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BUBBLE GROWTH IN AN EXPANDING UNIVERSE

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Candidate

Ornella Pantano

Supervisor

Prof S A Bonometto

TRIESTE

SISSA - SCUOLA
INTERNAZIONALE
SUPERIORE
DI STUDI AVANZATI
TRIESTE
Strada Costiera 11

CONTENTS

PREFACE

This thesis is concerned with the growth of a bubble in a first order phase transition in the early universe. We are interested, in particular, in a possible first order phase transition connected with the change of properties of strong interacting matter at energy densities higher than nuclear density. The relativistic hydrodynamical equation of a spherical symmetric fluid and the Gauss-Codazzi equations for a singular hypersurface are presented for describing the growth of a stable hadron bubble in an expanding background where the strong interacting matter is in super-cooled quark-gluon plasma state. The characteristic equations of motion for a relativistic fluid are also presented in this thesis.

The first chapter is a general introduction to the scenarios connected with a quark-hadron phase transition in the early universe. In the second chapter our present knowledge of confinement- deconfinement transition and the characteristics of a first order phase transition are reviewed. The third chapter is concerned with the relativistic hydrodynamic equations for a spherical symmetric fluid. The discontinuity in the fluid properties due to the transition are described using the Israel method for singular hypersurfaces. In the fourth chapter, the characteristic forms of the equations of motion of a relativistic fluid are also derived. They will be used in a successive numerical analysis for solving the junction conditions across the transition discontinuity.

ACKNOWLEDGEMENTS

I would like to thank my supervisor Prof. S.A. Bonometto, for introducing me to cosmological aspects of phase transitions and for his encouragement.

I am also grateful to Dr. J.C. Miller for his contribution and guidance to the hydrodynamical part of the work described here.

I also wish to thank all my colleagues who helped me in the preparation of this thesis.

NOTATION

The metric signature is $- + + +$.

Latin indices run over the four coordinate labels.

Repeated indices are summed.

The natural units $h = c = k = 1$ are used.

CHAPTER ONE

INTRODUCTION

In recent years, Quantum Chromodynamics (QCD) has become the leading theory for describing the strong interacting particles called hadrons. The idea of hadrons as composite particles, whose constituents are called quarks, was introduced in the early sixties as an attempt to classify the rich hadronic spectrum obtained in the experiments. The theory of QCD as a dynamical theory of the interaction of quarks was developed by Fritzch and Gell-Mann(1972).

QCD is able to explain two important characteristics of the particles forming hadrons: the observed absence of free quarks in nature (confinement); and the behaviour of quarks inside nucleons (asymptotic freedom). Deep inelastic scattering of electrons by protons shows, in fact, that protons behaves as if their constituents move independently of one another.

The asymptotic freedom property of QCD, i.e. the fact that at short distances the effective coupling strength goes to zero, has an important consequence on the state of strong interacting matter at high densities and temperatures: detailed analytic calculations show that at high energy densities strong interacting matter behave like a free gas of quarks and gluons (Kapusta 1979, Kalashnikov 1984).

At energy densities far below nuclear density strong interacting matter is composed of widely separated hadrons and, since the range of strong interactions is finite, these hadrons, on the average, interact weakly. Hadronic matter becomes an ideal hadron gas at these low energy densities.

The natural scale at which we expect a drastic change in the behaviour of strong interacting matter can be obtained on the basis of simple considerations. Because of their composite character, hadrons have a finite volume and at an energy density of the order of nuclear matter, they begin to overlap. Then quarks are no longer confined to individual hadrons and they may move through the system. At this point the properties of hadronic matter change drastically and a true phase change is not excluded. The energy density corrisponding to this possible phase change should be of the order of that inside a proton $e \simeq M/(4 r^3/3) \geq 500$ MeV fm^{-3} where we have used the value 1 Gev for the mass M of proton and 0.8 fm for the radius r corrisponding to the r. m.s. charge radius of the proton. This energy density is only few times that of nuclear matter ($e_{NM} \simeq 150$ MeV fm^{-3}).

Phase transition are often associated with a spontaneous breaking or realization of symmetry. In the confinement-deconfinement case the phase transition seems connected with the restoration at high density of the chiral symmetry (Fucito et al.1985, Gavai et al.1984). This is a symmetry of QCD interactions for massless up and down quarks (Shuryak 1981, Pisarski 1982). Numerical calculations show, in fact, a rapid change in the thermodynamical quantities at the same temperature at which chiral symmetry is restored (Polonyi et al.1984, Celik et al.1985 , McLerran 1985).

Different methods have been used until now in the numerical analysis, but because of computational difficulties, a complete answer is not yet available. It is likely that the transition is first order (Fucito et.al. 1984, Celik et al. 1985) as has been demonstrated for a Yang-Mills system, i.e. a system of only gluons (Borgs-Seiler

1983, Kogut et al. 1985, Celik et al. 1983). All numerical results agree on a rapid change in the energy density at a temperature T_c between 100 - 400 MeV. A definitive answer on the character of the transition is hoped not only for a complete understanding of the QCD theory but also for the description of several physically interesting situations dealing with very high density matter. Ultrarelativistic heavy ion collisions ($e \simeq 10 - 100\ e_{NM}$, for a review, see Cleymans et al. 1985) and cores of neutron stars are two widely studied systems where the effect of deconfinement is likely to be observed. According to the big bang theory, the early universe is another possible context where we can study the effects of a transition from a quark-gluon plasma to a gas of hadrons.

At a sufficiently early stage of the history of the universe ($T \gg 200$ MeV) the strong interacting matter is in the quark-gluon plasma state. As the temperature decreases the colour forces become more and more important and below some critical temperature T_c, the hadron phase becomes energetically favoured. In the case of a first order phase transition, we expect some supercooling, i.e. if the system evolves through equilibrium states, it does not change phase immediately at T_c, but it continue to stay in a metastable quark-gluon phase. The degree of supercooling depends on the mechanism causing the formation of seeds of the new hadron phase in the super-cooled plasma (DeGrand-Kajantie 1984, Van Hove 1985). First order phase transitions are usually mediated by impurities. In the early universe, however, if the hypothesis of homogeneity is well satisfied, it is most likely that hadron bubbles are nucleated by thermal fluctuations (Laudau-Lifshitz 1980). Moreover, because of the creation of an interphase region, only nuclei whose size is above a definite value $r_{cr}(T)$ can survive. Bubbles of critical radius r_{cr} are formed in equilibrium with the surrounding medium and, as the temperature further decreases, they begin to grow. The length of the transition and its effects are related to the way in which bubble nucleation and growth operates.

The purpose in studying the details of the cosmological quark-hadron transition is to see which kinds of consequences it can have on later evolution of the universe (Bonometto-Masiero 1985). First analyses, made by Olive (1981) and Suhonen (1982), were based on a two phase model and discussed the temperature of the transition and its length. The existence of a long "plateau" of temperature was then showed using QCD results on lattice (Bonometto-Pantano 1984, Bonometto-Sakellariadou 1984). The case of a first order phase transition without supercooling has been studied in several papers: Källman(1982), Kâmpfer-Schulz (1984), Bonometto-Matarrese (1983), Lodenquai-Dixit (1983). The creation of black holes or very collapsed structures due to density fluctuations or turbolence produced by the transition,was first examined by Crawford-Shramm(1982)(Shramm-Olive 1984).

Recently several authors have analysed another possible effect suggested by Witten (1984),i.e., the formation of nuggets of stable quark matter or, at least, the concentration of baryon in the region where, at the end of the transition, strong interacting matter is again in the quark phase. If quark nuggets survive until now, they can solve completely the dark matter problem in the present universe. It seems, however, that a simple baryon concentration is more likely (Bonometto et al.1985, Applegate-Hogan 1985, Iso et al. 1985, Alcock-Farhi 1985). If inhomogeneities persist until the nucleosynthesis, they can cause distorsions in cosmic elements aboundances (Madsen-Risager 1985), and also the present limit on baryonic matter density need to be rediscussed (Yang et al. 1984). It has been also proposed (Iso et al. 1985) that these concentrated baryon clouds may, eventually, form invisible stellar objects with planetary masses after recombination. The baryon concentration effect was obtained comparing the equilibrium density of baryon number in the two phases assuming $T \simeq T_c$ and equal chemical potential (thermodynamical equilibrium).

An hydrodynamical description of a first order quark-hadron transition was first made by Gyulassy et al. (1984) where two classes of processes were studied in 1+1 dimensions. They discussed separately the case in which the velocity of the transition front is supersonic (detonation front) or subsonic (deflagration front) respect to the medium ahead. In a successive paper Kurki-Suonio (1984) extended the analysis to a 3+1 dimensional deflagration process using similarity solution. In both these papers the transition is studied in a Minkowski space-time and surface effects related to the transition layer are neglected. The analysis of Gyulassi et al. shows that detonation requires significantly more supercooling than deflagration and therefore a subsonic front is more likely in a quark-hadron transition.

In order to consider also surface effects and expansion of the universe Maeda (1985) has applied Israel(1966)'s singular hypersurface method for deriving the equation of motion of a spherical bubble in an expanding universe. Using the hypothesis that the space is uniform inside and outside the shell the description of the system appears simplified. The equation of motion of the shell can be written explicitly.

In this thesis the complete set of hydrodynamic and junction condition equations governing the motion of a relativistic spherically symmetric fluid appearing in two different phases will be presented. These phases are separated by a layer of negligible thickness where a first order phase transition is transforming one phase into the other.

For the transition layer Israel's method is used and the junction conditions are derived in the limit in which surface tension is equal to the surface energy density. The equation are formulated in a comoving (Lagragian) coordinate system.

In order to solve the junction conditions across the time-like hypersuface describing the evolution of the transition layer, we must know the fluid variables just behind and ahead of the discontinuity front. In the finite difference method, that we will use for integrate numerically our hydrodynamic equations, the fluid variables are known only at some fixed points that do not, usually, coincide with the position membrane. For this purpose we derive the relativistic hydrodynamic equations also in the "characteristic form" This formulation allows to compute in a very elegant and accurate way the fluid variables near the transition hypersurface.

In the first analysis, we will use the bag model for describing the quark-gluon phase and we will consider the hadrons as an ideal gas of massless pions. The baryon number will be assumed to be zero. Notwithstanding the simplification of the equation of state, this model has the basic physical characteristics of a first order quark-hadron transition and we should be able to derive some important indications on the bubble growth velocity and on the possible formation of shocks. In a successive step we will modify the equation of state in order to introduce perturbative expansion in the quark phase and a wider spectrum with finite volume corrections in the hadron phase. Finally, a finite baryon number density bill also be introduced in order to see whether and in which way possible inhomogeneities in the baryon number are formed.

CHAPTER TWO

QCD AND CHARACTER OF CONFINEMENT-DECONFINEMENT TRANSITION

The present knwoledge of confinement-deconfinement transition and the properties of a first order phase transition are reviewed briefly in this chapter.

After an introduction on the fundamental thermodynamic relations for a many particles system, the transition temperature and the order of the transition are discussed mainly refering to the recent results on lattice.

Subsequently, two models are presented for describing the quark and the hadron phase near the transition point.

Finally, a general introduction on first order phase transitions and nucleation processes is presented.

2.1 THERMODYNAMICAL RELATIONS

The thermodynamical properties of a system governed by an Hamiltonian $\hat{H}$ may be derived from the knowledge of the partition function

$$Z = \mathrm{Tr}\; e^{-\beta[\hat{H} - \vec{\mu}\cdot\vec{\hat{N}}]} \tag{1}$$

where $\beta=1/T$ is the inverse of the temperature and μ_i is the chemical potential conjugate to the charge N_i. Knowing the partition function the mean value of the observable O is given by

(2) $$\langle O \rangle = \frac{1}{Z} \, \mathrm{Tr} \, \hat{O} \, e^{-\beta(\hat{H} - \vec{\mu}\cdot\hat{\vec{N}})}$$

A well known result of statistical mechanics is the relation between the thermodynamical potential and the partition function is

(3) $$\Omega = -T \ln Z$$

From eq. (1) and (3) we obtain the average number density associated with a conserved charge (for example the total baryon number), differentiating with respect to the conjugate chemical potential μ_i

(4) $$n_i = \frac{\langle N_i \rangle}{V} = -\frac{1}{V} \frac{\partial \Omega}{\partial \mu_i}$$

The average energy density is similarly given by

(5) $$e = \frac{\langle H \rangle}{V} = \frac{1}{V} \left\{ 1 - T \frac{\partial}{\partial T} - \vec{\mu} \frac{\partial}{\partial \vec{\mu}} \right\} \Omega$$

The same expressions could be obtained from the thermodynamical relations :

(7) $$\Omega(T, V, \mu) = E - TS - \sum_i \mu_i N_i = -PV$$

(8) $$d\Omega = -S\,dT - P\,dV - \Sigma_i N_i \, d\mu_i$$

where E is the energy, S the entropy and P the pressure. We have also:

(7) $$P = -\frac{T}{V} \ln Z$$

In the case of a free gas of particles and antiparticles with mass m, chemical potential μ (consider for simplicity only one kind of conserved charge) and degeneracy factor g in a large volume limit, the partition function is

(9) $$\ln Z(T,\mu,V) = \frac{gV}{6\pi^2 T} \int_0^\infty \frac{dk\, k^4}{(k^2+m^2)^{1/2}} \left[\frac{1}{e^{\beta[(k^2+m^2)^{1/2}-\mu]}+\eta} + \frac{1}{e^{\beta[(k^2+m^2)^{1/2}+\mu]}+\eta} \right]$$

with η =+1 for fermions and η=-1 for bosons.
For massless fermions we obtain

(10) $$(T \ln Z)_f = \frac{g_f V}{24} \left(\frac{7}{30} \pi^2 T^4 + \mu^2 T^2 + \frac{1}{2\pi^2} \mu^4 \right)$$

and for massless bosons with zero chemical potential

(11) $$(T \ln Z)_b = \frac{g_b V}{90} \pi^2 T^4$$

2.2 QUANTUM CHROMODYNAMICS

QCD is a non-abelian gauge theory describing the strong interacting elementary particles known as quarks. The whole spectrum of hadrons , in fact, can be explained introducing several "flavours" of quarks: at present there is experimental evidence of six distinct flavour : up, down, strange, charme, bottom and top. The strong interaction between quarks, however, is concerned with another attribute namely, their colour properties. Each quark is characterised by a colour index α, with α=1,2,3 (the three colour states are also called red, blue and green). If we think of the colour as forming a three-dimensional abstract space, QCD theory is constructing requiring that gauge transformations be invariant under SU(3) transformations in this space.

The Lagragian of QCD is

$$(1)\qquad \mathcal{L} = \Sigma_f \bar{\psi}_{Af}\left[\gamma^i\left(i\,\delta_{AB}\,\partial_i + \frac{1}{2} g\, \tau^\alpha_{AB}\, A_i^\alpha\right) - m_f\,\delta_{AB}\right]\psi_{Bf} - \frac{1}{4} F_{ij}^{\alpha} F^{ij}_{\alpha}$$

where ψ_{Af} is the quark field, f refers to the flavour and A to the colour index. A_i^α are the gluon fiels, with α= 1.. 8, mediating the strong interactions between quarks, g is the coupling constant, m_g the quark mass and

$$(2)\qquad F_{ij}^{\alpha} = \partial_i A_j^\alpha - \partial_j A_i^\alpha + g\, f_{\alpha\beta\gamma}\, A_i^\beta A_j^\gamma$$

Let τ^α be the eight generators of the SU(3) colour gauge group normalized so that $\mathrm{tr}\ \tau^\alpha \tau^\beta = 2\,\delta^{\alpha\beta}$, then $f_{\alpha\beta\gamma}$ are the structure constants of SU(3), defined by $[\tau^\alpha, \tau^\beta] = 2i\, f_{\alpha\beta\gamma}\, \tau^\gamma$

A term $-\mu\, \Sigma_f \bar{\psi}_f \gamma_0 \psi_f$ must be added to (1) in case of baryon number different from zero. Once the Lagrangian is given, the analysis of the thermodynamical properties of QCD at finite temperature is, at least in principle, a well defined problem.

The partition function in the path integration formulation is given by

$$(3)\qquad Z = \int [d\bar{\psi}][d\psi][dA_i]\; e^{+S}$$

where [ ] means the sum over all the possible configurations of the system and S is the euclidean QCD action.

(4) $$S = \int_0^{\beta} d\tau \int d^3x \; \mathcal{L}_E \left(\psi, \bar{\psi}, A_i \right)$$

where $\mathcal{L}_E$ is the euclidean Lagrangian density obtained from (1) by a Wick rotation in the complex plane t such that τ =it. The fields have to satisfy the periodicity conditions

$$A_i(\vec{x},0) = A_i(\vec{x},\beta)$$

$$\psi(\vec{x},0) = -\psi(\vec{x},\beta) \quad , \quad \bar{\psi}(\vec{x},0) = \bar{\psi}(\vec{x},\beta)$$

These conditions allow us to interpret ß as the inverse of the temperature. Then, all the thermodynamical quantities are derived from (3) using the relations we have seen in section 1.

a) Perturbative QCD

In practise the evaluation of Ω is not very simple. Perturbation theory involving weak coupling expansion associated with renormalization group method can be used with success only at high temperature and density when the effect of particle interactions are small by Kalashnikov-Klimov (1979) and Kalashnikov (1984),(see also Toimela 1984), in the lowest order of pertubation theory are

(5) $\frac{\mu}{T} < 1$ $$P = -\frac{\Omega}{V} = \frac{8}{45}\pi^2 T^4 + N_f\left[\frac{1}{2}T^2\mu^2 + \frac{7}{60}\pi^2 T^4\right]$$
$$+ 3\pi^2 \alpha_{eff}(T)\left[\frac{1}{6}T^4 + N_f\left(\frac{5}{72}T^4 + \frac{T^2\mu^2}{4\pi^2}\right)\right]$$

with $$\alpha_{eff}(T) = \frac{8}{(33-2N_f)\ln T/\Lambda}$$

where Λ is the QCD scale parameter, N_f is the number of quark flavours. For $\Lambda \approx 120$ MeV and $N_f = 3$, the critical temperature T_0 at which the pressure becomes negative is about 240 MeV.
In the high density region it results

$$(6) \quad T/\mu \qquad P = N_f \left[\mu^4/4\pi^2 + \frac{1}{2}\mu^2 T^2 \right]\left(1 - \frac{3}{2}\alpha_{eff}(\mu)\right)$$

with $$\alpha_{eff}(\mu) = \frac{8}{(33-2N_f)\ln \mu/\mu_0}$$

where $\mu_0 \simeq 3\Lambda$. The pressure becomes negative for n_θ 0.2 fm^{-3} (N_f=3). From the equations (5) and (6), it is possible to see immediately that for high temperatures and baryon densities, vanishes and the system behaves like a free gas of quarks and gluons. The appearance of a negative pressure for temperatures and densities below T_0 and n_θ ca n be interpreted as sign of a phase transition.

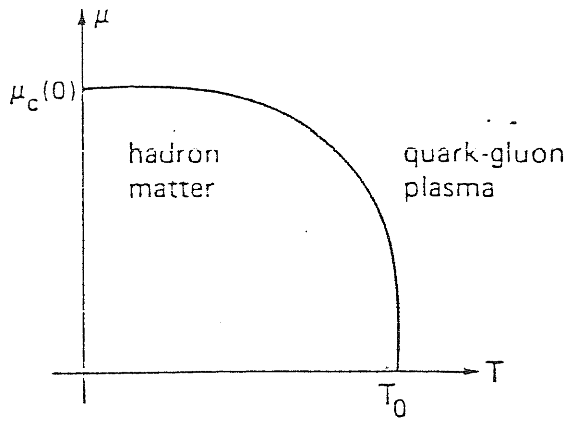

fig. 1. Phase transition diagram.

b) Numerical QCD

In order to know the behaviour of strong interacting matter for the whole range of temperature, pertubation theory cannot be used because its prevision are reliable only in the case of weak interaction. For this reason, in this last years numerical methods have been more and more widely used in studying QCD theory (for a review see Cleymans et al.1985). In numerical QCD, the partition function is evaluated on a lattice where point are separated by multiple of some spacing a, thus the possible momenta range between 1/a and 1/Na, where Na is the linear lattice size. The results are, hence, indipendant from infrared and ultraviolet divergences. Let N_t and N_s be the number of lattice sites in the time and space direction respectively, then the volume and the temperature of the system are equal to $V = N_s a^3$ and $1/T = N_t a$.

The action is rewritten in terms of link variables A_x^i and site variables $\psi(x)$, i.e., the gauge field is defined along the links between lattice points, while the fermionic field is defined in the lattice points. Moreover the lattice action must satisfy the local gauge invariance and in the limit $a \to 0$ it must reduce to the continuum form.

Physical observables must, of course, be indipendent of the choise of the lattice. This is garantee by renormalization group theory through a relation between lattice spacing and coupling g

$$a \Lambda_L = \left\{ \frac{11 \; N_f g^2}{48 \pi^2} \right\}^{-51/121} \exp \left\{ - \frac{24 \pi^2}{11 N_f g^2} \right\}$$

where Λ_L is a dimensionful parameter which characterised the interaction scale. All physical quantities are then calculated by computer simulations techniques in terms of the free parameter . Their value in physical units is obtained, at the end, fixing the arbitrary lattice scale Λ_L by calculating some known quantity, like the mass of proton.

The observables that have been most commonly used in finite temperature investigations of QCD are:

i) the total energy density: the results of lattice calculations are shown in fig. 2. We can see that energy density has an abrupt change from values comparable with which of a gas of free mesons to values approaching the energy density of an ideal gas of quarks and gluons. Thus, also in numerical analysis is evident the Stefan-Boltzman behaviour that we have already seen in pertubation theory.(Cleymans et al. 1985).

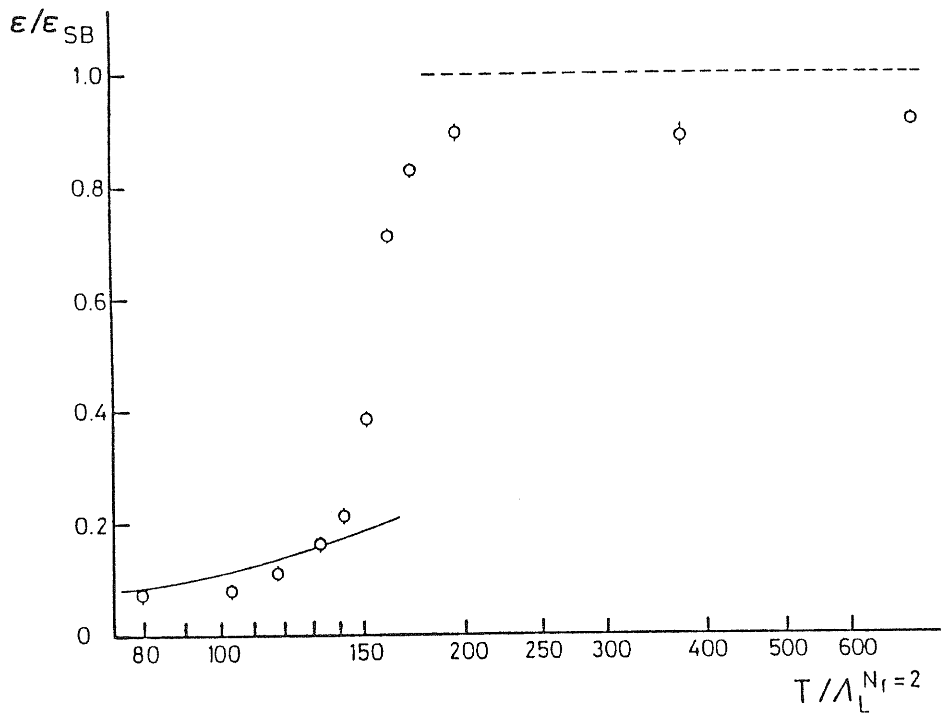

fig. 2. The total energy density, normalized to the ideal gas limit , as a function of the temperature. It is also shown the ideal gas limits for the quark-gluon plasma(dashed line) and for a system of , and mesons(Celik et al. 1985).

ii) the average quantity $\langle\bar{\psi}\psi\rangle$ which is an order parameter for the spontaneous breaking of chiral symmetry. This is a quasi-exact symmetry of QCD equation of motion restored at high temperature in the limit of zero mass for the flavours up, down and posssibly strange. (Fucito et al. 1985, Gavai et al. 1984).

iii) the thermal Wilson loop $\langle L\rangle$ which is related to the free energy of an isolated quark. Actually $\langle L\rangle$ represents a true order parameter of the phase transition in the case of a Yang-Mills system. Its expectation value is, in fact, zero in the confined phase and finite in the deconfined one according to an infinite and finite, respectively, free energy of an isolated quark (Polonyi et al. 1984, Celik et al. 1985, Fucito et al. 1984).

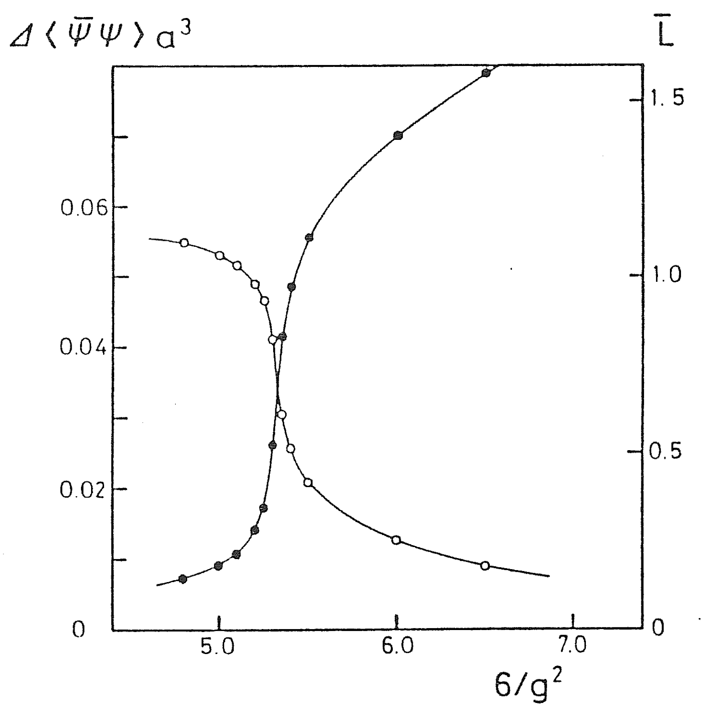

fig. 3. The chiral symmetry restoration measure $\langle\bar{\psi}\psi\rangle$ (open circles) and the deconfinement measure $\bar{L}$ (full circles) as function of $6/g^2 \sim T/\Lambda_L$. (Celik et al. 1985)

The critical temperature T_c is found to be of the order of 200 MeV within an uncertainty of a factor 2.

The problem of the order of the transition is completely solved for Yang-Mills systems in which analytic (Borgs-Seiler 1983) and numerical calculations agree on the existence of a first order phase transition arising as a consequence of the breaking of the symmetry of the system with respect to the centre Z_3 of the group SU(3). The latent heat released during this transition is evaluated to be 1.5 ± 0.5 GeV/fm^3 (Kogut et al. 1983b, Celik et al. 1983, Svetisky-Fucito 1983). The value of T_c is determined by computing $\langle L \rangle$ or the specific heat. For a Yang-Mills system, $T_c = (150-170) \pm 50$ MeV/fm^3. At present, however, a definitive answer is not yet available for the full QCD theory. The introduction of dynamical quarks causes numerical problems that are not completely solved up to now.

Some consideration can be made on the results obtained with different methods

a) at a temperature of the order of 200 MeV, the energy density presents a rapid variation and the Stefan-Boltzmann limit is approached at higher temperatures.

b) a rapid variation of both $\langle L \rangle$ and $\langle \bar{\psi}\psi \rangle$ quantities at almost the same temperature (see fig. 3) suggested the possibility of a deconfinement transition connected with chiral symmetry restoration.

Most of the studies of QCD on lattice refers to the case $\mu=0$. Recently the case $\mu \neq 0$ has also been analysed. As we expect transition temperature decreases for increasing value of μ (Engels-Satz 1985, Damgard 1985; see fig. 4).

The results for a pure Yang-Mills theory and the behaviour of physical quantities like ε, $\langle \bar{\psi}\psi \rangle$ and $\langle L \rangle$ strongly suggests the existence of a first order phase transition in the full QCD theory. In view of this possibility, we think that it is worth studying the effect of this transition in the early universe.

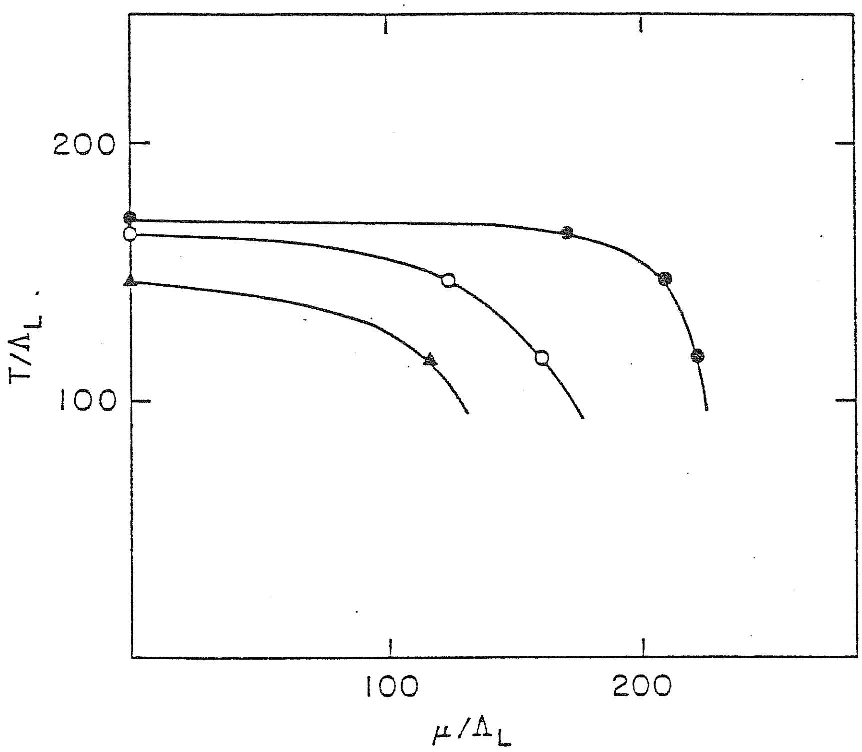

fig. 4. Deconfinement phase diagram, for dynamical quarks of mass m_q= 400 (●), 170 (o) and 20 MeV (▲); the curves are only to guide the eye(Engels-Satz 1985).

2.3 USE OF QCD RESULTS IN COSMOLOGY

The QCD results on lattice was first use in cosmology by Bonometto and Pantano (1983). On the bases of temperature dependence of energy density and pressure obtained in lattice computations. We worked out how the temperature and scale factor deviate from the one of a radiation-dominated universe as a result of confinement forces.

According to the standard model we assume that for temperature $T \gg 200$ MeV, the universe is homogeneous and isotropic. Thus, in a system of comoving coordinate (r, θ, φ), the metric is the Robertson-Walker one

$$(1) \qquad ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-k^2 r^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2\right]$$

where k=+1, 0 or -1 for a closed, flat or open universe respectively. t is the proper time of a comoving observer. Once one knows the equation of state of the cosmological fluid, the scale factor a(t) is calculated from the Einstein equations, that in this case reduce to the formulae

$$(2) \qquad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\, e + \frac{k}{a^2}$$

$$(3) \qquad -\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\,(e + p)$$

If no dissipative phenomenon presents and the system evolves through equilibrium states, the entropy S for a comoving volume is conserved, i.e.,

$$(4) \qquad S = a^3(t)\,\frac{(e+p)}{T} = \text{constant}$$

In the case of a radiation-dominated universe, this implies the well known relation $aT = \text{constant}$. Consider now the confinement period and assume that the transition from a state of an ideal quark-gluon plasma to an ideal hadron gas is continous, even if very rapid. The contribution of strong interacting matter to the total energy density and pressure is derived from lattice results, while the contribution of radiation-like particles is given by $e_r = 3\,p_r = \frac{\pi^2}{30}\, g_r T^4$ where g_r is the total degeneracy factor. When the energy and the pressure depend only on the temperature, eq. (4) is true also during the confinement period, then we can work out the behaviour of the quantity aT as a function of T over the temperature range $(80 \div 800)\Lambda_L$. The deviation from a radiation-dominated universe is very clear in fig. 5. As the confinement forces become important, the temperature rapidly stabilises at a value of the order of $80\,\Lambda_L$ ($\Lambda_L = 2.0 \pm 0.6$ MeV). During the whole process quarks and gluons are transformed in leptons and photons as it can be seen from the increase

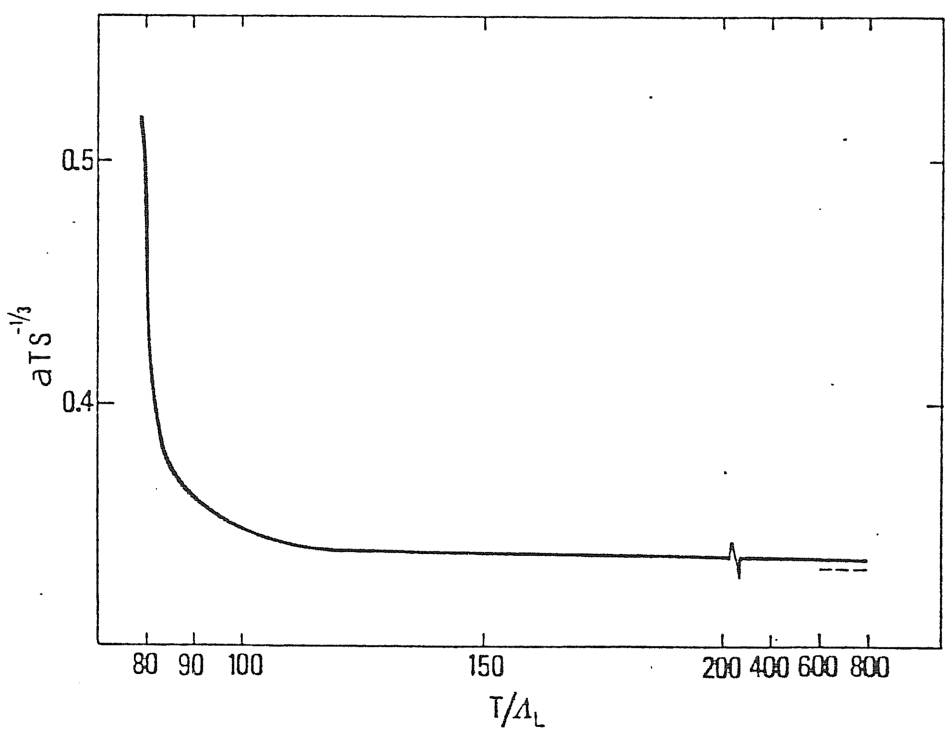

Fig. 5. The deviation of a(T) from its behaviour in a radiation dominated model is shown, by plotting the temperature dependence of a(T)T.

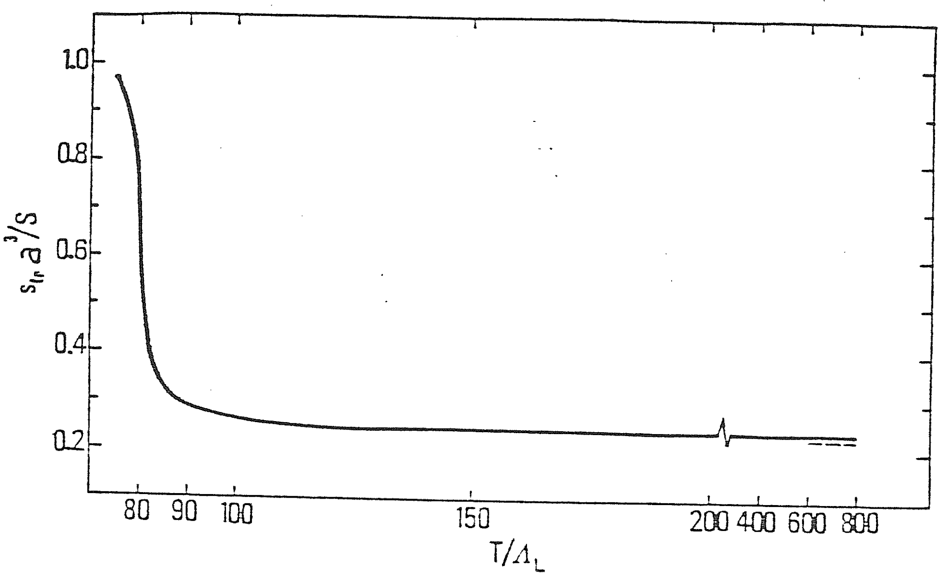

Fig. 6. The amount of entropy in photon-lepton form is plotted. Its sharp increase in correspondence with the onset of confinement is shown.

of entropy fraction of photons and leptons respect to the total entropy. (See fig. 6.)

The integration of Einstein equations gives a(t) and, using equation (4), T(t) (Bonometto-Sakellaradiou 1984). The plateau of temperature (see fig. 7) obtained in the integration is very long ($\gtrsim 10^{-5}$ s) compared with the age of the universe at the beginning of the ,nfinement ($t_{exp} \simeq 10^{-6}$ s).

If during this period the system goes through a first order phase transition, the evolution must be studied more carefully. In the case of negligible supercooling, i.e., in the case in which it is possible to neglect density fluctuations related to the nucleation process, the analysis of a first order phase transition can be made assuming the whole system in equilibrium at the critical temperature T_c and pressure p_c. For an isothermal and isobaric transition equation (4) is still valid and the Einstein equations can be integrated analytically (Bonometto-Matarrese 1983, Bonometto-Sakellaradiou 1984, Lodenquai-Dixit 1984). Under this assumption, the length of the actual transition appears very short($\simeq 10^{-7}$ s) compared with the length of temperature plateau. This comparision suggests that confinement forces may be important also before the actual phase transition. Their importance, however, depends on the detailed behaviour of both energy and pressure. Since our analysis only the data concerning

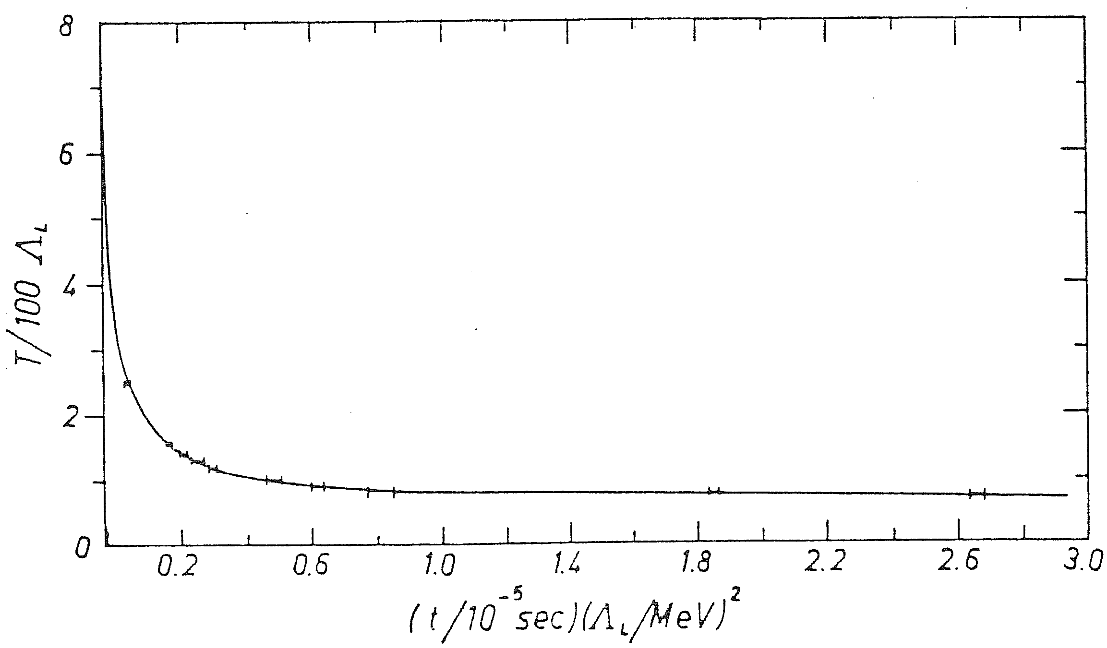

fig. 7. The time dependence of the temperature at the epoch of the quark-hadron phase transition is shown; the error bars are due to lattice QCD data.

the energy density have been improved while no more data on the pressure have been computed. Then accurate QCD computation on lattice appears to be important not only for knowing the character of the transition but also for a good description of the thermodynamical quantities near the critical temperature.

The existence of a long plateau of temperature can effect sensibly cosmological constraints on supersymmetric particles. Preliminary computation shows a shift at least of 20% in the predicted masses of a particle heavier than 200 MeV (Goldberg 1983, Lee-Weinberg 1977).

2.4 MODELS FOR QUARK AND HADRON MATTER NEAR THE TRANSITION

Numerical analysis of QCD allowes us to see the rapid change in some physical quantities, but, up to now, it has not been able to give complete description of the thermodynamics of QCD useful for studying the effect of the transition in a physical context. Except for a few cases, like the pressure computation made by Montvay-Pietarinen (1982), and the more recent paper on specific heat (Celik et al. 1983) and sound velocity (Gavai-Gocksch 1985), only the energy density has been repeatedly calculated. For this reason many models has been proposed in order to describe in a suitable way, the effect of strong interaction near the transition point. (Kàrsch-Satz 1980, Olive 1981, Suhonen 1982, Källman 1982, Kapusta 1982, Hagerdon 1985)

Bag model

A simple model for a first order confinement phase transition can be based on the M.I.T. bag model (Chodos et al. 1974). Although, as we will see, this model neglects certain QCD interactions and treats the hadronic phase in a simplified manner, it does contain the basic physical characteristics of strong interacting matter, i.e., the confinement and the asymptotic freedom of quarks at low and high temperature respectively. Moreover, the trnasition temperature and the latent heat per unit volume also agree with lattice QCD.

In the bag model, the interaction between quarks are neglected or treated in the lowest order of perturbation theory. The ground state of quark-gluon matter enclosed in a finite volume V is represented by a shift from the physical vacuum obtained adding a term $\ln Z_{vac} = BV/T$ to the partition function given by perturbation theory.

At the lowest order in perturbative expansion, for $\mu=0$, we have (Kalashnikov 1984)(see eqs (1.5) and (2.5)):

(1) $$p = \frac{8}{45}\pi^2 T^4 + \frac{7}{60}\pi^2 N_f T^4 + 3\pi^2 \alpha_{eff}(T)\left[\frac{1}{6}T^4 + \frac{5}{72}N_f T^4\right] - B$$

(2) $$\varepsilon = B + \frac{24}{45}T^4\pi^2\left[1 - \frac{45}{16}\alpha_{eff}\left(1 - \frac{1}{3\ln T/\Lambda}\right)\right] + \frac{7}{20}\pi^2 N_f T^4\left[1 - \frac{75}{42}\alpha_{eff}\left(1 - \frac{1}{3\ln T/\Lambda}\right)\right]$$

with

(3) $$\alpha_{eff}(T) = \frac{8}{(33 - 2N_f)\ln T/\Lambda}\ , \qquad 145\ \mathrm{MeV} \le B^{1/4} \le 235\ \mathrm{MeV}$$

From equations (1) and (2), we see that, neglecting perturbative corrections, the entropy is independent of the bag constant B and it is formally equal to the expression one gets for an ideal gas of quarks and gluons. This implies, from equation (3.5), that the product aT is constant until the actual phase transition occurs. In the last section, however, we saw that aT may deviate sensibly from the radiation-dominated case before T_c.

Finite volume model in the hadron phase

The approximation of pions as point-like particles is quite restrictive also for temperature of the order of pion mass. The mean distance between particles derived from point-like statistics ($d \simeq n^{-1/3} \simeq 1.4\,T^{-1}$) is slightly bigger than intrinsic pion size ($\sim 1/m_\pi$) also for temperature of the order of pion mass. Moreover, the situation is even worse if we consider hadrons with mass bigger than pion. Although at T_c contributions of heavier hadron species to the total particle number are suppressed by a Boltqmann factor $e^{-m/T}$, their spectrum is very rich and their importance increases with temperature (Pantano 1983).

Different models have been introduced for describing the interactions between hadrons (Karsh-Satz 1980, Olive 1981, Kapusta-Olive 1983).

An useful approximation for their interaction is the introduction of an infinite potential at radius $\sim \frac{1}{m_\pi}$ (Karsh-Satz 1980) that, physically, expresses the requirement of non-overlapping of hadrons. In this model, the partition function for $\mu = 0$ is

$$(1) \qquad Z(T,V) = \sum_{N=0}^{V/V_E} V(N) \prod_{i=1}^{N} \int d^3 q_i \, dm_i \, \rho(m_i) \, \exp\{-\varepsilon_i/T\}$$

($\varepsilon_i = q_i^2 + m_i^2$, m_i is the mass of i^{th} particle) where $\rho(m)$ yields the hadron spectrum

$$(2) \qquad \rho(m) = \sum_{k=1} g_k \, \delta(m - m_k) + c \, \vartheta(m - m_0) \, T_H^{5/2} \, m^{-7/2}$$

Here we distinguish between a discrete and a continuum part; $m_0 \simeq 1200$ MeV is the mass scale above which the continuum spectrum is supposed to start. T_H is the hagedorn temperature and in order to avoid a divergent result, it must be $T_H > T_c$. Moreover,

(3) $$V(N) = (V - N V_E)^N \quad , \qquad V_E = 4 V_0$$

where $V_0 = (4\pi/3) m_\pi^{-3}$ is the proper volume of hadrons which we consider as constant. In the thermodynamic limit, $V \to \infty$, we have

(4) $$p(T) = \frac{T}{V_E} b(x)$$

where (k_1 and k_2 are Bessel functions)

(5) $$b e^b = x = \frac{V_E T}{2\pi^2} \int dm \, \rho(m) \, m^2 \, k_2\left(\frac{m}{T}\right)$$

and

(6) $$\rho = \frac{T^4}{2\pi^2 (1+b) e^b} \int dm \, \rho(m) \left(\frac{m}{T}\right) \left[3 k_2\left(\frac{m}{T}\right) + \frac{m}{T} k_1\left(\frac{m}{T}\right)\right]$$

The generalization in the case $\mu \neq 0$ has been examined by Bonometto (1983) and Bonometto et al. (1985).

2.5 FIRST ORDER PHASE TRANSITION AND THERMAL NUCLEATION THEORY

A first order phase transition is characterised by a finite discontinuity in the first derivatives of the thermodynamic potential. In fig.8 the equilibrium curves for a system that undergoes a first order phase transition are drawn. At a temperature T_c, the two phases can coexist in equilibrium since their pressures are equal. For temperature different from T_c the system is in the phase of maximum pressure (minimum thermodynamic potential). A first order phase transition is not usually immediate, but, if the system evolves through equilibrium states, it is often associated with supercooling or superheating phenomena. In our specific case, as the temperature decreases, the strong interacting matter may continue to stay in the quark phase that becomes metastable.

In a homogeneous medium, the change from a metastable to a stable phase occurs as a result of fluctuations whose form that form small quantities of the new phase. Although the new phase is the stable one, the energetically unfavourable process of creation of an interface has the result that nuclei below a certain size are unstable and disappear.

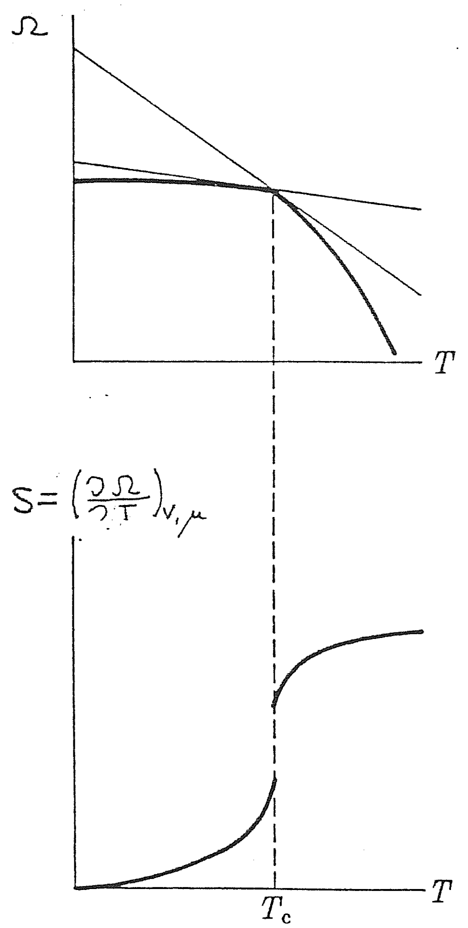

Fig. 8. Temperature dependence of the thermodynamical potential of fixed volume and chemical potential. The system undergoes a phase transition at $T = T_c$, accompanied by a latent heat (entropy discontinuity).

The thermodynamical properties of such an interface are entirely described by one quantitiy γ (function of state of the system) called surface tension. It is defined in the following way: let S be the area of the interface, there the work done in increasing the area of an amount dS is equal to

(1) $$dR = \gamma \, dS$$

From (1) we see that can be interpreted as a negative pressure $\alpha = -\gamma$. In a constant volume V, the differential $d\Omega$ becomes

$$d\Omega = -SdT - Nd\mu + \alpha\, ds$$

and Ω is now

(2) $$\Omega = \Omega_0 + \gamma S$$

where Ω_0 refers to the bulk matter.

A nucleus of the new phase can coexist in equilibrium with the old phase if the ocndition of thermodynamical equilibrium is satisfied. That means that the temperature and the chemical potential of the two phase are equal

(3) $$T_1 = T_2 = T$$

$$\mu_1(p_1, T) = \mu_2(p_2, T)$$

and the sum of the forces acting on each phase is zero. The presence in the two phases are different; The relation between them is obtained from the condition of minimum for Ω, at a fixed total volume V, keeping μ and T constant.

(4) $$\Omega = - P_1 V_1 - P_2 V_2 + \gamma S$$

where 1 refers to the nucleus of the new phase and 2 to the metastable phase. For a spherical nucleus the condition of mechanical equilibrium gives

(5) $$P_1 - P_2 = \frac{2\alpha}{r_{cr}}$$

From equation (1.7), we can derive the surface part of the entropy

(6) $$S_S = - \left(\frac{\partial \Omega_s}{\partial T} \right)_{\mu, T} = - S \frac{d\gamma(T)}{dT}$$

Then the surface energy σ_s density is (under the assumption that no particle is contained in the interface, equation (1.7)

(7) $$\sigma_s = \left(\gamma - T \frac{d\gamma(T)}{dT} \right)$$

Generally it results $\gamma/\sigma \leq 1$. Since we cannot know the exact behaviour of γ (T), as it depends on the detail interaction between particles in the transition layer, we will assume in our computation (Witten 1984, DeGrand-Kajantie 1983)

(8) $$\gamma = \sigma_s = \gamma_0 T_c^3$$

According to fluctuaiton theory (Landau-Lifshitz 1980, 1981), the probability of producing a nucleus is proportional to exp(-R_{min}/T) where R_{min} is the minimum work needed in forming the nucleus; Since the equilibrium condition requires that the chemical potential and the temperature are the same in the two phases, this work is equal to the variation of Ω . For the critical radius of equation (3) the probability per unit space and time is

$$p(T) \simeq p_0 T_c^4 \exp\left\{ - \frac{16\pi\gamma^3}{3T(p_1 - p_2)^2} \right\}$$

where p_0 is an unknown quantity that we will assume to be of the order one; the characteristic time and length of strong interaction appear in the factor T_c^4. (See also Hogan 1983.)

Using the bag model in its simplest form, the nucleation rate becomes (without perturbative correction and finite volume effect),

$$p(T) = p_0 T_c^4 \exp\left\{ - \frac{W_0}{\hat{T}(1-\hat{T}^4)^2} \right\} \tag{9}$$

$$W_0 = 16\pi\gamma_0^3 / 3\left[(g_i - g_f)\frac{\pi^2}{90} \right]^2$$

where $\hat{T} = T/T_c$; g_i and g_f are the degeneracy factors of particles before and after the transition. The nucleation rate has a maximum for $T^* = 0.58\, T_c$. The transition needs to be completed before T^*, otherwise it will never be completed (Guth-Weinberg 1981). Assuming that the space outside the bubble is not effected by the transition and neglecting the initial size of the bubble, the fraction of space in the hadron phase at temperature T is (DeGrand-Kajantie 1985)

$$\frac{V_H}{V_{tot}}(T) \simeq \exp\left[\log\left(\frac{T_c^4 v_{def}^3}{8\pi G B/3} \right) - \frac{W_0}{16(1-\hat{T})^2} - 6 \log \frac{1}{1-\hat{T}} \right]$$

where v_{def} is the velocity, considered constant, at which the bubble deflagrates. In this analysis, the transition appears completed at a temperature

$$T_{PT} \simeq \left(1 - \frac{\sqrt{w_0}}{50}\right) T_c$$

The main contribution in completing the transition comes from the bubbles nucleated with a critical size $6\gamma_0/(\sqrt{w_0}\,T_c)$ at a temperature $T_N = T_{PT} - \frac{w_0}{6000} T_c$.

For w_0 in the range (10^{-1} — 100), i.e., for the range of nucleation temperatures (0.80 — 0.99) T_c, the critical radius range between 8 and 4 fm. The hydrodynamical treatment can be done on all the range of temperature although it is very suitable for small supercooling ($w_0 < 10$).

CHAPTER THREE

BUBBLE GROWTH IN AN EXPANDING UNIVERSE

The aim of this thesis is to present a formalism for describing the growth of a spherical bubble of hadrons in a supercooled quark-gluon plasma in an expanding background.

In this chapter we will derive the hydrodynamic equations for a spherical symmetric relativistic fluid.

Succesively, the Gauss-Codazzi formalism will be presented as a suitable method for describing the evolution of the transition layer. The width of the shell where the transition is going on is of the order of the strong interaction length scale, i.e., $\simeq 1$ fm . When the dimensions of the bubble are sufficiently large, one can regard this membrane as a zero-thickness surface. A surface energy density and a surface tension must be associated to the surface, as a result of the change of bulk matter properties across it.

The complete set of junction conditions for the fluid variables and the metric components will be settled down explicitly across the timelike hypersurface describing the motion of the transition layer.

3.1 RELATIVISTIC HYDRODYNAMICS FOR A SPHERICAL SYMMETRIC PERFECT FLUID

The well known expression for the stress-energy tensor for a perfect fluid is

(1) $$T^{ij} = (p+e)\, u^i u^j + p\, g^{ij}$$

where p is the pressure, e the proper energy density and $u^i = dx^i/d\tau$ is the four- velocity of the fluid normalized so that

(2) $$u^i u_i = -1$$

In the case of spatial spherical symmetry the metric can be written in the general form

$$ds^2 = -\alpha^2\, dx^{0\,2} + \beta^2\, dx^{1\,2} + R^2\, d\Omega^2$$

with $$d\Omega^2 = d\theta^2 + \operatorname{sen}^2\theta\, d\varphi^2$$

where α, β and R are function of x and x only .

An usually conveniant choice of coordinates is the one in which the four-velocity of the fluid is proportional to $(\partial/\partial t)$ so that the spatial coordinates are constant along the world line of any fluid element. This choice determines a comoving or Lagrangian frame. If we denote with t and μ the time and the radial coordinates respectively,the metric can be written as

(3) $$ds^2 = -a^2\, dt^2 + b^2\, d\mu^2 + R^2\, dR^2$$

Using condition (2) the fluid four-velocity is

(4) $$u^i = (a^{-1}, 0, 0, 0)$$

The function R is called circumference coordinate since it measures the proper circumference of a sphere of radial coordinate μ at the time t

$$\int ds = \int g_{\theta\theta}^{1/2}\, d\vartheta = 2\pi R(\mu, t)$$

In the newtonian or special relativistic limit, R represents the position or eulerian coordinate at the time t of the shell of fluid labelled by the comoving coordinate μ.

Where one has to follow the propagation of disturbancies in the fluid, it is also convenient to consider an Eulerian frame. The closest relativistic analogue of the classical Eulerian frame is the Schwarzschild one in which R is taken to be the radial coordinate. In this coordinate system, the metric is

$$ds^2 = -A^2 dT^2 + B^2 dR^2 + R^2 d\Omega^2 \tag{5}$$

where A and B are functions of R and T only.
The fluid velocity normalized according to eq.(2),is

$$u^i = \left(A^{-1} \left(1 + B^2 u^2\right)^{1/2},\ u,\ 0,\ 0 \right) \tag{6}$$

Here the radial component of the four-velocity is denoted simply by u.

For a spherically symmetric fluid,described by the metric (3), the Einstein eqs reduce (May & White 1966, Misner 1968) to

$$4\pi G \varrho R^2 R_\mu = \frac{1}{2}\left[R + \frac{R R_t^2}{a^2} - \frac{R R_\mu}{b^2} \right]_\mu \qquad \left(T^0{}_0 \right) \tag{7}$$

$$4\pi G p R^2 R_t = -\frac{1}{2}\left[R + \frac{R R_t^2}{a^2} - \frac{R R_\mu}{b^2} \right]_t \qquad \left(T^1{}_1 \right) \tag{8}$$

$$4\pi G (\varrho + p) R^3 = \left[R + \frac{R R_t^2}{a^2} - \frac{R R_\mu^2}{b^2} \right] + \frac{R^3}{ab}\left[\left(\frac{a_\mu}{b}\right)_\mu - \left(\frac{b_t}{a}\right)_t \right] \tag{9}$$

$$\left(T^2{}_2 = T^3{}_3 \right)$$

(10) $$\frac{a_\mu}{a} R_t + \frac{b_t}{b} R_\mu - R_{\mu t} = 0 \qquad (T_0')$$

(the subscripts " " and "t" denote partial derivatives with respect to μ and t respectively)

If the fluid obeys a one-parameter equation of state

(11) $$p = p\,(e)$$

the state of the system is completely determined by set of eqs(7-11).

In general, however, the equation of state of the fluid depends on two parameters; for example the proper energy density and the baryon number density. In this case the set of eqs(7-11) is no longer sufficient to solve completely the problem and an additional equation must be considered. For a fluid with net baryon number different from zero, the local conservation of baryon number is expressed by the law

(12) $$(\, n\, u^i\,)_{;i} = 0$$

where n is the proper baryon number density and u^i is the fluid four-velocity, and a semicolon is used to denote covariant derivatives.

In the comoving frame eq.(12) reduces to

(13) $$\frac{\partial}{\partial t}\left(n\, b\, r^2 \,\mathrm{sen}\, \vartheta \right) = 0$$

As a consequence of this equation, the total baryon number A contained within a spherical surface of radius μ

(14) $$A = \int_0^\mu n \; 4\pi b\, R^2 \, d\mu$$

is conserved.

This property provides a natural way of defining the radial coordinate μ . In fact we can identify μ with A and so μ represents the total baryon number contained within the sphere labeled by μ .

From eq.(14) we can see that the choice of the radial coordinate fixes also the value of the metric coefficient b

$$(15) \qquad b = \frac{1}{4\pi n R^2}$$

We will see below that eq.(12) can be used in the computations also in the case of zero baryon number density, reinterpreting 1/n as the proper volume of an element of fluid and n as the proper number density of elements of fluid. Then the definition of the radial coordinate assumes also in this case a precise physical meaning : it represents the number of fluid elements contained within μ as it was defined at the initial time.

In order to leave the relativistic equations of motion in a form analogous to the classical ones, we shall write explicitly the laws of local conservation of energy and momentum, although they are already contained in the Einstein equations

$$(17) \qquad T_i{}^j{}_{;j} = 0$$

In the comoving frame we have

$$(18) \qquad T_o{}^j{}_{;j} = 0$$

$$(19) \qquad T_1{}^j{}_{;j} = 0$$

It is convenient to introduce the quantities

(20) $$\Gamma = R_\mu / b$$

(21) $$u = R_t / a$$

It can be easily checked that (R_t/a) is the radial component of the fluid four-velocity in the Schwarzschield frame.

Using relations (20) and (21) and eq.(10), eq.(18) can be rewritten as

(22) $$e_t \quad a\left(\frac{u_\mu}{R_\mu} + \frac{2u}{R} \right) (e + p) = 0$$

If we define a function m such that

(23) $$m_\mu = 4\pi e R^2 R_\mu$$

(24) $$m_t = -4\pi p R^2 R_\mu$$

using definitions (20) and (21), eq.(7) then becomes

(25) $$\Gamma^2 = 1 + u^2 - \frac{2m}{R}$$

with

(26) $$m(\mu, t) = \int_0^\mu 4\pi e R^2 R_\mu \, d\mu$$

From eqs (23),(24) and (26) we can interpret $m(\mu,t)$ as the total energy of a sphere of matter contained within μ.

Γ is a factor which corrects the local rest frame energy for the kinetic energy and the gravitational binding energy.

Using def.(21) and the constraint eqs(25) and (10), the equation of motion (8) can be written as

$$u_t = -a\left[\frac{\Gamma}{b}\,\frac{p_\mu}{(e+p)} + \frac{Gm}{R^2} + 4\pi G p R\right]$$

Finally, using the definition of u and equation (10), the baryon number conservation law (13) becomes

$$\frac{(nR^2)_t}{nR^2} = -a\,\frac{u_\mu}{R_\mu}$$

The complete set of equations we shall use for studying the dynamics of the fluids on the two sides of the confinement layer is the following :

(27) $$u_t = -a\left[\frac{\Gamma}{b}\,\frac{p_\mu}{(e+p)} + \frac{Gm}{R^2} + 4\pi G p R\right]$$

(28) $$R_t = a u$$

(29) $$e_t = -a\,(e+p)\left[\frac{u_\mu}{R_\mu} + \frac{2u}{R}\right]$$

(30) $$a_\mu = -a\,\frac{p_\mu}{(e+p)}$$

(31) $$m(\mu,t) = \int_0^\mu 4\pi e R^2 R_\mu\, d\mu$$

(32) $$p = p(e)$$

(33) $$\Gamma = R_\mu / b$$

(34) $$\frac{(nR^2)_t}{nR^2} = -a\,\frac{u_\mu}{R_\mu}$$

The boundary conditions are

$$u = 0 \,, \quad R = 0 \,, \quad \Gamma = 1 \qquad \text{at} \quad \mu = 0$$

$$a = 1 \qquad \text{at} \quad \mu_{max}$$

The second condition syncronizes the coordinate time with the proper time of the fluid at the boundary. The condition $\Gamma = 1$ at the origin represents the request of local flatness; it was used in integrating eq.(7) to obtain eq.(25) .

3.2 SURFACE LAYER IN GENERAL RELATIVITY

In the last section we have seen the hydrodynamic equation for a sphercally symmetric fluid. When a bubble of hadrons nucleates in the supercooled plasma of quarks and gluons,it appears a thin shell where the hydrodynamic quantities change rapidly between the equilibrium bulk values in the two fluids. The thickness of this layer will be of the order of the strong interaction length scale (fm) and,when the bubble radius is of the order of few fermi,we can approximate it as zero thickness surface. The evolution of the confinement region can then be described by a time like hypersurface which separates the four-dimensional space-time in two different regions that we will denote by V^+ and V^- .

We shall follow here Israel (1968) treatement of singular hypersurface in general relativity. In this method the history of such hypersurfaces is characterised in a geometrical way by the extrinsic curvature of their embeddings in space-time. If the hypersurface is embedded in a space-time V with metric tensor g_{ij}, curvature $R_{ijl}{}^{m}$ and Einstein tensor G_{ij}, then the main relations which describe the hypersurface are the Gauss-Codazzi equations

(1) $$\overset{(3)}{R} + K_{ij} K^{ij} - K^{2} = -2\, G_{ij}\, u^{i} u^{j}$$

(2) $$D_{j} K_{i}{}^{j} - D_{i} K = G_{lm}\, h_{i}{}^{l}\, u^{m}$$

Here K_{ij} is the extrinsic curvature three-tensor of Σ (it measures in which way Σ "bends" in V)

(3) $$K_{ij} = h_{i}{}^{l}\, h_{j}{}^{m}\, u(l;m)$$

n^{i} is the unit normal to Σ, which is space like

(4) $$n^{i} n_{i} = +1 \qquad \text{and}$$

(5) $$h_{ij} = g_{ij} - n_{i} n_{j}$$

is the induced metric on Σ which decribes intrinsic characters. Finally D_{i} denotes the intrinsic covariant derivative (i.e. associated with h_{ij})

(6) $$D_{i}\, T^{jl} = h_{i}{}^{m}\, h_{r}{}^{j} \qquad T^{rs}{}_{;m}$$

In our physical context the hypersurface divides the space-time in two regions in which the strong interacting matter is in a different phase; mathematically the change of phase is related to a δ-function singularity on Σ of the stress-energy tensor T_{ij} .

This implies that the two extrinsic curvatures $K_{ij}^{\pm}$ of associated with its embedding in V^+ and V^- are different (here after the labels "+" and "-" will refer to the quark-gluon phase and hadron phase respectively). The time-like hypersurface for which $K_{ij}^{-} \neq K_{ij}^{+}$ are usually called "surface layer".

Let's define two kinds of brackets $\{A\}^{\pm}$ and $[A]^{\pm}$ such that for every field variable A we have

$$\{A\}^{\pm} = A^+ + A^- \qquad\qquad [A]^{\pm} = A^+ - A^-$$

where the labels "$\pm$" refer to values immediately in front and behind the surface.

Accorrding to Israel we introduce $\tilde{K}_{ij}$ and S_{ij} as

(7) $$[k_{ij}]^{\pm} = -8\pi G\left[S_{ij} - \frac{1}{2} h_{ij} S\right]$$

(8) $$\{k_{ij}\}^{\pm} = 2\tilde{k}_{ij}$$

where $S = g^{ij} S_{ij}$. The symmetric tensor S_{ij} defined by eq. (7) is called surface energy-momentum tensor, in fact it is equal to

(9) $$S_{ij} = \lim_{\epsilon \to 0} \int_{-\varepsilon}^{\varepsilon} T_{\mu\ell}\, h_i^{\ \mu} h_j^{\ \ell}\, dx$$

where x is the proper distance through Σ in the direction of the normal vector $\pm n^i$.

The sum and difference of eqs (1) and (2) , using Einstein equation $G_{ij} = 8\pi h T_{ij}$ and relations (7) and (8) , yields the set of equations

(10) $$\overset{(3)}{R} + \tilde{K}_{ij}\, \tilde{K}^{ij} - \tilde{K}^2 = -16\pi^2 G^2 \left(S_{ij} S^{ij} - \frac{1}{2} S^2 \right) - 8\pi G \left\{ T_{\Sigma}^{ij}\, n_i n_j \right\}^{\pm}$$

(11) $$\tilde{K}_{ij}\, S^{ij} = \left[T_{\Sigma}^{ij}\, n_i n_j \right]^{\pm}$$

(12) $$D_i \tilde{K}^i_{\ j} - D_j \tilde{K} = 4\pi G \left\{ T_{\Sigma}^{lm}\, n_l\, h_{jm} \right\}^{\pm}$$

(13) $$S_i^{\ j}{}_{;j} = - \left[T_{\Sigma}^{lm}\, n_l\, h_{im} \right]^{\pm}$$

In the next section we will concentrate our attention on eqs(11) and (13) that,in highly symmetric problem as the one in which we are interested,describe completely the evolution of the shell,once they have been coupled to the Einstein equations in $V^{\pm}$. Eqs (10) and (12), instead,should be solved only if one is interested in the form of fictitious background space $\tilde{V}$,such that Σ is regular in it with extrinsic curvature $\tilde{K}_{ij}$.

3.3 COMPLETE SET OF JUNCTION CONDITIONS

Let's consider a perfect fluid type of matter,then S_{ij} and T_{ij} take the form

(1) $$S_{ij} = (\sigma + \alpha)\, v_i v_j + \alpha\, h_{ij}$$

(2) $$T_{ij}^{\pm} = (p^{\pm} + e^{\pm})\, u_i^{\pm} u_j^{\pm} + p^{\pm} g_{ij}^{\pm}$$

where σ and α on the surface energy density and the two-dimensional pressure; $p^{\pm}$, $e^{\pm}$ and $u^{\pm}$ are the pressure,the energy density and the four-velocities of the fluids in the two different phases; v^i is the unit time-like vector tangent to Σ ..

These relations should now be explicitly expressed for our problem. For a spherically symmetric system there exist two space-like Killing vectors $e^{i}_{(A)}$ (A=2,3) which span the space-time. Then the unit normal n^{i} ,the time-like tangent vector v^{i} and the two Killing vectors form an orthonormal tetrad system on Σ . They are related to the metric tensor g_{ij} through the relation

$$g_{ij} = \eta_{AB}\, e_i^{(A)}\, e_j^{(B)} \tag{3}$$

where η_{AB} = (diag -1,1,1,1) and $e_i^{(1)} \equiv v_i$, $e_i^{(2)} \equiv n_i$.

In our comoving frame let $\mu_\Sigma(t)$ be the location of the transition layer at the coordinate time t and $\dot\mu_s$ the "layer speed" $d\mu/dt$. The equation of the hypersurface Σ is then

$$f = \mu - \mu_s(t) = 0 \tag{4}$$

Therefore the normal n_i is given by

$$n_i = N \frac{\partial f}{\partial x^i} = N\left(-\dot\mu_s , 1, 0, 0\right) \tag{5}$$

where N is a constant obtained with the normalization condition (2.4)

$$N = \frac{ab}{\left(a^2 - b^2\dot\mu_s^2\right)^{1/2}}$$

The tangent vector to the hypersurface along the stream line is

$$v^i = \frac{dx^i_\Sigma}{d\tau} = \frac{1}{a^2 - b^2\dot\mu_s^2}\left(1 , \dot\mu_s , 0, 0\right) \tag{6}$$

$$v^i v_i = -1$$

where $d\tau^2 = (a^2 dt^2 - b^2 d\mu^2)^{1/2}$ is the proper time of the shell. Contracting eq.(2.13) with v^i and using the perfect fluid form for S_{ij} and T_{ij} we have

(7) $$D_i\left[(\sigma+\alpha)\, v^i\right] - v^i D_i \alpha = \left[(p_\Sigma + e_\Sigma)(u^i_\Sigma n_i)(u^\ell_\Sigma v_\ell)\right]^{\pm}$$

Contracting eq.(2.13) with the other vectors of the tetrad we have

$$(\sigma+\alpha)\, n_i\, v^i\, D_i v^j = 0$$

$$(\sigma+\alpha)\, e^{(A)}_i\, v^i\, D_i v^j = 0$$

Because of the symmetry $v^i D_i v^j = 0$, the last two eqs are trivial.
Eq.(7) represents the energy conservation law across Σ .
With assumption (2.4.8) eq.(7) reduces to

(8) $$\left[(p_\Sigma + e_\Sigma)(u^i_\Sigma n_i)(u^\ell_\Sigma v_\ell)\right] = 0$$

The assumption implies that there is no energy flux onto Σ from V and the increase of the total surface energy is due only to superficial stress forces.
Using the explicit form of n_i and v^i , eq.(8) reduces to

(9) $$\left[(p_\Sigma + e_\Sigma)\,\frac{a b}{a^2 - b^2 \dot{\mu}_s^2}\right]^{\pm} = 0$$

Eq.(2.11b) yields a second junction condition across Σ . Expressing the acceleration vector in terms of the extrinsic curvature

$$\frac{D v^i}{d\tau} = - n^i v^\ell v^m k_{\ell m}$$

and using the relation derived from (1) and (3)

$$n^i n^j \tilde{k}_{ij} = \frac{1}{\sigma}\left(s^{ij} - \alpha\, e^i_{(A)} e^{j(A)}\right) \tilde{k}_{ij} \qquad \sigma \neq 0$$

it is possible to rewrite (2.11) in the following form

$$(10) \qquad \alpha\, \tilde{k}_{ij}\, e^{i(A)} e^j_{(A)} - \frac{\sigma}{2}\left\{ n_i \frac{\Delta v^i}{d\tau} \right\}^{\pm} = -\left[\left(p_\Sigma + e_\Sigma\right)\left(u^i_\Sigma n_i\right)^2 + p_\Sigma \right]^{\pm}$$

For a spherically symmetric system

$$(11) \qquad k^{ij}\, e_i^{(A)} e_j^{(B)} = \left\{ n^i \left(\ln R\right)_{,i} \Big|_\Sigma \right\}^{\pm}$$

Using assumption (2.4.8) and relation (11), eq.(10) becomes

$$(12) \qquad \frac{\sigma}{2}\left\{ 2\, n^\ell \frac{R_{,\ell}}{R} + n_i \frac{\Delta v^i}{d\tau} \right\}^{\pm} = \left[\left(p_\Sigma + e_\Sigma\right)\left(u^i_\Sigma n_i\right)^2 + p_\Sigma \right]^{\pm}$$

In our comoving frame we have

$$(13) \qquad n^\ell \frac{R_{,\ell}}{R} = \gamma b \left(\frac{\dot\mu_s}{a^2} \frac{R_t}{R} + \frac{1}{b^2} \frac{R_\mu}{R} \right)$$

where $\gamma = a\, dt/d\tau = a / \left(a^2 - b^2 \dot\mu_s^2\right)^{1/2}$

The acceleration is

$$(14) \qquad n_i \frac{\Delta v^i}{d\tau} = \frac{b}{\gamma}\left[\frac{d^2\mu_\Sigma}{d\tau^2} + \frac{2 b_t}{a b}\gamma \frac{d\mu}{d\tau} + \left(\frac{b_\mu}{b} + \frac{a_\mu}{a}\right)\left(\frac{d\mu}{d\tau}\right)^2 + \frac{a_\mu}{a b^2} \right]$$

After some manipulations eq.(12) becomes

$$(15) \qquad \left[e\, b^2 \dot\mu_\Sigma^2 + p\, a^2 \right]^{\pm} = -\frac{\sigma}{2}\left\{ \frac{c^2}{a b} \frac{d}{dt}\left(\frac{b^2 \dot\mu_s}{c}\right) + c^2 \frac{c_\mu}{a b} + \frac{2c}{R}\left(b\dot\mu_s u_\Sigma + a \right]_\Sigma \right) \right]^{\pm}$$

where $$c = \left(e^2 - b^2 \dot{\mu}_s \right)^{1/2}$$

The two junction conditions we have seen up to now don't exaust the whole set of junction conditions that we need in order to specify completely the problem. Instead of manipulating the Gauss-Codazzi equation it is simpler to fix the junction condition on the metric, considering directly the interval ds separating two points on the world surface of the shell and imposing that it must be the same for a comoving observer just behind the membrane and a comoving observer just ahed of it. Thus

$$\left[-e^2 dt^2 + b^2 d\mu^2 + R^2 d\Omega^2 \right]^{\pm} = 0$$

Considering the particular case $d\mu = dt = 0$ we have

(16) $$\left[R \right]^{\pm} = 0$$

i.e. R is continuous as we expect according to its meaning. This implies

$$\left[e^2 dt^2 - b^2 d\mu_s^2 \right]^{\pm} = 0 \qquad \text{or}$$

(17) $$\left[e^2 - b^2 \dot{\mu}_s \right]^{\pm} = 0$$

The complete set of junction conditions is

$$\left[e b^2 \dot{\mu}_s^2 + p e^2 \right]^{\pm} = -\frac{\sigma}{2} c^2 \left\{ \frac{1}{eb} \frac{d}{dt} \left(\frac{b^2 \dot{\mu}_s}{c} \right) + \frac{c_\mu}{eb} + \frac{2}{Rc} \left(b \dot{\mu}_\Sigma u_\Sigma + e \Pi_\Sigma \right) \right]^{\pm}$$

$$\left[e b \left(p_\Sigma + e_\Sigma \right) \right]^{\pm} = 0$$

$$[R]^{\pm} = 0$$

$$[e^2 - b^2 \dot{\mu}_s^2]^{\pm} = 0 \qquad \text{or} \qquad [c]^{\pm} = 0$$

In the case of net baryon number different from zero,an additional junction condition must be considered guarantee the baryon number conservation across the membrane. Assuming that the net baryon number in the shell is equal to zero,the baryon flux conservation is expressed by the equation

(18) $$[n u^i n_i]^{\pm} = 0$$

It is not worth writing explicitly the previous relation in the comoving frame since we obtain a trivial relation. Let's consider the Schwarzschield frame. The equation of Σ in these coordinates is

$$f = R_s(T) - R = 0$$

and the normal

(19) $$n_i = \frac{\partial f}{\partial x^i} = (S, -1, 0, 0)$$

(20) $$S = \frac{dR_s}{dT}$$

Using (19) and (1.5), eq.(18) becomes

(21) $$[n u^0 S - n u]^{\pm} = 0$$

It can be shown that S and u^0 can be written the following way

$$S = \frac{a u + b \Gamma \mu}{a \Gamma + b u \mu} \quad A \sqrt{\Gamma^2 - u^2}$$

$$u^0 = \frac{\Gamma}{a \sqrt{\Gamma^2 - u^2}}$$

Using these relations relations eq.(21) becomes

$$\left[\frac{m b \left(\Gamma^2 - u^2 \right)}{a \Gamma + b u \mu} \right]^{\pm} = 0$$

Because of the definition of b (1.15) and by the continuity of R (16)

(22) $$\left[\frac{\left(\Gamma^2 - u^2 \right)}{a \Gamma^2 - b u \mu} \right]^{\pm} = 0$$

In the case of zero baryon number, eq.(22) represents the conservation of the fluid element across the membrane.

3.4 INITIAL CONDITIONS

During our analysis we plan to follow the evolution of the bubble since its formation. According to thermal nucleation theory the nuclei with critical size of the new phase are formed at rest respect to the surrounding medium. The junction conditions, that have to be verified during the expansion of the bubble, must also be true, as a limit, at the initial time. Then initial conditions have to be consistent with the junction condition equations.

We assume that before the beginning of bubble nucleation, the universe is well described by the Robertson-Walker metric

(1) $$ds^2 = -dt^2 + a^2(t)\left[d\mu^2 + \mu^2 d\Omega^2\right]$$

here, we have taken k, the curvature constant, to be zero, and the time coordinate represents the proper time of cosmic fluid.

The formation of a bubble cannot affect the medium far away from it, and therefore the metric (1) is still valid at long distance. Locally one could expect an input of energy in the medium ahead of the bubble, but one can easily show that the input is negligible with respect to the total energy contained within a radius ten times bigger than the radius of the bubble. Moreover, this surplus of energy can be rapidly spread out by neutrino diffusion before the bubble begins to grow. Then the metric (1) can be assumed valid up to the membrane. This implies

(2) $$a^+ = 1$$

$$a^+\mu = 1$$

Since at the initial time the bubble is at rest, the junction condition (3.17) for the g_{00} metric function gives

(3) $$a_+ = a_- = 1$$

Moreover, we assume that inside the bubble the condition $\sigma_\mu = 0$ is also true.

Under the condition equation (3.15) becomes

(4) $$\left\{ \alpha \frac{\Gamma}{R} \right\}^{\pm} = \left[p_\Sigma \right]^{\pm}$$

Before nucleation, as the universe is spatially flat, is equal to 1. The small dimension of the bubble ($\tau <$ 10 fm) implies that $u \sim 10^{-17}$, then u^2 and (2M/R) are so small that although they slightly changed because of thermal fluctuations We can assume $\Gamma = 1$ also after the nucleation of the bubble. Relation (2) reduces to the classical expression for a critical radius we have seen in section 2.4

(5) $$R_{cr} = \frac{2\alpha}{p_+ - p_-}$$

where $\alpha = -\gamma$, γ surface tension.

Another condition which must be satisfied is the energy flux equation (3.9), which reduces to

$$(e_+ + p_+)\, v_+ = (e_- + p_-)\, v_- \qquad v = 1/n$$

This equation gives the relation between the proper volume of an element of fluid in the quark and in the hadron phases respectively. Here $(e_+ + p_+)$ $(e_- + p_-)$ the confinement causes a growth of the comoving volume of an element of fluid. In fact, according to the bag model equation of state, it results $(v_-/v_+) \simeq 3$.

CHAPTER FOUR

CHARACTERISTICS METHOD AND EXACT JUNCTION CONDITIONS TREATMENT

The system of equations and junctions conditions we have presented in section 3.1 and 3.3 does not allow an analytic solution and we have to solve it numerically. The most widely used technique in this kind of problems is the finite difference methods in which a suitable grid is superimposed on the space-time. The discretization of space-time implies that the fluid variables are computed only at some particular points. This lack of knowledge does not usually prevent the treatment of possible discontinuities like shocks. In these cases, in fact, it is possible to introduce artificial dissipative terms into the original equations so that the discontinuities are spread out into narrow but finite regions across which the fluid variables change rapidly but continuously.

On the contrary the kind of discontinuity which we are dealing with need to be studied exactly because the transition layer separates two different phases of strong interacting matter whose thermodynamical properties are discontinous if the transition if first order. Therefore, in our system, the junction conditions must be applied exactly and this requires us to know at each time step the values of fluid variables immediately behind and ahead of the transition layer.

For this purpose it is presented in this chapter a different form of relativistic hydrodynamical equations that allows us to compute the fluid variables near the discontinuity .

4.1 HYDRODYNAMICAL EQUATIONS IN A CHARACTERISTICS FORM.

When in a fluid such as viscosity and heat conduction can be neglected, the partial differential equations of motion that in this case are hyperbolic and so possess real characteristics, can be combined so that the resulting equations contain derivative in one direction only. The one parameter families of curves defined by these directions are simply called characteristics (Courant-Friedrichs 1948, Hoskin 1963).

Consider the equation of motion of a relativistic perfect fluid we have written in the Lagrangian form:

(1) $$u_t = -a\left[\frac{\Gamma}{b}\frac{p_\mu}{(e+p)} + \frac{Gm}{R^2} + 4\pi G p R\right]$$

(2) $$\frac{(nR^2)_t}{nR^2} = -a\frac{u_\mu}{R_\mu}$$

(3) $$e_t + \left(\frac{b_t}{b} + \frac{2R_t}{R}\right)(e+p) = 0$$

These are the relativistic analogues of the classical Euler, mass conservation and energy conservation equations, respectively.
Using the relations

$$R_t = a u$$

$$R_\mu = \Gamma / 4\pi R^2 n$$

we can rewrite eq.(2) in the form

(4) $$\frac{n_t}{n} + \frac{2au}{R} + a u_\mu \frac{4\pi R^2 n}{\Gamma} = 0$$

It result, also, for an adiabatic fluid

$$\frac{\partial m}{\partial t} = \left(\frac{\partial m}{\partial p}\right)_s \frac{\partial p}{\partial t} \qquad \text{with} \qquad \left(\frac{\partial m}{\partial p}\right)_s = \frac{1}{c_s^2 w}$$

The last equation is derived from the first law of thermodynamics with $c_s^2 = \left(\partial p/\partial \rho\right)_s$ being the sound speed and $w = \frac{\rho + p}{m}$ the specific enthalpy. Then equation (4) becomes

$$(5) \qquad \frac{1}{w m c_s^2} \frac{p_t}{Q} + 4\pi R^2 \frac{m}{\Gamma} u_\mu = -\frac{2u}{R}$$

After multiplying equation (5) by $M^{1/2} W^{1/2}$ and equation (1) by $(W/\ )^{1/2}$, their sum and difference are respectively

$$(6) \qquad \frac{1}{m c_s}\left(\frac{p_t}{Q} + 4\pi R^2 m c_s p_\mu\right) + \frac{w}{\Gamma}\left(\frac{u_t}{Q} + 4\pi R^2 m c_s u_\mu\right) =$$

$$= -\frac{w}{\Gamma}\left(\frac{2u}{R} c_s \Gamma + \frac{G m}{R^2} + 4\pi G R p\right)$$

$$(7) \qquad \frac{1}{m c_s}\left(\frac{p_t}{Q} - 4\pi R^2 m c_s p_\mu\right) + \frac{w}{\Gamma}\left(\frac{u_t}{Q} - 4\pi R^2 m c_s u_\mu\right) =$$

$$= -\frac{w}{\Gamma}\left(\frac{2u}{R} c_s \Gamma \quad \frac{G m}{R^2} \quad 4\pi G p R\right)$$

Now equations (6) and (7) are in a characteristic form and may be rewritten as

$$(8) \qquad \frac{1}{m w c_s} dp \pm \frac{1}{\Gamma} du = -\frac{Q}{\Gamma}\left(2u \frac{\Gamma}{R} c_s \pm \frac{G m}{R^2} \pm 4\pi G p R\right) dt$$

$$(9) \qquad \text{along} \qquad d\mu = \pm 4\pi R^2 m c_s Q \, dt$$

where the sign + refers to positive or forward characteristics and - refers to the negative or backwards characteristics.

Consider finally equation (3) and combine it with baryon number conservation equation, we can write

(10) $$e_t - n_t w = 0$$

and there being $e_t = \frac{1}{c_s^2} p_t$ we have the advective equation

(11) $$dp - c_s^2 w \, dn = 0$$

along $$d\mu = 0$$

From the characteristic form of the equation of motion, it appears more clear how the state of the fluid at some time t is influenced by the state of the fluid of precedent times. Consider, for example, the two time steps t and t+dt and assume that the state of the fluid is completely known at time t.

In order to determine the state of the system at the point P at time t + dt, we have drawn from P the backward and forward characteristics, using quaiton (9) and have computed the space coordinates of points M and L where they intersect the time level t. As we know completely the state of the system at time t is now possible, using the equaton of motion (8), to compute the velocity and the pressure at P. Then the advective equation (11) and the equaton of state give the baryon number density and the energy density at the same point P. The last two quantities a and R are coumputed by the advective equation R_t=a u and by equation (3.1.30). Actually the whole system of equations has to be solved iteratively until all variables have converged. If the numerical calculation is done on a finite mesh defined in advance (see fig. 1 the points A, B, C and P), it is ussually necessary to perform interpolations at L and M as the calculation proceeds.

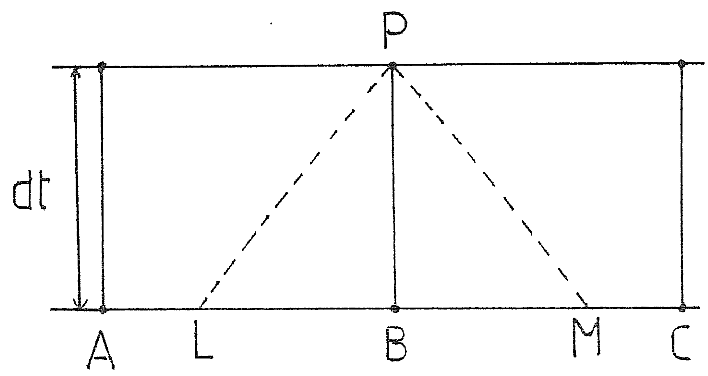

Fig. 1. Calculation of an ordinary point in the equal time mode. LP and MP are characteristics.

4.2 DISCONTINUITY POINTS

In the analysis made by Gyuloassy et al. (1984) in the case of a 1-dimensional flow in a Minkowski space time, it emerged that two possiblee situation can arrive in the transition front. For a strong supercooling, the transition is mediated by a shock wave (detonation front) and the hadronic matter is produced in a superheated state. In the case of a small supercooling, that is, in fact, the most likely situation, the transition proceeds as a slow conbustion sub-sonically with respect to the quark-gluon phase. This transition front is called deflagration front. We are going to see now the difference between the two situations in terms of characteristic curves.

a) Strong detonation front

According to the Jouguet's rule (Courant-Friedrichs 1976, pg. 215), a strong detonation front is supersonic with respect to the medium ahead and subonic with respect to the medium behind. This situation is shown in fig. 2.

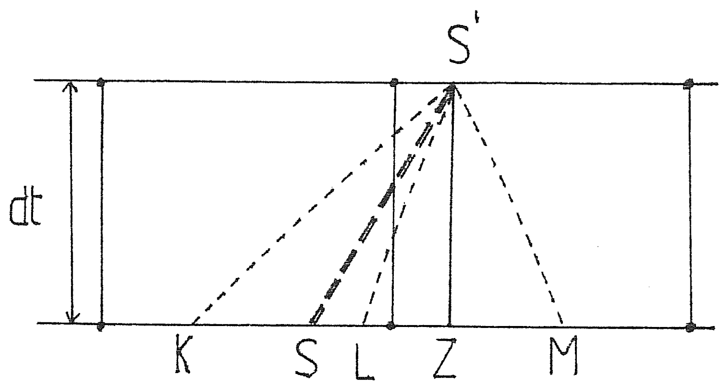

Fig.2. Calculation of a detonation in the equal-time mode. SS' is the path of the detonation front.

The unknown quantities on each side are the fluid velocity u, the presssure p, the baryon number density n, the Schwarzschild radius R and the g_{00} metric component a (remenber that the coefficient b in the metric (3.13) is expressed in terms of n and R because of our particular choice of comoving coordinates), and eventually detonation velocity. The energy density is derived from the equation of state.

As we have done before for an ordinary point of the fluid, we assume, again, the knowledge of all the quantities at the time level t. From fig. 2 we see that the state of the fluid can be completely determined ahead of detonation front because the discontinuity is supersonic with respect to this region and the fluid cannot be influenced by it. In the medium behind, on the contrary, only the forward characteristic is present: all the other information is given by the five junction conditions we have seen in section 3.3. (two for the metric and three due to momentum, energy and baryon number conservation). So, at the end, we are able to determine completely the state of the fluid ahead and behind the discontinuity and also the velocity of detonation front.

b) Weak deflagration front

In this case, the reaction front is subsonic with respect to the medium on both sides. The situation is illustrated in fig. 3.

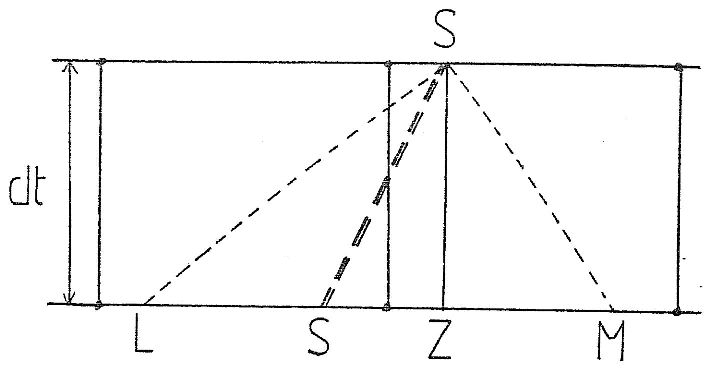

Fig.3. Calculation of a deflagration in the equal-time mode. SS' is the path of the deflagration front.

We see immediately that, compared to the previous case, thereis a lack of knowledge because there is not the forward characteristics in the medium ahead, there the state of the system cannot be completely determined without an additional information. For this purpose, it is necessary either to study in detail the dynamics in the transition layer, or to make some suitable assumption about one of the variables of the system.

In our particular case, it appears very reasonable to assume that the temperature is the same on the two sides of the layer. The fact that strong interacting matter is, in the early universe, at this temperature, in equilibrium with lepton and photons and, in particular, the presence of neutrinos that are not yet decoupled from the other particle species, should assure the thermal equilibrium between the two sides of the transition layer.

There, assuming $T_+ = T_-$, the characteristic equations and the junction conditions allow us to compute the fluid variables and the deflagraiton velocity.

Let us concentrate on the deflagration case that seems more likely in our problem. The most practical way for calculating the state of the system at S' is the following:

1) Estimate the position of S' at the time step t +dt using the deflagration velocity $\dot{\mu}_S$ at S.
2) Extrapolate the value of T at S from the value of T at the deflagration front in the two previous time steps.
3) For the element of fluid immediately ahead we can cumpute
   - i) $u_+(T)$ using the backward characteristic;
   - ii) $n_+(T)$ using the advective equation (1.11);
   - iii) $R = R(\mu)$ using the advective eqaution $R_t = a\,u$
   - iv) a_+ from the metric condition (3.1.30).

   For the fluid element behind
   - i) $u_-(T)$ using the forward characteristics.
4) The junction conditions presented in section 3.3 are then used for

   calculating a_-, $\dot{\mu}_{S'}$, b_- and a better estimate of T (actually appears more convenient to use T^4 as p and e depend only on T^4).

   For the numerical calculation it is more convenient to rewrite the junction condition in the following form

$$\eta^2\left(\Gamma_-^2 + \beta^2 u_+^2\right)x^4 - 2\eta\beta\left(u_+ u_- + \eta\,\Gamma_+\Gamma_-\right)x^3 + \left[\eta^2\beta^2\left(\Gamma_+^2 - u_+^2\right) - \left(\Gamma_-^2 - u_-^2\right)\right]x^2 + 2\beta\left(\eta\,u_+ u_- + \Gamma_+\Gamma_-\right)x - \left(u_-^2 + \beta^2\Gamma_+^2\right) = 0 \tag{1}$$

(2) $$y = \frac{1}{\eta x}$$

(3) $$\dot{\mu}_{s'} = \frac{e_+}{b_+} \frac{(x\Gamma_- - \beta\Gamma_+)\eta x}{(\eta x v_+ \beta - v_-)}$$

(4) $$T^4 = \frac{1}{\left[e b^2 \dot{\mu}^2/T^4 + p e^2/T^4\right]^{I}} \left(- \frac{\sigma c^2}{2} \left\{ \frac{1}{eb} \frac{d}{dt} \left(\frac{b\dot{\mu}}{c} \right) + \frac{c_\mu}{eb} + \frac{2}{cR} \left(e\Gamma_+ b u \dot{\mu} \right) \right\}^{\pm} \right.,$$

where we have introduced the notation

$$x = e_- / e_+$$

$$y = b_- / b_+$$

$$\eta = (e + p)_- / (e + p)_+$$

$$\beta = (\Gamma^2 - u^2)_- \, / \, (\Gamma^2 - u^2)_+$$

From equation (4), we have the new estimate of $T_{S'}$

5) $\dot{\mu}_{S'}$ allows a better estimate of S' and then the cycle is repeated from 2) until all the quantities have converged.

4.3 FUTURE RESEARCH

The system of equations we have presented in section 3.2, 3.3 and 4.1, we will allowed us to fallowed in detail the growth of a spherical bubble, once known the equations of state of the two phases and the surface tension of the separation layer. Our method does not give a full description of a first order phase transition, but the knowledge of the dynamics of a bubble is essential for a better understanding of the whole transition.

In our numerical analysis we plan to compute the velocity of the transition front and how this velocity depends on the degree of supercooling. We expect that the transition propagates as a deflagration, but the detonation case (Steinhardt 1982) is not, a priori, excluded. We want also to check if any shock discontinuity is formed ahead of the confinement front in the case of deflagration, The comparison between these velocities is important because, if shock discontinuity propagates rapidly, far away from the deflagration, their collisions can cause turbolent medium where the successive growth of the bubbles will be slowed down. Eventually, we will modify the equation of state of the two phases in order to consider a net baryon number density, $\mu \neq 0$.

From the numerical analysis, we should be able to see where any inhomogeneity in baryon number density is formed. We plan also to include in the hydrodynamical equations neutrino transport term as its effect should be important especially for a high temperature gradient. The knowledge of bubble growth velocity and nucleation probability will allow us to compute also the length of the tr sition.

The final part of the transition when bubbles meet and collide (Midorikawa 1985, Hawking et al. 1982) cannot be treated in our context but our method is again suitable for treating the shrinking of the regions left in the quark phase at the end of the transition.

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