ISAS - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

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" STELLAR EVOLUTION WITH OVERSHOOTING FROM CONVECTIVE CORES "

Thesis for the title of "Magister Philosophiae"

Candidate:

Alessandro BRESSAN

Supervisor:

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Contents

Introduction

In this thesis I will review some problems concerning the evolution theory of massive and intermediate mass stars.

Even if current stellar evolution theory predicts a number of results that are in good agreement with the observational scenario, some unexpected results arise when one tries to get a more detailed comparison with experimental data. This happens not only for the most advanced stages of stellar evolution, where the theory is more uncertain, but even in the early stages, like the Hydrogen or Helium burning ones, where the physical structure of the models is fairly well known.

Among other effects I will be mainly concerned with the problem of convective mixing in stellar cores. As summarized in the first chapter, it is actually possible that the usual Schwarzschild's criterion for determining the border of the convective core, greatly underestimates its true extension.

Some alternative criteria are also described in chapter I. The main outcoming result is that when the local treatement is given up, one is forced to use some arbitrary parameters for describing convection in a non-local way.

ChapterII deals with the effects of applaying a new criterion to the massive star evolution. Stringent observational data for these very luminous stars, suggest that the amount of "extra" mixing would be considerable.

In chapter III the same analysis is performed for intermediate mass stars, and a variety of problems which arise in comparing standard evolutionary results whit observations, is described.

Chapter IV briefly concernes with some aspects of stellar evolution theory for low mass stars.

Taking into account the complete scenario one can infer that an enlarged mixing inside stars, which have a convectively unstable core, can remove a series of discrepancies between theory and observations.

However to get a careful quantitative comparison, a systematic analysis of the new models is required together with complete sets of isochrones for a subsequent more significant statistical comparison with observations.

Chapter I

The standard criterion

The standard treatement of convection in stellar interiors rests upon the simple analysis which leads to the so-called Schwarzschild's criterion.

For a chemically homogeneous star model simple considerations (Cox and Giuli 1968 p. 262) show that a sufficient condition for stability against convection is given by :

$$\left.\frac{\partial T}{\partial r}\right|_{matter} > \left.\frac{\partial T}{\partial r}\right|_{adiabatic}$$

which may also be written as :

$$\nabla_{radiative} < \nabla_{adiabatic}$$

$$\text{where} \quad \nabla = \frac{\partial \ln T}{\partial \ln P}$$

It is important to realize that this criterion defines the zones that are convectively stable or unstable; but it cannot say anything about the nature of the convective medium. This requires the solution of the full system of fluidodynamic equations which is one of the greatest task of physic.

To avoid excessive complexity , some simple models are studied, and the one which is mostly employed in stellar evolution is the Mixing Lenght Theory (Bohm-Vitense 1958). In this approximation convection is described by means of eddies that mantain their own identity along a mean free path L ,before dissolving and mixing with the surrounding matter.

Usually, the mixing lenght L is expressed as a fraction of a characteristic scale height, say the pressure scale height Hp: $(H_p)^{-1} = P^{-1} \frac{\partial P}{\partial r}$

$$L = \lambda \cdot H_p$$

where λ is the so called mixing length parameter. When this theory is applied to the convective cores of stellar models it turns out that its efficiency (defined as the ratio between the energy losses during the motion of eddies and the excess heat content they have when they dissolve), is so high that convective elements may be treated as adiabatic (Cox and Giuli 1968)

Therefore in standard evolutionary computations one assumes the following scheme to deal with the convective cores:

i) the mean gradient of the convective medium is the adiabatic one

ii) convection extends and mixes over the whole unstable region.

Due to this simple description nothing can be said about the velocity field inside convective cores, and one usually makes the approximation that it coincides with the accelerating field (the unstable region). Thus it is implicitely assumed that the extension of convective motions inside stable regions, which in advance we will call "overshooting", is neglegibly small. However, as we will see, to neglect this phenomenon may lead to important consequences in stellar evolution.

Non local approaches.

In the last few years non-local approaches in the treatment of convective cores, have been the subject of a great amount of theoretical work.
First attempts to evaluate the extension of the overshooting region lead to the conclusion that overshooting was neglegibly small. (Roxburgh 1965, Saslow and Schwarzschild 1965). Contrary to what belived, recent studies about this phenomenon show that the scale length of the overshooting region is a non neglegible fraction of the mixing length itself (Shaviv and Salpeter 1973 , Maeder 1975 , Maeder and Mermillod 1981). Most of the recent studies rest upon some parameters that introduce a degree of arbitrariness which may be eliminated only through a careful comparison between model predictions and observations. Altough based on rather arbitrary assumptions about this parametrization, some works clearly illustrate the model responce to a variation of the mass size of the mixed region (Massevitch et al. 1979, Doom 1982 a,b) More realistic treatements rest upon some extension of the Mixing Length Theory (Maeder 1975, Bressan et al. 1981, Matraka et al. 1982) thus having as a free parameter the mixing

length itself. Further investigations describe convection in terms of turbulence, introducing the characteristic scale length of turbulence as a free parameter. However, Roxburgh (1975) and, more recently, Doom(1984) have derived a particular formulation to treat convective overshooting. Roxburgh's criterion for convective overshooting is intended to improve upon other previous descriptions of this physical fenomenon ,mainly because its final formulation does not contain arbitrary free parameters. Owing to this,a careful reexamination of the whole problem seems to be required, aimed to assess the validity of Roxburgh's criterion and its physical interpretation recently brought about by Doom (1984).

The Roxburgh criterion.

Roxburgh's criterion rests upon the averaged form of the thermal energy conservation equation of fluidodynamics;and its derivation proceedes as follows. The mass,momentum and total energy conservation equations in which gravity,nuclear energy sources,radiative flux are accounted for, while viscosity,conductive flux and composition changes are neglected can be written in cartesian coordinates (Ledoux and Walraven 1958)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho v_i) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}(\rho v_i v_j) = -\frac{\partial P}{\partial x_i} - \rho g_i \tag{2}$$

$$\frac{\partial}{\partial t}\rho\left(e + \frac{v^2}{2}\right) + \frac{\partial}{\partial x_i}\left[\rho v_i\left(W + \frac{v^2}{2}\right)\right] = \varepsilon\rho - \frac{\partial F_i}{\partial x_i} - \rho g_i v_i \tag{3}$$

where summation is intended on repeated indices,and symbols have the following meaning:

$W = e + P/\rho$ = enthalpy per unit mass

e =internal energy per unit mass

ε =rate of nuclear energy production per unit mass

P, ρ, T =pressure,density,temperature

F_i, g_i, v_i = i-component of radiative flux,gravitational acceleration and velocity.

Multiplying eq.(2) by v_i one obtains the mechanical energy conservation equation

$$\frac{\partial}{\partial t}\left(\rho\frac{v^2}{2}\right)+\frac{\partial}{\partial x_i}\left(\rho v_i\frac{v^2}{2}\right)=-v_i\frac{\partial p}{\partial x_i}-\rho v_i g_i \qquad (4)$$

Subtracting eq. (4) from eq. (3) and using the thermodynamic identity

$$dW=T\,dS+\frac{dp}{\rho} \quad ; \quad S=\text{specific entropy} \qquad (5)$$

one obtains the thermal energy conservation equation(Ledoux and Walraven 1958)

$$\frac{\partial}{\partial t}(\rho S)+\frac{\partial}{\partial x_i}(\rho S v_i)=-\frac{1}{T}\frac{\partial F_i}{\partial x_i}+\frac{\varepsilon\rho}{T}\equiv\frac{A}{T} \qquad (6)$$

Following Roxburgh(1978), I now specify the above equation for the case of a turbulent medium. If the characteristic scales of turbulence are much shorter than the characteristic scales of the convective medium ,it is possible to separate thermodynamic quantities $f(x_i,t)$ in a mean part independent of time $f_o(x_i)$ and a fluctuating time dependent part $f_1(x_i,t)$ in such a way that

$$f(x_i,t)=f_o(x_i)+f_1(x_i,t) \qquad (7)$$

$$f_1/f_o \ll 1$$

$$\overline{f(x_i,t)}=f_o(x_i)$$

where the bar denotes an average process over fluctuations. In particular note that conservation of mass requires, (no net transport of mass):

$$\overline{\rho v_i}=0 \qquad (8)$$

and then for any thermodynamic quantity f

$$\overline{\rho v_i f}=\overline{\rho v_i f_o}+\overline{\rho v_i f_1}=\overline{\rho v_i f_1}$$

Furthemore from eq. (1) one gets

$$\overline{\rho v_i \frac{\partial \rho}{\partial x_i}} = \frac{\partial}{\partial x_i}\overline{\rho v_i \rho_1} + \overline{\rho_1 \frac{\partial \rho_1}{\partial t}} \simeq \frac{\partial}{\partial x_i}\overline{\rho v_i \rho_1} \qquad (9)$$

With the aid of the above equations the Roxburgh condition can be easily derived. Equation (5) writes in the averaged form

$$\frac{\partial}{\partial t}\overline{(\rho_1 S_1)} + \frac{\partial}{\partial x_i}\overline{(\rho v_i S_1)} = \frac{A_0}{T_0} - \frac{\overline{A_1 T_1}}{T_0^2}\,, \qquad (10)$$

and neglecting the first and last term

$$\frac{\partial}{\partial x_i}\overline{T_0 \rho v_i S_1} - \overline{\rho v_i S_1}\frac{\partial T_0}{\partial x_i} = A_0 = -\frac{\partial F_i}{\partial x_i} + \varepsilon_0 \rho_0 \qquad (11)$$

Integrating eq. (11) in spherycal simmetry from r_1 to r_2 (the borders of the convective zone) one has

$$r^2 T_0 \overline{\rho v S_1}\Big|_{r_1}^{r_2} - \int_{r_1}^{r_2} r^2 \overline{\rho v S_1}\frac{\partial T_0}{\partial r}\,dr = -r^2 F_0\Big|_{r_1}^{r_2} + \int_{r_1}^{r_2}\varepsilon_0 \rho_0 r^2\,dr$$

Now $v(r_1)=v(r_2)=0$ by definition and defining $\varepsilon_0 \rho_0 r^2 = \frac{\partial}{\partial r}\left(\frac{L_N}{4\pi}\right)$ one gets $4\pi r^2 F_R\Big|_{r_1}^{r_2} = L_N\Big|_{r_1}^{r_2}$

because outside the convective region energy is supposed to be carried out by radiation alone. Then the Roxburgh condition is:

$$\int_{r_1}^{r_2} 4\pi r^2\,\overline{\rho v S_1}\,\frac{\partial T_0}{\partial r}\,dr = 0 \qquad (12)$$

This condition is integrable if one knows the temperature gradient,the run of s ,v and across the convective region. Since these quantities are not known a priori, the difficulty is overcome by Roxburgh defining the convective flux to be

$$F_c = T_0 \overline{\rho v S_1}$$

Thus equation (12) becomes

$$\int_{r_1}^{r_2} 4\pi r^2 \frac{F_c}{T_0} \frac{\partial T_0}{\partial r} dr = 0 \qquad (13)$$

and because $\frac{\partial T_0}{\partial r} < 0$, inside stars, Fc has to change sign (this corresponds to overshooting). Condition (13) may easily be integrated if one estimates Fc and $\frac{\partial T_0}{\partial r}$ (≃adiabatic one) in the convective region(Roxburgh 1978). The nature of the integrand in equation (12) has been however recently questioned by Eggleton(1983) who mainly arose doubts about its physical nature. Doom (1984) has identified this term with the divergence of the turbulent kinetic energy flux F_t and has reobtained the same condition. However both conditions(12) and (13) may be subjected to some criticism,as it will be discussed in the following ,which invalidate their correctness.

First I like to clarify the role of pressure fluctuations that explicitly are said to be accounted for in obtaining condition (12) and (13) in Roxburgh (1978). If pressure fluctuations are not neglegible, then from equation (3) one sees that the convective flux is given by

$Fc_i = \overline{\rho v_i W} = \overline{\rho v_i W_1} = T_o \overline{\rho v_i S_1} + \overline{v_i P_1}$

where $W_1 = T_o S_1 + \frac{1}{\rho_o} P_1$ for fluctuating quantities

and not simply $Fc_i = T_o \overline{\rho v_i S_1}$ as in Roxburgh.

The contribution to Fc by the turbulent kinetic energy flux Fk is not contained in the above expression,as it is accounted for separately in the basic equations. The presence of the term $\overline{v_i p_1}$ obviously invalidates the Roxburgh criterion because now the a priori knowledge of the structure variables v_i and p_1 is necessary to proceed further. On the other hand ,pressure fluctuations are neglegibly small when turbulent velocities are much less than the sound velocity (Cox and Giuly 1968).

As a consequence of this, the average kinetic energy flux is neglegible with respect to the thermal convective flux as it may be seen on the basis of the following estimates.

The turbulent kinetic energy flux Fk is (i-component)

$$F_{K_i} = \frac{1}{2} \overline{\rho v^2 v_i} \sim \rho_o \overline{v^2} \, \overline{v_i}$$

while the convective flux is

$$Fc_i = \overline{C_p \rho T v_i} \simeq C_p T_o \rho_o \overline{v_i} \simeq \rho_o v^2_{Sound} \overline{v_i}$$

C_p = specific heat at constant Pressure

(note that $\overline{v}$ is a very small quantity as log as ρ_1/ρ_o is << 1

$\overline{v_i} = - \overline{\frac{\rho_1}{\rho_o} v_i}$)

so that

$$F_{Ki}/F_{ci} \simeq \frac{\overline{v^2}}{v_{sound}^2} \ll 1 \quad \text{if } v \ll v_{sound}$$

Therefore, if pressure fluctuations are neglegibly small the averaged form of the total energy conservation equation (3) becomes

$$\frac{\partial}{\partial X_i} T_o \overline{\rho V_i S_1} = A_o = -\frac{\partial \overline{F_i}}{\partial X_i} + \mathcal{E}_o \rho_o \qquad (14)$$

This equation is about equal to the average form of the thermal energy equation (11) since mechanical terms in it turn out to be neglegibly small (note that I refer to averaged quantities). Equations (11) and (14) must differ within the order of approximation we have been using in the derivation procedure. In fact, subtracting eq. (11) from eq. (14) one gets (neglecting time derivatives)

$$\overline{\rho V_i S_1} \frac{\partial T_o}{\partial X_i} = \frac{\overline{A_1 T_1}}{T_o} \qquad (15)$$

which explicitly shows that Roxburgh's criterion follows from a*, while other of the same order have been neglected. Turning now to the interpretation given by Doom (1984) according to which the term of relation (15) identifies with the divergence of the turbulent kinetic energy flux it can be argued as

* Second order approximation term

follows.

Averaging eq (5) one gets

$$\frac{\partial}{\partial x_i} \rho v_i \frac{v^2}{2} = - v_i \frac{\partial p}{\partial x_i} \qquad (16)$$

which may be written using the thermodynamic relation (5)

$$\frac{\partial}{\partial x_i} \overline{\rho v_i \frac{v^2}{2}} = - \frac{\partial}{\partial x_i} \overline{\rho v_i W_1} + \frac{\partial}{\partial x_i} T_0 \overline{\rho v_i S_1} - \overline{\rho v_i S_1} \frac{\partial T_0}{\partial x_i} + \overline{\rho v_i T_1} \frac{\partial S_0}{\partial x_i}$$

inserting the following relations for the fluctuations $W_1 = T_0 S_1 + \frac{1}{\rho_0} P_1$ one gets

$$\frac{\partial}{\partial x_i} \overline{\rho v_i \frac{v^2}{2}} = - \frac{\partial}{\partial x_i} \overline{v_i P_1} - \overline{\rho v_i S_1} \frac{\partial T_0}{\partial x_i} + \overline{\rho v_i T_1} \frac{\partial S_0}{\partial x_i} \qquad (17)$$

This equation first shows that the divergence of Fk is not simply given by $\frac{\partial F_{k i}}{\partial x_i} = \overline{\rho v_i S_1} \frac{\partial T_0}{\partial x_i}$ but further terms have to be considered.

Second, if one integrates equation (17) from r_1 to r_2 (borders of convective region) the condition below is derived

$$\int_{r_1}^{r_2} r^2 \overline{\rho v S_1} \frac{\partial T_0}{\partial r} dr = \int_{r_1}^{r_2} r^2 \overline{\rho v T_1} \frac{\partial S_0}{\partial r} dr$$

which clearly contraddicts Roxburgh's condition since the two term are independent and thus cannot be set equal to zero separately.

Conclusions

The Roxburgh's criterion for determining the extension of stellar convective cores result from arbitrarily retaining in first order approximation of thermal energy conservation equation, the term

$$\overline{\rho v_i S_1} \frac{\partial T_0}{\partial x_i}$$

which in reality turns out to belong to the second order approximation of the same equation (involving also time derivatives of the fluctuations) so that further terms should be included for the sake of internal consistence. Consequently the identification of the above term with the divergence of the turbulent kinetic energy flux suggested by Doom(1984) is not correct. Therefore , despite of their simplicity and lack of any free parameter those criteria are not physically consistent.

Other criteria

In the previous section I was mainly devoted to treat those criteria that the authors claimed to be parameter independent.
We have seen that they are subjected to some severe criticism which invalidates their usage for stellar model computations. It is important to stress here the need of some arbitrary parameter for a non local description of convection; moreover, as we are dealing with something which, by itself is not directly "observable" from outside, we are left with the hard task of evaluating this parameter from the effects it produces as the models evolve.
Then a series of evolutionary sequencies has to be computed varying such a parameter, and model predictions have to be compared with observations. This requires a lot of computational effort, even if convection is treated locally; when a non local criterion is introduced the situation gets much worse.
In fact one of the most serious difficulties in dealing with overshooting is that iterative procedures are usually required which enormously increase the computing time (20 time longer using

Maeder's algorithm). Therefore when evolutionary sequences were calculated ,the amount of overshooting was tested only in some models and then fixed during subsequent evolution.

However recently, Bressan et al.(1981) have developed a new method which, in the framework of the mixing lenght theory, enables one to calculate overshooting from convective cores in a simple way and with a reasonable amount of computational effort.

New investigations on stellar evolution described in the following section will mainly rest upon this criterion.

The overshooting model of Bressan et al.(1981)

The method proposed for evaluating the layer reached by convective elements which overshoot from the unstable core rests on the following scheme. It seems resonable (Shaviv and Salpeter 1973, Maeder 1975) that, independently from the theory adopted to follow convective motions, the temperature distribution in deep interiors of the stars can be the one characterized by the adiabatic gradient up to the region where velocities of convective elements vanish. The run of the real gradient is schematized in fig.1).

To determine the border of convective region one may proceed in the following way.

In the framework of the mixing length theory the acceleration given to a convective element is expressed by(considering olny upward moving elements):

$$v_r \frac{\partial v_r}{\partial r} = - g \frac{\Delta \rho}{\rho} \qquad (18)$$

where g, ρ, v_r are the gravitational acceleration,density and velocity. $\Delta \rho$ is the density defect and it is given by:

$$\Delta \rho(r) = \int_{r_1}^{r} \left[\frac{\partial \rho^*}{\partial r'} - \frac{\partial \rho}{\partial r'} \right] dr' \qquad (19)$$

where r_1 is the starting level ,asterisk refers to convective elements and other quantities to the surrounding matter.

By means of equation of state eq.(19) may be written as

$$\Delta \rho = -\int_{r_1}^{r} \frac{\rho}{T} \frac{\chi_T}{\chi_\rho} \left(\frac{\partial T^*}{\partial r'} - \frac{\partial T}{\partial r'} \right) dr' + \int_{r_1}^{r} \frac{\rho}{\mu} \frac{\chi_\mu}{\chi_\rho} \frac{\partial \mu}{\partial r'} \, dr' \qquad (20)\ ;$$

here T and μ are the temperature and molecular weight and

$$\chi_T = \left. \frac{\partial \ln P}{\partial \ln T} \right|_{\rho,\mu} ; \quad \chi_\rho = \left. \frac{\partial \ln P}{\partial \ln \rho} \right|_{T,\mu} ; \quad \chi_\mu = \left. \frac{\partial \ln P}{\partial \ln \mu} \right|_{\rho,T}$$

The convective flux Fc (carried by elements originating at level r_1) at level r is given by

$$F_c = K c_p \rho v_r \int_{r_1}^{r} \left[\frac{\partial T^*}{\partial r'} - \frac{\partial T}{\partial r'} \right] dr \qquad (21)$$

where Cp is the specific heat at constant pressure and the factor K take into account some average between rising and falling elements.

From eq.(20) and eq.(21) one can see that it is possible to express the quantity $\Delta \rho$ in eq.(19), if some mean values $\left\langle \frac{\rho}{T} \frac{\chi_T}{\chi_\rho} \right\rangle$, $\left\langle \frac{\rho}{\mu} \frac{\chi_\mu}{\chi_\rho} \right\rangle$ over the distance $r - r_1$ are assumed, by means of the convective flux Fc and the molecolar weight .

Then equation (19) writes:

$$v \frac{\partial v}{\partial r} = \frac{g}{\rho} \left\langle \frac{\chi_T}{\chi_\rho} \frac{\rho}{T} \right\rangle \frac{F_c}{K C_p \rho v} - \frac{g}{\rho} \left\langle \frac{\chi_\mu}{\chi_\rho} \frac{\rho}{\mu} \right\rangle \Delta \mu \qquad (22)$$

where

$$\Delta \mu = \mu(r) - \mu(r_1)$$

The convective flux Fc can be derived from the equality

$$F_{tot}=F_r + F_c \qquad (23)$$

where Ftot is the total available energy flux (generated by nuclear sources).

It has to be stressed that this treatment only holds in convective cores, where the real gradient is known to be approximatly given by the adiabatic one; in this way the radiative flux Fr is known and then from eq. (23) one gets the convective flux Fc. This does not hold in outer convective layers in which the real gradient strongly depends on the convective flux itself.

All quantities being known, integration of eq.(22) is carried out from an initial level r_1 out to the level r_1 +l (where l is the mixing length parameter), or out to the level at which velocity vanishes (inside the stable region). The highest level at which velocity is not zero is chosen to be the border of the convective core.

Some comments are needed about eq.(22). As far as the term involving the molecular weight difference is concerned, numerical experiments show that when $\Delta\mu \neq 0$ the convective motions abruptly stop . In fact, the negative contribution given by this term is by several order of magnitude greater than that of the temperature excess.

However if the mixing process is assumed to be very efficient (the timescales of convective motions l/v are much smaller than nuclear timescales), then arguments invoked by Castellani(1971), indicate that in a very short time, barriers of molecular weight might be eroded. Thus eq.(22) may be used removing the last term as it is the case for chemical homogeneous models or for shrinking cores.

Viscosity has been neglected at all; this is because of the high Reynolds number characterizing stellar interiors (due to large dimensions of eddies ≃ l), which is a dimensionless measure of the ratio between inertial and viscosity terms in the momentum conservation equation of fluidodynamics. However, viscosity may be introduced in eq.(22) through a term proportional to some power of the velocity.

The average quantities in eq.(22) may also be substituted by local ones without affecting significantly the results (numerical experiments have been performed).

With all these simplifications eq.(22) is easily adapted to numerical computation on a Henyey scheme; the required time is only a factor of two or less greater than the one of standard models.

Chapter II

Results for a homogeneous model

Some results of the application of the new criterion for convective cores to homogeneous models are shown in fig.2) and they are compared with a model of different stellar mass obtained by Maeder(1975) Fig.3). In fig.2) the velocity curves for three omogeneous models of mass 20,60,100 and chemical composition X,Y,Z =0.7,0.28,0.02 are shown. The point where the usual Schwarzschild's criterion holds is also shown; the overshooting region extends far beyond this point The percentual amount in mass of this region for the above models is respectively 38%,14%,6%. No important differences in the internal structure between models with and without overshooting exist, as can be seen from fig.3) (Maeder 1975). The only main difference when overshooting is accounted for is in the size of the mixed region. This however will have important effects in the subsequent evolution.

Overshooting in massive stars.

Massive stars are defined to be those stars that never experience condition of degeneracy in the core and then undergo quiescent ignition of all nuclear fuels . In the standard theory the minimum mass for this range is of about 10-12 Mo. A number of evolutionary tracks for this range of masses may be found in the literature (Chiosi and Summa 197 ,Chiosi et al. 1978, Maeder 1982 a,b), however I will rest upon the work of Bressan et al. (1981) and Bertelli et al. (1984 a) . Tables 1) and 2) show some characteristic quantities of the models of 20,60,100 M⊙ and chemical composition X,Z=0.7,0.02 during the phase of central H burning ; on the zero age main sequence (ZAMS),point a) and at the minimum effective temperature ,point b). Also models with mass loss are included as will be discussed in the following section. Models are parametrized by the ratio

$$\lambda = L/H_p$$

Evolution at constant mass.

There are only slight differences in the luminosity and effective temperature (Teff) between models with overshooting (λ =1) and standard ones on the ZAMS (set A and A′ respectively). As evolution proceeds the effects of an enlarged size of the convective core cause the models with overshooting to be more cooler and luminous.

The 100 Mo model reaches the red zone in the H-R diagram while burning hydrogen in the center.

As more fuel is available, these models have a H-burning lifetime which is larger then that of the standard ones -about 30%,15%,11% for 20,60,100 M⊙ models. For more massive stars this fraction is lower because they are already characterized by a very large convective core.

Evolution with mass loss. When mass loss is included, according to the formalism of Castor,Abbott and Klein (1975) : $\dot{M} = \frac{L}{c V_{th}} \frac{\alpha}{\Gamma} \left| \frac{1-\alpha}{1-\Gamma} \right|^{\frac{1-\alpha}{\alpha}} (K\Gamma)^{1/\alpha}$

c = light velocity; V_{th} = thermal velocity; Γ = ratio of luminosity L to the Eddington luminosity; α, K parameters = 0.83, 0.01 (Chiosi et al. 1978)

the situation drastically changes with increasing mass. The main sequence band widens at lower masses more than in the case employing overshooting alone. On the contrary, for more massive stars (about 100 M⊙) the main sequence band shrinks toward the ZAMS. This behaviour can be attributed to the pealing off of the envelope which ,in the case of overshooting, is very efficient due to the increased lifetime.

The 100 Mo and 60 Mo models show, at their surfaces, highly CNO processed material (Xs=0.360 and 0.504 respectively) as the pealing has already reached the molecular weight profile generated by the receeding convective core.

He burning phase

As far as He burning is concerned, all models with overshooting alone, burn helium at their centers near the Hayashy line in the H-R diagram. For the models having both overshooting and mass loss the behaviour depends critically on the rate of mass loss in the red ; if this last is high enough the model will spent the last phase of He burning in the blue region. More massive stars, however, have already a low Hydrogen content and then experience He burning in the blue.

Comparison whit observations.

A great effort has been recently made to infer some constraints on evolution of massive stars from observational data.

A fairly complete set of luminous stars with Mb < -7 and within 2.5 Kpc around the Sun, is now available from the works of Humphreys(1978),Humphreys and Davidson(1979), Garmany et al.(1982)

The H-R diagram for supergiant stars in young clusters and associations in the solar vicinity shows the following mean features,fig.5):

i) a group of very luminous O and B stars with Mb<-10

ii)an envelope of decreasing temperature with declining luminosity for the hottest stars, with an upper limit for M supergiants at about -10<Mb<-9.5

iii) A continuous distribution of stars in the luminosity range -9.5<Mb<-7, up to spectral type A0 with a remarkable crowding in the range B0-B2, a gap for spectral types F,G,K and a clump of M type stars.

The standard theory of massive star evolution predicts a zone in the H-R diagram in which models evolve in a Kelvin time scale after the end of H burning and the beginning of He burning. However no gap of this type exists among stars more massive than about 15 M⊙ . The inclusion of mass loss somewhat relaxes this discrepancy for stars mor massive than about 40-50 M⊙, but the problem still remains for less massive stars.

As can be seen from fig.4) this situation may not be improved by models with overshooting alone because they predict too many bright stars in the region of more luminous supergiants (moreover mass loss is an "observed" phenomenon).

This discrepancy between theory and observations has been also noticed by Meyland and Maeder(1982). They have performed a comparative analysis of the colour-magnitude diagrams of 23 young clusters (age $<2*10^7$ years) in the Galaxy,Large and Small Magellanic Clouds. The observations of galactic clusters (11) were compared with theoretical evolutionary tracks accounting for a moderate mass loss rate but not for overshooting (Maeder 1981a,b).

In table 3) the lifetime ratios over the total one are compared with observed frequencies for each spectral type. The unescapable conclusion is that about 30% more stars are observed out of the predicted main sequence.

Models with overshooting and mass loss, on the contrary, greatly improve upon the agreement between theory and observations.

In the highest part of the H-R diagram they predict a narrow main sequence band (as it is observed), while in the lowest part they predict a widening up to Teff=4.3 with a lifetime in this extended region comparable with the He burning lifetime.

Furthermore Bertelli et al.(1984) isolated among the Humphreys catalog a strip at limiting magnitudes -7<Mb<-9 and improved this set with the help of the Garmany et al. (1982) catalog of bright O stars and with the Van der Hutcht et al. (1981) catalog of Wolf Rayet stars (which current scenario explains as post red supergiant stars). Table 4) shows observed absolute numbers N and number ratios N/Nt for three different grouping. Each of these follow a scenario proposed by theoretical evolution which rests upon the previous models; standard evolution,standard plus mass loss,overshooting plus mass loss. These scenarios predict a main sequence widening ,respectively, up to the spectral types O9.5,B0.5,B1. If a continuous star formation is assumed then the number ratio inside the MSB has to match the lifetime ratio of the corresponding phase. The last is given approximately by the Q values involved in the H and He nuclear reaction energy release (slightly modified by the core mass extent) ; one may guess that the He burning lifetime is at most 10% of the total lifetime (our models give about 6%).

This is the number of stars one expects to escape the observed main sequence band.

It is soon evident that a great discrepancy exists between theory and observation, as suggested from the number ratios of table 4). For the most favourable case still a 20% of stars are out of the MSB, which corresponds at more than two times the He burning lifetime. Moreover this case was obtained with an intermediate mass loss rate while current observations indicate that in the domain of this comparison a much lower mass is lost (Garmany et al.1981). This means that a overshooting parameter is required higher than λ =1 used for those computations, as shown by fig.6) (Bertelli et al. 1984).

However other effects might be responsible for the MSB widening. Bertelli et al.(1984) investigated the role played by radiative opacities together with non hydrostatic atmospheres as proposed by De Loore(1982). They came to the conclusion that opacities with features like those proposed by Carson(1976) could lead theoretical results in good

agreement with the observed scenario. These opacities, having a relative maximum around CNO ionization regions, provide more extended envelopes for stars which leave the ZAMS; however these features have benn shown to be physically unrealistic by more recent calculations (Carson et al.1984).
Stothers and Chin (1984) have provided further evidence of a main sequence widening . They compare theoretical isochrones which rest upon a rough parametrization of the overshooting, with very young star cluster turnups (the observed vertical part of the main sequence). The outcoming suggestion is that a substantial amount of overshooting (or another form of mixing) occours in cores of massive stars; the quoted value of λ corresponds to about 1.5 .

In conclusion different investigations based on a variety of observational material, agree in stating that standard evolutionary models, even with mass loss, cannot explain some basic features of the H-R diagram for the most luminous stars. In particular star counts for different spectral types coupled with observed turnup mophologies, indicate a much wider main sequence band than suggested by standard model computations. Theoretically such widening can be produced by a more extended internal mixing, or by enhanced opacities, like those proposed by Carson; the most realistic scenario is obtained when both are taken into account.

Chapter III

Intermediate mass stars.

Intermediate mass stars are defined those that ignite Helium in a non degenerate core , but develop a highly electron degenerate carbon-oxigen core following central He exhaustion.

Standard models confine intermediate mass stars in the range 2.2-8,9 Mo. The lower limit is set by the maximum mass which experiences central He flash . The upper limit corresponds to the stars in which carbon ignition starts off center in a middle degenerate core with an energy release that is high enough to remove degeneracy.

This nominal definition is however very dependent on details of stellar model computation and , ultimately ,it is the critical core mass for non degenerate ignition that defines this class of models. The critical mass is about 0.31 Mo for non degenerate He ignition and 1.06 Mo for non degenerate C ignition.

All stars that develop a He core of about 0.31 Mo before undergoing highly degeneracy and that undergo highly degeneracy before developing a C-O core of 1.06 Mo belong to this class.

The relation between these critical masses and the initial mass of the star Mi depends on a variety of inputs in model computations , like mass loss ,size of convective core , neutrino loss rates, etc.. . Then it is not surprising that changing one of the above conditions will primarily change the nominal definition of intermediate mass stars ,and of course of other mass ranges.

Here j will mainly concerne with the effect of overshooting during almost the entire evolution of these stars. These will be followed in the three major phases ; the Hydrogen burning, the Helium burning and the asymptotic giant branch (A.G.B.). Each phase will be treated separately and the major qualitative results of both the standard and non standard models will be outlined with the purpose of looking for observable effects which possibly may discriminate between the two proposed scenarios.

I will assume as representative of this mass range the stars of initial masses 5,7,9 M☉.

Among the several models already existing in the literature a few account for overshooting during H burning phase (Maeder 1976 ; Maeder and Mermillod 1981 ; Matraka et al. 1982 ; Huang and Weigert 1983) . When possible I will rest upon models provided by Bertelli et al. for sake of homogeneity.

The H-R diagram for model masses of 5,7,9 Mo is shown in fig. 7) for different values of the overshooting parameter λ .

The input physics is the following :

i) chemical composition X,Y,Z = 0.7, 0.28, 0.02

ii) opacity from tables of Cox and Stewart (1969) ;bilinear interpolation is used and ,for additional analysis, bicubic spline interpolation too.

iii)Nuclear reaction rates from Fowler et al. (1975)

iv) mixing length in the envelope λ_{m} =1.5

v) costant mass evolution

vi) overshooting treated as in Bressan et al. (1981) with λ =0,0.5,1

Core Hydrogen burning phase.

As in the case of massive stars, the zero age main sequence (Z.A.M.S.) is only slightly affected by convective overshooting.

The mass size of the convective core increases with increasing λ . Fig. 8) shows this quantity for homogeneous models and compares it with the results of Matraka et al. (1982) obtained for λ =0.25,0.5 but for a different chemical composition X,Z = 0.602,0.44 .

From the computations of Bressan et al.(1981) and Bertelli et al.(1984) it appears that the same overshooting parameter implies a relative growth of the size of the convective core which increases as the model mass decreases. For masses of 20,9,5 Mo and λ = 1 the relative fractionary mass increase Δ q/q is respectively 27%,49%,75% ($q = M_c/M_{tot}$).

On the other hand, a comparison with Matraka and al. results shows that with decreasing metallicity , both the standard and overshooting cores get larger : 19% for λ =0.5 and Mi= 5 Mo , while 15% for λ =0 and same mass .

During H-burning models with overshooting run at higher luminosities, this effect being growing during evolution , and extend to lower effective temperatures , causing the widening of the main sequence band.

In addition ,as more fuel is available , which largely overwhelms the effect of a greater luminosity , the core H-burning lifetime increases consistently.

Observational constraints.

During this phase the main characteristic of the models directly comparable with observational data is ,as for massive stars, the widening of the main sequence band.

A composite Mv-(U-B)o diagram of galactic clusters (from Maeder and Mermillod 1981)is shown in fig. 9) together with the main sequence band obtained from model computations of Bertelli et al. after transformation from Log L - Log Te to Mv-(U-B)o plane with relationships given by Buser and Kurucz (1978).

This diagram shows ,only qualitatively, that standard models cannot account for the widening of the main sequence band in the case of intermediate mass stars too. Quantitative results were not obtained here and are the purpose of a later work .

One may speculate about uncertainty in experimental data , when constructing composite diagrams,and in fact a more rigorous analysis has to base on systematic comparisons between observed and syntetic star clusters..

However here it is worth mentioning that when a comparison is made as in fig. 9) one has to keep in mind that about 50% of the H-burning lifetime is spent in a very narrow region along the Z.A.M.S. and this clearly stresses the discrepancy obtained from standard evolution.

Core Helium burning.

Following central H-exhaustion the stars move to the red region of the H-R diagram in a Kelvin time scale and then ,climb along the Hayashy line , while central He core heats due to the gravitational contraction and H burning proceeds in a shell.
At the same time the external convective envelope moves inward and eventually reaches layers left out by the receding H-burning core, thus starting the so called " first dredge up" of nuclear processed material.
When central density and temperature reach the critical value for He ignition this proceeds quiescentely because the inner core is not degenerate. Central convection is initiated ,driven by the high temperature dependence of He burning .
During subsequent evolution the He inert core continuously grows in mass ,because of the H shell burning, followed by the growth of the convective core.
The model luminosity is now slightly decreasing down to a minimum after which a blue loop is started in the H-R diagram.
In this phase standard models face with the so called problem of semiconvection. Due to the growth of the

convective core a steep discontinuity in molecular weight appears at its border , where opacity is very dependent on the He,C,O content. Region slightly outside the convective core are strongly unstable with respect to perturbation in molecular weight; if for some reason (penetration of convective elements, diffusion)this region is partially mixed with the core and the opacity considerably increases due to injection of carbon and oxigen rich material. The radiative gradient thus overcomes the adiabatic one and the region becomes completely convective.
This process propagates outward giving rise to a series of gradient profiles like in the fig.10), and arising the question of where the convective border actually is.

The difficulty is overcome by assuming that in the dashed region of fig. 10) , mixing is only partially efficient (semiconvection) giving rise to a molecular weight profile region in which the neutrality condition against convection holds.
This has been the subject of a great deal of theoretical work aimed to determine the true process

of mixing and the consequent neutrality condition. The problem is not yet solved and both the Scwarschild and Ledoux criteria for convective neutrality are found in the literature.

It is worth mentioning that some authors refer to this mixing process as to "overshooting during He burning" because ,in reality the inner full convective region moves slightly outward than what initially predicted; it is however a different kind of "overshooting", because no full penetration of convective motions in radiatively stable regions is considered.

On the contrary, models with overshooting parameter λ =1 never experience this phenomenon while, for λ =0.5 it is only marginally present only at the end of He burning phase. This is obviuos because when $\lambda \rightarrow 0$ one expects to recover the standard results.

Turning now to the H-R diagram morphology it is well known that when the loop crosses the Cepheid instability strip the star start pulsating with a well known relation between the period and the average density. Although one of the major goals of the stellar evolution theory is the prediction of the existence of these very typical objects still some problems exist ,as we will see in the following.

The Cepheid phase.

As noticed by Matraka et al.(1982) and by Huang and Weigert(1983) overshooting strongly affects the morphology of the H-R diagram during He burning.
The models evolve at higher luminosity ,with a Δ Log L which is about 0.5 for M=5Mo and 0.3 for M=9Mo and λ =1. Furthermore the extension of the blue loop is lowered as the size of the overshooting region increases.
Matraka et al.(1982) argued that this would constitute a strong difficulty for model with overshooting and ,in fact, their calculations showed that only stars more massive than about 6 M$_\odot$ would be able to cross the instability strip. This in turn, would mean that no cepheids with periods shorter than about 8 days could be explained by theory.
They were also able to show that the morphology of the loop is profoundly affected by other agents,among which the most important is the treatment of the external convective envelope. If the mixing length ratio decreases the Hayashy line gets redder but the hottest point of the loop moves to the blue.
A later work of Huang and Weigert (1983) confirmed this finding as may be seen in fig.11) ,where their

case for $M=5M_{\odot}$ and $\lambda=0.5$ is shown.

Even if a decrease in the external mixing length parameter (down to 0.5) may eliminate this problem for models with overshooting it has to be noticed that recently Vandenberg and Bridges(1984) claimed that the standard value of $\lambda_{em}=1.5$ should be preferred. Furthermore other important inputs in the model computations are known to modify the appearence of the loop in the H-R diagram. Apart from the effect of metal content (the less the Z the more extended the loop), or of the mass loss (which has been discussed by Lauterborn et al.(1971)),also the nuclear energy rates for He burning have a great impact on the H-R diagram morphology.

In particular the parameter $\delta_{\alpha}^2(7.12\ MeV)$ of the reaction $^{12}C(\alpha,\gamma)^{16}O$ is determined with an uncertainty of about one order of magnitude and its effect on the occurrence of loops has been reviewed by Iben(1972), and it is illustrated in fig. 12). The probable values of this parameter range from $\delta_{\alpha}^2(7.12)=0.045$ and δ_{α}^2 $=0.125$. An increase this parameter from the minimum value to about the maximum not only affects the chemical composition of the final C-O core (changing the C/ O ratio from about 1/2 to zero) but also produces more extended loops in the H-R diagram in the region of less massive intermediate mass stars , while the luminosity of the models is not affected.

Standard models ,the ones of Matraka et al. of Huang and Weigert and those of Bertelli et al. illustrated in fig. 7) were obtained assuming the value given by Flower(1975) of θ^2_α =0.078 . Recently however, new accurate measurements of cross section have increased its value of about three times Kettner et al.(1982). The models computed with this new value (Bertelli et al 1984) confirm the finding of Iben (1972) ; the loops have about the same luminosity than previous computations but they are more extended. In such a way models with overshooting and the new value for θ^2_α ,as proposed by Kettner et al. do not face the problem of the absence of blue loops for stars in the mass range 4 to 6 M☉.

The mass-luminosity relation for cepheid stars.

Another important problem arises when comparing theoretical predictions with observations in this phase.

In the last few years it becames evident that the mass of cepheids predicted by stellar evolution theory were larger than those inferred by pulsation theory by a non negligible factor. This so called "cepheid mass anomaly" appears more or less stressed in the different ways in which the comparison is made.

The linear pulsation theory predicts that a relation exists between the period and the mean density of a cepheid, which may be written in terms of its mass and radius (or equivalently luminosity and effective temperature) :

$$Q = \Pi \sqrt{M/R^3}$$

where Q is the pulsation constant and Π the period. Cogan (1970) studied the 13 cepheid stars employed by Sandage and Tamman (1969) for calibrating the period-luminosity-color relation. From the well known mean luminosity and effective temperature he was able to show that the masses inferred from pulsation theory (with the help of the above relation) were too low ,by a factor of 30%-40% , with respect to the masses inferred by evolutionary tracks crossing the instability strip at the same luminosity.

Furthermore since the first application of hydrodynamic calculations to cepheid pulsations ,Christy(1968,1974),Stobie (1969,a,b,c), it was evident that the so called "bump cepheids" could not be modeled unless the masses used in the calculations were about 30% or 40% less than the evolutionary masses of the same luminosity. Since then a great amount of theoretical work has been performed to clarify if this mass anomaly is only apparent. A number of different agents was invoked to lower the evolutive mass, like mass loss (whose amount however also modifies the extension of the loop and turns out to be too high); or increased envelope opacities, like those suggested by Carson (1976), whose effect is mainly directed to lower the luminosity of the loop (Carson and Sthoters 1984)

On the other hand improved hydrodynamical calculations (Fricke et al. 1971,1972,Adams et al.1980) have demonstrated that a "bump cepheid" mass anomaly of about 40% exists ,independently of numerical codes employed in the calculations.

As far as the "pulsation masses" are concerned, the discrepancy is schematized in fig. 13) (from Iben and Tuggle 1972). In that figure the dots indicate the pulsation masses obtained for the sixteen galactic cepheids and the shaded region represents the mass-luminosity relation of the blue loop for stars with chemical composition X,Z = 0.7,0.02 . The fitting line for cepheids has a slope (Log L=5.6 Log M) different from that drawn for the evolutive models (Log L=4 Log M). For a given luminosity the discrepancy Log M is about 15%.

It was noticed by Iben and Tuggle(1972 a,b,) that an underestimate of the intrinsic luminosity of the cepheid stars by a factor of Log L =0.1 and a slightly different slope of the P-L-C relation from that of Sandage and Tamman (used for the plot in fig.13) would remove the discrepancy.

This is not at variance with observations as the luminosity depends on the distance scale and for the sixteen cepheids of fig. 13) the last rests mainly on the Hyades distance modulus. An increase of Log L =0.1 implies an increase of (M-m)=0.3 mag.; in the last few years the Hyades distance modulus has grown from about 3.00 mag. to 3.18 mag (Flower1979) and recently to about 3.4 (Hanson 1979).

This constitutes a possible overcoming of the pulsation mass anomaly ,even if some problems remain as indicated by fig.13 a), which shows a plot of Mev (evolutionary masses) vs. Mpul (pulsation masses) calculated with the new distance for the Hyades (Hanson 1979) by Cox(1979).

On the other hand, bump mass anomaly may be eliminated postulating an overabundance of He (Y=0.75) in a very narrow region of mass inside the ionization region. This is not implausible, but strongly conflicts with the immediate mixing that an outward decreasing molecular weight would imply.

Turning now to overshooting models ,what can be said at this moment is that evolving at somewhat higher luminosity (Δ Log L =0.5 for M=5Mo and Λ =1), they offer an elegant possibility to overcome the difficulties of standard ones ,both with pulsation and bump mass anomalies. This can be seen from fig.13) when shifting the evolution dashed area of about Δ Log L =0.5.

Up to now no pulsation calculation has been made for models with overshooting but, on one side (pulsation mass) linear theory is expected to give about the same relation between period and average density; on the other (bump mass) they fruitfully obey the requirement of providing the same luminous envelopes with a total mass about 30%-40% lower than in standard models.

To conclude this section dealing with He burning phase, a remark on lifetimes is needing. Standard models predict that the He burning lifetime is about 30% 40% of H burning time. This is due primarily to the growth of the inert He core as the H shell advances during evolution.
On the contrary, in the new models evolution at higher luminosity exactly balances the effects of an enlarged size of the core. Therefore they predict a He burning lifetime which is about 6% for λ =1 or 15% for λ =0.5 of the H burning lifetime. Of course this has a very clear observational counterpart in the number ratio of evolved to main sequence stars and may be compared with theory, if one disposes of a good enough photometry for well populated young clusters.

The Asymptotic Giant Branch phase.

Following central He-exhaustion the C-O core undergoes gravitational contraction while central density and temperature grow.

In the H-R diagram the model is climbing again along the Hayashy line which,for intermediate mass stars ,asymptotically approaches the prolongation of the red giant branch for low mass stars, from which the name given to this phase.

The convective envelope expands and penetrates inward in mass.The model is initially burning in both H and He shells,but the first is immediately quenched by expansion cooling; in this way, convection penetrates inside the inert He core ,starting the so-called second dredge-up phase.

During this phase the luminosity is mostly provided by gravitational core contraction and by shell He burning.However when temperature and density enter the range in which neutrino losses become important, the core reaches an equilibrium configuration with neutrino losses exactly balancing gravitational energy input.

Therefore, the central temperature stops its increase while density continuosly grows up,driving the model toward central electron degeneracy.

In the outer C-O core regions, neutrino losses are much less efficient and then temperature continuously increases .However the point where neutrino losses just balance gravitational heating progressively shifts outward, thus producing an off center maximum temperature.

On the other hand, nuclear energy sources provided by carbon burning are not yet sufficient to overcome neutrino losses but grow whit temperature and experience the same off center shifting.

Here the model faces with two different fates.

If maximum temperature grows faster than being pushed outward by neutrino cooling (this happens when enough C-O material is outside the maximum to significantly contribute in gravitational heating),then carbon burning starts off-center with a power that immediately overwhelms neutrino losses and forces the core to leave degeneracy. The limiting mass for a middle degenerate C-O core to undergo this carbon ignition is about 1.06 $M_{\odot}$.

On the other hand if there is not enough heating from the gravitational pool ,neutrino losses make the temperature to drop,after a stage in which a maximum off center was reached;the central core becomes highly electron degenerate and only when the C-O mass reaches the critical value of 1.4 M⊙ carbon ignition may occur, however with a destroying power.

For the first class of models subsequent evolution was investigated in a series of papers by Nomoto et al. (see Nomoto 1984 and references therein). Briefly, after quiescent central C-burning the star develops an highly degenerate O-Ne-Mg core and in a short time scale reaches the stage in which electron capture by Ne and Mg is dominant. The star then implodes giving rise to a deflagration type 1/2 supernova.

Stars belonging to the second class (properly intermediate mass stars) follow a different evolution and, what is important, survive for a time significant to produce observable effects in the H-R diagram.

While C-O core is contracting, the luminosity input and the effect of cooling force the radiative gradient in the envelope to overcome the adiabatic one, in progressively more internal regions. Thus convection deeply penetrates inward and shifts the He-H discontinuity toward the He shell, as it may be seen in fig 14 from Becker and Iben(1979).

As the location in mass of this discontinuity (M_H) approaches the location of the He shell, heating forces the Hydrogen to reignite in a thin shell. This determines the end of the second dredge-up episode (or Early A.G.B. phase) and the star enters the double shell phase which is characterized by a series of thermal pulses of the He shell.

During this phase the C-O core is highly degenerate and its mass increases due to the effect of the burning shells which,alternately process H into He and He into C and O (Neon is present in a very small quantity).

However, the He shell is unstable to nuclear energy release and a little flash is started which generates convection and expansion of the surrounding layers.This expansion is so quick and mixing so efficient that He nuclear reactions

cannot reach equilibrium and the main product is ^{12}C. Expansion quenches out the He shell and subsequentely the H shell, forcing the external convective envelope to advance in regions ^{12}C enriched by previous internal mixing; thus ,by this mechanism newly made ^{12}C is brought to the surface during a series of flash episodes which constitute the so called third dredge up phase.

When and ,eventually, if surface carbon abundance overwhelms oxygen abundance ,the star shows up as a carbon star, to distinguish it from normal stars in which oxygen is more abundant.

The luminosity of the models along the A.G.B. is quite well represented by a relation involving the mass inside the H burning shell (Paczynski 1970)

$$L = 5.925 \times 10^4 (M_H - 0.495)$$

and it is mostly provided by H - burning. If gravitational energy is important (as it is the case for more massive intermediate mass stars)the relation is slightly different, involving also the total mass M (Iben 1977):

$$L = 6.34 \times 10^4 (M_H - 0.44) (M/7)^{0.19}$$

This luminosity refers to the interpulse phase which has a characteristic time duration given by

$$\log \Delta t (1/2) \simeq 3.05 - 4.5 (M_H - 1.0)$$

and grows until $\log \Delta t$ is about 25 % greater than the above value.

The number of pulses experienced by a model is an increasing function of the initial mass starting from 13 at log Mi =0,to 840 at log Mi =0.6 and ending at about 9000 for log Mi > 0.7 .

Then comparing the number of pulses with the interpulse time one can see that a considerable amount of time is spent during the thermal pulsing phase . The situation is summarized in fig. 15) (from Iben 1981). In that figure several mass-luminosity properties are shown for a set of evolutionary models (including low mass stars) with chemical composition X,Y,Z = 0.7,0.28,0.02.

The models begin the thermal pulsing phase along the line marked "begin" ,and end up in three different ways on the lines marked "end". For initial masses smaller than about 1.23 ,the models lose their envelope by stellar wind before developing a core mass of 1.4 M☉. Stars in the range 1.38 M☉ < M < 5 M☉ eject a planetary nebula and become white dwarfs. Stars more massive than 5 M☉ develope a degenerate C-O core of Mc=1.4 M☉ before losing their envelopes ,therefore presumably undergoing supernova explosion. Following a dredge-up law inferred by model computations, lines of constant ratio $^{12}C/^{16}O$ = 1,2,4 are drawn in fig. 15).

As can be seen from that figure, stars in the range 4 to 9 M☉ must appear as bright C stars with bolometric magnitudes in the range -6 < Mb <-7.2 . In addition ,more detailed calculations (Iben 1981) show that using a birthrate derived from Salpeter's mass function a theoretical distribution corresponding to models of fig. 15) for C and M stars may be obtained as summarized in fig 15 b).

C stars are normalized to a total number of 100.The number of C stars versus magnitude indicator M(0.81) defined by a study of Blanco et al. (1980) for Magellanic Clouds is also shown for comparison.

It is evident that theory is at variance with observations.

First the theoretical distribution of carbon stars ,which ranges from Mb=-5 to Mb=-7.5 and has a maximum at about Mb=-6.3 ,is completely "off center" with respect to the observed distribution which has a maximum at about Mb = -4.7 and extends from Mb=-3.5 to Mb =-6 .

This means that where theory predicts that C star stage may be easily reached (M > 5 Me)there are not real C stars ,and where theory predicts that dredge-up mechanisms are not able to produce C stars , they actually exist !. To make things worse, theory predicts a ratio of M to C stars of about 4, while abservations indicate a much smaller ratio ,at about 3/5 . Even if the dredge-up mechanism is supposed to be more efficient for low mass stars ,the theoretical distribution of C stars fails to predict the true one ,giving too many stars above Mb = -5.5 . In conclusion if,on one hand, it is fairly well established that the C star phenomenon occurs along the A.G.B.,above a certain luminosity, on the other hand,the observed distribution vs. magnitude is completely at variance with theoretical predictions.

Surveys of selected fields in the Magellanic Clouds confirmed that C stars are confined within the magnitude range $-4<Mb<-6$,and none is found brighter than this limit.

Recently Frogel and Blanco (1983) claimed that young clusters in the Magellanic Clouds which contain Cepheids (age < 10^8 years),do not have C stars ;on the contrary ,stars in the range 4-6 M☉ are expected to easily expose C rich material at their surfaces.

Several arguments have been invoked to explain such a paucity of bright C stars .The most convincing one is the so called "envelope burning" Iben (1975) . It is suggested that if the base of the convective envelope is sufficiently hot,then the fresh dredged-up C may be directly converted into N via the reaction $^{12}C(p,\gamma)\ ^{13}N(\beta,\nu)\ ^{13}C(p,\gamma)\ ^{14}N$.

If this is the case ,then the star should appear as a

M giant star. This however conflicts with the more surprising observational result that not only are C stars absent, but also M stars brighter than about Mb=-6 are completely missing (Frogel and Blanco 1983).

Therefore on the basis of observational evidences one is forced to conclude that real A.G.B. stars leave A.G.B. considerably before the critical mass of 1.4 Mo is reached. Stressing this point there is the fact that the number of observed non C stars long period variables in the Magellanic Clouds (= 100) with bolometric magnitude Mb < -6 ,is about a factor of twenty lower than the number of cepheid stars which are thought of to be their progenitors. Since the lifetime of a cepheid is about 10^6 ys,one can conclude that the time spent by a real A.G.B. star above Mb=-6 is about 10^5 ys.,or a factor of ten less than theoretical predictions.

The A.G.B. phase for models with overshooting.

A number of evolutionary computations has been provided by Bertelli et al.(1984) up to non degenerate C ignition. To follow the thermal pulsing phase is a hard task as it requires considerable amount of computing time and very detailed grids of models.

However a number of interesting results has been obtained for those models that undergo C ignition avoiding the thermal pulsing phase. The main results may be better understood by looking at fig. 16) and 17) (Bertelli et al. 1984). In fig. 16) the convective core size during He burning is plotted against central He content for two different values of the overshooting parameter (λ =0 :standard case; and λ =1). The core mass is much greater in models with overshooting, and this is due to overshooting in both H and He burning phases . Because of the continuous growth of the convective core the region is completely mixed until Yc is as low as 10^{-3} ; then the core starts receeding and leaves a very flat Y profile (Y= 10^{-3}) which extends outward where the maximum size of convective core generated the steep C-O He discontinuity (this maximum extension is showed in the right side of fig 16) for different initial masses).

When the central He burning terminates,the core contracts and He begins burning in a shell which covers the whole flat profile and has its maximum efficiency around the discontinuity. For initial masses greater than about 6 M☉ the mass size inside this discontinuity already overwhelms the critical value of 1.06 M☉. The central temperature and density runs are shown in fig. 17) for this contracting phase ,corresponding to the early A.G.B. phase.

All models down to the 6 M☉ ignite carbon in a non degenerate core (for the 6 M☉ model the ignition is off center and in a middle degenerate core as can be seen in fig.18 ,19). Thus when overshooting is taken into account (λ = 1),models of initial mass greater than about 6 M☉ behave like massive stars , and consequently do not experience thermal pulses in a phase of degenerate C-O core.

Moreover they climb up along the A.G.B. in a characteristic time which is about a factor of ten less than that predicted by standard models. It is clear that models with overshooting follow,at least approximatly, some requirements imposed by experimental evidences. The new models predict quite straightforwardly that young clusters (with turnoff mass at about say 5 or 6 M☉)show Cepheids without having C or M stars above Mb=-6. However the problem of C stars is far from being solved by overshooting alone. It suffices remembering that ,while on A.G.B. , the luminosity is strictly related to the core mass Mc. Thus one expects less massive stars to continuously populate the bright part of the A.G.B. even with overshooting.

For such stars the key mechanism may be the mass loss ,as suggested by a series of investigations also for standard models (see Iben and Renzini 1983 and references therein). It is worth mentioning that ,at the same mass loss rate ,two conflicting factors determine the amount of mass lost before A.G.B. in models with overshooting with respect to standard ones. One is the higher luminosity at which they evolve and the other is the shorter time of the He burning phase.

The last however is less important because a considerable fraction of time is spent in the loop phase ,thus far from the Hayashy line ,where mass loss is known to be more efficient. With the usual dependence of the mass loss rate on stellar luminosity and temperature (Reimers 1975)

$$\dot{M} = \eta \cdot 1.27\ 10^{-5}\ M^{-1}\ L^{1.5}\ T_{eff}^{-2} \quad (M_\odot\ Yr^{-1})$$

$$(1/3 < \eta < 1)$$

the run at higher luminosity largely overwhelms the effect of a shorter He lifetime. On the other hand during the A.G.B. phase one does not expect differencies in the amount of mass lost by thermally pulsating models , as the luminosity depends primarily on the core mass Mc.

The problem of mass loss is however far from being solved and,in fact, the Reimers's mass loss rate which is calibrated on low mass stars, probably underestimates the true rates for intermediate mass stars and, surely it fails in the domain of red supergiants. Recently Waldron (1984) has provided a new relation which is more sensitive to the luminosity of the star :

$$\dot{M} = 0.1084\ L^{1.96}\ T_{eff}^{-3.54} \quad (M_\odot\ Yr^{-1})$$

A calibration of this relation in the domain of low mass stars, combined with its steepness in the Log L - Log Te plane gives, for intermediate mass stars,rates corresponding to the Reimers's relation with η ranging from 1 to 2 and,for red supergiants ,the observed rates (Chiosi 1984).

Therefore this somewhat larger mass loss rate may prevent low intermediate mass stars from populating the bright portion of the A.G.B. , without conflicting with the well estimated rates for low mass stars.

Finally a particular attention deserves a result concerning the second dredge-up phase in new models. Some ^{12}C is brought to the surface during this phase,contrary to standard models ,because of the higher luminosity and extension of the C-O core at the beginning of A.G.B. . At this time the He shell is centered on the CO-He discontinuity and it is rather thick. When external convection goes down the He shell becomes thin around this discontinuity ,leaving a partially 3α processed region which after a while is reached by external convection. The amount of ^{12}C brought to the surface is very low ($Ys = 10^{-4}$) but it is comparable with the initial oxygen abundance of models with low metallicity.

If confirmed ,this new behaviour introduces a very simple way of obtaining ^{12}C enhanced atmospheres in metal poor stars and offers a new approach to explain why C stars are so easily produced in the Magellanic Clouds.

Chapter IV

Low mass stars

Low mass stars are characterized by the growth of a highly electron degenerate He core following the H-burning phase and by the occurence of the He flash. The upper mass limit for these stars is placed around 2.25 M⊙ ,Iben(1967),Sweigart and Gross (1978),for a solar chemical composition.

A detailed analysis of the properties of low mass stars is beyond the aim of this thesis. However some insights on the effects of overshooting may be found in the low mass stars that have a convective core, while burning Hydrogen. Barbaro and Pigatto(1984) performed a statistical analysis on a group of 38 among the oldest open clusters, with the aim of testing the agreement with the results of stellar evolution theory during H shell and central He burning . In their work they adopted the isochrones of Ciardullo and Demarque(1977) and evolutionary tracks by Sweigart and Gross (1976); both these sets of models do not account for convective overshooting. They were able to show that the sample of clusters might be divided into two groups, with respect to the age :

i) older clusters, which exhibit a well populated Red Giant Branch (RGB), are in good agreement with the theory.

ii) younger clusters that, inspite of a turnoff mass of about 1.4-1.5 $M_\odot$ (assuming a solar chemical composition), during the red phase show a morphology which is characteristic of stars that do not undergo the He flash. This second group is at variance with theory; in addition, the minimum mass for the development of a convective core during central H burning has been approximately extimated at 1.5 $M_\odot$, indicating that overshooting might be responsible for the "anomalous" behaviour.

Following this suggestion, Bertelli and Bressan (1984) performed a set of evolutionary computations for a 1.8 $M_\odot$ star of chemical composition X,Z = 0.7,0.02 and with three different values of the overshooting parameter (λ =0,0.5,1).

The run of this model in the T_c- ρ_c plane is shown in fig.20) together with Iben's (1974) models. In fig.20) it is clearly shown that overshooting greatly affects the evolution of the star and may explain the results of Barbaro and Pigatto. For λ =0 the model follows standard evolution and after central Hydrogen exhaustion, the inert He core degenerates before reaching the Schönberg-Chandrasekhar mass, limit above which an isothermal non degenerate core cannot exist. On the contrary, for different values of the parameter λ, the inert He core approaches this limit while being not yet degenerate: in this case it gets hotter, due to gravitational contraction. In particular, from fig.20) it seems that assuming the value 0.5 for the overshooting parameter the limiting mass for a quiescent He ignition is around 1.8 Mo and something smaller in the case of λ =1.

Finally, another problem concerning low mass stars deserves particular attention. Following Mould and Aaronson (1980) it is possible to determine the age of a star cluster, from the maximum luminosity of its A.G.B.. On the other hand, the age may be inferred from the main sequence fitting of isochrones, if chemical composition is known.
Recently Hodge(1983) performed a comparison among ages inferred using the above methods, for a group of clusters in the Magellanic Clouds. His result, shown in fig.21), clearly suggests that a trouble must lie in the theory. While for the oldest clusters both the methods give results which are in good agreement,

for the youngest ones the ages derived by fitting the main sequence at turnoff, are much smaller than those derived from the A.G.B. Because the age spread between A.G.B. stars and turnoff stars is much tighter than the discrepancy found by Hodge, and because the properties of the models during A.G.B. are fairly well known (the luminosity of these models depending mainly on the core mass), one may infer that the trouble concernes the turnoff.

This difficulty may be explained if overshooting exists, to a certain degree, but it is not accounted for in model computations. Then overshooting does not greatly modify the A.G.B. of low mass stars, while it increases the turnoff luminosity; in the fit with standard models one is forced to use a more massive object in the main sequence, with the obvious consequence of deriving a much younger age.

On the contrary, the oldest clusters have turnoff masses in correspondence with a radiative core, and do not show this discrepancy.

Conclusions.

In this work we discussed the possibility of removing unexpected features which arise when comparing theoretical predictions with observational data, assuming that convection in stellar interiors is to be treated in a more consistent way.

This requires the knowledge of the velocity field inside convective cores, while in the standard model computations only the determination of the acceleration field is usually performed.

At this purpose several criteria have been recently developed , but none of them is free from arbitrary parameters. In fact the two criteria which grant this requirements suffer from severe internal inconsistencies which invalidate their derivations.

Making use of a new criterion which can be easily applied to stellar model computations, some relevant results have been obtained, demonstrating that, if overshooting exists, then it strongly affects the evolution of the stars and produces a series of consequences which allow the comparison with experimental data.

On the other hand observations show some features that might be explained only invoking a more extended internal mixing ;in particular, the observed main sequence widening for massive and intermediate mass stars and the related star counts, suggest that the amount of mixing during H-burning phase, might be relevant. This mixing modifies the whole structure of the star as it evolves off the main sequence. One can say that the star will "remember" for ever this extra mixing occurred during its H-burning phase. Even overshooting is not neglegible during He-burning and in fact both effects cause the star to behave like a more massive object with respect to what predicted by the standard evolution.

However a careful determination of this mixing constitutes a hard task, because it is an internal property of the model which cannot be directly "seen" from outside.

Therefore its description in the framework of the mixing length theory may be too simple and may not lead to correct predictions.

In order to get a quantitative comparison between the results of these new models and the observed scenario, statistical tools are needed; one has to build up a grid of models for various chemical compositions, and once the sets of isochrones have been computed, one may get synthetic H-R diagrams to be compared with the observed ones. This will be the purpose of my future investigation.

On the other hand some specific properties of the new models may be already tested ,as in the case of cepheid masses, thus providing a new amount of additional information.

Finally a new scenario has been opened for the final phases of stellar evolution, which may be supported by observations better than the standard ones.

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Figure captions

Fig.1) The run of the real temperature gradient as assumed in the overshooting model.

Fig.2) Curves of maximum velocities for the Z.A.M.S. models of 20,60,100 M⊙ ; vertical dashed lines indicate the point where the Schwarzschild criterion holds $\nabla_R = \nabla_{ad}$.

Fig.3) Values of the various function v (velocity), f = Fr/(Fr+Fc) , and excess temperature ΔT inside a 2 Mo model (Maeder 1975).

Fig.4) Theoretical H-R diagram for the sets of models of tables 1) and 2):

A′ standard

A with overshooting

B′ whit mass loss

B with mass loss and overshooting

Fig.5) The "theoretical" H-R diagram for the most luminous stars in the solar vicinity

Fig.6) Theoretical widening for the main sequence band with various overshooting parameters without mass loss.

Fig.7) Evolutionary tracks of the 5,7,9 Mo models with various overshooting parameters ($\delta^2 \alpha$ =0.078)

Fig.8) Size of convective core (Mc/Mt) vs. model mass for the Z.A.M.S.

Fig.9) Composite Mv-(B-V)o diagram for galactic star clusters

Shaded line corresponds to points b of the models with overshooting parameter λ =0.5 (Maeder and Mermillod 1981)

fig.10) Temperature gradient profiles caused by mixing during He-burning.

Fig.11) Evolutionary tracks for a 5 $M_{\odot}$ model with λ =0.5 in the red giant region of the H-R diagram, for different values of the mixing length in the outer convective zone (Huang et al. 1983).

Fig.12) Blue limits during He burning for different values of the parameter $\theta^2_\alpha(7.12)$ of the nuclear reaction $^{12}C(\alpha , \gamma)^{16}O$.

Fig.13) Mass-luminosity relationships from standard evolutionary tracks and from observed Cepheids (Iben and Tuggle 1972)

Fig.14) Second dredge-up phase

Fig.15) Mass-luminosity properties during A.G.B thermal pulsing phase.

Fig.15b) Distribution in number versus bolometric magnitude for thermally pulsating M and C stars. Also shown is the Blanco and al. observed distribution of C stars in the Magellanic Clouds.

Fig.16) Mass size of the convective core during He burning for overshooting parameters λ =0,1.

Fig.17) Log ρc-Log Tc diagram for models with overshooting parameter λ =1; also shown is the case λ =0 M=9Mo.

Fig.18) Run of temperature,density,gravitational energy release,nuclear energy release from the He shell, neutrino energy losses for different models along the early A.G.B. phase of a 6 Mo model.

Also shown are the chemical profiles of H,He,C.

Fig.20) Log ρc-Log Tc diagram for the 1.8 Mo model with overshooting parameter λ =0,0.5,1 .

Also shoun for comparison are standard models from Iben

Fig.21) Comparison between the ages derived from the A.G.B. method by Mould and Aaronson (1982) and from the main sequence fitting by Hodge (1983).

Table 1 (Basic Quantities of Models)

Mass*	20	60	100	20	60	100
Set A						
Model	a			b		
$t(10^6 ys)$	0.	0.	0.	9.813	3.899	2.999
$Lg\ L/L_\odot$	4.611	5.702	6.101	5.077	6.029	6.376
$Lg\ T_e$	4.537	4.680	4.717	4.368	4.265	3.614
q_c	0.558	0.779	0.850	0.373	0.522	0.567
X_e	0.700	0.700	0.700	0.017	0.004	0.019
Set A'						
Model	a			b		
$t(10^6 ys)$	0.	0.	0.	7.459	3.378	2.69
$Lg\ L/L_\odot$	4.640	5.720	6.119	4.935	5.964	6.307
$Lg\ T_e$	4.542	4.683	4.721	4.444	4.506	4.507
q_c	0.438	0.684	0.813	0.241	0.403	0.455
X_c	0.700	0.700	0.700	0.032	0.017	0.012

* in solar units

q_c: fractionary mass of the convective core

X_c: central by mass abundance of hydrogen

Table 1

Mass	20	60	100	20	60	100
Set B						
Model	a			b		
$t(10^6 ys)$	0.	0.	0.	10.758	4.015	2.862
$Lg\ L/L_\odot$	4.594	5.967	6.098	4.977	5.945	6.235
$Lg\ T_e$	4.535	4.679	4.718	4.295	4.462	4.665
$\dot{M}$	2.1(-7)	3.1(-6)	8.5(-6)	8.4(-7)	9.7(-6)	1.6(-5)
M	19.74	59.62	99.32	16.00	40.83	67.81
q_c	0.558	0.772	0.852	0.412	0.662	0.799
X_c	0.700	0.700	0.700	0.011	0.014	0.120
X_e	0.700	0.700	0.700	0.700	0.504	0.360
Set B'						
Model	a			b		
$t(10^6 ys)$	0.	0.	0.	8.020	3.396	2.760
$Lg\ L/L_\odot$	4.625	5.717	6.120	4.798	5.839	6.220
$Lg\ T_e$	4.540	4.683	4.721	4.406	4.433	4.482
$\dot{M}$	2(-7)	3.3(-6)	1.(-5)	4.5(-7)	6.8(-6)	1.5(-5)
M	19.90	59.74	99.80	18.20	45.27	68.39
q_c	0.427	0.680	0.806	0.237	0.441	0.574
X_c	0.700	0.700	0.700	0.027	0.011	0.033
X_e	0.700	0.700	0.700	0.700	0.700	0.566

$\dot{M}$: rate of mass loss in units of $M_\odot/yr$

M : current total mass of the model

q_c : fractionary mass of the convective core

X_c : central by mass abundance of hydrogen

X_e : surface by mass abundance of hydrogen

Table 2

Table 3. Results of models and of counts for four "track-shaped" bands centered around the evolutionary tracks of 9, 15, 30, and 60 $\mathcal{M}_\odot$. The values of the ratios of the different lifetimes (respectively different number of stars) to the total lifetime (respectively total number of stars) are given

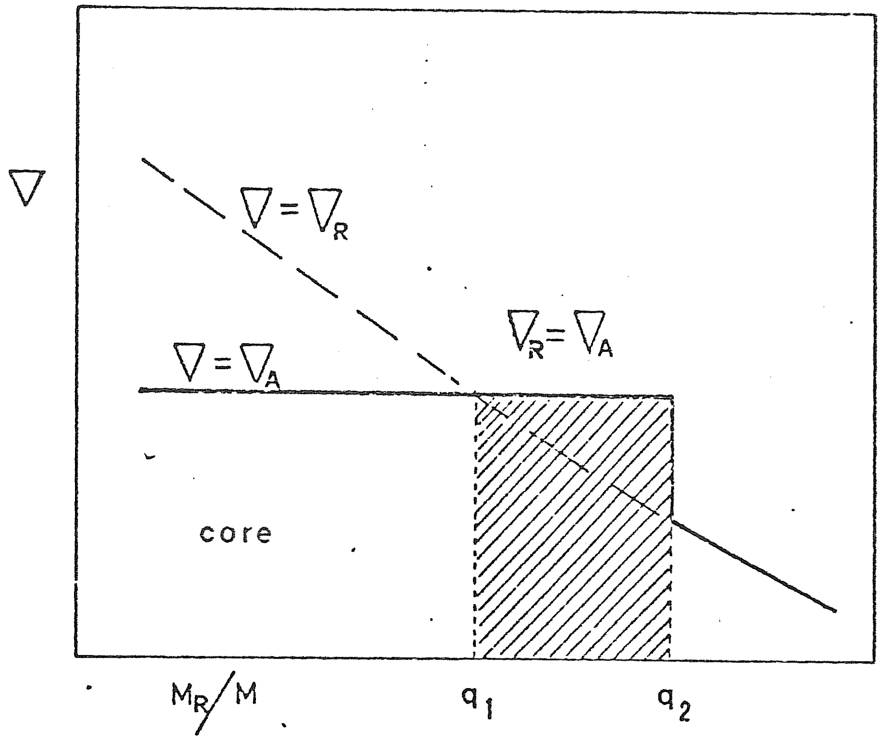
$t_{tot}=t_{OB}+t_A+t_{FG}+t_{KM}$ $N_{tot}=N_{OB}+N_A+N_{FG}+N_{KM}$			Models					Observations				
			A	B	C			GC	GAC	Galaxie	LMC	SMC
60 $M_\odot$	t_{KM}		0.015	0.007	0.002	N_{KM}		0.00	0.00	0.00	0.00	0.00
	t_{FG}		0.029	0.001	0.000	N_{FG}		0.00	0.00	0.00	0.00	0.00
	t_A		0.022	0.000	0.000	N_A		0.00	0.00	0.00	0.00	0.00
	t_{OBA}	/ t_{tot}	0.955	0.946	0.929	N_{OBA}	/ N_{tot}	1.00	1.00	1.00	1.00	1.00
	t_{OB}		0.934	0.946	0.929	N_{OB}		1.00	1.00	1.00	1.00	1.00
	t_{MS}		0.927	0.929	0.923	N_{MS}		0.50	1.00	0.73	1.00	1.00
30 $M_\odot$	t_{KM}		0.002	0.024	0.013	N_{KM}		0.00	0.00	0.00	0.00	0.00
	t_{FG}		0.000	0.027	0.002	N_{FG}		0.00	0.17	0.08	0.03	0.00
	t_A		0.000	0.018	0.001	N_A		0.06	0.17	0.11	0.00	0.00
	t_{OBA}	/ t_{tot}	0.998	0.949	0.930	N_{OBA}	/ N_{tot}	1.00	0.83	0.92	0.98	1.00
	t_{OB}		0.998	0.931	0.929	N_{OB}		0.94	0.67	0.82	0.98	1.00
	t_{MS}		0.921	0.928	0.921	N_{MS}		0.59	0.33	0.47	0.94	0.68
15 $M_\odot$	t_{KM}		0.014	0.036	0.098	N_{KM}		0.02	0.08	0.05	0.08	0.12
	t_{FG}		0.001	0.002	0.001	N_{FG}		0.02	0.04	0.03	0.03	0.12
	t_A		0.001	0.004	0.000	N_A		0.02	0.06	0.04	0.01	0.03
	t_{OBA}	/ t_{tot}	0.985	0.961	0.894	N_{OBA}	/ N_{tot}	0.96	0.88	0.92	0.89	0.76
	t_{OB}		0.984	0.958	0.894	N_{OB}		0.95	0.81	0.89	0.88	0.73
	t_{MS}		0.895	0.894	0.893	N_{MS}		0.70	0.35	0.54	0.75	0.54
9 $M_\odot$	t_{KM}		0.085	0.124	-	N_{KM}		0.07	0.08	0.07	0.08	0.20
	t_{FG}		0.010	0.032	-	N_{FG}		0.03	0.06	0.04	0.17	0.40
	t_A		0.013	0.000	-	N_A		0.00	0.05	0.02	0.06	0.00
	t_{OBA}	/ t_{tot}	0.905	0.844	-	N_{OBA}	/ N_{tot}	0.90	0.86	0.88	0.75	0.40
	t_{OB}		0.893	0.843	-	N_{OB}		0.90	0.81	0.86	0.68	0.40
	t_{MS}		0.843	0.835	-	N_{MS}		0.82	0.55	0.70	0.50	0.20

table 3

Star counts in different spectral types for stars with luminosity in the range $-7 \geqq M_b \geqq -9$

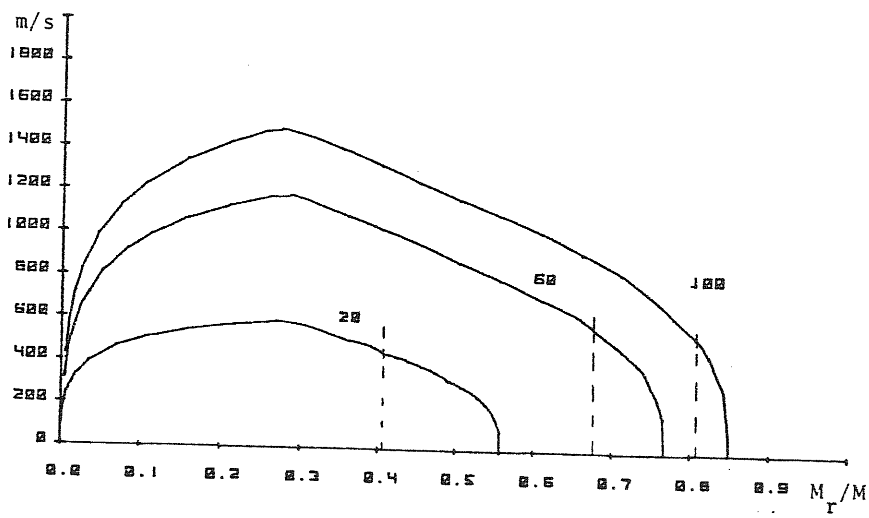
Sp [a]	O	B	A	F	G	K	M	WR
N N/Nt	138 0.342	201 0.498	10 0.025	4 0.010	1 0.002	1 0.002	29 0.072	20 0.050
Sp [b]	O - B0.5	B1 - B9	A	F	G	K	M	WR
N N/Nt	259 0.640	80 0.198	10 0.025	4 0.010	1 0.002	1 0.002	29 0.072	20 0.050
Sp [c]	O - B1	B2 - B9	A	F	G	K	M	WR
N N/Nt	290 0.718	49 0.121	10 0.025	4 0.010	1 0.002	1 0.002	29 0.072	20 0.050
Sp [a']	O	B	A	F	G	K	M	WR
N N/Nt	280 0.513	201 0.368	10 0.018	4 0.007	1 0.002	1 0.002	29 0.053	20 0.037
Sp [b']	O - B0.5	B1 - B9	A	F	G	K	M	WR
N N/Nt	401 0.734	80 0.147	10 0.018	4 0.007	1 0.002	1 0.002	29 0.053	20 0.037
Sp [c']	O - B1	B2 - B9	A	F	G	K	M	WR
N N/Nt	432 0.791	49 0.09	10 0.018	4 0.007	1 0.002	1 0.002	29 0.053	20 0.037

Table 4



∇
∇ = ∇R
∇R = ∇A
∇ = ∇A
core
MR/M
q1
q2

Fig 1



m/s
1800
1600
1400
1200
1000
800
600
400
200
0
20
60
100
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Mr/M

Fig 2

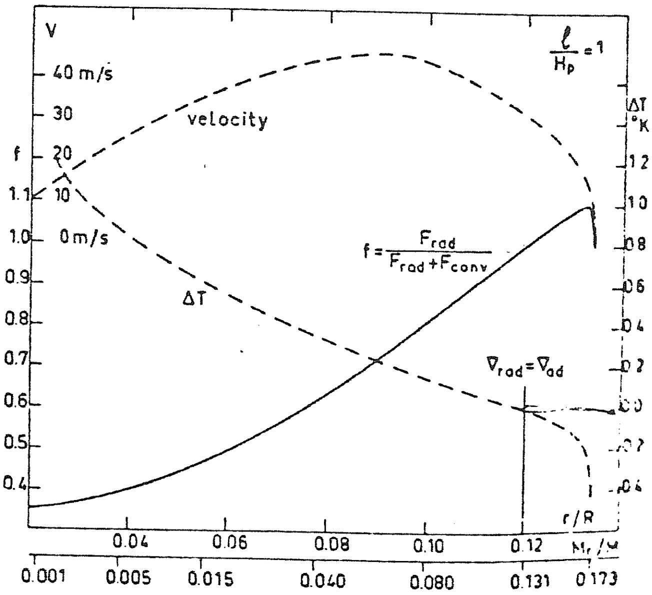


Fig. ▮ Values of the various functions v, f and ΔT according to the distance to the centre or to the mass fraction. The star is a chemically homogeneous $2\mathcal{M}_{\odot}$ star computed with $\ell/H_p = 1$, $k = 1.2$ and ... the initial central level r_i is at $M_r/M = 0.0007$. The limit ... is indicated. The distance of overshooting amounts here to 14% of the mixing-length

FIG 3 a)

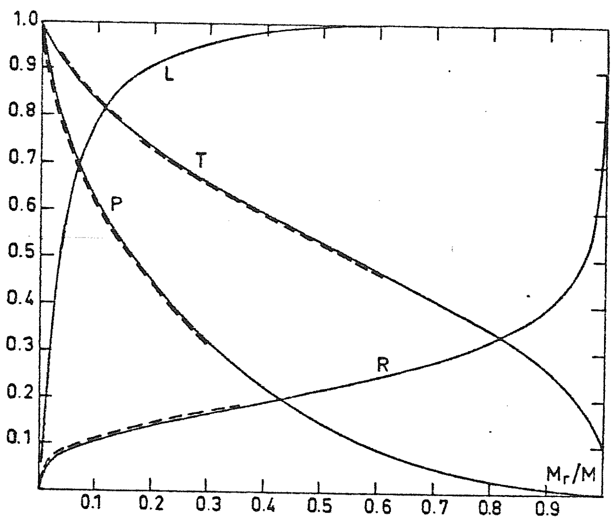
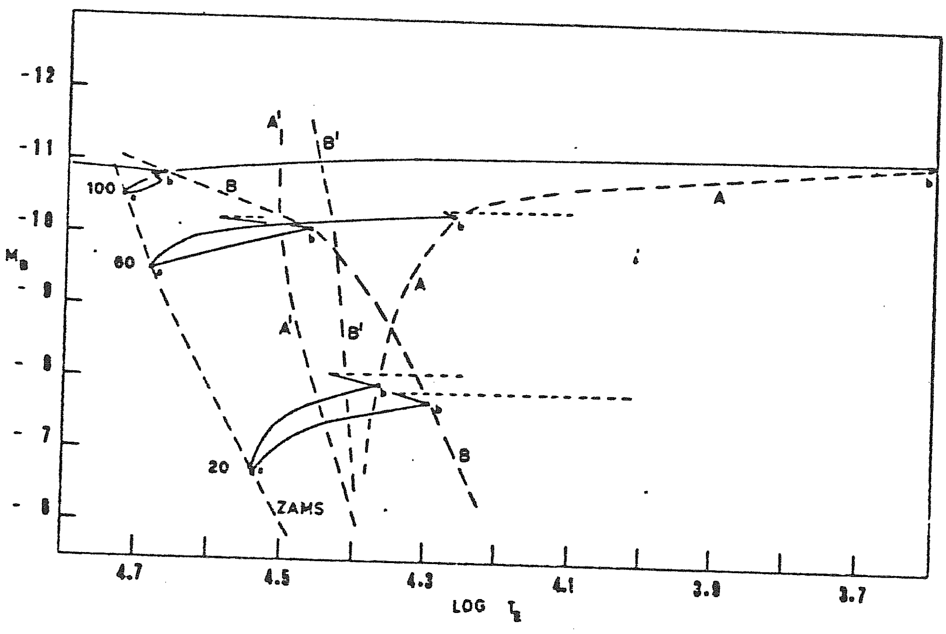


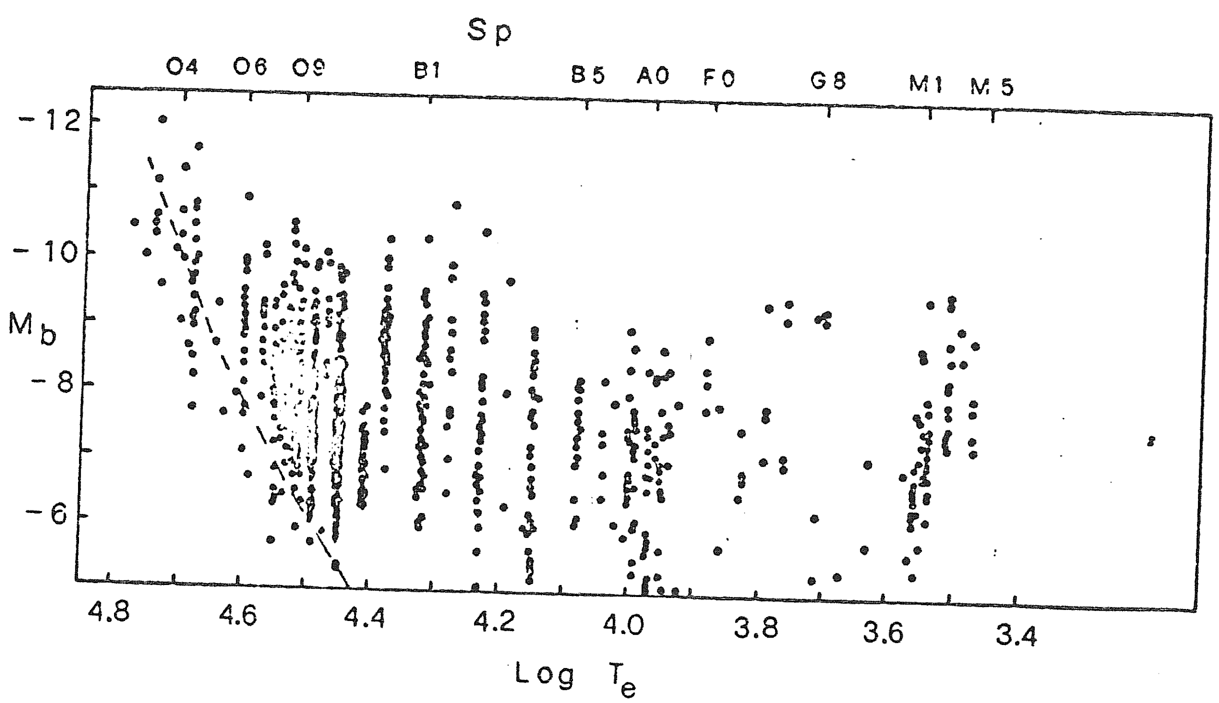
Fig. ▮ Changes between the structure of a homogeneous $2\mathcal{M}_{\odot}$ star computed with Schwarzschild's criterion (continuous line) and criterion $v = 0$ for $\ell/H_p = 1$ (broken line)

Fig 3 b)



-12
-11
-10
M B
-9
-8
-7
-6
100
60
20
A'
B'
B
A
ZAMS
4.7
4.5
4.3
4.1
3.9
3.7
LOG T E

Fig 4



Sp
O4
O6
O9
B1
B5
A0
F0
G8
M1
M5
-12
-10
M b
-8
-6
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
Log T e

Fig 5

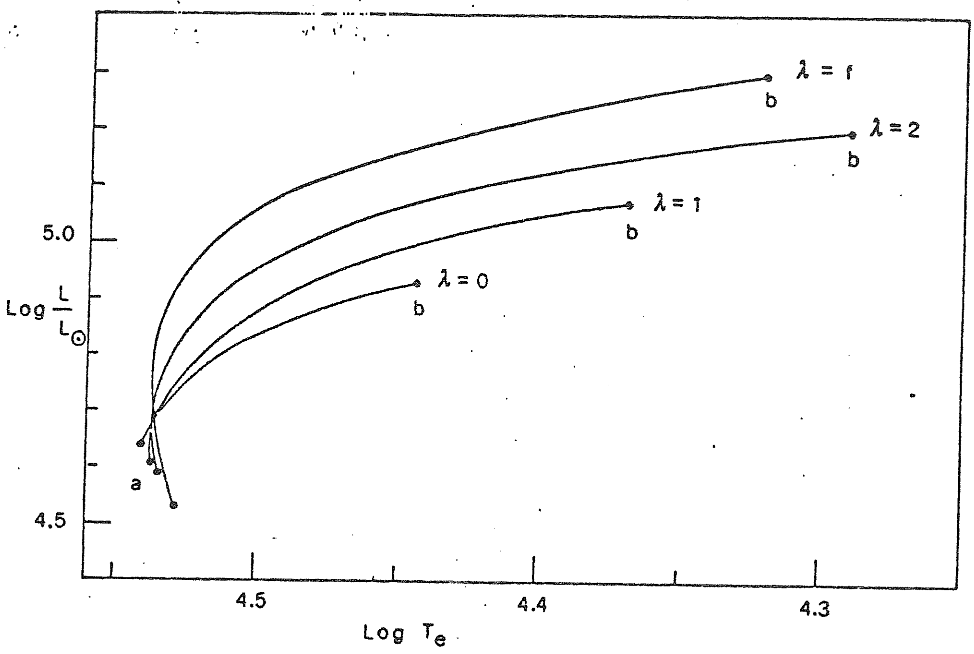


Fig 6

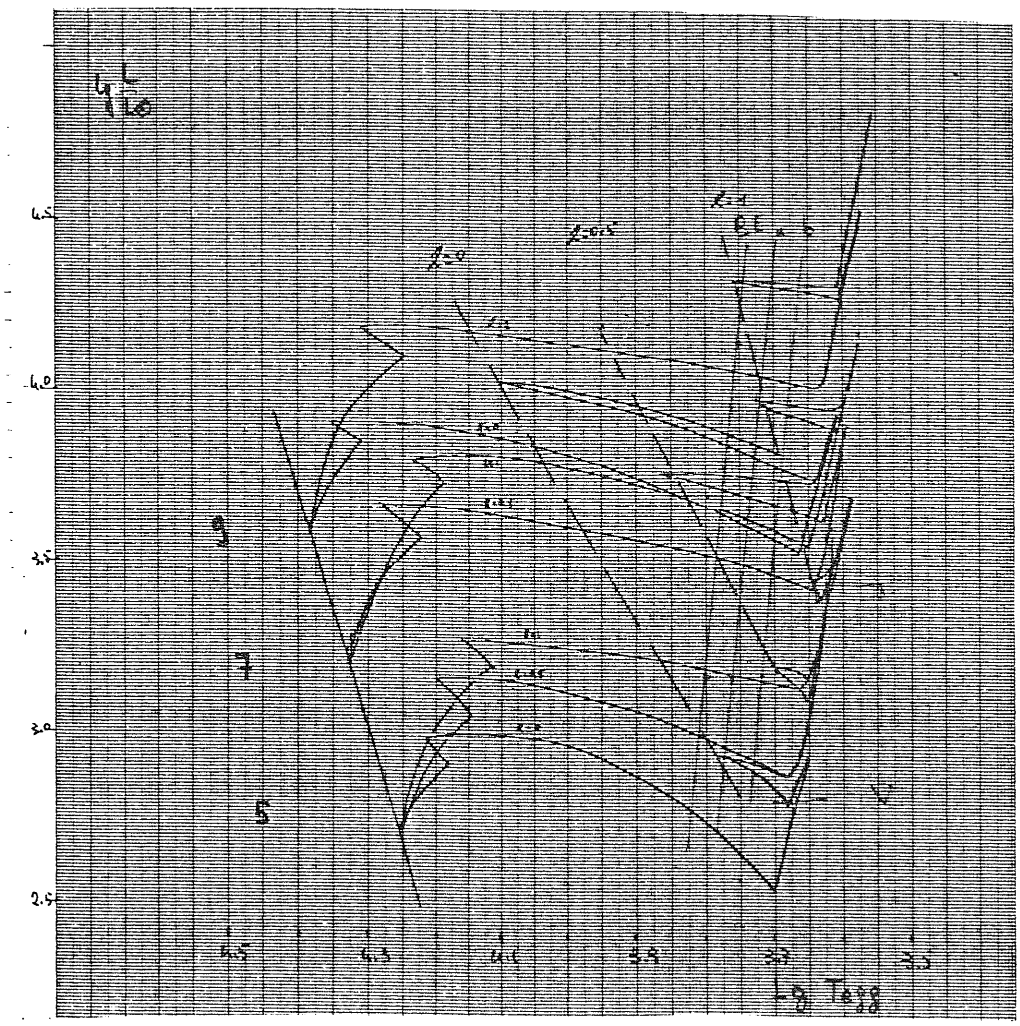


Fig 7

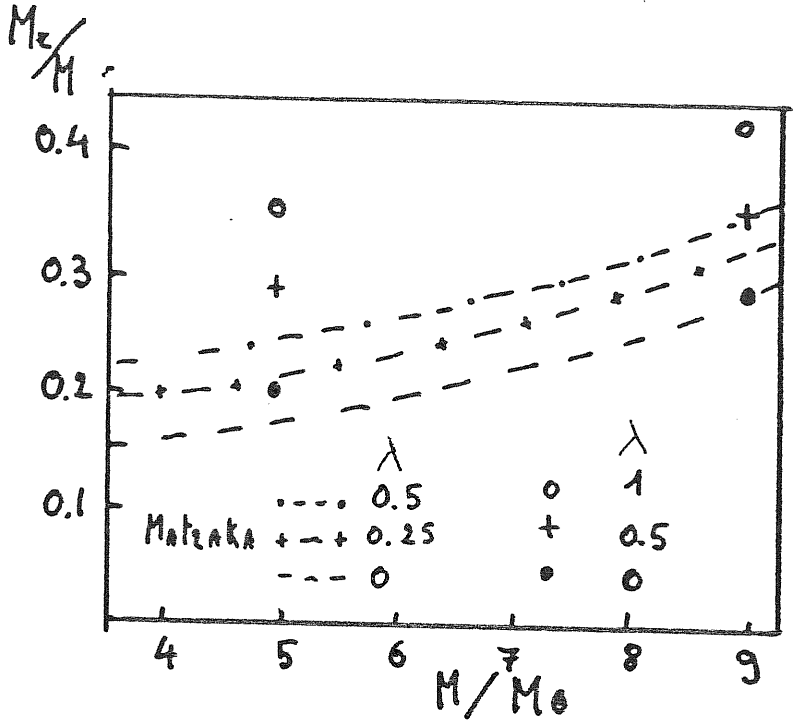


Fig 8.

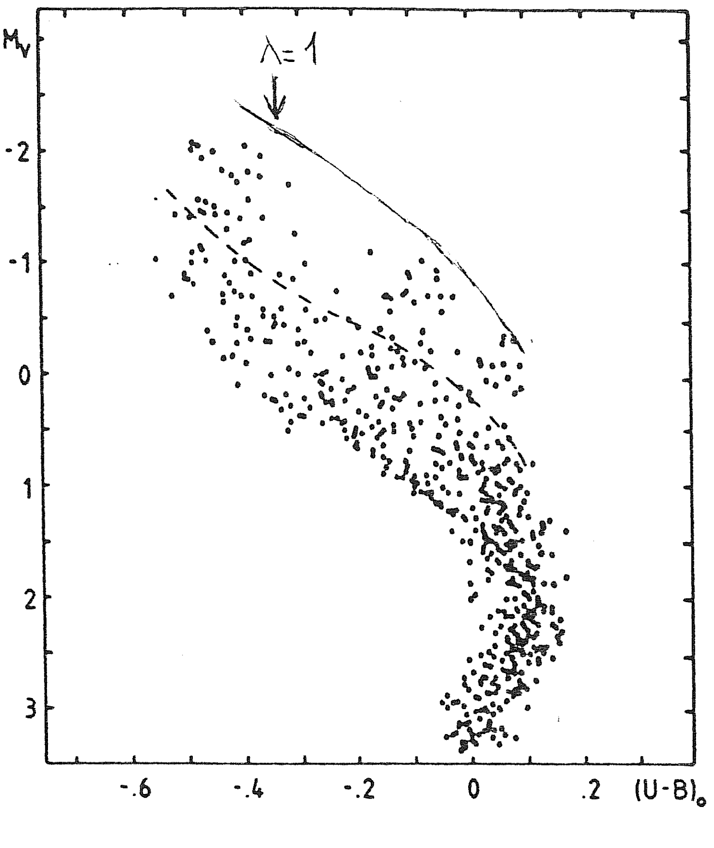


Fig 9

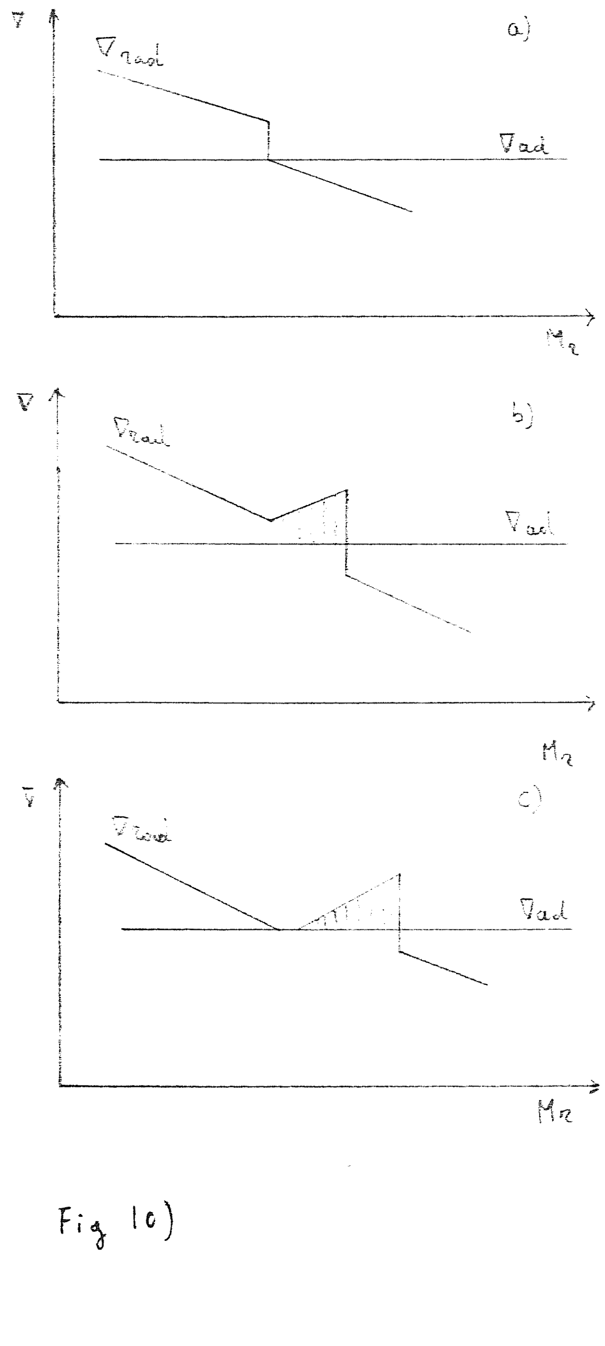


Fig 10)

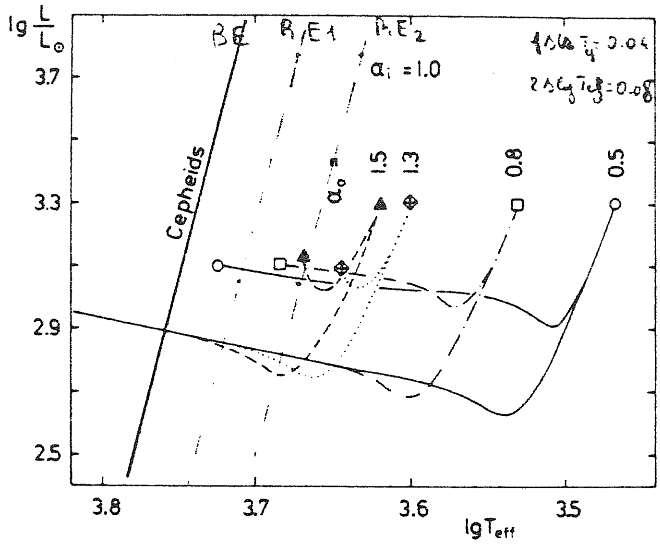


Fig. [redacted] Evolutionary tracks for a $5\,M_\odot$ star in the red giant region of the HR diagram; the overshooting in the main-sequence phase was here calculated using $\alpha_i = 0.5$. The ratio of mixing length to pressure scale height in the outer convective zone is varied from $\alpha_0 = 0.5$ to 1.5. For each of these tracks, equal symbols denote the highest point at the Hayashi-line (deepest penetration of the outer convective zone) and the bluest point of the loop

Fig 11

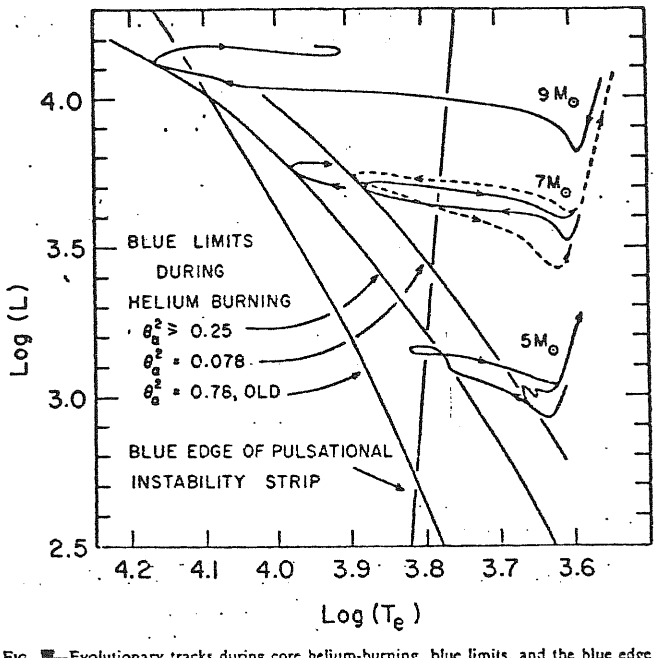


FIG. [redacted]—Evolutionary tracks during core helium-burning, blue limits, and the blue edge of a theoretical instability strip for radial pulsation in the fundamental mode. Initial composition parameters are $X = 0.7$, $Y = 0.28$, $Z = 0.02$. Three choices of reduced width θ_a^2 are represented. For the $5\,M_\odot$ models, $\theta_a^2 = 0.078$ and 0.25; for the $7\,M_\odot$ models, $\theta_a^2 = 0.078$ and 0.78; for the $9\,M_\odot$ model, $\theta_a^2 = 0.25$. The blue limits connect, for different masses, the bluest points reached during the major core helium-burning phase. A blue limit defined by an earlier set of models (see Iben 1967) is also shown.

Fig 12

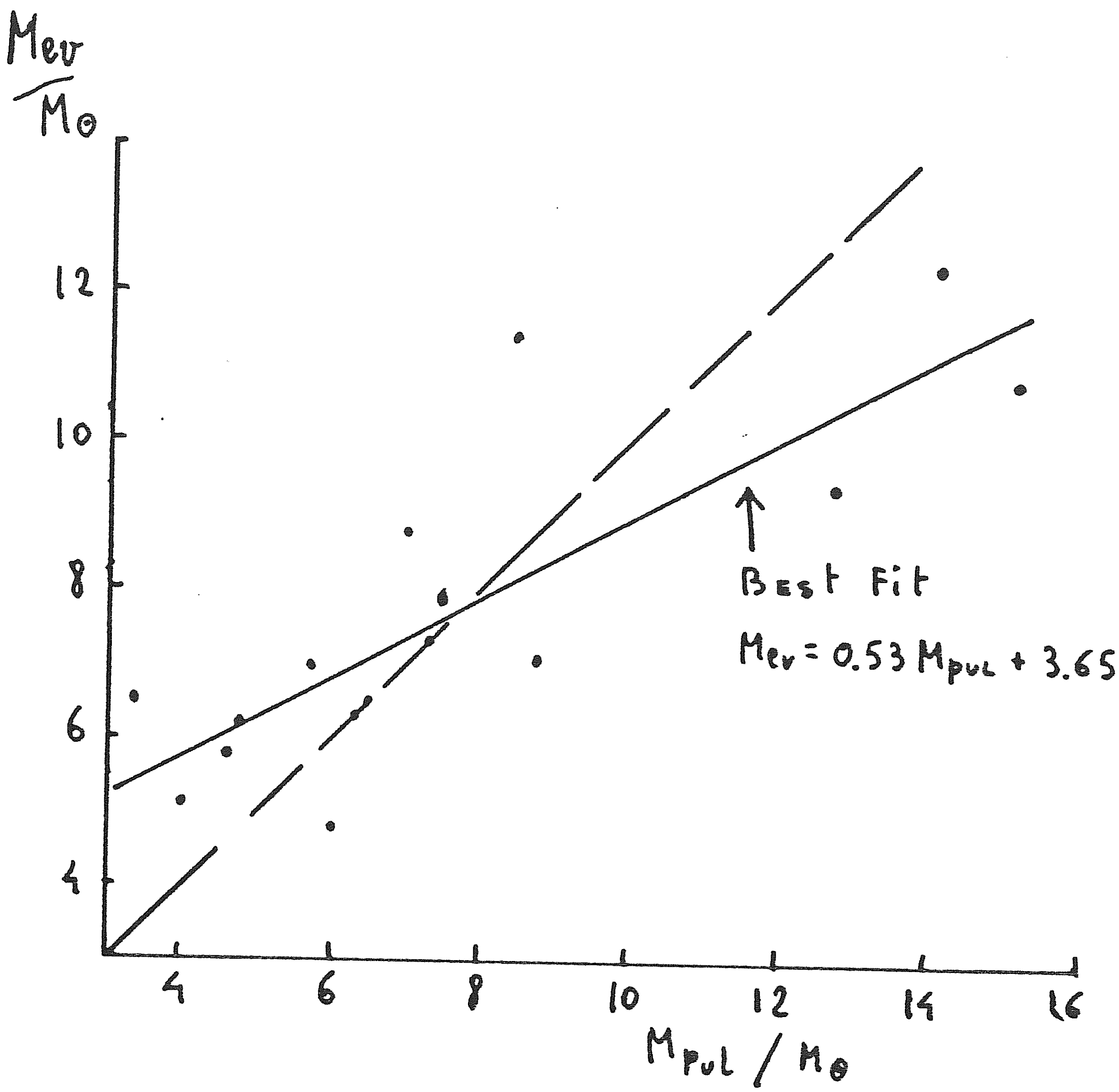


Fig. 13 a)

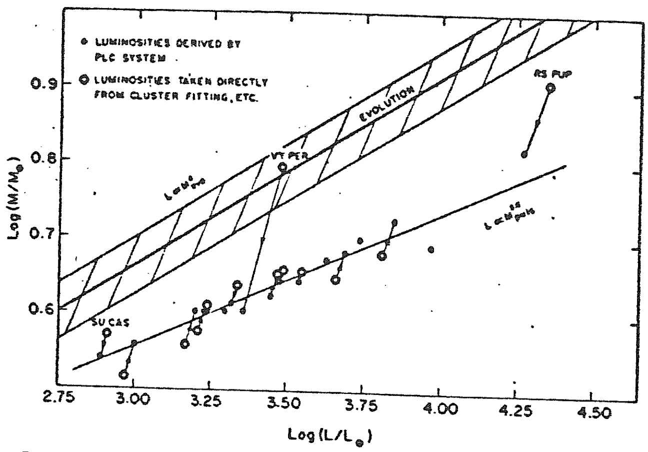


FIG. ▮—Mass-luminosity relationships. The shaded region represents the $\mathfrak{M}$-L relationship for stars of composition $(X, Z) = (0.71, 0.02)$ during the core-helium-burning stage. Circles are the consequence of comparing observed periods and colors and estimated luminosities with a theo-retical relationship between period, surface temperature, luminosity, and mass for pulsation in the fundamental mode. The luminosity coordinates of the 13 open circles have been estimated by the main-sequence-fitting technique. The luminosity coordinates of the filled circles have been esti-mated by using the period-luminosity-color (PLC) relationship given by Sandage and Tammann (1969).

Fig 13

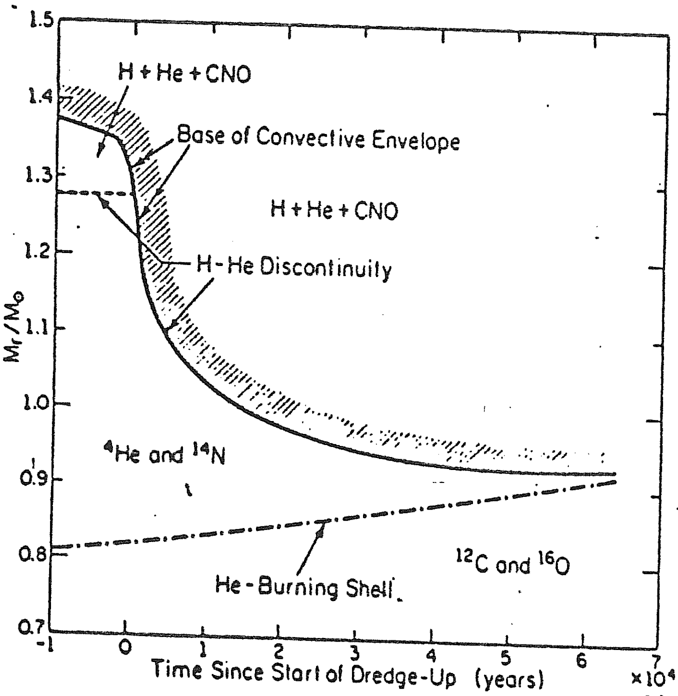


Figure 1 The variation with time of the location (in mass) of the base of the convective envelope and of the center of the helium-burning shell during the second dredge-up phase in a model of mass $5\ M_\odot$ and initial composition $(Y, Z) = (0.28, 0.001)$.

Fig 14

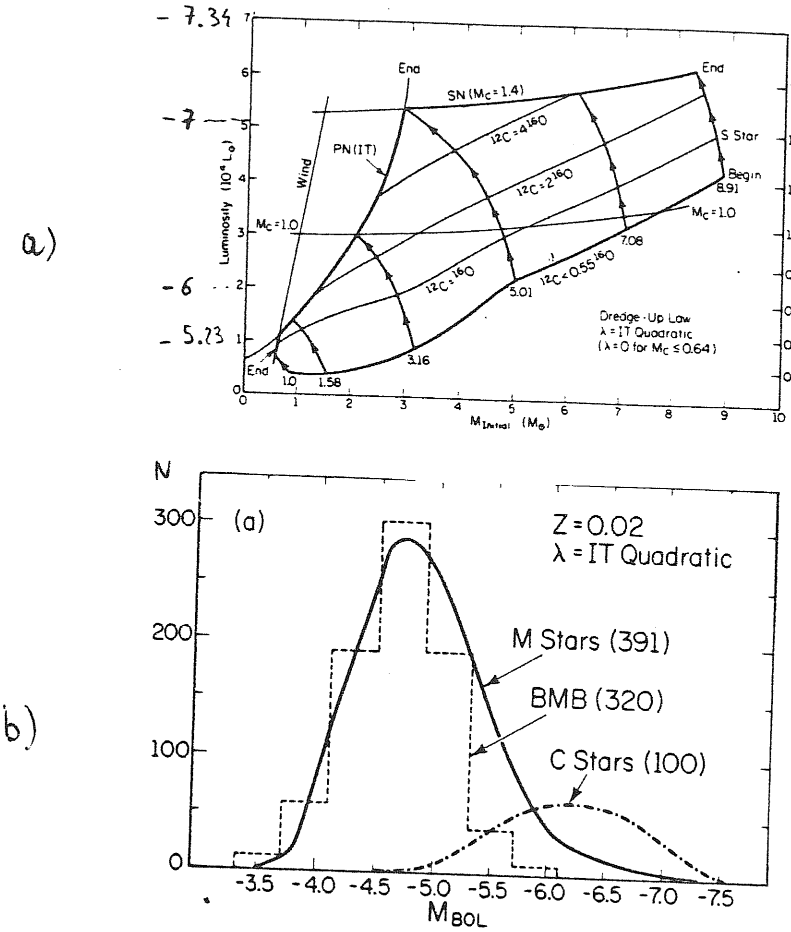

a)
b)
- 7.34
-7
-6
- 5.23
Luminosity ($10^4 L_\odot$)
$M_{Initial}$ ($M_\odot$)
M_c
End
End
SN (M_c = 1.4)
PN (IT)
Wind
S Star
Begin
8.91
M_c = 1.0
M_c = 1.0
7.08
5.01
3.16
1.58
1.0
End
Dredge - Up Law
λ = IT Quadratic
(λ = 0 for M_c ≤ 0.64)
N
(a)
Z = 0.02
λ = IT Quadratic
M Stars (391)
BMB (320)
C Stars (100)
M_{BOL}

Fig 15

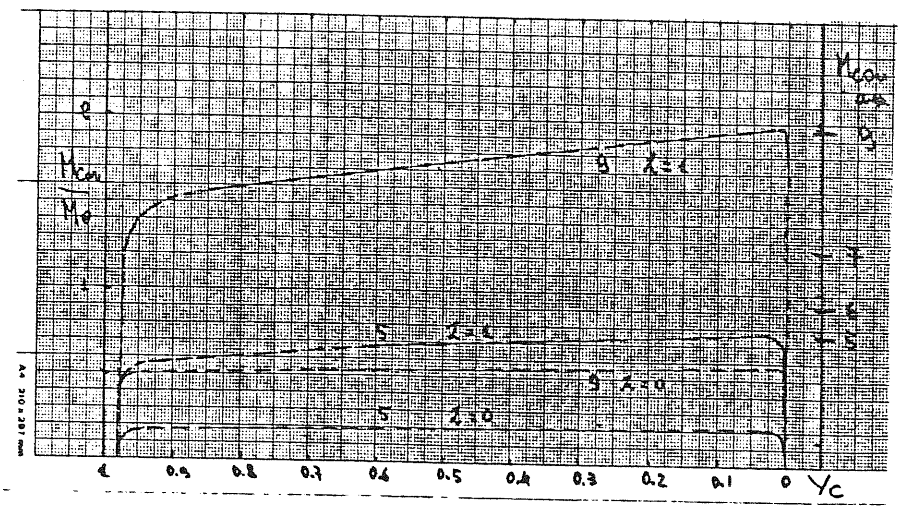

$\frac{M_{co}}{M_\odot}$
Y_c

Fig 16

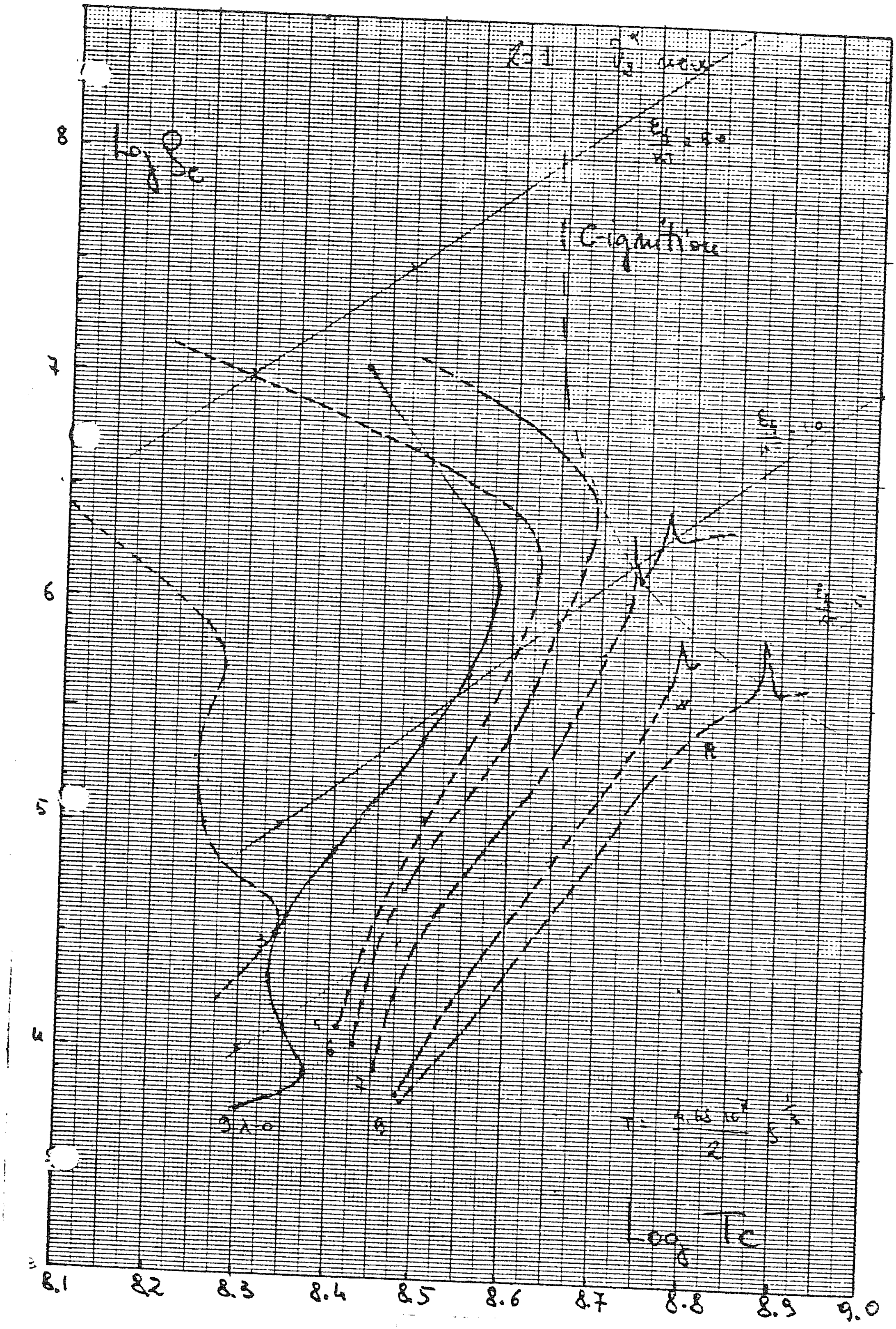
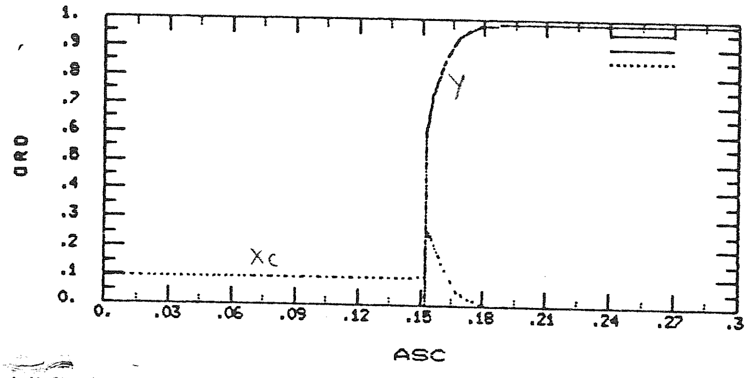
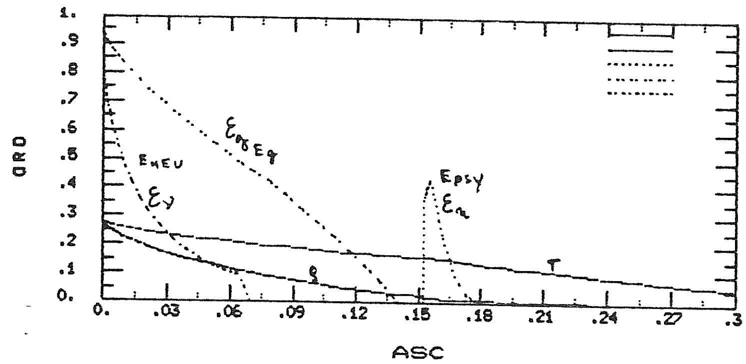


Fig. 17

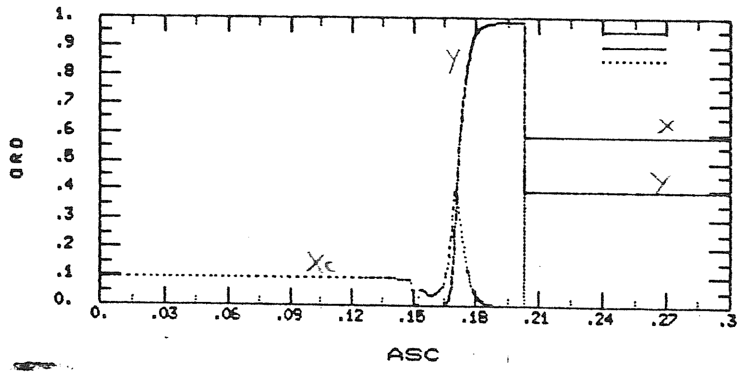


$ M=6Mo Mod.282

a)

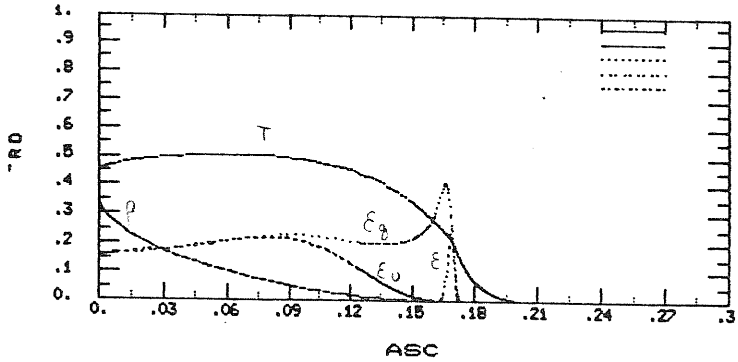


$ 6Mo Mod.282
Scale:T(1E9),Ro(1E5),Epsy(5E5),Eg(1E4),Eneu(1E3)



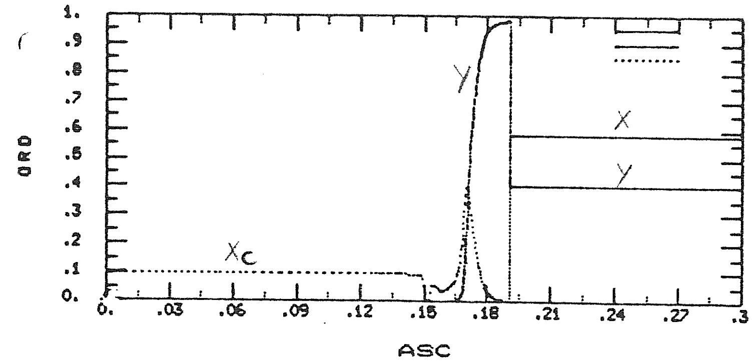
b)

$ M=6Mo mod.322



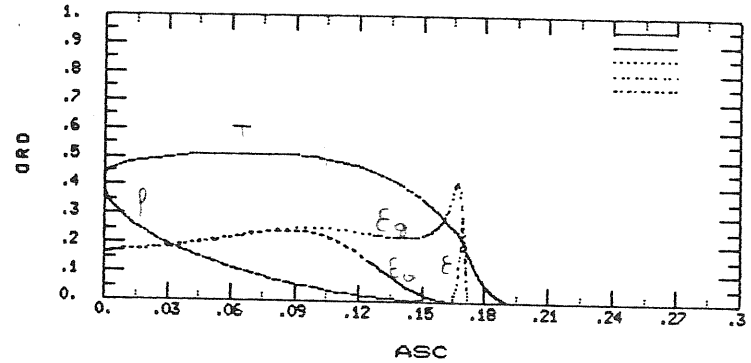
$ 6Mo Mod.322
Scale:T(1E9),Ro(1E7),Epsy(5E6),Eg=Eneu=2E5

Fig 18

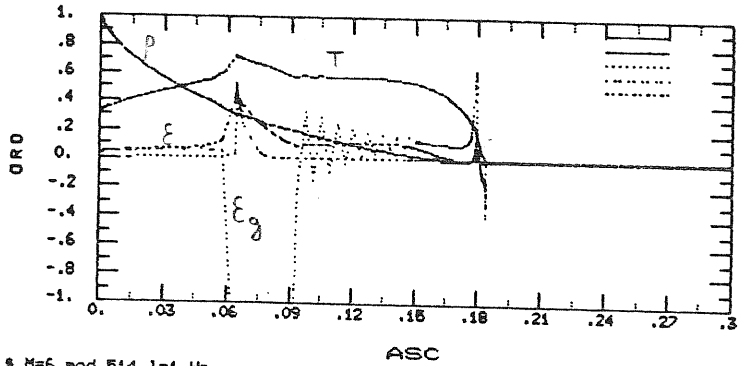


a)

M=6Mo Mod.362 l=1Hp

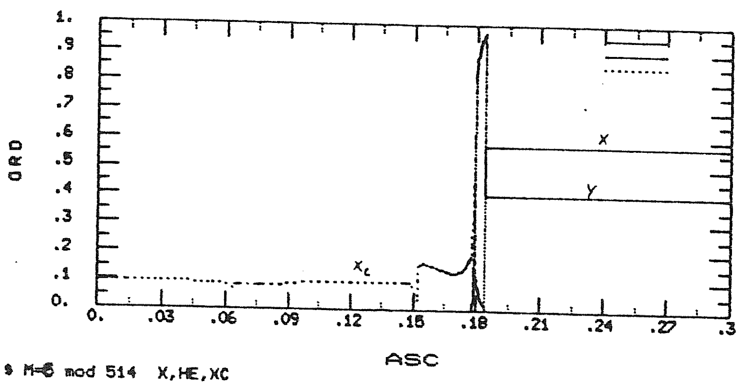


$ 6Mo Mod.362 Scale: T(1E9),Ro(1E7),Epsy(5E6),Eg=Eneu=2E5



$ M=6 mod 514 l=1.Hp
scale : T(1E9),Ro(1E7),En(1E7),Eg=Eneu(2E5)

b)



$ M=6 mod 514 X,HE,XC

Fig 19

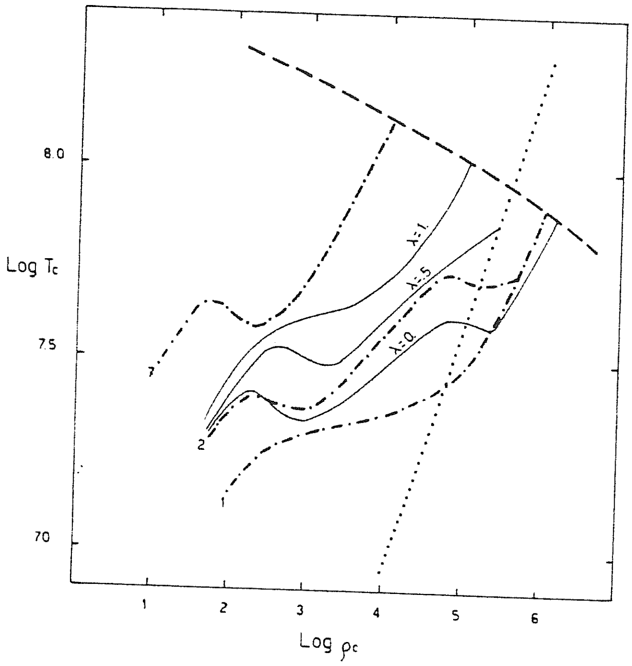


Fig. Evolution for the central conditions. Dashed line: locus of the He ignition; dotted line: boundary between non-degenerate region (left) and degenerate region (right) defined by the condition $E_f/kT=10$ (E_f is the Fermi energy); dashed-dotted lines: tracks taken from Iben (1974); full lines: tracks for models with 1.8 $M_\odot$ (Bertelli and Bressan, unpublished). $\lambda = l/H_p$

Fig. 20)

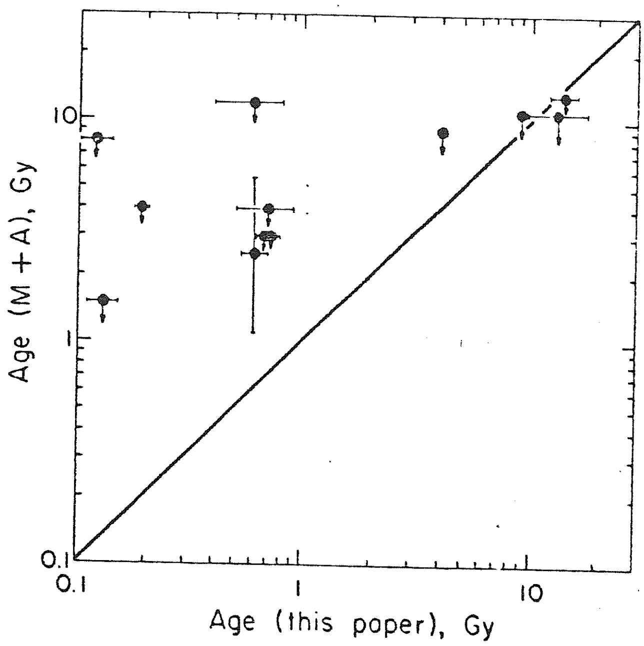


FIG. —A comparison of the ages derived from the main sequences with those given by Mould and Aaronson (1982) based on carbon star members. Upper limits are indicated by arrows. Data for NGC 1856's apparent carbon star member (Richer 1981) has been added using Mould and Aaronson's calibration.

Fig. 21)