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TESI

DIPLOMA DI PERFEZIONAMENTO

"MAGISTER PHILOSOPHIAE"

HIGHER ORDER CORRECTIONS TO ELECTROWEAK PROCESSES

CANDIDATO:
dott. S. Bellucci

RELATORE:
Prof. Nello Paver

Anno Accademico 1983/84

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1. Introduction

If one defines the parameters of the Salam-Weinberg theory at a momentum scale M^* of the order of the W-masses, the weak effective hamiltonian at a momentum scale $\mu \ll M^*$ has logarithmically enhanced corrections, of order $\alpha \ln \frac{M^{*2}}{\mu^2}$. A computation of these corrections has been presented in ref. [1] for that part of the hamiltonian which leads to detectable weak-electromagnetic interference effects. In the limit of negligible strong interaction effects, the order $\alpha \ln \frac{M^{*2}}{\mu^2}$ corrections can be divided into two classes.

A) Corrections proportional to a sum over quark and leptons flavours. These can be reabsorbed into the definition of a running Salam-Weinberg angle (see eq. (3.12) below).

B) The corrections not additive in quark and lepton flavours are reported in Table 1. The effect of these corrections turns out to be considerably smaller than the effect of the shift in $\sin^2\theta$.

Consideration of strong interaction correction makes very little change. Only the coefficients A_u and A_d of eq. (3.1) below, are affected. The relation which defines the running $\sin^2\theta$ in terms of $\sin^2\theta(M^*)$, eq. (3.12), is essentially unaffected; in fact, the strong corrections cancel exactly to order $\alpha_s \ln \frac{M^{*2}}{\mu^2}$ and higher order corrections make a negligible change. Strong interactions renormalize the corrections of type B). The effect is reported in Table 2, for $\mu^2 = 2$ GeV^2. The value $\alpha_s(1.4 \text{ GeV}) = 0.7$ is used and the results are given for the two extreme cases: four flavours only, which means m_t, $m_b > M^*$, and six fully active flavours, namely m_t, $m_b < \mu$. The effect is not particularly large for A_d and it is non-existent, in practice, for A_u, due to an accidental cancellation.

Deviations from the tree level pattern are very small for the coefficients $V\mu, u, d$ and $C\mu, u, d$ of eq. (3.1), but are sizeable for Au, d. From Table 2 one can extract the ratio, which vanishes at the tree level:

$$\frac{Au + Ad}{Au - Ad} = 0.05 \qquad (1.1)$$

It is interesting to determine the valud of $\sin^2\theta(M^*)$, in order to compare with the prediction of grand unified theories. In a first approximation, this can be obtained by fitting the SLAC data on deep inelastic electron scattering on deuterium [2] with the tree level formula. One obtains in this way $\sin^2\theta$ ($\mu \simeq 1.4$GeV) $= 0.224 \pm 0.020$ and the application of the correction given by eq. (3.12) gives $\sin^2\theta(M^*) = 0.217 \pm 0.020$, which is in very good agreement with the prediction of grand unified theories. However, the value of $\sin^2\theta(\mu)$ can be shifted upward by the effect of the strange quark sea and, although is unlikely that its contribution could be large at such low q^2, more precise data, where the strange quark distribution is directly disentangled from the data, are certainly needed.

Preliminary results are available from CERN NA4 experiment [3] on deep inelastic muon scattering on nucleons, which are relevant for the experimental investigation of the neutral current structure and for a check of the predictions of the Salam-Weinberg-GIM model. Measurements of the charge and polarization asymmetry, corresponding to a simultaneous reversal of both the helicity and the charge of the muon, depend on the vector and the axial-vector couplings of the muon to the neutral vector boson. The authors of ref. [3] find a preliminary value of $\sin^2\theta(\mu \simeq 7.7\ \text{GeV}) = 0.26 \pm 0.09$ and the application of the correction (3.12) gives us a value of $0.2\cdot 10^{-2}$ for the difference between $\sin^2\theta(7.7\ \mathrm{Gev})$ and $\sin^2\theta(M^*)$[24]. A complete calculation of the one-loop corrections to the charge and polarization asymmetry in $\mu^{\mp}(\pm\lambda) + N \to \mu^{\mp}(\pm\lambda)$ + anything, can be found in ref. [4].

There are two classes of non-logarithmically enhanced corrections which have potentially large coefficients, such as to compete with the $\ell u \frac{M^{*2}}{\mu^2}$ enhancement. For reactions not completely inclusive, such as the SLAC asymmetry, the infrared virtual divergencies associated to the vanishing photon mass cancel with the corresponding real photon divergencies, leaving logarithmic mass singularities, i.e. terms of order $\alpha \ell u(\frac{\mu^2}{m^2})$, m being a lepton or quark mass. These corrections can be computed in a way similar to that followed in ref. [5] for neutrino processes. They are needed for a complete analysis of $\sin^2\theta$, but we may observe that, unlike the renormalization of $\sin^2\theta$ these corrections are not additive in quark and lepton flavour, so that we may expect them not to change significantly the results on $\sin^2\theta(M^*)$.

A part of the remaining (i.e. with no mass singularity) corrections is actually of order $\alpha_W = \alpha(\sin\theta)^{-2} \approx 5\alpha$. In ref. [1] they were computed in a simple way, by taking the limit $\sin^2\theta \to 0$, α_W fixed (see Appendix F). These corrections alter significantly the leading logarithmic result, eq. (1.1), i.e. upon including them, one obtains:

$$\frac{Au + Ad}{Au - Ad} = 0.10 \qquad (1.2)$$

On the other hand, the analysis of $\sin^2\theta$ is essentially unaffected by such corrections.

In the case of the angular distribution asymmetry of the muon pair in the annihilation $e^+e^- \to \mu^+\mu^-$, the influence of the strong interactions turns out to be particularly suppressed. In fact, there are no leading corrections of the form $(\alpha \ell n \frac{M^{*2}}{\mu^2})^m \cdot (\alpha_s \ell n \frac{M^{*2}}{\mu^2})^n$, for $m = 1,2$ and n positive integer. Therefore it is meaningful to compute the $(\alpha \ell n \frac{M^{*2}}{\mu^2})^2$ correction. In ref. [6] these corrections plus the whole of the order α corrections have been computed(1). The order α correction were already computed numerically in ref. [8]; in ref. [6] the analytic expression of the correction is reported. The order α corrections are given in the kinematic region $m^2, \mu^2 \ll |s|,|t|,|u| \ll M^2, M_0^2$ where m, μ are the electron and muon masses and M, M_0 are the masses of the vector bosons; the invariants s, t, u are defined in Appendix A. A complementary region 1600 GeV$^2 \leqslant s \leqslant$ 19·600 GeV^2 is considered in ref. [9] where the radiative corrections to polarized e^+e^- annihilation are computed in the standard model.

The asymmetry, defined in eq. (5.2) below, is given by eq. (5.18) where the corrected values of the coefficients $d_{1,2}(\theta_s)$ are expressed by eqs. (5.41) and (5.40). The virtual one-loop corrections, after the renormalization of the UV divergences, still contain the IR ones. Including the Bremsstrahlung corrections, in the soft photon approximation, one verifies that the IR divergence cancels out. Still missing is the analytic expression of the hard photon corrections, which however are free from UV and IR divergence, well-defined and numerically calculable by integrating over the characteristics of the experimental set-up. In the $e^+e^- \to \mu^+\mu^-$ case, these corrections do not affect the dominant part of the order α real corrections, namely the mass-singular terms, which diverge in the limit of vanishing lepton masses.

The thesis is organized as follows. In Sect. 2 we sketch the theoretical framework. In Sect. 3 we present the structure of the leading corrections and discuss the strong interaction effects. Sect. 4 is devoted to determine the corrections in the cases of electron-deuterium asymmetry and of charge and polarization asymmetry in $\mu^{\pm}$ N deep inelastic scattering; in the case of the backward-forward μ^{+} asymmetry in $e^{+}e^{-} \to \mu^{+}\mu^{-}$ we determine the leading correction up to the second order in α. In Sect. 5 the order α corrections to the $e^{+}e^{-} \to \mu^{+}\mu^{-}$ asymmetry are considered. There are six Appendices: Appendix A contains the conventions and notations; in Appendices B and C details on the virtual order α correction, and in Appendix D the finite part of the soft Bremsstrahlung, are illustrated for the $e^{+}e^{-} \to \mu^{+}\mu^{-}$ asymmetry; Appendix E contains the Feymnan rules for the Salam-Weinberg Lagrangian; Appendix F contains an evaluation of the order α_W corrections to the weak hamiltonian, in the limit $\alpha \to 0$.

2. The Salam-Weinberg Theory of Electroweak Interactions

The main feature of the Salam-Weinberg model[10] is the renormalizability of the resulting theory that was conjectured and realized by the authors by means of the Higgs mechanism. We will assume as a background for this chapter the notion of the spontaneous symmetry breakdown of non-abelian gauge theories and the Higgs phenomenon.

In the original model the gauge group is $SU(2)_L \times U(1)_Y$; each lepton and its neutrino transform as a spinoral representation of the weak isospin group $SU(2)_L$:

$$\ell_i = \binom{\nu_i}{\ell_i}_L \qquad i = e, \mu, \tau, \dots$$

where $\nu_L = \frac{1}{2}(1+\gamma_5)\nu$ and $\ell_L = \frac{1}{2}(1+\gamma_5)\ell$ are the left-handed components of the lepton spinor fields; their introduction is natural because the weak interactions involve only the projections with positive chirality. Let $\vec{t} \equiv (t_1,t_2,t_3)$ be the weak isospin quantum numbers and $\overset{1}{W}_\lambda$, $\overset{2}{W}_\lambda$, $\overset{3}{W}_\lambda$ the Yang-Mills fields representing the gauge vector bosons; they belong to the regular representation of the $SU(2)_L$ group and transform as a triplet. The vector field B_λ and the quantum number Y of the weak hypercharge which together with the third component of the weak isospin defines the electric charge operator $Q = t_3 + \frac{1}{2}Y$, are associated to the gauge group U(1). By means of the previous definition of the Q operator we can reconstruct the quantum numbers of Table 1, where e_R, μ_R represent the right-handed components (defined by $\ell_{iR} = \frac{1}{2}(1-\gamma_5)\ell_i$, $i = e, \mu \dots$) which are $SU(2)_L$ singlets yet enter the definition of the e.m. current because the latter conserves parity. In Table 1 appears the Higgs field ϕ introduced in order to give a mass to the particles of the theory. Let us suppose to spontaneously break the $SU(2)_L$ symmetry preserving the symmetry of the gauge group $U(1)_Q$ associated with the electric charge; the $U(1)_Q$ massless vector field, namely the photon field A_λ, is expressed as a linear combination of W_λ^3 and B_λ, while the orthogonal combination W_λ^o indicates a spin-1 neutral massive particle:

$$A_\lambda = c_\theta B_\lambda + s_\theta W_\lambda^3$$

$$W_\lambda^o = -s_\theta B_\lambda + c_\theta W_\lambda^3 \qquad (2.1)$$

where $s_\theta = \sin\theta_W$, $c_\theta = \cos\theta_W$ and θ_W is the mixing angle between the neutral fields. In addition we define:

$$W_\lambda = \frac{1}{2}(W_\lambda^1 + iW_\lambda^3)$$

$$W_\lambda^+ = \frac{1}{2}(W_\lambda^1 - iW_\lambda^2) \qquad (2.2)$$

W_λ and W_λ^+ represent spin-1 charged massive particle.

The coupling of the $SU(2)_L \times U(1)_Y$ vector bosons to leptons is given by

$$g \sum_{i=1}^{3} (J_\lambda^1 W_\lambda^1) + \frac{1}{2} g' J_\lambda^Y B_\lambda \qquad (2.3)$$

where g, g′ are the coupling constants and $\vec{J}_\lambda \equiv (J_\lambda^1, J_\lambda^2, J_\lambda^3)$ and J_λ^Y are the currents associated to the $SU(2)_L$ and $U(1)_Y$ subgroups; they obey the relations:

$$\int d^3x\, \vec{J}_o(x) = \vec{t}$$

$$\int d^3x\, J_o^Y(x) = Y \qquad ;$$

Using the (2.1) in the (2.3), the latter becomes:

$$g(J_\lambda^1 W_\lambda^1 + J_\lambda^2 W_\lambda^2) + (g s_\theta J_\lambda^3 + \frac{1}{2} g' c_\theta J_\lambda^Y) A_\lambda + (g c_\theta J_\lambda^3 - \frac{1}{2} g' s_\theta J_\lambda^Y) W_\lambda^o \qquad (2.4)$$

The e.m. interaction is:

$$e J_\lambda^{e.m.} A_\lambda = e(J_\lambda^3 + \frac{1}{2} J_\lambda^Y) A_\lambda$$

where e is the proton charge.

Comparing with (2.4) we get:

$$e = g\, s_\theta = g' c_\theta \tag{2.5}$$

and the Salam-Weinberg angle appears to be nothing but the inverse tangent of the singlet and triplet coupling constant rate.

By means of (2.2) and (2.5) we can give to (2.4) the form:

$$eJ_\lambda^{e.m.} A_\lambda + \frac{1}{2}g\,(J_\lambda W_\lambda^{+} + J_\lambda^{+} W_\lambda) + \frac{g}{c_\theta} J_\lambda^{z} W_\lambda^{o}$$

where $J_\lambda^z = c_\theta J_\lambda^3 - \frac{1}{2} s_\theta^2 J_\lambda^Y$, $J_\lambda = J_\lambda^1 + iJ_\lambda^2$, $J_\lambda^{e.m.} = J_\lambda^3 + \frac{1}{2} J_\lambda^Y$ are the weak neutral current, the weak charged current and the e.m. current; their expressions in terms of the lepton fields look like:

$$J_\lambda^z = \frac{1}{2} \bar{\nu}_{eL}\gamma_\lambda \nu_{eL} - \frac{1}{2}(c_\theta^2 - s_\theta^2)\, \bar{e}_L\gamma_\lambda e_L + s_\theta^2\, \bar{e}_R\gamma_\lambda e_R + (e \to \mu)$$

$$= \frac{1}{4}[\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\,\nu_e + \bar{e}\gamma_\lambda(v_\theta - \gamma_5)e] + (e \to \mu)$$

$$J_\lambda^{+} = \frac{1}{2}\, e\gamma_\lambda(1+\gamma_5)\,\nu_e + \frac{1}{2}\,\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu$$

$$J_\lambda^{e.m.} = -\bar{e}\gamma_\lambda e - \mu\gamma_\lambda\mu$$

being $v_\theta = 4s_\theta^2 - 1$.

Let us now consider the complex scalar field:

$$\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} H + \sqrt{2}\,F + i\phi^3 \\ -\phi^2 + i\phi^1 \end{pmatrix},$$

whose vacuum expectation value $\langle 0|\Psi|0\rangle$ produces the spontaneous breakdwon of the symmetry; it is a multiplet with isospin $\frac{1}{2}$ and hypercharge 1 which couples to the quantities $\bar{e}_L e_R$ and $\bar{\mu}_L\mu_R$ in order to give masses to the charged leptons.

By making use of these assumptions one finds the relations $M = c_\theta M_o$ between the $W^\pm$ mass and the W° mass. The H field represents a spinless neutral Higgs particle, while the fields $\phi^i (i=1,3)$ are the would be Goldstone bosons associated to the longitudinal component of the propagators of the fields $W_\lambda^i (i=1,3)$; we define $\phi^o = \phi^3$, $\phi^\pm = \frac{1}{\sqrt{2}}(\phi^1 \mp i\phi^2)$. In the quantization procedure we fix the gauge by introducing a term $\mathcal{L}_{g.f.}$ in the Lagrangian and at the same time introduce the Faddeev-Popov ghost fields X^i, $i=0,1,2,3$ having a dynamics described by the Lagrangian $\mathcal{L}_{ghost}$. We make the choice:

$$\mathcal{L}_{g.f.} = -\frac{1}{2}(-\partial_\mu W^a_\mu + M\phi^a)^2 - \frac{1}{2}(-\partial_\mu B_\mu - \frac{s_\theta}{c_\theta} M\phi^o)^2$$

which corresponds to the particular renormalizable gauge usually named Feynman-'t Hooft gauge.

By defining

$$Y^A = s_\theta X^3 + c_\theta X^o$$

$$Y^o = s_\theta X^o + c_\theta X^3$$

$$X^\pm = \frac{1}{\sqrt{2}}(X^1 \mp iX^2)$$

we can write down the complete Lagrangian in terms of the fields W^o_μ, A_μ, $W^\pm_\mu$, H, $\phi^\pm$, ϕ^o and the ghost fields:

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_\phi + \mathcal{L}_{g.f.} + \mathcal{L}_{fermions} + \mathcal{L}_{ghost}$$

where $\mathcal{L}_{YM}$ is the sum of the free Lagrangian of the Yang-Mills fields $\vec{W}_\lambda$ and the one of the B_λ field:

$$\mathcal{L}_{YM} \; \mathcal{L}_{YM}(W^1_\lambda, W^2_\lambda, W^3_\lambda) + \mathcal{L}_{YM}(B_\lambda) .$$

The Lagrangian for the Φ field also contains the Higgs-lepton couplings that we systematically ignore so as we possibly do for the the fermion mass term in $\mathcal{L}_{fermions}$. The expression of $\mathcal{L}$ in terms of diagonal fields reads:

$$\mathcal{L} = -W^+_{\mu,\nu} W^-_{\mu,\nu} - M^2 W^+_\mu W^-_\mu - \frac{1}{2} W^o_{\mu,\nu} W^o_{\mu,\nu} - \frac{1}{2}\frac{M^2}{c^2_\theta} W^o_\mu W^o_\mu -$$

$$-\frac{1}{2}A_{\mu,\nu}A_{\mu,\nu} - \phi^+_{,\mu}\phi^-_{,\mu} - M^2\phi^+\phi^- - \frac{1}{2}\phi^0_{,\mu}\phi^0_{,\mu} - \frac{1}{2}\frac{M^2}{c_\theta^2}\phi^0\phi^0 - \frac{1}{2}H_{,\mu}H_{,\mu} -$$

$$-\frac{1}{2}M_H^2H^2 - ic_\theta\left\{W^0_{\mu,\nu}W^{[+}_\mu W^{-]}_\nu - W^0_\nu W^{[+}_\mu W^{-]}_{\mu,\nu} + W^0_\mu W^{[+}_\nu W^{-]}_{\mu,\nu}\right\} -$$

$$-is_\theta\left\{A_{\mu,\nu}W^{[+}_\mu W^{-]}_\nu - A_\nu W^{[+}_\mu W^{-]}_{\mu,\nu} + A_\mu W^{[+}_\nu W^{-]}_{\mu,\nu}\right\} - \frac{1}{2}\left(W^+_\mu W^-_\mu\right)^2 +$$

$$+\frac{1}{2}\left(W^+_\mu W^-_\nu\right)^2 + c_\theta^2\left\{W^0_\mu W^+_\mu W^0_\nu W^-_\nu - W^0_\mu W^0_\mu W^+_\nu W^-_\nu\right\} +$$

$$+s_\theta^2\left\{A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu\right\} + s_\theta c_\theta\left\{A_\mu W^0_\nu\left(W^+_\mu W^-_\nu + W^+_\nu W^-_\mu\right) -\right.$$

$$\left.-2A_\mu W^0_\mu W^+_\nu W^-_\nu\right\} - 2\beta M^2 - 2\beta MH - \frac{1}{2}\beta\left(H^2 + \phi^{0\,2} + 2\phi^+\phi^-\right) + 2M^4\alpha -$$

$$-\alpha MH\left\{H^2 + \phi^{0\,2} + 2\phi^+\phi^-\right\} - \frac{1}{8}\alpha\left\{H^4 + \phi^{0\,4} + 4\phi^+\phi^-\phi^+\phi^- + 4\phi^{0\,2}\phi^+\phi^- +\right.$$

$$\left.+4H^2\phi^+\phi^- + 2\phi^{0\,2}H^2\right\} - MHW^+_\mu W^-_\mu - \frac{1}{2}\frac{M}{c_\theta^2}HW^0_\mu W^0_\mu -$$

$$-\frac{1}{2}i\left\{W^+_\mu\left(\phi^0\phi^-_{,\mu} - \phi^-\phi^0_{,\mu}\right) - W^-_\mu\left(\phi^0\phi^+_{,\mu} - \phi^+\phi^0_{,\mu}\right)\right\} +$$

$$+\frac{1}{2}\left\{W^+_\mu\left(H\phi^-_{,\mu} - \phi^-H_{,\mu}\right) + W^-_\mu\left(H\phi^+_{,\mu} - \phi^+H_{,\mu}\right)\right\} + \frac{1}{2}\frac{1}{c_\theta}W^0_\mu\left(H\phi^0_{,\mu} - \phi^0H_{,\mu}\right) -$$

$$-i\frac{s_\theta^2}{c_\theta} M W^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + i s_\theta M A_\mu (W^+_\mu\phi^- - W^-_\mu\phi^+) -$$

$$-\frac{i}{c_\theta}\left(\frac12 - c_\theta^2\right) W^0_\mu(\phi^+\phi^-_{,\mu} - \phi^-\phi^+_{,\mu}) + i s_\theta A_\mu(\phi^+\phi^-_{,\mu} - \phi^-\phi^+_{,\mu}) -$$

$$-\frac14 W^+_\mu W^-_\mu (H^2+\phi^{0\,2}+2\phi^+\phi^-) - \frac18 \frac{1}{c_\theta^2} W^0_\mu W^0_\mu \left[H^2+\phi^{0\,2}+2(2s_\theta^2-1)^2\phi^+\phi^-\right] -$$

$$-s_\theta^2 A_\mu A_\mu \phi^+\phi^- - \frac12 \frac{s_\theta^2}{c_\theta} W^0_\mu \phi^0 (W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac12 i \frac{s_\theta^2}{c_\theta} W^0_\mu H (W^+_\mu\phi^- - W^-_\mu\phi^+) +$$

$$+\frac12 s_\theta A_\mu \phi^0 (W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac12 i s_\theta A_\mu H (W^+_\mu\phi^- - W^-_\mu\phi^+) - \frac{s_\theta}{c_\theta}(2c_\theta^2-1) W^0_\mu A_\mu \phi^+\phi^- -$$

$$-\bar\nu_e \partial\!\!\!/ \nu_e - \bar e(\partial\!\!\!/ + m) e + \frac{i}{4c_\theta}\bar\nu_e\gamma^\mu(1+\gamma_5)\nu_e W^0_\mu + \frac{i}{2\sqrt2}\bar\nu_e\gamma^\mu(1+\gamma_5) e W^+_\mu +$$

$$+\frac{i}{2\sqrt2}\bar e\gamma^\mu(1+\gamma_5)\nu_e W^-_\mu - \frac{i}{2\sqrt2}\frac{m}{M}\bar\nu_e(1-\gamma_5)e\phi^+ + \frac{i}{2\sqrt2}\frac{m}{M}\bar e(1+\gamma_5)\nu_e\phi^- -$$

$$-\frac12\frac{m}{M}\bar e e H - \frac12 i \frac{m}{M}\bar e\gamma_5 e\phi^0 + \frac{i}{4c_\theta}\bar e\gamma^\mu(v_\theta - \gamma_5) e W^0_\mu - i s_\theta \bar e\gamma^\mu e A_\mu +$$

$$+\bar X^+\partial^2 X^+ - M^2\bar X^+ X^+ + \bar X^-\partial^2 X^- - M^2 \bar X^- X^- + \bar Y^0\partial^2 Y^0 - \frac{M^2}{c_\theta}\bar Y^0 Y^0 +$$

$$+\bar Y^A\partial^2 Y^A - i s_\theta W^+_\mu(\partial_\mu \bar X^+ Y^A - \partial_\mu \bar Y^A X^-) + i c_\theta W^+_\mu(\partial_\mu \bar Y^0 X^- - \partial_\mu \bar X^+ Y^0) -$$

$$-is_\theta W^-_\mu(\partial_\mu \bar{Y}^A X^+ - \partial_\mu \bar{X}^- Y^A) + ic_\theta W^-_\mu(\partial_\mu \bar{X}^- Y^0 - \partial_\mu \bar{Y}^0 X^+) -$$

$$-is_\theta A_\mu(\partial_\mu \bar{X}^- X^- - \partial_\mu \bar{X}^+ X^+) - ic_\theta W^0_\mu(\partial_\mu \bar{X}^- X^- - \partial_\mu \bar{X}^+ X^+) -$$

$$-\frac{1}{2} MH\left(\bar{X}^+ X^+ + \bar{X}^- X^- + \frac{1}{c_\theta^2} \bar{Y}^0 Y^0\right) + \frac{i}{c_\theta}\left(\frac{1}{2} - c_\theta^2\right) M(\bar{X}^+ Y^0 \phi^+ - \bar{X}^- Y^0 \phi^-) +$$

$$+i \frac{M}{2c_\theta}(\bar{Y}^0 X^- \phi^+ - \bar{Y}^0 X^+ \phi^-) - is_\theta M(\bar{X}^+ Y^A \phi^+ - \bar{X}^- Y^A \phi^-) -$$

$$-i \frac{M}{2}(\bar{X}^- X^- \bar{X}^+ X^+)\phi^0 \qquad (2.6)$$

We defined $x_{,\mu} \equiv \partial_\mu x$, $x^{[a}y^{b]} = x^a y^b - x^b y^a$ and, according to ref. [8], $M = \frac{g}{\sqrt{2}} F$, $\alpha = \frac{\lambda}{g^2}$, $\beta = \mu + \lambda F^2$, $M_H^2 = 4\alpha M^2$; λ, μ are the parameters entering the definition of $\mathcal{L}_\phi$:

$$\mathcal{L}_\phi = -(D_\mu \phi)^+ (D_\mu \phi) - \mu \phi^+ \phi - \frac{1}{2} \lambda (\phi^+ \phi)^2 .$$

In eq. (2.6) we have to associate a factor g to each term containing three fields, a factor g^2 to terms with four fields and a factor g^{-1} to term with just one field.

In Appendix E we write explicitly the Feynman rules for the Lagrangian (2.6).

So far we described a theory for weak interactions between leptons; to introduce hadrons we must add quark doublets $\binom{u}{d}_L = \frac{1+\gamma_5}{2}\binom{u}{d}$ together with the singlets: $u_R = \frac{1-\gamma_5}{2}u$, $d_R = \frac{1-\gamma_5}{2}d$, to the lepton weak isospin multiplets previously defined . The doublet fields are mixed unitarily with fields of higher masses by the matrix of the generalized Cabibbo angles. In the following we ignore the effects of the mixing.

Also, the strong interactions between quarks are described as gauge interaction based on the gauge group $SU(3)_{colour}$: this symmetry is not spontaneously broken and therefore there are no new Higgs fields. Taking into account all gauge interactions one ends with a unified gauge theory based on the group $SU(3)_{col} \times SU(2)_L \times U(1)_Y$.

3. Leading Logarthimic Corrections to the Weak Leptonic and Semi-Leptonic Low-Energy Hamiltonian

In this chapter we restrict ourselves to that part of the weak hamiltonian which leads to detectable weak-electromagnetic interference effects:

$$H = \frac{G_F}{2\sqrt{2}} [\bar{e}\gamma_\lambda\gamma_5 e\,(V_\mu \bar{\mu}\gamma_\lambda\mu + V_u\,\bar{u}\,\gamma_\lambda u + V_d\,\bar{d}\gamma_\lambda d)$$

$$+ \bar{e}\gamma_\lambda e(A_\mu \bar{\mu}\gamma_\lambda\gamma_5\mu + A_u\,\bar{u}\gamma_\lambda\gamma_5 u + A_d\,\bar{d}\gamma_\lambda\gamma_5 d)$$

$$+ \bar{e}\gamma_\lambda\gamma_5 e\,(C_\mu\,\bar{\mu}\gamma_\lambda\gamma_5\mu + C_u\bar{u}\gamma_\lambda\gamma_5 u + C_d\bar{d}\gamma_\lambda\gamma_5 d)] \qquad (3.1)$$

where G is the Fermi constant to be taken from muon mean-life and ρ is given in terms of the Salam-Weinberg parameters, defined at an energy scale $M^*=O(M,M_o)$ of the order of the vector boson masses:

$$\rho = \frac{M^2}{M_o^2 \cos^2\theta(M^*)}$$

The hamiltonian (3.1) gives rise to the following effects:

i) The parity-violating asymmetry in the deep inelastic scattering of polarized electrons [2]:

$$e_{R,L} + N \rightarrow e_{R,L} + \text{anything}$$

determined by the couplings $A_{u,d}$ and $V_{u,d}$.

ii) Parity violations in atoms [11], related to the same couplings.

iii) The backward-forward μ^+ asymmetry [12,13] in $e^+e^- \rightarrow \mu^+\mu^-$ related to the coefficient C_μ in eq. (1).

iv) The charge and polarization asymmetry in the deep inelastic scattering of muons [14]:

$$\mu \mp (\pm\lambda) + N \rightarrow \mu(\pm\lambda) + \text{anything}$$

where λ is the μ^- longitudinal polarization; this effect is related to the couplings $A_{u,d}$ and $C_{u,d}$. Following ref. [1] we want to compute the leading logarithmic corrections to all weak processes involving charged leptons only; these are the corrections which logarithmically diverge in the limit

$$M^2,\ M_o^2 \rightarrow \infty, \qquad G = \text{const}.$$

What happens to the theory when we make arbitrarily large the W-masses? In this limit, as a consequence of a well-known theorem [15], the vector bosons decouple from the other particles of the theory and all possible diagrams in which the W's are exchanged give no contribution to the corrections; we are left essentially with a four-fermion interactions plus the e.m. corrections which are U.V. divergent, as a consequence of our choice of infinite values for M^2, M_o^2. The higher order corrections to the four fermion interactions will be computed with a U.V. cut-off Λ to be identified with the ratio between the vector boson mass scale $M = O(M, M_o)$ and the momentum scale μ where our experiments are measured. The renormalization group equations allow us to compute the corrections to the bare hamiltonian in terms of the anomalous dimensions of four fermion operators. We can rewrite the uncorrected hamiltonian relevant to our problem as:

$$H^{(o)} = \frac{G\rho}{2\sqrt{2}} \{\frac{1}{2} J^z_\mu J^z_\mu + \frac{2}{\rho}[\bar{u}\gamma_\mu(1+\gamma_5)d\ \bar{d}\gamma_\mu(1+\gamma_5)u] + u,d \to c,s \to t,b\} \tag{3.2}$$

where the total neutral weak current J^z_μ reads:

$$J^z_\mu = \bar{f}\gamma_\mu[\tau_3 - 4Q\sin^2\theta(M^*) + \tau_3\gamma_5]f$$

In the fermion matrix space Q is the electric charge and τ_3 is twice the weak isospin of the left-handed component of the fermion (i.e. $\tau_3 = +1$ for up-quarks and $\tau_3 = -1$ for down-quarks and charged leptons). From eqs. (3.2) and (3.3) we find the uncorrected values of the coefficients in (3.1):

$$V_\mu^{(o)} = -[1 - 4\sin^2\theta(M^*)] \qquad V_q^{(o)} = \tau_3 - 4\sin^2\theta(M^*)Q$$

$$A_\mu^{(o)} = -[1 - 4\sin^2\theta(M^*)] \qquad A_q^{(o)} = \tau_3[1 - 4\sin^2\theta(M^*)]$$

$$C_\mu^{(o)} = 1 \qquad C_q^{(o)} = -\tau_3$$

We expand both the bare hamiltonian $H^{(o)}$ and the renormalized hamiltonian H into a basis of four fermion operators:

$$H^{(o)} = \sum_i C_i^{(o)} O_i$$
$$H = \sum_i C_i O_i(\mu) \tag{3.3}$$

where, as we have indicated explicitly, the operators in H are renormalized in such a way to have matrix elements equal to those of the bare operators, at a momentum scale of order μ.

The coefficients of eq. (3.3) obey the renormalization group equation:

$$\left(-\frac{\partial}{\partial t} + \beta\frac{\partial}{\partial \alpha} + \beta_s \frac{\partial}{\partial \alpha_s}\right)C_i + C_k \Gamma_{ki} = 0 \tag{3.4}$$

where $t = \ln\left(\frac{M^2}{\mu^2}\right)$ and Γ is defined by:

$$O_i^{unren} = (\delta_{ij} + \Gamma_{ij} \ln\Lambda^2) O_j$$

which relates the unrenormalized operators O_i^{unren} to the corrected ones. For non-leptonic operators, Γ contains terms of order α_s; for leptonic and semi-leptonic operators Γ is of order α. We define:

$$\vec{\alpha} = (\alpha, \alpha_s) \quad , \qquad \vec{\beta} = (\beta, \beta_s)$$

and let $\vec{\alpha}(t)$ be the solution of the equation:

$$\frac{d\vec{\alpha}}{dt} = \vec{\beta}[\vec{\alpha}(t)] \tag{3.5}$$

which fulfills the condition $\vec{\alpha}(o) = \vec{\alpha}$. Then, by using the identity:

$$\left(\vec{\beta} \cdot \frac{\partial}{\partial \vec{\alpha}}\right)\vec{\alpha}(t) = \vec{\beta}[\vec{\alpha}(t)]$$

we can give the expression of the solution of (3.4):

$$C_i = C_k^{(o)}[t = o, \vec{\alpha}'(t)]\left[\bar{T}\exp \int_o^t dt'\Gamma(\vec{\alpha}(t'))\right]_{ki} \tag{3.6}$$

we have indicated by $\bar{T}$ the anti-ordering operator in the t variable; in a matrix product $\bar{T}$ prescribes to order the matrices in such a way that the corresponding values of t decrease from left to right.

First, we neglect the strong interactions; thus, in the one-loop approximation, the following expressions hold:

$$\Gamma = \alpha\gamma \quad , \qquad \beta = \frac{1}{3\pi} b_Q \alpha^2 \equiv \frac{1}{3\pi} (\sum Q^2) \alpha^2$$

In eq. (3.6) we can now neglect the $\vec{T}$ operator

$$C_i = C_k^{(o)} [t = o, \overset{+}{\alpha}(t)] [\exp \int_o^t \alpha(t')\gamma \, dt']_{ki} \qquad (3.7)$$

Recalling eq. (3.5) we can rewrite the exponential factor of eq. (3.7) as:

$$\exp \int_o^t \alpha(t')\gamma \, dt' = \exp \int_{\alpha(\mu)}^{\alpha(M^*)} d\alpha \, \frac{\gamma\alpha}{\beta(\alpha)} = \exp\{\frac{3\pi\gamma}{b_Q} \ln [\frac{\alpha(M^*)}{\alpha(\mu)}]\}$$

We have, up to the second order in α:

$$\exp \int_o^t \alpha(t')\gamma \, dt' = 1 + \frac{3\pi\gamma}{b_Q} \ln [\frac{\alpha(M^*)}{\alpha(\mu)}] + \frac{1}{2} \frac{9\pi^2\gamma^2}{b_Q^2} \ln^2 [\frac{\alpha(M^*)}{\alpha(\mu)}] \qquad (3.7)$$

From (3.5) we have:

$$\frac{d\alpha}{dt} = \frac{b_Q}{3\pi}\alpha^2$$

Integrating between the values $\ln(\frac{M^2}{M^2}) = o$ and $\ln(\frac{M^2}{\mu^2}) = t$ we get

$$\alpha(\mu) = \alpha(M^*) \frac{1}{1 + \alpha(M^*)\frac{b_Q t}{3\pi}} \qquad (3.8)$$

We expand the logarithm of eq. (3.7), up to terms $[\alpha(M^*)]^2$:

$$\ln[\frac{\alpha(M^*)}{\alpha(\mu)}] = \frac{b_Q}{3\pi} t\alpha(M^*) [1 - \frac{1}{2} \frac{b_Q}{3\pi} t\alpha(M^*)]$$

and obtain:

$$\exp \int_o^t \alpha(t')\gamma \, dt' = 1 + \gamma\alpha(M^*)t + \frac{1}{2}\alpha^2(M^*)t^2(\gamma^2 - \frac{b_Q}{3\pi}\gamma)$$

Recalling eqs. (3.7) and (3.3) we write the first order and the second order leading logarithmic corrections as:

$$H^{(1)}(\mu) = C_i^{(o)} \gamma_{ij} O_j(\mu)\alpha(M^*)t$$

$$H^{(2)}(\mu) = C_i^{(o)} (\gamma^2 - \frac{b_Q}{3\pi}\gamma)_{ij} O_j(\mu) \frac{1}{2}\alpha^2(M^*)t^2$$

In order to determine the leading logarithmic corrections up to the second order we just need to compute the renormalization of the whole hamiltonian up to the first leading logarithmic order:

$$\frac{1}{\alpha(M^*)\ln(\frac{M^{*2}}{\mu^2})}\,\delta(H^{(o)}) = C_i^{(o)}\gamma_{ij}O_j(\mu) \qquad (3.9)$$

Indeed, by applying again the first order correction to eq. (3.9):

$$\frac{1}{\xi}\,\delta[\frac{1}{\xi}\delta(H^{(o)})] = C_i^{(o)}\gamma_{ij}\gamma_{jk}O_k(\mu)$$

where $\xi \equiv \alpha(M^*)\ln(\frac{M^{*2}}{\mu^2})$, we can express the second order correction as follows:

$$H^{(2)}(\mu) = \{\frac{1}{\xi}\delta[\frac{1}{\xi}\delta(H^{(o)})] - \frac{b_Q}{3\pi}\,\frac{1}{\xi}\delta(H^{(o)})\}\frac{1}{2}\,\xi^2 \qquad (3.10)$$

We now compute the divergent part of the one photon correction to $H^{(o)}$, eq. (3.2). The relevant classes of diagrams are illustrated in fig. 1; for the charged current contribution to $H^{(o)}$, only the diagrams of fig. 1b are present. The diagram of fig. 1a gives a correction to $H^{(o)}$ proportional to a sum quark and lepton flavours:

$$\delta_a(H^{(o)}) = \frac{G\rho}{2\sqrt{2}}\,\delta_a(\frac{1}{2}J_\mu^z J_\mu^z) = \frac{G\rho}{2\sqrt{2}}\,\frac{\alpha}{3\pi}\,\ln\Lambda^2[-4(b_2 - b_Q\sin^2\theta(M^*))] \cdot J_\mu^{e.m.}J_\mu^z \qquad (3.11)$$

where $b_2 = \frac{1}{4}\sum\tau_3 Q$. On the other hand, the change of $H^{(o)}$ under a variation δs^2 of $\sin^2\theta$ is:

$$\delta(H^{(o)}) = \frac{G\rho}{2\sqrt{2}}(-4\delta s^2)J_\mu^{e.m.}J_\mu^z$$

Therefore, the corrections of fig. 1a are equivalent to a shift of $\sin^2\theta$ in the coefficients $V_\mu^{(o)}$, $V_q^{(o)}$, $A_\mu^{(o)}$, $A_q^{(o)}$ by the amount:

$$\sin^2\theta(\mu) - \sin^2\theta(M^*) = \frac{\alpha}{3\pi}\,[t_2 - t_Q\sin^2\theta(M^*)] \qquad (3.12)$$

where we have defined:

$$t_Q = \sum_i Q_i^2 t_i(\mu) \quad , \quad t_2 = \frac{1}{4}\sum_i (Qt_3)_i t_i(\mu)$$

The sum is extended to all fermions; $t_i = \ln\frac{M^{*2}}{\mu^2}$ if the fermion exchanged has $m_i < \mu$ and $t_i = \ln(\frac{M^{*2}}{m_i^2})$ if $m_i > \mu$. It is interesting to note that by extrapolating the running SU(2) coupling, α_W, below the W and Z mass, dropping thus the contribution of W and Z but keeping the contribution of light fermions to the β-function, one would get precisely eq. (3.12). Using the quark mass values (in GeV) $m_{u,d} = 0.14$, $m_s = 0.25$, $m_c = 1.8$, $m_b = 5.0$, $m_t = 20$, and the measured lepton masses we find:

$$\sin^2\theta(1.4\ \mathrm{GeV}) - \sin^2\theta(M^*) = 0.7\cdot 10^{-2} \tag{3.13}$$

$$\sin^2\theta(1\mathrm{MeV}) - \sin^2\theta(M^*) = 0.9\cdot 10^{-2} \tag{3.14}$$

for $\sin^2\theta$ between 0.21 and 0.22. The value in eq. (3.14) is relevant for parity violation in atoms.

Corrections from the diagrams of figs. 1b,c are summarized in table 1.

Strong interactions modify the contribution of the diagrams of figs. 1a,b in the case where quarks run into the loop, as we will see later. Let us turn now to consider eq. (3.6) in presence of strong interactions. We have now, in the one-loop approximation:

$$\Gamma = \alpha\gamma + \alpha_s\gamma_s \quad , \quad \beta_s = \frac{1}{3\pi} b_s \alpha_s^2 = -\frac{33-2n}{12\pi}\alpha_s^2$$

Defining:

$$U_s(t,o) = \bar{T}\exp\int_o^t dt'\alpha_s(t')\gamma_s$$

and expanding eq. (3.6) up to second order in α and in the leading logarithmic approximation in α_s, one gets:

$$C_i = C_k^{(o)}[U_s(t,o) + \alpha \int_0^t dt' U_s(t,t')\gamma U_s(t',0)$$
$$+ \frac{\alpha^2}{2} \int_0^t dt' \int_0^{t'} dt'' U_s(t,t')\gamma U_s(t',t'')\gamma U_s(t'',0)]_{ki}$$

and, recalling

$$H(\mu) = C_i^{(o)}[U_s(t,0)]_{ij} O_j(\mu) + \alpha \int_0^t dt' C_i^{(o)}[U_s(t,t')\gamma U_s(t',0)_{ij} O_j(\mu)$$
$$+ \frac{\alpha^2}{z} C_i^{(o)}[\int_0^t dt' \int_0^{t'} dt'' U_s(t,t')\gamma U_s(t',t'')\gamma U_s(t'',o)]_{ij} O_j(\mu) \qquad (3.15)$$

In the end, we shall be interested in the semileptonic and leptonic components of $H(\mu)$, so that we may replace U_s by one, whenever it acts on the $O_i(\mu)$; thus eq. (3.15) simplifies to:

$$H(\mu) = H^{(o)'} + \alpha C_i^{(o)}[\int_0^t dt' U_s(t,t')]_{ij}\gamma_{jk} O_k(\mu)$$
$$+ \frac{\alpha^2}{2} C_i^{(o)}[\int_0^t dt' \int_0^{t'} dt'' \; U_s(t,t')\gamma U_s(t',t'')]_{ij}\gamma_{jk} O_k(\mu) \qquad (3.16)$$

This equation can be interpreted as follows. We must let the effective hamiltonian evolve from M^* to μ' under the influence of the strong interactions ($t' = \ln\frac{M^{*2}}{\mu'^2}$). Then the leectromagnetic interaction acts, transforming for instance non-leptonic into semileptonic operators. Let again the strong interaction act between μ' and μ'' and so on, at the various orders of the expansion of eq. (3.16). The final result is obtained by integrating over all the intermediate variables t', t'', ... We can already draw some conclusions from eq (3.16):

i) We may divide $H^{(o)}$ into a non-leptonic and into a leptonic and semileptonic part. The contribution of the latter to eq. (3.16) is free from strong interaction effects, in the first order approximation in α.

ii) The matrix γ describes the exchange of one photon, i.e. it can change only one quark pair into a lepton pair. Therefore, in computing the leading logarithmic corrections of the first order in α to any purely leptonic process, non vanishing contributions are obtained only by starting with a semileptonic or purely leptonic four fermion operator: in the calculation of purely leptonic terms we may thus drop the non-leptonic part from $H^{(o)}$ and the result is not affected by the strong interactions, which enter only to order $[\alpha \ln M^{*2}]^2$.

iii) From the above observation it also follows that we may obtain semileptonic operators from the non-leptonic terms, to first leading logarithmic order in α, but the lepton pair must be in a vector state; therefore only A_u and A_d are influenced by the strong interactions.

We may also develop U_s in powers of α_s. Fist-order terms describe the contribution to H from the non-leptonic operators obtained by applying first-order QCD corrections to $H^{(o)}$. The latter operators are products of quark colour octet bilinears. Since the photon is a colour singlet, it cannot produce a lepton pair from a $q\bar{q}$ pair of the same bilinear. It thus follows that:

iv) Corrections proportional to a sum over quark flavours are not affected by strong interactions, to first order in $\alpha_s \ln M^{*2}$.

Defining u(x), d(x), s(x), c(x) to be the parton densities in the proton and $\bar{u}(x)$, ..., $\bar{c}(x)$ the corresponding antiparton densities, and setting:

$$q(x) = u(x) + d(x) + s(x) + c(x)$$

$$D(x) = u(x) + d(x) + \bar{u}(x) + \bar{d}(x) + \frac{2}{5}[s(x) + \bar{s}(x)] + \frac{8}{5}[c(x) + \bar{c}(x)]$$

$$= q(x) + \bar{q}(x) - \frac{3}{5}[s(x) + \bar{s}(x)] + \frac{3}{5}[c(x) + \bar{c}(x)] \qquad (3.17)$$

we can write the flavour non-changing, parity-violating part of the non-leptonic hamiltonian as follows:

$$H^{(o)}_{NL} = \frac{G\rho}{2\sqrt{2}}[-(1 - 2\sin^2\theta(M^*))\bar{q}\gamma_\mu\gamma_5\tau_3 q\bar{q}\gamma_\mu\tau_3 q$$

$$+ \frac{2}{3}\sin^2\theta(M^*)\bar{q}\gamma_\mu\gamma_5\tau_3 q\bar{q}\gamma_\mu q - \frac{1}{\rho}\bar{q}\gamma_\mu\gamma_5\tau_1 q\bar{q}\gamma_\mu\tau_1 q$$

$$- \frac{1}{\rho}\bar{q}\gamma_\mu\gamma_5\tau_2 q\bar{q}\gamma_\mu\tau_2 q] \qquad (3.18)$$

The last two terms correspond to the charged current hamiltonian. We will consider the two extreme cases of four and six quark flavours and we will find that the results are very similar, indicating the the b and t contributions are irrelevant. In this simplified case, the four-fermion operator basis is fixed once for all, and we may directly use the results of ref. [16] for the anomalous dimension matrix. Let us define $O(A,B) = \bar{q}\gamma_\mu\gamma_5 Aq\bar{q}\gamma^\mu Bq$, where A and B act on flavour and colour indices; the four-fermion operators of the basis are:

$$O_s(\tau) = \frac{1}{6}[O(\tau,\tau) - \frac{O(1,1)}{n+1}] + \frac{1}{16}[O(\tau t^A,\tau t^A) - \frac{O(t^A,t^A)}{n+1}]$$

$$O_A(\tau) = \frac{1}{12}[O(\tau,\tau) + \frac{O(1,1)}{n-1}] - \frac{1}{16}[O(\tau t^A,\tau t^A) + \frac{O(t^A,t^A)}{n-1}]$$

$$O(1,1) \quad , \quad O(t^A,t^A) \quad , \quad O(\tau_3,1) \quad ,$$

$$O(\tau_3 t^A,t^A) \quad , \quad O(1,\tau_3) \quad , \quad O(t^A,\tau_3 t^A)$$

where $\tau = \tau_1, \tau_2, \tau_3$, the colour matrices are indicated by t^A, summation over A is understood and n = 4 or 6 is the number of quark flavours. In the n = 4 case the operator $O_s(\tau)$ transforms according to the "84" representation of SU(4), the operator $O_A(\tau)$ according to the "20" representation, the operator $O(1,1)$ and $O(t^A,t^A)$ according to the singlet representation and the last four operators according to the regular representation. The anomolous dimension matrix is block diagonal in the set of the basis operators and so is the matrix $U_s(t,o)$, which can be explicitly written as:

$$U_s(t,o) = \bar{T}\exp \int_0^t \alpha_s(t')\gamma_s dt' = \left[\frac{\alpha_s(M^*)}{\alpha_s(\mu)}\right]^d$$

In the basis given above the matrix d is given by (see [16]):

$$d_s = \frac{6}{33 - 2n}$$

$$d_A = -2d_s$$

$$d_1 = \frac{9}{66 - 4n}\begin{bmatrix} 0 & \frac{11}{9} \\ \frac{32}{9} & \frac{1}{27}(12n - 40) \end{bmatrix}$$

$$d_R = \frac{9}{66 - 4n}\begin{bmatrix} 0 & \frac{2}{9} & 0 & 1 \\ 0 & \frac{1}{27}(12n - 85) & \frac{32}{9} & \frac{5}{3} \\ 0 & \frac{11}{9} & 0 & 0 \\ \frac{32}{9} & \frac{41}{27} & 0 & -3 \end{bmatrix}$$

The expansion of (3.18) into the basis is:

$$H_{NL}^{(o)} = -\frac{G\rho}{2\sqrt{2}}\{(1 - 2\sin^2\theta(M^*))\, O_s(\tau_3) + \frac{1}{\rho}[O_s(\tau_1) + O_s(\tau_2)]$$

$$+ (1 - 2\sin^2\theta(M^*))O_A(\tau_3) + \frac{1}{\rho}[O_A(\tau_1) + O_A(\tau_2)] + (1 + \frac{2}{\rho} -$$

$$- 2\sin^2\theta(M^*))[\frac{n-3}{3(n^2-1)}O(1,1) + \frac{n}{2(n^2-1)}O(t^A,t^A)] -$$

$$- \frac{2}{3}\sin^2\theta(M^*) \cdot O(\tau_3,1)\} \equiv \sum_i C_i^{(o)}O_i$$

The correction of order α in eq. (3.16) can now be expressed as:

$$\delta H = \sum_{ij} C_i^{(o)} X_{ij}(\delta O)_j \tag{3.19}$$

where $\delta O_i = \alpha\gamma_{ik}O_k \ln(\frac{M^{*2}}{\mu^2})$ is the result of the insertion of O_i into the diagrams of figs. 1a,b (with Λ^2 replaced by M^{*2}); the X matrix is given by:

$$X = \frac{1}{t}\int_0^t dt' \left[\frac{\alpha_s(M^*)}{\alpha_s(\mu')}\right]^d$$

with $t' = \ln(\frac{M^{*2}}{\mu'^2})$, and can be computed numerically, recalling that:

$$\frac{\alpha_s(\mu)}{\alpha_s(M^*)} = 1 + \frac{33-2n}{12\pi}\alpha_s(\mu)\ln(\frac{M^{*2}}{\mu^2})$$

Inserting the operators of the basis into the diagram of fig. 1a and according to eq. (3.19) we find

$$\bar{\delta}_a(A_q) = -\frac{4\alpha}{3\pi}\overline{A(t_2 - \sin^2\theta(M^*)t_Q)\tau_3} + \frac{4\alpha}{3\pi}\{\frac{2}{3}\sin^2\theta(M^*) \cdot$$
$$D\bar{t}_2 - \overline{(t_2 - \frac{1}{2}t_Q)} \cdot [B\sin^2\theta(M^*)\tau_3 + C(1 + \frac{2}{\rho} - 2\sin^2\theta(M^*))]\} \tag{3.20}$$

with:

$$A = \frac{2X_s + X_A}{3}$$

$$B = -2[A - (X_R)_{11}]$$

$$C = \frac{1}{n^2 - 1}\left[2(n - 1)X_s - (n + 1)X_A - (n - 3)(X_1)_{11} - \frac{3}{2}n(X_1)_{21}\right]$$

$$D = (X_R)_{13}$$

In eq. (3.20) the bar indicates that quark contribution only are retained. For comparison, we recall that the previously found contribution from quark loop diagrams of type a was (see eq. (3.11):

$$\bar{\delta}_a(A_q)|_{\alpha_s = 0} = -\frac{4\alpha}{3\pi}(t_2 - \sin^2\theta(M^* t_Q)\tau_3 \qquad (3.21)$$

In estimating the value of $\bar{\delta}_a(A_q)$ in eq. (3.20) we have to use, for consistency:

$$\bar{t}_2 = \ln(\frac{M^{*2}}{\mu^2}) \sum_i \frac{1}{4}Q_i\tau_{3i} = \frac{3}{8}n \ln(\frac{M^{*2}}{\mu^2})$$

$$\bar{t}_Q = \ln(\frac{M^{*2}}{\mu^2}) \sum_i Q_i^2 = \frac{5}{6}n \ln(\frac{M^{*2}}{\mu^2})$$

A numerical evaluation of the relevant matrix elements of X gives, for $\mu^2 = 2\text{GeV}^2$, $\alpha_s(\mu) = 0.7$ and four (six) active flavours:

$A = 1.033$ (1.041) , $B = 0.007$ (0.010)

$C = -0.021$ (−0.025) , $D = 0.004$ (0.004)

The smallness of (A−1), B, C and D reflects their vanishing to first order in α_s that we anticipated in the remark iv) for eq. (3.16) and can be now explicitly verified by the expression of A, B, C, D given above. After the redefinition of $\sin^2\theta$ according to eq. (3.11), we are left with negligible corrections (i.e. smaller than 10^{-3}). Indeed, to all practical purposes, we can identify the first term in eq. (3.20) with the $\alpha_s = 0$ contribution (3.21), so that eq. (3.12) is uninfluenced by leading logarithmic strong corrections. As for the second term in eq. (3.20), it gives a contribution less than 10^{-3} to A_q and can be safely neglected.

Inserting the operators of the basis into the diagram of fig. 1b leads to the correction:

$$\delta_b(A_q) = \frac{\alpha}{\pi}\ln(\frac{M^{*2}}{\mu^2})\{H[-\frac{1}{3}Q(1 - 4\sin^2\theta(M^*)Q\tau_3) - \frac{2}{3\rho}\tilde{Q}$$

$$+ E \cdot \frac{1}{3}Q(1 + \frac{2}{\rho} - 2\sin^2\theta(M^*)) - F\frac{2}{9}\sin^2\theta(M^*)Q\tau_3\} \tag{3.22}$$

where $\tilde{Q} = \tau_1 Q \tau_1 = Q - \tau_3$ and with:

$$H = 2X_s - X_A$$

$$E = \frac{2X_s}{n+1} + \frac{X_A}{n-1} - \frac{n-3}{3(n^2-1)}[(X_1)_{11} + \frac{16}{3}(X_1)_{12}]$$

$$- \frac{n}{2(n^2-1)}[(X_1)_{21} + \frac{16}{3}(X_1)_{22}]$$

$$F = H - \{(X_R)_{11} + (X_R)_{13} + \frac{16}{3}[(X_R)_{12} + (X_R)_{14}]\}$$

In the limit $\alpha_s = 0$, $H = 1$ and $E = F = 0$, while with the same assumptions as before:

$$H = 0.395(0.318) \quad , \quad E = 0.127\ (0.132) \quad , \quad F = 0.136\ (0.151)$$

Therefore the strong interactions replace $\delta_b(A_q)$ in table 1, with the expression in eq. (3.22); we have reported in table 2 the numerical evaluation of the corrections to $V^{(o)}_{u,d}$ and $A^{(o)}_{u,d}$ induced by the renormalization of $\sin^2\theta$ eq. (3.12) (second column) and by the other corrections (third to fifth columns), with or without strong interaction effects and with the assumption $\rho = 1$.

4. Physical Applications of the Leading Corrections.

We begin by considering the electron-deuteron left-right asymmetry that can be expressed [17] in terms of the parton densities and of the coefficients of eq. (3.1) as follows:

$$A = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = - |q^2| \frac{G}{4\sqrt{2}\ \pi\alpha} [a(x) + \frac{1 - (1-y)^2}{1 + (1-y)^2} b(x)] \tag{4.1}$$

where q^2 is the electron momentum transfer, x and y are the usual scaling variables and we define:

$$a(x) = \frac{\sum_i f_i(x) V_i Q_i}{\sum_i f_i(x) Q_i^2} \tag{4.2}$$

$$b(x) = \frac{\sum_i f_i(x) A_i Q_i}{\sum_i f_i(x) Q_i^2} \tag{4.3}$$

$f_i(x)$ is the density of the partons of type i in the target, and the sum runs over partons and antipartons. Recalling the definitions (3.17) and assuming u, d ↔ c, s universality, we can express a and b in the case of electron-deuteron deep inelastic scattering according to:

$$a(x) = \frac{9}{5}[\frac{2V_u - V_d}{3} - \frac{s(x) + \bar{s}(x) - c(x) - \bar{c}(x)}{D} \frac{4}{15}(2V_d + V_u)]$$

$$b(x) = \frac{9}{5}[\frac{q(x) - \bar{q}(x)}{D}(\frac{2A_u - A_d}{3}) - \frac{s(x) - \bar{s}(x) - c(x) + \bar{c}(x)}{D} \cdot (\frac{2A_u + A_d}{3})]$$

It is reasonable to assume $s(x) = \bar{s}(x)$ and, in discussing the SLAC data, $c(x) = \bar{c}(x) = 0$. Furthermore, as it follows from the previous chapter, most of the leading corrections are accounted for by the use of a running $\sin^2\theta(\mu)$. This is certainly true for $V_{u,d}$ and to a lesser extent, for $A_{u,d}$. Therefore it should be quite adequate to fit the experimental data with eq. (4.1) and with:

$$a(x) = \frac{9}{5}\left[1 - \frac{20}{9}\sin^2\theta(\mu) + \frac{4}{15}\,\frac{s(x) + \bar{s}(x)}{D(x)}\right]$$

$$b(x) = \frac{9}{5}\left[1 - 4\sin^2\theta(\mu)\right]\,\frac{q(x) - \bar{q}(x)}{D(x)}$$

An estimate of $\sin^2\theta(\mu)$ and therefore of $\sin^2\theta(M^*)$ can be achieved by evaluating $a(x)$ at large x. The SLAC data have been analyzed under the additional hypotesis $s(x) = \bar{q}(x) = 0$. In ref. [2] one can find the one standard deviation contour in the ab plane. In the Salam-Weinberg theory, b is small and it has the same sign as a. If one imposes the latter constraint on the data, a rather precise evaluation of a is obtained, yielding:

$$\sin^2\theta(1.4\ \mathrm{GeV}) = 0.224 \pm 0.020$$

From eq. (3.13) we get:

$$\sin^2\theta(M^*) = 0.217$$

in good agreement with the prediction of grand unfed theories [18, 19]. However the importance of the previous result is lessen by a certain amount of uncertainty in it.

i) To see if $s(x)$ is reasonable, we have examined the CDHS data on neutrons scattering [20]. For $0.1 \leqslant x \leqslant 0.2$, which is the same range as in the SLAC data, they find:

$$\frac{4}{15}\,\frac{s + \bar{s}}{D} = (1.6 \div 2.6) \cdot 10^{-2}$$

Taking a mean value of 2.10^{-2} this would correspond to a shift of $\sin^2\theta(\mu)$ of + 0.01 and a similar shift for $\sin^2\theta(M^*)$. It has to be stressed, however, that this estimate is rather pessimistic in that one expects a consistent decrease of the strange sea from CDHS ($-q^2 = 10$ GeV^2) to SLAC ($-q^2 = 2\mathrm{GeV}^2$): a factor of 2 to 3 would not be unreasonable. Of course, in using the CDHS data, one should be aware that the strange sea for deep inelectric scattering of polarized electrons may differ considerably from the strange sea in neutrino scattering even at the same q^2, because of corrections of order α_s times the gluon density (see ref. [21]).

ii) The effect of the antiquark sea ($\bar{q} \neq 0$) amounts to changing the slope of the Salam-Weinberg line in the ab plane, thereby decreasing the value of $\sin^2\theta(\mu)$. By making use again of CDHS data, in the same x range as above, we get the value:

$$\frac{q - \bar{q}}{q + \bar{q}} = 0.6$$

and, in correspondence, we estimate from fig. 2 of ref. [2], with $s(x) = \bar{s}(x) = 0$, a shift in $\sin^2\theta(\mu)$ of $- 0.004$. As in the case of the strange sea i), this should be considered as an upper bound to the effect.

iii) The modification of the value of $\sin^2\theta(\mu)$ obtained by taking into account the radiative corrections of the last three columns of table 2 as well as the correction computed in App. F, eq (F.5), amounts to less than 0.002.

Summarizing, we can say that the largest uncertainties arise from the strange and the antiquark sea: more precise data on deuterium asymmetry are clearly needed to clarify the important issue of the precise value of $\sin^2\theta(M^*)$.

Next, we turn our attention to the deep inelastic muon scattering on nucleons. The charge and polarization asymmetry is defined by the expression

$$B(\lambda) = \frac{d\sigma^+(-\lambda) - d\sigma^-(\lambda)}{d\sigma^+(-\lambda) + d\sigma^-(\lambda)} \qquad (4.4)$$

where λ is the longitudinal polarization of the incident muon and $d\sigma^{\mp}$ are the differential inclusive cross sections of the processes:

$$\mu^{\mp}(\pm\lambda) + N \rightarrow \mu^{\mp}(\pm\lambda) + \text{anything}$$

Using the quark-parton model and the Salam-Weinberg model the asymmetry (4.4) for an isoscalar target, can be expressed as (see ref. [14]):

$$B(\lambda) = \frac{\rho}{2} \frac{G}{2\sqrt{2}\,\pi\alpha} |q^2| \, \frac{1 - (1-y)^2}{1 + (1-y)^2} \, (g_A^{\mu^-} + \lambda g_V^{\mu^-}) \, \frac{\sum_i f_i(x) Q_i \, g_A^i}{\sum_i f_i(x) Q_i^2}$$

where q^2 is the muon momentum transfer; the coefficients $g_V^{\mu^-}$ and $g_A^{\mu^-}$ are the vector and axial-vector neutral couplings of the μ^- to the Z°; g_A^i is the axial-vector coupling of the parton of type A_i to the Z°. In terms of the coefficients of eq. (3.1), we have:

$$B(\lambda) = \frac{\rho}{2} \, \frac{G}{2\sqrt{2}\,\pi\alpha} \, |q^2| \, \frac{1 - (1-y)^2}{1 + (1+y)^2} \, [g(x) + \lambda b(x)] \qquad (4.5)$$

with:

$$b(x) = \frac{\sum_i f_i(x) A_i Q_i}{\sum_i f_i(x) Q_i^2}$$

$$g(x) = \frac{\sum_i f_i(x) C_i Q_i}{\sum_i f_i(x) Q_i^2}$$

Recalling the definitions (3.17), we obtain:

$$b(x) = \frac{9}{5}\Big[\Big(\frac{2}{3}A_u - \frac{1}{3}A_d\Big)\frac{q(x) - \bar{q}(x)}{D(x)} - \Big(\frac{2}{3}A_u + \frac{1}{3}A_d\Big)$$

$$\cdot\ \frac{s(x) - \bar{s}(x) - c(x) + \bar{c}(x)}{D(x)}\Big]$$

$$g(x) = \frac{9}{5}\Big[\Big(\frac{2}{3}C_u - \frac{1}{3}C_d\Big)\frac{q(x) - \bar{q}(x)}{D(x)} - \Big(\frac{2}{3}C_u + \frac{1}{3}C_d\Big)$$

$$\cdot\ \frac{s(x) - \bar{s}(x) - c(x) + \bar{c}(x)}{D(x)}\Big] \qquad (4.6)$$

To take into account the corrections discussed in the previous chapter we have to use a running $\sin^2\theta(\mu)$ and fit the experimental data with eq. (4.5) and with [24]:

$$b(x) = \frac{9}{5}\{[1 - 4\sin^2\theta(\mu) + \frac{1}{27}(\frac{17}{2} + \frac{8}{\rho} - 30\sin^2\theta(\mu)\frac{\alpha}{\pi}\ln\frac{M^{*2}}{|q^2|}$$

$$+ \frac{3}{16}\frac{\alpha_W}{\pi}]\ \frac{q(x) - \bar{q}(x)}{D(x)} - [\frac{1}{3}(1 - 4\sin^2\theta(\mu)) + \frac{1}{27}(\frac{39}{2} - \frac{134}{3}\sin^2\theta(\mu))$$

$$\cdot\ \frac{\alpha}{\pi}\ln\frac{M^{*2}}{|q^2|} + \frac{9}{16}\frac{\alpha_W}{\pi}]\ \frac{s(x) - \bar{s}(x) - c(x) + \bar{c}(x)}{D(x)}\} \qquad (4.7)$$

$$g(x) = \frac{9}{5}\{[-1 + (1+ 4\sin^2\theta(\mu))(-\frac{1}{2} + \frac{14}{9}\sin^2\theta(\mu)\frac{\alpha}{\pi}\ln\frac{M^{*2}}{|q^2|}$$

$$- \frac{3}{16}\frac{\alpha_W}{\pi}]\ \frac{q(x) - \bar{q}(x)}{D(x)} - [-\frac{1}{3} + (1 - 4\sin^2\theta(\mu))(-\frac{5}{6} + 2\sin^2\theta(\mu))$$

$$\cdot\ \frac{\alpha}{\pi}\ln\frac{M^{*2}}{|q^2|} - \frac{9}{16}\frac{\alpha_W}{\pi}]\ \frac{s(x) - \bar{s}(x) - c(x) + \bar{c}(x)}{D(x)}\} \qquad (4.8)$$

where $\alpha_W = \alpha(\sin^2\theta)^{-2} \approx 5\alpha$.

We recall what we stated in the introduction, namely that contributions of order $\alpha \ln \frac{|q^2|}{m^2}$ (m being a lepton or quark mass) are needed for a complete analysis of the potentially large corrections to the coefficients (4.6), and have not been computed; these contributions are not expected to affect the estimate of $\sin^2\theta(M^*)$. The preliminary value from CERN NA4 experiment [3]:

$$\sin^2\theta(7.7\mathrm{GeV}) = 0.26 \pm 0.09$$

should be shifted by the amount $0.2 \cdot 10^{-2}$. In fact, using the same quark mass values as in computing eq. (3.13) and (3.14), and the measured lepton masses, we find:

$$\sin^2\theta(7.7\mathrm{GeV}) - \sin^2\theta(M^*) = 0.2 \cdot 10^{-2}$$

for $\sin^2\theta(M^*)$ between 0.21 and 0.22. From eqs. (4.5), (4.7), and (4.8) we obtain:

$$B(\lambda) = \frac{\rho}{2}\frac{G}{2\pi\alpha\sqrt{2}}|q^2|\frac{1-(1-y)^2}{1+(1-y)^2}\frac{9}{5}\{[-1+(1-4\sin^2\theta(\mu))$$

$$\cdot(-\frac{1}{2}+\frac{14}{9}\sin^2\theta(\mu))\frac{\alpha}{\pi}\ln\frac{M^*}{|q^2|}-\frac{3}{16}\frac{\alpha_W}{\pi}]\cdot\frac{q(x)-\bar{q}(x)}{D(x)}-[-\frac{1}{3}+(1-4\cdot$$

$$\cdot\sin^2\theta(\mu))(-\frac{5}{6}+2\sin^2\theta(\mu))\frac{\alpha}{\pi}\ln\frac{M^*}{|q^2|}-\frac{9}{16}\frac{\alpha_W}{\pi}]\frac{s(x)-\bar{s}(x)-c(x)+\bar{c}(x)}{D(x)}$$

$$+\lambda[(1-4\sin^2\theta(\mu)+\frac{1}{27}(\frac{17}{2}+\frac{8}{\rho}-30\sin^2\theta(\mu))\frac{\alpha}{\pi}\ln\frac{M^*}{|q^2|}+\frac{3}{16}\frac{\alpha W}{\pi})$$

$$\cdot\frac{q(x)-\bar{q}(x)}{D(x)}-(\frac{1}{3}(1-4\sin^2\theta(\mu))+\frac{1}{27}(\frac{39}{2}-\frac{134}{3}\sin^2\theta(\mu))\frac{\alpha}{\pi}\ln\frac{M^*}{|q^2|}$$

$$+\frac{9}{16}\frac{\alpha_W}{\pi})\frac{s(x)-\bar{s}(x)+c(x)-\bar{c}(x)}{D(x)}]\}$$

Taking $\rho = 1$ and assuming the so-called valance quark approximation: $s(x) = c(x) = \bar{q}(x) = 0$, we evaluate the asymmetry at $\lambda = -1$, as a function of q^2 and y:

$$B(-1) = -1.62\cdot 10^{-4}\frac{|q^2|}{\mathrm{GeV}^2}\frac{1-(1-y)^2}{1+(1-y)^2}\cdot\{2-4\sin^2\theta(\mu)$$

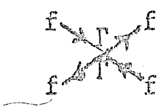
$$+\frac{\alpha}{\pi}\ln\frac{M^*}{|q^2|}\cdot[(1-4\sin^2\theta(\mu)0(\frac{1}{2}-\frac{14}{9}\sin^2\theta(\mu))+\frac{1}{9}(\frac{11}{2}-10\cdot$$

$$\sin^2\theta(\mu))]+\frac{3}{8}\frac{\alpha}{\pi\sin^2\theta(\mu)}\} \qquad (4.9)$$

where we assume for the Fermi constant the value $G = 1.165 \cdot 10^{-5}\ GeV^{-2}$. If we take $\sin^2\theta = 0.22$, $M^{*2} = 10^4 GeV^2$, $\alpha = \frac{1}{137}$, $|q^2| = 60\ GeV^2$, the percentage corrections to the asymmetry are of order of 0.6%. Notice that the effect of the antiquark sea ($\bar{q} \neq 0$) would be simply to introduce in eq. (4.9) an overall factor $\frac{q(x) - \bar{q}(x)}{D(x)}$ without changing the percentage correction.

We conclude this chapter by discussing the muon backward-forward symmetry in $e^+e^- \to \mu^+\mu^-$. In this case the influence of strong interactions turns out to be particularly suppressed and therefore it is meaningful to compute the leading logarithmic corrections up to the second order in α; we will consider also the whole of the order α corrections. We will devote Section 5 to the latter topic.

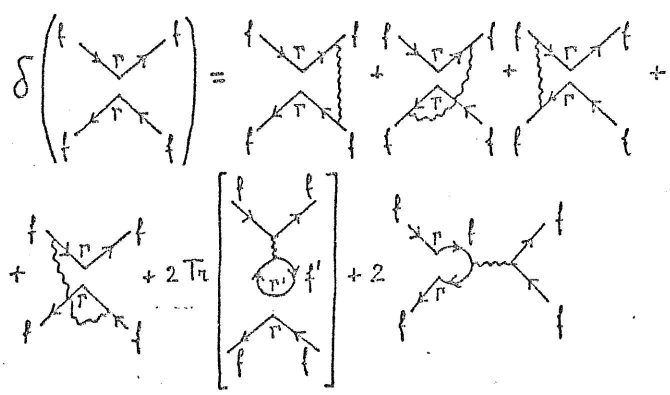
The uncorrected hamiltonian relevant to $e^+e^- \to \mu^+\mu^-$ is (compare with eq. (3.2)):

$$H^{(o)} = K\frac{1}{2}J^z_\mu J^z_\mu = K\frac{1}{2}$$  (4.10)

where

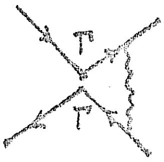
$$K = -(2\pi)^4 i \frac{G}{2\sqrt{2}}, \qquad \Gamma = \gamma_\mu(-4\sin^2\theta(M^*) + \tau_3 + \tau_3\gamma_5)$$

Let us neglect for a while the strong interaction and consider the electromagnetic corrections to diagram (4.10):

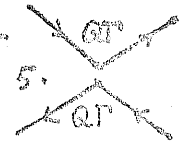


(4.11)

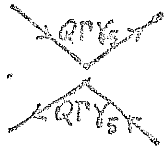
where the trace is carried out in the fermion space. The factor of 2 takes into account that in closing the fermion lines of (4.10) one has two choices which give the same result. We show explicitly how the simple computation of the correction (4.11) works:



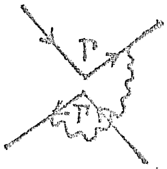
$$= K \frac{e^2}{16\pi^2} \frac{1}{2} \ln\left(\frac{M^{*2}}{\mu^2}\right) \Big[5 \cdot$$



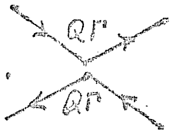
$$- 3 \cdot$$



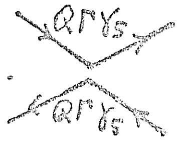
$$\Big]$$



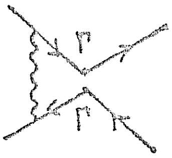
$$= K \frac{e^2}{16\pi^2} \frac{1}{2} \ln\left(\frac{M^{*2}}{\mu^2}\right) \Big[-5 \cdot$$



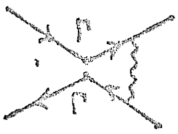
$$- 3 \cdot$$

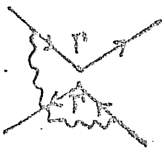


$$\Big]$$

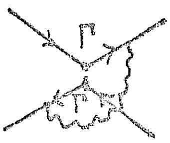


$$=$$

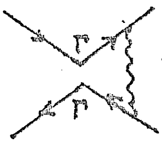




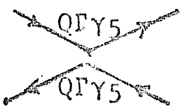
$$=$$



The total contribution of the box diagrams is:



$$+ \ldots = K \frac{e^2}{16\pi^2} \ln\left(\frac{M^{*2}}{\mu^2}\right)(-6)$$

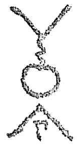


$$= -\frac{3}{2}(J'_\mu J'_\mu)\frac{\alpha(M^*)}{\pi} \ln\left(\frac{M^{*2}}{\mu^2}\right) K$$

where we introduce the current:

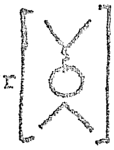
$$J'_\mu = \bar{f}\gamma_\mu Q(-4\sin^2\theta(M^*)Q + \tau_3 + \tau_3\gamma_5)\gamma_5 f$$

The contribution of the diagram containing the fermion loop is:



$$= -\frac{1}{3}(J_\mu^{e.m.} J_\mu^{z}) Q'(-4\sin^2\theta(M^*)Q' + \tau_3') \frac{\alpha(M^*)}{\pi} \ln \frac{M^{*2}}{\mu^2} \cdot K$$

where $J_\mu^{e.m.} = \bar{f}Q\gamma_\mu f$; taking the trace in the fermion space, one gets(3) :

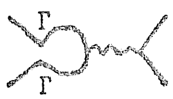


$$\mathrm{Tr}\left[\;\cdot\;\right] = -\frac{4}{3}(b_2 - \sin^2\theta(M^*)b_Q)(J_\mu^{e.m.} J_\mu^{'''}) \frac{\alpha(M^*)}{\pi} \ln \frac{M^{*2}}{\mu^2} \cdot K$$

The current $J_\mu^{'''}$ is defined as:

$$J_\mu^{'''} = \bar{f}\gamma_\mu(-4\sin^2\theta(M^*)Q + \tau_3)f$$

The last diagram has the value:



$$= K(-\frac{1}{6})(J_\mu^{e.m.} J_\mu^{''}) \frac{\alpha(M^*)}{\pi} \ln \frac{M^{*2}}{\mu^2}$$

where:

$$J_\mu^{''} = \bar{f}\gamma_\mu[(-4\sin^2\theta(M^*)Q + \tau_3)^2 + 1]Qf$$

So, the leading logarithmic first order correction reads:

$$\delta H^{(o)} = K \cdot \frac{1}{2}[-\frac{3}{2}(J_\mu^{'} J_\mu^{'}) - \frac{8}{3}(b_2 - \sin^2\theta(M^*)b_Q)(J_\mu^{e.m.} J_\mu^{'''})$$

$$-\frac{1}{3}(J_\mu^{e.m.} J_\mu^{''})] \frac{\alpha(M^*)}{\pi} \ln \frac{M^{*2}}{\mu^2} \qquad (4.12)$$

Specializing this equation to our particular process we find:

$$\delta H^{(o)} = K(-\frac{3}{2})(4\sin^2\theta(M^*) - 1) \frac{\alpha(M^*)}{\pi} \ln \frac{M^*}{\mu^2} (\gamma^\mu\gamma_5 \times \gamma^\mu\gamma_5) \qquad (4.13)$$

Recalling eq. (3.10), we can now, by iteration determine the leading logarithmic second order correction $H^{(2)}(\mu)$; we need only to renormalize the current products contributing to eq. (4.12). We explicitly notice that the renormalization of the current products of eq. (4.12) due to penguin-diagrams (the last two diagrams in eq. (4.11)) is negligible, because at the end these diagrams give the product of some current times the electromagnetic current, and the latter does not contain $\gamma^{\mu}\gamma_5$ terms (see chapter 5). By applying the correction due to one photon exchange between each current pair in eq. (4.12) and specializing our formulae to the case $e^+e^- \to \mu^+\mu^-$, we obtain (escaping intermediate passages):

$$\delta[\delta H^{(o)}) = K[\frac{5}{2} - 2(4\sin^3\theta(M^*) - 1)(b_2 - \sin^2\theta(M^*)b_Q)$$
$$+ \frac{1}{4}(4\sin^2\theta(M^*) - 1)^2][\frac{\alpha(M^*)}{\pi} \ln \frac{M^{*2}}{\mu^2}]^2(\gamma_\mu\gamma_5 \times \gamma_\mu\gamma_5)$$

Recalling eq. (3.10) we get:

$$H^{(2)}(\mu) = \frac{K}{2\pi^2}[\frac{5}{2} - 2(4\sin^2\theta(M^*) -1)(b_2 - \sin^2\theta(M^*)b_Q)$$
$$+ \frac{1}{4}(4\sin^2\theta(M^*) -1)^2(1 + 2b_Q)]\alpha^2(M^*) \ln^2 \frac{M^{*2}}{\mu^2}(\gamma_\mu\gamma_5 \times \gamma_\mu\gamma_5)$$

It is convenient to express $H^{(2)}(\mu)$ in terms of $\alpha(o)$ which is the directly measurable quantity:

$$\alpha = \frac{g^2\sin^2\theta(o)}{4\pi} = (137.03604)^{-1} \qquad (4.14)$$

The relation:

$$\alpha(\mu) = \alpha(o) \frac{1}{1 - \frac{\alpha(o)}{3\pi} \sum_{m_i<\mu} (Q_i^2 \ln \frac{\mu^2}{m_i^2})}$$

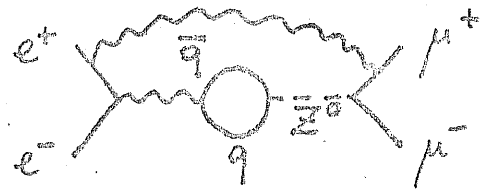
where m_i are the fermion masses fulfilling the condition $m_i < \mu$ on which the sum is carried, leads to:

$$H^{(2)}(\mu) = \frac{K}{2\pi^2}\{[\frac{5}{2} - \frac{v_\theta}{2}\sum_i Q_i(\tau_{3i} - 4s_\theta^2 Q_i) + \frac{1}{4}v_\theta^2(1 - 2\sum_i Q_i^2)]$$

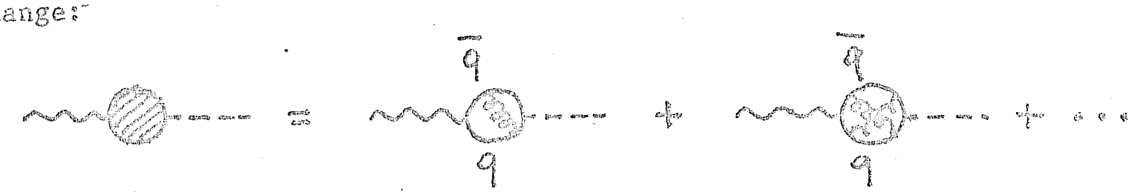
$$\cdot \ln^2\frac{M^{*2}}{\mu^2} \sum_{m_i<\mu} Q_i^2 \ln\frac{\mu^2}{m_i^2}\ln\frac{M^{*2}}{\mu^2}\}\alpha^2(o)(\gamma^\mu\gamma_5\times\gamma^\mu\gamma_5) \qquad (4.15)$$

where we introduce the notation $s_\theta^2 = \sin^2\theta(M^*)$, $c_\theta^2 = \cos^2\theta(M^*)$, $v_\theta^2 = 4s_\theta^2 - 1$. In the first order correction $\delta(H^{(o)})$, the coupling constant α can be expressed as $\alpha(o)$ without any modification on eq. (4.13), because the difference between the various choices of α, is of order $(\alpha\ln M^{*2})^2$. The numerical analysis of the corrections (4.13) and (4.15) is demanded to the next section.

Let us consider now the influence of strong interactions. It follows from remark ii) after eq. (3.16) that the first leading logarithmic order correction $\delta(H^{(o)})$ is not affected by leading logarithmic α_s corrections. To the l.l. second order in α, we can obtain leptonic operators starting from non-leptonic terms, but the two lepton pairs which are obtained in this way must be both in a vector state and their contribution to the correction is proportional to $\gamma^\mu\times\gamma^\mu$ and independent from the kinematic invariant s, t, u: therefore its effect is completely neglibible (see next section). Thus, the order $(\alpha\ln\frac{M^{*2}}{\mu^2})^2$ correction will be obtained starting from a vector which has no component in the non-leptonic vector; then, similarly to the case of $\delta(H^{(o)})$ we can argue that the correction (4.15) is free from leading logarithmic strong corrections. The influence of strong interactions is limited to α_s^m terms (with m positive integer) on $\alpha\ln\frac{M^{*2}}{\mu^2}$ corrections, and to $\alpha_s^m\ln^{m-1}(\frac{M^{*2}}{\mu^2})$ on $\alpha^2\ln^2\frac{M^{*2}}{\mu^2}$ corrections. Indeed, a diagrammatic analysis of these corrections shows that the strong interactions enter only through the vacuum polarization diagrams, such as for instance the diagram:



where the strong interactions dress the quark loop by means of gluon exchange:



These corrections can be expressed in terms of the ratio between e^+e^- inelastic cross sections:

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

A similar problem, in a different context, is discussed in ref. [22].

5. One-loop corrections to the μ^{+} asymmetry in $e^{+}e^{-} \to \mu^{+}\mu^{-}$.

We now discuss all the order α corrections to the backward-forward asymmetry, in the hypotesis:

$$m^2,\ \mu^2 << |s|,\ |t|,\ |u| << M^2,\ M_o^2 \tag{5.1}$$

where m, μ are the electron and muon masses and M, M_o are the masses of the vector bosons; the invariants s, t, u are defined in Appendix A. The asymmetry is defined as:

$$A = \frac{d\sigma(\theta_s)}{d(\cos\theta_s)} - \frac{d\sigma(\pi - \theta_s)}{d(\cos\theta_s)} \tag{5.2}$$

where θ_s is the scattering angle. The transition amplitude of the process $e^{+}e^{-} \to \mu^{+}\mu^{-}$

shown in fig. 2 can be expressed in the following general way:

$$\mathcal{A} = M_a + \delta M_a + M_b + \delta M_b \tag{5.3}$$

In this formula M_a and M_b are the tree amplitudes corresponding to the diagrams in

fig. 3; they are given by:

$$M_a = (2\pi)^4 ig^2 \frac{s_\theta^2}{s}\ \gamma^\mu \times \gamma^\mu \tag{5.4}$$

$$M_b = -(2\pi)^4 ig^2 \frac{1}{16c_\theta^2}\ \frac{1}{M_o^2}(v_\theta^2\gamma^\mu \times \gamma^\mu + \gamma^\mu\gamma_5 \times \gamma^\mu\gamma_5) \tag{5.5}$$

In eq. (5.5) we have neglected terms proportional to $\gamma^\mu \times \gamma^\mu \gamma_5$ or $\gamma^\mu \gamma_5 \times \gamma^\mu$; in fact they give a vanishing contribution to the physical effect which we are interested in, since the asymmetry (5.2) is parity conserving.

δM_a is the pure QED correction to M_a and δM_b the full one-loop correction to M_b. The expressions of δM_a and δM_b will be given later; for the time being we just remark that they are respectively of order $g^4 \cdot \frac{1}{s}$ and $g^4 \cdot \frac{1}{M_o^2}$.

The differential cross section can be divided into two parts containing respectively, the tree level value plus the virtual corrections, and the real corrections:

$$\frac{d\sigma}{d(\cos\theta_s)} = \frac{d\sigma^V}{d(\cos\theta_s)} + \frac{d\sigma^R}{d(\cos\theta_s)} \tag{5.6}$$

As for the virtual part, we have:

$$\frac{d\sigma^V}{d(\cos\theta_s)} = \frac{1}{s}\,\frac{1}{32\pi}\,\frac{1}{4} \sum_{s_1 s_2 s_3 s_4} \left|\frac{}{(2\pi)^4 i}\right|^2 \tag{5.7}$$

where s_1 and s_2 are the spins of the incoming e^+ and e^-, respectively, and s_3, s_4 are the spins of the outgoing μ^+,μ^-. The contributions to the differential cross section we are interested in arise from the interference term:

$$\frac{2}{s}\mathrm{Re}\left[M_a^*(M_b + \delta M_b) + M_b^* \cdot \delta M_a\right] \tag{5.8}$$

In eq. (5.7) we shall neglect the contribution of the term $\frac{1}{s}|M_b + \delta M_b|^2$ since it is of order $\frac{s}{M_o^4}$ while the interference term (5.8) is of order $\frac{1}{M_o^2}$.

Being the fermion masses much smaller than the W masses, the theory has an approximate chiral symmetry, so that terms different from products of the type VV, VA, AV, AA appear in the amplitude only with a factor $\frac{m^2_{\text{fermion}}}{M_W^2}$ or

$\frac{m^2_{fermion}}{s}$ and can be neglected. Neglecting the parity violating terms, we can write:

$$M_b + \delta M_b = -(2\pi)^4 i g^2 \frac{1}{16c_\theta^2} \frac{1}{M_o^2}(c_1\gamma^\mu\gamma_5 \times \gamma^\mu\gamma_5 + c_2\gamma^\mu \times \gamma^\mu) \quad (5.9)$$

$$\delta M_a = (2\pi)^4 i g^2 \frac{s_\theta^2}{s}(\eta\gamma^\mu\gamma_5 \times \gamma^\mu\gamma_5 + \xi\gamma^\mu \times \gamma^\mu) \quad (5.10)$$

The coefficients η and ξ are of order g^2, while c_1 and c_2 are given by the sum of their tree parts plus the corrections:

$$c_1 = 1 + \delta_1 \quad (5.11)$$

$$c_2 = v_\theta^2 + \delta_2 \quad (5.12)$$

where δ_1 and δ_2 are of order g^2. From eqs. (5.6)÷(5.10) we obtain:

$$\frac{d\sigma^V}{d(\cos\theta_s)} = \frac{-g^4}{128\pi} \frac{s_\theta^2}{c_\theta^2} \frac{1}{M_o^2}\{(1 + 2\frac{t}{s})c_1 + (1 + 2\frac{t}{s} + 2\frac{t^2}{s^2})c_2 + \eta[v_\theta^2(1 + 2\frac{t}{s}) + 1 + 2\frac{t}{s} + 2\frac{t^2}{s^2}] + \xi[1 + 2\frac{t}{s} + v_\theta^2(1 + 2\frac{t}{s} + 2\frac{t^2}{s^2})]\}$$

Using eq. (A.1) we have:

$$\frac{d\sigma^V}{d(\cos\theta_s)} = \frac{-g^4}{128\pi} \frac{s_\theta^2}{c_\theta^2} \frac{1}{M_o^2}\{\cos\theta_s \cdot c_1(\theta_s) + \frac{1}{2}(1 + \cos^2\theta_s) \cdot c_2(\theta_s) + [v_\theta^2\cos\theta_s + \frac{1}{2}(1 + \cos^2\theta_s)] \cdot \eta(\theta_s) + [\cos\theta_s + v_\theta^2 \frac{1}{2}(1 + \cos^2\theta_s)] \cdot \xi(\theta_s)\} \quad (5.13)$$

where we explicitly indicated that the coefficients c_1, c_2, η, ξ depend in general upon the scattering angle θ_s (besides, of course, the dependence from the centre of mass energy).

From eqs.(5.2) and (5.I3) we obtain the following (virtual) contribution to the backward-forward asymmetry of the angular distribution:

$$\frac{d\sigma^V(\theta_s)}{d(\cos\theta_s)} - \frac{d\sigma^V(\pi-\theta_s)}{d(\cos\theta_s)} = \frac{-g^4}{64\pi}\frac{s_\theta^2}{c_\theta^2}\frac{1}{M_0^2}\Big\{\cos\theta_s\Big[\frac{c_1(\theta_s)+c_1(\pi-\theta_s)}{2} +$$

$$+ \frac{\xi(\theta_s)+\xi(\pi-\theta_s)}{2} + v_\theta^2\,\frac{\eta(\theta_s)+\eta(\pi-\theta_s)}{2}\Big] + \frac{1}{2}(1+\cos^2\theta_s)\cdot$$

$$\cdot\Big[\frac{c_2(\theta_s)-c_2(\pi-\theta_s)}{2} + v_\theta^2\,\frac{\xi(\theta_s)-\xi(\pi-\theta_s)}{2} + \frac{\eta(\theta_s)-\eta(\pi-\theta_s)}{2}\Big]\Big\} \qquad (5.I4)$$

Let us make a remark about eq.(5.I4).The coefficient $\eta(\theta_s)$ arises from two photon exchange diagrams so that, by charge conjugation invariance, η is symmetric under $\theta_s \to \pi-\theta_s$. Thus only the sum $\frac{1}{2}[\eta(\theta_s)+\eta(\pi-\theta_s)]$ contributes to eq.(5,I4). Furthermore, since only the difference $\frac{1}{2}[c_2(\theta_s)-c_2(\pi-\theta_s)]$ enters eq.(5.I4), we may drop all the contributions to $c_2(\theta_s)$ which are indepe- ndent from ϑ_s (such as, for instance, those coming from the γ - W° mixing diagram).

We shall consider the real corrections to the cross secti- on in the soft photon approximation. Therefore we can write:

$$\frac{d\sigma^R(\theta_s)}{d(\cos\theta_s)} = b(\theta_s)\,\frac{d\sigma^{(0)}(\theta_s)}{d(\cos\theta_s)} \qquad (5.I5)$$

where $\frac{d\sigma^{(0)}(\theta_s)}{d(\cos\theta_s)}$ is the tree level value of the differential cross section:

$$\frac{d\sigma^{(0)}(\theta_s)}{d(\cos\theta_s)} = \frac{-g^4}{128\pi}\frac{s_\theta^2}{c_\theta^2}\frac{1}{M_0^2}\Big[\cos\theta_s + v_\theta^2\,\frac{1}{2}(1+\cos^2\theta_s)\Big] \qquad (5.I6)$$

The real contribution to the angular asymmetry is, then:

$$\frac{d\sigma^R(\theta_s)}{d(\cos\theta_s)} - \frac{d\sigma^R(\pi-\theta_s)}{d(\cos\theta_s)} = \frac{-g^4}{64\pi}\frac{s_\theta^2}{c_\theta^2}\frac{1}{M_0^2}\Big[\cos\theta_s\,\frac{b(\theta_s)+b(\pi-\theta_s)}{2} +$$

$$+\frac{v_\theta^2}{2}(1+\cos^2\theta_s)\,\frac{b(\theta_s)-b(\pi-\theta_s)}{2}\Big] \qquad (5.17)$$

Collecting together eqs.(5.2), (5.6), (5;I4), (5.I7) we can finally express the asymmetry in the following way:

$$A = \frac{-g^4}{64\pi}\frac{s_\theta^2}{c_\theta^2}\frac{1}{M_0^2}\left[\cos\theta_s \cdot d_1(\theta_s) + \frac{1}{2}(1+\cos^2\theta_s)\cdot d_2(\theta_s)\right] \qquad (5.I8)$$

where d_1, d_2 are given, up to order α, by the expressions:

$$d_1(\theta_s) = \frac{c_1(\theta_s) + c_1(\pi-\theta_s)}{2} + \frac{\xi(\theta_s)+\xi(\pi-\theta_s)}{2} + v_\theta^2\,\eta(\theta_s) + \frac{b(\theta_s)+b(\pi-\theta_s)}{2} \qquad (5.I9)$$

$$d_2(\theta_s) = \frac{c_2(\theta_s) - c_2(\pi-\theta_s)}{2} + v_\theta^2\frac{\xi(\theta_s)-\xi(\pi-\theta_s)}{2} + v_\theta^2\,\frac{b(\theta_s)-b(\pi-\theta_s)}{2} \qquad (5.20)$$

To compute the coefficients in eqs.(5.9), (5.I0) we use the method of dimensional regularization [23]. The two infinite parameters Δ, Δ_{IR}, representing the UV and IR divergence, have the common expression, in terms of the pole $\frac{1}{\varepsilon} = \frac{1}{n-4}$:

$$\Delta = -\frac{2}{\varepsilon} + \Gamma'(1) - \ln\pi$$

where n is the space-time dimensionality and $\Gamma'(x)$ is the derivative of the Euler function.

We refer to the Appendix B for the individual values of the diagrams contributing to δM_a. Summing up eqs.(B.I), (B.2), we get eq.(5.I0) with:

$$\xi = \frac{-g^2 s_\theta^2}{16\pi^2}\Big\{4\Big(\ln\frac{t}{u} + \ln\frac{s}{m\mu} - 1\Big)(-\Delta_{IR} - \ln 4\pi^2 + \ln s) - \frac{s}{u}\Big[-\ln\Big(-\frac{t}{s}\Big) + $$
$$+ \Big(1+\frac{s}{2u}\Big)\ln^2\Big(-\frac{s}{t}\Big)\Big] + \frac{s}{t}\Big[-\ln\Big(-\frac{u}{s}\Big) + \Big(1+\frac{s}{2t}\Big)\ln^2\Big(-\frac{s}{u}\Big)\Big] -$$
$$-2\ln\frac{s}{m\mu} - \ln^2\Big(\frac{s}{m^2}\Big) - \ln^2\Big(\frac{s}{\mu^2}\Big) - \frac{8\pi^2}{3} + 8 - \frac{4}{3}\sum_{m_i<\sqrt{s}}\Big[Q_i^2\Big(\ln\frac{s}{m_i^2} - \frac{5}{3}\Big)\Big]\Big\}$$

$$\eta = \frac{-g^2 s_\theta^2}{16\pi^2}\Big\{\frac{s}{u}\Big[-\ln\Big(-\frac{t}{s}\Big) + \Big(1+\frac{s}{2u}\Big)\ln^2\Big(-\frac{s}{t}\Big)\Big] + \frac{s}{t}\Big[-\ln\Big(-\frac{u}{s}\Big) + \Big(1+\frac{s}{2t}\Big)\ln^2\Big(-\frac{s}{u}\Big)\Big]\Big\}$$

Recalling eq.(A.I) we have, then:

$$\frac{\xi(\theta_s)+\xi(\pi-\theta_s)}{2} = \frac{\alpha}{\pi}\Big\{(\Delta_{IR} + \ln 4\pi^2 - \ln s)\Big(\ln\frac{s}{m\mu} - 1\Big) + \frac{1}{2}\ln\frac{s}{m\mu} +$$
$$+ \frac{1}{4}\ln^2\frac{s}{m^2} + \frac{1}{4}\ln^2\frac{s}{\mu^2} + \frac{2\pi^2}{3} - 2 + \frac{1}{3}\sum_{m_i<\sqrt{s}}\Big[Q_i^2\Big(\ln\frac{s}{m_i^2} - \frac{5}{3}\Big)\Big]\Big\} \qquad (5.21)$$

$$\frac{\xi(\theta_s)-\xi(\pi-\theta_s)}{2}=\frac{\alpha}{\pi}\Big\{(\Delta_{IR}+\ln 4\pi^2-\ln s)\ln\left(\frac{1-\cos\theta_s}{1+\cos\theta_s}\right)+\frac{1}{2}\frac{1}{\cos\theta_s+1}\cdot$$
$$\cdot\left[\ln\left(\frac{1-\cos\theta_s}{2}\right)-\frac{\cos\theta_s}{1+\cos\theta_s}\ln^2\left(\frac{1-\cos\theta_s}{2}\right)\right]+\frac{1}{2}\frac{1}{\cos\theta_s-1}\left[\ln\left(\frac{1+\cos\theta_s}{2}\right)+\frac{\cos\theta_s}{1-\cos\theta_s}\ln^2\left(\frac{1+\cos\theta_s}{2}\right)\right]\Big\} \quad (5.22)$$

$$\eta(\theta_s)=\eta(\pi-\theta_s)=\frac{\alpha}{\pi}\Big\{-\frac{1}{2}\frac{1}{\cos\theta_s+1}\left[\ln\left(\frac{1-\cos\theta_s}{2}\right)-\frac{\cos\theta_s}{1+\cos\theta_s}\ln^2\left(\frac{1-\cos\theta_s}{2}\right)\right]+$$
$$+\frac{1}{2}\frac{1}{\cos\theta_s-1}\left[\ln\left(\frac{1+\cos\theta_s}{2}\right)+\frac{\cos\theta_s}{1-\cos\theta_s}\ln^2\left(\frac{1+\cos\theta_s}{2}\right)\right]\Big\} \quad (5.23)$$

We can divide the corrections to M_b in three classes containing the self-energy diagrams, the vertex diagrams and the box diagrams. We call respectively δ_1^S, δ_1^V, δ_1^B the contributions to the correction δ_1 in eq.(5.II) which come from these classes, and similarly for δ_2. In fig.4 we sketch these classes of diagrams; they represent the sum of the graphs listed in Appendix C. Adding eqs. from (C.I) to (C.6) we obtain the following contributions $\tilde{\delta}_1$, $\tilde{\delta}_2$ to the corrections appearing in eqs.(5.II), (5.I2):

$$\tilde{\delta}_1=\delta_1^S+\delta_1^V+\delta_1^B=\frac{g^2}{16\pi^2}\Big\{\left(2+\frac{1}{c_\theta^2}\right)(\Delta-\ln M^2+\ln 4\pi^2)+\frac{1}{c_\theta^2}\ln c_\theta^2-$$
$$-\frac{1}{c_\theta^2}R_H\left(\frac{M_H^2}{M_0^2}\right)+\frac{7}{8c_\theta^2}-\frac{3}{16}\frac{(v_\theta^2+1)^2}{c_\theta^2}+1+s_\theta^2\Big[4(\Delta_{IR}-\ln s+\ln 4\pi^2)\cdot$$
$$\cdot\left(\ln\frac{st}{u\,m\mu}-1\right)+2\ln\frac{s}{m\mu}+\ln^2\frac{s}{m^2}+\ln^2\frac{s}{\mu^2}+\frac{8\pi^2}{3}-8+\left(3-4\ln\frac{\sqrt{tu}}{s}\right)\cdot$$
$$\cdot\ln\frac{t}{u}+v_\theta^2\left(6\ln\sqrt{tu}-6\ln M_0^2-7\right)\Big]\Big\} \quad (5.24)$$

$$\tilde{\delta}_2=\delta_2^B=\frac{g^2}{16\pi^2}\left[4s_\theta^2v_\theta^2\ln\frac{t}{u}\cdot\left(\Delta_{IR}+\frac{3}{4}-\ln\sqrt{tu}+\ln 4\pi^2\right)+6s_\theta^2\ln\sqrt{tu}\right] \quad (5.25)$$

The corrections (5.24) and (5.25) are divergent since they are expressed in terms of the bare parameters. The renormalizability of the Salam-Weinberg model allows us to absorb all the UV infinities into the definition of the renormalized parameters: g, s_θ, M. When $\sqrt{s} \ll M_W$, however, it is easy to see that in our case all the UV infinities are removed by a single renormalization, namely the renoramalization of the Fermi constant G, taking the muon mean-life as the experimental input to fix the one-loop corrected value of G:

$$\frac{G}{\sqrt{2}}=\frac{G^0}{\sqrt{2}}\left(1-\delta\right)$$

where G^0 is the bare Fermi constant:

$$\frac{G^0}{\sqrt{2}} = \left(g^{(0)}\right)^2 \Big/ 8\left(M^{(0)}\right)^2$$

The correction δ has the form [22]:

$$\delta = \frac{g^2}{16\pi^2}\left[\left(2+\frac{1}{c_\theta^2}\right)\left(\ln M^2 - \Delta - \ln 4\pi^2\right) + R_H\left(\frac{M_H^2}{M^2}\right) - \frac{5}{4} - \frac{7}{8c_\theta^2} - \frac{1}{2}c_\theta^2 + \left(\frac{3}{4s_\theta^2} - \frac{1}{c_\theta^2}\right)\ln c_\theta^2\right.$$

where

$$R_H(x) = \frac{1}{8}x + \frac{3}{4}\frac{x}{x-1}\ln x \tag{5.26}$$

and M_H is the mass of the Higgs particle. Adding to $\tilde{\delta}_1$ the infinite counterterm δ, we obtain:

$$\begin{aligned}\delta_1(\theta_s) = \tilde{\delta}_1 + \delta = \frac{g^2}{16\pi^2}\Big\{&4s_\theta^2\left(\Delta_{IR} - \ln s + \ln 4\pi^2\right)\left(\ln\frac{s}{m\mu} + \ln\frac{1-\cos\theta_s}{1+\cos\theta_s} - 1\right) + \\ &+ s_\theta^2\left[3 + 4\ln 2 - 4\ln|\sin\theta_s|\right]\ln\frac{1-\cos\theta_s}{1+\cos\theta_s} + 6v_\theta^2 s_\theta^2\left(\ln\frac{s}{2} + \ln|\sin\theta_s|\right) + \\ &+ s_\theta^2\left[2\ln\frac{s}{m\mu} + \ln^2\frac{s}{m^2} + \ln^2\frac{s}{\mu^2} + \frac{8\pi^2}{3} - 8 - v_\theta^2\left(6\ln M_0^2 + 7\right)\right] - \\ &- \frac{3}{16}\frac{(v_\theta^2+1)^2}{c_\theta^2} - \frac{1}{4} - \frac{1}{2}c_\theta^2 + \frac{3}{4s_\theta^2}\ln c_\theta^2 + \rho\left(\frac{M_0^2}{M_H^2}\right)\Big\}\end{aligned} \tag{5.27}$$

with

$$\rho(x) = \frac{3}{4}\frac{1}{c_\theta^2 x - 1}\left[\ln c_\theta^2 + \left(\frac{1}{c_\theta^2} - 1\right)\frac{1}{x-1}\ln x\right]$$

As for $\tilde{\delta}_2$, we have:

$$\begin{aligned}\delta_2(\theta_s) = \tilde{\delta}_2 = \frac{g^2}{16\pi^2}\cdot 2s_\theta^2\Big[&2v_\theta^2\ln\frac{1-\cos\theta_s}{1+\cos\theta_s}\cdot\left(\Delta_{IR} + \frac{3}{4} + \ln 4\pi^2 - \ln\frac{s}{2} - \right. \\ &\left.- \ln|\sin\theta_s|\right) + 3\ln\frac{s}{2} + 3\ln|\sin\theta_s|\Big]\end{aligned} \tag{5.28}$$

In the last passage of eqs.(5.27), (5.28) we have made use of eq.(A.I).

By putting together eqs.(5.II), (5.I2), (5.27), (5.28) we find in conclusion:

$$\begin{aligned}\frac{c_1(\theta_s) + c_1(\pi-\theta_s)}{2} = 1 + \frac{\alpha}{4\pi}\Big[&4\left(\Delta_{IR} + \ln 4\pi^2 - \ln s\right)\left(\ln\frac{s}{m\mu} - 1\right) + \\ &+ 6v_\theta^2\ln|\sin\theta_s| + 2\ln\frac{s}{m\mu} + \ln^2\frac{s}{m^2} + \ln^2\frac{s}{\mu^2} + \frac{8\pi^2}{3} - 8 + 6v_\theta^2\ln\frac{s}{2M_0^2} - 7v_\theta^2 - \\ &- \frac{3}{16}\frac{(v_\theta^2+1)^2}{c_\theta^2 s_\theta^2} - \frac{1}{4s_\theta^2} - \frac{1}{2}\frac{c_\theta^2}{s_\theta^2} + \frac{3}{4s_\theta^4}\ln c_\theta^2 + \frac{1}{s_\theta^2}\rho\left(\frac{M_0^2}{M_H^2}\right)\end{aligned} \tag{5.29}$$

$$\frac{c_2(\theta_s) - c_2(\pi-\theta_s)}{2} = \frac{\alpha}{\pi}v_\theta^2\ln\frac{1-\cos\theta_s}{1+\cos\theta_s}\cdot\left(\Delta_{IR} + \frac{5}{4} + \ln 4\pi^2 - \ln\frac{s}{2} - \ln|\sin\theta_s|\right) \tag{5.30}$$

Substituting eqs.(5.2I), (5.22), (5.23), (5.29), (5.30) into eq.(5.I4), one determines the contribution of the virtual diagrams to the angular asymmetry A.

Note that δ_1, eq.(5.27), goes to a finite limit as $s_\theta^2 \to 0$ with fixed g. Indeed (see Appendix F) it is possible to obtain directly the value of δ_1 in this limit, that is not so far from the physical situation ($s_\theta^2 \simeq 0.22$).

The IR divergences of the virtual part of the corrections are removed by the inclusion of the bremsstrahlung of a soft (i.e. with an energy $k_0 \ll m, \mu$) photon by the external lines. The amplitude for the process sketched in fig. 5 is:

$$\mathcal{A} = (-e)\, B\, \frac{\hat{p} - \hat{k} + im}{-2(p \cdot k)}\, \hat{\varepsilon}\, u(p)$$

where B represent the contribution of the shaded region in fig. 5 and $\varepsilon(k)$ is the polarization of the emitted photon. Using the transversality condition:

$$k \cdot \varepsilon = 0$$

we have:

$$\mathcal{A} = e\, B \left(\frac{p \cdot \varepsilon}{p \cdot k} + \frac{\hat{\varepsilon}\hat{k}}{2\, p \cdot k} \right) u(p)$$

The soft photon approximation consists in neglecting ($\hat{\varepsilon}\hat{k}$) with respect to ($p \cdot \varepsilon$). This is reasonable because: i) the term we retain is by itself gauge invariant, so that no cancellation is expected to occur with the contribution from higher energy photons; ii) we obtain in this way the dominant real corrections, namely those terms which are singular in the limit $m, \mu \to 0$, dropping terms of the type αm^2, $\alpha m^2 \ln m^2$ or definitely of order α.

In the following we compute first the divergent part of the correction due to emission of a soft real photon and then determine its finite part using the minimum detection energy of the photons as a cut-off. As expressed by eq.(5.15), the emission of a soft photon modifies the total transition probability by a factor:

$$b = \frac{e^2}{(2\pi)^{3+\varepsilon}} \sum_{pol.} \int d^4k\, \delta(k^2)\, \theta(k_0) \left| \frac{p_1 \cdot \varepsilon}{p_1 \cdot k} - \frac{p_2 \cdot \varepsilon}{p_2 \cdot k} + \frac{p_3 \cdot \varepsilon}{p_3 \cdot k} - \frac{p_4 \cdot \varepsilon}{p_4 \cdot k} \right|^2 f(k_0) =$$

$$= \frac{\alpha}{4\pi^2} \int \frac{d^{3+\varepsilon}k}{(2\pi)^{\varepsilon} k_0} \left[\frac{p_1^2}{(p_1 \cdot k)^2} + \frac{p_2^2}{(p_2 \cdot k)^2} + \frac{p_3^2}{(p_3 \cdot k)^2} + \frac{p_4^2}{(p_4 \cdot k)^2} - \frac{2(p_1 \cdot p_2)}{(p_1 \cdot k)(p_2 \cdot k)} + \frac{2(p_1 \cdot p_3)}{(p_1 \cdot k)(p_3 \cdot k)} - \right.$$

$$\left. - \frac{2(p_1 \cdot p_4)}{(p_1 \cdot k)(p_4 \cdot k)} - \frac{2(p_2 \cdot p_3)}{(p_2 \cdot k)(p_3 \cdot k)} + \frac{2(p_2 \cdot p_4)}{(p_2 \cdot k)(p_4 \cdot k)} - \frac{2(p_3 \cdot p_4)}{(p_3 \cdot k)(p_4 \cdot k)} \right] f(k_0)$$

We have introduced here a function $f(k_0)$ which represents the probability that a photon with an absolute value of the space-momentum equal to $|\vec{k}|$, is not detected. The function $f(k_o)$ represents approximately the resolution of the experimental apparatus and, for the moment, we are interested only in its properties:

$$f(0) = I \qquad\qquad f(\infty) = 0 \tag{5.3I}$$

Then it is easy to see that:

$$I_1(q) = \int \frac{d^{3+\varepsilon}k}{(2\pi)^\varepsilon k_0} \frac{q^2}{(q\cdot k)^2} f(k_0) = -\frac{4\pi}{\varepsilon} + O(1) \tag{5.32}$$

$$I_2(q_1, q_2) = \int \frac{d^{3+\varepsilon}k}{(2\pi)^\varepsilon k_0} \frac{(q_1 \cdot q_2)}{(q_1\cdot k)(q_2\cdot k)} f(k_0) = -\frac{4\pi}{\varepsilon} \ln\left(-\frac{2 q_1\cdot q_2}{m_1 m_2}\right) + O(1) \tag{5.33}$$

where O(I) indicates terms that are regular in the limit $\varepsilon \to 0$ and depend on the details of $f(k_0)$; in the above expressions we assume:

$$q_1 \neq q_2$$

and:

$$q^2_{1,2} = -m^2_{1,2}$$

with the condition (see eq.(5.I)):

$$|2(q_1 \cdot q_2)| \gg m_1^2,\ m_2^2$$

The IR divergent part of the b coefficient is then:

$$b_{IR} = \frac{2\alpha}{\pi}\left(1 - \ln\frac{st}{u m_\mu}\right)\Delta_{IR} \tag{5.34}$$

Recalling that $\vartheta_s \to \pi - \vartheta_s$ corresponds to $t \leftrightarrow u$, we have:

$$\frac{b_{IR}(\theta_s) + b_{IR}(\pi - \theta_s)}{2} = \frac{2\alpha}{\pi}\left(1 - \ln\frac{s}{m_\mu}\right)\Delta_{IR} \tag{5.35}$$

$$\frac{b_{IR}(\theta_s) - b_{IR}(\pi - \theta_s)}{2} = \frac{2\alpha}{\pi} \ln\left(\frac{1-\cos\theta_s}{1+\cos\theta_s}\right)\cdot\Delta_{IR} \tag{5.36}$$

From eqs.(5.I8), (5.I9), (5.20) and from the expressions (5.2I), (5.22), (5.23), (5.29), (5.30), (5.35), (5.36) we see that the complete asymmetry has no divergence anymore.

We refer to Appendix D for the computation of the finite part of $b(\vartheta_s)$. If $f(k_0)$ is approximated by a step function:

$$f(k_0) = \vartheta(\Delta - k_0)$$

we obtain (see eq.(D.I8)):

$$\frac{b(\theta_s)+b(\pi-\theta_s)}{2} = \frac{2\alpha}{\pi}\Big[\Big(1-\ln\frac{s}{m\mu}\Big)\big(\Delta_{IR}+\ln 4\pi^2-\ln\Lambda^2-\ln 4\big)+ + \ln\frac{s}{m\mu}-\frac{1}{4}\ln^2\frac{s}{m^2}-\frac{1}{4}\ln^2\frac{s}{\mu^2}-\frac{\pi^2}{3}\Big] \qquad (5.37)$$

$$\frac{b(\theta_s)-b(\pi-\theta_s)}{2} = \frac{2\alpha}{\pi}\Big\{\ln\Big(\frac{1+\cos\theta_s}{1-\cos\theta_s}\Big)\big(\Delta_{IR}-\ln\Lambda^2+\ln 4\pi^2-\ln 4\big)+ + \omega\Big[x=\frac{1}{2}(1+\cos\theta_s)\Big]\Big\} \qquad (5.38)$$

where the function $\omega(x)$ is given by eq.(D.I9) in an integral form.

By substituting eqs.(5.37), (5.38) into eq.(5.I7) one can obtain the contribution of the soft real corrections to the asymmetry. Thus, in the one-loop approximation, the coefficients $d_{1,2}(\theta_s)$ that appear in the asymmetry (5.I8) have the following expressions:

$$d_1(\theta_s) = 1+\frac{2\alpha}{\pi}\Big\{\ln\Big(\frac{s}{4\Lambda^2}\Big)\Big(1-\ln\frac{s}{m\mu}\Big)+\frac{3}{2}\ln\frac{s}{m\mu}+\frac{\pi^2}{3}-2+ + \frac{1}{6}\sum_{m_i<\sqrt{s}}\Big[Q_i^2\Big(\ln\frac{s}{m_i^2}-\frac{5}{3}\Big)\Big]-\frac{3}{128}\frac{(v_\theta^2+1)^2}{s_\theta^2c_\theta^2}-\frac{1}{32s_\theta^2}-\frac{1}{16}\frac{c_\theta^2}{s_\theta^2}+ + \frac{3}{32s_\theta^2}\ln c_\theta^2+\frac{1}{8s_\theta^2}\rho\Big(\frac{M_0^2}{M_H^2}\Big)+\frac{v_\theta^2}{4}\Big[3\ln\frac{s}{2M_0^2}-\frac{7}{2}+3\ln|\sin\theta_s|- - \frac{1}{\cos\theta_s+1}\Big(\ln\Big(\frac{1-\cos\theta_s}{2}\Big)-\frac{\cos\theta_s}{1+\cos\theta_s}\ln^2\Big(\frac{1-\cos\theta_s}{2}\Big)\Big)+ + \frac{1}{\cos\theta_s-1}\Big(\ln\Big(\frac{1+\cos\theta_s}{2}\Big)+\frac{\cos\theta_s}{1-\cos\theta_s}\ln^2\Big(\frac{1+\cos\theta_s}{2}\Big)\Big)\Big]\Big\} \qquad (5.39)$$

$$d_2(\theta_s) = \frac{2\alpha}{\pi}v_\theta^2\Big\{\Big(\frac{3}{8}+\frac{1}{2}\ln 2-\frac{1}{2}\ln|\sin\theta_s|-\ln\frac{s}{4\Lambda^2}\Big)\ln\Big(\frac{1-\cos\theta_s}{1+\cos\theta_s}\Big)+\frac{1}{4}\frac{1}{\cos\theta_s+1}\cdot \cdot\Big[\ln\Big(\frac{1-\cos\theta_s}{2}\Big)-\frac{\cos\theta_s}{1+\cos\theta_s}\ln^2\Big(\frac{1-\cos\theta_s}{2}\Big)\Big]+\frac{1}{4}\frac{1}{\cos\theta_s-1}\Big[\ln\Big(\frac{1+\cos\theta_s}{2}\Big)+\frac{\cos\theta_s}{1-\cos\theta_s}\ln^2\Big(\frac{1+\cos\theta_s}{2}\Big)\Big]+ + \omega\Big[x=\frac{1}{2}(1+\cos\theta_s)\Big]\Big\} \qquad (5.40)$$

The order α corrections to the asymmetry are logarithmically divergent in the limit M^2, $M_0^2\to\infty$, G = const. The divergent term from eq.(5.39) can be seen to be:

$$\delta_1 \xrightarrow[G=\mathrm{const}]{M^2,M_0^2\to\infty} -\frac{3}{2}v_\theta^2\frac{\alpha}{\pi}\ln M_0^2$$

which agrees with eq.(4.I3).

The inclusion of the leading logarithmic second order correction is straightforward because the form (5.I8) of the asymmetry does not change and the contribution (4.I5) simply adds a term to the coefficient d_1, with respect to eq.(5.39). We obtain, in conclusion:

$$d_1(\theta_s) = 1+\frac{2\alpha}{\pi}\Big\{\ln\Big(\frac{s}{4\Lambda^2}\Big)\cdot\Big(1-\ln\frac{s}{m\mu}\Big)+\frac{3}{2}\ln\frac{s}{m\mu}+\frac{\pi^2}{3}-2+ + \frac{1}{6}\sum_{m_i<\sqrt{s}}\Big[Q_i^2\Big(\ln\frac{s}{m_i^2}-\frac{5}{3}\Big)\Big]-\frac{3}{128}\frac{(v_\theta^2+1)^2}{s_\theta^2c_\theta^2}-\frac{1}{32s_\theta^2}-\frac{1}{16}\frac{c_\theta^2}{s_\theta^2}+ + \frac{3}{32s_\theta^2}\ln c_\theta^2+\frac{1}{8s_\theta^2}\rho\Big(\frac{M_0^2}{M_H^2}\Big)+\frac{v_\theta^2}{4}\Big[3\ln\frac{s}{2M_0^2}-\frac{7}{2}+3\ln|\sin\theta_s|-$$

$$- \frac{1}{\cos\theta_s + 1}\left(\ln\left(\frac{1-\cos\theta_s}{2}\right) - \frac{\cos\theta_s}{1+\cos\theta_s}\ln^2\left(\frac{1-\cos\theta_s}{2}\right)\right) +$$

$$+ \frac{1}{\cos\theta_s - 1}\left(\ln\left(\frac{1+\cos\theta_s}{2}\right) + \frac{\cos\theta_s}{1-\cos\theta_s}\ln^2\left(\frac{1+\cos\theta_s}{2}\right)\right)\Bigg]\Bigg\} +$$

$$+ \frac{\alpha^2}{2\pi^2}\left\{\left[\frac{5}{2} - \frac{v_\theta}{2}\sum_i Q_i\left(T_{3i} - 4s_\theta^2 Q_i\right) + \frac{1}{4}v_\theta^2\left(1 - 2\sum_i Q_i^2\right)\right]\cdot\right.$$

$$\left.\cdot \ln^2\left(\frac{M^{*2}}{s}\right) - \sum_{m_i < \sqrt{s}} Q_i^2 \ln\left(\frac{s}{m_i^2}\right)\ln\left(\frac{M^{*2}}{s}\right)\right\} + O\left[\alpha^2 \ln\left(\frac{M^{*2}}{s}\right)\right] \qquad (5.4I)$$

while the coefficient d_2 is unaffected and it is given by eq.(5.40) up to corrections of order $(\alpha \ln M_0^2)^2$ included.

Putting in eqs.(5.4I), (5.40), (5.I8) the values:
s = I000 GeV , Λ = I GeV , M^* = 10^2 GeV , s_θ^2 = 0.22 ,
M_0 = 90 GeV , M_H = 200 GeV , m_e = 0.0005II GeV , m_μ = 0.I0566 GeV ,
m_τ = I.7842 GeV , m_u = m_d = 0.I4 GeV , m = 0.25 GeV ,
m = I.8 GeV , m_b = 5.0 GeV , m_t = 20 GeV

we get the three plots in figg. 5, 6, 7. They represent the behaviour of $d_1(\theta_s)$, $d_2(\theta_s)$, $A(\theta_s)$ for $40° \leq \vartheta_s \leq 140°$. The two integrals of eq.(D.I9) have been evaluated numerically.

In order to show the order of magnitude of the different contributions to $d_1(\theta_s)$ we call:

$$d_1^{MS} = \frac{2\alpha}{\pi}\left[\ln\left(\frac{s}{4\Lambda^2}\right)\left(1 - \ln\frac{s}{m_\mu}\right) + \frac{3}{2}\ln\frac{s}{m_\mu} + \frac{1}{6}\sum_{m_i<\sqrt{s}} Q_i^2\left(\ln\frac{s}{m_i^2} - \frac{5}{3}\right)\right] = -0.25454$$

$$d_1^{LL} = \frac{2\alpha}{\pi}\frac{v_\theta^2}{4}\cdot 3\ln\frac{s}{2M_0^2} = -0.14\cdot 10^{-3}$$

$$d_1^{LL2} = \frac{\alpha^2}{2\pi^2}\left\{\left[\frac{5}{2} - \frac{v_\theta}{2}\sum_i Q_i\left(T_{3i} - 4s_\theta^2 Q_i\right) + \frac{1}{4}v_\theta^2\left(1 - 2\sum_i Q_i^2\right)\right]\ln\left(\frac{M^{*2}}{s}\right) - \right.$$

$$\left. - \sum_{m_i<\sqrt{s}} Q_i^2 \ln\left(\frac{s}{m_i^2}\right)\ln\left(\frac{M^{*2}}{s}\right)\right\} = -0.27\cdot 10^{-3}$$

The two latter equations show that a one-loop order computation of d_1 is incomplete in the sense that d_1^{LL2} is two times bigger than d_1^{LL}. At any rate, both corrections are beyond experimental reach, for the time being.

Finally we show in fig. 8 the behaviour of what we can call the genuine weak correction:

$$d_1^{WE} = \frac{2\alpha}{\pi}\left\{ \frac{\pi^2}{3} - 2 - \frac{3}{128}\frac{(v_\theta^2+1)^2}{s_\theta^2 c_\theta^2} - \frac{1}{32 s_\theta^4} - \frac{1}{16}\frac{c_\theta^2}{s_\theta^2} + \frac{3}{32 s_\theta^4}\ln c_\theta^2 + \frac{1}{8 s_\theta^2} f\left(\frac{M_0^2}{M_H^2}\right) + \right.$$
$$+ \frac{v_\theta^2}{4}\left[-\frac{7}{2} + 3\ln|\sin\theta_s| - \frac{1}{\cos\theta_s+1}\left(\ln\left(\frac{1-\cos\theta_s}{2}\right) - \frac{\cos\theta_s}{1+\cos\theta_s}\ln^2\left(\frac{1-\cos\theta_s}{2}\right)\right) + \right. \tag{5.42}$$
$$\left.\left. + \frac{1}{\cos\theta_s - 1}\left(\ln\left(\frac{1+\cos\theta_s}{2}\right) + \frac{\cos\theta_s}{1-\cos\theta_s}\ln^2\left(\frac{1+\cos\theta_s}{2}\right)\right)\right]\right\}$$

APPENDIX A

Following [8] and referring us to fig. 2 we use the following notations. We choose as positive the incoming momenta and adopt a metric such that the square of a time-like 4-vector be negative:

$$p_1^2 = p_2^2 = -m^2 \quad , \quad p_3^2 = p_4^2 = -\mu^2$$

We define the invariants:

$$s = -(p_1 + p_2)^2 = -(p_3 + p_4)^2$$
$$t = -(p_1 + p_4)^2 = -(p_2 + p_3)^2$$
$$u = -(p_1 + p_3)^2 = -(p_2 + p_4)^2$$

They are not independent variables:

$$s + t + u = 2m^2 + 2\mu^2$$

Our computations are discussed in the region:

$$s \geq 0 \quad , \quad t \leq 0 \quad , \quad u \leq 0$$

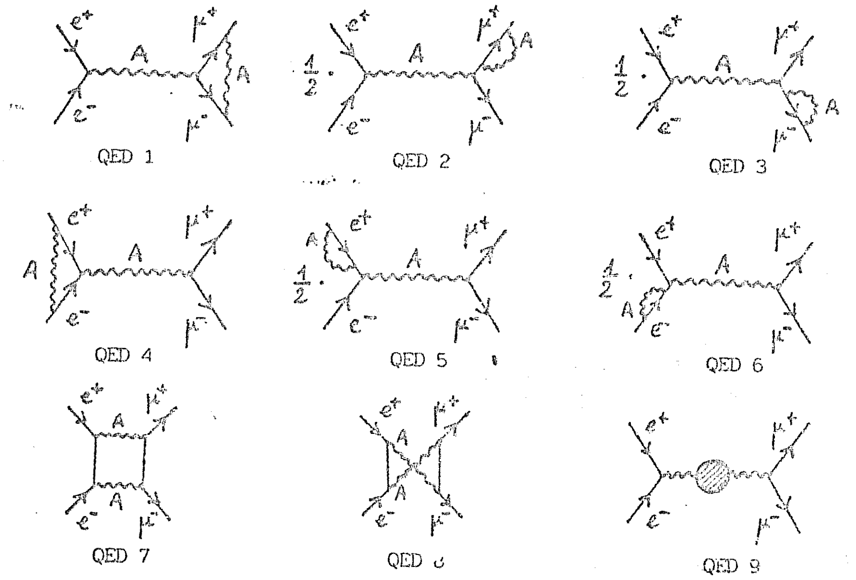
but the results can be analytically continued.

If E is the energy of the incoming particle in the centre of mass system of the two incoming particles, then

$$s = 4E^2 \quad , \quad t = -2E^2(1 - \cos\theta_s) \quad , \quad u = -2E^2(1 + \cos\theta_s) \qquad \text{(A.1)}$$

APPENDIX B

We list the diagrams providing the pure QED corrections to the graph a of fig. 3:



The blob in the graph QED 9 represents the sum of all possible particles of type i, each having an electric charge Q_i and a mass m_i, which can go round the loop. The diagrams from QED 1 to QED 6 have amplitude proportional to $\mathcal{M}_a$; by adding their contribution we get:

$$\sum_{k=1}^{6}(\text{QED K}) = \frac{\alpha(0)}{4\pi}\Big[4\left(\Delta_{IR} - \ln s + \ln 4\pi^2\right)\left(\ln\frac{s}{m_\mu} - 1\right) + 2\ln\frac{s}{m_\mu} + \ln^2\frac{s}{m^2} + \ln^2\frac{s}{\mu^2} + \frac{8\pi^2}{3} - 8\Big]\cdot\mathcal{M}_a \tag{B.1}$$

The sum of the box diagrams gives:

$$(QED\,7)+(QED\,8) = -(2\pi)^4 i\, g^2 s_\theta^2 \frac{\alpha(0)}{4\pi}\cdot\Big\{(\gamma^\mu\otimes\gamma^\mu)\Big\{\frac{4}{s}\ln\frac{t}{u}\left(-\Delta_{IR}-\ln 4\pi^2+\ln s\right)-$$

$$-\frac{1}{u}\left[-\ln\left(-\frac{t}{s}\right)+\left(1+\frac{s}{2u}\right)\ln^2\left(-\frac{s}{t}\right)\right]+\frac{1}{t}\left[-\ln\left(-\frac{u}{s}\right)+\left(1+\frac{s}{2t}\right)\ln^2\left(-\frac{s}{u}\right)\right]\Big\}+$$

$$+(\gamma^\mu\gamma_5\otimes\gamma^\mu\gamma_5)\cdot\Big\{\frac{1}{u}\left[-\ln\left(-\frac{t}{s}\right)+\left(1+\frac{s}{2u}\right)\ln^2\left(-\frac{s}{t}\right)\right]+$$

$$+\frac{1}{t}\left[-\ln\left(-\frac{u}{s}\right)+\left(1+\frac{s}{2t}\right)\ln^2\left(-\frac{s}{u}\right)\right]\Big\}\Big\} \qquad (B.2)$$

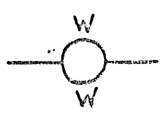
The photon self-energy diagrams are proportional to $\mathcal{M}_a$, and from their sum we get:

$$(QED\,9) = \frac{\alpha(0)}{3\pi}\sum_{m_i<\sqrt{s}}\left[Q_i^2\left(\ln\frac{s}{m_i^2}-\frac{5}{3}\right)\right]\cdot\mathcal{M}_a \qquad (B.3)$$

APPENDIX C

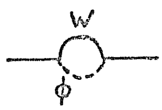
In the following we give the list of the diagrams symbolically represented in fig. 4 and the corresponding contributions to the quantities δ_1 and δ_2.

<u>Self-energy diagrams of the W°:</u>

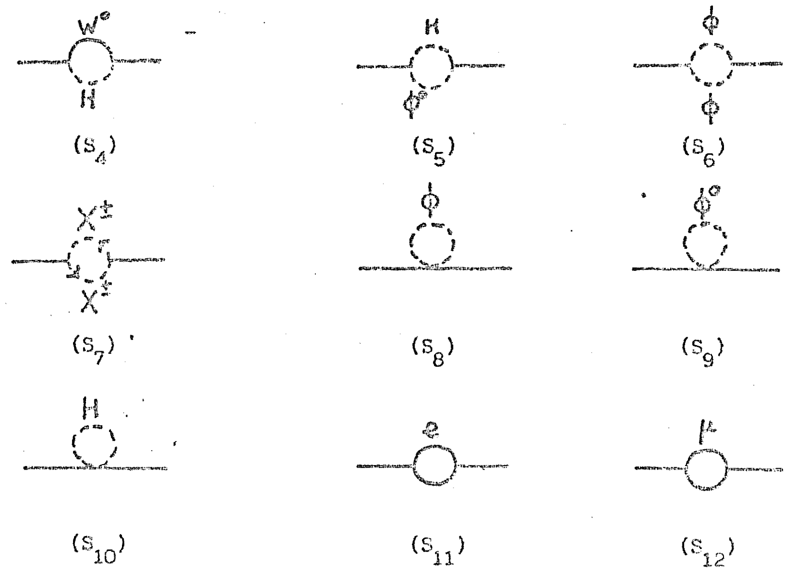


(S_1)

(S_2)



(S_3)



(s_4) (s_5) (s_6) (s_7) (s_8) (s_9) (s_{10}) (s_{11}) (s_{12})

We have indicated as ϕ^+, ϕ^-, ϕ^0 the would-be Goldstone bosons, $X^\pm$ the Higgs-Kibble ghosts, and H the Higgs particle.

The sum of these diagrams give the value of the W° self-energy for vanishing momentum transfer:

$$\Pi^0_{\mu\nu}(0) = \delta_{\mu\nu}\,\Pi^0(0)$$

By adding the contribution to $\Pi^0(0)$ coming from the diagrams, we get:

$$\frac{\tilde{\Pi}^0(0)}{M_0^2} = \frac{1}{(2\pi)^4 i}\,\frac{\Pi^0(0)}{M_0^2} =$$

$$= \frac{g^2}{16\pi^2}\left[\left(-4c_\theta^2+2+\frac{1}{c_\theta^2}\right)\left(\Delta-\ln M_0^2+\ln 4\pi^2\right)+\frac{7}{8c_\theta^2}-\left(2-4c_\theta^2\right)\ln c_\theta^2-\frac{1}{c_\theta^2}R_H\left(\frac{M_H^2}{M_0^2}\right)\right].$$

where $R_H(x)$ is given by eq.(5.25); M_H represents the Higgs boson mass. The expression of the corrections associated to the diagram a in fig.4 is, then:

$$\delta_1^s = \frac{\tilde{\Pi}^0(0)}{M_0^2} =$$

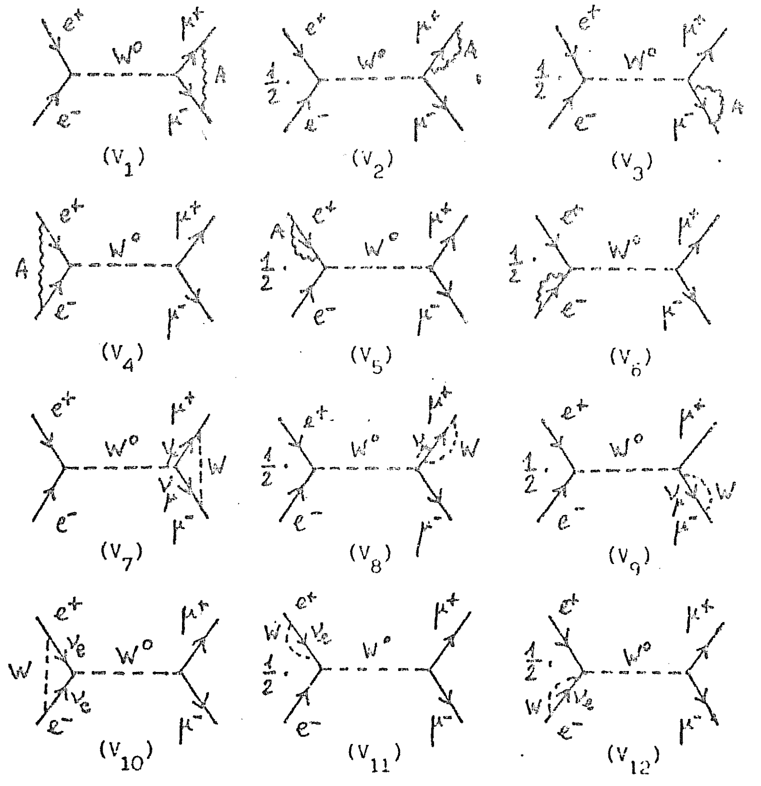
$$= \frac{g^2}{16\pi^2}\left[\left(-4c_\theta^2+2+\frac{1}{c_\theta^2}\right)\left(\Delta-\ln M_0^2+\ln 4\pi^2\right)+\frac{7}{8c_\theta^2}-\left(2-4c_\theta^2\right)\ln c_\theta^2-\frac{1}{c_\theta^2}R_H\left(\frac{M_H^2}{M_0^2}\right)\right.$$

$$\delta_2^S = v_\theta^2 \frac{\tilde{\Pi}^0(0)}{M_0^2} =$$

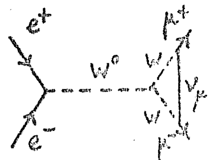
$$= v_\theta^2 \frac{g^2}{16\pi^2}\left[\left(-4c_\theta^2+2+\frac{1}{c_\theta^2}\right)\left(\Delta - \ln M_0^2 + \ln 4\pi^2\right) + \frac{7}{8c_\theta^2} - \left(2-4c_\theta^2\right)\ln c_\theta^2 - \frac{1}{c_\theta^2} R_H\left(\frac{M_H^2}{M_0^2}\right)\right] \quad \text{(C.2)}$$

Vertex diagrams:

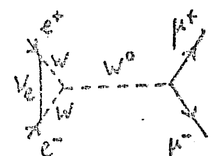
We include in this class of graphs the wave-function renormalization of the external legs. For convenience, we divide the vertex diagrams into two classes: those (diagrams from V_1 to V_6) in which a photon is exchanged between the charged external lines of the diagram b in fig. 3, and the others (diagrams from V_7 to V_{20}) with no photon exchanged.



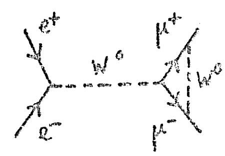
(C.1)



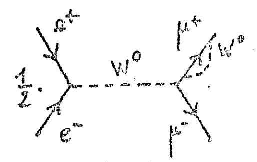
(V_{13})



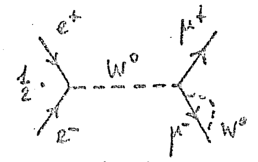
(V_{14})



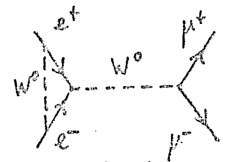
(V_{15})



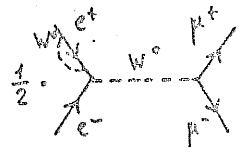
(V_{16})



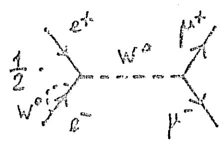
(V_{17})



(V_{18})



(V_{19})



(V_{20}) .

Calling $(\delta_1^V)_p$ and $(\delta_2^V)_p$ the contribution of the diagrams of the first class to δ_1 and δ_2 , and by $(\delta_1^V)_o$ and $(\delta_2^V)_o$ the corresponding contributions from the second class of diagrams, we obtain:

$$\left(\delta_1^V\right)_p = \frac{g^2}{16\pi^2} s_\theta^2 \left[4\left(\Delta_{IR} + \ln 4\pi^2 - \ln s\right)\left(\ln\frac{s}{m\mu} - 1\right) + 2\ln\frac{s}{m\mu} + \ln^2\frac{s}{m^2} + \ln^2\frac{s}{\mu^2} + \frac{8\pi^2}{3} - 8\right]$$

$$\left(\delta_2^V\right)_p = \frac{g^2}{16\pi^2} v_\theta^2 s_\theta^2 \left[4\left(\Delta_{IR} + \ln 4\pi^2 - \ln s\right)\left(\ln\frac{s}{m\mu} - 1\right) + 2\ln\frac{s}{m\mu} + \ln^2\frac{s}{m^2} + \ln^2\frac{s}{\mu^2} + \frac{8\pi^2}{3} - 8\right]$$

$$\left(\delta_1^V\right)_o = \frac{g^2}{16\pi^2} 4 c_\theta^2 \left(\Delta - \ln M^2 + \ln 4\pi^2\right)$$

$$\left(\delta_2^V\right)_o = \frac{g^2}{16\pi^2} 4 v_\theta c_\theta^2 \left(-\Delta + \ln M^2 - \ln 4\pi^2\right)$$

The total correction from the vertex diagrmas in fig. 3 is:

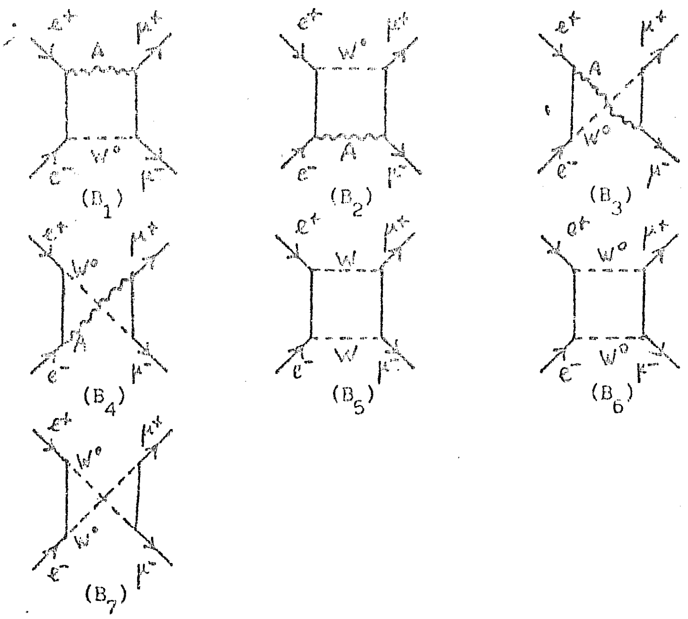
$$\delta_1^V = (\delta_1^V)_p + (\delta_1^V)_o = \frac{g^2}{16\pi^2}\Big\{4c_\theta^2(\Delta - \ln M^2 + \ln 4\pi^2) +$$

$$+ s_\theta^2\Big[4(\Delta_{IR} - \ln s + \ln 4\pi^2)\Big(\ln\frac{s}{m\mu} - 1\Big) + 2\ln\frac{s}{m\mu} + \ln^2\frac{s}{m^2} + \ln^2\frac{s}{\mu^2} + \frac{8\pi^2}{3} - 8\Big]\Big\} \qquad \text{(C.3)}$$

$$\delta_2^V = (\delta_2^V)_p + (\delta_2^V)_o = \frac{g^2}{16\pi^2}\Big\{4v_\theta c_\theta^2(-\Delta + \ln M^2 - \ln 4\pi^2) +$$

$$+ v_\theta^2 s_\theta^2\Big[4(\Delta_{IR} - \ln s + \ln 4\pi^2)\Big(\ln\frac{s}{m\mu} - 1\Big) + 2\ln\frac{s}{m\mu} + \ln^2\frac{s}{m^2} + \ln^2\frac{s}{\mu^2} + \frac{8\pi^2}{3} - 8\Big]\Big\} \qquad \text{(C.4)}$$

Box diagrams:



The contribution of the box diagrams, represented in fig. [illegible] c and listed here, is:

$$\delta_1^B = \frac{g^2}{16\pi^2}\Big[4s_\theta^2 \ln\frac{t}{u}\Big(\Delta_{IR} + \frac{3}{4} - \ln\sqrt{tu} + \ln 4\pi^2\Big) + v_\theta^2 s_\theta^2\big(6\ln\sqrt{tu} - 6\ln M_0^2 - 7\big) - \qquad \text{(C.5)}$$

$$- \frac{3}{16}\frac{(v_\theta^2+1)^2}{c_\theta^2} + 1\Big]$$

$$\bar{\delta}_2^{B} = \frac{g^2}{16\pi^2}\Big[4 s_\theta^2 v_\theta^2 \ln\frac{t}{u}\left(\Delta_{IR} + \frac{3}{4} - \ln\sqrt{tu} + \ln 4\pi^2\right) + s_\theta^2\left(6\ln\sqrt{tu} - 6\ln M_0^2 - 7\right) - \frac{3}{4}\frac{v_\theta^2}{c_\theta^2} + 1\Big] \tag{C.6}$$

APPENDIX D

We determine here the finite parts of the integrals I_1 and I_2 eqs.(5.32), (5.33).

Passing to angular co-ordinates and recalling that k_μ is a light-like 4-vector, we can write:

$$\int d^{3+\varepsilon}\vec{k} = \int d\Omega_{3+\varepsilon} \int_0^\infty dk_0 \, k_0^{2+\varepsilon} \tag{D.1}$$

By calling ϑ the angle between $\vec{k}$ and $\vec{q}$ and introducing the parameter:

$$\beta = \frac{|\vec{q}|}{q_0} \tag{D.2}$$

we get :

$$I_1 = \int_0^\infty dk_0 \, f(k_0) \, k_0^{\varepsilon-1} \, c_\varepsilon$$

where:

$$c_\varepsilon = \int \frac{d\Omega_{3+\varepsilon}}{(2\pi)^{\varepsilon}} \frac{1}{(1-\beta\cos\vartheta)^2} \frac{q^2}{q_0^2} \tag{D.3}$$

Integrating by parts and using the (5.31), we find:

$$I_1 = \frac{1}{\varepsilon} c_0 - \int_0^\infty dk_0 \, f'(k_0)(c_0 \ln k_0 + c_1)$$

where C_0 and C_1 are defined by the expansion:

$$c_\varepsilon = c_0 + \varepsilon c_1 + O(\varepsilon^2) \tag{D.4}$$

If Λ is the minimum detection energy of the photons we can approximate, for simplicity, the observed distribution $f(k_0)$ as a step-function:

$$f(k_0) = \vartheta(\Lambda - k_0) \quad , \quad f'(k_0) = -\delta(k_0 - \Lambda) \tag{D.5}$$

Thus, we have:

$$I_1 = \frac{1}{\varepsilon} c_0 + c_0 \ln \Lambda + c_1 \qquad (D.6)$$

From eqs.(D.3), (D.4) we obtain:

$$C_0 = -4\pi \qquad (D.7)$$

From eqs.(D.2), (D.3) one has:

$$C_\varepsilon = \int_0^\pi d\theta \, \frac{\Omega_{2+\varepsilon}(r = |\sin\theta|)}{(2\pi)^\varepsilon} \, \frac{(\beta^2 - 1)}{(1 - \beta\cos\theta)^2} \qquad (D.8)$$

where $\Omega_{2+\varepsilon}$ $(r = |\sin\theta|)$ is the area of the hypersphere with radius $r = |\sin\theta|$ in the $(2+\varepsilon)$-dimensional space; its expression is:

$$\Omega_{2+\varepsilon}(r = |\sin\theta|) = 2 \frac{\pi^{1+\varepsilon/2}}{\Gamma(1+\frac{\varepsilon}{2})} \left(|\sin\theta|\right)^{1+\varepsilon}$$

By substituting into eq.(D.8) and recalling eqs.(D.4),(D.7), we get:

$$C_1 = 2\pi J_\beta - \frac{C_0}{2} \left[\Gamma'(1) + \ln 4\pi\right]$$

where:

$$J_\beta = (\beta^2 - 1) \int_0^\pi d(\cos\theta) \, \frac{\ln(|\sin\theta|)}{(1 - \beta\cos\theta)^2} =$$

$$= \ln\left(\frac{E^2}{m^2}\right) \qquad (D.9)$$

where :

$$m^2 = -q^2 \quad , \quad E = q_0 \qquad (D.10)$$

Thus, by substituting into eq.(D.6), we have:

$$I_1(q) = 2\pi\left[\Delta_{IR} - \ln\Lambda^2 + \ln 4\pi^2 + \ln\left(\frac{E^2}{m^2}\right)\right] \qquad (D.11)$$

As for I_2, eq.(D.1) and the definitions:

$$\vec{\beta}_1 = \frac{\vec{q}_1}{q_{10}} \quad , \quad \vec{\beta}_2 = \frac{\vec{q}_2}{q_{20}} \quad , \quad \vec{n} = \frac{\vec{K}}{K_0} \qquad (D.12)$$

allow us to write:

$$I_2 = \int_0^\infty dK_0\, f(K_0)\, K_0^{\varepsilon-1} D_\varepsilon$$

where ,

$$D_\varepsilon = \int \frac{d\Omega_{3+\varepsilon}}{(2\pi)^\varepsilon} \frac{1}{1-\vec{\beta}_1\cdot\vec{n}} \frac{1}{1-\vec{\beta}_2\cdot\vec{n}} \frac{(q_1\cdot q_2)}{q_{10}\, q_{20}} \qquad (D.13)$$

Similarly to the I_1 case we define:

$$D_\varepsilon = D_0 + \varepsilon D_1 + O(\varepsilon^2)$$

and we have:

$$I_2 = \frac{1}{\varepsilon} D_0 + D_0 \ln\Lambda + D_1 \qquad (D.14)$$

Executing the Feynman's shift eq.(D.13) becomes:

$$D_\varepsilon = \int_0^1 dx \int \frac{d\Omega_{3+\varepsilon}}{(2\pi)^\varepsilon} \frac{1}{(1-\vec{v}\cdot\vec{n})^2} \frac{(q_1\cdot q_2)}{q_{10}\, q_{20}}$$

where :

$$\vec{v} = x\vec{\beta}_1 + (1-x)\vec{\beta}_2$$

Then:

$$D_0 = -4\pi \ln\left[\frac{2|(q_1\cdot q_2)|}{m_1 m_2}\right] \tag{D.15}$$

As for D_1, we get:

$$D_1 = 2\pi J - \frac{D_0}{2}\left[\Gamma'(1) + \ln 4\pi - \frac{1}{2}\ln 4\right]$$

where:

$$J = (1-\vec{\beta}_1\cdot\vec{\beta}_2)\int_0^1 dx\, \frac{1}{v}\,\frac{1}{v^2-1}\ln\left(\frac{1-v}{1+v}\right)$$

$$v = |\vec{v}|$$

By comparing with eq.(D.14) and using eq.(D.15) one has:

$$I_2 = 2\pi\left\{\ln\left[\frac{2|(q_1\cdot q_2)|}{m_1 m_2}\right]\left(\Delta_{IR} - \ln\Lambda^2 + \ln 4\pi^2 - \ln 4\right) + J\right\}$$

We have to discuss the expression of the function $J = J(q_1, q_2)$ in the following cases:

a)	$q_{1,2} = p_{1,2}$	or	$q_{1,2} = p_{3,4}$
b)	$q_{1,2} = p_{1,3}$	or	$q_{1,2} = p_{2,4}$
c)	$q_{1,2} = p_{1,4}$	or	$q_{1,2} = p_{2,3}$

In the a) cases we get:

$$J = \frac{1}{2}\ln^2\left(\frac{s}{m^2}\right) + \frac{\pi^2}{3}$$

and then:

$$I_2(p_1, p_2) = 2\pi\left[\ln\left(\frac{s}{m^2}\right)\left(\Delta_{IR} - \ln\Lambda^2 + \ln 4\pi^2 - \ln 4\right) + \frac{1}{2}\ln^2\left(\frac{s}{m^2}\right) + \frac{\pi^2}{3}\right] \tag{D.16}$$

$$I_2(P_3,P_4) = 2\pi\left[\ln\left(\frac{s}{\mu^2}\right)\left(\Delta_{IR} - \ln\Lambda^2 + \ln 4\pi^2 - \ln 4\right) + \frac{1}{2}\ln^2\left(\frac{s}{\mu^2}\right) + \frac{\pi^2}{3}\right] \tag{D.17}$$

In the b) and c) cases one has, respectively:

$$J = J\left(-\frac{u}{s}\right) = \frac{1}{4}\ln^2\left(\frac{s}{m^2}\right) + \frac{1}{4}\ln^2\left(\frac{s}{\mu^2}\right) + \eta\left(-\frac{u}{s}\right)$$

$$J = J\left(-\frac{t}{s}\right) = \frac{1}{4}\ln^2\left(\frac{s}{m^2}\right) + \frac{1}{4}\ln^2\left(\frac{s}{\mu^2}\right) + \eta\left(-\frac{t}{s}\right)$$

where:

$$\eta(x) = \int_{\sqrt{1-x}}^{1+\sqrt{x}} dt\left(2\sqrt{x}\,\frac{1}{t^2+2t+1-x} + \frac{1}{t-1+\sqrt{x}}\right)\cdot\ln\left(-\frac{t^2+2t+1-x}{t^2-2t+1-x}\right) -$$

$$-\frac{1}{2}\ln^2\left[\frac{1+\sqrt{x}-\sqrt{1-x}}{2\left(1+\frac{1}{\sqrt{x}}\right)}\right] - Li_2\left[\frac{1+\sqrt{x}-\sqrt{1-x}}{2(1+\sqrt{x})}\right] +$$

$$+ Li_2\left(\frac{1+\sqrt{x}-\sqrt{1-x}}{2\sqrt{x}}\right) - Li_2\left(\frac{1+\sqrt{x}-\sqrt{1-x}}{2}\right) \quad , \quad \text{for } 0<x<1 .$$

The dilogarithm - function is defined as:

$$Li_2(x) = -\int_0^x \frac{dt}{t}\ln(1-t)$$

Then:

$$I_2(P_1,P_3) = I_2(P_2,P_4) = 2\pi\left[\ln\left(\frac{-u}{m\mu}\right)\left(\Delta_{IR} - \ln\Lambda^2 + \ln 4\pi^2 - \ln 2\right) + J\left(-\frac{u}{s}\right)\right]$$

$$I_2(P_1,P_4) = I_2(P_2,P_3) = 2\pi\left[\ln\left(\frac{-t}{m\mu}\right)\left(\Delta_{IR} - \ln\Lambda^2 + \ln 4\pi^2 - \ln 2\right) + J\left(-\frac{t}{s}\right)\right]$$

In conclusion we get:

$$b = \frac{\alpha}{4\pi^2}\Big[I_1(p_1)+I_1(p_2)+I_1(p_3)+I_1(p_4)-2I_2(p_1,p_2)-2I_2(p_3,p_4)+$$

$$+2I_2(p_1,p_3)+2I_2(p_2,p_4)-2I_2(p_1,p_4)-2I_2(p_2,p_3)\Big]=$$

$$=\frac{2\alpha}{\pi}\Big[\left(\Delta_{IR}-\ln\Lambda^{\varepsilon}+\ln 4\pi^2-\ln 4\right)\left(1-\ln\frac{s}{m\mu}-\ln\frac{t}{u}\right)+$$

$$+\ln\frac{s}{\mu m}-\frac{1}{4}\ln^2\frac{s}{m^2}-\frac{1}{4}\ln^2\frac{s}{\mu^2}-\frac{\pi^2}{3}+\omega\left(-\frac{u}{s}\right)\Big]$$

(D.18)

where:

$$\omega(x)=\int_{\sqrt{1-x}}^{1+\sqrt{x}}dt\left(2\sqrt{x}\,\frac{1}{t^2+2t+1-x}+\frac{1}{t-1+\sqrt{x}}\right)\cdot\ln\left(-\frac{t^2+2t+1-x}{t^2-2t+1-x}\right)-$$

$$-\int_{\sqrt{x}}^{1+\sqrt{1-x}}dt\left(2\sqrt{1-x}\,\frac{1}{t^2+2t+x}+\frac{1}{t-1+\sqrt{1-x}}\right)\cdot\ln\left(-\frac{t^2+2t+x}{t^2-2t+x}\right)-$$

$$-\frac{1}{2}\ln^2\left[\frac{1+\sqrt{x}-\sqrt{1-x}}{2\left(1+\frac{1}{\sqrt{x}}\right)}\right]+\frac{1}{2}\ln^2\left[\frac{1-\sqrt{x}+\sqrt{1-x}}{2\left(1+\frac{1}{\sqrt{1-x}}\right)}\right]-\mathrm{Li}_2\left[\frac{1+\sqrt{x}-\sqrt{1-x}}{2(1+\sqrt{x})}\right]+$$

$$+\mathrm{Li}_2\left(\frac{1+\sqrt{x}-\sqrt{1-x}}{2\sqrt{x}}\right)-\mathrm{Li}_2\left(\frac{1+\sqrt{x}-\sqrt{1-x}}{2}\right)+\mathrm{Li}_2\left[\frac{1-\sqrt{x}+\sqrt{1-x}}{2(1+\sqrt{1-x})}\right]-$$

$$-\mathrm{Li}_2\left(\frac{1-\sqrt{x}+\sqrt{1-x}}{2\sqrt{1-x}}\right)+\mathrm{Li}_2\left(\frac{1-\sqrt{x}+\sqrt{1-x}}{2}\right)$$

(D.19)

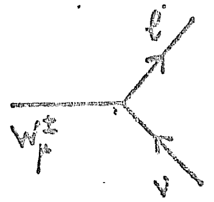
The formulae:

$$\frac{t}{u}=\frac{1-\cos\theta_s}{1+\cos\theta_s}\quad,\quad -\frac{u}{s}=\frac{1}{2}(1+\cos\theta_s)$$

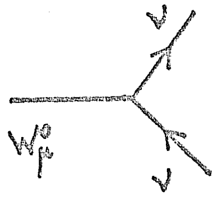
substituted into eq.(D.18) give the explicit dependence on θ_s of the coefficient b.

Appendix E

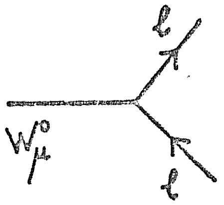
From the lagrangian (2.6) one gets the Feynman rules reported here. In $e^+e^- \to \mu^+\mu^-$ as well as in any process involving no external vector line, the coupling of the Higgs particles H, ϕ°, $\phi^\pm$ to a fermion current is proportional to the fermion mass and is then suppressed by a factor $\frac{m}{M}(\approx 10^{-5})$ for the electron and $\frac{\mu}{M}(\approx 10^{-3})$ for the muon; these couplings are negligible. The vertices relevant for the one-loop computation in $e^+e^- \to \mu^+\mu^-$ are:



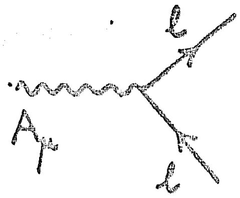
$$\frac{ig}{2\sqrt{2}}\gamma_\mu(1+\gamma_5)$$



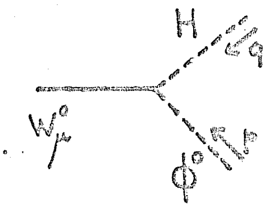
$$\frac{ig}{4c_\theta}\gamma_\mu(1+\gamma_5)$$



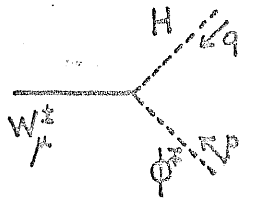
$$\frac{ig}{4c_\theta}\gamma_\mu(v_\theta-\gamma_5)\quad , \text{con}\quad v_\theta = 4s_\theta^2-1$$



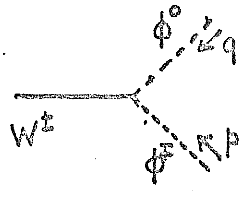
$$-igs_\theta\gamma_\mu$$



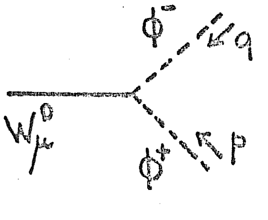
$$\frac{ig}{2c_\theta}(p-q)_\mu$$



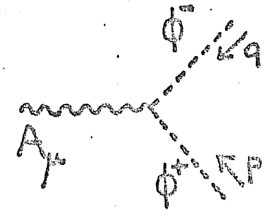
$$\frac{ig}{2}(p-q)_\mu$$



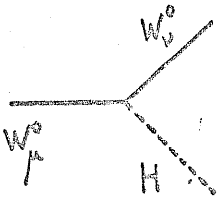
$$\pm\frac{g}{2}(p-q)_\mu$$



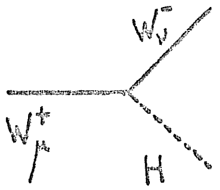
$$g\,\frac{c_\theta^2-s_\theta^2}{2c_\theta}(p-q)_\mu$$



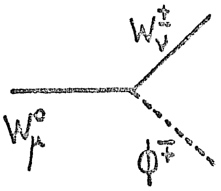
$g s_\theta (p-q)_\mu$



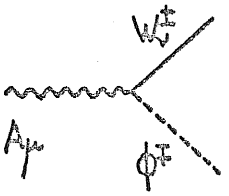
$-g \frac{M_0}{2c_\theta} \delta_{\mu\nu} \cdot (2!) = -g \frac{M_0}{c_\theta} \delta_{\mu\nu}$



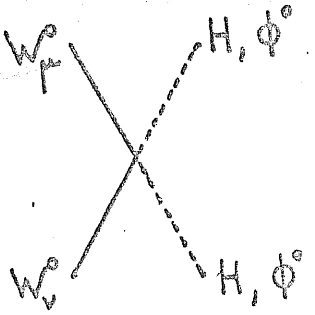
$-gM \delta_{\mu\nu}$



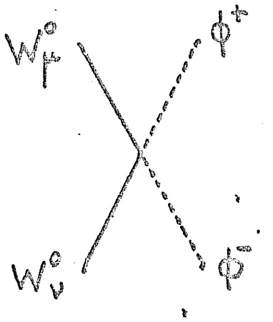
$\mp i g s_\theta^2 M_0 \delta_{\mu\nu}$



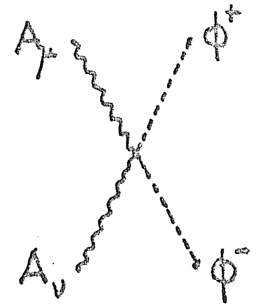
$\pm i g s_\theta M \delta_{\mu\nu}$



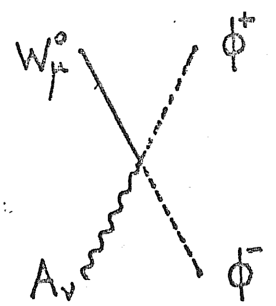
$$-\frac{g^2}{8c_\theta^2}\delta_{\mu\nu}\cdot(2!)\cdot(2!) = -\frac{g^2}{2c_\theta^2}\delta_{\mu\nu}$$



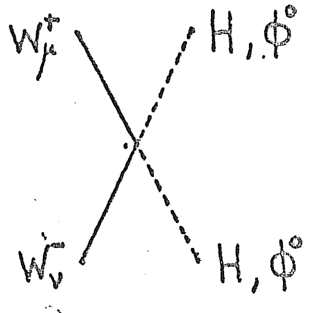
$$-\frac{g^2}{4c_\theta^2}(s_\theta^2-c_\theta^2)^2\delta_{\mu\nu}\cdot(2!) = -\frac{g^2}{2c_\theta^2}(s_\theta^2-c_\theta^2)^2\cdot\delta_{\mu\nu}$$



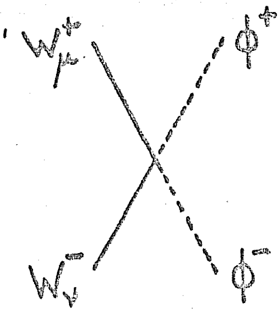
$$-g^2 s_\theta^2\delta_{\mu\nu}\cdot(2!) = -2g^2 s_\theta^2\delta_{\mu\nu}$$



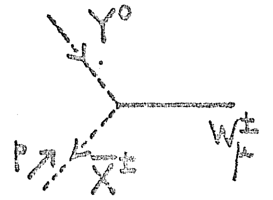
$$g^2\frac{s_\theta}{c_\theta}(s_\theta^2-c_\theta^2)\delta_{\mu\nu}$$



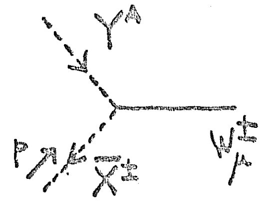
$$-\frac{g^2}{4}\delta_{\mu\nu}\cdot(2!) = -\frac{g^2}{2}\delta_{\mu\nu}$$



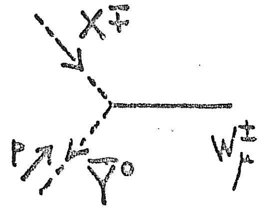
$$-\frac{g^2}{2}\,\delta_{\mu\nu}$$



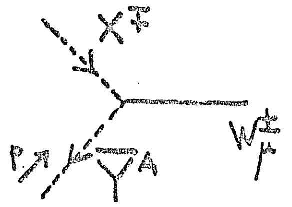
$$\pm g c_\theta p_\mu$$



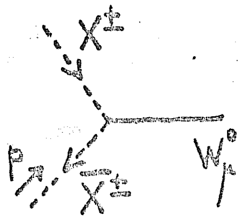
$$\pm g s_\theta p_\mu$$



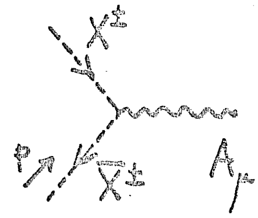
$$\mp g c_\theta p_\mu$$



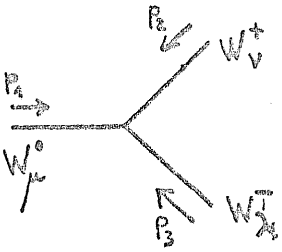
$$\mp g s_\theta p_\mu$$



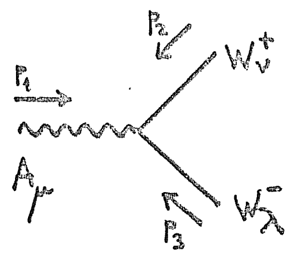
$$\mp g c_\theta p_\mu$$



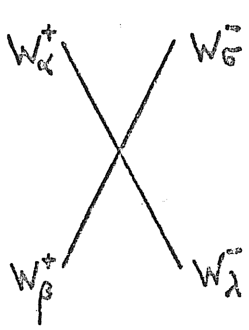
$$\mp g s_\theta p_\mu$$



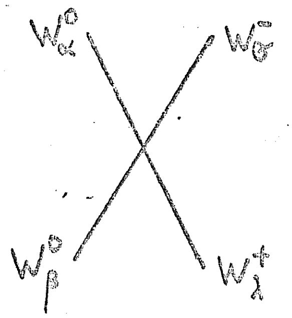
$$-g c_\theta \left[\delta_{\mu\lambda}(p_1-p_3)_\nu + \delta_{\lambda\nu}(p_3-p_2)_\mu + \delta_{\mu\nu}(p_2-p_1)_\lambda \right]$$



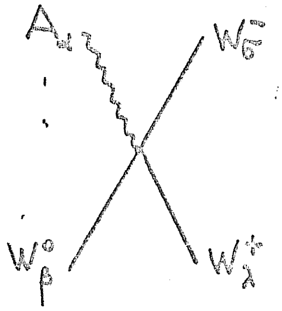
$$-g s_\theta \left[\delta_{\mu\lambda}(p_1-p_3)_\nu + \delta_{\lambda\nu}(p_3-p_2)_\mu + \delta_{\mu\nu}(p_2-p_1)_\lambda \right]$$



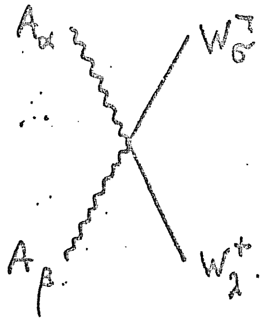
$$\frac{g^2}{2}(\delta_{\alpha\beta}\delta_{\lambda\sigma} - \delta_{\alpha\sigma}\delta_{\beta\lambda})\cdot(2!)^2 = 2g^2(\delta_{\alpha\beta}\delta_{\lambda\sigma} - \delta_{\alpha\sigma}\delta_{\beta\lambda}).$$



$$-g^2 c_\theta^2(\delta_{\alpha\beta}\delta_{\lambda\sigma}-\delta_{\alpha\sigma}\delta_{\beta\lambda})\cdot(2!) = -2g^2 c_\theta^2(\delta_{\alpha\beta}\delta_{\lambda\sigma}-\delta_{\alpha\sigma}\delta_{\beta\lambda})$$



$$-g^2 s_\theta c_\theta(2\,\delta_{\alpha\beta}\,\delta_{\lambda\sigma}-\delta_{\alpha\sigma}\,\delta_{\beta\lambda}-\delta_{\alpha\lambda}\,\delta_{\beta\sigma})$$

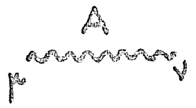


$$-g^2 s_\theta^2(\delta_{\alpha\beta}\delta_{\lambda\sigma}-\delta_{\alpha\sigma}\delta_{\beta\lambda})\cdot(2!) = -2g^2 s_\theta^2(\delta_{\alpha\beta}\delta_{\lambda\sigma}-\delta_{\alpha\sigma}\delta_{\beta\lambda})$$

A factor $(2\pi)^n i$ must be introduced for each vertex. The combinatory factors have been explicitly indicated. It is necessary to associate a minus sign to each Faddeev-Popov ghost loop.

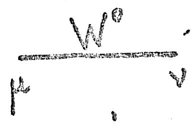
Propagators:

photon:



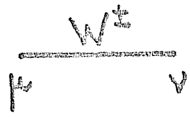
$$\frac{\delta_{\mu\nu}}{p^2-i\varepsilon}$$

neutral vector boson:



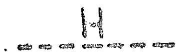
$$\frac{\delta_{\mu\nu}}{p^2+M_0^2-i\varepsilon}$$

charged vector bosons:



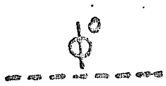
$$\frac{\delta_{\mu\nu}}{p^2+M^2-i\varepsilon}$$

Higgs particle:



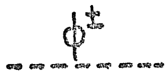
$$\frac{1}{p^2+M_H^2-i\varepsilon}$$

neutral would be Goldstone boson:



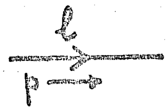
$$\frac{1}{p^2+M_0^2-i\varepsilon}$$

charged would be Goldstone bosons:



$$\frac{1}{p^2+M^2-i\varepsilon}$$

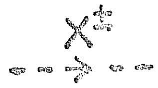
lepton:



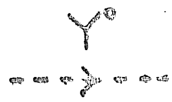
$$\frac{-i\hat{p}+m_\ell}{p^2+m_\ell^2-i\varepsilon}$$

where m_ℓ is the lepton mass ($m_\nu=0$, $m_e= m$, $m_\mu = \mu$);

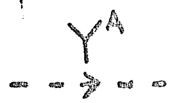
Faddeev-Popov ghosts:



$$\frac{1}{p^2+M^2-i\varepsilon}$$



$$\frac{1}{p^2+M_0^2-i\varepsilon}$$



$$\frac{1}{p^2-i\varepsilon}$$

A factor $\frac{1}{(2\pi)^n i}$ for each propagator is needed.

Appendix F

In this appendix we evaluate a particular class of non-logarithmically enhanced corrections to the effective hamiltonian, eq.(3.I), namely those corrections which do not vanish in the limit:

$$\alpha \to 0, \quad \sin^2\vartheta \to 0, \quad \alpha_W = \frac{\alpha}{\sin^2\vartheta} \text{ fixed,} \qquad (F.I)$$

and neglecting strong interactions. In this limit, the Salam-Weinberg theory simplifies considerably, in that:

i) the photon coincides with the U(I) gauge boson, and it decouples from the other particles;

ii) neglecting Higgs boson couplings to the fermions the theory has an exact, global SU(2) symmetry;

iii) the Z-boson coincides with W_3, its mass being degenerate with the W mass.

Under these conditions the zeroth-order low energy weak hamiltonian takes the form:

$$H = \frac{1}{\sqrt{2}} G^0 \vec{J}_\mu \cdot \vec{J}_\mu \qquad (F.2)$$

where G^0 is the bare Fermi constant and

$$\vec{J}_\mu = \sum_f \bar{f}_L \gamma_\mu \vec{\tau} f_L$$

(L=left-handed).

First-order corrections to eq.(I.2) arise from W-exchange diagrams and, because of the SU(2) symmetry, have the form:

$$\delta H = \frac{1}{\sqrt{2}} G^0 [\varepsilon_1 \vec{J}_\mu \cdot \vec{J}_\mu + \varepsilon_2 J_\mu^{(0)} J_\mu^{(0)} \qquad (F.3)$$

where we have introduced the isosinglet current:

$$J_\mu^{(0)} = \sum_f \bar{f}_L \gamma_\mu f_L$$

ε_1 has both divergent and finite contributions which, however, need not to be computed since they can be entirely absorbed into the definition of the renormalized Fermi coupling:

$$G = G^0 (1 + \varepsilon_1)$$

ε_2 is finite and, in fact, it receives contributions from box diagrams only, where two massive W's are exchanged between the fermion lines. A simple computation gives:

$$\varepsilon_2 = (\varepsilon_2)_{box} = -\frac{9}{16}\frac{\alpha_W}{\pi} \qquad (F.4)$$

We note that the correction ε_2 is well defined, provided one uses the Fermi constant G as one of the parameters which define the theory. Furthermore, since G, as taken from μ-decay, is unaffected by corrections of order $\alpha \ln M^2$, the use of G agrees with our philosophy of defining the parameters of the theory from amplitudes at a mass-scale M.

Eqs.(F.3), (F.4) give for the coefficients in eq.(3.I) the corrections:

$$\delta V_f = \delta A_f = -\delta C_f = \frac{9}{16\pi}\alpha_W = 0.006 \qquad (F.5)$$

for $\sin^2\vartheta = 0.22$. These corrections have to be added to the last three columns in table 2.

We note:

i) the corrections (F.5) are of the same order as the leading logarithmic corrections δ_b, δ_c; their inclusion increases the r.h.s. of eq.(I.I) to 0.IO, eq.(I.2);

ii) as discussed in section 4, the effect of (F.5) on the determination of $\sin^2\vartheta$ is small;

iii) since $\alpha_W \approx 5\alpha$, the corrections (F.5) are expected to give the largest part of the entire first order correction to eq.(3.I) (with the exception of the mass-singular terms).

Footnotes

(1) : The pure QED order α corrections are not given in ref. [6] ; for this part of the corrections one can see ref. [7].

(2) : We neglect, here and in the following, parity-violating terms $\gamma^\mu \otimes \gamma^\mu \gamma_5$ because they do not contrubute to the asymmetry (see section 5).

Table Captions

Table I: Leading logarithmic corrections to the coefficients of the effective hamiltonian from the diagrams of figg. Ib,c in absence of strong interactions. The entries of the table have to be multiplied by $\frac{\alpha}{\pi} \ln \frac{M^{*2}}{\mu^2}$. Q is the electric charge of the quark, $\tau_3 = +1, -1$ and $\tilde{Q} = -\frac{1}{3}, +\frac{2}{3}$ for u and d, respectively; $s^2 = \sin^2\vartheta(M^*)$.

Table 2: Uncorrected values of V_q and A_q and leading logarithmic corrections, without (third column) and with (fourth and fifth columns) strong interaction effects. A bar indicates a correction of absolute value less than 10^{-3}.

Figure Captions

fig. I: Examples of diagrams contributing to the leading e.m. corrections. The wavy line represents a photon.

fig. 2: The diagrammatic representation of the process $e^+e^- \to \mu^+\mu^-$.

fig. 3: The diagrams contributing to the $e^+e^- \to \mu^+\mu^-$ cross section, at the tree level.

fig. 4: The one-loop diagrams contributing to the corrections δ_1 (eq.(5.II)) and δ_2 (eq.(5.I2)). They represent the sum of the graphs listed in appendix C.

fig. 5: Bremsstrahlung of a soft photon by an external line for the process of fig. I.

fig. 6: Plot of d_1 given by eq.(5.4I) as a function of the scattering angle, fixing the values of the parameters as indicated in section 5.

fig. 7: Plot of $d_2(\vartheta_s)$ given by eq.(5.40).

fig. 8: A as a function of ϑ_s as stated in eq.(5.I8).

fig. 9: Behaviour of the genuine weak correction $d_1^{WE}(\vartheta_s)$ given by eq.(5.42).

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TABLE 1

	δ_b	δ_c
V_μ	$\frac{1}{3}(1-4s^2)$	$\frac{1}{2}(1-4s^2)$
V_q	$-\frac{1}{3}Q(1-4s^2)$	$\frac{1}{2}Q\tau_3(1-4s^2)$
A_μ	$\frac{1}{3}(1-4s^2)$	$\frac{1}{2}(1-4s^2)$
A_q	$-\frac{1}{3}Q(1-4s^2Q\tau_3)-\frac{2}{3\rho}\bar{Q}$	$\frac{1}{2}Q(\tau_3-4Qs^2)$
C_μ	0	$-\frac{1}{2}(1-4s^2)^2$
C_q	0	$-\frac{1}{2}Q(\tau_3-4s^2Q)(1-4s^2)$

TABLE 2

	Uncorrected value ($\sin^2\theta(M)=0.220$)	$\sin^2\theta$ renormalization	Others		
			$\alpha_s=0$	4 flav.	6 flav.
V_u	0.413	−0.018	0.002	0.002	0.002
V_d	−0.707	0.009	—	—	—
A_u	0.120	−0.027	0.010	0.010	0.010
A_d	−0.120	0.027	—	0.003	0.004

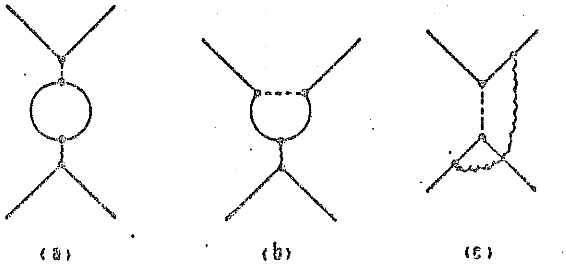

Fig. 1

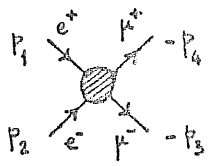

Fig. 2

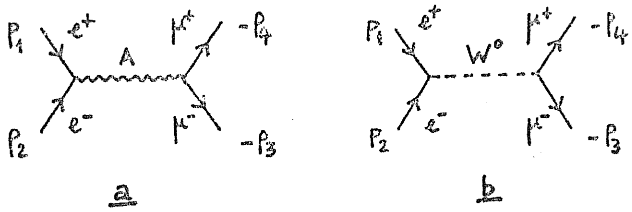

Fig. 3

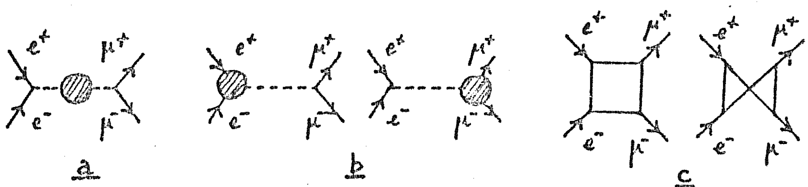

Fig. 4

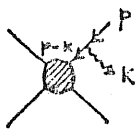

Fig. 5

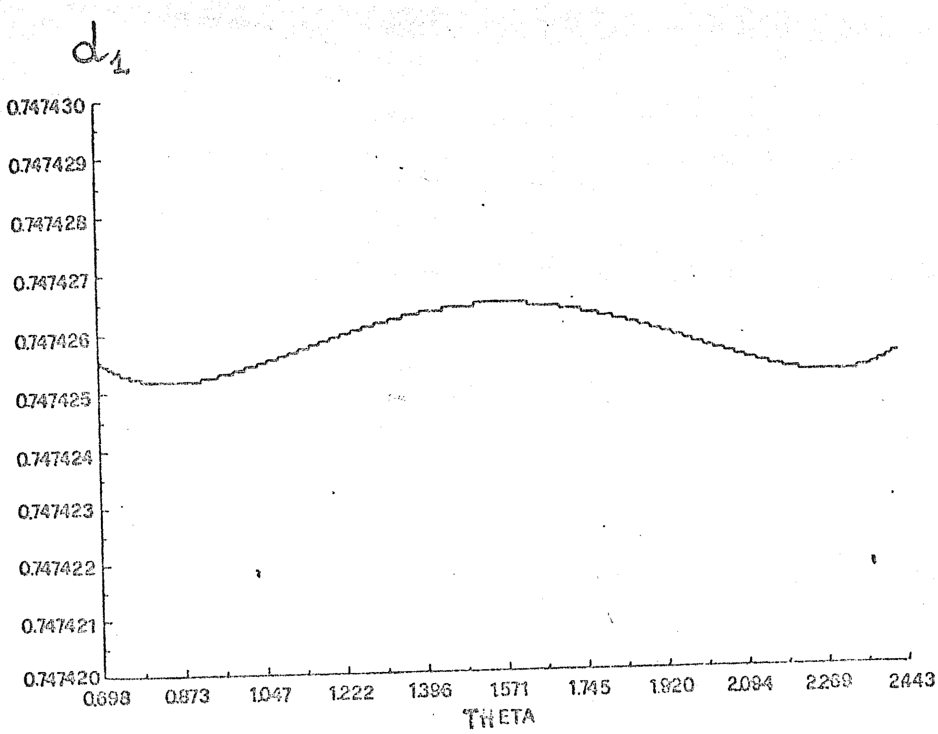
d1
0.747430
0.747429
0.747428
0.747427
0.747426
0.747425
0.747424
0.747423
0.747422
0.747421
0.747420
0.698
0.873
1.047
1.222
1.396
1.571
1.745
1.920
2.094
2.269
2.443
THETA

Fig. 6

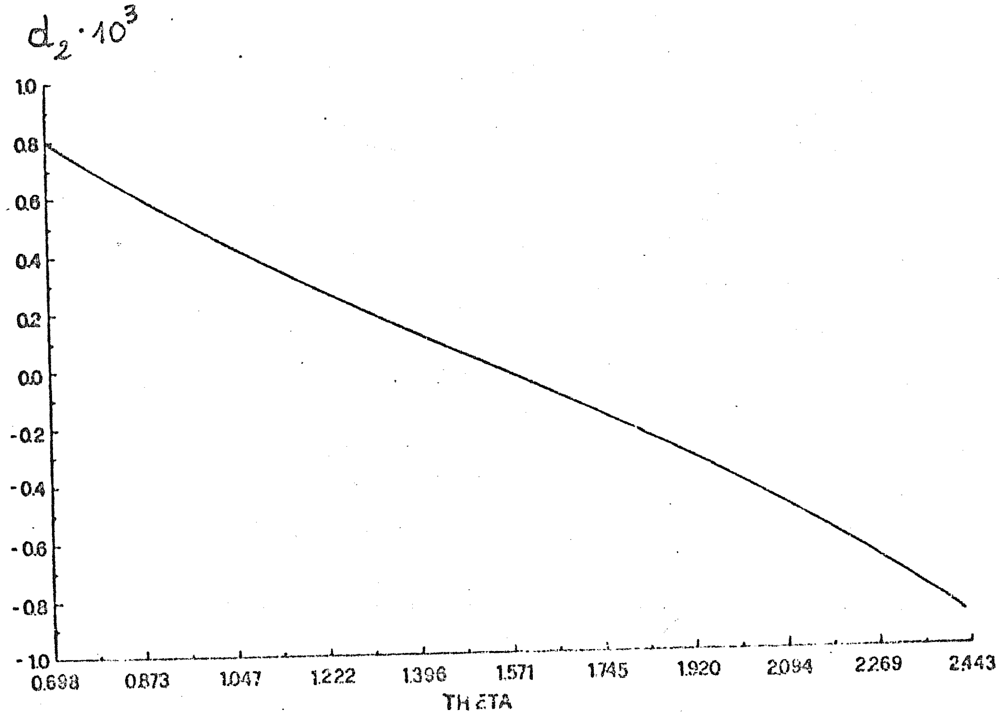
d2·10^3
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0.698
0.873
1.047
1.222
1.396
1.571
1.745
1.920
2.094
2.269
2.443
THETA

Fig.7

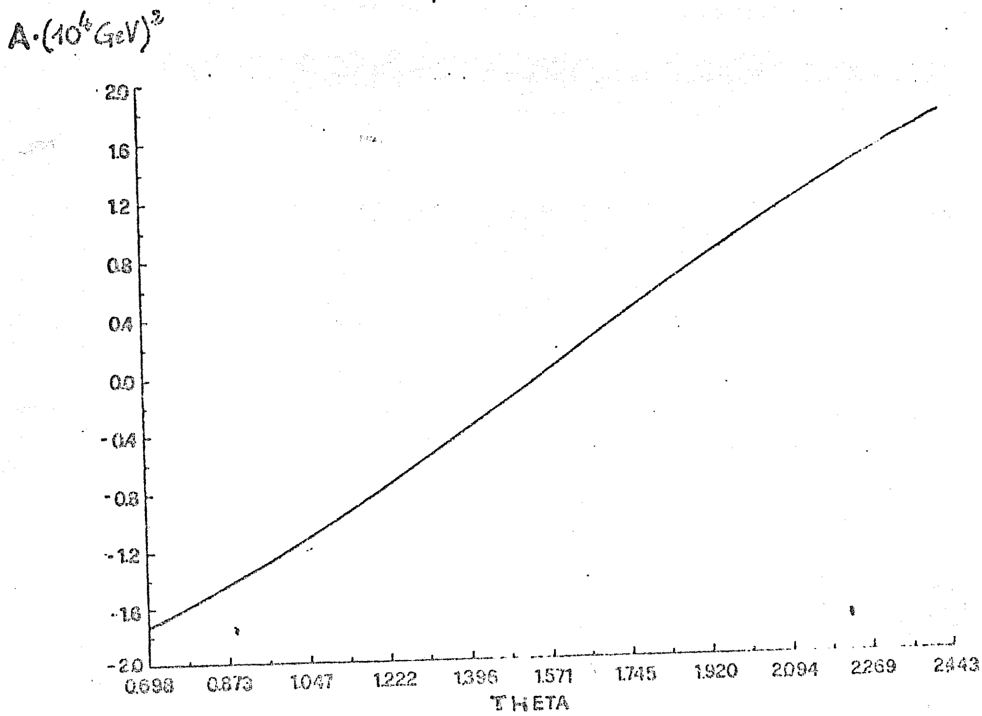
A·(10^4 GeV)^2
2.0
1.6
1.2
0.8
0.4
0.0
-0.4
-0.8
-1.2
-1.6
-2.0
0.698
0.873
1.047
1.222
1.396
1.571
1.745
1.920
2.094
2.269
2.443
THETA

Fig. 8

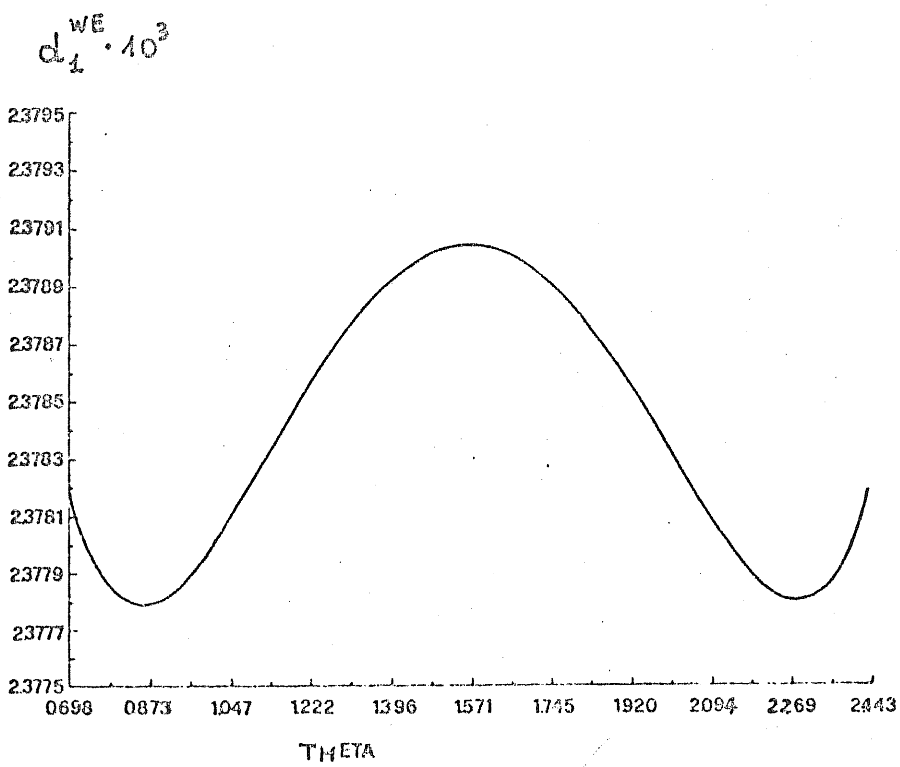
d_1^WE · 10^3
2.3795
2.3793
2.3791
2.3789
2.3787
2.3785
2.3783
2.3781
2.3779
2.3777
2.3775
0.698
0.873
1.047
1.222
1.396
1.571
1.745
1.920
2.094
2.269
2.443
THETA

Fig. 9