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1 Introduction

If we consider the path of a light ray in a gravitational field, it is well known that its trajectory will be bent by the gravitational force. This is a straightforward application of Einstein's theory (Landau, L. D. and Lifshitz, E. M. 1979). In terms of this theory, the gravitational lens idea has been created and is one of the most vigorous and fastest growing in modern extragalactic astronomy. In such gravitational lens systems, the line of sight to a distant quasar passes very near an intervening galaxy, cluster of galaxies, rich cluster, dark matter distribution, massive black hole or cosmic string. This results in a strong magnification, displacement, distortion and in some cases fragmentation of the quasar's image due to the gravitational deflection of photons from the quasar by the intervening object. A gravitational lens system consists of a source, a lens, an observer, and at least one image of the source formed by the lens and seen by the observer. A model of such a lens system consists of two parts: cosmological part, which describes the propagation of light rays from the source to the deflector and from there to the observer, and the deflector part, which describes the deflection of rays by a mass distribution in the plane of the reflector. The observable properties of a gravitational lens system are of two kinds: deflector and image observables. The observable properties of gravitational images are their redshifts, relative angular positions, ratios of magnifications, and relative delays of signals propagating from source to observer along the different paths for different images ("time delay"). There are no source observables, because objects in the source plane cannot be observed directly owing to the presence of the deflector. At present, the observable properties of the deflector consist, at most, of its redshift,

velocity dispersion, and centre-of-brightness position(Gorenstein, M.V. et al. 1988). The large red shifts(distance) of quasars make them very valuable tools for studying cosmology and the properties of intervening matter in the Universe through observations of absorption lines and gravitational lenses. We will discuss their astronomical significance in a later section.

On the other hand, as a constructive reaction to this trend, some people say that we in fact have the existence of true pairs of physically distinct quasars. For example, the two QSO's are gravitationally bound. We will obtain different observable quantities from each component. Such as different spectra, different colour magnitudes, different redshifts,etc, and we can obtain a physical relationship between the two components.An example would be if the light curve showed periodic variation or quasi-periodic variation at large intervals. We would know the parameters of evolution of the orbit and would also know that an interaction mechanism existed in the pairs of physically distinct quasars. A physical binary quasar should be an interesting object to study, as it may provide us with some clues about the origin and maintenance of QSO activity. Evidence for tidal interactions is often indicated in the low-redshift QSO-Galaxy associations, and has even been proposed as a possible trigger of the QSO activity(Balick and Heckman 1982; Hutching 1983; Stockton 1986). Tidal shocks may facilitate a runaway gravitational collapse of the central cluster in participating galaxies, and thus actually form the central“engine”, or feed more stars into it if it already exists. Galaxy collisions also provide plausible means of transporting the interstellar matter fuel to the central engines. In any case, the projected separation and the velocity difference between the two QSO's are consistent with a tidal encounter. If more such cases are established in the future, by comparing the counts of single and binary quasars, we may be

able to estimate the efficiency of tidal encounters in triggering or stimulating the QSO phenomenon, or a “quasar formation propensity” among the galaxies in general(Djorgovski, S., et al. 1988).

Since the first double quasar(Walsh, Carswell, and Weymann 1979) was observed, many astrophysicists have produced a lot of work on this subject. Both observation and theory were stimulated. Many binary quasars have been discovered in succession up until the present. People were stimulated to explain these discoveries. There are several such systems for which the gravitational lensing interpretation is now reasonably well established. For example,this is the case for 0957+561(Walsh, et al.1979), 1115+080(Weymann et al. 1980), 2016+112 (Lawrence et al. 1984), and 2237+030(Huchra et al.1985). It has been established quite firmly that gravitational lens phenomenon exists in the universe by these discoveries. But the observed data of other double quasars, with the exception of a few previous double quasars, are scarce(Djorgovski, S., et al.1988) and some data were indistinct. Even some data are misleading(Huchra, J. P. 1986). Then came a new difficulty and a dispute was created along with these un-certain data(Turner, E. L., Schneider, D. P., et al. 1986). Although the new theoretical discussions are flourishing, one of them is that some of the claimed “gravitational lens” systems are actually physical pairs of quasars with small separations(Phinney and Blandford 1986; Bahcall, Bahcall, and Schneider 1986). The level and methods of observation make endless progress. Many new candidates are expected from the VLA gravitational lens survey(see also Djorgovski, S., et al. 1988). But so far no definite conclusion has been arrived at(Huchra, J. P. 1986).

At first sight, it would seem that two theories of gravitational lenses

and pairs of quasars are in contradiction. Actually, these two ideas are independent from each other. There are some good examples for gravitational lenses in the sky. So far, there is a nice example of a pair of quasars (also see Djorgovski, et al. 1988). The dispute only occurred for some binary quasars in which there are many equivocal data (Bahcall, J. N., et al. 1986). Then, there is existence in the sky both for pairs of quasars and gravitational lenses. But there are several double quasars that need to be judged in detail. We have to wait for future, more exact, observations.

Even though the discovery of quasars (objects with the highest red shift and probably the most distant and luminous sources in the universe) is one of the most exciting events in the recent history of astronomy, in many ways their study has proven to be one of the least successful enterprises in astronomy during the past two decades. Despite intensive and systematic observations in every available wavelength range, few if any of even the simplest and most basic questions about quasars can be answered with certainty. Similarly, in spite of their large redshifts, quasars have shed little light on cosmological problems. The enormous investment of observational resources in quasar studies has led to the discovery of ~ 6000 quasars (Phinney, E. S. 1986) and has generated a large and complex body of empirical and phenomenological information, but even in this research effort the situation is far from satisfactory. No well-defined and generally accepted morphological classification system has appeared. Very few clear correlations or regularities in quasar properties have been observed. Notwithstanding these difficulties and disappointments, the enthusiasm of astronomers for the study of quasars has not declined. Observational work continues at an undiminished rate, and theoretical attention is increasingly focused on a particular "best guess" model (Shakura, N. I., Sunyaev,

R. A. 1973 and Pringle, J. E. 1981) for the nature of the central energy source. Despite the expenditure of large amounts of telescope time and other resources, most of the fundamental questions concerning quasi-stellar objects (quasars) remain unanswered. A complex phenomenology of radio, infrared, optical, and x-ray properties has accumulated but has not yielded even a satisfactory classification system. Perhaps the most dramatic results of quasar studies do not bear upon quasars at all but rather on the intergalactic gas clouds and gravitational lenses through which we view some quasars. In these studies, the quasar simply acts as a convenient background light source which allows observations of absorption lines or gravitational deflections due to intervening objects. Above all, despite the slow growth in our understanding of their physical nature, quasars still provide us with the best available probe of the most distant observable regions of the universe.

2 Double Quasars

To find actual physical pairs of quasars would be very interesting, as it would provide diagnostics of processes and insight into phenomena which are not probed by the gravitational lensing: the nature of clustering at large redshifts, the role of interactions in triggering and fuelling of galaxian nuclear activity, etc. A binary quasar may be a beacon pointing towards a high-redshift cluster, which could provide an important target for the studies of galactic evolution.

The radio source OM-076 was first identified as a blue, 17.5^m stellar object on the Palomar Sky Survey by Radovich and Kraus(1972), and then “rediscovered” as PKS 1145-071 by Bolton, Shimmins, and Wall(1975), who reaffirmed the optical identification. By spectroscopy on this object, Wilkes

et al.(1983) and Wilkes(1986) identified it as a QSO with $z = 1.345$.

Recently, several authors reported the discovery of a pair of quasars at a redshift of 1.345, separated by 4.2 arcsec in projection(Djorgovski, S. 1988), apparently associated with the radio source 1145-071. Their observational evidence seems reliable. They used the CCD mounted at the Cassegrain focus of (f/8.01) of the ESO 2.2-m telescope to obtain an initial image of the field. The separation of the two components was obtained as:

$$\Delta\alpha_{A-B} = -3.3 \pm 0.1 \text{ arcsec} \quad \Delta\delta_{A-B} = +2.6 \pm 0.1 \text{ arcsec}$$

which corresponds to a separation of 4.2 arcsec in the direction $PA = 128^\circ$. The optical intensity ratio of the two images is approximately 2.5. The image of the field is shown in Figure 1. Spectra of the two components were obtained, using the low resolution ($300 \text{ lines } mm^{-1}$) grating. The spectra confirmed immediately that both objects A and B are quasars, at apparently the same redshift. The spectra are shown in Figure 2. The spectra appear very similar, except for the C IV 1549 line, which clearly has a larger equivalent width in the QSO A, and all emission lines except for the C IV 1459 have the same equivalent widths. The redshift difference based on the C IV line alone, which is the strongest feature, and on the cross-correlation of the two spectra including the C IV line, is approximately $250 \pm 100 \text{ km/s}$ in the rest-frame. If this discrepant emission line is excluded, the redshift difference drops to approximately $100 \pm 100 \text{ km/s}$, and is consistent with zero. Thus, on the basis of the optical data alone, we would have concluded that this double QSO is another example of gravitational lensing, because the imaging and spectroscopic data are consistent with the interpretation of the pair as a gravitational lens. The difference in the equivalent widths of the C IV line, its shape, and thus the redshift

difference, could be easily attributed to the intrinsic variation (which can occur on a time scale of a few months), plus the time delay between the two images (which may be of the order of one or two years). The apparent optical properties of the system did not appear very different from the “classic” gravitational lens 0957+561. However, this evidence is neither compelling nor conclusive, and is also consistent with the interpretation of the pair as a physical binary quasar. But, the crucial evidence comes from radio maps: short observation of PKS 1145-071 were obtained with the Very Large Array (VLA). The result is dramatic (See Figure 3). The radio source is single and unresolved at all VLA frequencies, and positionally coincident with the brighter of the two optical images. The fainter of the two optical components is a factor of 500 fainter in the radio. This dramatic difference in L_{Opt}/L_{Radio} between the two components suggests that the system is a genuine binary quasar, and the first such well documented case.

Assuming that the two QSO’s are gravitationally bound, and that there are no other massive objects in their immediate proximity, they used their projected separation and the velocity difference to place a lower limit on the total mass of the system. From the equality of the kinetic and potential energies, and independently from the shape of the orbit, the following equation should apply: $M_A + M_B = Rv^2/2G$, where M_A and M_B are the masses of the quasars, R is their three-dimensional separation, and v is their three-dimensional mutual velocity. Now, the projected separation and the observed radial velocity difference are lower limits to R and v , independent of the inclination and phase of the orbit. They adopt $R_{min} \simeq 20kpc$, and $v_{min} \simeq 200km/s$, so obtaining $M_A + M_B \simeq 10^{11}M_{\odot}$ as a lower limit, which is a reasonable value for normal galaxies. Allowing for the projection effects would suggest a true value several times larger than this. Another

exciting possibility is that this pair indicates a high-redshift galaxy cluster: if quasars are rare events, then two quasars are suggestive of a high galaxy density. Mere existence of rich clusters at such large redshift provides an interesting timing constraint for the theories of large-scale structure formation. Studies of “normal” galaxies in this hypothetical cluster should be extremely valuable for the investigations of galaxy evolution at large look-back times.

The slight sore point is a statistical one. Assuming the quasars were placed randomly in the sky (Phinney, E. S., et al. 1986), the probability of finding another quasar of apparent magnitude m within an angle θ of a given quasar would be $P_r(\theta, m) \simeq n\pi\theta^2$, where $n(m)$ is the sky density of the quasars brighter than magnitude m . For PKS 1145-071 B,C, $\theta = 4.2$ arcs, $n(17.5) \leq 1$ per deg. we obtain $P_r \leq 3 \times 10^{-6}$. The expected number of random associations with the separation and magnitude of PKS 1145-071 B,C is $NP_r(4.2\text{arcs}, 17.5)/2 \leq 1.2 \times 10^{-2}$, where $N(17.5) < 8 \times 10^3$ is published, plus unpublished quasar catalogues that are brighter than 17.5^m . We see that the expected number is small. Thus, on the basis of the statistical data alone, we would have excluded that this double QSO was a pair of quasars. However, quasars are probably not distributed uniformly throughout the Universe, and there is evidence that they are clustered in much the same way as the galaxies (Shaver, 1984). Then, this statistical approach leads us only in a speculative direction. It is impossible to rule out the previous observational evidence.

3 An Equivocal Double Quasar

Of course, we meet the phenomenon that is nearly perfect, like PKS 1145-071. The other phenomena have brought disappointment. Almost

no one double quasar, except for a few apparent examples of gravitational lenses, can be interpreted as a pair of quasars or gravitational lens unequivocally. They have all been under dispute up till now (Phinney, E.S. and Blandford, R.D.1986, Huchra, J. P. 1986).

One of them, whose name is 1146+111B,C, is by far the most disputable. Much interest has been shown in a pair of quasars with nearly identical redshifts, $z \simeq 1.01$ (Hazard et al. 1979), and is a most likely candidate for a physical pair because its separation (157 arc s) is much greater than that of any of the confirmed gravitational lenses (Turner, E. L. 1986). In all of the observational data, only data of redshift and separation were accepted, other data were equivocal and incomplete, and completely contradictory consequences would result from different observational data.

Huchra (1986) has obtained moderate-resolution spectra of 1146+111B and C at several epochs with spectrographs on the Multiple Mirror Telescope (MMT), which span the wavelength range 3,200–7,000Å and include the strong C III] line, as well as several Fe II bands not covered in earlier spectra. The ultraviolet spectra of the two quasars are different. There are also different velocity shifts between the quasars as measured by the C III] and Mg II lines. Although it is impossible to rule out the lensing hypothesis, these observations increase the probability that these objects are just two quasars at nearly the same redshift. The observational data are as follows. The spectra of the two objects (Fig.4) in the vicinity of the Mg II line are similar. But the spectra of the two objects at $< 5000\text{Å}$, however, are clearly different. Quasar B has a strong C III] line of equivalent width(EW) -19Å ; quasar C has a much weaker C III] feature of EW -7Å . Even after correction for continuum contamination by Fe II emission, C's C III] line EW is

still only half of B's. In contrast, the Mg II features in the two objects are almost the same: $EW \simeq -18\text{\AA}$ in B and -13\AA in C: the equivalent widths if these lines do not vary significantly from these means in the individual spectra (see also Huchra, 1986). The larger scatter in the Mg II widths is due to the difficulty of subtracting the strong [OI] $5,577\text{-}\text{\AA}$ line, and the low efficiency of the Multiple Mirror Telescope spectrograph plus 300 lines per mm grating at 5600\AA . On the other hand, quasar C shows relatively strong Fe II bands, the $2,100\text{-}\text{\AA}$ band redshifted to 4180\AA , the $2,500\text{-}\text{\AA}$ at $5,000\text{\AA}$ and $2,950\text{-}\text{\AA}$ band at $5,900\text{\AA}$. These bands are present in quasar B but are much weaker. Quasar C also exhibits the $1860\text{-}\text{\AA}$ feature seen just shortward of C III], which is probably due to Al III or Fe emission (Grandi,S. 1981, Hartig, G. and Baldwin, J. 1986). This feature is weaker in quasar B. In addition to these results are the spectra in the near-infrared obtained at European Southern Observatory (Shaver, P. et al. 1986). Deep charge-coupled-device (CCD)imagery has been obtained at Lowell Observatory and at Kitt Peak National Observatory (see also Huchra 1986). Both the near-infrared spectra of the two objects and their optical broadband colours are different. It has been suggested that the spectral differences may be due to the presence of extended emission in quasar B. Neither object appears extended in the deep CCD images; the equivalent width differences would require even pure-emission-line objects to be visible at 24mag. In any case, a visible extent on arc second or larger scales in one object and not in the other is an argument against the lensing hypothesis. Huchra has also tried to improve the measurement of the velocity difference between B and C. A simple cross-correlation of the summed MMT-spectrograph spectra of B and C over the entire wavelength range yields a velocity difference ($V_B - V_C$) $\simeq +400 \pm 100\text{km/s}$. Cross-correlation of the Mg II line alone

yields $(V_B - V_C) \simeq -100 \pm 60 \text{ km/s}$. Cross-correlation of the region around C III] alone yields $(V_B - V_C) \simeq +100 \pm 60 \text{ km/s}$, although velocities derived in this region can be compromised by variations in the relative strengths of Fe and Al multiplets. Figure 5 shows the regions around the C III] and Mg II lines in both quasars. The different velocity difference between the carbon and magnesium lines is not surprising, especially if B and C are separate quasars. Velocity differences of 1000 km/s are often seen between the lines of different ionization states in the same quasar (Gaskell, C.M. 1982, Wilkes, B. J. 1984). The original arguments for suggesting that 1146+111B, C are images of a lens were based on their nearly identical redshifts and the similarity of their spectra at Mg II. These arguments have weakened as more data have accumulated. Under the double-quasar hypothesis, the similarity of the spectra at Mg II is not impossible. Quasars of similar luminosity (B and C have $m_B \simeq 18.5$ and 18.6 , respectively, at the same redshift) have similar Mg II equivalent widths (Richstone, D., et al. 1980). It is also well known that quasars are often associated with groups and clusters of galaxies (Yee, H., et al. 1984, Stockton, A. 1978). The velocity difference between the quasars is typical of the velocity dispersions of small groups of galaxies (Huchra, J., et al. 1982).

Recent calculations of the probabilities of chance associations of quasars, given the total number of quasars that have been observed to date, support the likelihood of finding the quasars clustered (Phinney, E. S. et al. 1986). These authors show that they are probably observing two distinct though neighbouring quasars, assuming that the quasars are probably not distributed uniformly throughout the universe. Some authors suggested that they are clustered in much the same way as galaxies (Shaver, P. A. 1984). So their two-point correlation function is $\xi(r) \simeq (r/r_0)^{-1.8}$, where

$r_0 = 5(1+z)^{-1}h^{-1}Mpc$, h = the Hubble constant $H_0/100kms^{-1}Mpc$ and the cosmological density parameter $\Omega_0 = 1$. The probability associated with this excess for a given quasar to have a companion within angle θ brighter than magnitude m is then

$$P_c(\theta, m, z) = 2\pi(1+z)^3 \int_0^{D\theta} dbb \times \int_{-\infty}^{\infty} dl\Phi\xi[(b^2+l^2)^{1/2}] \quad (1)$$

where $D(z)$ is the angular diameter distance of the quasar, and $\Phi(L, Z)$ is the integral luminosity function for quasars with luminosity $L(m, z)$. Now, $\Phi(L, z) = (dn/dz)/D^2(1+z)^{1/2}$ (Ref. Weinberg, S. 1972) and observations (Marshall, H. L. et al. 1984) indicate that for $0.5 < z < 2$, quasars of a given magnitude are uniformly distributed in redshift with $dn/dz \simeq 0.5n$. Substitution in equation (3-1) then gives $P_c(\theta, m, z) = 38(1+z)(\sqrt{1+z}-1)\theta^3\xi(\theta D)dn/dz$ for $D\theta \leq 2r_0$, where n is per steradian and θ in radians. Numerically, $P_c \sim 2 \times 10^{-4}$. For the 1146+111B,C, the expected number of closely clustered companions is $NP_c(157arcs, 18.5, 101)/2 \sim 1$. If the quasars were placed randomly on the sky, the expected number of random associations with the separation and magnitude of 1146+111B,C is $NP_r(157arcs, 18.5)/2 \sim 20$, which is roughly what is observed (Shaver, P.A. et al. 1984). Hence we would expect by now to have discovered of the order of one physically associated pair of magnitude 18.5 quasars separated in angle by less than 157 arc s and in velocity by the velocity dispersion of small groups which dominate the correlation function, $\sim 300km/s$.

Additional spectra of the two objects were obtained by Shaver (1986), covering the wavelength region $5,600\text{\AA} - 9,200\text{\AA}$. They failed to confirm the similarity of the objects; object B showed much stronger Balmer-line emission than object C. However, given the substantial dissimilarity of the spectra of B and C in the ultraviolet and infrared, the different relative

velocities of different spectral features and the lack of an identifiable lensing object, this system is, plausibly, simply two associated quasars at nearly the same redshift.

4. Quasar Pairs: illusion or reality?

Turner et al.(1986) obtained spectra of 1146+111B. C with a high signal-to-noise ratio using the cryogenic camera at the Kitt Peak National Observatory 4-m telescope. These spectra, which cover the region of the redshifted Mg II line, were quite similar. In addition, the redshift difference between the quasars as measured from the Mg II line was less than the observed error of $\sim 300\text{km/s}$. The quasar images were separated from each other on the sky by more than two minutes of arc (2.6 minutes to be precise). The significance of this report was quickly appreciated by many people.

Although since the first observation in 1979 there has accumulated a handful of cases in which the image of a single quasar is split into two or even three by the interposition of a massive galaxy along the line of sight to the distant object, the angular separations previously reported have been very much smaller than that of the pair of images measured by Turner et al., a few seconds of arc rather than a few minutes. The two authors Richard and Paczynski, explore the possibility that the very large separation may be caused by the interposition either of a very massive black hole or, alternatively, of one of the galactic structures called a cosmic string. We will note that their calculations, interesting in their own right, are at once tentative and suffused with a sense of what good luck it must be that observations should have led so simply to such striking inferences. Now, unfortunately, it appears that these endeavours may have been misplaced.

In a field where it is never easy to be sure that quasar images have been accurately identified with each other, it is too soon for certainty. But stimulated by the report of Turner et al., others have looked at the components of the supposed double quasar in spectral regions other than that on which the original identification was based, and they find enough discrepancies to suggest that the apparent identity of the two objects is in doubt. We should consider carefully what is involved in the identification of a gravitationally split quasar image. That an intervening massive object can split one image into two or more follows from the way in which, in general relativity, the gravitational field of a massive object distorts the usual straight-line paths followed by light. The magnitude of the effect and the apparent separation of the two images depend on the mass of the intervening object. The known cases of split quasar images require intervening masses comparable with those of galaxies. 1146+111B,C separated images would require more massive objects, hence the speculation about black holes and cosmic strings.

To establish that a close pair of quasar images has indeed been generated by the interposition of a gravitational lens is, in reality, quite difficult. The first test is that the two images should have the same measured red-shift (which is not affected by the bending of the gravitational lens). One difficulty is that the red-shifts are usually so large that it is hard to tell which atomic transitions are ultimately responsible for the spectral lines observed in the emission from the quasar. Another is that the lines, even if accurately identified, may be so broad that the accuracy of inferences about their displacement is bound to be limited. Two images of 1146+111B,C were reported to have a red-shift defined by $z=1.012$ on the strength of an emission line from magnesium which, while normally in the ultraviolet, has

been displaced by the recession of its source to the middle of the visible light spectrum.

Those who search for double-quasar images also pay close attention to the general characteristics of the spectra of the two candidate images, but there are some important complications. Thus the spectra of quasars can vary rapidly on quite short timescales: most of the variations show up in the continuum flux, but there may also be substantial variations in the strength of emission lines in the spectra. Because one of the consequences of the effect of the gravitational lens is that different images of the same quasar may reach the observer by paths which differ considerably in the traverse-time of light, there is room for some latitude in the comparison of the spectra of two images. In Turner's case, the two light paths may have differed by as much as 1,000 light years, ample to account for the spectral differences he and his associates reported in the two quasar images.

A new development is a set of measurements (P.A.Shaver and S.Christiani, 1986) in a part of the spectrum extending towards the infrared from the longer wavelength limit of the range covered by Turner et al. In this previously unexplored region of the spectrum of the Turner quasar, there appear to be pronounced differences in the intensity of certain hydrogen lines. When this evidence appears, we will know whether the quasar images are indeed produced by a gravitational lens from the same single distant object. Advance notice of these measurements is being provided now, with the consent of the authors, merely because of the excitement generated by the original measurements.

If Turner et al. were unlucky in their choice of a spectral range in which to carry out measurements of the components of the double quasar,

there will be a widespread sense of disappointment that a potentially fascinating phenomenon has so quickly vanished. But that does not imply that the two quasar images concerned are without interest. The supposition that they might be gravitationally split images of the same object seems first to have been raised by Paczynski(1986) on the basis of the measurements originally due to C.H.Hazard(1979).

If the two images are now gravitational artefacts, then they are most probably distinct quasars(with nearly identical redshift) which are also members of a distinct galactic cluster. But the measured difference between the velocities of the two images is less than 200km/s, which is less than the spread of velocities found in typical galactic clusters, and which may therefore suggest some special relationship between the two quasars. That would be an interesting if less spectacular development in its own right, one that would no doubt bring comfort to those who, like H.Arps(1980), have long argued for some systematic relationship between the positions of quasars in the sky.

Meanwhile, the urgent need is to further test the relationship between the two images of Turner's quasar. Detailed study of the relative intensity of the iron and magnesium lines in the two images will no doubt persuade most people one way or the other. When all this has been done, there will remain one logical ambiguity. If the travel times of widely split quasars may be of the order of 1,000 years, and very much greater than the timescale of substantial variability of individual quasars, will it ever be possible to prove(or disprove) that such widely split images are caused by gravitational lenses, however massive or peculiar?

But so far, it is unfortunately impossible to rule out the lensing hy-

pothesis with the available data because of the possibly large phase delay between images with such a large separation (Turner, E. et al. 1986). Then we have no idea what quasars do on thousand-year timescales. Because of the great variability of quasars in brightness, spectral shape and line profiles it is unlikely that any single observation of this pair of objects will give a clear answer to the competing hypotheses of a lens or binary system.

5. Gravitational Lenses

We have seen various comments previously. Could we ask which explanation is best? At present, we can not obtain a firm answer. However, though long predicted, the ongoing detailed study of gravitational lens systems among the quasars opens the door to a wide variety of qualitatively new types of observations and may provide a major new tool for extragalactic astronomy and cosmology. In addition to the uses of gravitational lenses discussed below, it is useful to recall that they do represent a fundamental physical phenomenon of some importance. Their existence was predicted by Einstein, and their astronomical significance discussed (Zwicky, F. 1937) during the same period when the advent of modern physics led to the widely celebrated predictions of degenerate dwarfs, neutron stars, black holes, dark matter, rich clusters, interstellar clouds, the microwave background radiation, and so forth. It is not impossible that gravitational lensing will someday also join this list in terms of its fundamental importance for astronomy.

A model of the lens system consists of several parts. The first major use of gravitational lensing is as a probe of the distribution of matter, particularly dark matter, both in the lensing objects themselves and along the line of sight out to the lenses. During recent years, it has become increas-

ingly clear that most of the mass density of the Universe and even much of the matter in individual galaxies must reside in some form that does not emit strongly (at least at any accessible wavelength) (Faber, S.M., et al. 1979). The factor by which this dark material exceeds the mass density in ordinary stars, interstellar matter, and other detectable objects is not known but could be ~ 10 to 100 (Turner, E.L., 1984). Discovery of the nature and detailed properties of this component of the Universe is generally recognized to be one of the most important problems in astronomy today. Gravitational lensing offers hope of addressing this problem in several new ways because it depends entirely on the space distribution of gravitating material between the source and the observer and not at all on any of its other properties. Especially, observations of individual lens systems and the statistical properties of samples of lenses can be used in principle to determine the total masses of lensing galaxies; to characterize the distribution of dark matter in rich clusters; to discriminate between very different models for the nature of the dark material (for example, neutrinos versus very-low-mass, nonburning stars versus massive black holes); to detect a uniformly distributed component of the Universe's mass density, and even to identify specific condensed dark objects if such things exist.

A second major application of gravitational lens observations will be the determination of cosmological parameters. This possibility arises because the cosmological background in which lensing occurs determines the geometrical optics of the situation. Measurements of differential time delays between images in lens systems and the statistics of the properties of samples of lenses can be used to determine all the classical cosmological parameters for example $H_0, q_0,$ and Λ (the value of H_0 gives the expansion rate and hence age of the Universe; q_0 measures the deceleration of the

expansion and determines whether the Universe is open or closed; the cosmological constant Λ may be regarded as a measure of the energy density of the vacuum), in principle. Moreover, the self-consistency of such determinations can be used to check the validity of the standard cosmological models. The possibility of measuring or at least placing limits on Λ is particularly good because lensing is much more sensitive to this parameter than other available astronomical observables.

A third possible application of gravitational lenses is to use them directly as lenses in a sort of natural telescope that will allow us to look much more deeply into space (in a few small patches of the sky) than would otherwise be possible. This possibility, which was first pointed out in 1937(Zwicky, F. 1937), may in a few special situations improve the effective performance of our telescopes by a factor of several(at no additional cost!).

We will have noticed the phrase “in principle” or the word “possible” in most of the critical sentences above and will note that many of these potential applications of lensing will be quite difficult to realize. Indeed, some may be completely impractical even in the long term. Gravitational lenses are rare and hard to locate; so far, only six cases have been found among the known quasars (though many could have been missed). Many of the idealized lensing tests such as determinations of the cosmological parameters may be confused by other astrophysical effects in reality. Many of the required observations will be practically difficult since they will require that faint objects be monitored with high resolution and good time coverage. Nevertheless, the questions that lenses might enable us to answer are so important and so difficult to attack by other means that the incentive for at least trying to overcome the problems will be great. One

probably should not be too discouraged yet that work on the six known examples of lensing has not produced any breakthroughs[in fact, it can be demonstrated from the properties of 0957+561 that the matter in the rich cluster that forms part of that lens can not be distributed like the galaxies (Shapiro, I.I. 1982)], for two reasons. Firstly, groundbased observations do not have the resolution or sensitivity to objects of low surface brightness needed to clearly disentangle the quasar images from the lensing galaxy and to determine the positional and structural parameters of the latter accurately: Space Telescope will decisively relieve this problem in a few years. Secondly, only six cases have been studied so far (and only two extensively); the first six pulsars discovered did not teach us much about neutron stars or general relativity (Turner, E. L. 1984), although larger samples, the Crab pulsar, and the binary pulsar eventually did. It may well be the same for gravitational lenses; the major discoveries may have to await statistical samples and particularly simple or otherwise special objects.

6. A Probable Judgement of Lens Hypotheses

It has been speculated (Turner, E.L., 1986) that the quasar pair 1146+111B,C are two bright images of a single quasar produced by a gravitational lens. They obtained the spectra of 1146+111B,C. The spectrograms are shown in Fig.6. These two spectra agree precisely in their significant features. Fig.7 gives the ratio B/C of the two spectra. The Mg II line ratio is ~ 1.02 which the continuum ratio varies from roughly 0.96 at the blue end to around 0.83 at the red end. These small variations do not inhibit a lens interpretation, as similar small differences in line strengths and continuum slopes are observed in such confirmed lens cases as 0957+561 (Young, P., et al. 1980) and 2016+112 (Lawrence, C. R. et al. 1984). They may be accounted for by

a variety of explanations including source variability plus differential time delays, gravitational mini-lensing events (Paczynski, B. 1986), extinction by intervening material, contamination of the source spectrum by light from the lens, and systematic observational errors resulting from differential refraction effects. Overall, the spectroscopic similarity of 1146+111B and C is as strong as for any of the known (few arc second separation) lens systems. They obtained the average images using the CCD camera with the same telescope. In an analysis of this data, they pointed at: (1) an image separation of 156.8 ± 0.5 arcs, (2) an R-band B/C brightness ratio of 0.93 ± 0.01 ; (3) a possible very distant rich cluster of galaxies between the two quasar images; and (4) an unusual number of fairly bright ($R \sim 18$ to 19), apparently low redshift ($z \leq 0.2$) galaxies in the field including one roughly between B and C. The quasar images themselves show no signs of resolution.

The data supporting the lens hypothesis are remarkably similar-high signal-to-noise, and moderate resolution spectra of the two components. Both spectra show strong Mg II $\lambda 2798$ emission at $z = 1.012 \pm 0.001$ with indistinguishable redshifts ($\Delta v = 126 \pm 309 \text{ km/s}$), widths ($FWHM = 64 \pm 4 \text{ \AA}$) and detailed profile shapes. Both spectra also show broad troughs at $\lambda 6180$ and several weaker continuum features. Neither object exhibits the [O II] $\lambda 3727$ line which is frequently strong in Mg II emission objects. Of course, the possibility of a physically distinct (not lensed) pair of objects with astonishingly similar spectroscopic properties is not logically excluded by the data. 1146+111B.C and the surrounding field warrant further study, and some attempts in this direction are under way. Verification of the spectral similarity of B and C at other wavelengths and searches for additional, multiply-imaged objects are of particular interest. other candidate large

splitting lenses should also be examined more carefully.

7. Various Gravitational Lens Hypotheses

7.1 The Model of Massive Rich Clusters of Galaxies

We will have noticed that the major dispute between binary quasars and gravitational lensing was focused on the quasar pair 1146+111B,C. The main reason is that its separation (157 arc s) is much greater than that of any of the confirmed gravitational lenses. The level of apparent separation of two images depends on the mass of the intervening object. The known cases of split quasar images require intervening masses comparable with those of galaxies. If the foreground galaxy clustering apparent in a deep R band charge-coupled device (CCD) image proves insufficient to explain the large image splitting, other possibilities such as massive dark objects (for example, $a \sim 10^{15} M_{\odot}$ black hole) or a cosmic string may be indicated.

The importance of 1146+111 B,C's lines in its unprecedented large image separation has been realised and the questions and possibilities thus raised, and various probable models of gravitational lensing, have been created.

The first question is that of the nature of the lensing object. The most conventional candidate would be a very massive rich cluster of galaxies. If such a cluster lay at the most probable redshift of 0.32 and had an isothermal sphere mass distribution, the required one-dimensional velocity dispersion would lie near 2,200km/s (depending weakly on q_0). If the cluster instead lay at a redshift of 0.81, which would minimize (for a constant M/L value) its visibility, assuming an elliptical galaxy k correction (Schneider, D. P., et al. 1983), the requirement would be $\sim 4,500 km/s$. The former value is extreme but not entirely beyond the observed range for rich clusters

(Mushotzky, R.F., et al.1978). Nevertheless, such a cluster would be easily detectable in deep CCD images at any redshift less than 1, so this candidate lens is subject to direct observational test. If no suitable rich cluster can be located, more exotic possibilities such as $\sim 10^{15} M_{\odot}$ black holes and cosmic strings will need to be considered. Indeed, lens systems much like 1146+111B, C are a predicted consequence of cosmic strings(Vilenkin,A. 1984, Gott, J. R., 1985).

Second is the possibility that additional objects in the 1146+111 field may be multiply imaged. The large angular splitting of the B,C pair implies a large multiple imaging cross-section for most reasonable lens models. Some of the other known(Arp,H.C., et al. 1980) quasars in the field could well have additional, so far undetected images; D, E and K are particularly likely candidates (Turner, E. L., et al. 1986). Even if they are not multiply imaged, lens magnification and hence flux enhancement could account for the statistically unlikely “concentration” of quasars which originally called attention to the field. In any case, discovery of additional multiply-imaged objects would offer the opportunity of modelling the lens mass distribution more precisely than for other known lenses and, should this distribution prove simple, to carry out powerful cosmological tests.

Third is the large differential time delay expected for large splitting image pairs. For reasonable lens models, the 1146+111B,C time delay is of the order of 10^3 years. This offers the opportunity of observing variations in the lensed objects taking place over time periods well in excess of those usually accessible. If the lensing geometry is highly symmetrical, as the roughly equal brightnesses of B and C might suggest, the time delay will be substantially smaller than this “natural” value.

7.2 A Hypothesis of a Double Cluster

Galaxies at large redshift can behave as gravitational lenses and produce multiple images of background quasars (Gunn, J. E. 1981). Five convincing examples of the lensing action have recently been discovered. They are Q0957+561, Q1115+080, Q2345+007, Q2016+112, Q1635+267 (Walsh, D., et al. 1979, Weymann, R. J. et al. 1980, Weedman, D. W., et al. 1982, Lawrence, C. R. et al. 1984, Djorgovsky, S. et al. 1983, Foltz, C. B., et al. 1984). However, the observed image separations, which range up to 7 arc s, are conspicuously larger than the 1-2 arc s separations expected from normal galaxies. Thus, it has generally been concluded that the galaxy lensing action is considerably assisted by the magnification of an associated cluster (Young, P., et al. 1981). The cluster also amplifies the quasar flux and so the distribution of observed image separations is biased towards large values (Narayan, R., et al. 1984). Then, clusters of galaxies can act as gravitational lenses which produce double quasars with splittings as large as 1-2 arc min. Of course, the potential well of the cluster must be deep enough on the scale of the image to focus the light, which means that the velocity dispersion within the cluster must be high and the core radius small. These requirements are reduced if there are two (or more) clusters along the line of the sight. This triple image produced by two clusters has been modelled without pursuing the probability of such an alignment in any detail (Paczynski, B., et al. 1984). Here Crawford predicts the number of double quasar images in the sky that are produced by double clusters (Crawford, C. S., et al. 1986). The clustering of clusters (Bahcall, N. A., et al. 1983) and the occurrence of binary clusters (Ulmer, M. P., 1981; Forman, W., et al. 1981) enhances the probability of double clusters well above that expected from a purely random spatial distribution. Several double

images are expected. Double clusters should be considered for apparent double quasar images such as 1146+111B,C before cosmic strings (Gott, J. R. 1986) or supermassive black holes (Paczynski, B. 1986) are required. The discovery of the actual wide double quasar image will be important in the study of the evolution of the clustering of clusters.

Several authors (Crawford et al. 1986) have used two approximate methods to estimate the number of wide double images. The first works from the number density of rich clusters (Bahcall, N.A., 1979) and the clustering of clusters. There is no strong richness/velocity dispersion relation (Bahcall, N. R. 1985, and Rivolo, A. R. 1985) but they assume that two clusters of Abell richness class 2 contribute an effective line-of-sight velocity dispersion, σ , of 2,000 km/s. The angular separation of the gravitational images is $\sim 2(\sigma/2000 \text{ km s}^{-1})^2$ arc min. This requires that they each have a δ of $\sim 1400 \text{ km/s}$. The present number density of clusters of this richness and greater is $n(0) \approx 6 \times 10^{-8} \text{ Mpc}^{-3}$ (Bahcall, N. A. 1979). They adopt $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$. A similar result is obtained using data from Piccinotti, et al. 1982. The fraction of the sky covered by double clusters aligned within an angle closer than the observed image separation θ (assuming no spatial correlation) is

$$f = [4c^3/H^3\pi\theta^2n(0) \int_{k_1}^{k_2} (k^{-3/2} + k^{-5/2} - 2k^{-2})dk]^2 \quad (7-1)$$

where $k = 1 + z$; and $[k_1, k_2]$ is the range of redshift considered for the cluster. The covering fraction f is enhanced by the clustering of clusters (Bahcall, N. A. 1983) which, with scaling to $z \sim 0.5$, corresponds to a factor of $19/\theta$, where θ is in arc minutes. Then if $\theta \approx 2$ arc min we have an f of 10^{-5} . This implies that there would be about four double images in the sky if there are more than 10 appropriate background quasars per

square degree. They have taken $z=0.5$, $k_1 = 1.25$ and $k_2 = 1.75$ so the values are appropriate to quasars of redshift ~ 1 . The velocity dispersion requirements are lessened if the clusters image the more numerous background quasars with redshifts of between 2 and 3. Including Abell richness class 1, the estimate data of f is on the order of 5×10^{-5} . Consequently, there are likely to be ~ 200 wide quasar images in the sky if the background quasar density is as high as 100 per square degree.

The two authors Bahcall, et al. 1983 remark that the clustering of clusters increases with richness so we may expect that it is most pronounced for the more massive objects required here. The above estimate involves a large extrapolation of their correlation function to small scales (it was measured on more nearby clusters on the scale of degrees rather than arc minutes). There is, however, a reasonable fraction of nearby clusters observed to be double (Forman, W. et al. 1981) with the separation required by a 2 min arc lens image. An excellent candidate is the A399/401 cluster system which has velocity dispersions (Jones, C. et al. 1984) of 1424 and 1294 km/s, (both with an uncertainty of ~ 400 km/s). The apparent separation of these clusters at $z=0.073$ is 33 arc min, which corresponds to 4.8 arc min at $z=0.5$. Simple geometry then indicates that the probability of observing them with a separation of 2 arc min is ~ 0.1 . For Crawford's second method, assume that the A399/401 pair is not unique and estimate its spatial density to be approximately one per volume corresponding to $z=0.073$. There are then ~ 70 pairs aligned to within 2 arc min over the sky out to $z=0.75$. Again, we expect about four double quasar images in the sky from 10 background quasars per square degree.

The timescale over which double clusters may last with such small

separations of a few megaparsecs is unknown, but it can take several crossing times (each $\sim 10^9$ yr) for the clusters to merge (McGlynn, T. A. et al. 1984).

But they have ignored the possible brightness enhancement of the background quasars by the gravitational lens, which depends on the precise light paths. Any Sunyaev-Zeldovich effect (Sunyaev, R. A., Zel'dovich, Ya. 1972) due to hot gas in the clusters is a factor of two smaller in a double cluster than from a single very massive cluster (and possibly even less if the beamsize is not matched to the cluster size). No single cluster has yet been reported with σ of 200 km/s (Crawford, C.S. et al. 1986). These estimates suggest that several quasars at $z \geq 1$ are gravitationally lensed into double images of 2 arc min separation by double clusters.

7.3 Lensing by Dark Matter.

The binary lensed QSO 2345+007(Weedman et al. 1982) has the largest image separation, 7.3 arc sec, of all seven confirmed lensed QSOs to date. Since the redshift of the QSO images is 2.15, the most likely redshift of a lensing galaxy is 0.4-0.7 (Turner et al. 1984). Previous searches for the lensing galaxy have been reported by Keel(1983). Keel's R band CCD data allowed him to place a limit of $R > 23$ for a 3σ detection of any object within $10''$ of the pair. However, nothing has yet been observed to such a light R magnitude as 25. Tyson et al.(1986) have described an imaging and spectroscopic search for faint galaxies in the neighborhood of the QSO pair, to look for a possible third lensed QSO image and to search for the lensing galaxy. Deep CCD images of the lensed QSO 2345+007 taken in the blue, red, and near infrared show no galaxy between the QSO images to $J=25.5$, $R=26$, and $I=24.5$ mag. There are a few 23.8 I mag. galaxies within 10

arc sec of the QSO. Even if one of these field galaxies was near the line of sight to the QSO, there must be other objects of very high M/L assisting the lensing. Two absorption-line systems at $z=1.5$, separated by 1000km/s, are seen in the QSO spectra. They would have detected images of galaxies at $z=1.5$, in the QSO vicinity. This is the strongest case, among the seven confirmed lensed QSO's, for lensing by dark compact objects. The QSO 2345+007 is the best candidate for dark matter lensing. A nonluminous mass of $10^{13}M_{\odot}$ is required (Tyson et al. 1986)

7.4 Gravitational Lensing Effects of Vacuum Strings.

We have seen that galaxies (Narayan et al. 1984), rich clusters of galaxies (Turner, et al. 1986), double clusters (Crawford, et al. 1986) and dark matter (Tyson, et al. 1986) can behave as gravitational lenses and produce multiple images of background quasars. But heated argument still focuses on the two quasars, 1146+111B,C, separated by 157 arc s. If they are in fact two images of the same quasar, this raises the question: what massive object is responsible for gravitational lensing ? A predicted consequence of cosmic strings is that they should produce equal-brightness double-lens images of distant quasars with separations of up to several minutes of arc (Vilenkin, 1981, Gott, J. R. 1985)

Zeldovich (1980) proposed that vacuum strings produced in the early Universe could provide the fluctuations necessary to produce galaxies. Kibble et al. (1982) has shown that in the symmetry breaking of SO(10) via SU(5), stable strings can appear which survive subsequent transitions. The string is characterized by a mass per unit length μ and a tension $t = \mu$ produced by the negative pressure (in a vacuum string the only component of pressure $P_z = -\rho$ is along the direction of the string), in units where

$G = c = \hbar = 1$. In general relativity mass has units of length, so mass per unit length is a dimensionless quantity; $\mu = 1$ corresponds to one Planck mass per Planck length = $1.35 \times 10^{28} g/cm$. If the double quasar 1146+111B,C is produced by a cosmic string, its mass per unit length is $\mu \geq 4.0 \times 10^{23} g/cm$, a value large enough to be interesting for string-assisted galaxy formation (Vilenkin, A. 1981, Turok, N. 1985) and near the upper limits implied by the isotropy of the cosmic microwave background (Kaiser, N. et al. 1984) and constraints on gravitational radiation (Hogan, C. et al. 1984). But note that four other QSO's, also close enough to be double-imaged, have thus far not been detected. Searches for these and for the microwave background temperature shift across the string (Gott, J. 1985) would be useful.

The exact metric for a straight cosmic string has been given elsewhere (Gott, 1985):

$$ds^2 = -dt^2 + dz^2 + dr^2 + (1 - 4\mu)^2 r^2 d\phi^2 \quad (7-2)$$

where μ is the mass per unit length in Planck masses per Planck length, and $t, z, r,$ and ϕ are the usual cylindrical coordinates. This is a conical space with angle deficit $D = 8\pi\mu$. Because the space-time is locally flat there is no image magnification. For convenience we adopt a value of $\Omega = 1$ for the cosmic density parameter, and the Hubble constant $H_0 = 50 km/s$ Mpc. The low-peculiar-velocity string produces double-lens images of equal brightness with separations: (Gott, 1985)

$$\Delta\theta = D \left[\cos \alpha - \frac{1 - (1+z_s)^{-1/2}}{1 - (1+z_q)^{-1/2}} \right] \quad (7-3)$$

where α is the angle between the string and the plane of the sky, z_q is the redshift of the QSO, and z_s is the redshift of the string at its point of closest approach. The formula applies only for those values of α, z_s, z_q which make the term in brackets positive. If 1146+111 is produced by a string, $\Delta\theta = 2.6$ arc min and the minimum value of μ is obtained in the limit $\cos\alpha = 1, z \ll 1$ then:

$$\mu \geq 3.0 \times 10^{-5} = 4.0 \times 10^{23} gcm^{-1} \quad (7-4)$$

If this string is moving relativistically with a velocity V_s perpendicular to its own length and to the line of sight, this will produce a temperature shift

$$\Delta T \simeq 2.7DV_s c^{-1} k \simeq 2.0V_s cm^{-1} k \quad (7-5)$$

in the microwave background across the string (Gott, 1985). A detailed map to look for this effect would be invaluable. This effect will also cause a velocity shift $\Delta V \simeq V_s D$ between the two spectra. The observations (Turner, et al. 1986) indicate $\Delta V \simeq 126 \pm 309 km/s$ in the rest frame of the QSO. A velocity shift $\simeq 120 km/s$ could be produced by a perpendicular string velocity $V_s \simeq 0.5c$. The value $\mu \geq 3 \times 10^{-5}$ already pushes the allowed (Gott, J.R. 1985) upper limits, so $\cos\alpha$ cannot be small, and over the region of $1^0 \times 1^0$ near 1146+111 $\cos\alpha \simeq$ constant. Thus, without loss of generality we shall adopt $\cos\alpha = 1$ in what follows.

Any QSO lying within 2.6 arc min of the string with $z_q > 1.01$ will be double-lensed (see equation (7-3)). For $0.2 \leq z_s \leq 0.6$ we would expect four of the other QSOs in the field to be double-lensed. No equal-brightness images of these QSOs have been seen. Thus, if this lensing event is caused by a string, it is surprising that there are no other easily detectable double-lensed QSOs in the field. One reason may be because of time delays between

images(Gott, J.1985):

$$t_2 - t_1 = H_0^{-1}(\theta_1 - \theta_2)D[1 - (1 + z_s)^{-1/2}] \quad (7-6)$$

where θ_1 and θ_2 are the distances of the two images from the string. If the string passes directly down the middle of the strip then $t_2 - t_1 = 0$ for 1146+111B,C and we are guaranteed seeing them at equal brightness. The time delays for the other pairs of QSO images are, for $z_s = 0.2$ and $z_s = 0.6$ respectively:190yr and 6,000yr for the quasar at $z_q = 2.22$; 240yr and 3,900yr for the quasar at $z_q = 2.12$;450yr and 1,800yr for the quasar at $z_q = 1.10$; 300yr and 6,500yr for the quasar at $z_q = 1.93$. These delays are sufficient that the quasars could vary considerably in brightness. Mini-lensing could also affect the brightness of the different images, and as most of the string length is in the form of closed loops(Vilenkin,A. 1981, Turok, N. 1983), it is always possible that the string is significantly curved, so that it misses making images of some of these other QSOs.

Thus, vacuum strings may arise naturally as a consequence of symmetry breaking in the very early Universe. Gott(1985) has derived exact interior and exterior solutions to Einstein's field equations for vacuum strings, and has examined the gravitational lensing properties of vacuum strings. Strings can cause temperature fluctuations in the cosmic microwave background which could in principle be detected by the COBE satellite(Gott,J.R. 1985), and they can produce double quasar images separated by up to several minutes of arc. Thus gravitational lensing offers a promising way to detect vacuum strings.

7.5 Gravitational Lensing by Black Holes

A supermassive black hole can produce the lens effect (Paczynski, B. 1986). No observational or theoretical argument can be presented in favour

of this possibility at present. Should future observations point to a compact supermassive lensing object, the presence of a black hole could be established by the unique property that it would appear against the microwave background as a black spot with a diameter of ≥ 0.1 arc s (Paczynski, B. 1986). Various candidates for a gravitational lens whose images are split by a few minutes of arc have been proposed (Turner, E.L. 1986), including an unusually dense cluster of galaxies, a cosmic string or a black hole. As yet there is no observational evidence for the cluster of galaxies option, and it is worthwhile exploring the consequences of the hypothesis that the lensing object is a supermassive black hole.

The angular distance between two objects with redshifts z_1 and z_2 in a universe with dimensionless density $\Omega = 1$ and cosmological constant $\Lambda = 0$ is given as (Weinberg, S. 1972):

$$D_{ang}(z_1, z_2) = 2c/H_0(1 + z_2)[(1 + Z_1)^{-1/2} - (1 + Z_2)^{-1/2}] \quad (7-7)$$

where H_0 is the Hubble constant. This formula may be used to calculate the angular distance between the observer and the deflector, D_d ; between the observer and the source, D_s ; and between the deflector and the source, D_{ds} , as well as the effective distance to the lens, $D \equiv D_d D_{ds} / D_s$. If the source, the point-mass deflector, and the observer are perfectly aligned, then the image forms a ring with radius r_0 in the deflector's plane (Paczynski, B. 1986). This may be calculated from

$$r_0^2 = 4GM D / c^2 = 2r_g D \quad (7-8)$$

The angular radius of the ring as seen by the observer would be

$$\alpha_0 = r_0/D_d \quad (7-9)$$

Another interesting quantity is the black hole mass. For a given radius of a circular image, α_0 , it may be calculated as follows:

$$\gamma_g = \frac{\alpha_0^2 D_d D_s}{2 D_{ds}} \quad (7-10)$$

$$M = 2.3 \times 10^{15} M_\odot h_{100}^{-1} \frac{[(1+z_s)^{-1/2} - 1][(1+z_d)^{-1/2} - 1]}{(1+z_d)[(1+z_s)^{-1/2} - (1+z_d)^{-1/2}]} \quad (7-11)$$

where h_{100} is the Hubble constant in units of 100 km/sMpc^{-1} . Note that at a very small observer-deflector distance D_d , the black hole mass is simply proportional to that distance:

$$M_{BH} = 8 \times 10^5 M_\odot \times D_d/1pc \quad (7-12)$$

The only bright background that is present everywhere is the microwave background. The only instrument capable of either resolving the black spot, or at least noticing it as a negative luminosity source is the Very Large Array (VLA).

8. Another Hypothesis of the Gravitational Lens

8.1 Introduction

We will have noted that some observational data of binary quasars is ambiguous, yet various theories and hypotheses are still discussed briefly. It is well-known that the magnitude of the effect and the apparent separation of the two images both depend on the mass of the intervening

object(Weinberg, S. 1972). From previous articles, we have learned many binary quasars' redshifts and separations: 1146+111, redshift $z=1.01$; pair separation $\Delta\theta = 157$ arcs; 1635+267, $z=1.96$, $\Delta\theta = 4$ arcs (Djorgovski, S. 1984); 2345+007, $z=2.15$, $\Delta\theta = 7$ arc s (Weedman, et al. 1982). A number of known pairs or triplets of quasars have almost identical redshifts and angular separations θ between 1 and 4 arc min: Q0952+698 A,B,C, $z=2.05$, $\Delta z = 0.014$, $\Delta\theta = 3.6$ arcmin(Burbidge, E. M. et al.1980), Q0307-195 A,B, $z=2.13$, $\Delta z = 0.022$, $\Delta\theta = 1.0$ arc min (Macalpine, G. et al.1982, Shaver, P.et al.1983), U2 and U3 near NGC 2683 $z=1.26$, $\Delta z = 0.010$, $\Delta\theta = 3.6$ arc min(Arp, H.1983), Q0107-025 A,B, $z=0.96$, $\Delta z = 0.004$, $\Delta\theta = 1.3$ arc min(Paczynski, B. 1986). The known cases of split quasar images require intervening masses comparable with those of galaxies, rich clusters of galaxies, double clusters, dark matter. The 1146+111B,C separated images would require a more massive object, hence the speculation about black holes and cosmic strings. We have already reviewed these hypotheses.

But, all the previous articles have something in common. There are no articles which discuss the role of the electromagnetic field in this phenomenon. Only one phenomenon, the dependence on mass, was discussed. Of course, that is an undeniable factor. The main purpose is to seek an explanation of the larger angular separation of double quasars, for example 1146+111B,C. Then, various candidates for a gravitational lens whose images are split by a few minutes of arc have been proposed(Turner, E. L.et al.1986). But unfortunately, no observational or theoretical argument can be presented in favour of these various possible hypotheses at present(Paczynski,1986). Suppose the proposed program is carried out, and we become convinced that some widely separated pairs of quasars are, in fact, gravitationally lensed. We still would not know if the lensing were

due to a cosmic string, to a cluster of faint galaxies, or to some other object. It is hard to think of a test that would demonstrate that a given pair of point-like quasars is, in fact, a single object lensed by a cosmic string or some other object, and not by something else.

However, we well know that any physical field should influence the vacuum except the gravitational field(Dirac, 1975). In fact, most celestial bodies possess electric or magnetic fields. Black holes can also possess an electric field. In general, we say “a black hole with three hairs” (Weinberg, S. 1972). Neutron stars possess strong magnetic fields, $\sim 10^{12}$ gauss(Shapiro, S. L. et al. 1983). Recently, the superconductive cosmic string which possesses an electromagnetic field has been studied. It is known that certain spontaneously broken gauge theories give rise to stable strings or vortex lines(Witten, E. 1985, Vilenkin, A. et al. 1987). There has been considerable interest in the cosmological effects of strings which could be produced at a phase transition in the early Universe. Superheavy strings can generate cosmologically interesting density fluctuations and can also produce a number of distinctive observational effects(Chudnovsky, E. M. et al. 1986). The gravitating dyon black hole was proposed by some authors(Kasuya, M.1982, Julia, B. et al. 1975). It has both a magnetic and electric field. Moreover, these celestial bodies with electromagnetic fields may intervene between two quasar images, and so produce a lens effect. Thus, a unified consideration of the influence of electromagnetic fields in gravitational lens systems will have a real significance. Here, I shall explore the gravitational lens effect arising from a gravitating dyon black hole with the aid of previous theory. I have found that the electromagnetic field will provide an influencing term to the gravitational lens system. It is pointed out that the main difference between classical general relativity and the gravitating dyon

black hole is this influence term. This electromagnetic term will suppress the effect of the gravitational lens - for example, the angular separation of two apparent images can shift towards a smaller value, as the time delay should be reduced. We recall that people used supermassive objects with all their might to explain the larger angular separation of quasars, as if the smaller angular separation had been neglected. In fact, we still would not be clear about what intervening objects can cause different angular separation. Thus, in principle, we can discover the new dark matter in the sky through observations of different gravitational lens systems. If the lensing object is a gravitating dyon black hole then it might be seen as a dark spot against a bright background (Paczynski, B. 1986). A gravitating dyon black hole captures microwave background radiation and therefore it may appear as a 'source' with a negative luminosity.

8.2 Gravitating Dyon Black Hole

Dirac (1931, 1948) pointed out that quantum mechanics does not preclude the existence of magnetic monopoles. The interest in this subject has been enhanced by a recent observation of 't Hooft that classical solutions having the properties of magnetic monopoles may be found in a Yang-Mills theory with spontaneous symmetry breaking and with a suitable identification of the electromagnetic field ('t Hooft, G. 1974, Yang, C. N. and Mills, R. 1954). Furthermore, Schwinger proposed the dyon, i.e., a pole possessing both electric and magnetic charges (Schwinger, J. 1966). This dyon exists in Abelian theory. On the other hand, in non-Abelian theory, the 't Hooft magnetic monopole and the Julia-Zee dyon were spherically symmetric classical solutions of $SO(3)$ Yang-Mills theory coupled with a triplet Higgs field (Julia, B; Zee, A. 1975). The solution of the Einstein-Maxwell

equations in Kerr space-time was obtained by Newman et al. 1965. This solution corresponds to a rotating ring of mass and electric charge. Tomimatsu and Sato(1972) discovered a series of solutions for the gravitational field of a rotating mass, following Ernst's(1973) formulation of an axisymmetric stationary field. Furthermore Yamazaki(1978) obtained the charged Kerr-Tomimatsu-Sato family of solutions with arbitrary integer distortion parameter for gravitational fields of rotating masses.

Kasuya(1982,a) presented an exact solution of a rotating dyon black hole in the Tomimatsu-Sato-Yamazaki space-time and studied a special case. The Lagrangian density which describes the electromagnetic field induced by a dyon in curved space-time ($\hbar = c = 1$) is as follows:

$$\mathcal{L} = -\frac{1}{16\pi G} \sqrt{-g} R + \mathcal{L}_D \quad (8-1)$$

The field equation for $g_{\mu\nu}$ is

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 8\pi GT^{\mu\nu} \quad (8-2)$$

where the energy-momentum tensor of the dyon is given by

$$T^{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_D}{\delta g_{\mu\nu}} \quad (8-3)$$

δ is an arbitrary integer distortion parameter(Yamazaki,1978). \mathcal{L}_D is for different dyon. In Abelian theory, it is Lagrangian density which describes the electromagnetic field induced by a Schwinger dyon(Kamata, M. 1981) is:

$$\mathcal{L}_D = -\frac{1}{4} \sqrt{-g} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \quad (8-4)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + G_{\mu\nu}^*$, and $G_{\mu\nu}^*$ is the Dirac string term (Kasuya, M. 1982). The field equation for A_μ is

$$\partial_\nu(\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}) = 0 \quad (8-5)$$

The energy-momentum tensor of the electromagnetic field is given by

$$T^{\mu\nu} = -(g^{\mu\alpha}g^{\rho\beta}F_{\alpha\beta}F_{\nu\rho} - \frac{1}{4}\delta_\nu^\mu g^{\rho\alpha}g^{\sigma\beta}F_{\rho\sigma}F_{\alpha\beta}) \quad (8-6)$$

In non-Abelian theory, \mathcal{L}_D is the Lagrangian density which describes the electromagnetic field induced by a Julia-Zee dyon (Kasuya, M. 1981):

$$\mathcal{L}_D = \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a - \frac{1}{2} g^{\mu\nu} D_\mu \phi^a D_\nu \phi^a - \lambda V(\phi) \right] \quad (8-7)$$

with

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + e\epsilon_{abc}A_\mu^b A_\nu^c$$

$$D_\mu \phi^a = \partial_\mu \phi^a + e\epsilon_{abc}A_\mu^b \phi^c$$

$$V(\phi) = 1/4(\phi^a \phi^a)^2 - 1/2\nu^2(\phi^a \phi^a), \quad \nu^2 = \mu^2/\lambda,$$

where $\mu \equiv GM$, $\nu^2 = \langle \phi^2 \rangle$ determines the vacuum expectation value of the triplet Higgs field ϕ^a . Since the gauge field acts as a source of the gravitational field, it is apparent that nontrivial topologies can be generated by the interaction of the Yang-Mills field, the Higgs field, and the associated gravitational field. The equations of motion for the Yang-Mills field A_μ^a and the Higgs field ϕ^a are

$$D_\nu(\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}^a) + \sqrt{-g}g^{\mu\nu}e\epsilon_{abc}\phi^b D_\nu \phi^c = 0 \quad (8-8)$$

$$D_\mu(\sqrt{-g}g^{\mu\nu}D_\nu \phi^a) - \sqrt{-g}\lambda[(\phi^b \phi^b) - \nu^2]\phi^a = 0$$

where the isospace indices $a, b, c = 1, 2, 3$. The energy-momentum tensor of this system is given by

$$T^\mu_\nu = -\left(g^{\mu\alpha} g^{\rho\beta} F_{\alpha\beta}^a F_{\nu\rho}^a - \frac{1}{4} \delta^\mu_\nu g^{\rho\alpha} g^{\sigma\beta} F_{\rho\sigma}^a F_{\alpha\beta}^a\right) - \left(g^{\mu\rho} D_\rho \phi^a D_\nu \phi^a - \frac{1}{2} \delta^\mu_\nu g^{\rho\sigma} D_\rho \phi^a D_\sigma \phi^a\right) + \delta^\mu_\nu \lambda V(\phi) \quad (8-9)$$

The stationary axisymmetric dyon solution is a straightforward extension of the Tomimatsu-Sato-Yamazaki solutions in Boyer-Lindquist coordinates and is given by (Yamazaki, M. 1978, Kasuya, M. 1982):

$$ds^2 = \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{B_\delta \Delta \sin^2 \theta}{A_\delta} d\varphi^2 - \frac{A_\delta}{B_\delta} \left[dt - \frac{2aC_\delta \sin^2 \theta}{\sigma A_\delta} d\varphi \right]^2 \quad (8-10)$$

where the relations among the Boyer-Lindquist coordinates (t, r, θ, φ) , the Weyl coordinates (t, ρ, z, φ) , and the prolate spheroidal coordinates (t, x, y, φ) are given by (Kasuya, M. 1982).

The stationary axisymmetric dyon solution in Kerr-Newman form is given for the case of $\delta = 1$ in (8-10) by Kasuya, M. (1982)

$$A_1 = \frac{1}{(GM\sigma)^2} \left\{ \Sigma - G \left[2Mr - \frac{1}{4\pi} (Q^2 + \Phi^2) \right] \right\}$$

$$B_1 = \frac{1}{(GM\sigma)^2} \Sigma \quad (8-11)$$

$$C_1 = -\frac{\sigma G}{2(GM\sigma)^2} \left[2Mr - \frac{1}{4\pi} (Q^2 + \Phi^2) \right]$$

where

$$\Delta = (r - GM)^2 - [(GM)^2 - a^2 - G/4\pi(Q^2 + \Phi^2)] \quad (8-12)$$

$$\Sigma = r^2 + a \cos^2 \theta$$

where M is the mass of the dyon black hole, Q is the electric charge, Φ is the magnetic charge, the parameter a is related to the angular momentum S and the mass M by $a = S/(GM)$.

Kasuya, M.(1982) proved that both the Abelian(i.e.,Schwinger) and the non-Abelian(i.e., Julia-Zee) dyon solutions have the same metric(8-11). From the metric(8-10),i.e.,(8-11),we can show, from(8-12),in Boyer-Lindquist coordinates that there exists an event horizon at

$$r = GM + [(GM)^2 - a^2 - G/4\pi(Q^2 + \Phi^2)]^{1/2}$$

Thus, the dyon solution represents a black hole provided that

$$a^2 + G/4\pi(Q^2 + \Phi^2) \leq (GM)^2 \quad (8-13)$$

We call it a dyon black hole. Furthermore the infinite red-shift surface under (8-13) is given by

$$r = GM \pm [(GM)^2 - a^2 \cos^2 \theta - G/4\pi(Q^2 + \Phi^2)]^{1/2} \quad (8-13,a)$$

The structure of the space-time is schematically shown in Figure 8. This is an exact solution of a rotating dyon black hole with the Kerr-Newman metric ($\delta = 1$). The solution represents the gravitational, electric, and magnetic fields of mass M , and angular momentum, i.e., the four physical parameters(mass M , angular momentum S , electric charge Q , and

magnetic charge Φ), represent a black hole. This dyon solution leads to “a black hole with four hairs”. Taking $G = 0$ in (8-10) and (8-11), the metric reduces to flat space-time (Shima, K and Kasuya, M. 1980). We then have a rotating dyon solution flat space-time. Furthermore, in the case $a = 0$, it reduces to the spherically symmetric dyon solution ('t Hooft, 1974, Julia, B; Zee, A. 1975). Various limiting cases are schematically shown in Figure 9.

It is pointed out that a constant cosmological term has been neglected in the previous derivations of the Julia-Zee dyon. In fact, when $a = 0, \delta = 1$, the solution reduces to the spherically symmetric static dyon black hole solution (Kasuya, M. 1982,b):

$$(g_{\mu\nu}) = \text{diag}(-e^X, e^Y, r^2, r^2 \sin^2 \theta) \quad (8-14)$$

where $\mu, \nu = t, r, \theta, \varphi$ and X and Y are functions of r only, and

$$e^X = e^{-Y} = \frac{1}{3} \Lambda r^2 + 1 - \frac{2GM}{r} + \frac{G(Q^2 + \Phi^2)}{4\pi r^2} \quad (8-15)$$

with

$$\Lambda \equiv 8\pi G v_0 = \text{const} \quad (8-16)$$

It is pointed out that the cosmological term Λ comes out of the Higgs field ϕ^a . The field energy due to the A_μ^a and the ϕ^a can be obtained by the volume integral of the T_t^t :

$$E = \iiint_{r_0}^{r_\infty} T_t^t (-g)^{-1/2} dr d\theta d\varphi = \frac{Q^2 + \Phi^2}{8\pi} \left[-\frac{1}{r} \right]_{r_0}^{r_\infty} - \frac{\Lambda}{6G} \left[r^3 \right]_{r_0}^{r_\infty} \quad (8-17)$$

We see that the cosmological term gives a universal repulsion, and that the first and the second term in (8-17) have singularities at $r_0 = 0$ and at $r_\infty = \infty$, respectively. Einstein called the cosmological term “the biggest blunder of my life”, and abandoned it. Kasuya takes a new Higgs potential $\tilde{V}(\phi)$, given by

$$\tilde{V}(\phi) = 1/4[(\phi^a \phi^a) - v^2]^2$$

for which $\tilde{V}(\phi) = 0$ with $v = \pm C \Sigma$ (kasuya,1982,b). Thus the cosmological term does not appear in previous theory. Since the cosmological term is a constant, it does not contribute to the field equations of the Yang-Mills field A_μ^a and the Higgs field ϕ^a . Therefore, it can be neglected from the beginning. Thus, the spherically symmetric static dyon black hole solution is

$$e^X = e^{-Y} = 1 - \frac{2GM}{r} + \frac{G(Q^2 + \Phi^2)}{4\pi r^2} \quad (8 - 18)$$

8.3 The Lensing Effect of the Gravitating Dyon Black Hole

The gravitational lens phenomenon is based on the following essential and indisputable fact, i.e., the light rays from the source can be deflected by intervening objects between two quasar images. Various effects which arise from this deflected light have brought us many ambiguous data. Thus, various theoretical hypotheses have been produced. But in this area of investigation, the first direct determination of the path of the light ray is yet to appear. Most hypotheses were on the basis of a statistical principle and observational phenomena. Here, I will discuss the path of a light ray from a source to an observer in a lens system. Like most people (Gorenstein, M. V., et al. 1988, Gott, J. 1985, Paczynski, B., et al. 1984, Crawford, C 1986, ect), I discuss the gravitational lens arising from a gravitating dyon

black hole but do not pursue the probability of the existence such a system.

For simplicity, we consider the gravitational field arising from a static spherically symmetric dyon black hole, assuming that the redshift of the gravitating dyon black hole is smaller than the redshift of the quasar. The light ray from the source to the observer should be deflected by the gravitational field of the black hole. We have the metric (8-10). Due to the condition of a static, spherically symmetric metric ($a = 0, \delta = 1$), from (8-10) and (8-11), we obtain:

$$g_{\mu\nu} = \begin{pmatrix} e^X & & & \\ & e^Y & & \\ & & r^2 & \\ & & & r^2 \sin^2 \theta \end{pmatrix} \quad (8-19)$$

where e^X and e^Y are represented in (8-18). We consider the path of a light ray in a centrally symmetric gravitational field. As in every centrally symmetric field, the motion occurs in a single "plane" passing through the origin; we choose this plane to be the plane $\theta = \pi/2$. This path is determined by the eikonal equation

$$g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi = 0 \quad (8-20)$$

where ψ is the eikonal. Substituting metric (8-19) into (8-20), we find the following equation:

$$\left[1 - \frac{2GM}{r} + \frac{G(Q^2 + \Phi^2)}{4\pi r^2} \right]^{-1} \left(\frac{\partial \psi}{\partial t} \right)^2 - \left[1 - \frac{2GM}{r} + \frac{G(Q^2 + \Phi^2)}{4\pi r^2} \right] \left(\frac{\partial \psi}{\partial r} \right)^2 - \frac{1}{r^2} \left(\frac{\partial \psi}{\partial \phi} \right)^2 = 0 \quad (8-21)$$

By the general procedure for solving the Hamilton-Jacobi equation, we look for a ψ in the form

$$\psi = -\omega_0 t + L\varphi + \psi_r(r) \quad (8-22)$$

where ω_0 is the frequency of the light, L is the moment of the impulse, and $\psi_r(r)$ is an undetermined function. From (8-22), we obtain

$$\frac{\partial \psi}{\partial t} = -\omega_0, \quad \frac{\partial \psi}{\partial r} = \frac{d\psi_r(r)}{dr}, \quad \frac{\partial \psi}{\partial \varphi} = L$$

Substituting these formulae into (8-21), we find the derivative $d\psi_r(r)/dr$, and thus:

$$\psi_r(r) = \int \left[1 - \frac{2GM}{r} + \frac{G(Q^2 + \Phi^2)}{4\pi r^2} \right] \sqrt{\omega_0^2 - \frac{L^2}{r^2} \left[1 - \frac{2GM}{r} + \frac{G(Q^2 + \Phi^2)}{4\pi r^2} \right]} \cdot dr \quad (8-23)$$

The trajectory itself is determined by the equation $\frac{\partial \psi}{\partial L}$, combined with (8-22) and (8-23), so that

$$\varphi = \int \frac{\frac{L}{r^2} dr}{\sqrt{\omega_0^2 - \frac{L^2}{r^2} \left[1 - \frac{2GM}{r} + \frac{G(Q^2 + \Phi^2)}{4\pi r^2} \right]}} \quad (8-24)$$

Thus, we obtain a differential equation:

$$\left(\frac{L}{r^3} \frac{dr}{d\varphi} \right)^2 = \omega_0^2 - \frac{L^2}{r^2} \left[1 - \frac{2GM}{r} + \frac{G(Q^2 + \Phi^2)}{4\pi r^2} \right] \quad (8-25)$$

Defining a new variable $y = 1/r$, and inserting it into (8-25), we

obtain:

$$\left(\frac{dy}{d\varphi}\right)^2 = \frac{\omega_0^2}{L^2} - y^2 + 2GM y^3 - \frac{G(Q^2 + \Phi^2)}{4\pi} y^4 \quad (8-26)$$

Defining $H = dy/d\varphi$, and differentiating with respect to φ :

$$\frac{d^2y}{d\varphi^2} = -y + 3GM y^2 - \frac{G}{2\pi} (Q^2 + \Phi^2) y^3 \quad (8-27)$$

The formula (8-27) determines the trajectory of the light in the gravitating field of the dyon black hole. If we neglect the relativistic corrections, i.e., neglect smaller quantities which are second term and third term in (8-27), this equation gives a solution of the zeroth order approximate:

$$y_0 = \frac{\cos\varphi}{R} \quad (8-28)$$

where $1/R = \omega_0/CL$, i.e. a straight line passing at a distance CL/ω_0 from the origin. To study the relativistic corrections, we define $y = y_0 + y_1$, insert this into (8-27), and neglect higher order small quantities. therefore, we find an equation with respect to y_1 :

$$\frac{d^2y_1}{d\varphi^2} + y_1 = 3GM \frac{\cos^2\varphi}{R^2} - \frac{G}{2\pi} (Q^2 + \Phi^2) \frac{\cos^3\varphi}{R^3} \quad (8-29)$$

Thereafter, we proceed with a series of tedious operations. We obtain

the trajectory equation of the light ray:

$$y = \frac{\cos\varphi}{R} + \frac{GM}{R^2} (1 + \sin^2\varphi) - \frac{G(Q^2 + \Phi^2)}{8\pi R^3} (\cos\varphi + \frac{3}{2}\varphi \sin\varphi + \frac{1}{2}\sin^2\varphi \cos\varphi) \quad (8-30)$$

Finally, going to the limit $r \rightarrow \infty$, and looking for a φ in the form

$$\varphi = \pm \frac{\pi}{2} + \frac{1}{2}\Delta\varphi$$

where $\Delta\varphi$ is a small quantity, and inserting φ into (8-30), and performing some straightforward algebra, one arrives at:

$$\frac{1}{2}\Delta\varphi = \pm \left[\frac{2GM}{R} - \frac{3G}{32R^2} (Q^2 + \Phi^2) \right] \quad (8-31)$$

This means that under the influence of the field of the dyon black hole the light ray is bent: its trajectory is a curve which is concave toward the center (the ray is "attracted" toward the center), so that the angle between its two asymptotes is

$$\Delta\varphi = \left[\frac{4GM}{R} - \frac{3G}{16R^2} (Q^2 + \Phi^2) \right] \quad (8-32)$$

In other words, the ray of light, passing at a distance CL/ω_0 from the center of the field, is deflected through an angle $\Delta\varphi$.

8.4 Discussion and Conclusion

We have obtained the trajectory of the light ray passing at a distance R from the center of the field. This field arises from the gravitating dyon

black hole. Like cosmic strings, it exists in those theories with spontaneous symmetry breaking. Both the Abelian and non-Abelian dyon solutions have the same metric (8-11) (Kasuya, M. 1982): we have used this metric to study the effect of the gravitational lens, and obtained the orbit equation of the light ray,(8-32). Further, in electromagnetic theory(Jackson, J. D. 1975), equation (8-32) becomes the formula we are accustomed to:

$$\Delta\varphi = \left[\frac{4GM}{R} - \frac{3GR^2}{16} (E_R^2 + B_R^2) \right] \quad (8-33)$$

This is the angle between the two asymptotes of a bent light ray under relativistic corrections. It is pointed out here that E_R and B_R are the electric field strength and magnetic field strength respectively at a distance R from the center of the field.

From equation (8-33), as we have seen, a light ray, passing at a distance R from the center of the field of the gravitating dyon black hole, is deflected through an angle $\Delta\varphi$. In addition, we should note clearly that when the black hole possesses an electromagnetic field, the deflective angle of the light ray is smaller than when the black hole does not possess an electromagnetic field. This difference arises from an influence term, $3GR^2/16(E_R^2 + B_R^2)$ in the formula (8-33). In other words, the effect of the gravitational lens was reduced by the existing electromagnetic field. The strength of this reducing effect of the gravitational lens is proportional to the electromagnetic field of the black hole. If we only consider the influence of the electric field and neglect the magnetic field, then equation (8-33) should become coincident with that of the Reissner-Nordstrom space-time

(Chandrasekhar, S. 1983). Of course, if we do not consider this influence term, i.e., if both E and B are zero, The equation (8-33) will reduce to :

$$\Delta\varphi = \frac{4GM}{R} \quad (8-34)$$

Very clearly, this coincides with classical general relativity theory (Weinberg, S. 1972). Thus, the main difference of the gravitational lens between the gauge theory and classical general relativity theory is further considering the contribution of the electromagnetic field. At first sight, the contribution of this correction term is a positive one, ($\propto E_R^2, B_R^2$). But in fact, we are not clear about the electric intensity and magnetic intensity of the black hole, i.e., the electromagnetic field intensity which is located at R is unknown. In addition, the gravitating dyon black hole is smaller than other black holes (Ref.Fig.8). So the spatial extent of its influence is smaller than for more massive objects, for example cosmic strings, black holes, rich galaxies, clusters of galaxies, etc, i.e., R is smaller here. However, this correction term is important. Obviously, $\Delta\varphi$ will decrease along with the increasing of the electromagnetic field strength. Thus, we can obtain an inequality:

$$E_R^2 + B_R^2 < 64M/3R^3 \quad (8-35)$$

This is an upper limit to the electromagnetic field strength. It restricts the range of validity of the gravitational lens arising from the gravitating dyon black hole. This upper limit will reduce rapidly along with the increasing of R, and is proportional to M. If we had:

$$E_R^2 + B_R^2 = 64M/3R^3$$

the gravitational lens would lose efficiency. If we had $E_R^2 + B_R^2 >$

$64M/3R^3$, the dyon black hole would produce the effect of a divergent lens instead of a convergent one, and so could not concur with observations.

Whenever we obtain a prediction from general relativity, the question always arises of whether the result obtained really refers to an objective physical measurement or whether it has folded it into some arbitrary subjective elements dependent on our choice of coordinate system. In the case at hand, we should ask ourselves what the predicted change in φ really has to do with the position of stellar images on the photographic plates. Fortunately, the answer here is quite simple, for this is really a scattering experiment. The light ray comes in from a very great distance, is deflected as it passes close to the black hole, and is detected on earth, much the same distance away from the black hole as is the quasar. At the points of origin and detection the metric is essentially Minkowskian, and at these distances there is no question about the meaning of φ ; it is the azimuthal angle in a system of coordinates within which light rays define lines that are essentially straight. Hence we can relate $\Delta\varphi$ to the shift of stellar images on photographic plates by the ordinary rules of geometric optics (we are here neglecting effects of the gravitational field of the earth itself, because this field is very much weaker than the black hole).

If the effect of gravitational lensing does arise from dyon black holes, it would bring about some consequences that are worthwhile exploring. The angular-size distances are proportional to H_0^{-1} (Gorenstein, M. V., et al. 1988). In the gravitational lens system, without loss of generality, the quantities D_d, D_s , and D_{ds} represent respectively the observer-deflector, observer-source, and deflector-source angular-size distances. These angular-size distances are given by (Refsdal 1966):

$$D = \frac{1+z_1}{2H_0} [(1+z_1)^{-2} - (1+z_2)^{-2}] \quad (8-36)$$

where $D = D_d$ for $z_1 = z_d, z_2 = 0$; $D = D_s$ for $z_1 = z_s, z_2 = 0$; and $D = D_{ds}$ for $z_1 = z_d, z_2 = z_s$, as well as the effective distance to the lens, $D \equiv D_d D_{ds} / D_s$. If the source, the point-mass deflector, and the observer are perfectly aligned, then the image forms a ring with radius r_0 in the deflector's plane. This may be calculated from (we always use units with $\hbar = c = 1$):

$$r_0^2 = 4GMD = 2r_g D \quad (8-37)$$

The angular radius of the ring as seen by the observer would be

$$\theta_0 = r_0 / D_d \quad (8-38)$$

If the lensing object is a black hole then it might be seen as a dark spot against a bright background. For a Schwarzschild black hole, the dark spot is a circle with a diameter $3^{3/2} r_g$. The angular diameter of the dark circle as seen by the observer is

$$d_{BH} = 3^{3/2} r_g / D_d \quad (8-39)$$

A black hole captures microwave background radiation with a cross-section of $27\pi r_g^2$, and therefore it may appear as a 'source' with a negative luminosity, $L_{BH} = -27\pi r_g^2 \sigma [2.7 \times (1+z)]^4$ (Paczynski 1986). Another question is that of time delay. From equation (8-33), we see that $\Delta\varphi$ will decrease because of the influence of the electromagnetic field. Clearly, the meaning of this is that the deflection of the light ray will decrease along with the decrease in $\Delta\varphi$. Thus, some supermassive lensing objects can produce larger time delays, for example vacuum strings can produce delays

of $\sim 10^4$ yr. Rich galaxies can produce $\sim 10^3$ yr,etc: we can say here that the time delay arising from a dyon black hole will be smaller than the time delays which arise from more massive objects.

A more important and interesting question is whether the gravitating dyon black hole can actually be observed. Maybe we can obtain an answer by considering the effect of the tidal field. Since from equation (8-33), we see that the deflected angle $\Delta\varphi$ will decrease along with the increase of E_R, B_R , this means that the observed apparent separation of the two images will be reduced. Thus in principle, if future observations should discover that the apparent separation of the two images is smaller than their critical tidal radius, and that their images should have the same properties, such as redshift, luminosity and spectra etc, we would say that this lens system arises from a dyon black hole. In other words, we will have discovered a gravitating dyon hole in the sky through observations of a special lens system. Then, the radius of the tidal is a lower limit. If the apparent separation of the binary quasar was smaller than this lower limit, we would have to consider some new form of dark matter. This is the natural inference that is based on the previous discussion. We can obtain the distances d between the two images from equation (8-33) and (8-36):

$$d = \frac{1}{4H_0(1+z_s)} \left[\frac{4GM}{R} - \frac{3GR^2}{16}(E_R^2 + B_R^2) \right] \quad (8-40)$$

where z_s is the redshift of the quasar. we have seen that this distance d can decrease along with the increase of the electromagnetic field. In the theory, so long as this electromagnetic field satisfies the inequality (8-35), this distance d can be reduced so that the apparent separation of the two images exceeds their actual critical tidal radius.

From the previous discussion, we have seen this theory add a correction term as compared with classical general relativity. The contribution of the correction term for the electromagnetic field does not increase the effect of the gravitational lens but reduces this effect. Unfortunately, it can produce a resolution limit to observation, i.e., there are non-resolvable sources in the sky. Higher resolution observations will be required. Future progressive instruments, for example the new Very Long Baseline (VLB), may realise this requirement. If there are dyon black holes, the only bright background that is present everywhere is the microwave background. The only instrument capable of either resolving the black spot, or at least noticing it as a negative luminosity source is the Very Large Array (VLA). In fact, the black-spot detection might be feasible with the present VLA instrumentation (Paczynski 1986). It would be very interesting to look for this spot if future analysis indicates the existence of a very compact supermassive object.

In addition, such a consequence would have a general significance. It is not only appropriate to gravitating dyon black holes, but also to other objects which possess electric or magnetic fields. In fact, we still would not be clear about the electric or magnetic intensities of various intervening objects, so that we do not exactly determine the apparent separation distances d . So, it is not necessary that every lens system should arise from a coupled gravitational and electromagnetic force to produce the apparent separation or the non-resolution. So long as the electromagnetic field is not intense, we can obtain this consequence from observation today. Thus, we can ask for quasar pairs which have been identified as the products of gravitational lenses whether we have included the contribution of the electromagnetic force? On the other hand, we can utilize this theory to discover the new

matter in the sky from observations of various lens systems.

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QQ 1145-071

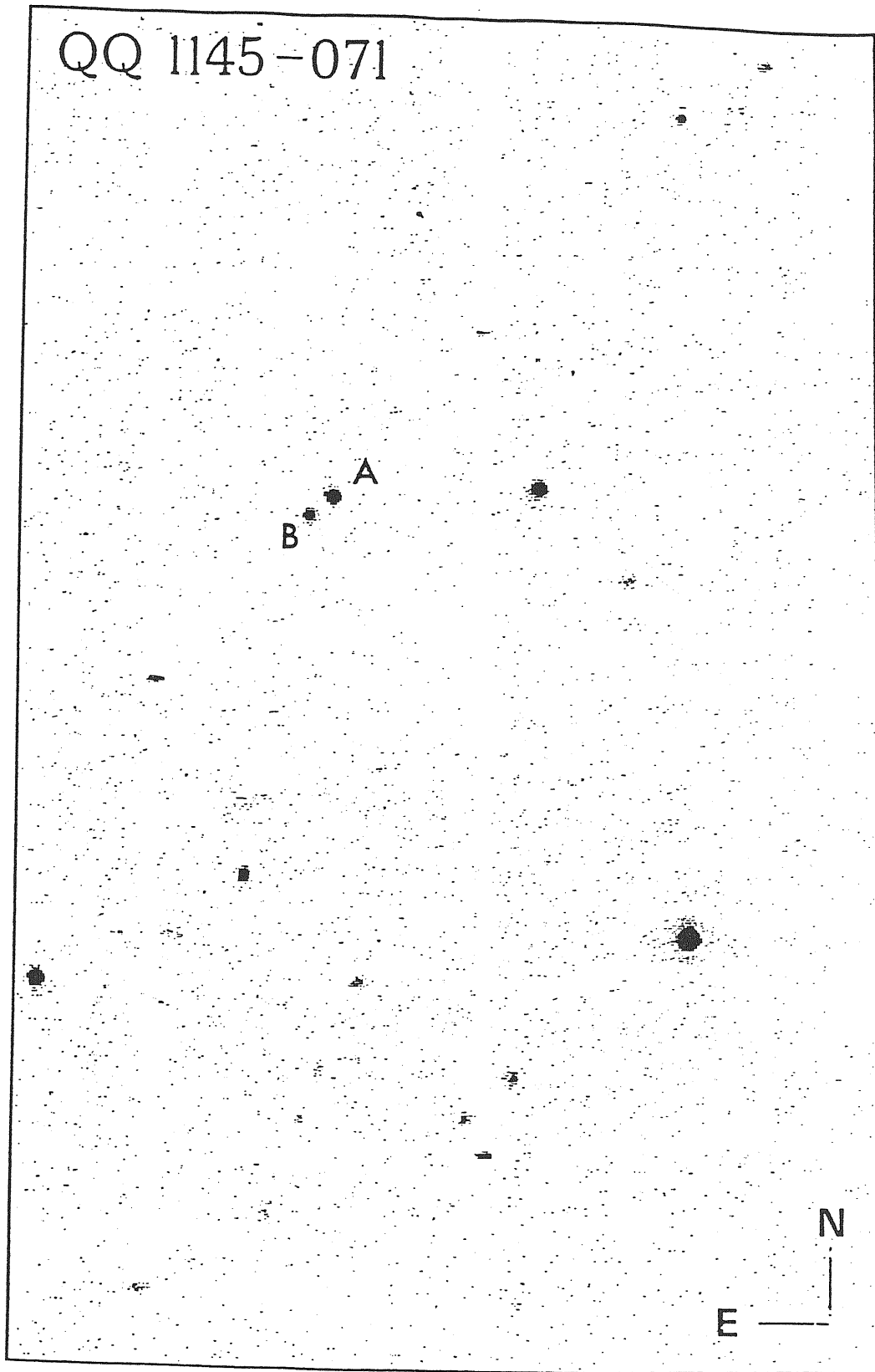


Fig.1 V-band CCD image of the PKS 1145-071 field, obtained at ESO. The field size is 115 by 184 arcsec, North to the top, East to the left. The two QSO's are labeled as "A" and "B".

(Adopted from Djorgovski, S. et al. 1988)

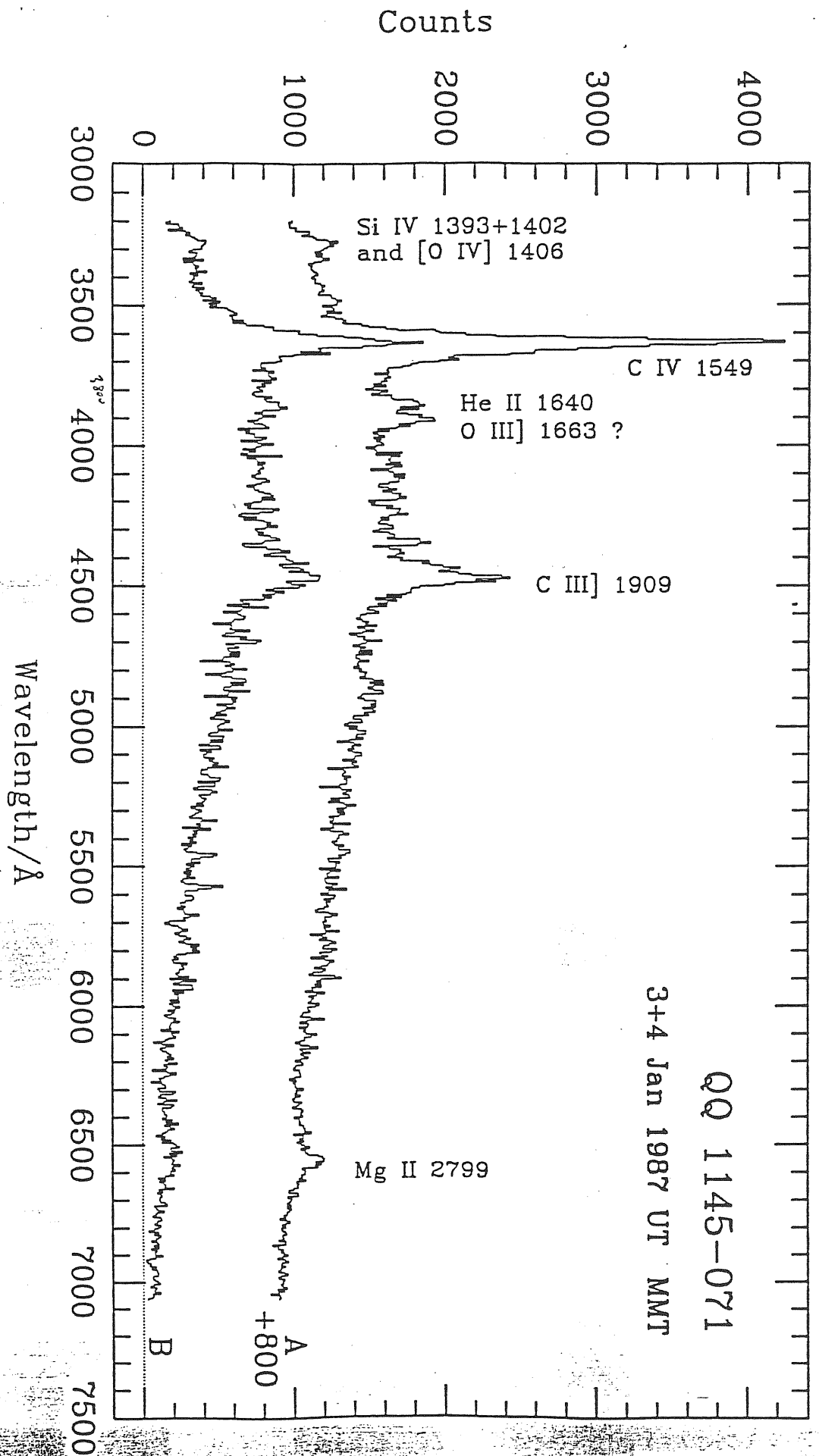
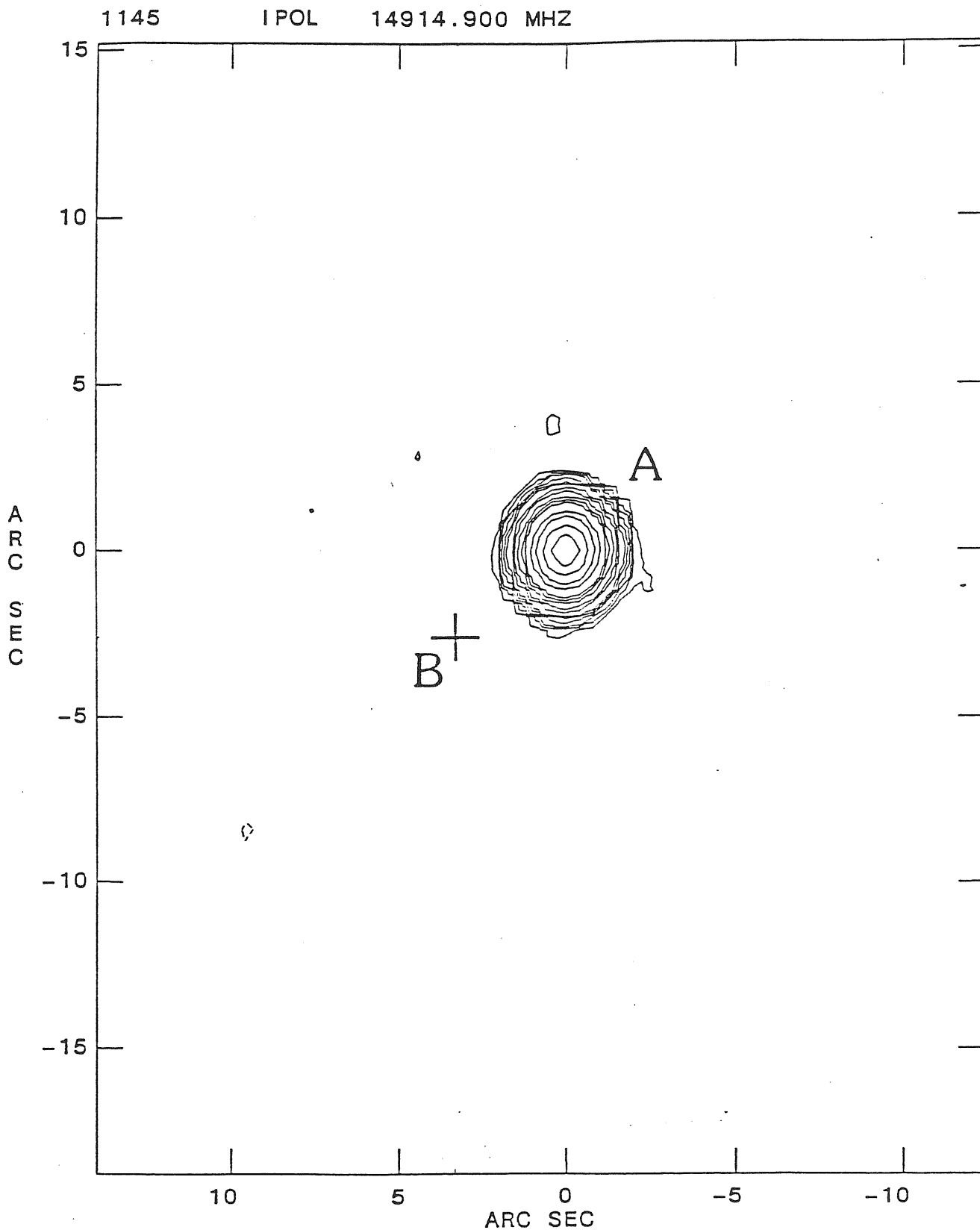


Fig.2 The spectra of the two quasars, obtained at the MMT: QSO A on the top, B on the bottom. The spectrum of the QSO A was shifted up by 800 units for clarity. Both spectra were rebinned to 6 \AA bins (the instrumental resolution is $\text{FWHM} \approx 12 \text{ \AA}$). The relative flux scale is arbitrary.



CENTER AT RA 11 45 18.300 DEC -07 08 1.00
 PEAK FLUX = 6.4447E-01 JY/BEAM
 LEVS = 6.4447E-03 * (-0.100, 0.100, 0.200,
 0.300, 0.500, 0.750, 1.000, 2.000, 3.000,
 5.000, 7.500, 10.00, 20.00, 30.00, 50.00,
 75.00)

Fig.3 Radio map of PKS 1145-071, obtained at VLA at 2cm, in the C configuration. The position of the QSO B is marked with the cross. The residual noise in this map is 0.23 mJy.

(Adopted from Djorgovski, S. et al. 1988)

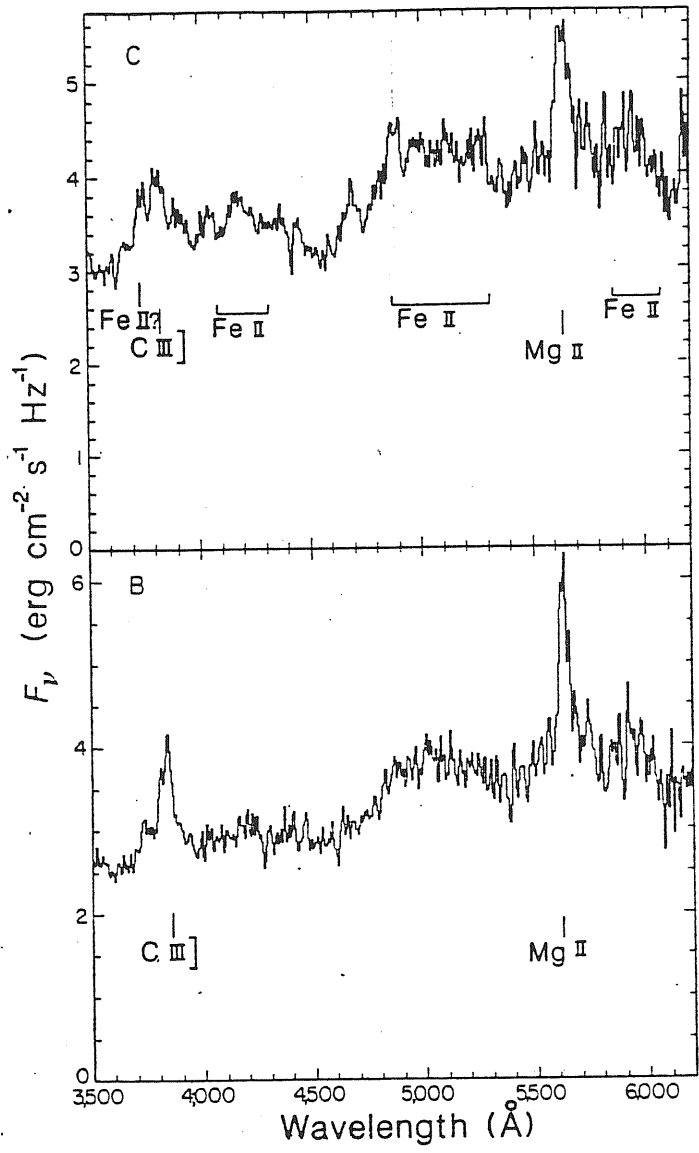


Fig.4 Summed observations with the MMT spectrograph of objects B and C with features identified.

(Adopted from Huchra, J. p. 1986)

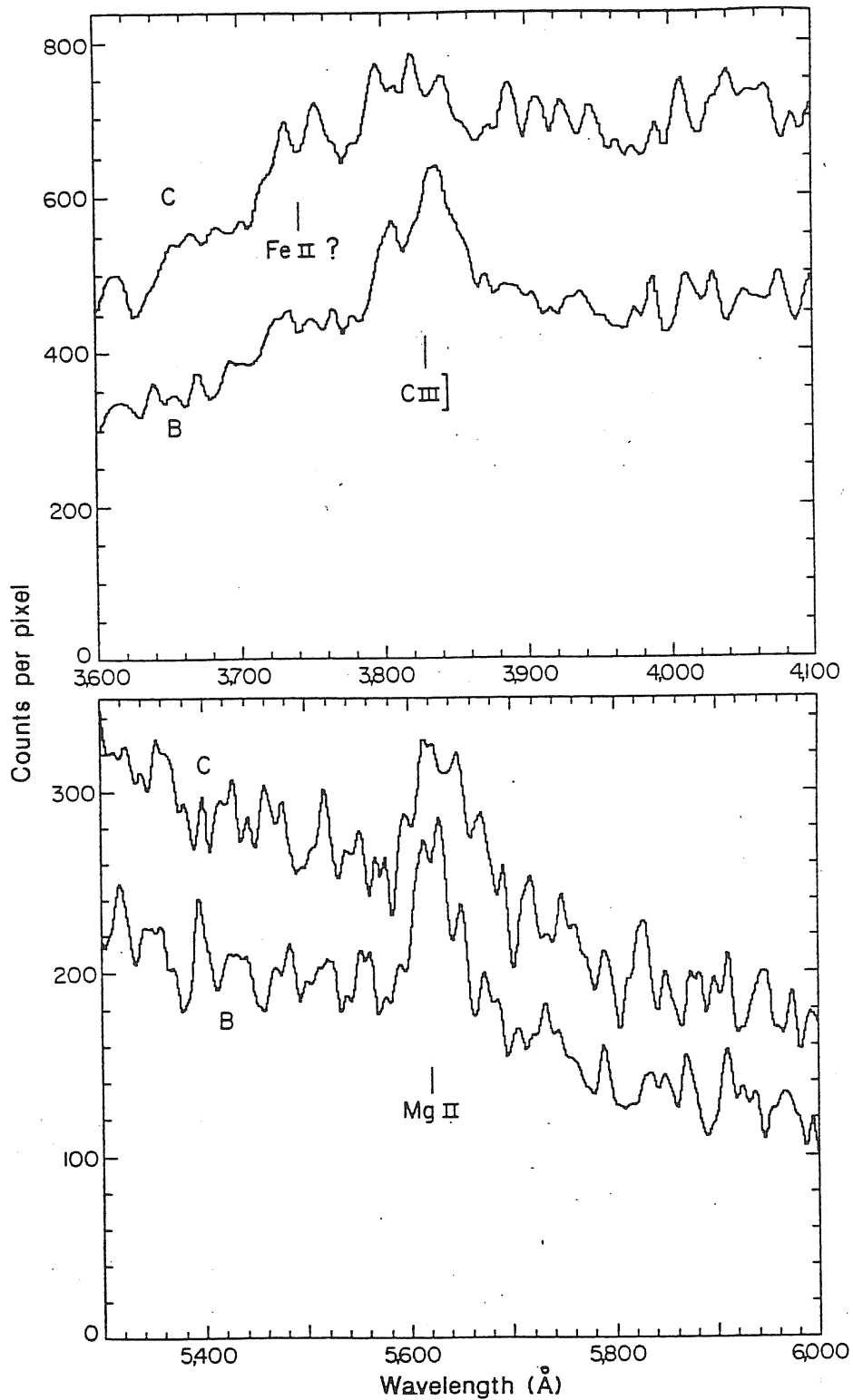


Fig. 5 Enlargements of the raw (counts per pixel) summed spectra of B and C in the regions of C III] (upper) and Mg II. The slight redshift of the Mg II line in B relative to the line in C, and the slight blueshift of the C III] line in B relative to C. The Mg II line in C also appears broader than that in B, but this may be an artefact given the low signal-to-noise ratio of the spectra at 5,600 Å.

(Adopted from Huchra, J. p. 1986)

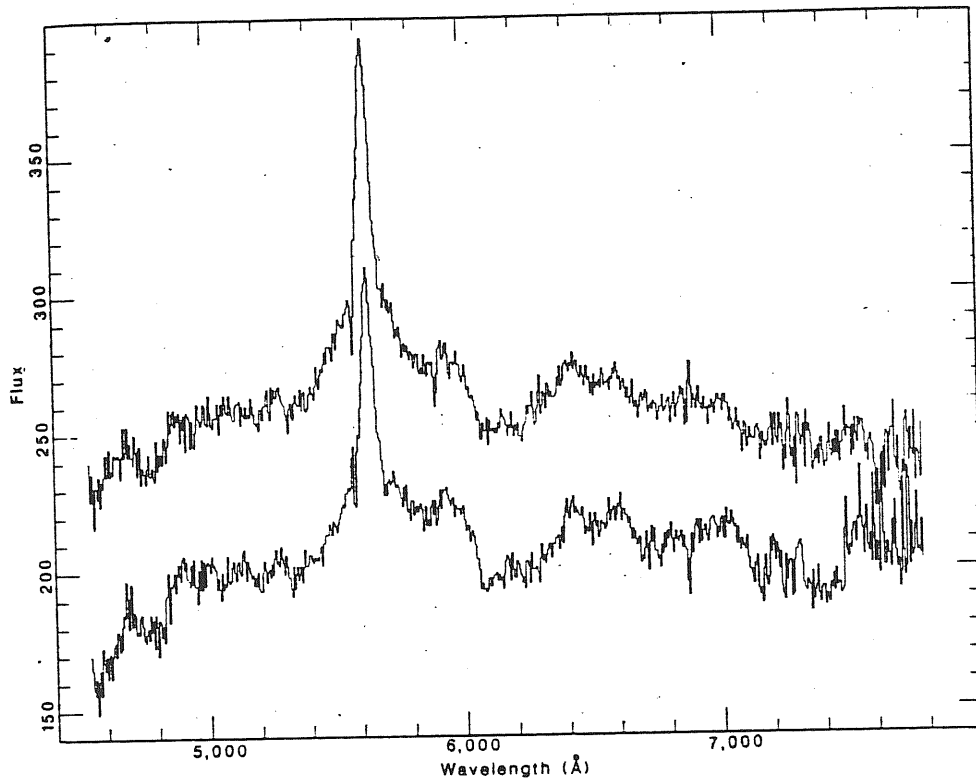


Fig.6 The spectra of 1146+111B and C. The vertical axis is a linear flux scale in arbitrary units. The upper spectrum is of object B; the lower, of object C (70 counts per channel were added to B to displace it from C). The identification of the strong emission feature as Mg II λ 2,798 is primarily based on Arp and Hazard's² detection of C III λ 1,909. Note the correspondence of the emission feature, the deep trough, and several weaker continuum dips and bumps in the two spectra.

(Adopted from Turner, et al. 1986)

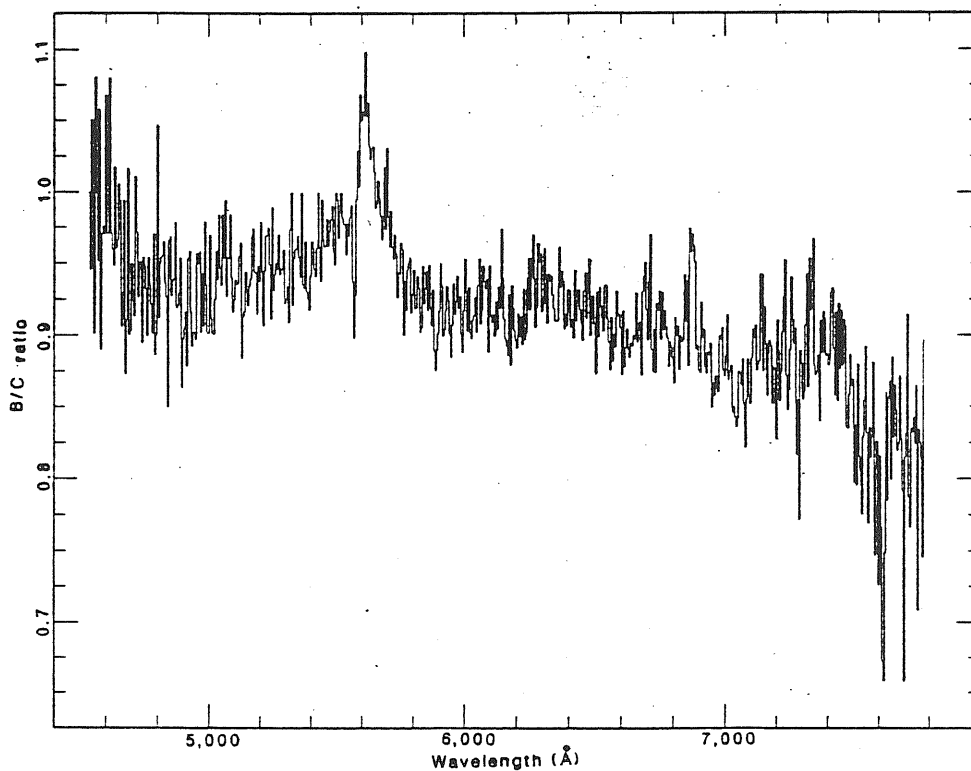


Fig. 7 The ratio of the spectrum of 1146+111B to that of 1146+111C. The feature in this ratio at the wavelength of the Mg II emission indicates that the line-to-continuum ratio in the two objects is slightly different. The smooth variation in the ratio from blue to red indicates that C has a redder detected continuum than B. The absence of other significant features in this ratio demonstrates the detailed match in shape and amplitude between respective emission and absorption features in the two spectra.

(Adopted from Turner, et al. 1986)

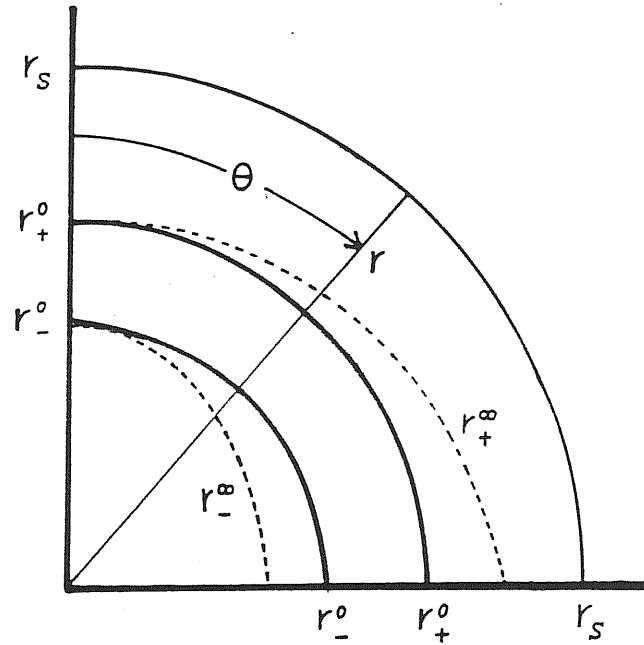


Fig. 8 The structure of the space-time in the Boyer-Lindquist coordinates. The dashed lines and the solid lines denote the infinite red-shift surfaces r_{\pm}^{∞} and the event horizons r_{\pm}^0 , respectively,

$$r_{\pm}^{\infty} = \mu \pm (\mu^2 - a^2 \cos^2 \theta - v^2)^{1/2},$$

$$r_{\pm}^0 = \mu \pm (\mu^2 - a^2 - v^2)^{1/2},$$

where $\mu \equiv GM$, $v^2 = G(Q^2 + \phi^2)/4\pi$, and $r_S (=2\mu)$ is the Schwarzschild radius.

(Adopted from Kasuya, M. 1982)

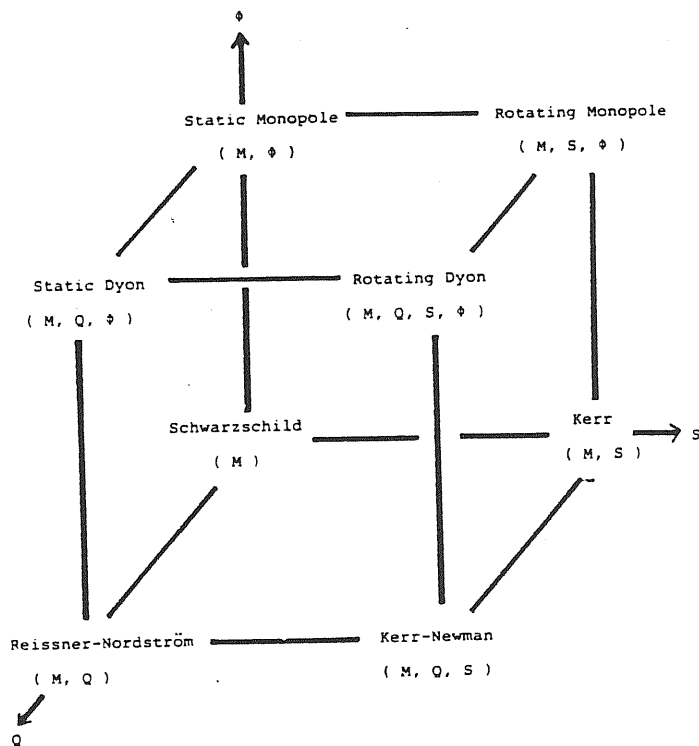


Fig.9 The relation of our gravitating and rotating dyon solution (characterized by four physical parameters; mass M , angular momentum S , electric charge Q , and magnetic charge Φ) to other solutions is illustrated.

(Adopted from Kasuya, M. 1982)