



ISAS - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

Texture Homogeneity and Symmetry Detection

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Introduction

Symmetry and texture perception have been investigated in order to understand the structure and processes of the visual system. The perception of various kinds of symmetry have been studied extensively in many aspects. Recently, works have focused on how the information in symmetrical displays is processed. Issues concerning the neurophysiology of symmetry perception and the processing models have been raised. Theories of texture perception are converging toward the point that preattentive texture discrimination is based on the local differences of stimulus features such as orientation, color, size, and brightness of textural elements.

This thesis is based on two experiments, designed to test if perception of symmetrical textures is affected by homogeneity of element sizes and shapes along the symmetry axis. Mirror symmetry about a vertical axis is used. Two kinds of patterns are studied: (1) in the first, the distribution of element sizes and shapes is completely homogeneous; (2) in the second, the distribution of element sizes and shapes is changed along the axis according to reflection. The dependent measures are sensitivity and reaction time of symmetry detection for human subjects. Results show that the lack of homogeneity makes symmetry more detectable but does not affect reaction time. It is hypothesized that heterogeneity of size and shape distributions makes the symmetry axis more salient and this saliency affects symmetry perception. Details about experiments are described in Chapter 5.

Our visual system consists of several separate subsystems whose func-

tions are quite distinct in early processing. The visual subsystems are considered to be responsible for extracting the functional primitives of early vision, such as edges, symmetries, etc.. Some anatomical, physiological, and psychophysical studies of the visual system are presented in Chapter 1. In Chapter 2, theories of preattentive texture perception are presented. Chapter 3 illustrates views of how visual information is reconstructed and visual patterns are represented. Chapter 4 discusses the perception of symmetry and Chapter 6 shows the conclusions from our experiments. The psychophysical and statistical methods used in data analysis are presented in Appendix 1 and Appendix 2.

Chapter 1

Visual System

1.1 Anatomical and physiological studies

Anatomical and physiological observations in monkeys indicate that the primate visual system consists of several separate and independent subsystems which analyze different aspects of the same retinal image. They can be conceived as independent parallel pathways from the eye to the cortex.

At the early levels, in lateral geniculate body, there are two subsystems. As shown in Fig. 1.1, the primate lateral geniculate body is composed of six layers. Single cells in the geniculate body receive input from one eye, right or left eye. The layers are stacked in an interdigitating way; e.g., in the left geniculate body, the sequence of the layers from the up downward is left, right, left, right, right, left. The geniculate body is composed of two subsystems: the upper four parvocellular layers and the bottom two magnocellular layers. Parvosystem receives input from retinal cell type A, which is large, and magnosystem receives input from retinal cell type B, which is small. These two types of cells are anatomically different. The two subsystems differ physiologically in four major ways: color, acuity, speed, and contrast sensitivity. (1) Most cells in parvocellular layers are strikingly sensitive to differences in wavelength, whereas cells in magnocellular layers

are not. (2) The magno system has better acuity than the parvo system, They are sensitive to orientation, size or spatial frequency. (3) Magno cells respond faster and more transiently than parvo cells. They are sensitive to the direction of movement. (4) Magno cells are much more sensitive to low-contrast stimuli than parvo cells. They are sensitive to edges.



Fig. 1.1. Six cell layers in the left lateral geniculate body of a macaque monkey (seen in a section cut parallel to the face).

The segregation of the two pathways is perpetuated in the primary visual cortex and other visual areas. Cells in magnocellular layers project to layer $4C\alpha$, which in turn projects to layer 4B. Layer 4B projects to visual area 2 and to cortex area MT, middle temporal lobe. Parvo cells project to layer $4C\beta$, then to layer 2 and 3, and from there to visual area 2. Physiological studies suggest that the segregation of functions is continued

to the highest levels so far studied. The segregation seems to become more and more specific at each successive level (Livingstone & Hubel, 1988). Fig. 1.2 (a) shows the main connections from the lateral geniculate body to the striate cortex and from the striate cortex to other brain regions.

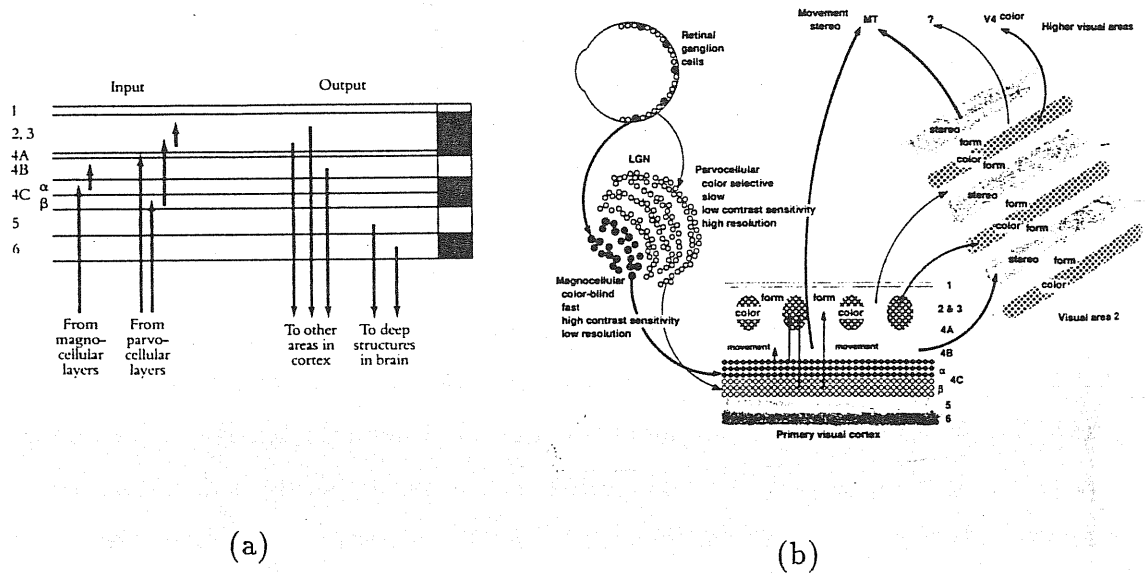


Fig. 1.2. (a) Main connections made by axons from the lateral geniculate body to the striate cortex and from the striate cortex to other brain regions. (b) Diagram of the functional segregation of the primate visual system.

Fig. 1.2 (b) shows the functional segregation from lateral geniculate body to high visual cortex and higher visual areas. One high visual area MT seems to be specialized for the analysis of movement and stereoscopic depth. Visual area 4 contains cells sensitive to color. So far, specific channels for color, orientation, brightness, stereo, movement, and size have been found.

1.2 Psychophysical Studies

Perceptual experiments have been done to establish the primitive features of visual system, to describe how these features are initially coded by the visual system and to understand how the features are related to the further processing and conscious experience of object perception.

Consistent with anatomical and physiological results, psychophysical experiments reveal the existence of different stages of perceptual processing. In early vision, image features, or primitives, such as color, orientation, spatial frequency, stereoscopic depth, and direction of movement, are coded by separate channels. Most theories suggest that in humans there are two visual systems. One is preattentive and another is attentive. The preattentive system is parallel in the sense that it can process visual information over a broad portion of the visual field at the same time. Features are coded simultaneously by the preattentive system. The attentive system is serial in the sense that the spatial extent of the stimuli that can be processed is limited. The attentive system is capable of identifying an object and establishing relationships between features by serial scanning.

Julesz (1984) indicates that the preattentive system can detect feature gradients but cannot identify which features create the gradients. He argues that the preattentive processing is limited to short range interactions. Differences of features can be coded in parallel only at high density. To know what features create the differences requires attentive processes operating over longer ranges. Attention is directed by the preattentive system

to the location where differences in features occur.

Julesz suggests that the operation of the preattentive system can be accounted for by a simple system of feature detectors. These feature detectors have local connections of inhibitory or excitatory type between similar detectors.

Treisman (1980), in her early feature-integration theory, accepted the idea of a dichotomy between preattentive and attentive. According to her point of view, the automatic grouping together of similar elements and separation of them from dissimilar ones, is allocated to preattentive processing. During the preattentive processing, features are registered simultaneously and independently on separate maps that are linked to a master map of locations. Thus, information about both *what* and *where* is offered by the preattentive system. Only the relation between the two is unspecified. When detail processing of an object perception is required, e.g., to identify the object, a further stage of attentional processes is accessed. It is proposed that focused attention allow for the linking together of information found in given spatial locations across the master map.

Treisman (1988) proposed a new explanation of perceptual processing. She hypothesizes that attention varies along a continuum during perceptual processing. Two extremes are (1) completely divided attention distributed widely over the whole display; (2) sharply focused attention to one item at a time.

Treisman suggests that the master map could correspond to area V_1 where many units appear to code several properties at once (Treisman, 1988). These properties are size or spatial frequency, orientation, color, binocular disparity, luminance, and contrast (Hubel & Wiesel, 1977; Thorell, De Valois & Albrecht, 1984). Areas beyond V_1 appear to be specialized in abstracting particular properties from the multidimensional array. Attention would gate the access to each of these specialized areas. Treisman suggests that the mechanism of selective attention is inhibiting inputs from all but a selected item or group.

Chapter 2

Texture Discrimination

As mentioned in Chapter 1, it is suggested that human vision operates in two modes, one is preattentive and the other is attentive. The preattentive system uses distributed attention that is mediated by a parallel process, while the attentive system uses focal attention scanned serially. The study of preattentive texture discrimination can serve as a model system with which to distinguish the role of local texture element detection from global (statistical) computation in visual perception.

Definition of Texture Discrimination

A texture is an aggregate of elements (that occur either at random or in semi-regular locations). When looking at textures, one can have the impression of either a unified texture or several separate subtextures, with little or no apparent effort. This is demonstrated by Figure 2.1. In Figure 2.1 (a) one can perceive effortlessly, even within a brief flash of presentation, that one quadrant of the texture differs from the rest. Julesz defined this effortless, spontaneous, and rapid performance of differentiating juxtaposed textures as visual texture discrimination (Julesz, 1962). On the other hand, in Figure 2.1 (b) there is no segregation between micropatterns \equiv and \equiv . Figure 2.1 (a) will be regarded as an example for texture discrimination since the texture can be easily discriminated in a

tachistoscopic flash. However, in Figure 2.1 (b) texture discrimination fails for tachistoscopic presentation times that yield good discrimination for Figure 2.1 (a). Hence, Figure 2.1 (b) will be regarded as a nondiscriminable texture.

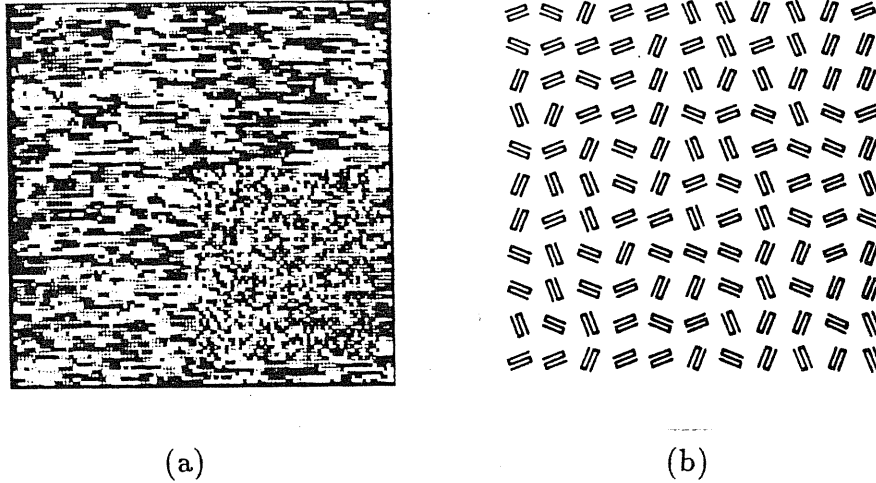


Fig. 2.1. (a) An example of discriminable textures. Two textures have identical first-order statistics but differ in second-order statistics. (b) An example of nondiscriminable textures.

Studies of texture discrimination have focused on the determination of the basic stimulus properties that mediate segregation (i.e., visual primitives), as well as on the nature of the segregation process itself.

Julesz's Conjecture

The problem of texture discrimination has been studied in two kinds of textures. One approach has used randomly generated black and white

dot textures, as shown in Figure 2.1 (a). Random dot textures can differ in their statistical properties: dot density (first-order statistic); joint spatial correlation of the dots (second-order statistics). The second approach has used textures composed of elements or primitives arranged either regularly or randomly (Beck, 1967; Olson & Attneave, 1970; Julesz et al., 1973).

Julesz has studied texture discrimination extensively with random dot textures. He uses the method of random geometry to describe textures (Julesz et al., 1973). In this method n-gons of arbitrary shape are thrown in a random way on the texture. The nth-order statistics is equivalent to

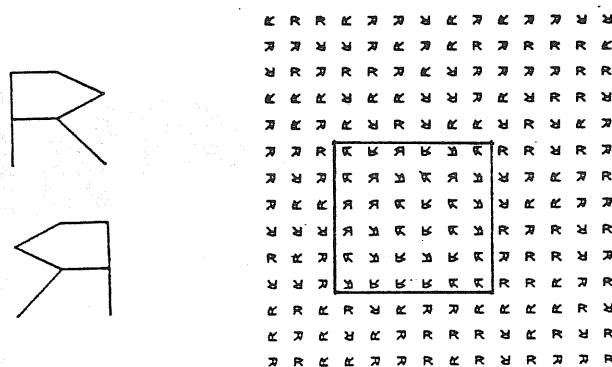


Fig. 2.2. Nondiscriminable iso-power-spectrum texture.

the n-gon statistics. This is the probability that the n vertices of n-gons will land on a certain color combination of the texture. For instance, the statistics that monopoles (the 1-gons) would fall on blacks is the first-order statistics. It determines the proportion of black and white areas. The second-order statistic is the probability that the vertices (end points) of randomly thrown dipoles (2-gons) of all possible lengths and orientations will fall on

certain colors of the texture, e.g., both on black. Similarly, the statistics of the three vertices of triangles (the 3-gons) falling on a specific color combination is the third-order statistics. Figure 2.1 (a) shows two subtextures with identical first-order statistics and different second-order statistics. A texture with identical second-order statistics (also called iso-dipole texture) but different third- and higher-order statistics is shown in Figure 2.2. Here one texture consists of identical micropatterns (R's) thrown at random in the peripheral area, while the second texture, embedded in a central square is composed of micro-patterns that are the mirrorimages of the others.

It has been shown (Julesz et al., 1973) that texture pairs composed of mirrorimage dual micropatterns are iso-dipole, regardless of the micropattern chosen. As demonstrated in Figure 2.2, such an iso-dipole texture pair cannot be discriminated without scrutiny, in spite of the fact that their third- and higher-order statistics differ. Indeed, from 1962 to 1978 many other kinds of iso-dipole textures were generated that could not be effortlessly discriminated (Julesz, 1962, 1971, 1975; Julesz et al., 1975; Schatz, 1978; Pratt et al., 1978). The observation that iso-dipole textures are usually not discriminable without scrutiny led to a conjecture. Julesz (1975) conjectured that texture discrimination does not occur for textures that have the identical global first- and second-order statistics. That is, textures which differ only in third- or higher-order statistics are not spontaneously segmented.

Julesz claims that such a conjecture is not just a mathematical game. After all, the second-order statistics determine the autocorrelation func-

tion. In turn, the Fourier transform of the autocorrelation function is the power spectrum. Therefore, iso-dipole textures are also iso-power spectra textures. In the light of this realization the conjecture is equivalent to the statement that in preattentive (effortless) perception of textures the phase (spatial position) spectra are ignored. Thus texture perception is very different from figure perception for which the slight distortion of phase spectra can render the figure unrecognizable. For example, in figure perception a shift of line arrangement makes a T different from an L. But, in texture perception there is no apparent segregation between these two textural elements as shown in Fig. 2.5. If Julesz' conjecture were corroborated, it would mean that the preattentive perceptual system operates quasi-linearly, in the sense that only the simplest nonlinear decision is made.

Counterexamples to Julesz's Conjecture

Julesz and his colleagues have constructed textural patterns which have the identical first- and second-order statistics but which give strong textural discrimination. Figure 2.3 shows some examples. (a), (b), and (c) were generated by the method described in Figure 2.4. The texture shown in (d) has identical third-order statistics (hence identical second-order statistics). The small squares in the first row and middle column are selected black and white at random, while all 2×2 squares on the left of the middle column contain even numbers of black squares and all 2×2 squares on the right of the middle column contain the odd numbers of black squares.

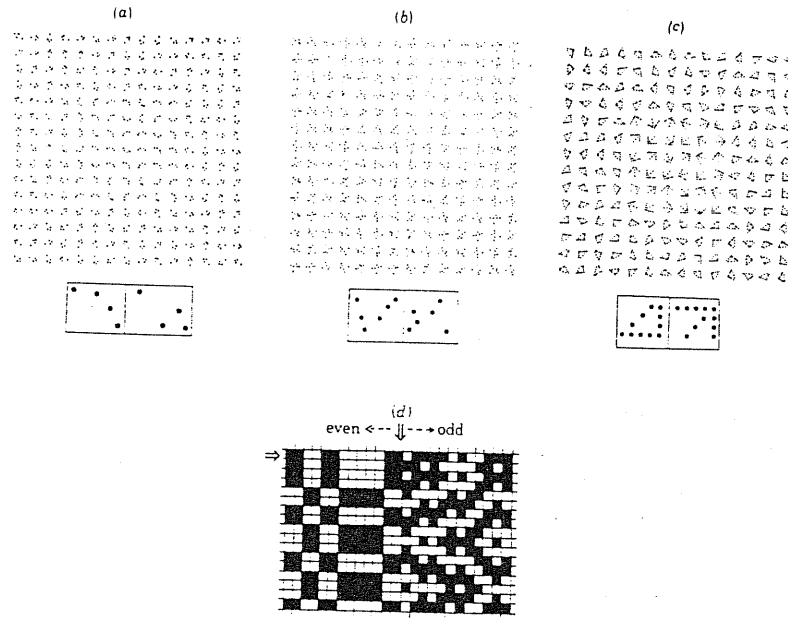


Fig. 2.3. Counterexamples to Julesz's conjecture. Discrimination is based on nonlinear local features of (a) connectivity, (b) corner, (c) closure, and (d) blobs.

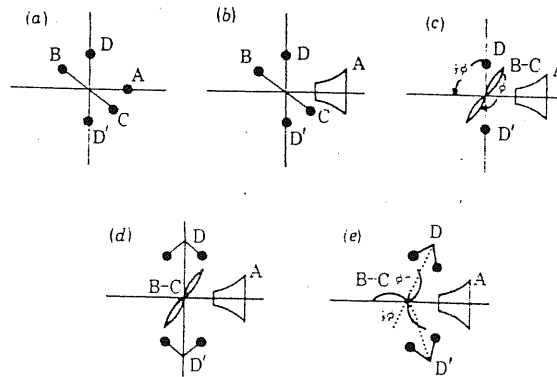


Fig. 2.4. General method for generating iso-power micro-patterns, seen as a generalization of the four-disk method by four steps. Step (b) involves the generalization of disk A to any bilaterally symmetric shape. Step (c) converts the disk B and C into 180° rotation invariant shape. Step (d) converts the disks D and D' into two shapes where each shape is invariant under reflections on the Y-axis and D' is the x-axis reflection of D. The

final step (e) demonstrates how B, C can be rotation invariant for $180^\circ/n$ rotations, while D and D' are symmetric with axes determined by $360^\circ/n$ rotations (from Caelli *et al.*, 1978).

Textons: Julesz's approach to preattentive perception

The existence of the counterexamples led Julesz to modify his conjecture. The modified conjecture proposed that the preattentive texture discrimination system cannot globally process third- and higher-order statistics, and that discrimination is the result of a few local conspicuous features, called textons. Only the first-order statistics of these textons have perceptual significance, and the relative phase between textons cannot be perceived without detailed scrutiny by focal attention.

Besides color, Julesz has so far identified three additional texton classes: (1) elongated blobs of given orientation, width and aspect ratio, (2) terminators, (3) line crossings. Corner, closure and connectivity can actually be simply described by differences in terminators. Preattentive texture discrimination is based either on the difference in the textons or the difference in the first-order statistics of the textons. In Figure 2.3 (a), (b), (c), (d), for instance, texture discrimination is based on the differences in connectivity, corner, closure, or blob, respectively.

Julesz concluded that preattentive texture perception is an early warning system which triggers the attentive perception system. If there is some discontinuity in the power spectra of adjacent areas, or there is some con-

spicuous local change in textons, the figure perception system is switched on. So the preattentive system can be regarded as the 'ground' perception system and the attentive system is the 'figure' system.

Beck's approach

Research has shown that texture discrimination occurs strongly in terms of simple properties such as brightness, color, size, and slope of texture elements (Beck, 1972), as shown in Figure 2.5.

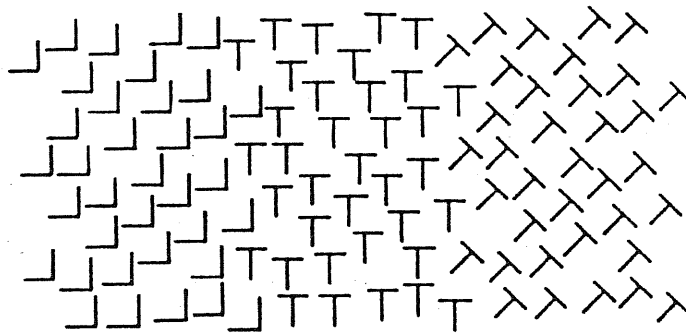


Fig. 2.5. The upright Ts are easily discriminated from the tilted Ts (a difference in line slope), but hardly discriminated from the Ls (a difference in line arrangement).

Beck (1972, 1973, 1982) hypothesized that texture discrimination is based on differences in the distribution of the slopes, sizes, colors, and brightnesses of texture elements and their parts. Discrimination occurs as a result of differences in the first-order statistics of local features rather than as a result of differences in the second-order statistics.

Beck's model

Beck's model hypothesized that the retinal intensity array is transformed into textural elements following the rules of proximity, similarity, and good continuity through the linking operations. Textural segmentation is a process operating on textural elements, the subpatterns which occur repeatedly within the texture, operated on directly in texture processing. The formation of elements is hierarchical, in the sense that elements may be features or aggregates of features. In Figure 2.5 elements are Ls upright Ts, and tilted Ts. Figure 2.6 shows the diagram of Beck's model

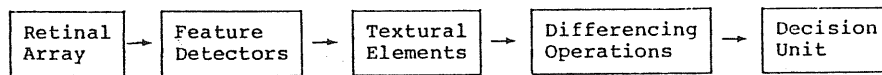


Fig. 2.6. The diagram of Beck's model.

There is an encoding of the brightness, color, size, slope, and the location of each textural element and its parts. Texture elements can differ with respect to more than a single value of a feature. For example, the upright Ts and tilted Ts in Figure 2.5 differ in two values of features (slope). The features belonging to textural elements in neighboring spatial regions are compared and the difference encoded. Difference detectors encode the total differences in brightness, color, size, and slope of texture elements in neighboring spatial regions. The response of a difference detector, such as the one for slopes, reflects the total difference between texture elements in

neighboring spatial regions.

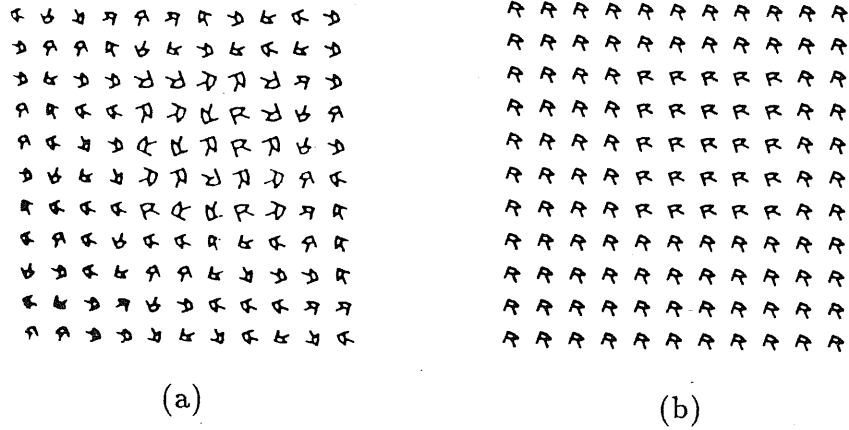


Fig. 2.7. Preattentively distinguishable textures (a) with different first-order statistics and difference in element size; (b) with different second-order statistics and different element orientation.

Not all discriminable textures have the same strength. As shown in Figure 2.7, the discrimination is stronger in the left panel than in the right panel. Beck proposed that the difference signals are proportional to the difference in activations of feature analyzers stimulated by textural elements belonging to neighboring spatial regions. It is further assumed that the difference signals are decreased by shared features that stimulate common analyzers. In a sense, similarity can be regarded as noise which reduces the discriminability. Difference signals arising from two different features, e.g., slope and brightness, summate and strengthen textural discrimination. The strength of a difference signal is a function of the size of the spatial region over which it is taken. The larger the spatial region for which a

difference signal occurs, the stronger the textural discrimination. This is illustrated in Figure 2.8. the difference signals become more concentrated in going from the top to bottom display.

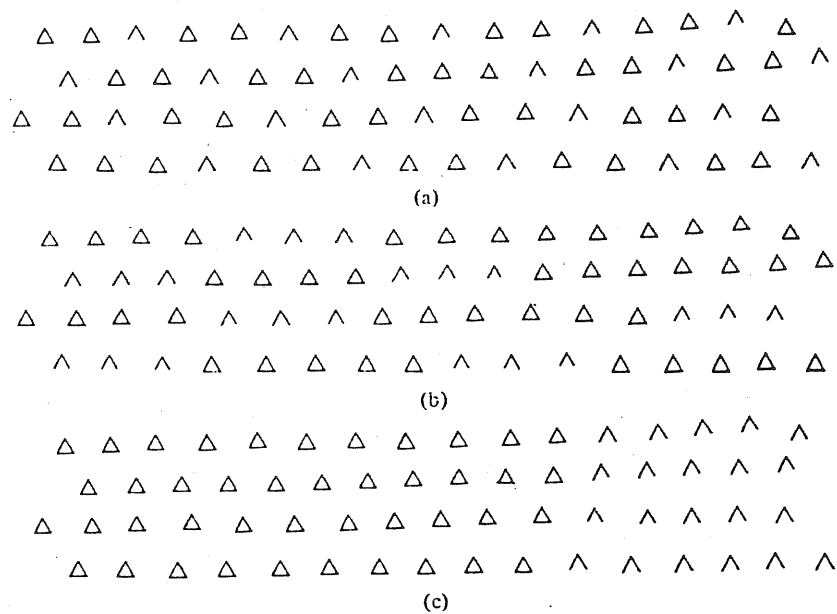


Fig.2.8. Example of how texture discrimination is a function of the size of the spatial region over which difference signals occur.

Beck assumes that there are decision units which segment a visual pattern into textural components on the basis of the magnitude and distribution of the difference signals. Textural discrimination occurs if the magnitude of difference signals between components of a visual pattern is sufficiently greater than that within each component.

Evaluations

By comparing Julesz's texton theory with Beck's hypothesis of texture discrimination, we may find that some of their views are similar. Two processes of decomposition and comparison are suggested. First, there is a decomposition of textures into more basic features in early visual processing. These features are detected automatically and in parallel. Second, texture discrimination is based on the differences in local first-order statistics of features of textural elements. When differences are large enough, segmentation occurs. Third, attention is called to the region where feature differences occur.

On the other hand, the textural elements suggested by Julesz and Beck are different. Julesz hypothesizes that textures are analyzed into elementary features which are called textons. According to Julesz, discrimination depends on the difference in texton density or identity between neighboring regions. Beck argues that stimulus features are grouped into textural elements through local linking operations, such as, grouping by proximity, similarity, or good continuation. According to Beck, discrimination depends on the feature differences between textural elements.

Several recent reports criticize the texton theory and some explanations have been proposed (Treisman & Gormican, 1988; Gurney & Browse, 1987, 1989; Bergen & Adelson, 1988). Treisman and Gormican, on the basis of their experiments on searching, argued against the preattentive detection of line-crossing and elongate blobs, two of the textons Julesz defined. They

think these are conjunctions of features, which require serial search. One example they used is the following: a vertical red bar among vertical blue bars and circular red blobs is unlikely to pop out, although each element would presumably count as a unique texton for Julesz.

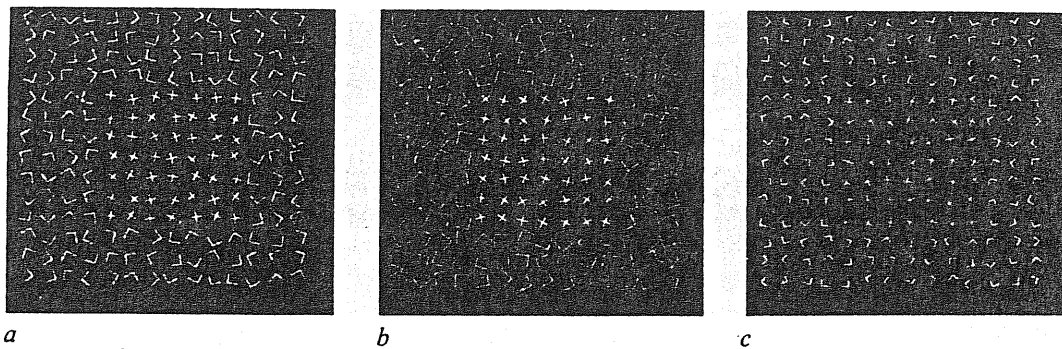


Fig. 2.9. Textures consisting of Xs within a texture composed of Ls. (a) the bars of the Xs have the same length as the bars of the Ls; (b) the bars of the Ls have been lengthened by 25%; (c) the bars of the Ls have been shortened by 25%.

Gurney and Browse (1989) claimed that textons are not necessary for texture discrimination. In their experiments on discriminability, they minimized the configurational differences between micropatterns. Only one micropattern pair with terminators, line-crossing, or line-segment differences was more discriminable than the L-T pair, which is claimed to be preattentively indiscriminable by Julesz and Beck. Thus, they hypothesize that something related to the notion of micropattern size has an effect on overall discrimination. They suggest that when two micropatterns that are

composed of the same line segments are enclosed by different circles, they will stimulate different sets of simple receptors. Similar discriminations have been studied also by Bergen and Adelson (1988). They tested the discriminability by varying the relative sizes of the Xs and +s, as shown in Fig. 2.9. They indicated in their report that lower-level mechanisms tuned for size may be sufficient to explain this discrimination. When the micropatterns produce equal responses in size-tuned mechanisms they are hard to discriminate, and when they produce different size-tuned responses they are easy to discriminate. Discriminability can be predicted without reference to more feature-like properties of the micropattern.

Bergen (1986) suggested that textures composed of Ls and +s elicited different responses from circular symmetric center-surround operators. Also, Beck (1986) showed that the responses of simple center-surround operators could form the basis for segmentation. Caelli (1985) has demonstrated computationally that L- and +- shaped micropatterns may be segmented on the basis of the responses of simple orientation selective filters. The similar ideas of center-surround operators or orientation selective filters are shared by other scientists (Marr & Hildreth, 1980; Fleet, Hallett & Jepson, 1985; Pollen & Ronner, 1983; Bergen & Julesz, 1983).

Recent results have shown that texture discrimination is an asymmetrical process, e.g., texture A within texture B may be much easier to discriminate than texture B within texture A (Julesz, 1981; Beck, 1982; Treisman & Souther, 1985; Enns, 1988; Treisman & Gormican, 1988; Gurnsey & Browse, 1989). An example is shown in Fig. 2.10. The region of Ls

embedded in +s is easier to discriminate than the region of +s embedded in Ls.

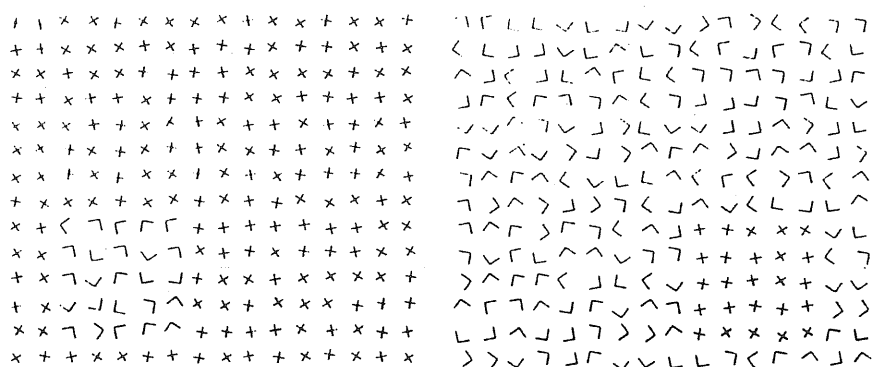


Fig. 2.10. (a) Discrimination of Ls embedded in a background of Xs. (b) Discrimination of Xs embedded in a background of Ls.

The asymmetry of discriminability cannot be explained by Julesz and Beck's hypotheses presented before. According to their hypotheses, discriminability is a function of the differences between two neighboring regions. In this case, asymmetry is not expected.

One explanation of such kind of asymmetry is "more is better" (Beck, 1982; Treisman & Gormican, 1988). It is supposed that a target stimulus having greater magnitude on some quantitative dimension (e.g., size) than the background distractors will be more easily detected than a weak stimulus in a background of strong distractors, where weak and strong refer

to the magnitude of the stimulus on some quantitative dimensions. The asymmetry shown in Fig.2.10 may due to the size difference, because L and + are enclosed by circles of different sizes.

The effect of organization was also considered and found to contribute to the asymmetry. Two textures differing only in the arrangement of identical micropatterns also elicit asymmetry discrimination. The irregular texture is more easier to discriminate. Fig. 2.11 shows such kind of asymmetry.

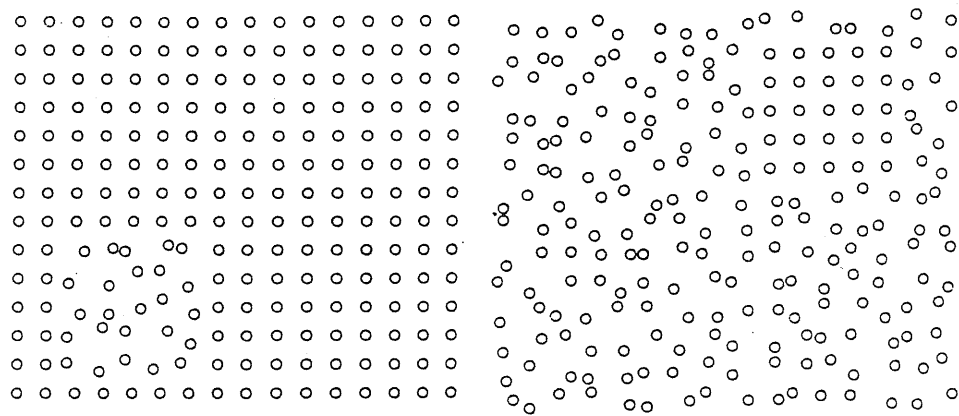


Fig. 2.11. The left panel shows irregularly placed circles embedded in regularity and the right panel shows regularly placed circles embedded in irregularity.

There are many computational approaches to texture discrimination (Caelli, 1985; Gurnsey & Browse, 1987, 1989; Malik & Perona, 1989; Laws, 1980; Levine, 1985; Field & Sagi, 1989; Kube, 1988; Poggio, 1988). Some of them involve nonlocal computations. They suggest that nonlocal factors play a role in texture discrimination. Details will not be presented here.

After all, which stimulus properties mediate texture discrimination is still in discussing and the nature of the discrimination is not yet completely explained.

Chapter 3

Pattern Perception

Pattern perception is one important branch in vision research. Though there has not been a strict definition yet, one thing is certain. A pattern has some extent of regularity; i.e. constituent elements are more or less in order. There are several ways of expressing this concept. One can say that patterns have structure, or internal organization; or that pattern elements are correlated.

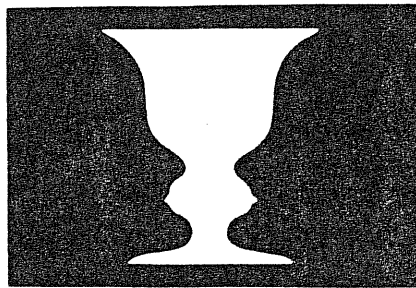


Fig.3.1. Both two profiles and a vase can be seen alternatively.

The overwhelming of information in our visual environment is proverbial. A fundamental process of visual perception is the extraction of information useful for separating the figure from the ground. Certain patterns tend to be seen as the thing, while the rest is perceived as the background. In essence, we divide the world into two categories; into something we shift our attention to, and into remaining items that form the not-thing and

become ignored. In some instances, figure-ground organization is bistable, as shown in Fig. 3.1, in which perception alternates between a white vase on a black background or two black profiles on a white background.

Factors influencing figure-ground organization have been studied by Rubin (1915) and other psychologists following the Gestalt tradition (Koffka, 1935; Kanizsa, 1979). Figure-ground organization is influenced, among other things, by stimulus factors like relative area, closure and symmetry.

Relative area. As shown in Fig. 3.2 (a), the smaller a region is the more it tends to be seen as figure; and conversely, the larger the area of a region is the more it appears to be the ground.

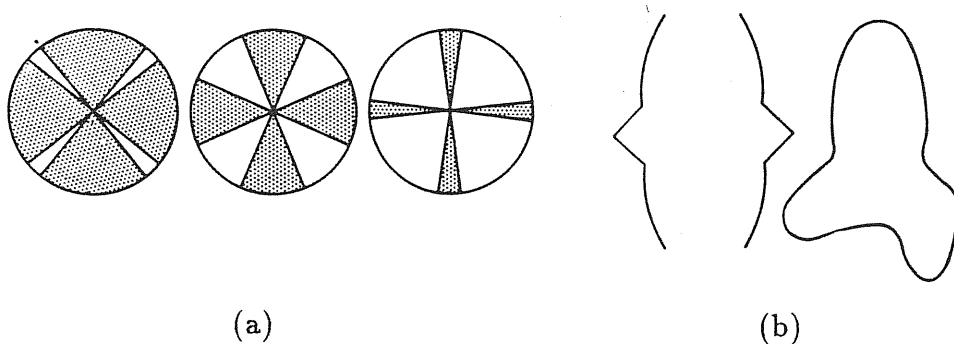


Fig. 3.2. Demonstrations showing (a) how the smaller area tends to become the figure; (b) how closure enhance figure perception.

Closure. Areas with closed contours are more likely to be seen as

figures than areas with open contours are, as shown in Fig. 3.2 (b).

Symmetry. The more symmetrical a region is the more likely it is to appear as the figure. The greater the amount of symmetry, the stronger is this tendency.

Pattern Simplicity

In principle, a pattern can be interpreted in many ways. Usually, only one interpretation is preferred. Even in the rather simple line drawing in Fig.3.3, several different interpretations are possible, but the preferred interpretation will be two squares.

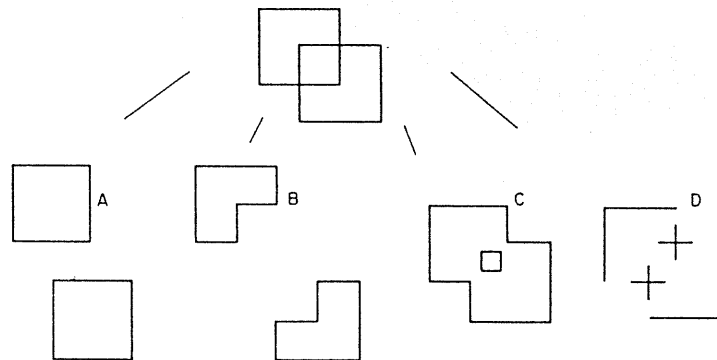


Fig. 3.3. Interpretations

In order to be able to predict such preference, scientists in shape perception have searched for the underlying principles that govern the human

interpretation of patterns.

Some scientists (e.g. von Helmholtz, 1867; Gregory, 1973), advocate the likelihood principle, which states that the preferred interpretation of a pattern is the one which reflects the most probable situation. Others, like the Gestaltists, advocate the minimum principle, which states that human pattern perception is guided by simplicity. For instance, grouping would occur according to proximity, similarity, continuity, and common fate (Wertheimer, 1912; Köhler, 1920).

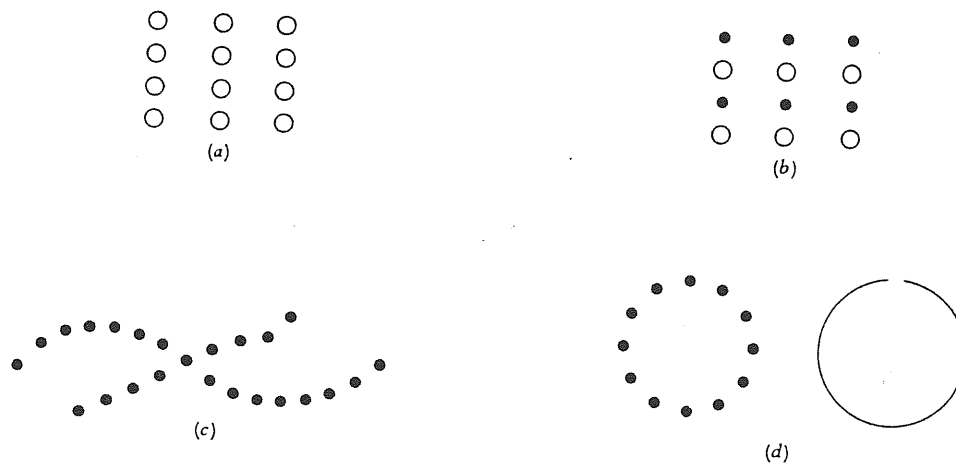


Fig. 3.4. The Gestalt laws in pattern organization. (a) the role of proximity; (b) the role of similarity; (c) the role of continuity; (d) the role of closure.

Proximity. Spatially adjacent elements are grouped, as shown in Fig. 3.4 (a).

Similarity. Elements of similar brightness, color, and shape are grouped, as shown in Fig. 3.4 (b).

Continuation. Among many possible perceptual interpretations those that will minimize changes or interruptions in the contours of the constituents will be perceived as figures, as shown in Fig. 3.4 (c).

Common fate. Elements which move or change together are seen as a unit or with a common fate.

Closure. Elements are grouped according to closure, as shown in Fig. 3.4 (d).

These Gestalt laws have great heuristic value, but it is impossible to find out a consistent priority order for them, i.e, sometimes one law is stronger, sometimes another law. Koffka (1935) suggested that all laws are specifications of a general minimum principle, the tendency towards *Prägnanz*.

Hochberg & McAlister (1953) and Leeuwenberg (1969, 1971) suggest that the figure *Prägnanz* maybe reduced to figure simplicity, which implies that perceptual interpretation process is guided by the minimum principle. The preferred interpretation of a pattern is reflected by the simplest description of that pattern. A pattern description can be seen as the formal counterpart of the way in which a pattern is represented by the perceptual system.

Many researches have been done to specify this minimum principle more exactly. Some scientists (Simon & Feigenbaum, 1964; Attneave, 1982; Biederman, 1987) regard simplicity as an internal aspect of the perceptual process. The idea of procedural simplicity implies that the preferred interpretation of a pattern corresponds to the pattern description generated from the most efficient procedure within some process model. Better in line with the Gestalt tradition, other scientists (e.g. Hochberg & McAlister, 1953) regard simplicity as being based on properties of a pattern itself. This starting point of phenomenal simplicity implies that the preferred interpretation of a pattern is thought to be reflected by the pattern description that expresses the largest amount of regularity in the pattern.

The transformational approach suggests, in different way, that the perceptual system reveals regularity in pattern by means of a fixed set of pattern transformations such as translation, rotation, and reflection. Regularity is specified as being constituted by certain arrangements of identical pattern parts. Garner (1970) takes the number of such invariant transformations, as allowed by a pattern, as a measure for the figure goodness of that pattern. Palmer (1983), on the other hand, describes pattern by means of a network in which pattern parts and their properties (revealed by invariant transformations) are stored and related to one another. Palmer suggests that the preferred interpretation is reflected by the "best" reference frame: each frame allows specific transformations, and the "best" frame is the one that reveals a maximum of symmetry in the pattern. Leyton (1986a, b) proposes a criterion for the internal structure of pattern descriptions which are formulated directly in terms of reference frames and invariant transformations. This criterion enables an explicit specification of perceptual

differences between possible interpretations of a pattern. Leyton, by the way, defines a description as a mapping from a set of transformations onto a pattern, i.e. the pattern is not converted into some description but just has to be reconstructable from that set of transformations.

In the encoding approach, descriptions of a pattern are obtained on the basis of a symbolic representation of the pattern, in which identical symbols represent identical pattern parts. By means of coding rules, the symbolic representation is reduced into a code. The encoding approach starts with Hochberg & McAlister (1953), who proposed a simple method to measure the complexity of 2- and 3-dimensional patterns. Later, more intricate coding systems have proposed. These systems are applied to the encoding of visual pattern (Vitz, 1966; Leeuwenberg, 1969, 1971; Restle, 1979).

Structural Information Theory

Leeuwenberg (1969) proposed hypothesis that pattern descriptions are meant to express regularity, where regularity is specified as being constituted by certain arrangements of identical pattern parts and is thought to be revealed by the perceptual system. His coding system is based on coding rules which prescribe the combining of single identities, and codes express the allowed combinations. On the basis of such combinations, the human system is thought to extract information. Leeuwenberg embodied the minimum principle in the coding rules, and claimed that preferred interpretation of a pattern is reflected by the simplest description of that

pattern.

By means of Leeuwenberg's coding approach (Leeuwenberg, 1968, 1971, 1989), pattern interpretations can be represented in a pattern code. The general procedure runs as follows. A pattern is first represented by a symbol series. Each symbol corresponds to an element of the pattern. For instance, in Fig. 3.5 the contour of the pattern consists of subsequent angles and line segments, each of which is labeled with a symbol, called pattern symbol, and angles or lines segments of equal size are labeled with an identical symbol. This pattern is represented by the symbol series *kalckalckalc*. Next, the symbol series is encoded into pattern code.

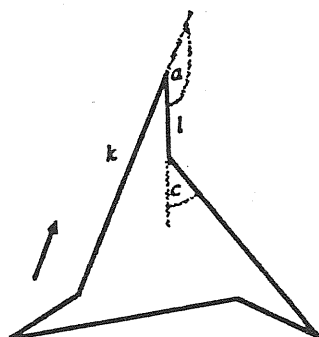


Fig. 3.5. Tracing the contour of the pattern, yields the symbol series 'kalckalckalc'. This symbol series represents the subsequent angles and line segments in the contour so that the pattern can be reconstructed.

The encoding consists of describing regularity in the symbol series which corresponds to regularity of the pattern. The symbol series is encoded as far as possible by applying onto it a number of coding rules. By means of these coding rules all redundant information is eliminated from the symbol

series so that after this encoding process the pattern is represented by a code which only contains information about the essential pattern elements and their interrelationships.

Coding rules

Leeuwenberg's structural information theory provides the concept of accessibility for the choice of appropriate coding rules. Regularity and hierarchy in a code of a pattern should correspond directly to regularity and hierarchy in the pattern. The criterion of accessibility is that a coding rule should be both holographic and transparent in order to be appropriate for the encoding of a symbol series that represents a visual pattern. This enables a differentiation between coding rules, on the basis of both regularity and hierarchy as described by coding rules. The coding rules that are in line with the concept of accessibility, not only account for an easy extraction of information from the pattern in order to construct a pattern code, but also account for an easy extraction, at higher cognitive levels, of a pattern information from the code. Based on the concept of accessibility there are three essential coding rules in Leeuwenberg's coding approach, each of which describes a specific class of regularity, namely iteration, symmetry, and alternation (ISA). The definitions of these rules are as follow:

Iteration rule:

$$kk \dots kk \rightarrow N * (k)$$

it is applied to express the series contains successive identical symbols.

Symmetry rule:

$$k_1 k_2 \cdots k_n p k_n \cdots k_2 k_1 \rightarrow S\langle(k_1)(k_2)\dots(k_n), (p)\rangle$$

it can be applied to express that a series contains pairs of identical symbols, nested around a so-called pivot.

Alternation rule:

$$k x_1 k x_2 \cdots k x_n \rightarrow \langle k \rangle / \langle (x_1)(x_2) \cdots (x_n) \rangle$$

$$x_1 k x_2 k \cdots x_n k \rightarrow \langle (x_1)(x_2) \cdots (x_n) \rangle / \langle k \rangle$$

it can be applied to express that a series contains successive subseries which either all begin or all end identically. According to these coding rules the pattern shown in Fig. 3.5 can be represented by the code $3 * (kalc)$.

Three aspects should be noted in applying the coding rules. First, the symbols are considered to be variables standing for arbitrary subseries. This implies that the coding rules can be applied not just to express identity of single symbols but, in general, to express identity of subseries in a series. For instance, $ababab \rightarrow 3 * (ab)$. Any subseries between parentheses in an ISA-form is called a chunk. Secondly, a code of a symbol series does not have to consist of just one ISA-form. In general, a code is a series consisting of single symbols and ISA-forms obtained by applying the coding rules to subseries of the symbol series. For instance: $akpkpfrstsrq \rightarrow a2 * (kp)fS\langle(r)(s), (t)\rangle q$. Thirdly, the subseries inside a chunk in an ISA-form can be encoded just like any symbol series. For instance: $bapabapa \rightarrow 2 * (bapa) \rightarrow 2 * (bS\langle(a), (p)\rangle)$. In such a case, the ISA-form is said to be hierarchically nested.

Minimum Principle

Leeuwenberg argues that a code provides a description of regularity in that series. The ultimate meaning of the code is constituted by the fact that a code provides a mean to obtain a classification and an organization of the series which is considered to reflect an interpretation of the pattern that is represented by the symbol series.

Because patterns can be partitioned into different groups of elements, and because coding rules can be applied to these sets of elements in different ways (e.g. by choosing different starting points or by applying the coding rules in different orders), different codes, representing different pattern interpretations, can be arrived at. Leeuwenberg coding approach uses the quantity structural information to measure the preference of different pattern interpretations. Structural information-load is defined as the number of pattern symbols in a code of a series plus the number of I-forms and S-forms in code. For instance, in Figure 3.4, structural information load about the interpretation we given is 5. Minimum code is a code that contains a minimum amount of structural information load. The preferred interpretation of a pattern is reflected by a minimum code of a symbol series that represent the pattern. As refer to Fig. 3.3, it is clear that interpretation (a) has the minimum code and it is true that this interpretation is preferred in perception.

Experimental validity of Leeuwenberg's structural information theory is controversial. There are evidences against this theory and there are

evidences support this theory.

Chapter 4

Symmetry Perception

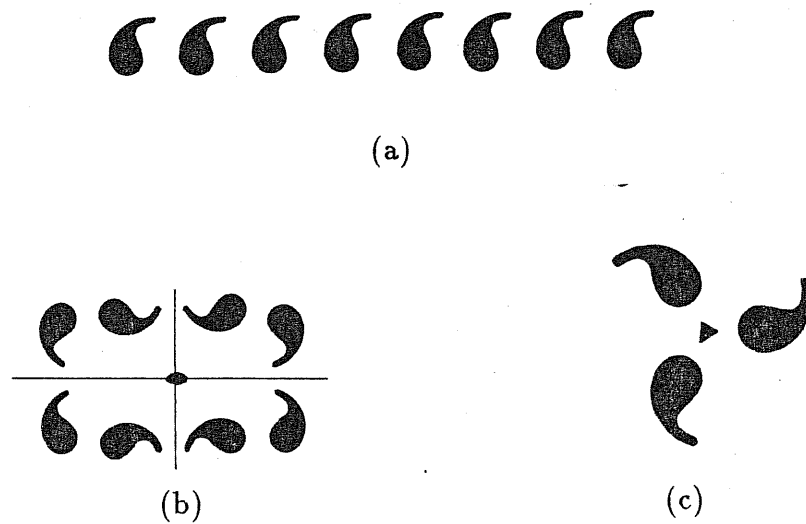


Fig. 4.1. Three basic forms of symmetry: (a) translation; (b) reflection; (c) rotation.

Symmetry is commonly classified as a simple harmony of proportions, of balance, of which there are three basic forms: (1) *translation*, in which the element moves up or down, left or right, or diagonally while keeping the orientation; (2) *reflection*, in which the element reflects as in a mirror; and (3) *rotation*, in which the element turns. Further more these forms can be combined to form the regular patterns. The general form of symmetry, however, requires the invariance of a configuration of elements under a

group of automorphic transformations (Weyl, 1952). In other words, symmetries are one of the simplest invariances of patterns that preserve their identities under certain specific transformations.

Symmetry is a very salient global property of pattern. Our visual system can perceive it efficiently and rapidly. Mach (1886) was the first one to draw attention to the importance of symmetry for our understanding of the visual processes and visual system. Symmetry reduces pattern complexity and is important in form perception. Perception of symmetry also affects other visual processes; e.g., encoding and representation (e.g., Garner & Sutliff, 1974; Enns, 1987; Howe, Powell, Jung, & Brandan, 1989; Leeuwenberg, 1988; Freyd & Trersky, 1984), recall, discrimination, and establishment of a reference frame.

Many researches have been done in various aspects of symmetry perception. Effects on symmetry detection and variable mediating symmetry detection have been studied. Several models or approaches have been suggested to account for experimental findings.

Effects and Explanations

Preference in detection

The most consistent finding about the perception of symmetry is that not all types of symmetry are equally salient. Symmetry about the vertical axis is the most salient one and is detected most quickly (e.g., Julesz, 1972;

Corballis & Reldan, 1975; Goldmeier, 1972; Palmer & Hemenway, 1978; Palsher, 1990). The order of horizontal and diagonal axes is debatable. One finding is that the horizontal is more salient than the diagonal (Palmer & Hemenway, 1978; Goldmeier, 1972). The other is that diagonal is more salient than the horizontal (Corballis & Roldan, 1975). Multiple symmetry is detected faster than single symmetry (Palmer & Hemenway, 1978).

Mach (1886) thought that the special salience of vertical symmetry was due to the structural bilateral symmetry of the visual system and that perception of horizontal symmetry was accomplished by mentally rotating the figure or by an intellectual act. This means that shape might be mentally rotated, or normalized before information about symmetry is extracted. This hypothesis is supported by Shepard and Metzler's (1971) experiment on similarity judgment, Corballis and Roldan's (1975) experiment on symmetry detecting in eight different orientations in 45 degrees steps. Decision times increased as the angle between the axis and the vertical axis increased. Corballis and Roldan also found that tilting the head shifted the decision time function in the direction of head tilts, suggesting that retinal coordinates are more important than gravitational ones. They concluded that the retinal information is mapped on the phenomenal coordinate system. Information can be rotated mentally on the phenomenal coordinates to test the information against a template for detecting vertical symmetry embedded symmetrically in the brain.

Palmer and Hemenway suggested a two stage model: (1) the observer selects a potential axis by a crude but rapid analysis of symmetry in all

orientations simultaneously; (2) if a given axis meets the selection criterion, a perceptual reference frame is established in the appropriate orientation. The observer then performs a detailed evaluation of symmetry about the selected axis by explicitly comparing the two halves for mirror-identity. The selection process is biased toward vertical and, to a less extent, horizontal rather than diagonal axes. Thus, symmetry about the vertical axis is easiest to detect, followed by horizontal and diagonal. To explain the effect that multiple symmetry is easier to detect than single, they suggested further that the order of the selection is variable. Otherwise, if the order were fixed, the preference for multiple symmetry should not occur. If the order were fixed, a figure with both vertical and horizontal symmetry should be detected as fast as a figure with only vertical symmetry, because vertical symmetry is detected before horizontal.

Attention to brain structure as a basis for the saliency of vertical symmetry is also presented in Julesz's work (1971). Julesz suggested that perception of symmetry requires a point-by-point comparison process based on neural anatomy that has a symmetrical organization around the center of the fovea.

Rock and Leaman (1963) argued against a simple structural explanation for the salience of vertical symmetry, on the ground that the advantage of vertical symmetry is not a matter of retinal orientation; e.g., if the observer tilts his head through 45 deg, a figure with true vertical symmetry is still more salient than a figure with true horizontal symmetry, even though both figures are equally tilted on the retina. Rock and Leaman suggest

that we have become sensitized to vertical symmetry simply because it is so common a characteristic of the environment. A great many objects, both natural and manmade, exhibit symmetry about the vertical. This sensitization, they note, could have come about either through learning or as a consequence of biological evolution. However, one could also interpret Rock and Leaman's data as further evidence for a process of mental rotation which normalizes the input before symmetry is perceived. Consequently, it is still possible to maintain that the perception of symmetry could depend on the structural symmetry of the nervous system.

There are preferences for detection of different kinds of symmetry. Julesz (1971) has shown examples which indicated that reflection symmetry is easiest to be perceived from complex patterns, followed by repetition (translation) and rotation symmetry.

Corballis and Roldan (1974) investigated rapid perceptual judgments about tachistoscopically presented patterns that were either symmetrical about or repeated across a vertical axis. Based on their results, they suggest that if the pattern are perceived holistically, reflection symmetry is more salient than repetition, but if they are perceived as two separate figures to be matched, then repetition is judged more rapidly than reflection symmetry. It is still conceivable that the perception of symmetry may depend on a point-by-point comparison between symmetrical regions of the two hemispheres.

Redundancy in Symmetrical Displays

By studying visual perception we can reveal how the brain processes information. For instance, one can distinguish between serial versus parallel processes; or study how the brain selects the relevant information. A general assumption regards the idea that visual information is redundant.

It has been claimed that the perception of bilateral symmetry in dot textures reflects the existence of processes which reduce redundancy in the image. Barlow and Reeves (1979) have suggested that the organism reconstructs an image on the basis of minimal information. They point out that one advantage of symmetry is that it allows the image to be described economically. For instance, if one half of an object is the mirror image of the other half, then one half need not to be described at all. The redundancy reduction in the perception of bilateral symmetry is achieved by ignoring the reflected half of the symmetrical pattern.

Julesz (1971) proposed that redundancy reduction is achieved by giving relatively greater weight to point close to the axis of symmetry. As he noted that, "... This point-by-point symmetrical representation is strongly weighted in favor of areas close to the axes of symmetry."

Jenkins (1982) studied the perception of bilateral symmetry in dynamic dot textures. He found that not all of the symmetry information available in a symmetrical texture is utilized by the visual system. The symmetry information utilized by the visual system fall within a stripe ap-

proximately 1 deg wide about the central axis of symmetry, irrespective of the retinal size of the texture. There is an increasingly efficient utilization of symmetry information from the outer boundary of the 1 deg stripe inwards, until, at .3 deg the symmetry information is utilized maximally. Outside the stripe, the symmetry information was found to be completely redundant. Barlow and Reeves found that symmetry is best detected when next to the axis, worst when in the middle of each half figure and higher again when lie near the edge of each half. These findings support Julesz's proposal.

According to known mammalian neurophysiology, Jenkins thought that a bilaterally symmetrical dot texture can also be described as a two-dimensional distribution of uniform orientation point-pair elements, of nonuniform size, which fall across the same axis evenly such that the uniformly oriented pairs have collinear midpoints. Jenkins studied such kind of bilateral symmetry perception. Based on his experimental results, he argued against the necessity to postulate the existence of a symmetrical neural organization centered about the fovea. He proposed that there are three processes involved in the perception of bilaterally symmetric dot textures: the detection of orientational uniformity of the different sized point-pair elements; the fusion of salient element point-pair into a salient feature; the detection of the symmetry of the resulting feature. The most likely candidates for the constituent elements in fusional process are the smaller, most salient, least redundant individual point-pair identified by Jenkins (1982) as subtending approximately 0.3 arc. There are interactions among these three processes.

Jenkins indicated additionally that the orientational uniformity process is most sensitive to vertical symmetry. The fusional process is indifferent to orientation whereas the detection of symmetry process seems to prefer the vertical axis.

Chapter 5

Experiment

Introduction

When a homogeneous texture is reflected around a vertical axis, a symmetrical texture is generated, as shown in Fig. 5.1. However, mirror symmetry disrupts texture homogeneity. In Fig.5.1, a large scale rectangle is formed by reflecting the square. It is easy to observe that the homogeneity is disrupted around the symmetrical axis due to the occurrence of large

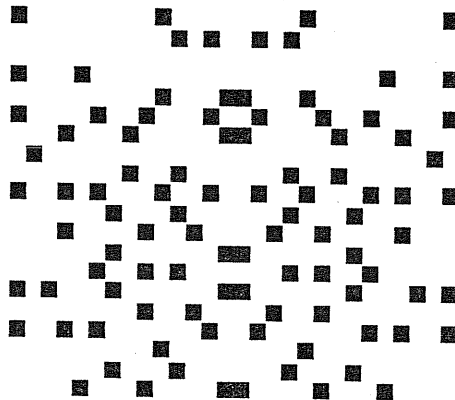


Fig. 5.1. An example of bilateral symmetry pattern got by reflecting the homogeneously distributed texture.

scale elements. Does heterogeneity along the symmetrical axis play a role in symmetry detection? This is the prime purpose of our experiments.

In our experiments two sets of symmetrical textures were tested and compared. The textures of both sets were generated by reflection around a vertical axis. In set A, the distribution of element sizes and shapes was heterogeneous. In set B, the distribution of element sizes and shapes in the original half-texture was heterogeneous, and designed to generate a homogeneous final texture. The sensitivity and the reaction time of symmetry detection were measured. The Signal Detection Theory (Appendix 1) was used in estimating the sensitivity. The t-Test (Appendix 2) method was used in statistical analysis.

Method

Experiment 1

Two sets of line textures were used in this experiment. Each set had both symmetrical textures and noise textures. The textures were $8.1\text{cm} \times 5.6\text{cm}$ and were shown on the computer screen which is $24\text{cm} \times 16\text{cm}$. Subjects sat viewing the screen binocularly at a distance about 80cm. The visual angle was about 5 deg. Stimulus luminance was kept constant through all the experiments.

Stimulus generation

Textures were generated by a computer plot program. We generated two sets of symmetrical patterns and two sets of noise (non symmetrical) patterns. The basic elements were lines 2.5mm or 5mm long. In set A some lines 10mm long could occur along the symmetry axis, as a consequence of reflection of 5mm lines. Some 5mm lines could occur after reflection of 2.5mm lines.

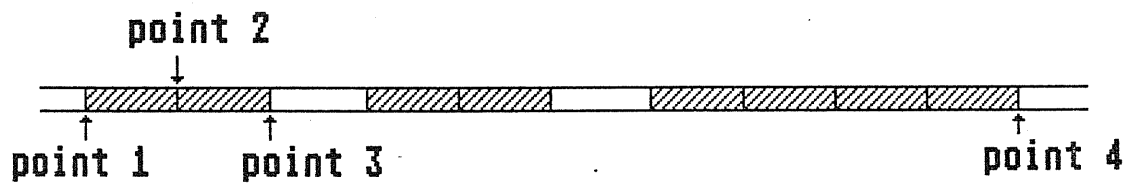


Fig. 5.2. Four possible cut points

There were 25 rows in each texture. For randomization purposes, textures were programmed as cylinders made of 25 circles. The elements in these circles were lines 2.5mm long and lines 5mm long. Lines were separated by a variable white space and randomly positioned by the computer, following a uniform distribution. The ratio of the black to white space was one to three.

Symmetrical textures: set A

As shown in Figure 5.2, there are four possible cut points: Point1, point2, point3 and point4. Each circle was cut at one of these points and then unfolded. The resulting stripe was reflected around a vertical axis in

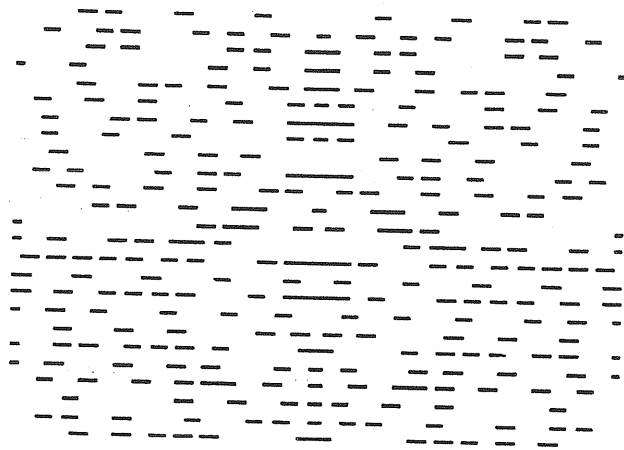


Fig. 5.3. An example of symmetry texture set A in experiment 1.
Set A is composed of heterogeneous textures.

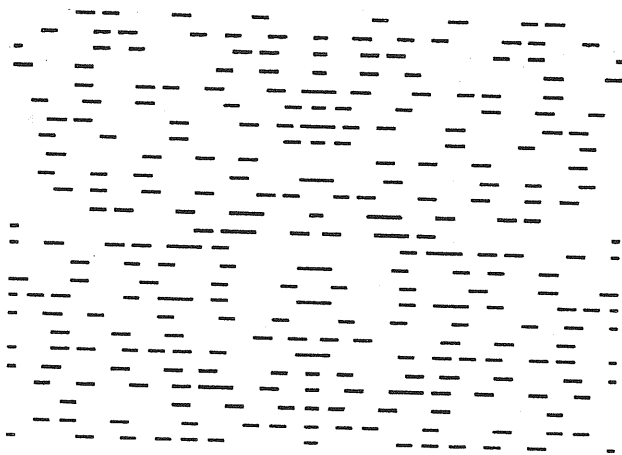


Fig. 5.4. An example of symmetry texture set B in experiment 1.
Set B is composed of homogeneous textures.

the right direction. This procedure produces heterogeneous textures made of black lines on a white background; lines along the symmetry axis could be 2.5mm, 5mm or 10mm long; lines far from the symmetry axis could be 2.5mm or 5mm long. Figure 5.3 shows this kind of texture.

Symmetrical textures: set B

Starting from the same basic circles used to generate set A, stripes were obtained by cutting at point 3 the circles those cut at point4 in set A, and at point2 those cut at point3 in set A. The remaining part of the texture was the same as in texture set A. Textures of set B were completely homogeneous. Figure 5.4 shows this kind of texture.

Noise textures: set A

To generate noise textures, each row was randomly shifted rightwards or leftwards in a range of 0 to 4mm. Noise textures were not completely uncorrelated, because all 10mm lines were not shifted in order to avoid the possibility that subject may detect the position of these elements instead of detecting symmetry. Figure 5.5 shows this kind of texture.

Noise textures: set B

Each row of the texture were shifted as in set A. The 5mm lines along the axis were not shifted, in analogy with 10mm lines of set A. Figure 5.6

shows this kind of texture.

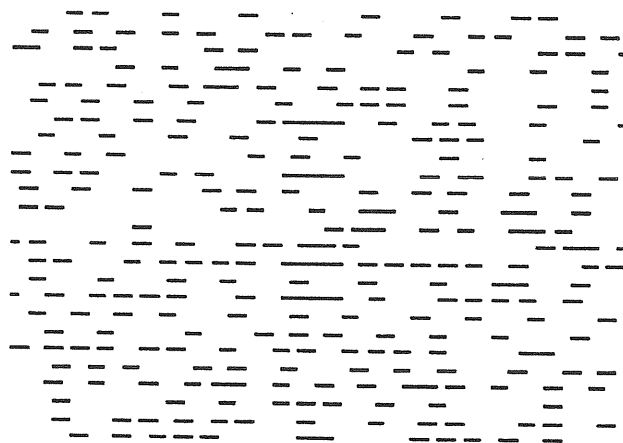


Fig. 5.5. An example of noise texture set A in experiment 1.
Set A is composed of heterogeneous textures.

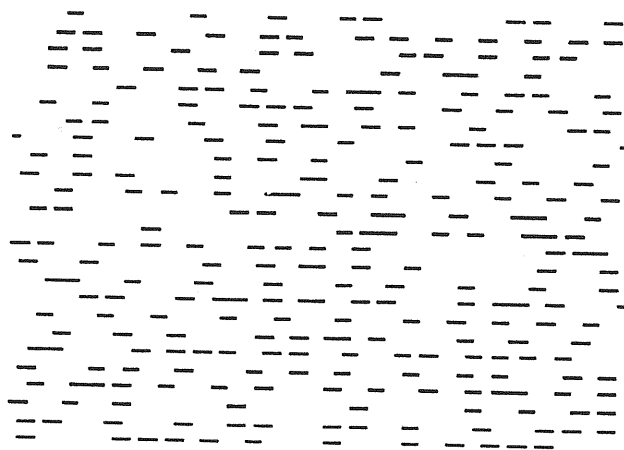


Fig. 5.6. An example of noise texture set B in experiment 1.
Set B is composed of homogeneous textures.

Subjects

Sixteen subjects participated in this experiment. Most of them were college students. All of them were naive with respect to the aim of the experiment.

Procedure

The experiment was composed of a training session and a test session. There were 80 trials in the test session, 20 trials of each set of either symmetrical textures or noise textures. The order of the trials was random. A training session contained a minimum number of 20 trials. Further training trials were presented, until the subject scored the critical level of 70% correct.

To initiate a trial, the subject pressed the "START" key whenever he/she was ready. A fixation point, located on the symmetry axis, was on during the intertrial interval. The stimulus texture was shown on the screen immediately after the subject pressed the "START" key and the exposure time was 500ms. The subject was instructed to indicate whether a texture was symmetrical or not by pressing the key "YES" or "NO". Keys corresponding to YES and NO were chosen by the subject freely from the keyboard at the beginning of the experiment. The subject was told to press the "YES" or "NO" as soon as he/she decided the texture was symmetrical or not. Responses and reaction times were registered by the computer.

Result

The time limit of 2 sec was chosen. Slower responses were eliminated from the data analysis.

Table 5.1: Sensitivity and reaction time of experiment 1

	$zP'(A)$ set A	$zP'(A)$ set B	RT set A	RT set B
	heterogeneity	homogeneity	heterogeneity	homogeneity
1	1.158	1.250	845	807
2	1.056	.947	1026	957
3	.983	1.174	1205	1256
4	3.090	1.863	1275	1136
5	1.419	1.217	1128	1148
6	1.915	1.628	1261	1262
7	.871	1.111	1107	1188
8	.998	.456	1223	1166
9	1.056	.947	858	892
10	3.090	1.863	1130	1125
11	1.915	1.628	1125	1155
12	.998	.456	1341	1332
13	1.915	1.458	1295	1350
14	.60	.637	688	666
15	1.579	1.748	1042	1050
16	.180	.973	1046	1037
average	1.489	1.210	1100	1095

Data were analysed according to SDT (Appendix 1). The sensitivity measure was $zP'(A)$. Each subject contributed with a pair of PHit and PF.A.

values. Table 5.1 shows the sensitivity measure and RT for the whole group.

Among the 16 subjects, 5 were more sensitive to set B (homogeneous textures) and 11 of them were more sensitive to set A (heterogeneous textures). There was no apparent difference in RTs in the two conditions. Sensitivities in the two conditions were compared by the t -test (Appendix 2).

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_x^2}{N} + \frac{\sigma_y^2}{N}}}$$

The results of the t -test is shown in table 5.2.

Table 5.2: Paired t -test

X: $zP'(A)$		Y: $zP'(A)$ set B	
DF	Mean X-Y	Paired t value	Prob. (2-tail)
15	.28	2.518	.0236

* Set A: heterogeneous textures; Set B: homogeneous textures.

The advantage due to heterogeneity around the axis is significant, $t=2.518$, ($p < 0.025$). This indicates that symmetry detection was better in the heterogeneous textures.

Experiment 2

Two sets of triangle textures were used in this experiment. Each set had both symmetrical and noise textures. The size of the stimulus texture was the same as those used in experiment 1. The logic of experiment 2 was identical to the logic of experiment 1. Shape homogeneity was the independent variable, instead of length homogeneity.

Stimulus generation

Symmetrical textures: set A

Isosceles right-angle triangles were generated in pairs on the computer screen. The middle points of these pairs were on the vertical symmetrical axis. The size of the pairs and the orientations of the triangles were random. Two triangles connected with their sides were allowed. The other configurations of the triangles were eliminated. Therefore, texture elements were small and large triangles in different orientations. On the symmetrical axis only the elements ▲ , ▼ , and ◆ were allowed to appear. The position of these elements were random and the probability was controlled. Figure 5.7 shows this kind of texture.

Symmetrical textures: set B

In set B all the parts of the texture were the same as in the texture

set A except of the elements on the symmetrical axis. The elements \blacktriangle , \blacktriangledown in texture set A were unchanged and the element \blacklozenge in texture set A were replaced by two elements \blacktriangle or \blacktriangledown decided by computer and were positioned randomly along the axis. Figure 5.8 shows this kind of texture.

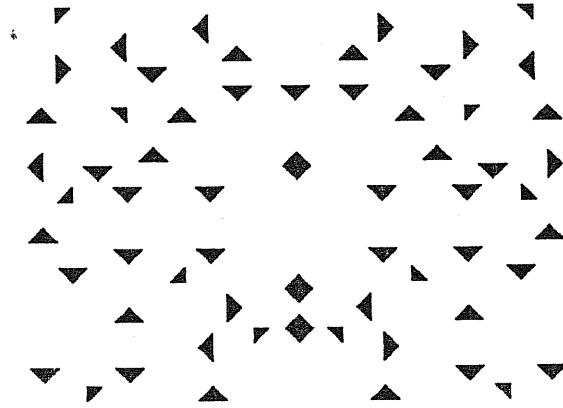


Fig. 5.7. An example of symmetry texture set A in experiment 2.
Set A is composed of heterogeneous textures.

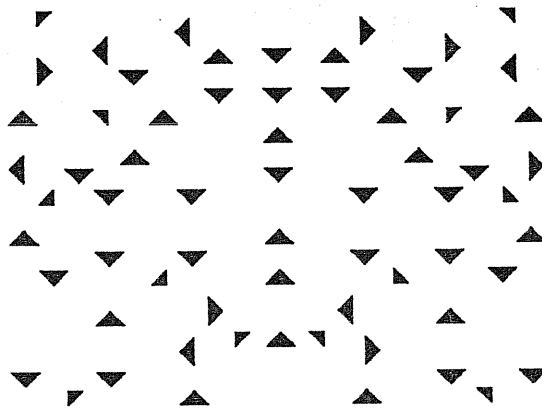


Fig. 5.8. An example of symmetry texture set B in experiment 2.
Set B is composed of homogeneous textures.

Thus we had two sets of stimulus texture which had differences only along the symmetrical axis. The distribution of element shapes was heterogeneous in texture set A and homogeneous in texture set B.

Noise textures: Set A and Set B

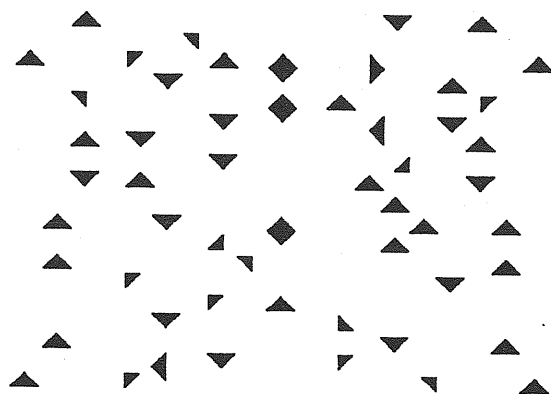


Fig. 5.9. An example of noise texture set A in experiment 2.
Set A is composed of heterogeneous textures.

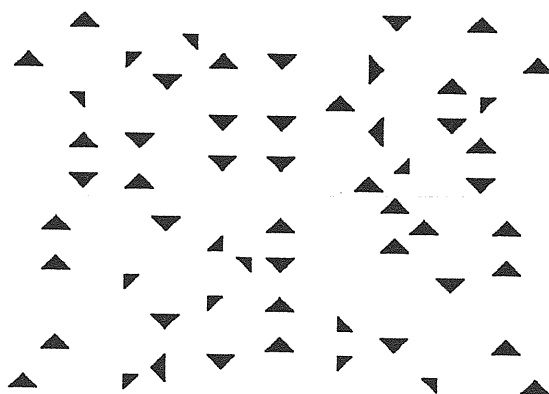


Fig. 5.10 An example of noise texture set B in experiment 2.
Set B is composed of homogeneous textures.

The elements allowed to appear in the noise texture were the same as those in the symmetrical texture. The elements on the central axis in the noise texture were the same as those on the symmetrical axis in the symmetrical texture. Elements on the boundary were generated in pairs, thus the noise textures had an symmetrical frames. The elements in other parts of the texture were generated randomly and had identical distribution. Figures 5.9 and 5.10 show this kind of textures.

Subjects

Sixteen subjects participated in the experiment. Two of them were involved in the experiment 1. All of them were naive with respect to the purpose of the experiment.

Procedure

The experiment was composed of a training session and a test session. There were 120 trials in the test session, 30 trials of each set of either symmetrical textures or noise textures. the order of the trials was random. A training session contained a minimum number of 20 trials. Further training trials were presented, until the subject scored the critical level of 70% correct. The procedure of experiment 2 was identical to that of experiment 1.

Results

The time limit of 2 sec was chosen. Slower responses were eliminated from the data analysis.

Table 5.3: Sensitivity and reaction time of experiment 2

	$zP'(A)$ set A	$zP'(A)$ set B	RT set A	RT set B
	heterogeneity	homogeneity	heterogeneity	homogeneity
1	.697	.646	1270	1137
2	2.382	1.701	956	940
3	1.434	1.229	1097	1191
4	2.128	1.810	958	940
5	2.311	2.114	1371	1392
6	1.281	1.361	657	703
7	.374	.024	1297	1567
8	1.001	.954	1254	1250
9	1.692	1.929	956	972
10	1.732	1.803	967	962
11	2.382	2.114	1060	964
12	1.834	1.489	897	872
13	1.426	1.107	919	906
14	1.169	1.043	1062	1113
15	.695	.356	926	964
16	2.114	1.810	747	757
average	1.541	1.343	1088	1163

Data were analysed according to SDT. The sensitivity measure was $zP'(A)$. Each subject contributed with a pair of PHit and PF.A. values.

Table 5.3 shows the sensitivity measure and RT for the whole group.

Table 5.4 Paired t-Test

X: $zP'(A)$ set A		Y: $zP'(A)$ set B	
DF	Mean X-Y	Paired t value	Prob. (2-tail)
15	.198	3.599	.0026

* Set A: heterogeneous textures; Set B: homogeneous Textures.

Among the 16 subjects, 3 were more sensitive to set B (homogeneous textures) and 13 of them were more sensitive to set A (heterogeneous texture). Difference in RTs was not significant. Sensitivities in the two conditions were compared by the *t*-Test. The results of the *t*-Test is shown in table 5.4. The advantage due to heterogeneity around the axis is significant, $t=3.599$, $p < 0.003$. This indicates that heterogeneous symmetrical textures are easier to discriminated from noise than homogeneous textures.

Chapter 6

Conclusion

Experimental results show that mirror symmetry is well detected in a brief exposure. Subjects were more sensitive to bilateral symmetry when the distribution of texture elements is heterogeneous along the axis. The heterogeneity of size and shape distribution does play a positive role in symmetry detection. It could be hypothesized that the differences of element size and shape make the axis more explicit and this make the symmetry more visible.

Size and shape are primitives processed in early vision. The differences of size and shape make discrimination or “pop-out” easier (see Chapter 2). When a homogeneous texture is reflected, the resultant symmetrical texture has large scale elements along the axis. These differences of size and shape make the axis salient, although this effect might be not so strong. Our experiments demonstrate that the explicitness of the axis enhances the sensitivity to symmetry.

Detection of symmetry in random textures may be affected by the properties of symmetric axis, e.g., orientation, location, and explicitness. Barlow and Reeves (1979) indicated that it is certainly more difficult to assess symmetry when the position of its vertical axis is unknown. Pashler (1990) showed that detection of symmetry is more efficient when subject

knows the location of the axis in advance. The RT is shorter and the error rate is lower. Corballis and Roldan (1975) showed that there were no differences in both RT and error rate when subjects knew the orientation of the axis or did not know it. The difference in their results is due to the usage of different kinds of displays. Fig. 6.1 (a) shows the display used by Pashler; (b) shows the display used by Corballis and Roldan, in which the

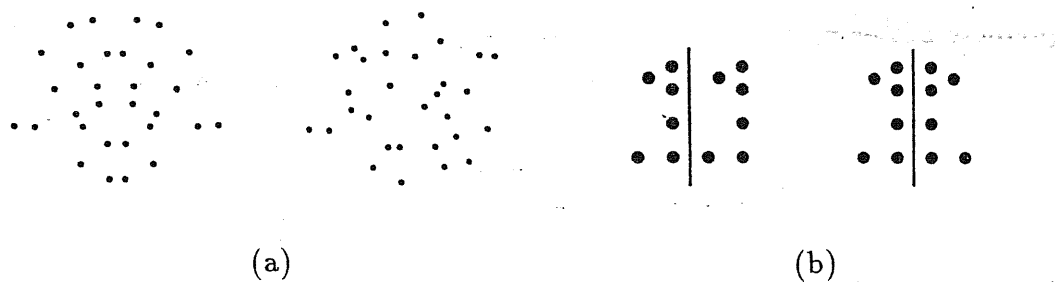


Fig. 6.1. (a) Symmetrical display used by Pashler; (b) Symmetrical display used by Corballis.

axis is marked by a line. As Pashler indicated, the display used by Corballis & Roldan explicitly cued the subject to the actual orientation of the axis. In our experiments, the orientation of the axis was always vertical and subjects knew it. The main experimental factor was appearance, or the explicitness of the axis. When the distribution of size and shape is heterogeneous, the axis is more explicit. The results showed that sensitivity is affected, although RT is not affected. On the basis of our experimental results, we can hypothesize that the explicitness of the axis is more useful for symmetry

detection than the preknowing of its orientation.

Previous researches converge toward the conclusion that symmetry detection is a redundancy reduction process. Not all information is equally used in detecting. In random-dot displays, information contained in the area around the axis is efficiently used in symmetry detection (Julesz, 1971; Jenkins, 1979, 1983; Barlow & Reeves, 1979). They indicated that the paired dots near the axis create a strong and vivid impression. The results that symmetry detection is fundamentally a short-range process also predicts better performance when the axis is explicit.

In Jenkins' three processes mode of symmetry detection in dot displays, he suggested that a process that fuses the most salient point-pairs into a salient feature precedes the process that determines whether this feature is symmetric. The salience of the central feature is dominant (Jenkins, 1983). Barlow and Reeves indicated that the salient feature, called outline, is important because this creates the impression of a vase, or a butterfly, or some other symmetrical object. They tested the efficiency of detecting mirror symmetry in random dot displays. They suggest that symmetry detection in their tasks requires nothing more than the comparison of dot densities measured over quite large areas symmetrically placed about the putative axis of symmetry (Barlow & Reeves, 1979). From their views, it seems that it is not necessary to postulate a point-by-point comparison.

The textures used in our experiments were different from dot textures. The textures were composed of several textural elements. Textural elements

were grouped into blocks according to proximity and continuity. In some instances central salient feature was formed. This enhanced the sensitivity of symmetry detection. Grouping into salient features is more difficult in triangle textures than in line textures. I think the comparison of element-pair and the comparison of global features are not exclusive. When the viewing angle is large, or the viewing distance is small, the comparison of element-pair is mainly used. When the viewing angle is small, or viewing distance is large, the comparison of global features is dominant. Further experiments are needed to clarify these issues.

Appendix 1

Signal Detection Theory

Classical psychophysics approach to detection centers upon the measurement of detection threshold. A detection threshold is the smallest amount of energy required for the stimulus to be reported 50% of the time. Signal detection theory, SDT, argues that the threshold obtained by classical psychophysical methods measures not only the observer's sensitivity but also his or her decision-making strategy, or criterion. Signal detection theory offered an improvement over classical psychophysics because it allows us to separate sensitivity from criterion.

When one is straining to "notice" a stimulus, particularly one of the small intensity, it is clear that there are two components to the process: the actual sensitivity of the receptor system to the particular properties of the stimulus and the decision process as to whether a stimulus change actually occurred or not. There is always noise inherent in any detection situation. The noise referred to by signal detection theory is an ever-varying level of neural activity of a type exactly like the nervous system's responses to the stimulus. There is background level of activity in the nervous system, and sensory signals are superimposed on the activity. Some of the noise is internal, related to the spontaneous activity level of the various neural process. Some of it is external, related to variation in signal strength, light from other source, and so forth.

In SDT it is assumed that the distribution of the noise is a normal distribution and the distribution of the signal stimulus is exactly like that of the noise distribution, but shifted to a higher mean by the amount of activation from the stimulus. Figure 1 shows the distribution of noise and signal stimuli. The distance of the two mean values determines the sensitivity. It depends upon stimulus intensity and the sensitivity of the observer. The symbol d' is used to represent sensitivity. The criterion x_c represents the observer's willing to report the stimulus, depends upon factors such as probability of stimulus occurrence and payoff. The signal is reported if $x > x_c$.

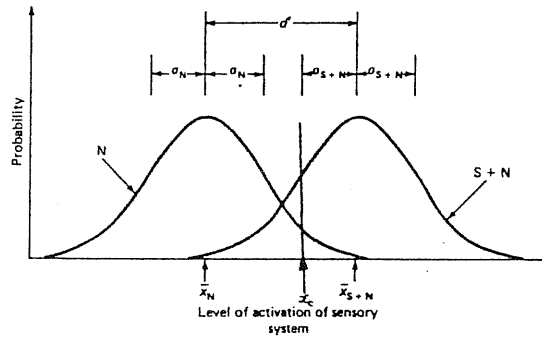


Fig. 1. The distributions of signal and noise stimuli.

In a signal detection experiment the experimenter presents the signal on some trials and does not present signal on other trials. As table 1 shows that there are four possible outcomes of a signal detection which are *Hit*, *False Alarm*, *Miss*, and *Correct Rejection*. The probability of each of these four outcomes depends upon the sensitivity measurement, d' , the distance of two means, and upon the criterion x_c .

Table 1: Four possible outcomes of SDT

trial report \	signal present	signal absent
Yes	hit (correct)	false alarm (mistake)
No	miss (mistake)	correct rejection (correct)

ROC curve

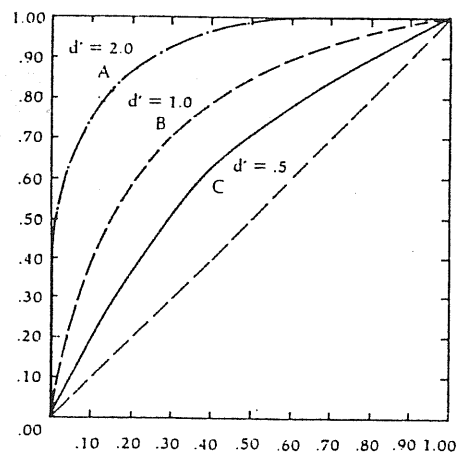


Fig.2. The receiver operating curves.

A receiver operating characteristic curve, ROC curve, shows the relationship between the probability of a hit and the probability of a false alarm. In an ROC curve, the sensitivity is a constant. Each point along

a given ROC curve represents a different criterion that the observer has adopted. Figure 2 shows a series of ROC curves.

The measure $P'(A)$

With respect to d' , another way of describing the observer's sensitivity would have been to have calculated $P'(A)$, the proportion of the area under the ROC curve. The advantage of this measure is that it is simple and easy to be translated into a computer program.

Area estimation with only a single pair of hit and false alarm rate

It seems almost axiomatic that $P'(A)$ can only be determined if we have a number of pairs of values of hit and false alarm rate from which the curve is constructed. However, it has been shown by Norman and Pollack (1964), Norman and Galanter (1964) that a single pair of PHit and PF.A. value can provide enough information to determine approximately the path of entire ROC curve. Fig. 3 is a brief illustration. Figure 3 (a) shows the plot of a single pair of PHit and PF.A. as the ROC curve, on which i lies, consists of a series of points each with equivalent sensitivity to i . The region, through which i 's ROC curve can not pass, will be those where performance is either clearly better than or worse than that at i . Figure 3 (b) shows the area which represents the better performance or worse performance. Suppose that the observer alters his or her response bias in favour of making more YES responses. This change of bias will increase the PHit and PF.A. by a proportion p and results in the point i

being moved along the straight line which joins i and $(1,1)$ in Figure 3 (c). Similarly, if the observer alters his or her response bias in favour of making more No responses, this will decrease the PHit and PF.A. by a proportion q and results in the point i being moved along the straight line which joins i and $(0,0)$ in Figure 3 (d).

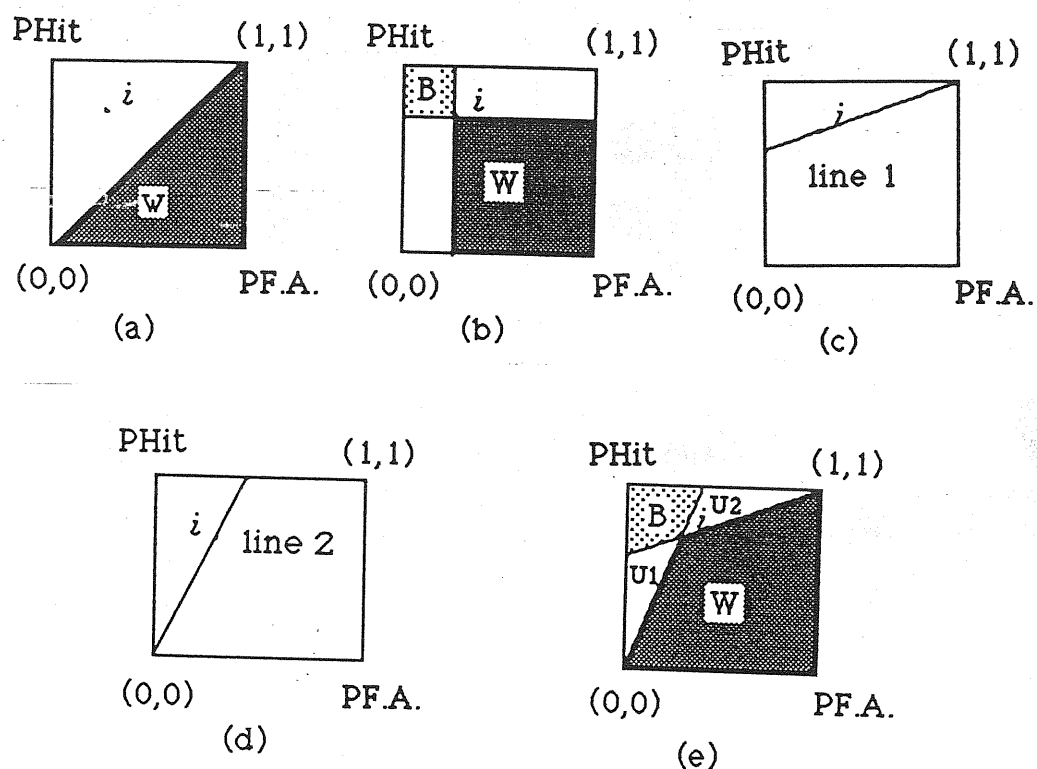


Fig. 3. Regions in which values of PHit and PF.A. will represent performance better or worse than that at point i .

All the points on line 1 and line 2 may have been obtained from an observer changing his willingness to respond YES or NO and hence may represent equivalent levels of sensitivity. Therefore any hit and false alarm

rates which give points in the unshaded areas U1 and U2 might have been obtained by a change in bias and need not represent greater sensitivity than i does. So the area under the ROC curve has an upper and lower bound.

$$P(A)(upper) = U1 + U2 + W$$

$$P(A)(lower) = W$$

The area under the real curve can be estimated as $P'(A)$

$$P'(A) = \frac{1}{2} \times (U1 + U2 + 2W)$$

z -Transformation of $P'(A)$

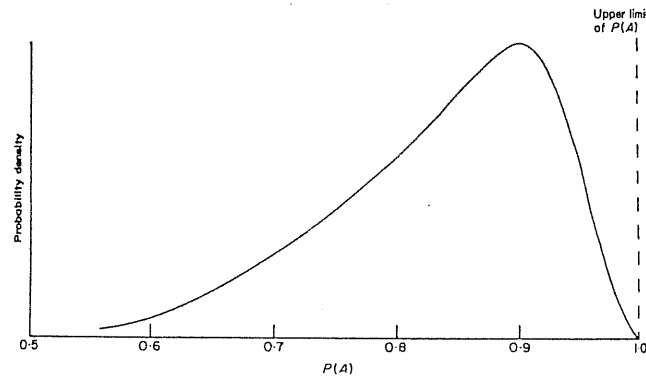


Fig. 4. The tendency of $P'(A)$ values representing high levels of sensitivity.

If $P'(A)$ is used as a sensitivity score, problems can arise. As this

measure is a probability score, it has a upper limit of 1. If some treatment conditions yield high sensitivity, the distribution of $P'(A)$ will bunch up at the top end as illustrated in figure 4. This skewness, if too extreme, can have unfortunately effects on the subsequent analysis of variance, so that it is better to transform it into other score before statistic analysis.

z -transformation is shown in Figure 5. The shaded area under the normal distribution curve is equal to $P'(A)$. So each $P'(A)$ has a corresponding standard score value, $zP'(A)$, which can be varied from $-\infty$ to $+\infty$.

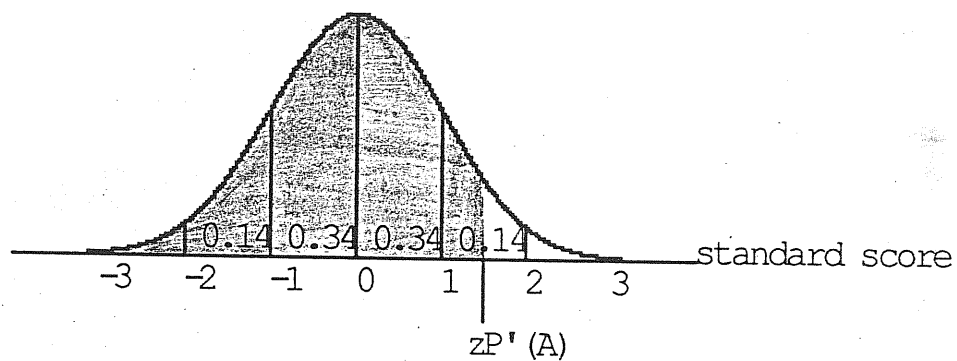


Fig. 5. z -transformation of $P'(A)$.

Appendix 2

t-Test

t-Test is a statistical method which can be used to justify if two experimental samples are taken from two different populations or they are taken from the same population. For example, we have measured the sensitivities of symmetry detection under two experimental conditions. Under one condition, homogeneous textures are used, and under another condition, heterogeneous textures are used. It is important to decide if the difference in sensitivities reflects the real difference in the two kinds of textures used or it is just a chance difference resulting from ordinary sampling error.

Apparently, the size of the difference in means and the variability in the scores will influence the decision. t-Test takes these two factors into account, where,

$$t = \frac{\text{difference in means}}{\text{standard deviation of the difference in means}}$$

A null hypothesis is set. It hypothesizes that two samples are taken from the same population. In this case, the sampling distribution of difference in means follows t-distribution, as showed in Fig. 6. Larger the difference in means, lower the probability to occur.

Conventionally, the probability $P=0.05$ is used as a significance level to disprove the null hypothesis. Now, test the probability that observed difference would occur if the null hypothesis were true. If the probability is less than the significance level, the null hypothesis will be rejected. This means that the observed difference is really due to the difference in two samples, i.e., the observed difference is due to the real difference in two experimental conditions.

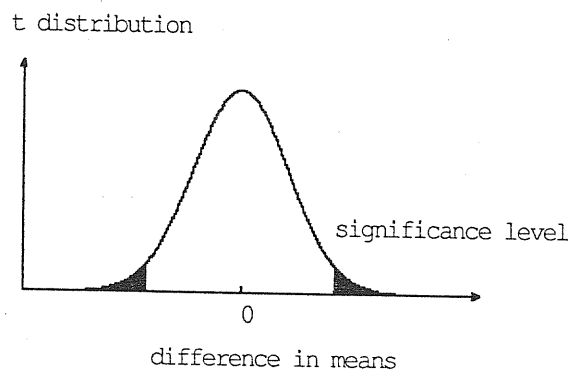


Fig.6. Sampling distribution of the difference means of two sample taken from the population.

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