



**ISAS - INTERNATIONAL SCHOOL
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**Phenomenological Constraints
on Supersymmetric
Grand Unification**

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Introduction

The aim of this thesis is the study of Supersymmetric Grand Unified models (SUSY GUT) from the phenomenological point of view. The present Introduction is an overview of the subjects studied, in which the lines of approach followed are explained.

In the first Chapter the basic theory of supersymmetric models is reviewed. Particular attention is devoted to settle a formalism suited for studying the general supersymmetric theory: derivation of lagrangians, analysis of the quantum structure, renormalisation group equations. This part can be considered a sort of “operating system” for the study of the phenomenology.

Then I pass to define the Minimal Supersymmetric Extension of the Standard Model (MSSM). I believe that some care is needed to allow readability of the results, and also to avoid misunderstandings. Important formulae for the masses of supersymmetric particles and for their interactions with the standard ones are derived.

The third Chapter is devoted to the study of possibility of embedding the MSSM in a SUSY GUT, or in other words to spelling out the most direct predictions of this kind of models. In this discussion virtues and vices of the model become apparent. It is noticeable the possibility to predict correctly α_s , to address successfully $b - \tau$ unification (with some proviso), to have a consistent picture of CP violation and a beautiful mechanism to explain the electroweak breaking; but the difficulty to predict for sure the observation of supersymmetric particles—except for the lightest higgs—in large regions of the parameter space is surely not a virtue of the model. In fact the key point is how to probe the space of supersymmetric parameters. Important technical tools needed for this aim are discussed.

The last Chapter can be considered a partial answer to the problem of the exploration of the parameter space, that uses interesting theoretical means and it is in a sense complementary to supersymmetric particle searches. Two processes that are forbidden at the tree-level in the Standard Model are studied in the context of SUSY GUT: the first (the electric dipole of the neutron) is related to the CP violating sector, the second (the $b \rightarrow s\gamma$ decay) to the Flavour-Changing Neutral Currents (FCNC) of the theory. While the first test is passed for what matter present day phenomenology, the latter is not automatically; in fact experimental observations require strong restrictions to the parameter space of the model, whereas SM, even if with large theoretical uncertainties, fits the data. This is clearly not a failure of the MSSM models, even if the question of

a natural explanation of such constraints is in my opinion important. Future theoretical progresses, restricting the parameter space or giving reasons of the elusiveness of SUSY world, or hopefully experimental findings are eagerly awaited.

1. Generalities of the supersymmetric models

In this Chapter we review the general formalism of $N = 1$ supersymmetric field theories, namely we discuss: supermultiplets, supersymmetric lagrangians, soft breakings, 1-loop quantum structure. Together with Appendices A and D this Chapter sets the notations followed; it is meant to expose the technical tools needed in phenomenological applications of supersymmetry.

1.1. Supermultiplets

Supersymmetry relates bosons and fermions in the same representation. Building the infinitesimal transformations, one realizes that to maintain the commuting character of (for instance) a boson fields, the parameters of supersymmetric transformations are necessarily fermionic in character. These parameters are in fact related with fermionic generators, that enlarge the Poincaré algebra; the resulting mathematical structures are called graded Poincaré algebras, or Poincaré superalgebras.

The representations of this superalgebra can be build as functions on the superspace, in much the same manner in which the functions on the ordinary 4-dim space are representations of the Poincaré group. In fact, regarding to the Poincaré group as a topological group, the 4-dim coordinates x_μ of the ordinary space can be regarded as the coordinates of the coset space, when the Minkowsky subgroup is factorized. The space spanned by the anticommuting coordinates and by the usual 4 parameter of the translations is called the superspace, and can be thought as the the analogous coset space: superspace = Poincaré supergroup / Minkowsky subgroup.

The functions defined on the superspace, called superfields, carry the SUSY representation obtained transforming the superspace coordinates. After expanding in series of the anticommuting parameters θ and $\bar{\theta}$, these functions are shown to be equivalent to a finite set of ordinary fields of different spin; these sets are called *supermultiplets*.

In fact, only the simplest superalgebra, with a minimum number of fermionic generators, can be relevant for the low energy phenomenology, because only in this case the supermultiplets can have a chiral structure, needed to describe properly the weak interactions. More specifically only two kind of superfields bear importance for low energy phenomenology: the vector and the chiral superfields. The vector superfield contains a

Majorana fermion $\lambda_M(x)$ and a real vector field $V_\mu(x)$, and it is obtained from the most general superfield imposing reality:

$$V(x, \theta, \bar{\theta}) = C(x) + [\theta\xi(x) + \theta^2 M(x) + \bar{\theta}^2 \theta(-i\lambda(x)) + h.c.] + \theta\sigma^\mu\bar{\theta}V_\mu(x) + \theta^2\bar{\theta}^2\frac{1}{2}D(x) \quad (1.1)$$

(the field λ in the previous equation is in fact a Weyl field, but also $\bar{\lambda}$ is present, and a Majorana field can be reconstructed). The chiral superfield contains a Weyl fermion $\psi(x)$ and a charged scalar $z(x)$:

$$\Phi(x, \theta) = z(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) \quad (1.2)$$

To build the generating functional, we have to represent supersymmetry also off-shell, that is without the constraints of the equations of motion. This requires the presence of the so called *auxiliary fields*, that is fields that are needed to represent supersymmetry, but that have no dynamic role. In particular let us note that the mass dimension of the fields $F(x)$ and $D(x)$ is two, assuming $V_\mu(x)$ in eq. (1.1) and $\psi(x)$ in eq. (1.2) to have canonical mass dimensions.

1.2. Supersymmetric lagrangians

1.2.1. Propagation of the gauge fields

The derivation of the gauge field lagrangian in the formalism of the superfields requires the use of the covariant derivatives, needed to build the supersymmetric and gauge invariant term $W^A W_A$. Taking the projection over θ^2 we build the lagrangian containing the term for the (covariant) propagation of the gauge fields and the gauginos propagating in the usual way:

$$\mathcal{L}_g = -\frac{G_\alpha^2}{4} + i\bar{\lambda}_\alpha(\hat{D}\lambda)_\alpha \quad (1.3)$$

$$\mathcal{L}_g^{aux} = \frac{D_\alpha^2}{2} \quad (1.4)$$

It is very important to resort to the Wess-Zumino gauge for a partial elimination of the auxiliary fields. This is a choice of the chiral superfields used as gauge parameters such that the auxiliary fields C, ξ, M of eq. (1.1) disappear; the usual gauge invariance of the vector field is left. The price to pay is that neither supersymmetry or generalized gauge invariance are separately present in the previous lagrangian.

A final observation to clarify the role of the D field in a renormalisable lagrangian. Since its mass dimension is 2, it cannot propagate. Besides that in the quadratic term in eq. (1.3), it may appear linearly, eventually multiplied by scalars with canonical dimension, but also alone, if it is connected to a $U(1)$ gauge group:

$$\mathcal{L}_\xi = \xi D(x) \quad (1.5)$$

1.2.2. Gauge interactions of matter fields

To find the interaction terms between matter and gauge fields we first have to render invariant the kinetic term of the chiral superfields; then we have to expand the supersymmetric expressions in term of the ordinary fields, taking the projection over the $\theta^2\bar{\theta}^2$ component (we will always work in the Wess-Zumino gauge).

The first step is realized building the term $\Phi^\dagger \exp(gV)\Phi$, where the superfield Φ belongs to some representation of the gauge group and V is the vector superfield. The component lagrangian can then be found by a straightforward application of the rules in Appendix A:

$$\mathcal{L}_{gm} = |\mathcal{D}_\mu z_a|^2 + i\bar{\psi}_a(\hat{D}\psi)_a + i\sqrt{2}g(z_a^*\lambda_{ab}\psi_b - \bar{\psi}_a\bar{\lambda}_{ab}z_b) \quad (1.6)$$

$$\mathcal{L}_{gm}^{aux} = F_a^*F_a + gz_a^*D_{ab}z_b \quad (1.7)$$

where the index a spans the gauge multiplet. I supposed to have a simple group (so only one gauge coupling constant is present) and I have included the group generator in the definition of the gauge superfields; that is in the vector field, in the gaugino λ and in the auxiliary gauge field D ($D_{ab} = D^\alpha T_{ab}^\alpha$; similarly for V_μ , λ and also $\bar{\lambda}$).

1.2.3. The SUSY flavour sector

The terms that extend the usual Yukawa sector in a SUSY fashion are built out of monomials in the chiral superfields of degree at most three; the projection over the θ^2 component produces in this way terms of mass dimension at most four. The polynomial of chiral superfields, that is a chiral (composite) superfield by itself, is called the *superpotential*:

$$f(\Phi) = l^a\Phi_a + \frac{1}{2}\mu^{ab}\Phi_a\Phi_b + \frac{1}{6}f^{abc}\Phi_a\Phi_b\Phi_c \quad (1.8)$$

As for any lagrangian, we need just to insure the global invariance of our SUSY flavour interactions to obtain also the local invariance, since no derivatives enter this part of the lagrangian.

The Φ_a are the component of the multiplet Φ , that contains all the chiral superfields in the model under consideration; Φ belongs to some representation (in general reducible) of the gauge group. The indices a, b, c identify the field components within this multiplet, e.g. $a = H_1^\sigma$; the coefficients f^{abc}, μ^{ab} are symmetric functions by definition.

Applying the formulae in Appendix A we derive

$$\begin{aligned} f(\Phi) &= \dots + \theta^2 \left(\sum_a F_a \frac{\partial}{\partial z_a} - \frac{1}{2} \sum_{a,b} \psi_a \psi_b \frac{\partial^2}{\partial z_a \partial z_b} \right) f(z) \\ &= \dots + \theta^2 \left(\sum_a F_a f^a(z) - \frac{1}{2} \sum_{a,b} \psi_a \psi_b f^{ab}(z) \right) \end{aligned} \quad (1.9)$$

where I used the notations

$$f^a \equiv \frac{\partial f}{\partial z_a}, \quad f^{ab} \equiv \frac{\partial^2 f}{\partial z_a \partial z_b}. \quad (1.10)$$

It is always apparent from the context when f is considered as a function of the scalar field z or of the corresponding superfield Φ . To summarize, the resulting terms in the lagrangian describes ordinary Yukawa terms and interactions of scalars:

$$\mathcal{L}_Y = -\frac{1}{2} \psi_a \psi_b f^{ab} + \text{h.c.} \quad (1.11)$$

$$\mathcal{L}_Y^{aur} = F_a f^a + \text{h.c.} \quad (1.12)$$

1.2.4. Elimination of the auxiliary fields

The last step to build the supersymmetric part of the lagrangian consists in the elimination of the D and F fields (eqs. (1.4), (1.7) and (1.12)). Since they appear quadratically in the lagrangian, they can be integrated away from the functional generator. The result is a contribution to the scalar potential of the form:

$$V_{susy} = \frac{1}{2} |d^\alpha|^2 + |f^a|^2 \quad (1.13)$$

where f^a is as in eq. (1.10) and

$$d^\alpha = g z_a^* T_{ab}^\alpha z_b. \quad (1.14)$$

with a possible addition of a ξ term. as in eq. (1.5). Summarizing the results of this section, formulae eq. (1.3), eq. (1.6), eq. (1.11) and eq. (1.13) we have

$$\mathcal{L}_{susy} = \mathcal{L}_g + \mathcal{L}_{gm} + \mathcal{L}_Y - V_{susy} \quad (1.15)$$

1.3. Low energy supergravity models

1.3.1. Soft breaking of supersymmetry

The supersymmetric lagrangian described in eq. (1.15) turns out to be too restrictive if we want to use it to describe the phenomenology of the electroweak-scale energies.

This lead theorists to consider the possibility to break the supersymmetry, without spoiling the cancellation of quadratic divergencies, attracting feature of supersymmetric model discussed in some details in the following. The soft terms have been catalogued by Girardello and Grisaru [1] and are of the form

$$m^2 z z^* \quad m^3 z \quad m^2 z^2 \quad m z^3 \quad \mu \lambda \lambda \quad (1.16)$$

where z is a scalar from a chiral superfield and λ is a gaugino, and I used m and μ to emphasize the fact that the couplings are dimensional. We can also notice that this terms are the lowest dimension component of the monomial of superfield used in the derivation of the supersymmetric lagrangian; the first comes from the kinetic term $\Phi^* \Phi$ of the chiral superfields, the last from the kinetic term $W^2 + \bar{W}^2$ of the vector superfields, the others from the “flavour” part (cfr. eq. (1.8)). When we consider a realistic model building an important requisite is to have soft terms that are gauge invariant; the previous observation connects the gauge invariance of the soft terms with the gauge invariance of the corresponding *supersymmetric* monomials. We will use:

$$\mathcal{L}_{soft} = \mathcal{L}_{\lambda\lambda} - V_{soft} \quad (1.17)$$

$$\mathcal{L}_{\lambda\lambda} = +\frac{1}{2} \mu_\alpha \lambda_\alpha \lambda_\alpha + h.c. \quad (1.18)$$

$$V_{soft} = [\eta(z) + h.c.] + V_{scalar} \quad (1.19)$$

where the two terms of the scalar part V_{soft} are:

$$V_{scalar} = m_{ab}^2 \tilde{z}_a^* \tilde{z}_b \quad (1.20)$$

$$\eta(z) = L^a z_a + \frac{1}{2} M^{ab} z_a z_b + \frac{1}{6} \eta^{abc} z_a z_b z_c \quad (1.21)$$

I defined the dimensionful couplings in η in analogy with the terms in eq. (1.8); notice also the apparently unconventional sign $+$ in the gaugino mass term (this is due to the fact that we use conventionally $-i\lambda$ in the vector superfield, as in eq. (1.1)).

The general form of the supersymmetric lagrangian with soft breaking terms is given by the sum of the lagrangians eq. (1.15) and eq. (1.17).

1.3.2. Low energy supergravity models

Soft terms arise naturally in the context of supergravity theories (for a review and an appropriate list of references see the basic work of Nilles, ref. [2]). The spontaneous breaking of supersymmetry is supposed to take place in the so-called hidden (*i.e.* non-observable) sector at a certain scale M_S , and the effects of the breaking to be communicated to the observable sector only via gravitational interactions. The partner of the graviton, the gravitino, takes mass “eating” the goldstino (super-Higgs effect); its mass is of the order of M_S times the suppression factor M_S/M_{Planck} (eventually squared, depending on the specific mechanism of SSB). In the flat limit $M_{\text{Planck}} \rightarrow \infty$, at fixed gravitino mass, we are left with soft breaking terms in the observable sector [3].

Spontaneously broken $N = 1$ supergravity, in its minimal formulation¹, allows also to reduce the number of couplings that appear in the previous equations; making reference to eq. (1.8) and eq. (1.21) we have:

$$m_{ab}^2 = m_{3/2}^2 \delta_{ab} \quad L^a = m_{3/2} C l^a \quad M^{ab} = m_{3/2} B \mu^{ab} \quad \eta^{abc} = m_{3/2} A f^{abc} \quad (1.22)$$

where $m_{3/2}$ is the gravitino mass and

$$B = A - 1, \quad C = A - 2. \quad (1.23)$$

For the following we will assume that at the grand unified scale $M_{\text{GUT}} \simeq 10^{16}$ GeV local supersymmetry is already broken and we have effectively a global supersymmetry theory with explicit soft breaking characterized by the TeV scale. To exploit the model it is necessary to rescale the parameters in the lagrangian with a renormalisation group analysis down to the low energy region; this situation is exactly the one that we meet studying GUT theories. This clearly entails an analysis of the quantum structure of the theory. This kind of analysis has been performed by different authors[4]; I will briefly discuss in the next section a very beautiful and simple method to obtain the 1-loop renormalisation group equations based on the effective potential.

¹Namely it is assumed that the matter superfields have minimal kinetic terms—a flat Kähler metric.

1.4. 1-loop quantum structure of supersymmetric theories

1.4.1. The effective potential method

For any renormalisable Yang-Mill theory with scalars, Weyl fermions and gauge vectors (not necessarily supersymmetric) we find, regularizing the theory with a momentum cutoff Λ , the following “structure of infinities” in the scalar sector at 1-loop[5] (using the Landau gauge)

$$V_\infty = \frac{1}{32\pi^2} \text{Str} \left[\Lambda^4 \left(\ln \Lambda - \frac{1}{4} \right) + \Lambda^2 \mathcal{M}^2(z) - \ln \Lambda \mathcal{M}^4(z) \right] \quad (1.24)$$

the mass matrices $\mathcal{M}^2(z)$ are the generalized mass matrices for each particle in the theory.

This formula allows us to extract a lot of informations about the dependence of the bare parameters of the scalar sector on the renormalised parameters, by performing elementary (but for the general case quite long) algebraic computations of supertraces of mass matrices, without computing Feynman diagrams; in fact one lacks only the informations about the wavefunction renormalisation. For the purpose of obtaining 1-loop the RGEs the best way to use this formula is the following. Consistently with the order of approximation used we write $V_\infty(z_R, \dots g_R) = V_\infty(z_B, \dots g_B)$, in terms of the bare and of the renormalised quantities respectively. Then we solve for the renormalised parameters the following equation

$$V(z_R^a, l_R^a, \dots g_R^\alpha) = V(z_B^a, l_B^a, \dots g_B^\alpha) + V_\infty(z_B^a, l_B^a, \dots g_B^\alpha) \quad (1.25)$$

This method is particularly suited for the supersymmetric lagrangians, ([6], [7]) since many parameters of the fermionic sector appear also as parameters of the scalar sector (for instance the Yukawa couplings and also the gauge couplings appear in 4-scalar interactions). In fact, we can obtain all the SUSY RGE with this method (except for the gaugino masses) just feeding some information about the field anomalous dimensions.

1.4.2. Absence of quadratic divergencies and mass matrices

When applying eq. (1.24) to SUSY theories, the fact that we have the same number of fermionic and of bosonic degrees of freedom implies that we have no quartic constant divergencies; that is the 0-point Green function, to be interpreted as a quantum-induced cosmological constant, is at most quadratically divergent.

From eq. (1.24) we can appreciate that the absence of quadratic divergencies is equivalent to the sum-rule $\text{Str}\mathcal{M}^2 = 0$. To show that this is indeed the case, for each supersymmetric theories, we just need to work out the generalized mass matrices.

Let us set some useful notation: we will distinguish a field from its conjugate with the position of the representation indices:

$$z^a \equiv (z_a)^*, \quad \Psi^a \equiv (\Psi_a)^*, \quad \lambda^a \equiv (\lambda_a)^*, \quad f_a(z) \equiv f^{a*}(z^*) \quad \dots \quad (1.26)$$

in the last example I recalled the notation of eq. (1.10) for the partial derivative of the superpotential $f(z)$, that I will use widely in the following.

Now we separate out the terms in the lagrangian that give rise to the mass matrices for the different fields, as follows

$$\mathcal{L}^{\text{"mass"}} = \frac{1}{2} \left\{ V_\mu^t \cdot M_1^2 \cdot V^\mu - \left[\Psi^t \cdot M_{1/2} \cdot \Psi + h.c. \right] - Z^\dagger \cdot M_0^2 \cdot Z \right\} \quad (1.27)$$

We use the quotation marks for “mass” to stress that we are treating generalized masses. Before proceeding it is convenient to list the relevant functions of the scalar fields we have encountered (equations (1.14),(1.8), (1.21)) using the convention eq. (1.26):

$$\begin{aligned} d^\alpha(z^a, z_a) &= g^\alpha z^a T_a^\alpha{}^b z_b \\ f(z_a) &= l^a z_a + \frac{1}{2} \mu^{ab} z_a z_b + \frac{1}{6} f^{abc} z_a z_b z_c \\ \eta(z_a) &= L^a z_a + \frac{1}{2} M^{ab} z_a z_b + \frac{1}{6} \eta^{abc} z_a z_b z_c \end{aligned}$$

The intermediate vector bosons take their mass after spontaneous symmetry breaking (SSB) from the first term in eq. (1.6); one verifies that the mass term can be written

$$M_1^2 = d^{\alpha a} d_a^\beta + d_a^\alpha d^{\beta a} \quad (1.28)$$

Collecting the bilinears in the fermions in the eqs.(1.6). (1.11), (1.18) we are lead to the following fermionic mass matrix:

$$M_{1/2} = \begin{pmatrix} f^{ab} & \sqrt{2} d^{\beta a} \\ \sqrt{2} d^{\alpha b} & \mu^\alpha \delta^{\alpha\beta} \end{pmatrix}, \quad \Psi = \begin{pmatrix} \psi_a \\ -i \lambda_\alpha \end{pmatrix} \quad (1.29)$$

Finally, using eqs. (1.13) and (1.19) we can write the scalar mass matrix as:

$$M_0^2 = \begin{pmatrix} f^{ac} f_{cb} + m_b^{2a} + d^{\alpha a} d_b^\alpha + d_b^{\alpha a} d^\alpha & f^{abc} f_c + \eta^{ab} + d^{\alpha a} d^{\alpha b} \\ f_{abc} f^c + \eta_{ab} + d_a^\alpha d_b^\alpha & f_{ac} f^{cb} + m_a^{2b} + d_a^\alpha d^{\alpha b} + d_a^{\alpha b} d^\alpha \end{pmatrix} \quad (1.30)$$

where the field $Z = (z^a, z_a)$ contains all the scalar fields in the theory.

Notice that the presence of a ξ term, eq. (1.5), leads to an additional constant contribute to the *scalar* mass matrix of the form:

$$\delta M_0^2 = \xi^{\alpha'} \begin{pmatrix} d_b^{\alpha'a} & 0 \\ 0 & d_a^{\alpha'b} \end{pmatrix} \quad (1.31)$$

where the index α' runs over the U(1) generators $Y^{\alpha'}$.

In terms of the generalized mass matrices, previously introduced, the supertrace reads:

$$\text{Str}(\mathcal{M}^2) = \text{tr}(M_0^2) - 2 \text{tr}(M_{1/2}^+ M_{1/2}) + 3 \text{tr}(M_1^2) \quad (1.32)$$

and can be easily computed to find

$$\text{Str}(\mathcal{M}^2) = 2 \left[\xi^{\alpha'} g^{\alpha'} \text{tr}(Y^{\alpha'}) + \text{tr}(m^2) - \sum_{\alpha} |\mu_{\alpha}|^2 \right] \quad (1.33)$$

We can now prove an important sum rule, first obtained by the authors of [8]; in the case in which the soft breaking terms m_b^a and μ_{α} and η are zero, and the trace of the U(1) generators is zero (it is the case of the standard model hypercharge, and it is necessary if we want to embed the group in a simple GUT) also the supertrace of the square of the mass matrix is zero. In the more general situation in which those contributions are present [9], the supertrace gives a constant term, meaning that only the cosmological constant, but no field depending term, is subject to quadratic divergencies [10]. This result is true in any supersymmetric theory.

Let us end this section with a phenomenological comment on this important sum rule. It is easy to convince oneself that a realistic spectrum cannot be reproduced, if one would suppose the ordinary particles and their SUSY partners to be the spectrum of the theory. On the other side it is clear that, in a softly broken supersymmetric theory, the phenomenological requests on the SUSY spectrum can be taken into account easily. Let us stress in any case the fact that this sum rule is valid only at the tree level. The alternative attempts to use the ξ terms to evade the phenomenological bound lead to a new U(1) with non zero trace; and one finds that the condition of absence of anomalies requires an enormous number of particles [11]; that is why this kind of models are not pursued.

1.4.3. Renormalisation group equations

The problem of deriving the RGE is reduced to the computation of $\mathcal{M}^4(z)$. In fact the task can be usefully divided in the computation of the RGE for the supersymmetric parameters (gauge, Yukawa, massive supersymmetric couplings), and that for soft breaking terms; further simplifications result from focussing time to time on the effect of a set of parameters (for instance the effect of the gauge+Yukawa couplings on the Yukawa couplings itself).

The reader interested is referred to Appendix D in which some details of the derivation can be found. The method is applied to derive the RGEs in the minimal supersymmetric extension of the Standard Model, with a particular attention to the matricial structure. The study of the renormalisation group equations is the subject of the third Chapter of this thesis.

2. From SM to MSSM

In this Chapter we focus on the minimal supersymmetric extension of the Standard Model (MSSM). The particles and the parameters present in the model are here discussed; mass matrices and the interactions are derived. The study of the parameter space of SUSY GUT model, requiring the use of renormalisation group techniques, is performed in the next Chapter; but let us remark that yet at this stage it is possible to start important phenomenological analysis of supersymmetric models.

2.1. Supersymmetric extension of the Standard Model

2.1.1. Notations

We will, after Haber and Kane, use a tilde to distinguish the non-standard fields in the supermultiplets: all the scalar quarks and leptons (called squarks and sleptons), and the fermionic partners of Higgs and gauge vector bosons fields (called higgsinos and gauginos) will be tilded in the Minimal Supersymmetric extension of the Standard Model (MSSM).

Let us set some notations useful for supersymmetric model building. As we have clarified in the discussion following eq. (1.8) the superpotential must be built with chiral superfields only; this kind of fields contains Weyl fermions of the same chirality of the θ parameter, by convention a left bispinor. This means that a right-handed fermion field enters the superpotential only after being conjugated. Let us exemplify the formalism considering the case of the up-quarks. The left component belongs to the superfields

$$U_L(x, \theta) = \hat{u}_L(x) + \sqrt{2} \theta u_L(x) + \theta^2 F_{u_L}(x) \quad (2.1)$$

The right component of the up-quark is instead a component of an antichiral superfield $U_R(x, \bar{\theta})$:

$$U_R(x, \bar{\theta}) = \hat{u}_R(x) + \sqrt{2} \bar{\theta} u_R(x) + \bar{\theta}^2 F_{u_R}(x) \quad (2.2)$$

We define another superfield $U_L^c(x, \theta)$ whose fermionic component is the left bispinor $u_L^c(x)$:

$$U_L^c(x, \theta) = \hat{u}_L^c(x) + \sqrt{2} \theta u_L^c(x) + \theta^2 F_{u_L^c}(x) \quad (2.3)$$

and then relate its conjugate to $U_R(x, \bar{\theta})$ (in Appendix A the conjugation in the θ space

is defined and discussed)

$$U_L^c(x, \theta) = \overline{U_R(x, \bar{\theta})} \quad (2.4)$$

This relation in components reads:

$$\begin{cases} \overline{u_R} &= u_L^c \\ \overline{\tilde{u}_R} &= \tilde{u}_L^c \\ \overline{F_{u_R}} &= F_{u_L^c} \end{cases} \quad (2.5)$$

Finally, we note that with this notations one can write

$$u_L^c = (u^c)_L \quad (2.6)$$

where in the right-hand-side we have the left-component of the charge-conjugate up-quark four component spinor, written in the chiral representation of the gamma matrices, that is regarding the left bispinor as a four-spinor with zero right-handed part.

2.1.2. *MSSM particle content*

Applications of supersymmetry, in the context of extending low energy gauge theories, respect the following principle: the gauge transformations must commute with the generators of supersymmetric transformations. This observation implies that the gauge assignments are given on supermultiplets; that is every ordinary particle must have a superpartner with the same gauge assignment. In fact let us suppose the contrary; considering the commutator of a local gauge transformation and of a global fermionic transformation, we would recognize the need to have in the theory a local fermionic transformations. That would imply a local translation, i.e. we would be lead to consider gravity, since the anticommutator of two fermionic generators in supersymmetry yields a translation. This is the reason why we believe that such gauge assignments cannot be of relevance whenever we consider scales of energies much lower to the Planck scale.

In the MSSM two Higgs superfields are present. An even number of Higgs superfield doublets with opposite hypercharge is in fact needed, since each higgsino contributes to the anomaly. Moreover, we will see in the following that we need at least two Higgs superfields to build a realistic *supersymmetric* lagrangian.

Let us finally list the particles of the MSSM, writing also their quantum numbers:

$$\begin{array}{lll}
q_L & \tilde{q}_L & (3, 2, +1/6) \\
u_L^c & \tilde{u}_L^c & (3^*, 1, -2/3) \\
d_L^c & \tilde{d}_L^c & (3^*, 1, +1/3) \\
l_L & \tilde{l}_L & (1, 2, -1/2) \\
e_L^c & \tilde{e}_L^c & (1, 1, +1) \\
h_1 & \tilde{h}_1 & (1, 2, -1/2) \\
h_2 & \tilde{h}_2 & (1, 2, +1/2) \\
G_\mu & \tilde{G} & (8, 1, 0) \\
A_\mu & \tilde{A} & (1, 3, 0) \\
B_\mu & \tilde{B} & (1, 1, 0)
\end{array} \tag{2.7}$$

These fields are *interaction eigenstates*, that is the ones that we use to write the gauge invariant lagrangian. The *mass eigenstates* are mixed states of these fields, as we will discuss in some detail in the following. Exactly as in the SM quarks and leptons are replied in three families.

The Higgs doublet h_1 has the same gauge numbers as the leptonic doublet $l_L = (\nu_L, e_L)$. The attempts to use the scalar sleptons as an Higgs doublet were not successful, firstly because the electroweak breaking would imply a violation of the lepton number as well.

It is also worthwhile to notice that the L,R suffixes of the scalar fields are not only conventional, but also useful; for instance the gaugino interactions can transform u_L in \hat{u}_L but not in \tilde{u}_R , in the same manner in which the gauge vector bosons interactions cannot flip the helicity of u_L .

The possible presence of three right handed neutrinos in the spectrum can be considered, in much the same way that in non-supersymmetric extensions of the SM:

$$\nu_L^c \quad \tilde{\nu}_L^c \quad (1, 1, 0) \tag{2.8}$$

2.2. The lagrangian of the MSSM

2.2.1. MSSM superpotential and R symmetry

Let us first discuss the other argument requiring the introduction of a second Higgs field in the MSSM. In the SM the mass terms of the fermions is generated coupling them

to the Higgs doublet. We have to use the Higgs doublet together with its conjugate in the lagrangian, since the up and down type of quarks require doublets with opposite hypercharge. This cannot be done in the MSSM, because the conjugation transforms a chiral superfield in a antichiral superfield (a different representation of the supersymmetry group), that would spoil the supersymmetric character of the superpotential: two different Higgs doublets must be introduced.

Knowing the representations in the MSSM, it is a simple group theoretical task to build all the possible low energy SUSY invariants. Since all chiral superfields can be a priori combined in gauge invariant monomials, and we have also an additional Higgs doublet, we understand a priori that we can obtain more invariants than in the SM.

Using the compact superfield notation the welcome invariants are (an antisymmetric $SL(2)$ -covariant matrix where needed is intended):

$$H_1 L E^c, H_1 Q D^c, H_2 Q U^c, H_1 H_2 \quad (2.9)$$

The first three terms are the supersymmetric extensions of the SM Yukawa terms, while the third is “new” and controls the global invariance in which the Higgs fields pick a common phase (that could generate a Goldstone boson after spontaneous symmetry breaking). On the basis of gauge invariance alone other terms are possible

$$L^2 E^c, H_1^2 E^c, L Q D^c, L H_2, \epsilon_{\alpha\beta\gamma} U^{c\alpha} D^{c\beta} D^{c\gamma} \quad (2.10)$$

All but the last term arise from the terms in eq. (2.9) because L and H_1 have the same quantum numbers. The conclusion is that in the MSSM, at difference that in the SM, lepton and/or baryon number conservation are not automatic.

Still we can conveniently study the behaviour of supersymmetric theories not only under global $U(1)$ rotations, but also under the so called R rotations. These are global transformations compatible with the supersymmetric ones; they act on a superfield as

$$\Phi'(x, \theta, \bar{\theta}) = \exp(-iR_\Phi \alpha) \cdot \Phi(x, \exp(i\alpha)\theta, \exp(-i\alpha)\bar{\theta}) \quad (2.11)$$

The integer R_Φ is the R charge of the field Φ ; notice that the component fields rotate in different manners. The reality constraint for the vector superfield implies $R_V = 0$. A monomial of the superpotential transforming with chiral charge 2 give rise to an R symmetric term in the lagrangian. Asking that the Higgs superfields have zero chiral charge (no goldstone bosons), the terms of eq. (2.9) are allowed iff $R_L + R_{E^c} = 2$, $R_Q + R_{U^c} = 2$, $R_{U^c} = R_{D^c}$; the first four terms of eq. (2.10) are excluded iff $R_L \neq R_H = 0$,

and the last iff $R_{U^c} \neq 2/3$. The most direct and symmetric solution of the previous constraints is to give charge 1 to each quark and lepton superfield. This is often referred as R symmetry (and it is a property of the superpotential that we will assume in the following).

In fact the most economical way to exclude this terms is just to impose a *matter parity*, under which only the matter superfields transform (not the higgs); in all unwanted terms the number of matter fields is odd, while is even in the others. Notice that this implies that the the lightest supersymmetric particle (LSP) is stable (or better can disappear only annihilating with its antiparticle), and also that SUSY particles must be produced in pairs in accelerators.

The superpotential for the SUSY SM reads:

$$f_{SM} = \epsilon^{\sigma\tau} [\Gamma_E^{ij} H_{1\sigma} L_{\tau i} E_j^c + \Gamma_D^{ij} H_{1\sigma} Q_{\tau i} D_j^c - \Gamma_U^{ij} H_{2\sigma} Q_{\tau i} U_j^c + \mu H_{1\sigma} H_{2\tau}] \quad (2.12)$$

where I have written the $SU(2)_L$ indices, and left the $SU(3)_c$ ones implicit.

With the notations of eq. (1.8) one writes:

$$\begin{aligned} f^{H_{1\sigma} L_{\tau i} E_j^c} &= \Gamma_E^{ij} \epsilon^{\sigma\tau} \\ f^{H_{1\sigma} Q_{\tau i}^{\alpha} D_j^{c\beta}} &= \Gamma_D^{ij} \epsilon^{\sigma\tau} \delta_{\alpha\beta} \\ f^{H_{2\sigma} Q_{\tau i}^{\alpha} U_j^{c\beta}} &= -\Gamma_U^{ij} \epsilon^{\sigma\tau} \delta_{\alpha\beta} \\ \mu^{H_{1\sigma}, H_{2\tau}} &= \mu \epsilon^{\sigma\tau} \end{aligned} \quad (2.13)$$

This form is particularly useful when deriving the RGE for the MSSM from the general expressions.

A final comment about the right-handed neutrinos. If gauge singlets are present we in general have to add two new terms in the superpotential, obtained by multiplying the singlets by the two bilinears $H_1 H_2$ and $L H_2$, together with the three monomial in the singlets. Matter parity, obviously extended, leaves only

$$f_{\nu_R} = -\epsilon^{\sigma\tau} \Gamma_N^{ij} H_{2\sigma} L_{\tau i} N_j^c + \frac{1}{2} M_R^{ij} N_i^c N_j^c \quad (2.14)$$

the last term gives rise to a Majorana mass for the right-handed neutrinos.

2.2.2. Masses and mixing angles of observed fermions

1. The angle β

The parameters Γ_X in the previous equations have to be identified with the conjugate of the usual Yukawa couplings, that is

$$\Gamma_X = Y_X^* \quad X = E, U, D \quad (2.15)$$

(we neglect for the moment the right-handed neutrinos) where $-Y_E^{ij} h \bar{L}_i e_{Rj}$ defines the leptonic Yukawa coupling, as it appears usually in the SM lagrangian. To prove this statement we extract the terms in the MSSM lagrangian in which an Higgs field interacts with two matter fermions; for instance from the first term in eq. (2.12), using the notation eq. (2.5) and the rules in Appendix A we find

$$\mathcal{L} \ni -\Gamma_E^{ij} h_1 l_{Li} \bar{e}_{Rj} + \text{h.c.} \quad (2.16)$$

The main difference between the SM and the MSSM Yukawa couplings is obviously due to the fact that in the MSSM we have two Higgs doublets, that a priori have two different vacuum expectation values:

$$\langle h_1^0 \rangle = v_1 \quad (2.17)$$

$$\langle h_2^0 \rangle = v_2 \quad (2.18)$$

The constraint due to measurement of the Z -mass,

$$M_Z^2 = \frac{g_1^2 + g_2^2}{2} v^2 \quad (2.19)$$

where $v^2 = v_1^2 + v_2^2$, leaves in fact undetermined the angle

$$\tan \beta = \frac{v_2}{v_1} \quad (2.20)$$

When asking the perturbativity of the Yukawa couplings up to GUT scale, as it is proved in the following, the angle β is restricted in the range $[\pi/4, \pi/2]$, that is: $\tan \beta \in [1, \infty]$.

2. Choice of basis in flavour space and the CKM matrix

Let us now discuss in details the structure of the Yukawa matrices in the MSSM. In close similarity with the SM arguments we perform redefinitions on the flavour multiplets, that is we choose a base in flavour space. By writing the matrices Γ_X in biunitary form,

$$\Gamma_X = L_X^t \gamma_X R_X \quad (2.21)$$

where γ_X are *diagonal* non-negative matrices; and making unitary redefinitions of the quark and lepton superfields, one finds that the parameters in eq. (2.12) can be chosen

to be:

$$\begin{cases} \Gamma_E = \gamma_E \\ \Gamma_U = K^t \gamma_U \\ \Gamma_D = \gamma_D \end{cases} \quad (2.22)$$

(*down-diagonal basis*) where

$$K = L_U L_D^\dagger \quad (2.23)$$

is the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) matrix [12].

The matrices γ_X are related to the (diagonal) mass matrices for leptons and quarks as following

$$\begin{cases} \gamma_E = M_E/(v \cos\beta) \\ \gamma_D = M_D/(v \cos\beta) \\ \gamma_U = M_U/(v \sin\beta) \end{cases} \quad (2.24)$$

This basis is very useful when performing the RGE analysis since the initial conditions are given in terms of masses and CKM mixings and it leaves the $SU(2)_L$ symmetry explicit, having applied the same unitary rotation to the U and D superfields.

To switch to the superfield basis in which the terms of the superpotential with the neutral Higgs superfields are flavour-diagonal (*i.e. the quark mass eigenstate basis*), we just have to redefine the U_L flavour multiplet of superfields as

$$U_L \rightarrow K^\dagger U_L . \quad (2.25)$$

In this basis the mixing matrix K appears

- i) in the interactions involving the charged vector superfields
- ii) in the interactions of the charged Higgs superfields in eq. (2.12),
- iii) and in the analogous terms for the charged Higgs fields in soft-terms interactions.

3. Final comments and remarks

Since the fermion masses are at present experimentally known, at least to a certain extent, the main indeterminacy on the Yukawa couplings comes from the angle β , that affects the overall size. More specifically, the behaviour with $\tan\beta$ is the following: if $\tan\beta \sim 1$, all the MSSM Yukawa coupling constants are $\sqrt{2}$ times larger than the corresponding ones in the SM, while if $\tan\beta$ is large the up Yukawa couplings in the MSSM are approximatively equal to those in the SM, while the other couplings are approximatively

$\tan \beta$ times larger. For these reasons a direct determination of these parameters would be of the maximum relevance.

Let us remark that the Cabibbo-Kobayashi-Maskawa matrix with three families in the MSSM, exactly as in the SM, contains a physical phase that may describe CP violating phenomena. In effect in the MSSM we could have others sources of CP violation, that we discuss in the next section.

Finally a comment on the choice of basis in family space if right-handed neutrinos are present, as in eq. (2.14). Starting from the previous choices of basis, in which Γ_E is diagonal, it is useful to further diagonalize the symmetric complex matrix $M_R = R^t m_R R$ redefining the neutrino superfields. It is remarkable that the number of physical parameters of the matrix Γ_N is not 18 (completely arbitrary matrix) but 15, since the L superfields can still absorb three unphysical phases (the E^c superfields must transform with the conjugate phases). In this manner we see that the leptonic part of the superpotential contains nine masses, six mixing angles and six physical phases².

2.2.3. Soft breaking terms and CP invariance

We focus now on the parameters of the supergravity-derived SUSY extension of the SM, concentrating in particular the considerations on their GUT-scale values. According to the discussion of formula (1.22), at the scale of supersymmetry breaking, that we assume to be the SUSY-GUT scale required by gauge couplings unification, all the scalars get an universal mass term m_G^2 (that in fact is the gravitino mass in the case of flat Kähler metric). Let μ_G be the value of the μ parameter at this scale. To each Yukawa interaction $\Gamma \Phi^3$ term in the superpotential correspond an interaction in the scalar potential $A_G \Gamma z^3$; similarly a term $B_G \mu_G h_1 h_2$ is induced. We will assume also that the three gaugino masses are equal at this energy scale: $M_\alpha = M_G$. Notice that, in accord with the discussion following eq. (1.16), the soft breaking parameters all break R symmetry. So for this lagrangian it is better to speak of R -rotation instead than of R -symmetry.

An important observation about the parameter B_G in the context of SUSY Grand Unification has to be done. The relation $B_G = A_G - 1$ (implied by the assumption of flat Kähler metric) has been widely used to reduce the number of free-parameters in studying

²A different choice would be: make M_R diagonal (redefining N_R), then render Γ_N hermitian (redefining L) and finally make also Γ_E hermitian (redefining E^c); the physical parameters are clearly only redistributed.

the phenomenology of the low-energy supergravity model. In fact this relation holds for the observable sector before the path integration of the heavy (GUT scale) degrees of freedom; see [13]. Giudice and Roulet in ref. [14] have recently reconsidered the issue, and concluded that the path integration in the interesting class of SUSY GUT models with $\mu'_G = 0$ gives an effective theory in which the original relation $B'_G = A'_G - 1$ translates to $|B_G| = 2$ (the prime indicates the parameters in the complete theory), and in all generality

$$|B_G| \geq 2 \tag{2.26}$$

A calculable model dependent μ_G term is then generated as a function of the original parameters, but this value depends on the detailed structure of the SUSY GUT theory. For these reasons the following analysis considers the case $|B_G| = 2$ leaving μ_G arbitrary, and compare it with the case $B_G = A_G - 1$.

The 4 parameters A_G , B_G , M_G , μ_G can be a-priori complex parameters. Using an appropriate R -rotation the gaugino mass parameter M_G can be made real; moreover, multiplying by a common phase the 2 Higgs superfields (with opposite hypercharge) $B_G \cdot \mu_G$ becomes real as well (notice that the argument invoked in the choice of flavour basis goes through after those redefinitions). We conclude that, in addition to the usual CKM phase, there are at most two typically supersymmetric phases that are physically relevant, say

$$\arg(A_G) \quad \text{and} \quad \arg(B_G) = -\arg(\mu_G). \tag{2.27}$$

These two parameters may have an important impact in the phenomenology of CP violation.

Let us finally remark that different specific and more restrictive choices of those supersymmetric parameters have been studied in the literature, like those models in which the gaugino masses M_α are very small (models with light gluinos), or models in which the parameters m^2 , M_α and A are related at the GUT scale (this noticeably happens in string-inspired scenarios).

2.3. Mass matrices and vertices

In the last part of this Chapter we set the basis for the study of physical processes, deriving and discussing mass matrices and interaction terms. Most of the results presented here are in effect valid in the generic supersymmetric model, sometimes called minimal supersymmetric extension of the SM; instead, the procedure to implement non trivial

constraints due to grand unification and supergravity, and the results, both numerical and analytics, will be discussed in details in the next two Chapters³. Let me remark that the selection of the material discussed here is done having in mind the physical applications of last Chapter.

2.3.1. Scalar quarks mass matrix

1. General formalism

Let us treat in some details the up squarks mass matrix. Defining the 6 dimensional vector

$$\tilde{u} = \begin{pmatrix} \tilde{u}_L \\ \tilde{u}_R \end{pmatrix} \quad (2.28)$$

we write the term in the lagrangian using the 6×6 scalar quark mass matrix $M_{\tilde{u}}^2$: $\mathcal{L} \ni -\tilde{u}^\dagger M_{\tilde{u}}^2 \tilde{u}$. It is useful to define the of 3×3 submatrices as follows:

$$M_{\tilde{u}}^2 = \begin{pmatrix} M_{\tilde{u},LL}^2 & M_{\tilde{u},LR}^2 \\ M_{\tilde{u},RL}^2 & M_{\tilde{u},RR}^2 \end{pmatrix} \quad (2.29)$$

The basis of squark mass eigenstates is reached using the unitary rotation $S_{\tilde{u}}$, defined by:

$$S_{\tilde{u}} M_{\tilde{u}}^2 S_{\tilde{u}}^\dagger = \text{diag}(m_{\tilde{u}_k}^2) \quad (2.30)$$

We can finally split $S_{\tilde{u}}$ into two 6×3 submatrices

$$S_{\tilde{u}} \equiv (S_{\tilde{u},L}, S_{\tilde{u},R}) \quad (2.31)$$

which relate the the scalar partners of the left and right-handed quarks to the scalar mass eigenstates.

Let us write for future use the form of the generalized mass matrices, keeping explicit the dependence of the mass matrices from the the neutral Higgs fields. The $L - L$ and $R - R$ blocks are determined by summing the (renormalised) direct contribution to the scalar mass⁴ $m_{Q_L}^2$ and $m_{U_R}^2$, with the tree-level and the D term contributions;

$$M_{\tilde{u},LL}^2 = m_{Q_L}^2 + M_U M_U^\dagger + \left(\frac{1}{2} g_2^2 - \frac{1}{6} g_1^2 \right) (|h_1^0|^2 - |h_2^0|^2) I \quad (2.32)$$

$$M_{\tilde{u},RR}^2 = m_{U_R}^2 + M_U^\dagger M_U + \frac{1}{3} g_1^2 (|h_1^0|^2 - |h_2^0|^2) I \quad (2.33)$$

³In particular the Higgs sector, being at the hearth of the low-energy supergravity model, is discussed at the end of next Chapter, together with the radiative symmetry breaking.

⁴The matrix $m_{U_R}^2$ is defined as the transpose of $m_{U_L}^2$, according to $(\tilde{u}_L^c)^* m_{U_L}^2 \tilde{u}_L^c = (\tilde{u}_R)^* m_{U_R}^2 \tilde{u}_R$.

The L - R block is instead

$$M_{\tilde{u},LR}^2 = (A_U h_2^0 + \mu h_1^0) Y_U \quad (2.34)$$

After spontaneous symmetry breaking the Higgs fields give rise to ordinary mass terms.

Let us comment on the L - R block of the matrix. This is important in particular since it is involved in the supersymmetric contributions to the phenomena in which the chirality is flipped, in particular to electromagnetic dipole amplitudes of quarks. Since at the GUT scale the A_U matrix is proportional to identity we easily realize that the larger element in the L - R sector is by far that connected to the top Yukawa coupling, that is we could have sizeable mixing effects for the stop squarks only. The accuracy of this approximation will be discussed in the following for the model under consideration, but let us remark here that, since only small out of diagonal contributions arise as a result of the renormalisation group evolution, only a departure from the universality assumption $Y_U^A = A_G Y_U$ at the GUT scale could imply large contributions. So we turn to spell out the consequences of this fact.

2. "Stop block" approximation of the mass matrix

It is useful to work out the approximation in which the three blocks of the matrix (2.29) are assumed diagonal, and in particular the block L - R has only the 3-3 entry non-zero, that is assumed to be real.

The only nontrivial block of the 6×6 matrix is the 2×2 stop part,

$$\begin{pmatrix} m_{iLL}^2 & m_{iLR}^2 \\ m_{iLR}^2 & m_{iRR}^2 \end{pmatrix} \quad (2.35)$$

The mixing matrices $S_{\tilde{u}L,R}$ are simply:

$$S_{\tilde{u}L} = \begin{pmatrix} \text{diag}(1, 1, \cos\theta) \\ \text{diag}(0, 0, -\sin\theta) \end{pmatrix} \quad ; \quad S_{\tilde{u}R} = \begin{pmatrix} \text{diag}(0, 0, \sin\theta) \\ \text{diag}(1, 1, \cos\theta) \end{pmatrix} \quad (2.36)$$

The angle θ obeys:

$$\tan 2\theta = \frac{2m_{iLR}^2}{m_{iLL}^2 - m_{iRR}^2} \quad (2.37)$$

$$\text{sign}(\sin 2\theta) = -\text{sign}(m_{iLR}^2) \quad (2.38)$$

The first condition guarantees the diagonalization of the mass matrix, that is eq. (2.35) becomes $\text{diag}(m_{i_1}^2, m_{i_2}^2)$; the second that $m_{i_1}^2$ is the lighter stop. The determination of θ

that I use is to have $\cos\theta$ positive, that is $\sin\theta$ with the sign of $\sin 2\theta$. The two stop masses:

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2} \left[(m_{\tilde{t}_{LL}}^2 + m_{\tilde{t}_{RR}}^2) \mp \sqrt{(m_{\tilde{t}_{LL}}^2 + m_{\tilde{t}_{RR}}^2)^2 - 4m_{\tilde{t}_{LR}}^4} \right] \quad (2.39)$$

Anticipating the results of next Chapter, $m_{\tilde{t}_{LL}}^2 - m_{\tilde{t}_{RR}}^2$ is usually smaller than $m_{\tilde{t}_{LR}}^2$ also after renormalisation group analysis (at GUT scale is zero). For this reason $\tan 2\theta$ is typically large number, so that $\theta \sim \pi/4$ if $m_{\tilde{t}_{LR}}^2 < 0$, or $\sim 3\pi/4$ in the opposite case. For future uses to write this result as:

$$\cos^2 \theta \sim \sin^2 \theta \sim \frac{1}{2} \quad (2.40)$$

$$\sin \theta \cos \theta \sim -\text{sign}(m_{\tilde{t}_{LR}}^2) \frac{1}{2} \quad (2.41)$$

Instead $2 m_{\tilde{t}_{LR}}^2$ compares with $m_{\tilde{t}_{LL}}^2 + m_{\tilde{t}_{RR}}^2$ in the stop mass formula; this latter must be larger than the mixing term. From eq. (2.34) we deduce in particular that $m_{\tilde{t}_{LR}}^2$ is larger if A_t and μ_R have the same sign; lower \tilde{t}_1 masses are then implied.

This observation is of some importance since the sign of μ is a renormalisation group invariant, (and at present a theoretically undetermined quantity) while the sign of A_t for large top Yukawa coupling is fixed to be that opposite to the gaugino mass:

$$m_{\tilde{t}_1}^2 \text{ lower (larger) if } \text{sign}(\mu_R) = -\text{sign}(M_G) \text{ (+sign}(M_G)) \quad (2.42)$$

It is worthwhile to specify these results in the large $\tan\beta$ scenario. In fact in this case the L - R mixing coincides with $m_t A_t$, since $v_1 \rightarrow 0$ as results from eq. (2.34); we can recast eq. (2.41) as

$$\sin \theta \cos \theta \sim \text{sign}(M_G) \frac{1}{2} \quad (\text{large } \tan\beta) \quad (2.43)$$

For the same reason we remark a substantial independence of the stop mass from the sign of μ .

2.3.2. Chargino mass matrix

1. General formalism

The charginos are the mass eigenstates of the charged higgsinos and winos mass matrix M_χ . The mixing is due to the fact that the wino-higgs-higgsino gauge interaction turns out in a mass term after spontaneous symmetry breaking. Then we have the direct

mass contributions due to the soft breaking gaugino masses and to the supersymmetric μ term. With our conventions of signs the matrix reads

$$-(\widetilde{W}^- \widetilde{h}_1^-) \begin{pmatrix} M_2 & gv_2 \\ gv_1 & -\mu_R \end{pmatrix} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{h}_2^+ \end{pmatrix} + h.c. \quad (2.44)$$

where following ref. [15] we define $\widetilde{W}^\pm \equiv -i(\widetilde{A}_1 \mp i\widetilde{A}_2)/\sqrt{2}$, and all the parameters are taken at the weak scale ($M_G \rightarrow M_2, \mu_G \rightarrow \mu_R$).

The matrix can be diagonalized using the biunitary transformation:

$$\begin{aligned} \widetilde{\chi}_L^+ &= V \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{h}_2^+ \end{pmatrix} \\ \widetilde{\chi}_L^- &= U \begin{pmatrix} \widetilde{W}^- \\ \widetilde{h}_1^- \end{pmatrix} \end{aligned} \quad (2.45)$$

The two mass eigenstates are the four dimensional spinors

$$\widetilde{\chi}_{1,2}^- = \begin{pmatrix} \widetilde{\chi}_{L1,2}^- \\ \widetilde{\chi}_{R1,2}^- \end{pmatrix} \quad (2.46)$$

where I defined $\widetilde{\chi}_R^- = \overline{\widetilde{\chi}_L^+}$.

Let us notice in particular that in the special case in which $\tan\beta = 1$ and $M_2 = \mu_R = 0$ the two charginos masses are determined by the wino-higgsino mixing term, and in fact coincide with M_W .

2. “Pure higgsino-wino” approximation of the mass matrix

The opposite situation, that considers the possibility that some of the supersymmetric masses be at the TeV scale, leads to the approximation of the chargino mass matrix in which the mixing terms are neglected:

$$M_\chi \approx \text{diag}(M_2, -\mu_R) \quad (2.47)$$

This approximation holds when $|M_2^2 - \mu_R^2| \gg m_W^2$ and $M_2^2, \mu_R^2 \gtrsim m_W^2$. Notice that these requirements are consistent with one of the eigenvalues, say $|\mu_R|$, being of the order of m_W , while the other remains much heavier. The approximate mass eigenvalues are simply given by the absolute values of the parameters M_2 and μ_R , and the two unitary rotations which “diagonalize” the chargino mass matrix are diagonal too:

$$\begin{aligned} U &\approx \text{diag}(\text{sign}(M_2), -\text{sign}(\mu_R)) , \\ V &\approx \mathbf{1} \end{aligned} \quad (2.48)$$

A similar approximation can be devised for the 4×4 neutralino mass matrix; notice that when it applies strong are the simplification in the dependence of the spectrum on the SUSY parameters.

3. Comment on the sign of μ in the literature

To compare our definitions and results with the literature it is worthwhile to comment on the sign of the parameter μ . This parameter appears also in the mass matrices of the neutralinos (zino, photino and neutral higgsino mixed states), of the scalar particles and in many interaction terms. Our conventions coincide with those of ref. [15], except for a minus sign in front of the μ entries in the chargino and neutralino mass matrices. Alternatively, one may want to keep the plus sign in the fermion mass matrices and change the sign of μ in the scalar mass matrices and Feynman rules.

2.3.3. The chargino-quark-squark vertex

As an important example of Feynman vertex of the theory let us consider the chargino-quark-squark vertex, that involves the mass matrices that we discussed. Two different contributions are present, namely that originating by the interaction with the wino (that according to the vector nature of the gauge interactions connect a left quark to a “left” squark), and those involving \tilde{h}_1^- and \tilde{h}_2^+ originating from the superpotential (that flip a left quark in a “right” squark and *viceversa*). A direct computation of the terms in eq. (1.6), and those in eq. (2.12) gives, in the fermions mass eigenstate basis defined by eq. (2.22) and eq. (2.25):

$$\mathcal{L}_{\tilde{W}\tilde{u}_L d_L} = -g\tilde{W}^+ \tilde{u}_L^* K d_L + h.c. \quad (2.49)$$

$$\mathcal{L}_{\tilde{h}_1^- \tilde{u}_L d_L} = \tilde{h}_1^- \tilde{u}_L^* K \gamma_D d_R + h.c. \quad (2.50)$$

$$\mathcal{L}_{\tilde{h}_1^- \tilde{u}_L d_L} = \tilde{h}_2^+ \tilde{u}_R^* \gamma_U K d_L + h.c. \quad (2.51)$$

Turning to the squark and chargino mass eigenstates, according to eq. (2.30) and eq. (2.45), and converting the bispinors in Dirac spinors according to the rules of Appendix A the lagrangian reads the following:

$$\begin{aligned} \mathcal{L}_{\tilde{\chi}\tilde{u}d} = \tilde{\chi}_a^- \tilde{u}^\dagger [& P_L(-V_{a1}^* S_{\tilde{u},L} g + V_{a2}^* S_{\tilde{u},R} \gamma_U) K \\ & + P_R(U_{a2} S_{\tilde{u},L} K \gamma_D)] d + h.c. \end{aligned} \quad (2.52)$$

The importance of this interaction resides in the fact that in many loop-induced processes involving the quarks as external particles, the W , the charged higgs and the

chargino exchange amplitudes are all present, and the supersymmetric contribution can behave very differently from the SM one, also because of the presence of the Yukawa couplings in the vertices. We present in the next section the vertex in which the quarks interact with the charged higgs to have a complete picture of the charged interactions in the supersymmetric extension of the SM.

2.3.4. The charged higgs-quark vertex

This vertex is in a sense non-supersymmetric: it is in common with the non-supersymmetric two Higgs doublet extensions of the Standard Model. Precisely, its supersymmetric counterpart is the higgsino-quark-squark vertex previously discussed, that, even if mixed with the wino in $\mathcal{L}_{\tilde{\chi}\tilde{u}d}$, can be “identified” as that part of in which the Yukawa couplings are entailed.

It is easy to derive the lagrangian in which the charged scalar higgs h_1^- , h_2^+ interact with the d quark, repeating the three steps done for the chargino: write the Yukawa couplings in the quark mass eigenstate basis, compute the bispinorial part that involves d_L and d_R , pass to four spinor notations. The result is:

$$\mathcal{L}_{h_1 h_2 \bar{u} d} = \bar{u} [P_R (h_1^-)^* K \gamma_D + P_L h_2^+ \gamma_U K] d + h.c. \quad (2.53)$$

In this case this is not the end of the story, since the unphysical degrees of freedom, that is the “Kibble” excitations have to be subtracted. Let us compute the directions in the weak isospin space in which this excitations lie. The Higgs multiplet $H = (h_2^+, h_0^2, h_1^0, h_1^-)$ (thought as a 8-dim real vector) develops the vacuum expectation in the direction $V = (0, v_2, v_1, 0)$. The three directions $i T^a V$ are those of the zero mass modes; the remaining five modes (orthogonal in the 8-dim space) are in general massive. Concerning the charged sector we can write

$$h_2^+ = \frac{v_1}{v} H^+ + \frac{v_2}{v} K^+ \quad (2.54)$$

$$h_1^- = \frac{v_2}{v} (H^+)^* - \frac{v_1}{v} (K^+)^* \quad (2.55)$$

where K^+ is the Kibble boson and H^+ the physical charged Higgs field. This form is particularly suitable to pass to the unitary gauge: just replace the two charged higgs with these expressions, and set $K = 0$. The interaction lagrangian expressed with the physical fields reads:

$$\mathcal{L}_{H^+ \bar{u} d} = H^+ \bar{u} [P_R K \frac{v_2}{v} \gamma_D + P_L \frac{v_1}{v} \gamma_U K] d + h.c. \quad (2.56)$$

Comparing this interaction and its supersymmetric counterpart, eq. (2.52) an important difference emerges. In the large $\tan \beta$ regime, the less Standard Model-like, the terms in which γ_D appears grow linearly, while those in which γ_U appears have two different behaviours: they tend to a non-zero limit in the charged higgsino lagrangian, while they tend to zero as $1/\tan \beta$ in the charged higgs lagrangian. This observation, done at this point, may suggest how the signatures of the MSSM can depart from those of the Higgs doublet extension of the SM; its actual relevance will be evident in the last Chapter, devoted to the study of physical processes.

3. Matching the Low and the High Energy Pictures

In this Chapter we begin the study of the SUSY GUT models. After exposing the strategy, the key subject of gauge couplings unification is discussed in details. Then we turn to masses of observed fermions, focussing in particular on the predictions of $b - \tau$ mass ratio. The main role of top quark coupling, and possible important role of tau neutrino are stressed. Mixing are described in general terms. Then we prove the consistence of “CP conserving” supersymmetric model, and finally we discuss the beautiful mechanism of radiative symmetry breaking: after an effort to enucleate the original idea from technicalities, we show how to improve the scenario considering the 1-loop corrections to the Higgs potential. The renormalisation group equations are listed in the last part of Appendix D.

3.1. Renormalisation Group Equations

It is interesting to notice that the RGE have a “modularity” character, in the sense that at 1-loop we have (formally)

$$\begin{aligned}\delta(\text{gauge}) &= \text{gauge} \\ \delta(\text{yukawa}) &= \text{gauge, yukawa} \\ \delta(\mu) &= \text{gauge, yukawa, } \mu\end{aligned}$$

while for the soft terms

$$\begin{aligned}\delta(M) &= \text{gauge, } M \\ \delta(A) &= \text{gauge, yukawa, } M, A \\ \delta(B) &= \text{gauge, yukawa, } M, A \\ \delta(m^2) &= \text{gauge, yukawa, } M, A, m^2\end{aligned}$$

Let us stress in particular that it is possible to study the gauge coupling renormalisation alone, but its study is in any case needed to proceed. The analysis of the parameters can be arranged in order of increasing number of RGE involved: gauge couplings (and gaugino masses); Yukawa couplings (and μ); A , B parameters; soft masses m^2 . This is reflected in the order of topics of the present Chapter: gauge coupling unification; quark and lepton masses; complex parameters; masses of the scalar sector, (in particular Higgs potential).

A possible objection is that for more sophisticated studies of the MSSM the “modularity” must be abandoned; for instance when considering threshold effects, the m^2 parameters could be “felt” yet by gauge couplings through the sparticles masses (similar is the case of 2-loop Yukawa couplings modifications of the gauge couplings running). In any case the simpler program of analysis above described is needed as a first step; if not conceptually, from the point of view of the practical approach. In fact I believe that this refinements should be judged case by case, since, from the phenomenological side, some of the parameters are totally unknown at present; the judgement should also consider the significance of these refinements in the whole theoretical picture.

3.2. Unification of the gauge couplings

3.2.1. Prediction of one gauge coupling constant

Let us consider the simple gauge unification group G , which undergoes a SSB at the scale M_{GUT} . When studying processes in which the typical momenta q are larger than M_{GUT} , the theory is in the symmetric phase and it is described by a single (running) gauge coupling constant.

The situation is different in the other regime, $q \ll M_{\text{GUT}}$, when the spontaneous symmetry breaking of the group G down to $H = \prod_i G_i$ has taken place. The running of the gauge coupling constant of subgroup G_i , related to IVBs that have remained light, is ruled by the fluctuations of the light particles only; this mechanism produces in general different coupling constants below M_{GUT} .

In this framework we can formulate the grand unification program for the gauge coupling constants in the SM, or in the MSSM: evolving two gauge coupling constants, their crossing point identify M_{GUT} and the value of the unified gauge coupling constant α_{GUT} ; starting from this point, the third gauge coupling constant can be predicted.

Since the generators of the residual symmetry group are the same that in the unified theory, they are normalized in the same way within each representation, in agreement with the universality of the charge. So the gauge coupling constants of different subgroups must be confronted after proper normalization of the generators. Specifically, in the context of $SU(5)$ grand unification (but also for $SO(10)$ and E_6), the SM hypercharge generator has to be multiplied by $\sqrt{3/5}$.

3.2.2. 1-loop prediction

Integrating the 1-loop RGEs⁵ we arrive to the prediction:

$$\sin^2 \theta_W = \frac{1}{18+n_d} \left[\left(3 + \frac{n_d}{2}\right) + \left(10 - \frac{n_d}{3}\right) \frac{\alpha}{\alpha_3} \right] \quad (3.1)$$

where n_d is the (even) number of Higgs doublets that we assume. The fact that in supersymmetry we have fermions and scalars in the same supermultiplet implies a strong dependence on n_d . Using the experimental inputs[16]:

$$\begin{aligned} \frac{1}{\alpha(M_Z)} &= 127.9 \pm 0.1 \\ \alpha_3(M_Z) &= 0.12 \pm 0.01 \end{aligned} \quad (3.2)$$

we derive

$$\sin^2 \theta_W(M_Z) = 0.230, 0.252, \dots \pm 0.003 \quad \text{for } n_d = 2, 4, \dots \quad (3.3)$$

where the errors come mainly from α_3 ; the SUSY SM with $n_d = 2$ is noticeably in agreement with the experimental value:

$$\sin^2 \theta_W(M_Z) = 0.2324 \pm 0.0003. \quad (3.4)$$

Also the unification mass in the MSSM \leftrightarrow $SU(5)$ GUT scheme (at variance with the non SUSY $SU(5)$) is large enough to agree with the the negative searches for decay of the proton; in fact

$$M_{\text{GUT}} = 2.4 \times 10^{16}, 1.2 \times 10^{15}, \dots \quad \text{for } n_d = 2, 4, \dots \quad (3.5)$$

This results are obtained using the central values of eq. (3.2). The inclusion of the error in α_3 can double or halve the prediction on M_{GUT} .

3.2.3. Refining the 1-loop predictions

Even if the large error on α_3 seems to swamp out the sensitivity of the predictions to a closer analysis, it is good policy to know and to keep under control finer details of the theory, also in view of better determinations of the strong coupling. Moreover to clearly disentangle the two issues (theoretical and experimental progresses), recent papers on the subject quote the prediction of the strong coupling α_3 .

⁵The β functions for the gauge coupling can be found in Appendix B.

We begin the analysis modifying the 1-loop formula for the running, by adding correction terms as follows (defining $T = \frac{1}{2\pi} \ln \frac{M_{\text{GUT}}}{M_Z}$):

$$\frac{1}{\alpha_i} - \theta_i^{2loop} + \Delta_i^{conv} + \Delta_i^{low} + \Delta_i^{high} = \frac{1}{\alpha_{\text{GUT}}} + b_i^{SUSY} T \quad i = 1, 2, 3 \quad (3.6)$$

The term θ_i in the previous equation is due to the 2-loop gauge coupling effects; consistently with the order of the approximation we can derive an analytical expression in which the 1-loop formula is modified by $\log(1\text{-loop})$ terms. The result for θ is

$$\theta_i = \sum_j \frac{b_{ij}}{4\pi b_j} \ln \left(\frac{\alpha_{\text{GUT}}}{\alpha_j} \right) \quad (3.7)$$

At 2-loop level also the Yukawa couplings enters the gauge coupling evolution; notice the estimation of this effect depends on $\tan \beta$. The term Δ^{conv} is the conversion factor to the renormalisation scheme \overline{DR} , that is a modification of the 't Hooft-Veltman scheme in which the γ matrix and the metric tensor are kept in 4 dimensions. Its use is due to the fact that supersymmetry is specific to the 4 dimensions. The third correction term in eq. (3.6) is a sum of two contributions: the first is due to the spread of supersymmetric particles above the M_Z scale (low energy thresholds effect). Their collective effect can be described as a ‘‘SUSY deficit’’ in the range between M_Z and M_i ,

$$\Delta_i^{SUSY} = \frac{1}{2\pi} (b_i^{SUSY} - b_i^{SM}) \ln \left(\frac{M_i}{M_Z} \right) \quad (3.8)$$

where the mass M_i is a geometrical average of the various thresholds, weighted proportionally to their contributions to the beta function of the i^{th} -gauge coupling. The second contribution to Δ^{low} is due to the top, since the value of $\sin^2 \theta_W(M_Z)$ is extracted from the experimental data assuming a reference value of the top mass (in formula (3.4) the value $\bar{m}_t = 138$ GeV is assumed). The residual error is due to systematics and to the uncertainty on the value of the Higgs mass. The last term is the high energy thresholds effect, that brings in a dependence of predictions on the spectrum of particles at the GUT scale, whose impact on the prediction can be analyzed similarly to that of low energy SUSY particles. We will assume for concreteness the spectrum of Georgi $SU(5)$: the vector superfield V , charged triplets of color; the physical Higgs Φ in the adjoint representation; the T and \bar{T} triplets, that appear together with the two light higgs doublets in the symmetric phase.

The result of the previous analysis can be summarized with the following formula:

$$\alpha_3 = \frac{7\alpha}{15 \sin^2 \theta_W - 3} (1 \pm \delta\alpha_3 \pm \delta\alpha) + 0.012 + \frac{28\alpha^2}{(60 \sin^2 \theta_W - 3)^2 \pi} \Delta \quad (3.9)$$

where the first line stays for the 1-loop results; the first term in the second line is the effect of the “pure” two-loop term, and the other corrections have been summarized in Δ :

$$\Delta = 1 - 19 \ln \left(\frac{M_{SUSY}}{M_Z} \right) + 18 \ln \left(\frac{M_T}{\sqrt[3]{M_V^2 M_\Phi}} \right) + H \left(\frac{m_t}{\tilde{m}_t} \right) \quad (3.10)$$

The unit constant is the effect of the conversion from \overline{MS} to \overline{DR} , the rest are threshold effects; in particular M_{SUSY} is a function, through the masses M_i defined in formulae (3.8), of the individual supersymmetric thresholds; the function $H(x)$ includes the top effects:

$$H(x) = 47.3(x^2 - 1) + 7.60 \ln x + 3.89 \quad (3.11)$$

Let us assume that the top mass is in the range $140 \div 190$ GeV; suppose also that the logarithm that describes the low energy threshold effects is smaller than two in module (this is implied if the supersymmetric spectrum does not exceed the TeV scale); estimate that the logarithm of the GUT masses have the same size.

The predictions take the following form:

$$\alpha_3(M_Z) = 0.128 \pm 0.001 + \left(\begin{array}{c} \pm 0.005 \quad +0.003 \\ \quad \quad \quad -0.004 \end{array} \pm 0.005 \right) \quad (3.12)$$

where the central value is for $m_t = 170$ GeV, $M_{SUSY} = M_Z$ and no GUT threshold effects; the first uncertainty reflects the experimental uncertainty on $\sin^2 \theta_W$. Those in brackets are theoretical uncertainties, that is: sparticles, top and GUT scale threshold corrections.

The accord with the experimental determinations of α_3 is fairly good and may be interpreted as a success of SUSY grand unification. The theoretical uncertainties and the experimental error on α_3 are of similar sizes; this fact points to the need of both better experimental inputs and theoretical improvements, in view of a finer test of the SUSY $SU(5)$ theory.

Let us discuss finally the “trends” of the prediction. The predicted values of α_3 decreases if *i*) $M_{SUSY} > M_Z$, *ii*) $m_t < 170$ GeV, *iii*) $M_T < \sqrt[3]{M_V^2 M_\Phi}$. To discuss properly the first feature we clearly wait for experimental discovery of SUSY particles, but indirect constraints can help. The second feature is at present interesting, in view of the possibility of a refinement of the experimental determinations of the top mass at CDF and at LEP. The control of the prediction is weaker for what matter the last contribution, but in any case it involves the mass of the higgs triplet, which may play a crucial role for proton decay in SUSY GUT, through effective dimension 5 operators, if lighter than

10^{16} GeV : in other words also GUT thresholds could be—to a certain extent—constrained by experiments.

3.3. Fermion masses

3.3.1. The IR quasi fixed point of the top Yukawa coupling

1. Top Yukawa coupling evolution

The top coupling evolution is substantially independent on the other Yukawa coupling constants if the value of $\tan\beta$ is small, say less than 10. Defining

$$t(T) = \frac{y_t^2(T)}{4\pi} \quad (3.13)$$

where $T = 1/2\pi \log(M_{\text{GUT}}/q)$ the top evolution is determined by

$$\begin{cases} \dot{t}(T) = (\sum_i b_u^i \alpha_i(T) - 6 t(T)) t(T) \\ t(0) = t_G \end{cases} \quad (3.14)$$

The integration of equation (3.14) gives us

$$t(T) = \frac{t_G E_u(T)}{1 + 6 t_G F_u(T)} \quad (3.15)$$

The function $E_u(T)$ summarizes the effect of the gauge coupling constants on the top running, namely we have $E_u(T) = \prod_i (\alpha_{\text{GUT}}/\alpha_i)^{b_u^i/b_i}$; its integral is the function $F_u(T)$. A useful auxiliary function is the denominator of the previous expression, $D_t(T) = 1 + 6 t_G F_u(T)$, that allows to express $t(T)$ as a derivative: $t = (\log D_t)'/6$.

The definition of the IR quasi fixed point (sometimes termed “pseudo-fixed point”) of the top [17] is obtained by letting formally $t_G \rightarrow \infty$ ⁶

$$t^{\text{IR}}(T) = \frac{E_u(T)}{6 F_u(T)} \quad (3.16)$$

This limit is physically interesting for the reason that the second term of the denominator of eq. (3.15) is large with respect to the first, even for moderate t_G ; namely, since $F_u(T_Z) \sim 23$, where $T_Z = 1/2\pi \log(M_{\text{GUT}}/M_Z)$, it is enough to have $t_G \gtrsim 0.1$.

⁶The curve for which the beta function is zero, that is the textbook definition of fixed point, is less interesting for phenomenological applications, but in this case, for $T \sim T_Z$, the two definitions give similar results.

From the previous considerations we understand the behaviour of a class of solutions of the top RGE: each RG trajectory starting at M_{GUT} with large values of t_G “converges” toward the value t^{IR} at lower energies. This gives in a sense a dynamical determination of the value of the top Yukawa coupling.

Alternatively we can look at the IR fixed point curve as the bound of the region in which the perturbativity approach can be pursued. This is most usefully done in the plane $(M_t, \tan\beta)$ (for large values of $\tan\beta$ the contributions of the other Yukawa couplings must be included, since the possibility of them to be large must be considered).

The value of the parameter t_G could be worked out formally comparing $t(T_Z)$ and $t^{\text{IR}}(T_Z)$ (t and t^{IR} for short). But from the formula for the ratio t/t^{IR} :

$$\frac{t}{t^{\text{IR}}} = \frac{6 t_G F_u}{D_t} \quad (3.17)$$

useful for following considerations, we derive $t_G \sim (1 - t/t^{\text{IR}})^{-1}$, that is an instability of this value. The UV instability is nothing more than the other face of the IR stability.⁷

Why so much attention to the parameter t_G ? The point is that it plays a key (indirect) role in the low energy supergravity models, influencing the evolution of the other parameters under renormalisation group. In the rest of this Chapter we will discuss the reasons why large values of t_G are welcomed and also to a certain extent expected in this kind of models.

2. “Determination” of $\tan\beta$

The recent determination of the top mass, [18]:

$$M_t = 174 \pm 10^{+13}_{-12} \text{ GeV} \quad (3.18)$$

allows to derive semi-quantitative (but crucially important) implications of this scenario. The IR quasi fixed point corresponds to have:

$$M_t^{\text{IR}} = (200 \pm 10) \sin\beta \text{ GeV} \quad (3.19)$$

where the variation is due to $\alpha_s(M_Z) = 0.12 \pm 0.01$; in the last formula 2-loops effects, that corresponds to an upward shift of $\sim 5\%$, are included. This means that if this picture is correct, that is if t_G is large, we are able to determine the value of $\tan\beta$, namely for

⁷For the actual computation of the ratio t/t^{IR} the measurement of the top mass is not sufficient: the determination of M_t^{IR} requires the knowledge of $\tan\beta$ as discussed below.

the central value of the top mass $\tan\beta = 1.5 \div 2.3$. It goes without saying that a better determination of the top mass is needed; but let us remark that lower values of the top mass are preferred by the analysis of the electroweak precision measurements.

3.3.2. Bottom-Tau Unification

1. Introduction

The unification of fermion masses, and in particular the masses of b -quark and τ -lepton [19] at the scale M_{GUT} where the gauge coupling unify,

$$m_b(M_{\text{GUT}}) = m_\tau(M_{\text{GUT}}) \quad (3.20)$$

is one of crucial issues of the Grand Unification. This relation is not only characteristic of $SU(5)$, with 5-plets of higgs originating the masses via a renormalisable operator: it holds also in $SO(10)$ and in E_6 with Higgs 10-plets and 27-plets respectively.

In any case this issue is more model dependent than gauge coupling unification, entailing the specific particle content in the theory. For this reason the study of the implications of this relation are physically meaningful in the MSSM, in which this part of the grand unification program can be successfully addressed.

Let me describe the procedure adopted. The parameters involved in the study are the Yukawa couplings of the third generation and $\tan\beta$. The three mass parameters related to the Yukawa couplings are constrained by the experiment, so that we have three experimental informations (considering known the top mass). In this way, enforcing the equality of y_b and y_τ at the GUT scale, we can fit all the parameters, or (eventually) we can discard the model (in fact the parameters are known only within the experimental errors, and the theoretical sources of uncertainties should be taken into account). More informations on technical details are presented in Appendix C.

2. Yukawa RGE

Since the mixing angles with the third family are small, and because of the hierarchy of the Yukawa couplings, the effect of flavour mixing can be neglected: we can consider the renormalisation effects of particles from the third family only.

It is convenient to write the renormalisation group equation (RGE) for the couplings:

$$\alpha_x = \frac{y_x^2}{4\pi} \quad (x = t, b, \tau), \quad (3.21)$$

being the analogous of the gauge coupling constants. In terms of α_x the masses of the quarks and leptons at the Z^0 -mass scale can be written as:

$$\begin{aligned} m_{t,\nu_\tau}(M_Z) &= \sqrt{4\pi\alpha_{t,\nu_\tau}(M_Z)} \frac{\tan\beta}{\sqrt{1+\tan^2\beta}} v \\ m_{b,\tau}(M_Z) &= \sqrt{4\pi\alpha_{b,\tau}(M_Z)} \frac{1}{\sqrt{1+\tan^2\beta}} v. \end{aligned} \quad (3.22)$$

Here $v \equiv \sqrt{v_1^2 + v_2^2}$.

Applying, for instance, the method of the effective potential of ref. [7], one finds the RGE for the couplings α_x :

$$\alpha'_t = \left(\sum_i b_u^i \alpha_i - 6\alpha_t - \alpha_b \right) \alpha_t \quad (3.23)$$

$$\alpha'_b = \left(\sum_i b_d^i \alpha_i - 6\alpha_b - \alpha_t - \alpha_\tau \right) \alpha_b \quad (3.24)$$

$$\alpha'_\tau = \left(\sum_i b_e^i \alpha_i - 4\alpha_\tau - 3\alpha_b \right) \alpha_\tau, \quad (3.25)$$

The signs in the eqs. (3.23-3.25) reflect the fact that during the evolution from the high to the low energy scale, the effect of the gauge coupling constants is to increase the Yukawa couplings, while the effect of the Yukawa itself is opposite.

Using (3.23-3.25) one derives the RGE for the mass ratio:

$$\left(\frac{m_b}{m_\tau} \right)' = \frac{1}{2} \left(\sum_i (b_d^i - b_e^i) \alpha_i - 3(\alpha_b - \alpha_\tau) - \alpha_t \right) \left(\frac{m_b}{m_\tau} \right). \quad (3.26)$$

The coefficient 1/2 reflects that $m \sim \sqrt{\alpha}$. The following conclusions can be drawn from eq. (3.26) immediately. (i) The increase of α_t , or of α_b tend to decrease the ratio. This is a key ingredient to achieve the $b - \tau$ unification. (ii) The gauge coupling α_s works in opposite direction: the predicted value of the m_b/m_τ ratio increases with α_s . (iii) The tau lepton gives only a partial compensation of the bottom effect, since in this case there is a factor 3 of enhancement (this is relevant for large $\tan\beta$ only).

3. Perturbative $b - \tau$ unification

The system eqs. (3.23-3.25) allows the study in the whole range of $\tan\beta$ of the perturbativity condition of the Yukawa couplings, that is the requirement that the Yukawa couplings stay perturbative (this study completes the treatment of the low $\tan\beta$ region done in previous section). The results are usefully presented in the plane $(m_t, \tan\beta)$, in which at each point corresponds a certain value of the Yukawa couplings at the grand

unification scale⁸. The curve of points in which they diverge marks the region in which the perturbative approach is valid; above this line the mass m_t blows up before grand unification scale. The increase of gauge coupling α_s results in a decrease of the α_t at large scales, and consequently, relaxes the divergency bound on $m_t(M_Z)$. The Yukawa couplings have an opposite effect. This causes the bending of the curves for large values of $\tan\beta$ (according to eqs. (3.23–3.25) an increase of $\tan\beta$ corresponds to increase of the bottom coupling α_b).

The $b - \tau$ unification curves lie very close to the divergency lines especially for low values of $\tan\beta$. This fact means that the ratio m_b/m_τ has to be dumped substantially by Yukawa renormalisation effects to achieve experimentally acceptable values. In fact for large values of the strong coupling constant, $\alpha_s \gtrsim 0.125$, it is possible that the unification and the divergency curves meet in a point, denying the possibility of perturbative unification for a certain range of values of $\tan\beta$ (namely lower values).

For relatively small values of $\tan\beta$, in the same approximation of eq. (3.14), it is possible to write the explicit expression for the mass ratio:

$$\frac{m_b}{m_\tau}(T_Z) = \left(\frac{E_d(T_Z)}{E_e(T_Z)} \right)^{1/2} \frac{1}{D_t(T_Z)^{1/12}}, \quad (3.27)$$

where the renormalisation effect of the top Yukawa coupling is summarized in the functions D_t , while the three functions E_x describe the gauge interaction effects:

$$E_x(T) = \Pi_i \left(\frac{\alpha_{\text{GUT}}}{\alpha_i(T)} \right)^{b_x^i/b_i} \quad (x = u, d, e) \quad (3.28)$$

For $\tan\beta < 10$ the approximate solution coincides with the result of numerical integration of the system (3.23–3.25) within few percent.

Let us come to conclusions and comment. The main result is that $b - \tau$ unification is possible, but at the large Yukawa coupling of top (and/or of bottom) quark only, so that the renormalisation effect due to the Yukawa interaction appreciably suppresses the value of m_b at low scale [20]. This issue is particularly interesting at present, since in this context we have the possibility to predict the value of $\tan\beta$ from the mass of the top. Moreover, from the theoretical point of view it furnishes physical motivations to consider seriously the IR fixed point scenario for the top quark.

⁸From the point of view of the procedure this problem is equivalent to $b - \tau$ unification, once the maximum value of the Yukawa coupling at the GUT scale is stated, that is once we have an input at the GUT scale. The actual procedure I adopted is discussed in Appendix C.

3.3.3. Massive neutrinos in SUSY GUT

1. Introduction

Studies of the solar, atmospheric, as well as relic (hot dark matter, large scale structure of the universe) neutrinos give some hints of the existence of nonzero neutrino masses [21]. The required values of masses can be naturally generated by the see-saw mechanism [22] if the Majorana masses of the RH neutrinos are at the intermediate scale: $M_R \sim (10^{10} - 10^{12})$ GeV. In particular, for the tau neutrino to be in the cosmologically interesting domain, $m_\nu \sim 3 - 10$ eV, one needs

$$M_R \lesssim m_t^2/m_\nu \sim 10^{11} \text{ GeV.} \quad (3.29)$$

The same scale of masses is required by mechanism of the baryon asymmetry generation based on the decay of the RH neutrinos [23]. Much lower masses: $M_R < 10^7$ GeV, are implied by the Primordial nucleosynthesis in the supersymmetric models with spontaneous violation of lepton number [24].

The possibility of having a scale M_R much lower than the GUT scale is particularly interesting when considering renormalisation group effects in the region of momenta $M_R < q < M_{\text{GUT}}$, that is when neutrinos are coupled to the other particles [25]. In the following analysis the Yukawa coupling of the neutrino are not supposed to be small, being related naturally to the up quarks Yukawa couplings in such schemes. This entails naturally the possibility of large renormalisation effects not only on the tau neutrino Yukawa itself, but also, indirectly, on the other parameters of the theory⁹. We will discuss in the following the impact of these considerations in the prediction of the $b - \tau$ mass ratio.

2. Renormalisation Group Equations with RH neutrinos

The renormalisation group equations are easily computed with the method of the effective potential. The system eqs. (3.23–3.25) is modified in the following manner:

$$\alpha'_t = \left(\sum_i b_u^i \alpha_i - 6\alpha_t - \alpha_b - \alpha_{\nu_\tau} \theta_R \right) \alpha_t \quad (3.30)$$

$$\alpha'_b = \left(\sum_i b_d^i \alpha_i - 6\alpha_b - \alpha_t - \alpha_\tau \right) \alpha_b \quad (3.31)$$

$$\alpha'_{\nu_\tau} = \left(\sum_i b_\nu^i \alpha_i - 4\alpha_{\nu_\tau} \theta_R - \alpha_\tau - 3\alpha_t \right) \alpha_{\nu_\tau} \quad (3.32)$$

⁹The right-handed neutrinos, being gauge singlets, do not influence the gauge coupling constants evolution at 1-loop level; the 2-loop effect is estimated to be small.

$$\alpha'_\tau = \left(\sum_i b_e^i \alpha_i - 4\alpha_\tau - \alpha_{\nu_\tau} \theta_R - 3\alpha_b \right) \alpha_\tau, \quad (3.33)$$

where $\theta_R(T)$ is the step function $\theta(T - T_R)$, which describes the effect of the ν_R decoupling at M_R ($T_R \equiv 1/2\pi \log(M_{\text{GUT}}/M_R)$)¹⁰.

At $M_{\text{GUT}} = M_R$ the system reduces to the system eqs. (3.23–3.25), decoupled from the neutrino influence, but influencing the evolution of the neutrino itself. It is worthwhile to notice that this evolution coincides with that of the mass operator, in the supersymmetric case, found recently in [26].

3. Large neutrino Yukawa coupling and $b - \tau$ unification

Using (3.30–3.33), or also, modifying formula (3.26) keeping into account the quark-lepton symmetry of the lagrangian, one finds the RGE for the mass ratio:

$$\left(\frac{m_b}{m_\tau} \right)' = \frac{1}{2} \left(\sum_i (b_d^i - b_e^i) \alpha_i - 3(\alpha_b - \alpha_\tau) - (\alpha_t - \alpha_{\nu_\tau} \theta_R) \right) \left(\frac{m_b}{m_\tau} \right). \quad (3.34)$$

The neutrino renormalisation results in increase of m_b/m_τ . Therefore to achieve the mass unification one should take even larger values of α_t to compensate the effects of neutrino. To quantify the effect we assume in the following that at the GUT scale

$$y_t(M_{\text{GUT}}) = y_{\nu_\tau}(M_{\text{GUT}}) \quad (3.35)$$

This relation holds for instance in $SO(10)$ GUT schemes if only Higgs ten-plets give mass to the third family.

Let us first review the perturbativity region in the $(\tan \beta, m_t)$ -plane. Fig. 1 shows clearly that the effect of the right-handed neutrinos is that of reducing the allowed parameter space, or in other words to lower the value of the IR quasi fixed point. This effect is numerically of the same order of magnitude of that due to ranging α_s up to lowest allowed values (discussed in previous section).

Fig. 2 shows the unification curves in the same plane. For RH masses in the cosmologically interesting range, these curves extend for large values of $\tan \beta$ only; it is still possible to realize $b - \tau$ mass unification, but not at low values of $\tan \beta$. This is due to the fact that the unification curves meet for a certain value of $\tan \beta$ the perturbativity curves, that is they exit from the perturbativity domain. An alternative point of view

¹⁰It is suggested that the effect of flavour mixing is negligible, and there is a hierarchy of the Yukawa couplings, so that the renormalisation effects of particles from the third family is important only.

is the following: the predicted value of the mass ratio is expected to be a decreasing function of $\alpha_t(M_{\text{GUT}})$; its steepness must diminish if neutrino renormalisation effects are considered. This is presented in Fig. 3, where a comparison with the experimental value is also made. We remark also that in the same manner as for eq. (3.27), it is possible to devise an analytic expression of the ratio that, in the low $\tan\beta$ region, encodes the neutrino renormalisation effects:

$$\frac{m_b}{m_\tau}(T_Z) = \left(\frac{E_b(T_Z)}{E_\tau(T_Z)} \right)^{1/2} \frac{D_{\nu_\tau}(T_Z)^{1/8}}{D_t(T_Z)^{1/12}}, \quad (3.36)$$

where the function D_{ν_τ} is defined as:

$$\begin{aligned} E_\nu(T) &= \prod_i \left(\frac{\alpha_i(T)}{\alpha_i(0)} \right)^{-b_i^i/b_i} \\ E_{\nu_\tau}(T) &= \frac{E_\nu(T)}{D_t(T)^{1/2}} \\ D_{\nu_\tau}(T) &= 1 + 4 \alpha_{\nu_\tau}(0) \int_0^T E_{\nu_\tau}(x) \theta_R(x) dx \end{aligned} \quad (3.37)$$

For $\tan\beta < 10$ the approximate solution eq. (3.36) coincides with the results of the numerical integration of eqs. (3.30–3.33) within $2 \div 3\%$.

Finally we present in Fig. 4 the boundaries of the regions, in the plane $(\tan\beta, M_R)$, in which perturbative unification can be realized. The lower bounds on $\tan\beta$ (and M_R) are obtained from the $b-\tau$ unification condition ($M_b^{\text{pole}} \leq 5.2 \text{ GeV}$) and the convergency limit for α_t for different values of α_s . Let us remark a strong dependence of this result on the strong coupling constant.

4. Prediction of tau neutrino mass

As a final issue we quote the prediction of the neutrino mass at $m_t = 174 \pm 10 \text{ GeV}$:

$$m_{\nu_\tau} = \left(10_{-2.2}^{+3.5} \right) \text{ eV} \left(\frac{10^{12} \text{ GeV}}{M_R} \right) \quad (3.38)$$

In previous formula $\tan\beta$ is supposed to be large, consistently with perturbative $b-\tau$ unification also for lower values of M_R .

An approximate analytical expression can be still derived:

$$m_{\nu_\tau} = \frac{E_\nu(T_Z)}{E_u(T_Z)} \frac{D_t(T_Z)^{1/2}}{D_{\nu_\tau}(T_Z)} \frac{m_t^2}{M_R} \quad (3.39)$$

the effect of gauge couplings renormalisation is $\approx 0.145 \mp 0.01$ for $\alpha_s(M_Z) = 0.120 \pm 0.005$. That due to Yukawa renormalisation (top and tau neutrino together) amounts to

a factor larger than two¹¹. For largest m_t allowed in the model we find that the Yukawa contribution is ~ 3 at $M_R = 10^{-4} M_{\text{GUT}}$; and ~ 8 for a decoupled neutrino, $M_R = M_{\text{GUT}}$.

The conclusions of this long section are the following. The presence of right-handed neutrinos with masses in the intermediate region disfavours $b - \tau$ unification, especially at low $\tan \beta$ (in particular the IR quasi fixed point region). It is important to know if this is the case, since this may testify for large GUT threshold effects, or even the absence of $b - \tau$ unification, but in any case non-trivial modifications of the assumptions considered. The answer could come from experiments [27] projected for the observation of $\nu_\mu - \nu_\tau$ neutrinos oscillation.

3.3.4. Renormalisation group evolution of the mixing matrix

Let us first write the renormalisation group equations for the Yukawa matrix in full matrix form. In analogy with the gauge couplings α_i we define the matrices

$$\alpha_x = \frac{1}{4\pi} Y_x \cdot Y_x^\dagger \quad x = U, D, E \quad (3.40)$$

A simple computation gives us:

$$\dot{\alpha}_U = b_U^i \alpha_i \alpha_U - 3\alpha_U^2 - \text{tr}(3\alpha_U) \alpha_U - \frac{1}{2}\{\alpha_U, \alpha_D\} \quad (3.41)$$

$$\dot{\alpha}_D = b_D^i \alpha_i \alpha_D - 3\alpha_D^2 - \text{tr}(3\alpha_D + \alpha_E) \alpha_D - \frac{1}{2}\{\alpha_U, \alpha_D\} \quad (3.42)$$

Notice that with this choice it is apparent that only the “left” matrices L_U, L_D , defined in eq. (2.21), matter in the Yukawa evolution:

$$\alpha_U = L_U^\dagger \text{diag}(U) L_U \quad (3.43)$$

$$\alpha_D = L_D^\dagger \text{diag}(D) L_D \quad (3.44)$$

where in passing I set the notation for U and D , useful in the following

$$\text{diag}(U) = \frac{\gamma_U^2}{4\pi} \quad (3.45)$$

$$\text{diag}(D) = \frac{\gamma_D^2}{4\pi} \quad (3.46)$$

that is $U_1 = \alpha_u, U_2 = \alpha_c, \text{etc.}$. In fact the previous RGEs still contain non-physical informations, since only the combination $K = L_U L_D^\dagger$, the Cabibbo-Kobayashi-Maskawa matrix, is observable (up to rephasing invariance).

¹¹For solutions corresponding to low value of $\tan \beta$, when they exist, the Yukawa contributions is approximatively doubled.

It is in fact possible to derive a simple renormalisation group equation for the physical variables only [28], [29] (even if it is possible to work with –in present context– unphysical Yukawa matrices: [30], [31]). We first write eq. (3.43) in the useful form

$$U = L_U \alpha_U L_U^\dagger \quad (3.47)$$

Then we derive this equation with respect to T and use eq. (3.41). Considering the diagonal matrix elements we find the evolution of the elements U_a ,

$$\dot{U}_a = (b_U^i \alpha_i - 3 U_a - 3 \sum_b U_b - \sum_b |K_{ab}|^2 D_b) U_a \quad (3.48)$$

while the out-of-diagonal elements give informations on the L_U evolution:

$$(L_U \dot{L}_U^\dagger)_{ab} = \frac{1}{2} \frac{U_a + U_b}{U_a - U_b} (K D K^\dagger)_{ab} \quad (3.49)$$

In eq. (3.41) only the anticommutator term gives rise to an evolution of the matrix structure; this tracks back the origin of the factor 1/2 in previous equation. These steps can be repeated for eq. (3.44).

The only information that we can get for the diagonal elements $(L_x \dot{L}_x^\dagger)_{aa}$ ($x = U, D$) is that they are pure imaginary numbers, as it results from unitarity of L_x . In any case this is enough to perform the last step, that is to derive the RGE for the squares $|K_{ab}|^2$ (obviously rephasing invariant quantities) by direct computation:

$$\begin{aligned} |\dot{K}_{ab}|^2 &= (U_a^2 - \sum_c |K_{cb}|^2 U_c + D_b^2 - \sum_c |K_{ac}|^2 D_c) |K_{ab}|^2 \\ &- \sum_{c \neq a} \frac{U_a}{U_a - U_c} \sum_d D_d V_{ac,db} - \sum_{d \neq b} \frac{D_b}{D_b - D_d} \sum_c U_c V_{ac,db} \end{aligned} \quad (3.50)$$

The rephasing invariant $V_{ac,db}$, termed also “quartet” is defined:

$$V_{ac,db} = K_{ad} K_{cd}^* K_{cb} K_{ab}^* + h.c. \quad (3.51)$$

In the case of three families it can be expressed as a simple function of $|K_{ab}|^2$. It is noticeable that an element which is zero does not change under RG evolution.

It is particularly useful to reduce this formula taking into account the strong hierarchies $U_1 \ll U_2 \ll U_3$, $D_1 \ll D_2 \ll D_3$. Neglecting all but the third family couplings in the right-hand side, and neglecting terms $O(|K_{23}|^2)$ with respect to 1 a strong simplification takes place:

$$|\dot{K}_{ab}|^2 = \begin{cases} (\alpha_t + \alpha_b) |K_{ab}|^2 & \text{if } (a, b) = (1, 3), (2, 3), (3, 2), (1, 3) \\ 0 & \text{otherwise} \end{cases} \quad (3.52)$$

where we come back to the notation $U_3 = \alpha_t$, $D_3 = \alpha_b$. This means that only the elements connected to the third family evolve in this approximation. This can still be recast in a more striking form resorting to the notations of Wolfenstein:

$$\dot{A} = \frac{1}{2} (\alpha_t + \alpha_b) A \quad (3.53)$$

$$\dot{\lambda} = 0 \quad (3.54)$$

$$\dot{\rho} = 0 \quad (3.55)$$

$$\dot{\eta} = 0 \quad (3.56)$$

Three final observations: 1) for comparison with experiments the set of variables most useful is K_{us} , $|K_{ub}|/|K_{cb}|$, $|K_{cb}|$ and the Jarlskog parameter J of CP violation (whose evolution is easily found from $J \sim A^2 \lambda^6 \eta$); in the approximation considered only the latter two evolve; 2) these RGEs are of the same form of the Yukawa RGEs, eqs. (3.23–3.25); only the Yukawa contributions are present, but in this case they *enhance* the mixings at low momenta (would we have studied the evolution in the SM, we would have found an opposite behaviour) 3) in the low $\tan \beta$ region the equations can be solved analytically; in this second form $A(T_Z) = A(T_{\text{GUT}}) D_t^{1/12}$.

The real problem in giving predictions for the mixing matrices is in fact that of assigning the GUT scale conditions on the mixing elements. This is at moment one really interesting field of researches, but the approaches are very far one another. For instance it has been proposed that some entries in the Yukawa matrices are zero as result of discrete flavour symmetries in the GUT scale lagrangian; and also that all the Yukawa interactions involving the light particles result from non-renormalisable interactions dictated by Planck scale physics. We will not push further this study in this thesis, but simply quote the interesting trend of the predictions, for which $|K_{ub}|/|K_{cb}|$ tends to lie in the lower part of the experimentally allowed range, whereas $|K_{cb}|$ follows the converse trend.

3.4. Complex parameters in the MSSM

The most conservative and natural assumption for CP violation in the soft sector is that the two phases in eq. (2.27) vanish identically at the GUT scale. This could be due to CP conservation in the sector responsible for SUSY breaking. However this case requires a closer analysis, since an independent source of CP violation (the CKM phase) is present in the model and it is involved in the renormalisation of the soft breaking parameters [32].

3.4.1. Renormalisation group equations

The phases of the gaugino masses and of the parameter μ do not evolve under renormalisation group, since the variation of these parameter is of the form

$$\dot{m} = R(g, Y) \cdot m \quad (3.57)$$

where R is a real function involving the gauge and the Yukawa couplings (the Yukawa enter as traces of hermitian combinations). In particular this implies that, in the case at hand

$$\text{sign}(\mu), \text{sign}(M_i) \text{ are RG - invariant} \quad (3.58)$$

So we concentrate the study on the coupled system of the Yukawa couplings Y_x and of the massive couplings Y_x^A and B .

It is worthwhile to use the following definition ($x = D, E$ or U):

$$A_x = Y_x^A \cdot Y_x^{-1} \quad (3.59)$$

useful in measuring the mismatch between Y_x^A and Y_x . Using the scale

$$T \equiv \frac{1}{2\pi} \log\left(\frac{M_{\text{GUT}}}{q}\right) \quad (3.60)$$

we derive the following RGE:

$$\begin{aligned} \dot{A}_E &= - (3\alpha_2 M_2 + 3\alpha_1 M_1) I \\ &\quad - \text{Tr}(A_E \alpha_E + 3A_D \alpha_D) I \\ &\quad - (5\alpha_E A_E + A_E \alpha_E) / 2 \\ \dot{A}_U &= - \left(\frac{16}{3}\alpha_3 M_3 + 3\alpha_2 M_2 + \frac{13}{9}\alpha_1 M_1\right) I \\ &\quad - \text{Tr}(3A_U \alpha_U) I \\ &\quad - (5\alpha_U A_U + A_U \alpha_U + \alpha_D A_U - A_U \alpha_D + 2A_D \alpha_D) / 2 \\ \dot{A}_D &= - \left(\frac{16}{3}\alpha_3 M_3 + 3\alpha_2 M_2 + \frac{7}{9}\alpha_1 M_1\right) I \\ &\quad - \text{Tr}(A_E \alpha_E + 3A_D \alpha_D) I \\ &\quad - (5\alpha_D A_D + A_D \alpha_D + \alpha_U A_D - A_D \alpha_U + 2A_U \alpha_U) / 2 \\ \dot{B} &= - (3\alpha_2 M_2 + \alpha_1 M_1) \\ &\quad - \text{Tr}(A_E \alpha_E + 3A_D \alpha_D + 3A_U \alpha_U) \end{aligned} \quad (3.61)$$

The A_x are a-priori generic 3×3 matrices, and the form of the initial conditions can be rewritten as:

$$\begin{aligned} M_i &= M_G \\ A_x &= A_G I \\ B &= B_G \end{aligned} \quad (3.62)$$

Notice that since the system is linear one can study independently the two cases a) $M_G = 0$, $A_G \neq 0$ and b) $M_G \neq 0$, $A_G = 0$. In particular the A_G contribution is renormalised by Yukawa couplings only. The GUT value for B , namely B_G , is always additive.

3.4.2. Numerical analysis of the solutions

The study of this system has been performed with numerical techniques. The result is that for any choice of the parameters at the GUT scale and of the Yukawa couplings at the low scale the only complex entries of the matrix $A_{U,D}$ are the off-diagonal ones, and that those matrices are hermitian up to a part per $10^{-3} - 10^{-4}$ (clearly, since there is no source of lepton number violation in this model the A_E matrix remains diagonal). The complex elements obey to the approximate hierarchy $|A_{13}|/|A_{12}| \sim |A_{23}|/|A_{13}| \sim 10$. Their behaviour with $\tan \beta$ is interesting. For small values of $\tan \beta$, the $A_{D,ij}$ are larger but decrease up to $\tan \beta \sim 5$, then they stay approximatively constant; the $A_{U,ij}$ are very small for small $\tan \beta$, then they increase roughly as $\tan \beta^2$. This is due to the fact that the evolution of the A_D off-diagonal elements is driven by the up Yukawa couplings, while the down Yukawa elements drive the evolution of the A_U .

An important conclusion is that the hypothesis of CP conservation at the GUT scale implies the reality of the diagonal elements of the Y_x^A matrices, that are the larger ones; the only induced CP violating terms change the flavour of the squarks (in a sense there is a link between CP and the flavour changing phenomena). These last terms enter the interactions of the squarks with the higgs scalars, but also the L - R squark mass term, as in eq. (2.34). As such the processes involving squarks exchange are important in the determination of the phenomenology of the CP violation in the supersymmetric standard model; we will come back on this issue in the last Chapter, when studying the supersymmetric contributions to the electric dipole moments of elementary particles.

3.5. Large Top Yukawa Coupling and Radiative SSB

The discussion of the spontaneous symmetry breaking (SSB) of the electroweak symmetry through radiative corrections requires the study of the full system of RGE. Deep insight on the nature of the phenomenon can be obtained studying the system in the approximation in which the gauge couplings and the top Yukawa coupling only drive the

equations, since analytical formulae can be in this case derived [33]: even if the mechanism of radiative electroweak symmetry is general, it is much less apparent when studied with numerical techniques only. In this section we will work out the analytic approximation, paying particular attention to the case in which the top is close to the IR fixed point.

3.5.1. The evolution of the massive parameters

At 1-loop order the gauge couplings α_i and the gaugino masses M_i can be treated as forcing terms in the other RGEs. The analytic integrals can be easily written with the help of the auxiliary functions $I_{N,x}(T)$, where $x = u, d, e, \mu, Q, U, H$:

$$\begin{aligned} \int \sum_i b_{x,i} \alpha_i M_i^N &= M_G^N \sum_i \frac{b_{x,i}}{b_i} \begin{cases} \ln(\alpha_{\text{GUT}}/\alpha_i) & N = 0 \\ \frac{1 - (\alpha_i/\alpha_{\text{GUT}})^N}{N} & N > 0 \end{cases} \\ &\equiv M_G^N I_{N,x} \end{aligned} \quad (3.63)$$

After showing the structure of the integrals I will come back to the notations of eq. (3.15).

Let us rewrite the integral top integral with this new notations:

$$t = \frac{t_G \exp(I_{0,u})}{1 + 6 t_G \int \exp(I_{0,u})} \equiv \frac{1}{6} (\ln D)' \quad (3.64)$$

The supersymmetric mass parameter μ has the same kind of RGE of the b and τ Yukawa couplings:

$$\mu = \mu_G \frac{\exp(I_{0,\mu}/2)}{D^{1/4}} \quad (3.65)$$

For the RGE of A_t (and of m^2) in which t enters as forcing term it is useful to follow this procedure of integration: first, to isolate the t depending part, to arrive at the form $y' = 6t(F - y)$ (where F is a known function), whose formal integral is $y = 1/D [y_0 + \int F dD]$. Second, to extract the dependence on t_G reconstructing in the function F the derivative with respect to D of the function $(\int I_{1,u} dD)/D$ -or of $(\int I_{1,u} dD)^2/D$ - whenever needed.

The expressions obtained for the A_t and B RGEs integrals are

$$A_t = \frac{A_G}{D} - M_G \left[I_{1,u} - \frac{6t_G}{D} \int \exp(I_{0,u}) I_{1,u} \right] \quad (3.66)$$

$$B = B_G - A_G \frac{3t_G \int \exp(I_{0,u})}{D} - M_G \left(I_{1,\mu} - \frac{3t_G}{D} \int \exp(I_{0,u}) I_{1,u} \right) \quad (3.67)$$

The three equations for $m_{Q_3}^2, m_{U_3}^2, m_{H_2}^2$ can be decoupled,

$$m_{Q_3}^2 = m_G^2 + M_G^2 I_{2,Q} - \frac{\delta m^2}{6} \quad (3.68)$$

$$m_{U_3}^2 = m_G^2 + M_G^2 I_{2,U} - \frac{\delta m^2}{3} \quad (3.69)$$

$$m_{H_2}^2 = m_G^2 + M_G^2 I_{2,H} - \frac{\delta m^2}{2} \quad (3.70)$$

and the term δm^2 , a pure top renormalisation effect reads:

$$\begin{aligned} \delta m^2 = & 3m_G^2 \frac{6t_G \int \exp I_{0,u}}{D} + A_G^2 \frac{6t_G \int \exp I_{0,u}}{D^2} - 2A_G M_G \frac{6t_G \int \exp(I_{0,u}) I_{1,u}}{D^2} \\ & + M_G^2 \left[\frac{6t_G \int \exp(I_{0,u}) (2 I_{2,u} + I_{1,u}^2)}{D} - \left(\frac{6t_G \int \exp(I_{0,u}) I_{1,u}}{D} \right)^2 \right] \end{aligned} \quad (3.71)$$

(I used $I_{N,Q} + I_{N,U} + I_{N,H} = 2 I_{N,u}$ in the last line).

It is particularly interesting to study the case in which the top is close to the IR quasi fixed point (that is the formal limit $t_G \rightarrow \infty$). To this end we notice that the dependence on t_G comes together with the factor $1/D$, so that it is possible to eliminate both t_G and D using formula eq. (3.17). We can rewrite the previous set of integrals in a suggestive form:

$$\mu = \mu_G \exp(I_{0,\mu}/2) \left(1 - \frac{t}{t_{\text{IR}}}\right)^{1/4} \quad (3.72)$$

$$A_t = A_G \left(1 - \frac{t}{t_{\text{IR}}}\right) - M_G \left[I_{1,u} - \frac{t}{t_{\text{IR}}} G_u \right] \quad (3.73)$$

$$B = B_G - \frac{A_G}{2} \frac{t}{t_{\text{IR}}} - M_G \left[I_{1,\mu} - \frac{t}{t_{\text{IR}}} \frac{G_u}{2} \right] \quad (3.74)$$

$$m_{Q_3}^2 = m_G^2 + M_G^2 I_{2,Q} - \frac{\delta m^2}{6} \quad (3.75)$$

$$m_{U_3}^2 = m_G^2 + M_G^2 I_{2,U} - \frac{\delta m^2}{3} \quad (3.76)$$

$$m_{H_2}^2 = m_G^2 + M_G^2 I_{2,H} - \frac{\delta m^2}{2} \quad (3.77)$$

where

$$\begin{aligned} \delta m^2 = & \frac{t}{t_{\text{IR}}} \left[3 m_G^2 + \left(1 - \frac{t}{t_{\text{IR}}}\right) (A_G^2 - 2A_G M_G G_u) \right. \\ & \left. + M_G^2 \left(H_u - \frac{t}{t_{\text{IR}}} G_u^2 \right) \right] \end{aligned} \quad (3.78)$$

The two functions $G_u(T)$ and $H_u(T)$ that have been introduced are defined as:

$$G_u = \frac{\int \exp(I_{0,u}) I_{1,u}}{\int \exp(I_{0,u})} \quad (3.79)$$

$$H_u = \frac{\int \exp(I_{0,u}) (2 I_{2,u} + I_{1,u}^2)}{\int \exp(I_{0,u})} \quad (3.80)$$

(as a matter of fact G_u can be expressed as $TE_u/F_u - 1$). The approximate numerical values at $T = T_Z \sim 5.3$:

$$\begin{aligned}
\exp(I_{0,\mu}) &\sim 2 \\
\exp(I_{0,u}) &\sim 15 \\
I_{1,\mu} &\sim 0.6 \\
I_{1,u} &\sim 4.2 \\
I_{2,Q} &\sim 7.2 \\
I_{2,U} &\sim 6.7 \\
I_{2,H} &\sim 0.5 \\
F_u &\sim 23 \\
G_u &\sim 2.2 \\
H_u &\sim 14
\end{aligned}$$

In the limiting case $t \rightarrow t^{\text{IR}}$ any reference to A_G and to B_G disappears, but in the equation for B itself (the parameter for the bilinear Higgs coupling), where the specific combination $B_G - A_G/2$ enters. This is a typical IR fixed point behaviour: in the same manner in which the top Yukawa coupling remains determined, whichever the value of t_G is, the massive couplings become insensitive to some initial conditions, namely to A_G or to B_G separately¹².

3.5.2. Radiative Electroweak Symmetry Breaking

An exciting possibility that derives from eqs. (3.68–3.70) is that $m_{H_2}^2$ only be driven negative, by the larger coefficient with which the negative top contribution enters (and by the fact that the gauge contribution for h_2 , that is $M_G^2 I_{2,H}$, is much smaller than the corresponding other two. This has attracted a lot of attention on these kind of model, since in a sense the SUSY SM supplemented with the the soft breaking terms has a built-in mechanism to explain the breaking of the gauge symmetry.

1. The Z mass as a function of the SUSY masses

Consider the tree level Higgs potential in the MSSM:

$$V_0 = m_1^2 |h_1^0|^2 + m_2^2 |h_2^0|^2 - (m_3^2 h_1^0 h_2^0 + h.c.) + \frac{1}{8} (g^2 + g'^2) (|h_1^0|^2 - |h_2^0|^2)^2 \quad (3.81)$$

¹²Formally the parameter μ follows the same destiny, but in practise it is much less influenced; *e.g.* $t/t^{\text{IR}} \sim 95\%$ implies $\mu/\mu_G \sim 70\%$.

where the $m_{1,2}^2$ (not necessarily positive!) and m_3^2 are

$$m_1^2 = m_{H_1}^2 + \mu^2 \quad (3.82)$$

$$m_2^2 = m_{H_2}^2 + \mu^2 \quad (3.83)$$

$$m_3^2 = -B\mu \quad (3.84)$$

As discussed previously the parameter m_3^2 can always be chosen to be real; in fact, by a proper choice of the sign of B (renormalised) can be chosen positive. Let us fix now the angle β defined as in eq. (2.20). The location of the non-trivial extremum, expressed with the parameters in the scalar potential is:

$$\tan^2\beta = \frac{m_1^2 + M_Z^2/2}{m_2^2 + M_Z^2/2} \quad (3.85)$$

$$\sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2} \quad (3.86)$$

This conditions are sufficient to assure the SSB if the parameters respect two inequalities:

$$m_1^2 + m_2^2 > 2 m_3^2 \quad (3.87)$$

$$m_1^2 m_2^2 < (m_3^2)^2 \quad (3.88)$$

The first derives from the stability of the potential in the direction $v_1 = v_2$, the second is needed to exclude the symmetric vacuum.

Considering eqs. (3.82–3.83) we can give a convenient alternative form to the conditions expressed in eqs. (3.85–3.86):

$$\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{M_Z^2}{2} \quad (3.89)$$

$$B = -\frac{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2}{2\mu} \sin 2\beta \quad (3.90)$$

The first condition can be used to compute the value that is of μ , the second that of B . So we succeed in the “net” elimination of 1 unknown parameter ($\tan\beta$ has been introduced), essentially thank to the M_Z mass condition¹³. In fact the elimination can eventually be performed for the GUT scale parameters, that is inserting in the minimization conditions the results of the RGE scaling. The general conclusion is that the 5-dim parameter space $(\mu_G, A_G, B_G, M_G, m_G^2)$ reduces to a 4-dim one, or to a 3-dim parameter space if a boundary condition for B_G is taken into account.

¹³Notice in passing that the eq. (3.89), written in the form $M_Z^2 = \dots$, requires upper bounds on the supersymmetric massive parameters if large cancellations have to be avoided [34]; analogous bounds are obtained if the lightest supersymmetric particle is the cold component of dark matter [35].

Even more interesting is the specification of this result in the case of the IR quasi fixed point scenario for the top quark: two parameters, for instance M_G and m_G as in the previous discussion, are sufficient to describe the spectrum and the interactions, without the need to consider *ansatzen* for B_G .

2: 1-loop corrections

It has been shown that the 1-loop corrections to the Higgs potential are important in determining the spectrum of the physical Higgs fields. For instance, from the potential of eq. (3.81) one derives the important tree level prediction for the lighter neutral Higgs particle h :

$$m_h^2 = \frac{1}{2} \left[m_Z^2 + m_A^2 - \sqrt{(m_Z^2 + m_A^2)^2 - 4m_Z^2 m_A^2 \left(\frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} \right)^2} \right] \quad (3.91)$$

where the CP odd scalar A has the mass:

$$m_A^2 = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 \quad (3.92)$$

Since m_h^2 is an increasing function of m_A and/or of $\tan \beta$ ¹⁴, fixed the other variable, we would conclude that m_h is in any case lighter than the Z boson.

The situation is particularly tough in the low $\tan \beta$ region, since the tree level bound reduces to

$$m_{h,\max} = m_Z \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1}. \quad (3.93)$$

Notice in passing that this is the case in the IR fixed point scenario, since low values of $\tan \beta$ are selected; it is also interesting to observe that in this case the tree level bound is actually saturated, due to the formal limit $\mu \rightarrow \infty$ in formula (3.89) and to eq. (3.92).

Life is in fact easier for supersymmetrist: once radiative corrections are considered, m_h^2 receives large corrections. More precisely, in the approximation in which only the top and the two stops (in first approximation degenerate), that is the particles more strongly coupled to the higgs, modify at 1-loop level the higgs 2-point function the correction terms to eq. (3.93) have a very simple form:

$$\delta m_{h,\max}^2 = \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \log \left(\frac{m_t^2}{m_t^2} \right) \quad (3.94)$$

¹⁴In fact this second property holds only if $\tan \beta \geq 1$, that is needed if the top has to remain perturbative; generic analysis of the model may release this constraint.

This correction actually dominates on the tree level prediction on some regions of the parameter space. Notice also that, at difference of the tree level contribution, it is independent of $\tan \beta$; so the region in which m_h is more strongly bound is still that of low $\tan \beta$. This conclusion is confirmed by more complete analysis in which stop mixings are kept, [36]; for instance if the IR fixed point scenario is correct, (and supposing $M_t \lesssim 170$ GeV) $m_{h,\max} \sim 100$ GeV. This should allow LEP-200 to observe 1 higgs particle or to reject the IR fixed point scenario¹⁵ that we recall is linked to the promising “low $\tan \beta$ solution” of $b - \tau$ unification.

3. The Effective Potential method

Having recalled the importance of 1-loop corrections for the physical spectrum, it is easy to appreciate the fact that these corrections modify also the minimization conditions eqs. (3.85–3.86). These are actually important when analysing the parameter space. So we come back to the main subject, and describe a method to derive the correction terms based on the Effective Potential. The resulting formulae are those used in the numerical analysis described in next Chapter.

The 1-loop corrected scalar potential

$$V_1 = V_0 + \Delta V \quad (3.95)$$

is the finite counterpart of the result (1.24), and can be expressed in general terms using the generalized mass matrix \mathcal{M} . Specifically in the \overline{MS} renormalisation scheme each particle contributes according to

$$\Delta V = \frac{1}{64\pi^2} \text{Str} \left[\mathcal{M}^4 \left(\log \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) \right] \quad (3.96)$$

where Q is the renormalisation scale. Let us stress that the Q -dependence of the lagrangian is twofold; the explicit Q -dependence in the previous formula and the implicit Q -dependence of the parameters of the lagrangian due to the renormalisation group evolution; these two effects combine into a higher order dependence on Q of the resulting effective lagrangian, and determine the greater stability of the predictions. Whenever needed I will take $Q = m_Z$; the numerical results will include the 1-loop contributions of the third family of quarks, leptons, squarks and sleptons.

The arguments exposed after eqs. (3.89–3.90) can be applied also for the 1-loop

¹⁵The same token applies to futuristic XXX-300 e^+e^- collider, but without restrictions on the top mass and replacing “IR fixed point scenario” with “MSSM”; see for instance [37].

scalar potential, with the only proviso that the elimination of the parameters needs in the general case the use of numerical techniques.

Finally let us remark that there are alternative ways to study the effects of the 1-loop corrections. There is obviously the possibility of a full 1-loop analysis of the potential. This method should be clearly the “first choice”, but at the same time it leads to very complicated expressions. It has been used in very interesting phenomenological analysis of the MSSM Higgs potential [37], but not yet in the more restrictive context of low energy supergravity model. Then we shortly comment on the Renormalisation Group method. Considering the case of the heavy coloured sparticles of a similar mass, we could integrate them out from the theory at their threshold, and then consider the effects of the breaking of supersymmetry using an extended form of the tree level potential of eq. (3.81), as we would do in a two Higgs doublets extension of the standard model. This approach has been pursued in ref. [38]; future null searches of supersymmetric particles could soften the assumption that superparticles are not close to the M_Z scale, even if at present this could be questioned (hopefully by some experiment!).

4. Rare processes

The last Chapter is devoted to the study of rare processes: the electric dipole moment of the neutron, CP violating form factor for scattering on external em field, and the FCNC process $b \rightarrow s\gamma$. Since these process are loop induced in the SM their value is sensitive through quantum fluctuations also to yet non-observed particles exchange¹⁶. Namely, even if supersymmetric contributions dominate the electric dipole of the neutron this effect is at present experimentally inaccessible in the scenario considered. The $b \rightarrow s\gamma$ process is instead observed; this leads to constraints on the parameter space of the model stronger than those coming from direct searches of supersymmetric particles.

4.1. Electric dipole of the neutron

4.1.1. Introduction

The electric dipole of an elementary particle Q is a CP violating effect, since it must be proportional to the spin, and a coupling spin-electric field violates the T-invariance;

$$H_I = d_Q \vec{E} \vec{\sigma} \quad (4.1)$$

A measurement of such an effect could provide a crucial check of the mechanism of CP violation.

At date no electric dipole has been yet measured. The better experimental limits come from the electron in the leptonic sector, while the neutron is the better hadron for this kind of measurements, both for its neutrality that for wide availability:

$$d_e \lesssim 10^{-26} e \text{ cm} \quad (4.2)$$

$$d_n \lesssim 10^{-25} e \text{ cm} \quad (4.3)$$

The value of the electric dipole of the neutron can be related to the electric dipoles of the u and d quarks in a non-relativistic model

$$d_n = \frac{4}{3}d_d - \frac{1}{3}d_u. \quad (4.4)$$

The reliability rests on the fact that this kind of models successfully predict the hadrons magnetic moments.

¹⁶The same token applies also for very well measured quantities, that are only *corrected* at 1-loop.

In a quantum field theoretical context the electric dipole of the elementary particles can be computed, in much the same manner as for the $g - 2$ muon factor. An important property in common with the magnetic moment is that both these effects need a chirality flip, or in other words a mass insertion of the particle involved is required. More specifically:

$$\mathcal{L}_{eff} = -\frac{d_Q}{4} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}(x) \bar{Q}(x) \sigma_{\rho\sigma} Q(x) \quad (4.5)$$

where $F_{\mu\nu}$ is the external field, or also, in a form more convenient for the actual computation with Feynman graphs

$$d_Q = -\text{Im}(A_{LR}) \quad (4.6)$$

where A_{LR} is the coefficient of the amplitude ($S_{fi} = 1 + i(2\pi)^4 \delta^4(p_f - p_i) \mathcal{A}_{fi}$) for the chirality flipping process $Q_R \rightarrow Q_L$, in an external em field:

$$A_{LR} = A_{LR} [iq^\nu A^\mu(q) \overline{u(p_f)} \sigma_{\mu\nu} P_R u(p_i)] \quad (4.7)$$

we define as usual $q_\mu = (p_i - p_f)_\mu$.

A naive guess of the electric dipole of an elementary particle in the SM on dimensional grounds is $e_Q m_Q J G_F$, where J parametrizes the amount of CP violation, e_Q is the charge of the particle and m_Q its mass. In fact such contributions are absent in the SM, since the only source of CP violation, the complex phase contained in the Cabibbo-Kobayashi-Maskawa matrix, enters the charged interactions only, but is of no matter in the 1-loop transition up quark \rightarrow virtual down quark \rightarrow up quark, in the sense that the CKM matrix enters as $|K_{ud}|^2$. The first non zero effect arises at three-loop level: two-loop in the electroweak and one in the strong interactions:

$$d_n^{SM}(\text{quarks}) = O(10^{-34}) e \text{ cm}. \quad (4.8)$$

As a matter of fact, it is likely that the neutron EDM is dominated by long-distance (LD) effects [39], which lead to:

$$d_n^{SM}(LD) = O(10^{-32}) e \text{ cm}. \quad (4.9)$$

For a review on the predictions of the electric dipole moment in the SM see the beautiful work of Shabalin [40].

4.1.2. Radiatively induced electric dipole in MSSM

We will discuss in this section the prediction of electric dipole moments in the SUSY SM, in which the soft parameters are assumed real at the GUT scale. Recalling the results

of analysis done in the previous Chapter (“Complex parameters in the MSSM”), we see that no electric dipole of the leptons can be generated at 1-loop level: squarks exchange is needed. The squarks are instead entailed at 1-loop level for quarks EDM; as a result an electric dipole of the neutron is induced.

From a qualitative mass insertion analysis of the relevant diagrams it is easy to convince oneself that a non vanishing (and possibly complex) L - R mass insertion in the squark line is needed. In fact, concentrating on the d quark dipole, the dominant contribution is the chargino exchange: the L - R block of the up squark matrix is larger than that of the d squarks (it is proportional to the quark masses), the gluinos (or neutralinos) give a down-quark EDM proportional at the leading order to $\text{Im}((A_D)_{dd})$, which is zero in the scenario here considered.

Considering the chargino amplitude, it gives an effect proportional to the off-diagonal elements of the A_U matrix and to complex combinations of the relevant CKM mixings. In particular we may expect that the dominant contribution arises through the exchange of the “higgsino” component of the chargino field, via the quark-squark “flavour-chain”

$$d_R \rightarrow (\tilde{u}_L, \tilde{c}_L) \rightarrow \tilde{t}_R \rightarrow d_L \quad (4.10)$$

since we can take advantage of the presence of the large top Yukawa coupling. For large $\tan\beta$, then, also the presence of the down Yukawa coupling becomes important as an enhancement factor. Computing the relevant Feynman graph we find that the d -quark EDM amplitude can be written as

$$d_d = \frac{1}{(4\pi)^2} \sum_{a=1}^2 \frac{1}{m_{\tilde{\chi}_a^-}} V_{a2} U_{a2} \text{Im} \left[K^\dagger \gamma_U \left(S_{\tilde{u},R}^\dagger \mathcal{F} \left(\frac{m_{\tilde{u}}^2}{m_{\tilde{\chi}_a^-}^2} \right) S_{\tilde{u},L} \right) K \gamma_D \right]_{dd} \quad e \text{ cm} \quad (4.11)$$

The function $\mathcal{F}(x)$ is given by

$$\mathcal{F}(x) = \frac{1}{6(1-x)^3} (5 - 12x + 7x^2 + 2x(2-3x)\ln(x)) \quad (4.12)$$

In formula (4.11) $\mathcal{F}(m_{\tilde{u}}^2)$ must be interpreted as a 6×6 diagonal matrix. Notice that the presence of the Yukawa couplings signals that this effect entails higgsino vertices only (see also Fig. 5).

The estimate of the maximum dipole moment that one could expect in this model is obtained considering the lightest chargino to be close to the present experimental limit, say $m_{\tilde{\chi}_1} \simeq 50$ GeV, the masses of the lightest squarks to be $O(100)$ GeV. A numerical evaluation yields the following result: the SUSY contribution to the neutron EDM

generated by the elementary EDM of the quarks can be as large as

$$d_n^{SUSY} = O(10^{-30}) \left(\frac{\tan \beta}{10} \right) e \text{ cm} \quad (4.13)$$

where the linear dependence on $\tan \beta$ for large β comes from the presence of the down quark Yukawa coupling.

4.2. The $b \rightarrow s\gamma$ process

4.2.1. Introduction

Recently CLEO group reported the observation of the radiative decay $b \rightarrow s\gamma$. The measurement of BR for exclusive $B \rightarrow X_s \gamma$ decay, namely [41]

$$\text{BR}(B \rightarrow K^*(892) \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5} \quad (4.14)$$

and, more important, that of the inclusive branching ratio [42]

$$1.0 \times 10^{-4} < \text{BR}(B \rightarrow X_s \gamma) < 4.0 \times 10^{-4} \quad (4.15)$$

(at 95% confidence level) opened the way to crucial tests of a delicate sector of the SM; in fact in flavour changing neutral current processes the 1-loop structure of the SM compares in a sensitive way with any source of non-standard physics, as proved by the authors of ref. [43]. Many papers appeared to review and clarify the predictions for this process in the standard model and beyond [46–56].

Non supersymmetric two Higgs doublet models [46–47, 58], are in fact sharply constrained, since the charged higgs amplitude interferes constructively with the SM one, whereas in the SUSY extension of the SM there is the possibility of having destructive interference, as we will discuss in the following. The main part of this section is based on the paper [56].

4.2.2. Numerical analysis of the parameter space

The numerical analysis of the parameter space can be performed as follows. For each choice of the top mass, of $\tan \beta$ and of the soft parameters m_G and M_G the parameters of the theory are determined (that is we can compute the spectrum, the vertices and the

relevant amplitudes). The soft breaking parameters m and M are spanned in the ranges $[0, 800]$ GeV, and $[-400, 400]$ GeV respectively, and both the assignments $B = A - 1$ and $B = 2$ are considered.

An interesting feature that emerges is that the mechanism of radiative breaking imposes a maximum value of $\tan \beta$, and this value shows a dependence on the top mass. In the case under consideration we roughly have $\tan \beta_{max} \approx 45$, while for $m_t = 140$ $\tan \beta_{max} \approx 40$ and for $m_t = 180$ $\tan \beta_{max} \approx 50$. This statement is dependent on the window of values of (m, M) considered, (and also on the value of B , being the bound stronger in the $B = 2$ case) but the previous choice of range cannot be considered restrictive, if the aim of the analysis is to test the possibility of SUSY particles in the nearby of electroweak energy scale.

Moreover, if $|B| = 2$ instead of $B = A - 1$ is imposed, the region of parameter space for which the radiative breaking of the electroweak symmetry can be realized diminishes considerably. For $|B| > 2$ the allowed regions become even smaller. This observation is in agreement with the results of ref. [52], where the authors noticed that B lying in the neighbourhood of zero represents the most favourable case. The constraint $B = A - 1$, to be imposed if one does not require the existence of a grand unified scenario, includes naturally this region.

4.2.3. Amplitude for the process

The $b \rightarrow s\gamma$ decay can proceed in the MSSM via five different intermediate particles exchanges:

1. W^- + up-quark
2. H^- + up-quark
3. $\tilde{\chi}^-$ + up-squark
4. \tilde{g} + down-squark
5. $\tilde{\chi}^0$ + down-squark

The first subprocess is the only one present in the SM; the second one is present in its two-higgs doublets extension; the last three are genuinely supersymmetric in nature.

In Figs. 6 and 7 the relative size of the various amplitudes to the SM one are shown. Although the figures are drawn for given values of m_t and $\tan\beta$ ($m_t = 160$ GeV and $\tan\beta = 8$) they exhibit the general features of the various contributions for a wide range of parameters. In particular, we observe that gluino and neutralino exchange can be neglected in comparison with charged higgs and chargino amplitudes. The behaviour of the gluino contribution, despite of an α_s/α_w enhancement factor, can be understood considering that the FC effects in the gluino vertex are only radiatively induced, and that the squarks and the gluino are the particles more constrained by direct searches (with the possible exception of a light gluino that we will not consider).

In Figs. 8, 9 and 10 the ratios of charged higgs and chargino induced amplitudes with the SM contribution are shown for $\tan\beta = 2, 20, 40$, while the top mass is kept at 160 GeV. The higgs amplitude is shown as a function of the charged higgs mass, while the chargino amplitude is shown both as a function of the lightest chargino mass and the lightest stop mass. No substantially different features appear by varying m_t in a few ten GeV interval from our preferred value.

By inspection of Figs. 8–10, the interference patterns are evident: the charged higgs amplitude interferes always constructively with the SM one, while the chargino amplitude can give rise to destructive interference with the SM and H^+ amplitudes, becoming for large $\tan\beta$ the dominant contribution. This effect softens the impact of the CLEO upper limit in eq. (4.15) on the MSSM. In spite of that, the full inclusion of the present experimental limits is already enough to exclude a large portion of the tested parameter space.

The reason why the chargino exchange amplitude, but not that of charged higgs, is so important in the large $\tan\beta$ limit emerges yet before computing the amplitudes, simply comparing the couplings involved. The leading part of the chargino exchange amplitude is

$$b_R \xrightarrow{\sim \tan\beta} \text{virtual } \hat{u}_L \rightarrow \text{virtual } \hat{u}_R \xrightarrow{\sim 1} s_L \quad \sim \tan\beta \quad (4.16)$$

while for the charged higgs exchange we have

$$b_R \xrightarrow{\sim \tan\beta} \text{virtual } u_L \rightarrow \text{virtual } u_R \xrightarrow{\sim \frac{1}{\tan\beta}} s_L \quad \sim 1 \quad (4.17)$$

The reasons of the different behaviours of the couplings have been discussed in detail in the derivation of eq. (2.56). In next section a close look to this amplitude is given.

4.2.4. The chargino exchange

Let us enter in some more details on the structure of the chargino amplitude. Suppose that we can apply approximation (2.48); this is the case in particular if the wino mass M_2 is large with respect to the W mass while the mass of the higgsino $|\mu_R|$ is of the order of M_W , or *viceversa*. In this case we can derive a simple expression for the amplitude A_{LR} for the chirality flipping process $b_R \rightarrow s_L \gamma$ (defined in strict analogy with the amplitude (4.7)):

$$A_{LR}^{\tilde{\chi}^-} \approx -\frac{\epsilon}{(4\pi)^2} \left[\frac{m_b}{M_2^2} K^\dagger g S_{\tilde{u},L}^\dagger \mathcal{G}\left(\frac{m_{\tilde{u}}^2}{M_2^2}\right) S_{\tilde{u},L} g K \right. \\ \left. + \frac{m_b}{\mu_R^2} K^\dagger \gamma_U S_{\tilde{u},R}^\dagger \mathcal{G}\left(\frac{m_{\tilde{u}}^2}{\mu_R^2}\right) S_{\tilde{u},R} \gamma_U K \right. \\ \left. - \frac{1}{\mu_R} K^\dagger \gamma_U S_{\tilde{u},R}^\dagger \mathcal{F}\left(\frac{m_{\tilde{u}}^2}{\mu_R^2}\right) S_{\tilde{u},L} K \gamma_D \right]_{sb} \quad (4.18)$$

where the (positive and decreasing) function $\mathcal{G}(x)$ is defined as:

$$\mathcal{G}(x) = \frac{1}{36(1-x)^4} (7 - 12x - 3x^2 + 8x^3 + 6x(2-3x)\ln(x)) \quad (4.19)$$

In the specific form of eq. (4.18) the wino and the higgsino contributions are clearly disentangled. Each part corresponds to the terms we would write in a naive mass insertion analysis of the relevant Feynman graphs.

The wino contribution is of the form of the W and of the charged higgs contribution. The GIM mechanism [57] operates for diagonal squark mass matrix; in sketchy notations

$$\sum_{a=u,c,t} K_{as}^* K_{ab} \mathcal{G}_{\tilde{a}_L} \\ = K_{ts}^* K_{tb} (\mathcal{G}_{\tilde{t}_L} - \mathcal{G}_{\tilde{c}_L}) + K_{us}^* K_{ub} (\mathcal{G}_{\tilde{u}_L} - \mathcal{G}_{\tilde{c}_L})$$

The second term in previous expression can be neglected in very good approximation, while the first term is slightly modified if the stop mixing is considered: $\mathcal{G}_{\tilde{t}_L} \rightarrow \cos^2 \theta \mathcal{G}_{\tilde{t}_1} + \sin^2 \theta \mathcal{G}_{\tilde{t}_2}$. The only large contribution in the second line of eq. (4.18) is the one in which γ_t is selected. In this case there is no GIM suppression, but again the combination $K_{ts}^* K_{tb}$ can be factorized as for the SM contribution. These two first terms can be simplified in the approximation of eq. (2.40); using $\gamma_t^2 = g^2 \cdot (m_t^2/2M_W^2 \sin^2 \beta)$ and $g^2 = 8G_F M_W^2/\sqrt{2}$:

$$A_{I+II}^{\tilde{\chi}^-} = -m_b G_F \sqrt{\frac{\alpha}{(2\pi)^3}} K_{ts}^* K_{tb} \left\{ \frac{M_W^2}{M_2^2} \left[\mathcal{G}\left(\frac{m_{\tilde{t}_1}^2}{M_2^2}\right) + \mathcal{G}\left(\frac{m_{\tilde{t}_2}^2}{M_2^2}\right) - 2 \mathcal{G}\left(\frac{m_{\tilde{u}_L}^2}{M_2^2}\right) \right] \right. \\ \left. + \frac{m_t^2}{2 \sin^2 \beta \mu_R^2} \left[\mathcal{G}\left(\frac{m_{\tilde{t}_1}^2}{\mu_R^2}\right) + \mathcal{G}\left(\frac{m_{\tilde{t}_2}^2}{\mu_R^2}\right) \right] \right\} \quad (4.20)$$

Both these contributions have opposite phase compared to the charged higgs amplitudes; this is reminiscent of the exact cancellation that would take place in supersymmetric limit.

A short comment to specify this result in the low $\tan\beta$ scenario, for instance as implied by $b - \tau$ unification with $m_t \lesssim 180$ GeV. In this case $M_2 \ll \mu_R$, so the wino exchange dominates the amplitude, since $O(1/\mu)$ terms decouple. Despite to the appearances there is still a dependence on the sign of μ : this comes about from the effect of (2.42); in the case in which $\text{sign}(\mu) = -\text{sign}(M_G)$ the chargino amplitude is larger and the total amplitude smaller.

Let us finally come to the contribution of the last line. The presence of only one SUSY mass (that is μ_R) in the denominator, in contrast with the other two terms that are “doubly SUSY suppressed” does not enhance a priori this contribution, since the L - R structure implies an insertion of m_t (compare with eq. (2.34)); an enhancement may instead come from the fact that the function \mathcal{G}/\mathcal{F} is a decreasing function, and in zero its value is ~ 0.23 . It is worthwhile to notice that it corresponds exactly (in this approximation) to the contribution that gives rise to the dipole electric moment, eq. (4.11); at the same time this same term is the one that dominates for large $\tan\beta$.

We can proceed in simplifying this expression; referring to the approximation of eq. (2.41) we find

$$A_{III}^{\tilde{\chi}^-} = \sigma \cdot m_b G_F \sqrt{\frac{\alpha}{(2\pi)^3}} K_{ts}^* K_{tb} \left\{ \frac{1}{\sin 2\beta} \frac{m_t}{|\mu_R|} \left[\mathcal{F}\left(\frac{m_{\tilde{t}_1}^2}{\mu_R^2}\right) - \mathcal{F}\left(\frac{m_{\tilde{t}_2}^2}{\mu_R^2}\right) \right] \right\} \quad (4.21)$$

where σ , that is the phase relative to the W exchange amplitude is

$$\sigma = -\text{sign}(\mu_R) \text{sign}(m_{\tilde{t}_{LR}}^2) \quad (4.22)$$

This analytic expression presents some interesting features, and suggests some considerations:

1. *Size* (dependence on stop sector parameters):

The value of the amplitude is crucially dependent on the splitting of the two stop mass eigenstates $m_{\tilde{t}_{1,2}}$, \tilde{t}_1 being the lighter stop quark. Since $\mathcal{F}(x)$ is a positive, monotonically decreasing function of x in the interval $x \in [0, \infty]$ ($\mathcal{F}(0) = 5/6$, $\mathcal{F}(\infty) = 0$) the term in square brackets is maximized when $m_{\tilde{t}_1}$ is light. The splitting of the two mass eigenstates depends on the size of the L - R entry in the stop mass matrix, and corresponds to the L - R mass insertion on the squark line in the interaction representation of Fig. 11 (the term $\text{sign}(m_{\tilde{t}_{LR}}^2)$ of σ is the sign of this L - R mixing).

2. *Size* (dependence on μ parameter):

On the other side $|\mu_R|$ should be as light as possible in order not to suppress the contribution. Some words are needed to precise this assertion. Let us first observe that this part of the chargino amplitude vanishes in the limit $\mu_R \rightarrow 0$ (as $\mu_R \log(\mu_R^2)$) but also for $\mu_R \rightarrow \infty$ (as $1/\mu_R$). The first behaviour has to be expected, since from the diagram of Fig. 11 it appears that this component of the chargino amplitude is proportional to the $\tilde{h}_1 - \tilde{h}_2$ higgsino mixing; the second is simply the decoupling. We conclude that there is a maximum in between these two extrema, precisely for $|\mu_R| \sim 2/5 m_{\tilde{t}_1}$. Recalling that in this approximation $|\mu_R|$ is the mass of one chargino—or more precisely the mass of the higgsino—, we can see that the amplitude is maximally enhanced, keeping the stop masses fixed, for $|\mu_R|$ equal to the maximum between the experimental limit on lightest chargino mass and $2/5$ of the lightest stop mass. Since light stops are compatible with present experimental data the absolute minimum is for maximally light higgsino mass, $|\mu_R| \sim 47 \text{ GeV}$.

3. Sign (the large $\tan \beta$ limit):

In the large $\tan \beta$ limit the sign σ depends on the product of the parameter μ_R , that is the same of μ_G and on the sign of the gaugino mass at the GUT scale M_G (see eq. (2.43)). This implies that the region in which the chargino amplitude gives rise to a destructive interference effect corresponds to the region in which μ is negative and M_G positive or *viceversa*. This observation is important since in the large $\tan \beta$ limit the term eq. (4.21) dominates the chargino amplitude¹⁷.

4.2.5. The inclusive branching ratio

1. Introduction

Once the amplitude is known at the M_Z scale (in the SM or MSSM) we must resort to renormalisation group to keep into account the QCD effects up to energy scales relevant for the process, of the order of the M_b mass. In fact sizeable operator mixing effects must be considered: this has been first pointed in [44], and then thoroughly considered in refs. [45]. The present experimental error on α_s implies an uncertainty in the predicted $BR(b \rightarrow s\gamma)$ of about 15–20%; but before that, it is estimated that Non-Leading-Corrections could bring up to 30% of uncertainty in the prediction (for a recent and complete discussion

¹⁷In the IR fixed point scenario the sign σ of this term is instead typically minus, since the large value of μ_R dominates $m_{\tilde{t}_{LR}}^2$ (2.34).

on these issues see A.J. Buras *et al.* in ref. [45]). A way to consider such uncertainties is simply to compare the predictions with bounds on the inclusive branching ratio slightly larger than the experimental ones.

2. The predicted inclusive branching ratio

In Figs. 12–15 the numerical results for the branching ratio of the process under consideration are presented. The reference value $m_t = 160$ GeV is chosen, while $\tan\beta$ takes the value 2, 8, 20, 40 in figures 12, 13, 14, 15 respectively. In each figure the total branching ratio is plotted versus three relevant SUSY masses: that of the charged Higgs, of the lightest chargino and of the lightest top squark. The cases $B = A - 1$ and $B = 2$ are compared. The horizontal solid line represents the SM result, which depends only on the top quark mass and the value of the strong coupling, $\alpha_s(m_Z)_{\overline{MS}} = 0.12$ in all the figures to simplify comparison between the SM and the MSSM predictions.

When considering values of $\tan\beta$ of $O(1)$ (Fig. 12) the destructive interference effect, discussed in the previous subsection, is quite small. In this case, already the present CLEO inclusive upper bound restricts in a sizeable way the area in parameter space allowed by the model. For instance, a lower bound of about 200 GeV on the charged Higgs mass is obtainable for both choices of B (recall that the corresponding LEP-I lower bound is 47 GeV). However, as soon as $\tan\beta$ is of $O(10)$ and larger the destructive interference effect of the chargino amplitude becomes substantial and the CLEO upper bound would not be by itself very effective in constraining the low energy SUSY spectrum. This is dramatically evident from the strong similarity of the plots of the branching ratio and the single chargino amplitude when $\tan\beta = 20$ or 40 (regions of chargino dominance can be also identified for lighter particle masses). Finally, it has to be noticed that for large $\tan\beta$ values and $B = 2$ the allowed parameter space for the model becomes very small and the model becomes correspondingly quite predictive.

3. The allowed parameter space

The experimental determination of the branching ratio for the inclusive decay is crucial in excluding large portions of the available parameter space of the model; this can be already seen by overlapping both the upper and lower bounds of eq. (4.15) to Figs. 12–15. In a more suggestive way the outcomes of imposing the full constraint of eq. (4.15) are shown in Figs. 16 and 17. In Fig. 16 the shaded areas show the implementation of the CLEO inclusive limits in the plane of the mass of the lightest Higgs boson h and the mass of the CP odd scalar A (denoted as H_1^0 and H_3^0 in figures). For a wide range

of $\tan\beta$ the masses of h and A allowed by internal consistency of the model and the experimental $b \rightarrow s\gamma$ bounds are higher than present LEP limits ($m_{h,A} \gtrsim 50$ GeV, for a recent review see for instance ref. [59]). Notice that the effect of radiative corrections on the Higgs masses that bring the lightest Higgs above M_Z , and in particular the fact that the mass increases with $\tan\beta$ as expected. We conclude that this complete numerical analysis confirms the expectations that the low $\tan\beta$ scenario guarantees higgs detection at LEP-200. In Fig. 17 the corresponding regions in the gluino mass - μ_R plane are shown. It is evident that, apart that for low $\tan\beta$ values, gluino masses must be heavier than ~ 200 GeV; this bound is stronger than those from direct searches.

APPENDICES

A. Conventions

Invariant antisymmetric tensors in $SL(2, C)$:

$$\begin{aligned}\epsilon^{12} &= -\epsilon_{12} = 1 \\ \epsilon^{i\dot{2}} &= -\epsilon_{i\dot{2}} = 1\end{aligned}\tag{A.1}$$

Bispinorial representations of Lorentz group ($A, B = 1, 2$ are $SL(2, C)$ indices):

$$\begin{aligned}\chi_A &= \epsilon_{AB} \cdot \chi^B \\ \chi^A &= \epsilon^{AB} \cdot \chi_B \\ \chi_{\dot{A}} &= \epsilon_{\dot{A}\dot{B}} \cdot \chi^{\dot{B}} \\ \chi^{\dot{A}} &= \epsilon^{\dot{A}\dot{B}} \cdot \chi_{\dot{B}}\end{aligned}\tag{A.2}$$

Conjugation on bispinors:

$$\begin{aligned}\overline{\chi^A} &= \bar{\chi}^{\dot{A}} \\ \overline{\chi_A} &= \bar{\chi}_{\dot{A}}\end{aligned}\tag{A.3}$$

Scalars in the tensor product:

$$\begin{aligned}\theta\psi &= \theta^A \psi_A \\ \bar{\theta}\bar{\psi} &= \bar{\theta}_{\dot{A}} \bar{\psi}^{\dot{A}}\end{aligned}\tag{A.4}$$

Fierzing:

$$\begin{aligned}\theta^A \theta^B &= -1/2 \cdot \theta^2 \\ (\theta\psi)(\theta\phi) &= -1/2 \cdot \theta^2 \cdot (\psi\phi)\end{aligned}\tag{A.5}$$

Vectors in the tensor product:

$$\begin{aligned}\sigma_{\dot{A}\dot{B}}^\mu &= (1, \sigma)_{\dot{A}\dot{B}} \\ \sigma_{\mu\dot{A}\dot{B}} &= (1, -\sigma)_{\dot{A}\dot{B}} \\ \bar{\sigma}^{\mu\dot{A}\dot{B}} &= (1, -\sigma)^{\dot{A}\dot{B}} \\ \bar{\sigma}_\mu^{\dot{A}\dot{B}} &= (1, \sigma)^{\dot{A}\dot{B}}\end{aligned}\tag{A.6}$$

Useful relations

$$\begin{aligned}\psi\theta &= \theta\psi \\ \bar{\psi}\bar{\theta} &= \bar{\theta}\bar{\psi} \\ \bar{\psi}\bar{\sigma}^\mu\phi &= -\phi\sigma^\mu\bar{\psi}\end{aligned}\tag{A.7}$$

Conjugation on bilinears:

$$\begin{aligned}\overline{\theta^A \psi^B} &= \bar{\psi}^{\dot{B}} \bar{\theta}^{\dot{A}} \\ \overline{\theta\psi} &= \bar{\theta}\bar{\psi} \\ \overline{\theta\sigma_\mu\psi} &= \psi\sigma_\mu\bar{\theta}\end{aligned}\tag{A.8}$$

Gauge transformations (latin indices are gauge indices; ϵ is real):

$$\begin{aligned}\delta\psi^a &= i[\epsilon_\alpha(+T^\alpha)]^{ab}\psi^b \\ \delta\bar{\psi}^a &= i[\epsilon_\alpha(-\bar{T}^\alpha)]^{ab}\bar{\psi}^b\end{aligned}\tag{A.9}$$

Covariant derivatives:

$$\mathcal{D}_\mu = \partial_\mu + igV_\mu\tag{A.10}$$

Gauge lagrangians for the bispinors:

$$\begin{aligned}i\psi\hat{\mathcal{D}}\bar{\psi} &= i\psi\sigma^\mu\mathcal{D}_\mu\bar{\psi} \\ i\bar{\psi}\hat{\mathcal{D}}\psi &= i\bar{\psi}\bar{\sigma}^\mu\mathcal{D}_\mu\psi\end{aligned}\tag{A.11}$$

Four component spinors (greek indices are Lorentz indices):

$$\Psi_\alpha^D = \begin{pmatrix} \lambda_A \\ \bar{\chi}^{\dot{A}} \end{pmatrix}\tag{A.12}$$

$$\bar{\Psi}_\alpha^D = (\chi^A, \bar{\lambda}_{\dot{A}})\tag{A.13}$$

Gamma matrices and bispinors:

$$\gamma_{\alpha\beta}^\mu = \begin{pmatrix} 0_A^B & \sigma_{A\dot{B}}^\mu \\ \bar{\sigma}^{\mu\dot{A}B} & 0_{\dot{B}}^A \end{pmatrix}\tag{A.14}$$

Chirality matrix:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\tag{A.15}$$

Chirality projectors:

$$\begin{aligned}P_L &= \frac{1-\gamma^5}{2} \\ P_R &= \frac{1+\gamma^5}{2}\end{aligned}\tag{A.16}$$

Relations between some ‘‘Weyl’’ bilinears and the ‘‘Dirac’’ bilinears:

$$\begin{aligned}\bar{\Psi}_D\gamma^\mu\Psi'_D &= \bar{\lambda}\bar{\sigma}^\mu\lambda' + \chi\sigma\chi' \\ \bar{\Psi}_DP_L\Psi'_D &= \lambda\lambda' \\ \bar{\Psi}_DP_R\Psi'_D &= \bar{\chi}\bar{\lambda}'\end{aligned}\tag{A.17}$$

Conjugation matrix:

$$C_{\alpha\beta} = \begin{pmatrix} \epsilon_{AB} & 0 \\ 0 & \epsilon^{\dot{A}\dot{B}} \end{pmatrix}\tag{A.18}$$

Majorana spinors:

$$C_{\alpha\beta}\bar{\Psi}_\beta^M = \Psi_\alpha^M\tag{A.19}$$

$$\Psi_\alpha^M = \begin{pmatrix} \chi^A \\ \bar{\chi}^{\dot{A}} \end{pmatrix}\tag{A.20}$$

B. Beta function coefficients

We list in this Appendix the beta function coefficients relative to the gauge sector in the SM and in its supersymmetric extension; the two-loop coefficients are taken from ref. [60].

Coefficients of the **SM 1-loop** running, assuming N_g families and n_l light Higgs doublets (in square brackets the case $N_g = 3, n_l = 1$):

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_g \cdot \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + n_l \cdot \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix} \quad \left[= \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix} \right] \quad (\text{B.1})$$

List of b -differences and the factor d (in square brackets the case $N_g = 3, n_l = 1$):

$$\begin{pmatrix} b_2 - b_3 \\ b_3 - b_1 \\ b_1 - b_2 \end{pmatrix} = \begin{pmatrix} 11/3 \\ -11 \\ 22/3 \end{pmatrix} + n_l \cdot \begin{pmatrix} 1/6 \\ -1/10 \\ -1/15 \end{pmatrix} \quad \left[= \begin{pmatrix} 23/6 \\ -111/10 \\ 109/15 \end{pmatrix} \right] \quad (\text{B.2})$$

$$d = (b_1 - b_3) + \frac{3}{5} (b_2 - b_3) = \frac{3}{5} (22 + n_l/3) \quad [= 67/5] \quad (\text{B.3})$$

Coefficients of the **SM 2-loop** running, assuming N_g families and n_l light Higgs doublets (in square brackets the case $N_g = 3, n_l = 1$):

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -136/3 & 0 \\ 0 & 0 & -102 \end{pmatrix} + N_g \cdot \begin{pmatrix} 19/15 & 3/5 & 44/15 \\ 1/5 & 49/3 & 4 \\ 11/30 & 3/2 & 76/3 \end{pmatrix} \\ + n_l \cdot \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 13/6 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \left[= \begin{pmatrix} 3.98 & 2.7 & 8.8 \\ 0.9 & 35/6 & 12 \\ 1.1 & 4.5 & -26 \end{pmatrix} \right] \quad (\text{B.4})$$

Coefficients of the **SUSY SM 1-loop** running, assuming N_g families and n_d Higgs doublets; in n_d there are light and heavy doublets; $n_d = n_l + n_h$ (in square brackets the case $N_g = 3, n_d = 2$):

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_g \cdot \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + 3 \cdot n_d \cdot \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix} \quad \left[= \begin{pmatrix} 66/10 \\ 1 \\ -3 \end{pmatrix} \right] \quad (\text{B.5})$$

List of b -differences and the factor d (in square brackets the case $N_g = 3, n_d = 2$):

$$\begin{pmatrix} b_2 - b_3 \\ b_3 - b_1 \\ b_1 - b_2 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \\ 6 \end{pmatrix} + 3 \cdot n_d \cdot \begin{pmatrix} 1/6 \\ -1/10 \\ -1/15 \end{pmatrix} \quad \left[= \begin{pmatrix} 4 \\ -48/5 \\ 28/5 \end{pmatrix} \right] \quad (\text{B.6})$$

$$d = (b_1 - b_3) + \frac{3}{5} (b_2 - b_3) = \frac{3}{5} (18 + n_d) \quad [= 60/5] \quad (\text{B.7})$$

Coefficients of the **SUSY SM 2-loop** running, assuming N_g families and n_d Higgs doublets (in square brackets the case $N_g = 3, n_d = 2$):

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + N_g \cdot \begin{pmatrix} 38/15 & 6/5 & 88/15 \\ 2/5 & 14 & 8 \\ 22/30 & 6/2 & 68/3 \end{pmatrix} \\ + n_d \cdot \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 7/2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \left[= \begin{pmatrix} 7.96 & 5.4 & 17.6 \\ 1.8 & 25 & 24 \\ 2.2 & 9 & 14 \end{pmatrix} \right] \quad (\text{B.8})$$

C. Input quantities at different scales

It is worthwhile to give here some technical details on a specific numerical procedure useful in typical GUT situations, like in the study of $b - \tau$ unification, and in general whenever some input quantities are known at different energy scales.

The mathematical problem in both cases can be described in the following general terms: given a system of N ordinary differential equations (namely a set of RGE) and given n initial and $N - n$ final conditions, find the set of the solutions of the system (in particular the set of $N - n$ initial conditions consistent with the given final conditions). This kind of problems are called boundary problems.

The numerical technique proposed is known as “shooting” algorithm. It requires to make a guess for the lacking variables at the initial time, and after having evolved the system to the final time, to assess the merit of the guess and to optimize the procedure. More concretely one can notice that from the algorithmic point of view, using of a routine to solve the differential equations and another one to solve nonlinear simultaneous equations, the problem can be tackled solving in the $N - n$ unknowns y_{in} the simultaneous $N - n$ equations

$$y(x_{in}, y_{in}; t_{in}, t_{fin})|_{RGE} - y_{fin} = 0 \quad (C.1)$$

where the vectors x_{in} and y_{fin} are known and the vector $y(x, y; t_{in}, t_{fin})|_{RGE}$ is the evolved of the y -vector. The convergence of the procedure depends in general not only on the system and on the given conditions, but also on the starting point y_{in} .

D. RGE for Softly Broken SUSY Gauge theories

In this Appendix I apply the method described in Chapter 1 to derive the RGE for the general $N=1$ supersymmetric theory; then the results are specified to case the supersymmetric extension of the SM.

D.1. RGE for the Wess-Zumino model

Let us first illustrate how the method works for a simple supersymmetric model, the Wess-Zumino model with N -chiral superfield (no vector superfield). We can compute the supertrace using the mass matrices of eq. (1.30) and eq. (1.29), in which we set to zero the gauge and the soft breaking parameters.

Referring to the superpotential f given in eq. (1.8) the result of the computation is:

$$V_\infty = -2k f^a X_a^b f_b \quad (\text{D.1})$$

where I used $k = \ln \Lambda / 32\pi^2$, and defined the constants X as

$$X_a^b = f_{acd} f^{bcd} . \quad (\text{D.2})$$

In this case eq. (1.25) reads,

$$\hat{f}^a \hat{f}_a = f^a f_a - 2k f_a X_b^a f^b + O(X^2) \quad (\text{D.3})$$

which is solved by

$$\hat{f}^a = f^a - k X_b^a f^b + O(X^2) \quad (\text{D.4})$$

that is, neglecting the $O(X^2)$ terms

$$\begin{cases} \hat{l}^a & = l^a - k X_b^a l^b \\ \hat{\mu}^{ab} \hat{z}_b & = \mu^{ab} z_b - k X_b^a \mu^{bc} z_c \\ \hat{f}^{abc} \hat{z}_b \hat{z}_c & = f^{abc} z_b z_c - k X_b^a f^{bcd} z_c z_d . \end{cases} \quad (\text{D.5})$$

At this point one needs some further informations to solve without ambiguity for the renormalised parameters. We can prove, just looking at the relevant Feynman graphs, that the anomalous dimension of the scalar fields depends on f^{abc} only through X_b^a ; more explicitly

$$\hat{z}_a = (\delta_a^{a'} + \alpha k X_a^{a'}) z_{a'} \quad (\text{D.6})$$

where α is an undetermined constant. With this supplementary information we get from eq. (D.5) the following equation for the parameter μ^{ab} :

$$\hat{\mu}^{ac} z_c = \mu^{ac} z_c - k X_b^a \mu^{bc} z_c - \alpha k \mu^{ab} X_b^c z_c \quad (\text{D.7})$$

Using the symmetry $\mu^{ab} = \mu^{ba}$ we conclude that $\alpha = 1$. The same procedure for f^{abc} allows us to obtain

$$\begin{cases} \hat{l}^a &= (\delta_{a'}^a - k X_{a'}^a) l^{a'} \\ \hat{\mu}^{ab} &= (\delta_{a'}^a - k X_{a'}^a)(\delta_{b'}^b - k X_{b'}^b) \mu^{a'b'} \\ \hat{f}^{abc} &= (\delta_{a'}^a - k X_{a'}^a)(\delta_{b'}^b - k X_{b'}^b)(\delta_{c'}^c - k X_{c'}^c) f^{a'b'c'} \\ \hat{z}_a &= (\delta_a^{a'} + k X_a^{a'}) z_{a'} \end{cases} \quad (\text{D.8})$$

This equations are known as *non-renormalisation* of the superpotential, because they imply the equation $\hat{f}(\hat{z}) = f(z)$; we see that, in a sense, we have only the (logarithmic) wavefunction renormalisation.

We get the corresponding RGE deriving with respect to the logarithmic “divergence” factor $-2k$, that is with respect to the momentum scale $t = \frac{1}{(4\pi)^2} \ln q$, (q appears implicitly in the logarithm together with Λ : $\ln(\Lambda/q)$)

$$\frac{d}{dt} l^a = \frac{1}{2} \{ X_{a'}^a l^{a'} \} \quad (\text{D.9})$$

$$\frac{d}{dt} \mu^{ab} = \frac{1}{2} \{ X_{a'}^a \mu^{a'b} + X_{b'}^b \mu^{ab'} \} \quad (\text{D.10})$$

$$\frac{d}{dt} f^{abc} = \frac{1}{2} \{ X_{a'}^a f^{a'bc} + X_{b'}^b f^{ab'c} + X_{c'}^c f^{abc'} \} \quad (\text{D.11})$$

For notational convenience we do not write the hats denoting renormalised parameter in the renormalisation group equations.

D.2. RGE in the general case

Next we consider the full scalar potential in presence of soft breaking terms: (eqs (1.13), (1.19) and (1.20)):

$$V = V_{SU5Y} + V_{soft} + V_{scalar} \quad (\text{D.12})$$

It is good policy in the general case to organize the computation of the supertrace of $\mathcal{M}^4(z)$ using the “modularity” of the mass matrices; *i.e.* computing separately the effect of *i*) the parameters in the superpotential – as we have done explicitly in the example of the Wess-Zumino model –, *ii*) the gauge parameters, *iii*) the soft breaking parameters and

iv) the “interference” terms (for instance those in which the gauge and the soft breaking parameters appear). The relevant mass matrices are listed in eq. (1.28), eq. (1.29) and eq. (1.30). In order to simplify the calculation it is very useful to take advantage also of the gauge covariance of the various terms in the scalar potential, as in the following example:

$$\delta_\alpha f(z) = \frac{\partial f}{\partial z_a} \delta_\alpha z = \frac{\partial f}{\partial z_a} T_a^{\alpha b} z_b = 0 \quad \Rightarrow \quad f^a d_b^\alpha = 0 \quad (\text{D.13})$$

Moreover, turning to the solution of eq. (1.25), it is convenient to compute first the effect of the supersymmetric parameters alone (gauge couplings+parameters in the superpotential), solving for the supersymmetric renormalised parameters and then compute the effect of the soft breaking parameters:

$$\hat{V}_{SUSY}(\hat{z}) = V_{SUSY} - k \text{Str } \mathcal{M}_{SUSY}^4 \quad (\text{D.14})$$

$$\hat{V}_{soft}(\hat{z}) = V_{soft} - k(\text{Str } \mathcal{M}^4 - \text{Str } \mathcal{M}_{SUSY}^4) \quad (\text{D.15})$$

Notice that the soft breaking parameters are all massive; therefore they cannot change the renormalisation of z_a, g_α, f^{abc} ; one can also prove that they cannot change the renormalisation of μ^{ab} ; this implies that the previous splitting is correct when solving for these parameters. On the other hand it is worth mentioning that l^a plays only a formal rôle if the soft breaking parameters L^a and M^{ab} are also present. In fact, as one can prove by expanding in components eq. (D.12), l^a appears always in the combinations $C^a \equiv L^a + \mu^{ab} l_b$, $C^{ab} \equiv M^{ab} + f^{abc} l_c$ in the the scalar potential. This means that when these parameters are all present at once the splitting $V_{SUSY} - V_{soft}$ is somewhat arbitrary. However, since l^a, L^a and M^{ab} appear linearly in C^a and C^{ab} the superposition of the RGE’s that we separately obtain from eq. (D.14) ($\rightarrow l^a$) and eq. (D.15) ($\rightarrow L^a, M^{ab}$) reproduce correctly and unambiguously the RGE we would obtain for C^a and C^{ab} from eq. (1.25).

Let us finally list the RGE for the general SUSY theory with soft breakings. The variable t is defined in accord with eq. (D.31).

$$\frac{d}{dt} l^a = \frac{1}{2} X_{a'}^a l^{a'} \quad (\text{D.16})$$

$$\frac{d}{dt} \mu^{ab} = \frac{1}{2} \{ X_{a'}^a \mu^{a'b} + X_{b'}^b \mu^{ab'} - 8g_\alpha^2 C_\alpha(a) \mu^{ab} \} \quad (\text{D.17})$$

$$\begin{aligned} \frac{d}{dt} f^{abc} &= \frac{1}{2} \{ X_{a'}^a f^{a'bc} + X_{b'}^b f^{ab'c} + X_{c'}^c f^{abc'} \\ &\quad - 4g_\alpha^2 [C_\alpha(a) + C_\alpha(b) + C_\alpha(c)] f^{abc} \} \end{aligned} \quad (\text{D.18})$$

$$\frac{d}{dt} L^a = \frac{1}{2} \{ X_{a'}^a L^{a'} + 2\eta^{acd} f_{bcd} l^b + 4m_b^{2c} f^{abd} \mu_{cd} \}$$

$$+2\mu^{ab}f_{bcd}M^{cd} + 2\eta^{abc}M_{bc}\} \quad (\text{D.19})$$

$$\begin{aligned} \frac{d}{dt}M^{ab} &= \frac{1}{2}\{X_{a'}^a M^{a'b} + X_{b'}^b M^{ab'} \\ &\quad -8g_\alpha^2 C_\alpha(a)M^{ab} + 16\mu^{ab}\mu_\alpha g_\alpha^2 C_\alpha(a) + \\ &\quad +2(f^{abc}M^{de} + \mu^{ac}\eta^{bde} + \mu^{bc}\eta^{ade})f_{cde}\} \end{aligned} \quad (\text{D.20})$$

$$\begin{aligned} \frac{d}{dt}\eta^{abc} &= \frac{1}{2}\{X_{a'}^a \eta^{a'bc} + X_{b'}^b \eta^{ab'c} + X_{c'}^c \eta^{abc'} \\ &\quad +4g_\alpha^2(2\mu_\alpha f^{abc} - \eta^{abc})[C_\alpha(a) + C_\alpha(b) + C_\alpha(c)] \\ &\quad +2(f^{abf}\eta^{cde} + f^{acf}\eta^{bde} + f^{bcf}\eta^{ade})f_{def}\} \end{aligned} \quad (\text{D.21})$$

$$\begin{aligned} \frac{d}{dt}m_a^{2b} &= \frac{1}{2}\{X_{a'}^{a'} m_a^{2b} + X_{b'}^b m_a^{2b'} + 4g_\alpha d_a^{\alpha b} \text{Tr}(T^\alpha m^2) \\ &\quad +4m_c^{2d} f_{ade} f^{bce} + 2\eta_{acd}\eta^{bcd} - 16g_\alpha^2 |\mu_\alpha|^2 C_\alpha(a)\delta_a^b\} \end{aligned} \quad (\text{D.22})$$

$$\frac{d}{dt}g_\alpha = \frac{1}{2}\{T_2(\alpha) - 3C_\alpha(\text{Adj})\}g_\alpha^3 \quad (\text{D.23})$$

$$\frac{d}{dt}\mu_\alpha = \{T_2(\alpha) - 3C_\alpha(\text{Adj})\}g_\alpha^2 \mu_\alpha \quad (\text{D.24})$$

In all but the two last equations repeated indices are summed. The last equation represents the gaugino mass renormalisation, and must be obtained with a different method, for instance computing the relevant Feynman graphs. The Dynkin index $T_2(\alpha)$ is computed for the reducible representation that contains *all* the scalar fields of the theory; the eigenvalue of the Casimir operator $C_\alpha(a)$ (resp. $C_\alpha(\text{Adj})$) is computed for the representation of the scalar fields z_a (resp. for the adjoint representation); the index α labels the group.

D.3. RGE in the SUSY SM

The previous set of RGE's can be applied to the case of the SUSY SM. We will identify the scalar fields in the SUSY SM using the index a of the multiplet z_a in the following way:

$$a \in \{H_{1\sigma}, H_{2\sigma}, L_{i\sigma}, E_i^c, Q_{i\sigma}^\alpha, U_i^{c\alpha}, D_i^{c\alpha}\} \quad (\text{D.25})$$

The function $f(z)$ for the SUSY SM is chosen as

$$f_{SM} = \epsilon^{\sigma\tau}[\Gamma_E^{ij} H_{1\sigma} L_{i\tau} E_j^c + \Gamma_D^{ij} H_{1\sigma} Q_{i\tau} D_j^c + \Gamma_U^{ij} H_{2\sigma} Q_{i\tau} U_j^c + \mu H_{1\sigma} H_{2\tau}] \quad (\text{D.26})$$

where I have written only the $SU(2)_L$ indices. The soft terms contained in $\eta(z)$ are:

$$\eta_{SM} = \epsilon^{\sigma\tau}[\Gamma_E^{Aij} H_{1\sigma} L_{i\tau} E_j^c + \Gamma_D^{Aij} H_{1\sigma} Q_{i\tau} D_j^c + \Gamma_U^{Aij} H_{2\sigma} Q_{i\tau} U_j^c + B\mu H_{1\sigma} H_{2\tau}] \quad (\text{D.27})$$

Considering the fact that soft breaking scalar mass terms have to be gauge invariant we write:

$$\begin{aligned}
& m_{H_1}^2 (H_{1\sigma})^* H_{1\sigma} + m_{H_2}^2 (H_{2\sigma})^* H_{2\sigma} \\
& + m_L^{2ij} (L_{i\sigma})^* L_{j\sigma} + m_Q^{2ij} (Q_{i\sigma}^\alpha)^* Q_{i\sigma}^\alpha \\
& + m_E^{2ji} (E_i^c)^* E_j^c + m_U^{2ji} (U_i^{c\alpha})^* U_j^{c\alpha} + m_D^{2ji} (D_i^{c\alpha})^* D_j^{c\alpha}
\end{aligned} \tag{D.28}$$

Notice that the fact that the mass matrices m_b^{2a} of the previous section commute with the gauge group generators does not imply that they are completely diagonal, since there are family replicas. I wrote m_E^{2ji} (resp. m_U^{2ji}, m_D^{2ji}), instead of m_E^{2ij} to refer to the mass matrix of the particle with charge -1 (resp. $+2/3, -1/3$). This point can be formulated more explicitly with the notation described in section 2.1.1, that is restoring the indices L, R : m_E^2 is the mass matrix of E_R , that is $\mathcal{L} \ni -E_R^* m_E^2 E_R$. We choose to identify E_L^c as a component of the multiplet $\{z_a\}$; that is we are working with $E_L^c = E_R^*$; and $E_R^* m_E^2 E_R = (E_L^c)^* (m_E^2)^T E_L^c$.

By comparison with the notations of the previous section one writes for instance

$$\begin{aligned}
f^{H_{1\sigma} Q_{i\tau}^\alpha D_j^{c\beta}} &= \Gamma_D^{ij} \epsilon^{\sigma\tau} \delta_{\alpha\beta} \\
\mu^{H_{1\sigma} H_{2\tau}} &= \mu \epsilon^{\sigma\tau}
\end{aligned} \tag{D.29}$$

The formulae for the terms in $\eta(z)$ are completely analogous. Some care is required when we consider the soft breaking scalar mass term; for instance

$$\begin{aligned}
\{m^2\}_{E_j^c}^{E_i^c} &= m_E^{2ji} \\
\{m^2\}_{Q_{i\sigma}^\alpha}^{Q_{j\tau}^\beta} &= m_Q^{2ij} \delta_\alpha^\beta \delta_\sigma^\tau
\end{aligned} \tag{D.30}$$

The computation of the matrices X_b^a is a little bit cumbersome; it is important to remember that f^{abc} are symmetric by definition and from this some factors two result, as for instance in $X_{H_{1\sigma}}^{H_{2\rho}} = 2 \text{tr}(\Gamma_E^\dagger \Gamma_E) \delta_\rho^\sigma + 2 \cdot 3 \text{tr}(\Gamma_D^\dagger \Gamma_D) \delta_\rho^\sigma$.

At this point we can apply the results of eqs. (D.16–D.24) to the model under consideration. Last step consists in passing from Γ_x couplings to Yukawa couplings Y_x , as in eq. (2.13); similar definitions are given for the couplings Y_x^A . We list in the following the whole set of the SUSY SM renormalisation group equations.

For notational convenience, the scale variable t in the following equations is defined

as

$$t = \frac{1}{(4\pi)^2} \ln \left(\frac{q}{q_0} \right) \quad (\text{D.31})$$

Yukawa couplings Y_E, Y_U, Y_D and μ -parameter:

$$\begin{aligned} \frac{d}{dt} Y_E &= -3(g_2^2 + g_1^2)Y_E \\ &\quad + 3Y_E Y_E^\dagger Y_E \\ &\quad + \text{Tr}(Y_E^\dagger Y_E + 3Y_D^\dagger Y_D)Y_E \end{aligned} \quad (\text{D.32})$$

$$\begin{aligned} \frac{d}{dt} Y_U &= -\left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{9}g_1^2\right)Y_U \\ &\quad + 3Y_U Y_U^\dagger Y_U + Y_D Y_D^\dagger Y_U \\ &\quad + 3\text{Tr}(Y_U^\dagger Y_U)Y_U \end{aligned} \quad (\text{D.33})$$

$$\begin{aligned} \frac{d}{dt} Y_D &= -\left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{9}g_1^2\right)Y_D \\ &\quad + 3Y_D Y_D^\dagger Y_D + Y_U Y_U^\dagger Y_D \\ &\quad + 3\text{Tr}(Y_D^\dagger Y_D)Y_D + \text{Tr}(Y_E^\dagger Y_E)Y_D \end{aligned} \quad (\text{D.34})$$

$$\begin{aligned} \frac{d}{dt} \mu &= [-(3g_2^2 + g_1^2) \\ &\quad + \text{Tr}(Y_E^\dagger Y_E + 3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D)]\mu \end{aligned} \quad (\text{D.35})$$

Gauge coupling constants:

$$\frac{d}{dt} g_3 = [-9 + 2N_g]g_3^3 \quad (\text{D.36})$$

$$\frac{d}{dt} g_2 = [-5 + 2N_g]g_2^3 \quad (\text{D.37})$$

$$\frac{d}{dt} g_1 = \left[1 + \frac{10}{3}N_g\right]g_1^3 \quad (\text{D.38})$$

Soft breaking parameters Y_E^A, Y_U^A, Y_D^A and B .

$$\begin{aligned}
\frac{d}{dt}Y_E^A &= -3(g_2^2 + g_1^2)Y_E^A \\
&+ 2 \cdot 3(g_2^2\mu_2 + g_1^2\mu_1)Y_E \\
&+ 5Y_E Y_E^\dagger Y_E^A + 4Y_E^A Y_E^\dagger Y_E \\
&+ \text{Tr}(Y_E^\dagger Y_E + 3Y_D^\dagger Y_D)Y_E^A \\
&+ 2 \text{Tr}(Y_E^\dagger Y_E^A + 3Y_D^\dagger Y_D^A)Y_E
\end{aligned} \tag{D.39}$$

$$\begin{aligned}
\frac{d}{dt}Y_U^A &= -\left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{9}g_1^2\right)Y_U^A \\
&+ 2\left(\frac{16}{3}g_3^2\mu_3 + 3g_2^2\mu_2 + \frac{13}{9}g_1^2\mu_1\right)Y_U \\
&+ 5Y_U Y_U^\dagger Y_U^A + 4Y_U^A Y_U^\dagger Y_U + 2Y_D^A Y_D^\dagger Y_U + Y_D Y_D^\dagger Y_U^A \\
&+ 3 \text{Tr}(Y_U^\dagger Y_U)Y_U^A \\
&+ 2 \cdot 3 \text{Tr}(Y_U^\dagger Y_U^A)Y_U
\end{aligned} \tag{D.40}$$

$$\begin{aligned}
\frac{d}{dt}Y_D^A &= -\left(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{9}g_1^2\right)Y_D^A \\
&+ 2\left(\frac{16}{3}g_3^2\mu_3 + 3g_2^2\mu_2 + \frac{7}{9}g_1^2\mu_1\right)Y_D \\
&+ 5Y_D Y_D^\dagger Y_D^A + 4Y_D^A Y_D^\dagger Y_D + 2Y_U^A Y_U^\dagger Y_D + Y_U Y_U^\dagger Y_D^A \\
&+ \text{Tr}(Y_E^\dagger Y_E + 3Y_D^\dagger Y_D)Y_D^A \\
&+ 2 \text{Tr}(Y_E^\dagger Y_E^A + 3Y_D^\dagger Y_D^A)Y_D
\end{aligned} \tag{D.41}$$

$$\begin{aligned}
\frac{d}{dt}B &= 2[3g_2^2\mu_2 + g_1^2\mu_1 \\
&+ \text{Tr}(Y_E^\dagger Y_E^A + 3Y_U^\dagger Y_U^A + 3Y_D^\dagger Y_D^A)]
\end{aligned} \tag{D.42}$$

Gaugino masses:

$$\frac{d}{dt}\mu_3 = 2[-9 + 2N_g]g_3^2\mu_1 \tag{D.43}$$

$$\frac{d}{dt}\mu_2 = 2[-5 + 2N_g]g_2^2\mu_2 \tag{D.44}$$

$$\frac{d}{dt}\mu_1 = 2\left[1 + \frac{10}{3}N_g\right]g_1^2\mu_3 \tag{D.45}$$

Soft breaking scalar masses:

$$\begin{aligned}
\frac{d}{dt}m_{H_1}^2 &= 2m_{H_1}^2 \text{Tr}(Y_E^\dagger Y_E + 3Y_D^\dagger Y_D) \\
&\quad + 2 \text{Tr}(Y_E^\dagger m_L^2 Y_E + 3Y_D^\dagger m_Q^2 Y_D + Y_E m_E^2 Y_E^\dagger + 3Y_D m_D^2 Y_D^\dagger) \\
&\quad + 2 \text{Tr}(Y_E^{A\dagger} Y_E^A + 3Y_D^{A\dagger} Y_D^A) \\
&\quad - 2(3g_2^2 |\mu_2|^2 + g_1^2 |\mu_1|^2)
\end{aligned} \tag{D.46}$$

$$\begin{aligned}
\frac{d}{dt}m_{H_2}^2 &= 2m_{H_2}^2 3 \text{Tr}(Y_U^\dagger Y_U) \\
&\quad + 2 \cdot 3 \text{Tr}(Y_U^\dagger m_Q^2 Y_U + Y_U m_U^2 Y_U^\dagger) + 2 \cdot 3 \text{Tr}(Y_U^{A\dagger} Y_U^A) \\
&\quad - 2(3g_2^2 |\mu_2|^2 + g_1^2 |\mu_1|^2)
\end{aligned} \tag{D.47}$$

$$\begin{aligned}
\frac{d}{dt}m_L^2 &= m_L^2 Y_E Y_E^\dagger + Y_E Y_E^\dagger m_L^2 \\
&\quad + 2(Y_E m_E^2 Y_E^\dagger + m_{H_1}^2 Y_E Y_E^\dagger) + 2(Y_E^A Y_E^{A\dagger}) \\
&\quad - 2(3g_2^2 |\mu_2|^2 + g_1^2 |\mu_1|^2) \cdot \text{I}
\end{aligned} \tag{D.48}$$

$$\begin{aligned}
\frac{d}{dt}m_E^2 &= 2(m_E^2 Y_E^\dagger Y_E + Y_E^\dagger Y_E m_E^2) \\
&\quad + 4(Y_E^\dagger m_L^2 Y_E + m_{H_1}^2 Y_E^\dagger Y_E) + 4Y_E^{A\dagger} Y_E^A \\
&\quad - 2 \cdot 4g_1^2 |\mu_1|^2 \cdot \text{I}
\end{aligned} \tag{D.49}$$

$$\begin{aligned}
\frac{d}{dt}m_Q^2 &= m_Q^2 (Y_U Y_U^\dagger + Y_D Y_D^\dagger) + (Y_U Y_U^\dagger + Y_D Y_D^\dagger) m_Q^2 \\
&\quad + 2(Y_U m_U^2 Y_U^\dagger + Y_U Y_U^\dagger m_{H_2}^2 + Y_D m_D^2 Y_D^\dagger + Y_D Y_D^\dagger m_{H_1}^2) \\
&\quad + 2(Y_D^A Y_D^{A\dagger} + Y_U^A Y_U^{A\dagger}) \\
&\quad - 2\left(\frac{16}{3}g_3^2 |\mu_3|^2 + 3g_2^2 |\mu_2|^2 + \frac{1}{9}g_1^2 |\mu_1|^2\right) \cdot \text{I}
\end{aligned} \tag{D.50}$$

$$\begin{aligned}
\frac{d}{dt}m_U^2 &= 2(m_U^2 Y_U^\dagger Y_U + Y_U^\dagger Y_U m_U^2) \\
&\quad + 2 \cdot 2(Y_U^\dagger m_Q^2 Y_U + m_{H_2}^2 Y_U^\dagger Y_U) + 2 \cdot 2(Y_U^{A\dagger} Y_U^A) \\
&\quad - 2\left(\frac{16}{3}g_3^2 |\mu_3|^2 + \frac{16}{9}g_1^2 |\mu_1|^2\right) \cdot \text{I}
\end{aligned} \tag{D.51}$$

$$\begin{aligned}
\frac{d}{dt}m_D^2 &= 2(m_D^2 Y_D^\dagger Y_D + Y_D^\dagger Y_D m_D^2) \\
&\quad + 2 \cdot 2(Y_D^\dagger m_Q^2 Y_D + m_{H_1}^2 Y_D^\dagger Y_D) + 2 \cdot 2(Y_D^{A\dagger} Y_D^A) \\
&\quad - 2\left(\frac{16}{3}g_3^2 |\mu_3|^2 + \frac{4}{9}g_1^2 |\mu_1|^2\right) \cdot \text{I}
\end{aligned} \tag{D.52}$$

Figure Captions

Fig. 1: The divergency curves on the $(m_t(M_Z) - \tan \beta)$ -plot for different values of M_R and α_s . The curves bound the region in which $\alpha_t > 0.5$ at Grand Unification-scale ($M_{\text{GUT}} = 3 \cdot 10^{16}$ GeV). The pole mass $M_b^{\text{pole}} = 5.2$ GeV was used as an input. $M_R = M_{\text{GUT}}$ (thin lines), $M_R = 10^{-3} M_{\text{GUT}}$ (medium lines), $M_R = 10^{-6} M_{\text{GUT}}$ (thick lines).

Fig. 2: The $b - \tau$ unification curves in the $(m_t(M_Z) - \tan \beta)$ -plane for different values of M_R and α_s (as in fig. 1). The curves correspond to the pole mass $M_b = 5.2$ GeV.

Fig. 3: The dependence of m_b/m_τ ratio at M_Z predicted from $b - \tau$ unification on $\alpha_t(0)$ for different values of M_R , $\tan \beta$ and α_s . The values of M_R and α_s as in fig. 1. The upper set of curves corresponds to $\tan \beta = 3$ and the lower ones to $\tan \beta = 50$. The horizontal lines show the upper experimental bounds on the ratio. They correspond to maximally admitted pole masses of b -quark: $M_b^{\text{pole}} = 4.85$ GeV (lower dotted line) and $M_b^{\text{pole}} = 5.2$ GeV (upper dotted line).

Fig. 4: Lower bounds on $\tan \beta$ and M_R from $b - \tau$ unification and convergency condition for $\alpha_t(0)$ ($\alpha_t(0) = 0.5$) for different values of $\alpha_s(M_Z)$ (figures at the curves). The value $M_b^{\text{pole}} = 5.2$ GeV was used.

Fig. 5: The leading SUSY contribution to the elementary EDM of the quarks in the class of models described in the paper is shown in the interaction eigenstate basis. The photon is attached in all possible ways.

Fig. 6: Ratios of SUSY induced over SM $b \rightarrow s\gamma$ amplitudes, for $m_t = 160$ GeV, $\tan \beta = 8$ and $B = A - 1$. The charged Higgs (*a*), chargino (*b*), gluino (*c*) and neutralino (*d*) components of the total amplitude are plotted versus the masses of the charged Higgs, lightest chargino, gluino and lightest neutralino respectively.

Fig. 7: Same as in Fig. 6 for $B = 2$.

Fig. 8: Ratios of SUSY induced over SM $b \rightarrow s\gamma$ amplitudes, for $m_t = 160$ GeV, $\tan\beta = 2$. Both choices $B = A - 1$ (a, b, c) and $B = 2$ (d, e, f) are shown. The charged Higgs (a, d) component of the total amplitude is plotted versus the mass of the charged Higgs boson which runs in the loop together with up-type quarks, whereas the chargino induced amplitude is plotted both versus the mass of the lightest chargino (b, e) and the lightest top squark (c, f).

Fig. 9: Same as in Fig. 8 for $\tan\beta = 20$.

Fig. 10: Same as in Fig. 8 for $\tan\beta = 40$.

Fig. 11: The leading component of the chargino exchange to the $b \rightarrow s\gamma$ amplitude in the limit of large $\tan\beta$ is shown in the higgsino-squark interaction basis. The photon is attached in all possible ways. The crosses indicate the presence of higgsino and stop mass insertions.

Fig. 12: The total inclusive branching ratio in the MSSM is shown for $m_t = 160$ GeV, $\tan\beta = 2$, $B = A - 1$ (a, b, c) and $B = 2$ (d, e, f), as a function of the masses of the charged Higgs boson (a, d), the lightest chargino (b, e), and the lightest top squark (c, f). The SM prediction for $m_t = 160$ GeV and $\alpha_s(m_Z)_{\overline{MS}} = 0.12$ is also shown (horizontal solid line) for comparison.

Fig. 13: Same as in Fig. 12 for $\tan\beta = 8$.

Fig. 14: Same as in Fig. 12 for $\tan\beta = 20$.

Fig. 15: Same as in Fig. 12 for $\tan\beta = 40$.

Fig. 16: The scattered dot areas represent the allowed MSSM regions in the plane of the lightest Higgs boson H_1^0 and the CP odd scalar H_3^0 masses after inclusion of the $b \rightarrow s\gamma$ bounds in eq. (4.15). Different values of $\tan\beta$ are shown.

Fig. 17: Same as in Fig. 16, in the gluino mass $-\mu_R$ plane (the latter is the $\tilde{h}_1 - \tilde{h}_2$ supersymmetric mixing parameter renormalised at the weak scale).

Figure 1

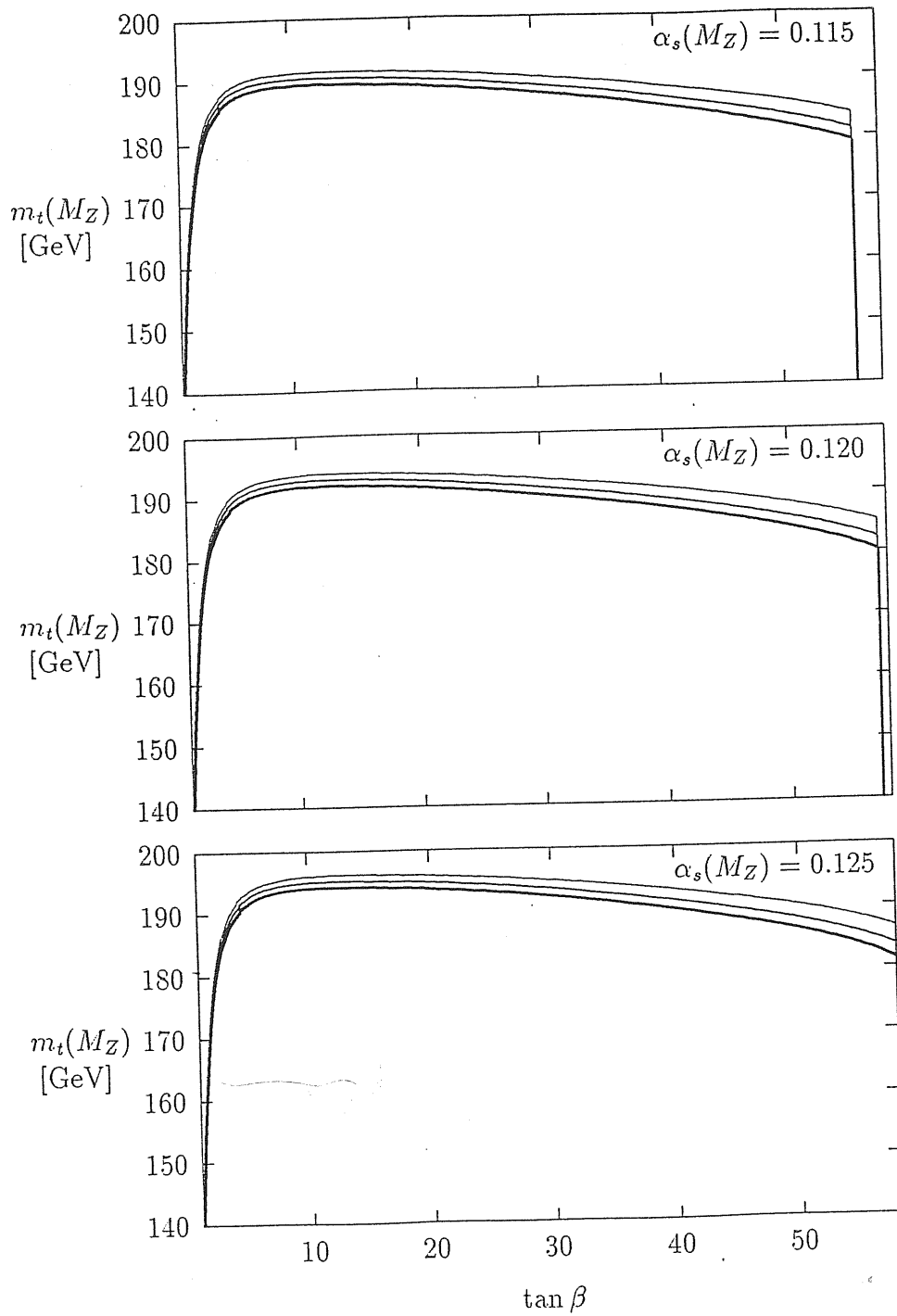


Figure 2

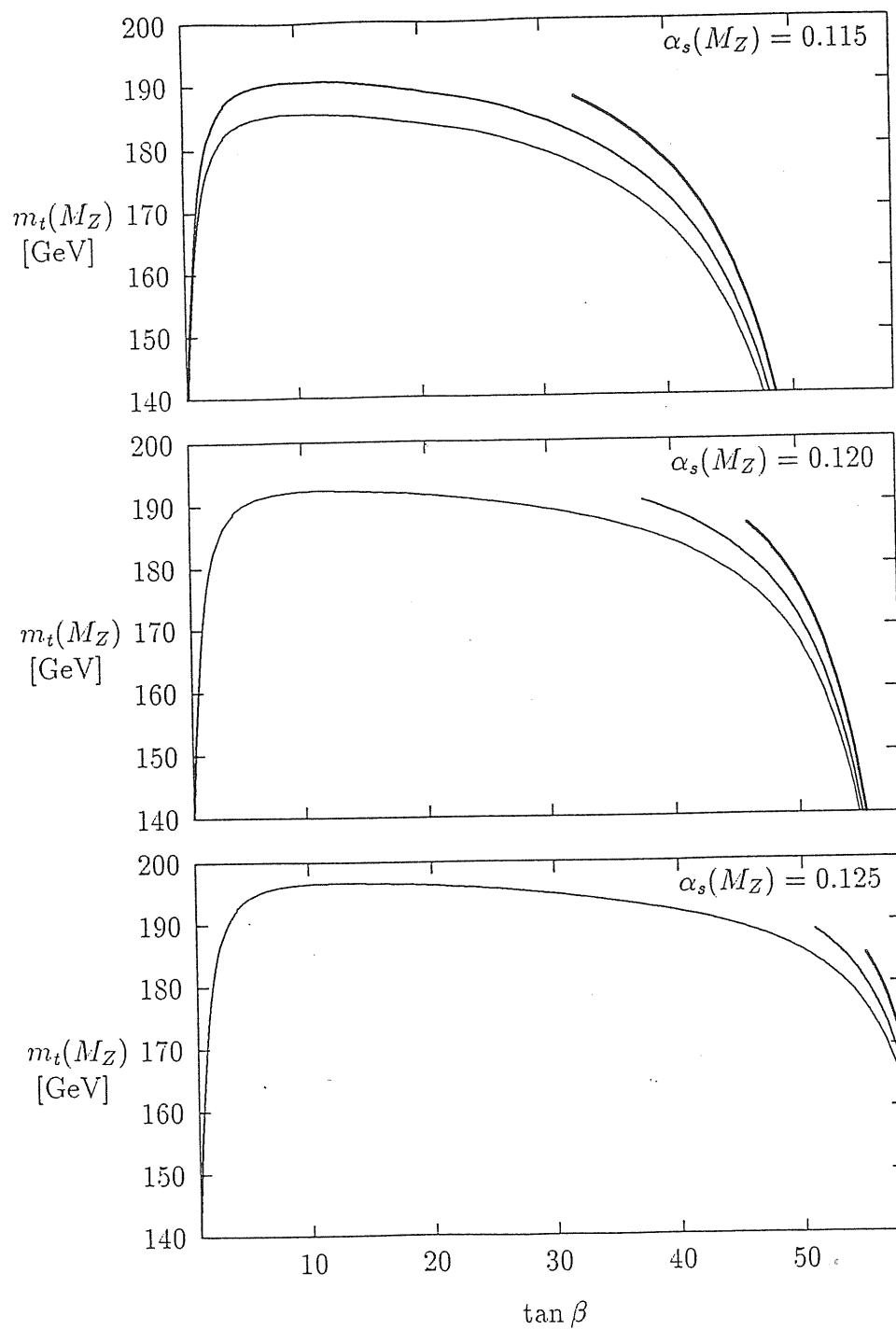


Figure 3

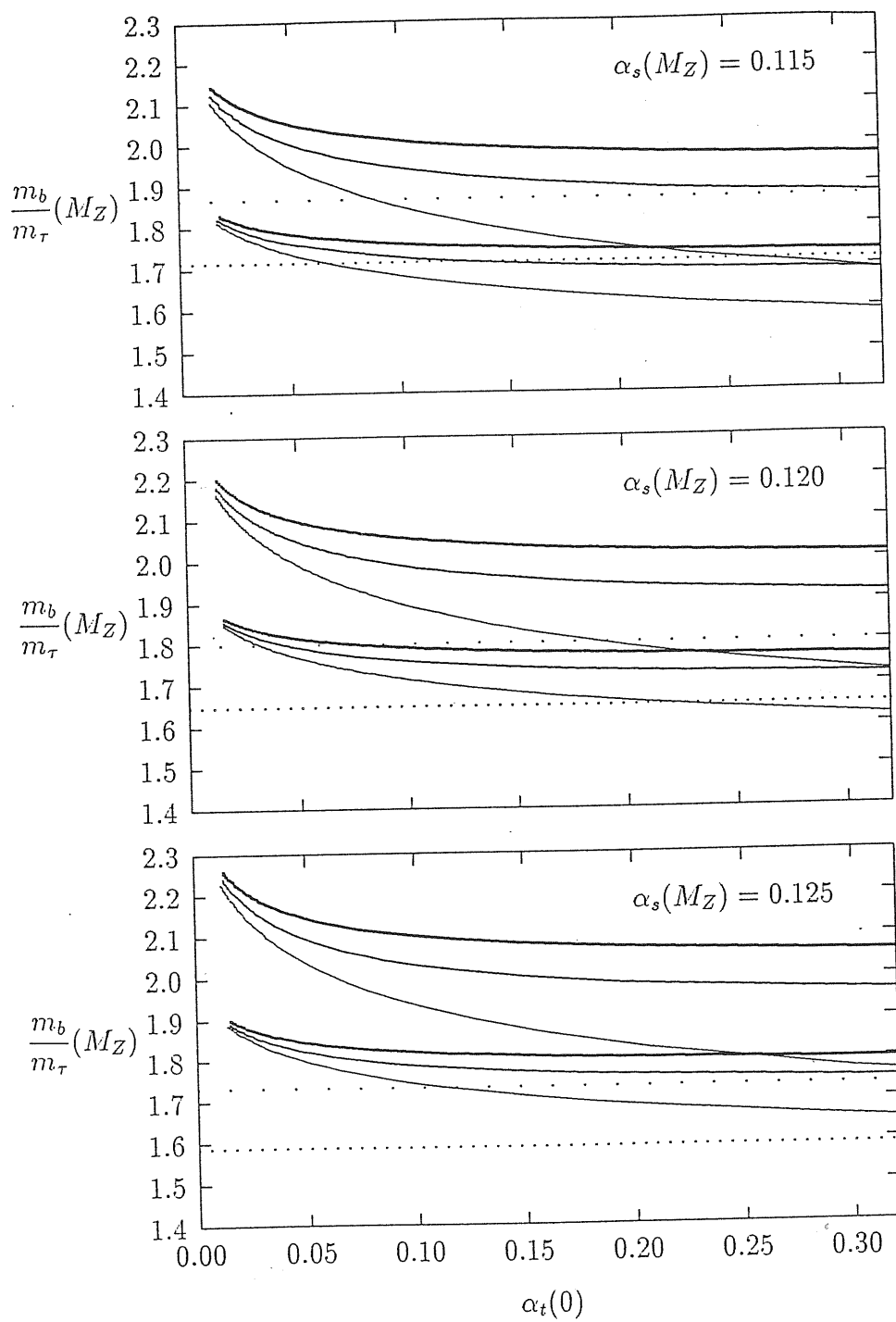


Figure 4

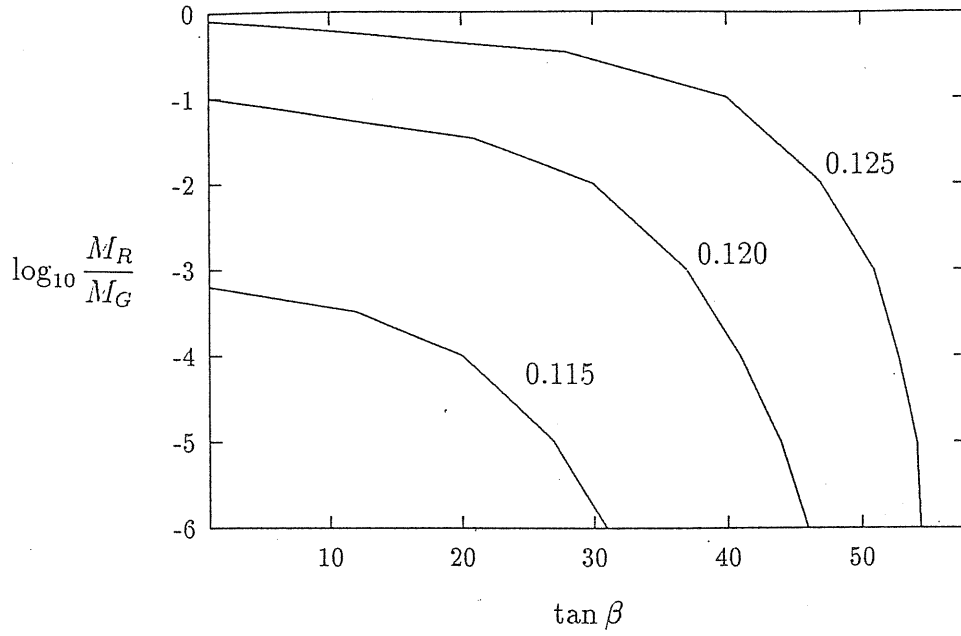


Figure 5

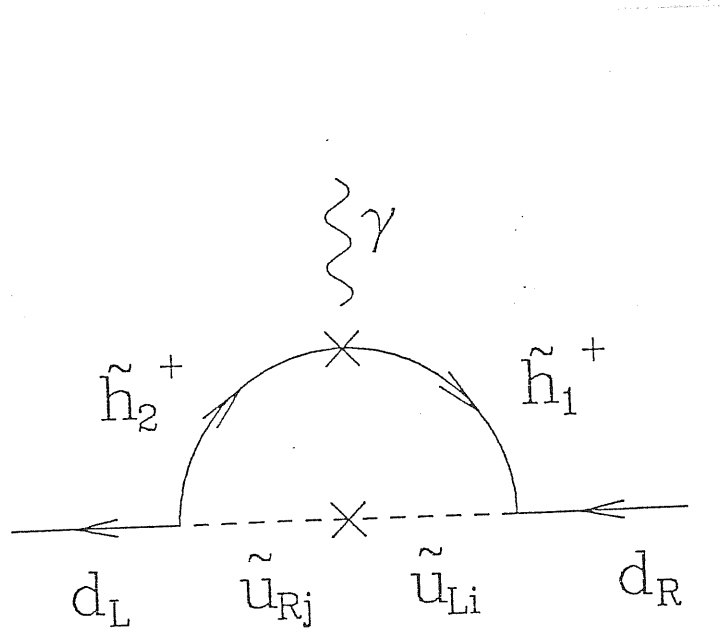


Figure 6

$$m_t = 160 \text{ GeV}, \quad v_2/v_1 = 8, \quad B = A-1$$

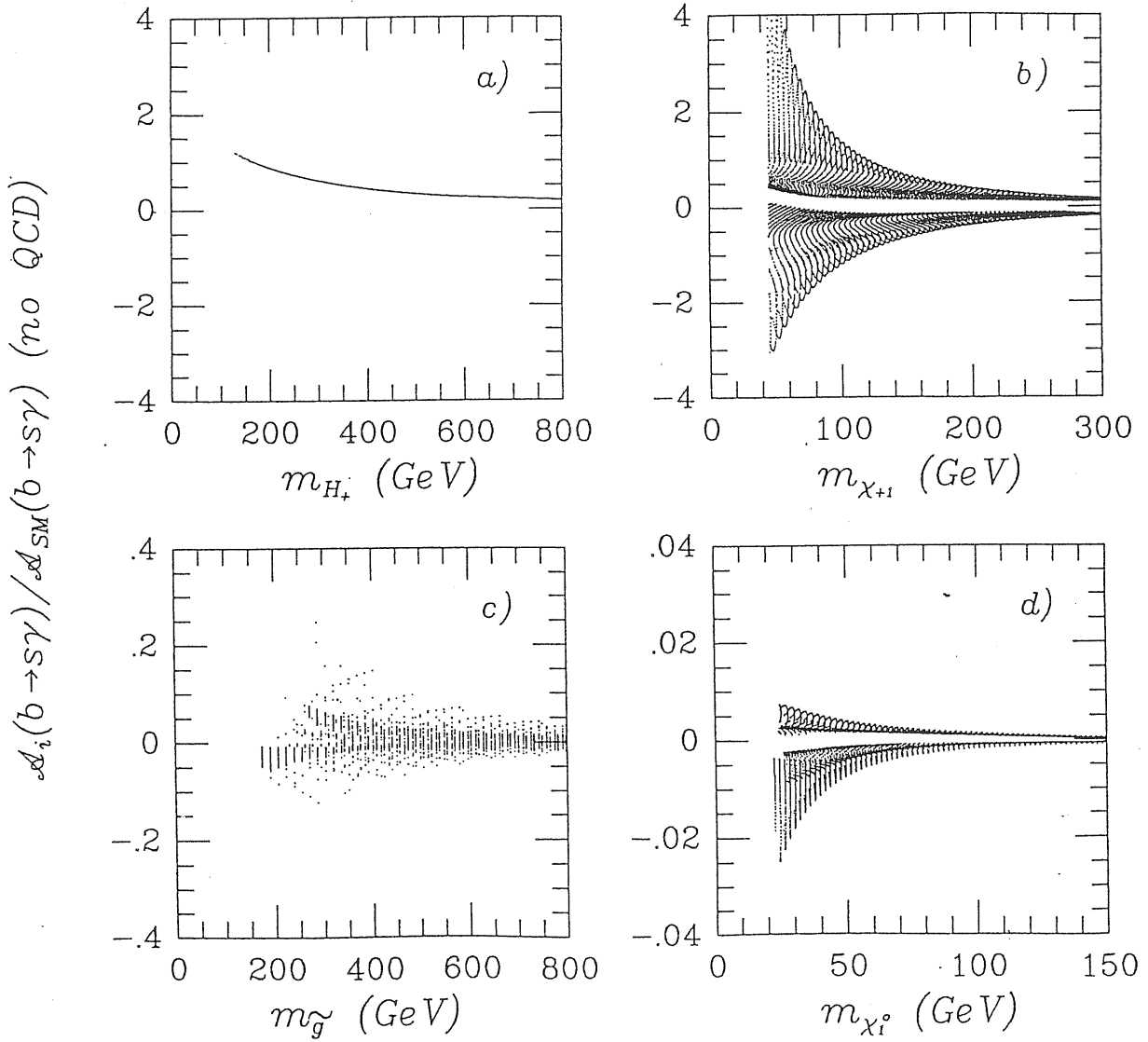


Figure 7

$$m_t = 160 \text{ GeV}, \quad v_2/v_1 = 8, \quad B = 2$$

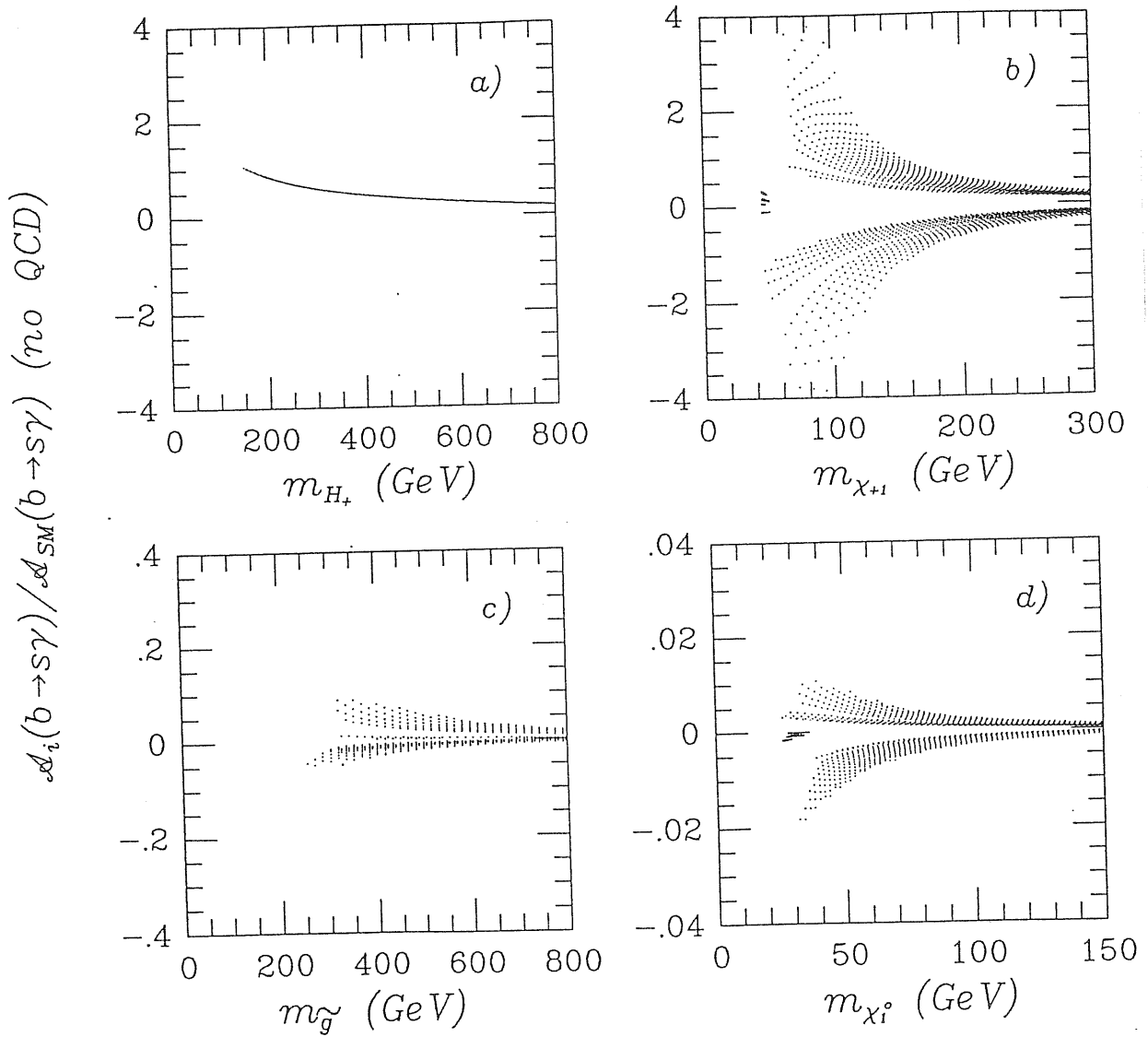


Figure 8

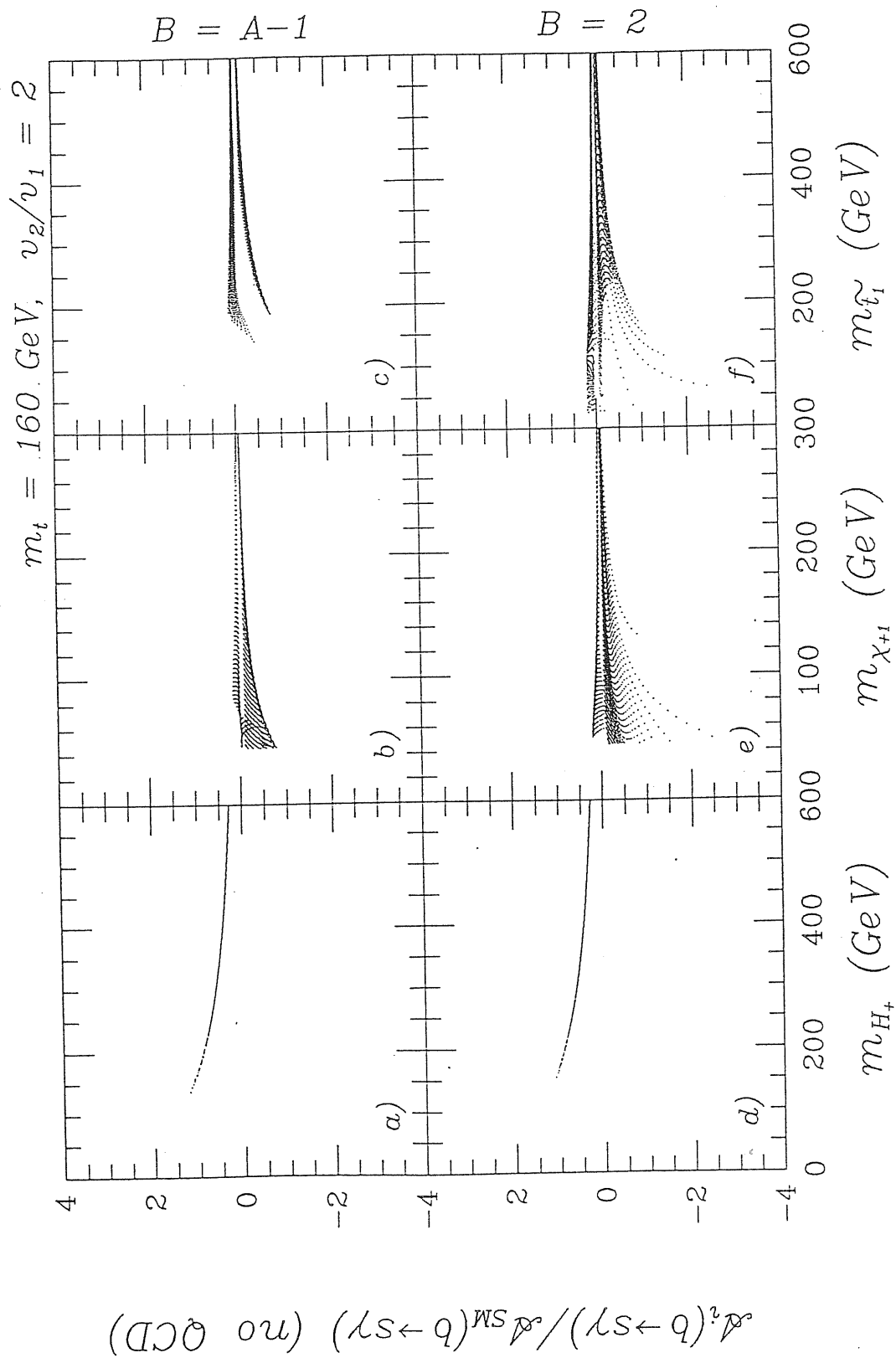
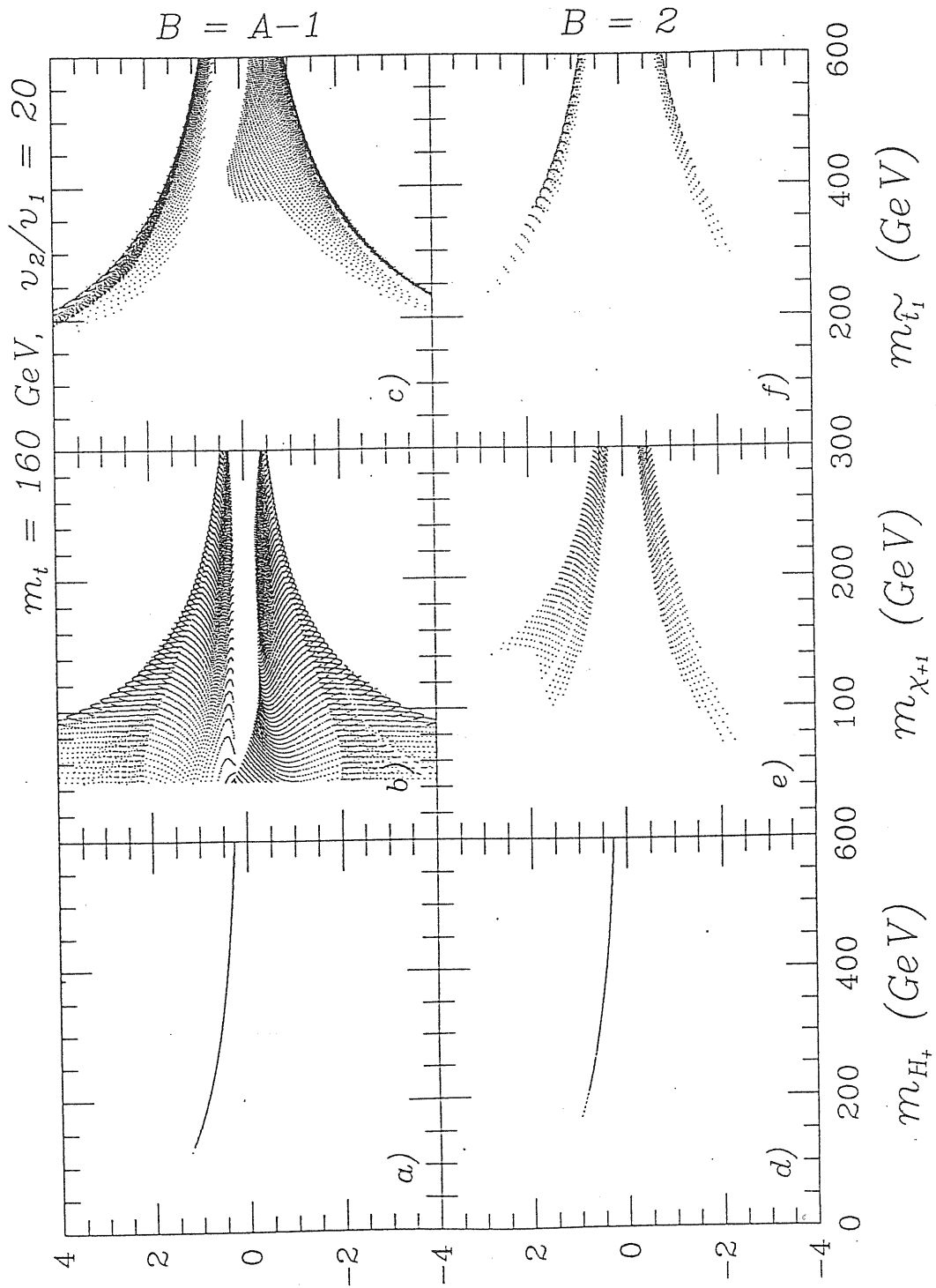


Figure 9



$$\mathcal{A}^2(b \to s\gamma) / \mathcal{A}^{SM}(b \to s\gamma) \text{ (no QCD)}$$

Figure 10

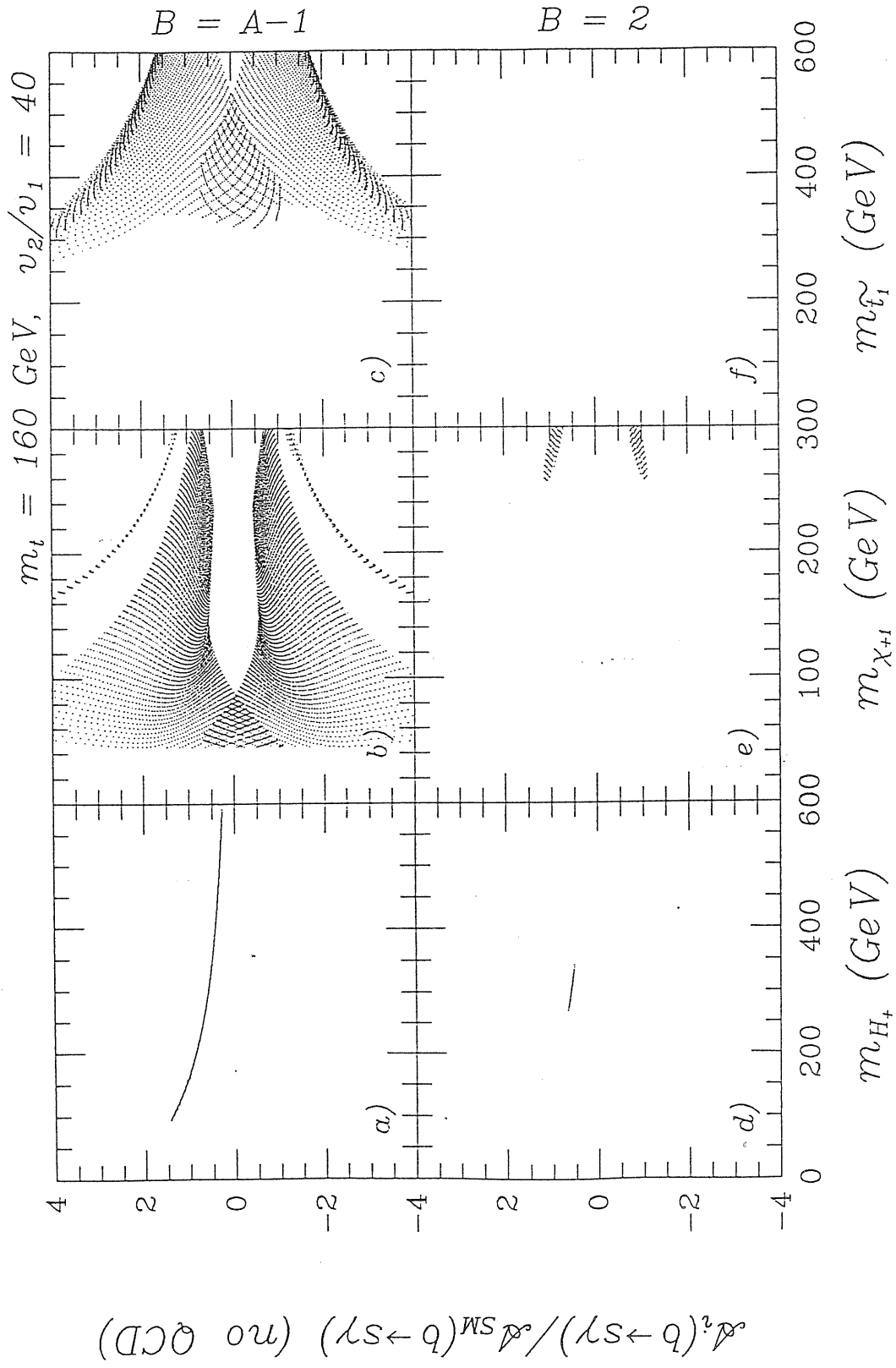


Figure 11

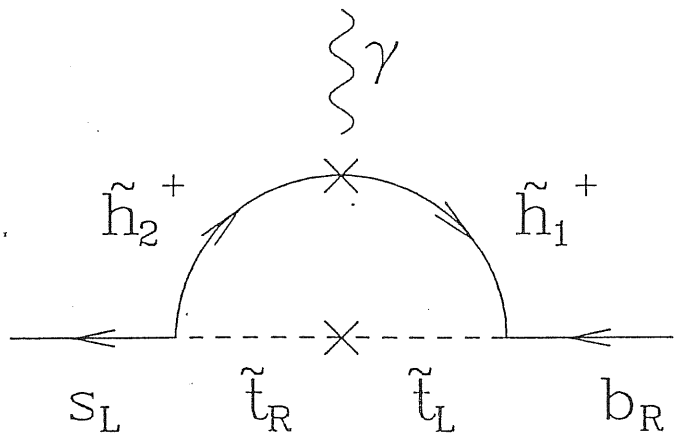


Figure 12

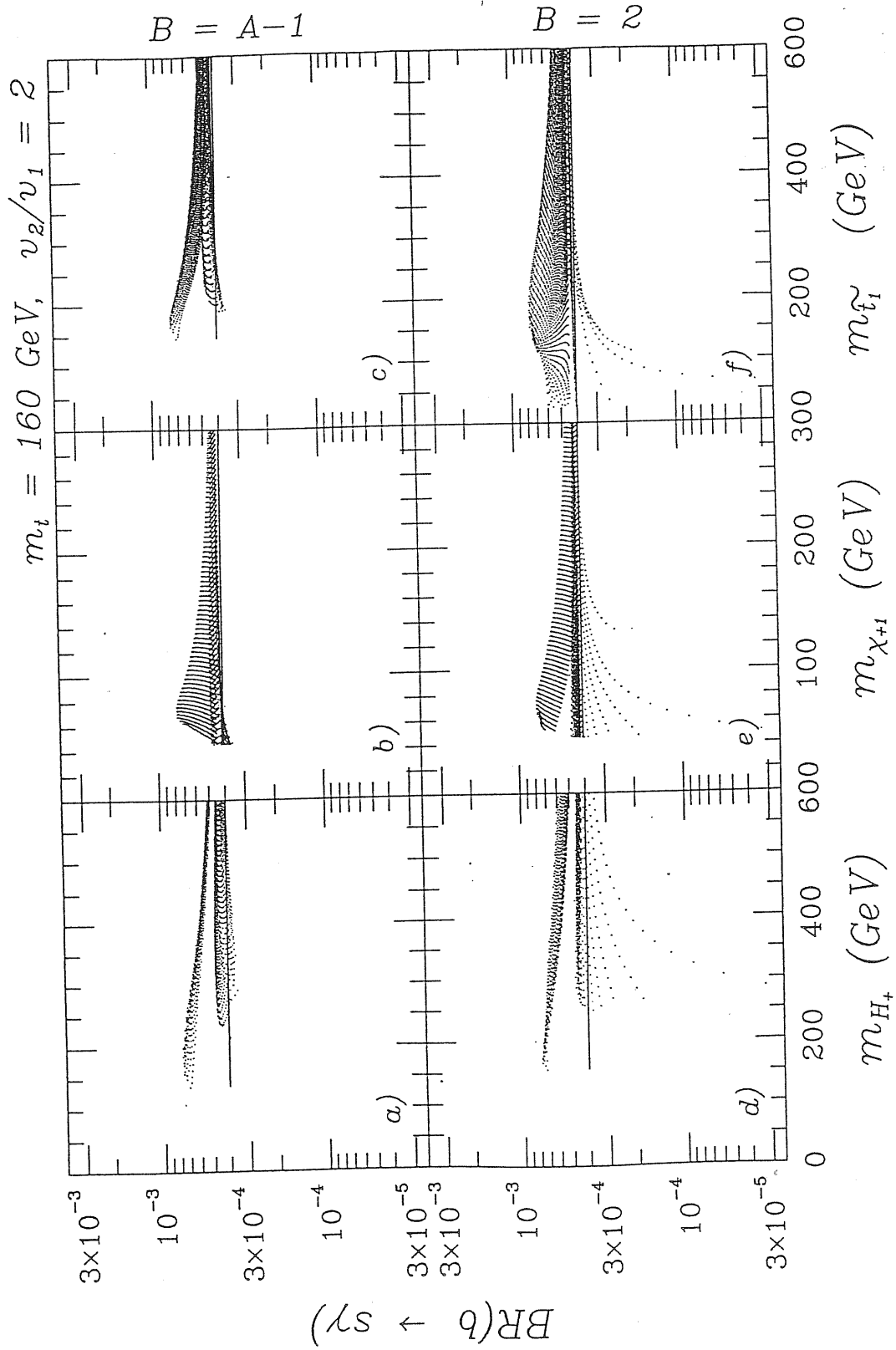


Figure 13

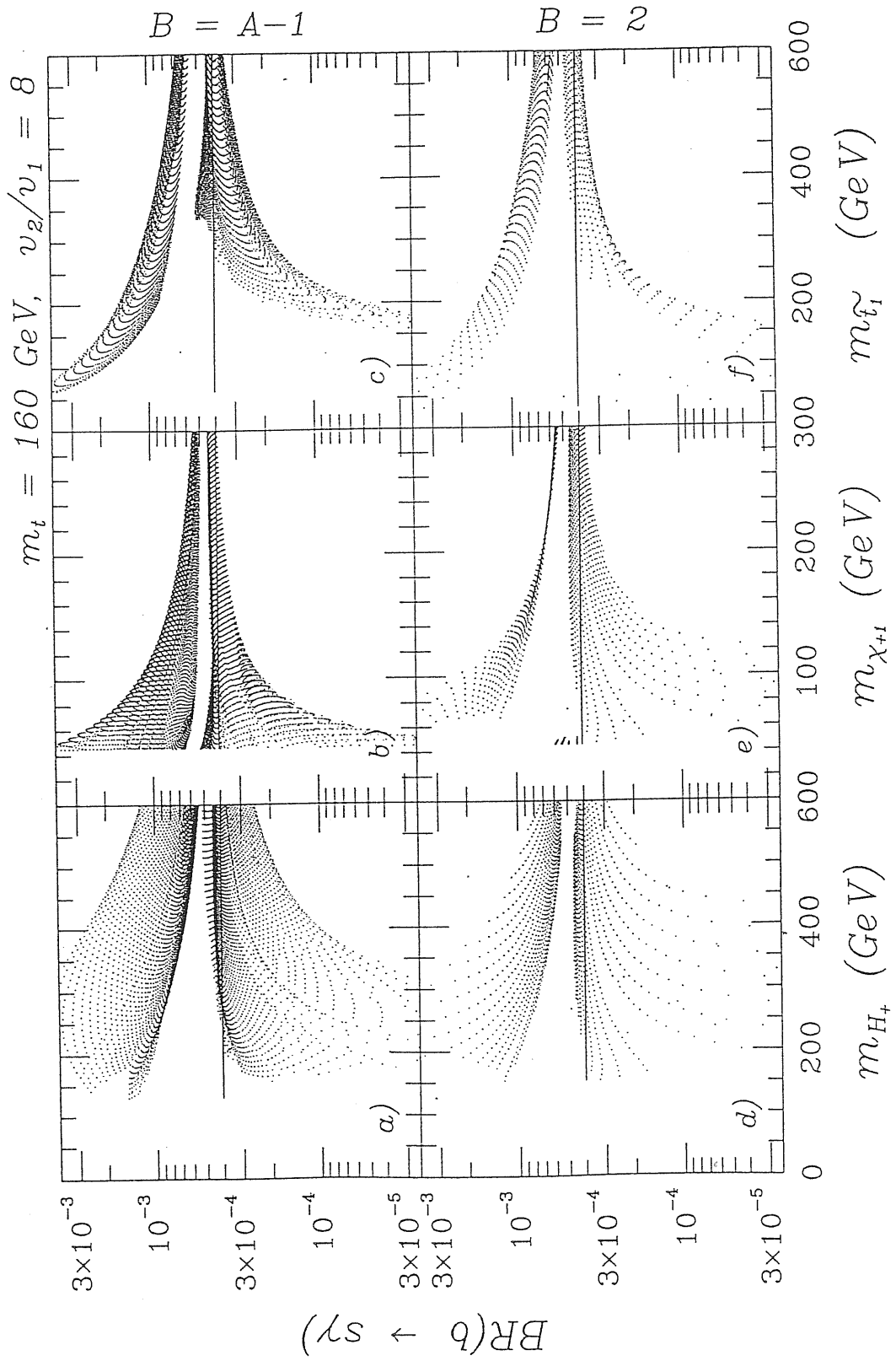


Figure 14

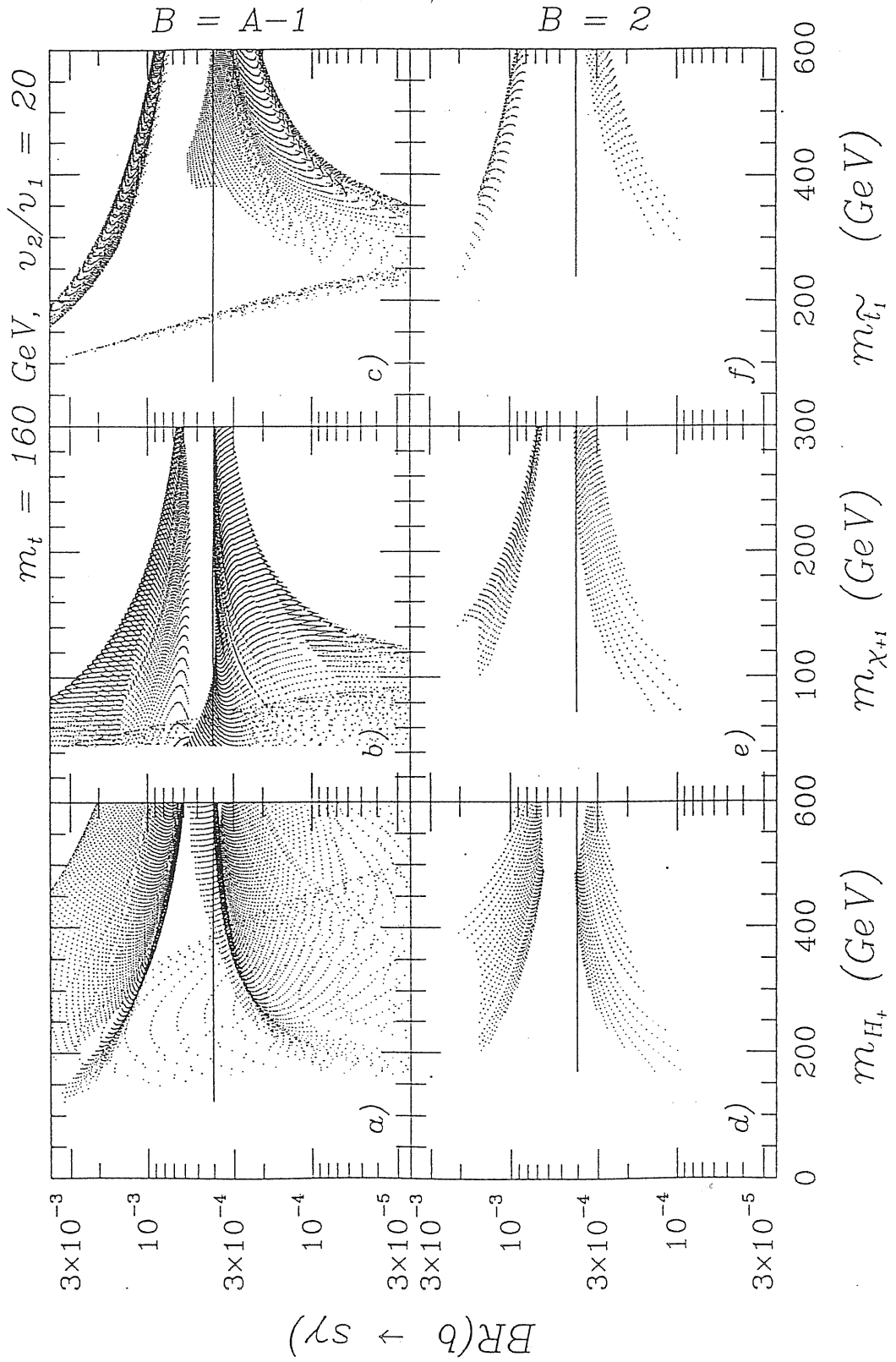


Figure 15

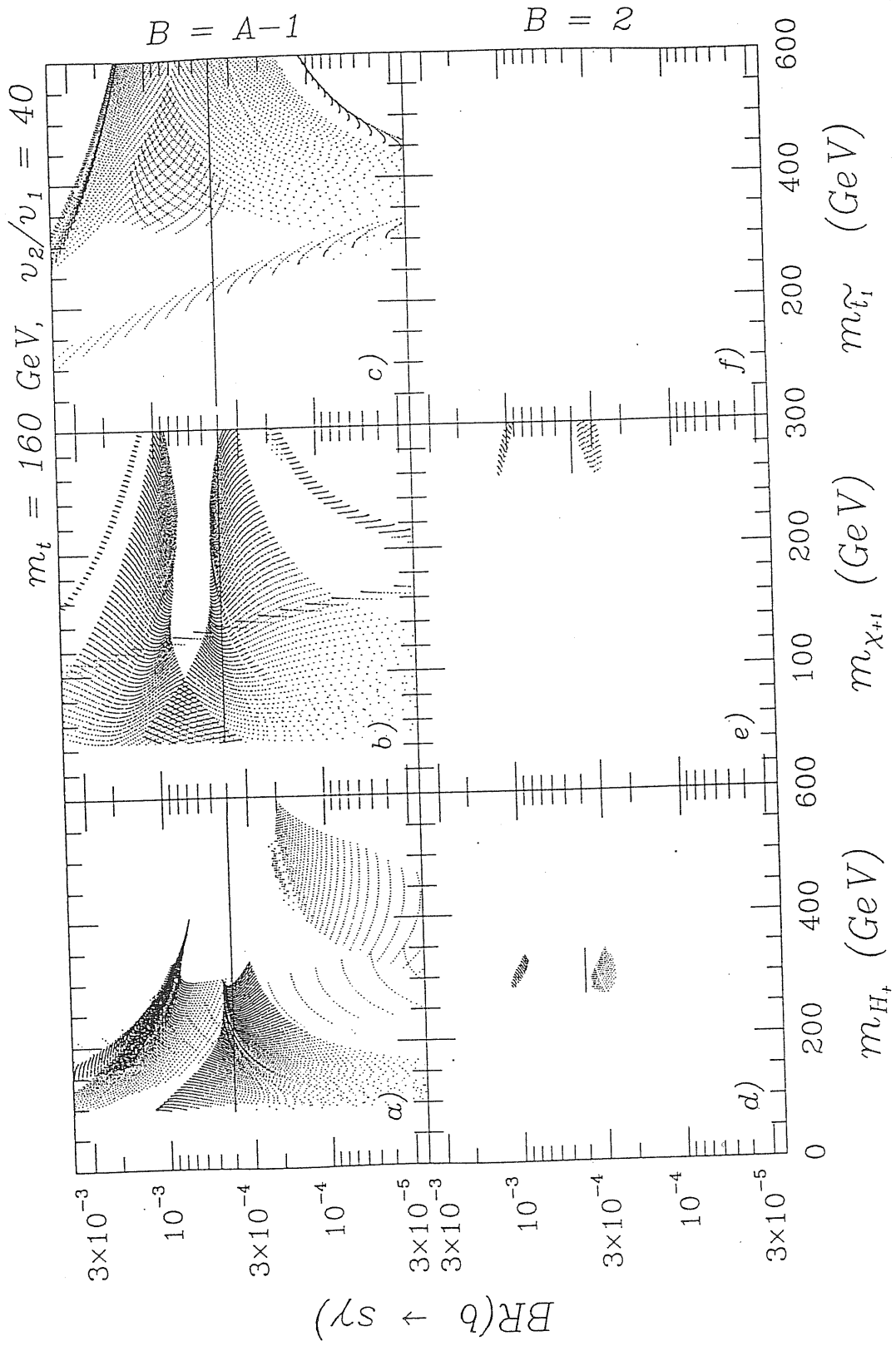


Figure 16

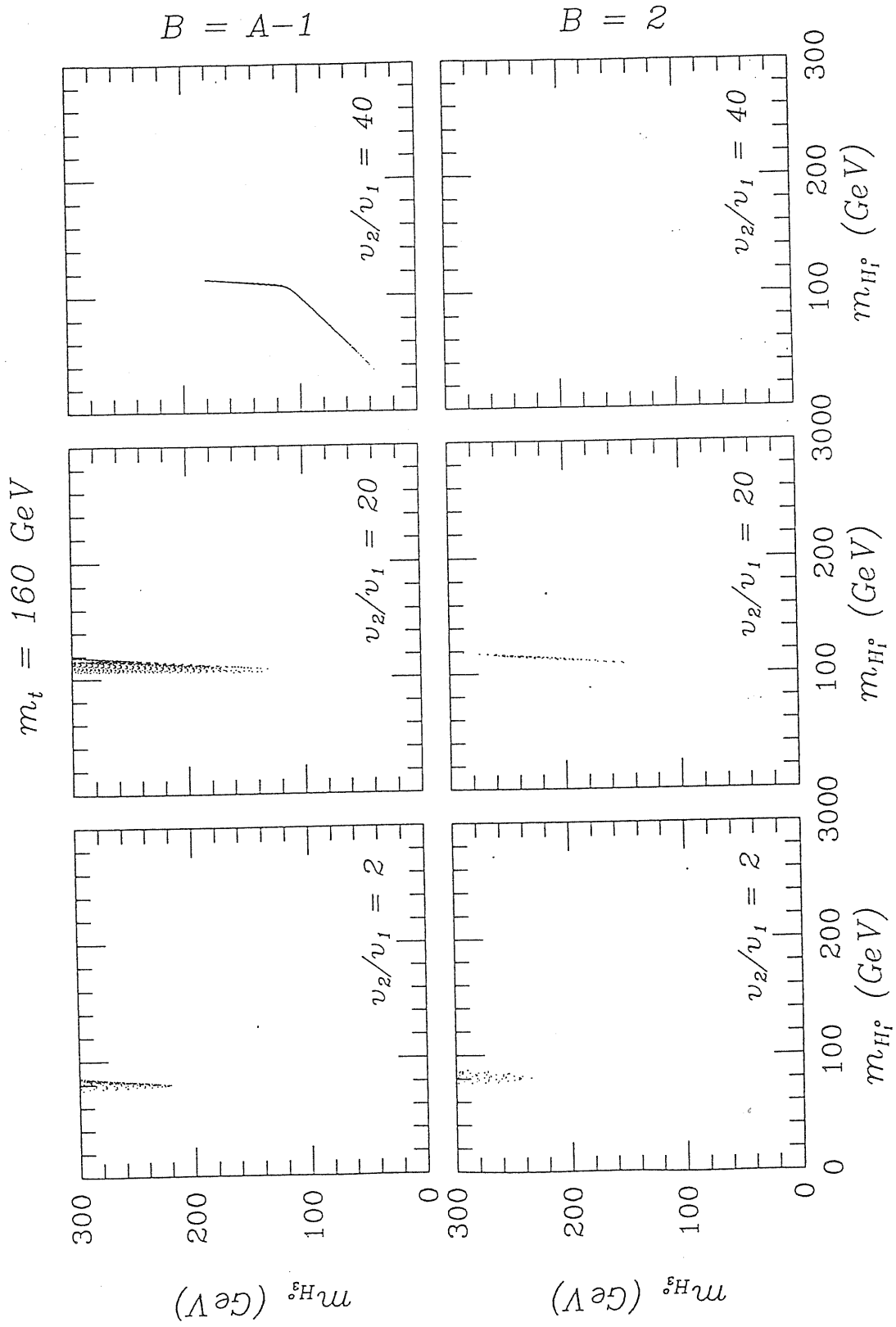
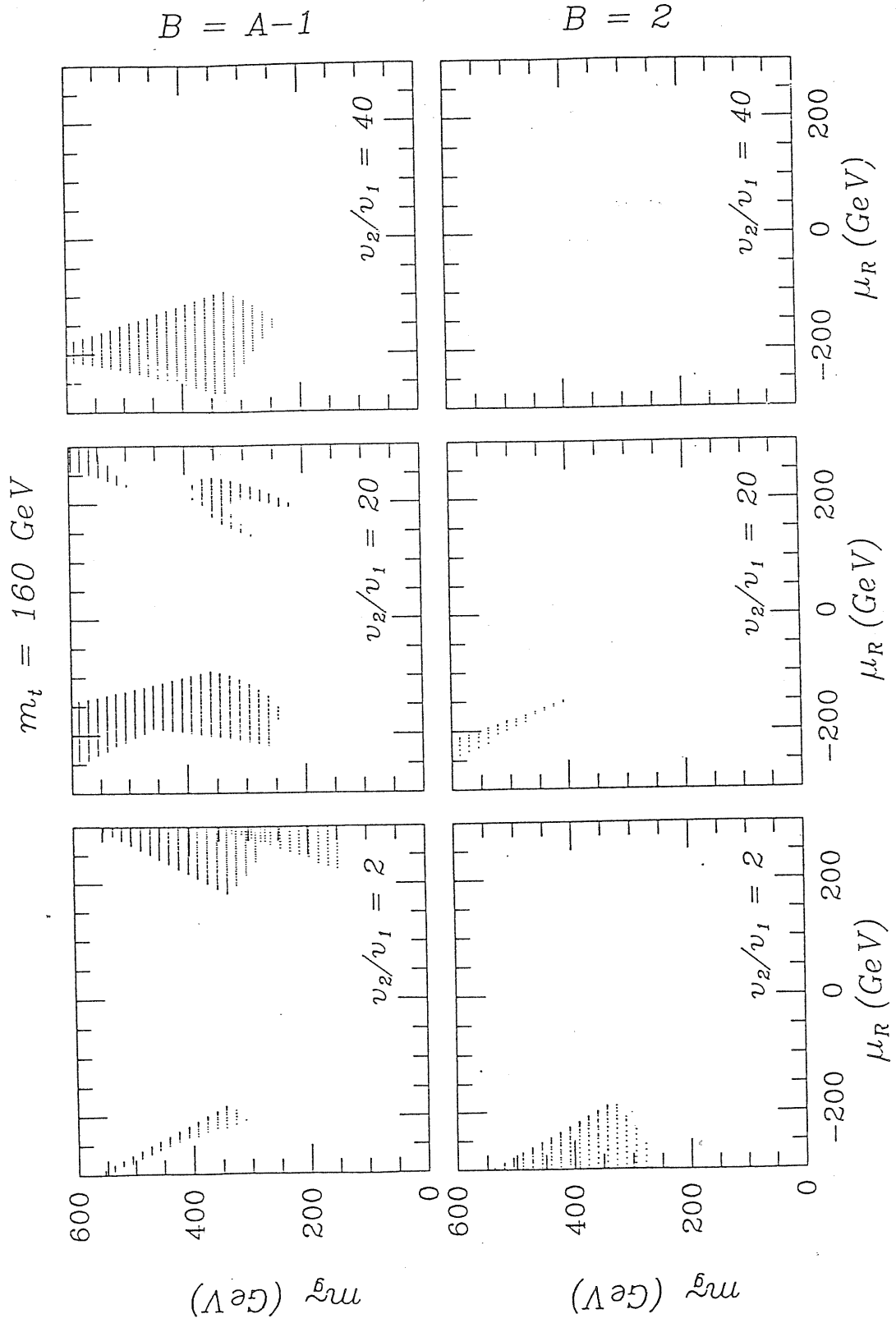


Figure 17



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