SISSA - SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI

TRIESTE
Strada Costiera 11

ISAS - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

Thesis submitted for the degree of

Doctor Philosophiae

Theoretical and phenomenological aspects of neutrino physics

Candidate: F. Giuliani

Supervisor: R. Barbieri

Academic Year 1988/89

TRIESTE

Theoretical and phenomenological aspects of neutrino physics

I wish to aknowledge R. Barbieri for having suggested the topics of my thesis, for many fruitful discussions and for a critical reading of the manuscript. I also thank the Physics Department of Pisa University for the kind hospitality.

Contents

Introduction

Neutrinos are at the moment the most unknown among the elementary particles up to now discovered. We do not know if they are massive or not and if, due to the possibility of mixing, they give rise to neutrino oscillations. We do not know if the right-handed partner exists and if neutrinos are Dirac or Majorana fermions. On the other hand, this elusive particle is able to *dominate* events of cosmological and astrophysical domain. These considerations strongly motivate physicists, both experimentalists and theoreticians, to better understand neutrino physics from the theoretical and phenomenological point of view.

For a long time people believed in a zero mass left-handed neutrino, but some years ago this idea has been reconsidered. In fact there is no reason, from the theoretical point of view, to have an exact conservation of the lepton number, a global symmetry of the Standard Model. In general extensions of the Standard Model this symmetry is broken and Majorana neutrino mass terms appear. This opens the possibility of mixing in the leptonic sector giving rise to the phenomena of generational neutrino oscillations. On the other hand, the hypothesis of the existence of right-handed neutrinos, restoring a sort of parity symmetry on the fermionic states, is also reasonable. In such a case it is possible to have Dirac mass terms and in general, except for the case of exact Dirac neutrinos, we have to expect mixings, and then oscillations, between left- and right-handed neutrinos, giving rise to an unitarity violation in the Z° neutrino couplings.

On the other hand, the values of the neutrino physics parameters are greatly relevant in cosmology and astrophysics. Stable light neutrinos affect both the ^4He nucleosynthesis and the total density of the Universe, giving rise to interesting cosmological bounds on their number of flavours and masses. They also could be good Dark Matter candidates. Moreover, neutrinos are relevant in a lot of astrophysical phenomena, such as the supernova explosions. From SN1987A it is possible to derive interesting and useful informations on their physics. Finally, the Solar neutrino puzzle is a possible signal of a non-standard physics in the neutrino sector. Different kinds of theoretical hypothesis have been elaborated, based on vacuum or matter resonant oscillations, or on the effects of a non-negligible neutrino magnetic moment. It will be particularly interesting in the near future to compare Davis and Kamiokande data, to better understand if a problem is really present, with particular attention to the anticorrelation between the detected neutrino flux and the cyclic variation of the sunspot number which Davis data seem to display.

Neutrino physics is very complex, a lot of different possibilities are opened, and at the moment almost completely unknown. However, neutrinos are able to play a fundamental role in a lot of relevant physical phenomena. We are then motivated to look for the phenomenological consequences that different theoretical hypothesis can induce. This kind of analyses partially enlights the neutrino nature being able to give us useful limits on some relevant neutrino properties, and interesting consequences also in physical sectors different from neutrino physics. In particular we have pointed out our attention on the effects of a sizeable mixing between the left-handed neutrino and some heavy *sterile* neutral lepton (we have in mind its right-handed partner) [1]. We derive that consistently with the present bounds the τ-system is the best place to look for a violation of lepton universality. The mixing of $\nu_{\tau L}$ with a *sterile* neutrino with a mass in the range of tens of GeV gives rise to an interesting phenomenology at the Z° resonance. We discuss how SLC and LEP will be able to test such a mixing. Our results are illustrated in Fig.s 4, 5 and 6.

Another way to derive informations on a possible universality violation in the leptonic sector is to examine the physics of the supernova SN1987A explosion [2]. In such a case from the total amount of the energy emitted we derive new stringent limits for the $\nu_{eL}-\nu_{eR}$ mixing, taking into account the effects of matter resonant neutrino oscillations [3]. In particular if the ν_{eR} mass is less than $10\,MeV$, right-handed neutrinos can be produced inside the supernova. From the requirement that the cooling of the star does not become too fast (less than 1 sec in contrast with the experimental observations), we derive an excluded interval $2.3\times 10^{-10}\leq \sin^2 2\theta \leq 1.1\times 10^{-3}$ for the $\nu_{eL}-\nu_{eR}$ mixing angle, if $\Delta m^2 = m_N^2 - m_\nu^2$ ($N\simeq \nu_{eR}$ and $\nu \simeq \nu_{eL}$ are the mass eigenstates) is placed outside the small window $2\times 10^5\,eV^2 \leq \Delta m^2 \leq 4\times 10^7\,eV^2$. The effects of matter resonant oscillations reduce the excluded interval to $1.6 \times 10^{-6} \leq \sin^2 2\theta \leq 1.1 \times 10^{-3}$ for $2 \times 10^5\, eV^2 \leq \Delta m^2 \leq 4 \times 10^7\, eV^2$. These results are illustrated in Fig.s 8 and 9.

On the other hand, as previously discussed, it is also possible that the ν_R exists as the right-handed component of a Dirac neutrino. In particular we consider that one of the possible solutions to the Solar neutrino puzzle assumes that the electron neutrino is a Dirac fermion with a magnetic moment of the order $(10^{-11} \div 10^{-10}) \cdot \mu_B$ [4]. If the time variation of Davis data would be confirmed by the last measurements, this hypothesis would receive a great phenomenological input. A minimal extension of the Standard Model, based on the gauge group $SU(3)_L \times U(1)_X$, which *naturally* gives rise to a *large* electron neutrino magnetic moment, has been recently proposed by R. Barbieri and R. N. Mohapatra (BM) [5]. Then we explore the phenomenological consequences of this model with particular attention to the predicted extra Z' boson [6]. Motivated by the previous considerations we will study the consequences of the $SU(3)_L \times U(1)_X$ neutral sector, comparing them first with the experimental determination of $\rho = M_W/M_{Z_1} \cos\theta_w$, where M_{Z_1} (the lowest eigenvalue of the $Z - Z'$ mass matrix) is the mass of the Z° boson, and secondly with the present neutral current data. Concerning the future prospects we will examine the effects of the $Z - Z'$ mixing at the Z° peak at LEP, and the direct Z' production at the hadron colliders SPS+Acol, TEVATRON, LHC and SSC.

From the present ρ measurements and the neutral current data we derive $M_{Z_2} \geq 170\, GeV$ and $-0.03 \leq \mathrm{tg}\chi \leq 0.08$, where M_{Z_2} is the highest eigenvalue of the $Z - Z'$ mass matrix and χ is the relative mixing angle. In general we find an allowed window depicted by the solid contour of Fig. 12 inside which the BM model lies, except only for extreme values of its parameters. The analysis at the Z° peak at LEP is performed in terms of a particular set of *experimental* variables, which depend on the lepton and hadron Z° decay rates, among which the model predicts an interesting sum rule illustrated with a straight line in Fig. 13 (a similar prediction is working for the E_6 superstring inspired models). Finally, in Fig.s 14 and 15, we report the results of the analyses of the Z' mediated e^+e^- production respectively at SPS+Acol and TEVATRON (both with an integrated luminosity of $10\, pb^{-1}$), LHC and SSC (both with integrated luminosity of $10^4\, pb^{-1}$). The number of the e^+e^- expected events at the Z' peak is plotted in terms of M_{Z_2}. Choosing 5 events as a minimum significant number for a discovery claim, we derive the following sensitivity limits for the previous machines and luminosities: $M_{Z_2} \geq 190\,GeV$ at SPS+Acol, $M_{Z_2} \geq 350\,GeV$ at TEVATRON, $M_{Z_2} \geq 3.5\,TeV$ at LHC and $M_{Z_2} \geq 6\,TeV$ at SSC.

The plan of the thesis is as follows. In chapter 1 we discuss the general aspects of neutrino physics; in section 1.1 its fundamental theoretical properties and the present experimental bounds. After a very brief review of neutrinos in cosmology in section 1.2, in 1.3 we point out our attention on the connections with astrophysics. That is particularly interesting for the succesive developments for the Solar neutrino puzzle and SN1987A.

In chapter 2 we discuss the phenomenology of a possible $\nu_L - \nu_R$ mixing. After a review of the present limits in section 2.1, we discuss the effects of a sizeable $\nu_{\tau L} - \nu_{\tau R}$ mixing in 2.2. In section 2.3 an example of a theoretical model is indicated.

In chapter 3 we study the problem of a $\nu_L - \nu_R$ mixing in the electron sector with the help of the supernova SN1987A event. After some general arguments about the supernova physics in section 3.1 and 3.2, we extract limits on the $\nu_{eL} - \nu_{eR}$ mixing in absence of matter resonant oscillations in section 3.3. In 3.4 and 3.5 we take into account the effects of the MSW mechanism and, after having studied in section 3.6 also the antineutrino sector, we are able in section 3.7 to get limits in the presence of matter resonant oscillations.

In chapter 4 the magnetic moment solution to the Solar neutrino puzzle and its theoretical and phenomenological consequences are discussed. In section 4.1 the phenomenological motivations for a large electron neutrino magnetic moment are explained, and in section 4.2 the corresponding theoretical problems and their solutions are studied. The BM model is introduced in section 4.3 , in 4.4 and 4.5 we extract the consequences of the predicted extra Z' boson. In section 4.6 the effects of the $Z - Z'$ mixing at the Z° peak at LEP are derived. Finally, prospects at the future hadron colliders are discussed in section 4.7.

After having reported our conclusions, at the pages 66, 67 and 68, we have inserted the five Tables mentioned in chapter 4. Figure captions with the corresponding Figures follow and finally, from page 80 on References are enlisted.

1. General considerations on neutrino physics

Before going to discuss the research topics previously illustrated, let us briefly recover the general neutrino theoretical aspects and their more relevant phenomenological consequences. We also give the present experimental, cosmological and astrophysical bounds on the various neutrino parameters.

1.1 Neutrino particle physics

• *Neutrino masses.* There is no strong theoretical argument that forbids neutrino to get a mass. The situation is different with respect to the photon case, where an unbroken gauge symmetry implies an exactly zero photon mass. However, if neutrino mass terms are present, they must be very small with respect to the other typical charged fermion masses, as indicated by the present laboratory limits [7]:

$$m_{\nu_e} < 11\, eV\,, \quad m_{\nu_\mu} < 250\, KeV\,, \quad m_{\nu_\tau} < 35\, MeV\,. \tag{1.1.1}$$

The Zurich bound $m_{\nu_e} < 18\, eV$ has been recently improved up to $11\, eV$ at the 95% C.L.. Also the Los Alamos experiment gives a more stringent bound: $m_{\nu_e} < 13.4\, eV$. These last experiments are in conflict with the ITEP result $17\, eV < m_{\nu_e} < 40\, eV$ [8], about which some doubts had been formulated due to the possibility that ^3H decays into an excited state of ^3He rather than to the ground state, with an effect similar to that of a finite neutrino mass, and for the complexity of the valine molecule used in the Lyubimov experiment.

The neutrino so far discovered is a left-handed fermion corresponding to the $\nu_L \sim (1/2, 0)$ representation of the Lorentz group. If its right-handed partner $\nu_R \sim (0, 1/2)$ exists, then Dirac mass terms $m_D \nu_L \nu_L^c$ (in Weyl notation $\nu_L^c = (\nu_R)^c = -\sigma_2 \nu_R^*$) would be possible, similarly to the charged fermion case. On the other hand, since neutrino has zero electric charge, no gauge symmetry forbids Majorana mass terms $m_L \nu_L \nu_L$ to appear, violating in such a case the lepton number conservation. One of the most spectacular consequences of Majorana masses is the occuring of neutrinoless double beta decay: $(A, Z) \rightarrow (A, Z+2) + 2e^-$. The current best limit is derived from UCSB-LBL [9], which is searching for $^{76}Ge \rightarrow {}^{76}Se + 2e^-$. From $T_{1/2}(^{76}Ge, 0\nu) > 5 \times 10^{23}\, yr\, (1\sigma)$ it is extracted:

$$< m_{\nu_e} > < 0.7 \div 1.8\, eV\,, \tag{1.1.2}$$

where $< m_{\nu_e} > = \sum_\alpha \xi_\alpha U_{e\alpha}^2 m_\alpha \cdot F(m_\alpha, A)$. m_α are the mass eigenvalues, U the mixing matrix and $\xi_\alpha = \pm 1$ is the CP parity of the neutrino mass eigenstate χ_α. $F(m_\alpha, A)$ is a nuclear factor such that $F \sim 1$ if $m_\alpha \ll 10\, MeV$, and $F \ll 1$ if $m_\alpha \gg 10\, MeV$. If all m_α are small enough, then $< m_{\nu_e} > = \mathcal{M}_{ee}$, the (11) element of the general neutrino mass matrix in the current eigenstate basis defined in eq. (1.1.5). However, this limit suffers from uncertainties of nuclear matrix element calculations. It is in any case interesting to observe that if UCSB-LBL and ITEP limits had been both confirmed, then we should have excluded a Majorana mass for the electron neutrino, unless to take into account an ad hoc cancellation between $U_{e1}^2 m_1$ and the other terms of $< m_{\nu_e} >$.

From the theoretical point of view, one of the most interesting aspects of neutrino physics is that, if they have non zero masses, the mass terms must be hierarchically suppressed with respect to the corresponding typical charged fermion masses. A possible solution to this kind of hierarchical problem is based on the so called Gell-Mann-Ramond-Slansky (GRS) or *see-saw* mechanism [10]. We suppose that the neutrino mass matrix on the basis (ν_L, ν_L^c) is the following (we ignore for semplicity the generational mixing):

$$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}, \tag{1.1.3}$$

where the Dirac term is of the typical order of magnitude of a charged fermion mass, while $M_R \gg m_D$ because ν_R is a singlet under $SU(2)_L \times U(1)_Y$, and then it is natural to expect that it acquires a mass bigger than the Fermi scale in agreement with the *survival hypothesis* [1]. In such a case the low mass eigenstate (mostly the ν_L) has mass

[1] On the basis of this hypothesis it is in general expected that if a gauge group G is broken to a residual subgroup H, the H singlets acquire masses whose typical order is the $G \rightarrow H$ breaking scale.

$$m_L \simeq \left(\frac{m_D}{M_R}\right) \cdot m_D \,. \tag{1.1.4}$$

It is suppressed with respect to the typical charged fermion mass by the mixing factor m_D/M_R. For example if $m_D \sim 1\,MeV$ and $M_R \sim 1\,TeV$, it follows $m_L \sim 1\,eV$. This scheme is completely natural in left-right extensions of the Standard Model and in the $SO(10)$ grand unification group (in E_6 the situation is richer due to the presence of other heavy neutral leptons). M_R is related to the scale of the $B-L$ and $SU(2)_R$ symmetry breaking. Then the *see-saw* mechanism connects the smallness of the neutrino mass to the high energy physics beyond the Fermi scale. A stringent naturality requirement should remove the zero in the first entrance of (1.1.3). However, it has been shown that the fundamental result of the GRS mechanism does not change [11]. Finally, people studied the radiative effects on light fermions of the presence of heavy fermions [12], with special attention to the radiative corrections on the up quark masses that the GRS mechanism can induce if M_R is next to the grand unification scale [13].

This is not the only possible solution to the problem of neutrino mass suppression. Another suggestive idea, more complicated to realize in detail, is the radiative generation of neutrino masses. A suitable symmetry, succesively spontaneously broken, could be useful in the protection of the neutrino mass, which in such a case may be of the Dirac type. An example of this mechanism will be discussed in the last section, in the context of the large neutrino magnetic moment problem.

• *Neutrino mixings.* If neutrino masses are not zero, then in general we have to expect non trivial mixings in the leptonic sector. For pure Dirac or pure Majorana neutrinos (without $\nu_L - \nu_L^c$ mixing terms) the situation is completely similar to the quark sector. The diagonalization of the charged lepton and neutrino mass matrices will give unitary redefinitions of the leptonic and neutrino states producing an unitary interaction matrix in the charged lepton currents and the flavour conservation in the neutral lepton current.

The situation in which Dirac and Majorana mass terms are present produces substantial differences with respect to the hadron sector. The general case, due to $\nu_L - \nu_L^c$ mixings, allows universality violation and flavour changing in the leptonic neutral current sector, due to the fact that the Z° boson does not interact with right-handed neutrinos. The most general mass matrix in the basis (ν_{aL}, ν^c_{aL}), with $a = e, \mu, \tau$, is [1]:

$$\mathcal{M}_\nu = \begin{pmatrix} M_L & {M_D}^T \\ M_D & M_R \end{pmatrix} . \tag{1.1.5}$$

M_L and M_R are respectively left- and right-handed Majorana neutrino mass matrices in the generational space and M_D is the Dirac mass matrix. The diagonalization of $\mathcal{M}_\nu$, by a suitable congruent transformation, yields six Majorana mass eigenstates χ_α $(\alpha = 1, \ldots, 6)$. In terms of the χ_α's the standard current eigenstates are:

$$\nu_a = \sum_{\alpha=1}^{6} U_{a\alpha}\, \chi_\alpha, \tag{1.1.6}$$

U is a 3×6 matrix such that $UU^\dagger = 1$, but $U^\dagger U \neq 1$. The diagonalization of the charged-lepton mass matrix gives:

$$l_a = \sum_{b=1}^{3} \Omega_{ab}\, e_b \ , \tag{1.1.7}$$

where e_a are the mass eigenstates. Then the charged-current weak interactions read

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}} {W_\mu}^- \sum_{a,\alpha} \bar{e}_{La} \gamma^\mu K_{a\alpha}\, \chi_{L\alpha} + h.c., \tag{1.1,8}$$

where $K = \Omega^\dagger U$ is a 3×6 matrix and $KK^\dagger = 1$, but in general $K^\dagger K \neq 1$. Another new feature, with respect to the quark case, is the existence of *non-diagonal* neutral-current interactions:

$$\mathcal{L}_{nc} = -\frac{g}{4 \cos\theta_W} Z_\mu \sum_{\alpha,\beta=1}^{6} \bar{\chi}_\alpha \gamma^\mu (1 - \gamma_5)\, P_{\alpha\beta}\, \chi_\beta, \tag{1.1.9}$$

where $P = U^\dagger U = K^\dagger K$ is not unitary, but has the properties of a projector: $P^2 = P$ and $P^\dagger = P$. We will study in detail the experimental consequences at the Z° peak of such a case in chapter 2 [1]. In chapter 3, using the SN1987A data, we will able to put very stringent limits on the $\nu_{eL} - \nu^c_{eL}$ mixing, if the ν_{eR} mass is enough light $(< 10\, MeV)$ [3].

Mixings in the leptonic sector give rise to the phenomena of neutrino oscillations. For a simple two neutrino system $\nu_e - \nu_X$, with $\nu_X = \nu_\mu, \nu_\tau, \nu_4$ or some sterile neutral lepton, the non oscillation $\nu_e - \nu_e$ probability after a time t is

$$| < \nu_e(t)|\nu_e(0) > |^2 = 1 - \frac{1}{2}\sin^2 2\theta \cdot \left(1 - \cos\frac{\Delta m^2}{2k}t\right) , \qquad (1.1.10)$$

with k the neutrino momentum, θ the mixing angle and $\Delta m^2 = m^2_{\nu_e} - m^2_{\nu_X}$. We need $\Delta m^2 \neq 0$ and $\theta \neq 0$ for the oscillation to be present. In such a case the oscillation length is:

$$L = \frac{4\pi k}{\Delta m^2} = 2.5 \times 10^2 \frac{k/MeV}{\Delta m^2/eV^2}\, cm . \qquad (1.1.11)$$

The reactor experiment which gives the most stringent limits on the parameters $\sin^2 2\theta$ and Δm^2 has been performed at Gosgen in Switzerland [14]. A big value for the mixing angle is compatible only for $\Delta m^2 \leq 4 \times 10^{-2}\, eV^2$, similarly a big value for Δm^2 requires a small mixing angle: $\sin^2 2\theta \leq 0.1$. The initial evidence at the Bugey reactor [15] for $\Delta m^2 = 0.2\, eV^2$ and $\sin^2 2\theta = 0.25$, is inconsistent with the Gosgen results and has recently been withdrawn. At the moment also the accelerator experiments have no evidence of neutrino oscillations. As an example we report in Fig. 1 the FNAL (E531 Collaboration, 1986) results [16]. The phenomenon of neutrino oscillations acquires a particular interest in the context of the matter resonant amplifications and the Solar neutrino puzzle, which we will discuss in section 1.3.

- *The neutrino magnetic moment.* If neutrino has a Dirac nature we have to expect in general a non zero neutrino magnetic moment. However, it is very small in the Standard Model with the addition of the ν_R [17]:

$$\mu_\nu = \frac{3eG_F}{8\sqrt{2}\pi^2} \cdot m_\nu = 3 \times 10^{-19} \cdot \mu_B \left(\frac{m_\nu}{1\, eV}\right) . \qquad (1.1.12)$$

In left-right symmetric models it is possible to gain some orders of magnitude because the radiative contribution becomes proportional to the l lepton exchanged mass [18],

$$\mu_\nu = \frac{G_F}{\sqrt{2}\pi^2} m_e m_l \sin 2\xi \cdot \mu_B , \qquad (1.1.13)$$

with ξ the left-right mixing angle ($|\xi| \leq 0.05$). Then we can obtain $\mu_{\nu_e} \leq 10^{-14} \cdot \mu_B$ in absence of generational mixings ($m_l = m_e$). In the general case m_l can be the μ or τ mass, but then the (1.1.13) becomes suppressed by the leptonic mixing angles and no significant improvement is expected. With an opportune charged scalar particle η it is possible to get a larger value: $\mu_{\nu_e} \sim 10^{-10} \cdot \mu_B$ [19]. The possibility of a *large* electron neutrino magnetic moment is very appealing from the phenomenological point of view, and from the theoretical point of view it produces some naturality problems which can be solved in a very interesting way with the help of a suitable symmetry acting on the neutrino system. We will treat in detail this kind of problems in the last chapter.

A single Majorana neutrino cannot have a magnetic moment. However, it is not excluded a non diagonal magnetic moment between generational different Majorana neutrinos (for example $\nu_e - \nu_\mu$) [20], although in such a case one has to expect problems because we have to require a strong degeneracy between the electron and μ-neutrino masses for an efficient working of the magnetic moment mechanism. Then does not seem simple and natural to accomodate the well known big ratio $m_\mu / m_e \simeq 200$ with the strong degeneracy $m_{\nu_\mu} \simeq m_{\nu_e}$, since (ν_e, e) and (ν_μ, μ) are both weak doublets. Moreover in general we have also to expect problems for flavour changing.

The present experimental limits from reactor data are [21]:

$$\begin{cases} \mu_{\nu_e} < 1.5 \times 10^{-10} \cdot \mu_B \,, \\ \\ \mu_{\nu_\mu} < 9.5 \times 10^{-10} \cdot \mu_B \,. \end{cases} \tag{1.1.14}$$

The astrophysical and cosmological bounds are of comparable level. From the absence of additional stellar cooling through magnetic moment induced neutrino emissions, expecially for helium burning stars in open clusters, we get $\mu_{\nu_e} < 10^{-11} \cdot \mu_B$ [22]. The cosmological bound is based on the nucleosynthesis constraint. In fact the induced magnetic moment scattering $\nu_L + e^- \rightarrow \nu_R + e^-$, before the neutrino decoupling, would double the effective number of neutrino species in the early Universe, causing an excess abundance of ^4He. Then it follows $\mu_\nu < 1.5 \times 10^{-11} \cdot \mu_B$, which should not be violated by more than one of the neutrino species [23]. These limits do not forbid the magnetic moment solution of the Solar neutrino puzzle, which requires $\mu_{\nu_e} \sim (10^{-11} \div 10^{-10}) \cdot \mu_B$ for $B_\odot = 1 \div 10\, KG$.

Very recently a stronger limit has been obtained from SN1987A [24]. It is $\mu_{\nu_e} < (0.3 \div 1) \times 10^{-11} \cdot \mu_B$, depending on the core temperature, and becomes even stronger, $\mu_{\nu_e} < (10^{-13} \div 10^{-12}) \cdot \mu_B$, if the number of high energy neutrino events ($E_\nu > 100\, MeV$), deriving from the rotation $\nu_R \rightarrow \nu_L$ in the galactic magnetic field, are taken into account. However, as we will see in the details in chapter 4, the theoretical production of a *large* magnetic moment requires the existence of some new physics not too much beyond the Fermi scale. Then in such a case the ν_R's are affected by new interactions with strength not completely negligible (the ν_R is not *sterile* now) and the supernova limit does not apply in such a case. In fact now the ν_R may be trapped in a similar way as the ν_L, with no dramatic change in the supernova evolution [25].

1.2 Neutrinos in cosmology

Before going to discuss some astrophysical aspects of neutrino physics, in particular the Solar neutrino puzzle and the SN1987A explosion, about which in the following we will point the attention for our research purposes, let us briefly recover some relevant connections between neutrinos and the physics of the early Universe [26].

• *The nucleosynthesis constraint.* The cosmological evolution of the neutrino number density n_ν is described by the Boltzmann equation

$$\dot{n}_\nu(t) + 3\frac{\dot{R}}{R}\, n_\nu(t) = \gamma_{int}\,, \tag{1.2.1}$$

where R is the scale factor of the Universe and $\gamma_{int} = (n_\nu^{eq\,2} - n_\nu^2)\cdot < \sigma v >$ takes into account the neutrino interactions. n_ν^{eq} is the neutrino number density at the thermal equilibrium, σ is the neutrino cross section for the interactions with the thermal background and v is the relative velocity in the interactions. A thermical mean value has been considered. For $t \leq 10^5\, yr$ the Universe was radiation dominated (RD) and $R \sim t^{1/2}$, while succesively the matter dominated era (MD) is realized and $R \sim t^{2/3}$. The temperature scales as $T \sim R^{-1}$ and the time scales for the two eras are roughly given by $t \sim 1\, sec/(T/1\, MeV)^2$ in the RD era, and $t \sim 13\, Gyr(T/2.7°K)^{-3/2}$ in the MD era. At high temperatures the rate of the reaction $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$, of order $G_F^2 T^5$, dominates over the expansion rate

$$\frac{\dot{R}}{R} = H = 1.66\ (g_{eff}(T))^{1/2}\ \frac{T^2}{M_P}\,, \tag{1.2.2}$$

where $M_P = 1.2 \times 10^{19}\, GeV$ is the Planck mass and $g_{eff}(T)$ is the number of degrees of freedom for relativistic particles at the temperature T. Neutrinos are in equilibrium up to $T \simeq 3\, MeV$. Then the expansion rate becomes bigger than the collision rate, giving rise to the neutrino decoupling from the radiation.

However, neutrinos still remain in equilibrium with nucleons (β-equilibrium) up to $T_* \simeq 1\,MeV$, at which temperature the neutrons over protons ratio is frozen to be

$$\frac{n}{p} \sim \exp\left(-\frac{m_n - m_p}{T_*}\right) . \tag{1.2.3}$$

The following evolution is controlled by the factor $e^{-t/\tau}$, with $\tau = 898\,sec$, up to the time of the nuclei formation $t_N \simeq 3.5\,min$, which crucially depends on the value of the barion abundance [27]:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = (3 \div 10) \times 10^{-10} . \tag{1.2.4}$$

The derived primordial helium abundance is $Y_p \simeq 0.24$.

From the nucleosynthesis process we can obtain an important bound on the number species of light stable neutrinos [28]. If N_ν increses then also $g_{eff}(T)$ increases, leading to an increasing of T_* and then to an earlier decoupling. Now if T_* increases also n/p and Y_p increase. In particular the relation $\delta Y_p = 0.012\,\delta N_\nu$ has been calculated, and from the observational upper limit $Y_p \leq 0.25$, together with the lower limit on η, it follows the constraint

$$N_\nu \leq 4 \tag{1.2.5}$$

for the number of neutrino species.

• *The cosmological density constraint.* If neutrinos decouple while they are relativistic ($m_\nu \leq 3\,MeV$), the effective temperature of the relic neutrinos to day is $T_\nu = (4/11)^{1/3} \times 2.74\,°K = 1.96\,°K$, and their number density is $n_{\nu+\bar\nu} = 110\,cm^{-3}$ per one Majorana neutrino species.

If they are massive their contribution to the mass density of the Universe is

$$\rho_{\nu+\bar\nu} = \sum_i \left(\frac{g_i}{2}\right) \left(\frac{m_i}{1\,eV}\right) 110\,cm^{-3} , \tag{1.2.6}$$

where $g_i = 2$ if Majorana and $g_i = 4$ if Dirac, is the number of degrees of freedom. Let us define $\Omega \equiv \rho/\rho_c$, where

$$\rho_c = \frac{3H_\circ^2}{8\pi G} = 10.5\,h_\circ^2\,KeV\,cm^{-3} , \tag{1.2.7}$$

$H_\circ = (\dot{R}/R)_\circ$ is the Hubble constant

$$H_\circ = 100\, h_\circ\, km\, sec^{-1}\, Mpc \tag{1.2.8}$$

and $h_\circ$ is a parameter observationally $0.5 \leq h_\circ \leq 1$. If now we require, as indicated by different kinds of observations, $\Omega \leq 1$, we derive an upper limit on the neutrino masses [29]:

$$\sum_i \left(\frac{g_i}{2}\right) m_i \leq 100\, h_\circ^2\, eV\,. \tag{1.2.9}$$

We also remember that the inflationary scenario [30] predicts $\Omega = 1$. Then from Cosmology we derive interesting indications for a non zero neutrino mass, whose typical value is placed around the eV scale. That is also in agreement with some possible solutions of the Dark Matter problem, which however seem to show some difficulties.

• *Neutrinos and Dark Matter.* Different arguments exist in favour of the Dark Matter, in particular: the existence of the galactic halo deduced from the rotation curve [31]; the large $M/L \simeq 100 \div 500$, mass to luminosity, ratio for the clusters [32]. If $\sum_i (g_i/2)\, m_i \geq 5\, eV$, the neutrino contribution to the cosmological mass density is $\Omega_\nu \geq 0.05\, h_\circ^{-2}$, and exceeds that from barions: $\Omega_B < 0.04\, h_\circ^{-2}$. Then neutrinos gravitationally dominate the Universe and a massive neutrino could be a good candidate for Dark Matter.

Such a neutrino could give rise to structures whose typical size is given by the Jeans mass [33]

$$M_J \sim \frac{M_P^3}{m_\nu^2} \sim 10^{15}\, M_\odot \left(\frac{m_\nu}{30\, eV}\right)^2 . \tag{1.2.10}$$

Therefore a neutrino with a $30\, eV$ mass may trigger the formation of superclusters, which should be the first collapsed astrophysical object. A difficulty of this scenario is that galaxies form too late. However, very recently, large structures at the scale of superclusters have been observed, large voids, with galaxies in walls and filaments [26]. Thus neutrinos are reconsidered as Dark Matter candidates, and other mechanisms, such as the cosmic strings, could be active in a faster formation of the galaxies.

• *Unstable neutrinos.* The constraint (1.2.6) is a quite strong condition on the neutrino masses. However, a neutrino could escape this condition if it were unstable with a lifetime shorter than the age of the Universe. The constraint in this case is [34]

$$m_\nu^2 \tau_\nu < 3.8\, h_\circ^4 \times 10^{21}\, sec\, eV^2\,, \tag{1.2.11}$$

instead of (1.2.6). However, if neutrino emits radiation within the lifetime of the Universe, the number of photons becomes extremely large. If the neutrino lifetime is $10^{15}\, yr \geq \tau_\nu \geq 10^5\, yr$, the emitted photon should be observed as a diffuse UV- or X-ray background radiation [34]. If $10^6\, sec \leq \tau_\nu \leq 10^5\, yr$, the emitted photons should have distorted the spectrum of cosmic microwave radiation [35]. Therefore these ranges are excluded. Additional constraints may also be derived from the nucleosynthesis argument, X-ray emission from the Sun, white dwarfs, supernova and also others [36].

These strong constraints are evaded if the decay does not involve the photon. An interesting possibility is that a neutrino with a $1\, KeV$ mass could trigger the formation of galaxies with $M \sim 10^{12}\, M_\odot$, and then it decays so that the mass density constraint is satisfied [37]. This would be an example of the neutrino as a cold Dark Matter particle.

• *Heavy neutrinos.* Finally, If neutrinos are heavy, they decouple after becoming non relativistic ($T < m_\nu$). In such a case the number density is suppressed by the Boltzmann factor $e^{-m_\nu/T}$ and it decreases as $n_\nu \sim 1/m_\nu^3$. Now the cosmological density constraint requires [38] $m_\nu > 2 \div 3\, GeV$, much larger than the upper bound on the τ neutrino mass. Therefore this argument is meaningful only for a fourth generation, or some neutral lepton similar to the neutrino. This argument usually applies to weakly interacting massive particles (WIMP's), which are thought to be good candidates for cold Dark Matter.

1.3 Astrophysical neutrinos

• *Solar neutrinos.* What is known as the Solar neutrino puzzle consists in a strong discrepancy between the Davis measurements [39] of the solar neutrino flux and the Bahcall-Ulrich theoretical predictions [40] based on the Standard Solar Model. This discrepancy has been recently confirmed by the Kamiokande data [41]. The Davis detector is based on the neutrino capture $^{37}Cl + \nu_e \rightarrow {}^{37}Ar + e^-$. The reaction is identified by the subsequent decay of $^{37}Ar\,(t_{1/2} = 35\, d)$. The experiment has detected from 1970.3 up to 1988.3 an average value of $2.33 \pm 0.25\, SNU$ (1 ν-capture per sec per 10^{36} target nuclei), where $7.8 \pm 0.9\, SNU$ are expected by Bahcall and Ulrich. Moreover, as we can see from Fig.s 2 and 3, Davis data seem to display an anticorrelation between the detected neutrino flux and the cyclic variation of the sunspot number, which follows an 11 years period.

The dominant part of Davis neutrinos is due to the ${}^8B \to {}^8Be^* + e^+ + \nu_e$ reaction, since the threshold of Davis detector is $0.81\, MeV$ and then the $p + p \to d + e^+ + \nu_e$, which would give a 10^4 bigger contribution to the flux, are cutted away because their maximum energy is $0.42\, MeV$. This sensibility to the ${}^8B$ neutrinos is very important. In fact the ${}^8B$ neutrino flux is expecially sensitive to the temperature at the centre of the Sun. One of the initial motivations of Davis experiment was to check this central temperature. It is easy to understand that a small change in the temperature may substantially modify the predicted ${}^8B$ neutrino flux. However, this possibility would not enlight the phenomena, if confirmed, of the time oscillation of Davis data.

This deficiency of the ${}^8B$ neutrino flux with respect to the Standard Model predictions has been recently confirmed by Kamiokande, whose detector is based on the electron neutrino scattering, which takes into account only ${}^8B$ neutrinos since its threshold is around $10\, MeV$. These measurements give us a reduction factor of $0.46^{+0.13}_{-0.12}$ with respect to the Standard Solar Model prediction. In particular it is interesting to observe that if we accept the Bahcall prediction for the non-${}^8B$ neutrinos (essentially ${}^7Be$ and ${}^{15}O$), we can deduce that Davis observes $0.53 \pm 0.32\, SNU$ from ${}^8B$. Then, for Kamiokande, we have to expect a reduction factor with respect to the Standard Solar Model of 0.09 ± 0.05, which is inconsistent with the Kamiokande measurement unless to choose a 3σ level. So, the Standard Solar Model theoretical predictions for the ${}^7Be$ and ${}^{15}O$ neutrinos, seem to give rise to a real problem independently from any theoretical assumption on the ${}^8B$ neutrinos.

One explanation of the Solar puzzle is the existence of mixings in the leptonic sector which produce vacuum oscillations $\nu_e \to \nu_X$. These could be relevant if $\Delta m^2 \equiv m^2_{\nu_e} - m^2_{\nu_X} \geq (10^{-11} \div 10^{-10})\, eV^2$ and if the mixing angles are very large. In such a case the neutrino vacuum oscillation length is smaller than $1.5 \times 10^{13}\, cm$, the distance between the Sun and Earth. Then if the mixing is maximal the average neutrino flux turns out to be reduced in a considerable way. Another interesting possibility is the presence of a large magnetic moment $\mu_{\nu_e} \sim (10^{-11} \div 10^{-10}) \cdot \mu_B$ for a Dirac ν_e. In such a case the transition, in the solar magnetic field, into a sterile right-handed ν_e, on one hand would reduce the expected neutrino flux, on the other could explain the anticorrelation with the sunspot number, since this number is correlated with the intensity of the Solar magnetic field. Another interesting effect which such a solution is able to produce is seasonal oscillation of the flux, due to the inclination of the Earth axis, which seems to be confirmed by the Davis measurements in the period of maximum solar activity. We will treat in detail this topic and its theoretical and phenomenological implications in the last chapter.

• *Matter resonant oscillations.* Recently S. P. Mikheyev and A. Yu. Smirnov have proposed an elegant solution (MSW) [42] in which, even in presence of small mixing angles, we can have a resonant amplification of the neutrino oscillations due to the matter interactions of ν_e. Let us consider for simplicity a $\nu_e - \nu_\mu$ system with mixing angle θ:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} , \tag{1.3.1}$$

ν_1 and ν_2 are the vacuum mass eigenstates. The time evolution of the state $\psi = a(t) \cdot \nu_e + b(t) \cdot \nu_\mu$ in the presence of matter is given by the Schrodinger equation

$$i \frac{d}{dt} \psi = H\psi , \tag{1.3.2}$$

where

$$H = \frac{\Delta m^2}{4k} \cdot \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} \sqrt{2} G_F n_e & 0 \\ 0 & 0 \end{pmatrix} . \tag{1.3.3}$$

k is the neutrino momentum and $\Delta m^2 = m_2^2 - m_1^2$. The second matrix, where n_e is the electron number density in the matter, takes into account the effects of the coherent forward scattering $\nu_e + e^- \to \nu_e + e^-$ via the charged current. The effects of neutral current scattering have been dropped because they are the same for ν_e and ν_μ, and so they only contribute to an overall phase. The situation would be different for $\nu_e - N$ oscillations, with N a *sterile* neutrino, in such a case also the neutral current interactions would be relevant [43]. In terms of the matter eigenstates ν_1^m and ν_2^m, we have

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta^m & \sin\theta^m \\ -\sin\theta^m & \cos\theta^m \end{pmatrix} \cdot \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} , \tag{1.3.4}$$

where

$$\sin^2 2\theta^m = \frac{\sin^2 2\theta}{\sin^2 2\theta + \left(\frac{A}{\Delta m^2} - \cos 2\theta\right)^2} , \tag{1.3.5}$$

and $A = 2\sqrt{2}kG_F n_e$. For $\Delta m^2 > 0$ $(m_1 < m_2)$, there is a critical density $n_e^c = \Delta m^2 \cos 2\theta/(2\sqrt{2}G_F k)$ for which the diagonal elements of H are equal, the mixing is maximum, and then a resonance occours. In such a case we do not have a resonance effect in the antineutrino sector, where viceversa it is present if $\Delta m^2 < 0$. If in the center of the Sun, where ν_e are produced, $n_e > n_e^c$, neutrinos cross the resonance point going out of the Sun. θ^m is minimum outside $(\theta^m = \theta \simeq 0)$ but it is maximum in the center of the Sun $(\theta^m = \pi/2)$, so $\nu_e = \nu_2^m$ in the center becomes $\nu_2 = \nu_\mu$ outside the Sun if the propagation is adiabatic:

$$\frac{1}{n_e}\frac{dn_e}{dr} \ll \frac{\Delta m^2}{k} \cdot \frac{\sin^2 2\theta}{\cos 2\theta} . \tag{1.3.6}$$

In such a case the resonant convertion probability $P(\nu_e \to \nu_\mu)$ is unity. In general a violation of the adiabatic condition is taken into account [44] by the Landau-Zenner term P_x in $P(\nu_e \to \nu_\mu) = 1 - P_x$, where:

$$P_x = \exp\left(-\frac{\pi}{4}\frac{\Delta m^2}{k}\frac{\sin^2 2\theta}{\cos 2\theta}\frac{n_e}{|dn_e/dr|}\right) . \tag{1.3.7}$$

The so called Bethe solution [45] is adiabatic and corresponds to $\Delta m^2 \sim 5 \times 10^{-5}\, eV^2$ and $\sin^2 2\theta \geq 4 \times 10^{-4}$. In such a case only high energy ^8B neutrinos are converted. The measured Kamiokande reduction factor $0.46^{+0.13}_{-0.12}$ seems to exclude this solution. In fact in such a case if the high energy ^8B neutrinos oscillate into a *sequential* neutrino (ν_μ or ν_τ), we should find a reduction factor of 0.14, due to the neutral current scattering in the Kamiokande detector. On the other hand if the oscillation is into a *sterile* neutrino, the reduction factor must be 0, since no relevant interaction is expected. Also a non adiabatic solution [46] corresponding to $\Delta m^2 \sin^2 2\theta \sim 10^{-7.5}\, eV^2$ is possible. Here all neutrino energies are affected but the conversion probability is less than one.

• *Neutrinos from SN1987A.* A new very relevant connection between astrophysics and neutrino properties has been established for the first time on February 23, 1987. On that date light and neutrinos from the explosion of the supernova SN1987A, sited in the Large Magellanic Cloud 50 kpc away, reached Earth. Neutrino signals have been detected by Kamiokande and IMB principally, and also by Mt.Blanc and Baksan: extrasolar neutrino astronomy was born [47]. There is no doubt that Kamiokande [48] and IMB [49] have observed burst signals from SN1987A, on the other hand maybe the Mt.Blanc [50] and Baksan [51] candidates are background fluctuations, and an absence in the Davis detector is consistent with our expectations. This phenomenon has confirmed on one hand the theoretical expectations on the stellar core collapse, on the other hand it is of significant importance in deriving new relevant limits on the various aspects of neutrino physics.

Kamiokande has detected 11 events, where the first 2 point back to the direction of the star, while the others are almost isotropic. IMB has reported 8 events. The first 2 Kamiokande events may be identified with $\nu_e + e^- \rightarrow \nu_e + e^-$ and it could be possible to identify them as produced by the initial deleptonization burst, whose characteristic duration is about $10\,msec$. However, if this the case, these events seem to be too much, in fact the predicted deleptonization events are about 0.2, ten times smaller. The other 9 events are isotropically distributed and so are interpreted as $\bar{\nu}_e + p \rightarrow e^+ + n$ reactions, with a duration of a few sec. The recoil energy is consistent with a Fermi distribution with temperature $T = 2 \div 3\,MeV$ and zero chemical potential. From this data we can extract the total energy liberated by neutrino emission, which corresponds almost to the total binding energy of the core. In fact, from the theoretical point of view, the fraction liberated as kinetic energy of the explosion, optical energy and gravitational wave, is only some small percentage. Then we derive the binding energy to be:

$$E_B = (2.5 \div 3) \times 10^{53}\,erg\,, \tag{1.3.8}$$

where we have assumed equipartition between the various neutrino species. The observations of Kamiokande and IMB confirm very well the theoretical predictions, in particular for the duration of the burst, the mean energy of the neutrinos and the derived binding energy of the core, which corresponds to a mass of about $1.4\,M_\odot$. Two kinds of limits on non standard physics can be extracted from SN1987A.

a) First of all we consider the limits on neutrino physics derived from the direct observation of the neutrino flux.

i) Since all the expected $\bar{\nu}_e$ are observed, the $\bar{\nu}_e$ *lifetime* must satisfy

$$\gamma\,\tau_{\bar{\nu}_e} > 1.6 \times 10^5\,yr\,. \tag{1.3.9}$$

This excludes that ν_e decay could be a good solution of the Solar neutrino puzzle, since as we know $\gamma_{Sun} \sim 10^{-1}\,\gamma_{SN}$.

ii) We cannot extract the value of the neutrino *mass*, since we do not know the delay time at the emission. However, we can put interesting bounds on this value, obtaining

$$m_{\bar{\nu}_e} < 23 \div 30\,eV\,. \tag{1.3.10}$$

If a reasonable supernova model is invoked, this limit is reduced to $m_{\bar{\nu}_e} < 10\,eV$, next to the more recent laboratory bounds.

iii) From the equipartition of the emitted energy between the various neutrino species, it is possible to extract a limit on the number of *neutrino flavours* (if $m_\nu < 10\,MeV$). We obtain

$$N_\nu \leq 6\,, \tag{1.3.11}$$

not as good as the cosmological and the more recent accelerator bounds.

iv) If a *neutrino mixing* exists, with an oscillation length smaller than $50\,kpc$, the only effect of the corresponding vacuum oscillations $\nu_e \leftrightarrow \nu_\mu, \nu_\tau$ is an increasing of the ν_e mean energy. In fact the ν_μ and ν_τ only interact via the neutral, rather than the charged, current weak interactions. Then their trapping is weaker with respect to the ν_e and they can be emitted from deeper and hotter regions of the core. However, it is difficult to resolve this energy enhancement, thus no definite statements on mixing can occur. Let us now consider the MSW effect. If this is the solution to the Solar neutrino puzzle, then only $\nu_e \leftrightarrow \nu_\mu, \nu_\tau$ resonant oscillations are possible, without transitions in the antineutrino sector, and the possibility of seeing a neutronization scattering should be significantly reduced. On the contrary, if we take into account a resonant effect in the antineutrino sector, as in the case of vacuum oscillations, no definite statement can be done. Similarly not much can be extracted for the mixing with *sterile* neutrinos from the direct observation. However, we will see in chapter 3 how, using an energetic argument introduced in the following, we are able to obtain extremely stringent limits.

b) We can also deduce bounds on the couplings of non standard particles using a powerful energetic argument.

i) This argument, as we will see in the detail in chapter 3, is based on the fact that a sufficiently light new particle which couples to the standard elementary particles, opens a new channel for the energy emission from the supernova. In particular for the *axions*, where the coupling is connected with the mass, we derive [52]

$$m_a \leq 0.75 \times 10^{-3}\, eV \qquad \text{or} \qquad m_a \geq 3.7\, eV\,. \tag{1.3.12}$$

ii) If ν_{eR} has a mass $m_{\nu_{eR}} < 10\, MeV$, and it couples through right-handed currents to the standard fermions, we can derive bounds on the strength of the right-handed interactions [53]

$$M_{W_R} \leq (3.7 \div 6.7) \cdot M_{W_L} \qquad \text{or} \qquad M_{W_R} \geq 200\, M_{W_L}\,, \tag{1.3.13}$$

and so on the value of the *right-handed vector boson mass.*

iii) Also a bound on the *neutrino magnetic moment* can be extracted [24],

$$\mu_{\nu_e} \leq (0.3 \div 1) \times 10^{-11} \cdot \mu_B\,, \tag{1.3.14}$$

since it controls the strength of the magnetic transition $\nu_{eL} \rightarrow \nu_{eR}$. As already discussed, this bound is valid for a ν_R with no other relevant interactions than that induced by the magnetic moment itself.

In the following we will study the general energetic argument mentioned at the point *b*), and it will be applied in the concrete case of the $\nu_{eL} - \nu^c_{eL}$ mixing [3].

2. Phenomenological implications of $\nu_L - \nu_R$ mixings

As already explained in section 1.1, if Dirac and Majorana mass terms are present, then the induced $\nu_L - \nu_L^c$ mixings give rise to universality violation and flavour changing in the leptonic neutral currents. This phenomenon arises in general extensions of $SU(2)_L \times U(1)_Y$. In the Standard Model, as well as in Grand Unification models (GUT's) based on the gauge group $SU(5)$, $B-L$ is an exact global symmetry and neutrinos turn out to be massless, unless one introduces non standard Higgs fields and/or extends the fermionic sector of the theory. However, in left-right symmetric models, or in GUT's based on gauge groups with rank larger than four as $SO(10)$ and E_6, $B-L$ is a local gauge symmetry spontaneously broken at some large mass scale. Therefore neutrinos turn out to be massive in a natural way, leading to phenomena as the neutrino oscillations, rare decays such as $\mu^- \to e^-\gamma$, neutrinoless double-beta decay, and to important consequences both for the elementary particle physics and for cosmology and astrophysics.

In this chapter we will focus on the phenomenological consequences of a sizeable mixing between the usual left-handed neutrino and its right-handed partner (more generally a heavy *sterile* neutral lepton) [1]. In particular we will concentrate on the effects in e^+e^- experiments at the Z° peak. In the following chapter we will study the same problem for the electron sector with the help of SN1987A. Here we compare our results with the present limits and we analyse the predictions of a $SO(10)$ based model [54] which naturally admits a large $\nu_{\tau L} - \nu_{\tau R}$ mixing.

2.1 Present limits on $\nu_L - \nu_R$ mixings

First of all let us discuss the experimental constraints on the parameters describing the mixing between the standard light neutrinos and a heavy neutral state N.

i) If ν_μ has a small contamination of a heavy mass eigenstate N, then:

$$\nu_\mu \simeq \left(1 - \frac{1}{2}\,|U_{\mu N}|^2\right) \cdot \chi_2 \,+\, U_{\mu N} \cdot N\,, \tag{2.1.1}$$

where χ_2 is the light mass eigenstate along which ν_μ is mostly directed. From $\mu-$ and $\beta-$ decays, using the unitarity constraint for three generations, we obtain an upper bound on $|U_{\mu N}|^2$ of order 10^{-4}. This limit is more stringent than the actual error on the unitarity constraint, because its central value is smaller than one: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9979 \pm 0.0021$ [55], and the effect of the $|U_{\mu N}|^2$ term is to increase the experimentally measured quantity $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ with respect to 1.

ii) Using for ν_e a parametrization similar to eq. (2.1.1), and taking into account the experimental data on the branching ratios for $\pi \to e\,\nu$ and $\pi \to \mu\,\nu$ decays, together with the bound on $U_{\mu N}$ previously derived, we get:

$$|U_{eN}|^2 \,\leq\, 6 \cdot 10^{-2}\,. \tag{2.1.2}$$

iii) Another constraint on the mixing parameter between the light neutrinos and a heavy state N with mass in the range of tens of GeV, comes from the absence of anomalous monojet events at PETRA and PEP [56]. The process $e^+e^- \to N\bar{\nu}_e$ will be observed as an electron-positron collision which results in an event with missing energy and momentum (due to the $\bar{\nu}_e$) on one side, and a jet of decay products of N on the other side (monojet event). The corresponding limit on $|U_{\tau N}|^2$ is shown in Fig. 4, as a function of the N mass. The same constraint also holds for $|U_{\mu N}|^2$, although it is not better than the unitarity bound. Due to the extra contribution of W-exchange, $|U_{eN}|^2$ is about 4.5 times more severely constrained than $|U_{\mu N}|^2$ and $|U_{\tau N}|^2$.

iv) A bound on $|U_{\tau N}|^2$ can be derived from the $\tau \to e\,\bar{\nu}_e\,\nu_\tau$ decay [57]. By taking the *world average* on the experimental τ-lifetime

$$< \tau_\tau > = (3.04 \pm 0.09) \cdot 10^{-13}\; sec, \tag{2.1.3}$$

together with the standard model prediction for $\Gamma(\tau \to e\,\bar{\nu}_e\,\nu_\tau)$, we get:

$$\mathcal{BR}_{th}(\tau \to e\,\bar{\nu}_e\,\nu_\tau) \,=\, \Gamma_{th}(\tau \to e\,\bar{\nu}_e\,\nu_\tau) \cdot < \tau_\tau > \,=\, (19.0 \pm 0.6)\,\%\,. \tag{2.1.4}$$

This has to be compared with the result of the direct measurement:

$$\mathcal{BR}_{exp}(\tau \to e\,\bar{\nu}_e\,\nu_\tau) \;=\; (17.5 \pm 0.4)\,\%. \tag{2.1.5}$$

Thus, we find:

$$\frac{\Gamma_{th}(\tau \to e\,\bar{\nu}_e\,\nu_\tau) \;-\; \Gamma_{exp}(\tau \to e\,\bar{\nu}_e\,\nu_\tau)}{\Gamma_{th}(\tau \to e\,\bar{\nu}_e\,\nu_\tau)} \;=\; 0.08 \pm 0.04\,. \tag{2.1.6}$$

The indication of a possible discrepancy, suggested by previous data in Ref. [58], does not seem so evident now if we require at least a 2σ level for a clear signal. In any case there is still the interesting possibility for a sizeable violation of lepton universality in the τ-system. With a parametrization of $\nu_{\tau L} - \nu_{\tau R}$ mixing analogous to eq. (2.1.1):

$$\nu_\tau = \chi_3 \cdot \cos\theta \;+ N \cdot \sin\theta\,, \tag{2.1.7}$$

where $\sin^2\theta \;\equiv\; |U_{\tau N}|^2$, we find

$$\Gamma(\tau \to e\,\bar{\nu}_e\,\nu_\tau) \;=\; \Gamma_{th}\cos^2\theta\,, \tag{2.1.8}$$

where Γ_{th} is the theoretical width-decay as calculated in the Standard Model in absence of mixing. Hence, using eq. (2.1.8), we get the upper limit

$$\frac{\Gamma_{th} \;-\; \Gamma}{\Gamma_{th}} \;=\; |U_{\tau N}|^2 \;\leq 0.16\,, \tag{2.1.9}$$

at the 2σ level.

The experimental constraints on the τ-neutrino mixing turn out to be not so stringent as for ν_e and ν_μ. Therefore, we will study the possible signatures of a sizeable $\nu_{\tau L} - \nu_{\tau R}$ mixing at the future $e^+\,e^-$ colliders. We are also motivated by a particular model, based on $SO(10)$, recently proposed by R. Johnson, S. Ranfone and J. Schechter (JRS) [54], which naturally allows the possibility of such a large mixing.

2.2 Effects of a sizeable $\nu_{\tau L} - \nu_{\tau R}$ mixing

A sizeable $|U_{\tau N}|^2$ could give an important effect in the neutrino counting experiments in $e^+\,e^-$ collisions at $\sqrt{s}$ somewhat above the Z° peak [59]. The experiments will look for single photons of a few GeV, recoiling against an undetected system with invariant mass equal to M_Z. Due to the non-vanishing mixing, the measured effective number of neutrinos is not an integer, but it is given by

$$\mathcal{N}_\nu \;=\; \sum_{a,b=1}^{3} |P_{ab}|^2 \,, \tag{2.1.10}$$

where P_{ab} is the upper-left 3×3 block of the mixing matrix defined in section 1.1. In the case of a mixing only between $\nu_{\tau L}$ and $\nu_{\tau R}$ eq. (2.1.7) gives:

$$P \;=\; U^\dagger U \;=\; \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos^2\theta & \sin\theta\,\cos\theta & 0 & 0 \\ 0 & 0 & \sin\theta\,\cos\theta & \sin^2\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} . \tag{2.1.11}$$

Therefore, we get:

$$\mathcal{N}_\nu \;=\; 3 - 2\,\sin^2\theta + \sin^4\theta \,. \tag{2.1.12}$$

Since the expected LEP sensibility to $\mathcal{N}_\nu$ is roughly $15\,\%$, the neutrino counting experiments can probe $|U_{\tau N}|^2$ up to 0.08. The corresponding region is shown in Fig. 4, compatibly with the bounds previously derived from τ- decay and monojet events.

The non-diagonal couplings of Z° to the neutrino mass eigenstates, eq. (1.1.9), lead to *one-sided* events, with unbalanced transverse energy, due to the process $Z^\circ \to N\nu_\tau$, with N decaying into observable particles [1]. The corresponding branching ratio is:

$$\mathcal{BR}(Z^\circ \to N\nu_\tau) \;=\; \frac{1}{2}\,\sin^2 2\theta \,\cdot\, \mathcal{BR}(Z^\circ \to \nu_e\,\bar\nu_e) \,\cdot\, I\left(\frac{M_N^2}{M_Z^2}\right) , \tag{2.1.13}$$

$$I(x) \;=\; (1-x)\left(1 - \frac{x}{2} - \frac{x^2}{2}\right) . \tag{2.1.14}$$

The mass of ν_τ , being much smaller than M_Z and M_N, has been neglected in eq. (2.1.13). The region of parameters giving a branching ratio for $Z^\circ \to N\,\nu_\tau$ larger than 10^{-5} is shown in Fig. 4. Due to the clear events characteristic of such a process, this limit is well within the reach of LEP.

[1] The decay $N \to \nu\,\nu\,\nu$ gives a contribution to $\mathcal{N}_\nu$, as measured in neutrino counting experiments, which however we have estimated to be less than 10^{-4}.

It is worth mentioning that no standard model background for this decay is expected. A non diagonal Z° coupling to a possible heavy fourth generation neutrino can occur only at one-loop level. Actually, for reasonable values of inter-generational mixings and masses of the extra charged lepton, the coupling turns out to be too small to be visible at SLC or LEP [60]. A non-conventional source of background is supersymmetry. The relevant exotic neutral fermions (generally called *neutralinos*) can reproduce the above-mentioned one-sided events. The missing energy is, in this case, carried by the lightest neutralino, which is stable due to R-parity conservation [2]. Nevertheless, the comparison with other experimental results will help to understand the origin of a possible discrepancy with the Standard Model. In particular, supersymmetry predicts a result for $\mathcal{N}_\nu$ in neutrino counting experiments larger than 3 [62]. On the contrary, the neutrino mixing always gives a $\mathcal{N}_\nu$ less than 3, although visible effects are possible only in a small region of parameters (see Fig. 4). Moreover, supersymmetry provides a rate for heavy neutralino pair production, which, if allowed by phase space, is of the same order of the rate for one-sided events [62]. On the other hand, the heavy neutrino pair production is suppressed by an extra power of $|U_{\tau N}|^2$ with respect to the non-diagonal process (eq. (2.1.13)). If $M_N < M_Z/2$, the branching ratio for $Z^\circ \to N\,N$ decay is:

$$\mathcal{BR}(Z^\circ \to N\,N) = \sin^4\theta \cdot \left(1 - 4\,\frac{M_N^2}{M_Z^2}\right)^{3/2} \cdot \mathcal{BR}(Z^\circ \to \nu_e\,\bar{\nu}_e)\,. \tag{2.1.15}$$

In Fig. 4 we show the region corresponding to a branching ratio (eq. (2.1.15)) larger than 10^{-5}.

2.3 A large $\nu_{\tau L} - \nu_{\tau R}$ mixing from a theoretical model

Now we examine how the JRS model relates the mixing $|U_{\tau N}|^2$ and the ν_τ-mass to M_N. We anticipate our results, showing in Fig. 4 the prediction of the model. The astrophysical, cosmological and experimental constraints on the ν_τ-mass single out a special region of parameters, defined by the dashed-line. Spectacular events at the Z° peak are predicted, although no visible deviations in τ-decays and neutrino counting

---

[2] Even some supersymmetric models with R-parity breaking can lead to the same events, due *e.g.* to the exchange of a scalar electron neutrino in the s-channel [61].

experiments are expected. Absence of $Z^\circ \to NN$ events can provide a distinguishable feature from supersymmetry. Let us turn to discuss in more detail this model.

The JRS model, essentially an extension to leptons of the Fritzsch-Stech quark model [63], has the nice feature to be particularly predictive for the lepton sector. Neutrino mass ratios and the lepton mixing matrix can be computed, taking as input parameters the masses of the charged fermions and the phenomenologically known entries of the K-M mixing matrix ($|U_{us}| = 0.225,\ |U_{cb}| = 0.055$).

In the neutrino mass matrix, $\mathcal{M}_\nu$ of eq. (1.1.5), M_D has entries of order of the charged fermion mass scale (m_D), while M_R is of order of the large $B-L$ breaking scale (M_{B-L}). In the conventional approach to the neutrino masses in theories like $SO(10)$, if M_R is non-singular, the diagonalization of $\mathcal{M}_\nu$ gives an effective 3×3 light Majorana neutrino mass matrix [10]:

$$\mathcal{M}_\nu{}^{eff} \simeq M_L - M_D^T\, M_R{}^{-1}\, M_D. \tag{2.1.16}$$

The second term of this equation, the well known *see-saw term*, is usually assumed to dominate over M_L. In such a case, one gets *three* light neutrinos (mostly weak-isodoublet) with masses $m_\nu \simeq \mathrm{O}(m_D{}^2\,/\,M_{B-L})$, and *three* super-heavy (mostly isosinglet and *sterile*) neutrinos with masses $M_N \simeq M_{B-L}$. This is known as the *ordinary see-saw mechanism* [10]. In this case, it is difficult to get a sizeable mixing between left- and right-handed neutrinos, essentially because of the very large gap of masses.

However, as it has been shown in Ref. [54], under specific, but plausible assumptions on the form of the fermion mass matrices in the framework of $SO(10)$, the Majorana mass matrix M_R seems to be close to the singularity defined by $\det M_R \simeq 0$. In such a situation, of course, the approximated eq. (2.1.16) is no longer valid, and one needs to diagonalize the full 6×6 mass matrix $\mathcal{M}_\nu$. At the exact singularity (*singular see-saw*), this diagonalization yields *two* super-light neutrinos (mostly ν_{e_L} and ν_{μ_L}), *two* super-heavy (sterile) neutrinos, and *two* maximally mixed states, giving rise to a single Dirac ν_τ with a mass $\mathrm{O}(100 MeV)$. Because of the present experimental upper bound of $35\, MeV$ on m_{ν_τ} [7], one must deviate from the exact singularity. Nevertheless, in a narrow region of parameters (corresponding to the so called *magic canyon* of Ref. [54]), one gets two splitted intermediate-mass states substantially mixed.

In this condition of *quasi-singularity*, we have diagonalized the neutrino mass matrix through numerical methods. Then, the mixing parameter $|U_{\tau N}|^2$ and m_{ν_τ} have been computed in terms of the mass of the heavy state N mixed with the τ neutrino.

Once a phenomenologically plausible set of values for the input parameters of the model is chosen, we have observed that the results around the singularity do not crucially depend on the top quark mass, in the range 70 to $100\, GeV$. In our calculations, we have assumed $m_{top} = 80\, GeV$.

Fig. 5, giving m_{ν_τ} as a function of M_N, shows how these two mass eigenvalues split, as one moves away from the singularity. In order to avoid an overproduction of primordial ^4He and an excess of X-ray flux from decays of the neutrinos emitted in supernovae and white dwarfs, one excludes a neutrino mass between 1 and $10\, MeV$, because in such a case the ν_τ lifetime is too big with respect to the cosmological and astrophysical constraints (see e.g. [64]). These constraints, together with the experimental upper bound $m_{\nu_\tau} < 35\, MeV$, single out an allowed range for M_N, as shown in Fig. 5.

Given $|U_{\tau N}|^2$, we can now compute the expected rate for the process $Z^\circ \to N\nu_\tau$, discussed in section 2.2. For the allowed values of M_N, interesting Z° branching ratios between 10^{-3} and 10^{-5} are achieved, as shown in Fig. 6.

• *Conclusions.* Consistently with the present bounds, the τ-system is certainly the best place to look for violation of lepton universality. The mixing of $\nu_{\tau L}$ with a *sterile* neutrino with a mass in the range of tens of GeV gives rise to an interesting phenomenology at the Z° resonance. We have shown how SLC and LEP will be able to test such a mixing in an efficient way. Finally, we have also studied the consequences of a predictive model, based on $SO(10)$, which admits a large $\nu_{\tau L} - \nu_{\tau R}$ mixing.

3. Limits on the $\nu_{eL} - \nu_{eR}$ mixing from SN1987A

In the previous chapter, taking into account only the present experimental data from laboratories, we have seen how the violation of lepton universality in the electron sector can be bounded at a level of 10^{-2}. In this chapter, if the ν_{eR} mass is less than $10\,MeV$, we will derive a drastically more stringent bound on the $\nu_{eL} - \nu_{eR}$ mixing, by analysing the physics of the supernova SN1987A explosion [3].

The analyses of this kind of limit is complicated, with respect to the other typical bounds extracted from the supernova, because neutrino mixing not only generates a coupling between the ν_{eR} and the standard elementary particles, but also induces the phenomenon of matter resonant oscillations [42] in the supernova. We will take into account these effects in the derivation of the limit on the $\nu_{eL} - \nu_{eR}$ mixing.

3.1 General aspects of the supernova physics

• *Supernova characteristics.* The collapse of the iron core of a massive evolved star, at the end of its thermal life, gives rise to a type II supernova [65]. Successively heavier elements, following the chain H, He, C, Ne, O and Si, are ignited to compensate the gravitational pressure. A star with a mass bigger than $5 \div 8\, M_{\odot}$, can ignite also silicon for a few days. The iron core grows and when it exceeds the Chandrasekhar limit ($M_{ch} \simeq 5.8\, Y_e^2 M_{\odot} \simeq 1.4\, M_{\odot}$ if $Y_e \simeq 0.4$), at which the pressure of degenerate electrons is still able to contrast the gravitational pressure, the collapse of the star starts. It is generally supposed that a neutron star, or a black hole if the core is massive enough, remains at the end of the process.

The energy released during this process is simply the binding energy of the inner core:

$$\Delta E \simeq E_B = \frac{3}{5}\, G\, \frac{M_c^2}{R_c} = 3 \times 10^{53} \left(\frac{M}{1.4\, M_{\odot}}\right)^2 \left(\frac{R}{10\, km}\right)^{-1} erg\,. \tag{3.1.1}$$

Almost all this energy is released via *neutrino emission.* In fact the photodisintegration $^{56}Fe \rightarrow 13\,^4He + 4n - 124\,MeV$, the kinetic energy of the ejected mass, the optical energy and the gravitational wave emission, are responsible for not more than some small percentage of E_B.

• *Neutrinos and the supernova.* Let us describe very briefly the dynamics of the collapse. When the core density increases, the electron capture reduces Y_e, the number of electrons per nucleon, then reducing the Chandrasekhar mass. Neutrinos produced by $e + p \rightarrow n + \nu_e$ have a mean free path smaller than the core radius. Then they are trapped in the so called *neutrino-sphere* with radius $R_\nu \sim 70\,km$. When the core density exceeds the nuclear density $\rho_c = 10^{14} \div 10^{15}\,g\,cm^{-3}$, the collapse is stopped and a reaction shock wave propagating outward is produced. When this wave reaches the *neutrino-sphere*, neutrinos leave the star, carrying away in $1 \div 10\,msec$ the lepton number of the star and a small fraction of the core binding energy $(1 \div 10\,\%)$.

After this initial ν_e burst, all kinds of neutrinos are produced via the reaction [1]

$$e^+ + e^- \rightarrow \nu_i + \bar{\nu}_i\,, \tag{3.1.2}$$

in a characteristic time interval of a few seconds they are emitted as blackbody radiation. Almost all the binding energy of the system is carried away by these thermal neutrinos. Detailed theoretical models seem to find an equipartition of the energy between the different neutrino species. Since the average energy of ν_μ and ν_τ is about 2 times higher than the ν_e energy, because the ν_μ and ν_τ only interact via Z° then producing *neutrino-spheres* with smaller radius, their flux is expected to be smaller with respect to the electron neutrino flux.

3.2 SN1987A and the elementary particle physics

• *The general argument.* As already discussed in section 1.3, there is a general argument, based on the energetic balance of the supernova, which allows us to extract interesting bounds on the possible extensions of the Standard Model. In fact, since we know that almost the total binding energy of the system, $E_B \simeq 3 \times 10^{53}\,erg$, is carried

[1] Also $2n \rightarrow 2n + \nu_i + \bar{\nu}_i$ may give a relevant contribution, expecially for μ- and τ-neutrinos.

away by neutrinos in a time interval of a few seconds, if some new particle X exists, the corresponding luminosity L_X, emitted via the X channel, must be bounded by

$$L_X \leq Q_B \simeq 3 \times 10^{53} \, erg/s \,, \tag{3.2.1}$$

otherwise the X particle would cool the core of the star in less than one second, in contrast with the Kamiokande [48] and IMB [49] observations.

Depending on the strength g_X of the X coupling two cases are possible.

i) No-trapping. In such a case the coupling is small enough to have that the mean free path λ_X exceeds the core radius R_c, then the X particles are free to leave the star. We are in the presence of a volume emission and $L_X \sim g_X^2$, the luminosity increases as g_X increases.

ii) Trapping. Now the coupling is strong enough to give $\lambda_X \leq R_c$. The X particles are trapped inside the star and only a black body emission is possible, analogously to the ν_L case. If R_X and T_X are respectively the radius and the temperature of the X*-sphere*, $L_X \sim R_X^2 \, T_X^4$ decreases as g_X increases.

This completely general situation is well described in Fig. 7. From this figure it is evident how the condition $L_X \leq Q_B$ gives rise to a forbidden interval for the coupling constant g_X. All the limits reported in section 1.3 have been derived by this general argument.

- *The $\nu_{eL} - \nu_{eR}$ case.* Let us take into account a $\nu_{eL} - \nu_{eR}$ mixing

$$\begin{cases} \nu_{eL} = \nu \cdot \cos\theta \, + \, N \cdot \sin\theta \,, \\ \nu^c_{eL} = N \cdot \cos\theta \, - \, \nu \cdot \sin\theta \,, \end{cases} \tag{3.2.2}$$

where N and ν are the mass eigenstates. We will consider the ν_{eR} as a *sterile* particle (a singlet in the context of the Standard Model). In such a case the interaction of the mass eigenstate N only depends on $\sin\theta$, allowing us to adopt the general argument previously exposed to obtain a forbidden interval for $\sin\theta$. The case of a ν_{eR} interacting in a sensible way via right-handed currents have been recently explored by R. Barbieri and R. N. Mohapatra [53], giving rise to the bounds on the mass of the right-handed vector bosons reported in section 1.3. Finally, the ν_μ and ν_τ cases are very similar, but the production reactions, mediated only via neutral currents, are weaker than the ν_e

reactions, therefore leading in general to less stringent limits.

As already stressed, the standard procedure applied to the case of a *sterile* ν_{eR} interacting via mixing, is complicated and modified by the matter resonant oscillation effects, induced by the mixing itself.

3.3 Limits in absence of resonant oscillations

First of all let us study the no-resonant oscillation case. If $\Delta m^2 = m_N{}^2 - m_\nu{}^2 < 0$, resonant oscillations in the $\nu - N$ system are not possible [42] and we can follow the standard treatment. In fact in such a case resonant oscillations could be present only in the $\bar{\nu} - \bar{N}$ sector.

If the right-handed neutrinos are enough light ($m_N \leq 10 MeV$), they can be produced in the core of the supernova via the dominant reaction $e^- + p \to n + N$. The reaction $2n \to 2n + \bar{\nu} + N$ is less important, except for μ- and τ-neutrinos. Then the production cross section is

$$\sigma = \frac{G_F{}^2}{\pi} (g_V{}^2 + 3 g_A{}^2) \sin^2\theta \; E_e{}^2 \simeq \frac{6 G_F{}^2}{\pi} \sin^2\theta \; E_N{}^2 \, , \tag{3.3.1}$$

E_N being the energy of N ($E_N \simeq E_e$).

As already explained, if the mixing is small enough the N particles are *not-trapped*, on the contrary if it is big enough they becomes *trapped* . Let us examine in the detail these two cases. In the following we will use: $\rho_c = 8 \times 10^{14}\, g\, cm^{-3}$, $T_c = 50\, MeV$ and $R_c = 10^6\, cm$ for the core parameters, and

$$\begin{cases} \rho = \rho_c \cdot \left(\frac{R_c}{R}\right)^3 , \\ T = T_c \cdot \left(\frac{R_c}{R}\right) , \end{cases} \tag{3.3.2}$$

outside the core ($R \geq R_c$).

• *No-trapping case.* For $\sin^2 2\theta \leq 2.7 \times 10^{-5}$ the N particles are not trapped into the star, their mean free path exceeds the core radius and a *N-sphere* does not exist. We can also assume that N does not decay during its travel to Earth, since for $m_N \leq 10\, MeV$, a mixing $\sin^2 2\theta \geq 0.8$ should be needed for N to decay into three light neutrinos before arriving to Earth.

Let us consider that the overall number of protons of the star is not fixed at the initial value $n_p \simeq 1.4\, M_\odot/2m_p \simeq 10^{57}$, but it is decreasing in time via the neutronization process. Then the total energy carried away at the time t is [53]:

$$E_N(t) = -V_c \cdot \int_0^t < E_N >_t \frac{dn_p}{dt'}\, dt' \,, \tag{3.3.3}$$

where $V_c \simeq 4 \times 10^{18}\, cm^3$ is the core volume. From $< E_N >_t \simeq < E_e >_t \simeq \mu_e(t) \simeq \mu_e(0)\,(n_p(t)/n_p(0))^{1/3}$ and

$$\frac{dn_p}{dt} = -6 \sin^2 \theta \,\frac{G_F^2}{\pi}\, n_p^2 \, < E_e >_t^2 \,, \tag{3.3.4}$$

we derive

$$E_N(t) = E_N(\infty) \cdot \left[1 - \left(1 + \frac{t}{\tau}\right)^{-4/5}\right] \,, \tag{3.3.5}$$

where

$$E_N(\infty) = \frac{3}{4} V n_p(0) \mu_e(0) \,, \tag{3.3.6}$$

$$\tau = \frac{\pi}{10 \sin^2\theta \, {G_F}^2 {\mu_e}^2(0) n_p(0)} \,. \tag{3.3.7}$$

$n_p(0)$ and $\mu_e(0)$ are respectively the number density of protons and the electron chemical potential at $t = 0$. For our calculations we have used $\mu_e(0) \simeq 300\, MeV$ and $n_p(0) \simeq 2.5 \times 10^{38}\, cm^{-3}$. Now we must require that the right-handed neutrino emission does not saturate the core binding energy in less than 1 sec; so, using for greater convenience the mean luminosity $L_N = E_N(1sec)/1sec$, we have to impose the condition

$$L_N \leq Q_B = 3 \times 10^{53} erg/sec \,. \tag{3.3.8}$$

This leads to a bound for the $\nu_{eL} - \nu_{eR}$ mixing angle. In fact from (3.3.5) and (3.3.6)

$$\tau \geq 0.15\, sec \,. \tag{3.3.9}$$

Finally, from (3.3.7) we can derive

$$\sin^2 2\theta \leq 2.3 \times 10^{-10} \,, \tag{3.3.10}$$

which is the bound for the *no-trapping* case and for $\Delta m^2 = m_N^2 - m_\nu^2 < 0$ (absence of matter resonant oscillations).

• *Trapping case.* If $\sin^2 2\theta \geq 2.7 \times 10^{-5}$, the sterile neutrinos are trapped into the star through the reactions $N + n \to p + e^-$, $N + n \to \nu_e + n$, $N + p \to \nu_e + p$. The first one gives the most stringent limit. Given the N mean free path [53]

$$\lambda_N = \frac{1}{n(R)\,\sigma(R)}\,, \tag{3.3.11}$$

with $n(R) = n_c(R) \cdot (R_c/R)^3$ and the cross section

$$\sigma(R) = \frac{60}{\pi}\, G_F^2\, \sin^2\theta\, T^2(R)\,, \tag{3.3.12}$$

we can calculate the *N-sphere* defined as the value of R at which the optical depth

$$\tau_N(R) = \int_R^\infty \frac{dR}{\lambda_N(R)}\,, \tag{3.3.13}$$

is about unity (2/3). Then we derive

$$R_N = R_c \cdot \left(\frac{15}{\pi}\, G_F^2\, \sin^2\theta\, n_c R_c T_c^2\right)^{1/4}\,, \tag{3.3.14}$$

from which $R_N \simeq 19.7\, R_c\, (\sin^2\theta)^{1/4}$, with $R_N \geq R_c$ for the trapping case. What is relevant is the dependence $R_N \propto (\sin^2\theta)^{1/4}$, from which we derive $T_N \propto (\sin^2\theta)^{1/4}$, and then adopting the Stefan-Boltzmann law

$$\frac{Q_N}{Q_\nu} = \left(\frac{T_N}{T_\nu}\right)^4 \left(\frac{R_N}{R_\nu}\right)^2 = (\sin^2\theta)^{-1/2}\,, \tag{3.3.15}$$

where the subscript ν refers to the ν-sphere parameters. From the observed ν-luminosity, $Q_\nu \simeq 5 \times 10^{51} erg/sec$, we derive that the luminosity condition $Q_N \leq Q_B$ implies

$$\sin^2 2\theta \geq 1.1 \times 10^{-3} \tag{3.3.16}$$

for the *trapping* case, in absence of matter resonant oscillations.

The two limits (3.3.10) and (3.3.16), exclude an interval of possible values for $\sin^2 2\theta$, and since the condition (3.3.16) is next to be excluded by laboratory limits, we have to take seriously the (3.3.10) as the more relevant limit. By using the supernova in the case of an extremely light sterile right-handed neutrino, $m_N \leq m_\nu \leq 18\, eV$, we have been able to put a limit of the order 10^{-10} on the violation of lepton universality in the electron sector, extremely more stringent than the 10^{-2} obtained with laboratory analyses.

If $\Delta m^2 > 0$ we have to take into account the $\nu - N$ resonant oscillations inside the supernova, which in general modify the energy released through the N-channel, for example due to the flipping of the trapped ν's into the sterile N's or vice versa. Therefore, in such a case a careful analysis of the energy emitted must be carried out in the relevant regions of the $\Delta m^2 - \sin^2 2\theta$ plane.

3.4 The $N \to \nu$ resonant transition

We will consider only the matter resonant oscillations (the N oscillation length is much less than the interaction length at the resonance) because the vacuum oscillations are suppressed by the factor $\sin^2 2\theta$ which is known to be small [1] [58]. The resonance condition corresponding to the $N \to \nu$ transition in the supernova is:

$$\Delta_0 \equiv \frac{\Delta m^2}{2E} = \frac{G_F \rho}{\sqrt{2} m_p}(3Y_e + 4Y_\nu - 1) \equiv V(\rho) \; , \tag{3.4.1}$$

where E is the particle energy, ρ is the star density, m_p is the proton mass, and Y_e, Y_ν, are the relative abundances of electrons and neutrinos respectively; V defines the potential. The presence of the Y_ν term is due to the neutrino-neutrino scattering and in the derivation of eq. (3.4.1) [66] we have used that in the supernova $Y_p + Y_n = 1$ and $Y_p = Y_e$ [67].

a) If the N's are trapped, the condition for crossing the resonance for the particles emitted from the N-sphere is $\Delta_0 \leq V_N$, V_N being the potential at the surface of the N-sphere. For our calculations we have assumed $(3Y_e + 4Y_\nu - 1) \simeq 0.18$, and a small variation of the e and ν abundances up to $R \simeq 10R_c$ [67]. Then from $\Delta_0 \leq V_N$ we obtain:

$$\Delta m^2 \sin^2 2\theta \leq 4.3 \times 10^4\, eV^2 \; , \tag{3.4.2}$$

where we have assumed the energy distribution of the N's to be enough peaked on the statistical expectation value $< E > \simeq 3.15\, T_N = 11.3\, (\sin^2 2\theta)^{-1/4}\, MeV$. The limit on

Δm^2 depends on the mixing, since the surface of the N-sphere depends on the strength of the N interactions, which is proportional to $\sin^2\theta$. On the other hand, for the values of the mixing angle for which the N's are not trapped inside the star but are emitted from the entire core, the neutrino energy is of the order of the electron chemical potential during the time of emission, so we can choose $< E > \simeq < \mu_e > \simeq 150\, MeV$. As a consequence, now the condition for the resonance crossing is independent on the mixing similarly to the solar case. Then the condition that we find from $\Delta_0 \leq V_c$, V_c being the potential in the core, reads:

$$\Delta m^2 \leq 1.6 \times 10^9\, eV^2 \ . \tag{3.4.3}$$

b) If N crosses the resonance, the *non-adiabatic* solution for the $N \to \nu$ oscillation gives, for $\sin\theta \ll 1$, a probability of flipping $P(N \to \nu) = 1 - P_x$, where P_x is the Landau-Zenner term [44]:

$$P_x = \exp\left(-\frac{\pi}{2}\, \sin^2 2\theta\, \frac{\Delta_0\, \rho_*}{|d\rho/dR|_*}\right) , \tag{3.4.4}$$

the subscript $*$ indicates the resonance point. Therefore, in order to have $P(N \to \nu) \geq 50\%$, we must have [2] $\Delta m^2\, (\sin^2 2\theta)^{7/4} \geq 1.3 \times 10^{-9}\, eV^2$ in case of trapping, and $\Delta m^2\, (\sin^2 2\theta)^{3/2} \geq 1.7 \times 10^{-8}\, eV^2$ for no-trapping.

c) Finally, if we require that the resonance point lies inside the ν-sphere, we must have $\Delta_0 \geq V_\nu = 6.7 \times 10^{-4}\, eV$, where V_ν is the potential at the surface of the ν-sphere itself. This condition becomes:

$$\Delta m^2\, (\sin^2 2\theta)^{1/4} \geq 1.5 \times 10^4\, eV^2 \ , \tag{3.4.5}$$

for *trapping*, and:

$$\Delta m^2 \geq 2 \times 10^5\, eV^2 \ , \tag{3.4.6}$$

for *no-trapping*.

---

[2] For large mixing angles an exact calculation gives the constraint $\sin^2 2\theta < 0.5$, independent of Δm^2 inside the region where the oscillations occur.

The constraints (3.4.2), (3.4.3), (3.4.5) and (3.4.6) describe a particular region as one can see in Fig. 8. Above this region the N particles do not cross the resonance, so that the limits are those found in the absence of oscillations. Below this region the resonance point is outside the ν-sphere, so the limits given in section 3.3 are not modified by $N \to \nu$ flipping, since the energy released through the N-channel still leaves the star. On the contrary, inside this region the N's oscillate into ν, with P larger or less than 50% depending on the conditions illustrated at the point b). In this case the limits of section 3.3 do not hold because we have to take into account the reduction in the energy released through the N-channel due to the $N \to \nu$ flipping inside the ν-sphere.

3.5 Effects of the $\nu \to N$ resonant transition

The only effect produced by $\nu \to N$ transitions outside the ν-sphere is the modification of the energy spectrum of the neutrinos, experimentally not observable due to the poor statistics. On the contrary, we must be careful if the resonance point lies between the ν- and the N-sphere (or the core surface if the N's are not trapped), because in such a case a new channel for energy emission is available.

The resonance condition corresponding to this situation reads:

$$\frac{\Delta m^2}{2E_\nu} = V_c \left(\frac{R_c}{R_*}\right)^3 \equiv V_* \ , \tag{3.5.1}$$

where now the subscript * indicates the resonance point for the $\nu \to N$ transition. Since the ν's are trapped, the mean energy of the ones that can cross the resonance is $E_\nu \simeq 3.15\, T_* = 3.15\, T_c\, (R_c/R_*)$. Then, we find:

$$R_* = R_c \left(\frac{6.3\, T_c V_c}{\Delta m^2}\right)^{1/4} \ , \tag{3.5.2}$$

for the radius at the resonance point. From eq. (3.5.2), and adopting a Stefan-Boltzmann law for the luminosity $\widetilde{Q}_N$ emitted through the sterile neutrinos generated by the flipping of ordinary ν's inside the ν-sphere, we derive the ratio:

$$\frac{\widetilde{Q}_N}{Q_\nu} = \left(\frac{R_\nu}{R_*}\right)^2 = 9.5 \times 10^{-3} \sqrt{\frac{\Delta m^2}{1 eV^2}} \ . \tag{3.5.3}$$

The region of interest is described by $R_* \geq R_N$ if the N's are trapped, or $R_* \geq R_c$ in the no-trapping case, and $R_* \leq R_\nu$. More precisely, this region, indicated as A in

Fig. 8, is defined by the conditions (3.4.2), (3.4.3), and $\Delta m^2 \geq 1.1 \times 10^4\, eV^2$; as we see it contains the region discussed in section 3.4. However, not all the points of this region give actually a $\nu \rightarrow N$ resonant oscillation; in fact, for $\Delta m^2 > 9.2 \times 10^{18}\ \sin^2 2\theta\, eV^2$, the resonant oscillation length turns out to be larger than the interaction length, making meaningless the resonance itself.

Outside the region A the relevant limits are given by the results found in absence of oscillations, namely eq. (3.3.10) for no-trapping and eq. (3.3.16) for trapping. Inside, a deeper analysis of the energy flux is needed.

3.6 Limits from $\bar{N}$

The first limits on the mixing angle that we can get in the region A are derived by looking at the energy released through the $\bar{N}$-channel.

The sterile antineutrinos are produced, essentially, via the reaction $2\,n \rightarrow 2\,n + \nu + \bar{N}$, and the luminosity emitted in the case of *no-trapping* turns out to be [68]

$$Q_{\bar{N}} \simeq \frac{V n_n^2}{\pi^5}\, m_n\, < p >\, g_A^2\, G_F^2\, \cos^4\theta_w\, \sin^2 2\theta\, \frac{T^5}{m_\pi^4} \simeq 1.8 \times 10^{59}\ \sin^2 2\theta\, erg/sec\,. \quad (3.6.1)$$

Thus from $Q_{\bar{N}} \leq Q_B$ we derive:

$$\sin^2 2\theta \leq 1.6 \times 10^{-6}\,. \quad (3.6.2)$$

If the $\bar{N}$'s are *trapped* through the reactions $\bar{N} n \rightarrow \bar{\nu_e} n$ and $\bar{N} p \rightarrow \bar{\nu_e} p$, the limit we get for the mixing is:

$$\sin^2 2\theta \geq 5.6 \times 10^{-4}\,, \quad (3.6.3)$$

very close to the limit (3.3.16). As a consequence, as illustrated in Fig. 8, a first part of the region A can be ruled out.

3.7 Limits in presence of resonant oscillations

We can now analyze in detail the constraints on the energy released in the different parts (a,b,...,e) of the region A. As we have already shown in section 3.3, the vertical line corresponding to $\sin^2 2\theta = 2.7 \times 10^{-5}$ in Fig. 8 distinguishes the regions of trapping and no-trapping.

a) In the sub-region *a*, above the line $\Delta m^2 = 2 \times 10^5\, eV^2$, the oscillation length of the ν's is less than the interaction length, so that only $N \to \nu$ transitions are present. Therefore, the condition for the emitted luminosity is

$$P(N \to N) \cdot L_N \leq Q_B\,, \tag{3.7.1}$$

where $P(N \to N) \simeq P_x$ if $\sin\theta \ll 1$. This condition is trivially satisfied in *a*, since $P(N \to N) \leq 1$. Below the horizontal line corresponding to $\Delta m^2 = 2 \times 10^5\, eV^2$ no kind of oscillation is present inside the ν-sphere. Therefore, we can conclude that the entire sub-region *a* is allowed.

b) Let us now consider the sub-region *b*. Here both the transitions $N \to \nu$ and $\nu \to N$ are present. Then the condition to impose is

$$P_x \cdot L_N + \widetilde{Q}_N \leq Q_B\,. \tag{3.7.2}$$

From eq. (3.5.3) we find that all the points for which $\Delta m^2 \geq 4 \times 10^7\, eV^2$ are excluded, because in such a case $\widetilde{Q}_N \geq Q_B$; on the other hand, a numerical analysis shows that the region below this line is allowed as illustrated in Fig. 9.

c) In the sub-region *c* we have to take into account only the $\nu \to N$ transitions (here the $N \to \nu$ resonance occurs outside the ν-sphere), so that we must require

$$L_N + \widetilde{Q}_N \leq Q_B\,. \tag{3.7.3}$$

But from eq. (3.5.3) we can check that in such a region $\widetilde{Q}_N \leq 7.5 \times 10^{-2} \cdot Q_B$, so that it is sufficient to require $L_N \leq Q_B$. As a consequence, it turns out that the bound on the mixing angle corresponding to the sub-region *c* is given by eq. (3.3.10).

d) Both the transitions $\nu \to N$ and $N \to \nu$ are present in *d*, therefore the luminosity condition reads

$$P(N \to N) \cdot Q_N + \widetilde{Q}_N \leq Q_B\,, \tag{3.7.4}$$

where now we must use the exact formula $P(N \to N) = \sin^2 2\theta + P_x \cdot \cos^2 2\theta$, valid for any angle θ. Since $\widetilde{Q}_N \leq Q_B$ is satisfied everywhere in *d* and the exponential factor

P_x turns out to be negligible, from eq. (3.3.15) it is trivial to see that our condition is satisfied everywhere in this region.

e) Here, only the ν's cross the resonance inside the ν-sphere; then we must require

$$\widetilde{Q}_N + Q_N \leq Q_B \,. \tag{3.7.5}$$

Since $\widetilde{Q}_N$ is again much smaller than Q_B, in the sub-region e we find the limit (3.3.16) derived in section 3.3.

• *Conclusions.* From the total amount of the energy emitted in the explosion of the supernova $SN1987A$, we have derived new stringent limits for the $\nu_{eL} - \nu_{eR}$ mixing angle. In particular, if $\Delta m^2 < 0$ we find the bounds $\sin^2 2\theta \leq 2.3 \times 10^{-10}$ and $\sin^2 2\theta \geq 1.1 \times 10^{-3}$. For $\Delta m^2 > 0$ neutrino oscillations modify these limits in a certain range of values of Δm^2, as depicted in Fig. 9. In particular we find that the bound for the trapping region is essentially not modified, while the upper limit in the case of no-trapping becomes $\sin^2 2\theta \leq 1.6 \times 10^{-6}$ in the small window $2 \times 10^5 \, eV^2 \leq \Delta m^2 \leq 4 \times 10^7 \, eV^2$, but it remains $\sin^2 2\theta \leq 2.3 \times 10^{-10}$, or also better, for all the other values of Δm^2, as one can see from Fig. 9.

4. Consequences of a large neutrino magnetic moment

In the previous sections we have analysed the case of Majorana neutrinos with non trivial $\nu_L - \nu_R$ mixings. If the right-handed neutrino exists, this is the more general case from the theoretical point of view, as already explained in section 1.1. However, the more specific case of a ν_R as the right-handed component of a Dirac neutrino, is at the same time extremely interesting. As already anticipated in section 1.3, a Dirac neutrino with a large magnetic moment could explain a possible anticorrelation between the detected solar neutrino flux and the cyclic variation of the sunspot number, which recent analyses of the data seem to display (see Fig.s 2 and 3) [4] [69]. This suggestive hypothesis opens very interesting problems from the theoretical point of view.

In this chapter, after having exposed the phenomenological motivations for a Dirac neutrino with a large magnetic moment, we will study the theoretical implications of such an hypothesis. In particular we will derive the phenomenological consequences [6] of a model, recently proposed by R. Barbieri and R. N. Mohapatra (BM), which gives rise to a large electron neutrino magnetic moment with a minimal extension of the Standard Model [5].

4.1 Phenomenological motivations

A suggestive explanation of the Solar neutrino puzzle is based on the hypothesis of a Dirac electron neutrino with a *large* magnetic moment [4] [69]:

$$\mu_{\nu_e} \simeq (10^{-11} \div 10^{-10}) \cdot \mu_B \,. \tag{4.1.1}$$

In such a case, since the solar magnetic field $B_\odot$ in the convective zone is estimated to be $B_\odot \sim 1 \div 10\, KG$, the neutrino helicity rotation $\nu_{eL} \to \nu_{eR}$ in the convective zone (between $2 \times 10^{10}\, cm$ and $1 \times 10^{10}\, cm$ from the solar surface) would be responsible of the reduction in the expected flux observed by Davis. Moreover, this explanation, differently with respect to the typical MSW mechanism, gives rise to an 11 years cyclic variation of the neutrino flux, anticorrelated with the sunspot cycle. That is because the solar magnetic field follows the 11 years period of the Solar cycle. Fig. 2 and Fig. 3 seem to display a time oscillation in the Davis data, and its connection with the Solar cycle. In any case, in the near future, a more clear and definite situation will be realized. In fact the 1988 and 1989 data correspond to a maximum of solar activity, and then, on the basis of the previous ideas, we should expect a second clear minimum in the Davis data.

At the present we can find another very interesting information in favour of the magnetic moment hypothesis. Due to the inclination of the Earth axis with respect to the Solar equator, the $\nu_{eL} \rightarrow \nu_{eR}$ transition would give rise to a minimum of the flux around March 5 and September 5. Now, the data from 1979 to 1982 (the previous maximum for the solar activity) were 0.03 ± 0.04 Argon atoms per day in the 7 runs nearest to March 5 and September 5. On the contrary, they were 0.37 ± 0.05 Argon atoms per day in the 12 remaining runs. That seems to be another phenomenological indication for a magnetic moment mechanism.

4.2 Theoretical consequences

The hypothesis of a Dirac electron neutrino with a large magnetic moment is extremely interesting from the theoretical point of view. In fact it is not difficult to understand that a μ_{ν_e} as large as in eq. (4.1.1) is in general difficult to realise in a *well founded* model. First of all, as already discussed in section 1.1, if we look at the simple extension of the Standard Model with the addition of the singlet ν_{eR}, the natural value of μ_{ν_e} turns out to be very small:

$$\mu_{\nu_e} = \frac{3eG_F}{8\sqrt{2}\pi^2} \cdot m_{\nu_e} \simeq 3 \times 10^{-19} \cdot \mu_B \left(\frac{m_{\nu_e}}{1\, eV}\right) . \tag{4.2.1}$$

But the difficulty to obtain $\mu_{\nu_e} \sim (10^{-11} \div 10^{-10}) \cdot \mu_B$ is completely general and connected with the *naturality* requirement that any *well founded* theoretical model has to satisfy.

The argument is general [70]. The magnetic moment interaction has to be related to some new physics occuring at a scale Λ greater than the Fermi scale. Then it can be described by effective $SU(2)_L \times U(1)_Y$ invariant interaction operators of dimension six:

$$\frac{\lambda_1}{\Lambda^2}\,\bar{L}\sigma_{\mu\nu}\nu_R\phi\,B_{\mu\nu}\,, \qquad \frac{\lambda_2}{\Lambda^2}\,\bar{L}\sigma_{\mu\nu}\tau\phi\,\mathbf{W}_{\mu\nu}\,. \tag{4.2.2}$$

Where L is the left-handed lepton doublet, ϕ is the Higgs doublet and $B_{\mu\nu}$, $\mathbf{W}_{\mu\nu}$ are respectively the $U(1)_Y$ and $SU(2)_L$ field strengths. These interactions give rise to a neutrino magnetic moment $\mu_\nu \sim \lambda < \phi_\circ > /\Lambda^2$, after the symmetry breaking. However, both these interactions also produce a neutrino mass term

$$\delta m_\nu \sim g\,\mu_\nu\,\Lambda^2\,, \tag{4.2.3}$$

where g is the typical electro-weak coupling, through the diagram of Fig. 10, where a Z or W bosons are exchanged. Now, since $m_{\nu_e} < 20\,eV$, a magnetic moment $\mu_{\nu_e} \sim (10^{-11} \div 10^{-10}) \cdot \mu_B$ is compatible only with an unacceptable low value of the scale Λ ($\Lambda \sim 1\,GeV$), or with an unpleasant cancellation between δm_{ν_e} and other possible contributions to the neutrino mass.

A possible elegant solution to the problem of the *natural* suppression of the neutrino mass with respect to a *large* magnetic moment, has been recently proposed by M. B. Voloshin [71]. The idea is that the natural smallness of m_ν can be a consequence of a $SU(2)_\nu$ symmetry, under which (ν_L, ν_L^c) transforms as a doublet. In fact, since the mass term $\bar{\nu}_R\nu_L$ is a scalar under the Lorentz group, due to Fermi statistics it must be a triplet under $SU(2)_\nu$. Viceversa, the magnetic moment term $\bar{\nu}_R\sigma_{\mu\nu}\nu_L$ is a vector under the Lorentz group, and then it must be a singlet under $SU(2)_\nu$. Then the $SU(2)_\nu$ symmetry, under which (ν_L, ν_L^c) is a doublet, forbids the mass term, but allows the magnetic moment term. This symmetry must be broken, and its breaking could be mild enough to solve in a natural way the smallness of the ratio $\delta m_\nu/\mu_\nu$.

It is now very appealing to find a realistic model which put together the Standard Model and this new symmetry $SU(2)_\nu$. Different solutions have been elaborated, but the more natural seems to be the BM model [5]. It joins in a minimal way $SU(2)_L \times U(1)_Y$ and $SU(2)_\nu$ in the larger group $SU(3)_L \times U(1)_X$, in such a way that $SU(2)_L$ and $SU(2)_\nu$ are both non commuting subgroups of $SU(3)_L$. Another possibility [20] is to treat $SU(2)_\nu$ as an *horizontal* group, connecting the electron and muon doublets. In such a case the $SU(2)_\nu$ doublet is the *horizontal* doublet $(\nu_{eL}, \nu_{\mu L})$, instead of the vertical one (ν_L, ν_L^c) reported in the $SU(3)_L \times U(1)_X$ solution.

We consider the *vertical* solution more realistic than the *horizontal* one. In fact in the second case, as already stressed in section 1.1, we expect difficulties due to the constraints on the horizontal $e-\mu$ interactions, and on the strong degeneracy that one has to require for the Majorana electron and μ-neutrino masses, against the well known ratio $m_\mu \simeq 200\, m_e$. Then in the following we will study the $SU(3)_L \times U(1)_X$ model and its phenomenological consequences. In particular we will discuss the presence of a new Z' vector boson, which comes out from the enlargement of the rank of the gauge group. In some sense peculiar characteristics of neutrino physics may induce relevant consequences also for other physical sectors. The direct couplings of Z' with the standard fermions and the $Z-Z'$ mixing give rise to a richer phenomenology which will be interesting to explore [6].

4.3 A realistic theoretical model

• *General aspects.* In the BM model, as already anticipated, the standard gauge group $SU(2)_L \times U(1)_Y$ is extended to $SU(3)_L \times U(1)_X$, in such a way that $SU(2)_L$ and $SU(2)_\nu$ are both non commuting subgroups of $SU(3)_L$. The rank of the standard gauge group is enlarged and then we have to expect a new Z' vector boson. The known leptons, with the adding of the right-handed neutrinos, transform as

$$L_a \equiv \begin{pmatrix} \nu_L^c \\ \nu_L \\ e_L \end{pmatrix}_a \sim (\mathbf{3}, -1/3)\,, \qquad e^c_{aL} \sim (\mathbf{1}, 1)\,. \tag{4.3.1}$$

The first boldfaced number indicates the $SU(3)_L$ representation and the second one is the value of the $U(1)_X$ generator. $a = 1, 2, 3$ is a family index.

To include quarks, two different ways can be choosen. In both ones we have to add a new heavy quark per generation (a sort of hadron companion of ν_L^c with respect to the $SU(2)_\nu$ symmetry), either of charge $+2/3$ (g) or of charge $-1/3$ (D). In the first case the representation is

$$Q_a \equiv \begin{pmatrix} g_L \\ u_L \\ d_L \end{pmatrix}_a \sim (\mathbf{3}, 1/3)\,,$$

$$g^c_{aL} \sim u^c_{aL} \sim (\mathbf{1}, -2/3)\,, \qquad d^c_{aL} \sim (\mathbf{1}, 1/3)\,. \tag{4.3.2}$$

In the second case we have to use a $\mathbf{3}^*$ representation:

$$Q_a \equiv \begin{pmatrix} D_L \\ d_L \\ u_L \end{pmatrix}_a \sim (\mathbf{3}^*, 0),$$

$$D^c_{aL} \sim d^c_{aL} \sim (\mathbf{1}, 1/3), \qquad u^c_{aL} \sim (\mathbf{1}, -2/3). \tag{4.3.3}$$

A trivial way to make the theory anomaly free is to add mirror fermions to the previous representations. However, in the case of the (4.3.3) assignement, a more appealing way to take care of gauge anomalies is to complete the fermion content (4.3.1) and (4.3.3) to form a $\mathbf{27}$ of E_6. In fact E_6 contains $SU(3)_c \times SU(3) \times U(1)$ as a subgroup, and the $\mathbf{27}$ contains three lepton-like $SU(3)$ triplets: (e, ν, ν^c), (E, N, N^c) and (L, E^c, e^c), with $X = -1/3, -1/3$ and $2/3$ respectively. E, N and L are the usual E_6 heavy leptons. In such a case e^c_L is not a $SU(2)_\nu$ singlet and the corresponding lepton mass is proportional to the $SU(2)_\nu$ breaking. That gives rise to some complications for the generation mechanism, which we will discuss in the following, of a *large* neutrino magnetic moment. In fact μ_{ν_e} is in general proportional to the one-loop exchanged lepton mass and then, in the E_6-like case, to the same $SU(2)_\nu$ breaking scale. This difficulty can be solved by further adding two $SU(3)$ singlet right-handed leptons with charges $+1$ and -1. In such a case e^c_L can be chosen as a $SU(2)_\nu$ singlet, and μ_{ν_e} is not more proportional to the $SU(2)_\nu$ breaking.

From now on we will choose the first assignement (4.3.2), for which e^c_L is a $SU(2)_\nu$ singlet and then the *natural* generation of a *large* electron neutrino magnetic moment is more direct and simple. The main results of the model and its phenomenological consequences are essentially independent of the details of the mirror sector.

In the $(\mathbf{3}, x)$ representation it is useful to choose the following basis for the group generators, written in terms of the usual Gell-Mann matrices, in which the physical content of the model is particularly clear:

$$V^\circ = \frac{1}{2\sqrt{2}}(\lambda_1 + i\lambda_2), \qquad \bar{V}^\circ = \frac{1}{2\sqrt{2}}(\lambda_1 - i\lambda_2), \tag{4.3.4}$$

the corresponding V°_μ, $\bar{V}^\circ_\mu$ bosons couple ν_L and ${\nu^c}_L$ in the lepton sector, and g_L, u_L in the quark sector;

$$V^+ = \frac{1}{2\sqrt{2}}(\lambda_4 + i\lambda_5), \qquad V^- = \frac{1}{2\sqrt{2}}(\lambda_4 - i\lambda_5), \tag{4.3.5}$$

the V_μ^+, V_μ^- bosons couple $\nu^c{}_L$ and e_L, or g_L and d_L;

$$T_{+L} = \frac{1}{2\sqrt{2}}(\lambda_6 + i\lambda_7), \quad T_{-L} = \frac{1}{2\sqrt{2}}(\lambda_6 - i\lambda_7), \quad T_{3L} = -\frac{1}{4}(\lambda_3 - \sqrt{3}\lambda_8), \tag{4.3.6}$$

are the generators of the standard $SU(2)_L$ subgroup and finally

$$V^8 = -\frac{1}{4}(\sqrt{3}\lambda_3 + \lambda_8), \qquad X = x \cdot I, \tag{4.3.7}$$

are the last diagonal generators. The electric charge Q is a linear combination of the neutral generators: $Q = T_{3L} + X - (1/\sqrt{3})V^8$. The Standard Model $U(1)_Y$ is generated by $Y = 2X - (2/\sqrt{3})V^8$.

• *The scalar content.* The particle content of the theory is completed with the introduction of the scalars. The first triplet

$$\varphi \equiv \begin{pmatrix} \varphi_1^- \\ \varphi_2^- \\ \varphi^\circ \end{pmatrix} \sim (\mathbf{3}^*, -2/3), \tag{4.3.8}$$

whose only neutral component is the third one, gives masses to the charged leptons and to the down quarks with the non vanishing v.e.v. $< \varphi^\circ >= a$. Other two triplets

$$\sigma \equiv \begin{pmatrix} \sigma_1^\circ \\ \sigma_2^\circ \\ \sigma^+ \end{pmatrix} \sim (\mathbf{3}^*, 1/3), \qquad \rho \equiv \begin{pmatrix} \rho_1^\circ \\ \rho_2^\circ \\ \rho^+ \end{pmatrix} \sim (\mathbf{3}^*, 1/3), \tag{4.3.9}$$

give masses to the up and g quarks. The quark Yukawa Lagrangian is

$$\mathcal{L}_Y = \lambda^u_{ab}\, Q_a\, u^c_b\, \sigma + \lambda^g_{ab}\, Q_a\, g^c_b\, \rho + \lambda^d_{ab}\, Q_a\, d^c_b\, \varphi + h.c., \tag{4.3.10}$$

and with the v.e.v.s $< \sigma_2 >= b$ and $< \rho_1 >= c$, we can avoid unwanted flavour changing neutral current effects in the charged $+2/3$ sector. This breaking pattern is determined by un unbroken residual global symmetry K_e, which we will discuss in the following.

The last triplet

$$\eta \equiv \begin{pmatrix} \eta_1^- \\ \eta_2^- \\ \eta^\circ \end{pmatrix} \sim (\mathbf{3}^*, -2/3)\,, \tag{4.3.11}$$

does not acquire a non vanishing v.e.v. and it is responsible of the radiative generation of the electron neutrino magnetic moment and mass, which are produced at one-loop level by the η exchange. Finally, there are two sextets

$$T_2 \sim (\mathbf{6}^*, 2/3)\,, \qquad T_3 \sim (\mathbf{6}^*, 2/3)\,, \tag{4.3.12}$$

whose v.e.v.s $< T_{2_{11}} >= h_2$ and $< T_{3_{11}} >= h_3$ give Majorana masses to $\nu^c_{\mu L}$ and $\nu^c_{\tau L}$. The standard subgroup $SU(2)_L \times U(1)_Y$ is broken only by φ and σ, the other scalars are responsible for the $SU(2)_\nu$ breaking.

The Yukawa Lagrangian in the leptonic sector is

$$\mathcal{L}_Y = \lambda_a\, L_a\, e^c_a\, \varphi + f\, L_1\, L_3\, \eta^\dagger + f'\, L_1\, e^c_3\, \eta +$$

$$+ g_2\, L_2\, L_2\, T_2 + g_3\, L_3\, L_3\, T_3 + h.c.\,. \tag{4.3.13}$$

$\mathcal{L}_Y$ has two global $U(1)$ symmetries, K_2 and K_3 as enlisted in Table 1, associated respectively to the second and third leptonic generations. They prevent any other coupling from appearing in $\mathcal{L}_Y$.

K_2 and K_3 are spontaneously broken, but the linear combination

$$K_e \equiv K_2 + K_3 + 4T_{3L} + 3X - 4Q - B\,, \tag{4.3.14}$$

where B is the barionic number, remains as a global unbroken symmetry of the model. It is easy to understand that K_e is a sort of electron leptonic number. In fact the K_e quantum numbers are $K_e(\nu_{eL}, e_L) = K_e(\nu_{eR}) = K_e(e_R) = 1$, while $K_e(\nu_{\mu L}, \mu_L) = K_e(\nu_{\tau L}, \tau_L) = K_e(\mu_R) = K_e(\tau_R) = 2$ and $K_e(\nu_{\mu R}) = K_e(\nu_{\tau R}) = 0$. On the quark sector it is zero: $K_e(u_L, d_L) = K_e(u_R) = K_e(d_R) = 0$; except for the g quark where $K_e(g_L) = K_e(g_R) = -2$.

It is now easy to understand how the unbroken symmetry K_e fixes the breaking pattern previously discussed. Moreover, it is clear how the only possible mass term for the electron neutrino is a Dirac term, while the μ- and τ-neutrinos can have only Majorana masses for their right-handed components. In principle, K_e does not forbid a mixing in the $\mu - \tau$ sector. However, an unbroken discrete symmetry $\mathcal{D}$, acting as $L_2 \to -L_2$, $e_2^c \to -e_2^c$ and as the identity in all the other cases, forbids this generational mixing.

• *The electron neutrino magnetic moment.* To understand how μ_{ν_e} and m_{ν_e} arise in this model, let us write down the Yukawa interactions of ν_{eL} and ν_{eL}^c with the scalars η_1 and η_2:

$$\mathcal{L}_\eta = f\left(-\nu_{eL}^c\,\tau_L\,\eta_2^+ + \nu_{eL}\,\tau_L\,\eta_1^+\right) + f'\left(\nu_{eL}^c\,\tau_L^c\,\eta_1^- + \nu_{eL}\,\tau_L^c\,\eta_2^-\right) + h.c.. \tag{4.3.15}$$

The one-loop η exchange, as depicted in Fig. 11, gives rise both to a Dirac mass,

$$m_{\nu_e} = \frac{ff'}{16\pi^2}\,m_\tau\,\ln\frac{m_1^2}{m_2^2}\,, \tag{4.3.16}$$

and, with the adding of a photon line, to a magnetic moment for the electron neutrino:

$$\mu_{\nu_e} = e\,\frac{ff'}{16\pi^2}\,m_\tau\,\left[\frac{1}{m_1^2}\,\ln\frac{m_1^2}{m_\tau^2} + \frac{1}{m_2^2}\,\ln\frac{m_2^2}{m_\tau^2}\right]\,. \tag{4.3.17}$$

In the previous formulas m_1 and m_2 are respectively the η_1 and η_2 masses, and in general we expect $m_1, m_2 \gg m_\tau$. In the limit in which $SU(2)_\nu$ is unbroken, $m_1 = m_2$ since η_1 and η_2 are components of a $SU(2)_\nu$ doublet. Then in this limit $m_{\nu_e} = 0$, while $\mu_{\nu_e} \neq 0$, and the smallness of the ratio m_{ν_e}/μ_{ν_e} depends on the $SU(2)_\nu$ breaking. For quasi degenerate η fields

$$\mu_{\nu_e} \simeq e\,\frac{2m_{\nu_e}}{\Delta m_\eta^2}\,\ln\frac{m_\eta^2}{m_\tau^2}\,, \tag{4.3.18}$$

where $m_\eta^2 \simeq m_1^2 \simeq m_2^2$, and $\Delta m_\eta^2 \equiv |m_1^2 - m_2^2| \ll m_\eta^2$. Now, choosing $m_\eta \sim 1\,TeV$, demanding $m_{\nu_e} \leq 20\,eV$ and $\mu_{\nu_e} \geq 10^{-11}\cdot\mu_B$, we obtain the following condition:

$$\Delta m_\eta^2 \leq 60\,GeV^2\,. \tag{4.3.19}$$

That is the condition, in the BM model, for the *natural* suppression of m_{ν_e} in the presence of a *large* μ_{ν_e}.

Since η_1 and η_2 are degenerate in the limit of $SU(2)_\nu$ unbroken, we must expect $\Delta m_\eta^2 \propto M_\nu^2$, with M_ν the $SU(2)_\nu$ breaking scale. Even if the η triplet is not directly coupled to the scalars which break down $SU(2)_\nu$, the $\eta_1 - \eta_2$ degeneracy is removed at the one-loop level. Then adopting again a *naturality* requirement it follows

$$\Delta m_\eta^2 \geq \frac{\alpha_w}{4\pi} M_\nu^2 \,. \tag{4.3.20}$$

From (4.3.19) and (4.3.20) we derive a *naturalness* bound

$$M_\nu \leq O(150)\, GeV \,, \tag{4.3.21}$$

for the $SU(2)_\nu$ breaking scale in the BM model, which then results very close to the Fermi scale. We are then motivated to analyse the phenomenological consequences of the model, which we expect to be rich. In particular we will study if there are inconsistencies with the present experimental data, and the possible experimental signatures for the enlarged standard gauge group proposed by the BM model.

• *General phenomenological aspects.* We easily understand that the more interesting sector to analyse is the neutral current sector. In fact all the non-diagonal new gauge vector bosons connect standard fermions with non-standard ones, which, except for the electron right-handed neutrino, are heavy enough and little mixed with the light fermions. In particular the gauge bosons $V_\mu^\pm$, V_μ° and $\bar{V}_\mu^\circ$ cannot at present be produced in hadronic collisions, or mediate μ and τ decay, even if their masses are not far from the Fermi scale, due to the heaviness of the g quarks, $\nu_{\mu L}^c$ and $\nu_{\tau L}^c$. On the other hand, not much can be concluded either from flavour changing neutral current effects induced by $V_\mu^\pm$ and g quarks, since the corresponding mixing angles are unknown, or from neutrino counting experiments, as the additional contribution from $e^+ + e^- \to \nu_e^c + \bar{\nu}_e^c$ gives a very low bound: $M_{V^\pm} \geq 35\, GeV$ [5].

More interesting is the phenomenology of the extra Z' boson and of the corresponding $Z - Z'$ mixing, which we will analyse in the successive sections. It is useful to this end to derive the masses of the non-diagonal gauge bosons. In particular for the standard $W_\mu^\pm$ we find

$$M_{W^\pm}^2 = \frac{1}{2} g^2 (a^2 + b^2) \,. \tag{4.3.22}$$

The V boson masses depend also on the $SU(2)_\nu$ breaking:

$$\begin{cases} M^2_{V^\pm} = \frac{1}{2}\, g^2 \left[2\,(h_2^2 + h_3^2) + c^2 + a^2\right], \\ M^2_{V^\circ} = M^2_{\bar V^\circ} = \frac{1}{2}\, g^2 \left[2\,(h_2^2 + h_3^2) + c^2 + b^2\right]. \end{cases} \tag{4.3.23}$$

The mass difference $M^2_{V+} - M^2_{V^\circ}$ is proportional only to the $SU(2)_L \times U(1)_Y$ breaking since (V^+_μ, V°_μ) as like as $(\bar V^\circ_\mu, V^-_\mu)$, is an $SU(2)_L$ doublet [1].

4.4 The $Z - Z'$ mixing in the BM model

Let us go to study the diagonal gauge boson sector of the model. We will see that not all the points of the $M_{Z_2} - \mathrm{tg}\chi$ plane (χ is the $Z - Z'$ mixing angle and M_{Z_2} the heaviest mass eigenvalue) are theoretically allowed and we will derive a relation between M_{Z_2} and the $SU(2)_\nu$ breaking scale which gives a connection between the value of the Z' mass and the naturality of the model.

The neutral gauge boson mass terms can be written as:

$$\mathcal{L}_M = \frac{1}{2}\, M_\circ^2 \cdot (Z_\mu, Z'_\mu) \cdot \begin{pmatrix} 1 & \alpha' \\ \alpha' & \beta' \end{pmatrix} \cdot \begin{pmatrix} Z_\mu \\ Z'_\mu \end{pmatrix}, \tag{4.4.1}$$

where we have defined $M_\circ^2 \equiv M_W^2/c_w^2$ and $\alpha' \equiv \alpha/f_w$, $\beta' \equiv \beta/f_w^2$, with $f_w \equiv (3 - 4s_w^2)^{1/2}$. α and β are adimensional parameters functions of v.e.v.s ratios:

$$\begin{cases} \alpha = 1 - 2c_w^2 y, & y = \dfrac{b^2}{a^2 + b^2}, \\ \beta = 1 - 4s_w^2 c_w^2 y + 4c_w^4 r, & r = \dfrac{4\,(h_2^2 + h_3^2) + c^2}{a^2 + b^2}, \end{cases} \tag{4.4.2}$$

and it is trivially true that $0 < y < 1$, $r > 0$. In eq. (4.4.1) we have used that the $SU(3)_L$ and $U(1)_X$ gauge couplings, respectively g and $\tilde g$, can be written in terms of the electric charge e and the sinus of the Weinberg angle s_w as $g = e/s_w$ and $\tilde g = \sqrt{3}e/f_w$. Z_μ is the Standard Model neutral boson while Z'_μ is a new neutral boson, mass eigenstate in the limit in which $SU(2)_L \times U(1)_Y$ is unbroken,

$$Z'_\mu = \frac{1}{\sqrt{3}c_w}\, \left(f_w V^8_\mu + s_w X_\mu\right). \tag{4.4.3}$$

---

[1] If $a = b$ in a natural way, which however in general is unexpected, then $M^2_{V+} - M^2_{V^\circ} = 0$, and this mass difference becomes non zero only at highest loops.

After the $SU(3)_L \times U(1)_X \longrightarrow U(1)_{e.m.}$ breaking, the neutral gauge interactions can be written as $\mathcal{L}_I = \bar{\psi}_L \gamma_\mu \mathcal{K}_\mu \psi_L$, where ψ_L indicates any of the fermion representations, and

$$\mathcal{K}_\mu = eQ \cdot A_\mu - \frac{e}{s_w c_w}(T_{3L} - s_w^2 Q) \cdot Z_\mu + \frac{e}{s_w c_w}\left(\frac{f_w}{\sqrt{3}} V^8 + \frac{s_w^2}{f_w} X\right) \cdot Z'_\mu, \tag{4.4.4}$$

with A_μ the photon field. Now the presence of a $Z - Z'$ mixing modify the Standard Model relation $M_{Z_1}^2 = M_W^2 / c_w^2$, since, as it is easy to see from eq. (4.4.1), the mass eigenstates $Z_{1\,\mu}$ and $Z_{2\,\mu}$ are in general linear combinations of Z_μ and Z'_μ. With the following transformation

$$\begin{pmatrix} Z_{1\,\mu} \\ Z_{2\,\mu} \end{pmatrix} = \begin{pmatrix} c_\chi & s_\chi \\ -s_\chi & c_\chi \end{pmatrix} \cdot \begin{pmatrix} Z_\mu \\ Z'_\mu \end{pmatrix}, \tag{4.4.5}$$

which defines χ as the mixing angle, we diagonalize the $Z - Z'$ system. If M_1 and M_2 are respectively the Z_1 and the Z_2 masses, introducing the ratios $p_{1,2} = M_{1,2}^2 / M_\circ^2$, we obtain the conditions

$$\begin{cases} c_\chi^2 p_1 + s_\chi^2 p_2 = 1, \\ c_\chi s_\chi (p_1 - p_2) = \alpha', \\ s_\chi^2 p_1 + c_\chi^2 p_2 = \beta'. \end{cases} \tag{4.4.6}$$

Since $\beta' - \alpha'^2 > 0$ for any value of the y and r parameters in their respective domains, then $p_{1,2} > 0$ and if $\beta' > 1$ and $-\pi/4 < \chi < \pi/4$, as it is natural to assume, then $p_2 > 1$ and $p_1 < 1$. Then in general we have to expect a lowering of the value of the Z° mass with respect to the Standard Model case. The first one of eq.s (4.4.6) is a relation between the physical quantities χ, M_1 and M_2, which can be used to obtain the deviation with respect to the Standard Model prediction $\rho = 1$:

$$\rho \equiv \frac{M_W^2}{M_1^2 c_w^2} \simeq 1 + \left(\frac{M_2^2}{M_\circ^2} - 1\right) \cdot \mathrm{tg}^2 \chi. \tag{4.4.7}$$

Eliminating p_1 in the last two eq.s of (4.4.6) we find

$$\begin{cases} (1 - p_2)\, \mathrm{tg}\chi = \alpha', \\ p_2 + (1 - p_2)\, \mathrm{tg}^2 \chi = \beta', \end{cases} \tag{4.4.8}$$

and from the constraints $0 < y < 1$, $r > 0$, we derive that not all the points of the $M_2 - \mathrm{tg}\chi$ plane are allowed in the model. In particular in the region of physical interest, $M_2 > 100\, GeV$ and $-0.1 < \mathrm{tg}\chi < 0.1$, the following conditions are always satisfied:

$$\begin{cases} M_2 < M_\circ \cdot \left(1 + \dfrac{1 - 2s_w^2}{f_w \mathrm{tg}\chi}\right)^{1/2}, & 0 < \mathrm{tg}\chi < 0.1, \\ M_2 < M_\circ \cdot \left(1 - \dfrac{1}{f_w \mathrm{tg}\chi}\right)^{1/2}, & -0.1 < \mathrm{tg}\chi < 0. \end{cases} \tag{4.4.9}$$

Using $s_w^2 = 0.23$ and $M_W = 81\, GeV$, we have represented these limits in Fig. 12 by the two dot-dashed lines and then any point above these lines is not allowed in the model. We can now express a functional relation between M_2 and χ parametrized in terms of the v.e.v.s ratios y or r. Directly from the first one of eq.s (4.4.8):

$$M_2 = M_\circ \cdot \left(1 + \frac{2c_w^2 y - 1}{f_w \mathrm{tg}\chi}\right)^{1/2}, \tag{4.4.10}$$

and from $p_2 > 1$ follows that $\mathrm{tg}\chi > 0 (< 0)$ if $2c_w^2 y > 1 (< 1)$. The limiting values $y = 0, 1$ are obviously coincident with the limiting conditions (4.4.9), and their physical meaning is respectively $b \ll a$ or $b \gg a$. Only for $y = 1/2c_w^2$, which however in general is not expected to be a natural relation of the model, $\alpha' = 0$ and so no mixing is present.

An interesting parametrization of the $M_2 - \chi$ relation is given in terms of the $SU(2)_\nu$ breaking scale with respect to the Fermi scale,

$$R \equiv [4\,(h_2^2 + h_3^2) + c^2]\, G_F = r/2\sqrt{2}. \tag{4.4.11}$$

Now from (4.4.8) we obtain

$$M_2 = M_\circ \cdot \left(1 + \frac{2c_w^2 - 8\sqrt{2}c_w^4 R}{f_w^2 \mathrm{tg}^2\chi - 2s_w^2 f_w \mathrm{tg}\chi - f_w^2}\right)^{1/2}, \tag{4.4.12}$$

in the range $-0.1 < \mathrm{tg}\chi < 0.1$. We have plotted this relation in Fig. 12 with three dashed lines for the values $R = 2, 4, 9$, taking into account the limiting conditions (4.4.9). M_2 is essentially constant as a function of χ, and it is increasing with R. However R cannot become too large, otherwise the $SU(2)_\nu$ breaking scale would become too big with respect to the Fermi scale, putting in trouble the naturality of the model. This is natural for R of order unity. We consider R less than 10 as an upper bound of the *naturality* region, so that, from Fig. 12, it follows $M_2 \leq 500 \div 600\, GeV$.

That concludes the theoretical analysis of the $Z - Z'$ mixing in the $SU(3)_L \times U(1)_X$ model. In the next section we will compare the theoretical predictions with the present bounds that we can derive from the measurements of ρ and from the Neutral Current (NC) experimental data.

4.5 Bounds on Z' from the present data

First of all let us discuss the deviation with respect to $\rho = 1$ induced by the Z' vector boson. From eq. (4.4.7) the deviation

$$\delta\rho \equiv \rho - 1 = \left(\frac{M_2^2}{M_\circ^2} - 1\right) \cdot \mathrm{tg}^2\chi > 0, \tag{4.5.1}$$

is also the same in other models such as E_6 [72], in such a way that the limits we will derive from the measurements of ρ are essentially model independent. Using the measured values of M_W, M_{Z_1} and collecting all the experimental data [73]:

$$\rho_{exp} = 0.998 \pm 0.0086. \tag{4.5.2}$$

Allowing now a 2σ level, we extract the limit

$$\delta\rho \leq 1.5 \times 10^{-2}, \tag{4.5.3}$$

which gives the experimental constraint

$$M_2 \leq M_\circ \cdot \left(1 + \frac{0.015}{\mathrm{tg}^2\chi}\right)^{1/2} . \tag{4.5.4}$$

This constraint is plotted, in the range $-0.1 < \mathrm{tg}\chi < 0.1$, in Fig. 12 by the solid lines, which turns out to be symmetric with respect to $\mathrm{tg}\chi = 0$. It follows that the model is well inside the limits (4.5.4) apart from extreme values of the parameters: y next to 0 or 1 together with a small value of R, as it is clear by looking at Fig. 12.

Next we will compare the model predictions with the present NC data [73]. This kind of analyses has been previously performed for other models, in particular for the E_6 [74] superstring inspired models. It is obvious however that, in contrast to the ρ case, the results are model dependent since they depend on the different Z' gauge couplings. Let us write eq. (2.10) in terms of the mass eigenstates $Z_{1\mu}$ and $Z_{2\mu}$:

$$\mathcal{K}_\mu = \frac{g}{c_w}\left[-(T_{3L} - s_w^2 Q)\, c_\chi + \frac{1}{f_w} Y_E\, s_\chi\right]\cdot Z_{1\mu} + \frac{g}{c_w}\left[\frac{1}{f_w} Y_E\, c_\chi + (T_{3L} - s_w^2 Q)\, s_\chi\right]\cdot Z_{2\mu}, \tag{4.5.5}$$

where we have not taken into account the photon term and we adopt the parametrization

$$Y_E \equiv \frac{f_w^2}{\sqrt{3}} V^8 + s_w^2 X. \tag{4.5.6}$$

Now the neutrino-hadron scattering is mediated by Z_1 and Z_2, in such a way that the effective Lagrangian is:

$$-\mathcal{L}_{eff}^{\nu_L h} = \frac{G_F}{\sqrt{2}}\, \bar{\nu}\gamma_\mu(1-\gamma_5)\nu \cdot [\epsilon_L\, \bar{q}\gamma_\mu(1-\gamma_5)q + \epsilon_R\, \bar{q}\gamma_\mu(1+\gamma_5)q], \tag{4.5.7}$$

where q is an u or d field, and

$$\epsilon_{L,R}(q) = \frac{1}{1+\mathrm{tg}^2\chi}\,\frac{1}{1-(p_2-1)\mathrm{tg}^2\chi}\,(1-0.37\mathrm{tg}\chi)\,(\epsilon^\circ_{L,R}(q) - 0.69 Y_E^{L,R}(q)\,\mathrm{tg}\chi) +$$

$$+\frac{1}{1+\mathrm{tg}^2\chi}\,\frac{1}{p_2}\,(0.37+\mathrm{tg}\chi)\,(\epsilon^\circ_{L,R}(q)\,\mathrm{tg}\chi + 0.69 Y_E^{L,R}(q)), \tag{4.5.8}$$

where we have used $s_w^2 = 0.23$; $\epsilon^\circ_L = T_{3L} - s_w^2 Q$ and $\epsilon^\circ_R = -s_w^2 Q$ are the Standard Model ϵ coefficients of the neutrino-hadron scattering. The first term of eq. (4.5.8) is due to the Z_1 exchange, while the second one to the Z_2 and it goes to zero in the limit $M_2 \to \infty$. The Z_1 exchange term coincides with the Standard Model case only if $\mathrm{tg}\chi \to 0$. Let us now parametrize the deviation with respect to the Standard Model by

$$z(q_{L,R}) \equiv \frac{\epsilon_{L,R}(q) - \epsilon^\circ_{L,R}(q)}{\epsilon^\circ_{L,R}(q)} = G(p_2,\chi) + 0.69\,\frac{Y_E^{L,R}(q)}{\epsilon^\circ_{L,R}(q)}\,H(p_2,\chi), \tag{4.5.9}$$

where

$$\begin{cases} H = \frac{1}{p_2} \frac{0.37 + (1-p_2)\mathrm{tg}\chi}{1 + (1-p_2)\mathrm{tg}^2\chi}, \\ G = (1-p_2)\mathrm{tg}\chi \cdot H. \end{cases} \tag{4.5.10}$$

The measurements of $\epsilon_{L,R}(q)$ [73] are listed in Table 2, together with the theoretical Standard Model [2] and $SU(3)_L \times U(1)_X$ values. From these data we obtain the experimental determination of the parameters $z(q_{L,R})$ which characterize the BM model. In Table 3 we list the theoretical, in terms of p_2 and χ, and experimental values of these parameters. The last three z theoretical values are essentially equal because the BM model predicts $Y_E^{L,R}(q) = \epsilon^\circ_{L,R}(q)$ for $q_{L,R} = d_{L,R}, u_R$. With a linear fit we obtain for the G and H parameters: $G = 0.002 \pm 0.028$, $H = -0.023 \pm 0.038$. From these data it follows that the simple function $G/H = (1-p_2)\,\mathrm{tg}\chi$ is experimentally determined to be: $G/H = -0.087 \pm 1.226$. Now neglecting $(p_2 - 1)\,\mathrm{tg}^2\chi$ in the denominator of H, because this term is less than 1.5% as derived from the ρ analyses, and allowing a 2σ level, it follows from the determination of H that M_2 and χ must satisfy the following experimental bounds:

$$M_2 \geq M_\circ \cdot \left(\frac{0.37 + \mathrm{tg}\chi}{0.053 + \mathrm{tg}\chi}\right)^{1/2}, \qquad -0.053 < \mathrm{tg}\chi < 0.1. \tag{4.5.11}$$

On the other hand the limits in the region $100\,GeV < M_2 < 1000\,GeV$ and $-0.1 < \mathrm{tg}\chi < 0.1$ deduced from the numerical value of G/H are less stringent than the bounds (4.5.4) derived from the ρ analyses, so taking into account the (4.5.4) and the (4.5.11) we individuate a 2σ allowed window for the BM model, indicated by solid lines in Fig. 12. As already explained, the two symmetric upper solid lines represent the experimental limits from the ρ analyses, while the lower solid line reproduces the limit (4.5.11) extracted from the NC data. The lowest allowed value of the Z_2 mass is $M_2 = 170\,GeV$ for $\mathrm{tg}\chi = 0.08$, and the mixing angle χ must satisfy the bounds $-0.03 \leq \mathrm{tg}\chi \leq 0.08$. At present the model is well inside the experimental window apart from extreme values of its parameters, and an $SU(2)_\nu$ breaking scale not far from the Fermi scale is in agreement with the present experimental bounds.

[2] The values of $\epsilon^\circ_{L,R}(q)$ listed in Table 2 take into account one-loop orders, this modify eq. (4.5.9) only for very small $O(\alpha)$ terms.

4.6 Limits on the mixing by looking at the Z° peak

Another useful way to study the phenomenological effects of a $Z - Z'$ mixing is to look directly at the Z° peak at LEP, in consideration of the fact that a lot of Z° will be produced in the near future at this machine. Due to the mixing, the mass eigenstate corresponding to the Z° boson is $Z_{1\mu}$ whose couplings to fermions are now modified with respect to the standard ones by order $O(s_\chi)$, as it is clear from eq. (4.5.5). Then we have to expect modifications of $O(s_\chi)$ in the various Z° decay rates. To isolate the pure $Z - Z'$ mixing effects with respect to the ones coming at one loop level from heavy top and Higgs, or at tree level from new exotic particles, it is useful to follow a general strategy recently proposed [75].

The relevant variables to use are essentially the Z_1 mass, the $Z_1 \to e^+e^-$, or $\mu^+\mu^-$, and the $Z_1 \to$ hadrons decay rates. Let us now calculate, in agreement with the standard treatment of Ref. [75], suitable combinations of these variables in terms of the parameter

$$v_\circ = 2\left(1 - \frac{4\mu^2}{M_1^2}\right)^{1/2} - 1, \tag{4.6.1}$$

where $\mu^2 \equiv \alpha\pi/\sqrt{2}G_F$ and $v_\circ$ is a small quantity such that we can neglect $O(v_\circ^2)$ terms at the 1% order. Now straightforward calculations give us:

$$\xi \equiv \frac{M_W^2}{M_1^2} \frac{2}{1 + \left(1 - \dfrac{4\mu^2}{M_1^2}\right)^{1/2}} \simeq \left[1 + \frac{3}{2}\nabla\rho\right] + \delta\xi^\chi, \tag{4.6.2}$$

$$\gamma \equiv \frac{9}{\alpha} \frac{\Gamma(Z_1 \to e^+e^-)}{M_1} \simeq \left[1 + \frac{2}{3}v_\circ + \nabla\rho\right] + \delta\gamma^\chi, \tag{4.6.3}$$

$$R_z \equiv \frac{3}{59} \frac{\Gamma(Z_1 \to u, d, s, c, b)}{\Gamma(Z_1 \to e^+e^-)} \simeq \left[1 + \frac{20}{59}v_\circ + \frac{1}{2}\nabla\rho\right] + \delta R_z^\chi, \tag{4.6.4}$$

where the square bracket terms are the sum of the tree order Standard Model contributions at $O(v_\circ)$ and $\nabla\rho$, which includes all the possible exotic non-mixing effects. The $\delta\xi^\chi$, $\delta\gamma^\chi$ and δR_z^χ terms take into account only the pure mixing effects (they are proportional to s_χ). Since the combinations $R_z - \frac{1}{2}\gamma$ and $R_z - \frac{1}{3}\xi$ are $\nabla\rho$ free, their possible deviations with respect to the Standard Model predictions will be a clear signal of a $Z - Z'$ mixing. It is then useful to introduce the variables:

$$\begin{cases} n \equiv [R_z - \frac{1}{2}\gamma] - [R_z - \frac{1}{2}\gamma]^\circ, \\ p \equiv [R_z - \frac{1}{3}\xi] - [R_z - \frac{1}{3}\xi]^\circ, \end{cases} \tag{4.6.5}$$

where the o superindex indicates the theoretical Standard Model values. From the eq.s (4.5.5) and (4.5.6) it is easy to derive in the BM model $\delta\xi^\chi = 0$, $\delta\gamma^\chi = (\sqrt{3}/f_w c_w)\, s_\chi$ and $\delta R_z^\chi = -(36/59)\,\delta\gamma^\chi$, and then

$$\begin{cases} n = \delta R_z^\chi - \frac{1}{2}\delta\gamma^\chi = -\frac{131}{118}\delta\gamma^\chi, \\ p = \delta R_z^\chi = -\frac{36}{59}\delta\gamma^\chi, \end{cases} \tag{4.6.6}$$

from which we derive the sum rule

$$\frac{p}{n} = \frac{72}{131} = \frac{6}{11} + O(1\%). \tag{4.6.7}$$

The $n = \frac{11}{6}p$ rule is a powerful prediction of the BM model. The situation is similar to other theoretical models such as E_6 where the prediction is $n = \frac{8}{5}p$ [75] making the E_6 and BM lines very close to one another as we can see from Fig. 13, although the $SU(3)_L \times U(1)_X$ line is shorter than the E_6 one since $-0.03 \leq s_\chi \leq 0.08$, as we know from Fig. 12, which together with (4.6.6) imply $-0.067 \leq p \leq 0.025$. So only for $p > 0.025$ we can have a clear E_6 signal, otherwise the two models give essentially the same prediction. The dot-dashed square in the centre of Fig. 13 indicates the expected experimental sensibility, so if the measurements fall inside this square no $Z - Z'$ mixing is revealed at the 1% sensibility level. In such a case the present allowed window of Fig. 12 would be sensibly reduced.

4.7 Prospects at future hadron colliders

An attractive possibility to detect the neutral boson Z_2 is to look for it via e^+e^- Drell-Yan [76] production in proton-(anti)proton colliders. More precisely, if the number of leptonic events is plotted versus their center of mass energy, the presence of Z_2 is revealed when this plot displays a Breit-Wigner peak, the mass M_2 of the new boson being the value of the invariant mass of the leptonic pair at the position of the peak.

Neglecting the background contribution, the total cross section is

$$\sigma = \frac{4\pi^2}{3M_2^3}\frac{\Gamma(Z_2 \to l\bar{l})}{\Gamma_{Z_2}} \sum_q U_q(\tau)\,\Gamma(Z_2 \to q\bar{q}), \tag{4.7.1}$$

where $\tau = M_2/\sqrt{s}$, $\sqrt{s}$ being the energy of the colliding hadrons in their center of mass frame, Γ_{Z_2} is the total Z_2 decay width, q runs over all quark and antiquark species of one of the two colliding hadrons and $U_q(\tau)$ is a function which gives a measure of the probability of finding a couple of partons $q\bar{q}$ being able to contribute to the Drell-Yan process,

$$U_q(\tau) = \int_{-1+\tau^2}^{1-\tau^2} \mathrm{d}y \; \frac{f_q(\frac{1}{2}y + \frac{1}{2}\sqrt{y^2+4\tau^2})\, f_{\bar{q}}(-\frac{1}{2}y + \frac{1}{2}\sqrt{y^2+4\tau^2})}{\sqrt{y^2+4\tau^2}}, \tag{4.7.2}$$

$f_{q,\bar{q}}$ being the parton distribution functions. In eq. (4.7.1) we have approximated the Breit-Wigner with a delta function in the limit in which Γ_{Z_2} is small enough and worked in the tree approximation. Within this assumption the partial width of Z_2 decaying into fermions $\psi\bar{\psi}$ is

$$\Gamma(Z_2 \to \psi\bar{\psi}) = \frac{M_2}{12\pi} A_\psi C_\psi \,(v_2^{\psi\,2} + a_2^{\psi\,2}), \tag{4.7.3}$$

where $A_\psi = 1(2)$ for Dirac (Majorana) fermions, $C_\psi = 1(3)$ if ψ is a singlet (triplet) of $SU(3)_c$ and $v_2^{q,l}$ ($a_2^{q,l}$) are the vector (axial) couplings of quarks and leptons to the gauge boson Z_2 which are defined by

$$\mathcal{L} = \bar{\psi}\,\gamma_\mu (v_2^\psi + a_2^\psi \gamma_5)\,\psi \cdot Z_{2\,\mu}, \tag{4.7.4}$$

where $\psi = q, l$. These coefficients can be deduced from eq. (4.4.4) and they are enlisted in Table 4 ($\lambda_{\mu,\tau}$ and $\chi_{\mu,\tau}$ indicate the Majorana spinors corresponding respectively to the right- and left-handed μ- and τ-neutrinos).

To compute the total width Γ_{Z_2} in eq. (4.7.1) the decay of Z_2 into charged gauge bosons must be taken into account. In general Z_2 will have an open channel to W^+W^- but not either to V^+V^- or to $V^\circ\bar{V}^\circ$. Indeed, the fact that $R \geq 1$, as understood from Fig. 12, and the approximation $p_2 \simeq \beta'$, true up to 1.5% (see eq. (4.4.8)), with eq.s (4.3.23) straightforwardly yield $M_2 < 2M_{V^+}$ and $M_2 < 2M_{V^\circ}$. The corresponding decay rate for W^+W^- is

$$\Gamma(Z_2 \to W^+W^-) = \frac{M_2}{12\pi}(1-4\eta_W)^{3/2}(1+20\eta_W+12\eta_W^2)\,\frac{e^2 c_w^2 s_\chi^2}{16 s_w^2 \eta_W^2}, \tag{4.7.5}$$

where $\eta_W \equiv M_W^2/M_2^2 = c_w^2/p_2$. A remarkable fact displayed by this expression is that although $\Gamma(Z_2 \to W^+W^-)$ is suppressed by the mixing angle s_χ^2, it is also enhanced by the term η_W^{-2}. Actually both effects compensate each other leading to a branching ratio $B(Z_2 \to W^+W^-)$ of the same order of magnitude than $B(Z_2 \to e^+e^-)$ [77]. Therefore, in contrast to other $O(s_\chi^2)$ terms appearing in the evaluation of Γ_{Z_2}, eq. (4.7.5) gives non negligible contributions. In the actual computation of Γ_{Z_2} we have assumed that Z_2 can decay freely into all conventional fermions (including the top quark), the new heavy g quarks, the Majorana neutrinos $\nu^c_{\mu L}$, $\nu^c_{\tau L}$, and $W^\pm$. In the latter we have put $s_\chi^2/\eta_W^2 \sim 1$. This altogether gives $\Gamma_{Z_2}/M_2 \simeq 3\%$, and a branching ratio

$$B(Z_2 \to e^+e^-) \simeq \frac{1}{77}. \tag{4.7.6}$$

Now with eq.s (4.7.1) and (4.7.6) we can extract predictions on the expected numbers of e^+e^- events for the future hadron machines SPS+Acol, TEVATRON, LHC and SSC, whose characteristics are listed in Table 5. A $10\,pb^{-1}$ integrated luminosity is assumed for SPS+Acol and TEVATRON, $10^4\,pb^{-1}$ for LHC and SSC. The type 1 Duke & Owens parton distribution functions [78] have been used together with the numerical values $e^2/4\pi = 1/127$ and $s_w^2 = 0.23$. The resulting number of events for the previous machines are plotted in Fig.s 14 and 15. We have not displayed the number of background events, mediated by Z° and γ, because it is negligible. If less than 5 events, which we assume to be the minimum significant Z' signal, are seen at SPS+Acol (TEVATRON, LHC, SSC), then $M_2 \geq 190\,GeV$ ($350\,GeV$, $3.5\,TeV$, $6\,TeV$).

• *Conclusions.* We have analysed the neutral current sector of the $SU(3)_L \times U(1)_X$ model [5] which predicts an extra Z' boson. The bounds on the mass and mixing of the new boson extracted from the present measurements of ρ and from the NC data are illustrated in Fig. 12 by solid contour lines. The model lies inside the experimentally allowed window except only for extreme values of its parameters. The minimal allowed mass value is $M_2 = 170\,GeV$ and the $Z - Z'$ mixing angle is delimited by $-0.03 \leq \mathrm{tg}\chi \leq 0.08$ at a 2σ level.

Concerning the future prospects, we have first studied the effects of the $Z - Z'$ mixing on the top of the Z° peak at LEP in terms of a suitable set of measurable variables, n and p defined by eq. (4.6.5), proportional to the mixing angle s_χ. The BM model lies on the straight line $n = \frac{11}{6}p$, close to the E_6 prediction, as depicted in Fig. 13. If the n and p measurements fall inside the dot-dashed square of Fig. 13 then no $Z - Z'$ mixing is revealed at the 1% sensibility level, and the bounds on s_χ are sensibly reduced. Secondly we have analysed the possibility of a direct Z' production at future hadron colliders deriving that less than 5 Z_2 events are expected at SPS+Acol, TEVATRON, LHC and SSC if M_2 is respectively bigger than $190\,GeV$, $350\,GeV$, $3.5\,TeV$ and $6\,TeV$, as can be seen in Fig.s 14 and 15. If no acceptable Z_2 signal were seen at TEVATRON (we have assumed more than 5 e^+e^- events for a clear signal), the explanation of the smallness of m_{ν_e}/μ_{ν_e} would still be *natural* in the BM model although closer to the naturality boundary $M_2 \leq 500 \div 600\,GeV$. However if no Z_2 signal were detected at LHC or SSC then the naturality of the model would become problematic.

Conclusion

One of the more relevant aspects of neutrino physics consists in the nature of the mass, Majorana or Dirac, if it is different from zero. We have explored the consequences that the two hypothesis can produce both from the theoretical both from the phenomenological point of view.

If the right-handed partner exists, and neutrinos are of Majorana type, in general we have to expect a $\nu_L - \nu_L^c$ mixing which give rise to a violation of lepton universality. We have derived that the τ-system is the best place to look for such an effect. We have shown how LEP and SLC can probe the mixing between the $\nu_{\tau L}$ and a *sterile* neutrino with a mass of order $10\,GeV$.

The experimental bounds on the violation of universality in the electron sector can be improved by looking at the SN1987A event. In fact the supernova luminosity constraints the $\nu_{eL} - \nu_{eR}$ mixing angle θ to be extremely small. Taking into account the matter resonant oscillation effects we have derived $\sin^2 2\theta \leq 2.3 \times 10^{-10}$, except for a value of the ν_{eR} mass around $1\,KeV$, where $\sin^2 2\theta \leq 1.6 \times 10^{-6}$.

On the other hand, the more specific case of a ν_{eR} as the right-handed component of a Dirac neutrino can give rise to extremely relevant physical consequences. In such a case a *large* neutrino magnetic moment can solve the Solar neutrino puzzle, moreover explaining a possible anticorrelation between the Davis data and the 11 year sunspot cycle. The requirement of a *large* μ_{ν_e} opens a *naturality* problem which can be solved by a minimal extension of the standard gauge group to $SU(3)_L \times U(1)_X$. The enlargement of the gauge group gives rise to relevant phenomenological consequences, in particular in what concern the prediction of an extra Z' boson. A very interesting connection between neutrino physics and the neutral current sector of the theory is established.

We have then explored the effects of this new vector boson. From the present ρ measurements and from the NC data we are able to put the bounds $M_2 \geq 170\,GeV$ and $-0.03 \leq \mathrm{tg}\chi \leq 0.08$ for the Z' mass and the $Z - Z'$ mixing. Also prospects at LEP and at the future hadron colliders have been explored. The *naturality* of a *large* neutrino magnetic moment in the BM model would become problematic only if no Z' signal were detected at LHC or SSC.

Neutrino physics is a physics of very small numbers, the same *large* μ_{ν_e} we have discussed above is hierarchically smaller than the electron neutrino magnetic moment. On the other hand, neutrinos could *dominate* events of cosmological and astrophysical size, and they could be a sort of key to partially enlight the elementary particle physics beyond the Fermi scale. Up to now a lot of theoretical work has been developed. However, an experimental determination of some neutrino parameters, difficult due to their smallness, would be needed. In this sense it is useful to look at the sky. SN1987A has been a lucky event, but a better understanding of the Solar neutrino puzzle would be necessary. Future prospects are very interesting and promising. In particular we expect that in the near future a comparison between Davis and Kamiokande experiments could enlight some relevant aspects of neutrino physics.

Table 1

ψ	K_2	K_3
L_1	0	0
L_2	1	0
L_3	0	1
e_1^c	0	0
e_2^c	-1	0
e_3^c	0	-1
φ	0	0
η	0	1
T_2	-2	0
T_3	0	-2
Q	0	0
ρ	0	-1
σ	0	1
g^c	0	1
u^c	0	-1
d^c	0	0

• The K_2 and K_3 quantum numbers of the fermion and scalar representations of the BM model are enlisted.

Table 2

parameter	$SU(2)_L \times U(1)_Y$	$SU(3)_L \times U(1)_X$	experiment
$\epsilon_L(u)$	$+0.345$	$+0.345(1+z(u_L))$	$+0.339 \pm 0.017$
$\epsilon_R(u)$	-0.152	$-0.152(1+z(u_R))$	-0.172 ± 0.014
$\epsilon_L(d)$	-0.427	$-0.427(1+z(d_L))$	-0.429 ± 0.014
$\epsilon_R(d)$	$+0.076$	$+0.076(1+z(d_R))$	-0.011 ± 0.057

• The experimental measurements, together with the theoretical Standard Model and $SU(3)_L \times U(1)_X$ values, of the $\epsilon_{L,R}(q)$ coefficients of the neutrino-hadron scattering are shown.

Table 3

parameter	th. value	exp. value
$z(u_L)$	$G(p_2,\chi) + 0.85H(p_2,\chi)$	-0.017 ± 0.049
$z(u_R)$	$G(p_2,\chi) - 0.70H(p_2,\chi)$	$+0.132 \pm 0.092$
$z(d_L)$	$G(p_2,\chi) - 0.68H(p_2,\chi)$	$+0.005 \pm 0.033$
$z(d_R)$	$G(p_2,\chi) - 0.70H(p_2,\chi)$	-1.145 ± 0.750

• The experimental and theoretical BM values of the $\epsilon_{L,R}(q)$ deviations with respect to the Standard Model determinations are reported.

Table 4

ψ	v_2^ψ	a_2^ψ
$e,\ \mu,\ \tau$	$\frac{e}{s_w c_w}\left(\frac{1}{4}-s_w^2\right)\left(\frac{1}{f_w}c_\chi - s_\chi\right)$	$\frac{e}{4s_w c_w}\left(\frac{1}{f_w}c_\chi - s_\chi\right)$
ν_e	$\frac{e}{4s_w c_w}\left(f_w c_\chi + s_\chi\right)$	$\frac{e}{4s_w c_w}\left(-\frac{1}{f_w}c_\chi + s_\chi\right)$
$\lambda_\mu,\ \lambda_\tau$	0	$-\frac{ec_w}{2s_w f_w}c_\chi$
$\chi_\mu,\ \chi_\tau$	0	$\frac{e}{4s_w c_w}\left(\frac{1-2s_w^2}{f_w}c_\chi + s_\chi\right)$
$u,\ c,\ t$	$\frac{e}{4s_w c_w}\left[\frac{3+2s_w^2}{3f_w}c_\chi + \left(1-\frac{8}{3}s_w^2\right)s_\chi\right]$	$\frac{e}{4s_w c_w}\left(\frac{1-2s_w^2}{f_w}c_\chi + s_\chi\right)$
$d,\ s,\ b$	$\frac{e}{4s_w c_w}\left[\frac{f_w}{3}c_\chi + \left(\frac{4}{3}s_w^2 - 1\right)s_\chi\right]$	$\frac{e}{4s_w c_w}\left(\frac{1}{f_w}c_\chi - s_\chi\right)$
$g_1,\ g_2,\ g_3$	$\frac{e}{6s_w c_w}\left(\frac{7s_w^2-3}{f_w}c_\chi - 4s_w^2 s_\chi\right)$	$-\frac{ec_w}{2s_w f_w}c_\chi$

• The vector and axial Z_2 fermion couplings are enlisted.

Table 5

experiment	$\sqrt{s}\ (TeV)$	$\int \mathcal{L}\,\mathrm{d}t\ (pb^{-1})$
SPS+Acol ($p\bar{p}$)	0.63	10
TEVATRON ($p\bar{p}$)	2	10
LHC (pp)	17	10^4
SSC (pp)	40	10^4

• The center of mass energy and the integrated luminosity of the future hadron colliders are given.

Figure captions

Fig. 1. The FNAL (E531 Collaboration) results on the $\nu_e - \nu_\tau$ and $\nu_\mu - \nu_\tau$ mixings are illustrated.

Fig. 2. The time dependence of Davis measurements is shown. The minimum at 1979-1980 corresponds to a maximum of Solar activity.

Fig. 3. The 11 year sunspot cycle is shown.

Fig. 4. The region of the plane $|U_{\tau N}|^2 - M_N$ which can be probed at SLC and LEP is illustrated. $|U_{\tau N}|^2 > 0.16$ is excluded from $\tau \to e\,\bar{\nu}_e\,\nu_\tau$ decay. The constraint derived from the absence of anomalous monojet events in e^+e^- collision experiments is also shown (as the solid line in the upper-left corner of the figure). The dot-dashed line gives the sensitivity limit on $|U_{\tau N}|^2$ from neutrino counting experiments. The other two solid lines define the regions where $\mathcal{BR}(Z^o \to N\nu_\tau)$ and $\mathcal{BR}(Z^o \to NN)$ are larger than 10^{-5}. Finally, the dashed line represents the prediction of the JRS model, with the constraints on m_{ν_τ} (see Fig. 5).

Fig. 5. The τ-neutrino mass versus the heavy (mostly *sterile*) neutrino mass, as predicted by the JRS model is reported. The region allowed by experimental and cosmological constraints ($10\,MeV < m_{\nu_\tau} < 35\,MeV$) is also shown.

Fig. 6. The branching ratio for the exotic $Z^o \to N\nu_\tau$ decay, plotted against M_N is illustrated. The allowed range of values of M_N derived from Fig. 5 is also indicated.

Fig. 7. The luminosity emitted by the supernova through the X channel, as a function of the X coupling g_X, is illustrated. On the left side of the L_X maximum is placed the *no-trapping* region, the *trapping* one is on the right. From the requirement $L_X \leq Q_B = 3 \times 10^{53}\,erg/s$ an excluded interval for g_X follows.

Fig. 8. The trapping and no-trapping regions are separated by the vertical dot-dashed line. Outside the region A, delimited by the diagonal and horizontal solid lines, eqs. (3.3.10) and (3.3.16) exclude the region between the vertical solid lines. Inside A matter resonant oscillations are relevant. The two vertical dashed lines inside A give us the limits derived from $\bar{N}$. The diagonal dashed line on the left side indicates the points where the oscillation length for $\nu \rightarrow N$ flipping becomes equal to the interaction length. Finally, the horizontal dashed line and the diagonal dashed line on the right side give the lower limit for which N flips into ν inside the ν-sphere. The sub-regions a, b, c, d, e, discussed in the text are also indicated.

Fig. 9. The limits for $\sin^2 2\theta$ are shown. The central region between the solid lines is excluded. It is represented how matter resonant oscillations reduce the excluded values of $\sin^2 2\theta$ in the window $2 \times 10^5 \; eV^2 \leq \Delta m^2 \leq 4 \times 10^7 \; eV^2$.

Fig. 10. The effective one-loop diagram (two-loop in reality) which gives a contribution to the neutrino mass is shown. The cross indicates the effective neutrino magnetic coupling.

Fig. 11. The one-loop diagrams of the BM model which contribute to the neutrino mass are depicted. They cancel in the limit of $SU(2)_\nu$ unbroken. The insertion of a photon line on the same diagrams gives rise to the magnetic moment contributions. In such a case the two diagrams do not cancel in the limit of $SU(2)_\nu$ unbroken, since the electric charge flows in opposite directions and the adding of a photon line has reversed the sign of one of the diagrams.

Fig. 12. Present experimental bounds on Z' are illustrated in the $M_2 - \mathrm{tg}\chi$ plane. The BM model lies between the two dot-dashed lines. The dashed lines correspond to model predictions for different values of the parameter R defined in the text. The two symmetric upper solid lines represent the experimental limits (at 2σ level) from the present ρ measurements and the lower solid line is the constraint from NC data.

Fig. 13. Predictions of the $SU(3)_L \times U(1)_X$ and E_6 models on the $n - p$ plane at the Z° peak at LEP are illustrated. These variables are proportional to the $Z - Z'$ mixing. The dot-dashed square indicates the expected 1% experimental resolution.

Fig. 14. Expected number of e^+e^- events versus the Z_2 mass M_2 at SPS+Acol (full line) and TEVATRON (dashed line) are indicated. A $10\,pb^{-1}$ integrated luminosity is assumed.

Fig. 15. Expected number of e^+e^- events versus the Z_2 mass M_2 at LHC (full line) and SSC (dashed line) are illustrated. A $10^4\,pb^{-1}$ integrated luminosity is assumed.

FNAL (E531 Collab.) - P.R.L. 57, 2897, (1986)

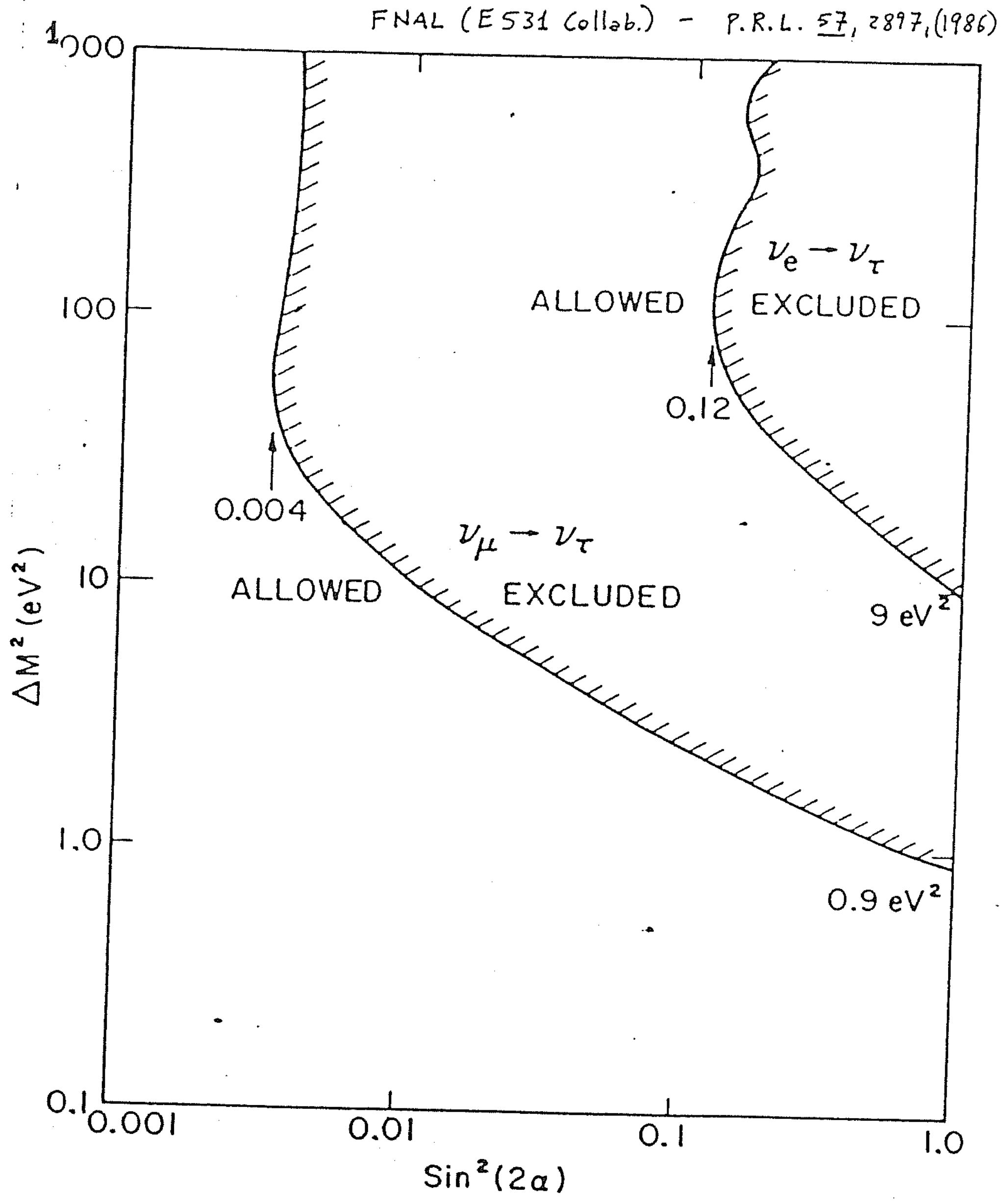


Fig. 1

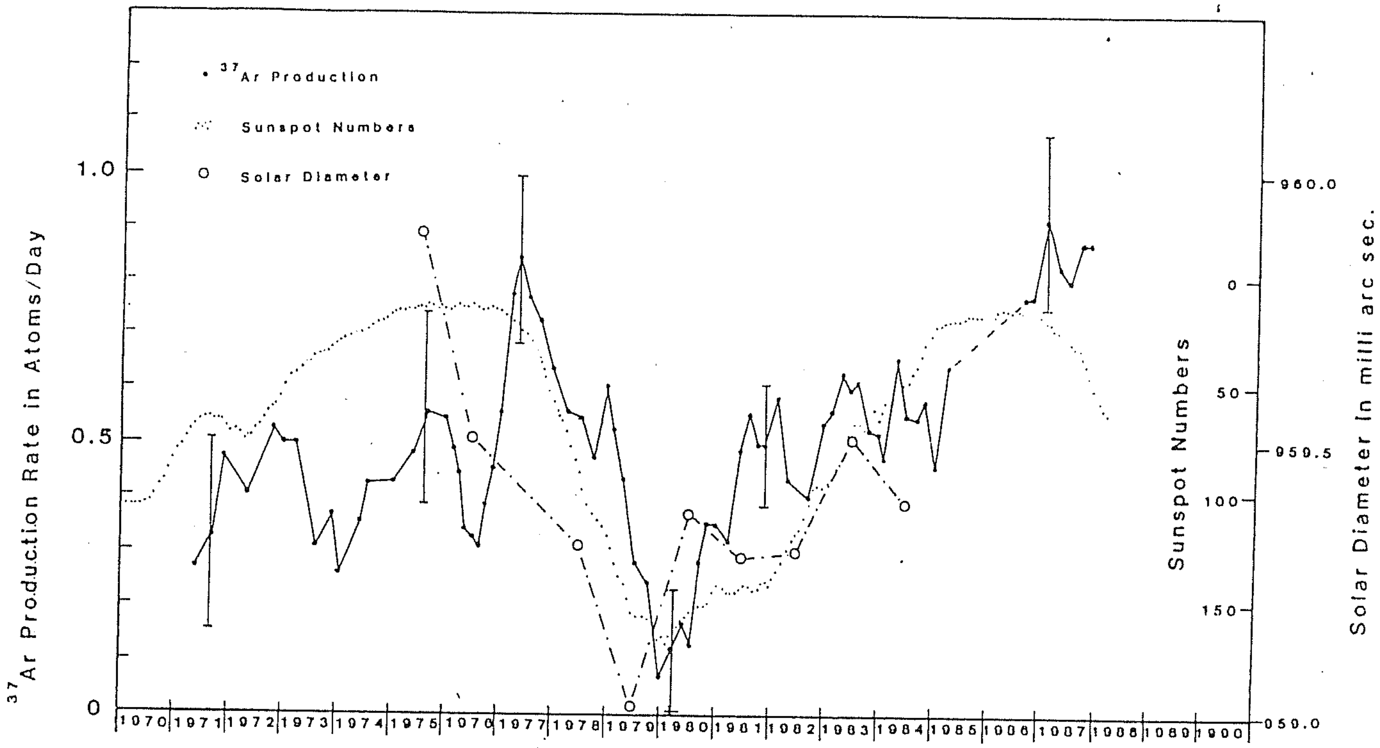


Fig. 2

MONTHLY MEAN ZURICH SUNSPOT NUMBERS
January 1944 – March 1983

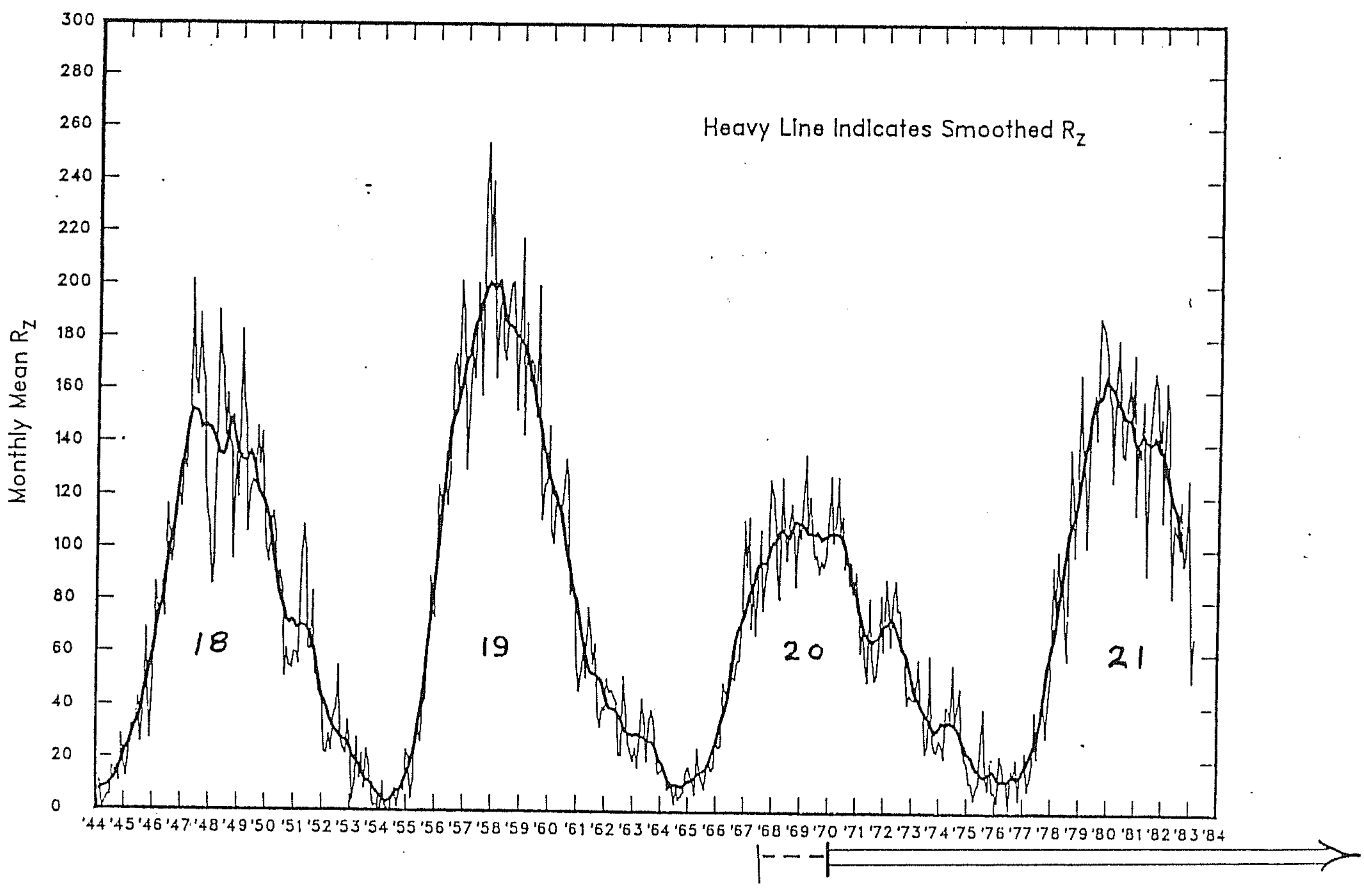


Fig. 3

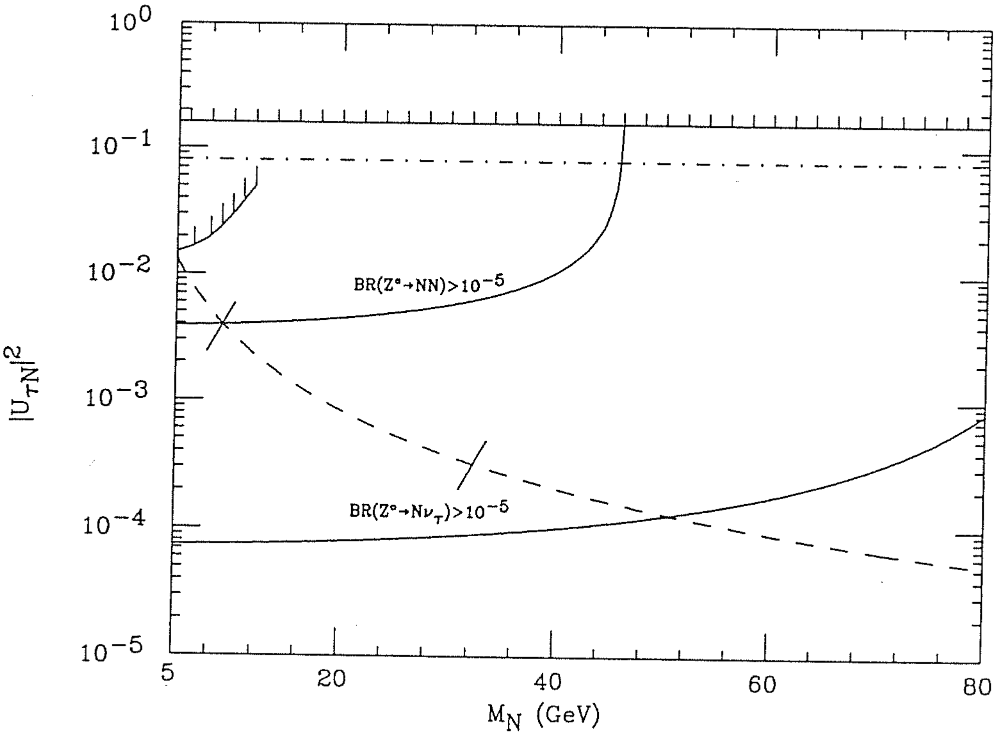

10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$|U_{\tau N}|^2$
BR(Z°→NN)>10^{-5}
BR(Z°→Nν_τ)>10^{-5}
5
20
40
60
80
M_N (GeV)

Fig. 4

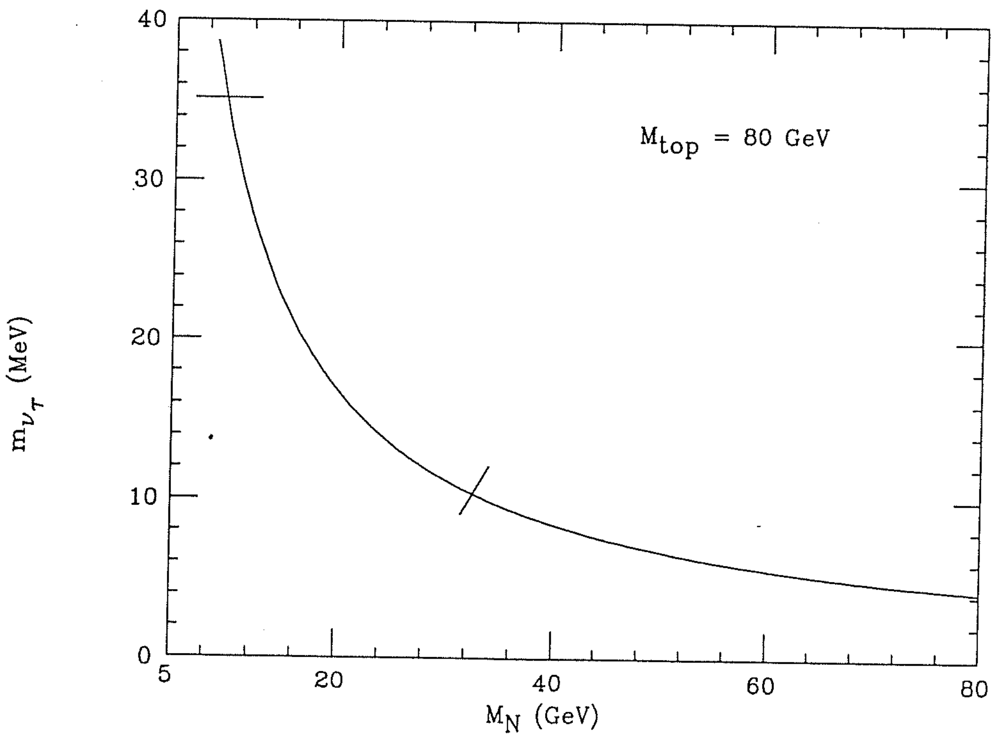
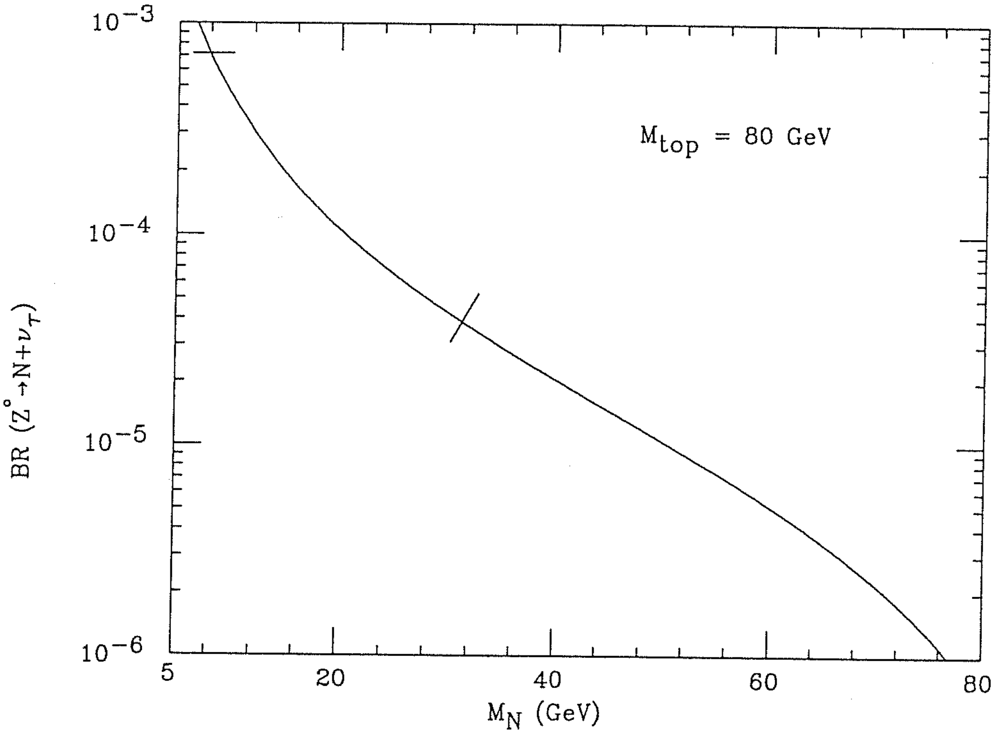
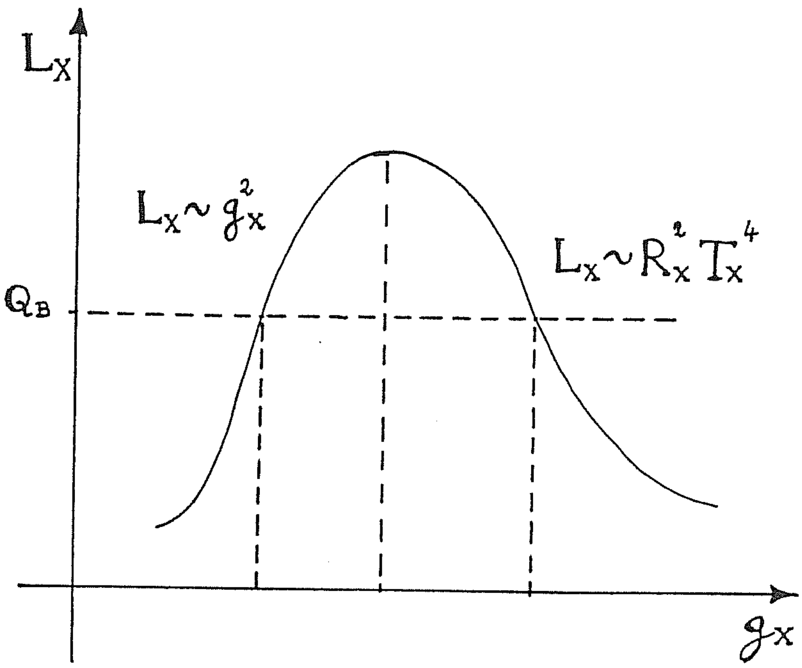

40
30
20
10
0
m_{ν_τ} (MeV)
M_{top} = 80 GeV
5
20
40
60
80
M_N (GeV)

Fig. 5



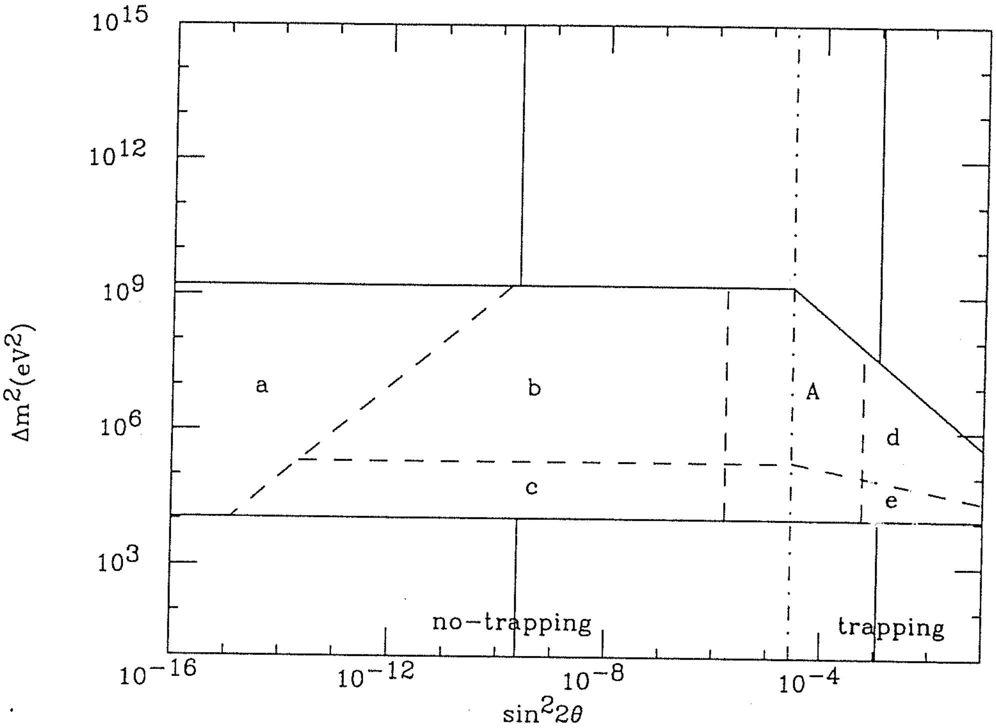
10^{-3}
10^{-4}
10^{-5}
10^{-6}
BR ($Z^{\circ} \to N+\nu_{\tau}$)
M_{top} = 80 GeV
5
20
40
60
80
M_N (GeV)

Fig. 6



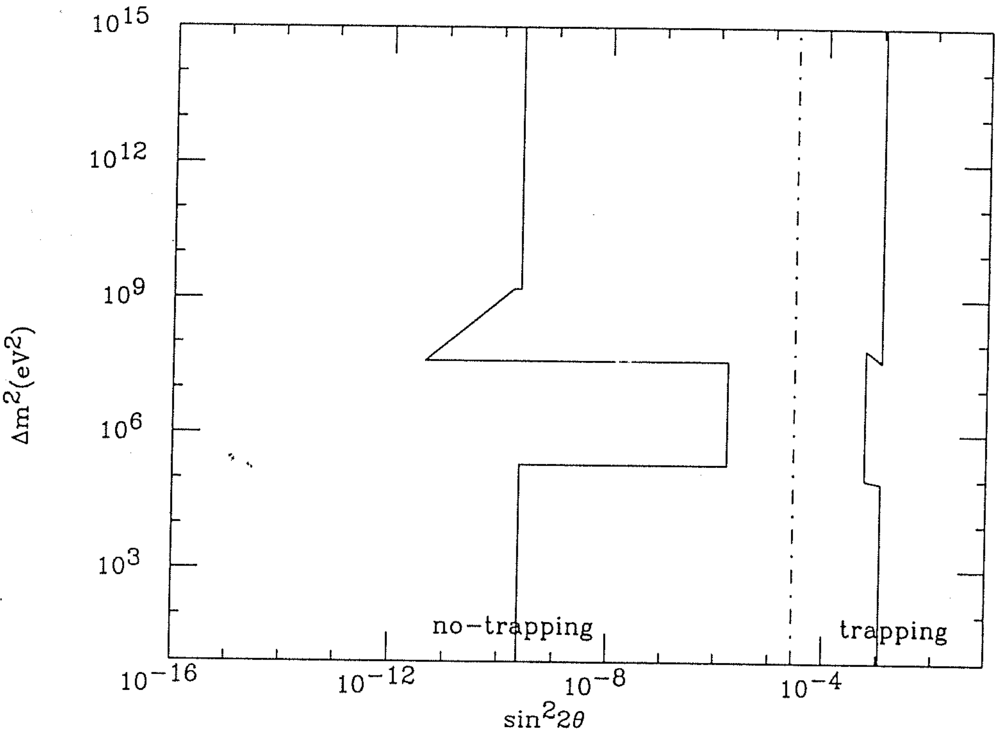
L_X
$L_X \sim g_X^2$
$L_X \sim R_X^2 T_X^4$
Q_B
g_X

Fig. 7



10^{15}
10^{12}
10^{9}
10^{6}
10^{3}
$\Delta m^2 (eV^2)$
a
b
A
d
c
e
no−trapping
trapping
10^{-16}
10^{-12}
10^{-8}
10^{-4}
$\sin^2 2\theta$

Fig. 8



10^{15}
10^{12}
10^{9}
10^{6}
10^{3}
$\Delta m^2 (eV^2)$
no−trapping
trapping
10^{-16}
10^{-12}
10^{-8}
10^{-4}
$\sin^2 2\theta$

Fig. 9

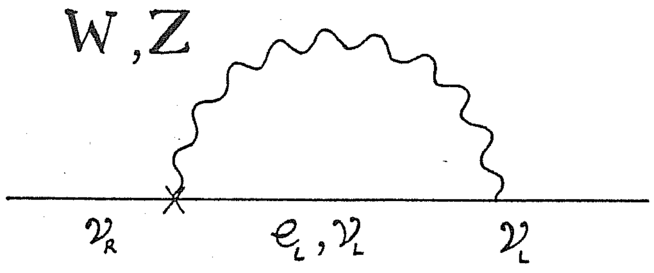

W,Z
ν_R
e_L, ν_L
ν_L

Fig. 10

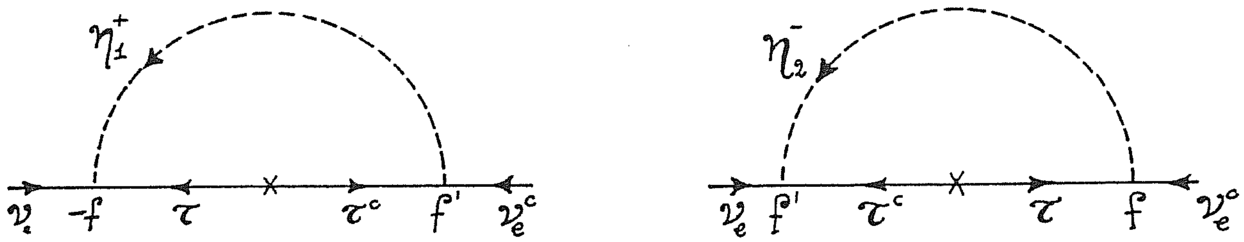

η_1^+
ν_e $-f$ τ τ^c f' ν_e^c
η_2^-
ν_e f' τ^c τ f ν_e^c

Fig. 11

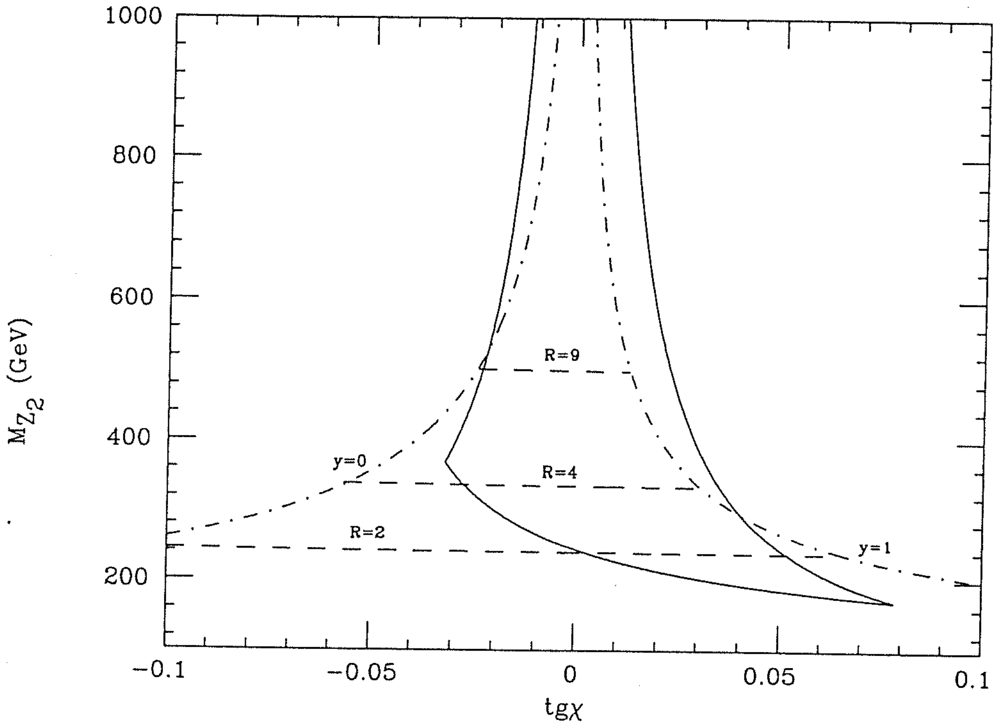

1000
800
600
400
200
M_{Z_2} (GeV)
R=9
R=4
R=2
y=0
y=1
−0.1
−0.05
0
0.05
0.1
tgχ

Fig. 12

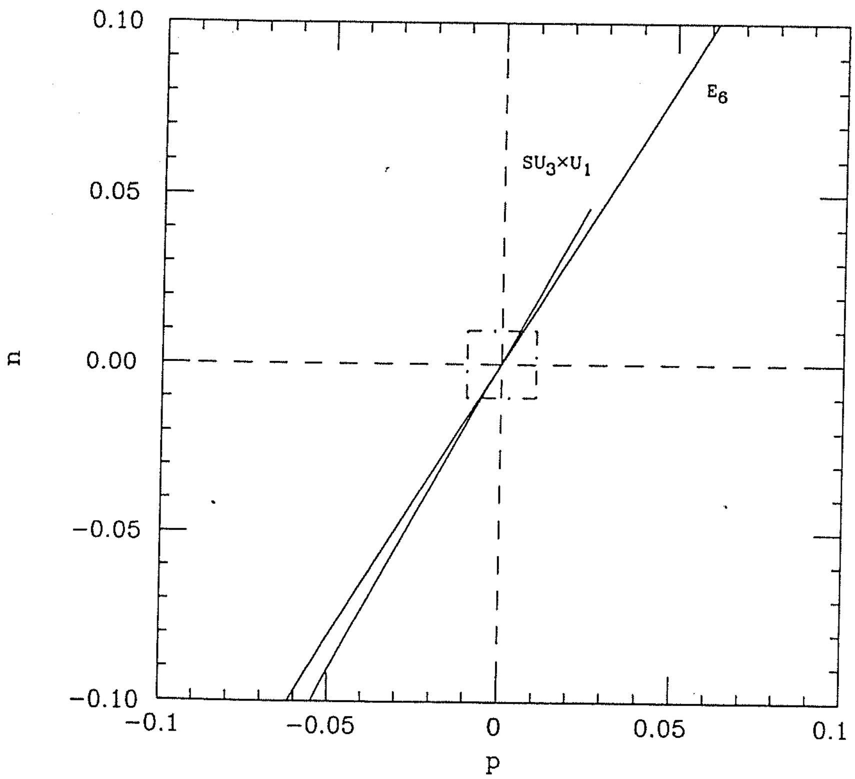
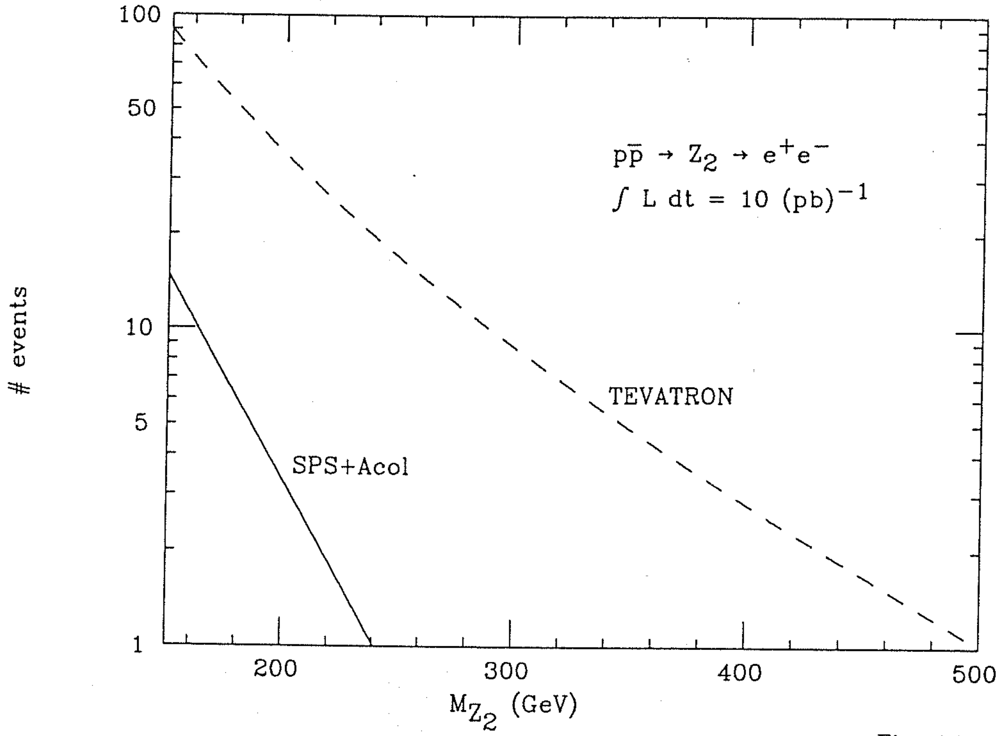
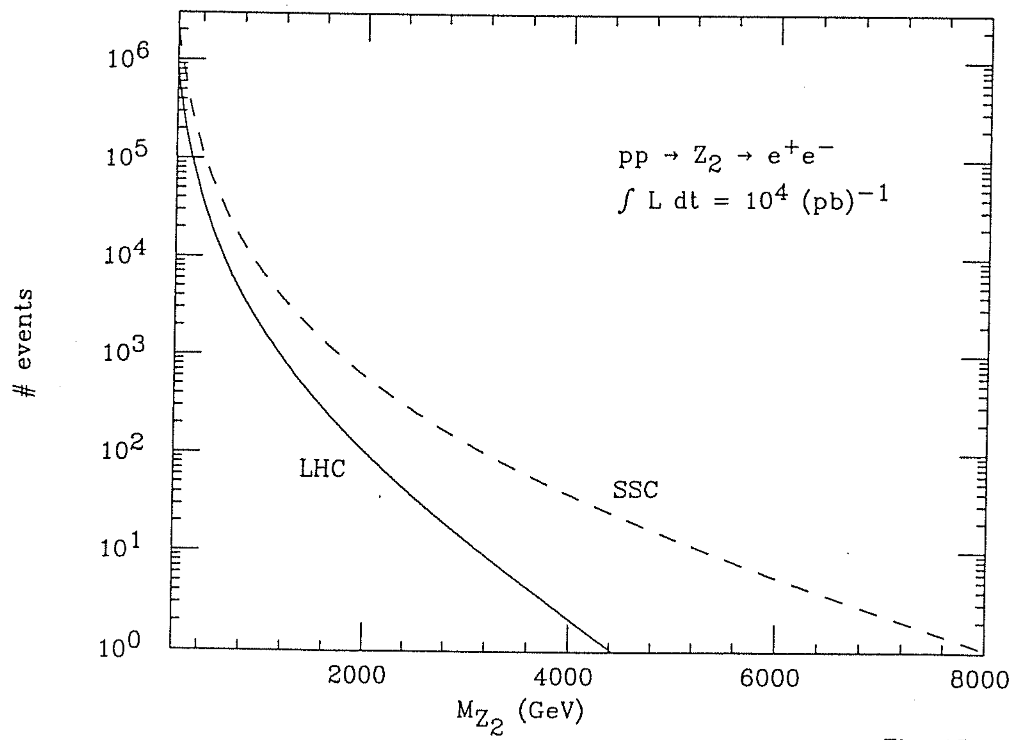

0.10
0.05
0.00
−0.05
−0.10
n
E_6
$SU_3 \times U_1$
−0.1
−0.05
0
0.05
0.1
p

Fig. 13



100
50
10
5
1
events
$p\bar{p} \rightarrow Z_2 \rightarrow e^+e^-$
$\int L\,dt = 10\ (pb)^{-1}$
TEVATRON
SPS+Acol
200
300
400
500
M_{Z_2} (GeV)

Fig. 14



10^6
10^5
10^4
10^3
10^2
10^1
10^0
events
$pp \rightarrow Z_2 \rightarrow e^+e^-$
$\int L\,dt = 10^4\ (pb)^{-1}$
LHC
SSC
2000
4000
6000
8000
M_{Z_2} (GeV)

Fig. 15

References

[1] G. F. Giudice, F. Giuliani and S. Ranfone, Phys. Lett. **B212** (1988) 181.

[2] B. G. Marsden, IAU Circular 4316 (1987).

[3] F. Giuliani and S. Ranfone, Pisa Preprint IFUP-TH 30/88 (1988), in press at Nucl. Phys. B.

[4] M. B. Voloshin, M. I. Vysotski and L. B. Okun, Sov. J. Nucl. Phys. **44** (1986) 440;
M. B. Voloshin and M. I. Vysotski, Sov. J. Nucl. Phys. **44** (1986) 544.

[5] R. Barbieri and R. N. Mohapatra, Phys. Lett. **218B** (1989) 225.

[6] F. Giuliani and B. Alles, Pisa Preprint IFUP-TH 10/89 (1989).

[7] 23^{rd} Yamada Conf., Japan, June 1989;
R. Abela et al., Phys. Lett. **146B** (1984) 431;
H. Albrecht et al., ARGUS Collab., Phys. Lett. **202B** (1988) 149.

[8] V. A. Lubimov et al., Phys. Lett. **94B** (1980) 266;
S. Boris et al., Phys. Lett. **159B** (1985) 217;
S. Boris et al., Phys. Rev. Lett. **58** (1987) 2019.

[9] D. O. Caldwell et al., Phys. Rev. Lett. **54** (1985) 281;
D. O. Caldwell et al., Phys. Rev. Lett. **59** (1987) 419.

[10] M. Gell-Mann, P. Ramond and R. Slansky, Supergravity Workshop at Stony Brook, ed. P. van Nieuwenhuizen (1979);
T. Yanagida, Proceedings of the Workshop on the Barion number of the Universe and Unified Theories, Tsukuba, ed. O. Sawada and A Sugamoto (1979).

[11] M. Magg and Ch. Wetterich, Phys. Lett. **94B** (1980) 61;
R. Barbieri, Erice Proc. on Unification of the Fundamental Particle Interactions, ed. A. Zichichi (1980).

[12] R. Barbieri and D. V. Nanopoulos, Phys. Lett. **95B** (1980) 43;
R. Barbieri, A. Masiero and D. V. Nanopoulos, Phys. Lett. **104B** (1981) 194.

[13] F. Giuliani, Master Thesis, ISAS (1985);
F. Giuliani and F. Strocchi, Phys. Lett. **154B** (1985) 48;
F. Giuliani and F. Strocchi, Phys. Lett. **177B** (1986) 82.

[14] G. Zacek et al., Phys. Rev. **D34** (1986) 2621.

[15] J. F. Cavaignac et al., Phys. Lett. **148B** (1984) 387.

[16] A. Gautnier, Proc. XXIII Int. Conf. on High Energy Physics, Berkeley (1986).

[17] K. Fujikawa and R. E. Shrock, Phys. Rev. Lett. **45** (1980) 963.

[18] J. E. Kim, Phys. Rev. **D14** (1976) 3000;
R. E. Shrock, Nucl. Phys. **B206** (1982) 359.

[19] M. Fukugita and T. Yanagida, Phys. Rev. Lett. **58** (1987) 1807;
R. Barbieri, R. N. Mohapatra and T. Yanagida, Phys. Lett. **213B** (1988) 69.

[20] K. Babu and R. N. Mohapatra, Maryland Preprint MDDP-PP-163 (1989).

[21] A. V. Kyuldjiev, Nucl. Phys. **B243** (1984) 387;
K. Abe et al., Phys. Rev. Lett. **58** (1987) 636.

[22] M. Fukugita and S. Yazaki, Phys. Rev. **D36** (1987) 3817.

[23] J. A. Morgan, Phys. Lett. **102B** (1981) 247.

[24] R. Barbieri and R. N. Mohapatra, Phys. Rev. Lett. **61** (1988) 27.

[25] R. Barbieri, Pisa Preprint IFUP-TH 21/89, to appear in a festschrift in honour of L. Radicati (1989).

[26] G. Gelmini, Neutrinos, ed. H. V. Klapdor (1988).

[27] K. A. Olive et al., Ap. J. **246** (1981) 557.

[28] J. Yang et al., Ap. J. **281** (1984) 493;
G. Steigman et al., Phys. Lett. **176B** (1986) 33.

[29] R. Cowsik and J. McClelland, Phys. Rev. Lett. **29** (1972) 669.

[30] A. H. Guth, Phys. Rev. **D23** (1981) 347.

[31] D. H. Rogstad and G. S. Shostak, Ap. J. **176** (1972) 315;
V. C. Rubin et al., Ap. J. **261** (1982) 439;
V. C. Rubin et al., Ap. J. **289** (1985) 81.

[32] S. M. Faber and J. S. Gallagher, Ann. Rev. Astr. Ap. **17** (1979) 135.

[33] R. Cowsik and J. McClelland, Ap. J. **180** (1973) 7;
D. N. Schramm and G. Steigman, Ap. J. **243** (1981) 1.

[34] D. A. Dicus, E. W. Kolb and V. L. Teplitz, Phys. Rev. Lett. **39** (1977) 168, 973E;
D. A. Dicus, E. W. Kolb and V. L. Teplitz, Ap. J. **221** (1978) 327.

[35] J. Silk and A. Stebbins, Ap. J. **269** (1983) 1.

[36] R. Cowisik, Phys. Rev. Lett. **39** (1977) 784;
S. W. Falk and D. N. Schramm, Phys. Lett. **79B** (1978) 511.

[37] M. S. Turner, G. Steigman and L. M. Krauss, Phys. Rev. Lett. **52** (1984) 2090;
G. Gelmini, D. N. Schramm and J. W. F. Valle, Phys. Lett. **146B** (1984) 311;
D. A. Dicus and V. L. Teplitz, Phys. Rev. **D34** (1986) 934.

[38] B. W. Lee and S. Weinberg, Phys. Rev. Lett. **39** (1977) 165.

[39] R. Davis, D. S. Harmer and K. C. Hoffman, Phys. Rev. Lett. **20** (1968) 1205.

[40] J. N. Bahcall et al., Ap. J. Lett. **292** (1985) L79.

[41] M. Koshiba, Accademia dei Lincei (1989), to appear in Phys. Rev. Lett..

[42] S. P. Mikheyev and A. Yu. Smirnov, N. Cim. **9C** (1986) 17.

[43] P. Langacker et al., Nucl. Phys. **B282** (1987) 589.

[44] S. J. Parke, Phys. Rev. Lett. **57** (1986) 1275.

[45] H. A. Bethe, Phys. Rev. Lett. **56** (1986) 1305.

[46] S. P. Rosen and S. M. Gelb, Phys. Rev. **D34** (1986) 969.

[47] D. N. Schramm, Comm. Nucl. Part. Phys. **17** (1987) 239.

[48] K. Hirata et al., Phys. Rev. Lett. **58** (1987) 1490.

[49] R. Bionta et al., Phys. Rev. Lett. **58** (1987) 1494.

[50] M. Aglietta et al., Europhys. Lett. (1987).

[51] A. Pomansky, Proc. XXII Rencontre de Moriond (1987).

[52] M. S. Turner, Phys. Rev. Lett. **60** (1988) 1797.

[53] R. Barbieri and R. N. Mohapatra, Phys. Rev. **D39** (1989) 1229.

[54] R. Johnson, S. Ranfone and J. Schechter, Phys. Lett. **179B** (1986) 355;
R. Johnson, S. Ranfone and J. Schechter, Phys. Rev. **D35** (1987) 282;
M. Gronau et al., Phys. Rev. **D37** (1988) 2597.

[55] A. Sirlin, Phys. Rev. **D35** (1987) 3423.

[56] F. J. Gilman and S. H. Rhie, Phys. Rev. **D32** (1985) 324.

[57] I. I. Bigi, SLAC Preprint SLAC-PUB-4439 (1987).

[58] S. L. Glashow, Phys. Lett. **187B** (1987) 367.

[59] E. Ma and J. Okada, Phys. Rev. Lett. **41** (1978) 287;
K. J. Gaemers, R. Gastmans and R.M. Renard, Phys. Rev. **D19** (1979) 1605;
G. Barbiellini, B. Richter and J. Siegrist, Phys. Lett. **106B** (1981) 414;
M. Caffo, R. Gatto and E. Remiddi, Nucl. Phys. **B286** (1987) 293.

[60] S. H. Rhie, SLAC Preprint SLAC-PUB-4270 (1987).

[61] S. Dimopoulos and L. J. Hall, Berkeley Preprint LBL-24738 (1988).

[62] R. Barbieri et al.,, Phys. Lett. **195B** (1987) 500;
R. Barbieri et al., Nucl. Phys. **B296** (1988) 75.

[63] M. Gronau, R. Johnson and J. Schechter, Phys. Rev. Lett. **54** (1985) 2176.

[64] E. W. Kolb, Proceedings of the 1986 Theoretical Advanced Studies Institute, Santa Cruz (1986).

[65] M. Fukugita, Kyoto Preprint RIFP-718 (1987).

[66] D. Notzold and G. Raffelt, Nucl. Phys. **B307** (1988) 924.

[67] D. Notzold, Max-Planck Preprint MPI-PAE/PTh 15/88 Preprint (1988).

[68] R. Barbieri, private communication.

[69] M. B. Voloshin, M. I. Vysotski and L. B. Okun, Sov. Phys. JETP **64** (1986) 446.

[70] R. Barbieri and G. Fiorentini, Nucl. Phys. **B304** (1988) 909.

[71] M. B. Voloshin, ITEP Preprint ITEP-87-215 (1987).

[72] F. del Aguila, M. Quiros and F. Zwirner, Yellow CERN-87-07 vol. II (1987) 165;
G. Costa et al., Nucl. Phys. **B297** (1988) 244.

[73] Particle Data Group, M. Aguilar-Benitez et al., Phys. Lett. **B204** (1988) 1.

[74] E. Cohen et al., Phys. Lett. **B165** (1985) 76.

[75] F. Boudjema, F. M. Renard and C. Verzegnassi,CERN Preprint CERN-TH.5049/88 (1988);
F. M. Renard and C. Verzegnassi, CERN Preprint CERN-TH.5204/88 (1988).

[76] S. D. Drell and T. M. Yan, Ann. Phys. **66** (1971) 578.

[77] F. del Aguila, M. Quiros and F. Zwirner, Nucl. Phys. **B284** (1987) 530.

[78] D. W. Duke and J. F. Owens, Phys. Rev. **D30** (1984) 49.