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1. INTRODUCTION

In the past year or so, the topic of supersymmetric theories in the background of four-dimensional anti-de Sitter space (AdS_4) has received much attention. The main reason is found in the occurrence of AdS_4 as a ground state solution of both grand unified theories and supergravity. In the case of gauged extended supergravities, for exemple, one can show that both the vacuum configuration and the perturbative dynamics in this background are invariant under the supersymmetry group OSp(N,4).[1] In this contest the simplest supersymmetric invariant theory, i. e. the generalization of the Wess-Zumino model to the fixed background of AdS_4, plays a central role.[2] In fact, the massless version of the model provides the small fluctuation action, while the massive scalar supermultiplet occurs in the coupling of supersymmetric grand unified theories to gravity.[1]

AdS_4 is not a globally hyperbolic space. Thus, the Cauchy problem and the definition of a quantum field theory require to impose boundary conditions, whose role is to control the flow of information through a timelike surface at spatial infinity, determining the convergence and conservation properties of the charges. Also, the possible vacua and therefore the two-point functions are determined by these boundary conditions.[3] The fully general and exhaustive study of the quantum properties of the theory of Refs[4,15] deals with the fixed background in an exact fashion, accounting for the choice of so-called reflective boundary conditions in solving the propagator equations in AdS_4. This reflective choice guarantees the conservation of the supersymmetry charges at spatial infinity.[1]

Quite naturally the boundary conditions are not expected to affect the calculation of the counterterms. This can be proven formally, as in Ref.[5]. In carrying the renormalization program one can resort to some kind of approximation, dealing with the effects of the background but neglecting the choice of boundary conditions imposed by supersymmetry invariance. The use of the adiabatic expansion of DeWitt-Schwinger, rigorously justified in Refs.[4,5], first allowed to renormalize supersymmetric theories in AdS_4. After the pioneer work of Refs.[6,7] , the task was accomplished for the theory quantized around one particular vacuum state by the independent work of Refs.[8,9]. Soon afterwards the independence of the renormalization coefficients from the choice of the vacuum was proven.[10] In Ref.[7] the validity of the no-renormalization theorem for the Wess-Zumino model in AdS_4 was investigated. Besides testing the dimensional reduction prescription as a supersymmetric regularization method to the one-loop order, the absence of renormalization for the mass and interaction lagrangian was extended to the model in this non-trivial background space. However, a first attempt to compute all the local contributions to the effective action to the one-loop order by the same dimensional regularization procedure brought some doubts about its reliability in a two-loop computation [6]. These questions related to finite corrections at the one-loop level were answered subsequently in Ref.[11] by a careful study of the limits of validity fo the adiabatic expansion for massless theories.

By the above reasons, recovering the no-renormalization of the mass and interaction lagrangians to the one-loop order by means of a regulator that explicitely preserves supersymmetry turns out to be a significant task. This was accomplished in Ref.[9],where the Pauli-Villars regularization method was applied to the computation of the 1PI two and three-point functions of the model. At the same time, the use of a regularization procedure that explicitely preserves supersymmetry opened the possibility of a deeper understanding of other interesting features of the model. We refer at this point to the non-vanishing of the one-point functions for the scalar and auxiliary fields of the model at the one-loop level. Because of this fact, a linear superfield insertion in the action is needed in order to make the theory renormalizable. Although the classical potential of the model results then modified in a way that does not induce the breaking of supersymmetry, the need for this insertion represents a violation of the no-renormalization theorem. On the other hand, the fact that the superspace integral thatgets renormalized is mathematically of the same type as the integrals of the quadratic and cubic superfield terms in the action opened the possibility of a similar violation of the theorem at higher loops for the mass and interaction lagrangians of the model.

The important fact that the reflective boundary conditions do not play any role in the calculation of the counterterms enables to implement a perturbative treatment of the effects of the background space, by exploiting the partial superconformal invariance of the model, as in Refs. [12,13] , where the superfield formulation of the model with interacting chiral and real gauge superfields in AdS_4 is consider ed, following the line suggested in Ref. [14]. This perturbative analysis, although completely independent from the DeWitt-Schwinger expansion, allowed to recover in Ref. [12] the results of the one-loop renormalization of the chiral self-interacting model . Implementing the abovementioned perturbative treatment enabled to analyze in Ref. [13], to any order in the loop expansion, the generation of a mass or interaction counterterm. The conclusion is that there is no renormalization of the mass nor of the cubic interaction action to all orders in perturbation theory and that the theorem holding in flat space-time is also valid in AdS_4 background except for the linear superfield renormalization, already present at the one-loop level. The proof is easily extended to the model with gauge interactions.

The presentation of the results in the present work makes close reference to the chronological order of the discoveries, for the sake of clarity in the exposition, as well as completeness. In chapter 2 we apply the renormalization program to the massless Wess-Zumino model in AdS_4 background, using the

version of Bunch and Parker of the adiabatic expansion. 16
This enables us to evaluate the local one-loop quantum corrections (both divergent and finite) to the effective action. We use an extrapolation of the dimensional reduction prescription taking only the momentum of the expansion in D dimensions while all the other tensors are considered in D = 4. We show that the Ward identities are satisfied by the divergent one-loop corrections but are violated by finite local terms proportional to the curvature of the space, indicating that a more appropriate choice of the regularization prescription is needed in order to maintain supersymmetry for these terms. Then, the adiabatic expansion of the free-field propagators is implemented together with a manifestly supersymmetric Pauli-Villars prescription and found to be inadequate to compute finite quantum corrections to the effective action.

Next, we calculate the 1PI two and three-point functions of the massive model and check that our dimensional reduction prescription preserves supersymmetry for the counterterms. It turns out that the no-renormalization theorem valid in flat space-time still holds in the AdS_4 background space, for what concerns the mass and the interaction lagrangians. The common renormalization factor that, multiplied times the kinetic action, gives the effective action is calculated. Then, we study the renormalization properties of the model in the adiabatic approximation, using supersymmetric Pauli-Villars regularization. We find that the one-loop effects proportional to the contraction parameter of the space force the insertion of a linear superfield term in the superpotential in order to renormalize the non-vanishing 1PI one-point functions of the model. We discuss the implications of this violation of the no-renormalization theorem induced by the fixed background space.

In chapter 3 we perform the computation of the effective potential of the Wess-Zumino model to the one-loop order, by making use of the adiabatic expansion in the effective action of the model. This enables us to obtain the renormalization coefficients of the theory and to prove that they are independent from the choice of a particular extremum of the potential. In spite of the fact that the abovementioned linear superfield insertion is needed for renormalization, we show that supersymmetry is preserved to the one-loop order at each of the four classically supersymmetric extrema of the effective potential.

The fully general analysis of the quantum properties of the theory, dealing with the fixed background in an exact fashion, is reported in chapter 4. We show that the expansion in powers of the contraction parameter of AdS_4 of the exact scalar and spinor propagators reproduces the standard adiabatic expansion up to an order beyond which no contribution to divergent counterterms can be expected, dismissing the claim about the role played by boundary conditions in the renormalization of supersymmetric models. We come back to consider more carefully massless fields in AdS_4 and find that a supersymmetric mass term is produced radiatively. We discuss its implications in terms of the breakdown of the no-renormalization theorem, as well as the modification induced in the pattern of chiral symmetry breaking in AdS_4. Next, we compute the one-loop effective potential, determining the effect of the background geometry exactly and disentangling the effects produced by the supersymmetric boundary conditions on the quantum corrections to the classical theory. After the renormalization of the kinetic action and the insertion of the linear counterterm in the superpotential, we solve the quantum corrected equations of motion, obtaining the vacuum solutions in the semiclassical approximation. The vacuum expectation values of the A and B fields are shifted by finite terms which depend upon the boundary conditions for the field propafators. Despite this result, we show with complete generality that supersymmetry is preserved to the one-loop order at each classically supersymmetric extremum of the effective potential. The use of two independent regularization procedu re, i.e. dimensional reduction and point-splitting method, both preserving the supersymmetry invariance of the theory, guarantees the correctness of the results, which naturally agree. We conclude this chapter 4 with the renormalization of the pure gravitational tensors and the related calculation of the trace anomaly of the stress-tensor.

In the second part of the present work we adopt an approximated approach, neglecting the effects of the reflective boundary conditions. This is justified, since we want to carry the analysis of the divergent counterterms to higher loops.[5] We consider in chapter 5 the Wess-Zumino model in the superfield formalism. The partial superconformal invariance of the model is used to trea perturbatively the effects of the background curvature. This turns out to be enough for the purpose of renormalization and allows us to analyze the possibility of the breakdown of the no-renormalization theorem to the one-loop order.

In chapter 6 we consider the interacting model of chiral and real gauge superfields . The superconformal invariance of the massless model allows us, once again, to implement an expansion in the curvature effects in terms of the interaction verices of the quantum model. Then, simple power counting arguments suffice to prove the no-renormalization of the mass and chiral self-interaction actions to all orders in perturbation theory.

The notation used throughout this work, as well as details on the calculation of the fermionic contributions to the corrections presented in chapter 2, are given in appendix A. Appendix B contains some details about the calculation of the Pauli-Villars regularized integrals of chapter 2.

2. ONE-LOOP ORDER RENORMALIZATION OF THE WESS-ZUMINO MODEL

2.1. The Wess-Zumino model in AdS_4

It is known that the symmetry group O(3,2), which can be realized as the isometry group of AdS_4, admits as its supersymmetric extension the supersymmetry group OSp(1,4). This leads to a natural formulation of supersymmetric quantum theories in the curved background space of AdS_4. In the corresponding superspace OSp(4,1)/O(3,1), the spinorial extension of AdS_4 space O(3,2)/O(3,1), the realization of the supersymmetry group is obtained in the simplest way introducing a chiral superfield $\Phi_+(x,\theta)$ [2]

$$\Phi_\pm(x,\theta) = T^\pm(x,\theta_+)$$

$$T^\pm(x,\theta_+) = A_\pm(x) + \bar\theta_\pm \psi_\pm(x) + \frac{1}{2}\bar\theta_\pm\theta_\pm F_\pm(x) \qquad (2.1.1)$$

where $\theta_+ = \frac{1}{2}(1+\gamma_5)\theta$. In eq.(2.1.1) we define

$$\Phi_-(x,\theta) = \left[\Phi_+(x,\theta)\right]^*$$

We start considering the simplest supersymmetric theory in AdS_4, the Wess-Zumino model, obtained from the chiral supermultiplet in eq.(2.1.1). The action of the self-interacting model can be built up in the supersymmetric invariant form

$$\begin{aligned} S = &\int \mathcal{D}M\, \Phi_+(x,\theta)\Phi_-(x,\theta) \\ &+ \frac{1}{2}m\left[\int \mathcal{D}M^L\, T^{+2}(x_L,\theta_L) + \int \mathcal{D}M^R\, T^{-2}(x^R,\theta^R)\right] \\ &+ \frac{\sqrt{2}}{3}\lambda\left[\int \mathcal{D}M^L\, T^{+3}(x_L,\theta^L) + \int \mathcal{D}M^R\, T^{-3}(x^R,\theta^R)\right] \end{aligned} \qquad (2.1.2)$$

In eq.(2.1.2) the chiral basis $\begin{pmatrix} x_\mu^L \\ \theta^L \end{pmatrix}$ is given by

$$\begin{pmatrix} x_\mu^L \\ \theta^L \end{pmatrix} = \begin{pmatrix} \left[1+\frac{a}{16(-g)^{1/8}}(\bar\theta\theta)\right]x_\mu - \frac{1}{4}\bar\theta\gamma_\mu\gamma_5\theta\frac{1}{(-g)^{1/8}} \\ \theta_+ - \frac{1}{2}a\,\bar\theta\theta\left(\theta_+ + \frac{3}{4}ia\,x\cdot\gamma\theta_-\right) \end{pmatrix} \qquad (2.1.3)$$

The right-handed basis is defined as

$$\begin{pmatrix} x_\mu^R \\ \theta^R \end{pmatrix} = \begin{pmatrix} (x_\mu^L)^* \\ C\bar\theta^{LT} \end{pmatrix}$$

and results from eq.(2.1.3) simply by making the change $\theta_+ \leftrightarrow \theta_-$. The different measures of integration in the action (2.1.2) can be expressed in the form

$$\mathcal{D}M = d^4x\, d^4\theta\sqrt{-g}\left[1+\frac{3}{2}a\bar\theta\theta + \frac{3}{8}a^2(\bar\theta\theta)^2\right]$$

$$\mathcal{D}M^L = d^4x\, d^2\theta^L\sqrt{-g}\left(1+\frac{3}{2}a\,\bar\theta^L\theta^L\right)$$

$$\mathcal{D}M^R = d^4x\, d^2\theta^R\sqrt{-g}\left(1+\frac{3}{2}a\,\bar\theta^R\theta^R\right)$$

Going to real components A, B, F, G

$$A_+ = (A+iB)/\sqrt{2} \qquad , \qquad F_+ = (F+iG)/\sqrt{2}$$

one gets the action of the model as the sum of the three pieces

$$S_{kin} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(D_\mu A D^\mu A + D_\mu B D^\mu B + i\bar\Psi\not{D}\Psi + F^2 + G^2\right) + a(AF+BG) + \frac{3}{2}a^2(A^2+B^2)\right]$$

$$S_m = \int d^4x\sqrt{-g}\; m\left[AF - BG + \frac{3}{2}a(A^2-B^2) - \frac{1}{2}\bar\Psi\Psi\right]$$

$$S_{int} = \int d^4x\sqrt{-g}\;\lambda\left[A^2F - B^2F - 2ABG + a(A^3-3AB^2) - \bar\Psi(A+i\gamma_5 B)\Psi\right] \qquad (2.1.4)$$

The invariance of each one of these pieces under the su=

persymmetry transformations

$$\delta A = \bar{\varepsilon}\psi \quad , \quad \delta B = -i\bar{\varepsilon}\gamma_5\psi$$
$$\delta\bar{\Psi} = i\bar{\varepsilon}\not{\partial}(A+i\gamma_5 B) + \bar{\varepsilon}(F+i\gamma_5 G)$$
$$\delta F = -i\bar{\varepsilon}\not{D}\psi - a\bar{\varepsilon}\psi \; , \; \delta G = \bar{\varepsilon}\gamma_5\not{D}\psi + ia\bar{\varepsilon}\gamma_5\psi \qquad (2.1.5)$$

is implied by the superfield formulation of the model. In the above eqs.(2.1.4), (2.1.5) A, B, F, G are real scalar fields and ψ is a Majorana spinor. The contraction para= meter a has dimension $[\text{length}]^{-1}$ and the Killing spinor $\varepsilon(z)$ is defined by

$$D_\mu \varepsilon(z) = \frac{-ia}{2}\gamma_\mu \varepsilon(z)$$

Our purpose in this chapter is to compute the one-loop divergent quantum corrections to the model. The fact that in the above formulation supersymmetry is realized off-shell guarantees that the only possible infinite renorm= alizations must have the form of a global factor for each one of the supersymmetry invariant pieces S_{kin}, S_m and S_{int}. Thus, let us say that in the process of renormali= zation the effective lagrangian of the model becomes

$$L \rightarrow Z_{kin} L_{kin} + Z_m L_m + Z_{int} L_{int} \qquad (2.1.6)$$

being the coefficients Z_{kin}, Z_m and Z_{int} completely in= dependent. We then can state that our first goal is to compute each one of these renormalization coefficients and, by looking in particular to Z_m and Z_{int}, to check if the no-renormalization of L_m and L_{int} still holds for the model formulated in the curved background space. In all these considerations the existence of a supersymme= tric regularization procedure is postulated. Actually, we should stress that its (or their, if several exist) identification among the supersymmetric regulators in use for the flat space-time models is a necessary condition for the computation of the renormalization factors Z's. We will come back to this matter below, where dimensional reduction is proposed as a valid supersymmetric regularization procedure at the one-loop level for the divergent quantum corrections.

The next step is the derivation of the Ward identity for the effective action of the model. We must allow for a generating functional depending on the respective sources J_A, J_B, η, J_F and J_G.

$$Z(J)=\int\mathcal{D}\phi\, \exp\left\{i\int dx\,\sqrt{-g}\left[\mathcal{L}(\phi)+J_A A+J_B B+\bar{\Psi}\eta+J_F F+J_G G\right]\right\}$$

where the J's are supposed to form a supermultiplet in order to mantain the invariance of the generating functional under supersymmetry transformation. The transformation rules are easily found to be

$$\begin{aligned}
\delta J_A &= -iD_\mu(\bar{\eta}\gamma^\mu\varepsilon) \quad , \quad \delta J_B = -iD_\mu(\bar{\eta}i\gamma_5\gamma^\mu\varepsilon)\\
\delta\eta &= -J_A\varepsilon + J_B i\gamma_5\varepsilon + i\not{D}(J_F\varepsilon) + aJ_F\varepsilon\\
&\quad + i\not{D}(J_G i\gamma_5\varepsilon) - aJ_G i\gamma_5\varepsilon\\
\delta J_F &= -\bar{\eta}\varepsilon \quad , \quad \delta J_G = -\bar{\eta}i\gamma_5\varepsilon
\end{aligned} \tag{2.1.7}$$

The Ward identity for the generating functional expresses its invariance under a supersymmetry transformation

$$\begin{aligned}
&-iD_\mu(\bar{\eta}\gamma^\mu\varepsilon)\frac{\delta Z}{\delta J_A} - iD_\mu(\bar{\eta}i\gamma_5\gamma^\mu\varepsilon)\frac{\delta Z}{\delta J_B} + \frac{\delta Z}{\delta\eta}\cdot\\
&\cdot\left[-J_A\varepsilon + J_B i\gamma_5\varepsilon + i\not{D}(J_F\varepsilon) + aJ_F\varepsilon + i\not{D}(J_G i\gamma_5\varepsilon)\right.\\
&\left.-aJ_G i\gamma_5\varepsilon\right] - \bar{\eta}\varepsilon\frac{\delta Z}{\delta J_F} - \bar{\eta}i\gamma_5\varepsilon\frac{\delta Z}{\delta J_G} = 0
\end{aligned} \tag{2.1.8}$$

and, obviously, a similar identity holds for the generating functional of the connected Green functions $W=-i \log Z(J)$.

By performing the usual Legendre transformation

$$\frac{\delta W}{\delta J_i} = \hat{\phi}_i \quad , \quad \Gamma(\hat{\phi}) = W - \int dx \sqrt{-g}\, J_i \hat{\phi}_i$$

and taking into account that $\frac{\delta \Gamma}{\delta \hat{\phi}_i} = -J_i$, one can translate the previous Ward identity into the corresponding expression holding for the effective action $\Gamma(\hat{\phi})$

$$\begin{aligned} & i\frac{\delta \Gamma}{\delta \hat{\bar{\psi}}} \gamma^\mu \varepsilon \partial_\mu \hat{A} + i\frac{\delta \Gamma}{\delta \hat{\bar{\psi}}} i\gamma_5 \gamma^\mu \varepsilon \partial_\mu \hat{B} - \frac{\delta \Gamma}{\delta \hat{A}} \hat{\bar{\psi}} \varepsilon + \frac{\delta \Gamma}{\delta \hat{B}} \hat{\bar{\psi}} i\gamma_5 \varepsilon \\ & - i \hat{\bar{\psi}} \overleftarrow{\nabla\!\!\!/} \varepsilon \frac{\delta \Gamma}{\delta \hat{F}} + a \hat{\bar{\psi}} \varepsilon \frac{\delta \Gamma}{\delta \hat{F}} - i \hat{\bar{\psi}} \overleftarrow{\nabla\!\!\!/} i\gamma_5 \varepsilon \frac{\delta \Gamma}{\delta \hat{G}} \\ & - a \hat{\bar{\psi}} i\gamma_5 \varepsilon \frac{\delta \Gamma}{\delta \hat{G}} - \frac{\delta \Gamma}{\delta \hat{\bar{\psi}}} \varepsilon \hat{F} - \frac{\delta \Gamma}{\delta \hat{\bar{\psi}}} i\gamma_5 \varepsilon \hat{G} = 0 \end{aligned} \qquad (2.1.9)$$

This represents nothing but the statement of the invariance of the effective action with respect to a supersymmetry transformation, once we have made the assumption that the classical fields $\hat{\phi}_i$ transform in the same way as their quantum counterparts.

By taking functional derivatives of the fundamental identity (2.1.9) we can obtain successive Ward identities relating the differnt n-point functions of the model. By deriving with respect to $\hat{\bar{\psi}}(y)$ and $\hat{A}(x)$, for exemple, and then setting all the sources equal to zero we obtain

$$\begin{aligned} & i\frac{\delta^2 \Gamma}{\delta \hat{\bar{\psi}}(y) \delta \psi(z)} \gamma^\mu \left[\partial_\mu \frac{1}{\sqrt{-g}} \delta(x-z)\right] \varepsilon(z) - \frac{\delta^2 \Gamma}{\delta \hat{A}(x) \delta \hat{A}(z)} \frac{1}{\sqrt{-g}} \delta(y-z) \varepsilon(z) \\ & - \frac{i\, \delta^2 \Gamma}{\delta \hat{A}(x) \delta \hat{F}(z)} \left[\frac{1}{\sqrt{-g}} \delta(y-z) \overleftarrow{\nabla\!\!\!/}\right] \varepsilon(z) + \frac{a\, \delta^2 \Gamma}{\delta \hat{A}(x) \delta \hat{F}(z)} \frac{1}{\sqrt{-g}} \delta(y-z) \varepsilon(z) = 0 \end{aligned} \qquad (2.1.10)$$

The expression (2.1.10) can be casted in a more conveni=

ent form by making use of integration by parts (after replacing the integral symbol)

$$\int dz \left\{ -i\bar{\varepsilon}(y) \frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}(y)\delta\hat{\psi}(z)} \gamma_\mu \varepsilon(z) \overleftarrow{D}_z^\mu \delta(x-z) - \frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{A}(z)} \bar{\varepsilon}(y)\varepsilon(z)\delta(y-z) \right.$$

$$\left. -i\bar{\varepsilon}(y)\gamma^\mu\varepsilon(z) D_\mu\left[\frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{F}(z)}\right]\delta(y-z) + 3a\frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{F}(z)}\bar{\varepsilon}(y)\varepsilon(z)\delta(y-z)\right\} = 0$$

In the above equation we have made use of the definition of $\varepsilon(z)$, $D_\mu\varepsilon(z) = -i\frac{a}{2}\gamma_\mu\varepsilon(z)$. We finally obtain

$$-i\bar{\varepsilon}(y)\frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}(y)\delta\hat{\psi}(x)}\gamma_\mu\varepsilon(x)\overleftarrow{D}_x^\mu + \left[\frac{-\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{A}(y)} + 3a\frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{F}(y)}\right]\bar{\varepsilon}(y)\varepsilon(y) = 0 \quad (2.1.11)$$

We notice explicitely that this expression is satisfied at the tree level in the operatorial sense, with

$$\frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}(y)\delta\hat{\psi}(x)} = i\not{D}_y\frac{1}{\sqrt{-g}}\delta(y-x)$$

$$\frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{A}(y)} = (-\Box_x + 3a^2)\frac{1}{\sqrt{-g}}\delta(x-y)$$

$$\frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{F}(y)} = a\frac{1}{\sqrt{-g}}\delta(x-y)$$

In fact, by applying eq.(2.1.11) to a test function $\phi(x)$ one gets

$$\int dx\sqrt{-g}\left\{-i\bar{\varepsilon}(y)i\left[\not{D}_y\frac{1}{\sqrt{-g}}\delta(y-x)\right]\gamma_\mu\varepsilon(x)\overleftarrow{D}_x^\mu\phi(x) + \phi(x)\left[\left(\Box_x - \right.\right.\right.$$

$$\left.\left.\left. - 3a^2\right)\frac{1}{\sqrt{-g}}\delta(x-y) + 3a^2\frac{1}{\sqrt{-g}}\delta(x-y)\right]\bar{\varepsilon}(y)\varepsilon(y)\right\}$$

and this expression vanishes identically after integration by parts.

By deriving once more the original Ward identity (2.1.9) with respect to $\hat{A}(u)$ we obtain a relationship among the three-point functions of the model

$$i\frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{\bar{\Psi}}(y)\delta\hat{\Psi}(z)}\gamma^\mu\left[\partial_\mu\frac{1}{\sqrt{-g}}\delta(u-z)\right]\varepsilon(z)+i\frac{\delta^2\Gamma}{\delta\hat{A}(u)\delta\hat{\bar{\Psi}}(y)\delta\hat{\Psi}(z)}\cdot$$
$$\cdot\frac{1}{\sqrt{-g}}\delta(x-z)\varepsilon(z)-\frac{\delta^2\Gamma}{\delta\hat{A}(u)\delta\hat{A}(x)\delta\hat{A}(z)}\frac{1}{\sqrt{-g}}\delta(y-z)\varepsilon(z)-\frac{i\,\delta^2\Gamma}{\delta\hat{A}(u)\delta\hat{A}(x)\delta\hat{F}(z)}\cdot$$
$$\cdot\left[\frac{1}{\sqrt{-g}}\delta(y-z)\overleftarrow{\not\partial}\right]\varepsilon(z)+\frac{a\,\delta^2\Gamma}{\delta\hat{A}(u)\delta\hat{A}(x)\delta\hat{F}(z)}\frac{1}{\sqrt{-g}}\delta(y-z)\varepsilon(z)=0 \qquad (2.1.12)$$

In the rest of this thesis we will focus our attention on the identity (2.1.10), or (2.1.11). It can be shown that indeed all the three-point functions of the model are identically zero and this makes eq. (2.1.12) trivially satisfied. Also, we will come back to considering the fundamental Ward identity (2.1.9) below, where we will pay attention to the one point functions of our theory.

Before going into the computation of the quantum corrections we take under consideration the classical solutions around which the quantum theory can be quantized. In this sense the structure of the extrema of the classical action, depending on the values of the parameters a and m, turns out to be extremely rich [2]. To proceed with this analysis we start considering the classical potential

$$V(A,B,F,G)=-\tfrac{1}{2}F^2-\tfrac{1}{2}G^2-a(AF+BG)-\tfrac{3}{2}a^2(A^2+B^2)$$
$$-mAF+mBG-\tfrac{3}{2}am(A^2-B^2)$$
$$-\lambda A^2F+\lambda B^2F+2\lambda ABG-\lambda a(A^3-3AB^2) \qquad (2.1.13)$$

After equating the auxiliary fields to their on-shell values

$$F=-aA-mA-\lambda A^2+\lambda B^2$$

$$G = -aB + mB + 2\lambda AB \qquad (2.1.14)$$

we have an expression for the tree-level potential in terms of the A and B fields

$$V(A,B) = \frac{1}{2}(a+m)(m-2a)A^2 + \frac{1}{2}(m-a)(2a+m)B^2 + \lambda m A^3 + \lambda m AB^2 + \frac{1}{2}\lambda^2(A^2+B^2)^2 \qquad (2.1.15)$$

The study of the extrema of this function reveals that the origin A=B=0 is a minimum of the potential for the values $|m|>2a$, being a saddle-point in the range $2a>|m|>a$ and a maximum for $a>|m|$. The absolute minimum of the potential for $m>0$ is found at the supersymmetric extremum $A=-(m+a)/\lambda$, $B=0$, present in the range $m>0$, $m<-4$, while for the values $0>m>-2a$ it is attained at the supersymmetric extrema $A=(-m+a)/2\lambda$, $B=\pm(a-m)^{1/2}(3a+m)^{1/2}/2\lambda$. Also, a non-supersymmetric minimum at $A=(-m+2a)/2\lambda$, $B=0$, survives in the range $2a>m>0$, $-2a>m>-4a$. In remarkable difference with the flat space-time realization of the supersymmetry, the potential energy is not positive definite and the condition F=0 does not imply now that the corresponding point be the lowest energy point of the potential. Then, it turns out that the symmetric point A=B=0 is the absolute minimum of the potential only for the values of the mass $m<-2a$.

It is, perhaps worth to notice explicitely that, in a purely classical context, it has been observed that, for field theories in AdS_4 background space, the analysis of the classical solutions of the theory cannot be restrained to only the absolute minima of the classical potential and that, in fact, the classical solution provided by a stationary point or even a global maximum of the potential can correspond to a stable vacuum state.[1]

The problem of the positiveness of the energy in AdS_4 has been addressed in ref.[17]. There, it has been suggested the possibility of defining the generators of the model from the supersymmetry algebra, what would give a positive definite expression of the energy. Furthermore, this choice would lead to a situation in which the supersymmetric minima are the only points with zero energy, in clear analogy with the flat space-time realization of supersymmetry [18]. At this point we are not going to go into the analysis of this improved expression of the energy, stressing the fact that the only modification with respect to our previous discussion would be the change in the values of the potential in what the supersymmetric minima are concerned. On the other hand, the proof of the persistence of supersymmetry when the quantum corrections are taken into account, to be carried in what follows for every one of these minima, is completely independent of these two different choices in the definition of the energy.

The above analysis points in the direction that only for a definite range of the values of the m and a parameters the symmetric point A=B=0 can be taken as an acceptable classical solutionaround which the model admits quantization. In the rest of this chapter we specialize our study to this symmetric phase, understanding that a convenient choice of the m and a parameters has been made (let us say $m<-2a$) so that the classical solution A=B=0 is, in fact, the solution with the lowest energy of the potential. This assumption simplifies this first attempt to compute the quantum corrections of the model placed in its non-trivial background that is performed in the next section 2.2.

Finally, we want to mention that the action in eq. (2.1.2) is not the most general action that one can write in terms of superfields, as long as one can allow for a linear superfield insertion of the form

$$S_\Lambda = \Lambda\left[\int \mathcal{D}M^L\, T^+(x^L,\theta^L) + \int \mathcal{D}M^R\, T^-(x^R,\theta^R)\right]$$
$$= \Lambda' \int d^4x \sqrt{-g}\,(3aA+F) \quad , \quad \Lambda' = \Lambda/\sqrt{2} \qquad (2.1.16)$$

As we will see in section 2.3, an insertion of this type is needed to make the theory renormalizable, even in the case in which this term is not included in the starting action. In principle, this fact could be interpreted as a first exemple of a violation of the no-renormalization theorem. With respect to what concerns our former discussion of the classical potential of the model, the inclusion of a term of this kind does not modify the supersymmetric character of the minima of the potential, leading only to the disappearance of some of them when varying the value of Λ'. Thus, we remain with, in general, supersymmetric extrema at $A=\{-(a+m)\pm[(a+m)^2-4\lambda\Lambda']^{1/2}\}/2\lambda$, $B=0$, and $A=(-m+a)/2\lambda$, $B=\pm[(a-m)(3a+m)+4\lambda\Lambda']^{1/2}/2\lambda$, while the position of the non-supersymmetric extremum at $A=(-m+2a)/2\lambda$, $B=0$ remains unchanged. In section 2.3 we will recover this persistence of supersymmetry through the dynamical computation of the 1PI one-point functions of the model, in the particular case in which the theory is quantized around the $A=B=0$ minimum. Also, in what follows we will take advantage of the above considerations by not making any particular assumption about the value of the Λ parameter. Our approach, then, will consist on using a linear insertion of the above type as a counterterm needed to renormalize the theory, and computing the radiative corrections to the classical equations of mo= tion independently of the choice of any particular sup= ersymmetric minimum. As we will see in chapter 3, the functional formalism applied to the computation of the effective potential of the model will be conveniently suited for this purpose.

2.2. The adiabatic expansion of the free-field propagators

First, we assume $m < -2a$ so that A=B=0 is the solution with the lowest energy of the potential. From S_{Kin} in eq.(2.1.4) we can then extract the equations of motion for the free-field propagators

$$(\Box + m^2 - 2a^2 \mp am)\langle A_\pm(x) A_\pm(x')\rangle_0 = -i(-g)^{-1/2}\delta(x,x')$$

$$(\Box + m^2 - 2a^2 \mp am)\langle A_\pm(x) F_\pm(x')\rangle_0 = i(a \pm m)(-g)^{-\frac{1}{2}}\delta(x,x')$$

$$(\Box + m^2 - 2a^2 \mp am)\langle F_\pm(x) F_\pm(x')\rangle_0 = i(\Box + m^2 - 2a^2 \mp am)\cdot$$
$$\cdot(-g)^{-1/2}\cdot\delta(x,x') - i(a \pm m)^2(-g)^{-1/2}\delta(x,x')$$

$$(i\not{\partial} - m)\langle\psi(x)\bar{\psi}(x')\rangle_0 = i(-g)^{-1/2}\delta(x,x') \qquad (2.2.1)$$

$$A_+ = A\ ,\quad A_- = B\ ,\quad F_+ = F\ ,\quad F_- = G$$

In dealing with the free-field propagators of the model one has to make use of some kind of perturbative expansion in the effects of the curvature of the space. As stated in the introduction, in order to apply the renormalization program to the 1PI functions of the model, we have decided to use an adiabatic expansion for the propagators. This expansion provides an adequate tool to calculate all the local quantum corrections to the model. To determine these local terms only the first adiabatic order in the expansion is needed. One of the ways in which the expansion can be understood consists in expanding the expressions in eq.(2.2.1) in normal coordinates around a given point x' of the background space by making use of

$$g_{\mu\nu}(x) = \eta_{\mu\nu} - \frac{1}{3} R_{\mu\rho\nu\sigma}(x') y^\rho y^\sigma \dots$$

$$\Gamma^\mu{}_{\rho\sigma}(x) = -\frac{1}{3}\left[R^\mu{}_{\rho\sigma\alpha}(x') + R^\mu{}_{\sigma\rho\alpha}(x')\right] y^\alpha \dots$$

$$e^a{}_\mu(x) = \delta^a{}_\mu - \frac{1}{6} R^a{}_{\nu\mu\sigma}(x') y^\nu y^\sigma \dots$$

$$\Gamma_\mu(x) = \frac{1}{8}\left[\gamma_{\hat a}, \gamma_{\hat b}\right]\left[\frac{1}{6} R^{\hat b}{}_\mu{}^{\hat a}{}_\nu(x') y^\nu + \frac{1}{3} R^{\hat b \hat a}{}_{\mu\nu}(x') y^\nu - \frac{1}{6} R^{\hat b}{}_\nu{}^{\hat a}{}_\mu(x') y^\nu\right] \dots$$

Then, we can obtain a systematic (orderby order) series expansion for $\langle A(x)A(x')\rangle_0$, etc., in which higher order terms contain tensors built from higher derivatives of the metric tensor. Specifically, we have

$$\langle A_\pm(x)A_\pm(x')\rangle_0 = i\int\frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-x')}\Big[\frac{1}{k^2-m^2+i0} - \frac{2a^2 \pm am}{(k^2-m^2+i0)^2}$$
$$+\frac{1}{3}R\frac{1}{(k^2-m^2+i0)^2} - \frac{2}{3}R_{\mu\nu}(x')\frac{k^\mu k^\nu}{(k^2-m^2+i0)^3} + O(R^2)\Big] =$$
$$= i\int\frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-x')}\cdot\Big[\frac{1}{k^2-m^2+i0} \mp am\frac{1}{(k^2-m^2+i0)^2} + O(a^4)\Big]$$

$$\langle A_\pm(x)F_\pm(x')\rangle_0 = -i\int\frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-x')}\cdot(a\pm m)\Big[\frac{1}{k^2-m^2+i0} + \frac{\mp am}{(k^2-m^2+i0)^2} + O(a^4)\Big]$$

$$\langle F_\pm(x)F_\pm(x')\rangle_0 = i\int\frac{d^4k}{(2\pi)^4} e^{ik(x-x')}\Big\{1 + (a\pm m)^2\Big[\frac{1}{k^2-m^2+i0} \mp am\frac{1}{(k^2-m^2+i0)^2} + O(a^4)\Big]\Big\}$$

$$\langle\psi(x)\bar\psi(x')\rangle_0 = -i\int\frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-x')}\Big[\frac{k^{\hat a}\gamma_{\hat a} - m}{k^2-m^2+i0} + \frac{R}{12}\frac{k^{\hat a}\gamma_{\hat a} - m}{(k^2-m^2+i0)^2}$$
$$-\frac{2}{3}R_{\mu\nu}(x')k^\mu k^\nu\frac{k^{\hat a}\gamma_{\hat a} - m}{(k^2-m^2+i0)^3} + \frac{1}{2}\gamma^{\hat a}k^\nu R_{\hat a\nu}(x')\frac{1}{(k^2-m^2+i0)^2} + O(R^2)\Big] \qquad (2.2.2)$$

where $R_{\mu\nu\alpha\beta} = a^2(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})$, $R_{\mu\nu} = 3a^2 g_{\mu\nu}$ and $R = 12a^2$.
In the propagators (2.2.2) the Feynman boundary condition is denoted by adding to k^2 a small positive imaginary

part. This is understood as a prescription in momentum space needed to make contact with the quantum model in the flat space-time. It has no relation with the boundary conditions that the large scale features of AdS_4 impose on the Green functions defined in x-space, which are extensively discussed in the literature[3]. In this sense the adiabatic expansion of the curved space-time propagators cannot reflect the choice of boundary conditions imposed by the global structure of AdS_4, but it is an adequate tool to investigate the short-distance behaviour of our model. Although the adiabatic expansion cannot reflect the global structure of the space-time, the ultraviolet behaviour of a quantum model is notsensitive to these global features nor to any particular choice of the vacuum in the curved space-time background. The adiabatic expansion is then adequate to our purposes as long as we are interested in renormalization coefficients that depend on the short distance behaviour of the model.

In fact, only the first adiabatic order is needed for the purpose of studying the renormalization properties of the model, as can be seen by the following dimensional argument. The key point is that we are intersted in obtaining the divergent quantum corrections that come from the computation of two and three-point 1PI diagrams. The maximum dimension of any of these objects is 2, in energy units, so that the only possibility is to have terms of order R (but not higher orders in the adiabatic expansion) contributing to them * . On the other hand, terms of this kind induce the renormalization of the operator $\frac{3}{2}a^2(A^2+B^2)$ already present in the kinetic lagrangian. The match of this contribution with the renormalization of the operator $\frac{1}{2}(D_\mu AD^\mu A+D_\mu BD^\mu B)$ to form the common renormalization factor Z_{kin} is, as shown below, a

* At this point, the assumption of the locality of the divergent quantum corrections in the model is implicit.

non-trivial check of the validity of the adiabatic expansion in this supersymmetric context.

In the one-loop calculations described in this section we are going to take a regularization procedure in which the momenta of the adiabatic expansions are analytically continued to D dimensions, while keeping all the remaining tensors in 4 dimensions. This is, in fact, an extrapolation of the usual dimensional reduction prescription adopted in flat space-time background [14], since we have to give now a definite prescription for the gravitational tensors $R_{\mu\nu\alpha\beta}$, $R_{\mu\nu}$. While in our case it is not completely clear that the counting of the fermionic degrees of freedom equals that of the bosonic ones, the solution adopted here in which these gravitational tensors are considered in 4 dimensions while the momenta are considered in D seems to us the closest in spirit to the usual dimensional reduction method, connecting at the same time with the classical invariance of the lagrangian for the 4-dimensional Anti-de Sitter background space.

In the following we calculate the divergent part of the 1PI two-point functions. In figure 1 we give the list of the Feynman diagrams contributing to the two-point functions Γ_{AF}, Γ_{AA}, Γ_{FF}, Γ_{BG}, Γ_{BB}, Γ_{GG} and $\Gamma_{\psi\bar\psi}$. Introducing the integral

$$I=\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-m^2)^2}=\frac{2i}{\varepsilon}\frac{1}{16\pi^2}+(\text{finite terms}) \qquad (2.2.3)$$

where $\varepsilon=4-D$, we have, from diagrams 1(a) and 1(b),

$$1(a)=12\lambda^2 a I$$

$$1(b)=-8\lambda^2 a I$$

Then,

$$\Gamma^{div}_{AF}(x,x') = \frac{4\lambda^2 a}{16\pi^2}\,\frac{2i}{\varepsilon}\,\delta(x,x') \tag{2.2.4}$$

For the calculation of Γ^{div}_{AA} we need, in addition to eq. (2.2.3), the integral

$$J = \int \frac{d^Dk}{(2\pi)^D}\,\frac{1}{k^2-m^2} = \frac{2i}{\varepsilon}\,\frac{1}{16\pi^2}\,m^2 + (\text{finite terms}) \tag{2.2.5}$$

Then,

$$1(c) = 36\lambda^2 a^2 I$$

$$1(d) = 8\lambda^2\left[(m^2+a^2)I + J\right]$$

$$1(e) = -48\lambda^2 a^2 I$$

$$1(f) = 8\lambda^2(m^2+a^2)I$$

Let us turn to the calculation of the divergent part of diagram 1(g) when both external lines represent the fi= eld A

$$1(g) = -2\lambda^2\,\mathrm{Tr}\int\frac{d^Dk}{(2\pi)^D}\left[\frac{\gamma^{\hat a}k_{\hat a}-m}{k^2-m^2}\,\frac{\gamma^{\hat b}(k_{\hat b}-p_{\hat b})-m}{(k-p)^2-m^2} + \frac{R}{12}\,\frac{\gamma^{\hat a}k_{\hat a}}{k^2}\cdot\frac{\gamma^{\hat b}(k_{\hat b}-p_{\hat b})}{(k-p)^4}\right] = -\frac{4\lambda^2}{16\pi^2}\left(-p^2+6m^2+\frac{1}{6}R\right)\frac{2i}{\varepsilon} + (\text{finite terms})$$

Using the relation between $\partial^\mu\partial_\mu$ and $D^\mu D_\mu$ in normal coo= rdinates [29]

$$\eta^{\mu\nu}\frac{\partial^2}{\partial y^\mu\partial y^\nu}\,\delta(y) = \left(D^\mu D_\mu - \frac{1}{3}R\right)\delta(x,x') = \left(\Box_x - 4a^2\right)\delta(x,x') \tag{2.2.6}$$

and adding together the contributions from diagrams 1(c) to 1(g) we obtain

$$\Gamma^{div}_{AA}(x,x') = \frac{-4\lambda^2}{16\pi^2}\left(\Box_x - 3a^2\right)\frac{2i}{\varepsilon}\,\delta(x,x') \qquad (2.2.7)$$

From diagram 1(h) we have

$$\Gamma^{div}_{FF}(x,x') = \frac{4\lambda^2}{16\pi^2}\,\frac{2i}{\varepsilon}\,\delta(x,x') \qquad (2.2.8)$$

The diagrams contributing to Γ^{div}_{BG} give

$$1(a) = 12\lambda^2 a I$$

$$1(b) = -8\lambda^2 a I$$

and

$$\Gamma^{div}_{BG}(x,x') = \Gamma^{div}_{AF}(x,x') \qquad (2.29)$$

For Γ^{div}_{BB} we have the contributions

$$1(c) = 36\lambda^2 a^2 I$$

$$1(d) = 8\lambda^2\left[(m^2+a^2)I + J\right]$$

$$1(e) = -48\lambda^2 a^2 I$$

$$1(f) = 8\lambda^2(m+a)(-m+a)I$$

$$1(g) = -2\lambda^2 \,\mathrm{Tr}\int\frac{d^Dk}{(2\pi)^D}\left[i\gamma_5\,\frac{\gamma^{\hat a}k_{\hat a}-m}{k^2-m^2}\,i\gamma_5\,\frac{\gamma^{\hat b}(k_{\hat b}-p_{\hat b}-m)}{(k-p)^2-m^2} + \frac{R}{12}\,i\gamma_5 \cdot\right.$$

$$\left.\cdot\,\frac{\gamma^{\hat a}k_{\hat a}\,i\gamma_5\,\gamma^{\hat b}(k_{\hat b}-p_{\hat b})}{k^2(k-p)^4}\right] = -\frac{4\lambda^2}{16\pi^2}\left(-p^2+2m^2+\frac{R}{6}\right)\frac{2i}{\varepsilon} + (\text{finite terms})$$

Using again eq.(2.2.6) and taking the sum of the above contributions we get

$$\Gamma^{div}_{BB}(x,x') = \Gamma^{div}_{AA}(x,x') \tag{2.2.10}$$

Diagram 1(h) gives

$$\Gamma^{div}_{GG}(x,x') = \Gamma^{div}_{FF}(x,x') \tag{2.2.11}$$

In computing diagram 1(i) the integrals proportional to m cancel and we are left with

$$\Gamma^{div}_{\psi_\alpha \bar{\psi}_\beta}(x,x') = -\frac{4\lambda^2}{16\pi^2}(\gamma_{\hat{a}})_{\alpha\beta}\,\frac{2}{\varepsilon}\,D^{\hat{a}}_x\,\delta(x,x') \tag{2.2.12}$$

The above results show that the divergent part of the 1PI two-point functions for the action (2.1.4) of the Wess -Zumino model in AdS_4 background does not provide any renormalization of L_m,

$$Z_m = 1$$

exactly in the same way as it happens in flat space-time. From the above results we can extract the common factor Z_{kin} renormalizing L_{kin}

$$Z_{kin} = 1 + \frac{4\lambda^2}{16\pi^2}\,\frac{2}{\varepsilon}$$

The fact that our regularization prescription preserves supersymmetry for the divergent one-loop corrections to the two-point functions is then manifest, since we have obtained a common renormalization of the various terms in L_{kin} and no renormalization at all for L_m. This can be checked equivalently by deriving the supersymmetry Ward identities of the theory. We have checked that the one-loop corrections calculated in this section satisfy all the supersymmetric Ward identities between the 1PI two-point functions. For example, by substituting (2.2.12) into (2.1.11) and using the relation

$$i\bar{\varepsilon}(x')\Gamma_{\psi_\alpha\bar{\psi}_\beta}(x,x')\gamma_{\hat{b}}\,\varepsilon(x)\overleftarrow{D}_x^{\hat{b}} = -8\frac{i\lambda^2}{16\pi^2}\frac{1}{\varepsilon}\Big[\bar{\varepsilon}(x')\gamma_{\hat{a}}\gamma_{\hat{b}}\,\varepsilon(x)\cdot$$
$$\cdot\,\delta(x,x')D_x^{\hat{a}}D_x^{\hat{b}}\Big]_{\alpha\beta} = -8\frac{i\lambda^2}{16\pi^2}\frac{1}{\varepsilon}\,\delta_{\alpha\beta}\,\delta(x,x')\Box_x$$

one readily checks that the Ward identity is satisfied for the divergent parts of the two-point functions.

Next we illustrate the calculation of the divergent part of the 1PI three-point functions. In figure 2 we list the Feynman diagrams for Γ_{FAA}, Γ_{AAA}, $\Gamma_{A\psi\bar{\psi}}$, Γ_{FBB}, Γ_{GAB}, Γ_{ABB} and $\Gamma_{B\psi\bar{\psi}}$. We make use once again of the integral (2.2.3).

Diagram 2(a) gives

$$\Gamma^{div}_{FAA}(x,x') = 0 \tag{2.2.13}$$

In the case of external A-fields, diagrams 2(b)-2(d) give

$$2(b) = 0$$
$$2(c) = 96\lambda^3 m I$$
$$2(d) = -96\lambda^3 m I$$

and, therefore,

$$\Gamma^{div}_{AAA}(x,x') = 0 \tag{2.2.14}$$

From diagram 2(e) with the external wavy line repres=enting an A-field, it follows

$$\Gamma^{div}_{A\psi\bar{\psi}}(x,x') = 0 \tag{2.2.15}$$

Diagram 2(a) gives, for the two external lines of ty=pe B,

$$\Gamma^{div}_{FBB}(x,x') = 0 \tag{2.2.16}$$

From diagram 2(a) we have, identifying the external lines with the appropriate fields,

$$\Gamma^{div}_{GAB}(x,x') = 0 \tag{2.2.17}$$

From diagrams 2(b)-2(d) with one A and two B external fields

$$2(b) = 0$$

$$2(c) = 32\lambda^3 m I$$

$$2(d) = -32\lambda^3 m I$$

we obtain

$$\Gamma^{div}_{ABB}(x,x') = 0 \tag{2.2.18}$$

The calculation of diagram 2(e) when the wavy line represents the B-field furnishes

$$\Gamma^{div}_{B\psi\bar{\psi}}(x,x')=0 \qquad (2.2.19)$$

All the divergent parts of the three-point functions vanishing at the one-loop level, we can conclude, in agreement with the renormalizability of the model and the absence of renormalization of L_{int} in the AdS_4 background space, that

$$Z_{int} = 1$$

The check that the above one-loop corrections satisfy the supersymmetric Ward identities between the 1PI three-point functions (see eq.(2.1.12)) is now a matter of triviality, since they vanish by explicit cancellation between the virtual (B,G) contribution and the virtual (A,F) one, as we have seen in the above computation. Actually, the fermion-loop contribution to Γ_{AAA} vanishes by itself since $Tr(\gamma^\mu\gamma^\nu\gamma^\lambda)=0$. The cancellation of the various contributions to $\Gamma_{A\psi\bar{\psi}}$ requires the use of the relation $\{\gamma_5, \langle\psi_\alpha(x)\bar{\psi}_\beta(x')\rangle\}=0$. Both cancellations are peculiar of the form of the adiabatic expansion for the spinor propagator in eq.(2.2.2).

To, summarize, in this section we have intended to give a first answer to the question of the renormalization of the model under investigation, at the one-loop level. For the Wess-Zumino model in flat space-time, a well-known consequence of its supersymmetric invariance and of the form of the interaction lagrangian -or, more precisely, of its renormalizability- is that the mass lagrangian L_m and the interaction lagrangian L_{int} are not renormalized [19]. This result is expressed in the most simple way in terms of superfields [14]. Although it is a common belief that the renormalizability should be preserved in a curved background space, it is by no means a foregone conclusion that the no-renormalization theorem should still be valid when the supersymmetry of the Wess-Zumino model is realized in the background of AdS_4.

In studying the above question related to the one-loop renormalization of the model we have made a first use, as a technical tool, of the adiabatic expansion in momentum space [16]. We have seen that all what is needed in order to compute the one-loop divergent corrections are the terms of the expansion of the first order in the curvature. Actually, provided that the cancellation of the non-local divergent contributions depending on the curvature can be shown, as it is the case for a self-interacting scalar theory[20,21], then one can conceive to evaluate all the local quantum corrections using the first order term in the expansion and, in this way, to renormalize the model to higher loops.

In regularizing the one-loop divergent integrals the question of preserving supersymmetry arises. In our case we have solved this problem by choosing an extrapolation of the dimensional reduction prescription that is used in flat space-time. This choice presents the advantage of making the calculation rather straightforward, although its compatibility with supersymmetry in curved space-time is not foregone. The results of this section show that each of the three separately supersymmetric invariant pieces of the Wess-Zumino action is renormalized by a common factor, what proves that our regularization prescription preserves supersymmetry for the one-loop divergent corrections. In fact, the above argument is equivalent to check that the supersymmetry Ward identities are satisfied by the quantum corrections. We have performed this check explicitely. Of course, at higher-loops there is no reason to believe that our prescription -as well as any other version of regularization in dimension different from D=4- should still work. One can conceive the use of other regularization prescriptions, such as Pauli-Villars or the point-splitting method. They can be realized in a manifestly supersymmetric way [14], although their application to the actual computation appears less straightforward than the use of dimensional reduction.

Before to close this section we concentrate on the case in which the Wess-Zumino supermultiplet is massless and use the version of Bunch and Parker of the adiabatic expansion to evaluate the whole of the local one-loop quantum corrections (both divergent and finite) to the effective action. At first, we use our extrapolation of the dimensional reduction prescription taking only the momentum of the expansion in D dimension while all other tensors are considered in D=4. We show that the Ward identities, which are satisfied by the divergent one-loop corrections, are violated by finite local terms proportional to the curvature of the space, indicating either that our regularization prescription is not suitable to mantain supersymmetry at the level of those finite terms or that the expansion cannot be applied to compute them for the massless model.

The calculation of the Feynman diagrams contributing to the 1PI two-point functions Γ_{AF}, Γ_{AA} and $\Gamma_{\psi\bar{\psi}}$ to the one-loop order gives the result

$$\Gamma_{AF}(x,x') = \frac{4iu^2}{16\pi^2} a\left(\Delta+2\right)\delta(x,x') + \left[\text{non-local } O(a^3)\text{ terms}\right]$$

$$\Gamma_{AA}(x,x') = \frac{-4iu^2}{16\pi^2}\left[(\Box_x - 3a^2)\Delta + 2(\Box_x - 4a^2)\right]\delta(x,x') + O(a^4)$$

$$\Gamma_{\psi_\alpha\bar\psi_\beta}(x,x') = \frac{-4u^2}{16\pi^2}\left[(\gamma_{\hat a})_{\alpha\beta}(\Delta+2)D_x^{\hat a}\delta(x,x') - \langle\psi_\alpha(x)\bar\psi_\beta(x')\rangle a^2\right] + O(a^4) \quad (2.2.20)$$

where $\lambda = u\mu^{\varepsilon/2}$ with u dimensionless and

$$\Delta = \frac{2}{\varepsilon} + \left.\frac{d\Gamma(z)}{dz}\right|_{z=1} + \log 4\pi - \log \rho^2/\mu^2$$

being $\Gamma(z)$ the Euler function. Substituting (2.2.20) in= to (2.1.11) and using the relation

$$i\bar\varepsilon(x')\Gamma_{\psi_\alpha\bar\psi_\beta}(x,x')\gamma_{\hat b}\,\varepsilon(x)\overleftarrow{D}_x^{\hat b} = \frac{-4iu^2}{16\pi^2}\Big[(\Delta+2)\bar\varepsilon(x')\gamma_{\hat a}\gamma_{\hat b}\,\varepsilon(x)\cdot$$
$$\cdot\,\delta(x,x')D_x^{\hat a}D_x^{\hat b} - a^2\bar\varepsilon(x')\langle\psi_\alpha(x)\bar\psi_\beta(x')\rangle\gamma_{\hat b}\overleftarrow{D}_x^{\hat b}\varepsilon(x) + 2ia^3\bar\varepsilon(x')\langle\psi_\alpha(x)$$
$$\bar\psi_\beta(x')\rangle\varepsilon(x) = \frac{-4iu^2}{16\pi^2}\Big\{\left[(\Delta+2)\delta(x,x')\Box_x - a^2\delta(x,x')\right]\delta_{\alpha\beta} + 2ia^3\bar\varepsilon(x')\langle\psi_\alpha(x)\bar\psi_\beta(x')\rangle\varepsilon(x)\Big\}$$

one readily checks that the Ward identity, which is sa= tisfied for the divergent part including terms proporti= onal to a^2, is also verified for the flat space-time fi= nite terms but violated by finite terms of order a^2. In fact

$$i\bar\varepsilon(x')\Gamma_{\psi_\alpha\bar\psi_\beta}(x,x')\gamma_{\hat a}\,\varepsilon(x)\overleftarrow{D}_x^{\hat a} + \left[-\Gamma_{AA}(x,x') + 3a\,\Gamma_{AF}(x,x')\right]\delta_{\alpha\beta}$$
$$= -\frac{4iu^2}{16\pi^2}a^2\delta(x,x') + [\text{non-local } O(a^3) \text{ terms}]$$

This can be interpreted as a consequence of the regula= rization procedure involved in the calculation or can be alternatively considered as an indication that the expansion in the background effect which we have intro= duced is not suitable to calculate finite corrections for

the $m = 0$ model. In fact, on the one hand the regularization procedure used in Ref. [1] is not guaranteed to preserve supersymmetry for finite terms of order a^2; on the other hand, at the level of these terms, it is not clear whether the adiabatic expansion is a suitable approximation of the AdS_4 background correction to the flat space-time behavior, in the case of the massless model, where no scale is available to determine the behavior of the background curvature as a small perturbative effect.

At this point, a more appropriate regularization scheme is required in order to maintain supersymmetry up to the level of the finite terms proportional to the curvature of the Ward identity between 1PI-two-point functions. A natural choice is provided by Pauli-Villars regularization, although it would be appropriate also to consider the method of point splitting.[14]

In the case of our quite straightforward calculation of the two-point functions for the massless model, one Pauli-Villars regulating field turns out to be sufficient to guarantee the finiteness of the relevant integrals. Then, the expansion of the

Pauli-Villars regulated free-field propagators is easily found to be

$$\langle A_\pm A_\pm \rangle_\Lambda = \int \frac{d^4k}{(2\pi)^4}\, e^{ik(x-x')} \Big[\frac{1}{k^2+i0} - \frac{1}{k^2-\Lambda^2+i0} \pm \frac{a\Lambda}{k^2-\Lambda^2+i0)^2} + \frac{a^2\Lambda^2}{(k^2-\Lambda^2+i0)^3} \Big]$$

$$\langle A_\pm F_\pm \rangle - \langle A_\pm F_\pm \rangle_\Lambda = -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-x')} \Big[\frac{a}{k^2+i0} - \frac{\pm\Lambda + a}{k^2-\Lambda^2+i0} \pm \frac{a\Lambda(a\pm\Lambda)}{(k^2-\Lambda^2+i0)^2} \pm \frac{a^2\Lambda^3}{(k^2-\Lambda^2+i0)^3} \Big]$$

$$\langle F_\pm F_\pm \rangle - \langle F_\pm F_\pm \rangle_\Lambda = \int \frac{d^4k}{(2\pi)^4}\, e^{ik(x-x')} \Big[\frac{a^2}{k^2+i0} - \frac{(\pm\Lambda + a)^2}{k^2-\Lambda^2+i0} \pm \frac{a\Lambda(\pm\Lambda+a)^2}{(k^2-\Lambda^2+i0)^2} + \frac{a^2\Lambda^4}{(k^2-\Lambda^2+i0)^3} \Big]$$

$$\begin{aligned}\langle \psi\overline{\psi} \rangle - \langle \psi\overline{\psi} \rangle_\Lambda = -\int \frac{d^4k}{(2\pi)^4}\, e^{ik(x-x')} \Big\{ \not{k} \Big(\frac{1}{k^2+i0} - \frac{1}{k^2-\Lambda^2+i0} \Big) \\ + \frac{\Lambda}{k^2-\Lambda^2+i0} + \frac{a^2}{2}\not{k} \Big[\frac{1}{(k^2+i0)^2} - \frac{1}{(k^2-\Lambda^2+i0)^2} \Big] \\ - \frac{a^2\Lambda}{(k^2-\Lambda^2+i0)^2} + 2a^2\Lambda^2 \frac{\not{k}-\Lambda}{(k^2-\Lambda^2+i0)^3} \Big\}\end{aligned} \qquad (2.2.21)$$

In Eq. (21) we have defined

$$A_+ = A \;, \qquad A_- = B \;, \qquad F_+ = F \;, \qquad F_- = G$$

The propagators for the regulating fields are adequately represented by an expansion including $O(a^2)$ terms. They are propagators for massive fields in AdS_4 and their expansion in the parameter a, including $O(a^2)$ terms, is shown to coincide with an adiabatic expansion.[5] As for the massless part of the propagators (21), we have discussed in the above the degree of accuracy of the procedure.

In momentum space one immediately derives

$$\Gamma_{AF} = -\frac{4i\lambda^2}{16\pi^2} a \log\left(\frac{-p^2}{\Lambda^2}\right)$$

corresponding to

$$\Gamma_{AF}(x,x') = -\frac{4i\lambda^2}{16\pi^2} a \log\left(\frac{1}{\Lambda^2}\right) \frac{1}{\sqrt{-g}} \delta^4(x,x') \qquad (2.2.22)$$

It may be worthwhile to notice explicitly that in the expression of the finite part of the two point function in configuration space quoted , we neglect the Fourier transform of functions containing the factor $\log(p^2)$. In fact, the cancellation of this kind of finite contributions is obviously implied by the corresponding cancellation of divergences in the Ward identity.

Next, we evaluate Γ_{AA} which is given by the sum of two contributions

$$\Gamma_{AA} = \mathrm{BOS} + \mathrm{FER}$$

The first one represents the sum of all the Feynman diagrams contributing to Γ_{AA} in which bosonic fields are running around the loop, while the second one is the contribution of the fermion loop. We have the results

$$\mathrm{BOS} = I + 8\lambda^2 \frac{i}{16\pi^2}\left[-\left(\Lambda^2 + \frac{1}{3}p^2\right) - \frac{a^2}{2}\log\left(-\frac{p^2}{\Lambda^2}\right) + \frac{a^2}{6}\right]$$

$$\begin{aligned}\mathrm{FER} = -I + 8\lambda^2 \frac{1}{16\pi^2}\Bigg[&\Lambda^2 + \frac{1}{3}p^2 + a^2 \log\left(-\frac{p^2}{\Lambda^2}\right) \\ &- \frac{1}{2}p^2 \log\left(-\frac{p^2}{\Lambda^2}\right) + \frac{5}{6}a^2\Bigg]\end{aligned}$$

where the unregulated integral

$$I(p) \equiv 8\lambda^2\Lambda^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-\Lambda^2} \frac{1}{(k-p)^2-\Lambda^2}$$

cancels out when adding the two contributions to Γ_{AA}

$$\Gamma_{AA} = \frac{4i\lambda^2}{16\pi^2}\left[(-p^2+a^2)\log\left(-\frac{p^2}{\Lambda^2}\right) + 2a^2\right]$$

which corresponds to

$$\Gamma_{AA}(x,x') = \frac{4i\lambda^2}{16\pi^2}\left[\left(\Box_x - 3a^2\right)\log\left(\frac{1}{\Lambda^2}\right) + 2a^2\right]\frac{1}{\sqrt{-g}}\delta^4(x,x') \qquad (2.2.23)$$

Making use once again of the expansions (21) one can evaluate the Feynman diagrams contributing to $\Gamma_{\psi\overline{\psi}}$

$$\Gamma_{\psi\overline{\psi}} = \sum_{i=1}^{3} G_i$$

$$G_1 = -8\lambda^2 \int \frac{d^4k}{(2\pi)^4}\not{k}\left(\frac{1}{k^2} - \frac{1}{k^2-\Lambda^2}\right)\left[\frac{1}{(k-p)^2} - \frac{1}{(k-p)^2-\Lambda^2}\right]$$

$$= \frac{4i\lambda^2}{16\pi^2}\not{p}\log\left(-\frac{p^2}{\Lambda^2}\right)$$

$$G_2 = -4\lambda^2 a^2 \int \frac{d^4k}{(2\pi)^4}\not{k}\left[\frac{1}{k^4} - \frac{1}{(k^2-\Lambda^2)^2}\right]$$

$$\times\left[\frac{1}{(k-p)^2} - \frac{1}{(k-p)^2-\Lambda^2}\right] = -\frac{4i\lambda^2}{16\pi^2}\frac{a^2}{p^2}\not{p}$$

$$G_3 = -8\lambda^2 a\Lambda^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-\Lambda^2)^2}\frac{1}{(k-p)^2-\Lambda^2} = \frac{4i\lambda^2}{16\pi^2}a$$

Adding up the different terms, one has

$$\Gamma_{\psi\overline{\psi}} = \frac{4i\lambda^2}{16\pi^2}\left[\not{p}\log\left(-\frac{p^2}{\Lambda^2}\right) + a\left(1 - \frac{a\not{p}}{p^2}\right)\right]$$

The corresponding result in configuration space is

$$\Gamma_{\psi_\alpha\overline{\psi}_\beta}(x,x') = \frac{4\lambda^2}{16\pi^2}\left[\log\left(\frac{1}{\Lambda^2}\right)(\not{D}_x)_{\alpha\beta}\frac{1}{\sqrt{-g}}\delta^4(x,x') + a^2\left\langle\psi_\alpha(x)\overline{\psi}_\beta(x')\right\rangle + ia\delta_{\alpha\beta}\frac{1}{\sqrt{-g}}\delta^4(x,x')\right] \qquad (2.2.24)$$

Substituting (24), (23) and (22) into the Ward identity and using

$$i\overline{\epsilon}(x')\Gamma_{\psi_\alpha\overline{\psi}_\beta}(x,x')\gamma_{\hat{a}}\epsilon(x)\overline{D}^{\hat{a}}_x = \frac{4i\lambda^2}{16\pi^2}\overline{\epsilon}(x)\epsilon(x)\left[\log\left(\frac{1}{\Lambda^2}\right)\frac{1}{\sqrt{-g}}\delta^4(x,x')\Box_x + a^2\frac{1}{\sqrt{-g}}\delta^4(x,x')\right]\delta_{\alpha\beta} + (\text{non}-\text{local } O(a^3) \text{ terms})$$

one sees immediately that

$$i\bar{\epsilon}(x')\,\Gamma_{\psi_\alpha\overline{\psi}_\beta}(x,x')\gamma_{\hat{a}}\epsilon(x)\bar{D}^{\hat{a}}_{x} + \left[-\Gamma_{AA}(x,x') + 3a\Gamma_{AF}(x,x')\right]\bar{\epsilon}(x)\epsilon(x)\delta_{\alpha\beta}$$

$$= -\frac{4i\lambda^2}{16\pi^2}a^2\frac{1}{\sqrt{-g}}\delta^4(x-x') + \mathcal{O}(a^3)$$

The finite terms proportional to the curvature do not respect the supersymmetry Ward identity between 1PI two-point functions. Given the manifestly supersymmetric character of our choice of the regularization scheme, the above violation of the supersymmetric Ward identity is the signal that the adiabatic expansion is inadequate to calculate the finite one-loop corrections to the $m = 0$ model.

In conclusion, the calculations performed in this section by making use of the adiabatic expansion show that there is no one-loop renormalization neither of the mass nor of the interaction lagrangian, what guarantees that this part of the no-renormalization theorem retains its validity at the one-loop level, for the model in AdS_4 background space. Exploring the possibility to extend the validity of the theorem to higher loops, by making use of the same technical tools of this section, would require the crucial check that the contributions of the non-local parts of the expansion of the propagators in momentum space cancel explicitely. In order to accomplish this rather formidable task, the use of a manifestly supersymmetric regulator -of the kind of those mentioned in the last part of this section- is a must.

2.3. Pauli-Villars regularized vacuum expectation values of the fields

Our intention in the present section is to proceed to the complete study of the renormalization properties of the Wess-Zumino model in AdS_4 background space at the one-loop level using the adiabatic expansion in momentum space. We regularize the one-loop divergent integrals by means of a manifestly supersymmetric invariant Pauli-Villars procedure [14]. These tools are sufficient to co= nclude that the one-loop effects proportional to the co= ntraction parameter of the space force the insertion of a linear superfield term in the superpotential in order to renormalize the non-vanishing 1PI one-point functions of the model. Also, by the same techniques, we calcula= te again the 1PI two and three-point functions and ver= ify in this way that the no-renormalization theorem ho= lds at the one-loop level for the mass and the interact= ion lagrangians, as anticipated in the previous section.

In the formalism of superfields the Pauli-Villars reg= ulated action of the model reads

$$S = \int \mathcal{D}M \sum_{j=0}^{N} c_j^{-1} \Phi_{+j}(x,\theta) \Phi_{-j}(x,\theta) + $$
$$+ \left\{ \int \mathcal{D}M^L \left[\frac{1}{2} \sum_{j=0}^{N} c_j^{-1} m_j \left(T_j^{+}(x^L,\theta^L) \right)^2 + \frac{\sqrt{2}}{3} \lambda \left(\sum_{j=0}^{N} T_j^{+}(x^L,\theta^L) \right)^3 \right] + h.c. \right\} \quad (2.3.1)$$

where m_j is the mass of each chiral superfield and c_j^{-1} its normalization factor. We have seen that the adiabat= ic expansion in momentum space, which we use to deal wi= th the free-field propagators of the model, provides an adequate tool to calculate all the local quantum correc= tions and, in order to determine these local terms, on= ly the first adiabatic order in the expansion is needed.

From eq.(2.3.1) one can easily obtain by projections the Pauli-Villars regulated free-fields propagators

$$\langle A_\pm(x)A_\pm(x')\rangle_0 = i\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\sum_{j=0}^{N}c_j\Big[\frac{1}{k^2-m_j^2+i0}$$
$$\mp\frac{am_j}{(k^2-m_j^2+i0)^2}+O(a^3)\Big]$$
$$\langle A_\pm(x)F_\pm(x')\rangle_0 = -i\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\sum_{j=0}^{N}c_j(a\pm m_j)\Big[\frac{1}{k^2-m_j^2+i0}$$
$$\mp\frac{am_j}{(k^2-m_j^2+i0)^2}+O(a^3)\Big]$$
$$\langle F_\pm(x)F_\pm(x')\rangle_0 = i\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\sum_{j=0}^{N}c_j\Big\{1+(a\pm m_j)^2\Big[\frac{1}{k^2-m_j^2+i0}$$
$$\mp\frac{am_j}{(k^2-m_j^2+i0)^2}+O(a^3)\Big]\Big\}$$
$$\langle\psi(x)\bar\psi(x')\rangle_0 = -i\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\sum_{j=0}^{N}c_j\Big[\frac{\not k-m_j}{k^2-m_j^2+i0}+\frac{a^2}{2}\frac{\not k}{(k^2-m_j^2+i0)^2}$$
$$+a^2\frac{m_j}{(k^2-m_j^2+i0)^2}-2a^2\frac{(\not k-m_j)m_j^2}{(k^2-m_j^2+i0)^3}+O(a^3)\Big]$$
$$A_+=A\ ,\quad A_-=B\ ,\quad F_+=F\ ,\quad F_-=G \qquad (2.3.2)$$

The supersymmetric invariance of the above treatment is, at this level, manifest.

Next, we give the details about the computation of the one-point functions of the model to the one-loop order. As in the previous section, we explicitely assume here the model to be placed in the symmetric phase in which the solution A=B=0 is the lowest energy point of the classical potential, so that the Feynman rules can be read directly from the expression of the action in eq.(2.1.4). Then, the non-vanishing values of the one-point functions for the A and F fields will give us the result of a non-trivial one-loop order correction to the classical sol=

ution A=B=0 for the vacuum expectation values of the fields.

At this point, we make use of the Pauli-Villars regularized expressions of the propagators in eq.(2.3.2) without assuming for the moment a definite number of regulating fields. The diagrams corresponding to figure 3(a) (i.e. with either A or B going around the loop) give a contribution to the A-field one-point function, to the one-loop order

$$3(a) = 6a^2\lambda\int\frac{d^4k}{(2\pi)^4}\left[\sum_i \frac{c_i m_i}{(k^2-m_i^2)^2}\right] + O(a^4)$$

while the contribution from diagrams of figure 3(b) is

$$3(b) = 4\lambda\int\frac{d^4k}{(2\pi)^4}\left[\sum_i \frac{c_i m_i}{k^2-m_i^2} - a^2\sum_i \frac{c_i m_i}{(k^2-m_i^2)^2}\right] + O(a^4)$$

The spinor-loop contribution represented in figure 3(c) is

$$3(c) = -4\lambda\int\frac{d^4k}{(2\pi)^4}\left[\sum_i \frac{c_i m_i}{k^2-m_i^2} - a^2\sum_i \frac{c_i m_i}{(k^2-m_i^2)^2} - 2a^2\sum_i \frac{c_i m_i^3}{(k^2-m_i^2)^3}\right] + O(a^4)$$

By adding up all these contributions we obtain for the one-point function corresponding to the A field

$$\Gamma_A = 6a^2\lambda\int\frac{d^4k}{(2\pi)^4}\sum_i \frac{c_i m_i}{(k^2-m_i^2)^2} + 8a^2\lambda\int\frac{d^4k}{(2\pi)^4}\sum_i \frac{c_i m_i^3}{(k^2-m_i^2)^3} + O(a^4) \quad (2.3.3)$$

The evaluation of the one-loop order F-field one-point function is carried through the computation of diagram 3(d) with either A or B going around the loop,

$$3(d) = 2a\lambda \int \frac{d^4k}{(2\pi)^4} \sum_i \frac{c_i m_i}{(k^2-m_i^2)^2} + O(a^3)$$

so that

$$\Gamma_F = 2a\lambda \int \frac{d^4k}{(2\pi)^4} \sum_i \frac{c_i m_i}{(k^2-m_i^2)^2} + O(a^3) \qquad (2.3.4)$$

Finally, one can check that the one-point functions for the B and G fields are zero to all orders in perturbation theory $\Gamma_B = \Gamma_G = 0$. This fact derives from the specific form of the Feynman rules of the model obtained from eq.(2.1.4), as can be easily seen when they are expressed in terms of two-component Weyl spinors (see Appendix A), and it is therefore peculiar of the model quantized around the classical solution A=B=0. We will come back to this point in the next chapter.

The above integrals have to be regularized by giving some constraints on the parameters c_i and m_i that intro= duce the regulating fields in the lagrangian of the mo= del. The approach that we are going to follow here is to apply the minimal set of constraints

$$\sum_{i=0}^{N} c_i m_i^{\alpha} = 0 \quad , \qquad \alpha = 0, 1, \ldots, m \qquad (2.3.5)$$

that serve to regulate the expressions of Γ_A and Γ_F. This procedure is obviously consistent with any other calcu= lation of regularized n-point functions of the model as long as the same set of constraints is used for their regularization. Then, the minimal set of constraints tu= rns out to be

$$\begin{aligned} &1 + \sum_{i=1}^{N} c_i = 0 \\ &m + \sum_{i=1}^{N} c_i m_i = 0 \end{aligned} \qquad (2.3.6)$$

where the choice $c_0 = 1$ has been made. To solve this set of equations we assume that, for our purposes, the minimum set of regulating fields with which they can be solved is used, i.e. 2. Then, we are able to determine c_1 and c_2 in terms of the masses

$$c_1 = \frac{m - m_2}{m_2 - m_1}$$

$$c_2 = \frac{m - m_1}{m_1 - m_2} \tag{2.3.7}$$

Under the requirement that, in the limit in which the masses go to the infinity, $\lim c_1 \neq 0, \infty$ and $\lim c_2 \neq 0, \infty$, we can deduce the asymptotic behaviour $m_1 \approx O(\Lambda)$, $m_2 \approx O(\Lambda)$ and $\lim_{\Lambda \to \infty} (m_1/m_2) \neq 1$, that is same asymptotic order but differnt asymptotic values for the two regulating masses. This proves to be enough to solit the regulated integral

$$W \equiv \int \frac{d^4k}{(2\pi)^4} \sum_i \frac{c_i m_i}{(k^2 - m_i^2)^2} =$$

$$= \int \frac{d^4k}{(2\pi)^4} \left[\frac{m}{(k^2 - m^2)^2} + \frac{c_1 m_1}{(k^2 - m_1^2)^2} + \frac{c_2 m_2}{(k^2 - m_2^2)^2} \right] \tag{2.3.8}$$

into a divergent and a finite part (the second regulated integral in eq.(2.3.3) exactly vanishes by applying the second sum rule in eq.(2.3.6). The quantity W can be expressed as

$$W = 2(c_1 m_1^3 + c_2 m_2^3) W_6 + (-c_1 m_1^5 - c_2 m_2^5 + 4 m_1^2 m_2^2 m) W_4$$
$$- 2m (m_1^2 m_2^4 + m_2^2 m_1^4) W_2 + m m_1^4 m_2^4 W_0 \tag{2.3.9}$$

where the integrals

$$W_n = \int \frac{d^4k}{(2\pi)^4} \frac{k^n}{(k^2-m^2)^2(k^2-m_1^2)^2(k^2-m_2^2)^2}$$

are computed in Appendix B.

We have the final results for the Γ_A and Γ_F one-point functions

$$\Gamma_A = i6a^2\lambda Q + O(a^4) \tag{2.3.10}$$

$$\Gamma_F = i2a\lambda Q + O(a^3) \tag{2.3.11}$$

$$Q = \frac{1}{(4\pi)^2}\Big[\frac{m_1 m_2}{m_1-m_2}\log\frac{m_2^2}{m_1^2} - \frac{3mm_1^4m_2^2 - mm_1^6}{(m_1^2-m_2^2)^3}\log\frac{m_2^2}{m^2}$$
$$+\frac{3m_2^4mm_1^2 - mm_2^6}{(m_1^2-m_2^2)^3}\log\frac{m_1^2}{m^2} - m\frac{m_1^7-m_2^7-3m_1^5m_2^2+3m_1^2m_2^5}{(m_1^2-m_2^2)^3(m_1-m_2)}\log\frac{m_2^2}{m^2}\Big]$$

where the only restriction on the regulating masses is that $\lim_{\Lambda\to\infty}(m_1/m_2)\neq 0,1,\infty$.

The first remark that we want to make about the results (2.3.10) and (2.3.11) is that they respect the supersymmetric invariance of the model, as long as the supersymmetry Ward identity for the one-point functions is satisfied. This can be derived from the fundamental Ward identity for the effective action reported in eq.(2.1.9). By applying a functional derivative with respect to $\hat{\bar\Psi}(y)$ and putting the sources equal to zero in the expression derived from eq.(2.1.9) we get

$$-\frac{\delta\Gamma}{\delta\hat A}\varepsilon(y) + i\partial\!\!\!/\frac{\delta\Gamma}{\delta\hat F}\varepsilon(y) + a\frac{\delta\Gamma}{\delta\hat F}\varepsilon(y) - \int d^4x\,(-g)^{\frac{1}{2}}\frac{\delta^2\Gamma}{\delta\hat{\bar\Psi}(y)\delta\hat\Psi(x)}\varepsilon(x)\hat F = 0 \tag{2.3.12}$$

where it is understood that all the derivatives are computed at the absolute minimum of the effective potenti=

al. As long as in our case this point corresponds to $\langle A\rangle \approx O(\lambda)$ and $\langle F\rangle \approx O(\lambda)$, we have that, at the first order in perturbation theory, this Ward identity reduces to an identity between the quantum corrections Γ_A and Γ_F that we have computed above

$$-\Gamma_A \varepsilon(y) + i\Gamma_F \not{D}\varepsilon(y) + a\Gamma_F \varepsilon(y) = 0 \tag{2.3.13}$$

or, equivalently, by using the definition $D_\mu \varepsilon(y) = -i\frac{a}{2}\cdot\gamma_\mu \varepsilon(y)$,

$$-\Gamma_A + 3a\,\Gamma_F = 0 \tag{2.3.14}$$

This identity is obviously verified by our results, eqs. (2.3.10), (2.3.11). The interpretation of this fact is, as we suggested already in section 2.1, that these 1PI one-point functions are corrections to a linear superfield insertion in the superpotential of the model. On the other hand, an insertion of this kind is now needed to renormalize Γ_A, Γ_F and the vacuum expectation values of the fields of the model.

By considering the lagrangian $L=L_{kin}+L_m+L_{int}+L_\Lambda$ we can express, for instance,

$$\begin{aligned}\langle A(x)\rangle &= \int \Pi \mathcal{D}\phi_i \, A(x)\, e^{i\int d^4y\sqrt{-g}\,(L_{kin}+L_m+L_{int}+L_\Lambda)} = \\ &= \int \Pi \mathcal{D}\phi_i \, A(x)\, i\int d^4y \sqrt{-g}\,(L_{int}+L_\Lambda)\, e^{i\int d^4y\sqrt{-g}\,(L_{kin}+L_m)} + \cdots \\ &= \int d^4y\sqrt{-g}\,\langle A(x)A(y)\rangle_0 \Gamma_A(y) + i\frac{3}{\sqrt{2}}a\Lambda\int d^4y\sqrt{-g}\,\langle A(x)A(y)\rangle_0 \\ &+ \int d^4y\sqrt{-g}\,\langle A(x)F(y)\rangle_0 \Gamma_F(y) + \frac{i}{\sqrt{2}}\Lambda\int d^4y\sqrt{-g}\,\langle A(x)F(y)\rangle_0 + \cdots\end{aligned} \tag{2.3.15}$$

and, similarly,

$$\langle F(x)\rangle = \int d^4y\sqrt{-g}\,\langle F(x)F(y)\rangle_0\,\Gamma_F(y) + \frac{i}{\sqrt{2}}\Lambda\int d^4y\sqrt{-g}\,\langle F(x)F(y)\rangle_0$$

$$+\int d^4y\sqrt{-g}\,\langle F(x)A(y)\rangle_0\,\Gamma_A(y) + i\frac{3}{\sqrt{2}}a\Lambda\int d^4y\sqrt{-g}\,\langle F(x)A(y)\rangle_0 + \cdots \qquad (2.3.16)$$

All the divergent contributions coming from Γ_A and Γ_F can be cancelled at once in $\langle A(x)\rangle$ and $\langle F(x)\rangle$ by choos= ing $\Lambda/\sqrt{2}=-2\lambda a Q_{div}$, where Q_{div} is defined as the sum of the linear and logarithmic divergencies in Q. After co= -mputing from the equations of motion

$$\int d^4y\sqrt{-g}\,\langle A(x)A(y)\rangle_0 = -i\,\frac{1}{m^2-am-2a^2}$$

$$\int d^4y\sqrt{-g}\,\langle A(x)F(y)\rangle_0 = i\,\frac{m+a}{m^2-am-2a^2}$$

$$\int d^4y\sqrt{-g}\,\langle F(x)F(y)\rangle_0 = -3i\,\frac{am+a^2}{m^2-am-2a^2} \qquad (2.3.17)$$

the renormalized expressions for the vacuum expectation values of the A and F fields turn out to be

$$\langle A\rangle = -2\lambda a\frac{1}{m+a}Q_R + O(a^4) \qquad (2.3.18)$$

$$\langle F\rangle = 6\lambda\frac{am+a^2}{m^2-am-2a^2}aQ_R - 6\lambda\frac{m+a}{m^2-am-2a^2}a^2Q_R$$

$$+O(a^4) = O(a^4) \qquad (2.3.19)$$

where

$$Q_R \equiv Q - Q_{div}$$

Then, it is shown that, at least to order of approxima= tion to which the free-field propagators are given, su=

persymmetry is not broken by radiative corrections, to the one-loop order. This is nothing but another version of the statement given in section 2.1 about the impossi= bility of breaking spontaneously supersymmetry by incl= uding a linear superfield term of the type of the one in eq.(2.1.16) in the superpotential. The proof of the statement given at the classical level seems to be more general, in the sense that it applies to every supersy= mmetric minimum of the effective potential, and not on= ly to the origin A=B=0). It is obvious, though, that aw= ay from the point A=B=0 only the study of the radiative corrections to the equations of motion for the vacuum ex= pectation values of the scalar and auxiliary fields can give a definite answer about the preservation of super= symmetry at the different minima of the model. The ana= lysis of these radiative corrections and the answer to the above question are deferred to the next chapter. In what follows we want, once again, turn to the study of the 1PI two and three-point functions of the model and repete their calculation using a manifestly supersymme= tric regularization scheme.

In the previous section we introduced an extrapolation to the AdS_4 background of the dimensional reduction pro= cedure familiar in flat space-time and the use of the fi= rst adiabatic order of the momentum expansion in the free-field propagators allowed us to apply the renormalization program to our model. The mass and the interaction parts of the action (2.1.4) were unrenormalized, while for the kinetic part we had a common renormalization factor Z_{kin}. In particular, these result guarantee the supersymmetric invariance of the procedure used there for the diverge= nt one-loop quantum corrections.

In the following we confirm the above result of the validity of the no-renormalization of the mass and interaction lagrangians for the model in AdS_4 manifestly preserving supersymmetry with our regularization procedure. To satisfy this requirement we regulate the divergent integrals in the two and three-point functions by means of the supersymmetric Pauli-Villars method. In order to regulate the 1PI two-point functions Γ_{AF}, Γ_{AA}, Γ_{FF}, Γ_{BG}, Γ_{BB}, Γ_{GG}, $\Gamma_{\psi\bar\psi}$ and the 1PI three-point functions Γ_{FAA}, Γ_{AAA}, $\Gamma_{A\psi\bar\psi}$, Γ_{FBB}, Γ_{GAB}, Γ_{ABB}, $\Gamma_{B\psi\bar\psi}$, we need to introduce only one Pauli-Villars regulating field, together with the sum rule

$$1+c=0 \tag{2.3.20}$$

In fact, the unregulated integral I_3 (see below) occurring in the calculation of Γ_{AA} and Γ_{BB} cancels explicitely, by supersymmetry, between the bosonic and the fermionic virtual contributios, in both cases.

Recalling the expression of the Pauli-Villars regulated propagators in eq.(2.3.2) one has, for the divergent part of Γ_{AF},

$$\begin{aligned}\Gamma_{AF}^{div} &= 4\lambda^2 a\left\{\Lambda^4\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)[(k-p)^2-m^2](k^2-\Lambda^2)[(k-p)^2-\Lambda^2]}\right\}_{div} = \\ &= 4\frac{i}{(4\pi)^2}\lambda^2 a\log\Lambda^2\end{aligned} \tag{2.3.21}$$

where Λ is the mass of the regulating field, that ultimately will go to the infinity, $\Lambda\to\infty$. For details on the evaluation of the above integral the reader is invited to consult Appendix B. Thus, we have

$$\Gamma_{AF}^{div}(x,x') = 4\frac{i}{(4\pi)^2}\lambda^2 a \log\Lambda^2\,\delta(x,x') \tag{2.3.22}$$

The comparison of the above result with eq.(2.2.4) gives the correspondence between the Pauli-Villars regularization and the dimensional reduction method

$$\log\Lambda^2 \longleftrightarrow \frac{2}{4-D} \tag{2.3.23}$$

The bosonic virtual contribution to Γ_{AA}^{div} can be expressed as

$$\left(\Gamma_{AA}^{div}\right)_{BOSE} = 4\lambda^2 a^2 I_1^{div} + 8\lambda^2 I_2^{div} + 8\lambda^2 I_3^{div}$$

The integrals are defined as follows

$$I_1 = \int\frac{d^4k}{(2\pi)^4}\left(\frac{1}{k^2-m^2}-\frac{1}{k^2-\Lambda^2}\right)\left(\frac{1}{(k-p)^2-m^2}-\frac{1}{(k-p)^2-\Lambda^2}\right)$$

$$I_2 = \int\frac{d^4k}{(2\pi)^4}\left(\frac{m^2}{k^2-m^2}-\frac{\Lambda^2}{k^2-\Lambda^2}\right)\left(\frac{1}{(k-p)^2-m^2}-\frac{1}{(k-p)^2-\Lambda^2}\right)$$

$$I_3 = \int\frac{d^4k}{(2\pi)^4}\left(\frac{m}{k^2-m^2}-\frac{\Lambda}{k^2-\Lambda^2}\right)\left(\frac{m}{(k-p)^2-m^2}-\frac{\Lambda}{(k-p)^2-\Lambda^2}\right)$$

The contribution from the spinor loop is

$$\left(\Gamma_{AA}^{div}\right)_{FERMI} = -8\lambda^2\left(a^2 J_1^{div} + J_2^{div} + I_3^{div}\right)$$

where

$$J_1 = \int\frac{d^4k}{(2\pi)^4}\,k\cdot(k-p)\left(\frac{1}{(k^2-m^2)^2}-\frac{1}{(k^2-\Lambda^2)^2}\right)\left(\frac{1}{(k-p)^2-m^2}-\frac{1}{(k-p)^2-\Lambda^2}\right)$$

$$J_2 = \int\frac{d^4k}{(2\pi)^4}\,k\cdot(k-p)\left(\frac{1}{k^2-m^2}-\frac{1}{k^2-\Lambda^2}\right)\left(\frac{1}{(k-p)^2-m^2}-\frac{1}{(k-p)^2-\Lambda^2}\right)$$

Therefore, we have

$$\Gamma_{AA}^{div} = 4\lambda^2 a^2 \left(I_1^{div} - 2 J_1^{div} \right) + 8\lambda^2 \left(I_2^{div} - J_2^{div} \right) \tag{2.3.24}$$

From the Appendix B we can extract the expressions

$$I_1^{div} = J_1^{div} = \frac{i}{(4\pi)^2} \log \Lambda^2$$

$$I_2^{div} = \frac{i}{(4\pi)^2} \left(-\Lambda^2 + m^2 \log \Lambda^2 \right)$$

$$J_2^{div} = \frac{i}{(4\pi)^2} \left(-\Lambda^2 + m^2 \log \Lambda^2 - \frac{1}{2} p^2 \log \Lambda^2 \right)$$

Using the relation between $\partial^\mu \partial_\mu$ and $D^\mu D_\mu$ in normal coo= rdinates, eq.(2.2.6), we obtain

$$\Gamma_{AA}^{div}(x,x') = -4 \frac{i}{(4\pi)^2} \lambda^2 \log \Lambda^2 \left(\Box_x - 3a^2 \right) \delta(x,x') \tag{2.3.25}$$

Once again, the correspondence to the dimensional redu= ction case (see eq.(2.2.7)) is realized by eq.(2.3.23).

Γ_{FF}^{div} is given by

$$\Gamma_{FF}^{div} = 4\lambda^2 I_1^{div}$$

and, therefore

$$\Gamma_{FF}^{div}(x,x') = 4 \frac{i}{(4\pi)^2} \lambda^2 \log \Lambda^2 \, \delta(x,x') \tag{2.3.26}$$

It is easily seen that

$$\Gamma_{BG}^{div}(x,x') = \Gamma_{AF}^{div}(x,x') \tag{2.3.27}$$

and

$$\Gamma^{div}_{GG}(x,x') = \Gamma^{div}_{FF}(x,x') \tag{2.3.28}$$

The contributions to Γ^{div}_{BB} are

$$(\Gamma^{div}_{BB})_{BOSE} = 4\lambda^2 a^2 I_1 + 8\lambda^2 I_2 - 8\lambda^2 I_3$$

$$(\Gamma^{div}_{BB})_{FERMI} = -8\lambda^2 (a^2 J_1 + J_2 - I_3)$$

so that

$$\Gamma^{div}_{BB}(x,x') = \Gamma^{div}_{AA}(x,x') \tag{2.3.29}$$

The evaluation of the divergent part of $\Gamma_{\psi\bar\psi}$ gives

$$\Gamma^{div}_{\psi_\alpha\bar\psi_\beta} = -8\lambda^2 \Big[\int \frac{d^4k}{(2\pi)^4} (K)_{\alpha\beta} \Big(\frac{1}{k^2-m^2} - \frac{1}{k^2-\Lambda^2}\Big)\Big(\frac{1}{(k-p)^2-m^2} - \frac{1}{(k-p)^2-\Lambda^2}\Big)\Big]_{div} = -4\frac{i}{(4\pi)^2}\lambda^2 (\not p)_{\alpha\beta} \log\Lambda^2$$

The result is, then,

$$\Gamma^{div}_{\psi_\alpha\bar\psi_\beta}(x,x') = -4\frac{1}{(4\pi)^2}\lambda^2 \log\Lambda^2 (\gamma_{\hat a})_{\alpha\beta} D^{\hat a}_x \delta(x,x') \tag{2.3.30}$$

The above results (2.3.22), (2.3.25), (2.3.30) are in agreement with the ones obtained in the massless case and reported in eqs. (2.2.22), (2.2.23), (2.2.24).

Evaluating the 1PI three-point functions is extremely simple, since their divergent parts vanish by explicit cancellation between the virtual (A,F) and (B,G) contributions

$$\Gamma^{div}_{FAA}(x,y,z) = \Gamma^{div}_{A\psi\bar{\psi}}(x,y,z) = \Gamma^{div}_{FBB}(x,y,z) =$$
$$= \Gamma^{div}_{GAB}(x,y,z) = \Gamma^{div}_{B\psi\bar{\psi}}(x,y,z) = 0 \qquad (2.3.31)$$

The only non-trivial cancellations concern Γ_{AAA} and Γ_{ABB}. The bosonic virtual contribution to Γ^{div}_{AAA} is

$$(\Gamma^{div}_{AAA})_{BOSE} = -96\lambda^3 \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2-m^2} - \frac{1}{k^2-\Lambda^2} \right) \cdot$$
$$\cdot \left(\frac{m}{k^2-m^2} - \frac{\Lambda}{k^2-\Lambda^2} \right) \left(\frac{m^2}{k^2-m^2} - \frac{\Lambda^2}{k^2-\Lambda^2} \right)$$

From the spinor loop we have

$$(\Gamma^{div}_{AAA})_{FERMI} = 96\lambda^3 \int \frac{d^4k}{(2\pi)^4} \left(\frac{m}{k^2-m^2} - \frac{\Lambda}{k^2-\Lambda^2} \right) \cdot$$
$$\cdot \left(\frac{1}{k^2-m^2} - \frac{1}{k^2-\Lambda^2} \right)^2 \cdot k^2 =$$
$$= 96\lambda^3 \int \frac{d^4k}{(2\pi)^4} \left(\frac{m}{k^2-m^2} - \frac{\Lambda}{k^2-\Lambda^2} \right) \left(\frac{1}{k^2-m^2} - \frac{1}{k^2-\Lambda^2} \right) \left(\frac{m^2}{k^2-m^2} - \frac{\Lambda^2}{k^2-\Lambda^2} \right)$$

Therefore

$$\Gamma^{div}_{AAA}(x,y,z) = 0 \qquad (2.3.32)$$

The vanishing of the divergent part of Γ_{ABB} can be verified in a similar manner

$$\Gamma^{div}_{ABB}(x,y,z) = 0 \qquad (2.3.33)$$

It is manifest that supersymmetry is mantained in the above results for the 1PI two and three-point functions, since we have obtained common renormalization factors for the different supersymmetric invariant pieces of the effective action of the model. In the case of the two-point functions, for example, this is equivalent to verify the Ward identity

$$-i\bar{\varepsilon}(y)\frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}(y)\delta\hat{\psi}(x)}\gamma_\mu\varepsilon(x)\overleftarrow{\partial}^\mu_x-\frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{A}(y)}+3a\frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{F}(y)}$$
$$-\bar{\varepsilon}(y)\int d^4z(-g)^{1/2}\frac{\delta^3\Gamma}{\delta\hat{\bar{\psi}}(y)\delta\hat{A}(x)\delta\hat{\psi}(z)}\hat{F}\,\varepsilon(z)=0 \qquad (2.3.34)$$

that can be immediately derived from eq.(2.1.9). After the redefinition $\Gamma=\Gamma_0+\tilde{\Gamma}$ in terms of the tree-level action Γ_0 and the one-loop quantum corrections $\tilde{\Gamma}$, the expression of the above equation recovers, to the one-loop order, the naive form of the Ward identity (2.1.11)

$$-i\bar{\varepsilon}(y)\frac{\delta^2\tilde{\Gamma}}{\delta\hat{\bar{\psi}}(y)\delta\hat{\psi}(x)}\gamma_\mu\varepsilon(x)\overleftarrow{\partial}^\mu_x-\frac{\delta^2\tilde{\Gamma}}{\delta\hat{A}(x)\delta\hat{A}(y)}+3a\frac{\delta^2\tilde{\Gamma}}{\delta\hat{A}(x)\delta\hat{F}(y)}=0 \qquad (2.3.35)$$

This expression is satisfied by $\Gamma^{div}_{\bar{\psi}\psi}$, Γ^{div}_{AA} and Γ^{div}_{AF}. Also, the corresponding Ward identity among the three-point functions is trivially verified. Finally, it is also clear that the no-renormalization theorem is valid in what concerns the mass and the interaction lagrangians. We report our result in terms of the renormalization coefficients

$$Z_{kin}=1+4\frac{\lambda^2}{(4\pi)^2}\log\Lambda^2$$
$$Z_m=1$$
$$Z_{int}=1 \qquad (2.3.36)$$

These results correspond to the ones obtained in the pr= evious section working with dimensional reduction, the correspondence being given by eq.(2.3.23).

In closing this section we would like to mention an unexpected and rather amusing result that is obtained by performing the calculation of the 1PI two and three-point functions with a cut-off regulator. This regulari= zation procedure is a rather brutal one and naturally one would not expect to obtain a supersymmetric result for the divergent quantum corrections. Despite the expe= ctations, the result for the two and three-point functio= ns turn out to be the same as with the other two regula= tors (supersymmetric Pauli-Villars regularization and di= mensional reduction) that have been used to carry the re= normalization of the model. We consider this merely as a one-loop accident.

Introducing the momentum cut-off M and making use of the integral

$$I = \left[\int_{k^2<M^2} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k-p)^2} \right]_{div} = \frac{i}{(4\pi)^2} \log M^2$$

we have

$$\Gamma^{div}_{AF}(x,x') = 4\lambda^2 a I \,\delta(x,x') \tag{2.3.37}$$

$$\Gamma^{div}_{AA}(x,x') = -4\lambda^2 I \left(\Box_x - 3a^2\right) \delta(x,x') \tag{2.3.38}$$

$$\Gamma^{div}_{\psi_\alpha\bar\psi_\beta}(x,x') = 4i\lambda^2 I \left(\gamma_{\hat a}\right)_{\alpha\beta} D^{\hat a}_x \,\delta(x,x') \tag{2.3.39}$$

The calculation of the 1PI three-point functions, using a cut-off in the momentum k, gives again vanishing resu=

lts for all the divergent parts.

We have obtained, at this point, the supersymmetric result using three different regularization procedures -one of these being a momentum cut-off method- and we can say that the choice of the regulator does not affect the divergent renormalization of the model to the one-loop order. With every probability, a cut-off procedure will not provide a supersymmetric result for the finite one-loop corrections, for instance, or in a two-loop calculation, and in this sense the choice of a manifestly supersymmetric regulator like the one we use in the end of section 2.2, as well as in the present section, is opportune.

At this point we can summarize the purpose of the present section, which has been to study the renormalization properties of the Wess-Zumino model in the background of AdS_4. The method of regularization used, a Pauli-Villars regularization procedure, guarantees the preservation of supersymmetry in the computation of the one-loop radiative corrections to the effective action. This offers the possibility of a clear interpretation of the non-vanishing one-point functions Γ_A and Γ_F corresponding to the A and F fields, respectively. In fact, it has been shown above that they respect the supersymmetry Ward identity among the 1PI one-point functions of the model, what naturally leads to the conclusion that they correspond to a renormalization of a linear superfield term in the superpotential. This makes a clear distinction with respect to the model formulated in flat space-time, since one of the contents of the no-renormalization theorem valid there is that a non-vanishing tadpole cannot be generated by radiative corrections [14]. In this sen=se, our result obtained in this section can be viewed as a first example of violation of the mentioned theorem introduced by the effect of the non-vanishing curvature of the non-trivial background space.

The situation concerning the renormalization of the mass and interaction lagrangians remains unchanged with respect to the flat space-time model [19]. We have shown here that the no-renormalization theorem holds in the case of the mass and interaction lagrangians, thus rec=overing the results obtained in the previous section by means of a dimensional reduction method. In fact, there is a complete equivalence between Pauli-Villars regular=ization and dimensional reduction when they are applied to the computation of the one-loop divergent corrections to the effective action.

At this level, the possibility of a violation of the no-renormalization theorem for the mass and interaction lagrangians at higher loops remains certainly open. Ac=tually, the impression generated by the results obtain=ed in the present section is that the above example of renormalization of the linear superfield term in the su=perpotential makes plausible a similar renormalization of the quadratic and cubic superfield terms, since their superfield integrals are mathematically equivalent. How=ever, the investigation of this topic will require the use of superfield techniques, as well as a perturbative approach in the effects of the curvature completely dif=fernt from the one adopted in this chapter.In the seco=nd part of this thesis we will analyze and answer exha=ustively to the questions formulated above.

In concluding this chapter and as an introduction to the next one, we want to mention that our results at this point should be viewed as partial, as long as a similar analysis would have to be done for every minimum of the classical potential and not only for the origin A=B=O. Several issues as, for instance, the universality of the renormalization coefficients and the solution of the one-loop corrected equations of motion will emerge from the investigation to be carried in the next chapter.

3. SEMICLASSICAL VACUUM SOLUTIONS OF THE WESS-ZUMINO MODEL

3.1. The adiabatic expansion of the effective action

We have seen that the flat space-time supersymmetry algebra admits generalization to the case of the fixed background of the four-dimensional Anti-de Sitter space. This fact, which lies in the possibility of extending the isometry group O(3,2) of this space to the wider super= symmetry group OSp(4,1), constitutes the basis for the work giving the basis for the superfield formulation of the different supersymmetric quantum models in this back= ground space [2]. Until now, however, little analysis has been reported about interacting models, in particu= lar their renormalization properties studied in full ge= nerality, as well as the influence of radiative correct= ions on the realization of supersymmetry, although some study of the simplest theory, the Wess-Zumino model, has been carried out in the free-field case [17].

In the previous chapter our approach to the study of the interacting Wess-Zumino model in AdS_4 was that of using a perturbative expansion in the effects of the cu= rvature in order to study the renormalization properties of the model quantized arond one particular classically supersymmetric extremum of the potential of the theory. Also, the use of an adiabatic expansion in the free-fi= eld propagators up to terms proportional to second der= ivatives of the metric proved to be enough to determine all the counterterms of the model. Additionally, the test of dimensional reduction as a valid supersymmetric regu= larization scheme at the one-loop level was carried th= rough the verification of the Ward identities for the divergent parts of the 1PI two and three-point functi= ons.

The picture that arises from the study of the radia= tive corrections to the model seems to indicate that the no-renormalization theorem is valid to the one-loop ord= er in what the mass and coupling constant are concerned. One cannot avoid, however, the introduction of a linear superfield insertion in the superpotential for the sake of making the theory renormalizable at the one loop le= vel. We have observed that this fact can be interpreted as a first example of violation of the no-renormalization theorem, since from the superfield point of view this is equivalent to the generation of a non-vanishing tadpole by radiative corrections. Moreover, we have shown that this insertion does not induce the spontaneous breaking of the supersymmetry invariance of the model, although the proof has been carried in a rather restricted conte= xt, since it has been confined to one of the four super= symmetric extrema of the classical potential.

The object of the present chapter is to generalize the above analysis for every vacuum solution around whi= ch the model can be quantized. This task can be accompl= ished by studying the effective action of the model, that we compute to the one-loop order, in this section, fol= lowing the above mentioned lines of using the adiabatic expansion and a dimensional regularization scheme. The counterterms that were found in the previous renormaliz= ation of the model are now recovered and the powerful functional techniques allow us to show that they are un= iversal and independent of the choice of a definite va= cuum solution. Finally, in the next section, by correc= ting the classical equations of motion of the fields, we will able to show that the supersymmetric character of the minima of the classical potential is not altered by the one-loop radiative corrections.

The supersymmetry invariance of the model is realized at the quantum level in the form of Ward identities among the different n-point functions . From the fundamental Ward identity holding for the effective action Γ, eq. (2.1.9), by deriving with respect to $\hat{\bar{\psi}}(y)$ and putting the sources equal to zero, we obtain

$$
\begin{aligned}
&-\frac{\delta\Gamma}{\delta\hat{A}}\varepsilon(y)+i\overrightarrow{\not{D}}\frac{\delta\Gamma}{\delta\hat{F}}\varepsilon(y)+a\frac{\delta\Gamma}{\delta\hat{F}}\varepsilon(y)-\int d^4x(-g)^{1/2}\frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}(y)\delta\psi(x)}\varepsilon(x)\hat{F}\\
&+\frac{\delta\Gamma}{\delta\hat{B}}i\gamma_5\varepsilon(y)+i\overrightarrow{\not{D}}\frac{\delta\Gamma}{\delta\hat{G}}i\gamma_5\varepsilon(y)-a\frac{\delta\Gamma}{\delta\hat{G}}i\gamma_5\varepsilon(y)-\\
&-\int d^4x(-g)^{1/2}\frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}(y)\delta\psi(x)}i\gamma_5\varepsilon(x)\hat{G}=0
\end{aligned}
$$

or, after using the definition $D_\mu\varepsilon(y)=-\frac{ia}{2}\gamma_\mu\varepsilon(y)$,

$$
\begin{aligned}
&\bar{\varepsilon}(y)\varepsilon(y)\left(-\frac{\delta\Gamma}{\delta\hat{A}}+3a\frac{\delta\Gamma}{\delta\hat{F}}\right)-\bar{\varepsilon}(y)\int d^4x(-g)^{1/2}\frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}(y)\delta\hat{\psi}(x)}\varepsilon(x)\hat{F}\\
&+\frac{\delta\Gamma}{\delta\hat{B}}\bar{\varepsilon}(y)i\gamma_5\varepsilon(y)-3a\frac{\delta\Gamma}{\delta\hat{G}}\bar{\varepsilon}(y)i\gamma_5\varepsilon(y)\\
&-\bar{\varepsilon}(y)\int d^4x(-g)^{1/2}\frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}(y)\delta\hat{\psi}(x)}i\gamma_5\varepsilon(x)\hat{G}=0
\end{aligned}
\qquad (3.1.1)
$$

This expression leads, in the flat space-time limit, to the usual statement that the spontaneous breaking of supersymmetry implies the appearance of a massless fermion. However, our pretension here is to use the above formula to test the supersymmetric invariance of our computation, that we are going to exhibit in the following, of the radiative corrections to the derivatives of the effective action $\delta\Gamma/\delta\hat{A}$, $\delta\Gamma/\delta\hat{F}$, $\delta\Gamma/\delta\hat{B}$, $\delta\Gamma/\delta\hat{G}$. With this purpose, we split the effective action to the one-loop order into its tree-level part Γ_0 and radiative correction Γ_1, $\Gamma=\Gamma_0+\Gamma_1$. Then, it easily seen that he previous identity transforms into

$$\bar{\varepsilon}\varepsilon\left(-\frac{\delta\Gamma_1}{\delta\hat{A}}+3a\frac{\delta\Gamma_1}{\delta\hat{F}}\right)+\frac{\delta\Gamma_1}{\delta\hat{B}}\bar{\varepsilon}i\gamma_5\varepsilon-3a\frac{\delta\Gamma_1}{\delta\hat{G}}\bar{\varepsilon}i\gamma_5\varepsilon=0 \qquad (3.1.2)$$

This is the desired identity for the one-point functions of the model that will be used subsequently.

In analogous way, by deriving the fundamental identity (2.1.9) with respect to $\hat{\bar{\psi}}(y)$ and $\hat{A}(x)$ and then setting all the sources equal to zero, we can obtain

$$\begin{aligned}
&i\frac{\delta^2\Gamma}{\delta\hat{\bar{\psi}}(y)\,\delta\hat{\psi}(z)}\gamma^\mu\left[\partial_\mu\frac{1}{\sqrt{-g}}\delta(x-z)\right]\varepsilon(z)-\frac{\delta^2\Gamma}{\delta\hat{A}(x)\,\delta\hat{A}(z)}\frac{1}{\sqrt{-g}}\delta(y-z)\varepsilon(z)\\
&+\frac{\delta^2\Gamma}{\delta\hat{A}(x)\,\delta\hat{B}(z)}i\gamma_5\frac{1}{\sqrt{-g}}\delta(y-z)\varepsilon(z)-i\frac{\delta^2\Gamma}{\delta\hat{A}(x)\,\delta\hat{F}(z)}\left[\frac{1}{\sqrt{-g}}\delta(y-z)\overleftarrow{\not{D}}\right]\varepsilon(z)\\
&+a\frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{F}(z)}\frac{1}{\sqrt{-g}}\delta(y-z)\varepsilon(z)-i\frac{\delta^2\Gamma}{\delta\hat{A}(x)\,\delta\hat{G}(z)}\left[\frac{1}{\sqrt{-g}}\delta(y-z)\overleftarrow{\not{D}}\right]i\gamma_5\varepsilon(z)\\
&-a\frac{\delta^2\Gamma}{\delta\hat{A}(x)\delta\hat{G}(z)}\frac{1}{\sqrt{-g}}\delta(y-z)i\gamma_5\varepsilon(z)-\frac{\delta^3\Gamma}{\delta\hat{\bar{\psi}}(y)\delta\hat{\psi}(z)\delta\hat{A}(x)}\varepsilon(z)\hat{F}\\
&-\frac{\delta^3\Gamma}{\delta\hat{\bar{\psi}}(y)\,\delta\hat{\psi}(z)\,\delta\hat{A}(x)}i\gamma_5\varepsilon(z)\hat{G}=0
\end{aligned} \qquad (3.1.3)$$

Once again, by splitting the effective action in the form $\Gamma=\Gamma_0+\Gamma_1$, the Ward identity for the one-loop radiative corrections to the two-point functions of the theory turns out to be

$$\begin{aligned}
&-i\bar{\varepsilon}(y)\frac{\delta^2\Gamma_1}{\delta\hat{\bar{\psi}}(y)\delta\hat{\psi}(x)}\gamma_\mu\varepsilon(x)\overleftarrow{D}^\mu-\frac{\delta^2\Gamma_1}{\delta\hat{A}(x)\,\delta\hat{A}(y)}\bar{\varepsilon}(y)\varepsilon(y)\\
&+\frac{\delta^2\Gamma_1}{\delta\hat{A}(x)\delta\hat{B}(y)}\bar{\varepsilon}(y)i\gamma_5\varepsilon(y)+3a\frac{\delta^2\Gamma_1\,\bar{\varepsilon}(y)\varepsilon(y)}{\delta\hat{A}(x)\delta\hat{F}(y)}-3a\frac{\delta^2\Gamma_1}{\delta\hat{A}(x)\delta\hat{G}(y)}\bar{\varepsilon}(y)i\gamma_5\varepsilon(y)=0
\end{aligned} \qquad (3.1.4)$$

This is the expression that we will need, together with eq.(3.1.2), to check the invariance of the renormalization procedure for the one-loop effective action. A similar Ward identity for the three-point functions of the model can be derived in a straigthforward way, but we do not need to go into its consideration at this point, as long as one of the results that our computation of the effective action establishes is the no-renormalization of S_{int} to the one-loop order.

We now proceed to the computation of the effective action of the theory to the one-loop order. This will provide us the infinite counterterms needed to renormalize the theory at the given perturbative order, as well as finite terms that we will use to correct the classical equations of motion of the fields. Before to go into this computation, we want to describe the kind of perturbative expansion in the curvature effects that we are going to use. In this sense, we will show in this section that an adiabatic expansion applied to the Green functions of the model can be easily implemented in the functional computation and renormalization of the one-loop effective action.

It is well-known that, given the action of a quantum field theory $S(\phi)$, the one-loop order effective action of the model can be computed by performing a classical shift $\hat{\phi}$ of the fields in the original action and then integrating the lagrangian quadratic in the quantum fields $L_q(\phi;\hat{\phi})$ [22]

$$\Gamma_1(\hat{\phi}) = i \log \int \mathcal{D}\phi \, e^{i\int d^4x \sqrt{-g}\, L_q(\phi;\hat{\phi})} \qquad (3.1.5)$$

After performing the gaussian integrals over all the fields, this expression can be cast in the form

$$\Gamma_1(\hat{\phi}) = i \sum_i c_i \, \mathrm{Tr} \log(-G_i) \qquad (3.1.6)$$

where G_i are different Green functions corresponding to each one of the fields, and $c_i = 1, -1/2$ for spinors and scalars, respectively.

Given the DeWitt-Schwinger expansion of the matrix element [23,24]

$$G_i(x,x') = \frac{-i}{(4\pi)^{n/2}} \int_0^\infty d(is)\, e^{-iM_i^2 s} e^{-\frac{i}{4}\frac{(x-x')^2}{s}} F_i(x,x';is)(is)^{-\frac{n}{2}} \qquad (3.1.7)$$

one can compute, after discarding a curvature-independe= nt additive constant,

$$\mathrm{Tr}\log(-G_i) = i\int d^n x\sqrt{-g}\,\frac{1}{(4\pi)^{n/2}}\int_0^\infty d(is)\, e^{-iM_i^2 s} F_i(x,x';is)\Big|_{x=x'}(is)^{-\frac{n}{2}-1} \qquad (3.1.8)$$

The effective action, to the one-loop order, turns out to be

$$\Gamma_1(\hat{\phi}) = \int d^n x\sqrt{g}\sum_i(-c_i)\frac{1}{(4\pi)^{n/2}}\int_0^\infty d(is)\, e^{-iM_i^2 s} F_i(x,x;is)(is)^{-\frac{n}{2}-1} \qquad (3.1.9)$$

This is an exact expression in the effects of the curva= ture, since the function F (x,x';is) is intended to be the exact kernel of the integral representation of G_i. In general, though, one has to deal with some kind of perturbative expansion. For the sake of studying the re= normalization properties of the model it is convenient to choose an adiabatic expansion of the type

$$F(x,x;is) = a_0(x) + a_1(x)\,is + a_2(x)(is)^2 + \cdots \qquad (3.1.10)$$

In a dimensional regularization scheme, for instance, it is easily seen from eq.(3.1.9) that, with this kind of short distance expansion, all the divergent quantiti= es renormalizing the effective action are concentrated in the first three adiabatic orders. Alternatively, ta= king into account that higher orders of the adiabatic e= xpansion contain higher derivatives of the metric of the space, one can understand this power series as a pertur=bative expansion in the curvature effects and carry it to the desired order. This will be, in fact, our point of view, although, when dealing with the finite correc=tions to the effective action, we will only include ter=ms with up to the second derivatives in the metric.

In what follows we are going to renormalize the effe=ctive action by means of a dimensional regularization prescription. At the same time, we are going to show that this procedure respects the supersymmetric invariance of the model to the one-loop order, not only for the di=vergent counterterms, but also for the leading correctio=ns to the equations of motion of the fields derived from the effective potential. Working in an arbitrary dimens=ion n, one can express the one-loop effective action as follows

$$\Gamma_1(\hat{\phi}) = \int d^n x \sqrt{-g} \sum_i (-c_i) \frac{1}{(4\pi)^{n/2}} \int_0^\infty d(is)\, e^{-iM_i^2 s} F_i(x,x;is)(is)^{-\frac{n}{2}-1} (\nu^2)^{-\frac{n}{2}+2} =$$

$$= \int d^n x \sqrt{-g} \sum_i (-c_i) \frac{1}{(4\pi)^{n/2}} \sum_{j=0}^{\infty} a_j^{(i)}(x) \int_0^\infty d(is)\, e^{-iM_i^2 s} (is)^{j-\frac{n}{2}-1} (\nu^2)^{-\frac{n}{2}+2} =$$

$$= \int d^n x \sqrt{-g} \sum_i (-c_i) \frac{1}{(4\pi)^{n/2}} \sum_{j=0}^{\infty} a_j^{(i)}(x) (M_i^2)^{\frac{n}{2}-j}\, \Gamma\!\left(j-\frac{n}{2}\right) (\nu^2)^{-\frac{n}{2}+2} \qquad (3.1.11)$$

where we have introduced the mass parameter ν in order to mantain the above expression dimensionless. Then, by putting ε =4-n and taking the limit $\varepsilon \to 0$, we can compute all the divergent terms in the form

$$\Gamma_1(\hat{\phi}) = \int d^n x \sqrt{-g} \sum_i (-c_i) \frac{1}{(4\pi)^2} \left\{ \frac{2}{\varepsilon} - \left[\gamma_E + \log\left(\frac{M_i^2}{\nu^2}\right) \right] \right\} \cdot$$

$$\cdot \left(\frac{M_i^4}{2} a_0^{(i)} - M_i^2 a_1^{(i)} + a_2^{(i)} \right) + \text{finite } O(a_3) \text{ terms} \qquad (3.1.12)$$

As we will see below, the operators involved in our actual computation are the inverse of the Klein-Gordon operator

$$\left[\Box + M_{(0)}^2 - 2a^2\right] G_{(0)}(x,x') = -(-g)^{-1/2}\,\delta^n(x-x') \qquad (3.1.13)$$

and the inverse of the squared Dirac equation

$$\left[\Box + M_{(\frac{1}{2})}^2 - 3a^2\right] G_{(\frac{1}{2})}(x,x') = -(-g)^{-\frac{1}{2}}\,\delta^n(x-x') \qquad (3.1.14)$$

The expansions up to order $O(a^2)$ of the respective kernels $F_{(0)}$ and $F_{(1/2)}$ have been computed in Ref. [5] and will be illustrated in section 4.1, giving the result

$$F_{(0)}(\mu, s) = 1 + \frac{1}{4}\mu^2 a^2 + O(a^4)$$

$$F_{(1/2)}(\mu, s) = \mathbb{1} + \left(is + \frac{1}{4}\mu^2\right) a^2\,\mathbb{1} + O(a^3) \qquad (3.1.15)$$

μ^2 having the meaning of the square of the proper distance in AdS_4. These expressions have the degree of accuracy that is needed for the renormalization of the fields of the model. The order $O(a^4)$ in both expansions can give rise to divergent contributions to the effective action, but it is obvious that these should be interpreted as the renormalization of pure gravitational tensors. In this section we are interested in the determination of the counterterms of the matter model, that can be at most of dimension 2 in energy units, so that orders higher than $O(a^2)$ can be safely discarded. The coincidence limit of $F_{(0)}$ and $F_{(\frac{1}{2})}$ provides the respective adiabatic orders

$$a_0^{(0)} = 1$$

$$a_1^{(0)} = 0$$

$$a_0^{(1/2)} = 1$$

$$a_1^{(1/2)} = a^2 \qquad (3.1.16)$$

It is worth mentioning that it has also been shown in Ref. [5] (see section 4.1) that the expansions (3.1.15) for $F_{(0)}$ and $F_{(1/2)}$ satisfy the supersymmetry Ward identity that relates the scalar propagator to the spinor propagator in AdS_4 [17]. This clears any doubt about the fact that the use of the adiabatic representations for both propagators can be implemented in a way that preserves the supersymmetry invariance of the model, at least for the determination of divergent counterterms and finite $O(a^2)$ contributions to the effective action.

Following the steps anticipated above, we shift the scalar fields around their classical values

$$A \to A + \hat{A}\ , \quad B \to B + \hat{B}\ , \quad F \to F + \hat{F}\ , \quad G \to G + \hat{G} \tag{3.1.17}$$

and integrate over the quantum fields in order to evaluate the one-loop effective action of the model

$$\Gamma_1(\hat{A},\hat{B},\hat{F},\hat{G}) = i \log \int \mathcal{D}A\, \mathcal{D}B\, \mathcal{D}F\, \mathcal{D}G\, \mathcal{D}\psi\, e^{iS} \tag{3.1.18}$$

where S is given by

$$\begin{aligned}
S = \int d^4x \sqrt{g}\, \Big\{ & \tfrac{1}{2}\left(D_\mu A D^\mu A + D_\mu B D^\mu B + i\bar{\psi} \not{D} \psi + F^2 + G^2\right) + \\
& + a(AF + BG) + \tfrac{3}{2} a^2 (A^2 + B^2) \\
& + m\left[AF - BG + \tfrac{3}{2} a^2 (A^2 - B^2) - \tfrac{1}{2}\bar{\psi}\psi\right] \\
& + \lambda\big(A^2\hat{F} + 2AF\hat{A} - B^2\hat{F} - 2BF\hat{B} - 2AB\hat{G} - 2GA\hat{B} \\
& - 2BG\hat{A} + 3aA^2\hat{A} - 6aAB\hat{B} - 3aB^2\hat{A} - \bar{\psi}\psi\hat{A} \\
& - \bar{\psi} i\gamma_5 \psi \hat{B}\big)\Big\}
\end{aligned} \tag{3.1.19}$$

The result of the formal integration is

$$\begin{aligned}
\Gamma_1(\hat{A},\hat{B},\hat{F},\hat{G}) = {} & i\,\mathrm{Tr}\log\left[D^\mu D_\mu - 3a^2 + (m + 2\lambda\hat{A})^2 + 4\lambda^2\hat{B}^2\right] \\
& - \tfrac{i}{2}\mathrm{Tr}\log\left[D^\mu D_\mu - 2a^2 + (m + 2\lambda\hat{A})^2 + 4\lambda^2\hat{B}^2 - \alpha\right] \\
& - \tfrac{i}{2}\mathrm{Tr}\log\left[D^\mu D_\mu - 2a^2 + (m + 2\lambda\hat{A})^2 + 4\lambda^2\hat{B}^2 + \alpha\right]
\end{aligned} \tag{3.1.20}$$

being

$$\alpha = \left[(am + 2\lambda\hat{F} + 2\lambda a\hat{A})^2 + 4\lambda^2 (\hat{G} + a\hat{B})^2 \right]^{1/2}$$

The use of the adiabatic expansion up to the second derivatives of the metric allows us to evaluate the divergent part of Γ_1. At the same time, the finite part of the effective action can be determined within the same perturbative approach up to terms of order a^2. From eq. (3.1.12) we have

$$L_{eff} = \sum_i (-c_i) \frac{1}{(4\pi)^2} \left\{ \left(\frac{M_i^4}{2} a_0^{(i)} - M_i^2 a_1^{(i)} \right) \Delta + \left[-\log \frac{M_i^2}{\nu^2} \left(M_i^4 \frac{a_0^{(i)}}{2} - M_i^2 a_1^{(i)} \right) + O(a^4) \right] \right\} \tag{3.1.21}$$

where the divergent quantity Δ is given by

$$\Delta = \frac{2}{\varepsilon} - \gamma_E + \log 4\pi$$

In eq.(3.1.21) we have explicitely indicated the order of the remaining finite terms in the perturbative expansion, which will be neglected in what follows, sistematically.

We are now in position to calculate the divergent part of the effective action Γ. Reçalling eq.(3.1.20) and making use of eq.(3.1.21) with c_i, M_i and the coefficients a_0, a_1 appropriate for spin 0 and spin 1/2 particles, we have

$$\Gamma_1 \Big|_{div} = \frac{1}{(4\pi)^2} \frac{\Delta}{2} \int d^4x \sqrt{-g} \left[3a^2m^2 + 4\lambda a m (3a\hat{A} + \hat{F}) + 8\lambda^2 \left(a\hat{A}\hat{F} + a\hat{B}\hat{G} + \frac{3}{2}a^2\hat{A}^2 + \frac{3}{2}a^2\hat{B}^2 + \frac{1}{2}\hat{F}^2 + \frac{1}{2}\hat{G}^2 \right) \right] \tag{3.1.22}$$

The first remark that it is necessary for us to make about the above result is that it verifies the Ward ide=

ntities (3.1.2) and (3.1.4). This certifies again the already known result that dimensional reduction (and, in fact, any dimensional regularization method) is a supersymmetric regularization procedure for the model at the one-loop level (see the previous chapter).

Further inspection shows that L_{kin} is renormalized by

$$Z_{kin} - 1 = \frac{1}{(4\pi)^2} 4\lambda^2 \Delta \tag{3.1.23}$$

and that L_m and L_{int} are unrenormalized to the one-loop order. Thus, we obtain the result, advanced in section 2.2, that the no-renormalization of the mass and interaction lagrangians derived in flat space-time[14,19] still holds in this non-trivial background. Furthermore, the derivation given here is completely general, in the sense that it is independent of the choice of any particular vacuum state. This, in turn, can be seen as a manifestation of the more general result concerning the universality of the renormalization coefficients in the model, that is also made explicit in our computation.

Finally, we turn our attention to the terms in eq. (3.1.22) that are linear in the fields. They do not correspond to the renormalization of any term already present in the action (2.1.4), although we have seen that they verify the supersymmetric Ward identity (3.1.2). In fact, this points in the direction that they must correspond to the renormalization of a supersymmetric insertion in the action of the model, that turns out to be the linear superfield insertion mentioned in the previous chapter. Thus, we are led to the conclusion that, in order to achieve renormalizability, we have to include S_Λ in the original action (see (2.1.16)). Although this introduces a new parameter Λ in the discussion of the model, we have shown in section 2.3 that, at least when the model is quantized around the $A=B=0$ minimum, the supersymmetric invariance is not spontaneously bro= ken by this radiative effect. The discussion of what ha= ppens in the case of the remaining classically supersym= metric minima is the object of the next section, in whi= ch the general treatment of the equations of motion of the fields of the theory allows us to elucidate the iss= ue of the persistence or breaking of supersymmetry at every minimum.

3.2. The one-loop effective potential and the solution of the equations of motion

Our aim remains to perform the computation of the effective potential of the Wess-Zumino model in AdS_4 to the one-loop order, by making use of our tool of the adiabatic expansion in the effective action of the model. This has given us the possibility to obtain, in the above section, the renormalization coefficients of the theory and to prove that they are independent of the choice of a particular extremum of the classical potential (provided, of course, that the extremum is chosen to be classically supersymmetric). In spite of the fact that a linear superfield insertion in the superpotential has proved to be needed in order to renormalize the model, we are able now to show, with complete generality, that supersymmetry is preserved to the one-loop order at every one of the four classically supersymmetric minima of the effective potential.

The one-loop effective potential for the renormalized theory has the finite value

$$
\begin{aligned}
V_{eff}^{(1)} = \frac{1}{(4\pi)^2}\frac{1}{2}\Big\{ & -\left(M^4 - 2a^2 M^2\right)\log\frac{M^2}{V^2} + \\
& + \left[\frac{1}{2}\left(M^2-\alpha\right)^2 \qquad\qquad \right]\log\frac{M^2-\alpha}{V^2} \\
& + \left[\frac{1}{2}\left(M^2+\alpha\right)^2 \qquad\qquad \right]\log\frac{M^2+\alpha}{V^2}\Big\} + O(a^4)
\end{aligned}
\qquad (3.2.1)
$$

where

$$M^2 = \left(m + 2\lambda\hat{A}\right)^2 + 4\lambda^2\hat{B}^2$$

$$\alpha = [(am + 2\lambda\hat{F} + 2\lambda a\hat{A})^2 + 4\lambda^2(\hat{G} + a\hat{B})^2]^{1/2}$$

Our purpose is , specifically, to study the structure of the minima of the one-loop order corrected effective potential of the model. In order to accomplish this task, we need to calculate the derivatives of $V_{eff}^{(1)}$ with respect to the scalar and auxiliary fields. The following expressions hold

$$\frac{\partial V_{eff}^{(1)}}{\partial\hat{F}} = \frac{1}{(4\pi)^2}\Big\{\Big[-(M^2-\alpha)\qquad\Big]\log\frac{M^2-\alpha}{v^2} + \Big[(M^2+\alpha)\qquad\Big]\log\frac{M^2+\alpha}{v^2} + \alpha\Big\}\frac{\lambda(am+2\lambda\hat{F}+2\lambda a\hat{A})}{\alpha} \tag{3.2.2}$$

$$\begin{aligned}\frac{\partial V_{eff}^{(1)}}{\partial\hat{A}} = \frac{1}{(4\pi)^2}\Big[&-4\lambda(m+2\lambda\hat{A})(M^2-a^2)\log\frac{M^2}{v^2}\\ &+2\lambda\Big(m+2\lambda\hat{A}-a\frac{am+2\lambda\hat{F}+2\lambda a\hat{A}}{2\alpha}\Big)\Big(M^2-\alpha\qquad\Big)\log\frac{M^2-\alpha}{v^2}\\ &+2\lambda\Big(m+2\lambda\hat{A}+a\frac{am+2\lambda\hat{F}+2\lambda a\hat{A}}{2\alpha}\Big)\Big(M^2+\alpha\qquad\Big)\log\frac{M^2+\alpha}{v^2}\\ &+4\lambda a^2(m+2\lambda\hat{A})+\lambda a(am+2\lambda\hat{F}+2\lambda a\hat{A})\Big]\end{aligned} \tag{3.2.3}$$

In the following, we are going to assume that $\hat{F}\approx O(\lambda)$ and $\hat{G}\approx O(\lambda)$, since we are interested in determining the shift in the supersymmetric minima of the classical potential induced by the one-loop quantum corrections. Then, neglecting terms that are $O(a^4)$, we can rewrite the above derivatives as follows

$$\frac{\partial V_{eff}^{(1)}}{\partial\hat{F}} = \frac{1}{(4\pi)^2}\,2\lambda a(m+2\lambda\hat{A})\Big[\log\frac{(m+2\lambda\hat{A})^2+4\lambda^2\hat{B}^2}{v^2} + \frac{3}{2}\Big] + O\Big(\frac{\lambda a^3}{m}\Big) \tag{3.2.4}$$

$$\frac{\partial V_{eff}^{(1)}}{\partial \hat{A}} = \frac{1}{(4\pi)^2} 6\lambda a^2 (m+2\lambda\hat{A}) \left[\log \frac{(m+2\lambda\hat{A})^2 + 4\lambda^2\hat{B}^2}{\nu^2} + \frac{3}{2} \right] + O\left(\frac{\lambda a^4}{m}\right) \quad (3.2.5)$$

It should be noticed that, in working out the expressions (3.2.2) and (3.2.3), we have neglected contributions of order λ^3 that are higher order in the loop expansion. In complete analogy to eqs.(3.2.4), (3.2.5), we can determine the first derivatives of the expression (3.2.1) with respect to the classical fields $\hat{G}$ and $\hat{B}$,

$$\frac{\partial V_{eff}^{(1)}}{\partial \hat{G}} = \frac{1}{(4\pi)^2} 4\lambda^2 a \hat{B} \left[\log \frac{(m+2\lambda\hat{A})^2 + 4\lambda^2\hat{B}^2}{\nu^2} + \frac{3}{2} \right] + O\left(\frac{\lambda a^3}{m}\right) \quad (3.2.6)$$

$$\frac{\partial V_{eff}^{(1)}}{\partial \hat{B}} = \frac{1}{(4\pi)^2} 12\lambda^2 a^2 \hat{B} \left[\log \frac{(m+2\lambda\hat{A})^2 + 4\lambda^2\hat{B}^2}{\nu^2} + \frac{3}{2} \right] + O\left(\frac{\lambda a^4}{m}\right) \quad (3.2.7)$$

We have the result that supersymmetry is explicitely mantained from the above derivatives. In fact, they satisfy the supersymmetric Ward identity (3.1.2), what shows that the use of the dimensional regularization procedure is perfectly adequate to determine the finite part of the one-loop effective potential. We believe that the contraction limit operation on the propagators, implicit in the calculation of a local object such as the effective potential, be responsible for the above nice result.

The vacuum solutions of the model at the one-loop level are obtained from the equatios of motion for the scalar and the auxiliary fields

$$\frac{\partial}{\partial \hat{F}}\left(V_0 + V_{eff}^{(1)}\right) = 0 \quad , \quad \frac{\partial}{\partial \hat{G}}\left(V_0 + V_{eff}^{(1)}\right) = 0$$

$$\frac{\partial}{\partial \hat{A}}\left(V_0 + V_{eff}^{(1)}\right) = 0 \quad , \quad \frac{\partial}{\partial \hat{B}}\left(V_0 + V_{eff}^{(1)}\right) = 0 \quad (3.2.8)$$

At this point, we feel free to set the renormalized physical value of the Λ parameter equal to zero, since we have seen that it cannot lead to the breaking of supersymmetry at any of the minima of the potential. Then, we take V_0 as the tree-level potential in eq.(2.1.13) and obtain, by neglecting terms of order $O(a^4)$, the following set of one-loop order corrected equatios of motion

$$\begin{aligned}
&\hat{F} = -a\hat{A} - m\hat{A} - \lambda\hat{A}^2 + \lambda\hat{B}^2 - \lambda Q \\
&\hat{G} = -a\hat{B} + m\hat{B} + 2\lambda\hat{A}\hat{B} - \lambda R \\
&a\hat{F} + 3a^2\hat{A} + m\hat{F} + 3am\hat{A} + 2\lambda\hat{A}\hat{F} - 2\lambda\hat{B}\hat{G} + 3\lambda a\hat{A}^2 \\
&\quad - 3\lambda a\hat{B}^2 + 3a\lambda Q = 0 \\
&a\hat{G} + 3a^2\hat{B} - m\hat{G} - 3am\hat{B} - 2\lambda\hat{B}\hat{F} - 2\lambda\hat{A}\hat{G} \\
&\quad - 6\lambda a\hat{A}\hat{B} + 3a\lambda R = 0
\end{aligned} \tag{3.2.9}$$

where

$$Q \equiv -\frac{1}{(4\pi)^2}\, 2a(m + 2\lambda\hat{A})\left[\log\frac{(m+2\lambda\hat{A})^2 + 4\lambda^2\hat{B}^2}{v^2} + \frac{3}{2}\right]$$

$$R \equiv -\frac{1}{(4\pi)^2}\, 4\lambda a\hat{B}\left[\log\frac{(m+2\lambda\hat{A})^2 + 4\lambda^2\hat{B}^2}{v^2} + \frac{3}{2}\right]$$

While it is technically impossible to solve exactly these equations, it is also true that we only need to have a consistent solution of them to the one-loop order, i.e. up to order λ^2. Then, we can start from the supersymmetric minima of the classical potential given in section 2.1, that we quote generically as A_0, B_0, and compute the corresponding shift in the A-B plane, that we call A_1, B_1,

$$A_{vac} = A_0 \cdot \left(\frac{1}{\lambda}\right) + A_1 \cdot (\lambda) + \ldots$$

$$B_{vac} = B_0 \cdot \left(\frac{1}{\lambda}\right) + B_1 \cdot (\lambda) + \ldots$$

The vacuum solutions that we obtain in this way are

i) $A_0 = B_0 = 0$

$$A_1 = -\frac{\lambda Q|_{\hat{A}=\hat{B}=0}}{a+m} \quad , \quad B_1 = 0 \tag{3.2.10}$$

ii) $A_0 = -(m+a)/\lambda$, $B_0 = 0$

$$A_1 = \frac{\lambda Q|_{\hat{A}=\hat{B}=0}}{a+m} \quad , \quad B_1 = 0 \tag{3.2.11}$$

iii) and iv) $A_0 = (-m+a)/2\lambda$, $B_0 = \pm\sqrt{(a-m)(3a+m)}/2\lambda$

$$A_1 = -2a\lambda \frac{1}{(4\pi)^2}\left(\log\frac{4a^2-2am-m^2}{v^2} + \frac{3}{2}\right)$$

$$B_1 = \mp 6a^2 \frac{1}{\sqrt{(a-m)(3a+m)}} \lambda \frac{1}{(4\pi)^2}\left(\log\frac{4a^2-2am-m^2}{v^2} + \frac{3}{2}\right) \tag{3.2.12}$$

Moreover, the result that can be obtained from these expressions is that supersymmetry is not broken by rad= iative corrections at any one of these minima, $F_1 = G_1 = 0$. Of course, this was expected in the case of the classi= cal A=B=0 minimum, since this is what a dynamical comp= utation from the 1PI one-point functions of the model has shown in section 2.3. Our result can be seen as an exte=

nsion to every supersymmetric minimum of the potential of the proof of the persistence of supersymmetry when quantum corrections are taken into account. It is worth= while, though, to remark the impossibility of setting in eqs.(3.2.10), (3.2.11) and (3.2.12) all the minima at their original values A_0 , B_0 , by an adequate choice of the renormalization mass scale ν , since this would require a dependence of ν upon the parameter a which is purely a feature of the external background geometry and not of the quantum system. This seems to make a clear distinction with the models in which supersymmetry is realized in flat space-time, although, independently of this fact, the conditions for the explicit realization of the supersymmetric invariance, $F_1 = G_1 = 0$, are automati= cally fulfilled at every one of the minima.

Let us try to draw some conclusions at the end of the present chapter, in which we have carried out an analy= sis of the renormalization properties of the Wess-Zumino model in AdS_4 at the one-loop level. This analysis has been subsequently used in our main purpose of studying the structure of the vacuum solutions in the quantized theory and, in particular, the persistence or breaking of supersymmetry at these solutions, when quantum corre= ctions are taken into account.

A dimensional regularization method has been impleme= nted in the computation of the one-loop effective poten= tial of the model, which has provided the counterterms needed to renormalize the theory to the given perturba= tive order. It has been reassuring to check that the su= persymmetric invariance is presrved at the one-loop le= vel by dimensional regularization not only for the diver= gent counterterms but also for the renormalized effecti= ve potential, at least to the order in the effects of the curvature of the space to which is given in the pre=

sent calculation. In performing the above test, our fo= rmer derivation of the fundamental Ward identity for the effective action has been applied to the effective pote= ntial of the model.

One of the points to which we confer more significan= ce in our renormalization of the theory is the universa= lity of the renormalization coefficients, understanding by this term that these are independent from the quanti= zation of the model around any particular vacuum solution. This result comes in a natural way when the powerful fu= nctional techniques are used in the computation of the effective action of the model, and allows us to recover the counterterms that were obtained in the previous cha= pter from the choice of a particular vacuum solution. Now, within the context of a dimensional regularization procedure, we remark the appearence in the effective ac= tion of the model of a divergent supersymmetric invaria= nt term linear in the scalar and auxiliary fields, that has been recognized, already in section 2.3, as the re= normalization of a linear superfield insertion not pre= sent in the original superpotential of the model. In the superfield language, this corresponds to the generation of a non-vanishing tadpole by radiative corrections and could be viewed as a first example of violation of the no-renormalization theorem. Moreover, the no-renormaliz= ation of the mass and interaction lagrangians of the mo= del is also immediately noticed by inspection of the di= vergent structure of the effective action in eq.(3.1.22).

Obtaining the renormalized expression of the effecti= ve potential to the one-loop order has allowed us to per= form the analysis of the vacuum solutions to the quanti= zed model. Starting from every one of the four supersy= mmetric invariant extrema that the model possesses at the classical level, we have computed the shift in the=

ir position induced by the one-loop quantum corrections. This is a generalization of the computation performed for the model quantized around the A=B=O solution, con= sidered in the previous chapter, in which case we show= ed explicitely that supersymmetry cannot be spontaneous= ly broken by introducing a linear superfield insertion in the superpotential. Away from the origin A=B=O, thou= gh, one has to go into a careful examination of the eq= uations of motion for the arguments of the effective ac= tion. In this sense, one of the features of our calcula= tion has been the impossibility of setting all the extr= ema to their classical values by an adequate choice of the renormalization mass scale. This seems to make a di= stinction with respect to the flat space-time realizatio= ns of the supersymmetry invariance, in which usually the no-renormalization theorem fixes the position of the e= xtrema of the effective action at their tree-level valu= es. We have also shown, finally, that , in spite of th= is effect, some conspiracy exists so that there are no induced shifts in the F or G directions and the spontan= eous breaking of supersymmetry never takes place at any one of the vacuum solutions of the model, to the given perturbative order.

4. BOUNDARY CONDITIONS AND RENORMALIZATION IN ANTI-DE SITTER SUPERSYMMETRY

4.1. The role of the boundary conditions

It is our purpose to show that the expansion in powers of the contraction parameter of AdS_4 of the exact scalar and spinor propagators reproduces the standard adiabatic expansion described above, up to an order beyond which no contribution to divergent counterterms can be expected. Our following proof has the crucial relevance to dismiss the claim about the role played by boundary conditions in the renormalization of supersymmetric models.

In the Introduction we have seen how AdS_4 emerges as a ground state solution of extended supergravity theories and Kaluza-Klein theories. The relevance of choosing 'reflective' boundary conditions (RBC) in solving the propagator equation in AdS_4, as well as the importance of this choice for supersymmetric models, have been stressed. We study in this section the influence of RBC on renormalizability properties, an issue which is addressed in recent literature [8] with an emphasis which we prove being exaggerated. We study the two-point functions for massive scalar and spinor fields. An expansion in the contraction parameter a is possible and it is uniquely determined. This turns out to coincide with the standard adiabatic expansion, including terms of order a^2 in the propagators. This proves that the RBC do not affect at all either the counterterms nor the $O(a^2)$ finite corrections, to all orders in the loop expansion. This is because renormalizability in the curved background space implies the locality of the counterterms, which are at most of dimension 2 for the known supersymmetric theories. This also shows the formal correctness of our calculations in the above sections. The correspondence with the adiabatic expansion of the free field propagators indicates that the concept of adiabatic vacuum, which is in general an approximation to the exact vacuum of the theory, can be identified in AdS_4

with the vacuum associated to RBC provided orders higher than $O(a^2)$ can be neglected, as it is the case for the purpose ofrenormalizing the quantum theory. For supersymmetric massless theories the equivalence between the two vacua breaks down but it is still possible to show that the conformal vacuum maps into the vacuum satisfying RBC and, then, can be consistently used in the renormalization of these models.

We begin by expanding the scalar propagator. The propagator of a scalar field, sat isfying

$$\left(\Box + m^2 - 2a^2\right) G(x,x') = -i\left[-g\right]^{-1/2} \delta(x,x') \qquad (4.1.1)$$

with appropriate boundary conditions, can be extracted for exemple from Ref. [17]

$$G(x,x') = \frac{a^2}{16\pi^2} \frac{\Gamma(\lambda_\pm)\Gamma(\lambda_\pm - 1)}{\Gamma(2\lambda_\pm - 2)} (-z)^{\lambda_\pm} F(\lambda_\pm, \lambda_\pm - 1, 2\lambda_\pm - 2; z) \qquad (4.1.2)$$

where z is given in terms of the square of the proper distance $\mu^2(x,x')$ between x and x'

$$z = 2\left[1 - \cos\left(a\sqrt{\mu^2}\right)\right]^{-1} \qquad (4.1.3)$$

and the choice of the 'regular' or 'irregular' boundary conditions of Refs. [1,25] corresponds to the sign + or − , respectively, in the expression

$$\lambda_\pm = \frac{3}{2} \pm \sqrt{\frac{1}{4} + \frac{m^2}{a^2}} \qquad (4.1.4)$$

The dependence of the propagator G(x,x') in the parameter a is analytic and we can expand the expression (4.1.2) including terms up to order a^2. Expanding the hypergeometric function, according to Ref. [26]

$$\frac{\Gamma(\lambda)}{\Gamma(2\lambda-2)} F(\lambda,\lambda-1,2\lambda-2;z) = (-z)^{1-\lambda}\frac{1}{\Gamma(\lambda-1)}$$
$$+\frac{(-z)^{-\lambda}}{\Gamma(\lambda-1)}\sum_{n=0}^{\infty}\frac{\Gamma(\lambda+n)}{\Gamma(\lambda-1)}\frac{\Gamma(-\lambda+3+n)}{\Gamma(-\lambda+2)}\frac{1}{n!}\frac{1}{(n+1)!}z^{-n}\left[\log(-z)+h_n\right] \tag{4.1.5}$$

where

$$h_n = \psi(2+n)+\psi(1+n)-\psi(\lambda+n)-\psi(\lambda-2-n)$$

and using the formulae

$$\frac{\Gamma(\lambda+n)\Gamma(-\lambda+3+n)}{\Gamma(\lambda-1)\Gamma(-\lambda+2)} = (-\lambda^2)^{n+1}\Big[1-\frac{3}{\lambda}(n+1)-\frac{1}{6\lambda^2}(2n^3-21n^2-35n-12) + O(1/\lambda^3)\Big]$$

$$\psi(\lambda+n) = \psi(\lambda)+\sum_{k=0}^{n-1}\frac{1}{\lambda+k} = \log\lambda+\frac{2n-1}{2\lambda}-\frac{1}{12\lambda^2}(6n^2-6n+1)+O\left(\frac{1}{\lambda^3}\right)$$

$$\psi(\lambda-2-n) = \psi(\lambda)-\sum_{k=1}^{2+n}\frac{1}{\lambda-k} = \log\lambda-\frac{2n+5}{2\lambda}-\frac{1}{12\lambda^2}(6n^2+30n+37)+O(1/\lambda^3) \tag{4.1.6}$$

we find

$$G(x,x') = -\frac{4}{\mu^2}+m^2\sum_{n=0}^{\infty}\frac{1}{n!(n+1)!}\left(-\frac{m^2\mu^2}{4}\right)^n\left[\log\left(-\frac{m^2\mu^2}{4}\right)+\psi(2+n)-\psi(1+n)\right]-$$

$$- m^2 \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!} \left(-\frac{m^2\mu^2}{4}\right)^n \left[\log\left(-\frac{m^2\mu^2}{4}\right) - \psi(2+n) - \psi(1+n)\right] n(n+1)\frac{a^2}{m^2} -$$

$$- m^2 \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!} \left(-\frac{m^2\mu^2}{4}\right)^n (2n+1)\frac{a^2}{m^2} + O(a^3) =$$

$$= \left(1 - a^2 m^2 \frac{\partial^2}{(\partial m^2)^2}\right)\left(\frac{1}{16\pi^2}\right)\left\{-\frac{4}{\mu^2} + m^2 \sum_{n=0}^{\infty} \frac{1}{n!(n+1)!}\left(-\frac{m^2\mu^2}{4}\right)^n \times\right.$$

$$\left. \times \left[\log\left(-\frac{m^2\mu^2}{4}\right) - \psi(n+1) - \psi(n+2)\right]\right\} + O(a^3) \qquad (4.1.7)$$

The expression between brackets is just the usual massive scalar propagator in flat space-time. We have shown, then, that the expansion of the exact scalar propagator in powers of a admits an adiabatic representation in terms of derivatives with respect to the mass of the flat space-time propagator, and that the choice of boundary conditions in Eq.(4.1.2) has no influence on the terms included in Eq.(4.1.7).

We are now in the position to analyze the adiabatic vacuum. The contraction parameter of the space enters the differential equations for the scalar and spinor propagators only through dependences that are analytic in the neighborhood of a = 0. By this reason, it is tempting to recover the result obtained above , now in the form of a series expansion in the a parameter as solution of the differential equation for the scalar propagator, and to obtain a similar series representation for the spinor propagator. This procedure is closer in spirit to the introduction of a short distance expansion in the propagator equations by using normal coordinates around a given point of the space [27]. The result is again an adiabatic representation for both propagators, and the preservation of supersymmetry by this method is guaranteed by the verification of the Ward identity that relates the scalar and spinor propagators in supersymmetric theories [17].

The differential equation (4.1.1) for the scalar propagator

$G(x,x')$ can be implicitely solved by means of a DeWitt-Schwinger representation [23,24]

$$G(x,x') = \int_0^\infty ds \langle x| e^{-isH} |x'\rangle \quad , \qquad H \equiv \Box + m^2 - 2a^2 \tag{4.1.8}$$

In order to make contact with the flat space-time limit of $G(x,x')$ it is useful to define

$$\langle x| e^{-isH} |x'\rangle = \frac{-i}{(4\pi i s)^2} e^{-im^2 s} e^{-\frac{i}{4}\frac{\mu^2}{s}} F(x,x';s) \tag{4.1.9}$$

Then, the kernel $F(x,x';s)$ is constrained by the condition $\lim_{a\to 0} F(x,x';s) = 1$, and must satisfy the differential equation

$$\left(-\frac{1}{i}\frac{\partial}{\partial s} + \frac{3}{2is}\right)F = \frac{\mu}{2is}3AF + \frac{\mu}{is}\frac{\partial}{\partial\mu}F + 3A\frac{\partial}{\partial\mu}F + \frac{\partial^2 F}{\partial\mu^2} - 2a^2 F \tag{4.1.10}$$

that is obtained after the use of [28]

$$\nabla_\alpha \mu = n_\alpha$$

$$\nabla_\alpha n_\beta = A\left(g_{\alpha\beta} - n_\alpha n_\beta\right)$$

$$A(\mu) = a\cot(a\mu) \tag{4.1.11}$$

The introduction of a self-consistent solution of the type

$$F(\mu,s) = \sum_{n=0}^{\infty} F_n(\mu,s)\, a^n$$

provides the result

$$F(\mu,s) = 1 + \frac{1}{4}\mu^2 a^2 + O(a^4) \tag{4.1.12}$$

and, after integrating by parts,

$$G(x,x') = \int_0^\infty i ds \frac{(-1)}{(4\pi i s)^2} e^{-im^2 s} e^{-\frac{i}{4}\frac{\mu^2}{s}} \left[1 + \frac{1}{4}\mu^2 a^2 + O(a^4)\right] =$$

$$= \int_0^\infty i ds \frac{(-1)}{(4\pi i s)^2} e^{-im^2 s} e^{-\frac{i}{4}\frac{\mu^2}{s}} \left[1 - m^2 a^2 (is)^2 + O(a^4)\right] \tag{4.1.13}$$

which is the adiabatic representation obtained above.

We deal next with the spinor propagator $\mathcal{G}(x,x')$ satisfying the equation

$$(\nabla\!\!\!/\,\nabla\!\!\!/ + m^2)\, \mathcal{G}(x,x') = -i[-g]^{-\frac{1}{2}}\, \delta(x,x') \tag{4.1.14}$$

and with integral representation

$$\mathcal{G}(x,x') = \int_0^\infty i ds \frac{(-1)}{(4\pi i s)^2} e^{-im^2 s} e^{-\frac{i}{4}\frac{\mu^2}{s}} \mathcal{F}(x,x';s) \tag{4.1.15}$$

The kernel $\mathcal{F}$ (x,x') must now satisfy the differential equation

$$\left(-\frac{1}{i}\frac{\partial}{\partial s}+\frac{3}{2is}\right)\mathcal{F}=\frac{\mu}{2is}\,3A\mathcal{F}+\frac{\mu}{is}\,n\cdot\nabla\mathcal{F}+\nabla\!\!\!/\nabla\!\!\!/\mathcal{F} \qquad (4.1.16)$$

A convenient ansatz to deal with the spinorial structure of $\mathcal{F}$ is

$$\mathcal{F}(x,x';s)=f(\mu,s)S(x)\bar{S}(x')+g(\mu,s)\,i n\!\!\!/\,S(x)\bar{S}(x') \qquad (4.1.17)$$

where S(x) is a matrix that maps constant spinors ξ into Killing spinors ξ (x) [17]. For our purposes, we can define S(x)$\bar{S}$(x') through the equations

$$\nabla_\alpha S(x)=-\frac{ia}{2}\gamma_\alpha S(x)$$

$$\bar{S}(x)=S^{-1}(x) \qquad (4.1.18)$$

Then, it can be seen that the action of the differential operator in the right hand side of Eq.(4.1.16) reproduces again the spinorial structures S$\bar{\text{S}}$ and $n\!\!\!/$S$\bar{\text{S}}$, giving the system of equations

$$\left(-\frac{1}{i}\frac{\partial}{\partial s}+\frac{3}{2is}-\frac{3}{2}A\frac{\mu}{is}\right)f=\frac{\mu}{is}\left(f'+\frac{a}{2}g\right)+3Af'+f''-$$

$$-4a^2f+2ag'+3aAg-ag'$$

$$\left(-\frac{1}{i}\frac{\partial}{\partial s}+\frac{3}{2is}-\frac{3}{2}A\frac{\mu}{is}\right)g=\frac{\mu}{is}\left(-\frac{a}{2}f+g'\right)-af'$$

$$+g''+3Ag'+3A'g-a^2g \qquad (4.1.19)$$

where the notation $'$ has the meaning of derivation with respect to the proper distance. In the same way as before, it is possible to implement a series expansion solution

$$f(\mu,s) = \sum_{n=0}^{\infty} f_n(\mu,s)\, a^n$$

$$g(\mu,s) = \sum_{n=0}^{\infty} g_n(\mu,s)\, a^n$$

providing the result

$$\mathcal{F}(\mu,s) = \left[1 + \left(is + \tfrac{1}{8}\mu^2\right)a^2 + O(a^4)\right] S\bar{S} + \left[\tfrac{1}{2}\mu a + O(a^3)\right] i\not{x}\, S\bar{S} \tag{4.1.20}$$

Inside the integral representation, the kernel can also be written in the form

$$\begin{aligned} \mathcal{G}(x,x') &= \int_0^\infty i\,ds\, \frac{(-1)}{(4\pi i s)^2}\, e^{-im^2 s}\, e^{-\frac{i}{4}\frac{\mu^2}{s}} \Big\{\Big[1 + \left(is + \tfrac{1}{8}\mu^2\right)a^2 \\ &\quad + O(a^4)\Big] S\bar{S} + \left[\tfrac{1}{2}\mu a + O(a^3)\right] i\not{x}\, S\bar{S}\Big\} = \\ &= \int_0^\infty i\,ds\, \frac{(-1)}{(4\pi i s)^2}\, e^{-im^2 s}\, e^{-\frac{i}{4}\frac{\mu^2}{s}} \Big\{\Big[1 + a^2 is - \tfrac{1}{2} m^2 a^2 (is)^2 \\ &\quad + O(a^4)\Big] S\bar{S} + \left[\tfrac{1}{2}\mu a + O(a^3)\right] i\not{x}\, S\bar{S}\Big\} \end{aligned} \tag{4.1.21}$$

Although the expansion (4.1.21) is not entirely of adiabatic type in the sense that is not a power series in the s variable, it is also true that to carry out consistently the expansion in the a parameter one has to take into account the expansion of $S(x)\bar{S}(x')$

$$S(x)\bar{S}(x') =$$

$$= \left(S(x') + \mu n\cdot\nabla S(x') + \frac{\mu^2}{2!} n^\alpha n^\beta \nabla_\alpha \nabla_\beta S(x') + \dots \right) \bar{S}(x') =$$

$$= \mathbb{1} - \frac{\mu a}{2} i\cancel{n}\,\mathbb{1} - \frac{1}{8}\mu^2 a^2 \mathbb{1} + O(a^3) \tag{4.1.22}$$

which allows us to rewrite Eq.(4.1.21) in the form

$$\mathcal{G}(x,x') = \tag{4.1.23}$$

$$= \int_0^\infty i ds \frac{(-1)}{(4\pi i s)^2} e^{-im^2 s} e^{-\frac{i}{4}\frac{\mu^2}{s}} \left[1 + a^2 is - m^2 a^2 (is)^2 + O(a^4)\right] \mathbb{1}$$

This is the result provided by the more standard approach of performing a normal coordinate expansion in the equation defining $\mathcal{G}(x,x')$ [29]. We have shown in section 3.2 that, in the functional approach to the renormalization of the effective action, the adiabatic orders contained in Eq.(4.1.23) are the right orders to give supersymmetric counterterms for the Wess-Zumino model in AdS_4.

At this point, we are in the position to show that the two adiabatic expansions for the scalar propagator $G(x,x')$ and the spinor propagator $\mathcal{G}(x,x')$ match the requirements demanded for supersymmetry to be preserved. In fact, it is well-known that the spinor propagator S(x,x') of the Dirac equation is related to the scalar propagator G(x,x') through the supersymmetry Ward identity of Ref. [17]

$$S(x,x')\Big|_{m^2=\tilde{m}^2} = \Big[(i\nabla\!\!\!/ + \tilde{m} + a)\, G(x,x')\Big|_{m^2=\tilde{m}^2-a\tilde{m}}\Big]\, S(x)\,\bar{S}(x') \qquad (4.1.24)$$

Given the general relation that exists in any curved space-time [27] between S(x,x') and $\mathcal{G}(x,x')$

$$S(x,x') = (i\nabla\!\!\!/ + m)\,\mathcal{G}(x,x') \qquad (4.1.25)$$

the supersymmetry Ward identity can be cast for our purposes in the form

$$(i\nabla\!\!\!/ + \tilde{m})\,\mathcal{G}(x,x')\Big|_{m^2=\tilde{m}^2} =$$

$$= \Big[(i\nabla\!\!\!/ + \tilde{m} + a)\, G(x,x')\Big|_{m^2=\tilde{m}^2-a\tilde{m}}\Big]\, S(x)\,\bar{S}(x') \qquad (4.1.26)$$

It can be shown without difficulty by using Eq.(4.1.13) and Eq. (4.1.23) that the expansions in the a parameter of both sides of this equation match up to the order computed in the above, i.e. up to order $O(a^2)$. The result is, in fact, the expansion of the spinor propagator

$$S(x,x') = \int_0^\infty i\,ds \frac{(-1)}{(4\pi i s)^2}\, e^{-i\tilde{m}^2 s}\, e^{-\frac{i}{4}\frac{\mu^2}{s}} \Big\{ \Big[\tilde{m} + a + \tilde{m}^2 a i s + \tilde{m} a^2 i s - \frac{\tilde{m}^3}{2} a^2 (is)^2 + O(a^3) \Big] S\bar{S} + \frac{\mu}{2is}\Big[1 + \tilde{m} a i s - \frac{\tilde{m}^2}{2} a^2 (is)^2 + O(a^3) \Big] i \not{x} S \bar{S} \Big\} \qquad (4.1.27)$$

The verification of the Ward identity (4.1.26) is obviously independent of applying or not the short-distance expansion (4.1.22) of $S(x)\bar{S}(x')$ and constitutes a clear check that the adiabatic expansions (4.1.13) and (4.1.23) for $G(x,x')$ and $\mathcal{G}(x,x')$ respectively are compatible with supersymmetry. This is a reflection of the fact that the expansion in powers of a of the exact propagator $\mathcal{G}(x,x')$ up to order $O(a^2)$ can be cast in terms of an adiabatic representation of the same type as Eq.(4.1.7), as we will see in section 4.3.

We now turn our attention to the conformal vacuum. For massless quantum field theories in AdS_4 the adiabatic expansion loses its meaning, as long as there is no scale parameter in the theory to which the contraction parameter of the space can be compared. Yet, there is another approach that brings some light about the role of the boundary conditions in the renormalization of these models. As far as supersymmetric models are concerned, there exists a conformal invariance at the classical level [12,14] that can be used to express the free field propagators of the model in terms of the corresponding counterparts in the flat space-time theory. Since the deviation of these models from a confarmal invariant theory away from dimension D = 4 is proportional to 4 - D , it is obvious that, for supersymmetric theories with divergencies proportional at most to $(4 - D)^{-1}$, the counterterms of the flat space-time theory could eventually be identified with the respective counterterms of the original model.

In what follows we will focus our attention in a generic scalar propagator of these supersymmetric models. AdS_4 is a conformally flat space-time whose conformal weight can be determined from the expression of the gravitational action with cosmological constant $\Lambda = -3a^2$ for the special choice $g_{\mu\nu} = \Omega^2 \eta_{\mu\nu}$,

$$S_{grav} = \int d^4x \left(6\Omega\partial^\mu\partial_\mu\Omega + 6a^2\Omega^4\right) \qquad (4.1.28)$$

The solution of the differential equation

$$\partial^\mu\partial_\mu\Omega + 2a^2\Omega^3 = 0 \qquad (4.1.29)$$

gives the conformal weight

$$\Omega = \left(1 + \frac{1}{4}a^2x^2\right)^{-1} \qquad (4.1.30)$$

One important remark is that the projection of AdS_4 into Minkowski space-time is confined to the region between the two space-like hyperboloids $x^2 = -(4/a^2)$. This is a reflection of the fact that, because of the relation between x^2 and proper distance μ in the original space-time

$$tg\left(\frac{a}{2}\sqrt{\mu^2}\right) = \frac{a}{2}\sqrt{x^2} \qquad (4.1.31)$$

the space-like infinity is reached for finite values of the x-coordinate. Given the conformal weight of the scalar field $\phi(x)$ that relates it to a scalar field in Minkowski space-time $\phi_M(x)$ as in Ref. [27]

$$\Omega(x)\,\phi(x) = \phi_M(x) \qquad (4.1.32)$$

it is possible to write any Green function $\langle\phi(x)\phi(x')\rangle$, irrespective of the boundary conditions that it should satisfy, in terms of a Green function $\langle\phi_M(x)\phi_M(x')\rangle$, solution of the Klein-Gordon operator in Minkowski space-time

$$\langle\phi(x)\phi(x')\rangle = \Omega^{-1}(x)\langle\phi_M(x)\,\phi_M(x')\rangle\,\Omega^{-1}(x') \tag{4.1.33}$$

The key point now is to stablish which boundary conditions in the flat space-time conformal projection give rise to the enforcement of what have been called 'transparent' and 'reflective' boundary conditions for the scalar propagator in AdS_4 [1,3]. At first sight the analysis appears promising since, given the structure of the time-like and space-like regions at the infinity in the flat projection space, there have to exist at least two propagators with different physical interpretation, vanishing respectively at time-like and space-like infinity. The first of them is given by

$$\langle\phi_M(x)\,\phi_M(0)\rangle = -\frac{1}{4\pi^2}\,\frac{1}{x^2} \tag{4.1.34}$$

and corresponds to the propagator in AdS_4 satisfying transparent boundary conditions

$$\langle\phi(x)\,\phi(0)\rangle = -\left(1+\frac{1}{4}a^2x^2\right)\frac{1}{4\pi^2}\,\frac{1}{x^2} =$$

$$= -\frac{a^2}{2}\,\frac{1}{4\pi^2}\,\frac{1}{1-\cos(a\mu)} \tag{4.1.35}$$

On the other hand, the propagator vanishing at the space-like boundary $x^2 = -(4/a^2)$

$$\langle \phi_M(x)\,\phi_M(0)\rangle = -\frac{1}{4\pi^2}\,\frac{1+\frac{a^2}{4}x^2}{x^2} \tag{4.1.36}$$

corresponds to the propagator satisfying regular reflective boundary conditions in AdS_4

$$\langle \phi(x)\,\phi(0)\rangle = -\left(1+\frac{1}{4}a^2x^2\right)\frac{1}{4\pi^2}\,\frac{1+\frac{a^2}{4}x^2}{x^2} =$$

$$= -\frac{a^2}{2}\,\frac{1}{4\pi^2}\left[\frac{1}{1-\cos(a\mu)}+\frac{1}{1+\cos(a\mu)}\right] \tag{4.1.37}$$

Finally, we point out that irregular reflective boundary conditions, that are in fact required in combination to regular boundary conditions in massless supersymmetric theories [1], demand the choice of the propagator

$$\langle \phi_M(x)\,\phi_M(0)\rangle = -\frac{1}{4\pi^2}\,\frac{1-\frac{a^2}{4}x^2}{x^2} \tag{4.1.38}$$

The three given propagators in Minkowski space have in common that they are solutions of the inhomogeneous Klein-Gordon equation in flat space-time with different solution of the homogeneous equation for each of them. In fact, in the space-like region $x^2 \leq 0$, $x'^2 \leq 0$, the different solutions with source away from the origin can be written as the analytic continuation of the solution of an elliptic boundary-value problem

$$\langle \phi_M(x)\phi_M(x')\rangle = -\frac{1}{4\pi^2}\begin{cases} \dfrac{1}{(x-x')^2} & (4.1.39)\\[2ex] \dfrac{1}{(x-x')^2} + \dfrac{\frac{4}{a^2x'^2}}{\left(x+\frac{4}{a^2}\frac{x'}{x'^2}\right)^2} & (4.1.40)\\[2ex] \dfrac{1}{(x-x')^2} - \dfrac{4/a^2x'^2}{\left(x+\frac{4}{a^2}\frac{x'}{x'^2}\right)^2} & (4.1.41)\end{cases}$$

Once again, the correspondence

$$(x-x')^{-2} \longrightarrow [1-\cos(a\mu)]^{-1}$$

$$\frac{4}{a^2x'^2}\left(x+\frac{4}{a^2}\frac{x'}{x'^2}\right)^{-2} \longrightarrow [1+\cos(a\mu)]^{-1}$$

can be checked for $x^2 \leq 0$, $x'^2 \leq 0$. This compact form of the three propagators contains all the information that is needed for the perturbative formulation of the quantum field theory, that is unambigously carried out by analytic continuation to euclidean, as opposite to riemannian, AdS_4 . For euclidean AdS_4 , in fact, the expression (4.1.40) corresponds to the solution by the method of the images of an elliptic problem inside the sphere $x^2 = 4/a^2$ with potential vanishing at the boundary. The difference among the differnt solutions (4.1.39), (4.1.40), (4.1.41) is then, in the light of this interpretation, the potential created by the image source outside the sphere. It is therefore evident that the divergent structure of the three propagators in the coincidence limit $x \to x'$ is exactly the same. As long as in flat space-time supersymmetric models quadratic divergencies are absent [14] , we conclude as a corollary that the counterterms of a massless supersymmetric theory in AdS_4 and in flat space-time must be the same.

Let us discuss the results of the present section. The adiabatic expansions of $G(x,x')$ and $S(x,x')$ have been used to determine the one-loop [6-10] and two-loop [30] divergent renormalization of the Wess-Zumino model in AdS_4. We have described the one-loop calculations in the previous chapters. Thanks to the results of the present section, we can state that the procedure illustrated in the previous chapters is accurate to all orders in the loop expansion since the adiabatic expansion (4.1.13) coincides with Eq. (4.1.7) term by term as a series in the variable $\mu(x,x')$ and the supersymmetry Ward identity (4.1.24) is satisfied by the adiabatic expansions of the propagators up to order $O(a^2)$. In a general curved space-time an expansion of the type (4.1.13) is an asymptotic expansion that cannot take into account the effect of different choices of boundary conditions. We have shown that in AdS_4 the expansion in powers of a of the exact propagator satisfying RBC is analytic and unique, and it coincides with the adiabatic expansion, in the standard version of the latter, up to an order in the a parameter that allows its use for the computation of divergent counterterms and even finite corrections of order $O(a^2)$ to the effective action of the model [10].

The lack of the expansion in the a parameter of the geometrical object $S(x)\bar{S}(x')$ forces the authors of Ref. [8] to define a modified spinorial functional derivative

$$\frac{\delta \psi_\alpha(x)}{\delta \psi_\beta(x')} = S_{\alpha\delta}(x)\, \bar{S}_{\delta\beta}(x')\, \frac{\delta(x,x')}{\sqrt{-g}}$$

in order to enforce a supersymmetric invariant counterterm for the spinor part of the action. By the simple property $\bar{S}(x) = S^{-1}(x)$ it is obvious that no modification is needed

$$\frac{\delta \psi_\alpha(x)}{\delta \psi_\beta(x')} = \mathbb{1}_{\alpha\beta}\, \frac{\delta(x,x')}{\sqrt{-g}}$$

They obtain the divergent part of the self-energy of the spinor field in momentum space $\sum(q) \sim (\not{q} + 2a) \log M$. The presence of the term 2a, which produces an apparent inconsistency with supersymmetry and calls for the spinorial functional derivative given in Ref. [8] is removed by the expansion (4.1.22) of $S(x)\bar{S}(x')$

$$\int \frac{d^4q}{(2\pi)^4}\, e^{iq\cdot n^\mu} (\not{q} + 2a) \log M \; S(x)\,\bar{S}(x') =$$

$$= \int \frac{d^4q}{(2\pi)^4}\, e^{iq\cdot n^\mu} \not{q} \log M$$

The latter result is in agreement with supersymmetry and reproduces the result obtained in Eq.(2.3.30) above. The expansion (4.1.22) of $S(x)\bar{S}(x')$ in powers of the a parameter is a relation among intrinsic geometrical objects defined in the background space and clarifies, once again, that the choice of boundary conditions cannot play any role in the renormalization of AdS_4 supersymmetry.

4.2. Massless fields in AdS_4 : radiative generation of a mass

In considering more carefully massless fields in AdS , we are going to implement once more the adiabatic expansion of the free field propagators in AdS_4 background for the $m = 0$ Wess-Zumino model. A supersymmetric invariant mass term is produced raddiatively and we will discuss its implications in terms of the breakdown of the no-renormalization theorem. In the previous chapters the issue of the renormalization of the model has been the object of an extended investigation, starting from the one-loop analysis of the model formulated in terms of component field. In the next chapters this analysis will culminate in the proof of the no-renormalization theorem (for divergent counterterms) for supersymmetric theories formulated in AdS_4 in terms of superfields. There, the superconformal invariance of the $m = 0$ part of the Wess-Zumino action will be exploited to give a perturbative treatment of the effects of the non-vanishing curvature in terms of the interaction vertices of the quantum theory. Thus, it would seem as nothing more could arise our interest for what concerns the $m = 0$ model, since its superconformal invariance ensures that the divergent quantum corrections can be calculated along the line of the flat background case, after rescaling the fields according to a superconformal transformation. In fact, looking at the $m = 0$ model in its component field formulation it is interesting from the point of view of exploring the limits of applicability of the adiabatic expansion when the calculation of the finite one-loop corrections is carried using this technical tool. As we recall from section 2.2, this tool is found to be inadequate to compute finite quantum corrections to the effective action.

The applicability of the standard adiabatic expansion to massive fields in AdS_4 is the main result of our proof in the previous section. The $m = 0$ case, however, is exceptional. Contrary to the claim of Ref.[8] , the adiabatic approximation does not reproduce regular terms in the short-distance expansion of the massless scalar propagator of section 4.1. Indeed, since the application of dimensional reduction and the adiabatic expansion of section 2.2, doubts have arisen on the capability of this expansion to provide a correct tool in evaluating finite one-loop corrections to the m = 0 Wess-Zumino model, although these doubts were moderated by the uncertainties about the compatibility between the dimensional reduction prescription used there and the supersymmetry invariance of the theory. The subsequent adoption of a manifestly supersymmetric invariant Pauli-Villars regularization procedure has shown that the failure to preserve supersymmetry for the finite terms proportional to the curvature of AdS_4 is due to the fact that the expansion in the background effects is not suitable to calculate these finite terms. With this caveat in mind, it is clear that it would be gratifying to confirm the result of the radiative generation of a finit mass, that we obtain in the following using the adiabatic approximation, implementing the exact propagators for massless fields. Strictly speaking, this result is not a consequence of the boundary conditions imposed on the field propagators, since in this case it would not be possible to obtain it through a short-distance approximation, such as the adiabatic expansion. Rather, the effect that we are detecting here and that represnts a finite violation of the no-renormalization theorem, is a mere consequence of the presence of a non-vanishing curvature.

Before we turn our attention to the evaluation of the quantum corrections it is opportune to take under consideration the classical solutions around which the theory can be quantized. From the classical potential, eliminating the auxiliary fields by their classical equations of motion

$$F = -aA - \lambda A^2 + \lambda B^2$$

$$G = -aB + 2\lambda AB$$

we obtain the expression of the tree-level potential depending on the A and B fields alone

$$V(A,B) = -a^2A^2 - a^2B^2 + \frac{1}{2}\lambda^2\left(A^2+B^2\right)^2 \quad . \qquad (4.2.1)$$

The structure of the minima of this function is easily found to be

$$A = \frac{a}{\lambda}\cos\theta \ , \qquad B = \frac{a}{\lambda}\sin\theta \tag{4.2.2}$$

where the infinity of values assumed by the (A, B) coordinates of the minima is parameterized by the angular variable $0 \le \theta < 2\pi$.

The classically supersymmetric invariant minima are obtained by $\theta_1 = \pi$, $\theta_{2,3} = \pm\pi/3$

$$A_1 = -\frac{a}{\lambda} \ , \qquad B_1 = 0 \tag{4.2.3}$$

$$A_{2,3} = \frac{a}{2\lambda} \ , \qquad B_{2,3} = \pm\frac{\sqrt{3}}{2}\frac{a}{\lambda} \tag{4.2.4}$$

In our calculation we are going to restrict ourselves to the one-loop analysis of the theory quantized around the classically supersymmetric local maximum of the function

$$A_0 = B_0 = 0 \ . \tag{4.2.5}$$

In order to justify this specialization, we recall that, in a purely classical context, it has been remarked that, for field theories in AdS_4 background, the analysis of the classical solutions of the theory cannot be restricted to only the absolute minima of the classical potential

The above feature presented by the origin (1) which is a supersymmetric extremum of the classical potential but does not correspond to the lowest energy point of the potential, is reflecting a general difference with the flat space-time realization of supersymmetry.

In making use of an adiabatic approximation we should anticipate that this does not reproduce adequately the structure of the propagator for $m = 0$ scalar fields in AdS_4. The solution of the equation

$$\left(\Box - 2a^2\right) G_F(x, x') = -\left[-g(x)\right]^{-1/2} \delta^4(x - x') \qquad (4.2.6)$$

with the "reflective" boundary condition appropriate for AdS is[14]

$$G_F(x, x') = -\frac{i}{8\pi^2} a^2 \left(\frac{1}{-u} \mp \frac{1}{2-u}\right) \qquad (4.2.7)$$

where the sign $-$ (or $+$) corresponds to the choice of regular (or irregular) boundary conditions of Refs. [1, 25]. Recalling the relation connecting the variable $\sigma(x, x')$, defined as one-half times the square of the proper distance, with the chordal distance $u(x, x')$

$$u(x, x') = 1 - \cosh\left(a\sqrt{-2\sigma(x, x')}\right)$$

we can expand the scalar propagator in σ

$$G_F(x, x') = \frac{i}{8\pi^2} \left(\frac{1}{\sigma} + C\, a^2\right) + O(a^4 \sigma) \qquad (4.2.8)$$

with $C = 2/3$ $(-1/3)$ for the regular (irregular) propagator. In using the adiabatic expansion in momentum space of Bunch and Parker, one is approximating the $m = 0$ scalar propagator by the function

$$\int \frac{d^4k}{(2\pi)^4} e^{iky} \frac{1}{k^2 + i0} = \frac{i}{8\pi^2} \frac{1}{\sigma + i0}$$

thus neglecting $\mathcal{O}(a^2)$ terms in $G_F(x,x')$, which are finite in the limit $x \to x'$. This is justified for the calculation of counterterms.[6] That such an approximation be enough to determine $\mathcal{O}(a^2)$ finite quantum corrections is not clear at all. In the above we have indeed seen that the $\mathcal{O}(a^2)$ finite terms computed through such an approximation are inconsistent with the supersymmetry invariance of the Wess-Zumino model. However, by looking at the terms neglected with respect to $G_F(x,x')$, it is plausible that this approximation be suitable up to finite terms of order a included. To clarify this point, we look at the propagator for $m=0$ spin-1/2 fields, which is needed for our one-loop calculation. The exact propagator of a massless spinor in AdS_4 is given in terms of the scalar propagator

$$S_F(x,x') = aG_F(x,x')S(x)\overline{S}(x') + \frac{\partial G_F}{\partial \mu} i\gamma^\tau n_\tau(x,x')S(x)\overline{S}(x')$$

and satisfies the equation

$$i\gamma^\tau D_\tau S_F(x,x') = [-g(x)]^{-1/2}\delta^4(x-x')$$

The Dirac matrix $S(x)$ is considered in Refs. [1,31] and satisfies $D_\mu S(x) = -\frac{i}{2}a\gamma_\mu S(x)$. The unit tangent at x, $n_\tau(x,x')$, to the geodesic from x to x', reads as $n_\tau(x,x') = D_\tau\mu(x,x')$ in terms of the proper distance $\mu(x,x')$.[32] In the case of interest, the spin-1/2 field belongs to a scalar supermultiplet and S_F satisfies the supersymmetric Ward identity[1]

$$S_F(x,x') = [(i \not{D} + a) \qquad G_F(x,x')]\, S(x)\overline{S}(x') \qquad (4.2.9)$$

We expand $S(x)\overline{S}(x')$ in μ

$$[S(x)\overline{S}(x')]_{\alpha\beta} = \Bigg\{-\frac{i}{2}a(\gamma^\tau)_{\alpha\beta}n_\tau(x,x')\mu(x,x')\left[1-\frac{1}{24}a^2\mu^2(x,x')\right]$$

$$+\left[1-\frac{1}{8}a^2\mu^2(x,x')\right]\delta_{\alpha\beta}\Bigg\} + \mathcal{O}(a^4\mu^4) \qquad (4.2.10)$$

and obtain the expansion of the spinor propagator

$$[S_F(x,x')]_{\alpha\beta} = \frac{i}{8\pi^2}\Bigg\{-4i(\gamma^\tau)_{\alpha\beta}n_\tau(x,x')$$

$$\times\frac{1}{\mu^3(x,x')}\left[1+\frac{1}{8}a^2\mu^2(x,x')\right] \pm \frac{1}{2}a^3\delta_{\alpha\beta}\Bigg\} + O(a^4\mu)$$

This expression allows us to recover the result

$$\lim_{x\to x'} tr\, S_F(x,x') = \pm 2a^3\frac{i}{8\pi^2} \qquad (4.2.11)$$

obtained in Ref.[32] where its physical interpretation as the order parameter of chiral symmetry breaking in AdS_4 is provided.

Following the lines of Ref.[10] for the calculation of the effective potential, one can easily see that terms vanishing in the coincidence limit $x \to x'$ of the exact propagators G_F and S_F do not contribute to the one-loop ultraviolet divergencies. The adiabatic approximation for the spinor two-point function is

$$-\int\frac{d^4k}{(2\pi)^4}e^{iky}\frac{\gamma^\tau k_\tau}{k^2+i0} - \frac{1}{2}a^2\int d^4k\frac{\gamma^\tau k_\tau}{(k^2+i0)^2}$$

$$= -\frac{i}{8\pi^2}\frac{4}{\mu^3}i\gamma^\tau n_\tau - \frac{i}{8\pi^2}\frac{1}{2}a^2\cdot\frac{1}{\mu}i\gamma^\tau n_\tau$$

In this way, we are neglecting $O(a^3)$ terms with respect to $S_F(x,x')$. This shows that the concern expressed in Ref. [8] about the reliability of the adiabatic expansion of the spinor propagator in computing the ultraviolet divergent corrections to the model, as it is done in Refs. [6, 7, 9], is not well-founded. The adiabatic expansion differs from the exact propagator $S_F(x,x')$ by terms which, on dimensional grounds, cannot affect the local counterterms. There is no doubt that the divergent 1PI two-point functions computed in the adiabatic approximation are consistent with the supersymmetry Ward identity .[6]

In Eq. (61) or Ref. [8]. the bispinor $S(x)\overline{S}(x')$ appears with a factor $\delta^4(x-x')$ in the divergent part of the fermion self-energy. It is sufficient to recall the property

$$\overline{S}(x) = [S(x)]^{-1}$$

to see explicitly that the counterterms in Ref. [3] are not affected by the reflective boundary conditions.

What can be worrying about the use of approximated propagators has to do with the calculation of finite corrections of order a^2. For these terms, it is only the massless scalar propagator which can be sensible to the reflective boundary conditions. Contrary to the claim in Ref. [8], the regular terms in the short-distance expansion of this propagator do not match its adiabatic approximation.

This provides an explaination for the fact that the adiabatic approximation is inadequate to compute the finite corrections considered in section 2.2. In this case, in order to give a more concrete meaning to this statement, one can consider the evaluation of the one-loop effective potential implementing the adiabatic expansion of DeWitt-Schwinger following the line of chapter 3. Specializing the results contained there to the m = 0 case, one has

$$\frac{\partial V_{\text{eff}}^{(1)}}{\partial \hat{F}} = \frac{1}{(4\pi)^2}\left(-2\,a^2 \log \frac{2a^2}{\nu^2} + 6\,a^2 \log \frac{6a^2}{\nu^2} + 2a^2\right)\lambda f$$

$$\frac{\partial V_{\text{eff}}^{(1)}}{\partial \hat{A}} = \frac{1}{(4\pi)^2}\left(-24a^2 \log \frac{4a^2}{\nu^2} + 6\,a^2 \log \frac{2a^2}{\nu^2} + 30 \log \frac{6a^2}{\nu^2} + 10a^2\right)\lambda a f$$

where $f = -1,\ \frac{1}{2}$, for the supersymmetric minima of the classical potential, (3) and (4) respectively. These expressions do not satisfy the supersymmetric Ward identity . We see in the appearing of the above $(\log a^2)$ factors a strong indication that the finite one-loop corrections to the $m = 0$ model cannot be computed using the perturbative approach in the background effects provided by the adiabatic expansion.

In Eq.(2.2.24), a term of order a appears which does not contribute to the supersymmetric Ward identity. This term can be interpreted as a negative mass term for the spinor field

$$m = -4a\frac{\lambda^2}{16\pi^2} \qquad (4.2.12)$$

radiatively induced at the one-loop level. It is important to calculate the corresponding mass term induced for the A and B fields by the one-loop corrections. We have

$$\Gamma_{BG}(x-x') = -\frac{4i\lambda^2}{16\pi^2}a\log\left(\frac{1}{\Lambda^2}\right)\frac{1}{\sqrt{-g}}\delta^4(x-x') + 8a\lambda^2\frac{i}{16\pi^2}$$

Recalling Eq.(2.2.22) we obtain a correction of the form

$$\int d^4x(-g)^{1/2}m(AF-BG)$$

with m given by (12).

Feynman diagrams with boson fields running around the loop contribute to Γ_{BB} as follows

$$\mathrm{BOS} = -I + 8\lambda^2\frac{i}{16\pi^2}\left[-\left(\Lambda^2+\frac{1}{3}p^2\right) - \frac{a^2}{2}\log\left(-\frac{p^2}{\Lambda^2}\right) + \frac{5}{2}a^2\right]$$

The integral $I(p)$ is given in sec. 2. From the fermion loop we have

$$\mathrm{FER} = I + 8\lambda^2\frac{1}{16\pi^2}\left[\Lambda^2 + \frac{1}{3}p^2 + a^2\log\left(-\frac{p^2}{\Lambda^2}\right) - \frac{1}{2}p^2\log\left(-\frac{p^2}{\Lambda^2}\right) + \frac{3}{2}a^2\right]$$

The sum of the above contributions in configuration space gives

$$\Gamma_{BB}(x,x') = \frac{4i\lambda^2}{16\pi^2}\left[(\Box_x - 3a^2)\log\left(\frac{1}{\Lambda^2}\right) + 8a^2\right]\frac{1}{\sqrt{-g}}\delta^4(x-x')$$

We recall Eq.(2.2.23) and find the one-loop correction

$$\int d^4x\sqrt{-g}\cdot\frac{3}{2}am\left(A^2-B^2\right)$$

with m given once again by (12). Thus, the quantum corrections to the model generate a finite mass action for the scalar supermultiplet, at the one-loop level. The supersymmetry invariance of the model is preserved by the radiatively induced mass term. On the other hand, the renormalization theorem holding in flat space-time[19] is broken by this finite effect.

Let us turn to the spin-1/2 field. Chiral symmetry is broken in AdS_4, already in the tree-level approximation, for any field theory containing a spin-1/2 particle.[32] Our result gives the one loop corrected value of the order parameter of the breaking for the spin-1/2 component of the self-interacting scalar supermultiplet. The propagator for the scalar field A is given by[17]

$$G_A(x.x') = -\frac{ia^2}{(4\pi)^2}\frac{\Gamma(\lambda_A)\Gamma(\lambda_A-1)}{\Gamma(2\lambda_A-2)}(-2u^{-1})\cdot F(\lambda_A,\lambda_A-1,2\lambda_A-2,2u^{-1}) \qquad (4.2.13)$$

where

$$\lambda_{A\pm} = \frac{3}{2} \pm \left|\frac{m}{a}-\frac{1}{2}\right|$$

and the + (or -) sign corresponds to the choice of regular (or irregular) boundary conditions for $G_A(x,x')$. The mass (12) satisfies the condition

$$\frac{m}{a} << 1 \qquad (4.2.14)$$

Thus, we obtain

$$\lambda_{A_+} = 2-\frac{m}{a}\ , \qquad \lambda_{A_-} = 1+\frac{m}{a}$$

The propagator G_A of Eq. (13) can now be expanded in the perturbative regime (14). The results for different choices of boundary conditions are

$$G_{A_+} = -\frac{ia^2}{4\pi^2}\frac{1}{u(u-2)}\left\{1-\frac{m}{a}\left[\frac{u-2}{2}\log\left(1-\frac{2}{u}\right)+\log\left(-\frac{2}{u}\right)\right]+0\left(\frac{m^2}{a^2}\right)\right\}$$

$$G_{A_-} = \frac{ia^2}{4\pi^2}\frac{u-1}{u(u-2)}\left\{1+\frac{m}{a}\left[\log\left(-\frac{2}{u}\right)-\frac{1}{2}\frac{u-2}{u-1}\cdot\log\left(1-\frac{2}{u}\right)\right]+0\left(\frac{m^2}{a^2}\right)\right\}$$

These expansions agree with Eq. (7) in the limit $m \to 0$.

In the case of a massive spin-1/2 field we have

$$S_F(x.x') = (m+a)G_A(x.x')S(x)\overline{S}(x') + \frac{\partial G_A}{\partial \mu} i\gamma^\tau n_\tau(x,x')S(x)\overline{S}(x') \qquad (4.2.15)$$

for the propagator which solves the equation

$$(i\gamma^\tau D_\tau - m)S_F(x,x') = [-g(x)]^{-1/2}\,\delta^4(x-x')$$

with boundary conditions appropriate for supersymmetry (the reflective case in Ref. [3]). The spinor propagator (15) satisfies a supersymmetry Ward identity analogous to (9 17). In order to fix ideas, let us choose regular boundary conditions for the A-field (and, accordingly, irregular ones for the pseudoscalar B). Using the expression

$$\frac{\partial G_{A_+}}{\partial \mu} = \frac{ia^3}{2\pi^2}\frac{u-1}{[u(2-u)]^{3/2}}\left\{1 - \frac{m}{a}\left[\frac{1}{4}\frac{(u-2)^2}{u-1}\log\left(1-\frac{2}{u}\right)\right.\right.$$
$$\left.\left. + \log\left(-\frac{2}{u}\right)\right]\right\} + 0(m^2 a)$$

as well as the short-distance expansion of the product $S(x)\overline{S}(x')$ (10), it is not difficult to obtain the result

$$tr\ S_F(x,x') = \frac{ia^3}{4\pi^2}\left\{1 - \frac{\lambda^2}{4\pi^2}\left[\frac{4}{a^2\epsilon^2} + \log\left(-\frac{a^2\epsilon^2}{4}\right) + \frac{5}{6}\right]\right\} + O\left(a^5\epsilon^2\right) \qquad (4.2.16)$$

as a consequence of (12). This is the one-loop expression of the order parameter of chiral symmetry breaking. In (16) we make use of a point-splitting regularization procedure introducing a small proper distance ϵ measured along the geodesic between x and x'.

In closing this section, we want to present the calculation of the 1PI one-point functions of the model. The adiabatic expansion allows us to determine the divergent one-point functions. They vanish to the one-loop order. We show that a calculation using the exact field propagators in AdS_4 gives non-vanishing finite values for the one-loop 1PI one-point functions. Clearly, the derivation of this result does not require the use of any regularization procedure. Recalling the short distance expansion (8) we have

$$\lim_{x\to x'}[G_A(x,x')-G_B(x,x')]=\frac{i}{8\pi^2}(C_A-C_B)a^2$$

where $C_A-C_B\equiv\alpha$ takes the values +1 (or -1) when regular boundary conditions are selected for the A (or B) field. We have seen that

$$\lim_{x\to x'} tr\ S_F(x,x')=\frac{i}{8\pi^2}a^3\cdot 2\alpha$$

Introducing the propagators $\langle AF\rangle$ and $\langle BG\rangle$

$$G_{\langle AF\rangle}(x,x')=-aG_A(x,x')$$

$$G_{\langle BG\rangle}(x,x')=-aG_B(x,x')$$

it is almost immediate to see that

$$\begin{aligned}\Gamma_A&=3\lambda a\lim_{x\to x'}[G_A(x,x')-G_B(x,x')]\\&+2\lambda\lim_{x\to x'}\left[G_{\langle AF\rangle}(x,x')-G_{\langle BG\rangle}(x,x')\right]\\&+\lambda\lim_{x\to x'} tr\ S_F(x,x')\\&=\frac{i\alpha}{8\pi^2}\cdot 3\lambda a^3\end{aligned}\qquad(4.2.17)$$

and that

$$\Gamma_F=\lambda\lim_{x\to x'}[G_A(x,x')-G_B(x,x')]=\frac{i\alpha}{8\pi^2}\lambda a^2\qquad(4.2.18)$$

These expressions preserve supersymmetry, since the supersymmetry Ward identity (2.3.14) is satisfied. The remaining one-point functions are all vanishing.

There is a shift in the vacuum expectation value of the A field, to the one-loop order

$$\begin{aligned}\langle A\rangle &= -i\int d^4y\sqrt{-g}\left[G_A(x,y)\Gamma_A + G_{\langle AF\rangle}(x,y)\Gamma_F\right]\\ &= -2ia\Gamma_F\int d^4y\sqrt{-g}G_A(x,y)\end{aligned} \qquad (4.2.19)$$

For the auxiliary field we have

$$\begin{aligned}\langle F\rangle &= -i\int d^4y\sqrt{-g}\left[G_{\langle AF\rangle}(x,y)\Gamma_A + G_{\langle FF\rangle}(x,y)\Gamma_F\right]\\ &= -i\Gamma_F\left[1-2a^2\int d^4y\sqrt{-g}G_A(x,y)\right]\end{aligned} \qquad (4.2.20)$$

after using the relation

$$G_{\langle FF\rangle}(x,x') = (-g)^{-1/2}\delta^4(x-x') + a^2G_A(x,x')$$

which can be derived from the equation of motion for the F field. From Eq. (7) (6) one gets

$$(\Box_x - 2a^2)\int d^4y\sqrt{-g(y)}G_A(x,y) = -1$$

By integrating $G_A(x,y)$ over the y-variable we get a constant, since AdS_4 is an homogeneous space. Thus,

$$\int d^4y\sqrt{-g(y)}G_A(x,y) = \frac{1}{2a^2} \qquad (4.2.21)$$

We plug this result into Eqs. (19) and (20) and obtain

$$\langle A\rangle = \frac{\lambda}{8\pi^2}\alpha a \qquad (4.2.22)$$

$$\langle F\rangle = 0 \qquad (4.2.23)$$

This shows that the vacuum expectation value of the A field is shifted from its vanishing tree level value by a finite quantity by taking into account the one-loop quantum corrections to the model. This shift depends upon the choice of boundary conditions for the equation of motion of the A-field propagator. On the other hand, it is quite remarkable that $\langle F\rangle$ vanishes and the supersymmetry invariance of the model placed in its classical phase (5) is preserved to the one-loop order. The massless model is the most sensitive to the infrared region behavior of the propagators; it is the case in which the reflective boundary conditions are likely to play an important role in the finite quantum correction to this model. It is certainly interesting to look at the quantization of the $m = 0$ model around the other supersymmetric minima of the classical potential (3), (4) in search of a possible spontaneous breaking of supersymmetry at the quantum level. This is done by functional techniques in the next section. We notice that part of the results (22), (23) can be extracted from Ref. [8]. The authors of Ref. [8] discuss only divergent corrections. Nevertheless, we can formally put all the masses (both of the physical and the regulating fields) equal to zero in Eq. (17) of Ref. [8]. After introducing a factor $\frac{1}{2}$ which is clearly missing in the passage from Eq. (16) to Eq. (17) in Ref. [8], we obtain

$$\langle A_i\rangle_{tr} = \frac{a\tilde{\lambda}}{16\pi^2}(\mu_i - 2a)\sum_j c_j\mu_j\psi\left(\frac{m_j}{a}\right) = \frac{1}{8\pi^2}\cdot a^3\tilde{\lambda}$$

Relating the notation of Ref. [8] to ours ($\tilde{\lambda} = 2\lambda$) and using (21) we have

$$\langle A_i\rangle = -\frac{1}{2a^2}\langle A_i\rangle_{tr} = -\frac{\lambda}{8\pi^2}a$$

in agreement with our result (22). Since the work of Ref. [8] is specialized to the choice $\lambda_{A_-} = 1$ in the massless case, we cannot extract from their formulae the case $\alpha = 1$ in (22). It also proves to be impossible to verify the 1PI one-point function expressions as well as the check of the Ward identity , since the validity of the results of Ref. [8] is limited to on-shell fields.

In conclusion,

we are led to reexamine the massless Wess-Zumino model by the need to explore the limits of applicability of the adiabatic expressions as well as to focus on the effect of the reflective boundary conditions for the field propagators in AdS_4. In fact, following Refs. [13, 14], one can conceive a superformal transformation that, applied to the $m = 0$ model, will bring its action into the flat space-time form. The calculation of divergent quantum corrections then remains the same as in flat background. This is the same situation as already encountered in AdS_4 for the $N = 4$ extended supersymmetric Yang-Mills theory. 13

We have shown that the adiabatic expansion is not adequate at the level of finite corrections of $\mathcal{O}(a^2)$. The reflective boundary conditions play a significant role for terms of $\mathcal{O}(a^2)$ and $\mathcal{O}(a^3)$, in the short-distance expansion of the scalar and spinor propagator, respectively. As a consequence, the one-loop vacuum expectation value of the scalar field exhibits a finite shift with respect to its classical value $\langle A\rangle_0 = 0$, No finite spontaneous breakdown of the supersymmetry invariance of the phase $\langle A\rangle_0 = \langle B\rangle_0 = 0$ takes place as a consequence of the shift.

An interesting phenomenon is the production of a finite supersymmetric invariant mass term in the one-loop effective action, that represents a breakdown of the renormalization theory holding in flat space-time. This result is obtained in the adiabatic approximation using Pauli-Villars regulated fields and should be confirmed by calculating with the exact propagators. However, since this result appears as a correction of $\mathcal{O}(a)$ in the spinor self-energy, for example, while the failure of the adiabatic approximation in calculating finite corrections begins with $\mathcal{O}(a^2)$ terms, it is quite reasonable to maintain that the finite breakdown of the no-renormalization theorem presented here be a physical effect produced by the non-trivial background geometry. It is gratifying that this finite effect is compatible with supersymmetry, as well as with the known fact that chiral symmetry is broken in any theory in AdS_4 describing a spinor field [32].

Clarifying doubts raised in recent literature[8] about the legitimacy of the adiabatic approximation in calculating ultraviolet divergent corrections, we have confirmed that terms in the exact propagators depending on the reflective boundary conditions do not effect the calculation of the counterterms. The latter can be performed as in Ref. [6]. The finite mass term induced radiatively does not depend upon the boundary conditions. The only quantum feature of the massless model around its classical phase $\langle A\rangle_0 = \langle B\rangle_0 = 0$ which depends on the boundary conditions characterizing AdS_4 supersymmetry [3] is shown by the non-vanishing value of $\langle A\rangle$ to the one-loop order.

4.3. Spontaneous breaking of supersymmetry

The occurrence of AdS_4 as a ground state solution of both grand unified theories and supergravity constitutes the motivation to study field theories in AdS_4 background. In particular, in the case of gauged extended supergravities, it is known that both the vacuum configuration and the perturbative dynamics in this background are invariant under the supersymmetry group $OSp(N,4)$.[1] The small fluctuation action is provided by the generalization of the massless Wess-Zumino model to the fixed background of AdS_4.[2] On the other hand, the massive scalar supermultiplet occurs in the coupling of supersymmetric grand unified theories to gravity.[1] On these premises, it emerges the need for studying supersymmetric field theories in the fixed background of AdS_4 and, in particular, the self-interacting scalar multiplet. It is our purpose, in the following, to describe exhaustively the quantum features of the latter, accounting for the choice of reflective boundary conditions in solving the propagator equations.[3] We have already stressed the relevance of this choice in building supersymmetric models. The effect of the background geometry is going to be determined exactly in our computation of the one-loop effective potential of the model. After the renormalization of the kinetic action and the isertion of the linear counterterm in the superpotential, we will obtain the vacuum solutions in the semiclassical approximation by solving the quantum corrected equations of motion. The result is that the vacuum expectation values of the A and B fields are shifted by finite terms which depend upon the boundary conditions for the field propagators. Despite this result, we will see that the semiclassical theory quantized around any of the classically supersymmetric extrema of the effective potential maintains its supersymmetry invariance. This result is completely general and holds its validity, whatever the choice of boundary conditions adopted.

We begin by expanding the exact propagators in their analytic dependence from the contraction parameter a of the space. We include terms of order a^2 and show their correspondence to the adiabatic approximation of the field propagators. This completes the proof the the statement, advanced in sec. 4.1, of the irrelevance of the reflective boundary conditions for the purpose of renormalization. In fact,

we have neglected this feature of AdS_4 supersymmetry in obtaining renormalization coefficients, as well as in proving their independence from the choice of the vacuum. In the present section we are able to carry the renormalization program, dealing with the fixed background in an exact fashion. Our calculation confirms the validity of the formal argument presented in Sec. 4.1 in favor of the independence of the renormalization of field theories in AdS_4 from the boundary conditions.

Finite quantum corrections can play a role in the spontaneous breaking of supersymmetry in curved background space. We propose a point-splitting method to regulate the ultraviolet divergences of supersymmetric theories in AdS_4, proving explicitely that the supersymmetry Ward identities are satisfied by the finite corrections to the model under investigation. The occurrence of a one-loop radiatively induced breaking of supersymmetry has been excluded for the $m \neq 0$ model, in the adiabatic approximation. Since we include in our quantum calculation the effect of the background space exactly, we can claim with full generality that the supersymmetric character of the extrema of the classical potential persists to the one-loop order.

The $m = 0$ case is exceptional and the adiabatic expansion of the propagators is not suitable for computing finite quantum corrections. The above result holds true for the semi-classical vacuum solutions of the massless model as well. There is no spontaneous breaking of supersymmetry at the quantum level for the model placed in any of the classically supersymmetric minima of the potential. For the massless multiplet quantized around the local maximum $\langle A \rangle_0 = \langle B \rangle_0 = 0$ we recover the persistence of supersymmetry previously found by a dynamical calculation of the effective action. The important point is that the reflective boundary conditions do not induce any radiative breaking of AdS_4 supersymmetry.

We begin by recalling the propagator of a spin-0 field

$$G_F(x,x') = -\frac{i}{16\pi^2}a^2\frac{\Gamma(\lambda)\Gamma(\lambda-1)}{\Gamma(2\lambda-2)}(-z)^\lambda F(\lambda,\lambda-1,\ 2\lambda-2;\ z) \quad (4.3.1)$$

where z is given in terms of one-half of the square of the geodesic distance between x and x', $\sigma(x,x') \equiv \frac{1}{2}\left[\mu(x,x')\right]^2$, by

$$z = 2\left[1-\cosh\left(a\sqrt{-2\sigma}\right)\right]^{-1} \quad (4.3.2)$$

The choice of regular or irregular boundary conditions, corresponds to the sign $+$ or $-$, respectively, in the expression

$$\lambda = \frac{3}{2} \pm \sqrt{\frac{1}{4}+\frac{m^2}{a^2}} \quad (4.3.3)$$

The two-point function (1) satisfies the equation

$$\left(\Box - 2a^2 + m^2\right) G_F(x,x') = -\delta^4(x-x')\left[-g(x)\right]^{-\frac{1}{2}} \quad (4.3.4)$$

with reflective boundary conditions. In the massive case, Eq. (1) can be expanded analytically in a as follows

$$G_F(x,x') = -\frac{i}{(4\pi)^2}\left\{-\frac{2}{\sigma} + \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\sigma m^2\right)^k}{k!(k+1)!}\left\{\left[-\log\left(-\frac{1}{2}\sigma m^2\right)\right.\right.\right.$$
$$\left.\left.\left. + \psi(k+1) + \psi(k+2)\right]\left[a^2 k(k+1) - m^2\right] - a^2(2k+1)\right\}\right\} + O(a^4) \qquad (4.3.5)$$

Indeed, this expansion suffices to study renormalization in AdS_4. The maximum dimension of the local counterterms is such that, on dimensional grounds, only the $O(a^2)$ terms in the expansion of the field propagators are needed to determine the local counterterms.

For supersymmetric theories, we need the propagator of a spin-$\frac{1}{2}$ field, obeying the equation

$$(i\gamma^\tau D_\tau - m)\, S_F(x,x') = [-g(x)]^{-\frac{1}{2}}\, \delta^4(x-x') \qquad (4.3.6)$$

with boundary conditions appropriate for supersymmetry . We define the geometric tensors $\left[S(x)\overline{S}(x')\right]$ and $\left[i\gamma^\tau n_\tau(x,x') S(x)\overline{S}(x')\right]$ in terms of the solution of the equation for Killing spinors $\epsilon_\alpha(x)$

$$D_\mu \epsilon_\alpha(x) = -\frac{ia}{2}(\gamma_\mu)_{\alpha\beta}\epsilon_\beta(x)$$

$$\epsilon_\alpha(x) = S(x)_{\alpha\beta}\theta_\beta$$

Here θ_β is a constant spinor and the γ-matrices are flat. The unit tangent in x, $n_\tau(x,x')$, to the geodesic from x to x' reads as follows

$$n_\tau(x,x') = D_\tau \mu(x,x')$$

in terms of the proper distance $\mu(x,x')$. Making the ansatz

$$S_F(x,x') = f(\mu)S(x)\overline{S}(x') + g(\mu)i\gamma^\tau n_\tau(x,x')S(x)\overline{S}(x') \qquad (4.3.7)$$

the solution of Eq. (5) is readily found

$$S_F(x,x') = (m+a)G_A(x,x')S(x)\overline{S}(x') + \frac{\partial G_A}{\partial_\mu} i\gamma^\tau n_\tau(x,x')S(x)\overline{S}(x') \quad (4.3.8)$$

The scalar function $G_A(x,x')$ satisfies Eq. (1) with the mass term $m_A^2 = m^2 - am$.

In the case of the spin-1/2 component of the scalar supermultiplet, the propagator (8) satisfies the supersymmetric Ward identity

$$S_F(x,x') = [(i\not{D} + m + a)\, G_A\,(x,x')]\, S(x)\overline{S}(x')$$

The geometric object $S(x)\overline{S}(x')$ can be expanded as follows

$$\left[S(x)\overline{S}(x')\right]_{\alpha\beta} = \left\{-\frac{i}{2}a(\gamma^\tau)_{\alpha\beta}n_\tau(x,x')\mu(x,x') + \left[1 - \frac{1}{8}a^2\mu^2(x,x')\right]\delta_{\alpha\beta}\right\} + O(a^3\mu^3)$$

$$(4.3.9)$$

Using Eqs. (5) and (9), it is possible to show that the propagator $S_F(x,x')$ has the following expansion in the a parameter

$$[{}_0S_F(x,x')]_{\alpha\beta} = -\frac{i}{16\pi^2}m\Bigg\{-\frac{2}{\sigma} - m^2\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\sigma m^2\right)^k}{k!(k+1)!}\Bigg[-\log\left(-\frac{1}{2}\sigma m^2\right)$$

$$+\psi(k+1)+\psi(k+2)\Bigg]\Bigg\}\delta_{\alpha\beta} - \frac{i}{16\pi^2}i\,n_\tau(\gamma_\tau)_{\alpha\beta}\mu\left(\frac{2}{\sigma^2}+\frac{m^2}{\sigma}\right)$$

$$+\frac{1}{2}m^4\left(-\frac{i}{16\pi^2}\right)in^\tau(\gamma_\tau)_{\alpha\beta}\mu\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\sigma m^2\right)^k}{(k+1)!(k+2)!}$$

$$\left\{(k+1)\left[-\log\left(-\frac{1}{2}\sigma m^2\right)+\psi(k+2)+\psi(k+3)\right]-1\right\}$$

$$[{}_1S_F(x,x')]_{\alpha\beta} = -\frac{i}{16\pi^2}ma^2\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\sigma m^2\right)^k}{k!(k+1)!}\Bigg\{(k+1)^2\Bigg[-\log\left(-\frac{1}{2}\sigma m^2\right)$$

$$+\psi(k+1)+\psi(k+2)\Bigg]-2(k+1)\Bigg\}\delta_{\alpha\beta} - \frac{i}{16\pi^2}i\,n^\tau(\gamma_\tau)_{\alpha\beta}\frac{a^2}{4}\mu\Bigg\{\frac{2}{\sigma}$$

$$-m^2\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\sigma m^2\right)^k}{k!(k+1)!}\left\{(2k+1)\left[-\log\left(-\frac{1}{2}\sigma m^2\right)+\psi(k+1)+\psi(k+2)\right]-2\right\}$$

$$S_F(x,x') = {}_0S_F(x,x') + {}_1S_F(x,x') + O(a^3)$$

(4.3.10)

Considering, for example, the massive Wess-Zumino model in AdS_4, it is seen that its divergent renormalization to any loop order is completely determined by the expansions (5) and (10) of the propagators. The finite quantum corrections computed using (5) and (10) are only determined with an accuracy which includes $O(a^2)$ terms (see chapter 3).

In computing the effective potential of a theory incorporating spin-1/2 field(s) it is useful to consider the two-point function defined by

$$(i\gamma^\tau D_\tau + m)\,\mathcal{G}_F(x,x') = S_F(x,x') \qquad (4.3.11)$$

Making an ansatz analogous to (7) for $\mathcal{G}_F(x,x')$ and recalling Eq. (8), we can solve for $\mathcal{G}_F(x,x')$

$$\mathcal{G}_F(x,x') = \frac{1}{2m}\Big\{[(m+a)G_A(x,x') + (m-a)G_B(x,x')] + \left(\frac{\partial G_A}{\partial \mu} - \frac{\partial G_B}{\partial \mu}\right) i\gamma^\tau n_\tau(x,x')\Big\} S(x)\overline{S}(x') \tag{4.3.12}$$

The scalar function $G_B(x,x')$ satisfies Eq. (4) with the mass term $m_B^2 = m^2 + am$. The bispinor $\mathcal{G}_F(x,x')$ satisfies the equation

$$\left(D^\tau D_\tau - 3a^2 + m^2\right)\mathcal{G}_F(x,x') = -\left[-g(x)\right]^{-1/2}\delta^4(x-x') \tag{4.3.13}$$

with the reflective boundary conditions appropriate for AdS_4. If the spin-1/2 field forms a scalar supermultiplet with the scalar and pseudoscalar fields A and B, then one is forced by supersymmetry to choose regular boundary conditions for both spin-0 fields $\left(i.e.\ \lambda_{A+} = \frac{3}{2} + \sqrt{\frac{1}{4} + \frac{m_A^2}{a^2}},\ \lambda_{B+} = \frac{3}{2} + \sqrt{\frac{1}{4} + \frac{m_B^2}{a^2}}\right)$ when $|m| \geq \frac{1}{2}a$ (assuming $a \geq 0$), while for $-\frac{1}{2} < \frac{m}{a} < \frac{1}{2}$ one is left with the freedom to select regular or irregular boundary conditions for A, but correspondingly must solve Eq. (4) with irregular or regular choice, respectively, for B.[1]

With the help of Eq. (9) the reader may check that the expansion of $\mathcal{G}_F(x,x')$ in the parameter a is

$$[\mathcal{G}_F(x,x')]_{\alpha\beta} = -\frac{i}{16\pi^2}\left(-\frac{2}{\sigma}\right)\delta_{\alpha\beta} - \frac{i}{16\pi^2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\sigma m^2\right)^k}{k!(k+1)!} \times \Big\{\left[-m^2 + a^2(k+1)^2\right]\left[-\log\left(-\frac{1}{2}\sigma m^2\right) + \psi(k+1) + (k+2)\right] + 2a^2(k+1)\Big\}\delta_{\alpha\beta} + O(a^3) \tag{4.3.14}$$

Since no trace of the boundary conditions appears in the expansions (5), (10), (14) it is obvious that those can be put in correspondence with DeWitt-Schwinger adiabatic representations. This can easily be done for Eq. (5). It is not too difficult to show the same for the spin-1/2 two-point functions and we merely quote the results

$$S_{DS} = mG_0 + i\,\not{n}\,\frac{\partial G_0}{\partial \mu} + ia^2\,\not{n}\,\frac{\partial}{\partial m^2}\frac{\partial}{\partial \mu}G_0 - i\,\not{n}\,a^2 m^2 \frac{\partial^2}{(\partial m^2)^2}\frac{\partial}{\partial \mu}G_0 - ma^2\frac{\partial G_0}{\partial m^2} - m^3 a^2 \frac{\partial^2 G_0}{(\partial m^2)^2} + O(a^3)$$

$$G_{DS} = G_0 - a^2\frac{\partial G_0}{\partial m^2} - a^2 m^2 \frac{\partial^2 G_0}{(\partial m^2)^2} + O(a^2)$$

where G_0 is the flat space-time scalar propagator

$$G_0 = -\frac{i}{16\pi^2}\left\{-\frac{2}{\sigma} - m^2\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\sigma m^2\right)^k}{k!(k+1)!}\left[-\log\left(-\frac{1}{2}\sigma m^2\right) + \psi(k+1) + \psi(k+2)\right]\right\}$$

Recalling (10), (14) one has

$$S_F(x,x') = S_{DS}(x,x') + O(a^3)$$

$$\mathcal{G}_F(x,x') = \mathcal{G}_{DS}(x,x') + O(a^3)$$

We notice that the disagreement, claimed in Ref. [8], between the singular behavior of the short-distance expansion of the spinor propagator S_F and the corresponding behavior of the adiabatic approximated propagator S_{DS} is simply not there. This would come from an $O(a)$ term in S_F and we see that these terms cancel explicitely in (10).

In chapter 3 the adiabatic approximated propagators G_{DS}, $\mathcal{G}_{DS}$ are used to compute one-loop corrections independently from the vacuum solution around which the supersymmetric theory is quantized. We now get more ambitious and study quantum supersymmetry in AdS$_4$ using the exact field propagators. For this, we need a regularization procedure and we choose point-splitting. The supersymmetry invariance of this treatment must be insured when introducing the infinitesimal separation $\mu(x,x') = \epsilon$ in the propagators (1), (12) (see below). In the short distance limit of Eq. (1) we have [26]

$$G_F(x,x') = -\frac{i}{16\pi^2}\left\{-\frac{4}{\epsilon^2} - \frac{1}{3}a^2 - m^2\left[-\log\left(-\frac{1}{4}a^2\epsilon^2\right) + \psi(1) + \psi(2) - \psi(\lambda) - \psi(\lambda-2)\right]\right\} + O\left(\epsilon^2, \epsilon^2\log\epsilon\right) \tag{4.3.15}$$

Then, from (12) we obtain, recalling (9)

$$\mathcal{G}_F(x,x') = -\frac{i}{16\pi^2}\left\{-\frac{4}{\epsilon^2} - \frac{a^2}{3} - (m^2 - a^2)\left[-\log\left(-\frac{1}{4}a^2\epsilon^2\right) + \psi(1) + \psi(2) - \frac{1}{2}\psi(\lambda_A) - \frac{1}{2}\psi(\lambda_A - 2) - \frac{1}{2}\psi(\lambda_B) - \frac{1}{2}\psi(\lambda_B - 2)\right]\right\} + O(\epsilon, \epsilon\log\epsilon) \tag{4.3.16}$$

In the bispinor (16) we are neglecting finite terms (in the $\epsilon \to 0$ limit) which result from multiplying the expansion (9) of the geometrical object $S(x)\overline{S}(x')$ times the divergent terms of the propagators G_A, G_B. These terms would add to (16) the contribution $-\frac{i}{16\pi^2}\left(-\frac{a^2}{2}\right)$. Discarding the latter insures that supersymmetry be maintained to the one-loop order. We suggest here that, in multi-loop integrals involving the bispinor $\mathcal{G}_F$ expressed in terms of spin-0 propagators G_A and G_B, discarding in the final result all finite terms related to the expansion (9) be a supersymmetric invariant prescription suitable to maintain the balance between bosonic and fermionic degrees of freedom in the curved background. The above is a natural prescription as can be seen from the following argument. The natural variable to express field propagators between two points of AdS_4 is the chordal distance $u(x,x')$ measured on the hyperboloid [17]. This is related to Eq. (2) by

$$2u^{-1} = z = \left(\frac{4}{\epsilon^2} + \frac{1}{3}a^2\right)a^{-2} + O(a^2\epsilon^2)$$

In this way, we are led to regularize the short-distance limit of the field propagators by introducing a small covariant separation $u(x, x')$. Then, in order to preserve the supersymmetric Ward identities, the natural requirement is that the quadratically divergent terms in the $u \to 0$ limit of the propagators G_A, G_B and $\mathcal{G}_F$ be the same. The prescription leads to Eq. (16).

The short-distance expansions of the exact propagators (15) (16) allow to compute one-loop finite corrections to spin-0/spin-1/2 theories in AdS_4 with complete generality, incorporating the effect of the boundary conditions. With respect to the use of adiabatic approximations there are two advantages: (i) the finite corrections are determined as analytic functions in a, *i.e.* to all orders in a, rather than just up to $\mathcal{O}(a^2)$; (ii) since for massless fields any perturbative treatment in the curvature of AdS_4 is not suitable, given the absence of an alternative scale parameter in the theory, the exact calculation is a must for $m = 0$.

In studying the extrema of the classical potential we do not need to specify their nature. It has been shown that the classical solutions given by a stationary point or even a global maximum of the potential can correspond to a stable vacuum state.[1]. We consider the quantization of the model around each of the classically supersymmetric extrema of the potential, assuming they are all legitimate tree-level vacuum solutions representing different supersymmetric phases. The classical potential exhibits the following supersymmetric invariant extrema

(i) $\hat{A}_0 = \hat{B}_0 = 0$, for any real value of m;

(ii) $\hat{A}_0 = -(m+a)/\lambda$, $\hat{B}_0 = 0$, for $m \geq 0$ and $m < -4a$;

(iii)-(iv) $\hat{A}_0 = (-m+a)/2\lambda$, $\hat{B}_0 = \pm\left[(a-m)^{1/2}(3a+m)^{1/2}\right]/2\lambda$, for $-3a < m < a$.

The extrema are analyzed in section 4.2 for the $m \neq 0$ case. The massless potential is exceptional, since it has an infinite class of minima breaking supersymmetry at the tree-level. The breaking of the supersymmetry invariance of the model in curved space-time is determined unambiguously in terms of non-vanishing vacuum expectation values for the auxiliary fields F, G. For the above extrema $\hat{F}_0 = \hat{G}_0 = 0$.

We perform a classical shift of the fields in the action (17)

$$A \to A + \hat{A} \ , \qquad B \to B + \hat{B} \ , \qquad F \to F + \hat{F} \ , \qquad G \to G + \hat{G}$$

and then integrate the action quadratic in the quantum fields S_q to obtain the one-loop effective action of the model

$$\Gamma_1(\hat{A}, \hat{B}, \hat{F}, \hat{G}) = i \log \int \mathcal{D}A\mathcal{D}B\mathcal{D}F\mathcal{D}G\mathcal{D}\psi \, e^{iS_q} \qquad (4.3.17)$$

where

$$\begin{aligned} S_q = \int d^4x\sqrt{-g}\Big\{ & \frac{1}{2}(D_\mu A D^\mu A + D_\mu B D^\mu B \\ & + i\overline{\psi}\, \not{D}\psi + F^2 + G^2) + a(AF + BG) + \frac{3}{2}a^2(A^2 + B^2) \\ & + m\left[AF - BG + \frac{3}{2}a(A^2 - B^2) - \frac{1}{2}\overline{\psi}\psi\right] \\ & + \lambda(A^2\hat{F} + 2AF\hat{A} - B^2\hat{F} - 2BF\hat{B} - 2AB\hat{G} \\ & - 2AG\hat{B} - 2BG\hat{A} + 3aA^2\hat{A} - 6aA^2\hat{A} - 6aAB\hat{B} \\ & - 3aB^2\hat{A} - \overline{\psi}\psi\hat{A} - \overline{\psi}i\gamma_5\psi\hat{B})\Big\} \end{aligned}$$

We have used the classical equation of motion obeyed by the extrema of the classical potential in order to get rid of terms linear in the quantum fluctuations. Integrating over the quantum fields in (17) gives

$$\begin{aligned} \Gamma_1\left(\hat{A}, \hat{B}, \hat{F}, \hat{G}\right) = -\frac{i}{2}\Big[& \mathrm{tr}\,\log\left(D^\mu D_\mu - 2a^2 + m_2^2\right) \\ & + \mathrm{tr}\,\log\left(D^\mu D_\mu - 2a^2 + m_3^2\right) \\ & - 2\,\mathrm{tr}\,\log\left(D^\mu D_\mu - 3a^2 + m_1^2\right)\Big] \end{aligned} \qquad (4.3.18)$$

where

$$m_1^2 = \left(m + 2\lambda\hat{A}\right)^2 + 4\lambda^2\hat{B}^2$$

$$m_2^2 = m_1^2 - \alpha \qquad (4.3.19)$$

$$m_3^2 = m_1^2 + \alpha$$

$$\alpha = \left[\left(am + 2\lambda\hat{F} + 2\lambda a\hat{A}\right)^2 + 4\lambda^2\left(\hat{G} + a\hat{B}\right)^2\right]^{1/2}$$

To compute the divergent part of (18) it is convenient to follow a suggestion by Burgess and Lütken,[33] taking the functional derivative of the effective action with respect to any of its arguments $\hat{\phi}$

$$\frac{\partial\Gamma_1}{\partial\hat{\phi}} = -\frac{i}{2}\lim_{x\to x'}\left[\frac{\partial m_2^2}{\partial\hat{\phi}}G_2(x,x') + \frac{\partial m_3^2}{\partial\hat{\phi}}G_3(x,x') - 2\frac{\partial m_1^2}{\partial\hat{\phi}}\mathcal{G}_1(x,x')\right] \qquad (4.3.20)$$

The Green function $G_2(G_3)$ in Eq. (20) is the propagator of a spin-0 field, as given by Eq. (1) with mass equal to $m_2(m_3)$ and boundary conditions which coincide with those selected for the $A(B)$ component of the scalar supermultiplet. The bispinor $\mathcal{G}_1$ is provided by Eq. (12) with mass equal to m_1 and boundary conditions for G_A and G_B chosen to preserve supersymmetry for any value of the mass.[1]

Since a coincidence limit is involved in Eq. (20), it is quite natural to regularize the one-loop calculation by the method of point-splitting. Recalling Eqs. (15) and (16) we obtain the divergent part of (20)

$$\frac{\partial\Gamma_1^{\text{div}}}{\partial\hat{\phi}} = -\frac{1}{32\pi^2}\log\left(-\frac{1}{4}\nu^2\epsilon^2\right)\left[m_2^2\frac{\partial m_2^2}{\partial\hat{\phi}} + m_3^2\frac{\partial m_3^2}{\partial\hat{\phi}} - 2(m_1^2 - a^2)\frac{\partial m_1^2}{\partial\hat{\phi}}\right] \qquad (4.3.21)$$

after introducing the subtraction parameter ν. Notice that quadratic divergences cancel in (21) because of supersymmetry invariance. Using the definitions (19) we can rewrite (21) in the form

$$\frac{\partial\Gamma_1^{\text{div}}}{\partial\hat{\phi}} = -\frac{1}{32\pi^2}\log\left(-\frac{1}{4}\nu^2\epsilon^2\right)\frac{\partial}{\partial\hat{\phi}}\left(\alpha^2 + 2a^2 m_1^2\right)$$

Calculating the derivatives and integrating formally over the hatted fields we get

$$\Gamma_1^{\rm div} = -\frac{1}{32\pi^2}\int d^4x\sqrt{-g}\log\left(-\frac{1}{4}\nu^2\epsilon^2\right)\Big[4\lambda am(3a\hat{A}+\hat{F})$$
$$+8\lambda^2\left(a\hat{A}\hat{F}+a\hat{B}\hat{G}+\frac{3}{2}a^2\hat{A}^2+\frac{3}{2}a^2\hat{B}^2+\frac{1}{2}\hat{F}^2+\frac{1}{2}\hat{G}^2\right)\Big] \qquad (4.3.22)$$

after omitting a divergent term independent from the $\hat{\phi}$'s which is irrelevant for our purposes here. The result (22) exhibits the independence of the renormalization in AdS_4 from the choice of boundary conditions for the propagators in (20), confirming the proof of sec. 4.1. Eq. (22) agrees with the results obtained using the adiabatic approximated propagators. The correspondence with Eq. (3.1.22) , where the one-loop effective action is regularized by going in dimension D, is given by

$$-\frac{1}{2}\log\left(-\frac{1}{4}\nu^2\epsilon^2\right) \longleftrightarrow \frac{1}{4-D}$$

We renormalize $S_{\rm kin}$ and introduce also a linear supersymmetric insertion in the action as required by the counterterms (22). Next, we analyze the one-loop corrected equations of motion of the fields. This enables us to study the spontaneous breaking of supersymmetry at each (classically invariant) extremum of the effective potential.

In the renormalized theory the derivative of the one-loop effective potential has the finite value

$$\frac{\partial V_1}{\partial \hat{\phi}} = \frac{1}{32\pi^2}\Bigg\{ -\overline{m}_2^2\Big[-\log\left(\frac{a^2}{\nu^2}\right) + \psi(1) + \psi(2) - \psi(\overline{\lambda}_2) - \psi(\overline{\lambda}_2 - 2)\Big] \frac{\partial m_2^2}{\partial \hat{\phi}}\bigg|_0 - \overline{m}_3^2\Big[-\log\left(\frac{a^2}{\nu^2}\right) + \psi(1) + \psi(2) - \psi(\overline{\lambda}_3) - \psi(\overline{\lambda}_3 - 2)\Big] \frac{\partial m_3^2}{\partial \hat{\phi}}\bigg|_0 + 2(\overline{m}_1^2 - a^2)\Big[-\log\left(\frac{a^2}{\nu^2}\right) + \psi(1) + \psi(2) - \frac{1}{2}\psi(\overline{\lambda}_A) - \frac{1}{2}\psi(\overline{\lambda}_A - 2) - \frac{1}{2}\psi(\overline{\lambda}_B) - \frac{1}{2}\psi(\overline{\lambda}_B - 2)\Big] \frac{\partial m_1^2}{\partial \hat{\phi}}\bigg|_0 \Bigg\} \qquad (4.3.23)$$

This is obtained my making use in Eq. (20) of the regulated propagators (15) and (16). The local quantity (23) is evaluated at the extremum characterizing the particular classically supersymmetric phase under study. In any classically supersymmetric phase both quantities $\hat{F}$ and $\hat{G}$ are of order λ. Thus, recalling Eq. (19) we have

$$\overline{\alpha} = a\overline{m}_1 + O(\lambda^2) \qquad (4.3.24)$$

We neglect $O(\lambda^2)$ terms in (24) since they correspond to terms of order λ^3 in the derivative (23), *i.e.* higher order in the loop expansion. For each phase we specify the value of $\overline{m}_1$ and the corresponding values of the arguments of the ψ-functions in Eq. (23). The values of $\overline{m}_2$, $\overline{m}_3$, $\frac{\partial m_1^2}{\partial \hat{\phi}}\big|_0$ and $\overline{\alpha}$ are then obtained from Eqs. (19) and (24).

Let us begin with the phase characterized by the classical extrema (iii) – (iv) of section III. We have

$$\overline{m}_1 = \left[a^2 + (a-m)(3a+m)\right]^{1/2} > a \qquad (4.3.25)$$

where the condition $-3a < m < a$ is needed for the existence of the extrema (iii) – (iv). Because of the inequality (25) supersymmetry forces the choice of regular boundary conditions for the propagators in Eq. (20)

$$\overline{\lambda}_2 = \overline{\lambda}_A = 1 + \frac{\overline{m}_1}{a} \quad , \quad \overline{\lambda}_3 = \overline{\lambda}_B = 2 + \frac{\overline{m}_1}{a} \qquad (4.3.26)$$

In the phase (ii), we have $\overline{m}_1 = m + 2a$. Since $|\overline{m}_1| \geq 2a$, the boundary conditions compatible with the supersymmetry transformations are given by the regular choice. In the range $m \geq 0$ we must select the values of the parameters given in (26). For $m < -4a$ the supersymmetric boundary conditions are

$$\overline{\lambda}_2 = \overline{\lambda}_A = 2 - \frac{\overline{m}_1}{a} \quad , \quad \overline{\lambda}_3 = \overline{\lambda}_B = 1 - \frac{\overline{m}_1}{a} \qquad (4.3.27)$$

For phase (i) ($\overline{m}_1 = m$) we need to distinguish between two different ranges of values for the mass m of the supermultiplet. The supersymmetric choice is the regular one for $|m| \geq \frac{1}{2}a$, *i.e.* Eq. (26) for $m \geq \frac{1}{2}a$ and Eq. (27) for $m \leq -\frac{1}{2}a$. For $|m| < \frac{1}{2}a$ we have the choice of two combinations of regular and irregular boundary conditions. Selecting regular (irregular) boundary conditions for the field A and, accordingly, irregular (regular) for B, yields the values of the parameters in Eq. (27) (Eq. (26)). We can rewrite Eq. (23) in the phases (iii) – (iv), (ii), as well as in phase (i) for $|m| \geq \frac{1}{2}a$, as follows

$$\frac{\partial V_1}{\partial \hat{A}} = \frac{1}{(4\pi)^2} 12\lambda a^2 \left(m + 2\lambda \hat{A}_0\right) \left(\psi\left(\frac{|m|_1}{a}\right) + \frac{a}{2|\overline{m}_1|} + \frac{1}{2}\log\frac{a^2}{\nu^2}\right]$$

$$\frac{\partial V_1}{\partial \hat{F}} = \frac{1}{(4\pi)^2} 4\lambda a \left(m + 2\lambda \hat{A}_0\right) \left[\psi\left(\frac{|\overline{m}_1|}{a}\right) + \frac{a}{2|\overline{m}_1|} + \frac{1}{2}\log\frac{a^2}{\nu^2}\right]$$

$$\frac{\partial V_1}{\partial \hat{B}} = \frac{1}{(4\pi)^2} 24\lambda^2 a^2 \hat{B}_0 \left[\psi\left(\frac{|\overline{m}_1|}{a}\right) + \frac{a}{2|\overline{m}_1|} + \frac{1}{2}\log\frac{a^2}{\nu^2}\right]$$

$$\frac{\partial V_1}{\partial \hat{G}} = \frac{1}{(4\pi)^2} 8\lambda^2 a \hat{B}_0 \left[\psi\left(\frac{|\overline{m}_1|}{a}\right) + \frac{a}{2|\overline{m}_1|} + \frac{1}{2}\log\frac{a^2}{\nu^2}\right] \qquad (4.3.28)$$

with the mass $\overline{m}_1$ appropriate for each phase $\hat{A}_0$, $\hat{B}_0$. In the phase $\hat{A}_0 = \hat{B}_0 = 0$, for $\frac{|m|}{a} < \frac{1}{2}$ we obtain from (23)

$$\frac{\partial V_1}{\partial \hat{A}} = \frac{1}{(4\pi)^2} 12\lambda a^2 m \left[\psi\left(-\frac{m}{2}\right) - \frac{a}{2m} + \frac{1}{2}\log\frac{a^2}{\nu^2} \right]$$

$$\frac{\partial V_1}{\partial \hat{F}} = \frac{1}{(4\pi)^2} 4\lambda a m \left[\psi\left(-\frac{m}{a}\right) - \frac{a}{2m} + \frac{1}{2}\log\frac{a^2}{\nu^2} \right]$$

$$\frac{\partial V_1}{\partial \hat{B}} = \frac{\partial V_1}{\partial \hat{G}} = 0 \qquad (4.3.29)$$

provided regular boundary conditions are selected for A (and irregular ones for B). The choice of irregular boundary conditions for A (and regular ones for B) yields the results

$$\frac{\partial V_1}{\partial \hat{A}} = \frac{1}{(4\pi)^2} 12\lambda a^2 m \left[\psi\left(\frac{m}{a}\right) + \frac{a}{2m} + \frac{1}{2}\log\frac{a^2}{\nu^2} \right]$$

$$\frac{\partial V_1}{\partial \hat{F}} = \frac{1}{(4\pi)^2} 4\lambda a m \left[\psi\left(\frac{m}{a}\right) + \frac{a}{2m} + \frac{1}{2}\log\frac{a^2}{\nu^2} \right]$$

$$\frac{\partial V_1}{\partial \hat{B}} = \frac{\partial V_1}{\partial \hat{G}} = 0 \qquad (4.3.30)$$

Eqs. (28) – (30) preserve the supersymmetry invariance expressed by the Ward identities. This guarantees that our regularization procedure is sound.

It is clear at this point that the results obtained with the adiabatic expansion of the propagators are only an approximation of Eqs. (28), (29) and (30). In fact, expanding in the parameter a (if $m \neq 0$), using the asymptotic expansion of the ψ function[26]

$$\psi(x) + \frac{1}{2x} = \frac{1}{2}\log x^2 + O(x^{-2})$$

it is easy to verify the agreement of Eqs. (28) – (30) with Eqs. (3.2.4)— (3.2.7).

This confirms that finite corrections of order a^2 do not depend from the choice of boundary conditions for the field propagators.

Next, let us focus on the massless case. Taking the limit $m \to 0$ in Eqs. (29) and (30) and using the property $\lim_{x\to 0}[x\psi(x)] = -1$, one recovers the results in Eqs. (4.2.17) and (4.2.18) for the 1PI one-point functions Γ_A and Γ_F. Also, the arguments presented suggest that the approximated calculation of the finite one-loop corrections to the massless model using the adiabatic expansion is unsuitable. This problem is overcome by our exact approach, yielding the following results

$$\begin{aligned}\frac{\partial V_1}{\partial \hat{F}} &= \pm\frac{1}{(4\pi)^2}2\lambda a^2\\ \frac{\partial V_1}{\partial \hat{A}} &= \pm\frac{1}{(4\pi)^2}6\lambda a^3\\ \frac{\partial V_1}{\partial \hat{B}} &= \frac{\partial V_1}{\partial \hat{G}} = 0\end{aligned} \tag{4.3.31}$$

for the $m = 0$ model quantized around the origin $\hat{A}_0 = \hat{B}_0 = 0$;

$$\begin{aligned}\frac{\partial V_1}{\partial \hat{F}} &= -\frac{1}{(4\pi)^2}8\lambda a^2\left[\psi(2) + \frac{1}{4} + \frac{1}{2}\log\frac{a^2}{\nu^2}\right]\\ \frac{\partial V_1}{\partial \hat{G}} &= 0\end{aligned} \tag{4.3.32}$$

and

$$\begin{aligned}\frac{\partial V_1}{\partial \hat{F}} &= \frac{1}{(4\pi)^2}4\lambda a^2\left[\psi(2) + \frac{1}{4} + \frac{1}{2}\log\frac{a^2}{\nu^2}\right]\\ \frac{\partial V_1}{\partial \hat{G}} &= \pm\frac{1}{(4\pi)^2}4\sqrt{3}\lambda a^2\left[\psi(2) + \frac{1}{4} + \frac{1}{2}\log\frac{a^2}{\nu^2}\right]\end{aligned} \tag{4.3.33}$$

for the $m = 0$ model around the two classically supersymmetric minima of the potential $\hat{A}_0 = -\frac{a}{\lambda}$, $\hat{B}_0 = 0$ and $\hat{A} = \frac{a}{2\lambda}$, $\hat{B}_0 = \pm\frac{\sqrt{3}}{2}\frac{a}{\lambda}$, respectively, along with the additional results $\frac{\partial V_1}{\partial \hat{A}} = 3a\frac{\partial V_1}{\partial \hat{F}}$, $\frac{\partial V_1}{\partial \hat{B}} = 3a\frac{\partial V_1}{\partial \hat{G}}$.

In the semiclassical approximation the vacuum solutions of the theory are obtained from the equations of motion

$$\frac{\partial}{\partial \hat{\phi}}(V_0 + V_1) = 0$$

As in chapter 3, we set equal to zero the renormalized value of the parameter in front of the linear insertion in the superpotential required by Eq. (22). Thus, we can take for V_0 the tree level potential provided by the action . This yields the one-loop corrected equations of motion

$$
\begin{aligned}
&\hat{F} = a\hat{A} - m\hat{A} - \lambda\hat{A}^2 + \lambda\hat{B}^2 + \frac{\partial V_1}{\partial \hat{F}} \\
&\hat{G} = -a\hat{B} + m\hat{B} + 2\lambda\hat{A}\hat{B} + \frac{\partial V_1}{\partial \hat{G}} \\
&a\hat{F} + 3a^2\hat{A} + m\hat{F} + 3am\hat{A} + 2\lambda\hat{A}\hat{F} - 2\lambda\hat{B}\hat{G} + 3\lambda a\hat{A}^2 - 3\lambda a\hat{B}^2 - \frac{\partial V_1}{\partial \hat{A}} = 0 \\
&a\hat{G} + 3a^2\hat{B} - m\hat{G} - 3am\hat{B} - 2\lambda\hat{B}\hat{F} - 2\lambda\hat{A}\hat{G} - 6\lambda a\hat{A}\hat{B} - \frac{\partial V_1}{\partial \hat{B}} = 0
\end{aligned}
\qquad (4.3.34)
$$

where the corrections $\frac{\partial V_1}{\partial \phi}$ are given by Eqs. (28) – (30). We solve (34) to the one-loop order, *i.e.* to order λ^2. For the classically supersymmetric extrema we have

$$A_{\text{vac}} = \hat{A}_0 + \hat{A}_1 + \ldots \ , \qquad \hat{B}_{\text{vac}} = \hat{B}_0 + \hat{B}_1 + \ldots$$

$$F_{\text{vac}} = \hat{F}_1 + \ldots \ , \qquad G_{\text{vac}} = \hat{G}_1 + \ldots$$

where terms of order λ^3 are neglected. Since the one-loop corrections $\frac{\partial V_1}{\partial \phi}$ satisfy the supersymmetry Ward identities , it is easily seen that (34) admits the solution

$$\hat{F}_1 = \hat{G}_1 = 0$$

Supersymmetry invariance persists at any of the classically invariant extrema of the potential, when one-loop quantum corrections are taken into account. This result extends the analysis of chapter 3 , including all orders in a. The non-trivial background space does not induce a spontaneous breaking of supersymmetry by radiative corrections.

We can now compute, with complete generality, the vacuum solutions for the different phases

(i)

$$\hat{A}_0 = \hat{B}_0 = 0$$

$$\hat{A}_1 = \frac{1}{(4\pi)^2} 4\lambda a \frac{m}{a+m} \left[\psi\left(\mp\frac{m}{a}\right) \mp \frac{a}{2m} + \frac{1}{2}\log\frac{a^2}{\nu^2} \right] \qquad (4.3.35)$$

for $\frac{|m|}{a} < \frac{1}{2}$, choosing regular or, respectively, irregular boundary conditions for the field A;

$$\hat{A}_1 = \frac{1}{(4\pi)^2} 4\lambda a \frac{m}{a+m} \left[\psi\left(\frac{|m|}{a}\right) + \frac{a}{2|m|} + \frac{1}{2}\log\frac{a^2}{\nu^2} \right] \qquad (4.3.36)$$

for $\frac{|m|}{a} \geq \frac{1}{2}$. $\hat{B}_1$ vanishes for any value of m.

(ii) $\hat{A}_0 = -\frac{m+a}{\lambda}$, $\hat{B}_0 = 0$, for $m \geq 0$ and $m < -4a$

$$\hat{A}_1 = \frac{1}{(4\pi)^2} 4\lambda a \frac{m+2a}{m+a} \left[\psi\left(\left|2+\frac{m}{a}\right|\right) + \frac{a}{2\,|m+2a|} + \frac{1}{2}\log\frac{a^2}{\nu^2} \right]$$

$$\hat{B}_1 = 0 \qquad (4.3.37)$$

(iii)–(iv) $\hat{A}_0 = \frac{a-m}{2\lambda}$, $\hat{B}_0 = \pm\frac{(a-m)^{1/2}(3a+m)^{1/2}}{2\lambda} \equiv \pm\frac{a}{2\lambda}\beta$, for $-3a < m < a$

$$\hat{A}_1 = -\frac{1}{(4\pi)^2} 4\lambda a \left(1 \pm \frac{m}{a\beta}\right) \left[\psi\left(\sqrt{1+\beta^2}\right) + \frac{1}{2\sqrt{1+\beta^2}} + \frac{1}{2}\log\frac{a^2}{\nu^2} \right]$$

$$\hat{B}_1 = \mp\frac{1}{(4\pi)^2} 4\lambda a\, \beta^{-1} \left(3 \pm \frac{2m}{a\beta}\right) \left[\psi\left(\sqrt{1+\beta^2}\right) + \frac{1}{2\sqrt{a+\beta^2}} + \frac{1}{2}\log\frac{a^2}{\nu^2} \right]$$

$$(4.3.38)$$

In the $m \to 0$ limit, we have, in phase (i)

$$\hat{A}_1 = \pm\frac{1}{(4\pi)^2} 2\lambda a$$

$$\hat{B}_1 = 0 \qquad (4.3.39)$$

where the sign + (−) corresponds to the choice of regular boundary conditions for the field A (B). Eq. (39) agrees with Eq.(4.2.22). The two supersymmetric minima of the $m = 0$ classical potential are shifted by the amounts

$$\hat{A}_1 = \frac{1}{(4\pi)^2} 8\lambda a \left[\psi(2) + \frac{1}{4} + \frac{1}{2}\log\frac{a^2}{\nu^2}\right]$$
$$\hat{B}_1 = 0 \tag{4.3.40}$$

for the $\hat{A}_0 = -\frac{a}{\lambda}$, $\hat{B}_0 = 0$ minimum and

$$\hat{A}_1 = -\frac{1}{(4\pi)^2} 4\lambda a \left[\psi(2) + \frac{1}{4} + \frac{1}{2}\log\frac{a^2}{\nu^2}\right]$$
$$\hat{B}_1 = \mp\frac{1}{(4\pi)^2} 4\sqrt{3}\lambda a \left[\psi(2) + \frac{1}{4} + \frac{1}{2}\log\frac{a^2}{\nu^2}\right] \tag{4.3.41}$$

for the $\hat{A}_0 = \frac{a}{2\lambda}$, $\hat{B}_0 = \pm\frac{\sqrt{3}}{2}\frac{a}{\lambda}$ minima. A remarkable difference with respect to the flat background case appears in all of the results (35) – (41): since the value of the geometrical parameter a is fixed, no particular choice of the subtraction scale ν can set the semiclassical vacua A_{vac}, B_{vac} to their original (tree-level) values $\hat{A}_0$, $\hat{B}_0$. The validity of this statement is extended here to the massless theory as well.

The calculation of the effective potential of the Wess-Zumino model in AdS_4 presented here has the merits of: (i) allowing to investigate the quantum features of the massless theory which gives the small fluctuation action in gauged extended supergravity;[1] (ii) providing the general proof that the supersymmetric character of the extrema of the classical potential (for both massive and massless fields) is not altered by the one-loop corrections. Our calculation deals with the exact propagators in AdS_4, as opposed to the previous chapters, where some kind of approximation in the background geometry is introduced when studying the renormalization of supersymmetric theories.

We propose a point-splitting regularization procedure. The one-loop counterterms do not depend on the regularization scheme and we find agreement with the calculations which make use of the adiabatic approximated propagators along with both dimensional regularization and Pauli-Villars methods. In detail, we show that, as it is known, a linear counterterm has to be introduced for the sake of renormalizability in the superpotential. For the mass and interaction lagrangians the no-renormalization theory holds its validity in AdS_4, while the kinetic action has to be renormalized. All renormalization coefficients do not depend from the choice of the vacuum around which the theory is quantized. Our calculation confirms the explicit independence of the above results and all the ultraviolet divergences from the boundary conditions chosen for the field propagators, as anticipated in sec. 4.1.

The equations of motion obtained from the renormalized effective potential yield the solutions $\langle F \rangle = \langle G \rangle = 0$, showing that quantum corrections preserve the supersymmetry invariance of the extrema of the potential to the one-loop order. The vacuum expectation values of the A and B fields are shifted from their tree-level values by finite one-loop terms. The shifts depend from the choice of reflective boundary conditions imposed in solving the equation of motion for the scalar propagator in AdS_4.

The claim of the persistence of supersymmetry to the one-loop order at each classically supersymmetric extremum of the potential holds with complete generality. The reflective boundary conditions (and the presence of a curved background) do not produce any radiative breaking of the supersymmetry invariance of the model. Our results extend the validity of this statement beyond the $\mathcal{O}(a^2)$ accuracy of chapter 3, for the $m \neq 0$ case. The equations $\langle F\rangle = \langle G\rangle = 0$ incorporate all orders in an expansion in the parameter a. For the massless model, we have proven the validity of the above claim, when the model is quantized around any of the supersymmetric minima of the classical potential. We also confirm that the supersymmetry invariance of the phase determined by the local maximum $\langle A\rangle_0 = \langle B\rangle_0 = 0$ is preserved to the one-loop order (see section 4.2).

The non-vanishing quantum shifts $\hat{A}_1$ and $\hat{B}_1$, which mark a clear distinction from the theory realized in flat space-time background, are expressed as analytic functions of a. The agreement with partial results contained in chapter 3 shows, in particular, that the boundary conditions do not effect the $\mathcal{O}(a^2)$ finite corrections, in the massive theory. In this case, the exact propagators are expanded in a, including $\mathcal{O}(a^2)$ terms, showing their correspondence with terms of the same order in the adiabatic approximations. This provides the justification for the irrelevance of the role played by the choice of boundary conditions in determining the whole of the $\mathcal{O}(a^2)$ corrections to the effective action, to all loop orders.

The verification of the supersymmetric Ward identities for the 1PI one- (and two-) point functions, along with the fact that supersymmetry is preserved throughout our calculation, ensures that the regularization scheme proposed here is suitable, at least for one-loop purposes. Our prescription consists in dealing symmetrically with the (exact) spin-0 and spin-1/2 propagators in AdS_4. This implies dropping $\mathcal{O}(a^2)$ terms in the bispinor $\mathcal{G}_F$ which are finite in the contraction limit. These terms originate from the expansion of the geometrical tensor $S(x)\overline{S}(x')$ propagating a spinor field between two space-time points.

As a separate point, we have performed the calculation of the semiclassical vacuum solutions using Green functions which propagate fields in n-dimensional anti-de Sitter space.[15] This dimensionally regularized calculation, yields the same supersymmetry-preserving results obtained here.

This provides us an independent confirmation of the following facts: i) the above results are correct; ii) the point-splitting regularization scheme which allowed us to obtain those results constitutes a valid proposal. In addition, we anticipate that the dimensional regularization scheme that we discuss simplifies the quantum calculation. This regularization prescription represents a valid alternative to the point-splitting procedure for the task to extend the calculation of the quantum corrections, studying the influence of the reflective boundary conditions to higher loops.

In Ref. [17] an action for the Wess-Zumino model in AdS$_n$ was proposed

$$S_{\text{kin}} = \int d^n x\,(-g)^{1/2}\left[\frac{1}{2}\left(D_\mu A D^\mu A + D_\mu B D^\mu B + i\bar{\psi}\,\not{D}\psi + F^2 + G^2\right) + (n-2)\frac{a}{2}(AF + BG) + \frac{a^2}{4}(A^2 + B^2)\cdot(n-1)(n-2)\right]$$

$$S_m = \int d^n x\,(-g)^{1/2}\, m\left[AF - BG + \frac{1}{2}a^2(A^2 - B^2)(n-1) - \frac{1}{2}\bar{\psi}\psi\right]$$

$$S_{\text{int}} = \int d^n x\,(-g)^{1/2}\,\lambda\left[A^2F - B^2F + \frac{1}{3}(n-1)a\,(A^3 - 3AB^2) - \bar{\psi}\,(A + i\gamma_5 B\psi)\right]$$

$$S = S_{\text{kin}} + S_m + S_{\text{int}} \tag{4.3.42}$$

with the supersymmetry transformation

$$\delta A = \bar{\epsilon}\psi \ , \qquad \delta B = -i\bar{\epsilon}\gamma_5\psi$$

$$\delta\bar{\psi} = i\bar{\epsilon}\,\partial\!\!\!/\,(A + i\gamma_5 B) + \bar{\epsilon}(F + i\gamma_5 G)$$

$$\delta F = -i\bar{\epsilon}\,D\!\!\!\!/\,\psi - \frac{a}{2}\bar{\epsilon}\psi(n-2), \quad \delta G = \bar{\epsilon}\gamma_5\,D\!\!\!\!/\,\psi + \frac{ia}{2}\bar{\epsilon}\gamma_5\psi\cdot(n-2) \tag{4.3.43}$$

The different terms of the sum (42) are invariant (up to a total divergence) under the transformations (43), with the exception of one ambiguous term (i.e. the trilinear spinor term which, in flat space-time, is shown to vanish by Fierz rearrangement). The way this action was obtained is , perhaps, too naive for expecting it to be exactly supersymmetric invariant. Indeed, the expressions (42) and (43) were merely guessed by a procedure of "interpolation" between the corresponding quantities in D = 2 and D = 4. We will disregard these details and proceed straight to derive the supersymmetry Ward identities for the field propagators. It is at the level of the Green functions of the model that we can give a reasonable prescription in order to maintain the balance in n-dimensions between fermionic and bosonic degrees of freedom.

Let us begin by discussing the expression of the spin-0 propagator

$$G_F(x,x') = \frac{-i}{(4\pi)^{n/2}}\, a^{(n-2)}\, \frac{\Gamma(\lambda)\Gamma(\lambda+1-\frac{1}{2}n)}{\Gamma(2\lambda-n+2)}\cdot$$

$$\cdot F\left(\lambda, \lambda+1-\tfrac{1}{2}n, 2\lambda-n+2\,;\, z\right) \tag{4.3.44}$$

where the choice of regular (irregular) boundary conditions is reflected by the + (-) sign in the λ-parameter

$$\lambda = \frac{n-1}{2} \pm \sqrt{\frac{1}{4} + \frac{m^2}{a^2}} \tag{4.3.45}$$

The propagator $G_F(x,x')$ is the solution of the equation

$$\left[\Box - \frac{1}{4} n(n-2)a^2 + m^2\right] G_F(x,x') = $$
$$= -\delta^n(x-x')\left[-g(x)\right]^{-1/2} \tag{4.3.46}$$

which corresponds to reflective boundary conditions.

The spin-1/2 propagator obeys the equation

$$(i\not{D} - m) S_F(x,x') = \left[-g(x)\right]^{-1/2} \delta^n(x-x')$$

The solution with supersymmetric boundary conditions is given by

$$S_F(x,x') = \left(m + a\frac{n-2}{2}\right) G_A(x,x') S(x) \bar{S}(x')$$
$$+ \frac{\partial G_A}{\partial \mu} i\gamma^\mu S(x) \bar{S}(x') \tag{4.3.47}$$

where $G_A(x,x')$ is the solution of Eq.(46) with mass term $m_A^2 = m^2 - am$, with appropriate reflective boundary conditions. For our spinor field belonging to a scalar supermultiplet, the propagator (47) satisfies the supersymmetric Ward identity

$$S_F(x,x') = \left[\left(i\not{D} + m + a\frac{n-2}{2}\right) G_A(x,x')\right] \cdot$$
$$\cdot S(x) \bar{S}(x')$$

The calculation of the effective potential of the model involves the two-point function $\mathcal{G}_F(x,x')$, obeying the equation

$$(i\not{D} + m) \mathcal{G}_F(x,x') = S_F(x,x')$$

The solution reads as follows

$$\mathcal{G}_F(x,x') = \frac{1}{2m}\left\{\left[\left(m + a\,\frac{n-2}{2}\right)G_A(x,x') + \left(m - a\,\frac{n-2}{2}\right)G_B(x,x')\right] + \left(\frac{\partial G_A}{\partial\mu} - \frac{\partial G_B}{\partial\mu}\right) i\gamma\!\!/ \right\} S(x)\bar{S}(x') \qquad (4.3.48)$$

The scalar function $G_B(x,x')$ satisfies Eq.(46) with mass squared $m_B^2 = m^2 + am$, with appropriate reflective boundary conditions. The bispinor (48) satisfies the equation

$$\left(\gamma^\alpha\gamma^\beta D_\alpha D_\beta + m^2\right)\mathcal{G}_F(x,x') = -\left[-g(x)\right]^{-1/2}\delta^n(x-x') \qquad (4.3.49)$$

We add for completeness that Eq.(49) can be rewritten in a form similar to Eq.(46). In fact, we have in n-dimensions [29]

$$[D_\mu, D_\nu]\Psi = \frac{1}{4} R_{\mu\nu\hat{a}\hat{b}}\,\gamma^{\hat{a}}\gamma^{\hat{b}}\,\Psi$$

for a spinor Ψ . Recalling the idntity

$$R_{\hat{a}\hat{b}\hat{c}\hat{d}}\,\gamma^{\hat{a}}\gamma^{\hat{b}}\gamma^{\hat{c}}\gamma^{\hat{d}} = -2R$$

we obtain

$$\gamma^\mu\gamma^\nu D_\mu D_\nu\Psi = D^\mu D_\mu\Psi - \frac{R}{4}\Psi$$

This allows to rewrite Eq. (49) as follows

$$\left[D^\mu D_\mu - \frac{1}{4}n(n-1)a^2 + m^2\right]\mathcal{G}_F(x,x') = -\left[-g(x)\right]^{-1/2}\delta^n(x-x')$$

The advantage of using dimensionally regularized propagators is that we can take the zero separation limit in a straightforward way. From Eq.(44) we obtain

$$\lim_{x'\to x} G_F(x,x') = \frac{-i}{(4\pi)^{n/2}}\, a^{(n-2)}\, \frac{\Gamma(1-\frac{n}{2})\,\Gamma(\lambda)}{\Gamma(\lambda-n+2)} \tag{4.3.50}$$

$$\lim_{x'\to x} \mathcal{G}_F(x,x') = \frac{1}{2m}\Big[\Big(m + a\,\frac{n-2}{2}\Big) G_A(x,x') + \Big(m - a\,\frac{n-2}{2}\Big) G_B(x,x')\Big] \tag{4.3.51}$$

Introducing $n = 4 - \varepsilon$, along with the definition $\lambda = \lambda^{(0)} - \varepsilon/2$, we can rewrite Eq.(50) as follows

$$G_F(x,x') = \frac{i}{16\pi^2}\,\frac{2}{\varepsilon}\, m^2 \Big\{ 1 + \frac{\varepsilon}{2}\big[\log 4\pi - \psi(1)\big] + \frac{\varepsilon}{2}\big[-\log a^2 + \psi(1) + \psi(2) - \psi(\lambda^{(0)}) - \psi(\lambda^{(0)}-2)\big]\Big\} \tag{4.3.52}$$

where we have used the property $(\lambda^{(0)} - 1)(\lambda^{(0)} - 2) = m^2/a^2$. As for the bispinor $\mathcal{G}_F$, it is consistent with the spirit of the dimensional reduction scheme proposed in Ref.[6], to put $n = 4$ in the geometric part of the coefficient $(a/2)(n-2)$ multiplying the propagators G_A and G_B in Eq.(51). This prescription, which corresponds to the idea of keeping the geometric tensors in 4-dimensions, while continuing the x-coordinates to dimension n, maintains the balance between bosonic and fermionic degrees of freedom. This balance would be upset by retaining the above coefficient in $n \neq 4$ when deriving the bispinor $\mathcal{G}_F$ from the scalar propagators G_A and G_B. Thus, we have

$$\mathcal{G}_F(x,x') = \frac{i}{16\pi^2}\,\frac{2}{\varepsilon}\,(m^2-a^2)\Big\{1 + \frac{\varepsilon}{2}\big[\log 4\pi - \psi(1)\big] + \frac{\varepsilon}{2}\Big[-\log a^2 + \psi(1) + \psi(2) - \frac{1}{2}\psi(\lambda_A^{(0)}) - \frac{1}{2}\psi(\lambda_A^{(0)}-2) - \frac{1}{2}\psi(\lambda_B^{(0)}) - \frac{1}{2}\psi(\lambda_B^{(0)}-2)\Big]\Big\} \tag{4.3.53}$$

With respect to the dimensionally regularized expression (50), we are neglecting, on the basis of our dimensional reduction prescription above, finite terms (in the $n \to 4$ limit). These terms would add to Eq.(53) the contribution $(-i/16\pi^2)(-4a^2)$. We show that discarding the latter guarantees that supersymmetry is maintained explicitely in our calculation. The prescription we suggest, in order to extend this result to higher loops, is to discard, in constructing the spinorial functions using as building blocks the scalar ones, the finite terms originating from the multiplication of the geometrical quantities times the divergent part of the scalar functions. This supersymmetry preserving dimensional regularization scheme represents the analogue of the supersymmetric point-splitting method proposed in Ref.[4] . Also, our view-point here is in agreement with the dimensional reduction prescription implemented for the one-loop calculation in Ref.[6] ,where the geometrical tensors are kept in D = 4, while continuing the coordinate to dimension n. It is worth recalling that the latter prescription has proven suitable for carrying the two-loop order renormalization of supersymmetry in AdS.

In the quantization of the model around any of its classically supersymmetric extrema, we integrate over the quantum fields the quadratic action S_q :

$$S_q = \int d^n x \sqrt{-g}\; L_q$$

obtained after shifting the fields in the action (2.1.4) around their classical values. The result of this integration reads as follows

$$\Gamma_1(\hat{A},\hat{B},\hat{F},\hat{G}) = -\frac{i}{2}\Big\{ \mathrm{tr}\log\big[D^\mu D_\mu - \tfrac{1}{4}n(n-2)a^2 + m_2^2\big] + \mathrm{tr}\log\big[D^\mu D_\mu - \tfrac{1}{4}n(n-2)a^2 + m_3^2\big] - 2\,\mathrm{tr}\log\big[\gamma^\mu\gamma^\nu D_\mu D_\nu + m_1^2\big]\Big\} \tag{4.3.54}$$

with masses m_1, m_2, m_3 as in Eq.(19). Taking the functional derivative of Γ_1 with respect to the classical field $\hat{\phi}$, we get

$$\frac{\partial \Gamma_1}{\partial \hat{\phi}} = -\frac{i}{2}\left[\frac{\partial m_2^2}{\partial \hat{\phi}} G_2(x,x') + \frac{\partial m_3^2}{\partial \hat{\phi}} G_3(x,x') - \right.$$

$$\left. - 2\frac{\partial m_1^2}{\partial \hat{\phi}} \mathcal{G}_1(x,x')\right] \qquad (4.3.55)$$

Plugging the regularized coincidence limits (52) and (53), the divergent part of Eq.(55) reads

$$\frac{\partial \Gamma_1^{div}}{\partial \hat{\phi}} = \frac{1}{16\pi^2} \cdot \frac{1}{\varepsilon} \frac{\partial}{\partial \hat{\phi}} \left(\alpha^2 + 2a^2 m_1^2\right)$$

Here we have used the definitions (19). After computing the derivatives with respect to the arguments of the effective action and carrying the subsequent formal integration, we finally obtain

$$\Gamma_1^{div} = \frac{1}{32\pi^2} \Delta \int d^n x \sqrt{-g}\left[4\lambda a m\left(3a\hat{A} + \hat{F}\right)\right.$$

$$+ 8\lambda^2\left(a\hat{A}\hat{F} + a\hat{B}\hat{G} + \frac{3}{2}a^2\hat{A}^2 + \frac{3}{2}a^2\hat{B}^2\right.$$

$$\left.\left. + \frac{1}{2}\hat{F}^2 + \frac{1}{2}\hat{G}^2\right)\right] \qquad (4.3.56)$$

with

$$\Delta = \frac{2}{\varepsilon} + \log 4\pi - \gamma_E$$

as required by minimal subtraction. In Eq.(56) we have omitted a divergent term which does not depend upon the hatted fields. We will come back to the evaluation of this term at the end of this section, when considering the renormalization of the gravitational tensors and the anomaly. The result (56) is consistent with our previous Eq.(3.1.22) obtained in the adiabatic approximation. There is also agreement with our result (4.3.21) obtained by supersymmetry invariant point-splitting regularization.

The finite value of the one-loop effective potential after renormalization is given by the equation

$$\frac{\partial V_1}{\partial \Phi} = \frac{1}{32\pi^2}\Big\{ -\bar{m}_2^2\Big[-\log\frac{a^2}{\nu^2} + \psi(1)+\psi(2)-\psi(\lambda_2^{(0)}) - \psi(\lambda_2^{(0)}-2)\Big]\frac{\partial m_2^2}{\partial \Phi}\Big|_0 - \bar{m}_3^2\Big[-\log\frac{a^2}{\nu^2} + \psi(1)+\psi(2)-\psi(\lambda_3^{(0)}) - \psi(\lambda_3^{(0)}-2)\Big]\frac{\partial m_3^2}{\partial \Phi}\Big|_0 + 2\left(\bar{m}_1^2 - a^2\right)\Big[-\log\frac{a^2}{\nu^2} + \psi(1)+\psi(2) - \frac{1}{2}\psi(\lambda_A^{(0)}) - \frac{1}{2}\psi(\lambda_A^{(0)}-2) - \frac{1}{2}\psi(\lambda_B^{(0)}) - \frac{1}{2}\psi(\lambda_B^{(0)}-2)\Big]\frac{\partial m_1^2}{\partial \Phi}\Big|_0\Big\} \tag{4.3.57}$$

We have introduced the mass parameter ν with the purpose of maintaining the effective action in Eq.(54) dimensionless. Since the effective potential in Eq.(57) is a local quantity to be evaluated at the extremum which characterizes the particular phase chosen, we have

$$\lambda^{(0)} = \bar{\lambda}$$

in the notation of Eqs.(26) and (27). Then, the analysis of the solutions of the one-loop corrected equations of motion of the fields proceeds along the line of the above calculation with point-splitting method, providing the results obtained there for the vacuum expectation values of the fields in Eqs. (35) - (38). We must stress the remarkable check of the results for the semiclassical vacuum solutions of the model obtained taking into account the reflective boundary conditions imposed by supersymmetry invariance on the Green functions, and regularizing the divergencies of the effective action by means of two independent schemes. These results agree among themselves and are consistent with the invariances of the theory.

It is worth noticing that the dimensional regularization scheme suggested by Burges, Gibbons, Freedman and Davis in Ref. [17] does not preserve supersymmetry at the one-loop level, contrary to their claim. In fact, including the contribution of the geometric tensors in n- dimensions which, according to our discussion above, spoils the balance between fermionic and bosonic degrees of freedom, one is adding to the right-handside of Eq. (57) the term

$$\frac{1}{32\pi^2}\left(2a^2 \frac{\partial m_1^2}{\partial \hat{\phi}}\Big|_0\right) \tag{4.3.58}$$

Recalling the definition of m_1 in Eq.(19) clarifies that the extra-term (58) makes impossible to satisfy the Ward identity (3.1.2). Thus, a naive dimensional regularization prescription, such as the one implemented in the calculations of Ref. [17], gives a non-supersymmetric invariant finite correction to the one-loop order. This can be shown also by looking at the effect of including the term (58) on the vacuum expectation values of the fields. It is easy to verify that the auxiliary fields acquire a non-vanishing vacuum expectation value, if one insists in regularizing dimensionally the effective action in a naive way

$$\langle F\rangle \neq 0$$

$$\langle G\rangle \neq 0$$

In closing this section, we want to present the renormalization of the pure gravitational tensors and the calculation of the trace anomaly. For this purpose, we have to resort back to the help of the adiabatic expansion of the Green functions used in chapter 3. Now we keep terms of order a^4. Recalling Eq.(3.1.12) and plugging into the effective action Γ_1 (3.1.20), we have

$$\Gamma_1^{div} = \frac{1}{(4\pi)^2}\,\frac{\Delta}{2}\int d^n x\,\sqrt{-g}\,\Big[\frac{m_2^4}{2} + a_2^{(0)} + \frac{m_3^4}{2} + a_2^{(0)} - m_1^4 + 2\,m_1^2 a^2 - 2\,a_2^{(1/2)}\Big] \qquad (4.3.59)$$

We have used the values of the adiabatic coefficients of Eq. (3.1.16). The $O(a^4)$ terms $a_2^{(i)}$ can be obtained expanding in series of powers of the contraction parameter of AdS the field propagators, following the lines of the proof exhibited in section 4.1. Here we merely quote the results

$$a_2^{(0)} = -a^4/15$$

$$a_2^{(1/2)} = \frac{11}{60}\,a^4 \qquad (4.3.60)$$

These values coincide with the second adiabatic order coefficients of the DeWitt-Schwinger expansion of Ref.[27]. Combining Eqs.(60) and (59) we get

$$\Gamma_1^{div} = \frac{1}{32\pi^2}\,\Delta\int d^n x\,\sqrt{-g}\,\Big[-\frac{1}{2}a^4 + 3a^2 m^2 + 4\lambda a m(3a\hat{A} + \hat{F}) + 8\lambda^2(a\hat{A}\hat{F} + a\hat{B}\hat{G} + \frac{3}{2}a^2\hat{A}^2 + \frac{3}{2}a^2\hat{B}^2 + \frac{1}{2}\hat{F}^2 + \frac{1}{2}\hat{G}^2)\Big] \qquad (4.3.61)$$

The first and second terms in the right-hand side of Eq.(61) can be reexpressed in terms of the scalar curvature of AdS_4 as follows

$$\frac{\Delta}{32\pi^2}\int d^n x\sqrt{-g}\left(-\frac{a^4}{2}\right) = -\frac{1}{32\pi^2}\frac{1}{\varepsilon}\frac{1}{144}\int d^n x\sqrt{-g}\,R^2 \tag{4.3.62}$$

$$\frac{\Delta}{32\pi^2}\int d^n x\sqrt{-g}\left(3a^2m^2\right) = \frac{1}{64\pi^2}\frac{1}{\varepsilon}m^2\int d^n x\sqrt{-g}\,R \tag{4.3.63}$$

The first remarkable fact in our interpretation above is that there is no renormalization of the cosmological constant Λ,

$$\Lambda = \Lambda_B \tag{4.3.64}$$

dismissing the claim of Ref.[34]. Nor a renormalization of the radius of AdS_4 is admissible according to us, as opposed to the findings of Ref.[34]. The counterterm (63) is proportional to the gravitational action and it produces the renormalization of the bare gravitational constant G_B

$$G = \frac{G_B}{1+16\pi G_B B}$$

$$B = \frac{1}{\varepsilon}m^2\frac{1}{64\pi^2} \tag{4.3.65}$$

The divergent quantity (62) is to be interpreted as giving rise to the anomaly in the trace of the renormalized stress-energy tensor. To see this we obtain the contribution of the counterterm W_{div} of Eq.(62)

$$W_{div} = \frac{1}{\varepsilon}\int d^n x\sqrt{-g}\,AR^2$$

$$A \equiv -\frac{1}{32\pi^2}\frac{1}{144} \tag{4.3.66}$$

to the trace of the stress-tensor

$$\langle T_\mu{}^\mu \rangle_{div} = g^{\mu\nu} \frac{2}{\sqrt{-g}} \frac{\delta W_{div}}{\delta g^{\mu\nu}}$$

This contribution reads, taking the variation of Eq.(66) with respect to $g^{\mu\nu}$

$$\langle T_\mu{}^\mu \rangle_{div} = 2 g^{\mu\nu} H_{\mu\nu} \frac{1}{\varepsilon} A$$

where

$$H_{\mu\nu} = \frac{1}{2} g_{\mu\nu} R^2 - 2 R R_{\mu\nu}$$

The final result of the renormalized anomalous trace is, then

$$\langle T_\mu{}^\mu \rangle_{div} = -(4-n) R^2 \frac{1}{\varepsilon} A =$$

$$= \frac{1}{32\pi^2} \frac{R^2}{144} \tag{4.3.67}$$

The anomaly (67) can be found in Refs. [17,30] which only consider the particular case of the model quantized around the origin $\hat{A}_0 = \hat{B}_0 = 0$. Our derivation in the present section has the merit of being general, extending to the renormalization of the gravitational tensors the proof of the universality of the renormalization coefficients, i.e. of their independence from the choice of the vacuum, given in chapter 3 for the matter lagrangian counterterms.

5. SUPERFIELD FORMULATION OF THE WESS-ZUMINO MODEL IN ANTI-DE SITTER SPACE

5.1. The superfield formulation

After the formulation of the different supersymmetric models in the background of AdS_4, which was given in ref. [2], and after some work devoted to study the properties of this realization of supersymmetry at the quantum level [17], the efforts have been focused in carrying out the renormalization of the supersymmetric models to the one-loop order [6,7,8,9]. In particular, the analogue of the Wess-Zumino model in this non-trivial background has been the object of much attention and the final status of the research that we have described in chapters 2 and 3 leads to the conclusion that the no-renormalization theorem [19] remains valid for the mass and interaction lagrangians of the model, while a linear superfield insertion in the effective action is generated by quantum corrections at the one-loop level. As it is the case in the flat space-time version of the model, an insertion of the above kind does not prevent the supersymmetry invariance to be preserved at the classical level. Furthermore, an exhaustive analysis has shown, through the computation of the effective potential of the model, that the one-loop quantum effects do not lead to the breaking of supersymmetry at any of the four different supersymmetric vacua that the model possesses at the classical level (see sections 3.1 and 3.2). [10]

In the present chapter we attempt to perform the analysis of the structure of the divergent one-loop quantum corrections to the model, in the framework of the superfield formalism [14]. In proceeding this way, we hope to open the possibility of extending the computation to hi=

gher orders in perturbation theory and, perhaps, giving some insight about the general structure of the effective action. In this sense, one should not forget that dealing with component fields does imply considering the model as a quantum field theory in curved space-time and, hence, the unavoidable fact of having to introduce the exact free-field propagators through some perturbative expansion in the effects of the curvature [16]. For orders higher than the first in coupling constant perturbation theory, non-local divergencies are present at intermediate steps of the calculation and their cancellation requires a careful check [16,20,21]. On the contrary, the superfield formalism allows us, as we show in the present chapter, to device an approach in which the perturbative formulation of the quantum model is understood only in terms of its interaction vertices.

From the physical viewpoint the main interest of applying the superfield formalism to the model in AdS_4 comes from the possibility of studying the breakdown of the no-renormalization theorem [14] for the mass or the interaction lagrangian of the model. We have already determined in this thesis that the renormalization of a linear superfield term in the superpotential,* found at the one-loop level, can already be interpreted as such a breakdown of the theorem in the curved space background. Moreover, it has been pointed out before that, in the context of the work of Ivanov and Sorin [2], the superfield integrals for the linear, quadratic and cubic terms in the superpotential are mathematically equivalent, what makes plausible a similar renormalization in the last two cases. The approach that we adopt in this second part of our work for the superfield formulation of the model follows the line suggested in ref. [14], instead of the work of Ivanov and Sorin. While we are able to show that both formulations are equivalent through the definition of the model in terms of component fields, as well as in superspace, the approach of ref. [14] is the best suited for the perturbative analysis of the model. It also stresses obvious differences in the structure of the effective action with respect to the flat space-time model and shows why the compelling argument that leads to the no-renormalization theorem in flat space-time does not work in the case of AdS_4, as we discuss below.

With complete generality, our approach to the study of Anti-de Sitter supersymmetry will consist in considering the supergravity multiplet coupled to the scalar multiplet of our model. We remember that, in the covariant approach to supergravity, the covariant derivatives ∇_A can be determined in terms of the prepotential H^m and the density compensator Ψ for the superconformal transformations. A convenient way of breaking the superconformal invariance is to introduce a covariantly chiral tensor-type compensator Φ

$$\bar{\nabla}_{\dot{\alpha}} \Phi = 0 \qquad (5.1.1)$$

Choosing a gauge for the Φ compensator fixes a relation between the Ψ and H^m superfields. Then, the supermultiplet is described in terms of H^m and a flat space-time chiral superfield φ (in the chiral representation) and the only superspace gauge freedom left is that of super-Poincaré transformations.

The transformation rules of the superdeterminant E made out of the supervierbein $E_A{}^M$

$$E^{-1} \longrightarrow E^{-1} e^{iK} \qquad (5.1.2)$$

$$K = K^M i D_M + K_\alpha{}^\beta i M_\beta{}^\alpha + \bar{K}_{\dot\alpha}{}^{\dot\beta} i \bar{M}_{\dot\beta}{}^{\dot\alpha}$$

and of the φ field

$$\varphi^3 \rightarrow \varphi^3 e^{i\overleftarrow{\Lambda}_{ch}} \,, \quad \overleftarrow{\Lambda}_{ch} = \Lambda^{\underline{m}} i \overleftarrow{\partial}_{\underline{m}} + \Lambda^\mu i \overleftarrow{D}_\mu \qquad (5.1.3)$$

make possible to write invariant actions for a real scalar superfield L_{gen}

$$S_1 = \int d^4x\, d^4\theta\, E^{-1} L_{gen} \qquad (5.1.4)$$

and for a chiral superfield L_{chiral}

$$S_2 = \int d^4x\, d^2\theta\, \varphi^3 L_{chiral} \qquad (5.1.5)$$

In particular, the action for supergravity is obtained by setting $L_{gen} = -3/k^2$. There exists also the possibility of adding a cosmological term with $L_{chiral} = \alpha/k^2$.

The choice of Anti-de Sitter space as the background space for the matter model implies taking H equal to zero and introducing the background only through the chiral compensator φ. Its equation of motion has to be obtained from the action of supergravity with cosmological term

$$S = -\frac{3}{k^2}\int d^4x\, d^4\theta\, E^{-1} + \left(\frac{\alpha}{k^2}\int d^4x\, d^2\theta\, \varphi^3 + h.c.\right) \qquad (5.1.6)$$

In the present case we have, after the choice of the gauge $\Phi = 1, \Psi = \varphi^{-1}\bar\varphi^{1/2}$ and $E^{-1} = \bar\varphi\varphi$. The equation of motion for the φ field turns out to be the same as for the chiral superfield of the massless Wess-Zumino model

$$\bar{D}^2 \bar\varphi = \alpha\, \varphi^2 \qquad (5.1.7)$$

The solution of this equation with regular behaviour at infinity is

$$\varphi = \frac{1}{1-\alpha\bar{\alpha}x^2} - \frac{\bar{\alpha}\theta^2}{(1-\alpha\bar{\alpha}x^2)^2} \qquad (5.1.8)$$

Then, by applying this solution in the construction of invariant actions of the general and chiral type, we can formulate different supersymmetric matter models in the given background. It is useful to remember, at this point, the expression of the covariant derivatives in terms of the φ field

$$E_{\dot{\alpha}} = \varphi^{-1}\bar{\varphi}^{\frac{1}{2}}\bar{D}_{\dot{\alpha}} \quad , \quad E_{\alpha} = \bar{\varphi}^{-1}\varphi^{1/2}D_{\alpha} \qquad (5.1.9)$$

At this level, the correspondence between the two versions of the superspace approach to Anti-de Sitter supersymmetry given in refs. [2] and [14] is understood as a gauge transformation of the compensator Φ. In ref. [2] the choice of the gauge in which $\Psi=1$ leads to the expression of the covariant derivatives in terms of the prepotential $H^{\underline{m}}$

$$E_{\dot{\alpha}} = e^{\frac{1}{2}H}\bar{D}_{\dot{\alpha}}\, e^{-\frac{1}{2}H} \,..$$

$$E_{\alpha} = e^{-\frac{1}{2}H}D_{\alpha}\, e^{\frac{1}{2}H}$$

with $H=H^{\underline{m}}iD_{\underline{m}}$. Thus, the effects of the non-trivial background appear as arguments of exponentials in the expression of the covariant derivatives, rather than simple factors of powers of φ and $\bar{\varphi}$ in front of the flat derivatives, as in eqs.(5.1.9). This feature of the description in ref.[14] makes the corresponding superfield approach preferable for calculating quantum corrections and will be used in the following, combined with a partial super=

conformal invariance of the Wess-Zumino model in AdS_4 space, to simplify essentially the description of its quantum features.

In what follows we are going to focus our attention on the model built out of a massive self-interacting chiral field η. This is just the Anti- de Sitter analogue of the flat space-time Wess-Zumino model. By using the two prescriptions given above, we can write its action in the form

$$S = \int d^4x\, d^4\theta\, E^{-1}\bar{\eta}\eta + \int d^4x\, d^2\theta\, \varphi^3\left(\frac{m}{2}\eta^2 + \frac{\lambda}{3!}\eta^3\right) + \int d^4x\, d^2\bar{\theta}\, \bar{\varphi}^3\left(\frac{m}{2}\bar{\eta}^2 + \frac{\lambda}{3!}\bar{\eta}^3\right) \tag{5.1.10}$$

To end this section, we remark that it is possible to show, by going to component fields, that this formulation of the model is completely equivalent to the one given by Ivanov and Sorin [2], that we have used throughout the first part of this thesis. This requires, however, a redefinition of the field components of the η superfield

$$\eta = A + \psi\theta - F\,\theta\theta \tag{5.1.11}$$

along with a conformal transformation of the metric in order to convert the derivatives in the flat space-time mapping of AdS_4 space into covariant derivatives. The transformation turns out to be

$$A \rightarrow A$$
$$\psi \rightarrow \Omega^{1/2}\psi$$
$$F \rightarrow \Omega F$$

$$\delta_{\mu\nu} \rightarrow \Omega^{-2} g_{\mu\nu} \qquad (5.1.12)$$

where

$$\Omega = \frac{1}{1 - \alpha\bar{\alpha} x^2}$$

It can be shown that, after this redefinition, the exp= ression of the action (5.1.10) recovers the usual compo= nent action obtained from the coset formulation of ref. [2].

5.2. One-loop renormalization of the superspace action

We have considered an approach to the Wess-Zumino model in AdS_4 background, in the superfield formalism. In what follows, a partial superconformal invariance of the model is used to treat perturbatively the effects of the background curvature. This proves to be enough for the purpose of renormalization and allows us to analyze the possibility of the breakdown of the no-renormalization theorem, that we carry to the one-loop order in the present section.

Thus, our aim is to calculate the divergent contributions to the one-loop effective action of the model. In order to accomplish this task, we adopt the functional integral approach in the quantization of the model. We define, as usual, the generating functional

$$Z(J) = \int \mathcal{D}\eta \mathcal{D}\bar{\eta}\, e^{S(\eta,\bar{\eta})}\, e^{\left(\int d^4x\, d^2\theta\, \varphi^3 J \eta + h.c.\right)}$$

but consider the mass term

$$\left(\frac{m}{2} \int d^4x\, d^2\theta\, \varphi^3 \eta^2 + h.c. \right)$$

as an interaction term, when performing the splitting of the action into a free-field part

$$S_0 = \int d^4x\, d^4\theta\, E^{-1} \bar{\eta}\eta$$

and an interaction part

$$S_{int} = \frac{m}{2} \int d^4x\, d^2\theta\, \varphi^3 \eta^2 + \frac{\lambda}{3!} \int d^4x\, d^2\theta\, \varphi^3 \eta^3 + h.c.$$

The usefulness of this splitting comes from the fact that, in the equivalent expression of the generating functional

$$Z(J) = e^{S_{int}\left(\frac{\delta}{\delta J}, \frac{\delta}{\delta \bar{J}}\right)} \int \mathcal{D}\eta \mathcal{D}\bar{\eta}\, e^{S_0} e^{\left(\int d^4x d^2\theta \varphi^3 J\eta + h.c.\right)}$$

all the dependence on the φ field in the free-field functional integral can be removed by performing a superconformal transformation according to the canonical weights of the fields

$$\hat{\eta} = \varphi\eta \qquad , \qquad \hat{J} = \varphi^2 J \tag{5.2.1}$$

This points in the direction that an easier description of the quantum model can be given in terms of the transformed fields, in view of which we decide to carry out the computation of the effective action defined un terms of $\hat{\eta}$ and $\hat{\bar{\eta}}$. For this purpose, we derive the Feynman rules of the model from the generating functional

$$\hat{Z}(\hat{J}) = Z(J) =$$
$$= e^{\left(\frac{m}{2}\int d^4x\, d^2\theta\, \varphi^7 \left(\frac{\delta}{\delta \hat{J}}\right)^2 + h.c.\right)} e^{\left(\frac{\lambda}{3!}\int d^4x d^2\theta\, \varphi^9 \left(\frac{\delta}{\delta \hat{J}}\right)^3 + h.c.\right)} \cdot$$
$$\cdot \int \mathcal{D}\hat{\eta}\mathcal{D}\hat{\bar{\eta}}\, e^{\int d^4x d^4\theta \hat{\bar{\eta}}\hat{\eta}} e^{\left(\int d^4x d^2\theta \hat{J}\hat{\eta} + h.c.\right)} \tag{5.2.2}$$

It is worth noticing explicitely that the transformation (5.2.1) is linear in η and J, since $\varphi(x)$ is a constant functional. Therefore, the Jacobian of the transformation (5.2.1) is trivial and $\hat{Z}(\hat{J})$ can be identified with Z(J) modulo an infinite constant of proportionality which is by all means irrelevant.

From the expression (5.2.2) it can be seen that the calculation of the free-field functional integral follows the same line as in flat space-time[14], so that the vacuum expectation values $\langle T\,\hat{\eta}(x)\,\hat{\eta}(x')\rangle$ and $\langle T\,\hat{\eta}(x)\,\hat{\eta}(x')\rangle$ have to solve respectively the equations

$$\Box \langle T\,\hat{\eta}(x)\,\hat{\bar{\eta}}(x')\rangle = i\,\bar{D}^2 D^2\,\delta^4(x-x')\,\delta^4(\theta-\theta')$$

$$\Box \langle T\,\hat{\eta}(x)\,\hat{\eta}(x')\rangle = 0$$

However, the resolution of these equations in the flat conformal projection space is not straightforward because one has to take the solutions of the homogeneous problems so that the boundary conditions required by supersymmetry for the scalar and spinor propagators in AdS_4 are enforced.[5] One way to identify these solutions consists in considering the conformal transformation that maps the chiral supermultiplet $(\hat{\mathcal{A}}, \hat{\psi}, \hat{\mathcal{F}})$ in flat space-time into the chiral supermultiplet in AdS_4 $(\mathcal{A}, \psi, \mathcal{F})$, according to the canonical weights of the fields

$$\Omega^{-1}\hat{\mathcal{A}} = \mathcal{A}$$

$$\Omega^{-3/2}\hat{\psi} = \psi$$

$$\Omega^{-2}\hat{\mathcal{F}} = \hat{\mathcal{F}} + a\,\mathcal{A}$$

Since we are dealing with the massless model, we have to choose the combination irregular-regular reflective boundary conditions, respectively, for the real and imaginary parts of the $\mathcal{A}$ field.[1] We have, then, from the work of Refs.[3, 7, 32]

$$\langle T\hat{A}(x)\hat{A}^*(0)\rangle = \Omega[x(\mu)]\left(-\frac{a^2}{16\pi^2}\frac{1}{\sin^2\frac{\mu a}{2}}\right) = -\frac{1}{4\pi^2}\frac{1}{x^2}$$

$$\langle T\hat{A}(x)\hat{A}(0)\rangle = \Omega[x(\mu)]\left(\frac{a^2}{16\pi^2}\frac{1}{\cos^2\frac{\mu a}{2}}\right) = \frac{a^2}{16\pi^2}$$

$$\langle T\hat{\psi}(x)\bar{\hat{\psi}}(0)\rangle = \Omega^{3/2}[x(\mu)]\left(\frac{ia^3}{16\pi^2}\frac{\sigma\cdot\hat{x}}{\sin^3\frac{\mu a}{2}}\right) = \frac{i}{2\pi^2}\frac{\sigma\cdot\hat{x}}{x^3}$$

$$\langle T\hat{\psi}(x)\hat{\psi}(0)\rangle = \Omega^{3/2}[x(\mu)]\left(\frac{a^3}{16\sqrt{2}\,\pi^2}\frac{1}{\cos^3\frac{\mu a}{2}}\right) = \frac{a^3}{16\sqrt{2}\,\pi^2}$$

$$\langle T\hat{A}(x)\hat{F}(0)\rangle = \langle T\hat{A}^*(x)\hat{F}^*(0)\rangle = \langle T\hat{F}(x)\hat{F}(0)\rangle = 0$$

$$\langle T\hat{F}(x)\hat{F}^*(0)\rangle = i\,\delta^4(x)$$

Going back from components to the $\hat{\eta}$ superfield[18], we can build the final result

$$\langle T\hat{\eta}(x)\hat{\eta}(0)\rangle = -\frac{1}{4\pi^2}\bar{D}^2 D^2\delta^4(\theta-\theta')\frac{1}{x^2}$$

$$\langle T\hat{\eta}(x)\hat{\eta}(0)\rangle = \frac{a^2}{16\pi^2} + \sqrt{2}\,\frac{a^3}{16\pi^2}\,\theta'\theta$$

We can read the contribution of the quadratic vertex, naively, as

$$\frac{m}{2}\,\varphi^{7} + h.c.$$

and the naive contribution of the cubic vertex as

$$\frac{\lambda}{3!}\,\varphi^{9} + h.c.$$

In practice, however, the power of the φ field is decreased when one takes into account the covariant functional derivative[14]

$$\frac{\delta\hat{J}(z)}{\delta\hat{J}(z')} = \frac{\varphi^2(z)}{\varphi^2(z')}\,\frac{\delta J(z)}{\delta J(z')} = \frac{\varphi^2(z)}{\varphi^2(z')}\left(\bar{\nabla}^2 + R\right)\left(E\,\delta^8(z-z')\right)$$

$$= \frac{1}{\varphi^3(z)}\,\bar{D}^2\,\delta^8(z-z')$$

The net result is that, when computing a generic derivative of the generating functional $\hat{Z}(\hat{J})$, there are no φ's at every cubic vertex, while the quadratic vertex becomes proportional to φ. This is nothing but a reflection of the fact that the deviation of the theory from a superconformal theory is given by the introduction of the mass term. Finally, we notice that the usual flat space-time D-algebra remains intact and that we can consider the Feynman rules of our model as equivalent to the ones for the flat space-time massless Wess-Zumino model with the addition of the quadratic vertex $(m/2)\varphi$. A remarkable feature of this approach is that, after the rescaling (5.2.1) that actually removes φ from the superconformal invariant part of the superfield action, taking into consideration the need to resort to some perturbative approach in the effects of the curvature of the background space, we are naturally led to introduce the above implicit expansion in the compensator φ.

The graph in figure 4 represents the divergent contribution of the one-point function to the effective action. The graph with no $\bar{\varphi}$ insertion, namely the tadpole present for the model in a trivial background, gives a vanishing contribution for the known reason that, after the D-algebra has been carried out, one is left with the integral of a chiral (or antichiral) superfield over the whole superspace. On the other hand, more φ-insertions give a finite result, as it is seen by the graphical equation in figure 5. This graph contains n φ-field and n+1 $\bar{\varphi}$-field insertions and the contribution drawn on the right-hand side of the equation is clearly finite for every $n>0$. In the manipulation of the D's we use the rule $\bar{D}^2 D^2 \bar{D}^2 = \Box \bar{D}^2$, with the definition $\Box \equiv (1/2)\partial^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}$. For n=0, carrying the flat space-time D-algebra following the rules given in ref.[14], we obtain the divergent contribution from figure 4

$$\mathcal{J}_1 = \frac{\lambda m}{2} \int d^4\theta \int \frac{d^4 p}{(2\pi)^4} \bar{\varphi}(-p,\theta)\hat{\eta}(p,\theta) I^{div} + h.c. \qquad (5.2.3)$$

where I^{div} is the divergent part of the integral

$$I = \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} \frac{1}{(k+p)^2} = \frac{2}{\varepsilon} \frac{1}{16\pi^2} + \text{finite terms} \qquad (5.2.4)$$

The above integral in momentum space is dimensionally regularized. The choice of a particular regularization procedure rather than another is actually ininfluent for the one-loop divergent renormalization of the effective action. The latter could be carried in the context of a Pauli-Villars regularization procedure, or even using a cut-off in momentum space as it has been remarked in the case of the renormalization of the effective action for component fields (see section 2.3).

The covariance of eq.(5.2.3) in the background space becomes manifest when expressing the result in terms of the original functions φ, η in x-space, rather than of their Fourier representations

$$\mathcal{J}_1 = \frac{\lambda m}{2} \int d^4x\, d^4\theta\, \bar{\varphi}(x,\theta)\hat{\eta}(-x,\theta) I^{div} + h.c.$$

$$= \frac{\lambda m}{2} \int d^4x\, d^4\theta\, \varphi(x,\theta)\bar{\varphi}(x,\theta)\eta(x,\theta) I^{div} + h.c. \qquad (5.2.5)$$

In the above equation the property $\varphi(x,\theta) = \varphi(-x,\theta)$ has been used. The counterterm $\mathcal{J}_1$ can be conveniently expre= ssed as a chiral integral by making use of the equation of motion for the compensator, eq.(5.1.7)

$$J_1 = \frac{\lambda\alpha m}{16\pi^2}\int d^4x\, d^2\theta \left[\varphi(x,\theta)\right]^3 \eta(x,\theta)\cdot\frac{1}{\varepsilon} + h.c. \qquad (5.2.6)$$

This is our main result. We are showing in this way that the inclusion of a counterterm of the form

$$\Lambda \int d^4x\, d^2\theta\, \varphi^3 \eta + h.c.$$

in the classical action (5.1.10) is needed in order to cancel the divergent contribution of the one-loop quantum corrections and attain renormalizability for the model. This represents a failure of the no-renormalization theorem in its flat space-time form. Precisely, as we have already mentioned, what is failing is the capability of supersymmetry invariance to restrict the counterterms actually produced by quantum corrections, among the ones preserving the symmetries of the tree-level model, to only those counterterms which are already present in the classical action. We show below that the counterterm does not produce a spontaneous breaking of the supersymmetry invariance of the model.

The analysis of the higher-point functions is very simple. From figures 6 and 7 we have the result that the minimum number of insertions of compensator superfields in the two-point functions gives a finite contribution to the effective action to the one-loop order. Proceeding in analogy to the case of the contribution represented graphically in figure 5, one can show in general that, in serting a higher number of compensators into the two-point functions produces graphs which give finite corrections. The only divergent two-point function contribution is therefore given by the massless correction

$$\frac{\lambda^2}{2}\int d^4\theta \int \frac{d^4p}{(2\pi)^4}\, \hat{\hat{\eta}}(-p,\theta)\,\hat{\eta}(p,\theta)\, I^{div} =$$

$$= \frac{\lambda^2}{2}\int d^4x\, d^4\theta\, \varphi(x,\theta)\bar{\varphi}(x,\theta)\eta(x,\theta)\bar{\eta}(x,\theta) I^{div} \qquad (5.2.7)$$

which corresponds to the graph in figure 8. Eq.(5.2.7) provides the renormalization coefficient in front of the kinetic action

$$Z_{kin} - 1 = \frac{\lambda^2}{16\pi^2} \cdot \frac{2}{\varepsilon} \qquad (5.2.8)$$

Obviously, the divergent contribution to the two-point function $\Gamma_{\eta\eta}$ vanishes and therefore the mass action does not get renormalized, at the one-loop level, namely Z_m= 1.

In figure 9 we have a sample contribution to the three-point functions obtained by inserting the minimum number of compensators. The result is clearly finite. Also, hi= gher numbers of compensator insertions give finite corre= ctions. It is obvious that the corrections vanish in the m=0 case, simply because there are no $\langle\hat{\eta}\hat{\eta}\rangle$ nor $\langle\hat{\bar{\eta}}\hat{\bar{\eta}}\rangle$ prop= agators, and thus we conclude that Z_{int} =1. The no-renorm= lization theorem remains valid also in the non-trivial background, in what the mass and interaction lagrangians are concerned.

With the results derived above,we have obtained the renormalization of the Wess-Zumino superfield action in AdS_4 space. The technical simplification of the calcula= tion of the quantum corrections in the superfield formu= lation of the theory can be appreciated by comparing the derivation of the results of the present section with the evaluation of the one-loop effective action in component fields, reported in chapters 2 and 3. Using the explicit form of the chiral compensator (5.1.8) into eq.(5.2.6), we get

$$J_1 = \frac{\lambda \alpha m}{16\pi^2} \int d^4x\, \Omega^4 \left[3\bar{\alpha} A(x) + \right.$$
$$\left. + \Omega^{-1} F(x) \right] \cdot \frac{1}{\varepsilon} + h.c. \qquad (5.2.9)$$

where the component fields A(x), F(x) of the chiral multiplet in eq.(5.1.11) as well as the determinant of the metric tensor must be redefined according to the transformation (5.1.12) and the correct identification $\bar{\alpha} = a$ must be made, in order to put J_1 in agreement with the results obtained in the formalism of component fields. In fact, from eq.(5.2.9) and taking eq.(5.2.8) into account, we recover the divergent one-loop contribution to the effective action, formerly obtained in chapter 3

$$\Gamma_1\big|_{div} = \frac{1}{16\pi^2}\frac{1}{\varepsilon}\int d^4x \sqrt{-g}\,\Big\{\lambda\sqrt{2}\, am\left(3a\hat{A}+\hat{F}\right)$$
$$+\lambda^2\Big[a\left(\hat{A}\hat{F}+\hat{B}\hat{G}\right)+\frac{3}{2}a^2\left(\hat{A}^2+\hat{B}^2\right)+\frac{1}{2}\left(\hat{F}+\hat{G}\right)\Big]\Big\} \qquad (5.2.10)$$

The above equation is to be compared with eq.(3.1.22). In the above equation the fields $\hat{A}$, $\hat{F}$ and $\hat{B}$, $\hat{G}$ are the classical values of the fields defined in chapter 3. These fields are introduced here as the real and imaginary parts, respectively, of the component fields A, F transformed by (5.1.12)

$$A = \frac{1}{\sqrt{2}}\left(\hat{A} + i\hat{B}\right) \quad , \quad F = \frac{1}{\sqrt{2}}\left(\hat{F} + i\hat{G}\right)$$

Here, we need to examine the possibility that the counterterm linear in the scalar fields in Γ_1 could produce a spontaneous breaking of the supersymmetry invaria=

nce of the action. In fact, as a direct consequence of the appearing of the chiral compensator in the expression of the superfield action of the model

$$\int d^4x\, d^4\theta\, \varphi\bar{\varphi}\,\eta\bar{\eta} + \left[\int d^4x\, d^2\theta\, \varphi^3 P(\eta) + h.c.\right] \qquad (5.2.11)$$

with the superspace potential

$$P(\eta) = \Lambda\eta + \frac{m}{2}\eta^2 + \frac{\lambda}{6}\eta^3 \qquad (5.2.12)$$

it is clear that the derivation holding in flat background of the condition

$$\frac{\partial P}{\partial \eta} = 0$$

for supersymmetry not to be broken [14] cannot be applied to the model in AdS_4 space-time. In principle, given the way in which φ enters the Feynman rules in our perturbative approach, we must allow for quantum corrections which are, in general, polinomial in φ and $\bar{\varphi}$, as it is the case of the finite non-local one-loop corrections, for instance. The flat space-time argument for the absence of spontaneous breaking of supersymmetry, induced by quantum corrections to the model quantized around a classically supersymmetric vacuum solution, breaks down at this point, since in its premises the bilinearity of the quantum corrections in the auxiliary fields F, G is postulated [14] .

Because of the lack, in the non-trivial background, of a general treatment of the supersymmetry breaking holding to all orders in perturbation theory, as well as independently from the classically supersymmetric vacuum state around which the model is quantized, the possibility of breaking supersymmetry by quantum corrections must be analyzed case by case, in the formalism of component fields. It turns out that the one-loop quantum corrections preserve the classical supersymmetry invariance of each one of the four vacua of the model (see chapter 3). At this point we can borrow the argument from our component-field functional-integral treatment of chapter 3, to show that the counterterm $\Lambda\eta$ in the superspace potential (5.2.12) does not break spontaneously supersymmetry neither at the classical nor at the one-loop level and we can refer the reader to the same chapter for the solution of the equations of motion modified by the above counterterm, as well as for the evaluation of the one-loop shifting in all the possible classical vacua of the theory.

We have seen that dealing with globally supersymmetric quantum systems in AdS_4 space can be best accomplished by using the machinery of superfield techniques. This allows us to recover the one-loop results obtained formerly in the formalism of component fields and represents a concrete indication that this will be the suitable way of renormalizing the Wess-Zumino model in the non-trivial background, to higher-loops. We have insisted already on the relevance of this higher-loops corrections, from the point of view of the search for possible non-vanishing renormalization of the mass and the interaction lagrangians, due to background effects.

In flat space-time the no-renormalization theorem is a direct consequence of the Feynman rules in superspace [14]. In AdS_4 space the situation does not appear so simple any more and we are not aware , at this point, of any general argument showing that chiral integrals cann= ot be generated by calculating quantum corrections to the classical action. In the perturbative approach in the compensator φ that we have proposed here we have expli= citely obtained, already at the one-loop level, the only chiral structure covariant with respect to the AdS_4 back= ground that can possibly be generated, namely

$$\alpha \int d^4x\, d^2\vartheta\, \varphi^3 \eta = \int d^4x\, d^4\vartheta\, \bar{\varphi}\, \hat{\eta}$$

This can be interpreted as a failure of the no-renormal= ization theorem induced by the presence of a non-trivial background geometry and, in principle, seems to support, at this stage our investigation, the possibility that also the other chiral structures allowed in the effecti= ve action have non-vanishing renormalization coefficien= ts, to some order in perturbation theory. To the one-loop order, our superfield calculation confirms that the no-renormalization theorem is valid for the mass and the interaction lagrangians and they remain unrenormalized.

The question of what can this approach tell us about the renormalization of the chiral integrals in the effec= tive action of the model to higher orders in the loop expansion remains open and will be answered by our anal= ysis of the next chapter. In our present perturbative treatment of the term $(m/2)\varphi\hat{\eta}^2$ a renormalization of the mass lagrangian would manifest itself through the signal of a singular behaviour at some stage in the calculation of higher-order corrections to the 1PI two-point functio= ns. In this sense, an alternative treatment of the mass lagrangian is conceivable and could be more suitable for a straightforward determination of the higher-order re= normalization of the mass lagrangian. This can be obta=

ined by defining $\varphi=1+\xi$, where the chiral superfield ξ vanishes in the flat space-time limit. The term $(m/2)\xi\hat{\eta}^2$ is to be considered as an interaction lagrangian in a theory in which the $\langle\hat{\eta}\hat{\eta}\rangle$ and $\langle\hat{\bar{\eta}}\hat{\bar{\eta}}\rangle$ propagators are not vanishing any more. Whetherthe counterterm

$$\alpha\int d^4x\, d^2\theta\,[\varphi(x)]^3[\eta(x)]^2 =$$

$$= \int d^4x\, d^4\theta\,[\varphi(x)]^{-1}\,\bar{\xi}(x)\cdot[\hat{\eta}(x)]^2$$

that could contribute to the renormalization of the mass lagrangian could be produced to higher-orders in the loop expansion, through a perturbative series of graphs with ξ-insertions reproducing the expansion $\varphi^{-1}=\sum_{n=0}^{\infty}(-\xi)^n$, is unclear, at this level, and will be the object of our investigation in the next chapter. This ξ-approach clearly provides a correct treatment of the divergencies of the theory, removing explicitely any trace of the presence of infrared divergencies in the quantum corrections to the classical model. The φ-approach is in this sense to be taken with a bit of care, since using it for higher-loop calculations could produce contributions to the quantum corrections which can be both infrared and ultraviolet divergent, at the same time. This is a general feature of any procedure in which a mass term in a massive theory is considered as a perturbation of the corresponding massless one.

The complete equivalence between the perturbative approach in ξ and the one in φ that we have adopted in the present section, for what concerns the one-loop ultraviolet divergent renormalization of the effective action of the model, is obvious and we do not need to discuss it in further detail here.

6. THE NO-RENORMALIZATION THEOREM FOR ANTI-DE SITTER SUPERSYMMETRY

6.1. Interacting chiral and real gauge superfields in AdS_4

In this chapter we turn our attention to considering the interacting system of chiral and real gauge superfields in Anti-de Sitter space. Also in this more extended physical system, the superconformal invariance of the massless part of the action allows us to implement a perturbative expansion in the curvature effects in terms of the interaction vertices of the quantum model, as we show below. Then, simple power counting arguments, which we will expose in the next section, will suffice to prove the no-renormalization of the mass and the chiral self-interaction parts of the action, to all perturbative orders.

We devote this section to giving the superfield formulation of the theory and introducing the expansion in the background effects. In the functional integral approach, the quantum Wess-Zumino model is described by the generating functional which we have already encountered in the previous chapter

$$Z(J) = \int \mathcal{D}\eta \mathcal{D}\bar{\eta}\, e^{S(\eta,\bar{\eta})} e^{\left(\int d^4x\, d^2\theta\, \varphi^3 J\eta + h.c.\right)}$$

$$S(\eta,\bar{\eta}) = \int d^4x\, d^4\theta\, \varphi\bar{\varphi}\, \eta\bar{\eta} + \left(\frac{m}{2}\int d^4x\, d^2\theta\, \varphi^3\eta^2 \right.$$
$$\left. + \frac{\lambda}{3!}\int d^4x\, d^2\theta\, \varphi^3\eta^3 + h.c.\right) \qquad (6.1.1)$$

The chiral compensator φ is introduced in the action

(6.1.1) to make it covariant with respect to the backgr= ound. The equation of motion for the φ field in the chi= ral representation is given by eq.(5.1.7) and possesses the regular solution reported in eq.(5.1.8).

The model possesses a partial superconformal invaria= nce, what makes useful the introduction of the supercon= formal transformation (5.2.1). This removes the explicit presence of the background in the description of the qua= ntum model in terms of the transformed fields, except for a term proportional to m which represents the deviation from an exactly superconformal invariant theory. Defining $\xi \equiv \varphi - 1$ and splitting the action in a free-field part

$$S_0 = \int d^4x\, d^4\theta\, \hat{\bar{\eta}}\, \hat{\eta} + \left(\frac{m}{2} \int d^4x\, d^2\theta\, \hat{\eta}^2 + h.c. \right)$$

and an interaction part

$$S_{int} = \frac{m}{2} \int d^4x\, d^2\theta\, \xi\, \hat{\eta}^2 + \frac{\lambda}{3!} \int d^4x\, d^2\theta\, \hat{\eta}^3 + h.c.$$

we can derive a consistent description of the quantum theory from the generating functional

$$\hat{Z}(\hat{J}) = Z(J) =$$

$$= e^{\left[\frac{m}{2} \int d^4x\, d^2\theta\, \varphi^6 \xi \left(\frac{\delta}{\delta \hat{J}} \right)^2 + h.c. \right]} e^{\left[\frac{\lambda}{3!} \int d^4x\, d^2\theta\, \varphi^9 \left(\frac{\delta}{\delta \hat{J}} \right)^3 + h.c. \right]} \cdot$$

$$\cdot \int \mathcal{D}\hat{\eta}\, \mathcal{D}\hat{\bar{\eta}}\; e^{\int d^4x\, d^4\theta\, \hat{\eta}\, \hat{\bar{\eta}} + \left(\frac{m}{2} \int d^4x\, d^2\theta\, \hat{\eta}^2 + h.c. \right)} e^{\left(\int d^4x\, d^2\theta\, \hat{J} \hat{\eta} + h.c. \right)} \qquad (6.1.2)$$

The effect of the reflective boundary conditions in the internal lines $\langle T\eta(x)\bar{\eta}(0)\rangle$, $\langle T\eta(x)\eta(0)\rangle$ considered in the previous section can be neglected for the evaluation of divergent counterterms. Then, the calculation of the free-field propagators can be car=

ried following the flat space-time case [14] to obtain the momentum-space expressions

$$\langle \hat{\bar{\eta}}\, \hat{\eta} \rangle = \frac{1}{p^2+m^2}\, \delta^4(\theta-\theta')$$

$$\langle \hat{\eta}\, \hat{\eta} \rangle = \frac{-mD^2}{p^2(p^2+m^2)}\, \delta^4(\theta-\theta')\ ,\quad \langle \hat{\bar{\eta}}\, \hat{\bar{\eta}} \rangle = \frac{-m\bar{D}^2}{p^2(p^2+m^2)}\, \delta^4(\theta-\theta')$$

After taking into account the covariant functional derivative

$$\frac{\delta \hat{J}(z)}{\delta \hat{J}(z')} = \frac{\varphi^2(z)}{\varphi^2(z')}\, \frac{\delta J(z)}{\delta J(z')} = \frac{1}{\varphi^3(z)}\, \bar{D}^2 \delta^8(z-z') \qquad (6.1.3)$$

the naive contribution from eq.(6.1.2) to the quadratic vertex

$$\frac{m}{2}\, \varphi^6 \xi \ + h.c.$$

and to the cubic vertex

$$\frac{\lambda}{3!}\, \varphi^9 \ + h.c.$$

in computing a generic derivative of the generating functional $\hat{Z}(\hat{J})$, reduce to $(m/2)\xi$ and to $\lambda/3!$, respectively. We recall here our remark of section 5.1 which is applicable to the ξ -approach that we are describing here, as well as to the perturbative analysis that we introduced there. The fact that, with our formulation, the D-algebra remains to be carried in the usual flat space-time context and that the Feynman rules of the theory turn out to be equivalent to the ones established for the flat space -time version of the quantum theory, with the addition of the quadratic vertex ((m/2)ξ +h.c.), not only constitute a remarkable simplification of the whole description in view of the demonstration of the extension of the validity of the no-renormalization theorem to quantum supersymmetric models in the non-trivial background, but also provides naturally an answer to our need to resort to some perturbative description of the effects of the curvature, through the above implicit expansion in the chiral superfield ξ . This possibility relies upon the rescaling (5.2.1) that allows indeed to remove the compensator φ from the m=0 part of the superfield action, which is invariant under a superconformal transformation.

Our one-loop calculation of the previous chapter shows that, among the possible counterterms

$$\alpha \int d^4x\, d^2\theta\, \varphi^3 \eta = \int d^4x\, d^4\theta\, \bar{\xi}\, \hat{\eta} \tag{6.1.4}$$

$$\alpha \int d^4x\, d^2\theta\, \varphi^3 \eta^2 = \int d^4x\, d^4\theta\, \varphi^{-1} \bar{\xi}\, \hat{\eta}^2 \tag{6.1.5}$$

$$\alpha \int d^4x\, d^2\theta\, \varphi^3 \eta^3 = \int d^4x\, d^4\theta\, \varphi^{-2} \bar{\xi}\, \hat{\eta}^3 \tag{6.1.6}$$

only the term (6.1.4), corresponding to a linear superfield insertion in the superpotential P(η) in eq.(5.2.12), is actually generated by one-loop quantum corrections. The possibility of generating the counterterms (6.1.5) and(6.1.6) through a perturbative series of Feynman diagrams containing a number of the ξ (and $\bar{\xi}$) superfield reproducing the expansions $\varphi^{-1} = \sum_0^\infty (-\xi)^n$ and $\varphi^{-2} = \sum_0^\infty (n+1)(-\xi)^n$ is not realized, to the one-loop order. The term (6.1.5) represents a renormalization of the mass action which in flat space-time is prevented by the no-renormalization theorem [19]. The possibility of a renormalization of the mass action is then the only interesting possibility to explore, to higher order in the loop expansion. On the one hand, the occurrence of a breaking of the no-renormalization theorem in what concerns the introduction of a linear superfield is found already to the one-loop order. On the other hand, a renormalization of the cubic interation as a result of having a non-trivial background curvature, is to be excluded on dimensional grounds.

It is straightforward to generalize the above treatment to the case of interacting chiral and real gauge superfields. This system is described (in the gauge-chiral representation) by the action [14]

$$S(\eta,\bar\eta,V)=\int d^4x\, d^4\theta\, \varphi\bar\varphi\, \bar\eta_j (e^V)^j{}_i\, \eta^i + tr\int d^4x\, d^2\theta\, \varphi^3 W^2 + \left(\frac{m}{2}\int d^4x\, d^2\theta\, \varphi^3\eta^2 + \frac{\lambda}{3!}\int d^4x\, d^2\theta\, \varphi^3\eta^3 + h.c.\right) \qquad (6.1.7)$$

where $V^i{}_j = V^A (T_A)^i{}_j$ and $(T_A)^i{}_j$ is a matrix representation of the generators of the gauge group of invariance of the action (6.1.7). With respect to eq.(6.1.1) we have only introduced new terms which are superconformally invariant and we can introduce the transformation

$$\hat W_\alpha = \varphi^{3/2} W_\alpha \quad , \quad \hat J_V = \varphi\bar\varphi J_V \qquad (6.1.8)$$

together with eq.(5.2.1), in order to reduce the m=0 part of the action (6.1.7) to its flat space-time form. The definition of $\hat W_\alpha$ in terms of the familiar derivatives in flat background is $\hat W_\alpha = i\bar D^2 D_\alpha V$; the covariantization of this expression in the Yang-Mills chiral representation

$$W_\alpha = i\left(\bar{\nabla}^2+\alpha\right)e^{-V}\nabla_\alpha e^V$$

gives W_α in terms of the background covariant derivatives

$$\nabla_\alpha = \bar{\varphi}^{-1}\varphi^{1/2}D_\alpha \quad , \quad \nabla_{\dot\alpha} = \varphi^{-1}\bar{\varphi}^{1/2}\bar{D}_{\dot\alpha}$$

In writing the transformation (6.1.8), the relation in the background chiral representation

$$\varphi^3\left(\bar{\nabla}^2+\alpha\right)f = \bar{D}^2\varphi\bar{\varphi}f$$

has been taken into account.

In eq.(6.1.8) J_v is the source in the term

$$\int d^4x\, d^4\theta\, \varphi\bar{\varphi}\, V J_v$$

After the superconformal transformation, the gauge-fixing procedure can be carried in the line of the flat background theory; the gauge propagator (in the Fermi-Feynman gauge, α =1) reads

$$\langle VV\rangle_0 = -\frac{1}{p^2}\,\delta^4(\theta-\theta')$$

In splitting the action we have now in the interaction part the extra-term

$$\int d^4x\, d^4\theta\, \hat{\bar{\eta}} \left(e^{V} - 1 \right) \hat{\eta} = \int d^4x\, d^4\theta\, \hat{\bar{\eta}} \sum_{m=1}^{\infty} \frac{V^m}{m!} \hat{\eta}$$

The corresponding modifications introduced in the generating functional $\hat{Z}(\hat{J})$ involve the operator

$$e^{\int d^4x\, d^4\theta\, \varphi^2 \bar{\varphi}^2 \frac{\delta}{\delta \hat{\bar{J}}} \sum_{m=1}^{\infty} \frac{1}{m!} \left(\varphi \bar{\varphi} \frac{\delta}{\delta \hat{J}_V} \right)^m \frac{\delta}{\delta \hat{J}}}$$

Recalling eq.(6.1.3), together with the covariant functional derivative [14]

$$\frac{\delta \hat{J}_V(z)}{\delta \hat{J}_V(z')} = \frac{\varphi(z)\bar{\varphi}(z)}{\varphi(z')\bar{\varphi}(z')} \frac{\delta J_V(z)}{\delta J_V(z')} = \frac{1}{\varphi(z)\bar{\varphi}(z)} \delta^8(z-z')$$

we conclude that there are no modifications of this vertex, with respect to the corresponding one in the flat theory. It is worthwhile to notice that, as a consequence of superconformal invariance, the ghost propagators and vertices of the flat space-time theory remain intact in AdS_4. The conclusion is that the quantum system of N=1 super Yang-Mills coupled to matter scalar superfields in AdS_4 is equivalent to the corresponding flat theory, provided the extra quadratic vertex $(m/2)\xi$ is introduced in the expression of the Feynman rules.

6.2. The proof of the no-renormalization theorem in AdS_4

We describe in the following the proof of the no-renormalization theorem. From what we have said in the preceding section it is clear that the renormalization of the mass term in the lagrangian demands, in terms of our perturbative approach in the curvature effects, recovering a very definite structure in the ξ and $\bar{\xi}$ fields, when computing the radiative corrections to one of these terms in the effective action. To be more precise, what we know is that the presence of a covariant divergent contribution to the $\Gamma_{\hat{\eta}\hat{\eta}}$ two-point function, for instance, can be manifest only through a sum of ξ and $\bar{\xi}$ insertions

$$\frac{1}{\varphi}\bar{\xi} = \sum_{n=0}^{\infty} \bar{\xi}(-1)^n \xi^n$$

Then, to prove, to a given order in perturbation theory, that there is no divergent contribution with only one $\bar{\xi}$ insertion (which represents the first term in the expansion) is equivalent to show that the mass term cannot get renormalized.

For the sake of clearness in the exposition, we start giving the argumentation for the model with only chiral self-interactions, and then we extend the proof to the case in which gauge interactions are taken into account. It will be supposed, in what follows, that reference is made to a given order in the perturbative calculation of the $\Gamma_{\hat{\eta}\hat{\eta}}$ two-point function. The consideration of the $\Gamma_{\hat{\eta}\hat{\eta}\hat{\eta}}$ three-point function follows afterwards. We will assume throughout the demonstration the existence of a supersymmetry preserving regularization procedure, to every order in the loop expansion. This is a fu=

ndamental hypotesis which is assumed to be holding when proving the validity of the theorem in flat space-time as well.

Thus, we begin by remarking that, in general, inserting a ξ or $\bar{\xi}$ field in the internal line of a flat space-time Feynman diagram improves the ultraviolet behaviour of the graph, while the D, $\bar{D}$-structure of the diagram does not suffer any change. In fact, one can easily see that only when introducing a $\bar{\xi}$ (ξ) vertex in a $\hat{\eta}\hat{\eta}$ ($\bar{\hat{\eta}}\bar{\hat{\eta}}$) internal line, the degree of divergence of the graph remains unchanged. This leads to the conclusion that a sufficient condition for the no-renormalization of the mass to hold is the absence of any Feynman diagram without ξ or $\bar{\xi}$ vertices that, after having performed the D-algebra, could lead to a divergent structure in momentum space. It is obvious that the chirality of the integrand that is used in flat space-time in order to disregard these diagrams does not help in Anti-de Sitter space, since now one has the possibility of building diagrams with $\bar{\xi}$ insertions that would make the integrand not chiral.

At this point, we would like to pose the problem of the search of these divergent structures in momentum space in terms of a similar problem for an equivalent scalar quantum field theory with cubic self-interaction vertices. There are, however, two things that could prevent us from doing this identification. The first one is the presence of propagators for the $\hat{\eta}\hat{\eta}$ or $\bar{\hat{\eta}}\bar{\hat{\eta}}$ internal lines with abnormal ultraviolet behaviour, with respect to what one would expect in the case of a scalar field theory. The second point is the generation of some p^2 factors (corresponding to $\Box$ in x-space) when performing the D-algebra to contract the whole graph in question to a point in $\vartheta,\bar{\vartheta}$-space. Fortunately, one of the troubles helps in solving the other, since one can always do the D-algebra in such a way that every $\hat{\eta}\hat{\eta}$ or $\hat{\bar{\eta}}\hat{\bar{\eta}}$ internal line ends with the corresponding p^2 factor. This converts these propagators into the ones with the expected ultraviolet behaviour. Furthermore, we remain, in general, with some other p^2 factors placed over $\hat{\eta}\hat{\bar{\eta}}$ internal lines. This ultraviolet behaviour is equivalent to the one in which the internal line has been contracted to a point, leading to the generation of quartic vertices from the two cubic vertices that were originally joined by the internal line. Thus, we see that the ultraviolet behaviour of the original superfield diagram is equivalent to the corresponding graph of a scalar field theory with cubic and quartic vertices.

Then, the last part of the proof, for the model with chiral self-interactions, consists in checking that none of these diagrams can be divergent when the wave-function renormalization of the original superfield theory is taken into account. In principle, we can classify these graphs in two categories: primitive graphs, i.e. graphs which do not contain another n-point function as an insertion, and graphs containing an insertion. To check the finiteness of the first class of diagrams is a matter of triviality, since by assumption the corresponding contribution in momentum space has to be dimensionless, being a contribution to a term of the type

$$\int d^4\theta\, \hat{\eta}\hat{\eta}$$

in the effective action . We know that, in order to build a diagram contributing to the $\Gamma_{\hat{\eta}\hat{\eta}}$ two-point function, we need at least one $\hat{\eta}\hat{\eta}$ (or $\hat{\bar{\eta}}\hat{\bar{\eta}}$) internal line, and the latter carries a factor of m. The finiteness of these primitive graphs is then ensured by a naive power counting of dimensions in the momentum integrals. Furthermore, we know that the diagrams which are not primitive can be divergent only if they contain the insertion of a divergent n-point function. At this point, we have to recall the equivalent scalar theory with cubic and quartic vertices to conclude that the only primitive divergent insertions thay we can have are given by two, three and four-point functions. It is obvious, however, that, if the D-algebra was performed in a convenient way, the last two types of insertions could be considered as contributions to terms of the type

$$\int d^4\theta \; \hat{\eta}^3$$

and

$$\int d^4\theta \; \hat{\eta}^4$$

respectively, in the effective action. Once again, the power counting of dimensions shows that these contributions are finite. Thus, we are led to the conclusion that only the insertion of a two-point function can make a graph divergent and, moreover, by the argumentation given at the beginning of the paragraph, we see that this is possible only if the insertion was an original $\Gamma_{\hat{\eta}\hat{\eta}}$ two-point function. But this is precisely the kind of divergence that is removed by the presence of the same diagram in which the insertion in question has been replaced by the corresponding term of the wave-function renormalization counterterm, what leads to the end of the argument.

Up to this point, what we have given is the proof of the no-renormalization of the mass term

$$m\left[\int d^2\theta\, \varphi\, \hat{\eta}\hat{\eta} + \int d^2\bar{\theta}\, \bar{\varphi}\, \hat{\bar{\eta}}\hat{\bar{\eta}}\right]$$

to any order in perturbation theory, relying upon the fact that there are no divergent contributions to the $\Gamma_{\hat{\eta}\hat{\eta}}$ or $\Gamma_{\hat{\bar{\eta}}\hat{\bar{\eta}}}$ two-point functions in the flat space-time model (before performing the superspace integration, wh= ich would give a vanishing result).

A similar argumentation can be given for the $\Gamma_{\hat{\eta}\hat{\eta}\hat{\eta}}$ or $\Gamma_{\hat{\bar{\eta}}\hat{\bar{\eta}}\hat{\bar{\eta}}}$ three-point functions of the theory. In fact, the only difference is that now primitive divergent contri= butions are excluded on dimensional grounds, from the fact that they are contributions to a term of the type

$$\int d^4\theta\, \hat{\eta}\hat{\eta}\hat{\eta}$$

in the effective action. Apart from this, one is led, in the same way as before, to the conclusion that it is impossible to obtain a divergent expression in momentum space for any contribution to the $\Gamma_{\hat{\eta}\hat{\eta}\hat{\eta}}$ or $\Gamma_{\hat{\bar{\eta}}\hat{\bar{\eta}}\hat{\bar{\eta}}}$ three-poi= nt functions of the flat space-time theory, what in tu= rn implies the no-renormalization of the interaction term

$$\lambda\left[\int d^2\theta\, \hat{\eta}^3 + \int d^2\bar{\theta}\, \hat{\bar{\eta}}^3\right]$$

to any order in perturbation theory.

Next we give the extension of the proof of the no-re= normalization of the mass and chiral self-interaction lagrangians when gauge interactions are switched on. Our fundamental hypotesis, in this context, will be the existence of a gauge-invariant supersymmetric regulari= zation procedure, to all orders in perturbation theory. The argumentation follows the same way as before, up to the point where insertions in a $\Gamma_{\hat{\eta}\hat{\eta}}$ two-point function or a $\Gamma_{\hat{\eta}\hat{\eta}\hat{\eta}}$ three-point function are considered. In this sense, we have to take care now of insertions of n-point functions with external gauge fields, but there is the obvious remark that the wave-function renormalization both of the gauge supermultiplet and of the chiral supermultiplet cannot be affected by the external background , as long as

$$\int d^4\theta \, V D^\alpha \bar{D}^2 D_\alpha V \propto \int d^2\theta \, \hat{W}_\alpha \hat{W}^\alpha$$

and

$$\int d^4\theta \, \hat{\bar{\eta}} e^V \hat{\eta}$$

(where the superfields φ and $\bar{\varphi}$ do not appear) are the only covariant atructures, in the sense of both the gauge invariance of the action (6.1.7) and the gravitational background, that can be built. Even the possibility of generating a term of the type

$$\int d^4\theta \, E^{-1} V^2 = \int d^4\theta \, \varphi\bar{\varphi} V^2$$

is ruled out by the fact that the first term in the expansion

$$\int d^4\theta \, \varphi\bar{\varphi} V^2 = \int d^4\theta \left(1 + \xi + \bar{\xi} + \xi\bar{\xi}\right) V^2$$

cannot be present because of the cancellation of quadratic divergencies in the gauge supermultiplet self-energy which takes place in flat space-time. Therefore, we can argue, as we did before, that the divergent part of each one of these wave-function renormalization insertions can be cancelled by the inclusion of a similar diagram in which the insertion has been replaced by the corresponding term of the wave-function renormalization counterterm. Finally, we conclude again that, in the presence of the gauge superfield, there is no divergent structure in momentum space that can come out of a graph contributing to the $\Gamma_{\hat{\eta}\hat{\eta}}$ two-point function or the $\Gamma_{\hat{\eta}\hat{\eta}\hat{\eta}}$ three-point function, in flat space-time, what leads, by means of the same argumentation given in the case of the chiral model, to the no-renormalization of the mass and self-interaction lagrangians in Anti-de Sitter space, to all perturbative orders.

We can resume the work discussed in the present chapter by recalling that we have implemented an expansion in the effect of the AdS_4 background curvature, in terms of the interaction vertices of the quantum super Yang-Mills coupled to matter chiral supermultiplets, in the superfield formulation of the theory. This is made possible by the existence of a superconformal invariant part of the superfield action. Transforming both the scalar superfield and the field strength of the real gauge superfield (as well ws their corresponding sources) according to the canonical weights, reduces the action of the interacting model to the flat space-time one plus a deviation from the exact invariant behaviour which would be absent in the massless case. The proof of the no-renormalization of the mass and the chiral self-interaction actions is then attained, order by order in perturbation theory, by simple power counting analysis, as we have exposed in the present section. The linear superfield insertion needed to renormalize the one-loop effective action constitute then the only deviation (and a minor one) of the quantum features of the theory in AdS_4

from the flat case.

It is perhaps worth to notice that the superfield action of N=4 Yang-Mills theory [14] possesses an exact superconformal invariance. Thus, the transformations (5.2.1) and (6.1.8) reduce the action of the theory in AdS_4 to the one in flat background. The calculation of the quantum corrections to the theory can be carried following the flat case and the conclusions that the β-function vanishes to all orders in perturbation theory and that N =4 supersymmetric Yang-Mills theory is a finite four-dimensional field theory are holding in AdS_4 space as well.

What we have shown in the present section is that the no-renormalization theorem holds in AdS_4 space as well as in the flat background. Neither the mass nor the chiral interaction actions suffer any renormalization and the proof is carried to all perturbative orders. Thus, in the context of perturbation theory, the only warning is the same as it is usually given in flat space-time about the possibility of invalidating the theorem through a pathological infrared-type behaviour [14].

In fact, the no-renormalization of the mass lagrangian of the scalar multiplet interacting with a Yang-Mills superfield, in the non-trivial background, opens the possibility of implementing the solution of the hierarchy problem in the context of the AdS_4 supersymmetric field theories.

7. CONCLUSIONS

Supergravity theories and, in particular, gauged extended supergrvities [36-39] and supersymmetric Kaluza-Klein theories[40,41] constitute a framework for the unification of gravity with the physics of elementary particles at "low energy". These theories contain so much symmetry that their classical solutions consist of a AdS factor, with the possible addition of extra-dimensions. As we have seen, these leading approximations in a loop expansion to the ground state configuration of the metric maintain unbroken, entirely or in part, the supersymmetries. We have addressed the question of the survival of the classical supersymmetry invariance of the vacuum state when quantum corrections are taken into account. Our result is the persistence of supersymmetry to the one-loop order.

The renormalization of supersymmetric theories in AdS does not respect the "naturalness" features dictated by the no-renormalization theorem in the case of supersymmetric theories in flat space-time. All the counterterms allowed by the classical invariance of the theory are present in the structure of divergencies of the effective action, at the quantum level. The linear superfield insertion in the superpotential of the interacting system of chiral and real gauge superfields constitutes such a breaking of the no-renormalization theorem holding in flat space-time. We have obtained the above results in a completely general way, showing how to deal with the reflective boundary conditions at spatial infinity, imposed by classical supersymmetric invariance on the field propagators.

AdS provides a natural infrared cut-off proportional to the contraction parameter of the space, for the theory of massless matter supermultiplets. The mass term generated by the quantum corrections of chapter 4, which break the no-renormalization theorem for finite terms, indicates that quantum calculations in AdS are naturally regulated in the infrared region . This prediction is consistent with the known fact that chiral symmetry is broken at the tree-level for field theories in AdS . In the light of our results, it emerges that chiral symmetry breaking and the intrinsic infrared regularization of field theories in AdS are compatible with the quantum realization of supersymmetry invariance.

An appealing possibility is the generation of a supersymmetric invariant divergent mass term, to higher oreders in the loop expansion. The abovementioned introduction of a linear superfield provides an example of radiative generation of the superfield integral of a term of the superpotential. The generation of a quadratic term would be mathematically equivalent to that. This motivates our study to apply the superfield formalism to AdS supersymmetry. In our superspace analysis we use the simple and fundamental tools of power counting and the superconformal invariance of the massless theory, which is broken at the quantum level only by finite anomalies. The latter are irrelevant to our purposes. We conclude that there is no renormalization of the mass action, to all orders in the loop expansion. The linear superfield insertion, already present at one-loop, remain the only deviation of the divergent structure of the effective action in AdS from its flat version.

The investigation of the structure of the exact vacuum and the comparison with the adiabatic vacuum of the massive theory and with the conformal vacuum of the theory of massless fields in AdS completes our study of a topic which constitutes the epitome of the beautyful application of the mathematical subtleties of quantum field theory in curved space-time to the solution of the fundamental problems of our physical understanding.

APPENDIX A: NOTATION

Greek indices are used as general coordinate indices $\mu, \nu, \ldots = 0, \ldots 3$ whereas latin indices $\hat{a}, \hat{b}, \ldots = 0, \ldots 3$ are used as indices referred to the tangent space frame at each point of the manifold. We use the conventions (-++) in the notation of ref.[35].

In chapter 2 we have used the expansion in normal coo= rdinates of a number of tensors. We remember that, in general, the expansion of a tensor W in normal coordin= ates y=x-x' around a point x' is given by

$$W_{\alpha_1 \cdots \alpha_m}(x) = W_{\alpha_1 \cdots \alpha_m}(x') + W_{\alpha_1 \cdots \alpha_m ; \mu}(x') y^\mu + \frac{1}{2!}\Big[W_{\alpha_1 \cdots \alpha_m ; \mu\nu}(x') - \frac{1}{3} \sum_{j=1}^{m} R^{\sigma}{}_{\mu \alpha_j ; \nu}(x') W_{\alpha_1 \cdots \alpha_{j-1} \sigma \alpha_{j+1} \cdots \alpha_m}(x') \Big] y^\mu y^\nu \ldots \quad \text{(A.1)}$$

The expansion of the metric tensor is, for example,

$$g_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{3} R_{\mu\rho\nu\sigma}(x') y^\rho y^\sigma \quad \text{(A.2)}$$

and the expansion of the vierbein

$$e^a{}_\mu = \delta^a{}_\mu - \frac{1}{6} R^a{}_{\nu\mu\sigma}(x') y^\nu y^\sigma \ldots \quad \text{(A.3)}$$

where the additional constraint of parallel transport of the vierbein along the geodesics

$$e_{a\mu;\sigma} y^\sigma = e_{a\mu;\sigma\rho} y^\sigma y^\rho = 0$$

has been imposed to determine the expansion. From these two expressions the expansions for the Christoffel sym= bols and for the spinorial connection

$$\Gamma_\mu = \frac{1}{8}\left[\gamma_{\hat{a}}, \gamma_{\hat{b}}\right] e^{\hat{a}\sigma} D_\mu e^{\hat{b}}{}_\sigma$$

can be easily obtained.

We have used the conventions $\{\gamma^{\hat{a}}, \gamma^{\hat{b}}\} = 2\eta^{\hat{a}\hat{b}}$ and $\gamma_5^2 = 1$. Although for simplicity of notation we describe the Wess-Zumino model in terms of four component Majorana spinors, the right combinatorics in terms of Weyl spinors has been applied in the computation of the contributions of figures 1(g), 1(i), 2(d) and 3(c), as well as in evaluating the quantities $(\Gamma_{AA})_{FERMI}$, $(\Gamma_{BB})_{FERMI}$, $\Gamma_{\psi\bar{\psi}}$, $(\Gamma_{AAA})_{FERMI}$, Γ_{ABB}, $\Gamma_{A\psi\bar{\psi}}$ and $\Gamma_{B\psi\bar{\psi}}$ of section 2.3. This means, in detail, that the interaction vertices have been taken as $A\chi\chi$, $A\bar{\chi}\bar{\chi}$, $iB\chi\chi$ and $-iB\bar{\chi}\bar{\chi}$, in terms of Weyl spinors, while the corresponding propagators

$$\langle \chi(x)\chi(x')\rangle = \frac{-im}{\Box - 3a^2 + m^2}\,\delta(x,x')$$

$$\langle \bar{\chi}(x)\bar{\chi}(x')\rangle = \frac{-im}{\Box - 3a^2 + m^2}\,\delta(x,x')$$

$$\langle \chi(x)\bar{\chi}(x')\rangle = \sigma^{\hat{a}} D_{\hat{a}} \frac{1}{\Box - 3a^2 + m^2}\,\delta(x,x')$$

as well as the rule $\mathrm{Tr}\,\sigma^{\hat{a}}\bar{\sigma}^{\hat{b}} = 2\eta^{\hat{a}\hat{b}}$, have been taken into account. When the Feynman rules of the Wess-Zumino model are expressed in terms of the above formalism one can easily check that the vacuum expectation values $\langle B\rangle$ and $\langle G\rangle$ vanish to all orders in perturbation theory, for the model quantized around the origin A=B=0, as it is stated in section 2.3.

APPENDIX B: COMPUTATION OF THE REGULARIZED INTEGRALS

We start calculating the integrals W_n , n = 0, 2, 4, 6, occurring in section 2.3

$$W_n = \int \frac{d^4k}{(2\pi)^4} \frac{k^n}{(k^2-m^2)^2(k^2-m_1^2)^2(k^2-m_2^2)^2} \tag{B.1}$$

For n = 4 we have

$$W_4 = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m_1^2)^2(k^2-m_2^2)^2} + O\left(\frac{1}{\Lambda^6}\right) \tag{B.2}$$

where the terms omitted are irrelevant for the results of sect. 2.3, when the limit $m_1, m_2 = O(\Lambda) \to \infty$ is taken. Then,

$$W_4 = \frac{i}{(4\pi)^2} \frac{1}{(m_2^2-m_1^2)^2} \int_0^1 dx \int_0^{1-x} dy \frac{1-x-y}{(x+y+\mu)^2} \tag{B.3}$$

where we have set $\mu = m_2^2/(m_1^2 - m_2^2)$. It is easily seen that

$$\int_0^1 dx \int_0^{1-x} dy \frac{1-x-y}{(x+y+\mu)^2} = -2 + (2\mu+1)\log\left(1+\frac{1}{\mu}\right) .$$

so that

$$W_4 = \frac{i}{(4\pi)^2} \frac{1}{(m_1^2-m_2^2)^2} \left(-2 + \frac{m_1^2+m_2^2}{m_1^2-m_2^2} \log \frac{m_1^2}{m_2^2}\right) \tag{B.4}$$

Next, we evaluate the case n = 6

$$W_6 = \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2-m_1^2)^2(k^2-m_2^2)^2} + O\left(\frac{1}{\Lambda^4}\right) \tag{B.5}$$

Again, we are omitting terms that give vanishing contribution when the masses of the Pauli-Villars fields go to the infinity. We have

$$W_6 = W_a + m_1^2 W_b \tag{B.6}$$

where

$$W_a = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m_1^2)(k^2-m_2^2)^2}$$

$$W_b = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m_1^2)^2(k^2-m_2^2)^2} \tag{B.7}$$

Using the expression

$$\int_0^1 dx \int_0^{1-x} dy \frac{1}{x+y+\alpha} = \alpha \log \frac{\alpha}{1+\alpha} + 1$$

it is immediately seen that

$$W_a = -\frac{i}{(4\pi)^2} \frac{1}{m_2^2-m_1^2} \left(\frac{m_1^2}{m_2^2-m_1^2} \log \frac{m_1^2}{m_2^2} + 1 \right) \tag{B.8}$$

On the other hand, from the calculation of W_4 we have the result

$$W_b = \frac{i}{(4\pi)^2} \frac{1}{(m_2^2-m_1^2)^2} \left(-2 + \frac{m_1^2+m_2^2}{m_1^2-m_2^2} \log \frac{m_1^2}{m_2^2} \right) \tag{B.9}$$

Adding the two contributions we obtain

$$W_6 = -\frac{i}{(4\pi)^2}\left[\frac{m_1^2+m_2^2}{(m_2^2-m_1^2)^2} + \frac{2m_1^2m_2^2}{(m_2^2-m_1^2)^3}\log\frac{m_1^2}{m_2^2}\right] \qquad (B.10)$$

Let us now turn to the calculation of W_2

$$W_2 = -\frac{i}{(4\pi)^2}\frac{2}{m_2^6}\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz\int_0^{1-x-y-z} dw\,\frac{1}{[w+z+\beta(x+y)]^3} + O\left(\frac{1}{\Lambda^8}\right) \qquad (B.11)$$

where $\beta = m_1^2/m_2^2$. Carrying the first two integrations, we have

$$W_2 = -\frac{i}{(4\pi)^2}\frac{1}{m_2^6}\int_0^1 dx\int_0^{1-x} dy\left\{\frac{1}{\beta(x+y)} - \frac{1}{1-x-y+\beta(x+y)} - \frac{1-x-y}{[1-x-y+\beta(x+y)]^2}\right\} \qquad (B.12)$$

It is not difficult to show that

$$\int_0^1 dx\int_0^{1-x} dy\,\frac{1}{\beta(x+y)} = \frac{1}{\beta}$$

$$\int_0^1 dx\int_0^{1-x} dy\,\frac{1}{1-x-y+\beta(x+y)} = \frac{1}{\beta-1} - \frac{1}{(\beta-1)^2}\log\beta$$

$$\int_0^1 dx\int_0^{1-x} dy\,\frac{1-x-y}{[1-x-y+\beta(x+y)]^2} = \frac{1}{(1-\beta)^2}\left(-2+\frac{1+\beta}{1-\beta}\log\frac{1}{\beta}\right)$$

Thus

$$W_2 = -\frac{i}{(4\pi)^2}\left[\frac{m_2^2+m_1^2}{(m_2^2-m_1^2)^2}\frac{1}{m_1^2m_2^2} + \frac{2}{(m_2^2-m_1^2)^3}\log\frac{m_1^2}{m_2^2}\right] \qquad (B.13)$$

For W_0 we have the expression

$$W_0 = \frac{i}{(4\pi)^2}\,\frac{6}{(m_2^2-m_1^2)^4}\int_0^1 dx\int_0^{1-x} dy\int_0^{1-x-y} dz\int_0^{1-x-y-z} dw\,\frac{1-x-y-z-w}{\left[z+w+\gamma(1-x-y)+\delta(x+y)\right]^4} =$$

$$= \frac{i}{(4\pi)^2}\,\frac{1}{(m_2^2-m_1^2)^4}\int_0^1 dx\int_0^{1-x} dy\,\left\{\frac{1-x-y}{\left[(1-x-y)(1+\gamma)+\delta(x+y)\right]^2}+\right.$$

$$+\,3\left[\frac{1}{(1-x-y)(1+\gamma)+\delta(x+y)}-\frac{1}{\gamma(1-x-y)+\delta(x+y)}\right]+$$

$$\left.+\,\frac{(1-x-y)(1+\gamma)+\delta(x+y)}{\left[\gamma(1-x-y)+\delta(x+y)\right]^2}-\frac{(1-x-y)(1+\gamma)+\delta(x+y)}{\left[(1-x-y)(1+\gamma)+\delta(x+y)\right]^2}\right\} \tag{B.14}$$

In the above, we have set $\gamma = m_2^2/(m_1^2 - m_2^2)$, $\delta = m^2/(m_1^2 - m_2^2)$. After some work one can verify that

$$\int_0^1 dx\int_0^{1-x} dy\,\frac{1-x-y}{\left[(1-x-y)(1+\gamma)+\delta(x+y)\right]^2} = \frac{1}{(1+\gamma)^2}\left(\log\frac{1+\gamma}{\delta}-2\right)+O(\delta)$$

$$\int_0^1 dx\int_0^{1-x} dy\,\frac{1}{(1-x-y)(1+\gamma)+\delta(x+y)} = \frac{1}{1+\gamma}\left(\log\frac{1+\gamma}{\delta}-1\right)+O(\delta)$$

$$\int_0^1 dx\int_0^{1-x} dy\,\frac{1}{\gamma(1-x-y)+\delta(x+y)} = \frac{1}{\gamma}\left(\log\frac{\gamma}{\delta}-1\right)+O(\delta).$$

$$\int_0^1 dx\int_0^{1-x} dy\,\frac{(1-x-y)(1+\gamma)+\delta(x+y)}{\left[\gamma(1-x-y)+\delta(x+y)\right]^2} = -\frac{2}{\gamma^2}-\frac{1}{\gamma}+\frac{1+\gamma}{\gamma^2}\log\frac{\gamma}{\delta}+O(\delta)$$

$$\int_0^1 dx\int_0^{1-x} dy\,\frac{(1-x-y)(1+\gamma)+\delta(x+y)}{\left[(1-x-y)(1+\gamma)+\delta(x+y)\right]^2} = \frac{1}{1+\gamma}\left(\log\frac{1+\gamma}{\delta}-1\right)+O(\delta)$$

Thus

$$W_0 = \frac{i}{(4\pi)^2}\left\{\frac{1}{m_2^4}\,\frac{1}{(m_1^2-m_2^2)^3}\left[(m_1^2-3m_2^2)\log\frac{m_2^2}{m_1^2}+2(2m_2^2-m_1^2)\right]\right.$$

$$+ \frac{1}{m_1^4} \frac{1}{(m_2^2 - m_1^2)^3} \left[(m_2^2 - 3m_1^2) \log \frac{m_1^2}{m^2} + 2\,(2m_1^2 - m_2^2) \right] \Bigg\} \tag{B.15}$$

In the following we calculate the integrals needed in section 4. Defining $\alpha = p^2/\Lambda^2$, we have

$$I_1 = \frac{i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{1}{[z + y - \alpha(x+y)(1-x-y)]^2} =$$

$$= \frac{i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \left[\frac{1}{y - \alpha(x+y)(1-x-y)} - \frac{1}{1-y-\alpha(x+y)(1-x-y)} \right] =$$

$$= \frac{i}{(4\pi)^2} \int_0^1 dx \left[\log \frac{1-\rho_+}{x-\rho_+} - \log \frac{1-\rho_-}{x-\rho_-} - \left(\log \frac{1-x-\sigma_+}{-\sigma_+} - \log \frac{1-x-\sigma_-}{-\sigma_-} \right) \right] + O(\alpha) \tag{B.16}$$

where $O(\alpha)$ indicates terms of order p^2/Λ^2 that go to zero in the limit $\Lambda \to \infty$. We have defined

$$\rho_\pm = \frac{1}{2} \left\{ 1 - \frac{1}{\alpha} \pm \left[\left(1 - \frac{1}{\alpha}\right)^2 + \frac{4x}{\alpha} \right]^{1/2} \right\} \tag{B.17}$$

$$\sigma_\pm = \frac{1}{2} \left\{ 1 - \frac{1}{\alpha} \pm \left[\left(1 - \frac{1}{\alpha}\right)^2 - \frac{4x}{\alpha} \right]^{1/2} \right\} \tag{B.18}$$

The second and fourth terms in the last line of eq. (A.16) are $O(\alpha)$ and can be dropped. The third term gives a finite contribution in the limit $\Lambda \to \infty$. The only divergent contribution comes from the first term. Integrating over the remaining Feynman parameter gives

$$I_1^{div} = \frac{i}{(4\pi)^2} \log \Lambda^2 \tag{B.19}$$

The divergent part of the integral I_2 can be expressed as follows

$$I_2^{div} = I_a + I_b \tag{B.20}$$

where

$$I_a = \left[\Lambda^4 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 (k^2-\Lambda^2)((k-p)^2-\Lambda^2)}\right]_{div} = -\frac{i}{(4\pi)^2}\Lambda^2 .$$

$$I_b = -m^2\left[\Lambda^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 (k^2-m^2)((k-p)^2-\Lambda^2)}\right]_{div} = m^2\frac{i}{(4\pi)^2}\log\Lambda^2 \tag{B.21}$$

Next we turn to the divergent part of J_1

$$J_1^{div} = \left[\Lambda^4 \int \frac{d^4k}{(2\pi)^4} k\cdot(k-p) \frac{1}{k^4 (k-p)^2 (k^2-\Lambda^2)((k-p)^2-\Lambda^2)}\right]_{div} = .$$

$$= J_a^{div} + J_b^{div} \tag{B.22}$$

where

$$J_a = \frac{i}{(4\pi)^2}\, 2\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{1-x-y-z}{[x+z-\alpha(x+y)(1-x-y)]^2} .$$

$$J_b = \frac{i}{(4\pi)^2}\, 2\alpha \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{(x+y)(1-x-y)(1-x-y-z)}{[x+z-\alpha(x+y)(1-x-y)]^3} \tag{B.23}$$

The quantity α is defined as before. For J_a^{div} we have

$$J_a^{div} = \frac{i}{(4\pi)^2}\cdot 2\left[\int_0^1 dx \int_0^{1-x} dy \frac{1-x-y}{x-\alpha(x+y)(1-x-y)}\right]_{div} = .$$

$$= \frac{i}{(4\pi)^2}\, 2 \left\{ \int_0^1 dx \left[(-\rho_+ + 1) \log \frac{1-\rho_+}{x-\rho_+} - (-\rho_- + 1) \log \frac{1-\rho_-}{x-\rho_-} \right] \right\}_{div} \tag{B.24}$$

We have introduced $\rho_\pm$ defined as in eq. (A.17). The second term in eq. (A.24) is finite in the limit $\Lambda \to \infty$ and we drop it. From the first term, after integrating over the x-variable we obtain

$$J_a^{div} = \frac{i}{(4\pi)^2} \log \Lambda^2 \tag{B.25}$$

The integral J_b turns out to be finite in the limit $\Lambda \to \infty$ and

$$J_1^{div} = \frac{i}{(4\pi)^2} \log \Lambda^2 \tag{B.26}$$

Finally, we consider the integral J_2^{div}. Its divergent part reads

$$J_2^{div} = J_c^{div} + J_d^{div} \tag{B.27}$$

where

$$J_c = \Lambda^4 \int \frac{d^4k}{(2\pi)^4}\, k\cdot(k-p)\, \frac{1}{k^2(k^2-\Lambda^2)(k-p)^2((k-p)^2-\Lambda^2)} \cdot$$

$$J_d = -m^2\Lambda^2 \int \frac{d^4k}{(2\pi)^4}\, k\cdot(k-p)\, \frac{1}{k^2(k^2-\Lambda^2)(k-p)^2((k-p)^2-m^2)} = m^2 \frac{i}{(4\pi)^2} \log \Lambda^2 \tag{B.28}$$

We can express J_c as follows

$$J_c = J_e + J_f \tag{B.29}$$

where

$$J_e = -\frac{i}{(4\pi)^2}\cdot 2\Lambda^2 \int_0^1 dx \int_0^{1-x} dy \left\{ \log\left[1-x-\alpha(x+y)(1-x-y)\right] - \right.$$

$$\left. - \log\left[y-\alpha(x+y)(1-x-y)\right] \right\}$$

$$J_f = \frac{i}{(4\pi)^2} p^2 \int_0^1 dx \int_0^{1-x} dy \, (x+y)(1-x-y) \left[\frac{1}{1-x-\alpha(x+y)(1-x-y)} \right.$$

$$\left. - \frac{1}{y-\alpha(x+y)(1-x-y)} \right] \qquad \text{(B.30)}$$

It is easy to show that

$$\left\{ \Lambda^2 \int_0^1 dx \int_0^{1-x} dy \, \log\left[1-x-\alpha(x+y)(1-x-y)\right]\right\}_{div} = -\frac{1}{4}\Lambda^2 \, .$$

and that

$$\left\{ \Lambda^2 \int_0^1 dx \int_0^{1-x} dy \, \log\left[y-\alpha(x+y)(1-x-y)\right]\right\}_{div} = -\frac{3}{4}\Lambda^2 - \frac{1}{6}p^2 \log\Lambda^2$$

Thus

$$J_e^{div} = -\frac{i}{(4\pi)^2}\left(\Lambda^2 + \frac{1}{3}p^2\log\Lambda^2\right) \qquad \text{(B.31)}$$

The first term in the expression of J_f in eq. (A.30) turns out to be finite and we drop it here. Working out the first term one has

$$\left[\int_0^1 dx \int_0^{1-x} dy \, \frac{(x+y)(1-x-y)}{y-\alpha(x+y)(1-x-y)}\right]_{div} = \frac{1}{6}\log\Lambda^2 \, .$$

and then

$$J_f^{div} = - \frac{i}{(4\pi)^2} \frac{1}{6} p^2 \log \Lambda^2 \tag{B.32}$$

The divergent expression resulting for the integral J_c of eq. (A.29) is then

$$J_c^{div} = - \frac{i}{(4\pi)^2} \left(\Lambda^2 + \frac{1}{2} p^2 \log \Lambda^2 \right) \tag{B.33}$$

Collecting the relevant integrals and plugging in eq. (A.27) we finally obtain

$$J_2^{div} = \frac{i}{(4\pi)^2} \left(- \Lambda^2 + m^2 \log \Lambda^2 - \frac{1}{2} p^2 \log \Lambda^2 \right) \tag{B.34}$$

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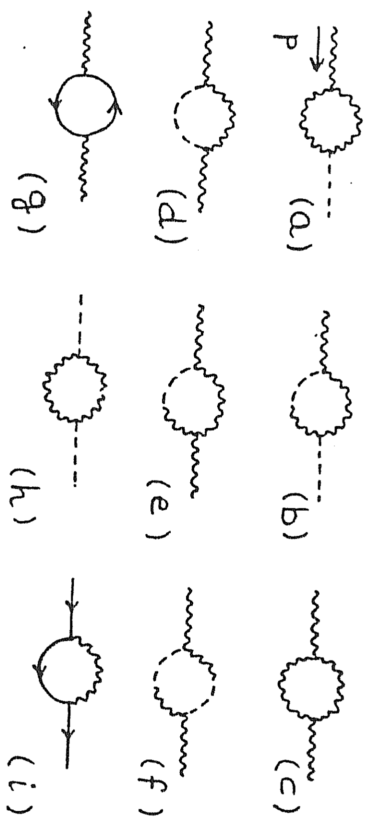
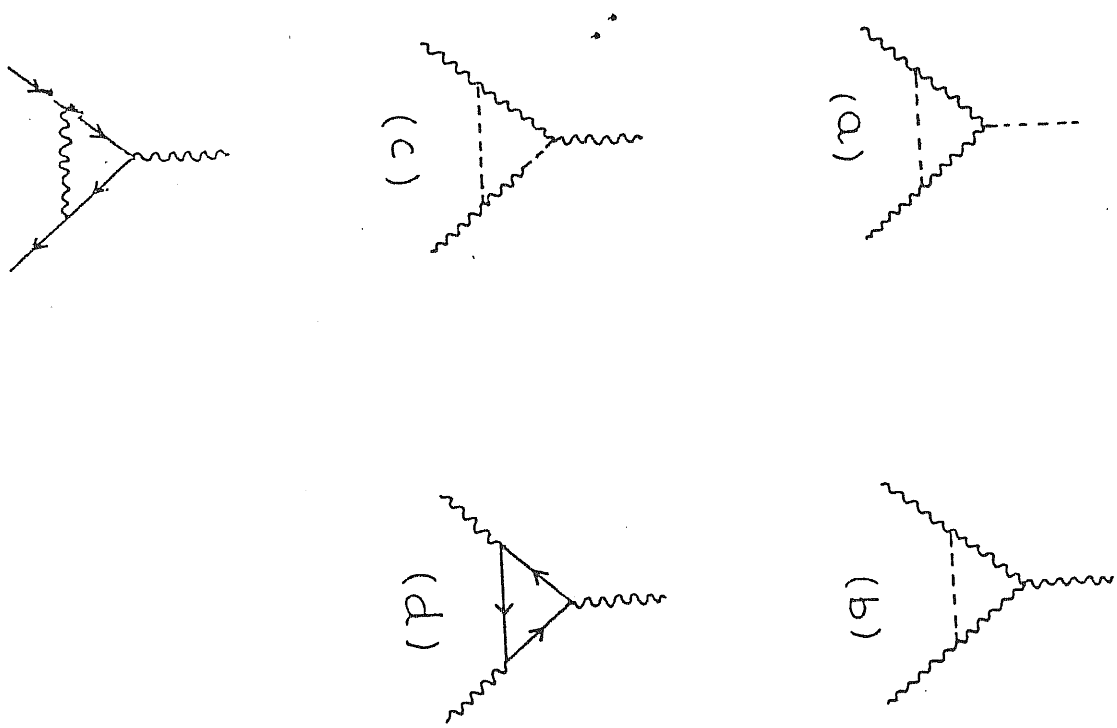


FIG. 1



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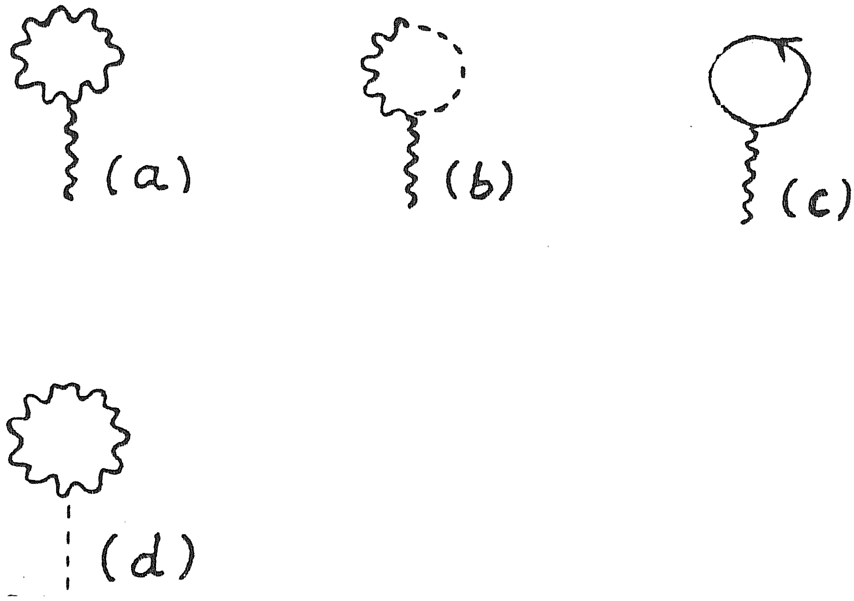

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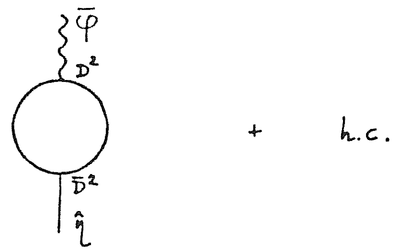

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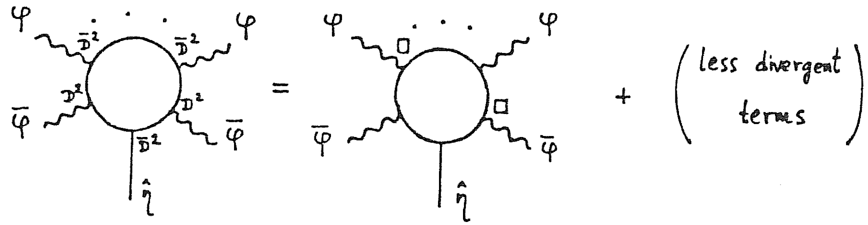

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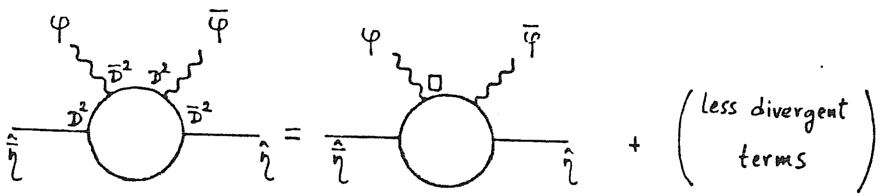

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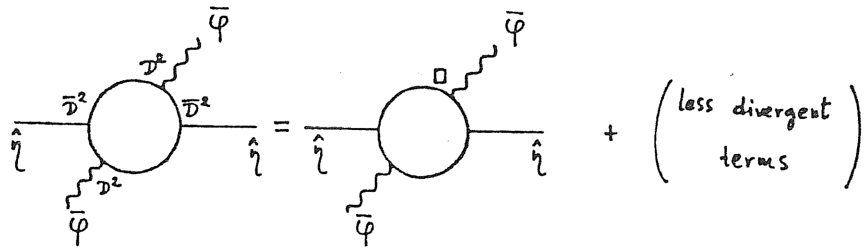

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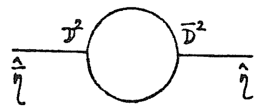

Fig. 8

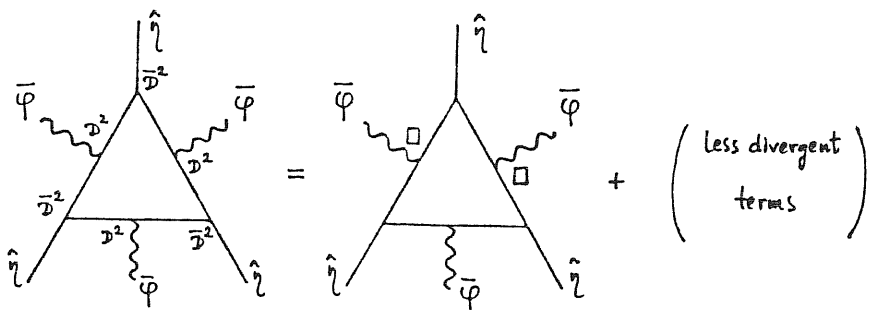

Fig. 9

FIGURE CAPTIONS

Fig. 1.-Feynman diagrams contributing to the 1PI two-point functions. Each diagram represents the sum over all the allowed combinations of fields going around the loop. A wavy line represents a scalar field (either A or B); a dashed line represents an auxiliary field (either F or G); a mixed line represents a transition (either $\langle AF \rangle$ or $\langle BG \rangle$). The solid lines represent spinor fields ψ.

Fig. 2.-Feynman diagrams contributing to the 1PI three-point functions. As in figure 1, each loop represents the sum of all the combinations of fields allowed by the Feynman rules. The meaning of the different types of lines is the same as in figure 1.

Fig. 3.-Feynman diagrams contributing to the 1PI one-point functions. The external wavy lines represent the scalar fields (A,B), while the external dashed line stands for the auxiliary fields (F, G). The internal wavy lines represent the sum over the scalar fields (either A or B) going around the loop; the mixed dashed-wavy line is intended for the sum over internal transitions (either $\langle AF \rangle$ or $\langle BG \rangle$) in the loop; the spinor field ψ is represented by a solid line.

Fig. 4.-The only divergent contribution of the 1PI one-point function to the effective action, in the superfield formalism.

Fig. 5.-Graphical equation for graphs with n φ-field and n+1 $\bar{\varphi}$-field insertions. The D-algebra has been carried in the right-hand side of the equ=

ation.

Fig. 6.-The result of the insertion of one φ and one $\bar{\varphi}$ into the two-point function is shown.

Fig. 7.-The insertion of two $\bar{\varphi}$'s into the two-point function gives the result depicted.

Fig. 8.-Divergent graph providing the wave-function renormalization of the superfield $\hat{\eta}$.

Fig. 9.-Contribution to the three-point functions obtained from the insertion of three $\bar{\varphi}$'s.