ISAS - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

Use of HNC and Scaling techniques on the study of FQHE

Thesis submitted for the degree of

"Master Philosophiæ"

CANDIDATE

Orion Ciftja

SUPERVISOR

Prof. Stefano Fantoni

October 1995

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1 Introduction

In 1982 Stormer and Tsui [?] while attempting to observe the Wigner crystal in a high-mobility $GaAs/Al_{0.3}Ga_{0.7}As$ heterojunction discovered the fractional quantum Hall effect (FQHE) at the fractional fillings $\nu = \frac{1}{3}$ and $\frac{2}{3}$ of the lowest Landau level(LLL) .

FQHE is observed in 2-d electronic systems in the extreme quantum limit of very high $B(> 5T)$, low $T(< 2K)$ and high-mobility electronic samples ($\mu > 10^5 cm^2/Vs$). Under these extreme conditions when the LLL is fractionally filled the FQHE resistencies appear quantized as :

$$\rho_{xy}(plateau) = \frac{h}{e^2}\frac{1}{\nu}$$

$$\rho_{xx}(plateau) = 0$$

where ν is the LLL filling factor.

The filling factors where FQHE was observed are such that they appear with odd denominators: $\nu = \frac{1}{3}, \frac{2}{3}$ ..., $\frac{1}{5}$,...$\frac{4}{9}$ ...etc ,but FQHE is not confined only to the lowest Landau level, he is observed also in the higher Landau levels (n=1,2...), where some surprises came.

A fractional Hall plateau $\rho_{xy} = (\frac{h}{e^2})/\frac{5}{2}$, corresponding to an even denominator filling

factor was found. There are indications that other even denominator filling factors ,like the very interestig case of $\nu = \frac{1}{2}$ exists and work in such direction is current research.

Why is the FQHE so exciting ?

Firstly the observation at the LLL of a quantized plateau at $\nu = \frac{1}{3}$ represents an unusual evidence of a fractional quantum number. Clearly, the single-particle density of states model cannot account for the existence of structures phenomenologically similar to the IQHE at fractional ν, where D(E) is structure-less.

It follows that the electronic state underlying the FQHE must be of many electronic origin, and the new quantized ρ_{xy} plateaus in FQHE result from transport of fractionally charged quasiparticles and quasiholes in the strongly correlated electronic system.

FQHE is actually met in 2-d electronic systems of very high mobility electrons. Impurity potentials are not expected to be very important in comparison with $e - e$ Coulomb interactions.

The first step in the FQHE explanation would be the study of the properties of a system of 2-d interacting electrons in a uniform positive background, with the magnetic field $\vec{B}$ so high and T so low, such that only the LLL would be partially filled. The filling factor of the lowest Landau level (LLL) ν can be written as :

$$\nu = 2\pi l_0^2 n$$

where $l_0 = \sqrt{\frac{\hbar}{eB}}$ is the so-called magnetic length and n is the 2-d electronic density.

The unit of Coulomb potential energy is $\frac{e^2}{\epsilon l_0}$ which is the energy scale throughout and ϵ is the background dielectric constant. Where FQHE has been generally observed can be

verified that :

$$\frac{e^2}{\epsilon l_0} \leq \hbar\omega_c$$

so the admixture of states in higher Landau levels can be ignored .

Magnetic field $\vec{B}$ is considered such that all electronic spins $\vec{S}$ are frozen along the applied field $\vec{B}$ and have no interesting dynamics .

Electrons with charge $-e(e > 0)$ are considered as usual to be confined in the x-y plane. They are subjected to a strong magnetic field perpendicular to the plane. Considering the symmetric gauge:

$$\vec{A} = \left[\frac{B}{2}y, \frac{-B}{2}x, 0\right]$$

the magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$ results to be of the form:

$$\vec{B} = [0, 0, -B]$$

The many-electron system is described by the hamiltonian:

$$H = \frac{1}{2m_e}\sum_{j=1}^{N}(-i\hbar\vec{\nabla}_j + e\vec{A}_j)^2 + \sum_{j=1}^{N}V(z_j) + \sum_{j<k}^{N}\frac{e^2}{|z_j - z_k|}$$

where $e > 0$, m_e - is the electron mass, $z_j = x_j + iy_j$ - is the location of the j-th electron in complex coordinates, $V(z_j)$-the potential generated by the uniform neutralizing background charge and N the number of electrons in the LLL.

For filling factor $\nu = \frac{1}{m}$ of the LLL with m=1.3.....odd. Laughlin [?] proposed the following variational ground state wave function :

$$\psi_m = \prod_{j<k}^{N}(z_j - z_k)^m \prod_{j=1}^{N} e^{\frac{-|z_j|^2}{4l_0^2}}$$

To determine which m minimazes the energy , he wrote the probability distribution of the 2-d electrons described by ψ_m as :

$$|\psi_m|^2 = e^{-\beta H_m}$$

with

$$\beta H_m = -2m \sum_{j<k}^{N} ln|z_j - z_k| + \sum_{j=1}^{N} \frac{|z_j|^2}{2l_0^2}$$

From here he identified $\beta = \frac{1}{m}$ and mapping the problem into a classical 2-d OCP plasma [?] he found that energy is minimazed when plasma 2-d electrons spread out uniformly in a disk where 2-d electronic density n_m corresponds to a filling factor $\nu = \frac{1}{m}$ where m is an odd integer.

Let us recall the 2-d OCP where the potential energy is the following :

$$V(r) = -e^2 \sum_{j<k}^{N} ln(\frac{r_{jk}}{L}) + N\frac{e^2}{2} \sum_{j=1}^{N} (\frac{r_j}{R})^2 + \frac{N^2e^2}{2}[ln(\frac{R}{L}) - \frac{3}{4}]$$

L -is an arbitrary scaling length and N -is the total number of particles. Taking the number density of the particles n , a convenient unit of length is the ion-disk radius : $a = \frac{1}{\sqrt{\pi n}}$ so in the future will be convenient to choose $L = a$, and use reduced distances $x = \frac{r}{a}$. If the N particles are confined to a disk of radius R uniformly filled by the neutralizing background , the total potential described previously is the sum of particle-particle , particle-background and background-background interactions .

Monte Carlo calculations of Caillol et al [?] have demonstrated that 2-d OCP is a hexagonal crystal when the dimensionless plasma parameter $\Gamma = e^2\beta$ is greater than 140 and a fluid otherwise. Laughlin maped his FQHE states to a 2-d OCP with coupling

parameters of the form :

$$\Gamma = 2m$$

The most interesting feature in the intermedite coupling regime of the 2-d OCP is that calculations can be done exactly for one special value of coupling constant , namely : $\Gamma = 2$.

The 2-d classical plasma provides a strong support that the Laughlin ground state function is indeed a translationally invariant incommpresible liquid up to $\nu_c \approx \frac{1}{7}$, where a 2-d hexagonal Wigner crystal ground state function is calculated to be more preferable.

For $\nu = \frac{1}{3}$ and $\nu = \frac{1}{5}$ Laughlin [?] , using a Modified Hypernetted Chain (MHNC) technique described by [?] found : $U(\frac{1}{3}) = (-0.4156 \pm 0.0012)\ \frac{e^2}{\epsilon l_0}$ and $U(\frac{1}{5}) = (-0.3340 \pm 0.0028)\ \frac{e^2}{\epsilon l_0}$.

Later , almost exact results for the energy per particle were obtained by Levesque et al [?] performing Monte Carlo calculations on 256 electrons and obtaining : $U(\frac{1}{3}) = (-0.410 \pm 0.0001)\ \frac{e^2}{\epsilon l_0}$ and $U(\frac{1}{5}) = (-0.3277 \pm 0.0002)\ \frac{e^2}{\epsilon l_0}$.

Laughlin's model accounts the FQHE at $\nu = \frac{1}{m}$ and $\nu = 1 - \frac{1}{m}$, (by electron-hole symmetry) [?] for m-odd .

The elementary charged excitations in a stable state $\nu = \frac{1}{m}$ are quasiparticles and quasiholes with fractional charge $\pm\frac{e}{m}$. If one electron is added to the system. it amounts to adding m elementary excitations. The wave function at $\nu = \frac{p}{q}$ changes by a complex phase factor $e^{i\pi\nu}$, upon the interchange of 2 quasiparticles. so quasiparticles in the Laughlin model obey fractional statistics , but they also can be described by wave functions obeying Bose or Fermi statistics.

When filling factor ν is slightly shifted from the stable value $\frac{1}{m}$, where m is odd, the ground state of the system is expected to consist of a small density of quasiparticles or quasiholes with fractional charge $\pm\frac{e}{m}$ interacting via Coulomb interaction.

When filling factor ν is slightly less than $\frac{1}{m}$, quasiholes are formed, while when is slightly higher than $\frac{1}{m}$, quasielectrons are formed . We study the second hierarchy states like $\nu = \frac{2}{i}$ ...using some non-conventional Ground State Wave Functions known with the name of Extended Shadow Wave Functions (ESWF).

Any further knowledges here may result useful for our future steps on studying the FQHE , namely to test the Ground State Wave Functions of Jain [?] , which seem the last credited ones of being the general . His Composite fermion idea looks fascinating , but further considerations and reliable tests are not available yet . We use the HNC/0 and HNC/0+E techniques throughout this work , which are very reliable in the thermodynamic limit . This has to do with the simple fact that even if people who make Monte Carlo can be very proud of their results and it is not easy to compete with them , in the case of the hierarchy of Jain , the thermodynamic limit is essential in order to materialize even the less understood filling factor $\nu = \frac{1}{2}$. Fortunately HNC guarantees that . What a HNC cannot generally guarantee is the high accuracy of MC simulations , because in a HNC we are forced to neglect the cumbersome so-called Elementary (Bridge) diagrams for whom until now there is not a systematic way of summing them .

We shall prove that , at least for the Laughlin states , this problem can be succesfully overcomed and the rather good results obtained with HNC/0 , become even better using a

Scaling technique , which I denote as : HNC/0+E .

2 Laughlin states

The Hypper-netted chain (HNC) method is a standart one used in the study of strongly interacting bosonic or fermionic systems. Let us make only a brief description of the 1-component HNC/0 method. It follows that multi-component HNC/0 method is only a straightforward generalization of the 1-component one.

Let suppose we have a 1-component non-ideal system in classical statistical mechanics and we want to calculate the pair-distribution function :

$$g(r_{12}) = \frac{N(N-1)}{\rho^2} \frac{\int exp[-\beta \sum_{i<j}^{N} u(r_{ij})] d\vec{r}_3 ... d\vec{r}_N}{\int exp[-\beta \sum_{i<j}^{N} u(r_{ij})] d\vec{r}_1 ... d\vec{r}_N} \tag{2.1}$$

at temperatures $T = (\beta k_B)^{-1}$ and density ρ. Let us define :

$$f(r_{ij}) = exp[-\beta u(r_{ij})] - 1$$

where $u(r_{ij})$ is the interaction between particles i and j .

Closely related to the pair distribution function is the long ranged pair correlation function defined as :

$$h(r_{12}) = g(r_{12}) - 1$$

which heals out to 0 as the interparticle distance grows very large. It turns out that the following formula is valid :

$$g(r_{12}) = [1 + f(r_{12})]e^{N(r_{12})+E(r_{12})}$$

where $N(r_{12})$ is the sum of all Simple Nodal 1-2 diagrams , while $E(r_{12})$ is the sum of all Simple Non-nodal diagrams . Now everything is reduced (nice word to describe the complicated things) on the ability to sum all possible diagrams which in this case can be grouped in the following classes and types :

- Composite (Non-nodal) diagrams , from now denoted as Non-nodal diagrams .
- Non-Composite (Simple Nodal) diagrams , of the type of Simple Chain or Netted Chain, from now denoted as Nodal diagrams .
- Non-composite (Simple Non-nodal) diagrams , of the type Elementary (known also with the name of Bridge) , from now denoted as Elementary diagrams .

To sum them is not an easy task. Returning to the previous formula if we have the Nodal function $N(r_{12})$ and the Elementary function $E(r_{12})$ the Non-nodal function $X(r_{12})$ is given as :

$$X(r_{12}) = g(r_{12}) - 1 - N(r_{12}) \tag{2.2}$$

and the generation of $g(r)$ must go through a self consistent procedure. Here comes the difficulty : its not known yet a systematic way to deal with the Elementary diagrams closing in the same time the self-consistency . And the best way to eliminate a problem is..... just

to neglect its existance . This is the reasion why always as a first approximation (and a good one) the contribution of the Elementary diagrams is put to 0 :

$$E(r_{12}) = 0$$

The subscript "0" at the HNC/0 reflects this approximation . The Nodal diagrams are easily summed in Furie space and we finish with

$$N(q) = \rho X(q)[X(q) + N(q)]$$

Finally within the 1-component HNC/0 scheme the pair distribution function $g(r_{12})$ is obtained by solving self-consistently the following set of coupled equations :

$$g(r_{12}) = [1 + f(r_{12})]e^{N(r_{12})} = e^{-\beta u(r_{12}) + N(r_{12})} \tag{2.3}$$

$$X(r_{12}) = g(r_{12}) - 1 - N(r_{12}) \tag{2.4}$$

$$N(q) = \frac{\rho X(q)^2}{1 - \rho X(q)} \tag{2.5}$$

We start the procedure puting $N(r) = 0$.

We calculate :

$$X(r) = g(r) - 1$$

Performing a Furie transform we find

$$X(q)$$

and from it we easily compute $N(q)$ which in turn by being Inverse Furie transformed is converted in the new $N(r)$. The process goes on until the desired convergency is achieved.

The 2-d Furie transforms are defined in the following way :

$$F(q) = \int e^{i\vec{q}\vec{r}} F(r) d\vec{r}$$

$$F(r) = \frac{1}{(2\pi)^2} \int e^{-i\vec{q}\vec{r}} F(q) d\vec{q}$$

Let us now apply the HNC/0 scheme to the Laughlin states :

$$\beta H_m = -2m \sum_{j<k}^{N} ln|z_j - z_k| + \sum_{j=1}^{N} \frac{|z_j|^2}{2l_0^2}$$

We can easily identify that : $\beta u(r)$ which enters the HNC/0 equations is the following : $-2mln(r)$.

To simplify notation let absorb β in the potential defining our pseudopotential as :

$$u(r) = -2mln(r)$$

In order to handle the logarithmic interaction , the standard procedure is to separate the short-range and the long-range part of the interactions as

$$u^s(r) = 2mK_0(Qr)$$

$$u^l(r) = -2mK_0(Qr) - 2mln(r)$$

where $K_0(x)$ is the modified Bessel function , and Q is the cutoff parameter of order unity. Defining the short-ranged Nodal and Non-nodal functions .

$$N^s(r) = N(r) - u^l(r)$$

$$X^s(r) = X(r) + u^l(r)$$

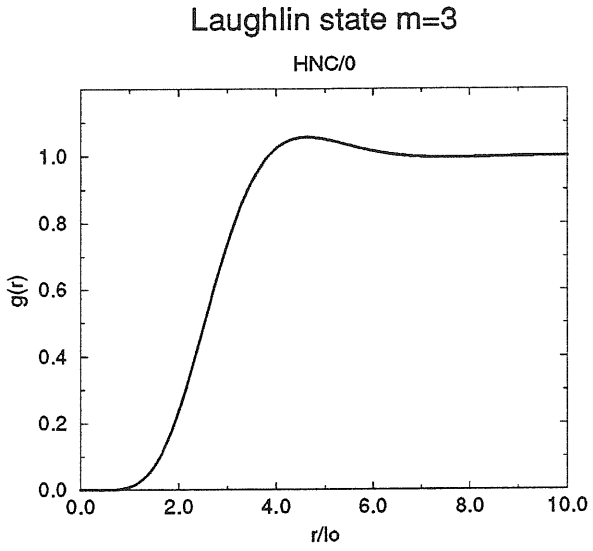

Fig.1

Figure 2.1: Pair distribution function $g(r)$ for m=3 obtained with a HNC/0 calculation

the HNC/0 final set of equations is solved with the same iteration scheme starting from $N^s(r) = 0$.

In Fig.1 we plot the resulting $g(r)$ for $\nu = \frac{1}{3}$ as a function of $\frac{r}{l_0}$, it shows clearly characteristics of a Liquid state.

For small $\frac{r}{l_0}$, $g(r)$ dies as $(\frac{r}{l_0})^6$, differently from the r^2 dependence of the crystal state . The ground state energy per particle results : $U(\frac{1}{3}) = -0.4055\ \frac{e^2}{\epsilon l_0}$ very close to the result $U(\frac{1}{3}) = -0.4056\ \frac{e^2}{\epsilon l_0}$ [?] , obtained with a similar technique .

It is widely belived that the HNC/0 treatment even if its very accurate lacks of Ther-

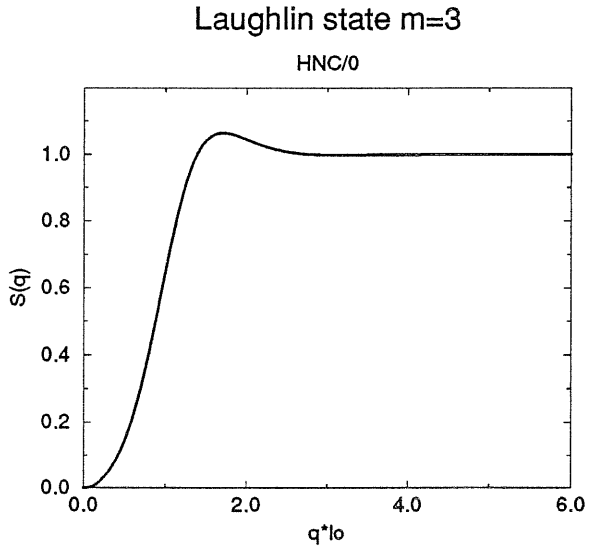

Fig.2

Figure 2.2: Structure factor $S(q)$ for m=3 obtained with a HNC/0 calculation

modynamic consistency . This is related to the fact that the sum rules that $g(r)$ must satisfy in the LLL are somehow violated . To my opinion this simply has to do with the fact that HNC/0 being an approximation cannot be accurate on everything . In my case the "Charge neutrality " sum rule and "Perfect screening" sum rule are satisfied in the limit of my numerical error , while the so-called "Compressibility " sum rule is violated .

In Fig.2 , we plot the structure factor of the $\nu = \frac{1}{3}$ state as a function of ql_0 . The same procedure applied to the $\nu = \frac{1}{5}$ state is a little bit more peculiar .

In Fig.3 we plot the resulting $g(r)$ for $\nu = \frac{1}{5}$ as a function of $\frac{r}{l_0}$ which still shows char-

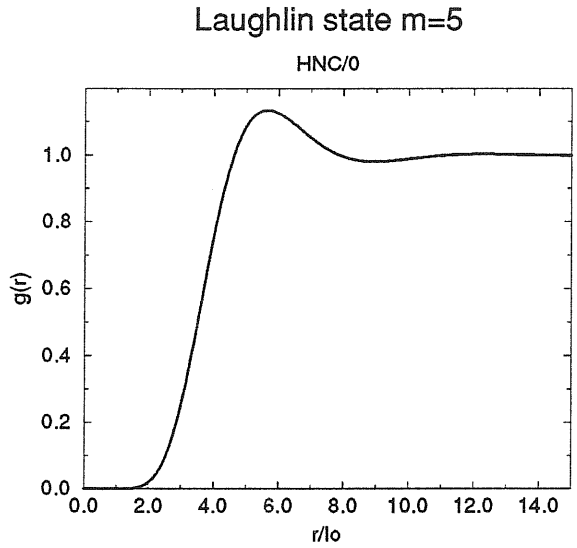

Fig.3

Figure 2.3: Pair distribution function $g(r)$ for m=5 obtained with a HNC/0 calculation

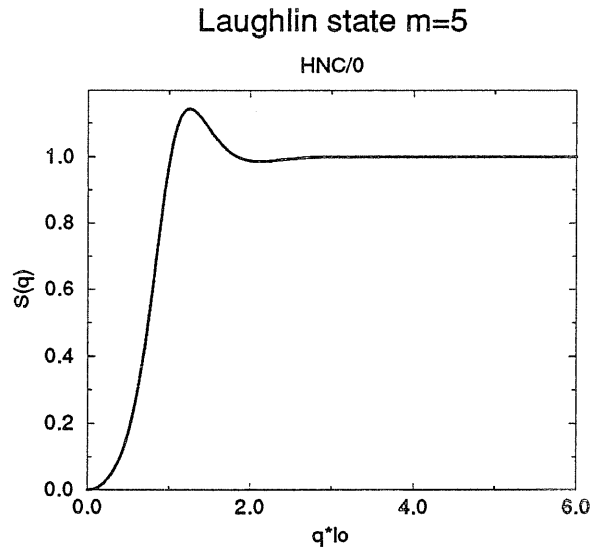

Fig.4

Figure 2.4: Structure factor $S(q)$ for m=5 obtained with a HNC/0 calculation

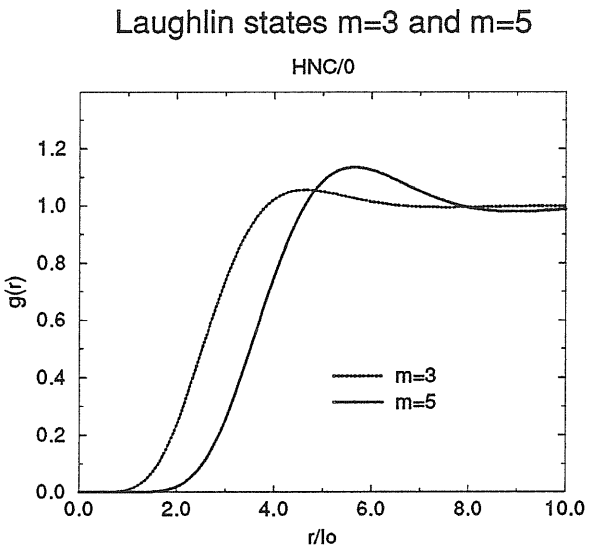

Fig.5

Figure 2.5: Pair distribution functions $g(r)$ for m=3 and m=5 obtained with a HNC/0 calculation

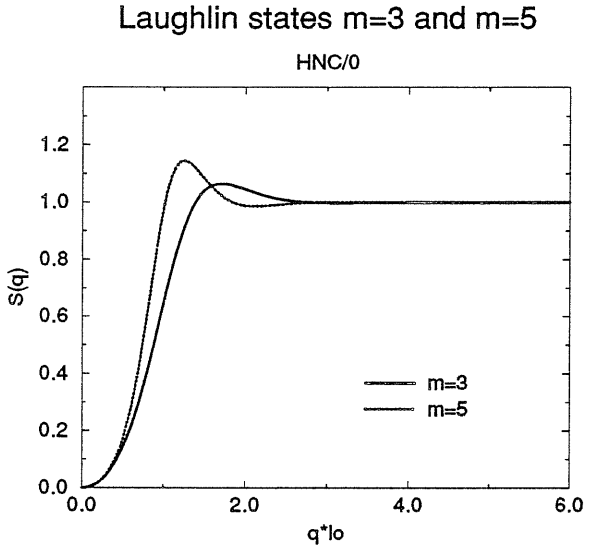

Fig.6

Figure 2.6: Structure factors $S(q)$ for m=3 and m=5 obtained with a HNC/0 calculation

acteristics of a Liquid state , but now $g(r)$ is much more structured than the $\nu = \frac{1}{3}$ case .

For small $\frac{r}{l_0}$, $g(r)$ dies as $(\frac{r}{l_0})^{10}$ in total agreement with the Laughlin theory .

The ground state energy per particle is : $U(\frac{1}{5}) = -0.3240\ \frac{e^2}{\epsilon l_0}$ very close to the almost exact result of [?] .

The resulting structure factor of the state $\nu = \frac{1}{5}$ is plotted in Fig.4 as a function of ql_0 .

With reference to the sum rules the same conclusions drawn for the $\nu = \frac{1}{3}$ case remain valid here .

In Fig.5 we compare both pair distribution functions and we can clearly see the tendency that for smaller filling factors the solid-like behaviour becomes more evident (in fact for

filling factors smaller than $\frac{1}{7}$ the electrons solidify in a Wigner crystal) .

In Fig.6 the same comparison between the respective structure factors is done.

The natural question coming after this treatment is :

How to improve it ? The magic answer seems to be "SCALING".

It is argued that a HNC/0 procedure must have a better performance if an intelligent way of envolving there the neglected Elementary diagrams is found . This is not easy and a systematic way of doing it is not known yet . However efforts were done by Laughlin [?] who used a MHNC treatment described by [?] . The ground state energies per particle he obtained are as good as the HNC/0 results in the sense that the absolute best estimation for $\nu = \frac{1}{3}$ is $U(\frac{1}{3}) = -0.410(1)\ \frac{\epsilon^2}{\epsilon l_0}$ and either HNC/0 , or MHNC have the same deviation from this value .

The main idea is based on the assumption that the shape of the Elementary function for a 2-d OCP for a general coupling constant Γ does not change significantly with Γ . In other words the following ansatz was done :

$$E_\Gamma(x) = \alpha E_{\Gamma=2}(x)$$

The $\Gamma = 2$ case is the only one where exact results are available [?] and what remains to be done is the fitting of coefficient α , to achieve thermodynamic consistency . In the Laughlin theory the states $\nu = \frac{1}{m}$ correspond to a 2-d OCP with coupling constant $\Gamma = 2m$ and this was exactly what he did .

Now let me think differently :

Is this ansatz the best one ?

I would like to judge in this way : there is no doubt that it works perfectly within the plasma framework , but to me it does not seem the most natural one in the framework of the FQHE . The only simple argument that I can state to support my claim is of physical nature :

The FQHE states $\nu = \frac{1}{m}$ and $\nu = 1$ are physically very different , the first one is an Incompressible liquid governed by strong correlations in the ground state , while the latter one is just the Integer Quantum Hall Effect (IQHE) where the LLL is completely filled.

For this reason I would like to pose the problem differently :

Am I able to calculate all Elementary diagrams and to sum them ?

The answer is : No

But what I am able to calculate numerically is the lowest order Elementary diagram $E4(r)$. It is not an easy task , but at the end is doable . The calculation of other Elementary diagrams seems to be a very difficult task and unless I am not able to sum all of them , essentially I dont need to think more .

Now I make the ansatz : the shape of the Elementary function for the Laughlin states $\nu = \frac{1}{m}$ does not change significantly from that of the lowest order Elementary diagram of that state :

$$E(x) = \alpha E4(x)$$

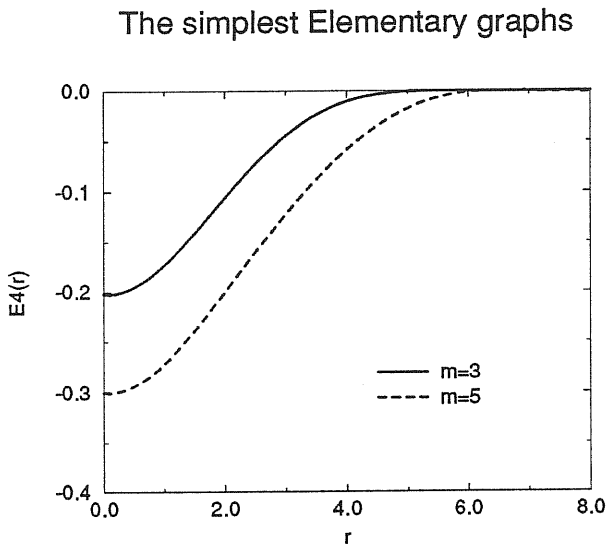

Figure 2.7: Lowest order Elementary function $E4(r)$ for m=3 and m=5

The next step is to fit α in order that $g(r)$ satisfies all the sum rules valid on the LLL reaching the thermodynamic consistency that was missing in the HNC/0 treatment . The lowest order Elementary diagram is the following :

$$E4(r_{12}) = \frac{\rho^2}{2}\int [h(r_{13})h(r_{14})h(r_{23})h(r_{24})h(r_{34})]d^2r_3 d^2r_4$$

where the long ranged pair correlation function $h(r)$ was previously obtained from the HNC/0 calculation .

In Fig.7 we plot the resulting $E4(r)$ functions for $m = 3$ and $m = 5$ respectively . After finding them the only thing which remains to be done is to run again the HNC/0

procedure with the difference that now we are including there also the Elementary function that was missing before .

Let me call this modified HNC/0 technique as : HNC/0+E .

The only equation that changes is the equation regarding the pair distribution function which now looks like :

$$g(r_{12}) = e^{-u(r_{12})+N(r_{12})+E(r_{12})}$$

As we knew before the HNC/0 pair distribution function lacks thermodynamic consistency, so my scaling parameter α must be fitted in such a way that the new $g(r)$ satisfies all sum rules where the "compressibility" sum rule was the bad-behaved one in the case of HNC/0

The scaling parameter α was found to be : $\alpha = 6.25$.

In Fig.8 we plot the pair distribution function $g(r)$ of the state $\nu = \frac{1}{3}$ obtained after fitting all the sum rules , as a function of $\frac{r}{l_0}$.

To my estimates it fits very well with the best calculations whom I know .

More surprisingly the ground state energy per particle for the Laughlin state $\nu = \frac{1}{3}$ is : $U(\frac{1}{3}) = -0.41005\ \frac{e^2}{\epsilon l_0}$, a value that fits perfectly with the MC exact result of Levesque et al [?] .

Let me mention here some of the best quoted results regarding this state , just to give a flavour of the multivariety of approaches used to study the Laughlin state $\nu = \frac{1}{3}$: Yoshioka et al. [?] put 6 electrons in a box obtaining : $U(\frac{1}{3}) = -0.4129\ \frac{e^2}{\epsilon l_0}$. Haldane and Rezayie [?] used the Spherical geometry to obtain : $U(\frac{1}{3}) = -0.415(\pm 0.0005)\ \frac{e^2}{\epsilon l_0}$. Morf and Halperin [?] performed a MC evaluation in Disk geometry for up to 144 electrons finding a $U(\frac{1}{3}) =$

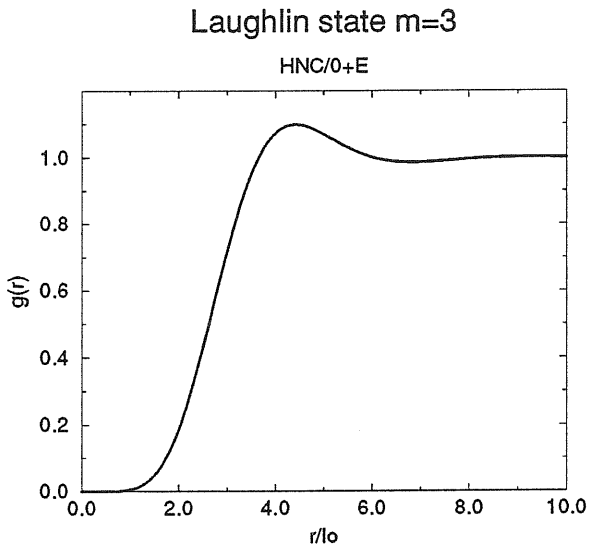

Fig.8

Figure 2.8: Pair distribution function $g(r)$ for m=3 obtained with a HNC/0+E treatment

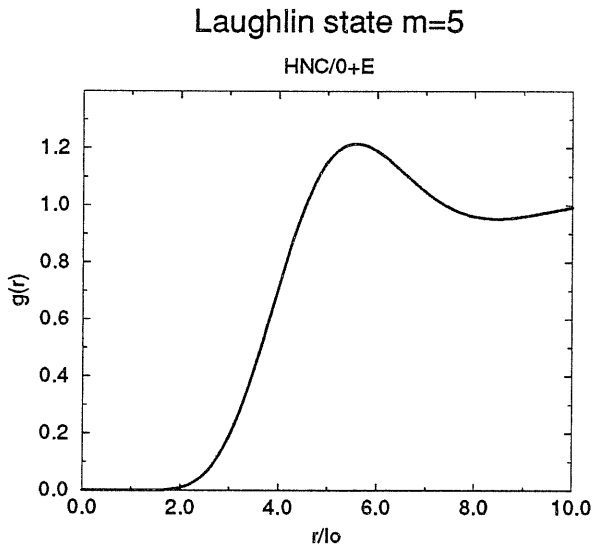

Figure 2.9: Pair distribution function $g(r)$ for m=5 obtained with a HNC/0+E treatment

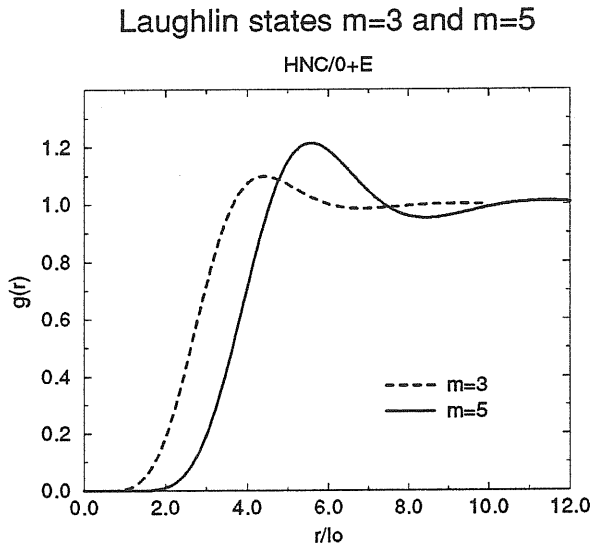

Fig.10

Figure 2.10: Pair distribution function $g(r)$ for m=3 and m=5 obtained with the HNC /0+E technique

$-0.4097\ \frac{e^2}{\epsilon l_0}$ and finally Girvin [?] using an analytic parametrization of $g(r)$ obtained $U(\frac{1}{3}) = -0.4087\ \frac{e^2}{\epsilon l_0}$.

In Fig.11 structure factors S(q) of the states $\nu = \frac{1}{3}$ and $\nu = \frac{1}{5}$. are plotted as a function of ql_0 .

The same scaling procedure (HNC/0+E) applied to the state $\nu = \frac{1}{5}$ results on finding a scaling parameter $\alpha = 5.2$ and the resulting pair distribution function is plotted in Fig.9 . The ground state energy energy per particle turns out to be : $U(\frac{1}{5}) = -0.3274\ \frac{e^2}{l_0}$ that fits

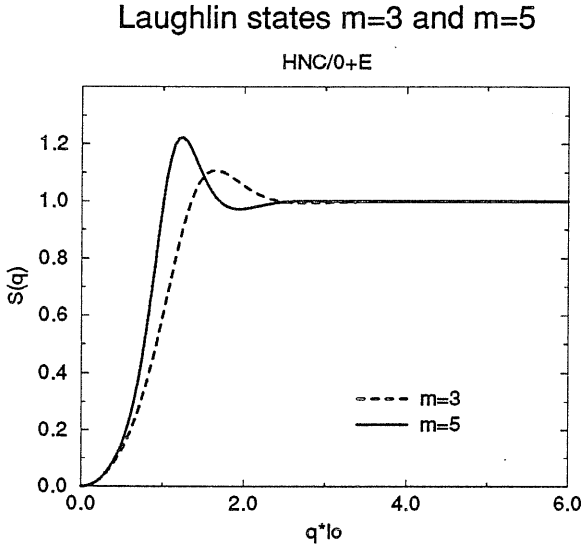

Fig.11

Figure 2.11: Structure factors $S(q)$ for m=3 and m=5 obtained with the HNC/0+E tech nique

very well with the almost exact results of [?] .

A comparison between the respective g(r)-s obtained from HNC/0+E is done in Fig.10 .

The respective structure factors are plotted in Fig.11 .

Fig.12 shows the influence of the Elementary function (HNC/0+E) on modifying the pair distribution function for the Laughlin state $\nu = \frac{1}{3}$.

Fig.13 regards the same . but for the state $\nu = \frac{1}{5}$.

In Fig.14 and Fig.15 we can respectively compare the HNC/0 and HNC/0+E structure factors of the Laughlin states $\nu = \frac{1}{3}$ and $\nu = \frac{1}{5}$.

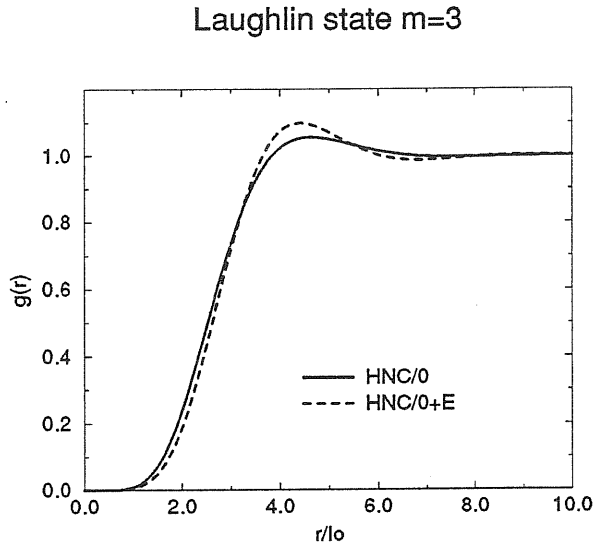

Figure 2.12: Pair distribution function $g(r)$ for m=3, a comparison between HNC/0 and HNC/0+E technique is done

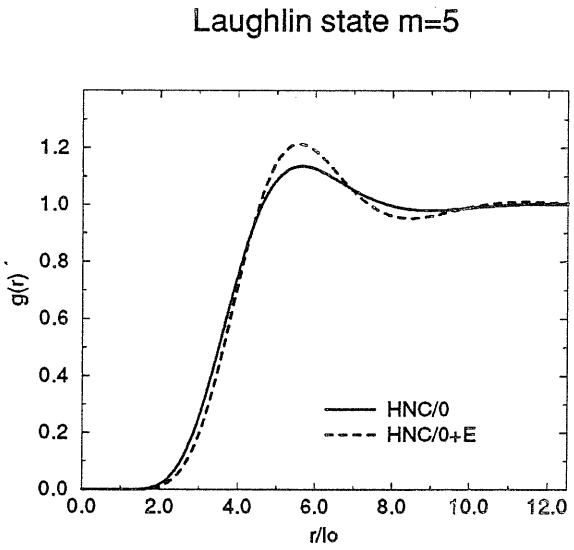

Figure 2.13: Pair distribution function $g(r)$ for m=5, a comparison between HNC/0 and HNC/0+E technique is done

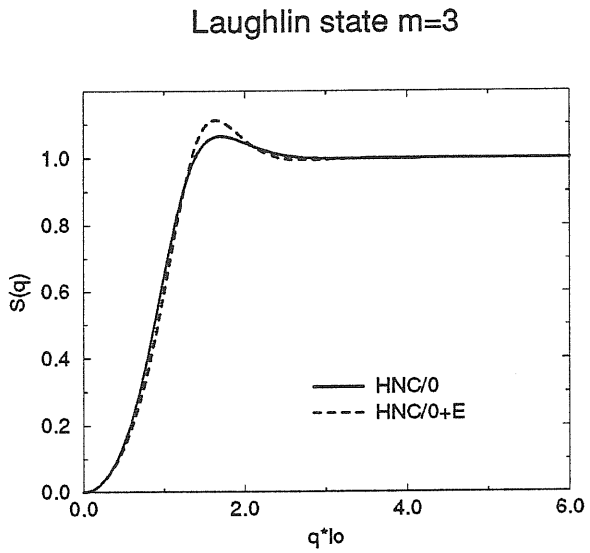

Fig.14

Figure 2.14: Structure factors $S(q)$ for m=3, a comparison between HNC/0 and HNC/0+E technique is done

Finally in Fig.16 and Fig.17 we compare the present result with the analytic form of $g(r)$ fited with 27 coefficients to MC data [?] .

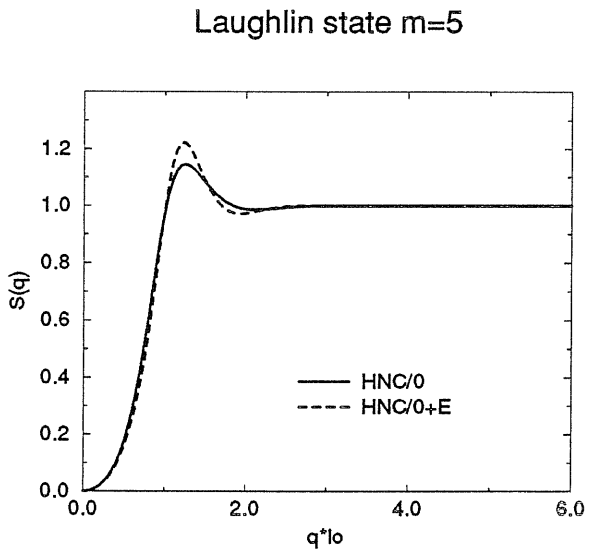

Figure 2.15: Structure factors $S(q)$ for m=5, a comparison between HNC/0 and HNC/0+E technique is done

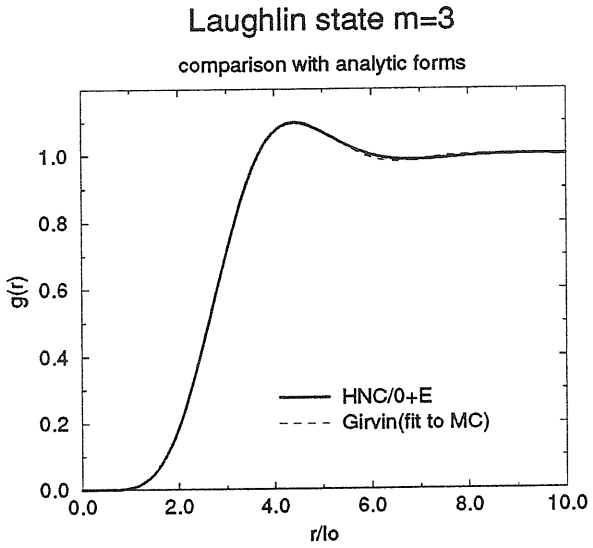

Fig.16

Figure 2.16: A comparison of our pair distribution function $g(r)$ for m=3 with an analyt c MC fitted one

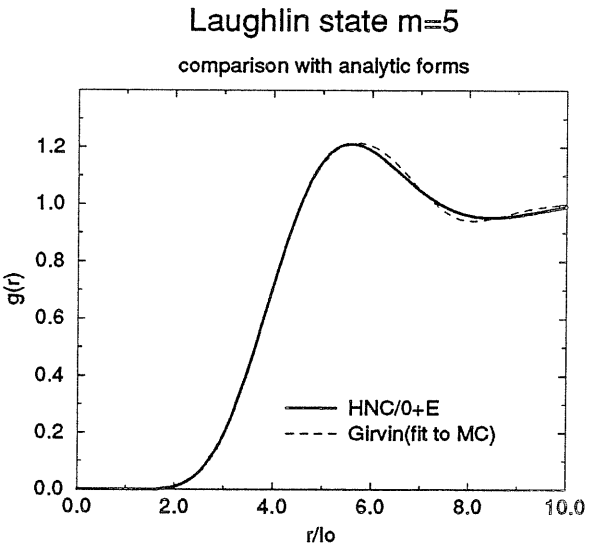

Fig.17

Figure 2.17: A comparison of our pair distribution function $g(r)$ for m=5 with an analytic M C fitted one

3 HNC/0 treatment for the ESWF

The description of strong interparticle correlations in both Bose and Fermi systems with continuous degrees of freedom is a long-standing problem of current interest. More recently the shadow wave function (SWF) has been proposed as a new variational ansatz to compute the properties of solid and liquid He^4 at T=0K.

The form of SWF in the bosonic case has been proposed to be as:

$$\Psi^B_{SWF}(\vec{r}_1...\vec{r}_N) = \prod_{i<j}^{N} f_{pp}(r_{ij}) \int \prod_{i=1}^{N} \theta(|\vec{r}_i - \vec{s}_i|) \prod_{i<j}^{N} f_{ss}(s_{ij}) d\vec{S} \tag{3.1}$$

$\vec{S}$ denotes all $[\vec{s}_i]$ and $\vec{s}_i$ are the so-called "shadow" variables associated to each particle,

$$f_{pp}(r_{ij}) = e^{\frac{-U_{pp}(r_{ij})}{2}}$$

$$f_{ss}(s_{ij}) = e^{-U_{ss}(s_{ij})}$$

where $U_{pp}(r_{ij})$ and $U_{ss}(s_{ij})$ represent respectively the particle-particle and shadow-shadow interaction. $f_{ss}(s_{ij})$ has the same structure as $f_{pp}(r_{ij})$ namely it heals out to 1 at large intershadow distances, whereas the "correlation" $\theta(x)$ between a particle and its associated shadow is taken of Gaussian form normalized to 1.

Physically, the shadow variables $\vec{s}_i$ can be thought of as mimicking the quantum correlation "holes" which the particles carry around themselves in the dense system.

In the fermionic case the SWF is assumed of the form:

$$\Psi^F_{SWF}(\vec{r}_1...\vec{r}_N) = \Psi^B_{SWF}(\vec{r}_1...\vec{r}_N)\Phi_{FG}(\vec{r}_1...\vec{r}_N) \tag{3.2}$$

where $\Phi_{FG}(\vec{r}_1...\vec{r}_N)$ is the uncorrelated Ideal Fermi wave function (Slater determinant). The physical interpretation of SWF as well as the request of more variational freedom and of a full symmetry under the exchange of any particle with any hole, suggests further extended forms for the SWF, so a new type of so-called extended shadow wave function (ESWF) was proposed.

The ESWF in the bosonic case is:

$$\Psi^B_{ES}(\vec{r}_1...\vec{r}_N) = \prod_{i<j}^{N} f_{pp}(r_{ij}) \int \prod_{i,j}^{N,M} f_{ps}(|\vec{r}_i - \vec{s}_j|) \prod_{i<j}^{M} f_{ss}(s_{ij}) d\vec{S} \tag{3.3}$$

where N and M are respectively the number of particles and shadows. The extension which (3.3) represents over the standard SWF of (3.1) concerns two aspects.

First, in the ESWF all shadows are correlated with all real particles, rather than being in one to one correspondence as in (3.1). This allows the possibility that the number and location of "holes" become different from those of the real particles.

The second aspect, which is however related to the first, is to assume a more general "correlation" function f_{ps}, than the original Gaussian form of (3.1). In (3.3) all three: $f_{pp}(x)$, $f_{ps}(x)$, $f_{ss}(x)$ are taken to heal out to 1 at large values of x.

The ESWF in the fermionic case is assumed of the form:

$$\Psi^F_{ES}(\vec{r}_1...\vec{r}_N) = \Psi^B_{ES}(\vec{r}_1...\vec{r}_N)\Phi_{FG}(\vec{r}_1...\vec{r}_N) \tag{3.4}$$

In the bosonic case of ESWF, the cluster diagrams are characterized by only 3 different types of points: p, s^R, s^L, where p denotes particle and $s^{R,L}$ denotes respectivly right or left shadow. In fact the normalization of ESWF in the bosonic case is:

$$< \Psi^B_{ES}|\Psi^B_{ES} >= \int [\prod_{i<j}^{N} f_{pp}(r_{ij})^2 \int \prod_{i,j}^{N,M} f_{ps}(|\vec{r}_i - \vec{s_j}^L|) \prod_{i<j}^{M} f_{ss}(s_{ij}{}^L) d\vec{S}^L$$

$$\int \prod_{i,j}^{N,M} f_{ps}(|\vec{r}_i - \vec{s_j}^R|) \prod_{i<j}^{M} f_{ss}(s_{ij}{}^R) d\vec{S}^R] d\vec{R} \tag{3.5}$$

and coincides with the partition function of a classical 3-component system $(p. s^R, s^L)$ interacting via the (pseudo)potentials:

$$U_{pp} = -lnf_{pp}{}^2$$

$$U_{ps^R} = U_{ps^L} = -lnf_{ps}$$

$$U_{s^R s^R} = U_{s^L s^L} = -lnf_{ss}$$

$$U_{s^R s^L} = 0$$

There have been several schemes [?],[?],[?] to explain the hierarchy of states for the FQHE. One of them for the second level of the hierarchy has been provided by Macdonald et al [?] in terms of the following wave function:

$$\Psi_\nu = \prod_{j<k}^{N} (z_j - z_k)^{p+1} e^{-\sum_{j=1}^{N} \frac{|z_j|^2}{4l_0^2}}$$

$$\int \prod_{j,k}^{N,M} (z_j - s_k) \prod_{j<k}^{M} (s_j^* - s_k^*)^m (s_j - s_k) \epsilon^{-\sum_{j=1}^{M} \frac{|s_j|^2}{2l_0^2}} d\vec{S} \tag{3.6}$$

where $l_0 = \sqrt{\frac{\hbar}{\epsilon B}}$ is the magnetic length, $N + M = mM$ [?] where N, M are respectively the number of particles and shadows and m is the Laughlin odd integer number.

The respective particle and shadow coordinates z_k and s_k are given in complex notation $x_k + iy_k$ where p is an integer (0,2,4..) [?] number .

If we consider the Laughlin parent state $\nu_0 = \frac{1}{m}$ the the wave function Ψ_ν will correspond to the daughter state :

$$\frac{1}{\nu} = \frac{1}{1 - \nu_0} + p$$

For instance the Laughlin parent state $\nu_0 = \frac{1}{3}$ where $m = 3$, for p=2 with generate the daughter state $\nu = \frac{2}{7}$.

The Laughlin parent state $\nu_0 = \frac{1}{3}$ where $m = 3$, for p=0 with generate the daughter state $\nu = \frac{2}{3}$.

The Laughlin parent state $\nu_0 = \frac{1}{5}$ where $m = 5$, for p=0 with generate the daughter state $\nu = \frac{4}{5}$.

In this way by choosing different m and p we can go from the Laughlin state (first level of hierarchy) to the proposed wave functions for other FQHE states. The exponential factors in Ψ_ν are relevant only in the trivial long wavelength limit, so they may be not considered for our purposes.

In this case a simple comparison between Ψ_ν and the bosonic ESWF Ψ_{ES}^B, evidents that Ψ_ν is a bosonic ESWF with :

$$f_{pp}(jk) = (z_j - z_k)^{p+1}$$

$$f_{ps}(jk) = (z_j - s_k)$$

$$f_{ss}(jk) = (s_j^* - s_k^*)^m (s_j - s_k)$$

As in the bosonic ESWf case the normalization of Ψ_ν yields:

$$< \Psi_\nu | \Psi_\nu >= \Psi_\nu^* \Psi_\nu = \prod_{j<k}^{N} |z_j - z_k|^{2(p+1)} \int \prod_{j,k}^{N,M} (z_j^* - s_k^{L*}) \prod_{j<k}^{M} (s_j^L - s_k^L)^n$$

$$(s_j^{L*} - s_k^{L*}) \prod_{j,k}^{N,M} (z_j - s_k^R) \prod_{j<k}^{M} (s_j^{R*} - s_k^{R*})^m (s_j^R - s_k^R) d\vec{S}^L d\vec{S}^R \qquad (3.7)$$

where the detailed form of the exponential factors is not considered being irrelevant for our purposes .

Writing :

$$|z_j - z_k|^{2(p+1)} = e^{-U_{pp}}$$

$$(z_j - s_k^R) = e^{-U_{pR}}$$

$$(s_j^{R*} - s_k^{R*})^m (s_j^R - s_k^R) = e^{-U_{RR}}$$

we see that the normalization condition for Ψ_ν coincides with the partition function of a 3-component system (p, R, L) interacting via the (pseudo)potentials :

$$U_{pp} = -2(p+1) ln|z_j - z_k|$$

$$U_{pR} = U_{pL} = -ln|z_j - s_k^R| - i\theta_{jR,kR}$$

$$U_{RR} = U_{LL} = -(m+1) ln|s_j^R - s_k^R| + i(m-1)\theta_{jR,kR}$$

$$U_{RL} = 0$$

For simplicity we denoted s^R and s^L respectivily R (right) and L (left) shadow.

For our 3-component system of p, R, L "particles" we can apply a 3-component HNC/0 treatment, but with a major difference with the previously known HNC/0 treatment in the sense that in the present case various HNC/0 quantities are complex functions and depend on both x_{jk} and y_{jk} components of the interparticle distance. Both the correlations $f_{ps}(jk)$ and $f_{ss}(jk)$ depend on the polar angle θ_{jk} . As a consequence the long range parts $U^l_{ps}(jk)$ and $U^l_{ss}(jk)$ brings the angular dependence into all the HNC quantities . It turns out that the Nodal functions $N_{\alpha\beta}(jk)$ and Non-nodal functions $X_{\alpha\beta}(jk) = g_{\alpha\beta}(jk) - 1 - N_{\alpha\beta}(jk)$ have a long range behavior exactly given by $-U^l_{\alpha\beta}(jk)$ and $U^l_{\alpha\beta}(jk)$, so that all pair distribution functions $g_{\alpha\beta}(jk)$ are short ranged [?].

The multi-component HNC/0 method is only a generalization of the 1-component one described in the previous sections . An iteration scheme similar to that used in the 1-component case can be used here.

In the case of a multi-component system the corrisponding HNC/0 quantities are given by the following equations:

$$g_{\alpha\beta}(r_{12}) = e^{-U_{\alpha\beta}(r_{12})+N_{\alpha\beta}(r_{12})} \tag{3.8}$$

$$N_{\alpha\beta}(r_{12}) = \sum_{\gamma} \rho_{\gamma} \int X_{\alpha\gamma}(r_{13})[X_{\gamma\beta}(r_{32}) + N_{\gamma\beta}(r_{32})]d\vec{r}_3 \tag{3.9}$$

$$X_{\alpha\beta}(r_{12}) = g_{\alpha\beta}(r_{12}) - 1 - N_{\alpha\beta}(r_{12}) \tag{3.10}$$

Here ρ_γ represents the densities of the different types of particles of the multi-con ponent system.

Returning to the FQHE we remind that the wave function Ψ_ν that was essentially a bosonic ESWF, corresponds to a 3-component system where the different "particles" were

identified as p (particle), R (right shadow) and L (left shadow).

Due to the fact that Ψ^B_{ES} is symmetric under the exchange of shadow variables s_i irrespective of r_i there are only 4 indipendent HNC/0 quantities, i.e, N_{pp}, $N_{pR} = N_{pL}$, $N_{RR} = N_{LL}$ and N_{RL}.

So among the 3 components p, R, L there are only 4 indipendent pairings : pp), (pR), (RR) and (RL).

The respective densities of the 3 components p, R, L are ρ_p, ρ_s and ρ_s. The 4 possible indipendent pairings are: (pp), (pR), (RR) and (RL). Let us define the 2-d Furie transform as :

$$F_{pp}(q) = \rho_p \int e^{i\vec{q}\vec{r}} F_{pp}(r) d\vec{r}$$

$$F_{ps}(q) = \sqrt{\rho_p \rho_s} \int e^{i\vec{q}\vec{r}} F_{pr}(r) d\vec{r}$$

$$F_{ss}(q) = \rho_s \int e^{i\vec{q}\vec{r}} F_{ss}(r) d\vec{r}$$

where $\vec{r}$ is a 2-d vector ; $d\vec{r} = r dr d\phi$; $\vec{s}$ can be R or L .

The 2-d inverse Furie transforms are :

$$\rho_p F_{pp}(r) = \frac{1}{(2\pi)^2} \int e^{-i\vec{q}\vec{r}} F_{pp}(q) d\vec{q}$$

$$\sqrt{\rho_p \rho_s} F_{ps}(r) = \frac{1}{(2\pi)^2} \int e^{-i\vec{q}\vec{r}} F_{ps}(q) d\vec{q}$$

$$\rho_s F_{ss}(r) = \frac{1}{(2\pi)^2} \int e^{-i\vec{q}\vec{r}} F_{ss}(q) d\vec{q}$$

From the general formula :

$$N_{\alpha\beta}(r_{12}) = \sum_\gamma \rho_\gamma \int X_{\alpha\gamma}(r_{13}) [X_{\gamma\beta}(r_{32}) + N_{\gamma\beta}(r_{32})] d\vec{r}_3$$

by performing a Furie transform like in the convolution we have:

$$N_{\alpha\beta}(q) = \sum_{\gamma} X_{\alpha\gamma}(q)[X_{\gamma\beta}(q) + N_{\gamma\beta}(q)]$$

As R and L are hermitically conjugated we expect:

$$F_{pp}(q) = F^{*}_{pp}(q)$$

$$F_{pR}(q) = F^{*}_{pL}(q)$$

$$F_{RR}(q) = F^{*}_{LL}(q)$$

$$F_{RL}(q) = F^{*}_{LR}(q)$$

While :

$$F_{pR}(q) = F_{Rp}(q)$$

$$F_{pL}(q) = F_{Lp}(q)$$

Keeping this in mind after a long algebra we determine all relations between HNC/0 quantities in the Furie space for the 4 possible indipendent pairings: (pp), (pR), (RR) and (RL).

$$X_{pR}(q) + N_{pR}(q) = \frac{X_{pR}(q)[1 - X^{*}_{RR}(q)] + X^{*}_{pR}(q)X^{*}_{RL}(q)}{D(q)} \tag{3.11}$$

$$X_{RR}(q) + N_{RR}(q) = \frac{[1 - X_{pp}(q)][1 - X^{*}_{RR}(q)] - X^{*}_{pR}(q)^{2}}{D(q)} - 1 \tag{3.12}$$

$$X^{*}_{RL}(q) + N^{*}_{RL}(q) = \frac{X^{*}_{RL}(q)[1 - X_{pp}(q)] + |X_{pR}(q)|^{2}}{D(q)} \tag{3.13}$$

$$X_{pp}(q) + N_{pp}(q) = \frac{|1 - X_{RR}(q)|^{2} - |X_{RL}(q)|^{2}}{D(q)} - 1 \tag{3.14}$$

The form of the function D(q) that enters the denominator is the following :

$$D(q) = [1 - X_{pp}(q)][|1 - X_{RR}(q)|^2 - |X_{RL}(q)|^2] - |X_{pR}(q)|^2[X_{RL}(q) + X^*_{RL}(q) - X_{pR}{}^2(q)[1 - X^*_{RR}(q)] - X^*_{pR}{}^2(q)[1 - X_{RR}(q)] \quad (3.15)$$

As in the previous standard HNC/0 treatment our (pseudo)potentials of the form : $U(z) = aln(z) + \theta$ are separated in their short-ranged and long-ranged parts :

$$U^s(z) = -aK_0(Qz)$$

$$U^l(z) = a[ln(z) + K_0(z)] + \theta$$

where K_0 is the modified Bessel function and Q is the cutoff parameter of order unity.

Later on , all Nodal and Non-nodal functions for all our indipendent pairings (pp), (pR), (RR) and (RL) are splitted in their short and long range parts , and done this our 3-component HNC/0 procedure is ready to start.

The set of coupled equations is solved by an iteration scheme where the pair-distribution function $g(r)$ in the present case may depend on the angle θ too .

The 0-order step in the numerical iteration procedure is to set

$$N^s(r) = 0$$

From the respective HNC/0 formula, knowing the respective short-range (pseudo)potential $U^s(r)$ we calculate $X^s(r)$.

By performing a Furie transform we obtain

$$X^s(q)$$

Substracting from $X^s(q)$ the known long-range part of the respective potential $U^l(q)$ in Furie space we obtain :

$$X(q)$$

From the set of formulas where enter only $X(q)$ quantities and $D(q)$ we can obtain the desired $N(q)$.

Subtracting $U^l(q)$ from it we obtain

$$N^s(q)$$

Finally performing an inverse Furie transform on it we obtain the new value of $N^s\ r)$ and we restart the iteration circle once again until the desired convergency is reached .

Differently from other computations, the present treatment deals with HNC/0 quantities which are:

a) Complex functions depending on z_{jk} complex coordinates of the interparticle distance.

b) Have angular dependence through the polar angle θ_{jk}.

In our computations we considered the density of particles $(m-1)$ times greater than that of shadows [?] .

For instance the state $\nu = \frac{2}{7}$ has $\rho_p = 2\rho_s$ while the state $\nu = \frac{4}{5}$ has $\rho_p = 4\rho_s$.

In all cases the axis coordinates are scaled as dimensionless in the following form :

$$\sqrt{\rho_p}r$$

This form was chosen as the most convenient for our calculations and is different from the widely used $\frac{r}{l_0}$. In our case the distances are expressed not in magnetic length l_0 units ,

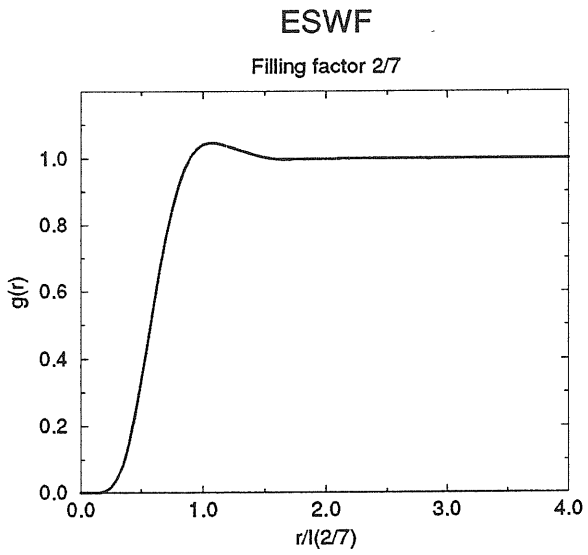

Fig.1

Figure 3.1: Pair distribution function for $\nu = \frac{2}{7}$, the axis coordinate is scaled as $L(\frac{2}{7}) = \sqrt{7\pi}l_0$

but in "kilometric" L units $\frac{r}{L}$ where :

$$L = \sqrt{\frac{2\pi}{\nu}}l_0$$

For $\nu = \frac{2}{7}$ we have the following dimensionless axis coordinate $\frac{r}{L(\frac{2}{7})}$ where $L(\cdots) = \sqrt{7\pi}l_0$ For $\nu = \frac{2}{3}$ and $\nu = \frac{4}{5}$ the "kilometric" unit length-s L will be : $L(\frac{2}{3}) = \sqrt{3\pi}l_0$ and $L(\frac{4}{5}) = \sqrt{2.5\pi}l_0$.

In Fig.1 we plot the pair distribution function $g(r)$ for $\nu = \frac{2}{7}$ as a function of $\frac{r}{L(\frac{2}{7})}$ which clearly shows characteristics of a Liquid state . It seems that $g(r)$ has no ev dentifiable

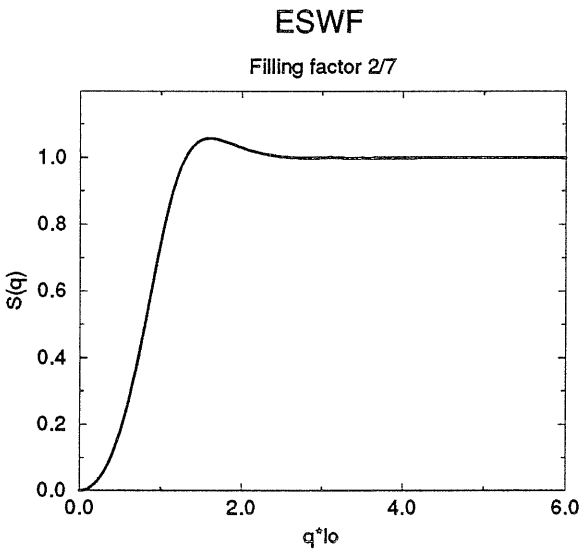

Fig.2

Figure 3.2: Structure factor $S(q)$ for $\nu = \frac{2}{7}$

angular dependency . The ground state energy per particle calculated from :

$$U(\nu) = \frac{\rho}{2} \int v(r)[g(r) - 1]d\vec{r}$$

where $v(r) = \frac{e^2}{\epsilon r}$ was found to be : $U(\frac{2}{7}) = -0.374\ \frac{e^2}{\epsilon l_0}$ in close agreement with the value $-0.377(3)\ \frac{e^2}{\epsilon l_0}$ of Morf and Halperin [?] being qualified as a reasonably good result in [?] .

In Fig.2 we plot the structure factor $S(q)$ for the state $\nu = \frac{2}{7}$ as a function of ql_0 .

Before presenting the results for the other states we anticipate that the pair distribution functions for the states $\nu = \frac{2}{3}$ and $\nu = \frac{4}{5}$ are different from each other only in the small-r regime. A good test of our ESWF and a useful source of information for future research seems to be the study of small -r behaviour of our pair distribution functions .

Yoshioka [?] diagonalized numerically the Hamiltonian for a finite system of 4 to 6 fermions [yosh1] and obtained the coefficients of expansion of g(r) for small r which seem to vary continuously as function of ν.

For small r around the origin $r = 0$ we can expand $g(r)$ as :

$$g(r) = \sum_{i=0}^{\infty} c_i (\frac{r}{l_0})^{2i}$$

We least-squared-fit the previously calculated $g(r)$-s in the region of $0 \leq r \leq 1.7 l_0$ to obtain the c_i-s.

For $\nu = \frac{2}{7}$ we found that :

$c_0 = 0$, $c_1 = 0$, $c_2 = 0$,$c_3 = 0.008185$,$c_4 = -0.001455$ and the others 0 within the limit of our numerical accuracy .

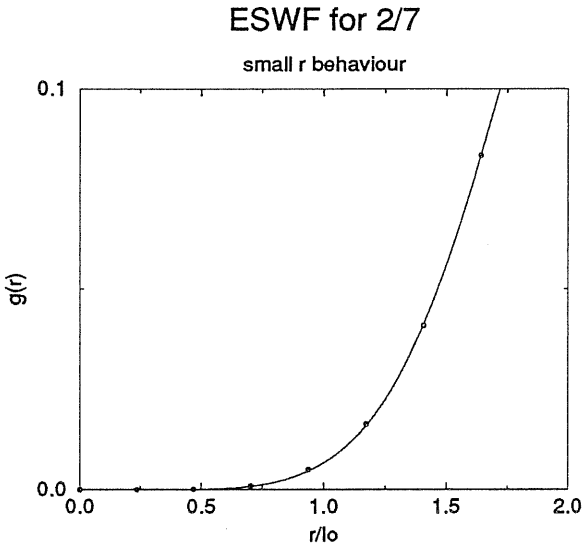

Fig.3

Figure 3.3: Small r behaviour for $\nu = \frac{2}{i}$.

We plot our fitting function in Fig.3 .

The values of the fitting coefficients seem to be in a quite good agreement with [?] .

The same procedure applied to the state $\nu = \frac{4}{5}$ gives :

$c_0 = 0.$, $c_1 = 0.481689$, $c_2 = -0.133997$, $c_3 = 0.024499$, $c_4 = -0.002222$.

The fitting function of the state $\nu = \frac{4}{5}$ is ploted in Fig.4 .

For the state $\nu = \frac{2}{3}$ we obtain :

$c_0 = 0.$, $c_1 = 0.435377$, $c_2 = -0.117703$, $c_3 = 0.021723$, $c_4 = -0.001994$.

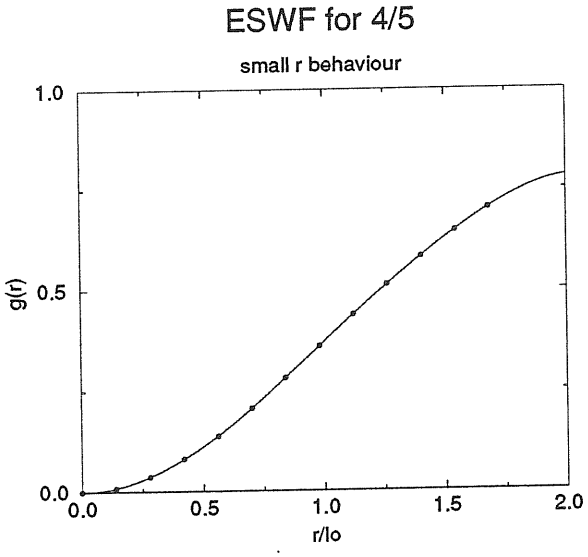

Figure 3.4: Small r behaviour for $\nu = \frac{4}{5}$

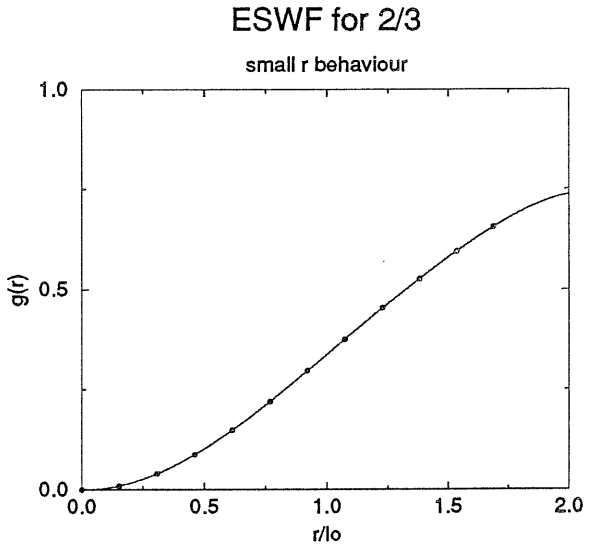

Figure 3.5: Small r behaviour for $\nu = \frac{2}{3}$

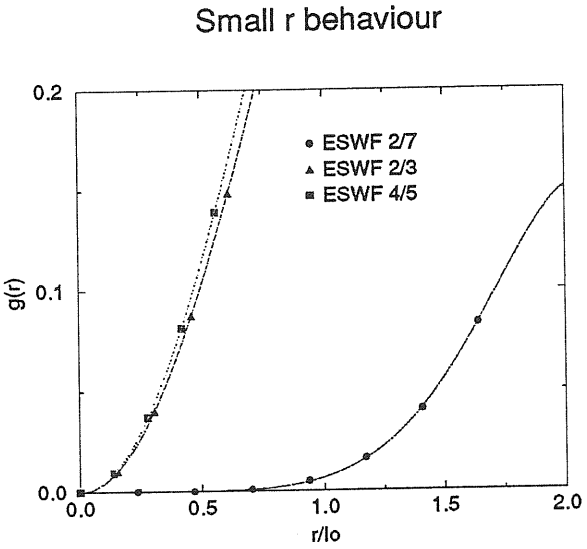

Figure 3.6: Small r behaviour for several states

In Fig.5 we plot the fitting function of the state $\nu = \frac{2}{3}$.

With some slight differences these results agree and support the finite size calculations of Yoshioka [?] .

The small -r behaviour of different FQHE states is plotted in Fig.6

There are only small differences between states $\nu = \frac{4}{5}$ and $\nu = \frac{2}{3}$.

Let me speculate that the small difference in their filling factors forces the fitting coefficients to be close to each other so that a continuous variation of c_i-s as functions of ν looks quite natural .

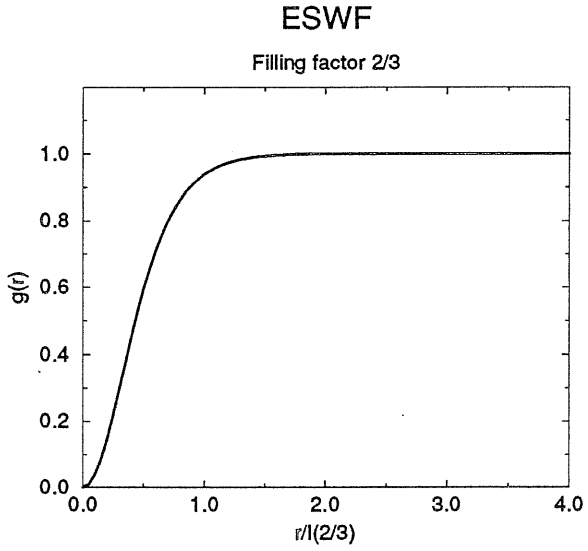

Fig.7

Figure 3.7: Pair distribution function $g(r)$ for $\nu = \frac{2}{3}$, axis coordinate is scaled in units $L(\frac{2}{3}) = \sqrt{3\pi}l_0$

In Fig.7 we plot the resulting $g(r)$ of the state $\nu = \frac{2}{3}$ as a function of $\frac{r}{L(\frac{2}{3})}$

In Fig.8 we plot the resulting $S(q)$ of the state $\nu = \frac{2}{3}$ The shape of $g(r)$ looks in a good agreement with that obtained by Morf and Halperin [?] using Non-antisymmetrized wave functions . In this case the ground state energy per particle was found to be : $U(\frac{2}{3}) = -0.510$ $\frac{e^2}{\epsilon l_0}$ close to the result $-0.509(5)$ $\frac{e^2}{\epsilon l_0}$ of [?] using Antisymmetrized wave functions .

Knowing the almost exact values of energy for the states $\nu = \frac{1}{3}$ and $\nu = \frac{1}{5}$ the exact energy values for the states $\nu = \frac{2}{3}$ and $\nu = \frac{4}{5}$ are found from the following formula of the

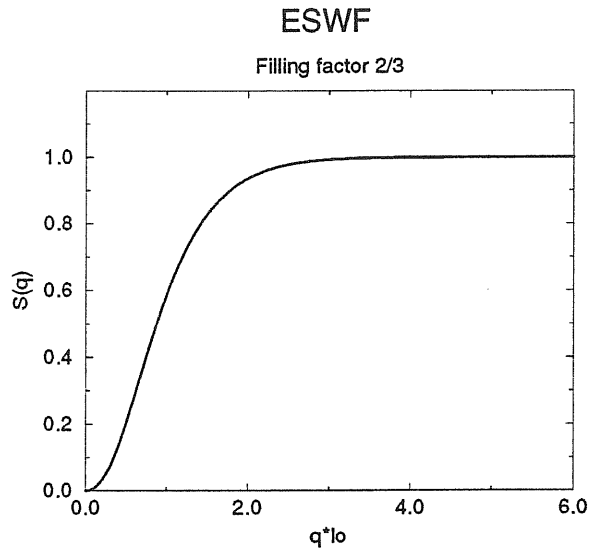

Figure 3.8: Structure factor $S(q)$ for $\nu = \frac{2}{3}$

particle-hole symmetry :

$$\nu U(\nu) = (1-\nu)U(1-\nu) + \sqrt{\frac{\pi}{8}}(1-2\nu)\frac{e^2}{\epsilon l_0}$$

The exact energies should be : $U(\frac{2}{3}) = -0.518\ \frac{e^2}{\epsilon l_0}$ and $U(\frac{4}{5}) = -0.5519\ \frac{e^2}{\epsilon l_0}$.

Finally pair distribution function and structure factor of the state $\nu = \frac{4}{5}$ were obta ned .

Ground state energy per particle of the state $\nu = \frac{4}{5}$ was found to be : $U(\frac{4}{5}) = -0\ 548\ \frac{e^2}{\epsilon l_0}$ close to the above exact result .

Shape of $g(r)$ for $\nu = \frac{4}{5}$ is similar with that of $\nu = \frac{2}{3}$, as a result the same can be said for their structure factors .

4 Conclusions

Among many interesting aspects of the FQHE , at first we studied the Laughlin states $\nu = \frac{1}{m}$ using the HNC/0 technique . Then we made a systematic study of the Laughlin states , using a modification of the HNC/0 technique (different from the MHNC of [?] and [?]) denoted here as HNC/0+E , to account the Elementary diagrams through a clever scaling .

A much better improvement on the HNC/0 results was observed . The HNC/0+E quantities look very accurate and fit very well even the best MC results for the energy per particle .

While for $\nu = \frac{1}{3}$ the results are impressive from all of point of views , the case $\nu = \frac{1}{5}$ is a little more peculiar , being more structered and longer tailed than the previous one .

Equally very good results were obtained also for this case , in the order of accuracy of the best available results . The HNC/0+E $g(r)$-s fit very well even with analytically parametrized MC-fitted functions . It may be speculated that such a treatment can be applied with success to other states , but right now I cannot strongly support my statement .

It is generally belived that HNC/0 underestimates structure . The present work , at least at the level of Laughlin states , guarantees that a clever way of bringing into the g: me the Elementary diagrams can overcome succesfully this weak point .

Hoping that HNC/0+E works well not only for the well-known Laughlin states , l ut also for the Jain's wavefunctions (how to apply it ?) it is expected that the problem of working on the thermodynamic limit can be overcomed without loosing the accuracy that a finite number Monte Carlo simulation is able to offer . Having not the guarantee for tl at , let have the hope...

Later , we switched on the second hierarchy states like $\nu = \frac{2}{i}$ ... for the FQI E . As ground state wave function for the strongly correlated electronic system , we usec a new one of the Extended Shadow form (ESWF). The ESWF were found to have a simil ır form with the wave functions proposed by Macdonald et al [?] . The problem was mape l into a 3-component mixture of particles and a multi-component HNC/0 technique was ap plied . Results regarding states $\nu = \frac{2}{7}$, $\nu = \frac{2}{3}$ and $\nu = \frac{4}{5}$ are reported.

The quantites obtained with this technique seem to agree quite well with s evera l other calculations , essentially confirming the validy of this treatment . The small -r be ıaviour that generally is believed to be rather "unfriendful" seems to be quite good for the ESWF , following with a very good accuracy (with only some slight differences) the coe ficients of expansion reported by Yoshioka [?] . In my opinion , a clever way of accounting the Elementary diagrams here , given the rather complicated form of the treatment , se ems , if

not impossible , a very difficult task .

Further , such a bet looks not so interesting , because it seems that these last times the most elegant and credited theory on dealing with the hierarchy scheme and accounting IQHE together with FQHE , is that of Jain [?] . The form of his ground state wave functions has not so big similarities with an ESWF form .

If we are able to perform a HNC/0+E or a FHNC treatment on the functions of Jain , with the favor of working on the thermodynamic limit , then things look more interesting .

In such situation even the less known $\nu = \frac{1}{2}$ state seems reachable with a good degree of accuracy . This is a difficult duty that , at present , regards only future .

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