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BARYON NUMBER NON-CONSERVATION IN SUPERSYMMETRIC MODELS

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SISSA - SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI

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CONTENTS

1. INTRODUCTION

That the two concepts mentioned in the title, "supersymmetry" and "baryon number non-conservation", have something to do with reality, it is far from being proven, in the sense that any clear experimental support for them is still lacking. I feel therefore obliged to sketch in this chapter the main motivations for : a) introducing supersymmetry into particle physics; b) taking into account the possibility that baryon number, B (and lepton number, L), might be only approximately conserved quantities. A plan of the thesis will close this introduction.

1.1: Why supersymmetry in particle physics?

The most obvious prediction of exact supersymmetry [1.1], namely the degeneracy of boson and fermion masses, is clearly in contrast with experiment. One is therefore led to envisage that supersymmetry, if realized in Nature, is badly broken at the scale of present accelerator energies. Thus the only reasons leading to the belief that supersymmetry may play a fundamental role in particle physics are theoretical ones. A tentative list is the following :

- It is the only non-trivial extension of the space-time Poincarè symmetry which allows particle multiplets of different spin and statistics : more precisely, the supersymmetry algebra is the only graded Lie algebra of symmetries of the S-matrix consistent with relativistic quantum field theory [1.2].
- It provides a "raison d'être" for the existence of elementary scalar fields in ordinary spontaneously broken

gauge theories (for instance, the standard model of strong and electro-weak interactions) .

- It unifies matter with radiation, to the extent that vector gauge fields (and the tensor gravitational field) necessarily have their associated fermionic partners.

- It may ultimately lead to the unification of all elementary forces of Nature; at least, supergravity [1.3] provides a possible theoretical framework for this achievement : in the context of N=8 extended supergravity [1.4] all elementary fields, including the graviton, sit in a single irreducible representation of the supersymmetry algebra.

- It implies a softening of quantum divergencies, due to mutual cancellations between boson and fermion loops in Feynman diagrams : supersymmetry not only survives quantization but improves the ultraviolet behaviour of relativistic quantum field theories. In particular, the vanishing of the β-function to all orders in perturbation theory for the N=4 supersymmetric Yang-Mills theory [1.5] and the greatly improved divergence structure of N=8 supergravity [1.6] open up the hope of achieving a finite field theory.

- It may solve (and this is the most popular argument in favour of supersymmetry) the so-called "hierarchy problem" [1.7] of ordinary interactions : there is no natural explanation of why the weak interaction scale ($m_w \sim 10^2$ GeV) is so small compared with the natural ultraviolet cutoff of the theory, i.e. the grand-unification scale ($M_x \sim 10^{15}$ GeV) or the Planck scale ($M_p \sim 10^{19}$ GeV). In the framework of perturbative unification the hierarchy problem has also a more technical aspect :

even if one adjusts the parameters of the theory to ensure the desired hierarchy at the tree level ($m_w \ll M_x$ or M_p), this hierarchy is not stable under radiative corrections. This problem is related to the fact that in a general renormalizable theory scalar masses are not protected by any symmetry (in contrast with fermion masses, which are protected by chiral symmetry), and receive quadratically divergent radiative corrections : even if a scalar field has zero mass at the tree level, after quantum corrections its mass will be naturally of order M_x (or M_p), unless an unnatural "fine-tuning" is made. Supersymmetry is the only known symmetry which prevents scalar self-energy diagrams to be quadratically divergent. Other problems of similar type, whose solution may come from supersymmetry, are the smallness of Einstein's cosmological constant [1.8] and of the CP-violating Θ-parameter of QCD [1.9] .

1.2: Why baryon number might be non-conserved ?

The conservation law of baryon number is certainly well satisfied : up to now there is no clear evidence of any process which can violate it [1.10] . Nevertheless, there are strong theoretical motivations for believing that B is not exactly conserved in Nature :

- It seems very unlikely that the apparent conservation of B may be linked to an exact local symmetry : a massless gauge boson coupled to baryon number would introduce discrepancies in the Eötvös experiment unless its coupling were incredibly weak [1.11].
- In the standard model of strong and electro-weak interactions the global symmetries associated to the conser-

vation of B and L in renormalizable interactions are incidental consequences of the choice of the fundamental fields and of the gauge invariance with respect to $SU(3)_c \times SU(2)_l \times U(1)_y$: there is no need to introduce independently a conservation principle for baryon and lepton number. In the extensions of the standard model this might be no longer true, due to the presence of additional particles which could participate in renormalizable B and/or L non-conserving interactions. Of course, one could simply impose B and L conservation laws as global symmetries of these models. This does not seem to be the most fruitful approach : global symmetries, like strangeness, isospin, etc., increasingly appear to us as incidental consequences of gauge symmetries and renormalizability, with no status as a priori constraints on a fundamental level.

- The existence of new interactions with $\Delta B \neq 0$ and/or $\Delta L \neq 0$ is naturally predicted by most of the grand unified models of strong and electro-weak interactions [1.12,1.13]. In these models there is also the intriguing possibility of baryon decay catalyzed by grand-unified monopoles [1.14].
- Even in the standard model B and L are not exactly conserved quantities, due to non-perturbative effects (which are however negligible from an experimental point of view) [1.15]. Non-perturbative quantum gravitational effects such as virtual black holes may also lead to baryon decay [1.16].
- A phenomenological explanation of the apparent excess of baryons over antibaryons in our universe seems to be possible, thanks to the interplay of baryon number violation, CP violation and disequilibrium in the very early universe [1.17].

1.3: Plan of the thesis

Baryon number violating processes have been widely investigated in the context of non-supersymmetric unified models [1.17]. In this thesis I shall try to review the different possibilities for B non-conservation which arise in supersymmetric models of strong and electro-weak interactions (without and with grand-unification or super-unification), limiting the discussion to the framework of a perturbative treatment.

Chapter 2 is devoted to a general analysis of the supersymmetric and gauge-invariant operators, carrying non-vanishing baryon and/or lepton number, that correspond to different choices of the gauge group. Chapters 3 and 4 are dedicated to a more detailed study of the mechanisms that can give rise to two particular processes : nucleon decay and neutron-antineutron oscillations. Some well-known results, frequently used but not explicitly mentioned in the text, are collected in the Appendices.

2. GENERAL OPERATOR ANALYSIS

This chapter is devoted to the construction (for different choices of the gauge group and of the associated set of fundamental superfields) of the lowest-dimensional supersymmetric and gauge-invariant operators with $\Delta B \neq 0$ and/or $\Delta L \neq 0$, corresponding to some possible selection rules. A preliminary analysis of their phenomenological implications, taking into account only the general features of supersymmetry and gauge-symmetry breaking, is carried out for some of them. The remaining ones, which can give rise to nucleon decay or neutron-antineutron oscillations in the context of specific models, will be studied in detail in the following chapters.

2.1: The standard gauge group $SU(3)_c \times SU(2)_l \times U(1)_y$

2.1.1: General form of the lagrangian and related troubles

To begin with, let us consider supersymmetric models of strong and electro-weak interactions based on the standard gauge group $G_o \equiv SU(3)_c \times SU(2)_l \times U(1)_y$. Such models will be regarded as effective low-energy theories (°), described by a renormalizable (°°) lagrangian, $\mathcal{L}$ (expressed in terms of low-energy fields only), which can be written as the sum of a globally supersymmetric part, $\mathcal{L}_{SUSY}$, plus

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(°) Here and in the following "low-energy" will stand for "$E \lesssim$ 1-10 TeV".

(°°) The possibility of non-renormalizable effective interactions between low-energy fields will be considered in subsection 2.1.4.

a soft supersymmetry-breaking part, $\mathcal{L}_{SOFT}$ (°).

In general, $\mathcal{L}_{SOFT}$ will contain mass terms for scalar bosons and gauge fermions, as well as particular trilinear couplings among scalar bosons. Several models have been constructed, based on N=1 supergravity [2.2], in which supersymmetry is broken by vacuum expectation values of scalar fields in a "hidden sector", and the effects of the breaking are transmitted to ordinary matter (the "observable sector") only through gravity : they give rise to a $\mathcal{L}_{SOFT}$ whose form is highly constrained, depending only on a few parameters of the underlying supergravity model. For the time being, however, I do not want to commit myself with the specific form of $\mathcal{L}_{SOFT}$, in order to keep the analysis as general as possible.

To construct the supersymmetric and gauge-invariant lagrangian, $\mathcal{L}_{SUSY}$, one has to decide :

1) the set of chiral superfields appearing in the low-energy theory and their transformation properties with respect to G_o;
2) the superpotential, i.e. an arbitrary gauge-invariant polynomial of at most degree three in the chiral superfields (or, equivalently, in their scalar components).

If one wants to give masses to quarks and leptons and break $SU(2)_l xU(1)_y$ down to $U(1)_{e.m.}$ according to the standard perturbative mechanism, avoiding at the same time $SU(2)_l$ $xU(1)_y$ anomalies, a minimal set of left-handed chiral superfields is that appearing in Table I [2.3]. To fix the notation, gauge vector superfields are also collected in Table II. Other conventions used in the following may be found in Apeendix A.

(°) Soft breaking of supersymmetry is studied in reference [2.1].

Table I

Minimal set of chiral superfields for supersymmetric models based on the standard gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. Chiral superfields are denoted by capital letters, their spin-0 components by capital script letters, their spin-1/2 components by lower case letters. Quantum numbers are indicated with respect to G_0, with the convention $Q_{em} = Y + T_{3L}$. Index $\alpha = 1,2,3$ refers to $SU(3)_c$, index i=1,2 to $SU(2)_L$, while a=1,2,3 is a generation index.

Superfield	Component fields	Quantum numbers
$Q_a^{\alpha i}$	$(\mathcal{Q}_a^{\alpha i}, q_a^{\alpha i}, F_{Q a}^{\alpha i})$	$(3,2,+1/6)$
$U^c_{a\alpha}$	$(\mathcal{U}^c_{a\alpha}, u^c_{a\alpha}, F_{U^c a\alpha})$	$(\bar{3},1,-2/3)$
$D^c_{a\alpha}$	$(\mathcal{D}^c_{a\alpha}, d^c_{a\alpha}, F_{D^c a\alpha})$	$(\bar{3},1,+1/3)$
L_a^i	$(\mathcal{L}_a^i, l_a^i, F_{L a}^i)$	$(1,2,-1/2)$
E_a^c	$(\mathcal{E}_a^c, e_a^c, F_{E^c a})$	$(1,1,+1)$
H^i	$(\mathcal{H}^i, h^i, F_H^i)$	$(1,2,-1/2)$
H'^i	$(\mathcal{H}'^i, h'^i, F_{H'}^i)$	$(1,2,+1/2)$

Table II

Gauge vector superfields associated to the standard group $G_0 \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$. Vector superfields are denoted by capital letters, their spin-1 components by the general symbol v^m, their spin-1/2 components by the general symbol λ. The Wess-Zumino gauge is adopted. Index A=1,2,...,8 refers to $SU(3)_c$, index I=1,2,3 to $SU(2)_L$. Quantum numbers are given with respect to G_0.

Superfield	Component fields	Quantum numbers
G^A	$(v_G^{mA}, \lambda_G^A, D_G^A)$	$(8,1,0)$
W^I	$(v_W^{mI}, \lambda_W^I, D_W^I)$	$(1,3,0)$
B	(v_B^m, λ_B, D_B)	$(1,1,0)$

In terms of the chiral superfields of Table I, the most general form of the superpotential is the following :

$$f = f_I + f_{II} + f_{III} \tag{2.1}$$

$$f_I = LE^cH + QD^cH + QU^cH' + HH' \tag{2.2}$$

$$f_{II} = LH' + LE^cL + QD^cL \tag{2.3}$$

$$f_{III} = U^cD^cD^c \tag{2.4}$$

In the above formulas, in order to make clear the physical content of each term, a synthetic notation has been used, omitting all coefficients as well as group and generation indices. The explicit expressions corresponding to the different terms have been collected, for future reference, in Table III. Note that terms of the form HE^cH would be identically zero, while terms $U^cD^cD^c$ and LE^cL vanish for superfields belonging to the same generation. Note also that, with the usual assignments of baryon and lepton number to chiral superfields, recalled in Table IV, f_I is characterized by $\Delta B=\Delta L=0$, f_{II} by $\Delta B=0, \Delta L=-1$ and f_{III} by $\Delta L=0, \Delta B=+1$.

Table III

Explicit expressions of the different terms appearing in the G_0-invariant superpotential f. Symbols $\epsilon^{\alpha\beta\gamma}$ $(\alpha,\beta,\gamma=1,2,3)$ and ϵ_{ij} $(i,j=1,2)$ are completely antisymmetric tensors of $SU(3)_c$ and $SU(2)_L$, respectively, normalized according to the following conventions : $\epsilon^{123}=+1$, $\epsilon_{12}=+1$.

Term	Explicit expression
LE^cH	$\Gamma^e_{ab}\varepsilon_{ij}L^i_a E^c_b H^j$
QD^cH	$\Gamma^d_{ab}\varepsilon_{ij}Q^{\alpha i}_a D^c_{b\alpha} H^j$
QU^cH'	$\Gamma^u_{ab}\varepsilon_{ij}Q^{\alpha i}_a U^c_{b\alpha} H'^j$
HH'	$\mu\varepsilon_{ij}H^iH'^j$

Table III (continued)

LH'	$m_a \varepsilon_{ij} L^i_a H'^j$
LE^cL	$\Gamma^{L1}_{abc} \varepsilon_{ij} L^i_a E^c_b L^j_c$
QD^cL	$\Gamma^{L2}_{abc} \varepsilon_{ij} Q^{\alpha i}_a D^c_{b\alpha} L^j_c$
$U^cD^cD^c$	$\Gamma^{B}_{abc} \varepsilon^{\alpha\beta\gamma} U^c_{a\alpha} D^c_{b\beta} D^c_{c\gamma}$

Table IV

Usual assignments of baryon and lepton number to the fundamental chiral superfields of Table I. Note that, in presence of the terms in f_{II}, which induce mixing between L and H, the assignment of lepton number is not obvious : this point is crucial in the discussion of lepton number violations [2.4,2.5], but will be skipped in the context of the present analysis, focused on baryon number non-conservation.

Superfield:	Q	U^c	D^c	L	E^c	H	H'
B :	+1	−1	−1	0	0	0	0
L :	0	0	0	+1	−1	0	0

As can be easily seen, the situation here is radically different from the standard (non-supersymmetric) model : there gauge invariance and representation content were sufficient to ensure B and L conservation in all renormalizable interactions; here, due to the presence of new particles (the scalar superpartners of quarks and leptons and the fermion superpartners of Higgs bosons), those accidental symmetries are lost, and one has to face B and L non-conservation already at the level of renormalizable interactions among low-energy particles [2.3,2.6].

Let us see immediately the most spectacular consequence of the new B and L violating interactions. The simultaneous presence of the two terms QD^cL and $U^cD^cD^c$ in the superpotential induces in the supersymmetric lagrangian $\mathcal{L}_{SUSY}$, among the others (°), the following Yukawa couplings :

$$q l \mathcal{D}^c \ , \text{ or :} \qquad -\Gamma^{L2}_{abc}\,\varepsilon_{ij}\,q^{\alpha i}_a\, l^j_c\, \mathcal{D}^c_{b\alpha} \tag{2.5}$$

$$\overline{u^c}\,\overline{d^c}\,\mathcal{D}^{c*} , \text{ or :} \qquad -\Gamma^{B}_{abc}\,\varepsilon_{\alpha\beta\gamma}\,\overline{u^c}^{\alpha}_a\,\overline{d^c}^{\beta}_b\,\mathcal{D}^{c*\gamma}_c \tag{2.6}$$

Those two interactions conspire, through the exchange of virtual scalar quarks of electric charge -1/3, to originate graphs like that in Fig.1, inducing $\Delta B=\Delta L$ nucleon decay at the tree level.

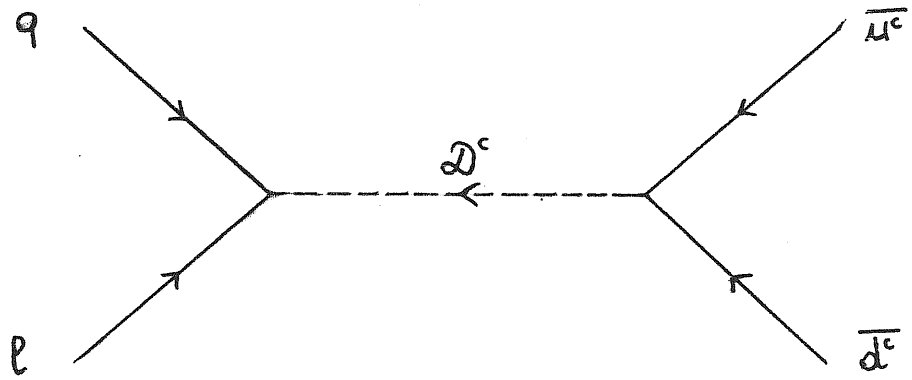

Figure 1 - Graph inducing $\Delta B = \Delta L$ nucleon decay at the tree level, mediated by the exchange of a virtual scalar quark of charge -1/3 and originated by the terms QD^cL and $U^cD^cD^c$ in the superpotential. Indices are omitted for simplicity.

Even neglecting the details of gauge and supersymmetry breaking, it seems natural to assume that scalar quarks have masses m_{sq} of order 10^2- 10^3 GeV : this turns out to

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(°) The general form of a renormalizable, supersymmetric and gauge-invariant lagrangian is recalled in Appendix B, together with its expression in terms of component fields (in the Wess-Zumino gauge).

be the case in many realistic models and is compatible with both the present experimental limits [2.7] and the solution of the technical aspect of the gauge-hierarchy problem. Then, putting for simplicity $\Gamma^B_{abc}=\Gamma^B$ ($b\neq c$) and $\Gamma^{L2}_{abc}=\Gamma^L$, the nucleon decay rate corresponding to the graph of Fig.1 is roughly given by :

$$\Gamma \simeq \frac{\Gamma^{B^2}\Gamma^{L^2}}{m^4_{sq}} m^2_p \;|\Psi(0)|^2 \;, \tag{2.7}$$

where m_p is the proton mass and $|\Psi(0)|^2 \simeq 2\times10^{-3}\,\mathrm{GeV}^3$ is a factor taking into account wave-function effects, which can be naively interpreted as the probability of finding two interacting quarks at the same point inside the nucleon ($^{\circ}$). Taking for the nucleon lifetime the conservative limit $\tau_N \gtrsim 10^{31}$ years, consistency with experiment requires the strong condition :

$$\sqrt{\Gamma^B\Gamma^L} \lesssim 10^{-12} - 10^{-13} \;. \tag{2.8}$$

In other words, to avoid conflict with nucleon decay experiments, at least one of the Yukawa coupling constants Γ^B, Γ^L must be fine-tuned to an incredibly small value, several orders of magnitude smaller than the typical values appearing in f_I.

($^{\circ}$) In formula (2.7) numerical factors of order unity, irrelevant for the present discussion, have been omitted for simplicity : for a detailed treatment see, for example, reference [2.8].

2.1.2: A possible way out : additional global symmetries

The situation outlined in the preceeding subsection, even if technically natural, is clearly unsatisfactory. One possible way out is well known : there must be a symmetry reason underlying the incredible smallness of the Yukawa coupling constants Γ^B or Γ^L. The most obvious choice is of course to impose the global symmetries associated to baryon and lepton number. This radical solution, however, is in contrast with the spirit of the present work, as explained in the introduction. On the other hand, leaving aside for the moment the possibility of extending the gauge group, there are many other global symmetries, weaker than B and L, that can forbid at least one of the dangerous terms QD^cL and $U^cD^cD^c$ in the superpotential. A tentative list is the following :

<u>Matter-parity</u> [2.3] : it is a discrete reflection symmetry under which quark and lepton superfields change their sign, while all the other superfields remain unchanged. Imposing matter-parity forbids all B and L violating terms in the superpotential (those contained in f_{II} and f_{III}), allowing only the $\Delta B=\Delta L=0$ terms contained in f_I.

<u>R-parity</u> [2.4] : it acts on superfields exactly in the same way as matter-parity, but in addition it changes the sign of the anticommuting coordinates θ and $\bar{\theta}$; operationally it is perfectly equivalent to matter-parity, since every vertex involves an even number of fermions; it must not be confused with the continuous global R-invariance.

<u>SY-parity</u> [2.6] : introduced in this context by Sakai and Yanagida, it is a multiplicative parity which leaves vector superfields unchanged and acts on chiral superfields in the following way :

$Q\to -Q$, $U^c\to -U^c$, $E^c\to -E^c$, $D^c\to D^c$, $L\to L$, $H\to -H$, $H'\to H'$;
these apparently strange assignments are clearly inspired by SU(5) grand unification; imposing SY-parity forbids all B and L violating terms contained in f_{II} and f_{III}, with the exception of the superrenormalizable term LH', and in addition it forbids the Higgs mass term HH' contained in f_I.

Lepton-parity [2.4,2.9] : weaker than matter-parity, it changes the sign only of lepton superfields (L,E^c), leaving all the other superfields unchanged; in the superpotential it forbids all the L violating terms contained in f_{II}, but it allows, in addition to f_I, also the B violating term $U^cD^cD^c$ of f_{III}.

Quark-parity [2.4,2.9] : weaker than matter-parity, it changes the sign only of quark superfields (Q,U^c,D^c), leaving all the other superfields unchanged; in the superpotential it forbids the B violating term f_{III}, but it allows, in addition to f_I, also the L violating terms contained in f_{II}.

Fiveness [2.6] : it is a global U(1) symmetry, equivalent to B-L through the relation B-L = ("fiveness"+4Y)/5 ; all vector superfields of G_o have zero fiveness, while the fiveness assignments of the chiral superfields are the following :
$Q(+1)$, $U^c(+1)$, $E^c(+1)$, $D^c(-3)$, $L(-3)$, $H(+2)$, $H'(-2)$;
such a symmetry is clearly suggested by ordinary SU(5) grand unification, and forbids all terms in f_{II} and f_{III}, while allowing all those in f_I.

R-invariance [2.6] : it is a global U(1) symmetry acting non-trivially on superfield coordinates; its action on a generic chiral superfield $\Phi_i(x,\theta)$ is the following :

$$\Phi_i(x,\theta) \longrightarrow e^{2ir_i\alpha}\,\Phi_i(x,e^{-i\alpha}\theta)$$

and the corresponding action on the antichiral conjugate :

$$\Phi_i^\dagger(x,\bar\theta) \rightarrow e^{-2ir_i\alpha}\,\Phi_i^\dagger(x,e^{i\alpha}\bar\theta),$$

where α is a real parameter and r_i is called the R-character of Φ_i. Clearly, R-invariance requires that all $\theta\theta$-terms have R-character r=+1, all $\bar\theta\bar\theta$-terms r=-1 and all $\theta\theta\bar\theta\bar\theta$-terms r=0. The R-characters of the chiral superfields of Table I are assigned as follows :

$r=+1/2$ for Q,U^c,D^c,L,E^c; $r=0$ for H,H'.

In this way R-invariance forbids all the terms contained in f_{II} and f_{III}, and, in addition, the term HH' contained in f_I.

The action of the global symmetries considered above on the chiral superfields of Table I, as well as their consequences on the terms appearing in the most general superpotential f, are summarized in Tables V and VI.

Table V

Transformation properties of the fundamental chiral superfields of Table I under some global symmetries, introduced to prevent renormalizable interactions leading to a too fast nucleon decay.

Chiral superfield \ Global symmetry	Matter parity	SY parity	Lepton parity	Quark parity	Fiveness	R invariance
Q	-1	-1	+1	-1	+1	+1/2
U^c	-1	-1	+1	-1	+1	+1/2
D^c	-1	+1	+1	-1	-3	+1/2
L	-1	+1	-1	+1	-3	+1/2
E^c	-1	-1	-1	+1	+1	+1/2
H	+1	-1	+1	+1	+2	0
H'	+1	+1	+1	+1	-2	0

Table VI

Action of the global symmetries of Table V on the terms appearing in the most general G_0-invariant superpotential f.

Term \ Global symmetry	Matter parity	SY parity	Lepton parity	Quark parity	Fiveness	R invariance
LE^cH	yes	yes	yes	yes	yes	yes
QD^cH	yes	yes	yes	yes	yes	yes
QU^cH'	yes	yes	yes	yes	yes	yes
HH'	yes	no	yes	yes	yes	yes
LH'	no	yes	no	yes	no	no
LE^cL	no	no	no	yes	no	no
QD^cL	no	no	no	yes	no	no
$U^cD^cD^c$	no	no	yes	no	no	no

A few comments are now in order. One can see from Table VI that all the global symmetries considered above achieve the desired goal, i.e. they forbid at least one of the dangerous terms QD^cL and $U^cD^cD^c$ in the superpotential, allowing at the same time the terms LE^cH, QD^cH and QU^cH', which are needed to give masses to quarks and leptons according to the standard mechanism of gauge-symmetry breaking. However, there are many other symmetries, both discrete and continuous, which can do that job : those listed above are considered the least artificial possibilities, in the sense that one can hope to explain their origin, in the context of a more fundamental theory (grand unification, compositeness, etc.), in terms of gauge symmetries and/or dynamical effects.

The symmetries considered above can be divided into two groups. In a first group one can put those symmetries (SY-parity, lepton-parity, quark-parity) which do not forbid all B and L violating terms in the superpotential : in this case there are still some possibilities for B or L non-conserving renormalizable interactions among low-energy particles, one of which will be examined in subsection 2.1.3.
In a second group one can put those symmetries (matter-parity, fiveness, R-invariance) which forbid all B and L violating terms in the superpotential, so that there are no B and L non-conserving renormalizable interactions among low-energy particles. This does not mean, however, that B and L are exactly conserved ; even in this case one can construct B and/or L violating non-renormalizable interactions (invariant under supersymmetry, gauge-symmetry and the additional global symmetry), which can be interpreted as effective interactions arising from the exchange of heavy particles of an underlying theory : this possibility will be examined in subsection 2.1.4.

2.1.3: A residual possibility for B violating renormalizable interactions between low-energy particles

Among the global symmetries considered in the preceeding subsection, there are some which still allow B or L violating interactions in renormalizable terms.

On one hand, the imposition of SY-parity or quark-parity makes B an exact symmetry of the renormalizable supersymmetric lagrangian, $\mathcal{L}_{SUSY}$. However, some terms appearing in the L violating part f_{II} of the superpotential are still allowed. The discussion of the corresponding phenomenology of lepton number violations [2.4,2.5] (which is, by the way, rather complicated and strongly model-dependent) is outside the purpose of the present work and will not be pursued further.

On the other hand, the imposition of lepton-parity results into a superpotential which conserves exactly lepton number L, but contains the B violating term f_{III}. What is the pattern of B non-conservation arising from such a context ? I have discussed this problem in a recent paper [2.9].

First of all, one has to examine the opportunities for nucleon decay. Since lepton number is exactly conserved in renormalizable terms, neglecting for the moment the possibilty of non-renormalizable interactions originated by the exchange of "exotic" heavy particles, nucleon decay can occur only according to the selection rule $\Delta B=-1, \Delta L=0$. Let us consider the general reaction :

$$\text{nucleon } (B=1, L=0) \longrightarrow \text{final state } (B=0, L=0).$$

Angular momentum conservation, combined with the $\Delta B=-1, \Delta L=0$ selection rule, requires a final state containing, among the other things, either a bosonic superpartner of an ordinary fermion (squark, slepton) or a fermionic superpartner of an ordinary boson (gaugino, higgsino).

At this point, in order to determine the kinematically allowed channels, some model-dependent information about the particle mass spectrum is needed. To keep the treatment as general as possible, let us consider two opposite possibilities :

1) at least one of the superpartners quoted above is sensibly lighter than the nucleon (as an illustrative example I shall take the case of a photino, λ_γ, with a mass of order 100 MeV);
2) all the superpartners quoted above have masses higher

than the typical nucleon mass, $m_{\mathcal{N}} \sim 1$ GeV.
Both cases can apparently occur in realistic models (in which, however, the possible existence of the terms f_{II} and f_{III} has been often ignored) [2.2].

In the first case $\Delta B=-1, \Delta L=0$ decay is therefore possible, being induced, for example, by the graph in Fig.2, which can give rise to reactions of the type : nucleon $\rightarrow$ photino + mesons.

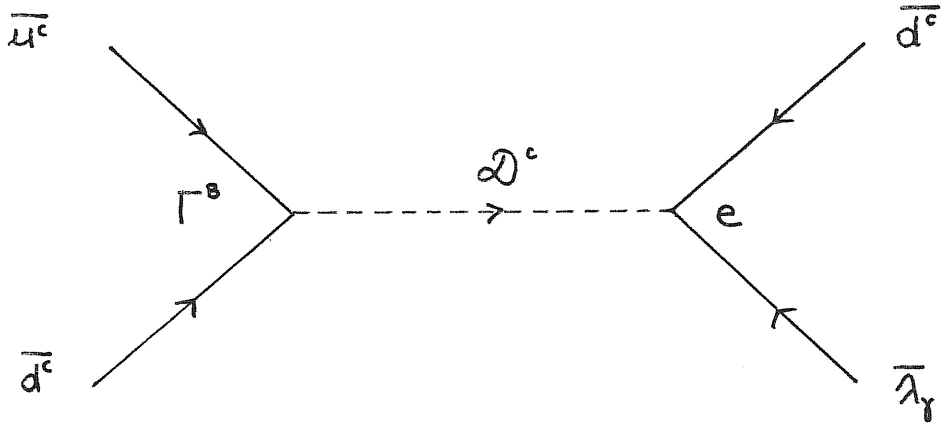


Figure 2 - Graph inducing $\Delta B=-1, \Delta L=0$ nucleon decay into photino (λ_γ) plus mesons, generated by the B violating term $f_{III} = U^c D^c D^c$ in the superpotential (e stands for the electromagnetic gauge coupling constant).

A rough order-of-magnitude estimate, based on dymensional analysis, is sufficient to see that, assuming squark masses not much higher than 1 TeV and Yukawa couplings in Γ^B not much smaller than 10^{-6}, the graph in Fig.2 induces nucleon decay at a catastrophic rate with respect to the experimental limits, even taking into account the unconventional selection rule under consideration.

The situation is radically different in the second case, where the process of Fig.2 is forbidden by energy conservation, and nucleon becomes stable, even if B is not a symmetry of the renormalizable lagrangian. However, $|\Delta B|=2, \Delta L=0$

transitions are still possible, in particular neutron-antineutron oscillations, described by graphs like that represented in Fig.3.

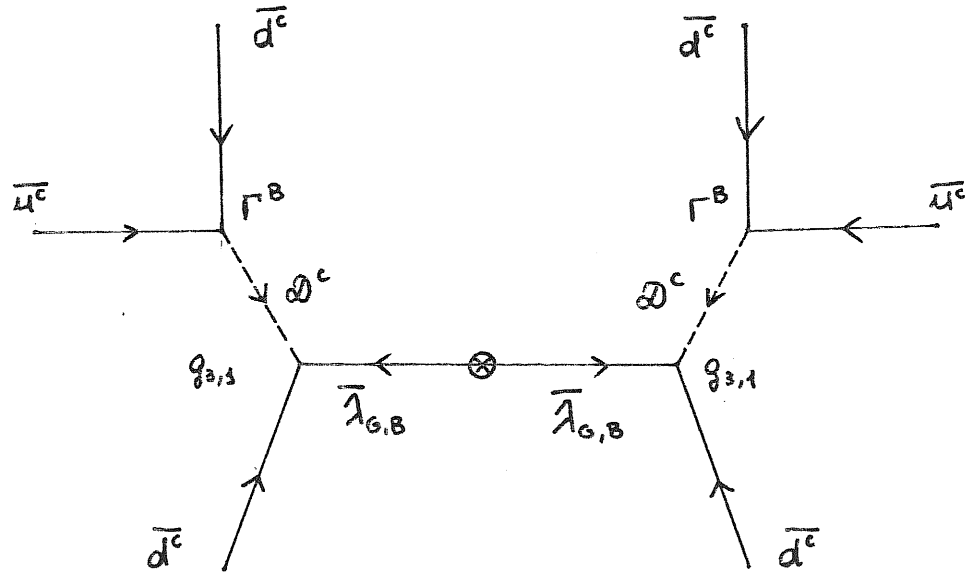


Figure 3 - Graph describing $|\Delta B|=2$ neutron-antineutron oscillations, induced by the B violating term $f_{III} = U^c D^c D^c$ in the superpotential. Symbols g_3 and g_4 stand for the gauge coupling constants of gluino and bino, respectively. The presence of a Majorana mass term for λ_G and λ_B in the soft supersymmetry-breaking part of the lagrangian, $\mathcal{L}_{SOFT}$, is assumed.

Let us give an estimate of the neutron-antineutron oscillation time, $\tau_{n\bar{n}}$, roughly given by the formula :

$$\tau_{n\bar{n}}^{-1} \simeq \frac{k^2 {\Gamma^B}^2 g^2}{m_{sq}^4 m_{g,b}} |\Psi(0)|^4 , \qquad (2.9)$$

where k represents the intergenerational mixing when one expresses interaction eigenstates in terms of mass eigenstates, while $|\Psi(0)|^4$ is a factor taking into account wave-function effects (corresponding to the factor $|\Psi(0)|^2$ appearing in formula (2.7) for the nucleon lifetime). Reasonable assignments of the parameters seem to be the following [2.9] : $k \sim 10^{-1}$, $\Gamma^{B} \sim 10^{-6}$, $g_3 \sim 1$, $g_1 \sim 10^{-1}$, $|\Psi(0)|^4 \sim 4\times 10^{-6}\,GeV^6$, $m_{\tilde{sq}} \sim m_{\lambda_6} \sim m_{\lambda_8} \sim 300$ GeV. This corresponds, in the most favourable case of gluino exchange, to an oscillation time $\tau_{n\bar{n}} \sim 10^{7-8}$ sec, which is just in the region of interest for experimentalists [1.10].

Let us summarize the main result of this subsection . There may exist supersymmetric models, based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, in which an unconventional choice of the global symmetry imposed to the superpotential in order to prevent a too fast $\Delta B = \Delta L$ nucleon decay gives rise to the following scenario : lepton number is exactly conserved and proton is stable for kinematical reasons, while n-$\bar{n}$ oscillation arise naturally.

A word of caution must be spent at this point about the choice of lepton-parity, which is not entirely appealing : discriminating between quarks and leptons, lepton-parity does not make much sense in a context of grand unification, where there are quarks and leptons sitting together in irreducible multiplets of the grand unification group G. However, one could hope to find a different framework, for example a supersymmetric composite model in which quarks and leptons have different substructures, where lepton parity can possibly arise.

2.1.4: B and/or L non-conserving effective interactions

Even if renormalizable interactions among low-energy particles automatically conserve B and L (and this is the case when matter-parity, fiveness or R-invariance are imposed to the supersymmetric lagrangian), nevertheless B and/or L violating processes among these particles can still take place, by means of the exchange of "exotic" heavy particles, not appearing in Tables I and II : such processes will be described in the effective low-energy theory by non-renormalizable operators, of dimension d, in units of mass, greater than four.

It is therefore useful to catalogue the different B and/or L non-conserving, supersymmetric and gauge-invariant effective interactions among the low-energy superfields (°) : I shall do that in the present subsection, following mainly a work done in collaboration with G.Costa and F.Feruglio [2.10]. As a general rule, each operator will be accompanied by an effective coupling constant proportional to M^{4-d}, where M is the typical mass of the heavy particles exchanged : for this reason only the lowest-dimensional interactions corresponding to the different selection rules will be considered. Moreover, I shall restrict my attention to the couplings among chiral superfields, imposing gauge invariance only under global G_o-transformations : the inclusion of gauge couplings with vector superfields (or extra derivatives) in an effective interaction would generally increase its dimension without leading to new opportunities for B and L non-conservation. Obviously, the possibility of constructing certain operators does not mean that they are actually realized in the more fundamental theory, which will eventually include possible heavy degrees of freedom : the occurrence of each operator should be checked everytime in the context of the specific model under consideration.

(°) The possibility that supersymmetry-breaking effective interactions might not be negligible with respect to the supersymmetric ones will be briefly commented in subsection 3.1.1, for the case of $\Delta B=\Delta L$ nucleon decay.

Let us proceed then with the list of the lowest-dimensional operators corresponding to the different selection rules.

$|\Delta B|=1, \Delta L=0$ - The lowest-dimensional non-renormalizable operators have d=5 and are of the form [2.6] :

$$[QQD^{c\dagger}]_{\theta\theta\bar\theta\bar\theta} + h.c.$$
$$[QQQH]_{\theta\theta} + h.c.$$

The remaining operators obeying this selection rule have all d>5. However, one can easily prove that all $|\Delta B|=1, \Delta L=0$ operators, of any dimension d, are strictly forbidden when matter-parity, fiveness or R-invariance are imposed to the supersymmetric effective lagrangian.

$\Delta B=0, |\Delta L|=1$ - Among the renormalizable operators, not included in the tree level lagrangian but possibly generated by radiative corrections, there are those of the form $[LH^{\dagger}]_{\theta\theta\bar\theta\bar\theta}$ + h.c., which are however forbidden by matter-parity, fiveness and R-invariance. Even for this selection rule one can construct supersymmetric and gauge-invariant operators of dimension d>4, but it is easily proven that they are all forbidden by matter-parity, fiveness or R-invariance, whatever dimension they have.

$\Delta B=\Delta L=\mp 1$ - The lowest-dimensional operators of this type have d=5 and are of the form [2.3,2.6] :

$$[QQQL]_{\theta\theta} + h.c.$$
$$[U^{c\dagger}U^{c\dagger}D^{c\dagger}E^{c\dagger}]_{\bar\theta\bar\theta} + h.c.$$

The above operators are allowed by matter-parity and fiveness, while they are forbidden by R-invariance. The lowest-dimensional operators allowed by R-invariance have d=6 :

$$[U^{c\dagger}D^{c\dagger}QL]_{\theta\theta\bar\theta\bar\theta} + h.c.$$
$$[QQU^{c\dagger}E^{c\dagger}]_{\theta\theta\bar\theta\bar\theta} + h.c.$$

The remaining operators obeying this selection rule have all d>6.

$\Delta B=-\Delta L=\mp 1$ - The lowest-dimensional operators of this type have d=6 and are of the form [2.10] :

$$[D^{c+}D^{c+}D^{c+}E^{c}]_{\theta\theta\bar{\theta}\bar{\theta}} + h.c.$$
$$[D^{c+}D^{c+}QL^{+}]_{\theta\theta\bar{\theta}\bar{\theta}} + h.c.$$
$$[D^{c+}D^{c+}D^{c+}L^{+}H^{+}]_{\bar{\theta}\bar{\theta}} + h.c.$$
$$[D^{c+}D^{c+}U^{c+}L^{+}H'^{+}]_{\bar{\theta}\bar{\theta}} + h.c.$$

All the above operators are allowed by matter-parity, but forbidden by fiveness and R-invariance. Moreover fiveness, being equivalent to B-L, forbids $\Delta B=-\Delta L=\mp 1$ operators of any dimension, while to get R-invariant operators of this kind one must go to d=7 (where there are several).

$\Delta B=0, |\Delta L|=2$ - The operators of minimum dimension have d=5 and are of the form [2.3] :

$$[LLH'H']_{\theta\theta} + h.c.$$

All the other operators of this type have dimension d>5. The above operators are allowed by matter-parity and R-invariance, but of course forbidden by fiveness, which, being equivalent to B-L, forbids every operator with $\Delta B=0, |\Delta L|=2$.

$|\Delta B|=2, \Delta L=0$ - The operators of minimum dimension have d=7 and are of the form [2.10,2.16] :

$$[U^{c+}U^{c+}D^{c+}D^{c+}D^{c+}D^{c+}]_{\bar{\theta}\bar{\theta}} + h.c.$$

They are allowed by matter-parity, forbidden by fiveness and R-invariance. Of course fiveness, being equivalent to B-L, forbids all the operators of this sort, of any dimension. On the other hand, to get R-invariant $|\Delta B|=2, \Delta L=0$ operators one has to go to d=8 (where there are several).

The list given above does not exhaust all the possible selection rules for B and/or L non-conservation. For instance, one could consider [2.10] $\Delta B=(\Delta L/3)=\mp 1$, $\Delta B=-(\Delta L/3)=\mp 1$, $\Delta B=\Delta L=\mp 2$, etc. : however, until now no convincing mechanism has been proposed that can generate such operators in supersymmetric models, so they will be omitted in the present discussion.

A remarkable feature of the supersymmetric operators listed above is that they have, in general, lower dimensionalities than the corresponding operators that one can construct in terms of ordinary fields (°) only [2.11,2.12]. To allow for a direct comparison, the situation is summarized in Table VII.

(°) "Ordinary fields" means here "quarks,leptons,gauge vector bosons and Higgs scalar bosons".

Table VII

Baryon and/or lepton number violating operators corresponding to the different selection rules. On the left there are the lowest-dimensional supersymmetric operators allowed by matter-parity. On the right there are the corresponding lowest-dimensional operators that one can construct in terms of ordinary fields only.

Selection rule	Operators	d	Operators	d
$\|\Delta B\|=1, \Delta L=0$	-	-	-	-
$\Delta B=0, \|\Delta L\|=1$	-	-	-	-
$\Delta B=\Delta L=-1$	$[QQQL]_{\theta\theta}$	5	$qqql$	6
	$[U^{c+}U^{c+}D^{c+}E^{c+}]_{\bar\theta\bar\theta}$	5	$\overline{u^c}\,\overline{u^c}\,\overline{d^c}\,\overline{e^c}$	6
	$[U^{c+}D^{c+}QL]_{\theta\theta\bar\theta\bar\theta}$	6	$\overline{u^c}\,\overline{d^c}\,ql$	6
	$[QQU^{c+}E^{c+}]_{\theta\theta\bar\theta\bar\theta}$	6	$qq\overline{u^c}\,\overline{d^c}$	6
$\Delta B=-\Delta L=-1$	$[D^{c+}D^{c+}D^{c+}E^{c}]_{\theta\theta\bar\theta\bar\theta}$	6	-	-
	$[D^{c+}D^{c+}QL^{+}]_{\theta\theta\bar\theta\bar\theta}$	6	-	-
	$[D^{c+}D^{c+}D^{c+}L^{+}H^{+}]_{\bar\theta\bar\theta}$	6	$\overline{d^c}\,\overline{d^c}\,\overline{d^c}\,\bar{l}\,\mathcal{H}^*$	7
	$[D^{c+}D^{c+}D^{c+}L^{+}H']_{\theta\theta\bar\theta\bar\theta}$	7		
	$[D^{c+}D^{c+}U^{c+}L^{+}H'^{+}]_{\bar\theta\bar\theta}$	6	$\overline{d^c}\,\overline{d^c}\,\overline{u^c}\,\bar{l}\,\mathcal{H}$	7
	$[D^{c+}D^{c+}U^{c+}L^{+}H]_{\theta\theta\bar\theta\bar\theta}$	7		
	$[QQD^{c+}L^{+}H]_{\theta\theta\bar\theta\bar\theta}$	7	$qq\overline{d^c}\,\bar{l}\,\mathcal{H}$	7
	$[QQD^{c+}L^{+}H'^{+}]_{\theta\theta\bar\theta\bar\theta}$	7		

Table VII (continued)

	$[D^{c+}D^{c+}QE^{c}H]_{\theta\theta\bar\theta\bar\theta}$	7	$\overline{d^c}\,\overline{d^c}qe^c\mathcal{H}$	7
	$[D^{c+}D^{c+}QE^{c}H'^{+}]_{\theta\theta\bar\theta\bar\theta}$	7		
$\Delta B=0, \Delta L=-2$	$[LLH'H']_{\theta\theta}$	5	$ll\mathcal{H}^*\mathcal{H}^*$	5
$\Delta B=-2, \Delta L=0$	$[U^{c+}U^{c+}D^{c+}D^{c+}D^{c+}D^{c+}]_{\bar\theta\bar\theta}$	7	$\overline{u^c}\,\overline{u^c}\,\overline{d^c}\,\overline{d^c}\,\overline{d^c}\,\overline{d^c}$	9
	$[QQU^{c+}D^{c+}D^{c+}D^{c+}]_{\theta\theta\bar\theta\bar\theta}$	8	$qq\overline{u^c}\,\overline{d^c}\,\overline{d^c}\,\overline{d^c}$	9
	$[QQQQD^{c+}D^{c+}]_{\theta\theta\bar\theta\bar\theta}$	8	$qqqq\overline{d^c}\,\overline{d^c}$	9

In principle, therefore, the supersymmetric context seems more favourable for the generation of baryon and lepton number violating processes than the non-supersymmetric one. To get a definite answer, however, one must carry out an explicit analysis, specifying how effective operators are generated and how one can pass from them to graphs in which all the external lines represent ordinary particles. For the selection rules $\Delta B=\Delta L=-1$ and $\Delta B=-\Delta L=-1$, corresponding to nucleon decay, this will be done in Chapter 3. For the selection rule $|\Delta B|=2, \Delta L=0$, corresponding to neutron-antineutron oscillations, this will be done in Chapter 4. Dimension five operators corresponding to the selection rule $\Delta B=0, |\Delta L|=2$ can originate, through graphs in which two scalar fields annihilate into the vacuum, Majorana neutrino masses : however, lepton number violations lie outside the subject of the present work, so they will not be discussed here.

2.2: Extensions of the standard gauge group

It has been shown in subsection 2.1.1 that dangerous B and/or L violating renormalizable couplings among low-energy particles tend to be present in the most general supersymmetric lagrangian based on the standard gauge group $G_o \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$. A possible way of eliminating the unwanted terms, imposing additional global symmetries, has been explored in subsection 2.1.2. From an aesthetical point of view, however, it would be preferable to understand B and L conservation in renormalizable interactions among low-energy particles in terms of gauge-invariance and representation content only, as in the case of the non-supersymmetric standard model. In this section I shall try to outline (without going into details) the possibility of forcing B and L conservation in the renormalizable low-energy lagrangian by means of an extension of the gauge group of the theory, from G_o to some larger group. Three cases will be considered, in one-to-one correspondence with the following three subsections.

2.2.1: Extra $\tilde{U}(1)$-factor

Let us consider supersymmetric models in which the gauge group is not just $G_o \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$, but contains an additional factor $\tilde{U}(1)_{\tilde{Y}}$. Historically these models were introduced by Fayet, in order to reconcile spontaneous breaking of global supersymmetry with an acceptable tree-level mass spectrum for the spin-0 partners of quarks and leptons [2.13]. It was subsequently emphasized by Weinberg [2.3] that this extra $\tilde{U}(1)$ may be used to forbid the unwanted B and/or L violating terms in the superpotential; it is sufficient to make the following assignments of the new quantum number $\tilde{Y}$ to the low-energy superfields of Table I : $Q(+1)$, $U^c(+1)$, $D^c(+1)$, $L(+1)$, $E^c(+1)$, $H(-2)$, $H'(-2)$.
In fact, gauge-invariance with respect to $\tilde{U}(1)_{\tilde{Y}}$ forbids all B

and/or L violating terms in the superpotential, and also the Higgs mass term HH', allowing only the terms LE^cH, QD^cH and QU^cH'. Moreover, $\tilde{U}(1)$-invariance forbids all the effective non-renormalizable interactions considered in subsection 2.1.4, with the only exception of the $\Delta B=\Delta L=\mp 1$ interactions of dimension six.

However, it must be pointed out that, while these models have no troubles for what concerns B and L non-conservation, they present many other serious drawbacks [2.14] : they tend to break $SU(3)_C$ and/or $U(1)_{e.m.}$ if supersymmetry is broken; they tend to have Adler-Bell-Jackiw anomalies associated to the new gauge interaction; they are not unifiable, i.e. $G_o \times \tilde{U}(1)_{\tilde{Y}}$ cannot be embedded in a simple gauge group G without restoring supersymmetry or losing asymptotic freedom; they also tend to have an approximate R-symmetry which keeps the gaugino masses too light with respect to the experimental bounds.

2.2.2: Left-right symmetry

Another class of models in which renormalizable B and/or L violating terms involving matter superfields are naturally suppressed in the gauge-invariant superpotential are those based on the left-right symmetric group $G_{LR} \equiv SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Neglecting Higgs superfields responsible for the breaking of $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$, matter and Higgs superfields are assigned to the following representations of G_{LR} (the notation used below is self-explanatory) :

$Q \sim (3,2,1,+1/3)$ $\quad$ $L \sim (1,2,1,-1)$ $\quad$ $H \sim (1,2,2,0)$

$Q^c \sim (3,1,2,-1/3)$ $\quad$ $L^c \sim (1,1,2,+1)$

In terms of the above chiral superfields, the most general renormalizable and gauge-invariant superpotential has the form $f_{LR} = QQ^cH + LL^cH + HH$, so that it conserves automatically B and L. In particular, dangerous trilinear couplings among matter superfields are completely forbidden, even in presence of a richer Higgs sector. Analogous considerations apply also to supersymmetric grand unified models based on the group SO(10) : matter superfields transform according to the spinorial representation $\underline{16}$ of SO(10), so that trilinear couplings among them are strictly forbidden by gauge-invariance [2.5].

2.2.3: Extra family-group

Let us imagine a supersymmetric model in which, in order to explain the generational structure of quarks and leptons, the standard gauge group $G_o \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$ is supplemented by an additional factor G_H, corresponding to an horizontal gauge symmetry. If matter superfields on the one hand, Higgs superfields on the other, are assigned to different representations of the family group G_H, chosen in such a way that none of the couplings in f_{II} and f_{III} are family-group invariant, one has automatically B and L conservation in renormalizable terms, as long as the family-group is unbroken [2.5,2.15]. When G_H is spontaneously broken at a certain scale m_F, one expects the appearance of the dangerous B and L violating terms, but they would be naturally suppressed by inverse powers of m_F.

2.3: Grand unification in SU(5)

This section is mainly devoted to the embedding of the results of section 2.1 into the framework of supersymmetric grand-unified models based on the simple gauge group SU(5), having in mind, even in this case, an effective theory described by a renormalizable lagrangian of the form $\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{SOFT}$.

First of all, in order to fix the notation, let us show in Table VIII a partial list of the fundamental superfields and their transformation properties with respect to SU(5); let us show then in Table IX their decomposition in terms of multiplets of the subgroup $G_o \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$.

Table VIII

Partial list of the fundamental superfields appearing in supersymmetric SU(5) models, together with their transformation properties. Indices x,y=1,2,...,5 refer to SU(5), while a=1,2,3 is a generation index. All superfields are chiral ones, with the exception of the gauge vector supermultiplet appearing in the last line : index R=1,...,24 spans the adjoint representation of SU(5).

Superfield	Component fields	SU(5) representation
M'_{ax}	$(\mathcal{M}'_{ax}, m'_{ax}, F_{M'ax})$	$\bar{5}$
M^{xy}_a	$(\mathcal{M}^{xy}_a, m^{xy}_a, F^{xy}_{Ma})$	10
H_x	$(\mathcal{H}_x, h_x, F_{Hx})$	$\bar{5}$
H'^x	$(\mathcal{H}'^x, h'^x, F_{H'}{}^x)$	5
A^R	$(v^{mR}_A, \lambda^R_A, D^R_A)$	24

Table IX

Decomposition of the superfields of Table VIII in terms of multiplets of the subgroup $SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5)$. A symbolic notation is used : for the detailed embeddings see, for example, reference [2.8].

$M'(\bar{5}) \to L(1,2,-1/2) + D^c(\bar{3},1,+1/3)$

$M(10) \to E^c(1,1,+1) + U^c(\bar{3},1,-2/3) + Q(3,2,+1/6)$

$H(\bar{5}) \to H_2(1,2,-1/2) + H_{\bar{3}}(\bar{3},1,+1/3)$

$H'(5) \to H'_2(1,2,+1/2) + H'_3(3,1,-1/3)$

$A^R(24) \to B(1,1,0) + W(1,3,0) + G(8,1,0) + X(3,2,-5/6) + X^+(\bar{3},2,+5/6)$

At this point an important remark is in order. The list of superfields given in Table VIII is certainly not complete, in the sense that those superfields are not sufficient to construct a consistent model. However, in order to realize the different stages of gauge-symmetry breaking in a way consistent with a realistic mass spectrum, various mechanisms have been proposed, each of which relies on a particular choice of additional Higgs superfields [2.2]. For this reason I have chosen to work only with those superfields, which are in practice determined by the low-energy content of the theory : this will be sufficient for the purpose of the present discussion.

We are now ready to look for those renormalizable couplings, appearing in the supersymmetric and gauge-invariant lagrangian $\mathcal{L}_{susy}$ and expressed in terms of the superfields of Table VIII, which can describe baryon and/or lepton number violations (the general form of a supersymmetric and gauge-invariant lagrangian is recalled in Appendix B).

Consider first the gauge couplings, which contain, among the various terms, those of the following form :

$[M'^{\dagger}M'A]_{\theta\theta\bar{\theta}\bar{\theta}}$, or : $[-g_5 M'^{\dagger x}_{a} A^R (T^R)^x{}_y M'_{ay}]_{\theta\theta\bar{\theta}\bar{\theta}}$,

$[M^{\dagger}MA]_{\theta\theta\bar{\theta}\bar{\theta}}$, or : $[-2g_5 M^{\dagger}_{axy} A^R (T^R)^y{}_z M^{zx}_a]_{\theta\theta\bar{\theta}\bar{\theta}}$,

where g_5 is the gauge coupling constant associated to SU(5) and $(T^R/2)^x{}_y$ are the generators in the fundamental representation 5. In terms of G_o-submultiplets, the above interactions decompose in the following way :

$[M'^{\dagger}M'A]_{\theta\theta\bar{\theta}\bar{\theta}} \rightarrow ([D^{c\dagger}LX^{\dagger}]_{\theta\theta\bar{\theta}\bar{\theta}} + h.c.)$,

$[M^{\dagger}MA]_{\theta\theta\bar{\theta}\bar{\theta}} \rightarrow ([U^{c\dagger}QX]_{\theta\theta\bar{\theta}\bar{\theta}} + h.c.) + ([E^{c\dagger}QX^{\dagger}]_{\theta\theta\bar{\theta}\bar{\theta}} + h.c.)$.

A detailed discussion of the resulting baryon and lepton number violations will be given in section 3.2.

Let us turn now to the most general form of the superpotential (or, better, of the part of the superpotential containing only chiral superfields of Table VIII). In the usual synthetic notation, it is given by :

$f_5 = f_A + f_B$,

$f_A = MM'H + MMH' + HH'$,

$f_B = M'MM' + M'H'$.

The explicit expressions corresponding to the above ones are collected in Table X. The content of each piece in terms of G_o-submultiplets is explicited in Table XI. Note that terms of the form HMH would be identically zero, while terms of the form M'MM' vanish for superfields belonging to the same generation.

Table X

Explicit expressions for the different terms appearing in the SU(5)-invariant superpotential f_5, constructed from the chiral superfields of Table VIII. Symbol ϵ_{xyuvw} stands for the completely antisymmetric tensor of SU(5), normalized according to ϵ_{12345} =+1.

Term	Explicit expression
MM'H	$\Gamma^1_{ab} M^{xy}_a M'_{bx} H_y$
MMH'	$\Gamma^2_{ab} \varepsilon_{xyuvw} M^{xy}_a M^{uv}_b H'^w$
HH'	$\mu H_x H'^x$
M'MM'	$\Gamma^{BL}_{abc} M'_{ax} M^{xy}_b M'_{cy}$
M'H'	$m_a M'_{ax} H'^x$

Table XI

Decomposition in terms of G_0-multiplets (as defined in Table IX) of the different terms appearing in the SU(5)-invariant superpotential f_5.

M'MH — $QD^cH_2 + LE^cH_2 + D^cU^cH_{\bar{3}} + QLH_{\bar{3}}$

MMH' — $QU^cH'_2 + U^cE^cH'_3 + QQH'_3$

HH' — $H_2H'_2 + H_{\bar{3}}H'_3$

M'MM' — $U^cD^cD^c + LE^cL + QD^cL$

M'H' — $D^cH'_3 + LH'_2$

Several terms, both in f_A and in f_B, can give rise to baryon and/or lepton number violations. The most dangerous of all are undoubtedly those of the form M'MM', which alone are sufficient to give rise to squark-mediated $\Delta B=\Delta L$ nucleon decay : the mechanism is completely analogous to that discussed the framework of $SU(3)_C \times SU(2)_L \times U(1)_Y$-models.

Even in this case a possible way out consists in imposing to the lagrangian some additional global symmetry, able to forbid the unwanted M'MM' terms in the superpotential. Suitable global symmetries are the trivial extensions of those encountered in subsection 2.1.1 (apart from lepton-parity and quark-parity, which, as pointed out before, do not make sense in a framework of grand unification). The action of such global symmetries on the fundamental superfields, as well as on the different terms of the superpotential, are summarized in Tables XII and XIII, respectively.

Table XII

Transformation properties of the fundamental chiral superfields of SU(5)-models under some global symmetries, introduced to prevent the unwanted terms M'MM' in the superpotential.

Chiral superfield / Global symmetry	Matter parity	SY parity	Fiveness	R invariance
M'	-1	+1	-3	+1/2
M	-1	-1	+1	+1/2
H	+1	-1	+2	0
H'	+1	+1	-2	0

Table XIII

Action of the global symmetries of Table XII on the terms appearing in the SU(5)-invariant superpotential f_5.

Term / Global symmetry	Matter parity	SY parity	Fiveness	R invariance
M'MH	yes	yes	yes	yes
MMH'	yes	yes	yes	yes

Table XIII (continued)

HH'	yes	no	yes	no
M'MM'	no	no	no	no
M'H'	no	yes	no	no

While all the symmetries considered above forbid the unwanted terms M'MM', they do not completely eliminate the sources of baryon and lepton number violation contained in the superpotential f_5. Leaving aside the terms M'H', which, allowed by SY-parity only, can give rise to Majorana neutrino masses, and will not be discussed here [2.4, 2.5], also the terms M'MH and MMH' can give rise to baryon and lepton number violations : such terms will be discussed in more detail in Chapter 3.

For what concerns the non-renormalizable interactions allowed by the above global symmetries, the situation in SU(5) closely reproduces that encountered in $SU(3)_C xSU(2)_L xU(1)_Y$: each G_o-invariant interaction considered in subsection 2.1.4 can be embedded in a corresponding SU(5)-invariant interaction, with the same transformation properties with respect to the additional global symmetries. A detailed list of the SU(5)-invariant interactions would be tedious; let us conclude this section giving only a few examples, relevant for the discussion which will be done in Chapter 3 :

$$[MMMM']_{\theta\theta} \rightarrow [QQQL]_{\theta\theta} + [U^c U^c D^c E^c]_{\theta\theta} ,$$

$$[MM'M^{\dagger}M'^{\dagger}]_{\theta\theta\bar{\theta}\bar{\theta}} \rightarrow [U^{c\dagger}D^{c\dagger}QL]_{\theta\theta\bar{\theta}\bar{\theta}} + h.c.,$$

$$[MMM^{\dagger}M^{\dagger}]_{\theta\theta\bar{\theta}\bar{\theta}} \rightarrow [QQU^{c\dagger}E^{c\dagger}]_{\theta\theta\bar{\theta}\bar{\theta}} + h.c..$$

3. NUCLEON DECAY

This chapter is devoted to the discussion of nucleon decay in supersymmetric models : the most part of it will concerne the selection rule $\Delta B=\Delta L=-1$, but a brief account will be also given of the possibility of $\Delta B=-\Delta L=-1$ nucleon decay. After writing down the explicit expressions for the supersymmetric interactions constructed in subsection 2.1.4, two main problems will be considered : how such operators can be generated in the context of specific models of supersymmetric grand unification and how one can pass from them to four-fermion graphs describing baryon decay. Some indications about decay rates and branching ratios will be given, but no detailed calculation of hadronic matrix elements will be performed : only the qualitative features that can characterize nucleon decay in supersymmetric models will be emphasized.

3.1: $\Delta B=\Delta L$ supersymmetric operators of dimension five

3.1.1: Universal features

First of all, let us write down the explicit expressions of the $\Delta B=\Delta L=-1$ supersymmetric interactions of dimension five [2.3,2.6], introduced in subsection 2.1.4 :

$$[QQQL]_{\theta\theta} \quad : \quad O^L_{abcd} \equiv \left[\varepsilon_{\alpha\beta\gamma}\,\varepsilon_{ij}\,\varepsilon_{kl}\,Q^{\alpha i}_a Q^{\beta j}_b Q^{\gamma k}_c L^l_d\right]_{\theta\theta} \qquad (3.1)$$

$$[U^{c\dagger}U^{c\dagger}D^{c\dagger}E^{c\dagger}]_{\bar\theta\bar\theta} \quad : \quad O^R_{abcd} \equiv \left[\varepsilon_{\alpha\beta\gamma}\,U^{c\dagger\alpha}_a U^{c\dagger\beta}_b D^{c\dagger\gamma}_c E^{c\dagger}_d\right]_{\bar\theta\bar\theta} \qquad (3.2)$$

A few comments are in order. Operators of the type O^L_{abcd} are symmetric in the indices a and b ($O^L_{bacd}=O^L_{abcd}$), vanish for quark superfields belonging to the same generation ($O_{aaad}=0$), and possess the additional symmetry property : $O^L_{abcd} + O^L_{cabd} + O^L_{bcad} = 0$. Operators of the type O^R_{abcd} are antisymmetric in the indices a and b ($O^R_{bacd} = -O^R_{abcd}$), so they also vanish for U^{c+} superfields belonging to the same generation : $O^R_{aacd} = 0$. Remember that, for the moment, quark superfields are to be taken in the interaction basis : we shall see later how to pass to mass eigenstates.

It is interesting to see how operators O^L and O^R can be generated, at the tree level, by the exchange of exotic particles : I shall do this in a diagrammatic way, making use of supergraphs.

Operators of the form O^L can arise as shown in Fig.4.

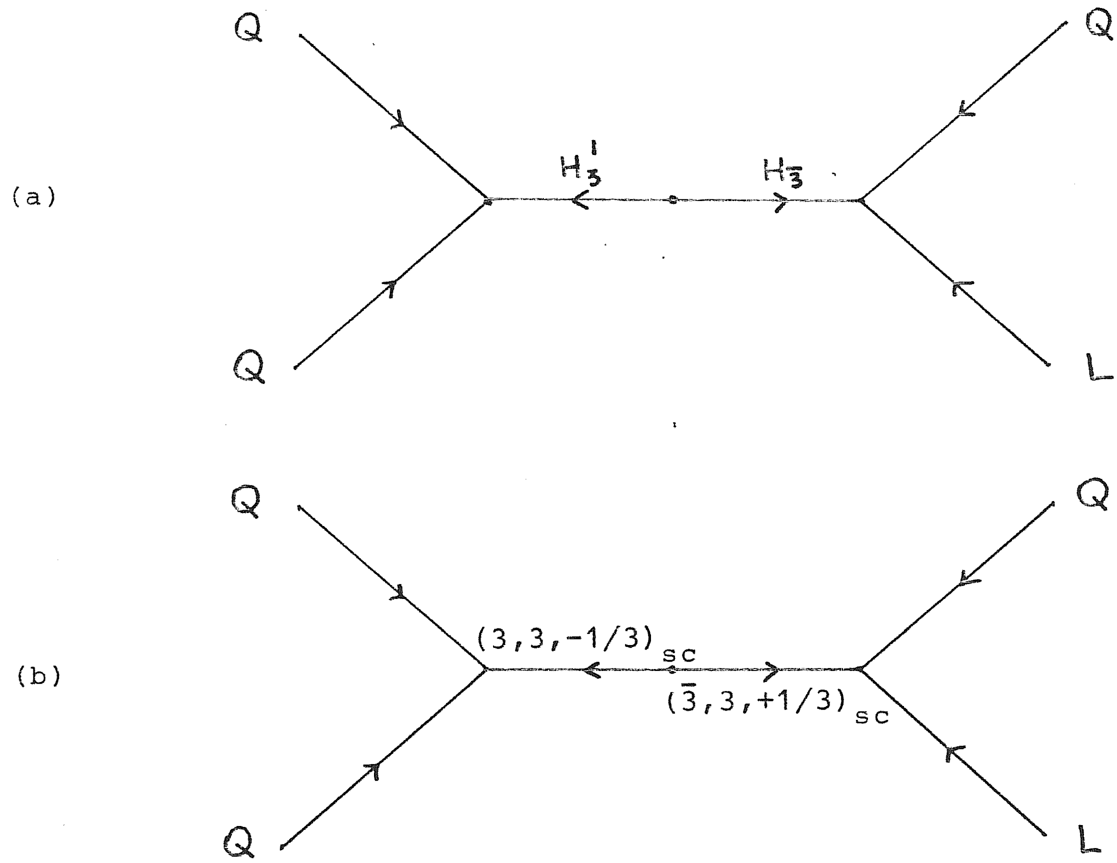


Figure 4 - Tree-level supergraphs giving rise to operators of the form O^L.

Supergraph of Fig.4a requires vertices of the type $[QQH_3']_{\theta\theta}$, $[QLH_{\bar{3}}]_{\theta\theta}$ and $[H_3'H_{\bar{3}}]_{\theta\theta}$, involving chiral scalar superfields $H_3' \sim (3,1,-1/3)_{sc}$ and $H_{\bar{3}} \sim (\bar{3},1,+1/3)_{sc}$: this possibility naturally arises in many SU(5)-models, as can be easily checked looking back at section 2.3.

Supergraph of Fig.4b requires similar vertices involving chiral scalar superfields with quantum numbers $(3,3,-1/3)_{sc}$ and $(\bar{3},3,+1/3)_{sc}$: to be realized in SU(5)-models, this possibility requires the addition of two Higgs superfields, transforming according to the representations 45 and $\overline{45}$, to those of Table VIII.

Operators of the form O^R can arise as shown in Fig.5.

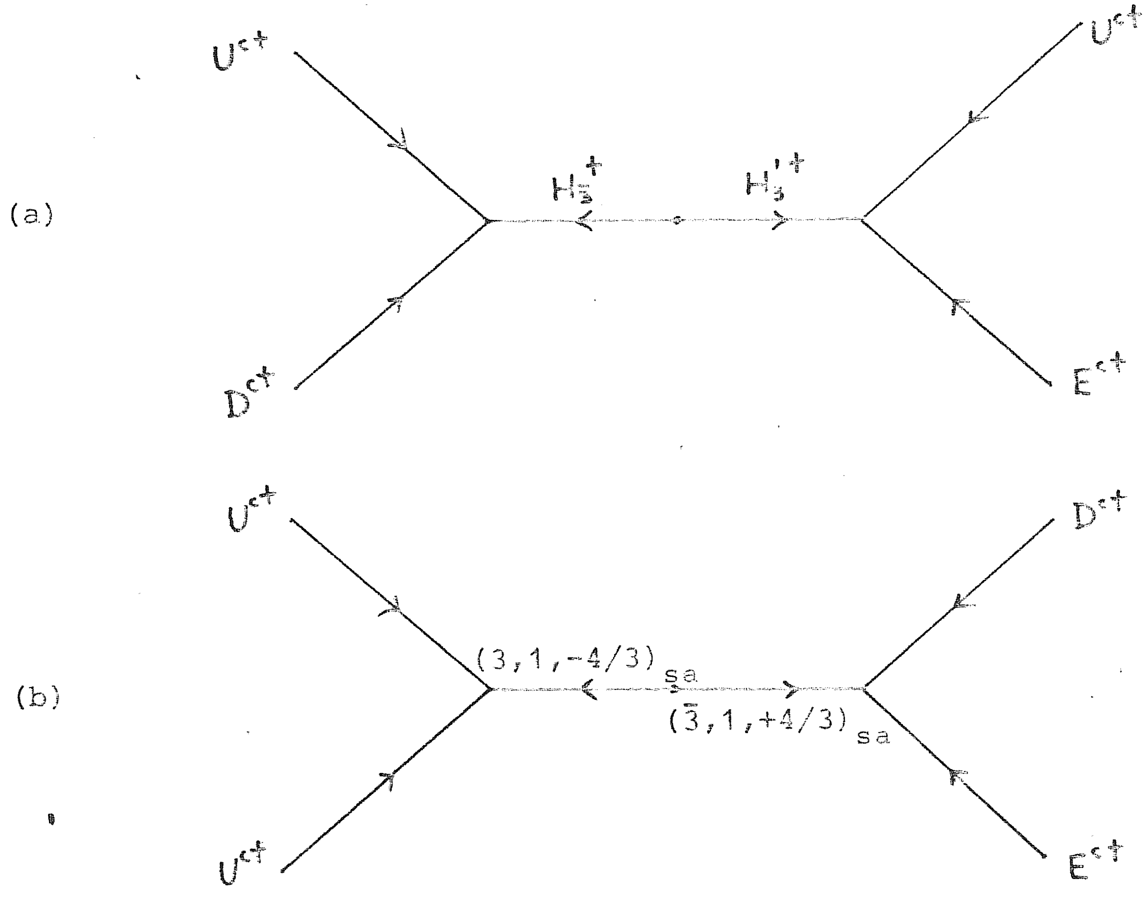


Figure 5 - Tree-level supergraphs giving rise to operators of the form O^R.

Supergraph of Fig.5a requires vertices of the type $[U^{c\dagger} D^{c\dagger} H_{\bar{3}}^{c\dagger}]_{\bar\theta\bar\theta}$, $[U^{c\dagger} E^{c\dagger} H_3^{'\dagger}]_{\bar\theta\bar\theta}$ and $[H_{\bar{3}}^{\dagger} H_3^{'\dagger}]_{\bar\theta\bar\theta}$, involving the antichiral conjugates of the chiral superfields $H_{\bar{3}}$ and H_3' : also this possibility naturally arises in many SU(5)-models (see again section 2.3).

Supergraph of Fig.5b requires similar vertices involving the antichiral conjugates of the chiral superfields $(3,1,-4/3)_{sc}$ and $(\bar{3},1,+4/3)_{sc}$, which can be embedded, for example, in the representations $\overline{45}$ and 45 of SU(5).

Note finally that, due to their chirality properties, d=5 operators O^L and O^R cannot be generated by the exchange of gauge vector superfields.

Further considerations, regarding the effective coupling constants of the operators O^L and O^R in the minimal supersymmetric SU(5) model [3.1], will be done in the following subsection.

Let us discuss at this point how supersymmetric operators of dimension five can induce four-fermion interactions describing nucleon decay.

As a first thing, one must get the expressions of O^L and O^R in terms of component fields, making use of the general formulas :

$$[ABCD]_{\theta\theta} = (-ab\mathcal{C}\mathcal{D} + \text{permutations}) + (-F_A\mathcal{B}\mathcal{C}\mathcal{D} + \text{permutations}),$$

$$[A^\dagger B^\dagger C^\dagger D^\dagger]_{\bar\theta\bar\theta} = (-\bar{a}\bar{b}\mathcal{C}^*\mathcal{D}^* + \text{permutations}) + (-F_A^*\mathcal{B}^*\mathcal{C}^*\mathcal{D}^* + \\ + \text{permutations}),$$

where $A=(\mathcal{A},a,F_A),\ldots,D=(\mathcal{D},d,F_D)$ are any four chiral scalar superfields. Remembering the general form of the renormalizable, supersymmetric and gauge-invariant low-energy lagrangian (recalled in Appendix C), and taking into account soft supersymmetry-breaking plus spontaneous gauge-symmetry breaking, it is easy to see that, in order to have four-fermion interactions describing nucleon decay, one must consider graphs of the two types shown in Fig.6.

(a)

(b)

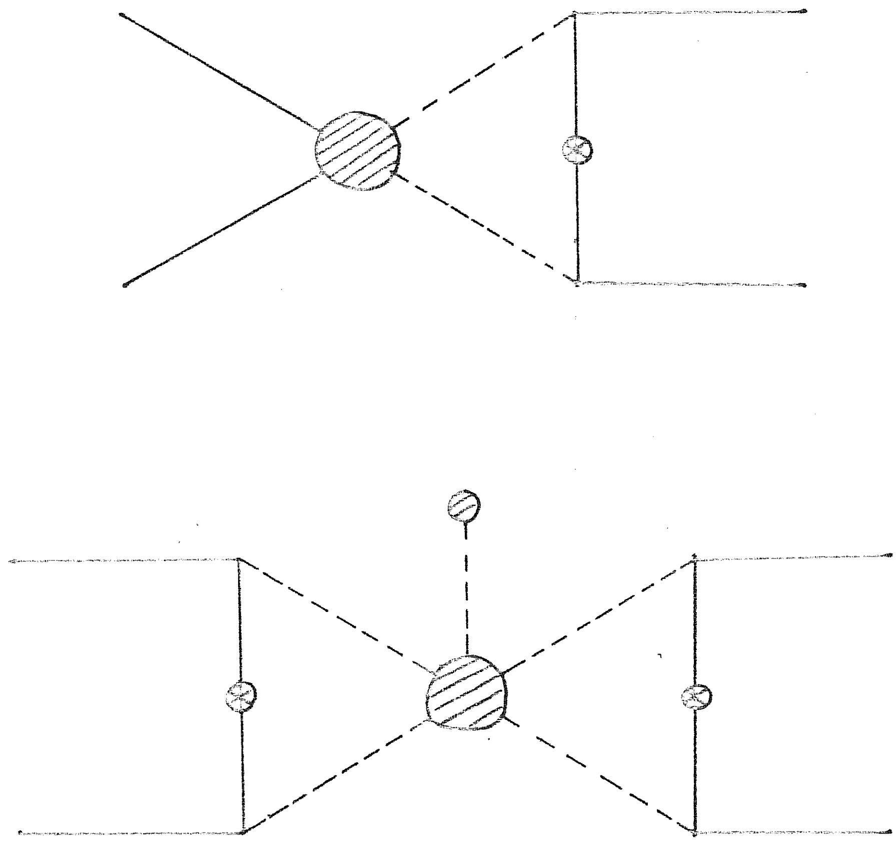


Figure 6 - Two ways of obtaining four-fermion graphs describing nucleon decay from supersymmetric effective interactions of dimension 5.

One-loop graphs of Fig.6a are obtained in the following way : one starts from an effective interaction involving two fermions and two scalars, represented by the big bubble; then the two scalars are transformed into two fermions, by the exchange of two gauginos ($\lambda_G, \lambda_W, \lambda_B$) or higgsinos (h,h'), which annihilate through a mass term, represented by the cross.

Two-loop graphs of Fig.6b are obtained in the following way : one starts from an effective interaction involving three scalar fields and one auxiliary field; then, eliminating the auxiliary field through the equations of motion, one passes to five-scalar interactions, represented by the big bubble; finally, by means of two loops like those considered above and the vacuum expectation value of a scalar Higgs field ($\mathcal{H}$ or $\mathcal{H}'$), one recovers the desired four-fermion interactions.

It can be easily seen [3.2] that, for a wide range of values of s-particles masses, the dominant contributions to nucleon decay come from one-loop graphs of Fig.6a : hence I shall restrict to them the following considerations.

Direct inspection reveals that all the mass terms appearing in graphs of Fig.6a must have one of the following forms :

$\lambda_G \lambda_G$ (or $\bar{\lambda}_G \bar{\lambda}_G$) (gluino exchange),
$\lambda_W \lambda_W$ (or $\bar{\lambda}_W \bar{\lambda}_W$) (wino exchange),
$\lambda_B \lambda_B$ (or $\bar{\lambda}_B \bar{\lambda}_B$) (bino exchange),
hh' (or $\bar{h}\bar{h}'$) (higgsino exchange).

Assuming, for simplicity, that all the above mass terms have coefficients of order m_W, one expects the higgsino exchange contributions to nucleon decay (containing Yukawa couplings) to be negligible with respect to the gaugino exchange contributions (containing gauge couplings) : thus from now on I shall concentrate on the four-fermion interactions involving gaugino exchange.

An important result is the following : operators of type O^R are irrelevant for nucleon decay [3.3]. Suppose in fact (in agreement with the strong suppression of flavour changing neutral currents) that super-Cabibbo mixing in squark propagators may be neglected and that squarks and sleptons are almost degenerate in mass [3.4]; remember then that operators of type O^R_{abcd} are antisymmetric in a and b, so that they must contain either a charm ($U_2^{c\dagger}$) or a top ($U_3^{c\dagger}$) superfield; observe also that operators of type O^R can be dressed only by gluino or bino exchange, which cannot change flavour; suppose finally that quark and lepton masses are generated through the usual mechanism of the standard non-supersymmetric model and of the minimal grand unification, so that one can pass from mass eigenstates to interaction eigenstates through the well-known Cabibbo-Kobayashi-Maskawa rotation. As a result, the corresponding four-fermion operators must contain either a charm ($\overline{u_2^c}$) or a top ($\overline{u_3^c}$) quark, making nucleon decay kinematically forbidden.

Let us concentrate, therefore, on the four fermion graphs originated by the supersymmetric interactions of type O^L. At first sight, one could think [3.2,3.5] that graphs involving gluino exchange must be dominant, because they contain the gauge coupling constant g_3 of the strong interactions. However, it can be shown [3.6,3.7] that graphs involving gluino and bino exchange give negligible contributions, so that only graphs involving wino exchange are relevant for nucleon decay. To see this, let us consider the explicit form of O^L_{abcd}, splitting the $SU(2)_L$-doublets into their components :

$$O^L_{abcd} = \left[\varepsilon_{\alpha\beta\gamma}\,(U^\alpha_a D^\beta_b U^\gamma_c E_d - U^\alpha_a D^\beta_b D^\gamma_c N_d - D^\alpha_a U^\beta_b U^\gamma_c E_d + D^\alpha_a U^\beta_b D^\gamma_c N_d)\right]_{\theta\theta}$$

Remembering that U_a=U,C,T and D_a=D,S,B, and denoting by primes the Cabibbo-rotated charge -1/3 quark superfields (linear combinations of the corresponding mass eigenstates), it is easy to see that the only terms that can contribute to nucleon decay, when dressed by gluino or bino exchange, are the following :

$$\left[\varepsilon_{\alpha\beta\gamma}\, S'^\alpha U'^\beta D'^\gamma N\right]_{\theta\theta}\,,\quad \left[\varepsilon_{\alpha\beta\gamma}\, B'^\alpha U^\beta D'^\gamma N\right]_{\theta\theta}\,,\quad \left[\varepsilon_{\alpha\beta\gamma}\, B'^\alpha U^\beta S'^\gamma N\right]_{\theta\theta}\,.$$

The above operators, when dressed by gluino or bino exchange, give rise to symmetric combinations of four-fermion interactions, such as :

$$\epsilon_{\alpha\beta\gamma}\left[(s'^{\alpha}u^{\beta})(d'^{\gamma}\nu) + (u^{\beta}d'^{\gamma})(s'^{\alpha}\nu) + (d'^{\gamma}s'^{\alpha})(u^{\beta}\nu)\right].$$

However, the combination inside the square brackets vanishes, because of the following simple algebraic identity :

$$\begin{aligned}&(\psi_1\psi_2)(\psi_3\psi_4) + (\psi_2\psi_3)(\psi_1\psi_4) + (\psi_1\psi_3)(\psi_2\psi_4) = \\ &= (\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta} + \varepsilon_{\gamma\alpha}\varepsilon_{\beta\delta} + \varepsilon_{\beta\gamma}\varepsilon_{\alpha\delta})\psi_1^{\alpha}\psi_2^{\beta}\psi_3^{\gamma}\psi_4^{\delta} = 0 .\end{aligned}$$

In the above formula ψ_1, ψ_2, ψ_3, ψ_4 are general left-handed Weyl spinors and $\alpha,\beta,\gamma,\delta=1,2$ are spinor indices (not to be confused with $SU(3)_C$-indices).

As a conclusion of the present subsection, let us summarize the results obtained so far : making only rather general assumptions, we have shown that the dominant contributions to nucleon decay from d=5 supersymmetric operators involve effective interactions with two fermions and two scalars, dressed by wino exchange.

Before passing to the next subsection, where more definite results will be obtained in the framework of the minimal supersymmetric SU(5) model, I must mention, for completeness, an observation made by Sakai [3.8,3.9] : in principle there is the possibility that four-scalar interactions, arising from spontaneous supersymmetry breaking at an intermediate mass scale $M_S \sim \sqrt{m_W M_P}$, may be competitive with the supersymmetric interactions considered above; however, a detailed analysis has shown that this can occur only in very particular cases, which will not be contemplated in the present discussion.

3.1.2: Predictions of minimal supersymmetric SU(5)

In the minimal supersymmetric SU(5) model [3.1], the basic baryon number violating interactions are those of the form $[M'MH]_{\theta\theta}$ and $[MMH']_{\theta\theta}$, whose explicit expressions have been written in Table X. In fact, a renormalization group analysis [3.10] shows that the unification scale (which in minimal non-supersymmetric SU(5) is $M_X=(1\text{to}4)\times 10^{14}$ GeV) moves up to $M_X \sim 10^{16}$ GeV, so that nucleon decay occurring through the exchange of gauge superfields, which has a rate proportional to M_X^{-4}, results in an unobservably long lifetime (for alternative possibilities see the following sections).

Combining the two terms $[M'MH]_{\theta\theta}$ and $[MMH']_{\theta\theta}$, with the aid of the mass mixing term $[HH']_{\theta\theta}$, one obtains an effective interaction of the form $[MMMM']_{\theta\theta}$, whose explicit expression can be easily worked out :

$$\frac{1}{M_H}\left[(\varepsilon_{xyuv\alpha} M_a^{xy}\Gamma^2_{ab}M_b^{uv})(M_c^{\alpha z}\Gamma^1_{cd}M'_{dz})\right]_{\theta\theta} . \qquad (3.3)$$

In the above formula M_H is the typical mass of the Higgs superfields $H_{\bar{3}}$ and H'_3, $\alpha=1,2,3$, $x,y,u,v,z=1,\ldots,5$ and numerical factors of order unity, irrelevant for the level of the present discussion, have been omitted for simplicity. The diagonal forms of the matrices Γ^1 and Γ^2 are related to the fermion masses through the formulas :

$$\Gamma^1_D = \frac{1}{\langle\mathcal{H}^o\rangle}\,\mathrm{diag}(m_d,m_s,m_b) \equiv \frac{1}{\langle\mathcal{H}^o\rangle}\, m^1_D \qquad (3.4)$$

$$\Gamma^2_D = \frac{1}{\langle\mathcal{H}'^o\rangle}\,\mathrm{diag}(m_u,m_c,m_t) \equiv \frac{1}{\langle\mathcal{H}'^o\rangle}\, m^2_D \qquad (3.5)$$

where the VEVs $\langle\mathcal{H}^o\rangle$ and $\langle\mathcal{H}'^o\rangle$ can be chosen real and must satisfy the condition :

$$<\mathcal{H}^o>^2 + <\mathcal{H}'^o>^2 = \frac{1}{2\sqrt{2}G_F} , \qquad (3.6)$$

and for notational convenience it is customary to define $\tan\theta_H \equiv <\mathcal{H}^o>/<\mathcal{H}'^o>$. Choosing a basis in generation space for matter fields M and M' such that Γ^1 is diagonal, Γ^2 takes the form :

$$\Gamma^2 = V^\dagger P \Gamma^2_D V , \qquad (3.7)$$

where V can be identified with the Cabibbo-Kobayashi-Maskawa mixing matrix and P is a diagonal matrix of phase factors, which is irrelevant for the present discussion and will henceforth be forgotten. From now on I shall denote with $D_a \equiv (D,S,B)$ and $U_a \equiv (U,C,T)$ the mass eigenstates, with $U'_a = V_{ab}U_b$ and $D'_a = V_{ab}D_b$ the so-called interaction eigenstates, which are, respectively, the $SU(2)_L$-partners of the former ones. Moreover, as a first rough approximation I shall assume for the CKM-matrix the following simplified structure :

$$V \simeq \begin{pmatrix} 1 & \varepsilon & \varepsilon^2 \\ -\varepsilon & 1 & \varepsilon \\ -\varepsilon^2 & -\varepsilon & 1 \end{pmatrix} \qquad (3.8)$$

where $\varepsilon \simeq \sin\theta_c$ (θ_c is the Cabibbo angle), i.e. $\varepsilon^2 \simeq (1/20)$.

It is now easy to extract from formula (3.3) the particular combination of the operators O^L_{abcd} and O^R_{abcd} which characterizes the minimal supersymmetric SU(5) model. Remembering the results of the former subsection, the only operator of interest is indeed the following :

$$\bar{O}^L \simeq \frac{G_F}{M_H \sin 2\theta_H} \left[\varepsilon_{\alpha\beta\gamma} (U^\alpha m^2_D D'^\beta)(U'^\gamma m^1_D E - D^\gamma m^1_D N)\right]_{\theta\theta} \qquad (3.9)$$

where numerical factors of order unity have again been omitted and $E_a \equiv (E,\mu,\tau)$, $N_a \equiv (N_e, N_\mu, N_\tau)$ are vectors in generation space. At this point, getting from $\bar{O}^L$ the explicit expressions of the corresponding four-fermion operators, obtained dressing the two fermions - two scalars terms by means of wino exchange, is only a matter of tedious algebraic calculations. Taking into account the spectrum of the quark masses and the given form of the V matrix, one arrives to the following naive prediction concerning the dominant decay mode [3.3] :

$$p \rightarrow K^+ \bar{\nu}_\tau \quad \text{and} \quad n \rightarrow K^0 \bar{\nu}_\tau .$$

This result is due to the fact that the presence in $\bar{O}^L$ of the quark mass matrices favours higher generations. For neutrinos, even the third generation $\bar{\nu}_\tau$ is allowed by phase space; for charged leptons, however, the second generation μ^+ is the highest one allowed. Detailed calculations of hadronic matrix elements [3.11] confirm the quoted result, giving the following hierarchy of decay modes, which uniquely characterizes minimal supersymmetric SU(5) :

$$\Gamma(\mathcal{N}\rightarrow\bar{\nu}_\tau K) > \Gamma(\mathcal{N}\rightarrow\bar{\nu}_\mu K) \simeq \Gamma(\mathcal{N}\rightarrow\bar{\nu}_\tau \pi) \gg$$
$$\gg \Gamma(\mathcal{N}\rightarrow\bar{\nu}_\mu \pi, \mu^+ K, \bar{\nu}_e K) \gg \Gamma(\mathcal{N}\rightarrow\mu^+\pi, \bar{\nu}_e \pi) \gg$$
$$\gg \Gamma(\mathcal{N}\rightarrow e^+ K) \gg \Gamma(\mathcal{N}\rightarrow e^+\pi) .$$

In particular, one finds the following branching ratios :

$$\Gamma(\bar{\nu}_\tau K) : \Gamma(\bar{\nu}_\mu K) : \Gamma(\mu^+ K) = 1 : 10 : 1000 .$$

For what concerns the total decay rate, let us simply mention the most relevant factors entering the expression of the decay amplitudes [3.3] : the effective coupling constant $G_F m_D^1 m_D^2 / M_H \sin 2\theta_H$ of the supersymmetric interaction $\bar{O}^L$ has already been mentioned; to the loop involving wino exchange is associated a factor of order $(1/16\pi^2)(g_2^2/2m_W)$, where g_2 is the $SU(2)_L$ gauge coupling constant and mass terms for squarks, sleptons and winos are assumed to be of order m_W; there is also a numerical factor, of order 10^{-1}, associated to the renormalization of the effective four-fermion operator from the scale M_H to the scale $m_p \sim 1$

GeV; finally one has to take into account the phase space factor of the specific matrix element. From the present experimental data on nucleon decay [1.10] :

$$\tau_N (p\rightarrow\mu^+K^o) \gtrsim 2x10^{31} years,$$
$$\tau_N (p\rightarrow\bar{\nu}K^+) \gtrsim 2x10^{30} years,$$
$$\tau_N (n\rightarrow\bar{\nu}K^o) \gtrsim 8x10^{30} years,$$

one finds [3.6,3.9] $M_H \gtrsim 10^{16-17}$GeV, a slightly higher value than the unification scale M_X.

3.2: $\Delta B=\Delta L$ supersymmetric operators of dimension six

If d=5 supersymmetric operators are suppressed by some additional symmetry, the main contribution to $\Delta B=\Delta L$ nucleon decay may come from d=6 operators, although on principle they are disfavoured by dimensional analysis.

The explicit expressions for the d=6 interactions introduced in subsection 2.1.4 can be easily written down :

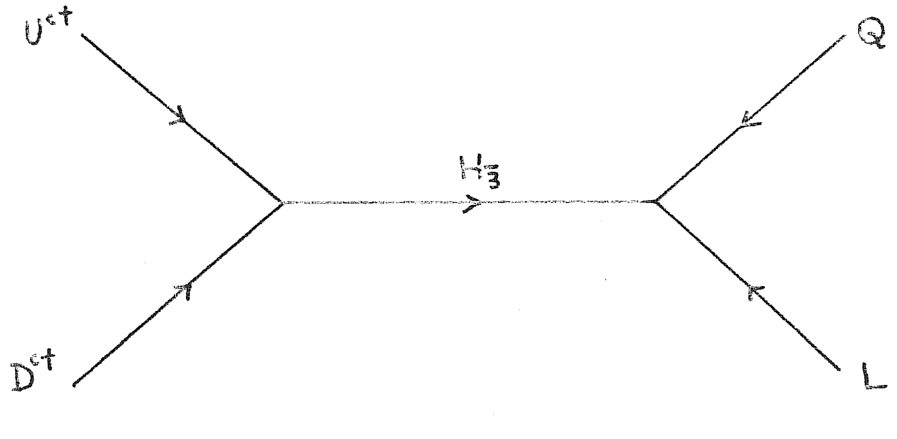
$$[U^{c\dagger}D^{c\dagger}QL]_{\theta\theta\bar{\theta}\bar{\theta}} \; : \; O^1_{abcd} \equiv [\varepsilon_{\alpha\beta\gamma}\,\varepsilon_{ij}U^{c\dagger\alpha}_a D^{c\dagger\beta}_b Q^{\gamma i}_c L^j_d]_{\theta\theta\bar{\theta}\bar{\theta}} \quad (3.10)$$
$$[QQU^{c\dagger}E^{c\dagger}]_{\theta\theta\bar{\theta}\bar{\theta}} \; : \; O^2_{abcd} \equiv [\varepsilon_{\alpha\beta\gamma}\,\varepsilon_{ij}Q^{\alpha i}_a Q^{\beta j}_b U^{c\dagger\gamma}_c E^{c\dagger}_d]_{\theta\theta\bar{\theta}\bar{\theta}} \quad (3.11)$$

Both O^1 and O^2 are nonvanishing, even if quark and lepton superfields all belong to the same generation.

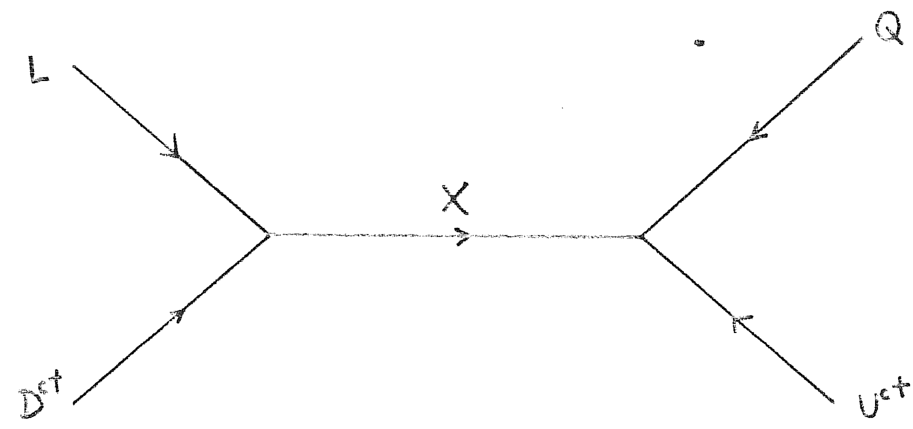
Let us see now, in the usual diagrammatic way, how operators of type O^1 and O^2 can be generated, at the tree level, by the exchange of exotic particles.

Operators of the form O^1 can arise as shown in Fig.7.

(a)



(b)



(c)

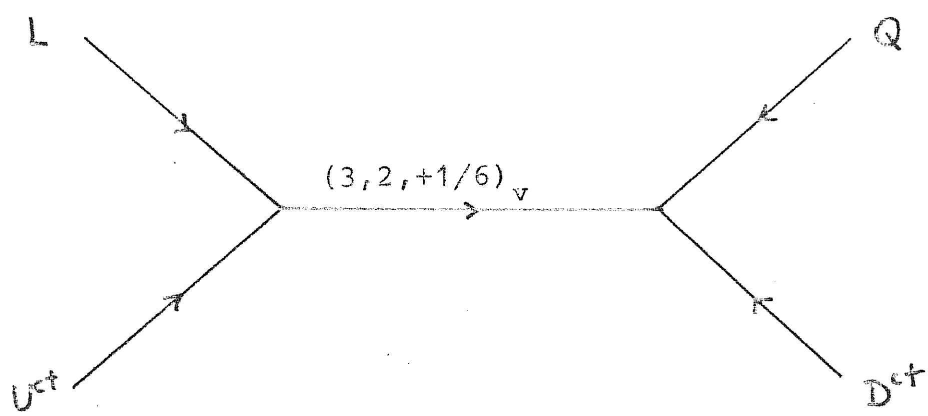
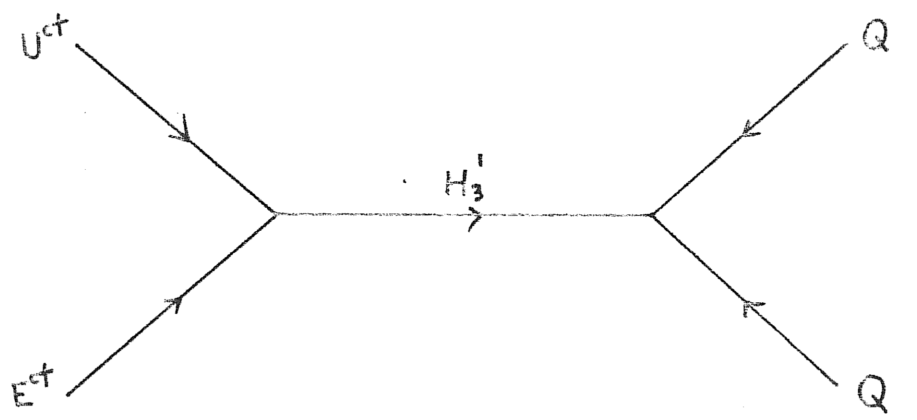


Figure 7 - Tree-level supergraphs giving rise to operators of the form O^1.

Operators of the form O^2 can arise as shown in Fig.8.

(a)



(b)

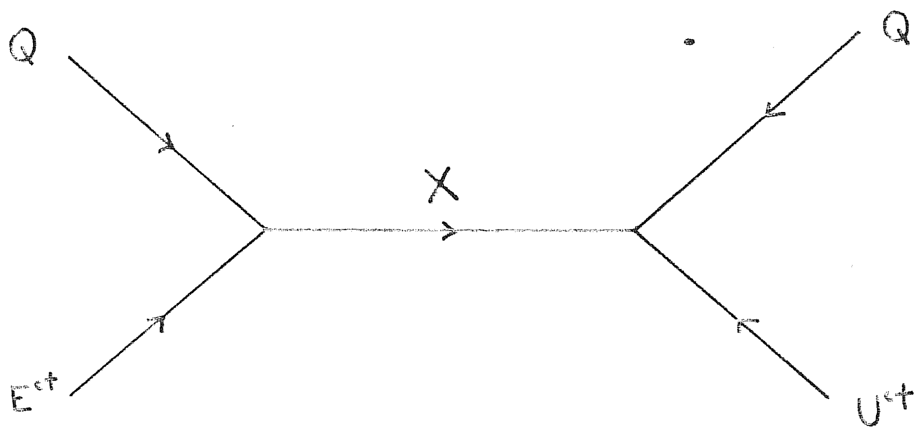


Figure 8 - Tree-level supergraphs giving rise to operators of the form O^2.

Supergraphs of Fig.7a and 8a involve the exchange of chiral scalar superfields $H_{\bar{3}}$ and H_3', respectively, but no mass mixing term between them : they arise naturally in most SU(5) models, as can be easily checked looking back at section 2.3.

Supergraphs of Fig.7b and 8b involve the exchange of the vector superfield $X\sim(3,2,-5/6)_v$, and are obviously present in any SU(5) model, being X one of the gauge vector superfields of SU(5).

Supergraph in Fig.7c involves the exchange of a gauge vector superfield with the quantum numbers $(3,2,+1/6)_v$: such a graph cannot arise in SU(5) models, since $(3,2,+1/6)_v$ does not belong to the gauge vector superfields of SU(5); however, it can be present in supersymmetric SO(10) models, since $(3,2,+1/6)_v$ can be embedded in the adjoint representation of SO(10).

Supersymmetric $\Delta B=\Delta L$ operators of dimension six do not need a "dressing" in order to give rise to four-fermion interactions describing nucleon decay. This can be evinced by the general formula :

$$[ABC^{\dagger}D^{\dagger}]_{\theta\theta\bar{\theta}\bar{\theta}} = ab\bar{c}\bar{d} + \dots ,$$

where the dots stand for additional terms containing, among the other things, scalar fields and derivatives.

3.2.1: Higgs superfield exchange

Cosmological considerations, combined with the desire of improving the values of $\sin^2\theta_W$ and m_b/m_τ given by the minimal supersymmetric SU(5) model, have led some authors [3.12] to construct models in which the triplets H_3' and $H_{\bar{3}}$ have masses of order 10^{10}GeV. In these models $\Delta B=\Delta L=-1$ operators of dimension five, which would cause an unacceptably large nucleon decay rate, are forbidden; on the other hand, the unification mass M_X is sufficiently high to preclude any observable effect of the supergraphs in Fig.7b and 8b : hence nucleon decay is dominated by graphs in Fig. 7a and 8a.

The explicit form of the supersymmetric operator describing nucleon decay is given (in the same notation of subsection 3.1.2 and neglecting even in this case numerical factors of order unity) by :

$$O^{LR} \simeq \frac{1}{M_H^2}\Big[(M_a^{\alpha y}\Gamma^1_{ab}M'_{by})(M_c^{\alpha x}\Gamma^1_{cd}M'_{dx})^\dagger + (\varepsilon_{xyuv\alpha}M_a^{xy}\Gamma^2_{ab}M_b^{uv})(\varepsilon_{rswz\alpha}M_c^{rs}\Gamma^2_{cd}M_d^{wz})^\dagger\Big]_{\theta\theta\bar{\theta}\bar{\theta}} \quad (3.12)$$

Passing to $SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplets, one gets :

$$O^{LR} \simeq \frac{1}{M_H^2}\Big\{\varepsilon_{\alpha\beta\gamma}\Big[(V^\dagger U^\alpha)\Gamma^1_D E - D^\alpha\Gamma^1_D N\Big]\Big[(VU^c_\beta)\Gamma^1_D D^c_\gamma\Big]^\dagger + \varepsilon_{\alpha\beta\gamma}\Big[U^\alpha\Gamma^2_D(VD^\beta)\Big]\Big[U^c_\gamma\Gamma^2_D(VE^c)\Big]^\dagger\Big\}_{\theta\theta\bar{\theta}\bar{\theta}} \quad (3.13)$$

It is already evident from the former expression that no $\bar{\nu}_\tau$ emission can occur, since ν_τ is always accompanied by a bottom quark. Exploiting the four-fermion interactions, and excluding all kinematically irrelevant terms, one ends up with :

$$\bar{O}^{LR} \simeq \frac{1}{M_H^2} \Big\{ m_d^2 (ued^c u^c) + \varepsilon m_d m_s (ues^c u^c) + \varepsilon m_s m_d (u\mu d^c u^c) + \\ + \varepsilon^2 m_s^2 (u\mu s^c u^c) - m_d^2 (d\nu_e u^c d^c) - \varepsilon m_d m_s (d\nu_e u^c s^c) - \\ - m_d m_s (s\nu_\mu u^c d^c) + m_u^2 (\mu d u^c e^c) + \varepsilon m_u^2 (d\nu u^c \mu^c) + \\ + \varepsilon m_u^2 (usu^c e^c) + m_u^2 \varepsilon^2 (usu^c \mu^c) \quad . \qquad (3.14)$$

It is evident from the above formula that the dominant nucleon decay modes are $\mathcal{N} \to \bar{\nu}_\mu K$ and $\mathcal{N} \to \mu^+ K$. More specifically, the following characteristic hierarchy of decay modes is predicted :

$$\Gamma(\mathcal{N} \to \bar{\nu}_\mu K, \mu^+ K) \; : \; \Gamma(\mathcal{N} \to \mu^+ \pi, e^+ K, \; \bar{\nu}_e K) \; : \; \Gamma(\mathcal{N} \to e^+ \pi, \bar{\nu}_e \pi) = \\ = \; 1 \; : \; \varepsilon^2 \; : \; \varepsilon^4 \; .$$

Although the above naive analysis does not take into account hadron wave-function effects, it seems reasonable to suppose that the latters should not upset the conclusions.

The calculation of the total decay rate, analogous to that outlined in the previous section, would give a total nucleon lifetime of order 10^{31} years, at the border of the present experimental limits.

3.2.2: Gauge superfield exchange

Supersymmetric models have been proposed [3.13] with an enlarged Higgs sector, containing, in addition to the superfields of the minimal SU(5) model, a $10 + \overline{10}$, whose components of the form $(1,1,\mp 1)$ remain light, while all the other components become superheavy. This enlargement, inserted into the renormalization group equations, gives interesting phenomenological consequences : the predicted values of $\sin^2\theta_W$ and m_b/m_τ are in accordance with experiment, and in particular the grand unification mass M_X is lowered to about the same value as in non-supersymmetric minimal SU(5). In these models d=5 operators are forbidden, while d=6 operators originated by the exchange of Higgs superfields do not give, in general, observable consequences. Hence nucleon decay, originated by graphs like those in Fig.7b and 8b, occurs in exactly the same way as in non-supersymmetric minimal SU(5) [1.13] : the dominant mode is expected to be $\mathcal{N}\to e^+\pi$, with a total nucleon lifetime of order 10^{31} years.

3.3: $\Delta B=-\Delta L$ nucleon decay

Motivated by the problem of the generation of the cosmological baryon asymmetry, a supersymmetric SU(5)-model has been proposed [3.14], in which the nucleon can decay according to the selection rule $\Delta B=-\Delta L=-1$. In such model d=5 supersymmetric operators are absent or strongly suppressed, while Higgs superfields H_3', $H_{\bar{3}}$ and gauge superfields X have sufficiently high masses to make $\Delta B=\Delta L$ operators of dimension six negligible. However, additional chiral superfields are present with respect to minimal SU(5) : an SU(5)-singlet N^c (not relevant to nucleon decay) and a superfield K, which transforms according to the representation 10 (plus $K^c\sim\overline{10}$ to cancel anomalies). The $SU(3)_C\times SU(2)_L\times U(1)_Y$-content of the Higgs superfield K is the following :

$$K(10) \to \Xi(1,1,+1) + \Phi(\bar{3},1,-2/3) + \eta(3,2,+1/6).$$

The relevant couplings of the new superfield are :

$$[M'M'K]_{\theta\theta} \to [D^cD^c\Phi]_{\theta\theta} + [LL\Xi]_{\theta\theta} + [D^cL\eta]_{\theta\theta} ,$$

$$[KKH']_{\theta\theta} \to [\eta\Phi H_2']_{\theta\theta} + [\Phi\Xi H_3']_{\theta\theta} + [\eta\eta H_3']_{\theta\theta} .$$

Starting from them, one can construct, for example, the supergraph of Fig.9, corresponding to the $\Delta B=-\Delta L=-1$ operator of dimension seven $[D^{c+}D^{c+}D^{c+}L^{+}H_2']_{\theta\theta\bar{\theta}\bar{\theta}}$:

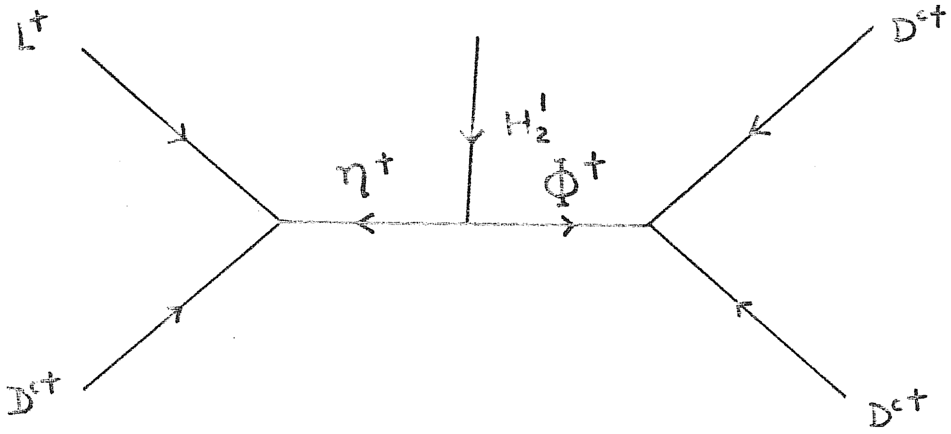


Figure 9 -Tree-level supergraph giving rise to a $\Delta B=-\Delta L=-1$ operator of dimension seven.

Note that $\Delta B=-\Delta L=-1$ operators of dimension six, which can potentially occur according to the model-independent analysis of subsection 2.1.4, do not arise in this model.

Decomposing the supersymmetric effective interaction into component fields, one can obtain from it the four-fermion graph of Fig.10, describing $\Delta B=-\Delta L$ nucleon decay.

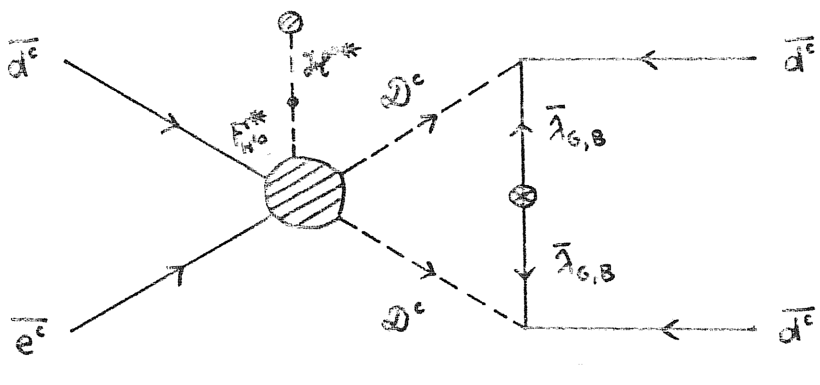


Figure 10 - Four-fermion graph describing $\Delta B=-\Delta L$ nucleon decay obtained from the d=7 supersymmetric operator corresponding to Fig.9.

Since the coupling $[D^{c\dagger} D^{c\dagger} \Phi^{\dagger}]_{\bar{\theta}\bar{\theta}}$ must be antisymmetric in generation indices, the dominant decay modes are expected to involve strange particles, and in fact they are claimed to be :

$$p \rightarrow \mu^- \pi^+ K^+ \quad \text{and} \quad n \rightarrow \mu^- K^+ .$$

An observable value for the total decay rate can be easily obtained assigning to Φ and η intermediate masses between the weak scale m_W and the GUT scale M_X.

4. NEUTRON-ANTINEUTRON OSCILLATIONS

Neutron-antineutron oscillations [4.1,4.2], and more generally $|\Delta B|=2, \Delta L=0$ processes, have received much attention in recent years : they can signal the presence of "oases" in the "desert" between the weak scale and the grand unification scale [4.2] and also play a relevant role in the generation of the cosmological baryon asymmetry [4.1,4.3]. From an experimental point of wiev [4.4], while $|\Delta B|=2, \Delta L=0$ transitions in nuclei can be revealed by the same experiments that are searching for nucleon decay, independent experiments have been done or proposed in order to detect free $n-\bar{n}$ oscillations.

In the framework of non-supersymmetric models, two mechanisms have been found, that can give rise to $n-\bar{n}$ transitions : one [4.2,4.5] involves the presence of scalar colour sextets having the quantum numbers of diquarks and masses $M_{\Delta_{qq}} \lesssim 10^5$GeV, the other one [4.6] requires the existence of a massive neutral lepton. It has been shown [4.6,4.7] that none of these two mechanisms can give rise to observable effects in the context of conventional SU(5) or SO(10) grand unification : to be definite, the first one could do that, but only at the price of making unnatural fine-tunings of the parameters in the scalar potential [4.8,4.9].

The supersymmetric scenario seems to offer new opportunities of getting observable $|\Delta B|=2, \Delta L=0$ transitions. Apart from the possibility of $n-\bar{n}$ oscillations mediated by squarks [2.9] , which has already been discussed in subsection 2.1.3, a promising aspect is the following : while in non-supersymmetric models the lowest-dimensional $|\Delta B|=2, \Delta L=0$ effective interactions have d=9, in supersymmetric models one can construct (as we have seen in subsection

2.1.4) $|\Delta B|=2, \Delta L=0$ operators with d=7 [2.10,2.16], so that, according to naive dimensional analysis, the corresponding effective coupling constant has a chance of being enhanced.

For the purpose of the present discussion, it is sufficient to recall the synthetic expression of the supersymmetric $|\Delta B|=2, \Delta L=0$ interactions of dimension seven, where group and generation indices are understood :

$$\left[U^{c\dagger}U^{c\dagger}D^{c\dagger}D^{c\dagger}D^{c\dagger}D^{c\dagger}\right]_{\bar{\theta}\bar{\theta}} + h.c.. \qquad (4.1)$$

In the following sections I shall briefly describe two different ways of generating them in supersymmetric grand unified models. For the moment, let us see how one can obtain from them the six-quark interactions describing neutron-antineutron oscillations. Rewriting the supersymmetric operator (4.1) in terms of component fields, one obtains, among the other things, effective interactions involving two quarks and four squarks, whose pictorial representation is shown in Fig.11.

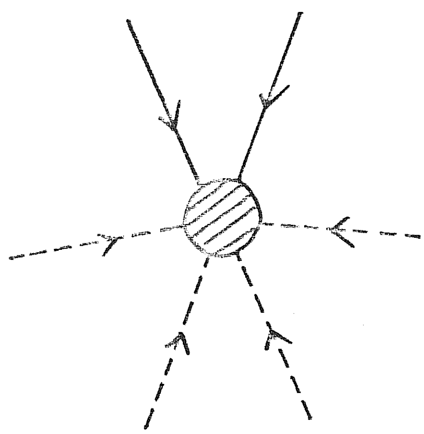


Figure 11 - Pictorial representation of the "two quarks - four squarks" interactions contained in d=7 supersymmetric operators of the form (4.1). Solid lines correspond to quarks, dashed lines to squarks.

Interactions like those in Fig.11 can be "dressed" as illustrated in Fig.12, giving rise in such a way to graphs with six quark external lines. As usual, one expects that gaugino exchange dominates over higgsino exchange : more specifically, the dominant contributions should come from gluino exchange, involving the strong gauge coupling constant g_3.

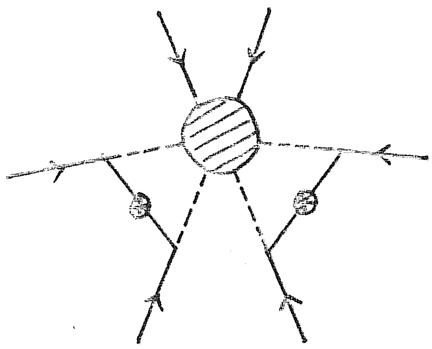


Figure 12 - Pictorial representation of the six-quark interactions resulting from the "dressing" of Fig.11. Crosses represent mass terms for gauginos (λ_G, λ_B, but not λ_W) or higgsinos (h,h').

Before passing to the following section, let us give an order-of-magnitude formula for the neutron-antineutron oscillation time, $\tau_{n\bar{n}}$, associated to graphs like that in Fig.12, in the case of gluino exchange :

$$\tau_{n\bar{n}}^{-1} \simeq A \times G_{eff}^{susy} \times \left[\left(\frac{1}{16\pi^2}\right)\left(\frac{8g_3^2}{3m_W}\right)\right]^2 \times |\Psi(0)|^4 \quad . \qquad (4.2)$$

In the above formula: G_{eff}^{susy} is the effective coupling constant [with dimension $(\text{mass})^{-3}$] associated to the supersymmetric interaction (4.1); the term inside the square brackets is a factor associated to each loop of Fig.12, under the assumption that squark and gluino masses are all of order m_W; $|\Psi(0)|^4$ is a factor taking into account hadron wavefunction effects; A is a factor taking into account renormalization effects. In the following sections, when estimating the values of $\tau_{n\bar{n}}$ associated to different expressions of G^{susy}_{eff}, the following reasonable assignments will be done : $A\sim1, m_W\sim80\text{GeV}, \alpha_3(m_W)\equiv g_3^2(m_W)/4\pi\sim0.12, |\Psi(0)|^4\sim4\times10^{-6}\text{GeV}^6$. So doing, in order to have an observable oscillation time $\tau_{n\bar{n}} \lesssim 10^{-10}$ sec, the following condition must be satisfied :

$$G^{susy}_{eff} \gtrsim 10^{-20}-10^{-21}\ \text{GeV}^{-3} . \qquad (4.3)$$

4.1: Mechanism involving colour sextets

One way of generating the effective interactions (4.1), which is nothing but the supersymmetrized version of the first of the two mechanisms quoted above, is shown in Fig. 13 : it involves colour-sextets chiral superfields, Δ_{qq}, having the right quantum numbers to be coupled to quark pairs.

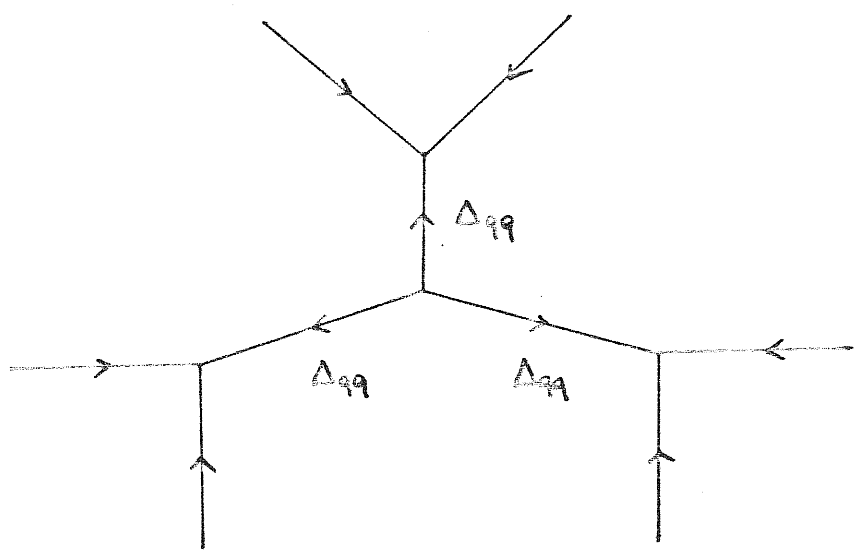


Figure 13 - A possible mechanism of generating the effective interactions (4.1). External lines represent SU(2) -singlet quark superfields.

From crude dimensional analysis, the effective coupling constant G^{susy}_{eff} associated to graphs like that in Fig.13

should be of order $1/M_{\Delta_{qq}}$. In principle, therefore, observable values of $\tau_{n\bar{n}}$ can be obtained for $M_{\Delta_{qq}} \lesssim 10^7$GeV. However, a renormalization group analysis [4.8] has shown that [at least in supersymmetric SU(5)] it is extremely difficult to reconcile the existence of colour sextets with a mass $M_{\Delta_{qq}} \lesssim 10^7$ GeV with the low-energy values of the Weinberg angle and of the ratio (m_b/m_τ), and with the requirement of perturbative grand unification at a scale $M_X < M_P$.

A variant of the above mechanism has been proposed by Lüst [4.10], in the framework of a supersymmetric SO(10) model. In that model the leading diagram for neutron-antineutron oscillations has the form shown in Fig.14, where $\Delta_{\ell\ell}$ is a scalar field having the quantum numbers of a dilepton and acquiring a non-vanishing vacuum expectation value.

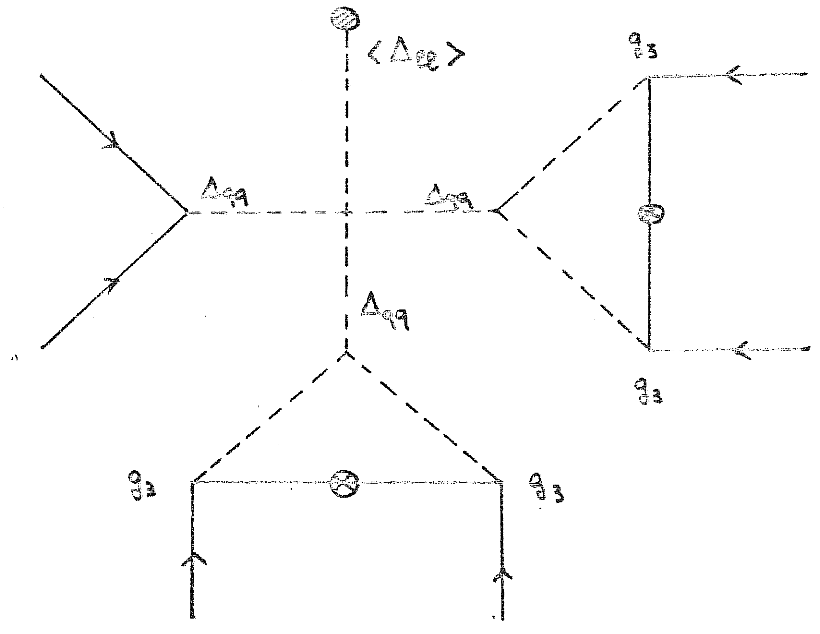


Figure 14 - Graph describing neutron-antineutron oscillations in a supersymmetric SO(10) model considered by Lüst.

The author of reference [4.10] claims that in his model :
1) an observable value of $\tau_{n\bar{n}}$ can be obtained for values

of $M_{\Delta_{49}}$ as high as 10^9 GeV; 2) such values of $M_{\Delta_{49}}$ can be naturally associated to the intermediate symmetry-breaking scale M_C, where $SU(4)_C \times SU(2)_L \times SU(2)_R$ breaks down to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, obtaining at the same time an acceptable low-energy value of $\sin^2\theta_W$. Both these claims can be questioned, but the discussion would be too long for the size of the present work, so it will not be tackled here.

4.2: Mechanism involving a massive neutral lepton

Another way of generating the effective interactions (4.1), which represents the supersymmetric version of the Kuo-Love mechanism [4.6], has been proposed by Kalara and Mohapatra [4.11] in the framework of a supersymmetric SU(5) model, and it is shown in Fig.15.

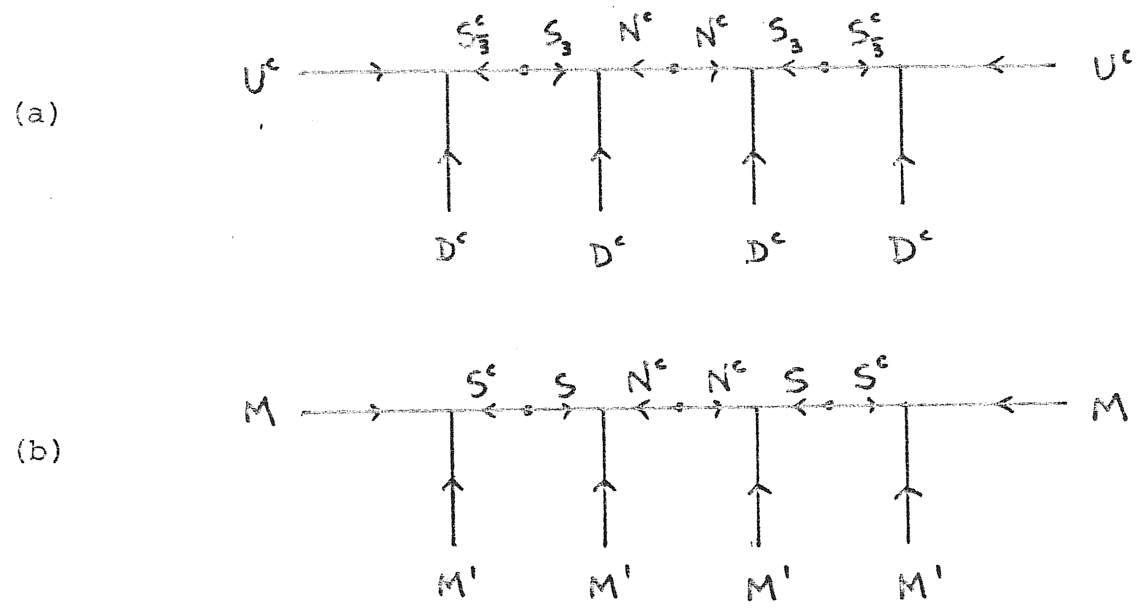


Figure 15 - Supergraph generating an operator of type (4.1) in the supersymmetric SU(5) model of reference [4.11] : (a) in terms of multiplets of $G_0 \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$; (b) in terms of the corresponding SU(5) multiplets.

The model of reference [4.11] contains, in addition to the usual matter superfields M and M', a chiral superfield N^c, transforming as an SU(5)-singlet, which can be associated to a left-handed antineutrino. In the Higgs sector (which is, by the way, rather complicated) the chiral superfields relevant for the present discussion are the following two :

$S(5) \longrightarrow S_2(1,2,+1/2) + S_3(3,1,-1/3),$

$S^c(\bar{5}) \longrightarrow S_2^c(1,2,-1/2) + S_3^c(\bar{3},1,+1/3).$

In the superpotential, couplings of the types S^cS, N^cN^c, $MM'S^c$, $M'SN^c$ are present, but couplings of type MMS are absent, so that superfields S and S^c cannot contribute to $\Delta B = \Delta L$ interactions of d=5, and for this reason are allowed to have masses of order $M_S \simeq 10^{10}$ GeV; the coefficient of the term N^cN^c is instead of order $M_N \simeq 10$ GeV. Thus, according to rough dimensional analysis, the effective coupling constant associated to the supersymmetric operators (4.1) is given in this model by :

$$G_{eff}^{susy} \simeq \frac{1}{M_N M_S^2} \simeq 10^{-21}\ \mathrm{GeV}^{-3},$$

so that observable $n-\bar{n}$ oscillations can take place.

APPENDICES

Appendix A: Notations, conventions and some useful results.

Notation and conventions have been chosen, apart from some minor changes, according to Wess [A.1]. Here is a list of those most frequently used in the text, supplemented by some useful results not included in [A.1].

Metric : $\eta_{mn} = \mathrm{diag}(-1,1,1,1)$

Weyl spinors: Two-component notation is adopted: spinors with undotted indices transform under the $(0,\frac{1}{2})$ representation of the Lorentz group, those with dotted indices transform under the $(1/2,0)$ conjugate representation.

The relevant conventions are the following :

$$\epsilon_{21} = \epsilon^{12} = +1 \qquad \epsilon_{12} = \epsilon^{21} = -1 \qquad \epsilon_{11} = \epsilon_{22} = 0$$

$$\psi^{\alpha} = \epsilon^{\alpha\beta}\psi_{\beta} \qquad \psi_{\alpha} = \epsilon_{\alpha\beta}\psi^{\beta}$$

$$\psi\chi = \psi^{\alpha}\chi_{\alpha} = -\psi_{\alpha}\chi^{\alpha} = \chi^{\alpha}\psi_{\alpha} = \chi\psi$$

$$\bar{\psi}\bar{\chi} = \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = -\bar{\psi}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}} = \bar{\chi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} = \bar{\chi}\bar{\psi}$$

$$(\chi\psi)^{\dagger} = \bar{\psi}\bar{\chi} = \bar{\chi}\bar{\psi}$$

Pauli matrices:

$$\sigma^0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar{\sigma}^{m\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\alpha\beta} \sigma^m_{\beta\dot{\beta}}$$

$$\bar{\sigma}^0 = \sigma^0 \qquad \bar{\sigma}^{1,2,3} = -\sigma^{1,2,3}$$

Dirac matrices:

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix} \qquad \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

Dirac spinors:

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}$$

Supersymmetry algebra:

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^m_{\alpha\dot{\alpha}} P_m$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

$$[Q_\alpha, P_m] = [\bar{Q}_{\dot{\alpha}}, P_m] = 0$$

Covariant derivatives (in the so-called "vector represen tation"):

$$D_\alpha = \partial_\alpha + i\sigma^m_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_m$$

$$\bar{D}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} - i\theta^\alpha \sigma^m_{\alpha\dot{\alpha}} \partial_m$$

Chiral (scalar) superfields:

They are characterized by the following condition:

$$\bar{D}_{\dot{\alpha}} \Phi = 0$$

In the so-called "chiral representation", the following decomposition holds:

$$\Phi(x,\theta) = A(x) + \sqrt{2}\,\theta\psi(x) + \theta\theta\, F(x)$$

Antichiral (scalar) superfields:

They are characterized by the following condition:

$$D_{\alpha} \Phi^{\dagger} = 0$$

In the so-called "antichiral representation", the following decomposition holds:

$$\Phi^{\dagger}(x,\bar{\theta}) = A^{*}(x) + \sqrt{2}\,\bar{\theta}\bar{\psi} + \bar{\theta}\bar{\theta}\, F^{*}$$

Vector (real) superfields:

They are characterized by the following condition:

$$V = V^{\dagger}$$

In the so-called "vector representation", and adopting the Wess-Zumino gauge, the following decomposition holds:

$$V(x,\theta,\bar{\theta}) = -\theta\sigma^{m}\bar{\theta}\, v_{m}(x) + i\,\theta\theta\bar{\theta}\bar{\lambda}(x) - i\,\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta} D(x)$$

Integration over a Grassmann variable:

$$\int d\eta = 0 \qquad \int d\eta\, \eta = 1$$

Integration in superspace :

$$d^2\theta = -\frac{1}{4} d\theta^\alpha d\theta_\alpha \qquad d^2\bar\theta = -\frac{1}{4} d\bar\theta_{\dot\alpha} d\bar\theta^{\dot\alpha}$$

$$d^4\theta = d^2\theta \, d^2\bar\theta$$

$$[R(x,\theta)]_{\theta\theta} \equiv \int d^2\theta \; R(x,\theta)$$

$$[S(x,\bar\theta)]_{\bar\theta\bar\theta} \equiv \int d^2\bar\theta \; S(x,\bar\theta)$$

$$[T(x,\theta,\bar\theta)]_{\theta\theta\bar\theta\bar\theta} \equiv \int d^4\theta \; T(x,\theta,\bar\theta)$$

$$\int d^4x \, d^2\theta \, R(x,\theta) = -\frac{1}{4} \int d^4x \, [D^2 R(x,\theta)]_{\theta=\bar\theta=0}$$

$$\int d^4x \, d^2\bar\theta \, S(x,\bar\theta) = -\frac{1}{4} \int d^4x \, [\bar D^2 S(x,\bar\theta)]_{\theta=\bar\theta=0}$$

$$\int d^4x \, d^4\theta \, T(x,\theta,\bar\theta) = \frac{1}{16} \int d^4x \, [D^2 \bar D^2 T(x,\theta,\bar\theta)]_{\theta=\bar\theta=0}$$

Appendix B: General form of a renormalizable, supersymmetric and gauge-invariant lagrangian.

I shall consider only the two extreme cases of an U(1) abelian gauge group and of a simple gauge group G, from which the most general case can be easily worked out.

1) U(1) abelian gauge group

Local gauge transformations of the chiral superfields ϕ_i and the vector superfield V can be written as:

$$\phi_i \rightarrow e^{-i y_i g \Lambda} \phi_i \qquad V \rightarrow V + \frac{i}{2}(\Lambda - \Lambda^+)$$

where Λ is a chiral superfield and Λ^+ its antichiral conjugate. The field strength is defined by the formula:

$$W_\alpha = -\frac{1}{4} \bar{D}\bar{D} D_\alpha V$$

and the most general gauge-invariant renormalizable superpotential must have the form:

$$f(\phi) = a + \lambda_i \phi_i + \frac{m_{ij}}{2} \phi_i \phi_j + \frac{g_{ijk}}{3} \phi_i \phi_j \phi_k$$

The most general renormalizable, supersymmetric and gauge-invariant lagrangian has the following form :

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_{FI}$$

$$\mathcal{L}_{YM} = \frac{1}{4} \left\{ [W^\alpha W_\alpha]_{\theta\theta} + [\bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha}]_{\bar\theta\bar\theta} \right\} \sim \frac{1}{2} [W^\alpha W_\alpha]_{\theta\theta}$$

$$\mathcal{L}_g = \left[\phi_i^+ e^{2 y_i g V} \phi_i\right]_{\theta\theta\bar\theta\bar\theta} \qquad \mathcal{L}_f = [f(\phi)]_{\theta\theta} + \text{h.c.}$$

$$\mathcal{L}_{FI} = [2\xi V]_{\theta\theta\bar{\theta}\bar{\theta}}$$

After some tedious algebraic calculations one gets, in the Wess-Zumino gauge, the following component-expressions:

$$\mathcal{L}_{YM} = -\frac{1}{4} v^{mn} v_{mn} - i\lambda\sigma^m \partial_m \bar{\lambda} + \frac{1}{2} D^2 \qquad (v_{mn} \equiv \partial_m v_n - \partial_n v_m)$$

$$\mathcal{L}_g = A_i^* \Box A_i - i\psi_i \sigma^m \partial_m \bar{\psi}_i + F_i^* F_i -$$
$$- y_i g v_m \psi_i \sigma^m \bar{\psi}_i + i y_i g v^m [A_i^* (\partial_m A_i) - (\partial_m A_i^*) A_i] -$$
$$- i\sqrt{2} y_i g [A_i \bar{\psi}_i \bar{\lambda} - A_i^* \psi_i \lambda] - y_i^2 g^2 v_m v^m A_i^* A_i +$$
$$+ y_i g D A_i^* A_i ;$$

$$\mathcal{L}_f = -\frac{1}{2} \left(\frac{\partial^2 f}{\partial A_i \partial A_j} \right) \psi_i \psi_j - \frac{1}{2} \left(\frac{\partial^2 f}{\partial A_i \partial A_j} \right)^* \bar{\psi}_i \bar{\psi}_j +$$
$$+ \left(\frac{\partial f}{\partial A_i} \right) F_i + \left(\frac{\partial f}{\partial A_i} \right)^* F_i^* ;$$

$$\mathcal{L}_{FI} = \xi D$$

Eliminating then the auxiliary fields through the equations of motion, one obtains :

$$\mathcal{L} = -\frac{1}{4} v_{mn} v^{mn} - i\lambda\sigma^m \partial_m \bar{\lambda} - i\psi_i \sigma^m \partial_m \bar{\psi}_i + A_i^* \Box A_i -$$
$$- y_i g v_m \psi_i \sigma^m \bar{\psi}_i + i y_i g v^m [A_i^* (\partial_m A_i) - (\partial_m A_i^*) A_i] -$$
$$- i\sqrt{2} y_i g [A_i \bar{\psi}_i \bar{\lambda} - A_i^* \psi_i \lambda] - y_i^2 g^2 v_m v^m A_i^* A_i -$$
$$- \frac{1}{2} \left(\frac{\partial^2 f}{\partial A_i \partial A_j} \right) \psi_i \psi_j - \frac{1}{2} \left(\frac{\partial^2 f}{\partial A_i \partial A_j} \right)^* \bar{\psi}_i \bar{\psi}_j -$$
$$- \left(\frac{\partial f}{\partial A_i} \right)^* \left(\frac{\partial f}{\partial A_i} \right) - \frac{1}{2} (y_i g A_i^* A_i + \xi)^2 .$$

2) simple gauge group G

Local gauge transformations for the chiral superfields Φ^i and the vector superfields V^a can be written as:

$$\Phi \to e^{-ig\Lambda}\Phi \qquad e^{2gV} \to e^{-ig\Lambda^\dagger} e^{2gV} e^{ig\Lambda}$$

where :

$$\Phi = \Phi^i \ , \ \Lambda = \Lambda^a (T^a)^i{}_j \quad , \ V = V^a (T^a)^i{}_j \ ;$$

matrices $(T^a)^i{}_j$ are the hermitian generators of G in the representation of the chiral superfields Φ^i , normalized according to $\mathrm{tr}(T^a T^b) = k\,\delta^{ab}$ and obeying the commutation rules $[T^a, T^b] = i c^{abc} T^c$, where c^{abc} are the completely antysymmetric structure constants of the group G. The field strength is defined in this case by the following formula:

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^{-gV} D_\alpha e^{gV}$$

while the superpotential has again the general form:

$$f(\Phi) = a + \lambda_i \Phi^i + \frac{m_{ij}}{2}\Phi^i\Phi^j + \frac{g_{ijk}}{3}\Phi^i\Phi^j\Phi^k \ .$$

The most general renormalizable, supersymmetric and gauge invariant lagrangian has the following form :

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_g + \mathcal{L}_f$$

$$\mathcal{L}_{YM} = \frac{1}{4g^2 k}\,\mathrm{tr}\left\{ [W^\alpha W_\alpha]_{\theta\theta} + [\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}]_{\bar\theta\bar\theta} \right\}$$

$$\mathcal{L}_g = [\Phi^\dagger e^{2gV}\Phi]_{\theta\theta\bar\theta\bar\theta} \qquad \mathcal{L}_f = [f(\Phi)]_{\theta\theta} + \mathrm{h.c.}$$

After some tedious algebraic calculations one obtains, in the Wess-Zumino gauge, the following component expressions :

$$\mathcal{L}_{YM} = -\frac{1}{4} v^{mn a} v^{a}_{mn} - i\lambda^{a}\sigma^{m}\partial_{m}\bar{\lambda}^{a} + \frac{1}{2} D^{a}D^{a} -$$

$$- \frac{ig}{2} c^{abc}\lambda^{a}\sigma^{m}\bar{\lambda}^{b}v^{c}_{m} \quad , \qquad \text{where :}$$

$$v^{mna} \equiv \partial^{m}v^{na} - \partial^{n}v^{ma} + \frac{g c^{abc}}{2} v^{ma}v^{nb} \; ;$$

$$\mathcal{L}_{g} = A^{*}_{i}\Box A_{i} + i(\partial_{m}\bar{\psi}_{i})\bar{\sigma}^{m}\psi^{i} + F^{*}_{i}F_{i} +$$

$$+ g v^{a}_{m}\bar{\psi}_{i}\bar{\sigma}^{m}(T^{a})^{i}{}_{j}\psi^{j} +$$

$$+ i\sqrt{2}g\,[A^{*}_{i}(T^{a})^{i}{}_{j}\psi^{j}\lambda^{a} - \bar{\lambda}^{a}\bar{\psi}_{i}(T^{a})^{i}{}_{j}A^{j}] +$$

$$+ i g v^{ma}[A^{*}_{i}(T^{a})^{i}{}_{j}(\partial_{m}A^{j}) - (\partial_{m}A^{*}_{i})(T^{a})^{i}{}_{j}A^{j}] -$$

$$- g^{2}v^{ma}v^{b}_{m}A^{*}_{i}(T^{a})^{i}{}_{j}(T^{b})^{j}{}_{k}A^{k} + gD^{a}A^{*}_{i}(T^{a})^{i}{}_{j}A^{j} \; ;$$

$$\mathcal{L}_{f} = -\frac{1}{2}\left(\frac{\partial^{2}f}{\partial A^{i}\partial A^{j}}\right)\psi^{i}\psi^{j} - \frac{1}{2}\left(\frac{\partial^{2}f}{\partial A^{i}\partial A^{j}}\right)^{*}\bar{\psi}_{i}\bar{\psi}_{j} +$$

$$+ \left(\frac{\partial f}{\partial A^{i}}\right)F^{i} + \left(\frac{\partial f}{\partial A^{i}}\right)^{*}F^{*}_{i} \; .$$

Eliminating then the auxiliary fields through the equations of motion, one gets :

$$\mathcal{L} = -\frac{1}{4}v^{mna}v^{a}_{mn} - \frac{ig}{2}c^{abc}\lambda^{a}\sigma^{m}\bar{\lambda}^{b}v^{c}_{m} +$$

$$+ i(\partial_{m}\bar{\lambda}^{a})\bar{\sigma}^{m}\lambda^{a} + i(\partial_{m}\bar{\psi}_{i})\bar{\sigma}^{m}\psi^{i} + A^{*}_{i}\Box A_{i} +$$

$$
\begin{aligned}
&+ \quad g v_m^a \, \bar{\psi}_i \bar{\sigma}^m (T^a)^i{}_j \, \psi^j \; + \\
&+ \quad i\sqrt{2} g \left[A_i^* (T^a)^i{}_j \psi^j \lambda^a - \bar{\lambda}^a \bar{\psi}_i (T^a)^i{}_j A^j \right] \; + \\
&+ \quad i g v^{ma} \left[A_i^* (T^a)^i{}_j (\partial_m A^j) - (\partial_m A_i^*)(T^a)^i{}_j A^j \right] \; - \\
&- \quad g^2 v^{ma} v_m^b \; A_i^* (T^a)^i{}_j (T^b)^j{}_k \; A^k \; - \\
&- \quad \frac{1}{2} \left(\frac{\partial^2 f}{\partial A^i \partial A^j} \right) \psi^i \psi^j \; - \; \frac{1}{2} \left(\frac{\partial^2 f}{\partial A^i \partial A^j} \right)^* \bar{\psi}_i \bar{\psi}_j \; - \\
&- \quad \frac{g^2}{2} \Sigma_a \left[A_i^* (T^a)^i{}_j \, A^j \right]^2 \; - \; \left(\frac{\partial f}{\partial A^i} \right)^* \left(\frac{\partial f}{\partial A^i} \right) \quad .
\end{aligned}
$$

Appendix C: Component-field expression of the $\Delta B = \Delta L = 0$, renormalizable, supersymmetric and gauge-invariant lagrangian in low-energy models based on the group $SU(3)_c \times SU(2)_L \times U(1)_Y$.

Consider the minimal set of chiral superfields listed in Table I, together with the B and L conserving superpotential :

$$f_I = \Gamma^e_{ab}\,\epsilon_{ij}\,\mathcal{L}^i_a\,\mathcal{E}^c_b\,\mathcal{H}^j + \Gamma^d_{ab}\,\epsilon_{ij}\,\mathcal{Q}^{\alpha i}_a\,\mathcal{D}^c_{b\alpha}\,\mathcal{H}^j + \Gamma^u_{ab}\,\epsilon_{ij}\,\mathcal{Q}^{\alpha i}_a\,\mathcal{U}^c_{b\alpha}\,\mathcal{H}'^j + \mu\,\epsilon_{ij}\,\mathcal{H}^i\,\mathcal{H}'^j \; .$$

Omitting from the beginning a Fayet-Iliopoulos term associated to the abelian factor $U(1)_Y$, the corresponding renormalizable, supersymmetric and gauge-invariant lagrangian is given (in the Wess-Zumino gauge) by :

$$\begin{aligned}\mathcal{L}_{susy} = \Big[& \left(\mathcal{L}^c_{K,gb} + \mathcal{L}^L_{K,gb} + \mathcal{L}^Y_{K,gb}\right) + \left(\mathcal{L}^c_{K,gf} + \mathcal{L}^L_{K,gf} + \mathcal{L}^Y_{K,gf}\right) + \\ & + \left(\mathcal{L}^c_{gbgf} + \mathcal{L}^L_{gbgf}\right) + \mathcal{L}_{K,ob} + \mathcal{L}_{K,of} + \left(\mathcal{L}^c_{g,of} + \mathcal{L}^L_{g,of} + \mathcal{L}^Y_{g,of}\right) + \\ & + \left(\mathcal{L}^c_{YUK,gf} + \mathcal{L}^L_{YUK,gf} + \mathcal{L}^Y_{YUK,gf}\right) + \left(\mathcal{L}^c_{3g,ob} + \mathcal{L}^L_{3g,ob} + \mathcal{L}^Y_{3g,ob}\right) + \\ & + \left(\mathcal{L}^c_{4g,ob} + \mathcal{L}^L_{4g,ob} + \mathcal{L}^Y_{4g,ob}\right) + \left(\mathcal{L}^c_{S,D} + \mathcal{L}^L_{S,D} + \mathcal{L}^Y_{S,D}\right)\Big] + \\ & + \Big[\, \mathcal{L}_{mass,of} + \mathcal{L}_{YUK,of} + \mathcal{L}_{S,F} \,\Big] \; .\end{aligned}$$

Terms in the first square bracket are independent of the choice of a specific superpotential; terms in the second square bracket depend on the superpotential. In the following I shall explain the meaning of each term, writing down the component-field expressions of those which are relevant for the discussion developed in the text.

$\mathcal{L}^{c}_{K,gb}$, $\mathcal{L}^{L}_{K,gb}$ and $\mathcal{L}^{Y}_{K,gb}$ are the "kinetic" terms involving the gauge bosons of $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$, respectively;

$\mathcal{L}^{c}_{K,gf}$, $\mathcal{L}^{L}_{K,gf}$ and $\mathcal{L}^{Y}_{K,gf}$ are the kinetic terms for the gauge fermions of $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$, respectively;

$\mathcal{L}^{c}_{gbf}$ and $\mathcal{L}^{L}_{gbf}$ are interaction terms between the gauge fermion and the gauge bosons of $SU(3)_c$ and $SU(2)_L$, respectively;

$\mathcal{L}_{K,sb}$ is a kinetic term for the spin-zero bosons;

$\mathcal{L}_{K,cf}$ is a kinetic term for the chiral fermions;

$\mathcal{L}^{c}_{g,cf}$, $\mathcal{L}^{L}_{g,cf}$ and $\mathcal{L}^{Y}_{g,cf}$ are the gauge couplings of the chiral fermions with the vector bosons associated to $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, respectively;

$\mathcal{L}^{c}_{YUK,gf}$, $\mathcal{L}^{L}_{YUK,gf}$ and $\mathcal{L}^{Y}_{YUK,gf}$ are the Yukawa couplings between scalar bosons, chiral fermions and the gauge fermions associated to $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, respectively; their explicit expressions are the following:

$$\mathcal{L}^{c}_{YUK,gf} = i\sqrt{2}\, g_3 \left[\mathcal{Q}^{*}_{a i\alpha}\, \lambda^{A}_{G} \left(\frac{t^A}{2}\right)^{\alpha}{}_{\beta}\, q^{\beta i}_{a} - \overline{q}_{a i\alpha}\, \overline{\lambda^{A}_{G}} \left(\frac{t^A}{2}\right)^{\alpha}{}_{\beta}\, \mathcal{Q}^{\beta i}_{a} \right] +$$
$$+\, i\sqrt{2}\, g_3 \left[u^{c}_{a\alpha}\, \lambda^{A}_{G} \left(\frac{t^A}{2}\right)^{\alpha}{}_{\beta}\, \mathcal{U}^{c*\beta}_{a} - \mathcal{U}^{c}_{a\alpha}\, \overline{\lambda^{A}_{G}} \left(\frac{t^A}{2}\right)^{\alpha}{}_{\beta}\, \overline{u^{c}_{a}}^{\beta} \right] +$$
$$+\, i\sqrt{2}\, g_3 \left[d^{c}_{a\alpha}\, \lambda^{A}_{G} \left(\frac{t^A}{2}\right)^{\alpha}{}_{\beta}\, \mathcal{D}^{c*\beta}_{a} - \mathcal{D}^{c}_{a\alpha}\, \overline{\lambda^{A}_{G}} \left(\frac{t^A}{2}\right)^{\alpha}{}_{\beta}\, \overline{d^{c}_{a}}^{\beta} \right],$$

$$\mathcal{L}^{L}_{YUK,gf} = i\sqrt{2}\, g_2 \left[\mathcal{Q}^{*}_{a\alpha i}\, \lambda^{I}_{W} \left(\frac{\tau^I}{2}\right)^{i}{}_{j}\, q^{\alpha j}_{a} - \overline{q}_{a i\alpha}\, \overline{\lambda^{I}_{W}} \left(\frac{\tau^I}{2}\right)^{i}{}_{j}\, \mathcal{Q}^{\alpha j}_{a} \right] +$$

$$+ \, i\sqrt{2}\, g_2 \left[\mathcal{L}^{*}_{a i}\, \lambda^I_W \left(\frac{\tau^I}{2}\right)^i{}_j\, l^j_a - \bar{l}_{a i}\, \bar{\lambda}^I_W \left(\frac{\tau^I}{2}\right)^i{}_j\, \mathcal{L}^j_a \right] +$$

$$+ \, i\sqrt{2}\, g_2 \left[\mathcal{H}^{*}_{i}\, \lambda^I_W \left(\frac{\tau^I}{2}\right)^i{}_j\, h^j - \bar{h}_{i}\, \bar{\lambda}^I_W \left(\frac{\tau^I}{2}\right)^i{}_j\, \mathcal{H}^j \right] +$$

$$+ \, i\sqrt{2}\, g_2 \left[\mathcal{H}'^{*}_{i}\, \lambda^I_W \left(\frac{\tau^I}{2}\right)^i{}_j\, h'^j - \bar{h}'_{i}\, \bar{\lambda}^I_W \left(\frac{\tau^I}{2}\right)^i{}_j\, \mathcal{H}'^j \right],$$

$$\mathcal{L}^{Y}_{int,ff} = i\sqrt{2}\, \frac{g_1}{6} \left[\mathcal{Q}^{*}_{a\alpha i}\, q^{\alpha i}_a\, \lambda_B - \mathcal{Q}^{\alpha i}_a\, \bar{q}_{a\alpha i}\, \bar{\lambda}_B \right] -$$

$$- \, i\sqrt{2}\, \frac{2g_1}{3} \left[\mathcal{U}^{c*\alpha}_a\, u^c_{a\alpha}\, \lambda_B - \mathcal{U}^c_{a\alpha}\, \bar{u}^{c\alpha}_a\, \bar{\lambda}_B \right] +$$

$$+ \, i\sqrt{2}\, \frac{g_1}{3} \left[\mathcal{D}^{c*\alpha}_a\, d^c_{a\alpha}\, \lambda_B - \mathcal{D}^c_{a\alpha}\, \bar{d}^{c\alpha}_a\, \bar{\lambda}_B \right] -$$

$$- \, i\sqrt{2}\, \frac{g_1}{2} \left[\mathcal{L}^{*}_{a i}\, l^i_a\, \lambda_B - \mathcal{L}^i_a\, \bar{l}_{a i}\, \bar{\lambda}_B \right] +$$

$$+ \, i\sqrt{2}\, g_1 \left[\mathcal{E}^{c*}_a\, e^c_a\, \lambda_B - \mathcal{E}^c_a\, \bar{e}^c_a\, \bar{\lambda}_B \right] -$$

$$- \, i\sqrt{2}\, \frac{g_1}{2} \left[\mathcal{H}^{*}_i\, h^i\, \lambda_B - \mathcal{H}^i\, \bar{h}_i\, \bar{\lambda}_B \right] +$$

$$+ \, i\sqrt{2}\, \frac{g_1}{2} \left[\mathcal{H}'^{*}_i\, h'^i\, \lambda_B - \mathcal{H}'^i\, \bar{h}'_i\, \bar{\lambda}_B \right] ,$$

where $(t^A/2)^\alpha{}_\beta$ and $(\tau^I/2)^i{}_j$ are the generators, in the fundamental representation, of $SU(3)_c$ and of $SU(2)_L$, respectively;

$\mathcal{L}^c_{3g,cb}$, $\mathcal{L}^L_{3g,cb}$ and $\mathcal{L}^Y_{3g,cb}$ are trilinear couplings involving two scalar bosons and the gauge bosons of $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, respectively;

$\mathcal{L}^c_{4g,cb}$, $\mathcal{L}^L_{4g,cb}$ and $\mathcal{L}^Y_{4g,cb}$ are quadrilinear couplings involving two scalar bosons and two gauge bosons of $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, respectively;

$\mathcal{L}^c_{S,D}$, $\mathcal{L}^L_{S,D}$ and $\mathcal{L}^Y_{S,D}$ are four-scalar interactions characterized by the squares of the gauge coupling constants of $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, respectively;

$\mathcal{L}_{mass,f}$ is a mass term for chiral fermions, given by:

$$\mathcal{L}_{mass,f} = -\mu \epsilon_{ij} h^i h'^j + h.c. ;$$

$\mathcal{L}_{YUK,f}$ contains Yukawa couplings between chiral fermions and scalar bosons, and is given by :

$$\mathcal{L}_{YUK,f} = -\Gamma^e_{ab}\epsilon_{ij}\left[\ell^i_a e^c_b \mathcal{H}^j + \ell^i_a h^j \mathcal{E}^c_b + e^c_b h^j \mathcal{L}^i_a\right] + h.c. -$$
$$-\Gamma^d_{ab}\epsilon_{ij}\left[q^{\alpha i}_a d^c_{b\alpha}\mathcal{H}^j + q^{\alpha i}_a h^j \mathcal{D}^c_{b\alpha} + d^c_{b\alpha} h^j \mathcal{Q}^{\alpha i}_a\right] + h.c. -$$
$$-\Gamma^u_{ab}\epsilon_{ij}\left[q^{\alpha i}_a u^c_{b\alpha}\mathcal{H}'^j + q^{\alpha i}_a h'^j \mathcal{U}^c_{b\alpha} + u^c_{b\alpha} h'^j \mathcal{Q}^{\alpha i}_a\right] + h.c. ;$$

$\mathcal{L}_{S,F}$ contains interactions involving only scalar bosons and, denoting with A_z the generic scalar boson, is given by :

$$\mathcal{L}_{S,F} = -\left(\frac{\partial f_I}{\partial A_z}\right)^* \left(\frac{\partial f_I}{\partial A_z}\right) .$$

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