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"SOME ASPECTS OF SPONTANEOUS AND SOFT BREAKING OF GLOBAL SUPERSYMMETRY"

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§ 1. INTRODUCTION

Supersymmetry [1] is the only non trivial extension of the Poincaré invariance consistent with general requirements of relativistic quantum field theory [2]. It connects states with different statistics or half-integral spin particles with integral spin particles.

Supersymmetry generators are spinorial charges Q_α, $\bar{Q}_{\dot\alpha}$ $(\alpha = 1,2)$, whose algebra is characterized by the following anticommutation relation:

$$\{Q_\alpha , \bar{Q}_{\dot\alpha}\} = 2\, \sigma^{\mu}_{\alpha\dot\alpha} P_\mu$$

P_μ being the generator of space-time translation, the energy-momentum four-vector. In a supersymmetric theory there can be one or more supercharges $Q^{(i)}_\alpha$ i=1,...N (generally $N \leq 8$). If $N > 1$ one speaks of N-extended supersymmetry.

Moreover the parameters of the transformations implemented by the supersymmetry generators may be or not functions of the space-time; in the former case we have to do with local supersymmetry, in the latter one we have to do with global supersymmetry.

There has been recently a growing interest in supersymmetry as a framework where some long standing problems of elementary particle physics might possibly find a solution.

These problems do not consist in conflicts between the standard (gauge) theoretical viewpoint and observation, but rather in difficulties concerning the theoretical picture itself.

Standard $SU(3) \times SU(2) \times U(1)$ model and grand unified theories [3] contain a high number (~ 20) of undetermined parameters. They do not provide an understanding of the spectrum of elementary particles and in particular of the existence of three (or more) families of quarks and leptons. They ignore gravity. Grand unified theories are affected by an additional problem, called the hierarchy problem, related to the existence of two widely separated mass scales in the theory: the grand unification mass $M_X \sim 10^{15}$ GeV and the energy scale at which $SU(3) \times SU(2) \times U(1)$ breaks into $SU(3) \times U(1)_{em}$, $M_W \sim 100$ GeV. Grand Unification do not provide an explanation of the origin of these two mass scales. Even worse, even if the scales are put by hand into the theory, this requires at any order of perturbation theory an incredible and innatural adjustment of the relevant parameters, which is technically unacceptable.

Can supersymmetry cure some of these troubles?

Since supersymmetry connects bosons with fermions and hence the corresponding sectors in a supersymmetric gauge theory, one may expect that this fact reduces the number of arbitrary parameters in the theory and puts some order in the particle spectrum. Actually this hope has not been realized (at least with N=1) because for each known particle, the supersymmetric theory requires a partner of opposite statistics with the same quantum numbers which unfortunately, cannot be identified among the particles at disposal: so, generally new degrees of freedom come into a supersymmetric theory.

Now come to the good points. One of the most appealing feature of globally supersymmetric theories is their remarkable ultraviolet behaviour [4]. In N=1 supersymmetric theories, for instance, a softening of the divergences originated by perturbation theory occurs due to cancellation between fermion and boson loops; actually no quadratic divergences arise in perturbation theory (with few expections). Less renormalization constants are required with respect to the generic quantum field theory and some parameters (tipically masses, scalar and Yukawa couplings) receive no renormalization (independent of a common wave function renormalization).

Ultraviolet properties of the extended supersymmetries are even more remarkable.

The technical aspects of the hierarchy problem finds a solution. Just because of the good renormalization properties of supersymmetric theories, relations between masses such as $\frac{M_W}{M_X} \sim 10^{-13}$ are stable against radiative corrections and, once they are set at tree level (why this has to happen is still an open problem) they hold true at any order of perturbation theory.

Local version of supersymmetry (or supergravity) [5] provides an appealing description of the gravitational interaction and with an even more ambitious program $N=8$ supergravity makes an attempt towards the unification of gravity with the other fundamental interactions.

In spite of these promising features, no direct experimental evidence exists that supersymmetry has something to do with particle physics. Scalars with the same quantum numbers of the known fermions have not been discovered at present energies and this indicates that if nature is described by a supersymmetric theory, supersymmetry has to be broken. According to the current attitude this has to happen spontaneously.

At this point one may think that, at energies much smaller than the Planck mass scale $(M_{Pl} \sim 10^{19}\ GeV)$, gravity could be neglected, al least in a first approximation.

In this perspective one can try to describe the other fundamental interactions with a spontaneously broken global supersymmetry.

Then the possible models must have $N=1$, that is only one supercharge. This follows from the fact that for $N>1$ only real representations are available for fermions, which is in contraddiction with the experience. Moreover a hierarchycal global supersymmetry breaking (from N to $N'<N$) is forbidden by the supersymmetry algebra. Extended supersymmetry may only be relevant at energies comparable with M_{Pl} where local supersymmetry is preferable. This is the origin of the interest in the spontaneous breaking of $N=1$ global supersymmetry, which is the main subject of this thesis.

It goes beyond the aim of this work ro review the status of the various unified models based on $N=1$ spontaneously broken global supersymmetry [6]. Anyway one must say that most of them run into non minor difficulties. A mass formula [7], which holds at tree level, creates some problems in giving the scalar supersymmetric partners of quarks and leptons an adequately high mass. In these models the breaking scale M_S of supersymmetry ranges from $M_S \sim M_W \sim 100\ GeV$ to $M_S^2 \sim M_W M_{Pl} \sim (10^{10}\ GeV)^2$ (This is possible because the phenomenological relevant quantity, namely the mass splitting between a boson and a fermion related by supersymmetry, does not depend only on M_S). Models with $M_S \sim M_W$ [8] suffer from serious deseases such as anomalies, unwanted flavour-changing neutral currents, ecc... The models which work better seem to require the presence of a mass scale near to the Planck mass M_{Pl}.

This fact may induce to think that gravity can be relevant to physics even at energy scales much below M_{Pl}.

Actually models which couple supergravity to a gauge theory has been produced [9] and it seems that they can overcome some of the difficulties which affect globally supersymmetric models of particle interactions. Suitable limits of these models for

M_{Pl} which tends to infinity have been taken [10]. An effective low energy theory arises from this procedure. It has the structure of a globally supersymmetric theory plus explicit but soft supersymmetry breaking terms as last surviving effects of the gravitational interaction. This motivates the study of such type of breaking [11] which is also discussed in the thesis.

The plan of the thesis is the following: the analysis restricts to $N=1$ globally supersymmetric theories.

In § 2 some general properties (definitions and criteria) of spontaneous supersymmetry breaking are reviewed.

In § 3 the spontaneous breaking of supersymmetry is studied in models describing only particles of spin 0 and spin $\frac{1}{2}$.

In § 4 spontaneous supersymmetry breaking in gauge models is investigated.

In § 5 some aspects of the ultraviolet behaviour of supersymmetric theories both with and without supersymmetry breaking are illustrated. Soft breaking of supersymmetry is also discussed.

Emphasis is given to those properties of supersymmetric theories which do not depend on the choice of a particular model. For instance: renormalization properties of supersymmetric theories both in discussing the impossibility of breaking supersymmetry through higher order corrections and in analyzing soft supersymmetry breaking; invariance of the superpotential under transformation of the complexification G_C of the gauge group G, in deriving a criterium for supersymmetry breaking in supersymmetric gauge theories.

All these features are then exemplified in two models now become "classical" O'Raifeartaigh model [12] for chiral theories and Fayet-Iliopoulos model [13] for gauge theories.

The notation used throughout the text is that of J. Wess and J. Bagger, "Supersymmetry and Supergravity" Princeton, Princeton

University Press, 1983. The formulae used most frequently are collected in a final appendix.

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* * *

§ 2 GENERAL FEATURES

In this paragraph we review some general results about spontaneous supersymmetry breaking. These results are independent, to a certain extent, of the particular model one can consider.

2.1 DEFINITION OF SPONTANEOUS SUPERSYMMETRY BREAKING.

Supersymmetry is spontaneously broken if there exists a field of theory $\varphi(x)$ with the property:

$$\int d^3x \left\langle \left[j_{0\alpha}(x), \varphi(0) \right]_{\pm} \right\rangle \neq 0 \tag{1}$$

where $j_{\mu\alpha}(x)$ is the supersymmetric current of the theory.

Comments:

(i) this definition excludes the possibility that supersymmetry is spontaneously broken through a non vanishing expectation value of the variation of a composite field, which is certainly interesting to investigate. However in the following we shall not commit ourselves with this possibility and so we retain the above definition.

(ii) Due to the spinorial character of the current $j_{\mu\alpha}(x)$ and the invariance of the vacuum under proper Lorentz transformation, the field φ must have spin one half and so in the above formula we can forget the commutator.

(iii) The supersymmetry charge $Q_\alpha = \int d^3x\, j_{0\alpha}(x)$ does not need in general to exist and make sense as an operator.

However in the following we shall make repeatedly use of expressions like:

$$\langle \{ Q_\alpha, \varphi(y) \} \rangle \quad , \qquad \text{ecc.}$$

which are to be understood as referring to corresponding meaningfull expressions like :

$$\int d^3x \langle \{ j_{0\alpha}(x), \varphi(y) \} \rangle$$

or some other appropriate limit.

2.2 SPONTANEOUS SUPERSYMMETRY BREAKING AND VACUUM EXPECTATION VALUES OF AUXILIARY FIELDS.

Let us consider a theory involving chiral (scalar) superfields $\Phi^{(i)}$ and vector superfields $V^{(k)}$.

A useful criterium for the spontaneous supersymmetry breaking of the theory can be derived looking at the susy transformation law of spinor fields:

$$i\,[\xi Q + \bar\xi \bar Q,\ \psi_\alpha^{(i)}]_- = \sqrt{2}\,\xi_\alpha F^{(i)} + \sqrt{2}\,i\,(\sigma^\mu \bar\xi)_\alpha \partial_\mu A^{(i)} \qquad (2)$$

$$i\,[\xi Q + \bar\xi \bar Q,\ \lambda_\alpha^{(k)}]_- = i\,\xi_\alpha D^{(k)} + (\sigma^{\mu\nu}\xi)_\alpha v_{\mu\nu}^{(k)}$$

Taking vacuum expectation values we find:

$$i\langle\{Q_\beta,\psi_\alpha^{(i)}\}\rangle = \sqrt{2}\,\epsilon_{\alpha\beta}\,\langle F^{(i)}\rangle$$
$$i\langle\{Q_\beta,\lambda_\alpha^{(k)}\}\rangle = i\,\epsilon_{\alpha\beta}\,\langle D^{(k)}\rangle \qquad (3)$$

From the above expressions we conclude that:

Supersymmetry is spontaneously broken if and only if some auxiliary field of the theory acquires a non vanishing vacuum expectation value.

We recall that spontaneous breaking of other symmetries (internal or gauge,...) is characterized by a non vanishing vacuum expectation value of physical scalar fields.

So we expect that all the possibilities (breaking of supersymmetry without breaking of internal symmetries, vice-versa and breaking of both supersymmetry and internal symmetries) can take place.

2.3 SUPERSYMMETRIC EXTENSION OF THE GOLDSTONE THEOREM [14].

Spontaneous supersymmetry breaking is accompanied with the presence of a massless state of spin ½ called Goldstone fermion or Goldstino.

This statement is the supersymmetric counterpart of the Goldstone theorem in the case of continuous (ordinary) symmetries. To the massless Goldstone boson of spin 0 it corresponds the massless Goldstone fermion of spin ½.

Let us briefly sketch the proof of the theorem. Suppose supersymmetry is spontaneously broken. Then an auxiliary field, say an F field for definiteness, must have a non vanishing vacuum expectation value. From (3) we can write:

$$i \left\langle \left\{ \int d^3x \; j_{o\beta}(x) \, , \, \psi_\alpha(0) \right\} \right\rangle = \sqrt{2}\, \epsilon_{\alpha\beta} \left\langle F(0) \right\rangle \neq 0$$

On the other hand making use of the local conservation of the supersymmetric current ($\partial^\mu j_{\mu\beta}(x) = 0$) we have :

$$i \left\langle \left\{ \int d^3x \; j_{o\beta}(x) \, , \, \psi_\alpha(0) \right\} \right\rangle =$$

$$= i \int d^4x \left\langle T \, \partial^\mu j_{\mu\beta}(x) \, \psi_\alpha(0) \right\rangle + i \int d^4x \left\langle j_{\mu\beta}(x) \, \psi_\alpha(0) \, \delta_0{}^\mu \, \delta(x_0) + \right.$$

$$\left. + \, \psi_\alpha(0) \, j_{\mu\beta}(x) \, \delta_0{}^\mu \, \delta(-x_0) \right\rangle =$$

$$= i \int d^4x \; \partial^\mu \left\langle T \, j_{\mu\beta}(x) \, \psi_\alpha(0) \right\rangle$$

From this we obtain:

$$i \int d^4x \; \partial^\mu \left\langle T \, j_{\mu\beta}(x) \, \psi_\alpha(0) \right\rangle = \sqrt{2}\, \epsilon_{\alpha\beta} \left\langle F(0) \right\rangle$$

or in another form:

$$\lim_{q_\mu \to 0} \; q^\mu \int d^4x \; e^{iqx} \left\langle T \, j_{\mu\beta}(x) \; \psi_\alpha(0) \right\rangle = \sqrt{2}\, \epsilon_{\alpha\beta} \left\langle F(0) \right\rangle \neq 0 \qquad (4)$$

The Fourier transform of the two point function $\langle T \, j_{\mu\beta}(x) \, \psi_\alpha(y) \rangle$ develops a <u>simple</u> pole at $q_\mu = 0$ with residue $\sqrt{2}\, \epsilon_{\alpha\beta} \; \langle F(0) \rangle$. This indicates that among the intermediate states contributing to the two point function $\langle T \, j_{\mu\beta}(x) \, \psi_\alpha(y) \rangle$ there must be a massless state of spin one half which is referred to as the Goldstone fermion or Goldstino.

2.4 SPONTANEOUS SUPERSYMMETRY BREAKING AND VACUUM ENERGY DENSITY.

In a N=1 globally supersymmetric theory the hamiltonian H and the supercharges Q_α (*) are related by the following relation:

$$H = \frac{1}{4} \sum_\alpha Q_\alpha^2 \tag{5}$$

which can be easily derived from supersymmetry algebra.

Since its appearance in the literature this relation gave rise to a popular criterium for the spontaneous breaking of supersymmetry.

The formal argument goes as follows:
relation (5) tell us that the hamiltonian is a non-negative definite operator. Now suppose that a supersymmetric state exists, namely a state Ω with the property: $Q_\alpha \Omega = 0$. Then from (5) it immediately follows that $H \Omega = 0$ and, since H is non-negative definite, Ω is the ground state of the theory. Clearly in this case supersymmetry is not spontaneously broken.

Vice versa suppose a supersymmetric state does not exist. Then supersymmetry is spontaneously broken (simply because $Q_\alpha |0\rangle \neq 0$) and the ground state $|0\rangle$ has an energy different from zero, since $H|0\rangle = 0$ would imply $\langle 0| \sum_\alpha Q_\alpha^2 |0\rangle = \sum_\alpha \| Q_\alpha |0\rangle \|^2 = 0$ that is $Q_\alpha |0\rangle = 0$.

So we can state that:

(i) Supersymmetry is spontaneously broken if and only if a supersymmetric state does not exist;

and

(*) Through this section the Majorana version of supercharges is used.

(ii) supersymmetry is spontaneously broken if and only if the energy of the vacuum is positive.

In this respect supersymmetry is completely different from ordinary symmetries. Spontaneous breakdown of the latters implies the existence of a symmetric state which however has a higher energy than other, non symmetric, states. In supersymmetric theories a supersymmetric state is always stable and forces supersymmetry to be realized algebrically.

One must say that the connection between the energy and the supersymmetric charges is still an open problem. Even if conclusion (i) and (ii) above seem to be confirmed in most of the perturbatively defined models, they encounter some non minor difficulties to be set in a more rigorous fashion. Let us briefly review some criticism.

Needless to say, if supersymmetry is good, relation (5) holds true and the supersymmetry invariant vacuum has zero energy.

But suppose supersymmetry is spontaneously broken. Then, according to the authors of reference [15] the vacuum expectation value of the operator $\sum_\alpha Q_\alpha^2$ (defined with a suitable limiting procedure since the global charges Q_α may not exist) diverges as a volume. Since translation invariance requires finite vacuum energy, the operator $\sum_\alpha Q_\alpha^2$ cannot converge to the generator of the time translation H. Therefore, in the case of spontaneous supersymmetry breaking, relation (5) breaks down. So one cannot use the spectrum of the hamiltonian to decide if supersymmetry is spontaneously broken or not. Indeed the Lorentz invariance of the vacuum always requires a vanishing vacuum expectation value of the hamiltonian.

Also Lopuszanski [16] with general arguments of quantum field theory reaches the conclusion that if supersymmetry is spontaneously broken the anticommutation relations of the broken supercharges are no more related to the energy-momentum vector.

On the other hand in most of N=1 global supersymmetric models criteria (i) and (ii) works very well. As we shall see the tree level scalar potential of a gauge invariant model with chiral (scalar) superfields Φ^i and vector superfields V^k reads:

$$V(\varphi) = F^i(\varphi)^* F^i(\varphi) + \frac{1}{2} D^k(\varphi) D^k(\varphi) \qquad (6)$$

where $F^i = F^i(\varphi)$ and $D^k = D^k(\varphi)$ are the equations of motion of the auxiliary fields; having denoted by φ, collectively, the physical scalar fields.

If susy is not spontaneously broken, then none of the auxilia ry fields F^i and D^k acquire a non vanishing vacuum expectation value and so, at least at tree level, the minimum of V is zero.

Conversely if supersymmetry is spontaneously broken, then at least one among the auxiliary fields F^i and D^k develops a non vanishing vacuum expectation value and relation (6) shows that in this case the minimum of V is greater than zero.

One possibility of explaining this remarkable property of perturbatively defined models is the following.

As suggested in reference [15], most likely in the models defined by perturbation theory (like gauge invariant N=1 globally supersymmetric ones) what one is really working out is not the hamiltonian H, meant as the generator of the time translation, but a functional which differs from H by some (divergent) c-number. So, even if, strictly speaking, the hamiltonian has always zero vacuum expectation value, nevertheless in the above mentioned models we can use the particular form of the potential to decide at least at classical level if supersymmetry is spontaneously broken or not.

§ 3. CHIRAL SCALAR SUPERFIELDS.

In this section we shall discuss the occurrence of spontaneous supersymmetry breaking in supersymmetric models describing only particles of spin 0 and spin ½.

3.1 GENERALITIES.

The supersymmetric action for n interacting chiral (scalar) superfields $\Phi_i \;\; (i = 1 \dots n)$ can be written in the following way:

$$A = \int d^4x\, d^4\theta\, \Phi_i^+ \Phi_i + \int d^4x\, d^2\theta\, f(\Phi) + \int d^4x\, d^2\bar\theta\, \bar f(\Phi) \qquad (7)$$

where $f(\Phi)$ is a function of the Φ_i called superpotential. Renormalizability requires $f(\Phi)$ to be at most a cubyc polynomial:

$$f(\Phi) = a + \lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \qquad (8)$$

(m_{ij} and g_{ijk} can always be taken completely symmetric).

In terms of component fields the action (7) can be thought as referring to the following lagrangian:

$$\mathcal{L} = i\partial_\mu \bar\psi_i \bar\sigma^\mu \psi_i + A_i^* \Box A_i + F_i^* F_i +$$
$$+ \left(\frac{\partial f(A)}{\partial A_i} F_i - \frac{1}{2} \frac{\partial^2 f(A)}{\partial A_i \partial A_j} \psi_i \psi_j + h.c. \right) \qquad (9)$$

+ a four divergence.

where the first line correspond to the first term on the right hand side in (7).

Since $\mathcal{L}$ does not contain derivatives of the (auxiliary) fields F_i, these can be eliminated from (9) through their equations of motion:

$$F_i = -\left(\frac{\partial f(A)}{\partial A_i}\right)^* \qquad F_i^* = -\frac{\partial f(A)}{\partial A_i} \tag{10}$$

The tree level scalar potential then reads:

$$V(A,A^*) = \frac{\partial f}{\partial A_i}\cdot\left(\frac{\partial f}{\partial A_i}\right)^* \tag{11}$$

This scalar potential is non-negative. Some of the auxiliary fields F_i acquire a non vanishing vacuum expactation value (that is supersymmetry is spontaneously broken) if and only if V at the minimum is positive (see (10) and (11)), or equivalently if and only if the system of equations:

$$\frac{\partial f}{\partial A_i} = 0 \qquad (i = 1, \dots n) \tag{12}$$

does not admit solutions.

3.2 SPONTANEOUS SYMMETRY BREAKING THROUGH RADIATIVE CORRECTIONS?.

Suppose supersymmetry is exact at tree level or in other words system (12) has solutions (it can have a discrete number of solutions or a continuous infinity of solutions in which case there are massless scalars in the theory).

The following questions rise:

(i) can high order corrections remove the degeracy of minima ?

(ii) can high order corrections lift the absolute minimum of the potential above zero so to induce spontaneous supersymmetry breaking ?

The answer to these questions is <u>no</u> as can be seen in the following way.

To any order of perturbation theory, the effective action for a supersymmetric theory involving only chiral (scalar) superfields, denoted collectively by ϕ , has the form:

$$\Gamma(\phi,\phi^{+}) = \sum_{n}\int d^4x_1 \ldots d^4x_n \int d^4\theta \; G(x_1 \ldots x_n) \cdot \\ \cdot \left[\text{Polinomial in} : \phi(x_i,\theta), D_\alpha \phi(x_j,\theta) \ldots \right] \tag{13}$$

This result can be proved very easily in the context of a manifestly supersymmetric formulation of the Feynman rules for supergraphs [17].

We see that the integrand in (13) is local in θ (a function of a single θ) and is integrated with $d^4\theta$, not $d^2\theta$.

We recover the effective potential from the effective action by setting spinor fields and spacetime derivatives to zero in (13). At this point the only θ's surviving are those of the F (F^*) components of chiral (antichiral) superfields ϕ (ϕ^+) : $\theta\theta F$ and $\bar\theta\bar\theta F^*$. Now imagine to performe the $d^4\theta$ integration. The only terms in the integrand giving a non zero contribution are those bilinear in F^*, F since they only can carry the correct $\theta^2\bar\theta^2$ factor.

Since we are interested in the classical x-independent minima of the effective potential, let us consider the restriction of the functional obtained with the above procedure to the class of x-independent fields A,F [18]. It will be a bilinear in F, F^*, namely:

$$F_i^* F_i \; h_{ij}(F, F^*, A, A^*) \tag{14}$$

where h_{ij} is a hermitean matrix.

So, for x-independent fields, to any order of perturbation theory, the effective potential has the form:

$$V_{eff} = - F_i^* F_i - F_i f_i(A) - F_i^* f_i(A)^* + \\ + F_i^* F_i \; h_{ij}(F, F^*, A, A^*) \tag{15}$$

where:

$$f_i(A) \stackrel{d}{=} \frac{\partial f(A)}{\partial A_i}$$

If we extremize the effective potential V_{eff} in (15) with respect to F_k, F_k^*, A_k and A_k^*, we obtain:

$$0 = \frac{\partial V_{eff}}{\partial F_k^*} = - F_k - f_k(A)^* + F_j h_{kj} + F_i^* F_j \frac{\partial h_{ij}}{\partial F_k^*} = 0 \tag{16'}$$

and:

$$0 = \frac{\partial V_{eff}}{\partial A_k} = - F_i f_{ik}(A) + F_i^* F_j \frac{\partial h_{ij}}{\partial A_k} = 0 \tag{16''}$$

+ their conjugates.

Now return to our original questions. Suppose supersymmetry is not spontaneously broken at the tree level, and let $F_k = 0$ $A_k = 0$ be a solution of the tree level counterpart of (16) ($h_{ij} = 0$):

$$\begin{cases} -F_k - f_k(A)^* = 0 \\ -F_i \, f_{ik}(A) = 0 \end{cases} \tag{17}$$

It is clear that $F_k = 0$, $A_k = 0$ is also a solution of equations (16).

Therefore:

(i) A possible vacuum at the tree approximation is a possible vacuum to all order of perturbation theory.

Moreover observe that the consider solution still leads to $V_{eff} = 0$. So, if we believe that vacuum energy is not negative, our solution is an absolute minimum of the effective potential and we conclude that:

(ii) Supersymmetry remains unbroken to all orders of perturbation theory.

Few comments are in order:

What about the positivity of the effective potential (which is indeed crucial for the statement (ii)) ? As we have seen in a previous section it seems difficult to prove this property on a general ground. Nevertheless there are strong indications in its favour for models defined in perturbation theory. Early works [19,20,21] find for a class of supersymmetric models an effective potential which, at the 1-loop approximation, vanishes at the symmetric minima of the classical potential and is complex elsewhere, complexity being interpreted as instability of the corrisponding field configurations. A later work [22] attributes the complexity of the potential to some oversemplification of the procedure followed for deriving it, and, redifining more carefully this procedure, obtains a 1 loop positive contribution to the effective potential in a simple supersymmetric model.

We have seen that higher order corrections do not remove the degeneracy existing between tree level vacua. This vacua may have very different physical properties, giving rise, for instance, to spontaneous breakdown of internal symmetries in different, inequivalent directions.

Moreover this vacua can be connected by continuous transformations in which case massless particles arise at tree level and remain massless to any order of perturbation theory.

In all these respects the situation for supersymmetric models is very different from the ordinary (non supersymmetric) ones.

3.3 SPONTANEOUS SUPERSYMMETRY BREAKING AT TREE LEVEL.

We have seen that for supersymmetric chiral models defined perturbatively, the only possibility for supersymmetry to be spontaneously broken is that this happens at tree level, since higher order corrections cannot perturb a supersymmetric situation.

Thus let us suppose that supersymmetry is spontaneously broken at tree level which means that the system (12):

$$\frac{\partial f}{\partial A_i} = 0 \qquad (18)$$

has no solutions.

A number of interesting properties of the tree level particle spectrum of such a model can be derived on general ground.

(i) Spontaneous supersymmetry breaking is associated with the presence of a massless fermion (goldstino).

We have already seen that this is a general feature of all theories with spontaneously broken supersymmetry. In the particular class of models we are dealing with we can reason as follows.

Let $\bar{A}_k$ be a minimum of the scalar potential V(A,A*). Then, from (11) the extremum condition gives:

$$\left(\frac{\partial f}{\partial A_i}\right)^* \frac{\partial^2 f}{\partial A_i \partial A_j}\bigg|_{A=\bar{A}} = 0 \qquad (19)$$

But, in two component language $\dfrac{\partial^2 f}{\partial A_i \partial A_j}\bigg|_{A=\bar{A}}$ is just the fermion mass matrix which, according to (19), must have a vanishing eigenvalue.

(ii) Spontaneous supersymmetry breaking is associated with the presence of a massless complex scalar in the tree level spectrum.

This is an interesting direct consequence of O'Raifeartaigh 1^{st} lemma [12] as observed in reference [23]. Let us sketch the proof.

Suppose only for simplicity $\bar{A}_k = 0$ and recall the explicit expression (8) of the superpotential.

Then condition (19) takes the form:

$$\lambda_i^* m_{ij} = 0 \qquad (20)$$

with $\lambda_i^* \neq 0$ (!) otherwise supersymmetry is not spontaneously broken.

O'Raifeartaigh 1st lemma says that if Ω_0 is the subspace on which the matrix m_{ij} is zero, a necessary condition for $\bar{A}_k = 0$ to be a local minimum of the scalar potential V is:

$$g_{ijk} \lambda_k^* \equiv 0 \qquad \text{on } \Omega_0$$

Since λ_i^* belongs to Ω_0, if 0 is a local minimum we can write:

$$g_{ijk} \lambda_i^* \lambda_k^* = 0 \tag{21}$$

From (20) and (21) we obtain:

$$V(A_i = c\lambda_i^*) = \sum_i \left| \lambda_i + c\, m_{ij} \lambda_j^* + c^2 g_{ijk} \lambda_j^* \lambda_k^* \right|^2 =$$

$$= \sum_i |\lambda_i|^2 = V(A_i = 0)$$

Thus along the line $A_i = c\lambda_i^*$ the potential is constant and equal to its minimum value. This implies the presence of a massless complex scalar at tree level.

(iii) A general mass formula holds at tree level [7]:

$$\sum_J (-1)^{2J} (2J+1)\, m_J^2 = 0 \tag{22}$$

where m_J is the mass of the (real) physical particle of spin J and the sum is understood over all particles.

Relation (22) follows from computing the traces of the squares of the boson (M_B) and fermion (M_F) mass matrices.

Now the square of the boson mass matrix for complex fields is:

$$\frac{1}{2}\begin{pmatrix} f_{ij}(\bar{A}^*)f_{ik}(\bar{A}) & f_{ijk}(\bar{A}^*)f_i(\bar{A}) \\ f_i(\bar{A}^*)f_{ijk}(\bar{A}) & f_{ik}(\bar{A}^*)f_{ij}(\bar{A}) \end{pmatrix} \qquad (23)$$

where $f_{ij}(A) = \dfrac{\partial^2 f}{\partial A_i \partial A_j}$, $f_{ij}(A^*) = \left(\dfrac{\partial^2 f}{\partial A_i \partial A_j}\right)^*$ ecc, ...

and the derivatives are taken at the minimum $\bar{A}$ of the scalar potential.

Therefore the trace of the square of the mass matrix M_B^2 for real scalar fields is:

$$\mathrm{tr}\, M_B^2 = 2 f_{ij}^*(A)\, f_{ij}(A) \qquad (24)$$

From the fermion mass terms:

$$-\frac{1}{2} f_{ij}(\bar{A})\,\psi_i\psi_j - \frac{1}{2} f_{ij}(\bar{A}^*)\,\bar{\psi}_i\bar{\psi}_j \qquad (25)$$

converting Weyl spinors ψ_α into Majorana ones $\psi_M \equiv \begin{pmatrix}\psi_\alpha \\ \bar{\psi}^{\dot{\alpha}}\end{pmatrix}$; choosing a basis for the γ's such that:

$$\gamma_0 = \begin{pmatrix} 1\!\!1 & 0 \\ 0 & -1\!\!1 \end{pmatrix} \qquad i\gamma_0\gamma_5 = \begin{pmatrix} 0 & 1\!\!1 \\ 1\!\!1 & 0 \end{pmatrix}$$

and defining :

$$M_{ij} \stackrel{d.}{=} \mathrm{Re}\, f_{ij}(\bar{A})$$

$$N_{ij} \stackrel{d.}{=} \mathrm{Im}\, f_{ij}(\bar{A})$$

(25) reads:

$$-\frac{1}{2}\,\Psi^{+}_{iM}\begin{pmatrix} M_{ij}\otimes 1\!\!1 & N_{ij}\otimes 1\!\!1 \\ N_{ij}\otimes 1\!\!1 & -M_{ij}\otimes 1\!\!1 \end{pmatrix}\Psi_{jM}$$

where $1\!\!1$ is the unity 2x2 matrix.

Therefore:

$$M_F^2 = \frac{1}{4}\begin{pmatrix} (M^2+N^2)\otimes 1\!\!1 & [M,N]\otimes 1\!\!1 \\ -[M,N]\otimes 1\!\!1 & (M^2+N^2)\otimes 1\!\!1 \end{pmatrix} \tag{26}$$

The trace of the square of the fermion mass matrix is:

$$\mathrm{tr}\, M_F^2 = f_{ij}(\bar{A}^*)\, f_{ij}(\bar{A}) \tag{27}$$

comparing (24) and (27), formula (22) immediately follows.

This derivation does not exploit the spontaneous supersymmetry breaking and in fact, in theory with unbroken supersymmetry, relation (22) is trivially satisfied for each supermultiplet.

Here the main point is that <u>the splitting among masses of particles</u> of a theory with spontaneously broken supersymmetry <u>cannot be arbitrary</u> but has to respect a pattern dictated by (22).

3.4 O'RAIFEARTAIGH MODEL.

As an interesting exemplification of the properties stated above, let us consider the O'Raifeartaigh model [12]. It consists of three chiral scalar superfields Φ_0, Φ_1, Φ_2, with r-characters(*):

$$n_0 = 1 \qquad n_1 = 0 \qquad n_2 = 1 \tag{28}$$

and the following discrete transformation law:

$$\begin{aligned} \Phi_0 &\longrightarrow \Phi_0 \\ \Phi_1 &\longrightarrow -\Phi_1 \\ \Phi_2 &\longrightarrow -\Phi_2 \end{aligned} \tag{29}$$

The most general superpotential giving rise to a renormalizable model and invariant under R and (29) transformations is:

$$f(\Phi) = c + \lambda \Phi_0 + m \Phi_1 \Phi_2 + g \Phi_0 \Phi_1^2 \tag{30}$$

For simplicity we shall take real parameters c, λ, m, g.

(*) R-transformation on a scalar superfield $\Phi(x,\theta,\bar\theta)$ is defined as follows:

$$\Phi(x,\theta,\bar\theta) \xrightarrow{R} e^{2ni\alpha}\,\Phi(x, e^{-i\alpha}\theta, e^{i\alpha}\bar\theta)$$

The F-term of the product of several chiral superfields is R-invariant if and only if the sum of the R-characters is 1.

In the model defined by (30), $\lambda \neq 0$, $m \neq 0$ imply spontaneous breaking of supersymmetry at tree level.

In fact the system (12) $\frac{\partial f}{\partial A_i} = 0$ reads:

$$\begin{cases} f_0(A) = \lambda + g A_1^2 & = 0 \\ f_1(A) = m A_2 + 2g A_0 A_1 & = 0 \\ f_2(A) = m A_1 & = 0 \end{cases} \tag{31}$$

which, for $\lambda \neq 0$ and $m \neq 0$ is clearly incompatible.
The minima of the scalar potential :

$$V = |\lambda + g A_1^2|^2 + |m A_2 + 2g A_0 A_1|^2 + m^2 |A_1|^2 \tag{32}$$

occur according to the following scheme:

A. In the region $|2g\lambda| < m^2$, the minima are:

$$\begin{cases} A_0 = y & y \text{ any value} \\ A_1 = 0 \\ A_2 = 0 \end{cases} \tag{33}$$

B. In the region $2g\lambda < -m^2$, the minima are:

$$\begin{cases} A_0 = y & y \text{ any} \\ A_1 = \pm \sqrt{\dfrac{-2g\lambda - m^2}{2g^2}} \\ A_2 = -\dfrac{2g}{m} A_0 A_1 \end{cases} \tag{34}$$

C. In the region $2g\lambda > m^2$ the minima are:

$$
\begin{aligned}
A_0 &= y \qquad\qquad \text{y any} \\
A_1 &= \pm i \sqrt{\frac{2g\lambda - m^2}{2g^2}} \qquad\qquad (35) \\
A_2 &= -2\frac{g}{m} A_0 A_1
\end{aligned}
$$

Let us concentrate for instance on the case A. We immediately observe the continuous degeneracy of the minima of the tree level potential. In this case:

$$
\begin{aligned}
\langle F_0 \rangle &= -\lambda \\
\langle F_1 \rangle &= 0 \\
\langle F_2 \rangle &= 0
\end{aligned}
$$

and the potential is constant on the plane:

$$A_i = t \langle F_i \rangle \qquad\qquad t \in \mathbb{C} \qquad\qquad (36)$$

as indeed we expect. (Degeneracy along (36) can be easily cheched in cases B and C, too). Thus there must be a complex scalar, massless at tree level.

Let us consider the tree level mass spectrum. As easily recogni zed expanding the scalar potential around one of the minima (33) and retaining only the quadratic terms, the square of the scalar mass matrix is:

$$M_B^2 = \frac{1}{2} \begin{array}{cc} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4g^2|y|^2 + m^2 & 2mgy^* & 0 & 2g\lambda & 0 \\ 0 & 2mgy & m^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2g\lambda & 0 & 0 & 4g^2|y|^2 + m^2 & 2mgy \\ 0 & 0 & 0 & 0 & 2mgy^* & m^2 \end{pmatrix} & \begin{array}{c} A_0^* \\ A_1^* \\ A_2^* \\ A_0 \\ A_1 \\ A_2 \end{array} \\ \begin{array}{cccccc} A_0 & A_1 & A_2 & A_0^* & A_1^* & A_2^* \end{array} & \end{array} \qquad (37)$$

From (37) we see that the complex scalar A_0 remains massless at the tree approximation.

The mass term for fermions is:

$$-\frac{1}{2} \cdot 2gy\,\Psi_1\Psi_1 - \frac{1}{2} m\,\Psi_1\Psi_2 - \frac{1}{2} m\,\Psi_2\Psi_1 + \qquad (38)$$

$+$ c.c.

The spinor field ψ_0 is massless (Goldstino), as expected since it is connected by a supersymmetry transformation to F_0 which develops a non vanishing vacuum expectation value, breaking supersymmetry.

With Majorana spinors the fermion mass matrix take the form:

$$M_F = \begin{array}{cc} \left(\begin{array}{c|c} g\begin{pmatrix}\mathrm{Re}\,y & \mathrm{Im}\,y\\ \mathrm{Im}\,y & -\mathrm{Re}\,y\end{pmatrix} & \frac{1}{2}\begin{pmatrix} m & 0\\ 0 & -m\end{pmatrix} \\ \hline \frac{1}{2}\begin{pmatrix} m & 0\\ 0 & -m\end{pmatrix} & 0\end{array}\right) & \begin{array}{c}\psi_1^+\\ \\ \\ \psi_2^+\end{array} \\ \begin{array}{cc}\psi_1 & \qquad\qquad \psi_2\end{array} & \end{array} \tag{39}$$

From (37) and (39) the mass formula (22) can be easily proved to be true.

One question arisesabout the degeracy of minima of the tree level potential: is the degeneracy unremovable as in the case of unbroken supersymmetry, or is it only a tree level phenomenon which disappears once higher order corrections are taken into account?

In this respect it is worthwhile to point out that although the energy is independent of $\langle A_0\rangle = y$, the masses of particles are not, as we can see from (37) and (39) above.

Since at tree level we can choose $|y|$ arbitrarily large and, in this limit, the particle A_0 has a mass of approximately $2g|y|$, the theory can have at tree level an arbitrarily large mass scale, completely unrelated to the parameters of the lagrangian.

This scenario was suggested by E. Witten [24] as an attempt to solve the so called hierarchy problem which affects grand unified theories, namely to exaplain the large ratio between the grand unification mass scale and the scale of ordinary particles.

As a matter of fact one loop corrections remove the degeneracy of classical minima [25] selecting one of them as the absolute one. So those scalars compelled to be massless at tree level by the valley of classical minima at one loop acquire a mass.

Moreover, in the absence of gauge interactions, the scalar potential increases with the scalar fields so that the minimum occurs at small values of the fields [24]. (This fact, by itself, does not rule out Witten's possibility since in a realistic model of elementary particles one has to include also a gauge sector).

A detailed analysis performed on a class of supersymmetric models describing chiral superfields of 0 and 1 R-characters, where R-invariance is used to trigger spontaneously supersymmetry breaking, shows that one loop corrections do not (spontaneously) break R-invariance [25]. This means that the expectation values of the physical scalar fieldswhich do not transform trivially under R must be zero at one loop level.

Hence one loop approximation for O'Raifeartaigh model considered here gives $A_0 = A_2 = 0$ at the minimum.

As far as higher order corrections to the mass formula (22) are concerned, they have been investigated in the O'Raifeartaigh model [26], and although the mass formula is expected to receive in this case finite corrections, nevertheless it remains exact to first order in the supersymmetry breaking parameter λ.

§ 4 SUPERSYMMETRIC GAUGE THEORIES

In this section we examine the occurence of spontaneous supersymmetry breaking in renormalizable supersymmetric gauge theories. The relevant formulae are collected in the appendix.

4.1 GENERALITIES

Let us indicate with $\mathcal{L}_S$ that part of a lagrangian density $\mathcal{L}$ which depends only on the scalar fields and moreover does not contain derivatives of these. For a supersymmetric gauge theory describing a vector supermultiplet V coupled to scalar superfields Φ_i which transform according to a certain, in general reducible, representation r of the gauge group G, we have:

$$\mathcal{L}_S = F_i^* F_i + f_i(A) F_i + f_i^*(A) F_i^* + \sum_{\substack{\text{simple}\\ \text{factors}}} \left(\frac{1}{2} (D^a)^2 + g D^a A^+ T^a A \right) + \sum_{\substack{U(1)\\ \text{factors}}} \left(\frac{1}{2} D^2 + g_Y D A^+ Y A + k D \right) \tag{40}$$

Some explanation is necessary. The gauge group G we have in mind is the product of several simple factors times several abelian $U(1)$ factors. f(A) is again a cubic polynomial:

$$f(A) = \lambda_i A_i + \frac{1}{2} m_{ij} A_i A_j + \frac{1}{3} g_{ijk} A_i A_j A_k \qquad (41)$$

where m_{ij} and g_{ijk} are totally symmetric invariant tensors with respect to G. ($\lambda_i \neq 0$ only for the components of ϕ_i which are singlet under G).

For each simple factor of G, T^a are the generators of that factor in the representation according to which ϕ_i transform; D^a are the auxiliary fields of the vector supermultiplet associa ted to that factor and g the corresponding coupling constant.

For each U(1) factor of G Y, D and g_Y have an analogous meaning.

We finally observe that, unlike simple factors, each abelian factor can contribute to $\mathcal{L}_S$ with a term of the kind kD which is simultaneously supersymmetric and gauge invariant.

As we shall see, the main novelty of the supersymmetric gauge theories with respect to the chiral ones is the presence of this D-term which is a possible source of spontaneous supersymmetry breaking.

As usual F and D (auxiliary) fields do not have kinetic terms and can be eliminated from $\mathcal{L}_S$ through their own equations of motion:

$$\begin{cases} F_i = - f_i^*(A) \\ D^a = - g A^+ T^a A \\ D = - g_Y A^+ Y A - k \end{cases} \qquad (42)$$

So one obtains the scalar potential of the theory:

$$V(A^*, A) = f_i^*(A) f_i(A) + \sum_{\substack{\text{simple}\\ \text{factors}}} \frac{1}{2}(g A^+ T^a A)^2 + \sum_{\substack{U(1)\\ \text{factors}}} \frac{1}{2}(g_Y A^+ Y A + k)^2 \tag{43}$$

The tree level scalar potential is non negative. Supersymmetry is spontaneously broken if and only if some auxiliary field develops a non vanishing vacuum expectation value and hence, at tree approximation, if and only if the scalar potential is positive at the minimum, which means that the system:

$$\begin{cases} f_i(A) = 0 \\ g A^+ T^a A = 0 \\ g_Y A^+ Y A + k = 0 \end{cases} \tag{44}$$

has no solution.

4.2 SPONTANEOUS SUPERSYMMETRY BREAKING THROUGH RADIATIVE CORRECTIONS ?

Let us begin by supposing that supersymmetry is not spontaneously broken at tree level or, equivalently, that the system (44) admits solutions.

Can this situation be modified by higher order corrections ?

As in the case of pure chiral supersymmetric theories we base our discussion on a general result of perturbation theory [17].

To any order of perturbation theory the effective action for a supersymmetric gauge theory has the form:

$$\Gamma(\Phi, \Phi^+, V) = \sum_n \int d^4x_1 \ldots d^4x_n \int d^4\theta \; G(x_1, \ldots x_n) \times \tag{45}$$

$$\times \left[\text{Polynomial in: } \Phi(x_i, \theta),\ D_\alpha \Phi(x_j, \theta), \ldots V(x_k, \theta) \ldots \right]$$

Note that also for a supersymmetric gauge theory the integrand is local in θ and it is integrated with $d^4\theta$.

We are interested in the effective potential V_{eff} for x-independent (scalar) fields A,F and D. So we set fields with non-zero spin and spacetime derivatives equal to zero in (45).

Now the main point we exploited in the chiral case to show that a tree level supersymmetric (F=0) solution of the extremum conditions and the equations of motion of auxiliary fields is also a supersymmetric solution of the corresponding equations corrected up to an arbitrary order, is that the corrections of the scalar potential $V_{eff}(F,A)$ are at least quadratic in the auxiliary fields F_i.

What does happen with supersymmetric gauge theories? The fields carrying θ's are now F $(\theta\theta)$, F^* $(\bar{\theta}\bar{\theta})$ and D $(\theta\theta\bar{\theta}\bar{\theta})$. So the terms which survive the $d^4\theta$ integration are again at least bilinear in the auxiliary fields (F,D) apart from a possible term linear in D (and without F's).

The "danger" of such a term is that D can multiply scalar physical fields with a non vanishing vacuum expectation value, and acquire itself (through equations of motion) a non vanishing v.e.v., thus breaking supersymmetry.

In the following we shall concentrate on this possibility which is the only one for supersymmetry to be spontaneously broken through radiative corrections.

Let us ask: which are the D fields which can possibly break supersymmetry through the mechanism suggested above ?

Only those which are connected by a supersymmetry transformation to a massless fermion, a possible candidate goldstino. Since supersymmetry is good at tree level, the entire vector supermultiplet will be massless; in particular the gauge vector boson V_μ has to be massless and so the subgroup G^k of the gauge group G, which is associated (through a generator T^k) to this vector supermultiplet, must be unbroken in lowest order.

The only D^k auxiliary fields which can cause supersymmetry breaking are those related to unbroken (at lowest order) gauge subgroups.

Now we distinguish two cases:

(i) The gauge group G is semisimple (i.e. it does not contain U(1) factors).

In this case the terms linear in D's generated by higher order corrections must have the following structure [27]: if D^k is associated to a generator T^k of the gauge group G then it always multiplies scalar physical fields which are not singlet under the subgroup of G generated by T^k.

Now, if T^k is the generator of an unbroken subgroup of G - the only case we are interested in - these physical scalar fields cannot develop a non vanishing vacuum expectation value, just because they are non singlet with respect to an unbroken symmetry. D cannot acquire a non vanishing expectation value or, stated in another way, the equations of motion of auxiliary fields and the extremum conditions of the effective potential are not affected by such a term.

In conclusion: if G is semisimple, then a term linear in D generated by higher order correction cannot cause supersymmetry breaking.

(ii) The gauge group G contains some U(1) factor (it is not semisimple).

Let us consider the possibility of a term linear in D, the auxiliary field of the vector multiplet associated to a U(1) which we suppose unbroken.

This term may have the same structure of those considered previously, in which case it will not modify a supersymmetric classical situation.

However since in this case D is itself gauge invariant a new possibility shows up. Namely, that of a term of the form:

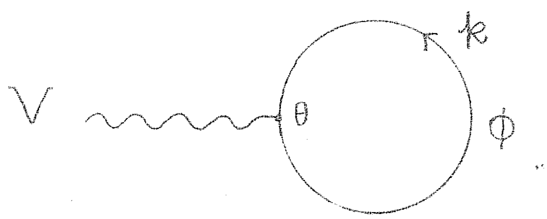
$$\xi D \tag{46}$$

(ξ is some constant coefficient). This term can be present already at classical level since it is supersymmetric and gauge invariant (see eq. (40)) and at that level may be a source of supersymmetry breaking as we shall see on a concrete example. In this discussion we are supposing that it is not present in the classical potential.

This term, originated by higher order corrections, modifies the equations of motion of the auxiliary field D possibly causing supersymmetry breaking. It happens that the classical supersymmetric solutions of the equations of motion of the auxiliary fields and the extremum conditions are no more solutions of the corresponding equations corrected by the new D-term.

Thus the possibility arises that there are no supersymmetric solutions at all !

In reference [28] a detailed study of the occurrence of a similar D-term is performed in a unbroken U(1) gauge theory. It is shown that this kind of term cannot be generated in any order of perturbation theory except at the one loop level. At this level the graph giving rise to such a term is the following:



Evaluating this graph one obtains :

$$g \sum_a Q_a \int \frac{d^4 k}{k^2} \int d^4\theta \, V(p=0,\theta,\bar{\theta}) \tag{47}$$

where Q_a is the U(1) charge of the chiral superfield Φ_a and the sum is extended over all chiral superfields.

From (47) we see that the coefficient is quadratically divergent and proportional to Tr Y (Y is the generator of U(1) in the representation according to which Φ_a transform).

Thus in any theory with a U(1) factor in the gauge group this D-term can be generated by one loop corrections even if it is not present at tree level, unless it is forbidden by some symmetry (a discrete parity, for instance).

However, we observe that, in the case of grand unification when our gauge group G containing a U(1) factor is embedded in a simple group $G^{(u)}$ (say an SU(N) or an SO(N) group), the U(1) generator Y becomes a generator of $G^{(u)}$. Since all generators of $G^{(u)}$ are traceless, Y is itself traceless and the corresponding D-term vanishes.

In conclusion:

if G contains some U(1) factor, a quadratically divergent D-term arises in perturbation theory. His contribution to the effective potential is such that supersymmetry can spontaneously break even if it is good at classical level. Models with grand unification at a higher energy scale or suitable discrete symmetries are protected against such an occurrence.

4.3 SPONTANEOUS SUPERSYMMETRY BREAKING AT TREE LEVEL.

Let us now investigate the possibility that in supersymmetric gauge theories the supersymmetry is spontaneously broken at tree level.

We can distinguish various situations:

1. Suppose that the system:

$$f_i(A) = 0 \tag{48}$$

admits no solution.

In this case, whatever can be of that part of the scalar potential which originates from the gauge interaction (last two lines in (43)), some auxiliary field F_i develops a non vanishing vacuum expectation value and supersymmetry is spontaneously broken.

2. Suppose that the system (48) admits a solution $\bar{A}$ and ask:

Is it possible for supersymmetry to be spontaneously broken through the non-vanishing vacuum expectation value of a D auxiliary field?

In other words: can the system

$$\begin{cases} g\, A^{+} T^{a} A & = 0 \\ g_Y A^{+} Y A + k & = 0 \end{cases} \qquad (49)$$

be inconsistent or incompatible with the equations $f_i(A)=0$?

Remarkably enough, in the case of gauge groups G <u>not containing U(1) factors</u>, this cannot occur: if equations (48) have a solution $\bar{A}$, then equations (48) together with the system (49) (without the last line) admit a common solution A' [18]. Let us briefly sketch the proof (*) for this.

For definiteness consider a simple gauge group G.

If the solution of equations (48) is $\bar{A}=0$, then this is also the solution for the system (49) and nothing must be proved. Hence suppose $\bar{A}\neq 0$.

The following property of supersymmetric theories reveals useful for our purpose:
the superpotential f(A) is invariant not only under the gauge group G but also under the complex estension G_c of G (same generators but parameters which can take complex values) [29].

We can use this property to find a new point A', obtained from $\bar{A}$ with a transformation of G_c , such that:

$$\sum_a \left(A'^{+} T^{a} A'\right)^2 = 0 \qquad (50)$$

On the other hand in A' $f_i(A)$ are still vanishing and so is the whole scalar potential.
Consider the quantity:

$$\Delta = \sum_a \left(A^{+} T^{a} A\right)^2 \qquad (51)$$

(*) This proof is due to GF. SARTORI, private communication.

It can also be expressed in the following way:

$$\Delta = tr\left(H^2(A)\right) \tag{52}$$

with:

$$H(A) = \sum_a T^a (A^+ T^a A) \tag{53}$$

(we take hermitian generators T^a with the property $tr\, T^a T^b = \delta^{ab}$).

Since H(A) belongs to the Lie algebra $\mathcal{G}$ of the Lie group G, it can be diagonalized by means of a similarity transformation realized by an element g of G; hence the following element of $\mathcal{G}$:

$$H'(A) = g\, H(A)\, g^{-1} \tag{54}$$

is diagonal.

It is clear that:

$$\Delta = tr\left(H'^2(A)\right)$$

On the other hand, as can be easily checked, for H'(A) the following expression holds:

$$\begin{aligned} H'(A) &= H(gA) = \\ &= H(A_g) = \\ &= \sum_a T^a (A_g^+ T^a A_g) \end{aligned} \tag{55}$$

having defined:

$$A_g = gA$$

Since H'(A) is a diagonal element of $\mathcal{G}$, the sum in (55) can be restricted to the diagonal elements C^a $(a=1,\dots \ell)$ of a suitable basis in a Cartan subalgebra $\mathcal{C}$ of $\mathcal{G}$ (ℓ is the dimension of $\mathcal{C}$):

$$H'(A) \doteq H(A_g) = \sum_{a=1}^{\ell} C^a (A_g^+ C^a A_g) \tag{56}$$

Let us now perform on A_g the following transformation of the complexification G_c of the gauge group G:

$$A'_g = g_c(\lambda)\, A_g$$

with:

$$g_c(\lambda) = \exp \sum_{b=1}^{\ell} \frac{\lambda^b}{2} C^b \tag{57}$$

where λ^b are <u>real</u> parameters. From (56) and (57) we get:

$$H(A'_g) = \sum_{a=1}^{\ell} C^a (A_g^+ \; C^a e^{\Sigma \lambda^b C^b} A_g) =$$

$$= \sum_{a=1}^{\ell} C^a \frac{\partial}{\partial \lambda^a} f(\lambda, A_g)$$

where:

$$f(\lambda, A_g) = A_g^+ \, e^{\sum \lambda^b C^b} A_g =$$
$$= \sum_i e^{\sum \lambda^b C^b_{ii}} |A_{gi}|^2$$

$f(\lambda, A_g)$ is function of the real variables λ , bounded from below. Hence at the point $\bar{\lambda}$ where it reaches its minimum value (if necessary at infinite λ) we have:

$$\frac{\partial}{\partial \lambda^a} f(\lambda, A_g)\Big|_{\lambda = \bar{\lambda}} = 0$$

(at least as a limit).

Here $H(A'_g)$ vanishes together with:

$$\Delta' = \mathrm{tr}\left(H^2(A'_g)\right) =$$
$$= \sum_a (A'^+_g T^a A'_g)^2 = 0$$

as we had promised.

The point A'_g solves contemporarily equations (48) and (49) and we can conclude that supersymmetry is unbroken.
Supersymmetric gauge theories whose gauge group contains U(1) factors do not share this property and, as an example of this fact, we shall describe the Fayet-Iliopoulos model [13] later on.
Other examples of supersymmetric gauge theories with U(1) factors in which supersymmetry is broken by the vacuum expectation value of a D auxiliary field can be found in reference [30] .

Let us summary the various possibilities of breaking spontaneously supersymmetry at tree level in a supersymmetric gauge theory.

(i) if no U(1) factor is contained in the gauge group G, as we have seen the only available mechanism in order to break spontaneously supersymmetry is that the system of equations $f_i(A)=0$ admits no solutions. So to speak, this is a breaking a là O'Raifeartaigh.

Moreover in this case the mechanism requires the presence of a singlet with respect to G among the fields A_i. Otherwise both the F and the D auxiliary fields would be quadratic in the A scalar fields (once expressed in terms of these) and $A_i=0$ would trivially solve equations (48) and (49).

(ii) if G contains some U(1) factor, all of the following mechanisms:

1. system (48) has no solution;
2. system (49) has no solution;
3. systems (48) and (49) are incompatible.

are in principle available to break spontaneously supersymmetry. Observe however that the situations 2. and 3. can only occur in the presence of some U(1) factor: it is just the k-term related to this factor which is responsible of 2. and 3.

We shall see this in detail working out the Fayet-Iliopoulos model.

* * *

4.4 MASS FORMULA.

In a large class of supersymmetric gauge theories with spontaneously broken supersymmetry the following mass formula holds at tree level [7]:

$$\sum_J (-1)^{2J} (2J+1) m_J^2 = 0 \tag{58}$$

where the sum is extended over all the particles, now including gauge bosons.

Let us refer for definitness to a U(1) gauge group. Then the condition which must be fullfilled for (58) to hold is that the U(1)-charges of all chiral superfields add up to zero.

The proof of (58) goes along the same lines as that of (22), the corresponding, identical formula for the chiral case. One has to compute the trace of the square of the mass matrices for the spin 0, spin $\frac{1}{2}$ and spin 1 sectors. At tree level these can be read from the action of the theory (see the appendix).

The square of the scalar mass matrix comes from the scalar potential, which, in the U(1) case is given by:

$$V = f_k^*(A) f_k(A) + \frac{1}{2}\left(k + \frac{t_k}{2} A_k^* A_k\right)^2 \tag{59}$$

where $\frac{t_k}{2}$ is the U(1)-charge of the field A_k and k comes from the $\int d^4x\, d^4\theta\, 2kV$ term of the action.

The square of the scalar mass matrix for complex fields is

$$\frac{1}{2}\begin{pmatrix} \dfrac{\partial^2 V}{\partial A_i^* \partial A_j} & \dfrac{\partial^2 V}{\partial A_i^* \partial A_j^*} \\[2ex] \dfrac{\partial^2 V}{\partial A_i \partial A_j} & \dfrac{\partial^2 V}{\partial A_i \partial A_j^*} \end{pmatrix} \tag{60}$$

where the derivatives are taken at the minimum $A=\bar{A}$ of the scalar potential. From (59) and (60) one can easily calculate the trace of M_0^2, the square of the mass matrix for real scalar fields (One must remember that, in order to refer the result to real particles, one has to double the trace of (60)). We obtain:

$$\mathrm{tr}\, M_0^2 = 2\, f_{ki}^*(\bar{A}) f_{ki}(\bar{A}) + 2\left(\frac{t_i}{2}\right)^2 \bar{A}_i^* \bar{A}_i + \\ + 2\left(k + \frac{t_k}{2}\bar{A}_k^* \bar{A}_k\right)\sum_i \left(\frac{t_i}{2}\right) \tag{61}$$

The mass terms for fermions are:

$$-\frac{i}{\sqrt{2}}\, t_k\left(\bar{A}_k \bar{\lambda}\bar{\psi}_k - \bar{A}_k^* \lambda \psi_k\right) - \frac{1}{2} f_{ij}(\bar{A})\psi_i\psi_j - \frac{1}{2} f_{ij}^*(\bar{A})\bar{\psi}_i\bar{\psi}_j \tag{62}$$

which can be written in the following form:

$$-\frac{1}{2}\Psi^T H \Psi + c.c. \tag{63}$$

with :

$$\Psi = \begin{pmatrix} \lambda' \\ \psi_j \end{pmatrix} \qquad \lambda' = i\lambda \tag{64}$$

$$H = \left(\begin{array}{c|c} 0 & -\frac{t_j}{\sqrt{2}}\bar{A}_j^* \\ \hline -\frac{t_i}{\sqrt{2}}\bar{A}_i^* & f_{ij}(\bar{A}) \end{array}\right) \tag{65}$$

Converting Weyl spinors into Majorana ones; choosing a basis for Dirac matrices such that:

$$\gamma_0 = \begin{pmatrix} 1\!\!1 & 0 \\ 0 & -1\!\!1 \end{pmatrix} \qquad i\gamma_0\gamma_5 = \begin{pmatrix} 0 & 1\!\!1 \\ 1\!\!1 & 0 \end{pmatrix} \tag{66}$$

and defining the new matrices:

$$\begin{aligned} m &= \mathrm{Re}\, H \\ n &= \mathrm{Im}\, H \end{aligned} \tag{67}$$

(63) reads :

$$-\frac{1}{2}\, \psi^{+}_{r_M} \begin{pmatrix} m_{rs} \otimes 1\!\!1 & n_{rs} \otimes 1\!\!1 \\ n_{rs} \otimes 1\!\!1 & -m_{rs} \otimes 1\!\!1 \end{pmatrix} \psi_{s_M} \tag{68}$$

where $1\!\!1$ is the unity 2x2 matrix.

Therefore the square of the mass matrix for fermions is:

$$M^2_{\frac{1}{2}} = \frac{1}{4} \begin{pmatrix} (m^2+n^2) \otimes 1\!\!1 & [m,n] \otimes 1\!\!1 \\ -[m,n] \otimes 1\!\!1 & (m^2+n^2) \otimes 1\!\!1 \end{pmatrix} \tag{69}$$

and its trace is:

$$\begin{aligned} \mathrm{tr}\, M^2_{\frac{1}{2}} &= (m^2+n^2)_{rr} = \\ &= \sum_{r,s} |H_{rs}|^2 = \\ &= f^{*}_{ij}(\bar{A})\, f_{ij}(\bar{A}) + 4\left(\frac{t_k}{2}\right)^2 \bar{A}^{*}_k \bar{A}_k \end{aligned} \tag{70}$$

The vector boson mass term is:

$$-\left(\frac{t_k}{2}\right)^2 \bar{A}_k^* \bar{A}_k v_\mu v^\mu \tag{71}$$

and so the mass of the <u>real</u> vector field is:

$$\mathrm{tr}\, M_1^2 = 2\left(\frac{t_k}{2}\right)^2 \bar{A}_k^* \bar{A}_k \tag{72}$$

From (61), (70) and (72) it follows:

$$\begin{aligned}
&\sum_J (-1)^{2J}(2J+1)\, m_J^2 = \\
&= \mathrm{tr}\, M_0^2 - 2\,\mathrm{tr}\, M_{\frac{1}{2}}^2 + 3\,\mathrm{tr}\, M_1^2 = \\
&= 2\left(k + \frac{t_k}{2}\bar{A}_k^* \bar{A}_k\right) \sum_i \left(\frac{t_i}{2}\right) = \\
&= -2\, D(\bar{A}) \sum_i \left(\frac{t_i}{2}\right)
\end{aligned} \tag{73}$$

So, in the U(1) case the right-hand side of (58) is equal to zero if and only if the U(1)-charges of the superfields add to zero. This is in particular true if there are no axial currents in the theory since in this case charged chiral superfields come in pairs of opposite U(1)-charges: one provides the left-handed component while the other provides the conjugate of the right handed component to a given Dirac spinor.

If the gauge group G is non abelian the proof of (58) is quite similar. The term which in the U(1) case contains the trace of the U(1)-charge now contains a combination of the traces of the generators T^a of G in the representation according to which A'_s transform. So the vanishing of $\mathrm{tr}\, T^a$ now guarantees the validity of (58).

4.5 THE FAYET-ILIOPOULOS MODEL.

In this paragraph we describe the Fayet-Iliopoulos model [13] as an example of supersymmetric gauge theory with supersymmetry spontaneously broken. The gauge group G of the model is U(1) and indeed the responsable of supersymmetry breaking is the k-term related to the U(1). This term produces an incompatibility between equations (48) and (49) which admit only separate solutions. Invariance of the superpotential under the complex extension of U(1) cannot be used to rotate away the D field at the minimum of scalar potential.

The model is discussed in various regions of the space of parameters where only supersymmetry is spontaneously broken or both supersymmetry and the gauge symmetry break.

The mass formula (58) is esplicitly verified in these different "phases". Also the Goldstino field is identified. Another interesting feature of supersymmetric gauge theories with spontaneously broken supersymmetry is presented: the degeneracy among classical minima of the scalar potential which is typical of purely chiral models is here removed simply by the occurrence of the gauge interaction.

The model describes two chiral superfields Φ_1 and Φ_2 of opposite U(1) charges +e and -e minimally coupled to a vector superfield V. The action A is:

$$
\begin{aligned}
A = \frac{1}{4} & \left(\int d^4x\, d^2\theta\, W^\alpha W_\alpha + \int d^4x\, d^2\bar\theta\, \overline{W}_{\dot\alpha} \overline{W}^{\dot\alpha} \right) + \\
& + \int d^4x\, d^4\theta \left(\Phi_1^+ e^{eV} \Phi_1 + \Phi_2^+ e^{-eV} \Phi_2 \right) + \\
& + m \int d^4x\, d^2\theta\, \Phi_1 \Phi_2 + m \int d^4x\, d^2\bar\theta\, \Phi_1^+ \Phi_2^+ + \\
& + \int d^4x\, d^4\theta\, 2kV
\end{aligned}
\qquad (74)
$$

A is manifestly supersymmetric; moreover it is invariant under U(1) gauge transformations. The corresponding lagrangian in component fields, evaluated in the Wess-Zumino gauge, reads (see formula A.18 of the appendix):

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4} v_{\mu\nu} v^{\mu\nu} - i\lambda\sigma^{\mu}\partial_{\mu}\bar{\lambda} + \frac{1}{2} D^2 + \\
& + A_1^* \Box A_1 + A_2^* \Box A_2 + i\partial_{\mu}\bar{\psi}_1\bar{\sigma}^{\mu}\psi_1 + \\
& + i\partial_{\mu}\bar{\psi}_2\bar{\sigma}^{\mu}\psi_2 + F_1^* F_1 + F_2^* F_2 + \\
& + e v^{\mu}\Big(\frac{1}{2}\bar{\psi}_1\bar{\sigma}^{\mu}\psi_1 - \frac{1}{2}\bar{\psi}_2\bar{\sigma}^{\mu}\psi_2 + \frac{i}{2}A_1^*\partial_{\mu}A_1 - \\
& - \frac{i}{2}A_2^*\partial_{\mu}A_2 - \frac{i}{2}(\partial_{\mu}A_1^*)A_1 + \frac{i}{2}(\partial_{\mu}A_2^*)A_2\Big) + \\
& - \frac{ie}{\sqrt{2}}\Big(A_1\bar{\lambda}\bar{\psi}_1 - A_2\bar{\lambda}\bar{\psi}_2 - A_1^*\lambda\psi_1 + A_2^*\lambda\psi_2\Big) + \\
& + \frac{e}{2} D\big(A_1^*A_1 - A_2^*A_2\big) - \frac{e^2}{4} v^{\mu}v_{\mu}\big(A_1^*A_1 + A_2^*A_2\big) + \\
& + m\,\big(A_1F_2 + A_2F_1 + A_1^*F_2^* + A_2^*F_1^* - \psi_1\psi_2 - \bar{\psi}_1\bar{\psi}_2\big) + \\
& + kD .
\end{aligned} \qquad (75)$$

Note the presence of the supersymmetric and gauge invariant kD term.

From (75) one easily derives the equations of motion for the auxiliary fields F_1, F_2 and D:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial F_1^*} &= F_1 + m A_2^* = 0 \\ \frac{\partial \mathcal{L}}{\partial F_2^*} &= F_2 + m A_1^* = 0 \\ \frac{\partial \mathcal{L}}{\partial D} &= D + \frac{e}{2}\left(A_1^* A_1 - A_2^* A_2 \right) + k = 0 \end{aligned} \tag{76}$$

These equations can be substituted in (75) giving rise to the following scalar potential:

$$V = F_1^*(A) F_1(A) + F_2^*(A) F_2(A) + \frac{1}{2} \overset{2}{D}(A) \tag{77}$$

or, esplicitly:

$$\begin{aligned} V = &\left(m^2 + \frac{ke}{2}\right) A_1^* A_1 + \left(m^2 - \frac{ke}{2}\right) A_2^* A_2 + \\ &+ \frac{e^2}{8}\left(A_1^* A_1 - A_2^* A_2\right) + \frac{k^2}{2} \end{aligned} \tag{78}$$

From (76) one can realized that, <u>owing to the presence of the kD term</u>, if k and m are different from zero, no choice of the scalar fields A leads contemporarily to $F_1(A) = F_2(A) = D(A) = 0$. Therefore supersymmetry is spontaneously broken; then (77) tell us that the (positive definite) scalar potential never vanishes.

We observe that the invariance of the superpotential $f(A) = m A_1 A_2$ under the complexification of U(1) is of no help here to rotate away the D(A) term, since the only solution to $F_1(A) = F_2(A) = 0$ is just $A_1 = A_2 = 0$, which cannot be mapped in anything else. Had we been in the non-abelian case, A=0 would have also solved D(A)=0, but the U(1) related k term makes this impossible.

<u>Minimization of the scalar potential</u>

The conditions for the extrema are:

$$\begin{cases} \left[\left(m^2 + \frac{ke}{2}\right) + \frac{e^2}{4}\left(A_1^* A_1 - A_2^* A_2\right)\right] A_1 = 0 \\ \left[\left(m^2 - \frac{ke}{2}\right) - \frac{e^2}{4}\left(A_1^* A_1 - A_2^* A_2\right)\right] A_2 = 0 \end{cases} \tag{79}$$

They have as solutions:

(i) $$\begin{cases} A_1 = 0 \\ A_2 = 0 \end{cases}$$

(ii) $$\begin{cases} A_1 = 0 \\ \frac{e^2}{4} A_2^* A_2 = -\left(m^2 - \frac{ke}{2}\right) \end{cases} \qquad \text{for } m^2 - \frac{ke}{2} < 0$$

(iii) $$\begin{cases} \frac{e^2}{4} A_1^* A_1 = -\left(m^2 + \frac{ke}{2}\right) \\ A_2 = 0 \end{cases} \qquad \text{for } m^2 + \frac{ke}{2} < 0$$

Let us concentrate on solutions (i) and (ii). A study of the stability of the extrema (i) and (ii) leads to the following conclusions:

The solution (i) is a local minimum of V for:

$$\left| \frac{ke}{2} \right| < m^2$$

The value of the scalar potential at (i) is:

$$V(i) = \frac{k^2}{2} > 0$$

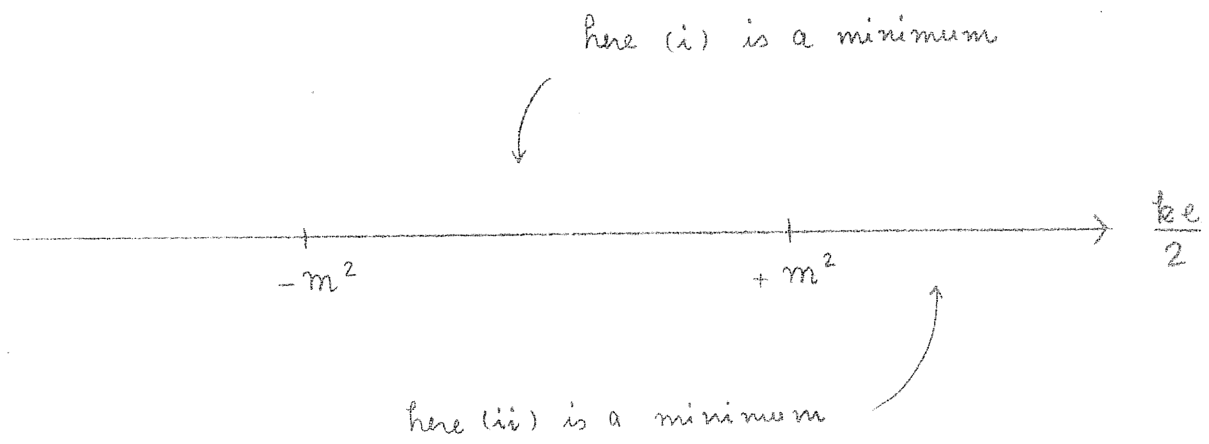
The solution (ii) which already requires:

$$m^2 - \frac{ke}{2} < 0$$

is a local minimum in this region with:

$$V(ii) = \frac{2m^2}{e^2}\left(ke - m^2\right) > 0$$

In summary:



Let us observe that in each of the regions considered here only one minimum occurs. In purely chiral models the spontaneous breaking of supersymmetry is accompained with a complex valley of degenerate minima which is removed only taking into account higher order corrections. Here no such degeneracy takes place. This is a common feature of supersymmetric gauge theories with spontaneously broken supersymmetry [23]. When supersymmetry is spontaneously broken the gauge interaction fixes unambiguously the vacuum expectation value of the scalar fields at tree level.

Let us now discuss the particle spectrum of the theory in the two cases in (i) and (ii).

(i)

We have no shift of scalar fields since $A_1 = A_2 = 0$ at the minimum. Therefore the vector field V_μ remains massless and the U(1) unbroken. Only supersymmetry is spontaneously broken in this phase.

The square of the scalar mass matrix is already diagonal in the basis chosen. If:

$$A \equiv \begin{pmatrix} A_1 \\ A_2 \\ A_1^* \\ A_2^* \end{pmatrix} \tag{80}$$

the mass term for scalar is:

$$- A^+ M_0^2 A$$

with :

$$M_0^2 = \mathrm{diag}\ \frac{1}{2}\left(m^2 + \frac{ke}{2},\ m^2 - \frac{ke}{2},\ m^2 + \frac{ke}{2},\ m^2 - \frac{ke}{2}\right) \tag{81}$$

So we have <u>two complex scalar</u> fields of masses $m^2 + ke/2$ and $m^2 - ke/2$ or four real scalar fields, two of masses $m^2 + ke/2$ and two of masses $m^2 - ke/2$. The absence of degeneracy among the minima reflects here in the absence of massless scalars.

Fermion masses come from:

$$-m\left(\psi_1\psi_2 + \bar{\psi}_1\bar{\psi}_2\right) \tag{82}$$

setting:
$$\psi_1 = \frac{\chi_1 + i\chi_2}{\sqrt{2}} \qquad \psi_2 = \frac{\chi_1 - i\chi_2}{\sqrt{2}} \tag{83}$$

and converting Weyl spinors into Majorana ones (82) becomes:

$$-\frac{m}{2}\left(\left(\bar{\chi}_1\chi_1\right)_M + \left(\bar{\chi}_2\chi_2\right)_M\right) \tag{84}$$

So we have two Majorana spinors - each of them has four real components - of mass $\frac{m}{2}$.

Note that λ has zero mass. Indeed:

$$\delta_{SUSY}\lambda = i\xi D + \sigma^{\mu\nu}\xi v_{\mu\nu} \tag{85}$$

and taking vacuum expectation values we have:

$$\langle \delta_{SUSY}\lambda \rangle = -i\xi k \tag{86}$$

which identifies λ as the Goldstino field.

Finally we observe that supersymmetry breaking has induced a splitting of the masses among the components of supermultiplets. Nevertheless this splitting obeys the mass formula (58), as now can be easily seen.

(ii)

Now the minimum of the scalar potential occurs for :

$$\begin{cases} A_1 = 0 \\ A_2 = v \end{cases} \tag{87}$$

with :

$$\frac{e^2 v^2}{4} = -\left(m^2 - \frac{ke}{2}\right) \tag{88}$$

where v is taken to be real (otherwise we pass to a real $\langle A_2 \rangle$ with a gauge transformation). Since $A_2 \neq 0$ we espect both supersymmetry and gauge symmetry to be spontaneously broken.

The mass term for the vector field v_μ comes from the shift of the scalar fields and is given by:

$$- \frac{e^2 v^2}{4} v_\mu v^\mu \tag{89}$$

Thus the real field v_μ has a mass:

$$\frac{e^2 v^2}{2} \tag{90}$$

Therefore the U(1) spontaneously breaks.

The mass term for scalars reads:

$$- A^{+} M_{0}^{2} A \tag{91}$$

with :

$$M_{0}^{2} = \begin{pmatrix} m^2 & 0 & 0 & 0 \\ 0 & \frac{e^2 v^2}{8} & 0 & \frac{e^2 v^2}{8} \\ 0 & 0 & m^2 & 0 \\ 0 & \frac{e^2 v^2}{8} & 0 & \frac{e^2 v^2}{8} \end{pmatrix} \tag{92}$$

and:

$$A \equiv \begin{pmatrix} A_1 \\ A_2 \\ A_1^{*} \\ A_2^{*} \end{pmatrix} \tag{93}$$

(92) can be put in a diagonal form:

$$-A^+ M_0^2 A = -A'^+ M_0'^2 A'$$

with:

$$M_0'^2 \cdot= \mathrm{diag}\left(0,\ m^2,\ m^2,\ \frac{v^2 e^2}{4}\right) \tag{94}$$

and:

$$A' = \begin{pmatrix} \frac{A_2 - A_2^*}{\sqrt{2}} \\ A_1 \\ A_1^* \\ \frac{A_2 + A_2^*}{\sqrt{2}} \end{pmatrix} \tag{95}$$

So we have one real massless scalar: $(A_2 - A_2^*)/\sqrt{2}$; one complex scalar of mass $2m^2$: A_1 ; and one real scalar of mass $\frac{v^2 e^2}{2}$: $(A_2 + A_2^*)/\sqrt{2}$.

Note the occurrence of a massless real field. It corresponds to a Higgs particle eaten by the gauge boson V_μ to reach the correct number of degrees of freedom for a massive vector field.

The fermion mass term is:

$$-m\left(\psi_1\psi_2 + \bar\psi_1\bar\psi_2\right) - \frac{ie}{\sqrt{2}}\left(-v\bar\lambda\bar\psi_2 + v\lambda\psi_2\right) \tag{96}$$

Defining:

$$\lambda' = i\lambda$$
$$\Psi = \begin{pmatrix} \lambda' \\ \psi_1 \\ \psi_2 \end{pmatrix} \tag{97}$$

(96) becomes:

$$-\Psi\, M_F\, \Psi \tag{98}$$

with :

$$M_F = \begin{pmatrix} 0 & 0 & +\frac{\nu e}{2\sqrt{2}} \\ 0 & 0 & +\frac{m}{2} \\ +\frac{\nu e}{2\sqrt{2}} & +\frac{m}{2} & 0 \end{pmatrix} \tag{99}$$

Diagonalization of M_F leads to:

$$-\Psi^T M_F \Psi = -\Psi'^T M_F' \Psi' \tag{100}$$

with :

$$M_F' = \mathrm{diag}\left(0, -\frac{1}{2}\sqrt{m^2 + \frac{e^2\nu^2}{2}}, +\frac{1}{2}\sqrt{m^2 + \frac{e^2\nu^2}{2}}\right) \tag{101}$$

and :

$$\psi' = \begin{pmatrix} \chi_0 \\ \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \dfrac{m}{\sqrt{m^2 + \frac{v^2e^2}{2}}}\lambda' - \dfrac{ve}{\sqrt{2}} \cdot \dfrac{1}{\sqrt{m^2 + \frac{v^2e^2}{2}}}\psi_1 \\ -\dfrac{ve}{2}\dfrac{1}{\sqrt{m^2 + \frac{v^2e^2}{2}}}\lambda' - \dfrac{m}{\sqrt{2}} \cdot \dfrac{1}{\sqrt{m^2 + \frac{v^2e^2}{2}}}\psi_1 + \dfrac{1}{\sqrt{2}}\psi_2 \\ \dfrac{ve}{2}\dfrac{1}{\sqrt{m^2 + \frac{v^2e^2}{2}}}\lambda' + \dfrac{m}{\sqrt{2}} \cdot \dfrac{1}{\sqrt{m^2 + \frac{v^2e^2}{2}}}\psi_1 + \dfrac{1}{\sqrt{2}}\psi_2 \end{pmatrix} \qquad (102)$$

Thus, in terms of the spinors:

$$\begin{cases} \chi_0 \\ \varphi_1 = i\,\chi_1 \\ \varphi_2 = i\,\chi_2 \end{cases} \qquad (103)$$

the fermion mass term (96) is:

$$-\frac{1}{2}\sqrt{m^2 + \frac{e^2v^2}{2}}\left(\varphi_1\varphi_1 + \bar{\varphi}_1\bar{\varphi}_1\right) - \frac{1}{2}\sqrt{m^2 + \frac{e^2v^2}{2}}\left(\varphi_2\varphi_2 + \bar{\varphi}_2\varphi_2\right) \qquad (104)$$

The fermion mass spectrum consists of:

(i) a massless spinor:

$$\chi_0 = \frac{i\,m}{\sqrt{m^2 + \frac{v^2 e^2}{2}}}\,\lambda - \frac{v e}{\sqrt{2}} \cdot \frac{1}{\sqrt{m^2 + \frac{v^2 e^2}{2}}}\,\psi_1 \tag{105}$$

χ_0 is the Goldstone spinor as one can check taking the vacuum expectation value of $\delta_{SUSY}\,\chi_0$.

(ii) two Majorana spinors each of mass:

$$\frac{1}{2}\sqrt{m^2 + \frac{e^2 v^2}{2}}$$

Also in this case the spontaneous breaking of supersymmetry results in a splitting of the masses among supermultiplet components. However the masses split according to the formula (58):

$$\sum_J (-1)^{2J}(2J+1)\, m_J^2 = 4m^2 + \frac{v^2 e^2}{4} +$$

$$- 2\left(m^2 + \frac{v^2 e^2}{2} + m^2 + \frac{v^2 e^2}{2}\right) +$$

$$+ 3\,\frac{e^2 v^2}{2} = 0$$

§ 5 ULTRAVIOLET BEHAVIOUR OF SUPERSYMMETRIC THEORIES. SOFT BREAKING OF SUPERSYMMETRY

In this paragraph we shall investigate some aspects of the quantum divergences of N=1 globally supersymmetric theories. The "softening" of these divergences with respect to those associated to an ordinary, non supersymmetric theory, will lead us to the concept of soft breaking of supersymmetry which we shall briefly discuss.

5.1 SUPERSYMMETRY AND QUANTUM DIVERGENCES

N=1 globally supersymmetric renormalizable theories can be formulated. A renormalization procedure exists which preserves the supersymmetric Ward identities. So one expects that some renormalization constants are related by supersymmetry. Indeed the renormalization properties of supersymmetric theories go beyond this fact. It turns out that supersymmetric theories are more convergent than the ordinary ones. Owing to the cancellation of divergences between diagrams with fermion loops and diagrams with boson loops, the divergences of a supersymmetric theory require fewer renormalization constant than the generic quantum field theory and, remarkably, some parameters of the theory receive no renormalization.

Take for instance the effective action for the chiral sector of the theory (13). No terms integrated with $d^2\theta$ can be generated in perturbation theory, and the non vanishing contributions which are bilinear in the auxiliary fields F's are logarithmic divergent (just for dimensional counting). This divergence can be eliminated with the wave function renormalization of the chiral superfields. The parameters of the superpotential do not receive any renormalization either infinite or finite, independent of the wave function renormalization. Observe, in particular that :

(i) At any order of perturbation theory, no quadratic divergence occurs.

This remains true also if we take into account the gauge sector of the theory as one can see by the same argument as above, with the possible exception of a quadratically divergent term linear in the D auxiliary field related to a $U(1)$ factor of the gauge group. However, as we have seen before, this quadratic divergence shows up only if the relevant generator is not traceless.

One must say that the above argument holds true only for theories with unbroken supersymmetry, since it is based on results obtained making use of the Feynman rules for that class of theories. Nevertheless statament (i) remains true also for theories with spontaneously broken supersymmetry.

It is noteworthy to verify explicitly this fact at one loop level. In a renormalizable theory the only possible quadratic divergences arise computing graphs with one or two external scalar lines, that is tadpole or self energy diagrams for scalars (we neglect the divergences associated to vacuum diagrams). Thus if quadratic divergences are produced in perturbation theory they must come out by computing the effective potential of the theory. Now, for a gauge theory the one loop contribution $\Delta V^{(1)}_{eff}$ to the effective potential is given by [31]:

$$\Delta V^{(1)}_{eff} \propto \sum_J (-1)^{2J}(2J+1)\, \mathrm{tr}\int d^4k\, \ln(k^2+M_J^2(\varphi_0)) \tag{106}$$

where $\Delta V^{(1)}_{eff}(\varphi_0)$ depends on the scalar, x-independent fields φ_0 through the tree level mass matrix $M_J(\varphi_0)$ for particles of spin J (φ_0 substitutes the point $\bar{\varphi}_0$ at which the tree level potential is stationary; (106) is derived in the Landau gauge and with

Majorana fermions). Expanding (106) in powers of $M_J^2(\varphi_0)/k^2$ and neglecting a possible quartic, field independent divergence, one finds:

$$\Delta V_{eff}^{(1)}(\varphi_0) \propto \Big[\sum_J (-1)^{2J}(2J+1)\, \mathrm{tr}\, M_J^2(\varphi_0)\int \frac{d^4k}{k^2} + \\ -\frac{1}{2}\sum_J (-1)^{2J}(2J+1)\, \mathrm{tr}\, M_J^4(\varphi_0)\int\frac{d^4k}{k^4} + \\ + \text{finite terms}\Big] \qquad (107)$$

If we introduce a cutoff Λ to handle the divergent quantities in (107) we recognize that the quadratic divergence, that is the $O(\Lambda^2)$ term in (107) is proportional to:

$$\sum_J (-1)^{2J}(2J+1)\, \mathrm{tr}\, M_J^2(\varphi_0) \qquad (108)$$

This is the expression which, at one loop, controls the presence of the quadratic divergences in a quantum field theory and hence in supersymmetric theories both with unbroken and with spontaneously broken supersymmetry. Expression (108) looks like the left-hand side of the mass formulae (22) and (73) a part from the fact that in those formulae φ_0 is identified with the value which extremize the tree level potential (and which we called $\bar{A}$ in that context). Now since this identification did not play any role in the proofs of the mass formulae, we can conclude that the formulae apply as well to expression (108). Hence:

(ii) Expression (108) vanishes in supersymmetric theories with unbroken supersymmetry and in most of supersymmetric theories with spontaneously broken supersymmetry.

Relevant exceptions, as we read from (73), are gauge theories whose gauge group contains some U(1) factors. In that case each of the U(1) factors contributes to the expression (108) with a term:

$$- 2 D(\varphi_0) \sum_i \left(\frac{t_i}{2} \right) \tag{109}$$

which is proportional to the trace of the U(1) generator. In this case a quadratic divergence proportional to (109) appears in the theory, as we already sow in (47) discussing the possibility of breaking supersymmetry through radiative corrections.

So, through an explicit analysis at one loop level we have seen that, apart from the above mentioned exception, whether or not supersymmetry is spontaneously broken, supersymmetric theories do not give rise to quadratic divergences.

5.2 SOFT BREAKING OF SUPERSYMMETRY.

Recently there has been some interest in studying models with explicit softly broken supersymmetry [10, 32] . Actually this kind of breaking can be done by hand [32] or it can correspond to the low energy description of a high energy spontaneous breaking of local supersymmetry [10] . In any case it can provide a phenomenologically viable alternative to the spontaneous breaking of global supersymmetry.

By definition an explicit breaking of supersymmetry is said to be soft when it preserves the good ultraviolet behaviour of supersymmetric theories, namely when it does not give rise to quadratic divergences in perturbation theory.

Two questions arise:

1. In a general globally supersymmetric theory what explicit breakings are soft ?
2. What are the counterterms required to cancel the (logarithmic) infinities induced by a given soft breaking term ?

In order to answer these questions, we shall adopt the following procedure [11].

We start from a supersymmetric theory (free from quadratic divergences) involving vector and chiral quantum superfields V and Φ and we couple, in a manifestly supersymmetric fashion, some external (spurion) superfields to the quantum superfields V and Φ .

Since supersymmetry consists in translational invariance in superspace, one can break supersymmetry by giving the spurion fields fixed x-independent, θ-dependent values.

The breaking term originated by this procedure will be soft if the considered coupling is consistent with the renormalizability criteria of superfield perturbation theory. Now, Feynman rules in superspace assign exactly four factors D_α , D_β , $\bar{D}_{\dot\alpha}$, $\bar{D}_{\dot\beta}$ to each vertex of a renormalizable globally supersymmetric theory [17] . (This can be seen simply functionally differentiating the term chosen in the action paying a little attention to the rules of functional differentiation in superspace when constraints are involved). More factors of $D's$ would signal the presence of a dimensional coupling constant which would destroy renormalizability. Thus only those breaking terms which give rise to no more of four factors of $D's$ in the corresponding Feynman rule, are soft.

Now each of the soft breaking terms selected as described above will give rise to some divergences which will require the appropriate counterterms.

Once again, to study these divergences the power counting rules reveal very useful. The effective action of the theory and hence its divergent part must be dimensionless. Moreover at any order of perturbation theory it must have the form:

$$\Gamma(\Phi,\Phi^+,V) = \int d^4x\, d^4\theta\, \mathcal{P}(\Phi,\Phi^+,V,D_\alpha\Phi,\dots) \qquad (110)$$

where $\mathcal{P}$ is a polynomial in the indicated variables always integrated with $d^4\theta$.

Each θ has (mass) dimension -1/2, so $d^4\theta$ has dimension 2 and $\mathcal{P}$ has dimension 2. Now think to $\mathcal{P}$ as the product of a possibly divergent integral and a part which can be factorized out of the integral. The degree of divergence of this integral depends on how many powers of mass have been factorized out of it. The factorizable quantities are: ϕ which has dimension 1, D_α which has dimension + 1/2, V which has dimension 0.

A propagator $\langle \phi\phi \rangle$ looks like:

$$\langle \phi\phi \rangle = \frac{M D^2}{p^2(p^2+M^2)} \delta^4(\theta-\theta') \tag{111}$$

and M, which has dimension 1, can be taken out of the integral contained in $\mathcal{P}$ thus lowering its degree of divergence. (This does not happens with $\langle \phi^+\phi \rangle$ or $\langle V V \rangle$ propagators).

If $\mathcal{P}$ includes only chiral fields ϕ, then the θ integration (with $d^4\theta$!) gives zero, unless an extra D^2 is present to make the integrand non chiral. In this case D^2 can be factorized, lowering again the degree of divergence of the integral.

Finally gauge invariance can require V to appear in $\mathcal{P}$ only in terms of its field strength $W_\alpha \sim \bar{D}^2 D_\alpha V$ and here again the $D's$ can be factorized out.

After having extracted all these factors we are able to compute the dimension of the integral included in $\mathcal{P}$. Divergences will occur for non negative dimensionalities. Actually having to do with soft breaking terms we expect at most a dimensionless logarithmic divergent integral.

Let us now list the allowed $\Delta\mathcal{L}$ breaking terms together with the induced counterterms (we shall not consider here divergences corresponding to insertions of only spurion fields into vacuum diagrams).

If we restrict momentarily ourselves to breaking terms made of chiral and antichiral fields only we at soon realized that the possibilities are few. Integrated with $d^4\theta$, only the combination $\phi^+\phi$ can give rise to soft divergences; the relevant spurion superfields must have dimension zero. Integrated with $d^2\theta$ the interesting combinations are ϕ^2 and ϕ^3 multiplied by spurion fields of dimensions 1 and 0 respectively.

The χ -independent, dimension 0 and 1 spurion fields at our disposal are:

$$\eta = \mu\theta^2 \qquad U = \mu^2\theta^2\bar{\theta}^2 \qquad (d=0)$$
$$\chi = \mu^2\theta^2 \qquad (d=1) \tag{112}$$

(μ is some mass)

Hence allowed soft breaking terms are :

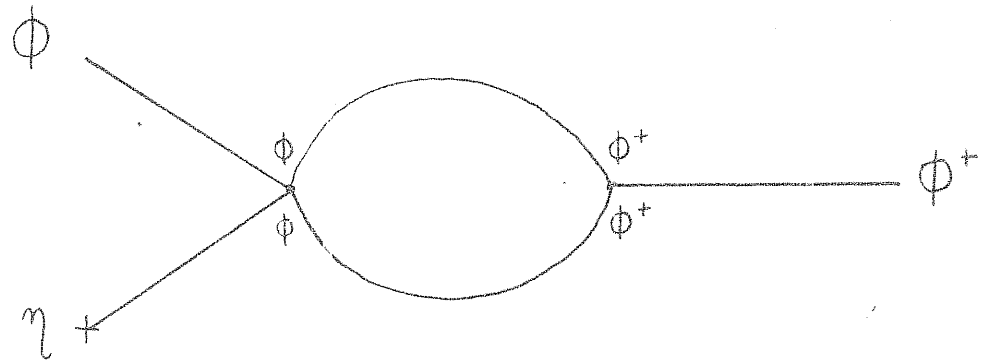
$$\Delta\mathcal{L}_0 = \int d^4\theta\, \eta\, \phi^+\phi$$

$$\Delta\mathcal{L}_1 = \int d^4\theta\, U\, \phi^+\phi$$

$$\Delta\mathcal{L}_2 = \int d^2\theta\, \chi\, \phi^2 \quad + h.c. \tag{113}$$

$$\Delta\mathcal{L}_3 = \int d^2\theta\, \eta\, \phi^3 \quad + h.c.$$

Actually $\Delta\mathcal{L}_0$ is just the counterterm required to cancel one of the infinities generated by $\Delta\mathcal{L}_3$;for instance in the graph:

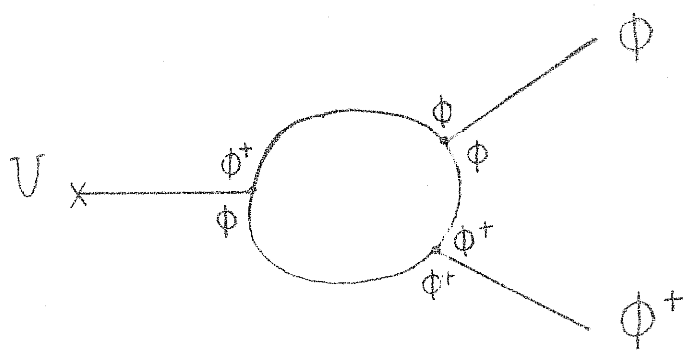


So we take as independent terms $\Delta\mathcal{L}_1$, $\Delta\mathcal{L}_2$ and $\Delta\mathcal{L}_3$. Let us briefly comment each of these terms.

1.
$$\Delta\mathcal{L}_1 = \int d^4\theta\, V\, \phi^+\phi =$$
$$= \mu^2 A^* A \tag{114}$$

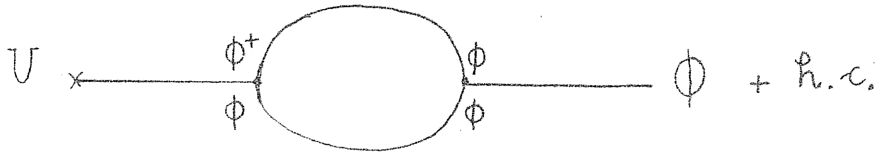
where the expression of $\Delta\mathcal{L}_1$ in terms of component fields is obtained by making the θ-integration (see also (A.7) in the ap pendix).

Then a renormalization of the term itself is required as one can understand by looking, for istance, at the following graph and applying the above explained power counting machinery:

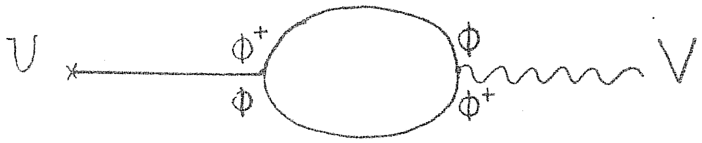


(115)

But other counterterms are required as one can see by the inspection of the following graphs:



(116)



(117)

Graph (116) requires a counterterm of the kind:

$$\Delta\mathcal{L} \sim \int d^4\theta\, M\, U(\phi+\phi^+) \sim \mu^2 M (A + A^*) \tag{118}$$

(M coming from the propagators $\langle\phi\phi\rangle$ and $\langle\phi^+\phi^+\rangle$)

Graph (117) requires a counterterm like:

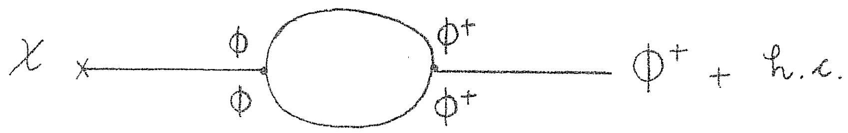
$$\Delta\mathcal{L} \sim \int d^4\theta\, U\, D^\alpha W_\alpha \sim \mu^2 D \tag{119}$$

(the combination $D^\alpha W_\alpha$ being present because of gauge invariance) .

2.

$$\Delta\mathcal{L}_2 = \int d^2\theta\, \chi\, \Phi^2 + h.c. = \\ = \mu^2 (A^2 + A^{*2}) \tag{120}$$

$\Delta\mathcal{L}_2$ generates the new divergence corresponding to the diagram:



which requires as counterterm:

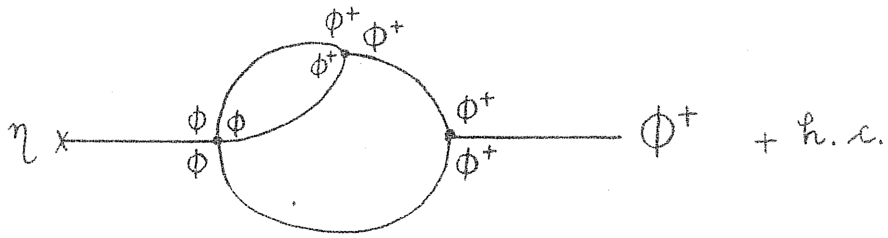
$$\Delta\mathcal{L} = \int d^4\theta(\chi\Phi^+ + \chi^+\Phi) \sim \tag{121}$$

$$\sim \mu^2(F+F^*)$$

3. $$\Delta\mathcal{L}_3 = \int d^2\theta\, \eta\, \Phi^3 + h.c. =$$

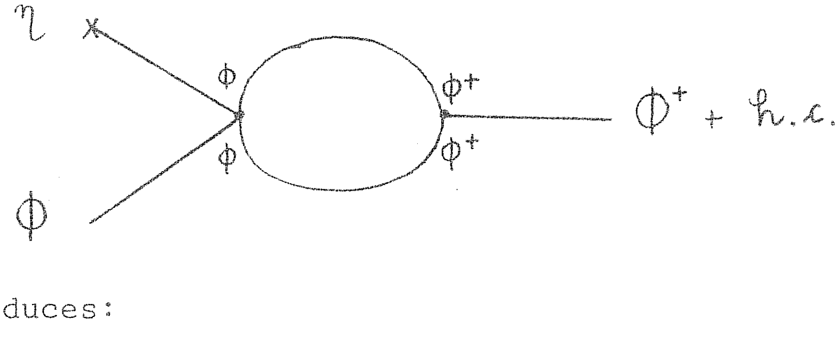
$$= \mu(A^3 + A^{*3})$$

New divergences and corresponding counterterms can be read off the following graphs:



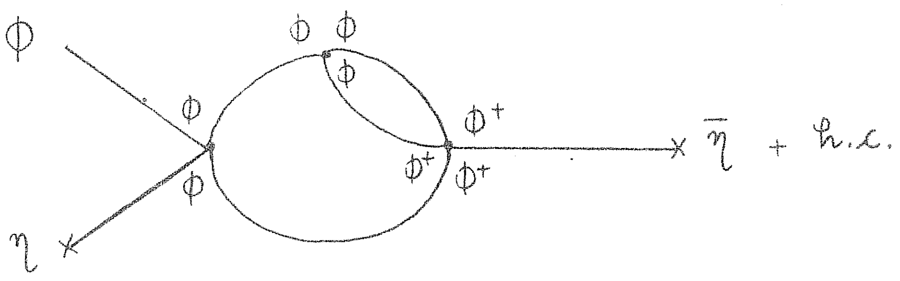
which induces:

$$\Delta\mathcal{L} = \int d^4\theta\, M\eta(\Phi^+ + \Phi) \sim \mu M(F^* + F) \tag{122}$$



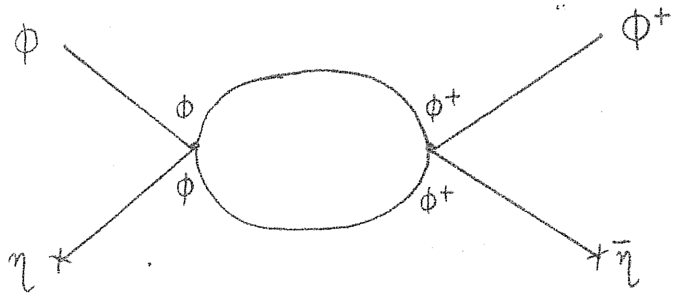
which induces:

$$\Delta\mathcal{L}_0 = \int d^4\theta\, \eta\, \Phi^+\Phi + h.c. \sim \mu\,(AF^* + A^*F) \tag{123}$$



which induces:

$$\Delta\mathcal{L} = \int d^4\theta\, \eta\bar{\eta}\, M(\Phi + \Phi^+) \sim \mu^2 M(A + A^*) \tag{124}$$



which induces:

$$\Delta\mathcal{L} = \int d^4\theta\, \eta\bar{\eta}\, \Phi^+\Phi \sim \mu^2 A^*A \tag{125}$$

The analysis for soft breaking terms containing vector superfields goes along the same line as described above and leads to these conclusions: the following soft breaking term is allowed:

4. $$\Delta \mathcal{L}_4 = \int d^2\theta \, \eta \, W^\alpha W_\alpha \sim \mu \lambda^\alpha \lambda_\alpha \tag{126}$$

(It is a mass terms for the gaugino λ). The new (logarithmic) divergences which originates from this term are of the same type as those induced by the term $\Delta \mathcal{L}_3$.
Thus the required counterterms are (122), (123), (124) and (125).

This concludes our discussion on soft breaking terms. We want to end this section with an example of a term which explicitly breaks supersymmetry but is not soft. A mass term for the spinor ψ of a chiral multiplet Φ provides such an example:

$$\Delta \mathcal{L} = \int d^4\theta \, \mu \, \theta^2 \bar{\theta}^2 (D^\alpha \Phi)(D_\alpha \Phi)$$

$$\sim \mu \, \psi^\alpha \psi_\alpha$$

The Feynman rule in superspace for this vertex consists in six factors of D , $\bar{D}$ thus leading to quadratic divergences.

* * *

APPENDIX:
SUPERSYMMETRIC GAUGE THEORIES

1. ABELIAN CASE

Consider a set of chiral (scalar) superfields $\Phi_k(x,\theta,\bar{\theta})$ together with a vector superfield $V(x,\theta,\bar{\theta})$. By definition they satisfy the relations:

$$\bar{D}_{\dot{\alpha}}\Phi_k = 0 \tag{A.1}$$

$$V^{+} = V \tag{A.2}$$

D_α , $\bar{D}_{\dot{\alpha}}$ are the spinorial covariant derivatives, whose expression in the so-called vector realization is given by:

$$D_\alpha = \partial_\alpha + i\,\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \tag{A.3}$$

$$\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\,\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{A.4}$$

Fundamental algebraic relations between $D's$ and $\bar{D}'s$ are:

$$\{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0 \tag{A.5}$$

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\,\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{A.6}$$

The expansions of the solutions of constraints (A.1) and (A.2) in powers of $\theta's$ look like:

$$\Phi_k = e^{i\theta\sigma^\mu\bar{\theta}\partial_\mu}\left(A_k + \sqrt{2}\,\theta\Psi_k + \theta\theta F_k\right) \tag{A.7}$$

$$V = \ldots - \theta\sigma^{\mu}\bar{\theta}v_{\mu} + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D \qquad (A.8)$$

where A_k , ψ_k , F_k , v_μ , λ , D are functions of x alone, called component fields.

U(1) gauge transformations on Φ_k and V are defined by:

$$\Phi_k' = e^{-it_k\Lambda}\,\Phi_k \qquad (A.9)$$

$$V' = V + i(\Lambda - \Lambda^+) \qquad (A.10)$$

t_k is the U(1) charge of Φ_k ; Λ is a chiral scalar superfield. A gauge exists, the Wess-Zumino gauge, where the terms indicated by dots in (A.8) vanish.

The most general renormalizable action involving these superfields and invariant under the U(1) gauge transformations (A.9), (A.10), as well as under N=1 global supersymmetry is given by:

$$A = A_{YM} + A_G + A_C + A_{FI} \qquad (A.11)$$

with:

$$A_{YM} = \frac{1}{4}\int d^4x\, d^2\theta\, WW + \frac{1}{4}\int d^4x\, d^2\bar{\theta}\, \bar{W}\bar{W} \qquad (A.12)$$

$$A_G = \int d^4x\, d^4\theta\, \Phi_k^+\, e^{t_k V}\,\Phi_k \qquad (A.13)$$

$$A_C = \int d^4x\, d^2\theta\, f(\Phi) + h.c. \qquad (A.14)$$

$$A_{FI} = \int d^4x\, d^4\theta\, 2\, k V \tag{A.15}$$

here:

$$W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V \tag{A.16}$$

is the field strength;

$f(\Phi)$, the superpotential, is forced by renormalizability to be at most a cuby polynomial:

$$f(\Phi) = a + \lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \tag{A.17}$$

In the Wess-Zumino gauge, the action (A.11) can be thought as referring to the following lagrangian density for the component fields:

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4} v_{\mu\nu} v^{\mu\nu} - i \lambda \sigma^\mu \partial_\mu \bar{\lambda} + \frac{1}{2} D^2 + \\
& + A_k^* \Box A_k + i \partial_\mu \bar{\Psi}_k \bar{\sigma}^\mu \Psi_k + F_k^* F_k + \\
& + t_k v^\mu \left(\frac{1}{2} \bar{\Psi}_k \bar{\sigma}^\mu \Psi_k + \frac{i}{2} A_k^* \partial_\mu A_k - \frac{i}{2} (\partial_\mu A_k^*) A_k \right) + \\
& - \frac{i}{\sqrt{2}} t_k \left(A_k \bar{\lambda} \bar{\Psi}_k - A_k^* \lambda \Psi_k \right) + \frac{t_k}{2} D A_k^* A_k \\
& - \frac{1}{4} t_k^2 v_\mu v^\mu A_k^* A_k + \\
& + f_i(A) F_i - \frac{1}{2} f_{ij}(A) \Psi_i \Psi_j + f_i^*(A) F_i^* - \frac{1}{2} f_{ij}^*(A) \bar{\Psi}_i \bar{\Psi}_j + \\
& + k D +
\end{aligned} \tag{A.18}$$

+ a four divergence.

here:

$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu \tag{A.19}$$

In discussing the Fayet-Iliopoulos model we have taken $t_1 = +e$ and $t_2 = -e$.
In discussing the general feature of a gauge supersymmetric theory - formulae (40) to (49) in the text - we have extracted from t_k the U(1) gauge coupling constant g_Y by defining the U(1) generator Y in the following way:

$$g_Y\, Y \equiv \frac{1}{2}\, \mathrm{diag}\,(t_1, \dots t_n) \tag{A.20}$$

2. NON ABELIAN CASE

Let G be a compact simple gauge group. Gauge transformations on a multiplet of chiral (scalar) superfields $\Phi \equiv \{\Phi_k\}$ and on the multiplet of vector superfields $\{V^a\}$ are now given by:

$$\Phi' = e^{-ig\Lambda}\,\Phi \tag{A.21}$$

$$e^{2gV'} = e^{-ig\Lambda^+}\, e^{2gV}\, e^{ig\Lambda} \tag{A.22}$$

here g is the gauge coupling constant;

$$\Lambda = \Lambda^a T^a \tag{A.23}$$

where T^a are the generators of G in the representation according to which the supermultiplet Φ transforms. They are propely norma lized:

$$\mathrm{tr}\, T^a T^b = k\, \delta^{ab} \tag{A.24}$$

Λ^a is a set of chiral scalar superfields:

$$V = V^a T^a \tag{A.25}$$

The most general renormalizable action involving these superfields and invariant under G as well as under N=1 global supersymmetry, is given by:

$$A = A_{YM} + A_G + A_C \tag{A.26}$$

with:

$$A_{YM} = \frac{1}{4g^2 k}\, \mathrm{tr}\left(\int d^4x\, d^2\theta\, WW + \int d^4x\, d^2\bar{\theta}\, \bar{W}\bar{W} \right) \tag{A.27}$$

$$A_G = \int d^4x\, d^4\theta\, \Phi^+ e^{2gV} \Phi \tag{A.28}$$

$$A_C = \int d^4x\, d^2\theta\, f(\Phi) + h.c. \tag{A.29}$$

here:

$$W_\alpha = -\frac{1}{4} \bar{D}^2 e^{-gV} D_\alpha\, e^{gV} \tag{A.30}$$

is the field strength; $f(\Phi)$ is the G-invariant superpotential (see (A.17)).

In the Wess-Zumino gauge, the lagrangian density giving rise to the action (A.26) reads:

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4} v^{a\mu\nu} v^{a}_{\mu\nu} - i \lambda^{a} \sigma^{\mu} \partial_{\mu} \bar{\lambda}^{a} + \frac{1}{2} D^{a} D^{a} \\
& - \frac{ig}{2} f^{abc} \lambda^{a} \sigma^{\mu} \bar{\lambda}^{b} v^{c}_{\mu} + \\
& + A^{*}_{k} \Box A_{k} + i (\partial_{\mu} \bar{\Psi}_{k}) \bar{\sigma}^{\mu} \Psi_{k} + F^{*}_{k} F_{k} + \\
& + g v^{a}_{\mu} \bar{\Psi} \bar{\sigma}^{\mu} T^{a} \Psi + \\
& + \sqrt{2}\, i g (A^{+} T^{a} \Psi \lambda^{a} - \bar{\lambda}^{a} \bar{\Psi} T^{a} A) + \\
& + i g v^{a\mu} (A^{+} T^{a} \partial_{\mu} A - (\partial_{\mu} A^{+}) T^{a} A) + \\
& - g^{2} v^{a\mu} v^{b}_{\mu} A^{+} T^{a} T^{b} A + g D^{a} A^{+} T^{a} A + \\
& + f_{i}(A) F_{i} - \frac{1}{2} f_{ij}(A) \Psi_{i} \Psi_{j} + f^{*}_{i}(A) F^{*}_{i} - \frac{1}{2} f^{*}_{ij}(A) \bar{\Psi}_{i} \bar{\Psi}_{j} +
\end{aligned} \qquad \text{(A.31)}$$

+ a four divergence

with:

$$v^{a}_{\mu\nu} = \partial_{\mu} v^{a}_{\nu} - \partial_{\nu} v^{a}_{\mu} + \frac{g}{2} f^{abc} v^{b}_{\mu} v^{c}_{\nu} \qquad \text{(A.32)}$$

Observe the absence of the Fayet-Iliopoulos term (A.15) in the non-abelian case.

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