6].

[6]In all this analysis we have not considered the gravitational corrections on the decay of the metastable vacuum. However, since they are of order $(RH)^2$ [35] , their contribution is completely negligible in all the cases of our interest.

Going on with the analysis, we specify to some particular values of the parameters. As anticipated in Section 1.1, we impose to the reheating temperature the upper bound $T_{rh} \leq 10^{12}\,\text{GeV}$, that is $\gamma \leq 3$. We see from eq. (5.16) that, for any fixed value of γ, it is always possible to obtain $\varepsilon = 1$ with arbitrarily low β/v, just allowing α to be large enough. However, for phenomenological reasons (see below) we restrict ourselves to $\alpha \lesssim 10$ and list[7] in Table 1 the cases for which $\beta/v \ll 1$.

γ	α	β/v	$R_{\text{an}}\,[GeV^{-1}]$	$R_{\text{num}}\,[GeV^{-1}]$	ϕ_{es}/v
1	8	$7.75 \cdot 10^{-3}$	$1.06 \cdot 10^{-10}$	$0.98 \cdot 10^{-10}$	1.30
2	10	$9.72 \cdot 10^{-3}$	$1.03 \cdot 10^{-11}$	$0.94 \cdot 10^{-11}$	1.23
3	8	$4.74 \cdot 10^{-4}$	$9.88 \cdot 10^{-13}$	$9.07 \cdot 10^{-13}$	1.25

Table 5.1: *Comparison between the analytical and numerical results for the radius of the tunneling bubble, for some values of the parameters of the model. In the last column, the escape point is given in units of v.*

We studied the tunneling also numerically and the solutions that we found are in good accordance with the previous semi-quantitative analysis. In particular, their shape is that of an instanton which interpolates between the initial value ϕ_{es} and the final one ϕ_m. The jump between the two values occurs at $x \equiv R_{num}$, very close to the analytical estimate R_{an} given by eq. (5.17), as can be checked in Table 1.

At this stage, it is easily understood that the initial conditions for quintessence are entirely determined by the tunneling and not given arbitrarily. In particular, if the present model is considered, when ϕ tunnels out at the escape point ϕ_{es} and starts rolling down the $V \sim \phi^{-\alpha-2}$ potential, it has an energy density given by

$$\begin{aligned} V(\phi_{es}) &\simeq V(\phi_m)\,\frac{4\,\alpha^\alpha}{(\alpha+2)^{\alpha+2}}\,\frac{(v/\phi_{es})^\alpha}{(\phi_{es}/v-1)^2} \simeq \\ &\simeq \rho_{rad}\,\frac{4\,\alpha^\alpha}{(\alpha+2)^{\alpha+2}}\,\frac{(v/\phi_{es})^{\alpha+2}}{(1-v/\phi_{es})^2} \end{aligned} \qquad (5.18)$$

[7]In order to avoid excessive fine tuning, we have not listed the cases for which $\beta/v < 10^{-5}$. However this is a somewhat arbitrary limit and nothing prevents from considering smaller values.

which is typically about 1% of the thermal one and which can be computed as a function of α and of the reheating temperature substituting the values of Table 1 in the last expression. Note also that the field ϕ at the beginning of the "quintessential" regime is of the order of v and, from eq. (5.9), can be easily estimated to be $\ll M_P$ (actually it tends to M_P when $\alpha \to \infty$).

These initial conditions naturally lay within the allowed range for quintessence and correspond to the *overshoot* case mentioned in Section 1.1. The field ϕ will then rapidly run to large values and its energy density will consequently drop down well below the tracker, as discussed in ref. [100, 147, 169]. Then, after a "freezing" phase of almost zero kinetic energy, it will eventually join the tracker path at more recent times. As a function of the exponent α in the potential (5.7), the equation of state of the scalar ϕ on the tracker is

$$w_\phi = -\frac{2}{\alpha+4} \ . \tag{5.19}$$

However it should be remembered that the present value of the equation of state is lower than the attractor value. When the scalar energy density ceases to be subdominant with respect to the matter one, the approximation in which the attractor was derived does not hold anymore [100, 147, 169]. The scalar then leaves the tracking path as soon as its energy density is comparable to that of matter and rapidly tends towards a cosmological constant–like behaviour with $w_\phi = -1$. For a present ratio $\Omega_\phi/\Omega_M \simeq 2$ we should restrict to $\alpha \lesssim 10$ to be compatible with the present data [160].

Hybrid quintessential inflation

The model that we will consider next was proposed in [82, 83] and involves a scalar potential arising from dynamical supersymmetry breaking, of the form $V = V_{Susy} + V_{S\not{u}sy}$, with

$$V_{Susy} = M^4 \left|1 - \lambda\frac{\chi^2\phi^2}{M^4}\right|^2 + \frac{\Lambda^{4+p}}{\phi^p} \quad , \qquad V_{S\not{u}sy} = \frac{1}{2}\beta M^2\chi^2 \ . \tag{5.20}$$

As extensively discussed in [82, 83], this potential can easily accomodate an early inflationary stage of the hybrid [103, 104] type[8]. We find that, quite surprisingly,

[8]The presence of a SUSY–breaking mass term for the scalar ϕ is cosmologically excluded if we require a late time quintessential behaviour, since it would induce a minimum in the ϕ–direction of the potential.

this model has already incorporated a late–time quintessential phase and this leads to important consequences in addition to those discussed in [82, 83]. The interesting point is that this class of potentials is the first example which allows for discussing quintessential inflation in a particle physics context. While this issue is discussed both in the purely inflationary (see [108] and references therein) or purely quintessential cases (see Chapter 3), it is still missing in the 'quintessential inflation' scheme. In the following we will then address this problem with the potential (5.20).

For $\phi < \phi_c = \sqrt{\beta M^2/2\lambda}$ the minimum of the potential is at $\chi = 0$ and inflation can occur if the term M^4 in (5.20) dominates. When ϕ rolls down to $\phi > \phi_c$ inflation is ended by instability in the χ direction, as typically occurs in hybrid models. In this case, however, the VEV of the scalar potential V does not almost instantaneously settle to zero but vanishes only after ϕ has run to infinity. This feature is very welcome if we want a quintessential component to be present in the subsequent evolution of the universe. In what follows we study for which range of the parameters this model can fulfill the double aim of accounting for both the inflationary and quintessential stages of our universe.

For $\chi = 0$ and $\phi < \phi_c$, the potential can be rewritten as

$$V = M^4 \left[1 + \alpha \left(\frac{M_P}{\phi}\right)^p\right] \tag{5.21}$$

with

$$\alpha \equiv \frac{\Lambda^{p+4}}{M_P^p\, M^4} \ . \tag{5.22}$$

We will see below that stringent upper limits apply to α for the model to fit observations. For the moment we only ask α to be small enough so that the constant term dominates eq. (5.21), leading to a first inflationary stage. This is naturally achieved if we require that the term Λ^{4+p}/ϕ^p in eq. (5.20) leads to a present energy density that does not exceed the critical one (see Section 1.1), that is if $\Lambda \leq \Lambda_c$ with

$$\Lambda_c^{p+4} = 10^{-123}\, M_P^{p+4} \ . \tag{5.23}$$

To estimate the starting point of inflation we consider the slow roll parameters

$$\varepsilon \equiv \frac{M_P^2}{4\pi}\left(\frac{H'(\phi)}{H(\phi)}\right)^2 \simeq \frac{M_P^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 = \left(\frac{\phi_0}{\phi}\right)^{2(p+1)} \tag{5.24}$$

$$\eta \equiv \frac{M_P^2}{4\pi}\left(\frac{H''(\phi)}{H(\phi)}\right) \simeq -\frac{M_P^2}{8\pi}\left[\frac{V''(\phi)}{V(\phi)} - \frac{1}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2\right] = \tag{5.25}$$

$$= \left(\frac{p+1}{2\sqrt{\pi}}\right)\left(\frac{\phi_0}{\phi}\right)^{p+1}\frac{M_P}{\phi} + \left(\frac{\phi_0}{\phi}\right)^{2(p+1)}$$

where a prime denotes differentiation w.r.t. ϕ and

$$\phi_0 = \left(\frac{p}{4\sqrt{\pi}}\alpha\right)^{1/(p+1)} M_P \ . \tag{5.26}$$

Since slow roll requires $\varepsilon, \eta \ll 1$, the accelerated expansion occurs for $\phi_0 \ll \phi \leq \phi_c$.

When inflation ends at $\phi = \phi_c$, the second scalar χ leaves zero (that becomes an unstable maximum of V) and oscillates about one of the two new ϕ–dependent minima that form at

$$\chi^2_{MIN} = \frac{2M^2}{\beta}\frac{\phi_c^2}{\phi^2}\left(1 - \frac{\phi_c^2}{\phi^2}\right) \ . \tag{5.27}$$

If we suppose that the scalar χ is coupled to the matter fields via terms like $h\chi^2\varphi^2$, its coherent oscillations would result in sufficient reheating of the universe by its decay products. The scalar χ eventually settles to χ_{MIN} and the reheating temperature is typically of order $\sim M$. After the reheating phase is completed the potential is given by

$$V(\chi_{MIN}, \phi) = M^4\frac{\phi_c^2}{\phi^2} + \frac{\Lambda^{4+p}}{\phi^p} \ . \tag{5.28}$$

It should be noted that, taking into account the bound (5.33) on Λ given below, it turns out that at late times ($\phi \gg \phi_c$) the term that will dominate in the above formula is the first one. That means that the tracking behavoiur will not be induced, as might be expected, by the explicit inverse power term present in the potential, but will instead come from the effective contribution $\sim \phi_c^2/\phi^2$ originated by the minimum of the second scalar χ.

The first term in eq. (5.28) constrains the ratio of ϕ_c to M, since we need that today $V(\chi_{MIN}, \phi \simeq M_P) \simeq \rho_c^0$. This translates to

$$\frac{\phi_c}{M} = \sqrt{\frac{\beta}{2\lambda}} \simeq \left(\frac{100\,\mathrm{GeV}}{M}\right)^3 \cdot 10^{-10} \ . \tag{5.29}$$

The smallness of ϕ_c constrains the other parameters of the model, once the two main requirements of inflation, namely that it lasts for enough e–folds and that it gives the spectrum of fluctuations observed by COBE, are taken into account. The number of e–folds of inflation, when $\eta \gg \varepsilon$ is given by [82, 83]

$$N_{\rm tot} \;=\; \frac{2\sqrt{\pi}}{M_P}\int_{\phi_{in}}^{\phi_c}\frac{d\,\phi'}{\sqrt{\varepsilon\,(\phi')}} \;\simeq\; \frac{8\pi}{p(p+2)}\frac{1}{\alpha}\left(\frac{\phi_c}{M_P}\right)^{p+2} \quad . \tag{5.30}$$

Since at least 50 or 60 e–folds are needed to solve the horizon and flatness problems, we impose $N_{\rm tot} \geq 50$. For what concerns the fluctuations, instead, we require that the curvature power spectrum[9] [82, 83]

$$P_R^{1/2} \equiv \frac{1}{\sqrt{\pi}}\frac{H\,(\phi_{50})}{M_P\,\sqrt{\varepsilon\,(\phi_{50})}} \;\simeq\; \frac{p+2}{\pi}\sqrt{\frac{2\,\pi}{3}}\left(\frac{M^2}{M_P\,\phi_c}\right)N_{\rm tot}\left(1-\frac{50}{N_{\rm tot}}\right)^{(p+1)/(p+2)} \tag{5.31}$$

matches the COBE normalization [23] $P_R^{1/2} = 5{\cdot}10^{-5}$, with spectral index [82, 83]

$$n-1 \;\equiv\; \frac{d\;\log(P_R)}{d\;\log k} \;=\; -4\varepsilon + 2\eta \;\simeq\; \left(\frac{p+1}{p+2}\right)\frac{2}{N_{\rm tot}(1-50/N_{\rm tot})} \quad . \tag{5.32}$$

The spectrum turns out to be blue ($n > 1$), but for $N_{\rm tot} > 50$ it quickly approaches scale invariance, $n \simeq 1$. The present limit $|n-1| < 0.2$ [108] translates to $N_{\rm tot} \gtrsim 60$.

Substituting eqs. (5.22) and (5.29) into eq. (5.30), we obtain the following upper bound on the mass scale[10] Λ:

$$\left(\frac{\Lambda}{\Lambda_c}\right)^{p+4} \;\simeq\; \frac{10^{-27p}}{p(p+2)2^{p+1}}\left(\frac{100\,{\rm GeV}}{M}\right)^{2p}\left(\frac{50}{N_{\rm tot}}\right) \quad . \tag{5.33}$$

Finally, from eq. (5.31) we get the rough estimate $\frac{M^2}{M_P\,\phi_c}\,N_{\rm tot} \sim 10^{-5}$, which translates into

$$M \;\simeq\; 100\,{\rm GeV}\left(\frac{50}{N_{\rm tot}}\right)^{1/4} \quad . \tag{5.34}$$

We thus understand that a sufficient amount of e–folds can be achieved only for a quite low reheating temperature (remember $T_{rh} \simeq M$). Anyhow, a low T_{rh} is also preferred since it weakens the upper bounds on ϕ_c and on Λ given by eqs. (5.29) and (5.33).

As an example of the orders of magnitude involved, for $M \simeq 50$ GeV we get $N_{\rm tot} \simeq 800$, $\phi_c \simeq 22$ eV , and Λ in the range 0.016 eV$(p=2) \div 5$ eV$(p=50)$.

---

[9] ϕ_{50} is the value of the inflaton field 50 e–folds before the end of inflation.

[10] Inserting this value for Λ in eq. (5.26) we can check that, consistently, $\phi_0 \ll \phi_c$.

The last mass scale is not very natural in a supersymmetric context, where one customarily expects values $\gtrsim$ GeV. However, the amount of fine tuning involved in Λ from eq. (5.33) is milder than in the case of the cosmological constant, where the mass scale is more than 30 orders of magnitude smaller than the "natural" value.

The last important point in our discussion is the "quintessential" evolution of ϕ after the reheating phase. As already noticed, the bound on Λ given by eq. (5.33) forces the second term in eq. (5.28) to be completely negligible during this last phase. Despite its shape is exactly the one required for the trackers, the only role it plays in this model is to drive ϕ towards ϕ_c during inflation. The term which dominates the potential (5.28) at late times comes instead from the dynamics of the field χ and the tracking behavior is guaranteed from the fact that it involves a negative power of the inflaton ϕ as well.

The initial conditions of this "quintessential" phase are fixed by the value ϕ^* of the field ϕ after reheating, when $\chi = \chi_{MIN}$ and eq. (5.28) starts holding. The precise value of ϕ^* depends on the details of the physics which governs the reheating, but it is reasonable to assume that it will not be much larger than ϕ_c. If this is the case, the initial energy of the quintessential field ϕ will be somewhat smaller (but not too smaller) than the one stored in the thermal background and, as in the previous model, we are again in the "overshoot" case.

The attractor equation of state for a potential $V \sim \phi^{-2}$ is simply $w_\phi = -1/2$, well within the observational bound [160].

In this Section we have discussed two possible schemes in which inflation and quintessence are unified [122]. In both cases it is the same field which at the same time plays the role of the inflaton and of the quintessence scalar. In this way we succeded to uniquely fix the initial conditions for quintessence from the end of inflation and have found that they are compatible with a late–time tracking [122].

In one example we studied first-order inflation with a potential going to zero at infinity like $\phi^{-\alpha}$. A bump in the potential at $\phi \ll M_P$ allows for an early stage of inflation while the scalar field gets "hung up" in the metastable vacuum of the theory. Nucleation of bubbles of true vacuum through the potential barrier sets the end of the accelerated expansion and starts the reheating phase. As it is well known, this scenario suffers from the so-called "graceful exit problem", but we briefly commented on possible ways out where (thanks to some auxiliary scalar field) the ratio of the tunneling rate to the Hubble volume, Γ/H^4, varies with time. After the reheating process is completed, the quintessential rolling of the scalar ϕ starts and its initial conditions (uniquely fixed by the end of inflation) are naturally within the range which leads to a tracking behavior in recent times.

As an alternative, we considered the model of hybrid inflation which, motivated by dynamical supersymmetry breaking, was proposed by the authors of [82, 83]. We showed that it naturally includes a late–time quintessential behavior. This result is very interesting since it is the first time that the quintessential inflation scheme is discussed in a particle physics motivated context. As typical of hybrid schemes, the potential is dominated at early times (that is until the inflaton field is smaller than a critical value ϕ_c) by a constant term and inflation takes place. Eventually the inflaton rolls above ϕ_c, rendering unstable the second scalar of the model, χ. This field starts oscillating about its minimum (whose position is determined by ϕ) and in this stage the universe is reheated. After χ has settled to the minimum, the inflaton continues its slow roll down the "residual" potential which goes to zero at infinity like ϕ^{-2}, thus allowing for a quintessential tracking solution. Also in this case the initial conditions for the quintessential part of the model do not have to be set by hand, but depend uniquely on the value of the inflaton field at the end of reheating.

5.3 Initial conditions from the interaction with the inflaton?

If we suppose that the inflaton ϕ and the quintessence scalar χ are two distinct fields, an interesting issue is whether their interaction can have any effect on the initial conditions for quintessence. In other words, we want to see if introducing an interaction potential $V(\phi, \chi)$ may help in setting uniquely the initial conditions for quintessence.

In this section we will discuss some speculations about the possibility that the inflaton ϕ sets the initial conditions for the quintessence scalar χ via an interaction potential. We will focus in particular on a toy model in which the minimum in the χ direction results to be ϕ-dependent. If this is the case, it may happen that – following the evolution of ϕ – at the beginning χ is trapped at a temporary finite minimum of V and then eventually escapes to infinity. In this scenario, the late–time rolling of χ would be determined by the conditions at the end of inflation.

Consider a potential of the form:

$$V(\phi,\chi) \;=\; \frac{\Lambda^{4+\alpha}}{\chi^{\alpha}} \;+\; \frac{\phi^2}{2}\,(g^2\chi^2+m^2) \tag{5.35}$$

where Λ and m are some mass scales, g is a dimensionless coupling and $\alpha > 0$. The coupled equations of motion for the two-field system are:

$$\ddot{\phi} \;+\; 3H\dot{\phi} \;+\; \phi\,(g^2\chi^2+m^2) \;=\; 0\,, \tag{5.36}$$

$$\ddot{\chi} \;+\; 3H\dot{\chi} \;+\; g^2\phi^2\,\chi \;-\; \alpha\,\frac{\Lambda^{4+\alpha}}{\chi^{\alpha+1}} \;=\; 0\,. \tag{5.37}$$

We want the first term in the potential (5.35) to become relevant only after inflation (when ϕ settles to zero) and the second one, instead, to dominate the dynamics of inflation. This is naturally achieved if, as it is usually done in quintessence models, we fix the scale Λ requiring that at present the scalar χ has already joined the tracker path and V is of order of the critical density. In this way we obtain, depending on the value of α :

$$\Lambda \;\simeq\; \left[\rho_c^0\, M_P^{\alpha}\right]^{\frac{1}{4+\alpha}} \;\simeq\; M_P\cdot 10^{-\frac{123}{4+\alpha}}\,. \tag{5.38}$$

It can be easily checked that, if chaotic initial conditions are considered (see below), the contribution of this first term in eq. (5.35) is negligible during inflation.

Let us now focus on the two-field dynamics with potential:

$$\overline{V} \;=\; \frac{\phi^2}{2}\,(g^2\chi^2+m^2)\,. \tag{5.39}$$

We choose the initial conditions (see also the discussion in [53]) in such a way that the Planck boundary $V \lesssim M_P^4$ is satisfied and the effective masses of the two fields do not exceed M_P:

$$|\chi| \sim\; M_P\,, \qquad |\phi| \sim g^{-1}\, M_P\;. \tag{5.40}$$

We also choose the coupling g to be large enough that at the very early stages of inflation the contribution of the term $\frac{m^2}{2}\phi^2$ can be neglected but also small enough to have $|\chi| \ll |\phi|$ at the beginning. Then a very short stage of χ-dominated inflation takes place, before χ itself settles down to its minimum $\tilde{\chi}$:

$$\tilde{\chi} \;=\; \left(\frac{\alpha\,\Lambda^{4+\alpha}}{g^2\phi^2}\right)^{\frac{1}{2+\alpha}}\;. \tag{5.41}$$

In this way we are also able to supress (by the same mechanism discussed in [53]) the long wavelenght fluctuations of χ that could spoil the model in absence of an interaction between the two fields.

While $\chi \simeq \tilde{\chi}$ the potential is given by

$$\overline{V}(\phi, \tilde{\chi}) \simeq \left(\frac{2g^2}{\alpha}\right)^{\frac{\alpha}{\alpha+2}} \left(1 + \frac{\alpha}{2}\right) \Lambda^{\frac{2(4+\alpha)}{2+\alpha}} \phi^{\frac{2\alpha}{2+\alpha}} + \frac{m^2}{2} \phi^2 . \tag{5.42}$$

If $m \sim 10^{-6}\, M_P$ as required by the COBE normalization, then the scalar energy density $\overline{V}$ in the last expression is dominated by $\frac{m^2}{2}\phi^2$. As a consequence ϕ evolves independently of χ and the usual chaotic inflation occurs.

During inflation, while ϕ rolls down, the minimum (5.41) of the potential will be slightly displaced from its original position. As far as $\tilde{\chi}$ increases slowly, the condition $\chi \simeq \tilde{\chi}$ will still hold. This is no longer true after inflation is ended and ϕ starts a regime of damped oscillations according to

$$\phi(t) = \Phi(t) \cdot \cos mt . \tag{5.43}$$

The amplitude of the oscillations, $\Phi(t)$, decreases exponentially (on a time scale longer than the period of oscillation $\sim m^{-1}$), due to both the expansion of the universe and the decay of the inflaton during the reheating process. The model with potential (5.35) cannot produce sufficient reheating by itself, because the decay rate $\Gamma(\phi\phi \to \chi\chi)$ never catches up with the expansion of the universe and reheating never completes [67, 84, 85]. Howewer it is easy to increase the efficiency of the mechanism by extending the particle content of the model.

If we suppose that the field χ is also coupled with fermions with an interaction term like $h\chi\bar{\psi}\psi$ the mechanism of "instant preheating" discussed in [52] could be invoked while ϕ passes through zero. As an alternative, one can couple some bosonic or fermionic field directly to the inflaton and then the usual perturbative reheating process [67, 84, 85] would work. This adds a decay rate Γ to the equation of motion (5.36) for ϕ.

During the oscillatory phase of the inflaton, $\tilde{\chi}$ quickly evolves down to infinity (when ϕ crosses zero) and backwards, but χ cannot catch up and makes small oscillation about the average position[11]

$$\chi_{av} = \left(\frac{\alpha\, \Lambda^{4+\alpha}}{g^2\, \Phi^2}\right)^{\frac{1}{2+\alpha}} . \tag{5.44}$$

[11] In practice, one can accurately enough sobstitue ϕ with Φ in the equation of motion (5.37) for the field χ.

While Φ is getting damped, the term $g^2\Phi^4\chi$ in eq. (5.37) will correspondingly shortly become negligible and χ will effectively start to feel just the $\sim \phi^{-\alpha}$ part of the potential. The moment when this happens can be considered as the starting point for quintessence. Note that this initial condition is, with good approximation, fixed by the position of the minimum (5.41) at the end of inflation, since reheating takes place almost instantaneously compared to the time it takes for χ to move. Substituting $\phi \simeq M_P/3$ and $\Lambda \simeq M_P \cdot 10^{-\frac{123}{4+\alpha}}$ in eq. (5.41) we find

$$\hat{\chi} \simeq \left(\frac{\alpha}{g^2}\right)^{\frac{1}{2+\alpha}} \cdot 10^{-\frac{123}{2+\alpha}} \cdot M_P \tag{5.45}$$

as the initial value for the field χ in the post-inflationay phase. Again, these initial conditions for the late time evolution of χ are within the allowed range for quintessence.

In conclusion, in this Section we have considered the possibility that the inflaton and quintessence scalar are not the same field. If this is the case, we have shown that the issue of fixing the initial conditions for quintessence is well posed also in this case and that the interaction with the inflaton is a promising mechanism for that.

Chapter 6

Conclusions

The observational evidence of a non–vanishing vacuum energy, presently dominating the cosmological expansion, is far from beeing conclusive. The Sn Ia data, which provide the only direct measurment of the acceleration of the universe, still await confirmation. Nevertheless, indirect evidence coming from a number of other cosmological constraints is mounting. Therefore the possibility of a non–vanishing cosmological constant should be taken seriously both from the particle physics and cosmology points of view.

In this thesis we have examined some issues related to the cosmological constant problem in particle physics and cosmology. In particular we have focussed on the so–called 'Quintessence models' which assume $\Lambda_{eff} = 0$ and study the cosmological evolution of a rolling scalar field. Shuch a field could indeed provide the required 'missing' energy density of the universe, $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$, and at the same time exibit a negative equation of state. It could then accelerate the universe as required by the SN Ia data. A particularly promising class of potential is given by inverse powers, $V(\phi) \sim \phi^{-\alpha}$, which admit a characteristic attractor solution, denominated 'tracker' in the literature. For a very wide range of the initial conditions the scalar field will end up on the tracker path before the present epoch.

We have then analyzed the role of Supersymmetric QCD as a possible model for quintessence [112, 134]. Our analysis completes the previous one by Binétruy [20] in two respects.

First, we have taken in full consideration the multi-scalar nature of the model, allowing for different initial conditions for the N_f independent scalar VEVs and studying the coupled system of N_f equations of motion. Starting with the same initial scalar energy density, but different fields' values we have shown that the tracking behavior becomes more difficult to reach the larger the difference among the initial conditions for the fields. Thus, an approximate flavor symmetry of the initial conditions is needed in order that SQCD may act as an effective quintessence model.

Secondly, we have sketched a possible way out to the common problem of all quintessence models considered so far, that is the presence of long-range interactions of gravitational strength mediated by the ultra-light scalar fields [28]. Our solution is inspired by the Damour-Polyakov relaxation mechanism for the superstring dilaton [42]. Basically, we postulate that all the couplings of the SQCD quark superfields with the SM ones are given by a unique function, which has a minimum close to today's values of the scalar fields' VEVs. Since all the deviations from Einstein gravity are parametrized by the slope of these couplings today, this could make the model phenomenologically safe with respect to limits on the weak equivalence principle and on the time dependence of the SM coupling constants. At the same time, during radiation domination the coupling with SM fields may have induced a SUSY-breaking -time dependent- mass to the scalar fields, with the effect of enhancing the initial configuration space leading to a late time tracking behavior.

We have also discussed two possible schemes in which inflation and quintessence are unified [122, 135]. In both cases it is the same field which at the same time plays the role of the inflaton and of the quintessence scalar. In this way we succeded to uniquely fix the initial conditions for quintessence from the end of inflation and have found that they are compatible with a late–time tracking.

In one example we studied first-order inflation with a potential going to zero at infinity like $\phi^{-\alpha}$. A bump in the potential at $\phi \ll M_p$ allows for an early stage of inflation while the scalar field gets "hung up" in the metastable vacuum of the theory. Nucleation of bubbles of true vacuum through the potential barrier sets the end of the accelerated expansion and starts the reheating phase. As it is well known, this scenario suffers from the so-called "graceful exit problem", but we briefly commented on possible ways out where (thanks to some auxiliary scalar field) the ratio of the tunneling rate to the Hubble volume, Γ/H^4, varies with time. After the reheating process is completed, the quintessential rolling of the scalar ϕ starts and its initial conditions (uniquely fixed by the end of inflation) are naturally within the range which leads to a tracking behavior in recent times.

As an alternative, we considered the model of hybrid inflation which, motivated by dynamical supersymmetry breaking, was proposed by the authors of [82, 83]. We showed that it naturally includes a late–time quintessential behavior. As typical of hybrid schemes, the potential is dominated at early times (that is until the inflaton field is smaller than a critical value ϕ_c) by a constant term and inflation takes place. Eventually the inflaton rolls above ϕ_c, rendering unstable the second scalar of the model, χ. This field starts oscillating about its minimum (whose position is determined by ϕ) and in this stage the universe is reheated. After χ has settled to the minimum, the inflaton continues its slow roll down the "residual" potential which goes to zero at infinity like ϕ^{-2}, thus allowing for a quintessential tracking solution. Also in this case the initial conditions for the quintessential part of the model do not have to be set by hand, but depend uniquely on the value of the inflaton field at the end of reheating.

In conclusion, the cosmological constant issue still leaves many open questions both in cosmology and particle physics.

On one hand we have to wait for a better understanding of the systematic errors possibly affecting the SN Ia data. Also the high precision CMB anisotropy information that will come with the MAP and PLANCK satellites will give a tremendous breakthrough in the measurment of the cosmological parameters. These last data should leave no more ambiguity in the determination of the total energy content of the universe, parametrized by Ω. Both these facts, together with the other cosmological measurments, should be able in a few years to settle once for all the issue of the vacuum energy contribution to the cosmological energy budget.

On the other hand, from the particle theory point of view much work is still to be done. At present we don't have any conclusive proposal which could set $\Lambda_{eff} = \Lambda + 8\pi G\varepsilon_0$ equal (or nearly equal) to zero. This problem will probably require a better understading of the unification of gravity with quantum field theory in order to be solved. As we have seen, the cosmological constant problem arises in particle physics when we face the quantum theory of fields with the cosmological setting. As a consequence, any resolution of the puzzle can only come from a better understanding of their deep relationship.

Nevertheless, as we have extensively discussed, if $\Lambda_{eff} = 0$ cosmological scalar fields ('Quintessence') might play a crucial role in accounting for the acceleration of the universe. In this respect, Supersymmetric QCD theories have proven to be workable particle physics candidates for Quintessence. A deeper investigation of this scenario would require the implementation of SQCD in a wider context, such as superstring theories or theories with large compact extra-dimensions, what clearly lies beyond the scope of the present work.

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