ISAS - INTERNATIONAL SCHOOL
FOR ADVANCED STUDIES

AN APPROACH TO SUBQUANTUM PHYSICS

Thesis submitted for the degree of

"Magister Philosophiae"

Astrophysics Sector

Candidate:

Antony Valentini

Supervisor:

Prof. D.W. Sciama

October 1990

TRIESTE

SISSA - SCUOLA
INTERNAZIONALE
SUPERIORE
DI STUDI AVANZATI
TRIESTE
Strada Costiera 11

ABSTRACT

We begin the development of a statistical mechanics based on the de Broglie–Bohm pilot-wave formulation of quantum theory, without making the assumption that the probability density P is equal to $|\Psi|^2$. Instead, this relation is shown to arise statistically from an "H-theorem", based on assumptions similar to those of classical statistical mechanics (rather than postulating subquantum "fluid fluctuations", as done by Bohm and Vigier). We construct a subquantum "entropy" which, when coarse-grained, increases with time, reaching a maximum when $P=|\Psi|^2$. Further, the relation $P=|\Psi|^2$ is shown to be necessary to avoid both instantaneous signals and violations of the uncertainty principle. An intimate relation is thus exhibited between the three "impossibility principles" of physics: The absence of instantaneous signals, the uncertainty principle, and the statistical law of entropy increase. Essentially, the first two principles may be seen as arising statistically from a subquantum version of the third. In addition to sketching some further developments, we discuss certain problematical aspects of the philosophy commonly associated with quantum theory.

CONTENTS

I. INTRODUCTION:

Quantum mechanics presents us with a set of abstract rules with which we may calculate the probability distribution associated with various physical processes. The predictions made are in remarkable agreement with experiment. However, for historical reasons, there is associated with the theory a general philosophy according to which a more detailed theory is impossible. This is perhaps the first time in the history of science that the founders of a major scientific theory have themselves confidently claimed the theory to be "finally valid", thereby erecting a dogmatic barrier to further research. ("Entgültig" was the word used by Heisenberg in reference to quantum mechanics, whose flavour is considered by Popper[1] to be well-described as representing an "end-of-the-road thesis").

By a "more detailed" theory, one might mean, for example, a theory with the aid of which one may acquire a certain "understanding" of Nature. As stressed by Bohm[2], science is, and always has been, concerned with understanding the world, in a broad sense, and not merely with the prediction of experimental data. From this viewpoint, the present quantum theory is a failure since, as stated by Feynman[3], "nobody understands quantum mechanics".

With regard to such "understanding", most formulations of quantum theory offer little, since they consist of little more than a set of abstract rules for calculating probability distributions. One is not, for example, given an image of the world as being "independently actual"[2], where such images have, in the past, been the real basis of physical understanding. Indeed, even today, many physicists find great value in "naive physical pictures", such as, for example, the image of the vacuum as containing virtual electron-positron pairs. Much of present-day discussion in physics is in fact carried out, in practice, in terms of such simple pictures (though always with the caveat that such pictures should not be taken "too seriously"). Further, realistic images are clearly of great value in suggesting new lines of research. (For example, the flowering of quantum field theory since the late 1960s has been based mainly on field theory, as opposed to the relatively imageless S-matrix scheme, where the former is far more suggestive with regard to further developments).

Remarkably, an "independently actual" image of quantum theory has existed in detailed form since 1952: The "pilot-wave", or "quantum potential", formulation, constructed in detail by Bohm[4]. The rudiments of this image were considered as early as 1927 by de Broglie[5,6], and presented at the Fifth Solvay Congress in Brussels However, despite Einstein's support (who had similar ideas regarding "ghost-waves" guiding photons, in the early 1920s), de Broglie's "realist" ideas were, as Bell[2] has put it, "simply trampled on". The prevailing attitude at the time was strongly opposed to the formulation of realistic images of the world, an attitude which largely reflects the anti-realist mood of post-World War I Europe.

Today, a more critical, questioning attitude towards quantum mechanics is reviving. It seems inevitable that human curiosity will eventually lead to the asking of questions which probe beyond the dogmatic barriers erected by Bohr and Heisenberg. As recent examples, one might consider the theory of "quantum jumps" by Ghirardi, Rimini and Weber[7], or the consideration of a nonlinear quantum mechanics by Weinberg[8]. That such proposals have been taken seriously by the mainstream physics community indicates that a subtle change in attitude towards quantum mechanics is taking place.

The present work is based on the de Broglie – Bohm pilot-wave formulation of quantum theory. As stressed repeatedly by Bell[9], this formulation is completely equivalent, as regards its predictions, to standard quantum mechanics. The fact that it is not widely accepted as an alternative formulation, as is Feynman's path-integral formulation, seems to be purely a matter of historical accident.

The pilot-wave formulation provides an "independently actual" image of quantum-mechanical processes. This in itself should be sufficient ground for giving the theory serious attention. There is a further (related) reason for taking the theory seriously: Contemplation of it leads immediately to the asking of probing physical questions, in contrast to the "don't-ask-questions" attitude encouraged by the other formulations. For the purposes of progressive research, the pilot-wave formulation is by far the most richly suggestive. We hope this will be shown in what follows.

II. AN APPROACH TO SUBQUANTUM PHYSICS:

A. The three "impossibility principles" of physics

Consider what might be termed the three fundamental "impossibility principles" of Physics: (i) The absence of instantaneous signals ("signal-locality"), (ii) The uncertainty principle, and (iii) The statistical law of entropy increase. Each of these principles may be put in the form of statements as to the impossibility of certain physical operations. Further, each involves, in one way or another, the notion of "information". Given these facts, it is natural to consider that there might be a relationship between these three principles. We show here that just such a relationship emerges, with no arbitrariness whatsoever, by straightforwardly developing a statistical mechanics based on Bohm's 1952 pilot-wave formulation of quantum theory (the beginnings of which were put forward by de Broglie in 1927).

B. The pilot-wave formulation and its mysterious features

Bohm's original (1952) formulation follows compellingly from standard wave mechanics. The essential laws are the Schrödinger equation, together with a "guiding equation" for the particle, $m\dot{\underset{\sim}{X}} = \underset{\sim}{\nabla} S$, where $\psi = R\, e^{(i/\hbar)S}$. (This latter guiding equation seems preferable to the more complicated and rather inelegant analogue of Newton's law for $\ddot{\underset{\sim}{X}}$, with a "quantum potential"). A detailed discussion of the theory of measurement has been given by Bohm[4]. For various reasons, the original formulation was later supplemented by Bohm and Vigier[10], by the introduction of subquantum "fluid fluctuations". In this paper, we consider only the original 1952 "pure" pilot-wave theory.

The original 1952 formulation contains the following mysterious (and, as we shall see, related) features: (a) The theory is fundamentally nonlocal in construction (consider the relation $m\dot{\underset{\sim}{X}} = \underset{\sim}{\nabla} S$, where $\underset{\sim}{X}$ represents the system in a multi-dimensional configuration space). And yet, the theory does not lead to instantaneous signals. This last fact surely deserves some explanation. (b) The probability density ρ should logically be independent of $|\psi|^2$ in this theory, as pointed out by Pauli[11]. One might then expect violations of the usual statistical dispersion ("uncertainty principle"). Indeed, as stated by Bohm[4] and discussed further below, the uncertainty principle is obtained only if $\rho = |\psi|^2$. One may then ask: Why, exactly, in the pilot-wave theory, should the uncertainty principle hold? (c) The field ψ enters as a guiding field for the motion of particles, but at the same time is required (by the experimental facts) to emerge as a probability density $|\psi|^2$. Given the logical independence of ρ and $|\psi|^2$, the relation $\rho = |\psi|^2$ is usually added as an additional postulate (which, however, will automatically be valid for all time if it is given as valid at some initial time). One may ask: Why does $\rho = |\psi|^2$ hold?

C. Subquantum statistical mechanics

These mysterious features of the pilot-wave theory will be clarified here to some extent, by the development of an analogue of classical statistical mechanics, based on the de Broglie-Bohm theory rather than on Newton's mechanics. We then derive the relation $\rho = |\psi|^2$ as the result of a statistical "subquantum H-theorem" (based on statistical assumptions analogous to those of classical statistical mechanics). We first show that, on a coarse-grained level, the relation $P = |\Psi|^2$ will be obtained for a complicated system with a large number of degrees of freedom (where $P = |\Psi|^2$ maximises a subquantum "entropy", whose coarse-grained value increases with time). It is then a simple matter to show that, on extracting a single particle from the system, the relation $\rho = |\psi|^2$ will be obtained. We also show that the relation $\rho = |\psi|^2$ is essential for the validity of signal-locality and of the uncertainty principle. The above three mysterious features of the pilot-wave theory are then explained as being the statistical results of a principle of increase of a subquantum "entropy". Further, this latter principle provides for the first time a definite link between the three (until now unrelated) "impossibility principles" referred to above.

III. SIGNAL-LOCALITY, UNCERTAINTY, AND THE SUBQUANTUM H-THEOREM:

A. Subquantum H-theorem

Consider a system with a large number of degrees of freedom, for example a gas of N particles ("electrons") which interact with eachother. The system has a wavefunction $\Psi(\underset{\sim}{X}_1,\ldots \underset{\sim}{X}_N,t)$, and a probability density $P(\underset{\sim}{X}_1,\ldots \underset{\sim}{X}_N,t)$ in configuration space. We wish to prove that, on a coarse-grained statistical level, we will observe $P = |\Psi|^2$. As discussed below, if one extracts a single particle from the system, and prepares it in a state with wavefunction ψ , one will then find $\rho = |\psi|^2$ for the particle, provided $P = |\Psi|^2$ held beforehand for the large system. Since all experiments deal with particles extracted from complex systems, a proof of $P = |\Psi|^2$ for the complicated system is sufficient to demonstrate $\rho = |\psi|^2$ for all single-particle laboratory experiments. (This is similar to a view expressed by Bohm[12]).

We represent the system by a single point $\underset{\sim}{X}$ in a 3N-dimensional configuration space. It is important to note that, given the wavefunction

$$\Psi(\underset{\sim}{X},t) = R(\underset{\sim}{X},t)e^{(i/\hbar)S(\underset{\sim}{X},t)}$$

then specifying the position $\underset{\sim}{X}$ automatically fixes the velocity $d\underset{\sim}{X}/dt$ by

$$m\dot{X}_i = \frac{\partial S(\underset{\sim}{X},t)}{\partial X_i} \qquad (1)$$

$(i = 1,\ldots,3N)$. For this reason, our statistical mechanics will take place in configuration space, rather than phase space.

We must consider, of course, an ensemble of such systems, so that the probability density $P(\underset{\sim}{X},t)$ is meaningful. Each element of the ensemble is taken to have the same wavefunction Ψ , but of course not necessarily the same $\underset{\sim}{X}$.

Let us begin then, at t=0, with a wavefunction $\Psi(\underset{\sim}{X},0)$, and a probability density $P(\underset{\sim}{X},0)$, where the latter has no <u>a priori</u> relation to $|\Psi(\underset{\sim}{X},0)|^2$. Following Bohm[12], it will be convenient to introduce a function $f(\underset{\sim}{X},t)$ defined by

$$P(\underset{\sim}{X},t) = |\Psi(\underset{\sim}{X},t)|^2 f(\underset{\sim}{X},t) \qquad (2)$$

which measures the ratio of P to $|\Psi|^2$ at the point $\underset{\sim}{X}$ at time t. (The definition (2) may be made rigorous by assuming that $|\Psi|^2$ is everywhere non-vanishing).

To fix our ideas, it may be helpful to point out clearly the dynamics: Firstly, $\Psi(\underset{\sim}{X},t)$ will evolve according to the Schrödinger equation. This evolution then depends only on the initial value $\Psi(\underset{\sim}{X},0)$ (and on the form of the classical inter-particle potential). It is completely independent of $P(\underset{\sim}{X},0)$. Secondly, being a probability density, $P(\underset{\sim}{X},t)$ necessarily evolves according to the continuity equation

$$\frac{\partial P(\underset{\sim}{X},t)}{\partial t} + \underset{\sim}{\nabla}\cdot(P(\underset{\sim}{X},t)\dot{\underset{\sim}{X}}) = 0 \tag{3}$$

Given $P(\underset{\sim}{X},0)$, this equation determines $P(\underset{\sim}{X},t)$ uniquely, once $\dot{\underset{\sim}{X}}(t) = (1/m)\underset{\sim}{\nabla}S(\underset{\sim}{X},t)$ is known (which of course follows from knowledge of $\Psi(\underset{\sim}{X},t)$). Essentially, the autonomous Schrödinger evolution of Ψ guides the particles via $m\dot{\underset{\sim}{X}} = \underset{\sim}{\nabla}S$, and hence determines the evolution of $P(\underset{\sim}{X},t)$.

Now it happens to follow from the Schrödinger equation that

$$\frac{\partial|\Psi|^2}{\partial t} + \underset{\sim}{\nabla}\cdot(|\Psi|^2(1/m)\underset{\sim}{\nabla}S) = 0 \tag{4}$$

In view of (1), this is precisely similar to (3). As pointed out by Bohm[12], the fact that P and $|\Psi|^2$ happen to obey the same continuity equation implies that the function $f(\underset{\sim}{X},t)$, being the ratio of P to $|\Psi|^2$, is preserved along the system trajectories $\underset{\sim}{X}(t)$:

$$\frac{df(\underset{\sim}{X},t)}{dt} \equiv \frac{\partial f}{\partial t} + \dot{\underset{\sim}{X}}\cdot\underset{\sim}{\nabla}f = 0 \tag{5}$$

This property is crucial in what follows.

Now, in classical statistical mechanics, one defines[13] a quantity H_{class} (essentially the negative of the entropy) by

$$H_{class} = \int d\Omega\, p \ln p \tag{6}$$

where $\int d\Omega$ is an integral over phase space, and p is the phase-space probability density. Since $dp/dt = 0$ along the system trajectories (Liouville's theorem), it follows from the continuity equation for p that $dH_{class}/dt = 0$, so that the exact H_{class} is constant in time. However, if one divides phase-space into small cells, one may define a coarse-grained density $\bar{p}$ by averaging over each cell (where $\bar{p}$ is defined as constant in each cell). The coarse-grained

$$\bar{H}_{class} = \int d\Omega \; \bar{p} \ln \bar{p}$$

may then be shown[13] to obey the classical H-theorem

$$d\bar{H}_{class}/dt \leq 0$$

The proof relies on just one crucial assumption: That $\bar{p}(0) = p(0)$, i.e. that the initial coarse-grained density is equal to the initial exact fine-grained value. This might be interpreted in terms of a "principle of uniform local probability", according to which the initial density may always be treated as uniform within a sufficiently small cell in phase-space (see the book by Waldram[13]). For our purposes, it is sufficient to consider the assumption $\bar{p}(0) = p(0)$ as representing an absence of detailed microstructure in the initial state. It is this assumption which introduces a distinction between past and future: Essentially, it is assumed that there is no special "conspiracy" in the initial conditions, which would lead to "unlikely" entropy-decreasing behaviour.

To construct a subquantum analogue H of H_{class}, we note that the quantities $d\Omega$ and p appearing in the definition (6) of H_{class} satisfy the Liouville property: They are both preserved along the system trajectories in phase space. What are the subquantum analogues of $d\Omega$ and p ? We have already noted that $df/dt = 0$ along the trajectories in configuration space. This suggests the replacement $p \longrightarrow f$. Further, the number of systems which occupy a comoving volume $d^{3N}\underline{X}$ must be constant in time, so that $Pd^{3N}\underline{X} = |\Psi|^2 f \, d^{3N}\underline{X}$, and therefore also $|\Psi|^2 d^{3N}\underline{X}$, is preserved along the system trajectories. We then choose to replace $d\Omega \longrightarrow |\Psi|^2 d\Sigma$, where $d\Sigma = d^{3N}\underline{X}$ is the volume element in configuration space. Thus, we are led to tentatively define the subquantum analogue of H_{class} as

$$H = \int d\Sigma\ |\Psi|^2 f \ln f \tag{7}$$

which may also be written as

$$H = \int d\Sigma\ P \ln(P/|\Psi|^2) \tag{8}$$

From the continuity equation for $|\Psi|^2$, and the fact that $df/dt = 0$, it follows that the exact H is constant; $dH/dt = 0$, as for the classical case.

Since all physical measurements have a finite accuracy, it is reasonable to work in terms of a coarse-grained probability density $\bar{P}$ (as suggested by P. and T. Ehrenfest for the classical case): We divide configuration space into cells of volume δV , and define

$$\bar{P} = (1/\delta V) \int_{\delta V} d\Sigma\ P$$

(taking $\bar{P}$ as constant in each cell). Performing an analogous coarse-graining of $|\Psi|^2$,

$$\overline{|\Psi|^2} = (1/\delta V) \int_{\delta V} d\Sigma\ |\Psi|^2$$

we define a coarse-grained H by

$$\bar{H} = \int d\Sigma\ \bar{P} \ln(\bar{P}/\overline{|\Psi|^2}) \tag{9}$$

Defining the ratio $\tilde{f} = \bar{P}/\overline{|\Psi|^2}$, we are now ready to prove that

$$\bar{H} = \int d\Sigma\ \bar{P} \ln\tilde{f}$$

decreases with time.

The proof rests on the assumption of no "microstructure" for the initial state, as for the classical case. Specifically, we assume the equality of coarse-grained and fine-grained quantities at the initial time $t = 0$, i.e. we assume

$$\begin{aligned} \bar{P}(0) &= P(0) \\ \overline{|\Psi(0)|^2} &= |\Psi(0)|^2 \end{aligned} \tag{10}$$

The proof now procedes along the lines of the classical case, but differs somewhat due to the presence of the factor $|\Psi|^2$ in (7).

We have

$$\bar{H}(0) - \bar{H} = \int d\Sigma\ \bar{P}(0) \ln\tilde{f}(0) - \int d\Sigma\ \bar{P} \ln\tilde{f}$$

(where $\bar{H} = \bar{H}(t)$, and so on). From (10), and the fact that the exact H is constant in time, it follows that

$$\int d\Sigma\ \bar{P}(0) \ln\tilde{f}(0) = \int d\Sigma\ P(0) \ln f(0) = \int d\Sigma\ P \ln f$$

Further, since $\tilde{f}$ is constant in each cell δV, it follows that

$$\int d\Sigma\ \bar{P} \ln\tilde{f} = \int d\Sigma\ P \ln\tilde{f}$$

From these equalities we find that

$$\bar{H}(0) - \bar{H} = \int d\Sigma\ P \ln(f/\tilde{f})$$

or

$$\bar{H}(0) - \bar{H} = \int d\Sigma\ |\Psi|^2 f \ln(f/\tilde{f})$$

Again using the fact that $\tilde{f}$ is constant in each cell δV, one easily shows that

$$\int d\Sigma\ |\Psi|^2(\tilde{f} - f) = 0$$

so that

$$\bar{H}(0) - \bar{H} = \int d\Sigma\ |\Psi|^2(f \ln(f/\tilde{f}) + \tilde{f} - f)$$

Since $x \ln(x/y) + y - x \geqslant 0$ for all x,y, we have the <u>subquantum H-theorem</u>

$$d\bar{H}/dt \leqslant 0 \tag{11}$$

The above relies on the continuity equation for both P and $|\Psi|^2$, leading to $df/dt = 0$, where $f = P/|\Psi|^2$. However, one could just as well have considered $g = |\Psi|^2/P$, where also $dg/dt = 0$, and one could then have made the alternative replacements in the classical definition (6), $p \longrightarrow g$ and $d\Omega \longrightarrow Pd\Sigma$, leading to the alternative definition

$$H = \int d\Sigma\ Pg \ln g = \int d\Sigma\ |\Psi|^2 \ln(|\Psi|^2/P) \tag{12}$$

whose coarse-grained value also decreases with time (the proof being similar to the above). Adding (8) and (12) leads to yet another definition,

$$H = \int d\Sigma \, (P - |\Psi|^2) \ln(P/|\Psi|^2) \tag{13}$$

which is symmetrical under $P \longleftrightarrow |\Psi|^2$, and of course also obeys the H-theorem, $d\bar{H}/dt \leqslant 0$.

In the present paper, the ambiguity in the definition of H does not concern us, and we choose the definition (13) on aesthetic grounds. As we shall see, the essential point is the existence of some function H of P and $|\Psi|^2$, whose coarse-grained value decreases with time, reaching a minimum when $\bar{P} = \overline{|\Psi|^2}$.

It is convenient to define a subquantum "entropy"

$$S = -k \int d\Sigma \, (P - |\Psi|^2) \ln(P/|\Psi|^2) \tag{14}$$

(where k is Boltzmann's constant) whose coarse-grained value increases with time. How this "entropy" relates to conventional notions of entropy will be explored elsewhere. Here we require only the fact that $d\bar{S}/dt \geqslant 0$.

B. $P = |\Psi|^2$ as an entropy maximum

We have shown that the coarse-grained quantity

$$\bar{S} = -k\int d\Sigma\ (\bar{P} - \overline{|\Psi|^2})\ \ln(\bar{P}/\overline{|\Psi|^2}) \tag{15}$$

increases with time. Since $(x - y)\ln(x/y) \geqslant 0$ for all x,y, we have

$$\bar{S} \leqslant 0 \tag{16}$$

so that $\bar{S}$ is bounded above. This fact, together with the fact that $\bar{S}$ cannot decrease, will be regarded here as proof that $\bar{S}$ eventually approaches its maximum value, i.e. $\bar{S} \longrightarrow 0$. From (15) we see that $\bar{S} = 0$ is attained if and only if $\bar{P} = \overline{|\Psi|^2}$ everywhere (so that the distribution $\bar{P} = \overline{|\Psi|^2}$ maximises the subquantum entropy). This leads us to the conclusion that the coarse-grained distribution $\bar{P}$ will eventually approach the value $\overline{|\Psi|^2}$.

This is our main result: <u>For purely statistical reasons, the coarse-grained probability density $\bar{P}$ will approach the value $\overline{|\Psi|^2}$</u>. Thus, provided one performs measurements of a sufficiently coarse accuracy with respect to configuration-space volumes, one will see a distribution $\bar{P} = \overline{|\Psi|^2}$.

This purely statistical derivation of $\bar{P} = \overline{|\Psi|^2}$, based on the original "pure" pilot-wave theory, thus obviates the need to introduce arbitrary stochastic subquantum fluctuations (e.g. "fluid fluctuations"), as done by Bohm and Vigier[10] and by many authors since, where the stochastic fluctuations were introduced (at least in part) in order to explain how the probability density approaches $|\Psi|^2$.

A heuristic understanding of how the result $\bar{P} = \overline{|\Psi|^2}$ arises may be given as follows: The exact (fine-grained) density is given by $P = |\Psi|^2 f$. Now, starting from an arbitrary $f(\underset{\sim}{X},0)$, the initial values of f are carried along the system trajectories in configuration space. If the system is sufficiently complicated, the chaotic wandering of the trajectories $\underset{\sim}{X}(t)$ will distribute the f values in an effectively random manner over the accessible region of configuration space. On a coarse-grained level, P will then be indistinguishable from $|\Psi|^2$. Another (equivalent) picture sees the increase of subquantum entropy as associated with the effectively random mixing of two "fluids", with densities P and $|\Psi|^2$, each of which obeys the same continuity equation, and is "stirred" by the same velocity field $\dot{\mathbf{X}} = (1/m)\nabla S$. If the pattern ∇S of flow lines is sufficiently complicated, the two "fluid" densities P and $|\Psi|^2$ will be thoroughly mixed, making them indistinguishable on a coarse-grained level.

C. $\rho = |\psi|^2$ from $P = |\Psi|^2$

We now show how, if a single particle is extracted from the large system, and prepared in a state with wavefunction ψ, then its probability density ρ will be equal to $|\psi|^2$, provided $P = |\Psi|^2$ holds for the large system (subject to appropriate coarse-graining).

The theory of state-preparation and of "measurement", in the pilot-wave formulation, has been discussed quite thoroughly by Bohm[4]. The article by Bell[14] contains a simple example, and notes that the "singling out of a "system" is a practical thing defined by circumstances, and is not already in the fundamental formulation of the theory".

We consider our complicated system above, consisting of N particles, with wavefunction Ψ, to be the "whole world". From the above we know that, on a coarse-grained level, one will have $\bar{P} = \overline{|\Psi|^2}$. We now need a mathematical representation of the following "practical thing defined by circumstances": The singling out of a particular particle (say i=1) from the system, prepared in an eigenstate $|q\rangle$ of some arbitrary "observable" Q. Quite generally, such a process may be said to have taken place when the wavefunction Ψ of the world takes the form

$$|\Psi\rangle = \sum_{q,\omega_i} c(q,\omega_i)\,|q, A_q, \omega_i\rangle \qquad (17)$$

Here A_q is a variable belonging to the so-called "apparatus" (which is assumed to be made up of particles i=2,...,M), where the value A_q corresponds to a "measured value" q of the "observable" Q. The variables ω_i (i=M+1,...,N) describe the rest of the world.

To see that this is a correct mathematical representation of the process, write

$$\Psi(\mathbf{X}_1,\dots,\mathbf{X}_N,t) = \sum_{q,\omega_i} c(q,\omega_i)\,\psi_q(\mathbf{X}_1)\,\psi_{A_q}(\mathbf{X}_2,\dots,\mathbf{X}_M)\,\psi_{\omega_{M+1}}(\mathbf{X}_{M+1})\cdots\psi_{\omega_N}(\mathbf{X}_N) \qquad (18)$$

The motion of the "extracted" particle i=1 is governed by

$$\dot{X}_1 = (1/m)\nabla_1 S = (\hbar/m)\nabla_1 \mathrm{Im}\ln\Psi(X_1,\ldots,X_N,t) \tag{19}$$

which depends on $X_2,\ldots,X_N$ (as well as on X_1). However, if an observed value $A_q = A_q^{meas}$ of the apparatus variable is specified, this constrains the values of $X_2,\ldots,X_M$ so that[14]

$$\dot{X}_1 = (\hbar/m)\nabla_1 \mathrm{Im}\ln\Psi\Big|_{A_q=A_q^{meas}}$$

$$= (\hbar/m)\nabla_1 \mathrm{Im}\ln \psi_{q^{meas}}\psi_{A_q^{meas}} \sum_{\substack{\omega_i \\ (i=M+1,\ldots,N)}} c(q^{meas},\omega_i)\psi_{\omega_{M+1}}\cdots\psi_{\omega_N}$$

which leads to

$$\dot{X}_1 = (\hbar/m)\nabla_1 \mathrm{Im}\ln\psi_{q^{meas}}(X_1)$$

since only the factor $\psi_{q^{meas}}$ depends on X_1 .

Thus, as far as $\dot{X}_1$ is concerned, the particle i=1 behaves as if it were "guided" purely by the wavefunction $\psi_{q^{meas}}(X_1)$.

Consider now the coarse-grained probability density for the particle,

$$\overline{\rho}(X_1,t) = (1/\delta V_1)\int_{\delta V_1} d^3X_1\,\rho(X_1,t)$$

$$= (1/\delta V_1)\int_{\delta V_1} d^3X_1\int d^3X_2\cdots\int d^3X_N\,P(X_1,\ldots,X_N,t)$$

where the coarse-graining volume δV in configuration-space is given by $\delta V=\delta V_1\ldots\delta V_N$. This may be rewritten as

$$\overline{\rho}(X_1,t) = \int d^3X_2\ldots\int d^3X_N\,\overline{P}(X_1,\ldots,X_N,t) = \int d^3X_2\ldots\int d^3X_N\,\overline{|\Psi(X_1,\ldots,X_N,t)|^2}$$

where we have used $\overline{P} = \overline{|\Psi|^2}$. This may then be recast as

$$\bar{\rho}(\mathbf{X}_1,t) = (1/\delta V_1)\int_{\delta V_1} d^3\mathbf{X}_1 \int d^3\mathbf{X}_2 \cdots \int d^3\mathbf{X}_N \, |\Psi(\mathbf{X}_1,\ldots,\mathbf{X}_N,t)|^2 \tag{20}$$

Again, specifying $A_q = A_q^{meas}$ leads to the vanishing of all terms in the superposition (18) for Ψ, except the term with $q=q^{meas}$. The right-hand-side of (20) then becomes

$$(1/\delta V_1)\int_{\delta V_1} d^3\mathbf{X}_1 \, |\psi_{q^{meas}}(\mathbf{X}_1)|^2 \int d^3\mathbf{X}_2 \cdots \int d^3\mathbf{X}_N \, |\psi_{A_q^{meas}}(\mathbf{X}_2,\ldots,\mathbf{X}_M)|^2$$
$$\times \Big| \sum_{\substack{\omega_i \\ (i=M+1,\ldots,N)}} c(q^{meas},\omega_i)\,\psi_{\omega_{M+1}}(\mathbf{X}_{M+1})\cdots\psi_{\omega_N}(\mathbf{X}_N) \Big|^2$$

so that

$$\bar{\rho}(\mathbf{X}_1,t) \propto \overline{|\psi_{q^{meas}}(\mathbf{X}_1)|^2} \tag{21}$$

The proportionality factor is removed upon multiplying $\psi_{q^{meas}}$ by an appropriate constant, where $\psi \longrightarrow c\,\psi$ for arbitrary constant c is a symmetry of the pilot-wave theory.

For all practical purposes, the extracted particle may then be treated as an independent system, with wavefunction $\psi_{q^{meas}}(\mathbf{X}_1)$, the equality $\bar{\rho}(\mathbf{X}_1) = \overline{|\psi_{q^{meas}}|^2}$, on a coarse-grained level, following immediately from $P=|\Psi|^2$ for the original system.

In the above, we have used the theory of measurement as outlined by Bohm[4,14], which relies on two crucial features. Firstly, the apparatus wavepackets ψ_{A_q} are taken as non-overlapping in configuration space, for distinct A_q. Secondly, in regions between such non-overlapping packets, the fact that $|\Psi|^2 = 0$ implies a vanishing probability density P. This last fact ensures that, once the system has entered one of the packets, it is "trapped" (being unable to cross the regions P=0). Now, in the present approach, we do not assume $P=|\Psi|^2$ <u>a priori</u>. Rather, we derive the coarse-grained relation $\bar{P} = \overline{|\Psi|^2}$. Nevertheless, the usual theory of measurement remains intact. This is so because $|\Psi|^2=0$ implies $\overline{|\Psi|^2} = 0$, and therefore also $\bar{P} = 0$. Since P is non-negative, we then have P = 0 in regions where $|\Psi|^2=0$, which is sufficient for the validity of the usual measurement theory.

D. Signal-locality and the uncertainty principle as consequences of $\rho = |\psi|^2$

(i) Signal-locality: The pilot-wave formulation is fundamentally nonlocal in structure. Having shown that, for statistical reasons, the observed probability density ρ will equal $|\psi|^2$, we now show that the absence of instantaneous signals depends crucially on the condition $\rho = |\psi|^2$. This will complete our demonstration that, in the pilot-wave formulation, effective locality on the statistical level arises for purely statistical ("thermodynamic") reasons.

We shall consider, for theoretical purposes, that ρ is known, even when $\rho \neq |\psi|^2$. How ρ could actually be known in practice will not be considered here, where we are occupied only with the general theoretical point that a known ρ would allow instantaneous signalling, only in situations where $\rho \neq |\psi|^2$. One might, if one wishes, consider that ρ is "known" by an imaginary subquantum "demon". Such a viewpoint is useful for the present theoretical purpose, though its practical content is as yet unknown.

Consider, then, two non-interacting spatially-separated systems A and B, which are quantum-mechanically "entangled". We mean, of course, that the total Hamiltonian is at all times a sum of two independent commuting Hamiltonians, while the total wavefunction may not be factorised as $\psi_A \psi_B$.

For definiteness, we consider two (one-dimensional) "boxes" A and B, separated by a large distance, each box containing a single particle with coordinate X_A and X_B respectively. For all $t \leq 0$, the total Hamiltonian is taken to be

$$H = H_A + H_B$$

where H_A and H_B are both independent of time. At $t \leq 0$, each box-plus-particle has a ground state $|E_0\rangle$ and an excited state $|E_1\rangle$. The total state-vector at $t = 0$ is taken to be

$$|\psi_0\rangle = (\alpha^2 + \beta^2)^{-1/2}(\alpha|E_0E_1\rangle + \beta|E_1E_0\rangle)$$

For simplicity we take α, β, $\langle X_A|E_i\rangle$ and $\langle X_B|E_i\rangle$ $(i = 0,1)$ to be real.

Having specified the initial Hamiltonian and wavefunction, it remains only to specify the initial probability density $\rho(X_A,X_B,0)$. We shall be interested in two special cases: (a) $\rho(X_A,X_B,0) = |\Psi_0(X_A,X_B)|^2$ and (b) $\rho(X_A,0)$ $(=\int dX_B\,\rho(X_A,X_B,0))$ is sharply peaked near some value $X_A=x_o$, while $\rho(X_B,0)$ is arbitrary.

The question of interest is the following: If, at $t>0$, the Hamiltonian H_B of system B is suddenly altered to $H_B' \neq H_B$ (for example, by suddenly moving the walls of box B), will the probability density $\rho(X_A,t)$ for the distant particle A be affected? For the above cases (a) and (b), the answers turn out to be no and yes respectively. The first case then shows that $\rho_o = |\Psi_o|^2$ is sufficient to prevent instantaneous signalling (as is already well-known from standard quantum mechanics). The second case shows that $\rho_o = |\Psi_o|^2$ is, generally speaking, necessary, as well as sufficient.

First (a): The condition $\rho_o = |\Psi_o|^2$ guarantees $\rho = |\Psi|^2$ for all $t>0$. For $t>0$ take

$$H = H_A + H_B'$$

where H_B' is independent of time for $t>0$ and $H_B' \neq H_B$ (so that the Hamiltonian of B changes suddenly across t=0). We then have

$$|\Psi(t)\rangle = e^{-(i/\hbar)(H_A+H_B')t}|\Psi_o\rangle$$

which gives

$$|\Psi(t)\rangle = (\alpha^2+\beta^2)^{-1/2}(\alpha\, e^{-(i/\hbar)E_0 t}|E_0\rangle\, e^{-(i/\hbar)H_B' t}|E_1\rangle + \beta\, e^{-(i/\hbar)E_1 t}|E_1\rangle\, e^{-(i/\hbar)H_B' t}|E_0\rangle\,) \tag{22}$$

(noting of course that $H_B'|E_i\rangle \neq E_i|E_i\rangle$). The fact that $e^{-(i/\hbar)H_B' t}$ is unitary then implies that

$$\int dX_B|\langle X_A X_B|\Psi(t)\rangle|^2 = (\alpha^2+\beta^2)^{-1}(\alpha^2|\langle X_A|E_0\rangle|^2 + \beta^2|\langle X_A|E_1\rangle|^2)$$

showing that $\rho(X_A,t)$ is completely independent of the value of H_B' . Thus, for this case, the probability density of particle A is independent of any operations on the distant system B, and no instantaneous signalling is possible. (It should be noted, however, that X_A itself is affected nonlocally by H_B' (see below). It is only the probability distribution of X_A , for an ensemble of systems, which is not so affected).

Now case (b): At t=0, $\rho(X_A,0)$ is sharply peaked near $X_A = x_o$. We wish to show that $\rho(X_A,t)$ depends strongly on H_B' . Note first that if $H_B' = H_B$, then

$$\psi(X_A,X_B,t) = \psi(X_A,X_B,0)e^{-(i/\hbar)(E_0+E_1)t}$$

which implies (taking ψ_o as real)

$$S(X_A,X_B,t) = -(E_0+E_1)t$$

so that $\dot{X}_A = \dot{X}_B = 0$. Clearly, the continuity equation for $\rho = \rho(X_A,X_B,t)$

$$\partial\rho/\partial t + \partial(\rho\dot{X}_A)/\partial X_A + \partial(\rho\dot{X}_B)/\partial X_B = 0$$

then implies that $\rho(X_A,X_B,t) = \rho(X_A,X_B,0)$ and $\rho(X_A,t) = \rho(X_A,0)$ for all t. Thus, if $H_B'= H_B$, then $\rho(X_A,t)$ is static, and remains sharply peaked at x_o for all $t>0$, so that a measurement of the particle position at $t>0$ would yield a value near x_o . In contrast, if $H_B' \neq H_B$, we would have (from (22))

$$\psi(X_A,X_B,t) = (\alpha^2+\beta^2)^{-1/2}(\alpha\langle X_A|E_0\rangle R_1(X_B,t)e^{(i/\hbar)(S_1(X_B,t)-E_0t)}$$
$$+ \beta\langle X_A|E_1\rangle R_0(X_B,t)e^{(i/\hbar)(S_0(X_B,t)-E_1t)})$$

where we have defined (for i=1,2)

$$\langle X_B|e^{-(i/\hbar)H_B't}|E_i\rangle = R_i(X_B,t)e^{(i/\hbar)S_i(X_B,t)}$$

with R_i , S_i real functions. Writing

$$\psi(X_A,X_B,t) = R(X_A,X_B,t)e^{(i/\hbar)S(X_A,X_B,t)}$$

we then have

$$\tan S = \frac{\alpha\langle X_A|E_0\rangle R_1\sin(S_1-E_0t)/\hbar + \beta\langle X_A|E_1\rangle R_0\sin(S_0-E_1t)/\hbar}{\alpha\langle X_A|E_0\rangle R_1\cos(S_1-E_0t)/\hbar + \beta\langle X_A|E_1\rangle R_0\cos(S_0-E_1t)/\hbar} \qquad (23)$$

The essential point is that, in general, the phase $S(X_A,X_B,t)$ will depend on both X_A and X_B, in a way that involves the functions $R_i(X_B,t)$ and $S_i(X_B,t)$ (where the latter depend on H_B'). Thus the value of $\dot{X}_A = (1/m)\,\partial S/\partial X_A \neq 0$ will depend on both X_A and X_B and, more importantly, will depend on R_i and S_i and therefore on H_B' . The distribution $\rho(X_A,t)$ will no longer be static, $\rho(X_A,t) \neq \rho(X_A,0)$, and measurement of X_A will then yield a value which may differ significantly from x_o , in a way which depends on the value of H'_B , i.e. on the sudden movement of the walls of the distant box B. Thus, if x_o (i.e. $\rho(X_A,0)$) were known, instantaneous signalling would be possible. (For this simple example, the evolution of $\rho(X_A,t)$ will clearly depend on the value of H'_B).

We conclude that, if the initial distribution ρ_o is considered as known, then the condition $\rho_o = |\psi_o|^2$ is crucial for the validity of signal-locality.

If ψ_o is not entangled (e.g. $|\psi_o\rangle = |E_0E_1\rangle$), then of course no instantaneous signalling is possible, whatever the relation between ρ_o and $|\psi_o|^2$. This follows immediately from (23): Putting, for example, $\beta = 0$ leads to

$$\tan S = \tan(S_1(X_B,t)-E_0t)/\hbar$$

which is independent of X_A . Thus $\dot{X}_A = (1/m)\,\partial S/\partial X_A$ will vanish, so that $\rho(X_A,t)$ will be static, and therefore independent of H'_B. (We note here that the continuity equation for $\rho(X_A,X_B,t)$ implies

$$\partial\rho(X_A,t)/\partial t + \partial\left(\int dX_B\,\rho(X_A,X_B,t)\dot{X}_A\right)/\partial X_A = 0$$

where in general $\dot{X}_A$ depends on X_B).

The above conclusion with regard to instantaneous signalling also applies for momentum "measurements". To discuss this, it is first of all crucial to note the distinction, in the pilot-wave formulation, between "actual" and "measured" values of momentum[4,9]. For the case of position, the following may be assumed: A so-called "measurement" of X yields a value which is equal to the actual value which existed prior to the "measurement". For the case of momentum, however, the result of what is usually termed a "measurement" generally <u>differs</u> from the actual value $m\dot{X}=\partial S/\partial X$ which existed prior to the "measurement".

For example, for a single particle in the ground state of a box, $S(X,t)= -E_0t$, and so $m\dot{X} = \partial S/\partial X = 0$. The particle is <u>at rest</u>. If one then "measures" the momentum by, for example, opening the box, this physical action alters the phase S of the wavefunction, and thereby alters (via $\partial S/\partial X \neq 0$) the position of the particle in such a way that, as $t \longrightarrow \infty$, the particle is found to be at

$X = \pm\, t(2E_0/m)^{1/2}$. The value (plus or minus) depends on the initial position of the particle in the box. Measurement of X at large t then yields a "measurement" of momentum, with result $\pm\,(2mE_0)^{1/2}$, unrelated to the actual vanishing value at t=0. As stressed by Bohm[4] and by Bell[9], the value of a momentum "measurement" is an outcome of the whole "apparatus-plus-system" set-up, and is not a pre-existing property of the system alone. The "measurement" must not be thought of as yielding a result which is related in any simple way to the true value prior to "measurement"[9]. Clearly, as stressed with great clarity by Bell[9], the word "measurement" is profoundly inappropriate in quantum theory, and should perhaps be replaced by the word "experiment". (The general confusion generated by mis-use of the word "measurement", in particular in leading to mistaken "impossibility proofs" regarding "hidden variables", has been clearly shown by Bell). For the present case, it might be more appropriate to regard the "measured" momentum value as being in a sense emitted by the whole process of opening the box. This emission has some associated probability distribution $\tilde{\rho}(p_A)$.

Returning to the question of signalling for the momentum case, the question becomes: Is the emitted distribution $\tilde{\rho}(p_A)$ affected by the distant H_B' ? For the case $\rho = |\psi|^2$, Bohm[4] has shown that such "measured" values have the usual distribution predicted by standard quantum mechanics (involving the Fourier transform of the wavefunction). Thus, again, no instantaneous signalling would be possible for this case, as in standard quantum mechanics. However, for our above example of $\rho \neq |\psi|^2$, the value of x_0 around which ρ_0 is peaked may be chosen so that, on opening the box at t=0, $X_A \longrightarrow -\infty$ rather than $+\infty$ (as $t \longrightarrow \infty$), yielding a "measured" value $p_A = -(2mE_0)^{1/2}$. But then, $H_B \longrightarrow H_B'$ for the distant box will alter X_A away from x_0 , so that opening the box would no longer necessarily yield $X_A \longrightarrow -\infty$ (i.e. $p_A = -(2mE_0)^{1/2}$). Thus, if $\rho_0 \neq |\psi_0|^2$, we find instantaneous signals also with regard to momentum-related experiments, and a lack thereof when $\rho_0 = |\psi_0|^2$, just as for the case of position.

(ii) Uncertainty principle: It is straightforward to show that the uncertainty principle holds if $\rho = |\psi|^2$, but is generally violated otherwise, noting again our viewpoint that ρ be regarded as theoretically known, independently of ψ.

Firstly, if $\rho = |\psi|^2$, it is known that[4] momentum-related experiments yield "measured values" with a distribution $|\tilde{\psi}|^2$, where $\tilde{\psi}$ is the Fourier transform of ψ. The standard deviation Δp of these values then necessarily obeys the usual statistical dispersion relation ("uncertainty principle")

$$\Delta p \Delta x \geqslant \hbar/2$$

where Δx is the standard deviation of the distribution $\rho = |\psi|^2$.

To show that $\rho \neq |\psi|^2$ may violate the uncertainty relation, consider the simple case of a single particle in a box, with ground-state wavefunction $\psi_0(x)$, where the particle position has a probability distribution (for an ensemble of similar systems) which is sharply peaked at some $x=x_0$. We again choose x_0 to be such that, on opening the box, the particle position $X \longrightarrow -\infty$ as $t \longrightarrow \infty$, yielding a "measurement" $p = -(2mE_0)^{1/2}$. For this simple case, the standard deviation of "measured" momentum values vanishes, while Δx is finite (and small compared to the size of the box). This clearly violates the usual Heisenberg statistical dispersion relation.

E. Discussion

We have shown how $P = |\Psi|^2$ arises from a statistical law of subquantum "entropy" increase. We have also seen how $P = |\Psi|^2$ is in general necessary for the validity of signal-locality and of the uncertainty principle. Thus, within the context of the de Broglie-Bohm pilot-wave formulation of quantum theory, one may take the view that signal-locality and the uncertainty principle emerge only as statistical laws, from an underlying subquantum version of the law of entropy increase.

The above development demonstrates a definite link between signal-locality, the uncertainty principle, and the statistical law of entropy increase. In this context, it is interesting to note that Einstein regarded his basing of special relativity on the constancy of the speed of light as analogous to basing classical thermodynamics on the impossibility of perpetual motion[15,16]. Our results suggest that something more than a mere analogy may be involved. It may also be worth noting that the principles of signal-locality and of entropy increase are linked by the fact that both play a crucial role in ensuring a sensible theory of communication: The first avoids the paradox of signalling into the past, while the second provides a unique time-direction which, as pointed out by Wiener[17], is necessary to give a sensible meaning to communication[18]. (Essentially, in the absence of a unique time-direction, one could signal into the past of a "time-reversed" observer, and thereby generate causal paradoxes similar to those associated with instantaneous signalling).

The central mystery of quantum theory, perhaps, is the nature of the field Ψ, which has more the character of a "guiding field of information" than of a conventional mechanical force field[19]. Essentially, Ψ somehow encodes information from the whole environment at each point in space. The particle then "reads" this information via the relation $m\dot{\underset{\sim}{X}} = \underset{\sim}{\nabla} S$. (We stress again that the quasi-mechanical interpretation in terms of a "quantum potential" seems unnaturally complicated and inelegant, for which reason we prefer the "guiding equation" $m\dot{\underset{\sim}{X}} = \underset{\sim}{\nabla} S$). The work of Wootters[20] gives a further hint that Ψ is indeed somehow information-related. From such an information-related view of the pilot-wave formulation, it is perhaps not so surprising to see the theory bringing forth a strong link between the three "impossibility principles" mentioned above, since these might also be termed the three "information-related" principles of physics.

The above link between the three "impossibility principles" may then be a hint of something deeper. We suggest that further consideration of this link may lead to a deeper understanding of quantum theory, especially as regards its relation to relativity and locality, and possibly to a deeper understanding of Ψ as a "guiding field of information".

IV. FURTHER DEVELOPMENTS:

A. Explanation of locality as a subquantum "heat death"

As Ballentine[21] has put it: "Perhaps what is needed is not an explanation of nonlocality, but an explanation of locality. Why, if locality is not true, does it work so well in so many different contexts?" In a sense to be defined below, our explanation of locality is that the world is in a state of subquantum "heat death", where the nonlocal connections of quantum theory may no longer be used for signalling.

To begin with, we introduce a subquantum analogue of Maxwell's demon. The Maxwell demon is an imaginary intelligent being who attempts to gain a detailed knowledge of the trajectories of molecules in a classical gas, usually with the intention of trying to violate the Second Law of Thermodynamics. In contrast, the subquantum demon attempts to gain knowledge of the de Broglie-Bohm particle trajectories, in an attempt to send instantaneous signals.

For the Maxwell demon, the process of gathering and using information regarding molecular trajectories leads to an additional entropy increase by which, statistically speaking, the Second Law is upheld[22]. We may expect that some analogous considerations will apply to the subquantum demon, preventing the demon from violating signal-locality.

The case of the subquantum demon turns out, in fact, to be closely analogous to that of a Maxwell demon whose equipment happens to be in thermal equilibrium with the gas which he is attempting to study. The latter demon's activities would be severely limited by the thermal fluctuations in his own equipment. The subquantum demon's activities are limited, ultimately, by the fact that his own equipment is subject to the relation $P = |\Psi|^2$, and therefore to the uncertainty principle.

As we have shown above, relaxing the condition $P = |\Psi|^2$ leads to the possibility of instantaneous signalling. In particular, we considered an example where the probability density ρ for a particle in a box was narrowly peaked relative to $|\psi|^2$. How could a subquantum demon ever construct such a situation? Clearly, only by making measurements with equipment which is not limited by the uncertainty principle. To see this, it is sufficient to consider Heisenberg's "microscope": As stressed by

Heitler[23], the result $\Delta x \Delta p \gtrsim \hbar$ for the "disturbed" electron depends crucially on the assumption of the same statistical spread $\Delta x \Delta p \gtrsim \hbar$ for the photons which are used to "see" the electron. Clearly, if the demon had access to photons which were not subject to the relation $P = |\Psi|^2$, so that the photon distribution could satisfy $\Delta x \Delta p \lesssim \hbar$, then the demon could use these to "measure" an electron's position, and thereby create an electron (ensemble) state of the form $\rho \neq |\Psi|^2$, as required for instantaneous signalling. This is clearly analogous to the advantages enjoyed by a Maxwell demon whose equipment is at a low temperature compared to that of the gas which he is studying.

Essentially, then, the situation seems to be as follows: The world presumably begins in a state $P \neq |\Psi|^2$, where instantaneous signalling would be possible. As the maximum subquantum-"entropy" state $P = |\Psi|^2$ is approached, the possibility of instantaneous signalling fades away. This situation is maintained by the fact that not only any "system", but also any "apparatus", is subject to $P = |\Psi|^2$ and therefore to the uncertainty principle. Once $P = |\Psi|^2$ is reached everywhere, one is "trapped" in a self-maintaining, self-consistent situation, and instantaneous signalling is impossible.

This situation is strikingly analogous to that of a world in a state of uniform thermal equilibrium, or "heat death". In the classical heat death, while individual molecules may continue their random motions, no more change takes place on the macroscopic level (ignoring the unlikely possibility of macroscopic fluctuations). In the subquantum heat death, while nonlocality is present for individual events, all measureable probability distributions are local, and no instantaneous signalling is possible.

According to the present approach our world is, to high accuracy, in such a state of subquantum heat death. From one point of view, this may seem a satisfying explanation of locality on the statistical level. From another point of view, the situation seems infuriating.

B. Interpreting the subquantum "entropy"

We have made use of the expression

$$S(P,\Psi) = -k \int d\Sigma \, (P - |\Psi|^2) \ln (P/|\Psi|^2)$$

for the subquantum "entropy". The precise relation, if any, of this quantity to conventional notions of entropy is as yet unknown. In particular, we have not yet found a precise mathematical understanding of it in terms of probability theory, as is done for the classical entropy. We wish to point out, however, what may be the starting point for such an understanding: From the viewpoint developed above, where signal-locality is seen as a state of maximum subquantum entropy, the subquantum entropy is closely associated with our inability to send instantaneous signals. Now we have shown that the latter is due to our lack of detailed knowledge of the de Broglie–Bohm trajectories. We see here the beginnings of an interpretation of our entropy in terms of a lack of "information".

The ability to send instantaneous signals requires knowledge of regions of configuration space where $P \neq |\Psi|^2$. We might then expect that only regions where $P \neq |\Psi|^2$ would contribute to the entropy. It is satisfying to see that our expression above fulfils this expectation. Nevertheless, much remains to be understood as regards the interpretation of what we have called the "subquantum entropy".

C. Beyond quantum theory?

Our derivation of the result $P = |\Psi|^2$ (on a coarse-grained level), together with its two corollaries of signal-locality and the uncertainty principle, is essentially statistical in nature. The probability distribution $P = |\Psi|^2$ does not arise as an exact law, but is only true as a "most likely" result, as is the case for the Second Law of Thermodynamics. From this viewpoint, signal-locality and the uncertainty principle also lose their status as exact laws, and become merely statistical in nature, again like the Second Law of Thermodynamics.

On the basis of our approach, it is then reasonable to expect the existence of small statistical deviations from the usual results of quantum theory.

First of all, however, it is reasonable to expect the coarse-grained result $P = |\Psi|^2$ to hold to high accuracy. This becomes clear if one considers what we believe to be the complicated and chaotic past history of our observable universe. One expects that our observable world has had ample opportunity to thoroughly "mix" the quantities P and $|\Psi|^2$ in configuration space, in the sense discussed above. It then seems likely that fine-grained deviations $P \neq |\Psi|^2$ would be inaccessible to present experimental measurements. However, since we cannot as yet estimate the scale below which fine-grained deviations would be present, the possibility should be kept in mind that these could be detected in practice. (For $P = |\Psi|^2$ to become valid on a coarse-graining scale δV, one presumably needs a time since the "beginning" which increases in magnitude as δV decreases. And presumably, for large enough times, δV may be taken as arbitrarily small).

To found a theory of statistical deviations of P from $|\Psi|^2$, and to then discuss the physical consequences of such deviations, requires careful consideration of certain subtle issues regarding the principles of statistical mechanics. An idea of these issues may be gained by considering the analogous problem of classical statistical mechanics: To derive corrections to the Boltzmann factor $e^{-E/kT}$, for a gas which is to high accuracy, but not exactly, in thermal equilibrium.

The subject will be developed elsewhere. Here we note only that, given the pilot-wave formulation, it seems sensible to follow the theory to its logical limits. If, on the basis of Bohm's 1952 "pure" pilot-wave theory, one may derive statistical deviations from the results of standard quantum theory, then one may be reasonably confident that such deviations actually exist in Nature. This is so because the existence of such deviations would be predicted without any arbitrary theoretical input - a rare situation in theoretical physics.

In the author's opinion, modifications to the "pure" pilot-wave formulation (such as subquantum stochastic fluctuations) should be considered only after subquantum statistical mechanics, based on the pure pilot-wave formulation, has been fully developed. For only then can we be sure as to what the <u>exact</u> predictions of the pure pilot-wave formulation are, so that only then can a comparison with precision experiments be made.

V. GENERAL REMARKS:

A. On the objective existence of electrons, molecules, and pollen grains

(i) Introduction: It might seem disappointing that our "subquantum statistical mechanics" leads, to high accuracy, to just the usual statistical predictions of standard quantum theory. One might have hoped for some easily-observable statistical deviations from the usual results, which may have served as convincing evidence for the objective existence of particle trajectories (if the predicted deviations were seen experimentally). However, the situation is extremely peculiar philosophically, and deserves comment.

(ii) Do fluctuations demonstrate objective reality? Early in this century, the existence of statistical fluctuations in liquids and gases was regarded as convincing proof of the objective existence of atoms and molecules. However, the form and magnitude of such fluctuations could be predicted essentially on the basis of just the Boltzmann factor $e^{-E/kT}$. While this factor was deduced, historically, on the basis of the atomic hypothesis, in applications one requires only the factor on its own. Its realistic basis is not needed.

Now, the quantum theory relation $P = |\Psi|^2$, which may be deduced, as we have seen, on a realistic basis, predicts statistical fluctuations in, say, the position of an electron in the ground state of Hydrogen - always with an ensemble in mind, of course. (Note that Ψ may be calculated for this case from the Schrödinger equation). From our point of view, these fluctuations are exactly analogous to those of classical statistical mechanics based on the factor $e^{-E/kT}$.

Our point, then, is this: In an earlier era, the observation of effects due to quantum fluctuations, which are accounted for in a realistic manner by the pilot-wave formulation, would have been taken as convincing evidence in favour of the assumed objective of, say, electrons. In our era, of course, this is not the case. Why not?

From the present viewpoint, accepting standard quantum theory is very analogous to accepting the results such as $e^{-E/kT}$ of classical statistical mechanics while ignoring the realistic mechanical basis for those results. Clearly, with such an attitude, the existence of quantum fluctuations, which may be predicted using only $P = |\Psi|^2$, is hardly going to be regarded as evidence for the objective existence of the neglected mechanical basis (such as electron trajectories). However, to be consistent, one must ask why fluctuation phenomena in liquids and gases are nevertheless widely accepted as evidence for the objective existence of atoms and molecules. One could "explain" such phenomena simply on the basis of the "fundamental law" $e^{-E/kT}$. The atomic hypothesis could be seen as "unnecessary", or "metaphysical" . (Many physicists would take just such a view with regard to the assumption of electron trajectories, which we have used to derive the "law" $P = |\Psi|^2$).

One might object that, in quantum theory, there exist fundamental limitations on the definable meaning of "complementary" quantities, embodied in the uncertainty relations, while no such limits exist in classical statistical physics. This, however, is not correct.

(iii) Complementarity and uncertainty in classical statistical physics: The analogy between quantum theory and classical statistical mechanics may be refined so as to incorporate, into classical statistical mechanics, a precise analogue of Bohr's "complementarity". Bohr's views against the objective existence of electron trajectories may then be used, in a precisely analogous manner, to "disprove" the objective existence of atoms, molecules, and even pollen grains.

We base our argument on a paper by Rosenfeld[24], which consists essentially of an outline of Bohr's viewpoints. In an attempt to show the wide application of the "logic of complementarity", Rosenfeld draws attention to the following remarkable fact: In statistical thermodynamics, there exists a precisely definable "complementarity" between (a) quantities such as the energy of a system, "the detailed motion of its parts"[24], and so on, and (b) macroscopic quantities such as pressure, temperature, and entropy. For example, energy and temperature are clearly complementary: To have a well-defined energy, a system must be thermally isolated, making its temperature meaningless. If the temperature is defined, by thermal contact, then the energy is not determined - it has instead a statistical distribution.

It is stated by Rosenfeld that this latter complementarity is precisely equivalent, logically speaking, to that of quantum theory. One may even derive "uncertainty relations", analogous to those of quantum theory, where Boltzmann's constant k plays a role analogous to Planck's constant $\hbar$. (For example, fluctuations in energy are $\sim N^{1/2}$, while those of temperature are $\sim N^{-1/2}$, where N is the number of independent elements in the system (atoms, molecules, or pollen grains - where the behaviour of the latter is just that of "large molecules"). The product $\Delta E \Delta T$ is then independent of N, and is found to be of the form kC, where C depends only on the physical properties of the system, and not on its size).

According to Bohr, the "uncertainty" in complementary quantities should not be seen as merely a statistical spread, or as representing a disturbance due to the measurement process (this latter point being made with particular reference to the EPR thought experiment). Rather, the uncertainty relations express fundamental "ambiguities" in the definition of complementary quantities within the context of an "entire experimental arrangement".

One might raise the following objection: One could measure the position and speed of each molecule to arbitrary accuracy, using feeble (classical) light, and thus determine the energy, without disturbing the system in any way. However, one may take the view that, by this act, the temperature necessarily becomes undefined. That this latter occurs without any physical "disturbance" is of no concern from the point of view of complementarity: For as Bohr argued for the EPR case, the changes in "ambiguity" of various spin components may occur without any physical disturbance. They occur rather by changes in the "whole experimental set-up", even if these changes take place far away from a given spin (so that the possibility of a direct disturbance is excluded). Thus, the situation in classical statistical physics is indeed governed by the "logic of complementarity", just as in quantum theory, as claimed by Rosenfeld.

Thus, it is clear that, by appropriately choosing the "whole experimental arrangement", we as experimenters are free to decide whether our system shall have a precisely defined energy, or a precisely defined temperature. Should one now follow Bohr and Heisenberg, and conclude from this that we are playing a part in "creating reality", even at the level of gases and liquids in the laboratory? Isn't the logic just the same as in quantum theory, as stated by Rosenfeld?

If so, then we arrive at the following remarkable conclusion: If one neglects quantum effects, and considers just the classical statistical mechanics of Brownian motion of pollen grains in a liquid, then the "detailed motion" of the liquid is "complementary" to its temperature. In other words, if the temperature of the liquid is well-defined, then it is "meaningless" to even think of the objective existence of the detailed motion of the molecules of the liquid, or of the pollen grains suspended in it (since a well-defined detailed motion would imply a definite energy value, and therefore a contradiction).

How one responds to this conclusion seems to be a personal choice, since the logic of complementarity is, as even Einstein admitted, quite self-consistent, and therefore irrefutable on purely logical grounds. One might agree with Bohr and Rosenfeld that complementarity reaches beyond the confines of quantum theory, having applications to thermodynamics, and even to biology[24]. But then one must give up, for certain situations, the idea of the objective existence of, for example, the motion of pollen grains, even in the "classical" ($\hbar \longrightarrow 0$) limit, just as many physicists long ago gave up the objective existence of electron motion in the quantum domain. Such a viewpoint would, from a logical standpoint, be perfectly consistent. On the other hand, the necessity of such a far-reaching renunciation of commonly-held notions of reality, at the macroscopic level, might instead be seen as evidence that the logic of complementarity is, while self-consistent, not scientifically credible. One would then have no choice but to regard the position of an electron in an eigenstate of momentum as being just as real, objectively, as the detailed motion of a pollen grain in a liquid which is in an "eigenstate" of temperature.

(iv) Conclusion: One might try to prove the existence of electron trajectories by means of some statistical deviations from the results of standard quantum theory. However, while such deviations may exist, such an approach would be like trying to demonstrate the existence of atoms by deriving deviations from the Boltzmann factor $e^{-E/kT}$.

Much of the problem really lies with the general philosophy associated with quantum theory, according to which any realistic basis for its statistical laws is to be regarded as "meaningless". This is a severe problem since, with such attitudes, experimental confirmation of the predicted statistical laws will never be regarded as evidence for the realistic basis of those predictions - unless the predictions differ from the results of standard quantum theory. But even then, such deviations, for example of the form $P \neq |\Psi|^2$, might be seen in terms of a new fundamental law, and the realistic basis again forgotten. This last actually has historical precedent: In an essay on Ostwald's "Energetics", published in 1896, Boltzmann points out how certain developments in electro-chemical theory originated in a purely molecular view due to Nernst, but that later "these propositions were severed from their molecular justification and presented as pure facts"[25].

In the face of the still widespread philosophy, it then seems a difficult task to demonstrate the objective existence of electrons. And also, perhaps, of pollen grains. In addition to working towards a realistic understanding of quantum theory, it is then important to carefully reconsider the philosophy of quantum theory as laid out by Bohr, Heisenberg, and others.

B. On the "instability" of the Bohr-Heisenberg philosophy

The present philosophy of quantum physics maintains that one should not try to look "behind the scenes", that it is not possible to go beyond the present statistical predictions of quantum mechanics. This philosophy rests essentially on Heisenberg's uncertainty principle. It is held that the impossibility of a simultaneous "measurement" of position and momentum renders the notion of a particle trajectory "meaningless". Closely related is Bohr's principle of "complementarity", which rests on the apparent fact that, in a certain sense, particle and wave aspects may not be observed simultaneously.

We wish to point out here the great danger in basing one's philosophy of science on principles whose basis is ultimately empirical.

Consider, for example, the linearity of the Schrödinger equation. The property of linearity may only be justified, ultimately, by experiment. It cannot be ruled out that, in the future, experiment will uncover small nonlinearities in the Schrödinger equation, as was hinted at by Wigner[26] in 1939, developed more recently by Weinberg[8], and recently tested experimentally by Bollinger et al.[27]. Now it has been shown[28] that such nonlinearities will in general allow a simultaneous measurement of particle paths and interference in a double-slit experiment, thus destroying Bohr's mutually-exclusive wave-particle complementarity. Thus, if even minute departures from linearity are present in Nature, the entire philosophy of Bohr and Heisenberg (as applied to Physics) will collapse.

One wonders if it is wise to believe in a philosophy of science whose validity depends so sensitively on the empirical content of that science. Any sensible philosophy of science must surely be capable of surviving at least small changes in the empirical content of science itself. The present situation in quantum theory seems similar to that of a physicist in the year 1700 basing his world-view on the linearity of Hooke's law.

Acceptance of such an unstable philosophy can only lead to dogmatism (for example the ruling out of nonlinearities <u>a priori</u>). It is striking to note that Bohr, while arguing against an attempt by Einstein to circumvent the uncertainty principle (at the Sixth Solvay Congress in 1930), is reported to have said that it would be "the end of Physics" if Einstein were right[29]. If the recent experimental search for nonlinearities in the Schrödinger equation, by Bollinger et al., had yielded a positive result, would this really have been "the end of Physics"?

C. Can realistic science be nonlocal?

Assuming the exact validity of the statistical predictions of quantum mechanics, Bell's theorem seems to rule out any local realistic theory lurking "behind the scenes". It seems that one must reject at least one of (a) locality, and (b) realism.

It is remarkable that many physicists seem to choose option (b) with little hesitation. It is claimed that the ideal of describing the world as "independently actual" is mistaken. Instead, one should only deal with "observations". One must not even think of "elements of reality". As pointed out below, there is little doubt that the current popularity of this choice may be traced back to anti-rational and anti-realist movements in early twentieth-century Europe.

If one forgets, for a moment, the historical debris by which we are still surrounded and influenced, one immediately asks: What really concrete arguments are there against <u>nonlocal realism</u>?

There seems to be only one clearly articulated argument against nonlocal realism, and this is the idea that a nonlocal reality could not be studied by scientific methods. It might be claimed that if, for example, the real state of affairs inside a Hydrogen atom in a laboratory in Rome depends on the situation in Venice, on Mars, and on the other side of our Galaxy, then application of the "scientific method", with its basis in experiment, will be hopeless. It is this claim which we wish to refute here.

The above claim states essentially that Physics can only develop if it is possible to isolate simple systems from the rest of the Universe. For only then will the experimentally-observed behaviour be simple enough for the discovery of physical laws to be possible. However, this rests on a naive view of the role of experiment in Physics. In particular, it rests on the mistaken idea that physical theories are constructed mainly on the basis of experimental facts.

While experiment is of course of fundamental importance, the above view misses the fact that physical theories are often constructed on the basis of appealing theoretical principles, which rely on just a very few experimental facts. The General Theory of Relativity is the classic example, as well as Schrödinger's construction of Wave Mechanics. As a more recent example, one might consider the theory of Quantum Chromodynamics, which was constructed on the basis of little more than local gauge-invariance. In this theory, the quantised quark and gluon fields are only very distantly related to the scattering cross-sections which are actually "observed" in laboratory experiments. Indeed, it is widely believed that quarks themselves will be forever hidden from "direct observation". Clearly, it would be absurd to claim that the theory of Quantum Chromodynamics was constructed mainly on the basis of experimental facts. Indeed, it is by now widely realised that experimental "facts" cannot even be stated or interpreted in the absence of some theoretical structure (see below).

Our point, then, is that the development of Physics in a nonlocal world would be possible by building theories on the basis of some (as yet unknown) general principles. Of course, one must then find some prediction of the theory which may be tested experimentally. But the <u>construction</u> of the theory cannot proceed on the basis of experimental facts alone. This situation is of course not new, and was clearly understood by Einstein, who stated that "A theory can be tested by experience, but there is no way from experience to the construction of a theory"[15].

In fact, the difficulties presented to the "scientific method" by nonlocality show a similarity to those presented by <u>nonlinearity</u>. As Einstein put it[15], regarding the nonlinear equations of the gravitational field: "No collection of empirical facts however comprehensive can ever lead to the setting up of such complicated equations...", which "...can be found only through the discovery of a logically simple mathematical condition that determines the equations completely or almost completely. Once one has obtained those sufficiently strong formal conditions, one requires only little knowledge of facts for the construction of the theory".

Clearly, the above argument against nonlocal realism is based on a misconception as to the role of experiment in Physics, and could just as well be aimed against nonlinear realism.

In a letter to de Broglie in 1954, Einstein[30] made some related remarks regarding the search for a "substructure" beneath quantum mechanics: "I have long been convinced that this substructure cannot be found in a constructive way, starting from the empirically established behaviour of objects, because the efforts necessary for this would exceed human possibilities. I have arrived at this conclusion....because of my experience in the theory of gravitation. The equations of the theory of gravitation could be discovered only on the basis of a purely formal principle (of general covariance)...". Einstein expected the substructure to be nonlinear in character, and saw clearly the implications against naive notions of "scientific method". Today, there are good reasons to expect that the substructure is nonlocal (and perhaps also nonlinear), which has rather similar implications for scientific method. One must then try to proceed, following Einstein, on the basis of some general formal principle or principles, which are as yet unknown. Certainly, there is no reason to believe that a nonlocal reality is inaccessible to science, no more than is a nonlinear one.

D. Quantum theory: A historical perspective

(i) General remarks: In his book "Physics and Beyond"[31], Heisenberg takes great pains to show how his work in physics was intimately bound up with the cultural developments of the time, a fact which Heisenberg clearly found satisfying. This attitude is indicated by the original German title "Das Teil und das Ganze" ("The Part and the Whole"). The mood of post-World War One Germany, and its influence on physics, is presented in considerable detail.

Since the present form of quantum theory reflects in part the mood of Europe in the 1920s, in particular, perhaps, of the German-speaking world, it is important to have some idea of the character of that mood.

Even a cursory survey shows a general anti-realist tendency, with roots in the late nineteenth century, strongly manifest in Art, Poetry, and Literature, a tendency which became more pronounced under the impact of the First World War. Regarding the latter: "The shock and disillusionment of war shook all faith in meaningful reality"[32]. The atmosphere is in fact well-documented by Heisenberg himself[31]. As in physics, the history of nineteenth and twentieth century Art is, more than that of any other era, "the visible token of the central problem of the modern age, the quest for a meaningful image of reality"[33]. The post-World War One atmosphere saw philosophers and writers calling for "the mobilisation of the spirit against the mechanism and determinism of the natural sciences"[34]. We do not wish to in any way question these tendencies from a purely humanistic point of view. It is their effect on physics which seems disturbing.

The effect of these tendencies on science itself is seen remarkably, for example, in the attitudes taken towards Einstein's theories of "relativity", not only by the general public, but by physicists themselves[35]. "Relativity" was widely taken to imply that reality is in some way subjective and observer-dependent, even by physicists such as Bohr, Heisenberg, and Born. To arrive at such views, the observer-dependence of space and time measurements, and of simultaneity, was stressed. However, the invariance of natural laws, their universality for all "observers", was ignored[36] (where such invariance shows that the theory of "relativity" could just as well have been called the theory of "invariants"). There is no doubt that relativity was widely perceived as indicating an end to the old order, and as discrediting the "materialist" ideals of an earlier generation[35]. This is seen in the fact that the popularity of "Einstein's" ideas led to a counter-reaction in Germany, where Einstein was accused, in a wave of anti-Semitism, of being a "scientific Dadaist"[35].

Such post-World War One trends of thought may not seem very relevant today (1990). However, the informal language of present-day physics is pervaded by a subtle observer-centred philosophy, whose roots lie in the late-nineteenth and early twentieth centuries. Regarding the effect of the latter era on contemporary thought, it is striking to note the extent to which present-day North American popular culture has its roots in the German philosophy of the 1920s and 1930s[37].

From our point of view, Heisenberg's book "Physics and Beyond" may be seen as a frank admission that, indeed, the approach to physics taken by himself and others was greatly influenced by the anti-realist, and even anti-rational, atmosphere of the times. In a letter to Heidi Born in 1927, Einstein pointed to the invasion of irrationalism in physics at the time, with its "tragic consequences"[38]. Given such an atmosphere, it is hardly surprising that Bohr, Heisenberg, and others, were so ready to conclude that, for example, spontaneous emission is a "fundamentally acausal process".

As discussed clearly by Heisenberg[31], Bohr's abandonment of the classical causal viewpoint, of objective processes taking place in space and time, was based largely on the puzzle of the "recurrence of forms": The fact that "even after a host of changes due to external influences, an iron atom will always remain an iron atom, with exactly the same properties as before"[31]. According to Bohr, this is "quite inexplicable in terms of the basic principle of Newtonian physics, according to which all effects have precisely determined causes"[31].

Given the "miracle of the stability of matter", and the fact that "we can see the inconstancies, the sudden jumps in atomic phenomena quite directly, for instance when we watch sudden flashes of light on a scintillation screen..."[31], given all that, one may understand the puzzlement felt by Bohr and others early in this century. Why, though, should all this lead one to the conclusion that it is "impossible to build up a descriptive time-space model of interatomic processes"?[31] Why should such a conclusion follow from the "discontinuous element.... of atomic phenomena"[31], as Heisenberg claimed (under the influence of Bohr)?

As stressed by Bell[9], the mere existence of the pilot-wave formulation demolishes such views. We might add further that the ability of classical deterministic physics to generate complex, recurrent, and stable structures has been vastly underestimated. This is shown by the development in recent years of the theory of complex dynamical systems, and of "dissipative structures" far from equilibrium, essentially on the basis of classical (i.e. nonquantum) theory[18].

(ii) <u>Against "historicism" in physics: The example of Boltzmann</u>: As stressed by Heisenberg, any creative scientific work is necessarily influenced by the mood of the times. However, it is dangerous to claim that science must always take its cue from attitudes prevailing in society at large. As stressed in another context by Popper[39], such "historicist" attitudes tend to undermine belief in human ingenuity. From the point of view of physics, the scientific researcher becomes, in the historicist view, a pawn subject to overpowering historical forces. For example, many contemporary physicists would say that the trend of our century is away from determinism, and even from realism, with the implication that a step backwards is unthinkable.

Such a historicist view of science is easily discredited by considering the case of Boltzmann, whose life's work was based on the (at that time undemonstrated) atomic hypothesis. Assuming the existence of atoms, Boltzmann (together with Maxwell, Gibbs, and others) was able to account for the observed properties of gases, and to deduce the Second Law of Thermodynamics (to within certain controversial points regarding time reversal, which persist to this day). However, it would be difficult to imagine a more hostile atmosphere into which Boltzmann's "materialist" ideas could have been introduced: In his book "Appearance and Reality", published in 1893, Bradley[32] maintained that "Nature by itself has no reality", and that the idea of Nature being "made up of solid matter interspaced with an absolute void" has to be abandoned. The "phenomenalism" of Mach, and the "energetics" of Ostwald, dominated the scene. As Bunge has put it: "Thermodynamics.... was regarded as the paradigm to be imitated. Physical theory was required to stick to observation,...... The hypothesizing of hidden entities and inner structures became taboo..."[36].

It is likely that this hostile atmosphere was partly responsible for Boltzmann's suicide in 1906. Ironically, shortly afterwards, the atomic hypothesis was confirmed by the observation of fluctuation phenomena predicted by the statistical mechanics whose development had begun with Boltzmann decades earlier. Remarkably, despite the experimental verification of the predicted properties of Brownian motion, Mach maintained his disbelief in the existence of atoms until his death in 1916.

The fact that, in spite of the hostile mood of the times, Boltzmann's ideas survived and conquered, should lay to rest any notions that the path taken by physical theory must be in accordance with the prevailing cultural atmosphere.

(iii) Boltzmann and Bohm, Mach and Heisenberg: There is of course a clear historical parallel between the work of Boltzmann, and that of Bohm, where Bohm has given a fully realistic account of the results of quantum theory, just as Boltzmann did for thermodynamics. The question of the real existence of electron trajectories recalls discussions concerning the existence of atoms at the turn of this century. Indeed, the "operationalist" arguments of Heisenberg against the meaningful existence of electron trajectories are strikingly reminiscent of the arguments of Mach et al. against the existence of atoms. The basic philosophy behind these arguments is, of course, that one should not speak of that which cannot be "directly observed". This philosophy evaporates when one realises that "direct observations" do not exist, since "observations" are impossible, and indeed meaningless, without some prior body of theory. For instance, even today, nobody has "directly observed" an atom.

One might point to the images generated by electron microscopes. But these are just patterns of light and shade on photographic paper. Their interpretation in terms of "atoms" depends on an extensive theoretical structure. As Einstein said to Heisenberg, "we must know the natural laws at least in practical terms, before we can claim to have observed anything at all"[31]. We would agree with Einstein that "The prejudice [of Mach and Ostwald] - which has by no means disappeared - consists in the belief that facts by themselves can and should yield scientific knowledge without free conceptual construction. Such a misconception is possible only because one does not easily become aware of the free choice of such concepts, which, through success and long usage, appear to be immediately connected with the empirical material"[15]. We might agree further that Mach was "a good mechanic but a deplorable philosopher"[35]. A sensible viewpoint was advocated by Boltzmann himself: "One ought not to say, with Ostwald, "Thou shalt not make any image", but only "Thou shalt admit in it as little of the arbitrary as is possible""[40].

Philosophical debates aside, it is a fact of history that, in the case of Boltzmann versus Mach, imaginative physical thinking of a realistic nature won the battle against naive operationalism.

It is the author's hope that a similar breakthrough will occur in quantum theory.

ACKNOWLEDGEMENTS

I am grateful to Prof. D.W. Sciama for moral support, to Sebastiano Sonego for playing the stimulating role of "doubting Thomas", and to Gabriela Ronkainen for her interest in the philosophy of quantum theory. This work was supported financially by the Science and Engineering Research Council of Great Britain.

REFERENCES

[1] K.R. Popper, Quantum Theory and the Schism in Physics (Unwin Hyman, London, 1982)

[2] The Ghost in the Atom, ed. P.C.W. Davies and J.R. Brown (Cambridge University Press, 1986)

[3] R.P. Feynman, The Character of Physical Law (MIT, Cambridge, MA, 1967)

[4] D. Bohm, Phys. Rev. 85 (1952) 166, 180

[5] Rapport au V'ième Congres de Physique Solvay (Gauthier-Villars, Paris, 1930)

[6] L. de Broglie, An Introduction to the Study of Wave Mechanics (E.P. Dutton and Company, New York, 1930)

[7] G.C. Ghirardi, A. Rimini, and T. Weber, Phys.Rev. D 34 (1986) 470

[8] S. Weinberg, Phys.Rev.Lett. 62 (1989) 485; Ann. Phys. 194 (1989) 336

[9] J.S. Bell, Speakable and Unspeakable in Quantum Mechanics (Cambridge University Press, 1987)

[10] D. Bohm and J.P. Vigier, Phys.Rev. 96 (1954) 208

[11] W. Pauli, in: Louis de Broglie , Physicien et Penseur, ed. A. George (Albin Michel, Paris, 1953), quoted in Ref. 10 above.

[12] D. Bohm, Phys.Rev. 89 (1953) 458

[13] J.R. Waldram, The Theory of Thermodynamics (Cambridge University Press, 1985); P.C.W. Davies, The Physics of Time Asymmetry (University of California Press, 1977)

[14] Ref. 9 above, chapter 14

[15] A. Einstein, in: Albert Einstein, Philosopher-Scientist, ed. P.A. Schilpp (Library of Living Philosophers, Evanston, Illinois, 1949)

[16] For a discussion of the role played by thermodynamics in Einstein's early thinking see M.J. Klein, Science 157 (1967) 509

[17] N. Wiener, Cybernetics (John Wiley, 1961)

[18] I. Prigogine and I. Stengers, Order out of Chaos (Flamingo, 1985)

[19] D. Bohm and B.J. Hiley, Found.Phys. 14 (1984) 255; D. Bohm and F. David Peat, Science, Order, and Creativity (Routledge, London, 1989

[20] W.K. Wootters, Phys.Rev. D 23 (1981) 357

[21]L.E. Ballentine, Am.J.Phys. 55 (1987) 785

[22]L. Szilard, Z. Phys. 53 (1929) 840 [English translation in *Quantum Theory and Measurement*, ed. J.A. Wheeler and W.H. Zurek (Princeton, 1983)]; C.M. Caves, Phys.Rev.Lett. 64 (1990) 2111

[23]W. Heitler, *The Quantum Theory of Radiation* (Dover, New York, 1984)

[24]L. Rosenfeld, Nature 190 (1961) 384

[25]L. Boltzmann, *Theoretical Physics and Philosophical Problems*, ed. B. McGuinness (D. Reidel, Dordrecht-Holland, 1974)

[26]E.P. Wigner, Ann.Math. 40 (1939) 149

[27]J.J. Bollinger, D.J. Heinzen, W.M. Itano, S.L. GIlbert, and D.J. Wineland, Phys.Rev.Lett. 63 (1989) 1031

[28]A. Valentini, Phys.Rev. A 42 (1990) 639

[29]*Quantum Theory and Measurement*, ed. J.A. Wheeler and W.H. Zurek (Princeton, 1983)

[30]*The Einstein-de Broglie Correspondence*, quoted in Ya.A. Smorodinskii and T.B. Romanovskaya, Sov.Phys.Usp. 31 (1988) 1080

[31]W. Heisenberg, *Physics and Beyond* (Harper and Row, New York, 1971)

[32]G. Barraclough, *An Introduction to Contemporary History* (Penguin Books, 1967)

[33]H.L.C. Jaffé, in: *20,000 Years of World Painting* (Greenwich House, 1983)

[34]A. Hauser, *The Social History of Art*, Vol.IV, Naturalism, Impressionism, The Film Age (Routledge and Kegan-Paul, London, 1962)

[35]R.W. Clark, *Einstein, The Life and Times* (Avon Books, New York, 1971)

[36]M. Bunge, in: *Quantum Theory and Reality*, ed. M. Bunge (Springer-Verlag, 1967)

[37]A. Bloom, *The Closing of the American Mind* (Penguin Books, 1987)

[38]*The Born-Einstein Letters* (Walker and Company,New York, 1971)

[39]R. Scruton, *A Dictionary of Political Thought* (Pan, 1983)

[40]Quoted in W. Moore, *Schrödinger, Life and Thought* (Cambridge University Press, 1989)