ISAS - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

BOUNDS ON SUPERSYMMETRIC PARTICLES AND ON E_6 FERMIONS FROM SEARCHES OF COSMOLOGICAL RELICS AND FROM LEP

Thesis submitted for the degree of

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Overview

The problem faced in this thesis is that of looking for ways to test (and constrain) some of the most accepted extensions of the Standard Model (SM) of elementary particles. It is divided in two parts: the first two chapters deal with the supersymmetric extension of the SM while the last two with phenomenological aspects of the exotic fermions that are present in the E_6 grand unified models.

I have considered some implications that the new results of the LEP collider have for these theories and analysed in detail the bounds that can be obtained from searches of cosmological relics, either of the stable lightest supersymmetric particle or of stable exotic quarks, that can in principle contribute to the missing mass that is known to exist in our universe.

In the first chapter, a brief sketch of how the supersymmetric SM can be obtained as the low energy limit of supergravity theories (possibly arising from superstrings) is presented, stressing why the naturalness problem associated to the fundamental Higgs scalars requires that supersymmetry should manifest at the weak scale. The bounds on the supersymmetric parameters resulting from accelerator searches and cosmological requirements is discussed. The nature of the lightest supersymmetric particle is analyzed and it is found that the most likely candidate is the neutralino, a neutral fermion combination of the superpartners of the neutral gauge bosons γ and Z and of the neutral Higgs bosons.

In the second chapter it is shown that for a very wide range of the parameter space the neutralinos make a significant contribution to the mass density of the universe. This makes them one of the most attractive dark matter candidates. I discuss the strategies for detecting dark matter neutralinos and concentrate on their indirect search through the observation of the neutrino flux induced by the annihilation of neutralinos trapped in the interior of the sun and of the earth underground proton decay experiments. It is shown that present experimental results already impose further constraints beyond those imposed by accelerators and that feasible improvements could allow to test a very interesting range not accessible to present colliders. Also the direct searches with Ge spectrometers is addressed.

I turn then to consider the phenomenology of the new fermions that appear in models based on E_6, the predilect unification group of superstrings. In these theories, each generation is assigned to a **27** representation that besides the standard fermions contains new ones: two neutral singlets, a new lepton doublet with its charge conjugate and a color triplet isosinglet quark of charge -1/3 together with its conjugate. The avoidance of terms inducing proton decay is usually obtained forbidding some dangerous couplings. This in turn can imply that the exotic quarks have no channels to decay. In the third chapter I consider the possibility of having a stable exotic quark and reject it on the light of the unsuccessful searches of anomalously heavy isotopes and on the astrophysical implications they could have. A natural way to make the exotic fermions unstable is to allow for their mixing with the ordinary ones. However, since ordinary and exotic fermions of the same colour and charge can be in different SU(2) representations, this mixing can induce deviations from the standard couplings of the ordinary fermions to the Z boson that are being measured at present at LEP. Using the results of the first run of LEP it is possible then to obtain important constrains on these mixings, as is shown in chapter 4.

CHAPTER 1

The Supersymmetric Standard Model and the Nature of the LSP.

Supersymmetric theories with R-parity symmetry predict that the Lightest Supersymmetric Particle (LSP) is stable and this feature gives the LSP especial relevance at least for two reasons. First, because as it often happens that the LSP is neutral, the characteristic signal of supersymmetric particle production in accelerators is the missing energy carried away by the LSP and second, because being the LSP stable, a significant amount of them could have been left over by the big bang and result of cosmological relevance today. Both consequences are important. They allow to constrain particular supersymmetric models and the second one can provide a solution to the problem of the missing dark mass in the universe if this is identified with the relic LSPs.

One of the supersymmetric models that has been more considered is the minimal supersymmetric extension of the standard model, arising as the low energy effective theory of N=1 supergravity [1.1]. In this frame, the spectrum and couplings of the supersymmetric particles can be computed in terms of a small number of parameters, allowing to identify the nature of the LSP in the different ranges of parameter values. From this analysis it results that the possible candidates are the neutralinos, the sneutrinos and the gravitinos, whose phenomenology is considered in some detail. The recent bounds obtained by LEP and proton colliders (Tevatron and CERN collider) as well as the possible cosmological implications of the supersymmetric particles lead to significant constraints to this supersymmetric models.

1.1-Sketch of low energy supergravity

Supersymmetry, being the unique non-trivial generalization of the Poincaré algebra [1.2] and having a lot of desirable features, has been widely applied. In particular, the supersymmetric extension of the standard model is a strong candidate for the low energy (E< M_{Planck}) particle physics model, probably arising from a more fundamental theory like superstrings. Among the nice features is that supersymmetry can solve the naturalness problem associated to the fundamental Higgs scalars[1]. This naturalness (or hierarchy) problem [1.3] arises because although one expects the renormalized Higgs mass to be $\sim M_W$, in order that the Higgs be related to the electroweak breaking, it receives at one loop quadratically divergent contributions $\delta m_H^2 \propto \Lambda^2$ (where Λ is the cutoff associated to the threshold of new-physics, typically M_{GUT} or M_{Pl}). Hence, the required counterterm should be extremely fine-tuned to produce the cancelation. Supersymmetry can solve this problem due to its nice ultraviolet behaviour: since the number of bosonic and fermionic degrees of freedom in multiplets with equal masses and couplings is equal, the quadratically divergent contributions of boson loops are canceled by the fermion loops. Clearly supersymmetry must be broken at low energies, but the cancelation of quadratic divergences partially subsists if supersymmetry is softly broken. This is what happens in supergravity induced models, where the loop contributions to scalar masses is proportional to the mass splitting between superpartners. Hence, to avoid unnatural fine-tunings the superpartner masses must be below the $\sim$ TeV scale. Thus, their consideration is unavoidable and one may hope that supersymmetry will be found in a not too far future.

The other nice feature of supersymmetry is that when it is made local (supergravity), due to the connection of supersymmetric and Poincaré generators, the theory automatically includes general relativity, providing a way to unify particle interactions in a gauge theory context. Local susy is usually thought to be broken at a large scale in

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[1] an alternative solution to this problem is compositeness (technicolor) in which case the Higgs boson is not elementary any more

a hidden sector that communicates with the 'ordinary' particles only through gravitational interactions (e.g. the E_8' sector of the $E_8 \times E_8'$ model obtained from superstrings, that is a singlet under the standard model interactions contained in E_8). When taking the low energy 'flat limit' one is left with a globally supersymmetric theory plus some soft susy breaking terms whose coefficients depend on the details of the hidden sector. When one rescales this theory from M_{Pl} to M_W using the renormalization group equations [1.4], different particle masses are renormalized differently according to their electroweak and strong interactions. In particular, this may induce the Higgs fields to acquire non-vanishing vevs, leading to the electroweak symmetry breaking as a final gift.

Global supersymmetric model

To construct a global super Yang Mills model coupled to matter one needs a real vector multiplet $V^a = V^{a\dagger}$ and chiral multiplets Φ^i (such that $\bar{D}_\alpha \Phi^i = 0$, where $\bar{D}$ is the covariant derivative in superspace (x, θ_α)). Making the usual polynomial expansion of the superfields in powers of the anticommuting variables θ and $\bar{\theta}$, the vector superfield has as components the vector field $v_\mu^a(x)$, a Weyl spinor $\lambda_a(x)$ (the gaugino, with adjoint gauge indices) and an auxiliary field $D(x)$ (coefficient of the $\theta\theta\bar{\theta}\bar{\theta}$ term ,i.e. $\int d^4\theta V = D$). Since under susy transformations auxiliary fields only gain a space-time derivative, $\int d^4x\ D$ is susy invariant. In addition, vector multiplets contain one real and one complex scalars and a Weyl spinor that can be gauged away (in the Wess Zumino gauge) in the massless case, while in the massive case they combine with the others to leave a Dirac fermion, a real scalar and the vector field. The chiral fields are composed of a Weyl spinor $\Psi_\alpha(x)$ and two complex scalars $A(x)$ and $F(x)$. The auxiliary field $F(x)$ is the coefficient of the $\theta\theta$ piece, i.e. $\int d^2\theta \Phi = F$, and can be used to construct susy invariants in the same way as the D terms of vector superfields.

The supersymmetric and gauge invariant lagrangian density is then

$$\mathcal{L} = \int d^4\theta \Phi^\dagger e^{2gV} \Phi + \int d^2\theta \frac{1}{4g^2}\ Tr W^\alpha W_\alpha + \int d^2\theta (g(\Phi) + h.c.) \tag{1.1.1}$$

where $V_j^i \equiv V^a T_j^{ai}$ with T^a the generators of the gauge group,

$$W_\alpha \equiv \bar{D}\bar{D} e^{-gV} D_\alpha e^{gV} \tag{1.1.2}$$

is the chiral superfield that contains the gauge field strength. Finally $g(\Phi_i)$ is a chiral function, called the superpotential. This function should be a polynomial of degree at most three in the observable fields in order that the low energy theory be renormalizable (supergravity is not renormalizable due to the gravitational effects, but one expects the low energy theory to be so). This lagrangian has, in the Wess-Zumino gauge, the usual kinetic terms for the gauge bosons, the gauginos, the matter fermions and scalars (with mass matrix $\frac{\partial^2 g(A)}{\partial A_i \partial A_j}$). The scalar potential is

$$V(A) = \sum_i F_i^\dagger F_i + \frac{1}{2}\sum_a (D^a)^2 \tag{1.1.3}$$

and the equations of motion for the auxiliary fields yield

$$F_i^\dagger = -\frac{\partial g(A)}{\partial A_i} \quad ; \quad D^a = -g\sum_i A_i^\dagger T^a A_i \ . \tag{1.1.4}$$

This global supersymmetric model implies the following tree-level mass supertrace relation

$$Str\ \mathcal{M}^2 \equiv \sum_J (-)^{2J}(2J+1)m_J^2 = 0. \tag{1.1.5}$$

Since variations under global susy of bosonic fields involve fermionic fields or derivatives of bosonic fields while those of fermionic fields involve auxiliary fields, to break susy preserving Poincaré invariance requires to give a vev to the variation under susy of a fermionic field ($\delta\Psi \propto F$, $\delta\lambda \propto D$). Hence, two kinds of global susy breakings have been studied, the Fayet Iliopoulos [1.5] ($\langle D\rangle \neq 0$) and the O'Raifearteigh [1.6] ($\langle F\rangle \neq 0$). In these attempts it is hard to give a positive contribution to $Str\ \mathcal{M}^2$ in order to lift the scalar superpartner masses to experimentally allowed values. However, our goal is to obtain a global susy breaking from the flat limit of broken supergravity where this is easily achievable.

Local supersymmetric model

In the usual Yang Mills theory the local gauge symmetry $\phi \to e^{ig\omega(x)}\phi$, with $\omega \equiv \omega^a T^a$, is obtained by making covariant the derivatives with the gauge connection $A_\mu \equiv A_\mu^a T^a$ satisfying $\delta A_\mu = \partial_\mu \omega - ig[A_\mu, \omega]$. In the same way, for the local supersymmetric transformations one has $\omega \equiv \alpha^m P_m - \frac{\lambda^{mn}}{2} M_{mn} + \bar{\epsilon} Q$, so that the connection becomes

$$V_\mu = e_\mu^m P_m - \frac{\omega_\mu^{mn}}{2} M_{mn} + \frac{1}{M_{Pl}} \bar{\Psi}_\mu Q \tag{1.1.6}$$

where e_μ^m is the vierbein ($g_{\mu\nu} = e_\mu^m e_\nu^n \eta_{mn}$) associated with translations, ω_μ is the spin connection associated to boosts and rotations and Ψ_μ is the spin 3/2 gravitino field related to the supersymmetry transformations. Local susy requires $\delta V_\mu = \partial_\mu \omega - ig[V_\mu, \omega]$. A realization of this is the Einstein plus Rarita Schwinger lagrangians

$$\mathcal{L} = -\frac{M_{Pl}^2}{2} eR - 2\epsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\sigma. \tag{1.1.7}$$

Here $e = \sqrt{g} = det\ e_\mu^m$ and $R = e_m^\mu e_n^\nu R_{\mu\nu}^{mn}$ is the scalar curvature, with

$$R_{\mu\nu}^{mn} \equiv \partial_\mu \omega_\nu^{mn} + \omega_\mu^{mk} \omega_{\nu k}^n - (\mu \leftrightarrow \nu). \tag{1.1.8}$$

To close the susy algebra off-shell one introduces vector, scalar and pseudoscalar auxiliary fields that complete the gravitational supermultiplet together with e_μ^m and Ψ_μ (from the eq. of motion, $\omega_\mu^{mn} = \omega_\mu^{mn}(e, \Psi_\mu)$).

The most general Lagrangian for the matter and gauge multiplets has the form

$$\mathcal{L} = \int d^4\theta E\{\ J(\Phi, \Phi^\dagger e^{2gV}) + \mathrm{Re}\left(\frac{1}{\mathcal{R}}(\frac{1}{4g^2} f_{ab}(\Phi)\ TrW^a W^b + g(\Phi))\right)\}. \tag{1.1.9}$$

Here g is the superpotential and the function f_{ab}, which transforms as a symmetric product of two adjoint gauge representations, allows for general non canonical gauge kinetic terms. E and $\mathcal{R}$ are the corresponding superspace determinant and chiral curvature.

After elimination of the auxiliary fields, rescaling and manipulation of the fields, the resulting lagrangian only depends on $d \equiv -3ln(-J/3)$ and g through the combination

$$\mathcal{G}(A_i, A^{i*}) = d(A, A^*) + ln|g(A)|^2 \tag{1.1.10}$$

called the Kahler potential (A is the scalar field in Φ).

The bosonic part of the lagrangian is

$$e^{-1}\mathcal{L}_B = -\frac{1}{2}R + \mathcal{G}_j^i D_\mu A_i D^\mu A^{i*} + e^{-\mathcal{G}}(3 + \mathcal{G}_k \mathcal{G}_l^{-1\ k} \mathcal{G}^l) \tag{1.1.11}$$

$$-\frac{1}{2} f_{ab}^{-1} D^{(a)} D^{(b)} - \frac{1}{4}\mathrm{Re}\ f_{ab} F^a F^b + \frac{1}{4} i\ \mathrm{Im}\ f_{ab} F^a \tilde{F}^b \tag{1.1.12}$$

where $D^{(a)} \equiv \mathcal{G}^i T_i^{a\ j} A_j$, with $\mathcal{G}^i \equiv \partial\mathcal{G}/\partial A_i$, $\mathcal{G}_i \equiv \partial\mathcal{G}/\partial A^{i*}$.

From eq. (1.1.11-12) we see that:

i) In order for the matter fields to have canonical kinetic terms we need $\mathcal{G}_j^i = \delta_j^i$ (e.g. $\mathcal{G} = A_i A^{i*} + ln|g|^2$). Since $\mathcal{G}_j^i$ is the metric of the Kahler manifold spanned by the fields A_i, this corresponds to a Kahler flat manifold. This condition is sometimes required, at least in the observable sector.

ii) The scalar potential is now

$$V = e^{-\mathcal{G}}(3 + \mathcal{G}_k \mathcal{G}_l^{-1\ k} \mathcal{G}^l) - \frac{1}{2} f_{ab}^{-1} D^{(a)} D^{(b)}. \tag{1.1.13}$$

iii) If $f_{ab} \neq \delta_{ab}$ the gauge kinetic terms are non-canonical. This is an interesting possibility because the gaugino mass term turns out to be

$$\frac{1}{4} e^{-\mathcal{G}/2} \mathcal{G}^l \mathcal{G}_l^{-1\ k} f_{ab,k}^* \bar{\lambda}_R^a \lambda_R^b + h.c. \tag{1.1.14}$$

and this allows for non zero tree-level gaugino masses if $f_{ab,k} \neq 0$. This feature is essential to break susy in some no-scale models [1.7] and happens in models derived from superstrings.

The mass term for the gravitino field is

$$e^{-\mathcal{G}/2} \bar{\Psi}_\mu \sigma^{\mu\nu} \Psi_\nu \tag{1.1.15}$$

The breaking of supergravity requires a non-vanishing vev for an auxiliary field. By the superhiggs effect, the gravitino eats the degrees of freedom of the associated goldstino acquiring the mass $e^{-\langle\mathcal{G}\rangle/2}$. To avoid a cosmological constant we should require a vanishing vacuum energy, i.e. $V(\langle A_i \rangle) = 0$ in the minimum $\partial V/\partial A_i|_{\langle A \rangle} = 0$. This condition means usually unnatural fine-tunings of the parameters in the superpotential (the solution of this problems is the aim of the no-scale models).

It is generally assumed that the fields breaking susy are singlets under the Standard Model group, i.e. that they are in a "hidden sector". The simplest example corresponds to only one hidden field z (from a chiral supermultiplet (z, χ, h)) with a Polonyi-type superpotential $\mathcal{G}(z) = m_S^2(z + \beta)$, which yields a scalar potential in the z direction (for canonical kinetic terms)

$$V(z) = m_S^4 e^{|z|^2} (3|z + \beta|^2 - |z(z^* + \beta) + 1|^2). \tag{1.1.16}$$

This potential is minimum at $\langle z\rangle = (\sqrt{3}-1)M$ and vanishes there only if $\beta = M(2-\sqrt{3})$ (we have reinserted the Planck mass $M = M_{Pl}/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV).

Susy is broken if the auxiliary field h acquires a non vanishing vev. Since its eq. of motion is $h(z) \propto g^*(z)$, the superpotential should be non-vanishing at the minimum. In Polonyi's example, $g(\langle z\rangle) = m_S^2 M$, so that susy is broken. The gravitino eats the degrees of freedom of χ becoming massive. The resulting gravitino mass, $m_{3/2} = e^{(\sqrt{3}-1)^2/2} m_S^2/M$, is of order M_W (as required by naturalness since $m_{3/2}$ sets the scale of the superpartners mass splittings in this model) if the susy breaking scale m_S is chosen in the range $10^{10} - 10^{11}$ GeV.

The superpotential for the observable fields y_i is usually

$$g = h_U H_2 Q U^c + h_D H_1 Q D^c + h_L H_1 L E^c + \mu H_1 H_2 \tag{1.1.17}$$

where Q_L, L_L, U^c_L, D^c_L, E^c_L and $H_{1,2}$ are the superfields associated to the standard model quark and lepton doublets, up quark, down quark and charged lepton right singlets and the two Higgs doublets of hypercharge $\pm 1/2$ required in susy models. Although other terms such as

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$$QLD^c + LLE^c + U^c D^c D^c + LH_2 \tag{1.1.18}$$

are in principle allowed by the gauge symmetries, they can lead to baryon number (B) or lepton number (L) violation that could produce baryon decay at catastrophic rates. The easiest way to avoid this is by invoking a multiplicatively conserved discrete symmetry, called R-parity, under which ordinary particles are even while the superpartners are odd ($R = (-)^{2s+3B+L}$, with s the spin of the particle). The consequences of imposing R-parity are that superpartners should be produced in pairs (and in particular the diagram with a single squark mediating proton decay is forbidden) and that in the decay products of a superpartner there should always be another superpartner, with the result that the lightest superpartner should be stable.

Low energy theory

In order to get the low energy effective theory, it is necessary to take the 'flat limit' of the complete supergravity Lagrangian written above. To take the flat limit means to shift the field z to his vev and to expand the lagrangian neglecting terms of order M_{Pl}^{-1} while keeping $m_S^2/M_{Pl} \simeq m_{3/2}$ fixed. The result is that z decouples from the observable sector, now characterized by the globally supersymmetric Lagrangian plus the soft-supersymmetry breaking terms

$$\mathcal{L}_{soft}(y_i, y_i^*) = m_{3/2}^2 \sum_{ij} S_{ij} y_i y_j^* + m_{3/2}[Ag^{(3)} + Bg^{(2)} + h.c.] + \sum_a M_a \bar{\lambda}_a \lambda_a + h.c. \quad (1.1.19)$$

In the case of a flat Kahler metric the coefficient A (of the terms in the superpotential trilinear in the fields $g^{(3)}$) and the coefficient B (of the bilinear terms) are related through $B = A - 1$ (for the Polonyi superpotential $A = 3 - \sqrt{3}$) and $S_{ij} = \delta_{ij}$, so that the scalars have a common mass $m_{3/2}$. In the more general case $\mathcal{G} = \Lambda_{ij}(z, z^*) y_i y_j^* + \ldots$ (where renormalizability of the observable sector requires $\langle \Lambda_{ij} \rangle = \delta_{ij}$) it is possible to have $S_{ij} \neq \delta_{ij}$ for a non trivial Λ_{ij} (see [1.8]) and hence, non-common scalar masses.

From this Lagrangian at the Planck scale we must now go down to the weak scale. The electroweak and strong interactions will renormalize all the parameters following the renormalization group equations [1.4]. One of the Higgs square masses becomes negative (pulled down by the top Yukawa coupling) inducing the electroweak symmetry breaking at a scale $\mu \simeq M \exp(-\mathcal{O}(1)/\alpha_t)$, with $\alpha_t \equiv h_t^2/4\pi$, i.e. the W boson mass is dynamically determined to be $\mathcal{O}(100 \text{ GeV})$ for reasonable values of the top Yukawa coupling h_t.

For simplicity, and to get predictability, one neglects intergenerational mixing and only retains the third generation Yukawa couplings (even only h_t). Furthermore, one generally assumes an underlying unification group (since in particular supersymmetric $SU(5)$ leads to a correct prediction of s_W^2 and an acceptable proton lifetime with $M_{GUT} \sim 10^{16} - 10^{17}$ GeV) what allows to relate the different gaugino masses at the unification scale. At this point, the model depends only on the unknown parameters h_t, $m_{3/2}$, A, B, $m_{1/2}$ and μ.

1.2- The nature of the LSP and the accelerator constraints.

Low energy mass spectrum

The resulting low energy mass spectrum is the following:

i) **Neutralinos:** the neutralino mass matrix in the base $(\tilde{W}_3, \tilde{B}, \tilde{H}_1, \tilde{H}_2)$ corresponding respectively to the $SU(2)$ and $U(1)$ gauginos and to the two higgsinos (these are the Majorana fermions superpartners of the neutral bosons in the model) is

$$\begin{bmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{bmatrix} \tag{1.2.1}$$

with M_1 and M_2 the bino and wino mass parameters, assumed to arise from a common gaugino mass $m_{1/2}$ at the GUT scale, so that at the weak scale they are related through $M_i = (\alpha_i/\alpha_G)m_{1/2}$ ($\alpha_G \simeq 1/24$ is the common coupling constant at the GUT scale). In particular, $M_1 = \frac{5}{3}M_2 \tan^2\theta_W$. The ratio of the two vacuum expectation values of the Higgs fields $\langle H_1^0 \rangle = v_1$ and $\langle H_2^0 \rangle = v_2$ is denoted by [2] $tg\beta \equiv v_2/v_1$ and μ is the mass parameter coupling the two higgses in the superpotential. We will denote χ the lightest of the four neutralino mass eigenvalues. There are three cases in which $m_\chi \simeq 0$: for $M_2 \to 0$ there is a massless photino $\tilde{\gamma} = c_W\tilde{B} + s_W\tilde{W}_3$; for $\mu \to 0$ there is a massless higgsino $\tilde{h} = s_\beta\tilde{H}_1 + c_\beta\tilde{H}_2$ and for $\mu M_2 = \frac{8}{5}M_W^2 \sin 2\beta$ there is another non-trivial massless combination of the four current eigenstates. For a given value of M_2, the largest values of m_χ result for large values of $|\mu|$, in which case the lightest neutralino is mainly a $\tilde{B}$ with mass $m_\chi \simeq \frac{5}{3}\,\mathrm{tg}_W^2 M_2$, so that as a general rule $m_\chi \lesssim M_2/2$.

We will take all the parameters of the lagrangian to be real, neglecting possible small CP violation effects. Without loss of generality we choose M_2 non negative since there is a symmetry in the renormalization group equations involved under a simultaneous change of the sign of μ and M_2. A correct electroweak breaking requires $v_2/v_1 \geq 1$ [1.9].

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[2] we have denoted $\sin\beta \equiv s_\beta$, $\sin\theta_W \equiv s_W$, etc.

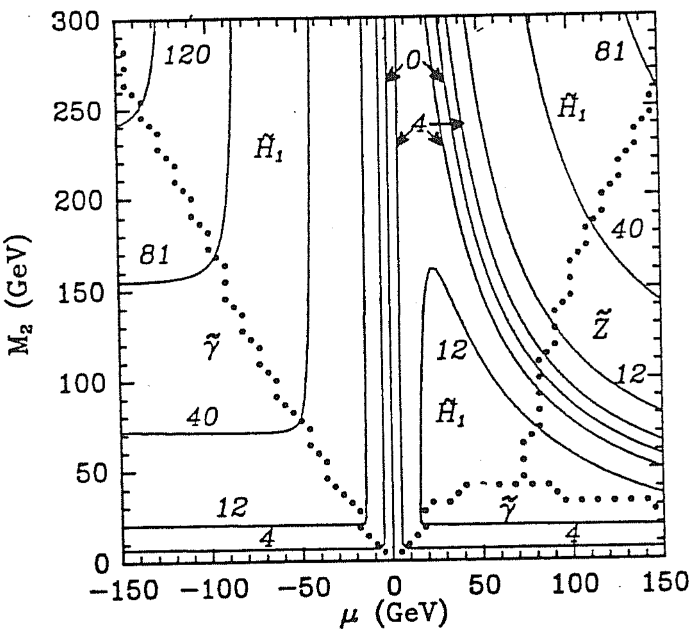

Fig. 1a.

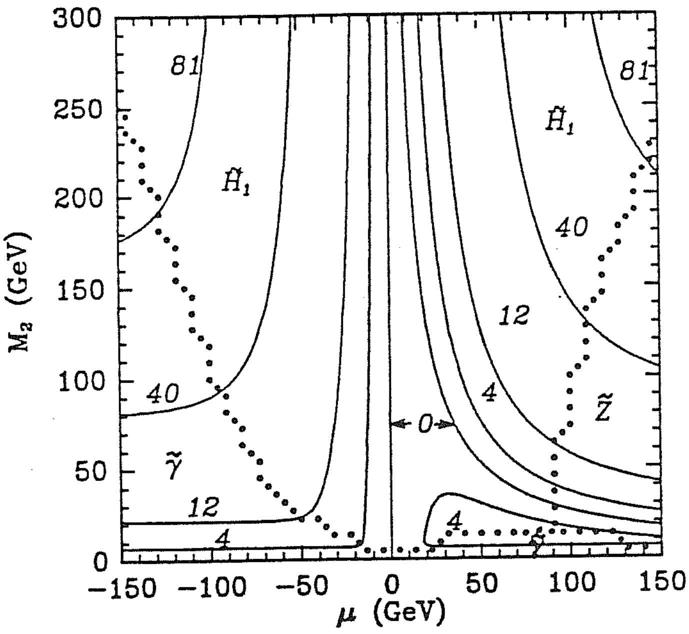

Fig. 1b.

Fig. 1: Contour maps of the mass of the lightest neutralino χ (in GeV) and its dominant component (in regions separated by dotted lines) for $v_2/v_1 = 2$ (1.a) and 8 (1.b).

Contour maps of m_χ and the dominant neutralino component of the χ (in regions separated by dotted lines) in the $\mu - M_2$ plane are shown in figs. 1a and 1b, for $v_2/v_1 = 2$ and $v_2/v_1 = 8$ respectively.

ii) **Charginos:** the masses of the charginos (Dirac fermions spartners of the charged W bosons and Higgses) can be also obtained in terms of the same parameters M_2, μ and v_2/v_1, diagonalizing the matrix

$$\begin{bmatrix} 0 & 0 & M_2 & M_W\sqrt{2}c_\beta \\ 0 & 0 & M_W\sqrt{2}s_\beta & \mu \\ M_2 & M_W\sqrt{2}s_\beta & 0 & 0 \\ M_W\sqrt{2}c_\beta & \mu & 0 & 0 \end{bmatrix}. \tag{1.2.2}$$

in the base $(\tilde{W}^+, \tilde{H}_2^+, \tilde{W}^-, \tilde{H}_1^-)$.

iii) **Sleptons:** The slepton masses are given by

$$m^2(\tilde{l}) = m_{3/2}^2 - M_Z^2 \frac{\mathrm{tg}^2\beta - 1}{\mathrm{tg}^2\beta + 1}(-Qs_W^2 + I_3) + \left[\frac{3}{2b_2}(1 - \frac{1}{F_2^2}) + \frac{3}{10b_1}(1 - \frac{1}{F_1^2})\right] m_{1/2}^2 \tag{1.2.3}$$

where Q and I_3 are the charge and the third component of the isospin, b_i are the coefficients of the β functions ($\partial\alpha_i/\partial\ln\mu^2 = b_i\alpha_i^2/4\pi$, $b_1 = 33/5$, $b_2 = 1$) while $F_i \equiv$

$1 + b_i\alpha_{GUT}\ln(M_{GUT}/Q_o)^2$ and Q_o should be taken as the weak scale. In models with non canonical kinetic terms for the scalar fields the common scalar mass could slightly differ from $m_{3/2}$ and even be not common (although this last feature could be constrained by the FCNC effects). Furthermore, in some no-scale models the common scalar mass term could be absent with the gravitino acquiring an arbitrary mass, the scale of susy breaking being set by the gaugino masses alone.

The sneutrino mass results then

$$m_{\tilde{\nu}}^2 \simeq m_{3/2}^2 - \frac{M_Z^2}{2}\frac{\mathrm{tg}^2\beta - 1}{\mathrm{tg}^2\beta + 1} + 0.72\ M_2^2 \tag{1.2.4}$$

while the charged slepton masses are always larger since the term proportional to M_Z^2 (arising from the D-terms) results positive for them.

iv) **Squarks:** the squark masses are given by formulae analogous to those of the sleptons with additional large positive contributions proportional to the gluino mass that push up their masses. Furthermore, for the top squarks neither the supersymmetric contribution m_t^2 nor the effect of the top Yukawa coupling in the renormalization group evolution can be neglected. Although the stops can be lighter than the other squarks[3], they do not turn out to be the LSP.

v) **Gluinos:** the gluino mass $M_3 = (\alpha_3/\alpha_2)M_2$ is always much larger than the χ mass so that they are not the LSP.

vi) **Higgs particles:** the Higgs spectrum in the minimal susy model corresponds to a quite constrained two doublet system [1.10]. There are three physical neutral Higgs particles, two scalars (usually called H_1 and H_2) and one pseudoscalar (H_3). Together with the would-be Goldstone boson eaten by the Z, they are the eigenstates of two 2×2 mass matrices, one for the real parts and the other for the imaginary parts of $H_{1,2}^0$. Both matrices depend on just two parameters, which we choose to be v_2/v_1 and the mass of the lightest one, m_{H_2}. The mass eigenvalues are given by

$$m_{H_a}^2 = r_a m_Z^2, \tag{1.2.5}$$

where

$$r_2 \leq c^2, \quad c \equiv \cos 2\beta, \tag{1.2.6}$$

[3] the $\tilde{t}_L - \tilde{t}_R$ mixing can give rise to some kind of seesaw that leads to a light stop state

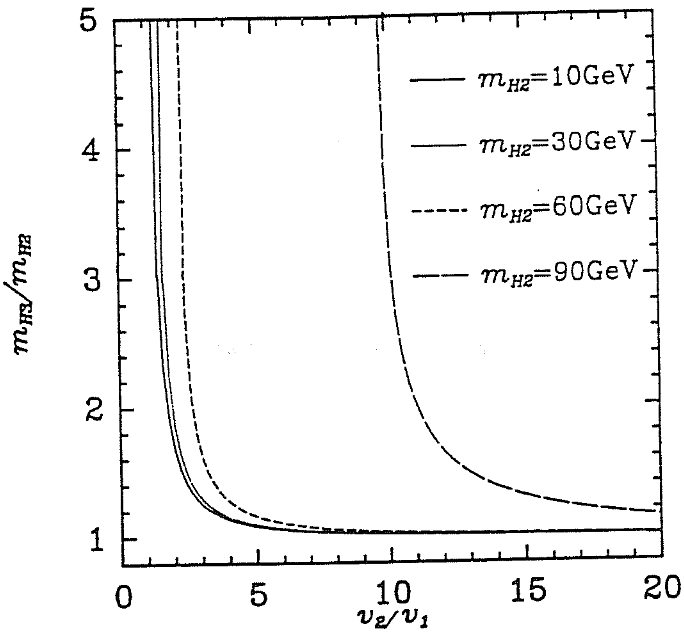

Fig. 2: Ratio of the mass of the pseudoscalar Higgs H_3 over the mass of the lightest H_2 as a function of v_2/v_1 for different values of m_{H_2}.

$$r_1 = c^2 \left(\frac{1 - r_2}{c^2 - r_2} \right), \tag{1.2.7}$$

$$r_3 = r_2 \left(\frac{1 - r_2}{c^2 - r_2} \right). \tag{1.2.8}$$

These equations imply $m_{H_2} \leq m_{H_3} \leq m_{H_1}$, with the lightest neutral scalar, H_2, lighter (and the heaviest, H_1, heavier) than the Z. For values of v_2/v_1 near one, the mass m_{H_3} is much larger than m_{H_2} but for large values of v_2/v_1 both masses become equal (see fig. 2). One can obtain the relation $m^2_{H_1} = m^2_Z + m^2_{H_3} - m^2_{H_2}$ that implies that when m_{H_3} becomes similar to m_{H_2} (for large v_2/v_1) then $m_{H_1} \simeq m_Z$. Finally, there are two charged Higgses $H^{\pm}$ with masses $m^2_{H^{\pm}} = m^2_W + m^2_{H_3}$.

To describe the low energy mass spectrum it is convenient to choose as independent free parameters the quantities h_t, tgβ, m_{H_2}, M_2 and μ (alternatively, instead of h_t one can use $m_{3/2}$ since all these parameters are related through the condition $\sqrt{v_1^2 + v_2^2} =$ 246 GeV for the electroweak breaking to reproduce M_W).

Experimental bounds on supersymmetric masses.

LEP has found a lower bound on the chargino mass of $m_{\chi^{\pm}} > 46$ GeV [1.11], that significantly constrain the values of M_2 and μ, parameters that determine also the neutralino masses. In the allowed region of the $M_2 - \mu$ plane, χ is always lighter than the charginos.

From the invisible Z-width Γ_{inv}, one obtains a number of equivalent massless ν species $N_\nu = 2.96 \pm 0.14$ (average of the four LEP experiments [1.12]). Since the $\tilde{\nu}$ contributes to Γ_{inv} in an amount

$$\Gamma_{\tilde{\nu}\tilde{\nu}} = \frac{1}{2}\left(1 - \frac{4m_{\tilde{\nu}}^2}{M_Z^2}\right)^{3/2} \Gamma_{\nu\bar{\nu}} \tag{1.2.9}$$

this leads to $m_{\tilde{\nu}} \gtrsim 43$ GeV if three degenerate sneutrino flavors contribute ($m_{\tilde{\nu}} \gtrsim 36$ GeV if just one $\tilde{\nu}$ contributes). This bound holds approximately even for unstable $\tilde{\nu}$, since the decay products are mainly invisible.

CDF and UA2 [1.13] have obtained a bound on the gluino mass of $M_3 \gtrsim 74$ GeV, that would translate in $M_2 \gtrsim$ 20 GeV using the unification relation of the gaugino masses.

Zen events at LEP [1.14], i.e. the production of $Z \rightarrow \chi\chi'$ of a heavier neutralino χ' that subsequently decays into the lightest χ plus observable particles that give rise to characteristic 'one-sided' events, exclude a small region with $\mu \simeq -50 \div -20$ GeV and $M_2 \simeq 10 \div 30$ GeV. LEP searches have also excluded Higgs masses $m_{H_2} \lesssim 25$ GeV for values of $v_2/v_1 \simeq 1$ and up to $m_{H_2} \simeq 40$ GeV for larger v_2/v_1 values [1.15].

Other bounds have been obtained at LEP but in general overlap with the previous ones. For instance, the bounds on the charged sleptons are taken into account when one uses the $\tilde{\nu}$ mass bound, the CDF bound and the mass relations.

Candidates for the LSP

It is now clear that in the minimal model the only candidates for the LSP are the neutralino, the sneutrino and the gravitino.

In non-minimal models one could imagine the LSP being a chargino, a charged slepton or a squark, but this would however be in conflict with the searches for exotic charged or strongly interacting particles in nature once their predicted cosmological abundance is taken into account.

From eq. (1.2.4) the following relation can be obtained:

$$m_{\tilde{\nu}} \lesssim m_{3/2} \qquad \text{if} \qquad M_2 \lesssim \sqrt{\frac{\mathrm{tg}^2\beta - 1}{\mathrm{tg}^2\beta + 1}}\, 75\mathrm{GeV} \tag{1.2.10}$$

In the case $m_{\tilde{\nu}} < m_{3/2}$ it results that the χ is actually the LSP, since the upper bound $m_\chi < M_2/2$ is in this case less than the present LEP bound on the $\tilde{\nu}$ mass. We arrive at the important conclusion that the $\tilde{\nu}$ can not be the LSP in the minimal model when the scale of susy breaking is set by a common gravitino mass contributing to the scalar masses.

1.3-Cosmological constraints and the gravitino problem

Besides the accelerator bounds previously discussed, there are further constraints on the supersymmetric parameters arising from cosmological arguments that should also be taken into account in this analysis. These are mainly the requirement that the universe not be overclosed by the relic LSPs and the avoidance of the dangerous effects of the decay products of long lived particles (gravitinos when they are not the LSP or relic χ or $\tilde{\nu}$ when gravitinos are the LSP) [1.16].

If the LSP is the neutralino, its relic cosmological mass density ρ_χ is determined by the rate of χ annihilations in the early universe and the constraint $\rho_\chi \lesssim \rho_c = 2 \times 10^{-29}\mathrm{g/cm}^3$ (ρ_c is the critical density necessary to close the universe) exclude neutralinos with masses below a few GeV (see next chapter). For what concerns the gravitinos, due to their gravitationally suppressed couplings they freeze-out at very early times surviving in large amounts until they decay at times $\tau \simeq M_{Pl}^2/m_{3/2}^3 \simeq 10^8$ s $\left(\frac{100\ \mathrm{GeV}}{m_{3/2}}\right)^3$, yielding an additional contribution to the LSP density since one χ is produced by each decaying gravitino.

In the case in which gravitinos are the LSP, they provide a candidate for hidden (undetectable?) dark matter. The 'Lee-Weinberg' [1.17] computation of the relic abundance of the χ (or $\tilde{\nu}$, whichever is the lightest) would yield their density after their 'freeze-out' in the early universe, but now these particles are unstable decaying into gravitinos (plus something) with a typical lifetime $\tau \simeq M_{Pl}^2/m^3$, where m is the particle mass. In both cases the extremely large primordial gravitino relic density turns out to be problematic (unless $m_{3/2} <$ few KeV [1.18]). The only way out is to assume a period of inflation, required also by other independent reasons (flatness, monopole, etc problems), that dilutes the gravitino density to acceptable values.

This is not the end of the story, because after the end of the inflation the universe is reheated by the decay of the inflaton. A large reheating temperature is required in order that the baryon asymmetry be generated through the usual GUT scenario, i.e. with the out of equilibrium decay of particles with mass $\gtrsim 10^{11}$ GeV (as is necessary to have simultaneously an acceptable proton lifetime). The correspondingly large reheating temperature leads to the regeneration of a large density of gravitinos in the thermal bath [1.19].

In both the cases in which χ is the LSP and the regenerated gravitinos decay $\tilde{G} \to \chi+$ 'something' or that the gravitino is the LSP and the relic χ (or $\tilde{\nu}$) decay $\chi \to \tilde{G}+$ 'something', the remaining decay products are very dangerous. They can destroy the agreement between nucleosynthesis yields and the observed light element abundances, produce excessive entropy release and distort the cosmic microwave background radiation [1.16].

In the case in which gravitinos decay, avoidance of these problems require the reheating temperature to be $T_R \lesssim 10^8 - 10^{10}$ GeV, in conflict with the generation of the baryon asymmetry in the usual way. Some possible ways to avoid this constraint have been proposed. If $m_{3/2} \gtrsim 10$ TeV the gravitinos decay before the elements are formed in the nucleosynthesis era, avoiding the bounds from photo and hadro-dissociation of the light elements. However, these gravitino masses are too large according to the naturalness criterion. A solution with $m_{3/2} \sim 100$ GeV has been proposed [1.20] in which gravitinos only decay into harmless ν and $\tilde{\nu}$. However, we have seen that in the case $m_{\tilde{\nu}} < m_{3/2}$ (small M_2) the LSP is necessarily the χ so that the decay $\tilde{G} \to \chi+$ 'something' are still dangerous[4]. Other solutions rely on assuming an acceptably low

[4] for small M_2 there is a significant $\tilde{\gamma}$ component leading to $\tilde{G} \to \tilde{\gamma}+\gamma$ unless $|\mu| \ll M_2$,

T_R ($< 10^8$ GeV) and looking for non standard baryogenesis scenarios at low energies: in models with R-parity breaking [1.21] (although in this case the LSP is no longer stable) or associated to the electroweak phase transition (with sphalerons). Finally, one proposed way out is to have large T_R but produce nucleosynthesis at the KeV scale induced by the decay of the gravitinos themselves (or other long lived particles) [1.22], taking benefit of the gravitino decay instead of having to worry about it.

In conclusion, even in the case in which the neutralinos are the LSP so that no relic gravitinos survive today, the cosmological implications of decaying gravitinos are problematic unless we give up the standard baryosynthesis or nucleosynthesis scenarios. The same kind of problems appear in the case $m_{3/2} < m_\chi < m_{\tilde{\nu}}$ due to the effects of the decays of the relic χ after nucleosynthesis (unless $\rho_\chi \ll \rho_c$ as happens for instance if m_χ is close to a pole in the annihilation cross section).

The gravitino problem is present mainly because one expects $m_{3/2} \sim M_W$ in this models where the scale of susy breaking is set by $m_{3/2}$. This condition can be avoided in the no-scale models, where the scale of susy breaking is set by gaugino masses and it is possible to have 'naturally' $m_{3/2} \gtrless M_W$.

1.4-No-scale models

An attempt to solve the fine tuning problem associated to the requirement of vanishing cosmological constant has been done with the so called no-scale models [1.7]. In these models, a symmetry under field transformations in the hidden sector, that makes the Kahler manifold Einstein flat and maximally symmetric, yields a scalar potential in the hidden direction completely flat (and vanishing), leaving the determination of the vev $\langle z \rangle$ (and the gravitino mass) to dynamical effects associated to the low energy physics.

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but this implies an excessively large ρ_χ

Since the scalar potential can be rewritten as

$$V(z) = -9e^{-4\mathcal{G}/3}\mathcal{G}^{-1}_{zz^*}\partial_z\partial_{z^*}e^{\mathcal{G}/3} \tag{1.4.1}$$

a Kahler potential satisfying

$$\partial_z\partial_{z^*}e^{\mathcal{G}/3} = 0 \tag{1.4.2}$$

automatically leads to a vanishing flat potential with zero cosmological constant. The solution to eq. (1.4.2) is

$$\mathcal{G} = 3\ ln(z + z^*) \tag{1.4.3}$$

that is clearly invariant under the $U_a(1)$ transformation $z \longrightarrow z + i\beta$. Actually, the whole Lagrangian is approximately invariant under the $SU(1,1)$ transformations

$$z \longrightarrow \frac{\alpha z + i\beta}{i\gamma z + \delta} \quad \text{with} \quad \alpha\delta + \beta\gamma = 1 \tag{1.4.4}$$

that, however, are broken by the gravitino mass term. Conversely, the requirement that the scalar potential be $SU(1,1)$ symmetric guarantees that it is flat and vanishing, so that the absence of a cosmological constant should be traced back to this symmetry.

The gravitino mass is not determined by the hidden sector superpotential and appears as a new degree of freedom of the low energy theory to be determined dynamically in the following way. The electroweak and strong interactions renormalize the potential of the observable sector, inducing the electroweak breaking when a non-trivial minimum in the 'Higgs direction' arises. The value of the potential at the minimum results to be a function of the variable gravitino mass, that is now fixed by requiring a minimal ground state energy, i.e.

$$\frac{\partial}{\partial m_{3/2}}V(\langle A\rangle) = 0. \tag{1.4.5}$$

If the observable sector is taken as before, with canonical kinetic terms and the superpotential (), the mass spectrum is also similar and the susy breaking scale is set by the gravitino mass (what provides the common scalar masses). In susy GUTs however, to avoid large contributions of $\mathcal{O}(m^2_{GUT}m^2_{3/2})$ to the cosmological constant, the heavy sector must remain supersymmetric at the tree level. Since the effects of the gravitino mass are common, a tree level susy breaking in the light sector can be achieved only with non-zero gaugino masses. It is possible to construct models in which the GUT breaking provides gaugino masses only to the light sector. In a more general context, this also allows to construct models in which the gravitino mass can have arbitrary values ($m_{3/2} \gtreqless M_W$, for specific choices of superpotential and of the function $f_{ab}(z)$) without affecting the low energy mass spectrum that depends now only on the gaugino masses. These are the ones that are dynamically determined to be $\mathcal{O}(M_W)$ in this case. An example of Kahler potential leading to a flat potential in the z direction with no susy breaking for the chiral fields in the observable sector (no common scalar mass terms) is $\mathcal{G} = ln(z + z^* - A_i A^{*i}) + F(A_i) + F^{\dagger}(A^{*i})$. However, renormalization from M_{Pl} down to M_W induces soft-breaking terms at low energies for the chiral fields.

It is very interesting to note that this kind of scenarios appear in superstrings, where Calabi-Yau compactification from 10 to 4 dimensions yield [1.23]

$$\mathcal{G} = \ln(S + S^*) + 3\ln(T + T^* - 2\phi_i \phi^{*i}) - \ln|W(\phi_i) + W(S)|^2 \tag{1.4.6}$$

and $f_{ab} = S\delta_{ab}$, where the hidden fields S and T are related to fields of the 10-dimensional supergravity field theory (the dilaton and antisymmetric tensor field) and to the field that scales the compactification manifold while ϕ_i are the fields in the 27 of E_6. One possibility that has been considered [1.24] is to trigger supergravity breaking with a gaugino condensation in the hidden E_8' sector that induces a potential $W(S)$ leading to the generation of ordinary gaugino masses that provide the seeds for susy breaking in the observable sector. Typically one obtains $m_{3/2} \sim M_{Pl}$ while the gaugino masses are dynamically determined to be $\mathcal{O}(M_W)$. It is clear that in this case the gravitino cosmological problem is also avoided.

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CHAPTER 2

Neutralino Dark Matter Searches

We are going to consider now in detail the case of neutralinos lighter than the W boson as the dark matter. We have seen in the previous chapter that the neutralino is very likely the LSP in the supersymmetric standard model. We will obtain the cosmological relic density of the lightest neutralino, estimate its contribution to the local halo density and study the possibility of its detection. We will present the bounds arising from present experiments (from indirect searches in underground detectors and direct searches with Ge spectrometers) and consider the prospects for future experimental searches of dark matter neutralinos [2.1-2.2].

2.1-The Dark Matter Problem

The matter content of the universe is usually measured in terms of $\Omega = \rho/\rho_c$, the cosmological density ρ in units of the critical density $\rho_c = 3H_o^2/8\pi G = 2\ h^2\ 10^{-29}$ gr/cm^3 necessary to close the universe. $H_o \equiv h \cdot 100\ Km/s\ Mpc$ is the present value of the Hubble constant, with $h \simeq 0.5 - 1$ the Hubble parameter (1 parsec=3.26 light years).

Although the visible mass accounts for $\Omega_{vis} \simeq 0.01$, several observations indicate that $\Omega_{tot} \geq 0.1 - 0.2$ [2.3]. One of the most striking examples is given by the measurements of the orbital velocities of stars or gas clouds orbiting around spiral galaxies. In fact, it is observed that these velocities remain approximately constant outside the regions where the light falls exponentially off and at least up to 2-3 optical radii [2.4]. Since $v(r)^2 = \frac{GM(r)}{r}$, the flatness of the rotational curves means that the mass contained within a radius r grows linearly with r. This is an indication of the existence of an halo of non luminous matter around the galaxy with a mass $M_{tot} > 3-10\ M_{vis}$ that provides the gravitational attraction necessary to bind objects of such high rotational velocities. In particular, the average halo density in the vicinity of the solar system is estimated to be $\rho^h \simeq 0.2 - 0.4$ GeV/cm^3 [2.25].

The existence of DM is also indicated by observations at larger scales, for example $\Omega \simeq 0.2$ is associated to the mass in groups and clusters of galaxies. The amount of DM at even larger scales is uncertain, in part because it is not known how this DM is distributed. If its distribution is the same as that of the galaxies, then $\Omega_{DM} \sim 0.2$[2.5]. But if a smooth component is assumed, it is possible that $\Omega_{DM} \simeq 1$. Tentative observations from the Infrared Astronomical Satellite, which is the largest scale redshift survey that has been used to estimate Ω, suggest that $\Omega \geq 0.7$ [2.6]. Finally, there is also a theoretical prejudice (inflation theory)[2.7] that favours $\Omega = 1$.

Which is the nature of this DM? Standard primordial nucleosynthesis calculations fit well with the observed abundances of the light nuclides (H, D, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li) only if the density Ω_b of ordinary ("baryonic") matter lies in the range $0.014\ h^{-2} \leq \Omega_b \leq 0.025\ h^{-2}$, with a firm upper limit $\Omega_b \leq 0.14$ [2.8]. If $\Omega \simeq 1$, the majority of the DM must be non baryonic, but if the cosmological density is actually at the lower end of the observationally allowed range, $\Omega \simeq 0.2$, it could be that the DM is mainly baryonic. However, in this case many constraints restrict the possible forms it may take to "Jupiters" or black holes, and it is difficult to invent schemes in which 90 % of the baryonic matter in galaxies is converted to these unusual forms rather than to stars. Though, it is quite plausible that the majority of the DM is nonbaryonic.

Many nonbaryonic elementary particles can provide the DM [2.9]. Among the most accepted candidates are τ-neutrinos with a small mass $\sim$ 30 eV (the GeV neutrino case has been ruled out by LEP), axions, monopoles, hidden matter, WIMPs and the lightest supersymmetric particle. In an early stage of the evolution of the universe, these particles are in thermal and chemical equilibrium[1] with the remaining ones, but as the universe expands and cools, the "freeze-out" temperature is reached when the annihilation rate becomes too low to allow the equilibrium condition to be preserved. If the particles are stable, after this decoupling their density diminishes only by their posterior (not very efficient) annihilation and because of the expansion of the universe. This leads, in general, to a significant contribution to Ω for weakly interacting particles.

2.2-The neutralino relic density

Let us now turn to the evaluation of the χ contribution to Ω. We have

$$\Omega_\chi = \frac{m_\chi n_\chi^o}{2\ h^2\ 10^{-29}\mathrm{g/cm}^3} \qquad (2.2.1)$$

so that we can obtain $\Omega_\chi h^2$ once the present relic χ number density n_χ^o is computed. The equation describing the evolution of the density n of a stable particle is

$$\frac{dn}{dt} = -3Hn - \langle\sigma_A v\rangle(n^2 - n_{eq}^2) \qquad (2.2.2)$$

where the first term is the dilution caused by the expansion of the universe while the second tell us that if the actual density is larger than the chemical equilibrium value n_{eq}, n will be reduced through particle annihilations at a rate proportional to the thermally averaged annihilation cross section σ_A times the relative velocity of the particles. When the neutralinos become non-relativistic, i.e. when $T < m_\chi$, their equilibrium density becomes Boltzmann suppressed ($n_{eq} \sim e^{-m/T}$) and as the universe cools by its expansion a lot of χ annihilations must take place in order to preserve the equilibrium situation. However, the χ dilution also affects the annihilation rate and

[1] the axion makes an exception since from its creation it is out of equilibrium

after the 'freeze out' temperature $T_{fo} \sim m/20$ the annihilations are no more sufficient to keep the actual density n_χ down to the equilibrium value n_{eq} and the neutralinos go out of chemical equilibrium. We can then integrate the evolution equation from the freeze out up to now neglecting n_{eq}. For this we assume an adiabatic expansion, i.e.

$$\frac{d(sR^3)}{dt} = 0 \quad \longrightarrow \quad \dot{s} = -3\frac{\dot{R}}{R}s \tag{2.2.3}$$

where the entropy density is $s = \frac{2\pi^2}{45} g_s T^3$, with g_s the effective number of degrees of freedom[2]

Defining $Y \equiv n/s$ and using that the Hubble constant is

$$H = \frac{\dot{R}}{R} = \sqrt{\frac{8}{3}\pi G\rho} \tag{2.2.4}$$

with the energy density being

$$\rho = \frac{\pi^2}{30} g_\rho T^4 \qquad \left(g_\rho \equiv g_{int} + g^i_{dec}\left(\frac{T_i}{T}\right)^4 \right) \tag{2.2.5}$$

[2] To compute g_s one adds 1 for each bosonic and 7/8 for each fermionic interacting relativistic degrees of freedom, while the decoupled relativistic particles contribute to s as [2.10]

$$s_i = \frac{2\pi^2}{45} g^i_{dec} T_i^3 \equiv \frac{2\pi^2}{45} g^i_s T^3$$

so that its contribution to g_s is $g^i_s = g^i_{dec}(T_i/T)^3$ with T being the photon temperature and T_i the temperature of the decoupled particles. For instance, the temperature of the neutrinos today differ from T due to the e^+e^- annihilations which reheat the photons at $T \sim$ MeV, once the ν have decoupled. From entropy conservation

$$g_{int}(T > \text{MeV})T_\nu^3 = g_{int}(T < \text{MeV})T^3 \quad \rightarrow \left(\frac{T_\nu}{T}\right)^3 = \frac{2}{2 + 4\cdot 7/8} = \frac{4}{11}$$

so that for three families of massless neutrinos $g_s(T <$ MeV$)=43/11$. For the remaining particles, we just add their contribution to g_s when they become relativistic ($T > m$) and count the gluons and quarks instead of the mesons and baryons above the QCD deconfinement temperature $T_c \simeq 200$ MeV.

the evolution equation for n can be cast in the form

$$\frac{dY}{dx} = -\frac{m}{x^2}\sqrt{\frac{b}{2\pi G}}\langle\sigma v\rangle(Y^2 - Y_{eq}^2) \tag{2.2.6}$$

with $x \equiv m/s^{1/3}$, resulting (since $Y_o \ll Y_{fo}$)

$$n_\chi^o \simeq \frac{\sqrt{2\pi}}{m_\chi M_{Pl}}\, s_o \left[\int_{x_{fo}}^{x_o} \frac{dx}{x^2}\, \sqrt{b}\, \langle\sigma v\rangle\right]^{-1} \tag{2.2.7}$$

where $b^3 \equiv \frac{2\pi^2}{45}\frac{g_s^{4/3}}{g_\rho}$. The freeze out value x_{fo} is given by[3]

$$x_{fo} \simeq \frac{1}{b}(\ln\ B - \frac{1}{2}\ln\ \frac{1}{b}\ln\ B)\Big|_{fo} \tag{2.2.8}$$

where

$$B \equiv \Delta\ m\frac{M_{Pl}}{2\pi^2 b^2}\ \langle\sigma v\rangle \tag{2.2.9}$$

with the numerical factor Δ, of order one, depending on the criterion used to define the freeze out temperature. (A fit to the numerical integration of eq. (1) leads to a preferred value $\Delta \simeq 1.5$ [2.11].)

In the case in which the particles undergo mainly s-wave annihilations, i.e. that $\sigma v \simeq$ constant, and taking $b(T) \simeq b(T_{fo})$ in the integration, we get the result

$$n^o = \frac{\sqrt{2\pi} s_o x_{fo}}{M_{Pl} m_\chi \sqrt{b}\langle\sigma v\rangle} \tag{2.2.10}$$

However, for the neutralinos there are channels of annihilation that happen only in p-wave, i.e. $\sigma v \propto v^2$. Hence, in the thermal average $\langle\sigma v\rangle \propto T$. In general, one expands σv in powers of v^2 retaining only the first contributions in the non relativistic limit. Replacing then $v^2 \to 6T/m$, the thermal average $\langle\sigma v\rangle$ results a function of T.

There are two reasons why the s-wave annihilation of neutralinos can be forbidden. The first [2.12] is that the identity of the two incoming Majorana neutralinos requires an antisymmetric wave function. Hence, if the interactions preserve chirality, the annihilation into fermion antifermion pairs must be p-wave in the $m_f \to 0$ limit. This is

[3] To deduce this we write $\delta \equiv Y - Y_{eq}$ and neglect $d\delta/dx$ before freeze-out, finding x_{fo} from the condition $\delta \simeq Y_{eq}$.

so because in this limit the outgoing fermions have opposite helicities and hence equal spins in the CM system. The total wave function can only be made antisymmetric in an L=1 p-wave state. This implies for instance that the s-wave part of $\chi\chi \to f\bar{f}$ through Z or H_3 exchange is proportional to m_f^2. The annihilation through s-wave can also be forbidden by a CP selection rule (we assume CP conservation in the interactions), since a χ pair in the initial state is, in s-wave, CP odd. Being H_2 CP even, annihilations through H_2 or into H_2H_2 (or H_3H_3) are not allowed in s-wave. Contrary, annihilation through the CP odd H_3 or into H_2H_3 can happen in s-wave [2.13].

Annihilation cross section

We can parametrize the relevant neutralino interactions as follows:

$$\begin{aligned}\mathcal{L} =& \frac{g}{2}\, a_2 H_2^0 \bar{\chi}\chi + \frac{g}{2}\, a_3 H_3^0 \bar{\chi}\gamma_5\chi - \frac{g}{2} k_f^{(2)} \frac{m_f}{m_W}\, H_2 \bar{f} f - \\ & \frac{g}{2} k_f^{(3)} \frac{m_f}{m_W}\, H_3 \bar{f}\gamma_5 f \,+ \frac{g}{4c_W} a_Z Z^\mu \bar{\chi}\gamma_\mu\gamma_5\chi + \,\dots\end{aligned} \tag{2.2.11}$$

where

$$k_f^{(2)} = \begin{cases} \cos\alpha/\sin\beta, & \text{for up–type fermions;} \\ -\sin\alpha/\cos\beta, & \text{for down–type fermions.} \end{cases} \tag{2.2.12}$$

$$k_f^{(3)} = \begin{cases} \cot\beta, & \text{for up–type fermions,} \\ \tan\beta, & \text{for down–type fermions,} \end{cases} . \tag{2.2.13}$$

The angle α coming from the diagonalization of the Higgs bosons mass matrix

$$\tan 2\alpha = \frac{1+r_3}{1-r_3}\tan 2\beta \qquad (-\frac{\pi}{2} \le \alpha \le 0) \tag{2.2.14}$$

and

$$a_z = Z_{13}^2 - Z_{14}^2 \tag{2.2.15}$$

$$a_2 = (Z_{13}s_\alpha + Z_{14}c_\alpha)(Z_{12} - Z_{11}tg_W) \tag{2.2.16}$$

$$a_3 = (Z_{13}s_\beta - Z_{14}c_\beta)(Z_{12} - Z_{11}tg_W). \tag{2.2.17}$$

The matrix Z_{ij} $(i,j=1,...,4)$ transforms the current neutralino eigenstates into mass eigenstates ($\chi_1 \equiv \chi$ being the lightest):

$$\chi_i = Z_{i1}\,\tilde{B} + Z_{i2}\,\tilde{W}_3 + Z_{i3}\,\tilde{H}_1 + Z_{i4}\,\tilde{H}_2. \tag{2.2.18}$$

At very low neutralino velocities, as for annihilations of neutralinos inside the sun or the earth, the thermal average of the product of the $\chi\chi$ annihilation cross section σ by their relative velocity v is only the s-wave piece:

$$\langle \sigma v\rangle = \sum_f \langle \sigma v\rangle_{f\bar{f}} + \langle \sigma v\rangle_{H_2 H_3}. \tag{2.2.19}$$

where the annihilation into $f\bar{f}$ is due to s–channel Z and H_3 exchange:

$$\langle \sigma v\rangle_{f\bar{f}} = c_f\,\frac{G_F^2 m_Z^4}{\pi}\,\frac{|\vec{p}_f|}{m_\chi}\,|\mathrm{Q}_Z^f + \mathrm{Q}_{H_3}^f|^2. \tag{2.2.20}$$

c_f is the number of colors ($c_f = 1$ for leptons, $c_f = 3$ for quarks). This channel is very important when the χ mass is close to a pole in the cross section, i.e. for $m_\chi \simeq M_Z/2$ or $M_{H_3}/2$. The annihilation into H_2H_3 is usually the dominant channel if kinematically allowed. It is due to t–channel and u–channel exchange of neutralinos χ_i $(i=1,...,4)$, and s–channel Z and H_3 exchange:

$$\langle \sigma v\rangle_{H_2 H_3} = \frac{G_F^2 m_Z^4}{\pi}\,\frac{|\vec{p}_H|}{m_\chi}\,|\sum_i \mathrm{Q}_{\chi_i}^H + \mathrm{Q}_Z^H + \mathrm{Q}_{H_3}^H|^2.$$

The reduced interaction amplitudes are:

$$\mathrm{Q}_Z^f = -m_f a_Z T_{3_f} P_Z \tag{2.2.21}$$

$$\mathrm{Q}_{H_3}^f = \epsilon_1 m_\chi F_{1i}^{(3)} 2\frac{m_f}{m_Z} k_f^{(3)} P_{H_3}, \tag{2.2.22}$$

$$\mathrm{Q}_{\chi_i}^H = -\sqrt{\frac{1}{2}}4F_{1i}^{(2)}F_{1i}^{(3)}\,(m_\chi \mu_- + \epsilon_1\epsilon_i m_{\chi_i})\,P_{\chi_i}, \tag{2.2.23}$$

$$\mathrm{Q}_Z^H = \sqrt{\frac{1}{2}}a_z\cos(\alpha-\beta)m_\chi\mu_- P_Z, \tag{2.2.24}$$

$$Q^H_{H_3} = \sqrt{\frac{1}{2}}\epsilon_1 F^{(3)}_{11} \cos(2\beta)\sin(\alpha+\beta) m_Z P_{H_3}; \qquad (2.2.25)$$

(ϵ_i is the sign of the χ_i mass eigenvalues) with

$$F^{(2)}_{ij} = \frac{1}{2}\left[\left(Z_{i3}\sin\alpha + Z_{i4}\cos\alpha\right)\left(Z_{j2}\cos\theta_W - Z_{j1}\sin\theta_W\right) + (i \leftrightarrow j)\right], \qquad (2.2.26)$$

$$F^{(3)}_{ij} = \frac{1}{2}\left[\left(Z_{i3}\sin\beta - Z_{i4}\cos\beta\right)\left(Z_{j2}\cos\theta_W - Z_{j1}\sin\theta_W\right) + (i \leftrightarrow j)\right], \qquad (2.2.27)$$

and the outgoing fermion and Higgs boson 3–momenta are:

$$|\vec{p}_f| = \left(m^2_\chi - m^2_f\right)^{1/2},$$
$$|\vec{p}_H| = \left(1 - 2\mu_+ + \mu^2_-\right)^{1/2},$$

with

$$\mu_\pm \equiv \frac{m^2_{H_3} \pm m^2_{H_2}}{4m^2_\chi}.$$

and the propagators are [4]:

$$P_Z = \frac{1 - 4m^2_\chi/m^2_Z}{4m^2_\chi - m^2_Z - i\Gamma_Z m_Z},$$
$$P_{H_3} = \frac{1}{4m^2_\chi - m^2_{H_3} - i\Gamma_{H_3} m_{H_3}},$$
$$P_{\chi_i} = \frac{1}{m^2_\chi(2\mu_+ - 1) - m^2_{\chi_i}}.$$

For the annihilation in the early universe, necessary to compute the relic density, the p-wave contributions must also be retained. There are channels that appear only in p-wave. For instance, the annihilation into $f\bar{f}$ through H_2 exchange leads to

$$\langle\sigma_A\left(\chi\chi \to f\bar{f}\right)\beta\rangle_{H_2} = \frac{g^4}{32\pi}\sum_f c_f \frac{a_2^2 k_f^{(2)\,2}}{m^2_\chi}\frac{m^2_f}{m^2_W}\frac{\left(1 - \frac{m^2_f}{m^2_\chi}\right)^{3/2}}{(4-x)^2}\beta^2 \qquad (2.2.28)$$

[4] Fictitious widths for the Higgs fields have been taken to regularize the cross sections, small enough so that our results are insensitive to them, *i.e.* $\Gamma \leq O(1\ \mathrm{GeV})$.

where $x \equiv m_{H_2}^2/m_\chi^2$. The β^2 factor ($\beta = v/c$) in this expression is the signal of the above–mentioned p–wave suppression. There is also the annihilation through H_1 exchange and its interference with the H_2 exchange. Also annihilations into two Higgses $\chi\chi \to H_2 H_2$, $H_3 H_3$ contribute, proceeding through χ exchange in the t-channel or H_2 and H_1 exchange in the s-channel.

Squarks and sleptons can mediate the process $\chi\chi \to f\bar{f}$ through t–channel exchange. Since sfermions are expected to be heavy, these channels are usually suppressed. They are only relevant in the regions of parameter space where the remaining channels do not contribute significantly. This is the case, for instance, for small M_2 values, for which the χ is mainly photino (small a_Z, a_2 and a_3) and, furthermore, the couplings of the (small) higgsino components are suppressed by factors of m_f/m_W. It is reliable to use the annihilation cross section for pure photinos in this case, whose s-wave part is (assuming that all squarks and sleptons have degenerate masses $\tilde{m}$)

$$\langle \sigma_A \left(\tilde{\gamma}\tilde{\gamma} \to f\bar{f}\right) \beta \rangle = \frac{8\pi\alpha^2}{\tilde{m}^4} \sum_f c_f \, Q_f^4 \, m_f^2 \sqrt{1 - \frac{m_f^2}{m_\chi^2}} \, . \tag{2.2.29}$$

Q_f is the electric charge of the fermions allowed by phase space. We consider the case $m_\chi < M_W$ since otherwise channels such as $\chi\chi \to WW$ should also be included [2.14].

Digression: There is a simplified way to compute the total annihilation cross section in the non-relativistic limit to $\mathcal{O}(v^2)$ that goes as follows:

In the process $\chi(p_1)\chi(p_2) \to A(q_1)B(q_2)$, with $p_i^2 = m^2$, $q_1^2 = m_1^2$ and $q_2^2 = m_2^2$, the following kinematical relations hold in terms of the Mandelstam variables $s = (p_1 + p_2)^2$ and $t = (p_1 - q_1)^2$

$$\begin{aligned} &2p_1.q_2 = s + t - m^2 - m_1^2 \quad , \quad 2p_2.q_1 = s + t - m^2 - m_2^2 \quad , \quad 2p_1.q_1 = m^2 + m_1^2 - t \\ &2p_2.q_2 = m^2 + m_2^2 - t \quad , \quad 2p_1.p_2 = s - 2m^2 \quad , \quad 2q_1.q_2 = s - m_1^2 - m_2^2 \end{aligned} \tag{2.2.30}$$

With the help of these expressions we can always write the modulus square of the scattering amplitude summed over final states and averaged over initial states as a function of t and s:

$$\overline{\sum} |A|^2 = f(t,s) \tag{2.2.31}$$

The differential cross section is

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s(s-4m^2)}\bar{\sum}|A|^2 \tag{2.2.32}$$

and since the variable t expressed in terms of the angle θ between $\vec{p_1}$ and $\vec{q_1}$ in the CM of the incident neutralinos is $t = t_o + \cos\theta\ \Delta t$, with

$$t_o \equiv m^2 + \frac{m_1^2+m_2^2-s}{2} \tag{2.2.33}$$

and

$$\Delta t = \frac{s}{2}\sqrt{1-\frac{4m^2}{s}}\sqrt{\left(1+\frac{m_1^2-m_2^2}{s}\right)^2 - \frac{4m_1^2}{s}} \tag{2.2.34}$$

we have that

$$\begin{aligned}\sigma v &= 2\sqrt{1-\frac{4m^2}{s}}\int_{t_o-\Delta t}^{t_o+\Delta t} dt\ \frac{d\sigma}{dt}(s,t) \\ &= \frac{1}{8\pi s\sqrt{s-4m^2}}\int_{-\Delta}^{+\Delta} dy\ f(t\equiv t_o+y,s)\end{aligned} \tag{2.2.35}$$

We can now Fourier expand the integrand

$$f(t,s) = f(t_o,s) + \frac{\partial f}{\partial t}(t_o,s)y + \frac{\partial^2 f}{\partial t^2}(t_o,s)\frac{y^2}{2} + \ldots \tag{2.2.36}$$

integrate and expand again using the non relativistic limit $s_{NR} \simeq 4m^2+m^2v^2$ neglecting terms $\mathcal{O}(v^4)$ to get finally

$$\sigma v \simeq \frac{\sqrt{k}}{32\pi m^2}\left\{f_o + v^2\left[\frac{\partial f_o}{\partial s}m^2 + \frac{\partial^2 f_o}{\partial t^2}\frac{m^4 k}{6} - \frac{f_o}{4k}\left(k+(1+\Delta\xi^2)\Delta\xi^2 - \frac{m_1^2}{2m^2}\right)\right]\right\} \tag{2.2.37}$$

where the subindex $_o$ means evaluated at $t = t_o$ and $s = 4m^2$, $\Delta\xi^2 = (m_1^2 - m_2^2)/4m^2$ and $k = (1+\Delta\xi^2)^2 - m_1^2/m^2$. Once this machinery is inserted in a reduce program, the task of the evaluation of σv up to p-wave terms is reduced to the evaluation of $f(t,s)$.

Relic and halo densities

Once we have the annihilation cross section, of which the expressions for the dominant channels have been shown, we can compute $\Omega_\chi h^2$ as a function of the parameters M_2 and μ for a given value of v_2/v_1 and fixing the Higgs mass m_{H_2}. The result is plotted in the figures 1.a-b for $v_2/v_1 = 2$ (1.a) and 8 (1.b) and $m_{H_2} = 50$ GeV (masses

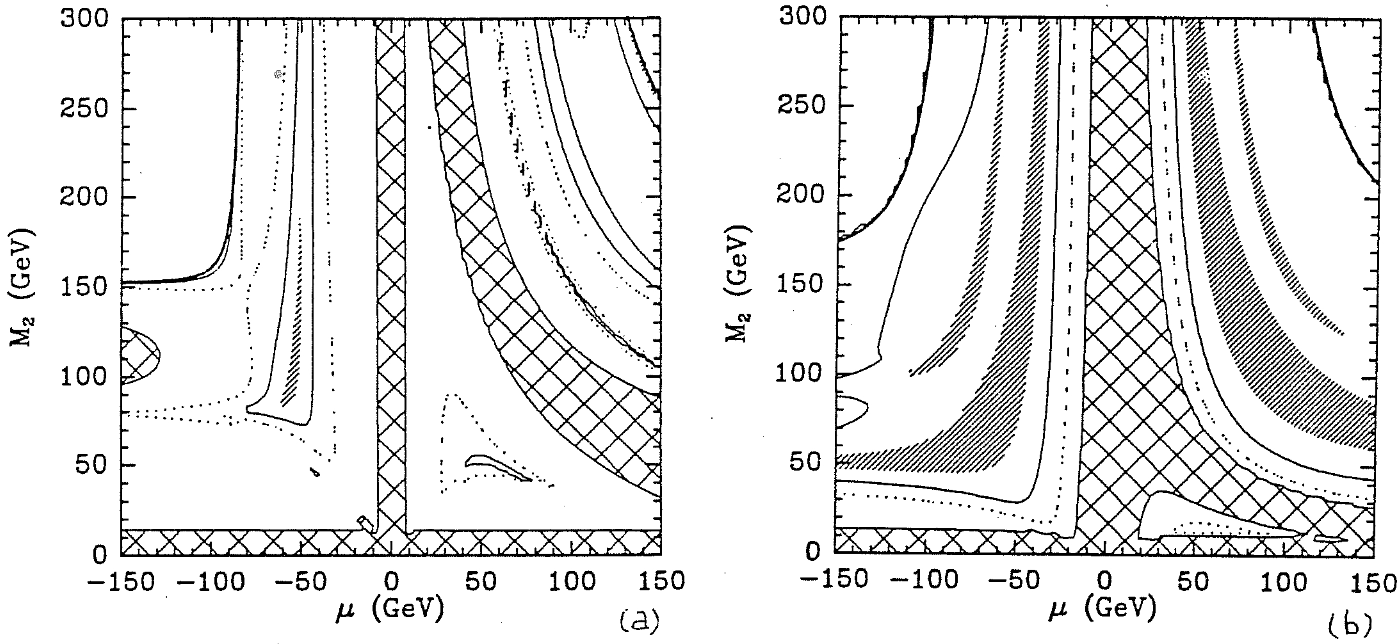

Fig. 1. Relic neutralino density for values of v_2/v_1 of 2 (1.a) and 8 (1.b) with $m_{H_2} = 50$ GeV. Corners of $m_\chi > m_W$ avoided (above the thick full lines). The cross-hatched regions are excluded because $\Omega_\chi h^2 \geq 1$. The levels $\Omega_\chi h^2 = 0.05$ (dotted lines), 0.01 (full lines) and the regions of $\Omega_\chi h^2$ <0.001 (hatched areas) are shown.

$m_\chi > m_W$ avoided) [5]. The $\Omega_\chi h^2 = 0.001$ level shows the poles of the Z and the H_3 in the annihilation cross section that can be identified by using fig. 1. In the cross-hatched area, that corresponds to $m_\chi \lesssim$ few GeV, the χ contributes $\Omega_\chi h^2 > 1$ overclosing the universe and contradicting the observed lifetime of the universe. In all the remaining region $0.001 < \Omega_\chi h^2 < 1$ neutralinos are of cosmological relevance.

The assumption that the neutralinos constitute the halo of our galaxy is not tenable if their cosmological abundance Ω_χ is not large enough to account for all the dark matter in the halos of galaxies, $\Omega_g \simeq 0.05$. The figures show where neutralinos may be the dark matter in the halos of galaxies, i.e. only in the regions of $0.01 \leq \Omega_\chi h^2 \leq 1$, between the full lines and the cross-hatched region. In this regions the relic density may be

[5] To estimate the relic abundance in the corners where the χ is almost a pure $\tilde{\gamma}$ we have made the simplifying assumption that all sfermions are degenerate at 100 GeV.

$\Omega_\chi \geq 0.05$ for an allowed value of h, so that neutralinos may constitute by themselves the halos of galaxies, and at the same time do not overclose the universe. Notice in fig. 1b how these areas reduce for large v_2/v_1.

2.3-Neutralino Detection

There are several ways in which one can try to search for the dark matter neutralinos. These are:

i) Direct detection: The direct search for neutralinos from the halo looking for the energy deposition they can produce in an interaction with a nuclei in a detector. These detectors are typically of Ge or Si, and the nucleus recoil energy is measured by the ionization it produces. There are several techniques in the stage of development that try to measure the energy transmitted to the crystal lattice by detecting the phonons generated at first or the subsequent increase of temperature. Since the typical halo neutralino velocity is $\sim$ 300 km/s, the maximum recoil energy of the nucleus N is (normalizing with N=Ge)

$$E_{recoil}^{max} = 4v^2 \frac{m^2 m_N^2}{(m+m_N)^2} \simeq 150 \text{ KeV} \frac{(m_N/m_{Ge})^2}{(1+m_N/m)^2} \tag{2.3.1}$$

so that it is essential to have very low detection thresholds in order to be able to test low mass dark matter particles (a 50 eV threshold is necessary to test m up to 1 GeV while the present threshold of a few KeV allows to test m up to $\sim$ 10 GeV). This is also why Si detectors, that have lower thresholds, are better to detect light dark matter particles whose energy transfer is small. However, even if the signal is above the detection threshold, for the experiments to be able to set any significant bound it is necessary to fight against the background sources, putting the detector deep underground, shielding it from radioactive sources and using cryogenics to avoid thermal noise. Since the χ rates are proportional to the χ halo density and to the χ–nucleus scattering cross section $\sigma_{\chi N}$ one can only constrain the product $n_\chi^h \cdot \sigma_{\chi N}$. Since $\sigma_{\chi N}$ is a function of the supersymmetric parameters, the experimental constraints on the signal set bounds on the parameter space (M_2, μ, v_2/v_1, m_H). The simplest working assumption is that the χ are the only particles that constitute the halo , i.e. that $\rho_\chi^h \simeq 0.3$ GeV/cm^3. Under this assumption bounds were obtained in [2.15-2.16]. However, as we have seen this assumption is not always tenable. For the values of the parameters for which this is not possible, the bounds will depend on the particular assumption made about the value of ρ_χ^h.

ii) Indirect detection: The indirect searches are based on the observation of the products of the annihilation of dark matter neutralinos. One possibility is to look directly to the annihilation of halo χs, searching in particular for the γ, $\bar{p}$, etc. so produced. However, this only seems to be competitive for the dark matter candidates that yield a monochromatic spectrum of γ since otherwise the signal over background ratio is small [2.17].

Another more promising possibility, that we will consider in detail, takes profit of the fact that during the lifetime of the solar system a large number of halo χ may have remained trapped in the sun or in the earth so that the large χ concentrations attained in their cores proportionally enhance the annihilation signal [2.18]. In this case, only the ν so produced can be searched for at underground detectors such as Fréjus [2.19], Kamiokande [2.20] or IMB [2.21]. The non observation at present of an excess flux already set bounds on the supersymmetric parameters.

We will now concentrate on the computation of the signal expected in this detectors.

The capture rate of neutralinos with mean velocity v_χ and mass density ρ_χ^h for a body (sun or earth) of mass M_B is given by [2.22]

$$C = \left(\frac{6}{\pi}\right)^{\frac{1}{2}} \frac{\rho_\chi}{v_\chi} M_B \sum_i \langle v_{esc}^2 \rangle_i \frac{\sigma_i}{m_\chi m_i} f_i S_i. \tag{2.3.2}$$

The sum runs over all the elements contained in the body. f_i is the fraction of M_B due to the element of mass m_i, σ_i is the cross section of the neutralino with the nucleus of kind i. $\langle v_{esc}^2 \rangle_i$ is the square escape velocity mediated over the distribution of the element. Both f_i and $\phi_i \equiv \langle v_{esc}^2 \rangle_i / v_o^2$, with v_o being the escape velocity at the surface, are shown in table I. Finally, S_i is a factor which takes into account the suppression due to the element mass mismatching with the neutralino and the possible lack of coherence of the interaction. The effect of the mismatching is particularly important for the earth since in this case the escape velocity is low, so that neutralinos are trapped only if they loose a sizable amount of their energy in the collisions. This is only possible in a head on collision of a neutralino with a nucleus of similar mass. This will imply that the capture of neutralinos by the earth is especially important for $m \simeq m_{Fe} = 56$ GeV. The effect of the lack of coherence becomes important when the transfer momentum is not negligible with respect to the inverse of the characteristic dimension of the nucleus. This has to be taken into account for heavy neutralinos. The analytical expression for S_i can be found in ref. [2.22].

The expression for the capture applies when the cross sections σ_i are smaller than the value for which the capture saturates (i.e. the one for which all the incoming particles remain trapped in the body). The large cross sections necessary for saturation, of approximately 10^{-36} cm^2 in the sun and two orders of magnitude larger in the earth, are achieved only for Higgs boson masses lower than 1 GeV, which are in conflict with experimental bounds.

The local halo density ρ_h (the halo density in the vicinity of the solar system) has been estimated to be 0.20 GeV cm^{-3} $\leq \rho_h \leq 0.43$ GeV cm^{-3} [2.25] and the expected value of the characteristic velocity of dark matter particles in the halo is 200 km s^{-1} $\leq v \leq 400$ km s^{-1} [2.26]. We choose values in the middle of the allowed ranges, $\rho_h = 0.3$ GeV/cm^3 and $v_\chi = 300$ km/s.

We consider capture by the earth as if it were a free body in space because nearly identical results are obtained from the real situation, in which both the earth and the neutralinos in the halo are moving deep within the potential well of the sun [2.27]. The χ's captured by the earth or the sun remain trapped provided they are heavy enough not to evaporate. The critical evaporation mass m_{ev} is $\simeq 12$ GeV for the earth and $\simeq 4$ GeV for the sun [2.27]. The number of trapped neutralinos with $m_\chi \lesssim m_{\text{ev}}$ decreases exponentially with decreasing χ mass. For $m_\chi > m_{\text{ev}}$ the evaporation is negligible and during the lifetime of the solar system an equilibrium is established between the

i	f_i	ϕ_i
O	0.30	1.2
Si	0.15	1.2
Mg	0.14	1.2
Fe	0.06	1.2
Ca	0.015	1.2
Al	0.011	1.2
Na	0.004	1.2
Fe	0.24	1.6
S	0.05	1.6
Ni	0.03	1.6

Table I.a

i	f_i	ϕ_i
H	$7.72\,10^{-1}$	3.16
He	$2.09\,10^{-1}$	3.40
O	$8.55\,10^{-3}$	3.23
C	$3.87\,10^{-3}$	3.23
Ne	$1.51\,10^{-3}$	3.23
Fe	$1.46\,10^{-3}$	3.23
N	$9.40\,10^{-4}$	3.23
Si	$8.13\,10^{-4}$	3.23
Mg	$7.39\,10^{-4}$	3.23
S	$4.65\,10^{-4}$	3.23

Table I.b

Table I: Mass fraction f_i of element i and normalized average 'potential energy' $\phi_i \equiv \langle v_i^2 \rangle / v_o^2$ for the elements present in the mantle ($\phi_i = 1.2$) and core ($\phi_i = 1.6$) of the earth [2.23] (table I.a) and in the sun [2.24] (table I.b).

capture and the annihilation of χs, so that the present annihilation rate is given by half the capture rate.

We will consider the neutrinos produced in the annihilation of the trapped χ's with energies $E_\nu \gtrsim 2$ GeV, for which the atmospheric neutrino flux from cosmic rays in the "neutrino observatories" is relatively low (see e.g. [2.28]). The production of neutrinos from the non–relativistic annihilation of pairs of χ's proceeds via the decay of intermediate heavy leptons and hadrons. The direct ν production is suppressed for

Majorana fermions in the non–relativistic limit. It is also possible to have 'tertiary' neutrinos produced by the decay of heavy leptons arising from Higgs decays in the annihilation channel $\chi\chi \rightarrow H_2H_3$.

The differential neutrino production rate is then:

$$\frac{dR_p}{dE_\nu} = \frac{C}{2}\sum_Y B_{\chi Y}\frac{dN_{Y\nu}}{dE_\nu}. \tag{2.3.3}$$

(the differential flux of neutrinos, at a distance R from the source, is given by $d\Phi/dE_\nu = dR_p/dE_\nu/(4\pi R^2)$). $B_{\chi Y}$ is the branching fraction for annihilation $\chi\chi \rightarrow Y$ into the channel Y, $dN_{Y\nu}/dE_\nu$ is the differential neutrino yield from the channel $Y \rightarrow \nu+$ *"anything"* and the sum runs over all intermediate states Y allowed by phase space (all standard $f\bar{f}$ pairs and H_2H_3). The annihilation into H_2H_3 is the dominant channel when open. Both bosons decay mainly into heavy fermions which in turn decay producing neutrinos. Thus, one intermediate step is added in the neutrino production chain with respect to the $f\bar{f}$ channel. When the H_2H_3 channel is open, the production of neutrinos from the $f\bar{f}$ channel is decreased and the final neutrino spectrum is softer, due to the lower energy of the fermions from H_2H_3.

Light hadrons (containing only u, d and s quarks) and light charged leptons (e, μ) will be stopped in the sun or the earth before they can decay. The subsequent neutrinos have an unobservable low energy spectrum [2.29]. Moreover, the production of light fermions is suppressed by the factor m_f^2. On the contrary, the effects of energy loss or neutrino absorption for hadrons containing heavy quarks and for the τ lepton are negligible for neutralino masses lower than 100 GeV [2.29]. Assuming that the annihilation in a t quark pair is kinematically forbidden, only $f = \tau, c, b$ contribute to the generation of energetic neutrinos.

To obtain the branching ratios and differential neutrino yields we write the production rate as

$$\frac{dR_p}{dE_\nu} = \frac{C}{2}\left[\sum_f B_{\chi,f\bar{f}}\frac{dN_{f\bar{f}}}{dE_\nu} + B_{\chi,H_2H_3}\frac{dN_{H_2H_3}}{dE_\nu}\right]. \tag{2.3.4}$$

The branching ratios are given by:

$$B_{\chi,f\bar{f}} = \frac{\langle\sigma v\rangle_{f\bar{f}}}{\langle\sigma v\rangle}$$
$$B_{\chi,H_2H_3} = \frac{\langle\sigma v\rangle_{H_2H_3}}{\langle\sigma v\rangle}. \tag{2.3.5}$$

with

$$\langle\sigma v\rangle = \sum_f \langle\sigma v\rangle_{f\bar{f}} + \langle\sigma v\rangle_{H_2H_3}. \tag{2.3.6}$$

We include hadronic fragmentation in $dN_{Y\nu}/dE_\nu$ by boosting the (approximate) energy distributions of neutrino yield per fermion in the rest frame of the decaying particle.

$$\frac{dN_{f\bar{f}}}{dE_\nu} = \frac{1}{E'_f}\int_{E_\nu/E'_f}^{1} \frac{dy}{y}\frac{dN^f}{dy}. \tag{2.3.7}$$

The distribution of neutrinos in the rest frame of the parent fermion f is given as a fit to a Monte Carlo simulation [2.29],

$$\frac{dN^f}{dy} = \sum_{i=0}^{4} a_i^f y^i \tag{2.3.8}$$

In these equations $y = 2E_\nu^0/m_f$, where E_ν^0 is the neutrino energy in the rest frame of f (thus $0 \le y \le 1$). The coefficients a_i^f are given in table II.

For quarks, the average energy of the hadrons in the f–generated jet, E'_f, is a fraction of the original fermion energy E_f, $E'_f = z_F^f E_f$. For a lepton, $E'_f = E_f$ if there is no loss of energy in the medium (as is the case for the τ). This reduces the naive estimates of the mean neutrino energy in the decay of b and c quarks by factors of 2-3, since the heavy quark must share the "beam" energy m_χ with the others jet constituents and also because there are generally more than three particles in the final state. The coefficients z_F^f and a_i^f are given in table II, taken from [2.29]. They were obtained with ν_μ, and we will assume an equal yield also for ν_e (and for antineutrinos). [6]

[6] Since the distributions for the τ and the b in [2.29] have unphysical tails, only the ranges $0 \le y \le 0.98$ for the τ and $0.02 \le y \le 0.83$ for the b should be considered.

f	Z_F	a_0	a_1	a_2	a_3	a_4
τ	1.00	0.0005	-0.0104	2.1250	-2.0750	-0.0669
c	0.55	0.0097	-0.3917	4.6487	-8.6226	4.3834
b	0.71	-0.0629	5.8391	-22.5507	32.5397	-16.5512

Table II: The coefficients Z_{Ff} and $a^{(n)}_{f\nu}$ for $f = \tau, c, b$, as taken from ref. [2.29].

The neutrino yield from a H_2H_3 pair has as intermediate step the production of fermions from each boson, with branching ratios B_{H_2f} and B_{H_3f},

$$\frac{dN_{H_2H_3}}{dE_\nu} = \sum_f \left(B_{H_2f}\frac{dN^f_{H_2}}{dE_\nu} + B_{H_3f}\frac{dN^f_{H_3}}{dE_\nu} \right), \tag{2.3.9}$$

where

$$\frac{dN^f_{H_i}}{dE_\nu} = \frac{1}{z^f_F E_{H_i}} \int_{E_\nu/z^f_F E_{H_i}}^1 \frac{du}{u} \int_u^1 \frac{dy}{y}\frac{dN^f}{dy}. \tag{2.3.10}$$

where kinematics gives:

$$E_{H_2} = m_\chi(1-\mu_-), \quad E_{H_3} = m_\chi(1+\mu_-). \tag{2.3.11}$$

The branching fractions B_{H_if} for $H_i \to f\bar{f}$ are:

$$\begin{aligned} B_{H_2f} &\propto m_f^2\left(1 - 4m_f^2/m_{H_2}^2\right)^{3/2}(k_f^{(2)})^2, \\ B_{H_3f} &\propto m_f^2\left(1 - 4m_f^2/m_{H_3}^2\right)^{1/2}(k_f^{(3)})^2, \end{aligned} \tag{2.3.12}$$

Let us turn to discuss the detection of the neutrino flux. The event rate for electrons (or muons) generated within the detector due to neutrino interaction is obtained by folding the flux with the neutrino cross section:

$$R_e = \int_{E_{th}}^{m_\chi} dE_\nu \frac{d\Phi}{dE_\nu} \int_{E_{th}}^{E_\nu} dE_e \frac{d\sigma_{\nu+\bar{\nu}}}{dE_e}(E_\nu, E_e) \tag{2.3.13}$$

where E_e is the electron (muon) energy and E_{th} is the minimum energy necessary to identify it. For E_ν larger than the proton mass, the differential charged current cross section for neutrinos and antineutrinos per nucleon is [2.28]:

$$\frac{d\sigma_{\nu+\bar{\nu}}}{dE_e}(E_\nu, E_e) \simeq \sigma \left(1 + a\frac{E_e^2}{E_\nu^2}\right), \tag{2.3.14}$$

with $\sigma = 0.81 \cdot 10^{-38}$ cm^2 GeV^{-1} and $a = 0.93$. Replacing eq. (4.3.5) in eq. (4.3.4) and taking into account the number of target nucleons, we find:

$$R_e = 154\frac{events}{Kt\ yr}\int_{E_{th}}^{m_X} dE_\nu \frac{d\Phi}{dE_\nu} E_\nu \mathcal{I}\left(\frac{E_{th}}{E_\nu}\right), \tag{2.3.15}$$

$$\mathcal{I}(x) = 1 + \frac{a}{3} - x - \frac{a}{3}x^3, \tag{2.3.16}$$

where the differential flux is expressed in cm^{-2} sec^{-1} GeV^{-1} and the energies in GeV. The same signal for electrons and muons is expected. However, the search for electrons is much more efficient, since the background is lower [2.28].

Energetic muon neutrinos can also be revealed through their interactions with the rock, producing muons which either pass through the detector or are stopped inside. The flux of muons with energy larger than E_{th} is given by [2.28]:

$$R_\mu = N_A \int_{E_{th}}^{m_X} dE_\nu \frac{d\Phi}{dE_\nu} \int_{E_{th}}^{E_\nu} dE_\mu \int_{E_\mu}^{E_\nu} dE'_\mu \frac{d\sigma_{\nu+\bar{\nu}}}{dE'_\mu}(E_\nu, E'_\mu)\frac{1}{\alpha(1 + E_\mu/\epsilon)}, \tag{2.3.17}$$

where $\alpha \simeq 2$ MeV cm^2 / g and $\epsilon = 510$ GeV take into account the muon energy loss and N_A is the Avogadro's number. Substituting eq. (4.3.5) in eq. (4.3.8), we obtain the rate of muon events:

$$R_\mu = 77\frac{events}{(10\ m)^2 yr}\int_{E_{th}}^{m_X} dE_\nu \frac{d\Phi}{dE_\nu} E_\nu^2 \mathcal{I}'\left(\frac{E_{th}}{E_\nu}\right), \tag{2.3.18}$$

$$\mathcal{I}'(x) = .73 - 1.3x + 0.5x^2 + .08x^4, \tag{2.3.19}$$

where a surface of 100 m^2 corresponds to a cubic detector of 1 kiloton of water. For the sun, eq. (2.3.18) has to be multiplied by the factor 1/2. This takes into account the fact that the signal is detectable only during the night, because of the large background of muons from the atmosphere. This factor is not present for the terrestrial signal, since it has been assumed that the neutralino annihilations occur in the core of the earth and the muons are always upgoing.

2.4-Neutralino-nucleus scattering cross section

To evaluate both the rate of neutralino direct detection and the capture of χ by the sun or the earth, that is necessary to know the rate of indirect χ detection, we need to know the neutralino-nucleus scattering cross section. Since the halo neutralino velocity is $\beta \simeq 10^{-3}$, we are only interested in the non-relativistic limit. The effective lagrangian at low energies for the χ has the form

$$\mathcal{L}_{eff} \sim \bar{\chi}\gamma_\mu\gamma_5\chi\bar{q}\gamma^\mu(V + A\gamma^5)q + N_{\tilde{h}}N_{\tilde{Z}}m_q(\bar{\chi}\chi\bar{q}q + ...) \tag{2.4.1}$$

where the first term is associated to the Z and $\tilde{q}$ exchange while the last one, proportional to the $\tilde{Z}$ and $\tilde{h}$ components of χ, is due to Higgs [2.16] and squark [2.15] exchange.

The currents appearing in the first term in $\mathcal{L}_{eff}$ have the following non-relativistic limits:

$$(\bar{\chi}\gamma_\mu\gamma_5\chi)_{NR} \;\rightarrow\; (0, \vec{S}_\chi) \tag{2.4.2}$$

with $\vec{S}_\chi$ being the neutralino spin,

$$\langle N|\bar{q}\gamma^\mu q|N\rangle_{NR} \sim J^\mu_{NR} \;\rightarrow\; (J^o, 0) \tag{2.4.3}$$

with J^o the density associated to the nucleon quark current J^μ.

$$\langle N|\bar{q}\gamma^\mu\gamma^5 q|N\rangle_{NR} \;\rightarrow (0, \Delta q\ \vec{S}_N) \tag{2.4.4}$$

with Δq, the quark contribution to the nucleon spin $\vec{S}_N$, being a quantity dependent on the structure functions of the quarks [2.30].

It is clear now that in the non-relativistic limit the vector quark current does not contribute, resulting a spin dependent interaction $\sim \Delta q\ \vec{S}_\chi \cdot \vec{S}_N$ from the first term in

$\mathcal{L}_{eff}$. In an interaction with a whole nucleus, in the regime of momentum transfer less than the inverse nuclear radius, using the Wigner-Eckart theorem this term leads to a χ scattering cross section from the nucleus proportional to the nuclear spin J

$$\sigma \propto \lambda^2 \; J(J+1) \left[\frac{N_{\tilde{h}}^2}{M_Z^4}, \frac{N_{\tilde{\gamma}}^2}{M_{\tilde{q}}^4} \right] \tag{2.4.5}$$

where the factor λ depends on the quark structure functions and on the nuclear shell model [2.30-2.31].

Turning now to the remaining scalar term in $\mathcal{L}_{eff}$, the evaluation of $\langle N|m_q\bar{q}q|N\rangle$ is quite subtle. Since the valence quark masses are very small one can naively expect a negligible coupling but there are, however, two important contributions:

One is from the coupling to the gluons present in the nucleon through a loop of heavy quarks q_h (c, b, t) [2.32], yielding to an interaction which dominant term (the only one surviving in the limit $m_{q_h} \to \infty$, i.e. in the so called heavy quark expansion [2.33]) is

$$\sum_{q_h} \langle N|m_q\bar{q}_h q_h|N\rangle \to -\frac{\alpha_s}{12\pi} n^h \langle N|G^{\mu\nu}G_{\mu\nu}|N\rangle \tag{2.4.6}$$

with $n^h = 3$ being the number of heavy quarks.

The other is the direct coupling to the sea of strange quarks that, as implied by the measurement of the pion nucleon sigma term, seems to be very important. In fact, a fit to the baryon octet mass spectrum leads [2.34], taking into account that $\sigma_{\pi N} \simeq 60$ MeV [2.35]

$$\langle N|m_s\bar{s}s|N\rangle \simeq 380 \text{ MeV } \bar{\Psi}_N\Psi_N \tag{2.4.7}$$

Both couplings are also related by their contribution to the nucleon mass through its expression as the trace of the energy momentum tensor (including the anomaly) between the nucleon states at zero momentum transfer:

$$m_N\bar{\Psi}_N\Psi_N = \langle N|\Theta_\mu^\mu|N\rangle = \langle N|\sum_q m_q\bar{q}q - b\frac{\alpha_s}{8\pi}GG|N\rangle \tag{2.4.8}$$

with $b = 11 - \frac{2}{3}n_q$. Using again the heavy quark expansion, the direct contribution of the heavy quarks cancel against their contribution to the anomaly, resulting

$$m_N\bar{\Psi}_N\Psi_N = \langle N|\sum_{q_l} m_q\bar{q}_l q_l - (11 - \frac{2}{3})n_l\frac{\alpha_s}{8\pi}GG|N\rangle \tag{2.4.9}$$

with $q_l = u,\ d,\ s$ being the light quarks (the scale is set by Λ_{QCD}). This implies that[7]

$$-\frac{9}{8}\frac{\alpha_s}{\pi}\langle N|GG|N\rangle \simeq 508 \text{ MeV } \bar{\Psi}_N\Psi_N \ . \tag{2.4.10}$$

The fact that the scalar interaction leads to spin independent couplings implies that in a low energy scattering from a heavy nucleus there will be a coherence enhancement factor A^2 (A is the number of nucleons in the nucleus) that generally makes this contribution dominant with respect to the spin-dependent one since most of the nucleon spins are aligned pairwise to zero so that J does not increase as A. Moreover, since squarks are experimentally (and theoretically) expected to be rather heavy while the Higgs boson mass is theoretically bounded to be $m_{H_2} \leq |\cos 2\beta| M_Z$, the Higgs boson exchange should give the dominant contribution to the scattering cross section from heavy nuclei at low energies. This amounts to:

$$\sigma_i^{H_2} = \frac{4\sqrt{2}}{\pi} G_F M_Z^2 \frac{m_i^2 m_\chi^2}{(m_i + m_\chi)^2} A^2 \frac{g_H^2}{m_{H_2}^4} \tag{2.4.11}$$

The Higgs-nucleon coupling g_H is

$$g_H = \left(g_{\phi NN}^{gg} \frac{2k_u^{(2)} + k_d^{(2)}}{3} + g_{\phi NN}^{s\bar{s}} k_d^{(2)} \right) (Z_{13}s_\alpha + Z_{14}c_\alpha)\ (-Z_{11}s_W + Z_{12}c_W) \tag{2.4.12}$$

where

$$g_{\phi NN}^{gg} = -(\sqrt{2}G_F)^{1/2} \frac{\alpha_s}{4\pi} \frac{\langle N|GG|N\rangle}{\bar{\Psi}_N\Psi_N} \simeq 5 \times 10^{-4} \tag{2.4.13}$$

is the coupling of a standard Higgs to the gluons inside the nucleon through one loop of heavy quarks (summing over c, b and t). The additional contribution proportional to

$$g^{s\bar{s}} = (\sqrt{2}G_F)^{1/2} \frac{\langle N|m_s \bar{s}s|N\rangle}{\bar{\Psi}_N\Psi_N} \simeq 1.5 \times 10^{-3} \tag{2.4.14}$$

accounts for the Higgs coupling to the sea of strange quarks. It is usually dominant, especially for large values of v_2/v_1. The factors $k_q^{(2)}$ take into account the different couplings of up and down type quarks to the Higgs bosons. Notice that the coupling

[7] the contribution of the u and d quarks to the nucleon mass are respectively ~ 20 and 30 MeV [2.34]

g_H is proportional to the product of a gaugino component of the χ (the $\tilde{Z}$ one) by a higgsino component. Thus, it vanishes for a pure photino or a pure higgsino. It depends on v_2/v_1 both explicitly through α and β and implicitly through the coefficients Z_{1i} and increases with v_2/v_1.

The following spin–dependent term arises from Z boson exchange and is important only in the scattering from hydrogen in the sun (where the heavy nuclei have very low abundances)

$$\sigma_{\mathrm{H}}^{SD} = 3\frac{G_F^2}{2\pi} a_Z^2 g_A^2 \frac{m_\chi^2 m_{\mathrm{H}}^2}{(m_\chi + m_{\mathrm{H}})^2}, \tag{2.4.15}$$

with $g_A = 1.25$.

Thus, the elastic scattering cross section is given by $\sigma_i = \sigma_{\mathrm{H}}^{SD} + \sigma_i^{H_2}$ in the sun and by $\sigma_i = \sigma_i^{H_2}$ in the earth.

2.5-Results

The expected event rate of neutrinos from the sun and the earth should be compared with the corresponding upper limits from underground detectors. At present, the event rate limits on neutrinos and antineutrinos of the electron and the muon type with energy $E_\nu \geq 2$ GeV at the 90% C.L. are 4.1 events kton^{-1} yr^{-1} for the sun and 6.4 events kton^{-1} yr^{-1} for the earth, resulting from charged current contained and vertex contained events at Fréjus [2.19]. The comparison with neutrinos of lower energy is also possible, but we do not attempt it here. The data from the center of the earth come from an aperture angle equal to 7° plus the resolution angle, which insures that essentially all neutrinos from χ annihilations should be collected.

The 90% C.L. limits from IMB [2.21] on the flux of up–going muons with energy larger than 2 GeV are 8.4 10^{-14} cm^{-2} s^{-1} from the sun and (conservatively) 2.65 10^{-13} cm^{-2} s^{-1} from the earth. The bound from the earth is quite conservative since it includes muons from all zenith angles larger than 98°, while the orbits of neutralinos trapped in the earth have a rather small radius, given by $r_\chi \simeq 0.12 R_\oplus (20\mathrm{GeV}/m_\chi)^{1/2}$ [2.22], with $R_\oplus$ the radius of the earth. So one would expect the neutralino signal to come from a much smaller solid angle and, since the angular resolution is good (3.5° [2.21]) at these muon energies, the experimental upper bound on the signal could be significantly improved (even by a factor between 10 and 100) by a further analysis of the experimental data.

In figs. 2a–c we present the areas of the $\mu - M_2$ plane that are not excluded by accelerator bounds and the part of them accessible to dark matter searches for the same values of v_2/v_1 and of m_{H_2} as in fig. 1. The large central hatched area is excluded mainly by the LEP bound on the chargino mass and the UA2 and CDF bounds on the gluino mass [2.36]. In figs. 2a and 2b we show the areas excluded by the limits of Fréjus and IMB under the assumption $\rho_\chi = \rho_h$ (hatched with positive slope lines). For $m_{H_2} = 50$ GeV, present experiments are sensitive only to the signal from the earth, both from contained neutrino events and upgoing muons, coming from neutralinos with masses in the range 50–70 GeV, close to the Fe nucleus mass were the capture rate by the earth is kinematically enhanced. Also indicated are the regions where $m_\chi > m_W$ (at the top–left and top–right corners), not analyzed here.

The areas truly rejected by DM searches are actually smaller than discussed up to now. It is obvious that neutralinos cannot constitute the halo if their relic density is too small to account for the dark matter in the halos of galaxies, but the relation between the relic abundance of neutralinos in the universe and its local abundance in the halo is not known and requires additional assumptions. Thus, we first presented the maximum extent that the exclusion areas may have to give a better idea of the actual meaning of these assumptions in terms of the possibility of restricting the different models.

In order to see the bounds that could be obtained taking into account the relic density of neutralinos, we have made the conservative hypothesis that in the regions of our parameter space where $\Omega_\chi h^2 \lesssim 0.05$ (see fig. 1) the χ's constitute only a fraction $\Omega_\chi h^2/0.05$ of the halo, i.e. their local density is only that fraction of ρ_h. This assumption is reasonable if halos consist of cold dark matter with cosmological density $\Omega_g h^2 = 0.05$ and with a spatial distribution similar to that of the χ's. Our procedure is conservative in the sense that we may be reducing the expected rate in a region of the $\mu - M_2$ plane by a factor larger than necessary if $h < 1$, since $\Omega_\chi = 0.05$ may correspond to $\Omega_\chi h^2 < 0.05$. In a matter dominated universe, the value $h = 1$ is compatible only with a very young universe. It corresponds to an age of only 9 Gyr, if $\Omega_{\rm total} h^2 = 0.05$. A more acceptable value for the age of the universe, 15 Gyr, corresponds to $h \simeq 0.6$, for the same $\Omega_{\rm total} h^2$.

As a result of this procedure, the excluded regions are appreciably reduced. For example, no exclusions remain for the values of v_2/v_1 and m_{H_2} chosen in fig. 2a, while for those in fig. 2b, only the small densely hatched areas remain excluded. In fig. 2a and 2c we also show the regions which would be accessible with a factor of 10 (hatched with horizontal lines) and 100 (dotted regions) improvement in the present bound from μ's from the earth. A factor between these is easily expectable (actually, the data to obtain it may already exist, as pointed out above).

The bounds from dark matter searches extend beyond the regions shown in the figures. They are in general weaker in the half-plane $\mu < 0$, where the χ couplings are generally suppressed. For instance, the present bounds in fig. 2b extend up to $M_2 \simeq 1$ TeV and up to $\mu \simeq 200$ GeV for $\mu > 0$, but they finish just outside the figure for $\mu < 0$. With a factor of 10 improvement the accessible regions extend up to M_2 and μ values of several hundreds GeV. With a factor of 100, the bounds would exclude most χ masses below m_W up to values of M_2 and μ of a few TeV.

In fig. 3 we plotted the maximum values of m_{H_2} as a function of v_2/v_1 for which bounds can still be obtained with the present experimental constraints from indirect searches (curve I1) taking into account the partial contribution of the neutralinos to the halo density. As m_{H_2} increases, the last bounds to survive are those from the earth, which exclude values of $m_\chi \simeq m_{\rm Fe}$ due to the enhancement of the capture rate of χ (this feature also compensates the decrease in the χ relic density near $M_Z/2$). It is very interesting to note that in the $\mu - M_2$ plane these bounds are complementary to those from accelerators. In fig. 3 we also show (solid lines) the region corresponding to the present LEP bounds (ALEPH curve) on the Higgs mass, and its maximum accessible value in the near future at LEP (LEPI curve), to which two different channels

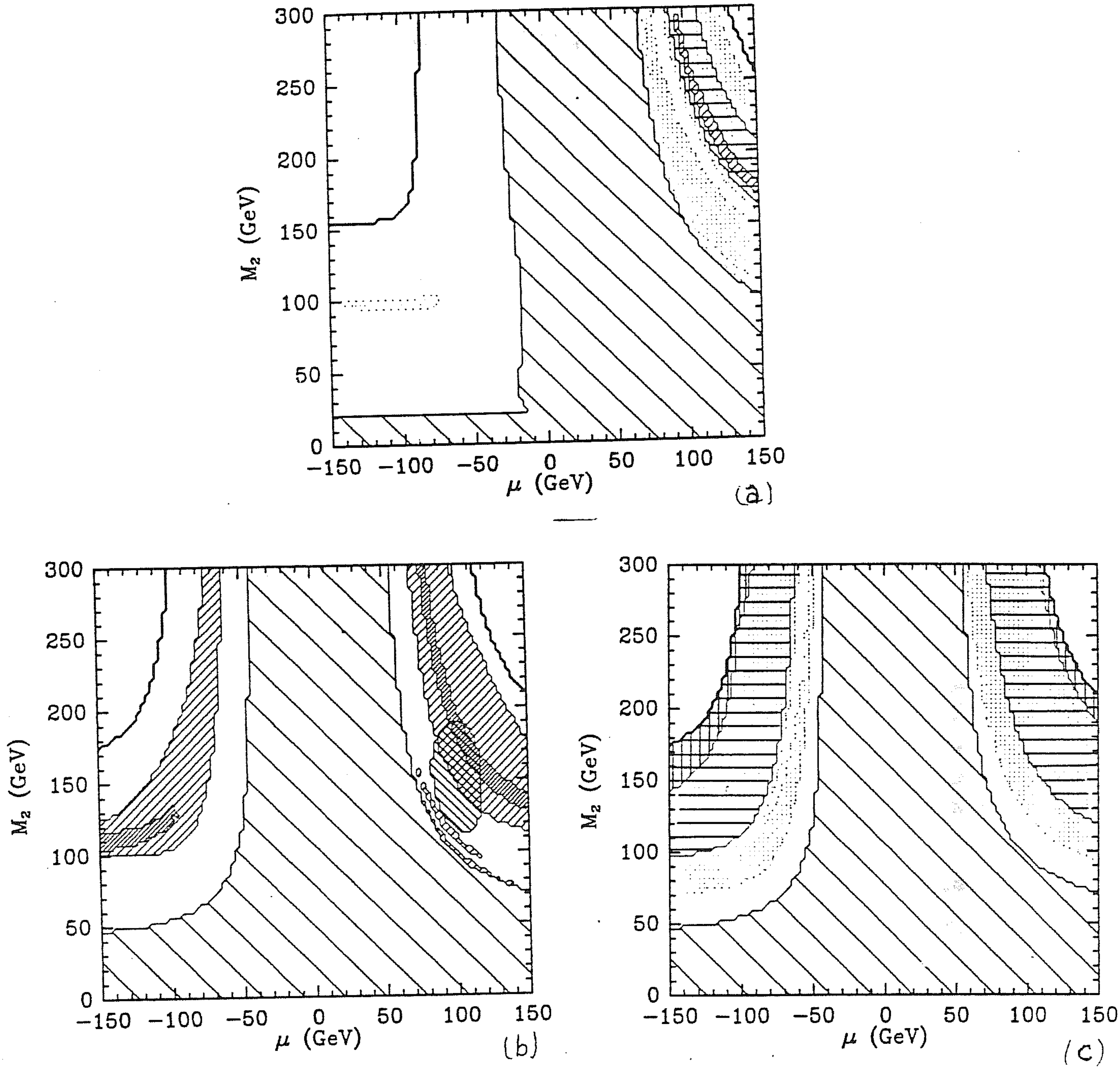

Fig. 2: Excluded regions in the $\mu - M_2$ plane, for $m_{H_2} = 50$ GeV and $v_2/v_1 = 2$ (4.a) and 8 (4.b-c). The central hatched area is excluded by the accelerator bounds (mainly from $m_{\chi^+} > 46$ GeV). Indirect searches under the assumption $\rho_\chi = \rho_h$ exclude the areas hatched with positive slope. Direct searches in Ge spectrometers exclude those with negative slope. Taking into acount the partial χ contribution to ρ_h only the densily hatched areas remain excluded. Improvements by factors of 10 and 100 in the limit from upgoing μ's from the earth should exclude the horizontally hatched and dotted regions respectively. Similar improvements in direct searches would exclude the vertically hatched area and a region comparable to the dotted one respectively.

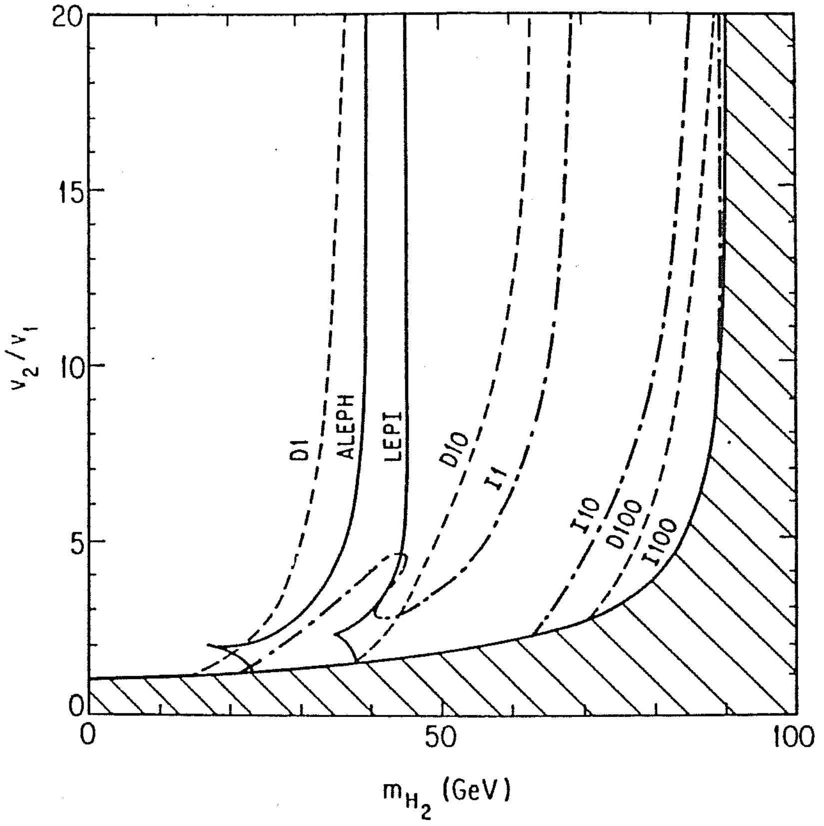

Fig. 3: Maximum values of the Higgs mass m_{H_2} as a function of v_2/v_1 for which there is a testable region in the $\mu - M_2$ plane accessible to present or future indirect (I) and direct (D) dark matter neutralino searches. I1 and D1 correspond to the present bounds while I10 and I100 are for improvements by factors of 10 and 100 in the bounds from upgoing μ's that we used and D10 and D100 are for the corresponding improvements in Ge spectrometer searches. The ALEPH curve is the present lower bound on m_{H_2} from LEP, while the LEPI curve is the kinematically accessible region for LEP.

contribute. For $v_2/v_1 \gtrsim 2$ the kinematical limit on m_{H_2} from $Z \to H_2 H_3$ is given. For smaller v_2/v_1, $Z \to H_2 Z^*$ provides a better bound (for the LEPI curve a statistics of 10^6 Z was assumed) [2.37]. The curves I10 and I100 correspond to an improvement by a factor of 10 and 100 respectively in the indirect bounds from upgoing μ's from the earth.

Although the scattering cross section diminishes as $m_{H_2}^{-4}$, reducing proportionally the χ capture rate for increasing Higgs mass, there is an important compensating effect that increases the event rate and explains the behaviour of the curve I1 for $m_{H_2} \simeq 50$ GeV. The threshold $(m_{H_2} + m_{H_3})/2$ for χ annihilation into $H_2 H_3$, which is the main channel if allowed, is beyond the interesting range $m_\chi \sim$ 50–70 GeV for $m_{H_2} \gtrsim$ 50 GeV. The absence of annihilations into $H_2 H_3$ for these masses has two consequences: a greater χ relic density and more energetic neutrinos from χ annihilations, because they are produced in the decay of primary heavy fermions instead of secondaries. Both factors increase the event rates, which are proportional to E_ν for contained events and to E_ν^2 for upgoing muons.

Notice also that the I100 curve coincides with the border of the dashed region that is theoretically excluded from the Higgs mass relation $m_{H_2} \leq |c_{2\beta}| M_Z$. This means that, with a factor 100 of improvement in μ searches, a neutralino with mass in the range 50–70 GeV should be found or definitely ruled out (within the minimal susy version of the standard model). Conversely, if the χ mass were around the Fe mass, the present experimental sensitivity of indirect dark matter searches would exclude Higgs masses below the I1 curve.

Figs. 2 and 3 contain still another independent piece of information. We have examined the bounds that direct searches of neutralinos with Ge spectrometers impose on the supersymmetric parameters. With the assumption $\rho_\chi = \rho_h$, the Ge bounds exclude the area hatched with negative slope lines in fig. 2b (there are no bounds for the parameters in fig. 2a). Taking into account the reduction in the signal due to the actual cosmological relic density there are no bounds on the parameters in fig. 2 with the present data [2.38], [8] but with a factor 10 of improvement the vertically hatched area in fig. 2c would be tested. With a factor 100 of improvement they would test a similar area as the upgoing μ's with a factor of 100 improvement which is shown in figs. 2a and 2c. This large improvement in direct searches is however not easily expected with the present techniques based on detecting ionization in Ge or Si crystals.

In fig. 3, the curves D1, D10 and D100 refer to the maximum m_{H_2} values for which there is an accessible region with $m_\chi < m_W$ to the searches with a Ge spectrometer, considering present bounds and factors 10 and 100 of improvement on them respectively.

2.6-Conclusions

We have examined experimental constraints on the parameters of the minimal

[8] We used slightly better unpublished data from the PNL-USC collaboration.

supersymmetric extension of the Standard Model of elementary particles from searches of dark matter neutralinos in the halo of our galaxy and detailed calculations of their relic abundance. We used bounds from Fréjus on contained and vertex contained neutrino events, from IMB on upgoing muons from the sun and from the earth and from Ge spectrometers. We have also shown the regions accessible to further dark matter searches.

The predicted relic abundance and signal depend on the values of four parameters that we have chosen to be μ, M_2, m_{H_2} and v_2/v_1. We give explicitly the results in a $\mu - M_2$ plane for two representative choices of v_2/v_1 (2 and 8) and for $m_{H_2} = 50$ GeV (out of reach for LEP I). As can be seen in fig. 1, just the requirement of an appropriate relic abundance largely restricts the parameter space: the neutralino could constitute the dark matter in the halos of galaxies only between the boundary of the excluded cross-hatched areas (in which neutralinos would overclose the universe, $\Omega_\chi h^2 > 1$) and the full lines ($\Omega_\chi h^2 < 0.01$). In fig. 2 we show the parameter ranges excluded by accelerators and dark matter searches. The accelerator bounds exclude most of the areas were neutralinos can be the main component of the halos of galaxies (see figs. 1 and 2). Still, we present the bounds assuming $\rho_\chi = \rho_h$ to show the maximum regions which could be tested (under any assumption on ρ_χ) with the present sensitivity of dark matter searches. Because the assumption $\rho_\chi = \rho_h$ is in general unrealistic, the points shown there as excluded by the IMB and Fréjus data might still correspond to a viable model. In fact, taking into account the actual χ relic density (by assuming that the χ contribution to the local halo density is proportional to its cosmological relic density as explained above), the excluded regions are sensibly reduced. In fig. 2 we also indicate the regions that would be accessible to dark matter searches with (reasonable) factors 10 and 100 of improvement in the bounds from upgoing muons from the earth taking into account the χ relic density.

The regions excluded become smaller for decreasing values of v_2/v_1 or increasing m_{H_2}. Before disappearing, the testable regions concentrate around the mass of the Fe nucleus that enhances the signal expected from the earth. At present, taking into account ρ_χ, the bounds disappear for $m_{H_2} > 25$ GeV for $v_2/v_1 = 2$, $m_{H_2} > 65$ GeV for $v_2/v_1 = 8$ and $m_{H_2} > 70$ GeV for $v_2/v_1 = 20$. This is shown in fig. 3, were we plotted the maximum values of m_{H_2} as a function of v_2/v_1 for which there are ranges of μ and M_2, outside those excluded by accelerators, accessible to indirect and direct dark matter searches, both at present and in the near future.

A factor larger than 10 of improvement in bounds from the upgoing muons from the earth should be easily achievable. We have shown such an improvement would allow to test the range $m_\chi \sim 50$–70 GeV for most of the Higgs boson masses allowed in the supersymmetric standard model. This is a very interesting range, because it can not be excluded by accelerators or by the condition of not overclosing the universe.

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CHAPTER 3

Are Exotic Stable Quarks Cosmologically Allowed ?

In this chapter we examine the possibility of the existence of new stable exotic quarks, as for example the $Q = -1/3$ charged quarks predicted by E_6 models. It is shown [3.1] that their cosmological consequences combined with bounds from superheavy element searches and the requirement that heavy particles captured by neutron stars do not induce their collapse into a black hole, exclude that possibility. Thus, in these models some mechanism must exist to allow the exotic quark decay.

3.1-Introduction

Several GUT extensions of the standard model enlarge considerably the particle content of the three standard generations, and some of them include heavy 'exotic' quarks among the new particles. An interesting example are the E_6 models [3.2], like those arising from ten dimensional superstring theories after compactification to the four physical dimensions [3.3]. In these models each generation of fermions is assigned to a **27**–dimensional (fundamental) representation that, together with the standard fifteen fermionic degrees of freedom, includes twelve additional new fields. The electromagnetic charge and colour quantum numbers of the new particles are univocally determined by the group structure. Besides the existence of new exotic charged and neutral leptons, each generation contains a new colour triplet and weak singlet quark Q of electric charge $-1/3$. The mass of this quark, being model dependent, is in principle arbitrary, but a lower bound on it comes from present collider experiments. The LEP results exclude the existence of new quarks lighter than $M_Z/2$.

Since under the unbroken $SU(3) \times U(1)$ gauge group the heavy quarks transform with the same quantum numbers as the standard 'down type' quarks, a mixing among them is allowed. If this mixing is present, the Q mass eigenstate should decay weakly into standard fermions. However, since the $SU(2) \times U_Y(1)$ quantum numbers are different, such a mixing induces deviations from the weak interactions of the down–type mass eigenstates predicted by the Standard Model and also flavor changing neutral currents. Present experimental data restrict considerably the allowed values for the mixing angle, specially for the first two generations for which it result typically $\sin\theta_{mix} \lesssim 0.05$ [3.4-3.5].

Here we want to analyse the consequences of assuming the exotic quark to be stable (or nearly so, *i.e.* with lifetime larger than the age of the universe). For some superstring-inspired E_6 models, a stable exotic quark is a natural consequence of the particular structure of the superpotential. In fact in order to avoid low–energy baryon and lepton number violation, it is necessary to require that some potentially dangerous couplings vanish, and this is most easily done by introducing certain discrete symmetries. In turn, these symmetries often imply that the mixing between the exotic quarks and the ordinary down type quarks vanishes [3.5]. Although we have in mind the E_6 candidate, the analysis can be extended with minor modifications to other coloured particles (charge 2/3 quarks, sextet quarks, ...).

If the heavy quark is stable, it can have important cosmological consequences. Its present density can be computed by following the thermal evolution of the universe. At very early stages its abundance is determined by the thermal and chemical equilibrium. Subsequently, the cooling of the universe reduces the annihilation rate of the heavy quarks until, at the freeze out temperature, the chemical equilibrium can no longer be mantained. However, as we will show the heavy particles remain in thermal equilibrium. In the confinement transition the exotic quarks hadronize together with the ordinary quarks. At this stage the relevant annihilation cross section associated with the disappearence of the heavy quarks increases and can reach a typical hadronic size, so that a significant reduction of the relic density of heavy hadrons takes place after confinement. (Our results on the present abundance of heavy quarks differ from a previous analysis that overestimated the annihilation cross section after confinement [3.6].) The superheavy hadrons are subject subsequently to primordial and eventually stellar nucleosynthesis, where heavy nuclei are also produced. The stringent experimental bounds from searches of superheavy elements are a powerfull test for the existence of these hypothetical stable exotic particles. We use also the bounds on very massive charged particles contributing to the cosmic dark matter that have been recently obtained from the study of their effects on the evolution of neutron stars [3.7]. These bounds, toghether with the cosmological requirements that the universe not be overclosed by these particles totally exclude the existence of a stable exotic quark.

In the following we assume that no particle-antiparticle asymmetry is present. This assumption does not affect the generality of the conclusions since an asymmetry can only increase the abundance of superheavies, giving more strength to the bounds obtained.

3.2-Relic density of stable quarks

The equation describing the evolution of the number density n of stable species is

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3\frac{\dot{R}}{R}\,n - \langle\sigma v\rangle(n^2 - n_{eq}^2) \tag{3.2.1}$$

where R is the scale factor of the universe, $\langle\sigma v\rangle$ is the thermally averaged annihilation cross section times the relative velocity and n_{eq} is the value of the density in chemical equilibrium (see chap. 2).

At temperatures above the confinement temperature $T_c \sim 200$ MeV, the relevant annihilation cross section of two heavy quarks Q involves the channels $Q\bar{Q} \to gg$,

$q\bar{q}$, where g is a gluon and q an ordinary quark (we neglect the contribution to the annihilation cross section coming from electroweak channels). For a colour triplet quark in the non-relativistic limit ($T \ll m$) annihilating into N_f lighter flavors, we obtain

$$\langle \sigma v \rangle_{Q\bar{Q}} = \frac{\pi \alpha_s^2}{m^2} \left(\frac{2}{9} N_f + \frac{7}{27} \right) \ . \tag{3.2.2}$$

In this expression, we take $\alpha_s(Q^2)$ renormalized to the scale $Q^2 \sim m^2$, since this is the relevant momentum transfer involved in the annihilation. Assuming only the standard physics at energies below m, we get for instance $\alpha_s((10 \text{ TeV})^2) \simeq \alpha_s(M_W^2)/2$. The appearence of new physics below the exotic quark mass could affect the value of α_s and could also open new channels for the annihilation. This is the case, for instance, if supersymmetry is present at the weak scale, since annihilations involving squarks and gluinos could contribute to eq. (5).

When confinement occurs, due to the presence of a relatively large number of ordinary quarks, the heavy quarks Q will hadronize mainly forming a system of 'superheavy kaons' ($Q\bar{q}$ and $\bar{Q}q$, with $q = u, \ d$) and, due to the baryon asymmetry, the $Q\bar{q}$ mesons will finish as superheavy baryons Qqq through annihilations with ordinary nucleons.

In a previous study of the survival of heavy quarks [3.5], the annihilation cross section below T_c was estimated to be equal to the ordinary nucleon-antinucleon cross section, $\sigma_{N\bar{N}} \sim 30 \text{ mb}/v$. However, several reasons indicate that this is an overestimate. In fact, in eq. (1) only the exclusive cross section that does not contain the two heavy quarks in the final state should be used. Since we are considering energies below Λ_{QCD}, the light quarks cannot be considered as spectators in the process of $Q\bar{Q}$ annihilation, which for instance could proceed through the hadronic process $\bar{Q}q + Qqq \to \bar{Q}Q + qqq$, with the formation of a $\bar{Q}Q$ bound state, which consequently decays into light particles. Although the associated cross section could be of hadronic size, since the Compton wavelength of TeV particles at MeV energies is $\lesssim$ fm, the total annihilation cross section cannot exceed the characteristic geometrical cross section associated to the range of the interactions ($\sim$ fm), *i.e.* $\sigma \lesssim 4\pi \text{ fm}^2 \sim 100$ mb. Instead, with the previously mentioned estimate of $\sigma \sim \sigma_{N\bar{N}}$ the very slow thermalized heavy hadrons (with $v =$

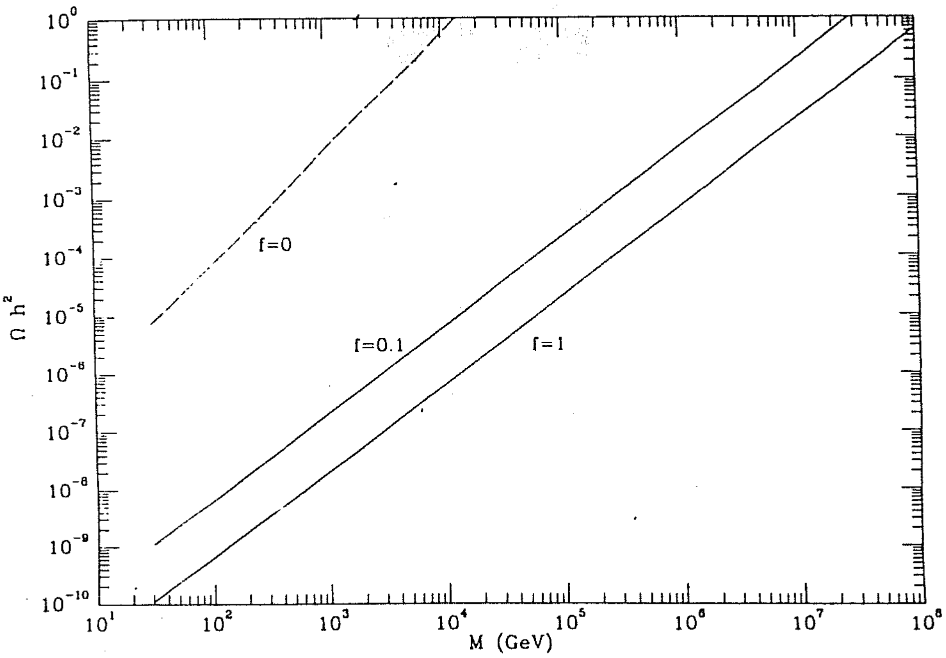

Fig. 1: Relic density of exotic stable quarks as a function of their mass for annihilation cross sections into light species after confinement of 100 mb ($f = 1$), 10 mb ($f = 0.1$) or negligible ($f = 0$).

$\sqrt{6T/m}$) would have a cross section much larger than one barn. The heavy hadrons, unlike the ordinary nucleons, enter the long wavelength regime where the annihilation cross section can grow beyond the geometric one only at temperatures ($<$ MeV) where the densities have been too diluted for the annihilation to be efficient. Furthermore, the exchange of ordinary mesons (π, ω, ...), that gives the main contribution to the total low energy annihilation of hadrons but does not affect the number of heavy quarks, should not be included in $\langle \sigma v \rangle$. Also, since the baryon asymmetry has washed out the ordinary antiquarks from the heavy hadrons, the vector exchange leads to repulsive interactions that further reduce the annihilation rate of $Q\bar{Q}$. In view of the previous discussion, we expect the relevant annihilation cross section to be much less than the corresponding geometrical cross section, and we will parametrize it as

$$\sigma \equiv f \cdot 100 \text{ mb} \qquad (3.2.3)$$

with $f < 1$.

In fig. 1 we show the resulting values of Ωh^2 for the upper value $f = 1$ and for $f = 0.1$. Here Ω is the present relic mass density of heavy quarks in units of the critical closure density, h is the Hubble constant in units of 100 km/s Mpc and, from observations, $0.4 \leq h \leq 1$. Due to the large uncertainty in the estimation of f that is related with the non-perturbative effects involved, we have also plotted the other extreme case in which the annihilation rate after confinement is negligible. This would correspond to an annihilation process $\bar{Q}q + Qqq \rightarrow$ *light hadrons* proceeding essentially through a partonic–like cross section. In this case, Ω is determined with good accuracy (since $T_{fo} >> T_c$) by the free quark annihilation rate before hadronization. Quantitatively, it is safe to neglect annihilations below T_c as long as

$$\langle \sigma v \rangle_{T<T_c} \lesssim \frac{T_{fo}}{T_c} \langle \sigma v \rangle_{Q\bar{Q}} \sim \frac{m}{5 \text{ GeV}} \langle \sigma v \rangle_{Q\bar{Q}} \tag{3.2.4}$$

where we have used a typical freeze out temperature $T_{fo} \sim m/30 - m/20$.

Figure 1 clearly implies that masses larger than $\sim 10^5$ TeV are cosmologically excluded, since they would yield $\Omega h^2 \gtrsim 1$, overclosing the universe. Moreover, the observed lifetime of the universe suggests $\Omega h^2 \lesssim 0.25$, making this constraint even stronger. For E_6 models, in the absence of any kind of mixing, each of the three flavours of exotic quarks contribute to Ω, and then the cosmological bounds should be applied to the sum of their contributions. This leads to un upper limit of $m \sim 3 \cdot 10^4$ TeV if the heavy quark masses are assumed to be similar.

If the heavy quark is not a triplet of colour (sextets of heavy quarks have been considered *e.g.* in [3.5]), or if new physics is present at energies below m, the annihilation cross section would differ (but in principle not drastically) from the case previously discussed, and the value of the relic density Ω, which is inversely proportional to it, will be correspondingly modified.

3.3-Thermalization

In the previous computation we have assumed that the heavy particles remain in thermal equilibrium. An argument to justify this assumption goes as follows: thermal equilibrium is mantained if the energy exchanged through collisions during an expansion time $\tau \simeq M_{Pl}/T^2$ is larger than the original energy $E \sim T$ of the heavy particle, *i.e.*:

$$n\, v\, \sigma\, \tau\, \Delta E \gtrsim E \tag{3.3.1}$$

where n is the number density of the scatterers, v is their mean velocity, and σ their typical cross section. Before confinement, thermalization proceeds mainly through scattering off termalized quarks and gluons through t–channel gluon exchange. Although the corresponding cross section has a Coulomb–like divergence associated with the exchange of soft gluons in the forward scattering, the relevant quantity for the thermalization is the energy transfer cross section

$$\sigma_{tr} = \int d\ \cos\theta\ \frac{d\ \sigma}{d\ \cos\theta}\ (1 - \cos\theta) \tag{3.3.2}$$

where θ is the center of mass scattering angle. Since inside the quark-gluon plasma the color charges undergo an 'electric' screening with a typical length [3.8] m_{el}^{-1} with $m_{el}^2 \simeq (gT)^2(N + N_f/2)/3$ playing the role of an effective gluon mass (N=3 is the number of colors), after taking into account this effect we obtain a finite result:

$$\langle\sigma_{tr}\rangle \sim \alpha^2 \frac{m^2}{T^4} \ln(\frac{9T^2}{m_{el}^2})\ . \tag{3.3.3}$$

Since in this case the momentum transfer is $\Delta p \sim T$, $v \simeq 1$ and $n \sim T^3$, eq.(3.3.1) is always satisfied.

After confinement, taking into account only the scattering off nucleons, we can derive from eq. (3.3.1) an upper bound for the mass of a heavy hadron for it to be in thermal equilibrium: the baryonic asymmetry yields $n_N \sim 10^{-9}T^3$, while for non relativistic particles $v \sim \sqrt{T/m_N}$, and $\Delta E \sim T\sqrt{m_N/m}$, with m_N the mass of the nucleon. We then obtain:

$$m[\mathrm{GeV}] \lesssim 10^{12}\, (\sigma\, [\mathrm{mb}])^2 \left(\frac{T}{\mathrm{MeV}}\right)^3 \tag{3.3.4}$$

We see that for a typical hadronic elastic cross section, the assumption of thermalization is correct in the whole range of masses that we have considered. Moreover, for $T \sim m_\pi$ the large number of pions present will further contribute to the thermalization of the heavy hadrons, and for the charged ones also the scattering off photons and electrons will contribute, leading in general to a bound higher than (3.3.4).

3.4-Fate of the stable quarks

Now, in order to see what kind of superheavy elements we should expect to find at the present time, and where we should look for them, we will follow the evolution of the heavy quarks from the confinement transition until now.

From T_c and up to $T \sim 1$ MeV, the electroweak interactions among the different heavy hadrons (for instance $\bar{Q}d+\nu \leftrightarrow \bar{Q}u+e$) will determine, due to the mass difference of some MeV's between the up and down quarks, an excess of $\bar{Q}u$ over $\bar{Q}d$. Their abundances should have the ration

$$\frac{n_i}{n_j} = e^{-(m_i - m_j)/T} \tag{3.4.1}$$

and we expect their mass difference to be comparable to the ordinary kaon or B-meson mass splitting.

For the heavy baryon, we will assume that the neutral isosinglet particle state $(Qud)^{I_3=0}_{I=0}$ is lighter than the positively charged member of the isotriplet $(Quu)^{I_3=1}_{I=1}$. This assumption is based on the same kind of analysis that explains qualitatively the mass relationship $m_{\Lambda^0} < m_{\Sigma^+}$: for s–wave baryons, the antisymmetry of the isosinglet $(ud)_A$ state in the internal isotopic–spin space forces the spins of the two light quarks to be antiparallel, and the energy of this configuration is lower with respect to the energy of the symmetric $(ud)_S$ triplet diquark state, that implies aligned spins [3.9].

At $T \sim 1$ MeV, electroweak interactions freeze out and primordial nucleosynthesis has begun. At these temperatures we expect that most of the heavy mesons will be positively charged $\bar{Q}u$, while most of the heavy baryons should be $Q(ud)_A$ neutral isosinglets. The surviving neutral $\bar{Q}d$ mesons, if they do not bind to any nucleus, will weakly decay with a typical meanlife of a few seconds. If they bind to nuclei, their decay rate depends on the Coulomb barrier that they feel inside the positively charged nuclei, but in all cases they finish as charged elements. In contrast, due to the large mass splitting (O(100 MeV)), the few surviving isotriplet baryons should decay electroweakly into the neutral isosinglet even if they bind to nucleons. During nucleosynthesis, a fraction of these neutral baryons may bind to protons (and neutrons) giving rise to superheavy positively charged elements.

This fact has an important consequence, because when galaxies form, after the so called 'violent relaxation', all the atoms that have fallen into the potential well of the galaxy become ionized and, as shown in ref. [3.10], the charged elements lighter than ~ 20 TeV fall into the disk together with the ordinary baryons. So for $m \lesssim 20$ TeV we expect most of the charged heavy elements to be found in the disk, while for larger masses they should originally remain mainly in the halos of galaxies. However, larger concentrations of superheavy elements in the disk are to be expected also for masses $>$ 20 TeV, since these particles can be captured by the disk during the following evolution of the galaxy. It was recently suggested [3.11] that this could happen efficiently for particle masses up to 10^5 TeV. Clearly the heavy baryons that do not form charged nuclei during nucleosynthesis will for the most part remain in the halo, leading to an asymmetry between Q and $\bar{Q}$ concentrations in the disk. The heavy elements that had fallen into the disk will be subject to stellar nucleosynthesis forming also superheavy nuclei of large Z.

For $m \lesssim 20$ TeV the proportion of heavy hadrons H with respect to ordinary nucleons present in the stars (and in the earth) should be of the order of the ratio of their cosmological densities: $n_H/n_{bar} = (\Omega_H/m)/(\Omega_{bar}/m_p)$, with m_p the proton mass. The resulting concentrations are enormously large ($\gtrsim 10^{-9}$) and exceed by several orders of magnitude the existing experimental bounds. For instance, searches of superheavy water exclude concentrations of heavy hadrons with respect to ordinary hydrogen in water larger than $\sim 10^{-28}$ for masses $\lesssim 1$ TeV [3.12], larger than $\sim 10^{-24}$ for $m \leq 10$ TeV [3.13], and with less certainty $> 10^{-15}$ for larger masses [3.12]. Actually, in the earth, we expect an even larger concentration of elements containing H hadrons, since they did not evaporate away as most of the ordinary light elements did during the earth lifetime.

For $m > 20$ TeV the heavy elements remain as dark matter or are captured in the disk, giving a contribution to the density of the galactic halo ρ_h in the neighborhood of the disk of at least $\Omega_H \cdot \rho_h$. However, the existence of large amounts of heavy CHArged Massive Particles, (CHAMPS, ref. [3.10]) has been recently shown to be in contradiction with the observed long life of several neutron stars [3.6]. This is due to the fact that CHAMPS captured by the protostellar cloud should collapse into the interior of the stars forming a black hole that would destroy the star in a time scale $\sim$ yr. For the black hole to form it is necessary that the total mass of the captured heavy particles exceeds the Chandrasekhar mass, so that degeneracy does not prevent the gravitational collapse. Since the capture by the protostellar cloud depends on the electromagnetic cross section off hydrogen [3.6], although the bounds were deduced for leptonic CHAMPS, they also hold for the charged hadronic superheavies under consideration. In the range of masses 20 TeV $\lesssim m \lesssim 10^5$ TeV, which is less constrained by searches of superheavy elements, contributions to the halo densities larger than $10^{-7} - 10^{-8}$ are ruled out by this argument. (This should be compared with our prediction of more than 10^{-6} in this mass range.)

For E_6 models, since nearly all the $\bar{Q}$'s form charged elements (*e.g.* $\bar{Q}u$), while a sizeable fraction of the Q's give rise to neutral states that are not efficiently captured by the protostellar cloud, we expect that a $\bar{Q} - Q$ asymmetry will be present inside the neutron star. This should be a general feature of models for which Q and $\bar{Q}$ belong to hadrons of different charge, or even both neutral, since in these cases we expect that they should bind differently with nuclei [3.5]. As a consequence, they should be captured by the protostellar cloud at different rates due to their 'chemical' difference, and since even a tiny asymmetry ($\lesssim 1\%$) between the concentrations of Q and $\bar{Q}$ would leave, after eventual $\bar{Q} - Q$ annihilation inside the star, enough superheavies to produce a black hole, the same conclusions deduced in ref. [3.6] still hold in this case.

In the case in which $\bar{Q} - Q$ were to form hadrons of equal charge, the large Coulomb barrier will prevent them from annihilating at the typical temperatures of neutron stars.

The previous analysis then leads to the conclusion that the existence of a stable exotic heavy quark can be safely ruled out.

We also note that in the presence of a particle–antiparticle cosmic asymmetry between the heavy quarks, all the bounds would be stronger: the relic abundance would clearly result larger, and hence the maximum cosmologically allowed mass would be smaller. For instance, an asymmetry $n_Q - n_{\bar{Q}}/n_\gamma \sim 10^{-10}$, which is comparable to the ordinary baryonic one, implies $m \lesssim 250$ GeV, and the corresponding larger density of relic superheavies would enhance the contradiction with the bounds previously discussed.

The results obtained have been based on the very accurate determination of the concentration of heavy hadrons in water, and on the observation of long lived neutron stars, but it should be mentioned that several other experimental data (*e.g.* concentrations of heavy isotopes of different elements, bounds from satellite detectors, etc.) as well as other theoretical considerations (*e. g.* the possible influence of heavy hadrons on stellar evolution) also constrain the exotic quark mass. For instance, the stringent bounds on strongly interacting dark matter that come both from detector searches near the top of the atmosphere [3.14] as well as from underground experiments [3.15], will apply to the neutral superheavy baryons, restricting thus their possible contribution to the local density of the halo (the charged component is stopped before it can reach the detectors mentioned).

The conclusion is that a stable or very long lived quark would be present with too large a density to be compatible with the cosmological requirements, with the bounds obtained from anomalous element searches and with some astrophysical implications. Hence, models with exotic quarks must include also a mechanism to allow for their decay. In the case of E_6 models this can be achieved by allowing for the presence of nonvanishing couplings of Q with other (scalar) particles that could mediate their decay or induce, through a non zero vacuum expectation value, a sizeable mixing among exotic and ordinary quarks.

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CHAPTER 4

Bounds on ordinary–exotic fermion mixing from LEP

In this chapter it is shown that recent measurements of the partial widths of the Z boson at LEP significantly improve previous bounds on the mixing between ordinary fermions and possible heavy fermions with exotic $SU(2) \times U(1)$ assignments, especially for the μ and τ leptons and for the b quark [4.1]. It is also stressed that in some extensions of the standard model with an additional $U(1)$ factor (as in E_6 models that are analysed in some detail), the effects of a $Z - Z'$ mixing can produce similar effects to those due to fermion mixing and then both should be taken into account. We constrain s_f^2, the square of the mixing between ordinary and exotic fermions, to $s_\tau^2 < 0.025$, $s_b^2 < 0.077$ and $s_\mu^2 < 0.024$, improving the previous bounds by almost a factor 10 for the τ lepton, by a factor 5 for the b quark and by a factor 2 for the μ.

4.1-Introduction

One of the striking results of the first few months of run of the LEP-1 and SLC machines is that no new particles have been produced. The bounds on the masses of many new particles that are predicted by a large class of models (SUSY, Composites, GUT's,...) are already near the kinematic limit accessible with this two machines and thus it seems that the search for direct evidences of new physics must be delayed to the time when a larger center of mass energy will be available.

On the other hand, the experimental data agree moderately well with the Standard Model (SM) and, even if for the moment there are no clear evidences of deviations from the theoretical predictions, it is possible that in the near future measurements at the Z–peak might reveal the existence of physics beyond the SM through indirect effects.

The increase in statistics, together with a better understanding of the systematical errors, will provide us with a set of high precision measurements that will be quite effective for the search of tiny new effects or, at least, to put stricter bounds on the relevant parameters that are generally introduced in any extension of the electroweak theory.

For example, deviations from the SM predictions are expected if gauge groups larger than $\mathcal{G}_{SM} = SU(2)_L \times U(1)_Y$ $[\times SU(3)_C]$ underlie the standard electroweak theory. In particular, these deviations could be due to a mixing among the standard fermions and new exotic ones (that often occur in models with enlarged gauge groups) as well as to a mixing of the standard Z_0 with additional neutral vector bosons. Both these effects will modify the fermion couplings to the gauge bosons, and most of the quantities that are measurable at the Z–peak are particularly good to detect possible deviations from the standard neutral current couplings.

With respect to the mixing among gauge bosons, we will assume that only one new neutral Z_1 mixes appreciably with the Z_0. Then, we are lead to investigate the phenomenological consequences of an effective gauge group $\mathcal{G}_{SM} \times U(1)'$. Since the direct product structure leaves the $U(1)'$ fermion quantum numbers, as well as the g' coupling constant completely arbitrary, a second assumption has to be made in order to obtain predictions: namely that our effective low energy gauge group originates from a *simple* group G_S, broken by some mechanism at a higher energy scale. Then, since $U(1)'$ belongs to the Cartan subalgebra of G_S, only few choices for the quantum numbers of the particles present in the model will be allowed, and the possible range for the value of the coupling constant g' will also be constrained.

For the sake of definiteness we will carry out our investigation in the frame of a class of E_6 models. The consequences of the presence of a new Z_1 of E_6 origin on Z–resonance physics has been deeply investigated by many authors [4.2]. However, to consider the modifications of the Z-couplings due to a $Z_0 - Z_1$ mixing alone is not totally consistent in these models, since similar effects can arise also from fermion mixing. In particular, since each fermion generation is assigned to a **27** representation of E_6, besides the 15 standard fields 12 additional 'exotic' particles per generation are predicted to exists. These are: a weak doublet of leptons (N, E^-) with its charged conjugate doublet (E^+, N^c) , a color triplet weak singlet quark h of charge -1/3 together with h^c, and two neutral singlets ν^c and S. In general a mixing among particles that have the same quantum numbers under the unbroken $U(1)_Q \times SU(3)_C$ group will be allowed, modifying the standard couplings of the fermions.

Bounds on the mixing of a Z_1 of E_6 origin with the standard neutral boson have been derived in ref. [4.3]. The results of that analysis constrain the mixing to $\tan\Theta_{mix} \lesssim 0.22$ for a Z_1 almost decoupled from neutrinos, and to a much lower value ($\tan\Theta_{mix} \lesssim 0.05$) in the other cases. However, the analysis in [4.3] does not take into account the possibility of fermion mixing, so that a combined analysis of these two effects should turn out in slightly worst bounds than the ones quoted.

On the other hand, the implications of the fermion mixing alone in a very large variety of observables have been used to constrain the mixing angles between ordinary and exotic fermions [4.4], resulting in $s^2 \equiv \sin^2\xi \lesssim 0.030 - 0.050$ for the first generation and for ν_μ, $s^2_\mu < 0.055$ while the bounds for the fermions in the third generation and for the second generation quarks are much worst. For instance, for the $b - h_b$ mixing the bound is $s^2_b < 0.43$ and for the $\tau - E_\tau$ mixing it is $s^2_\tau < 0.22$ (all at 90% c.l.).

Although these mixings can in principle vanish, there are good reasons to believe that they are nonzero. In fact, recently it has been shown using cosmological and astrophysical arguments, together with experimental bounds from heavy isotope searches, that new charged leptons [4.5] and new colored particles [4.6] can not be stable. Clearly, the mixing of the exotic particles with the ordinary ones provides a natural channel for their decay.

It is our purpose here to show that the present LEP results on partial widths of the Z boson already improve the previously mentioned bounds on s^2_b by a factor of 5, taking into account also the possible effects of a $Z_0 - Z_1$ mixing, while for s^2_τ the bound is improved almost by a factor 10 and this last result is essentially model independent.

In particular, it is important to constrain the b and τ mixings not only because they are poorly bounded at present, but also because they are theoretically expected to be the largest ones since, if the masses arise from a seesaw mechanism, one has

$$\begin{aligned} \sin^2\xi &\simeq (m/M) \qquad \text{linear}-\text{seesaw} \\ \sin^2\xi &\simeq (m/M)^2 \quad \text{quadratic}-\text{seesaw} \end{aligned} \tag{4.1.1}$$

where m and M are the light and heavy fermion masses respectively and for a large class of models one typically expects that the mixings will fall within the range suggested in (4.1.1). This argument also leads us to expect tiny mixings in the first two generations ($\lesssim 10^{-3}$), since unsuccessful searches of exotic particles that couple to the Z-boson suggest $M > M_Z/2$. We will concentrate on the consequences of the mixing between the ordinary and exotic charged leptons and between h_b and b quarks. A general analysis of the effects of lepton mixing in the charged and neutral sectors, of the quark mixing and $Z_0 - Z_1$ mixing in several quantities measurable at LEP (partial widths and asymmetries) is under consideration [4.7].

4.2-The formalism

The exceptional group E_6 [4.8] is one of the most interesting candidates for a unifying group. The reason is at least twofold: first E_6 contains as subgroups the symmetry groups of the most popular grand unified and left-right symmetric theories, like *e.g.* $SO(10)$, $SU(5)$, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and second, it is the only phenomenologically acceptable group that can arise from ten–dimensional superstring theories after Calabi Yau compactification down to the 4 physical dimensions [4.9]. A nice feature of the E_6 algebra is that the embedding of the color and weak isospin subgroup $SU(2)_L \times SU(3)_C$ is unique, but clearly in going from rank 6 down to rank 3, three Cartan generators are left, and it follows that the identification of the hypercharge axis is not unique. Here we will consider the embedding of $\mathcal{G}_{SM}$ in E_6 through the maximal subalgebras chain:

$$
\begin{aligned}
E_6 \longrightarrow\; & U(1)_\psi \times SO(10) \\
& \quad \hookrightarrow\; U(1)_\chi \times SU(5) \\
& \qquad\qquad \hookrightarrow\; \mathcal{G}_{SM}\,.
\end{aligned}
\tag{4.2.1}
$$

The most general form for $U(1)'$ compatible with this symmetry breaking chain will then be a linear combination of the $U(1)_\psi$ and $U(1)_\chi$ generators that will be parametrized in terms of an angle α. Correspondingly, the couplings of the fermions to the Z' boson will depend on both the ψ and χ quantum numbers through the combination ($c_\alpha = \cos\alpha$, $s_\alpha = \sin\alpha$):

$$
Q' = c_\alpha Q_\psi + s_\alpha Q_\chi . \tag{4.2.2}
$$

For the left handed fermions belonging to the **27** fundamental representation of E$_6$, the values of the abelian Q_ψ and Q_χ charges are listed in Tab. I.

	S_L	$\binom{E^+}{N_{E^+}}_L$ h_L	$\binom{N_{E^-}}{E^-}_L$ h^c_L	ν^c_L	$\binom{\nu}{e^-}_L$ d^c_L	e^c_L u^c_L $\binom{u}{d}_L$
$6\sqrt{\frac{2}{5}}Q_\psi$	4	−2		1		
$6\sqrt{\frac{2}{3}}Q_\chi$	0	2	−2	−5	3	−1

Table I: Quantum numbers for the left handed fermions of the fundamental **27** representation of E$_6$. Abelian charges are normalized to the hypercharge axis according to: $\sum_{f=1}^{27}(Q^f)^2 = \sum_{f=1}^{27}(\frac{Y^f}{2})^2 = 5$.

The multiplicative factors have been chosen in order to have the same normalization for the three abelian axes: $\mathrm{Tr}\, Q^2_\psi = \mathrm{Tr}\, Q^2_\chi = \mathrm{Tr}\,(\frac{Y}{2})^2$, so that at the unification scale the same coupling constant g_Y is associated to both Y and Q' charges. Possible deviations that could arise at the 100 GeV scale, as a consequence of a different running of the couplings, can be taken into account by writing:

$$g' = \kappa \frac{e}{c_w} \tag{4.2.3}$$

where the SM relation $e = g_Y c_w$ has been used.

We will denote with Z_1 the $U'(1)$ vector boson gauge eigenstate that in general has a non diagonal mass matrix with the standard Z_0. The mass eigenstates Z and Z' are related to Z_0 and Z_1 through an orthogonal transformation, parametrized in terms of a mixing angle Θ,

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} Z_0 \\ Z_1 \end{pmatrix}. \tag{4.2.4}$$

Then, the physical Z boson couples to fermions via the effective lagrangian:

$$\mathcal{L}_{NC}(Z) = -\left(\frac{G_\mu \cos^2\Theta\ M_Z^2}{2\sqrt{2}}\right)^{1/2} \sum_f \bar{\psi}^o_f \gamma_\mu [\tilde{v}_f - \tilde{a}_f \gamma_5] \psi^o_f \cdot Z^\mu \tag{4.2.5}$$

where the superscript "o" reminds that for the moment we are considering unmixed fermions. In this expression the effective couplings $\tilde{v}_f$ and $\tilde{a}_f$ correspond to the SM couplings v_f and a_f shifted by a quantity proportional to the $Z_0 - Z_1$ mixing and dependent on the Q' fermion quantum numbers:

$$\tilde{v}_f = v_f + s_w \hat{t}_\odot\ v'_f \qquad \tilde{a}_f = a_f + s_w \hat{t}_\odot\ a'_f \tag{4.2.6}$$

with

$$\begin{aligned} v_f &= 2T_3^f - 4Q^f\ s_w^2 = 2T_3^f - Q^f(v+1) & a_f &= 2T_3^f \\ v'_f &= 2s_\alpha(Q_\chi^f - Q_\chi^{f^c}) & a'_f &= 4c_\alpha Q_\psi^f + 2s_\alpha(Q_\chi^f + Q_\chi^{f^c}) \end{aligned} \tag{4.2.7}$$

where T_3^f is the left-handed fermion weak isospin, Q^f the electric charge, $s_w^2 = sin^2\vartheta_w$ with ϑ_w the weak mixing angle and $v = 4s_w^2 - 1$ is the charged lepton vector coupling. Since v is a small quantity that can be used as an expansion parameter for truncating expressions ($v \simeq -0.08$), it is useful to express all the fermions couplings as a function of v, as we have done. The ratio $g'/g_Y \equiv \kappa$ has been absorbed by rescaling the mixing angle: $\hat{t}_\Theta = \kappa \cdot \tan\Theta$. On resonance, the shifts of the standard couplings are by far the most important effects of the $Z_0 - Z_1$ mixing. $Z - Z'$ interference, as well as Z' exchange diagrams, are suppressed at least by a factor $\Gamma_Z\Gamma_{Z'}/M_Z M_{Z'}$ and can then be safely neglected. We will discuss later the effects of the shift on the physical Z mass due to the mixing.

The next step is to allow for a mixing among the standard and exotic fermions that will further modify the couplings. We note that since the electromagnetic (and color) quantum numbers of the exotic quarks and leptons are the same as those of the ordinary ones (as it must be since otherwise no mixing would be allowed), the electromagnetic current is unchanged. Moreover, table I shows that the $SU_L(2)$ transformation properties of the right handed $Q = -1/3$ quarks and left handed leptons also coincide with the ordinary ones, so that only the couplings of left handed down-type quarks (weak isospin doublet) and right handed ordinary leptons (weak singlets) will be modified by the mixing since their heavy partners are respectively singlets and doublets of weak isospin. Following ref. [4.4] we introduce two vectors for the ordinary and exotic left and right handed weak eigenstates $\psi^o_{L(R)} = (\psi^o_{ord}, \psi^o_{ex})^T_{L(R)}$, and other two vectors for the light (i.e., standard) and heavy mass eigenstates $\psi_{L(R)} = (\psi_l, \psi_h)^T_{L(R)}$, where for example for the down-type light quarks $\psi_l = (d, s, b)^T$. The weak and mass eigenstates are related by unitary transformations

$$\psi^o_L = U_L \psi_L \,; \qquad\qquad \psi^o_R = U_R \psi_R \tag{4.2.8}$$

with

$$U_{L(R)} = \begin{pmatrix} A & E \\ F & G \end{pmatrix}_{L(R)} \tag{4.2.9}$$

and from the unitarity of U

$$A^\dagger A + F^\dagger F = A^\dagger A + E^\dagger E = I \tag{4.2.10}$$

The 3×3 matrices E and F describe the mixing between the light and heavy states.

The part of the weak neutral current that gets modified by the mixing can be written:

$$\begin{aligned}\frac{1}{4}J_Z^\mu &\sim \sum_f \bar{\psi}_f^o \left[t_3 I_L^f P_L + t_3 I_R^f P_R - Q s_w^2\, I\right] \psi_f^o \\ &= \sum_f \bar{\psi}_f \left[t_3 U_L^{f\dagger} I_L^f U_L^f P_L + t_3 U_R^{f\dagger} I_R^f U_R^f P_R - Q s_w^2\, I\right] \psi_f\end{aligned} \tag{4.2.11}$$

where the sum involves only the $Q = -1/3$ quarks $q = d, h$ and the charged leptons $\ell = e, E$, $P_{L(R)} = \frac{1}{2}(1 \mp \gamma_5)$ and

$$I_L^q = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} ; \qquad\qquad I_R^\ell = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \tag{4.2.12}$$

while

$$I_R^d = I_L^\ell = I = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \tag{4.2.13}$$

Using (4.2.11) and the unitarity relations (4.2.10) it is easy to see how the couplings of the light states d and ℓ are further modified with respect to eq. (4.2.6):

$$\begin{aligned} &\tilde{v}_d \to \hat{v}_d = \tilde{v}_d - 2t_3(F_L^{d\dagger} F_L^d) \qquad && \tilde{a}_d \to \hat{a}_d = \tilde{a}_d - 2t_3(F_L^{d\dagger} F_L^d) \\ &\tilde{v}_\ell \to \hat{a}_\ell = \tilde{v}_\ell + 2t_3(F_R^{\ell\dagger} F_R^\ell) && \tilde{a}_\ell \to \hat{a}_\ell = \tilde{a}_\ell - 2t_3(F_R^{\ell\dagger} F_R^\ell) \end{aligned} \tag{4.2.14}$$

The matrices $F^\dagger F$ are in principle 3×3 non diagonal matrices that describe intergenerational mixing too. However, the off–diagonal terms that would induce flavor changing neutral currents at the tree level are severely constrained by experiments [4.4]. We will assume that these terms are negligibly small so that the light–heavy mixing occurs essentially between particles belonging to the same generation. We will then parametrize:

$$\mathrm{diag}(F_{R(L)}^\dagger F_{R(L)}) = \left((s_1^{R(L)})^2,\, (s_2^{R(L)})^2,\, (s_3^{R(L)})^2\right) \tag{4.2.15}$$

with $s_i^2 = \sin^2 \xi_i$.

Clearly, in our procedure to define the effective coupling of the fermions to the Z-boson, second order effects proportional to $\hat{t}_\Theta \cdot \sin^2 \xi$ have been neglected. In the following we will consider as 'first order terms' the following set of small parameters: $\hat{t}_\Theta$, $\sin^2 \xi$ and v, and we will neglect terms involving products or higher powers of them.

4.3-Theoretical expectations

Before beginning the analysis of the bounds that can be derived from the measurements of the partial widths $Z \to b\bar{b}$ and $Z \to \ell^+\ell^-$, we want to discuss an indirect effect of the fermion mixing that will enter as a theoretical uncertainty in any prediction for electroweak quantities. The set of electroweak parameters that is known with the best experimental accuracy is α, M_Z and G_μ (the Fermi constant measured in μ decay). In particular, G_μ is introduced to replace the W mass, whose experimental value is still affected by a large error. G_μ is related to M_W through the equation:

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2 \, \sin^2\theta_W(1-\Delta r)} \tag{4.3.1}$$

where Δr is a radiative correction that (taking into account only the leading contributions) can be written as $\Delta r \simeq \Delta\alpha - (c_w^2/s_w^2)\Delta\rho$. Here, the effect of $\Delta\alpha$ ($\simeq 0.06$) is to renormalize the electromagnetic charge to the scale M_Z

$$\alpha(M_Z^2) = \frac{\alpha(0)}{(1-\Delta\alpha)} \tag{4.3.2}$$

while $\Delta\rho$ [4.10] contains potentially large corrections that in the SM are essentially due to the top–bottom mass splitting, but that in general can arise from a mass difference between the components of any additional isodoublet of fermion [4.11] or scalar [4.12] particles that is present in the model. In a theory that allows for a $Z_0 - Z_1$ mixing, it is possible to take into account this effect by replacing $M_Z^2 \to \rho_{mix}M_Z^2$ since ρ_{mix} enters any expression in the same way as a $\rho_o \neq 1$ generated by non-standard Higgses [4.2,4.13]. In such a theory, a possible (and useful) definition of the Weinberg angle is $c_w^2 = M_W^2/(\rho_{mix}M_Z^2)$. Allowing now also for a fermion mixing, eq. (4.3.1) will be modified into

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha(M_Z^2)}{M_W^2(1-\frac{M_W^2}{\rho M_Z^2})}(1-\Delta s) \tag{4.3.3}$$

with

$$\rho \equiv \rho_{mix}(1+\Delta\rho)$$

and where

$$1-\Delta s \equiv c^L_\mu c^L_{\nu_\mu} c^L_e c^L_{\nu_e} \simeq 1-\frac{1}{2}({s^L_\mu}^2+{s^L_{\nu_\mu}}^2+{s^L_e}^2+{s^L_{\nu_e}}^2) \tag{4.3.4}$$

takes into account the effect of fermion mixing in μ–decay [4.4]. Two remarks are in order: first, both a $Z_0 - Z_1$ mixing and a heavy top produce positive deviations of ρ from the SM tree level value $\rho = 1$. Since the theoretical upper bound on m_t comes from measurements of the ρ parameter, in general allowing the top mass to vary in the range 80 GeV$< m_t <$230 GeV automatically takes into account the uncertainty related to a $Z_0 - Z_1$ mixing. This is not the case for the partial width into b–quarks, since this quantity receives an additional m_t–dependent contribution from the $Zb\bar{b}$ vertex correction that also involves the top mass and that almost cancels against the $\Delta\rho^{top}$ correction [4.14]. As a result, $\Gamma_{b\bar{b}}$ turns out to be nearly unsensitive to the value of the top mass. Thus, in this particular case the uncertainty coming from $\Delta\rho_{mix} \equiv \rho_{mix} - 1$ must be separately taken into account. A second point that we need to discuss is the effect of e, ν_e, μ and ν_μ mixing in the measured Fermi constant. We can include this effect simply by replacing $G_\mu \to G_\mu(1+\Delta s)$ in all the expressions, but in so doing Δs will induce an additional theoretical uncertainty. However, we expect this correction to be quite small since the mixings involved in μ–decay should be negligible ($\Delta s < 10^{-3}$ according to eq. (4.1.1)). In the forthcoming expressions we will keep trace of both these effects but, as we will see, they will not affect very much our numerical analysis since the overall error is largely dominated by the experimental uncertainty.

From the lagrangean (4.2.5), after the replacement $\tilde{v} \to \hat{v}$ and $\tilde{a} \to \hat{a}$, we can write the tree level expressions for the partial widths $Z \to f\bar{f}$ as:

$$\Gamma_{f\bar{f}} = \frac{\sqrt{2}\rho_{mix} G_\mu (1+\Delta s) M_Z^3}{48\pi}(\hat{v}_f^2 + \hat{a}_f^2) \tag{4.3.5}$$

Then, from the expression for the effective neutral couplings (4.2.6-4.2.7) and (4.2.14) we obtain for the partial decay width into b–quarks:

$$\Gamma_{b\bar{b}} = \Gamma_{b\bar{b}}^{SM}\left[1+\frac{19}{13}\left(\Delta\rho_{mix}+\Delta_S\right)-\frac{3}{13}\hat{t}_\Theta\left(\sqrt{10}\,c_\alpha - \sqrt{\frac{2}{3}}\,s_\alpha\right)-\frac{30}{13}(s^L_b)^2\right] \tag{4.3.6}$$

with, at the tree level

$$\Gamma_{b\bar{b}}^{SM} = \frac{\sqrt{2}G_\mu M_Z^3}{48\pi}\frac{(13-4v)}{3} \tag{4.3.7}$$

Including the 1–loop electroweak and QCD corrections [1] we have in the frame of the SM

$$\Gamma_{b\bar{b}}^{SM} = 377 \cdot (1 \pm 0.012)\mathrm{MeV} \tag{4.3.8}$$

where the theoretical uncertainty corresponds to the variation of the top mass, Higgs mass and $\alpha_s(M_Z^2)$ in the ranges

$$80\ \mathrm{GeV} < m_t < 230\ \mathrm{GeV}; \qquad 25\ \mathrm{GeV} < M_H < 1\ \mathrm{TeV}; \qquad 0.10 < \alpha_s < 0.14 \tag{4.3.9}$$

According to our previous discussion we will neglect the effect of Δs in eq. (4.3.5). To estimate the uncertainty due to $\Delta\rho_{mix}$, we use again eq. (4.3.3) with the experimental value of the $W-Z$ mass ratio averaged over the UA2 [4.15] and CDF [4.16] experiments: $M_W^2/M_Z^2 = 0.775 \pm 0.007$, and $M_Z = 91.170 \pm 0.033$ from LEP [4.17]. Neglecting again Δs, and subtracting the contribution of an 80 GeV top quark, we obtain at 90% c.l. $\Delta\rho_{mix} < 0.007$. We note that although the experimental value of ρ obtained in this way is slightly less precise than what could be obtained from low energy neutral to charged current ratio [4.3], this estimation is insensitive to possible Z' exchange diagrams.

To estimate the uncertainty induced in $\Gamma_{b\bar{b}}$ by the $Z_0 - Z_1$ mixing we have evaluated the values of the corresponding term $\delta_\Theta^b = \frac{3}{13}\hat{t}_\Theta\left(\sqrt{10}\,c_\alpha - \sqrt{\frac{2}{3}}\,s_\alpha\right)$ in the range experimentally allowed for Θ as a function of α that is quoted in ref. [4.3]. We obtain for this effect $-0.174 \le \delta_\Theta^b \le 0.013$. Although the bounds obtained in [4.3] ignored the effects of fermion mixing, we think that they should be reliable since they are derived from deep inelastic ν scattering off nucleons and from $e^+e^- \to \mu^+\mu^-$ data that involve only fermions for which mixing effects are expected to be negligible.

In conclusion, our numerical prediction for the partial decay width of the Z boson into b quarks is the following:

$$\Gamma_{b\bar{b}} = 377 \cdot \left(1 \pm .012 \ {}^{+.010}_{-.0} \ {}^{+.013}_{-.174} - \frac{30}{13}(s_b^L)^2\right)\mathrm{MeV} \tag{4.3.10}$$

[1] W. Hollik is acknowledged for providing the fortran program WIDTH.

where the first error comes from SM uncertainties, the second from ρ_{mix} and the third from $Z_0 - Z_1$ mixing. Since the effect of the $b - h_b$ mixing tends to decrease the decay rate, we have to compare the experimental data with the maximum allowed value of (4.3.10). Moreover, since also the effect of $Z_0 - Z_1$ mixing in this quantity turns out to be negative for almost all values of α, and the possible positive shift is bound to be quite small, the inclusion of this effect in our analysis does not change the bounds from what one would obtain assuming $\Theta = 0$. Clearly if any nonzero $Z_0 - Z_1$ mixing is experimentally found, this will generally result in a better bound on $(s_b^L)^2$.

For what concerns the partial width into τ and μ leptons, it looks convenient to get rid of the overall multiplicative coefficient in (4.3.5) by defining a quantity normalized with the electron width

$$R_\tau \equiv \frac{\Gamma_{\ell^+\ell^-}}{\Gamma_{e^+e^-}} = \frac{\hat{v}_\ell^2 + \hat{a}_\ell^2}{\hat{v}_e^2 + \hat{a}_e^2} \tag{4.3.11}$$

for which we obtain:

$$R_\ell = R_\ell^{SM} - 2(s_\ell^R)^2 + 2(s_e^R)^2 \tag{4.3.12}$$

and, neglecting the tiny effect of m_ℓ, $R_\ell^{SM} = 1$. Clearly the quantity R_ℓ is exactly one even in the presence of a $Z_0 - Z_1$ mixing, since the lepton couplings to the Z_1 boson also obey universality. In fact, the bound on the quantity $(s_\ell^R)^2 - (s_e^R)^2$ can be effectively thought as a bound on any source of violation of universality, of which the mixing among fermions that we are considering now is probably one of the most obvious. In comparing (4.3.12) with the experimental data we will again neglect $(s_e^R)^2$, obtaining thus an upper limit on the $\tau - E_\tau$ and $\mu - E_\mu$ mixing.

4.4-Experimental constraints

We now discuss the experimental data. To obtain the partial width $Z \to b\bar{b}$ the decay mode $b \to \mu$ has been used in ref. [4.18] (L3), while the ALEPH collaboration [4.19] uses both $b \to \mu$ and $b \to e$ decay modes to tag the b quark. Their results are, respectively

$$\begin{aligned}
\mathrm{Br}(b \to \mu)\,\Gamma_{b\bar{b}} &= 41.7 \pm 2.9(stat) \pm 3.0(syst)\ \mathrm{MeV} \\
\mathrm{Br}(b \to \mu)\,\Gamma_{b\bar{b}} &= 42.6 \pm 5.0(stat) \pm 2.1(syst)\ \mathrm{MeV} \\
\mathrm{Br}(b \to e)\,\Gamma_{b\bar{b}} &= 38.8 \pm 3.4(stat) \pm 1.8(syst)\ \mathrm{MeV}
\end{aligned} \tag{4.4.1}$$

where the second and third results are obtained by multiplying the ALEPH results for $\mathrm{Br}(b \to \ell)\,\Gamma_{b\bar{b}}/\Gamma_{had}$ by the hadronic width averaged over the four LEP experiments $\Gamma_{had} = 1788 \pm 23$ MeV [4.17]. Then we take the weighted average of the three measurements and using the value quoted in [4.19] for the branching ratio: $\mathrm{Br}(b \to \ell) = 0.102 \pm 0.010$ we finally obtain

$$\Gamma_{b\bar{b}} = 399 \pm 46\ \mathrm{MeV} \tag{4.4.2}$$

In this equation the uncertainty in the branching ratios for the decay of the b quark into e and μ leptons dominates the overall error. The ALEPH [4.20] and OPAL [4.21] collaborations have published data for the $Z \to \tau^+\tau^-$ and $Z \to \mu^+\mu^-$ partial widths. The average of their measurements gives

$$\begin{aligned}
\Gamma_{\tau^+\tau^-} &= 85.9 \pm 4.4\ \mathrm{MeV} \\
\Gamma_{\mu^+\mu^-} &= 85.2 \pm 4.2\ \mathrm{MeV}
\end{aligned} \tag{4.4.3}$$

Finally, for the partial width into electrons, we use the average of the four LEP experiments [4.20–4.23]

$$\Gamma_{e^+e^-} = 81.7 \pm 1.6\ \mathrm{MeV} \tag{4.4.4}$$

In this average we have assumed that the errors quoted in [4.20–4.23] are uncorrelated. We have neglected a possible correlation in the uncertainty that arises from the procedure for subtracting the t–channel Bhabha scattering from the data since this effect cannot be larger than a relative 2% [4.24], and in R_ℓ the errors that arise from the τ and μ partial widths largely dominate.

With this figures, the experimental value of our quantities at the 90 % c.l. are found to be

$$\begin{aligned}
\Gamma_{b\bar{b}} &= 399\ (1 \pm 0.19)\ \mathrm{MeV}; \\
R_\tau &= 1.051\ (1 \pm 0.09) \\
R_\mu &= 1.043\ (1 \pm 0.09)
\end{aligned} \tag{4.4.5}$$

We note that in R_τ and R_μ the error is probably overestimated since we expect that the systematic uncertainty that originates from the measurement of the luminosity should cancel in this two ratios.

From (4.3.10), (4.3.12) and (4.4.5) we get the following bounds for the b–h_b, τ–E_τ and μ–E_μ mixing angles:

$$\begin{aligned}(s_b^L)^2 &\leq 0.077\\ (s_\tau^R)^2 &\leq 0.025\\ (s_\mu^R)^2 &\leq 0.024\end{aligned} \tag{4.4.6}$$

It is interesting to note that the first two bounds are already comparable with the maximum values for the mixings that can be derived from eq. (4.1.1) given the present lower limits on the mass M of exotic particles.

In conclusion, we have analysed the consequences of a mixing among the ordinary fermions and new heavy exotic ones that are predicted to exist in E_6 models, centering the attention on quantities relevant for experiments at the Z peak. We have taken into account several effects that occur in this kind of theories, such as a Z_0–Z_1 mixing that will induce deviations from the SM couplings of the fermions and will also influence the value of the ρ parameter. We have compared the results with recent LEP data, obtaining new and improved bounds on the mixing angles of the τ and μ leptons and of the b quark.

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