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THEORETICAL APPROACH TO THE PHYSICS OF PROTON BEAMS IN THE SOLAR CORONA

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INTRODUCTION

The role of protons in the solar environment is getting reconsidered during recent years and surely the physics of protons will become one of the dominant topics of solar research during the next cycle.

Usually disregarded because of their great mass and inertia, recently they were proposed as major carriers of the flare energy. This means that the mind of the researchers is slowly changing towards an interest in developing new theories for these particles.

Surely improved observations during the coming cycle will augment our knowledge and new features will be discovered. Till that moment we can benefit of the good observations made by the SMM and other satellite experiments other than ground-based instruments.

These suggest that proton beams are accelerated in the solar corona and can possibly radioemit. In the present work we will review the observations which support such an hypothesis and we will develop a preliminary theoretical approach to the analysis of the electrostatic instabilities, which a proton beam can undergo.

It is understood that the topic is not limited to the solar physics but can find an application in other objects as well, provided a source of proton beams is active as the flare is in the sun. It is in fact not unreasonable to suppose that similar phenomena occur in solar-like stars as well as in different objects such as the accreting magnetic cataclysmic variables in the accretion column or the dwarf novae, etc.

The fact that we applied the theory by using the sun as a reference point depends on the detailed observations we can get about this star, which is on the contrary a difficult task in the case of more distant objects. In this way we can hope to have a more direct confirmation of the hypotheses.

CHAPTER I

THE SOLAR CORONA

I. 1 The Solar Corona as a Plasma

The solar corona is a highly ionized gas consisting mainly of electrons and protons plus a small percentage of heavy ions and a negligible percentage of neutral atoms.

In Table I are reported the coronal parameters within to orders of magnitude (average values for non-active regions).

There are however three fundamental requirements for an ionized gas to be a plasma (e.g. Chen, 1984), i.e. to exhibit a collective behaviour, and these are:

1 - the particle density must be so high that the Debye length is much smaller than the characteristic length of the gas:

$$n_\alpha : \quad \lambda_{D\alpha} = \left[\frac{2\ k\ T_\alpha}{4\ \pi\ n_\alpha\ q_\alpha^2} \right]^{1/2} \ll L$$

with n_α - number density of the particle species α; k - Boltzmann constant; T_α - absolute temperature; q_α - particle charge; L - characteristic length.

This means that local concentrations of charge or external potentials are shielded out on a distance scale much shorter than the characteristic size of the system. In such a case the plasma is "quasi-neutral", i.e.

neutral enough to allow the assumption ($n_i \sim n_e \sim n_o$) but not enough to have all the relevant electromagnetic forces suppressed.

In the solar corona we have:

$\lambda_D \sim 10^{-1}$ cm << L ~ 10^9 cm, and the requirement is satisfied;

2 - the number of particles in the Debye sphere (plasma parameter) must be much greater than unity:

$$N_{D\alpha} = n_\alpha \frac{4}{3} \pi \ \lambda_{D\alpha}^3 >>> 1$$

to get the Debye shielding mechanism working properly. In the case of the corona, we get $N_D = 10^{11}$ >>>1 and the condition is satisfied;

3 - the frequency of typical plasma oscillations must be greater than the collision frequency with neutral atoms to prevent the gas to behave like a neutral gas, governed by hydrodynamical forces, rather than a plasma, governed by electromagnetic forces:

$$\omega > 1/\tau$$

with ω-plasma oscillation frequency and τ - mean time between collisions with neutral atoms.

This condition is surely satisfied in the solar corona, as the gas is highly ionized.

Moreover, the low β-parameter ($\beta \sim 10^{-6}$) ensures that the diamagnetic effect is very small: the local value of the magnetic field B is not greatly reduced by the plasma and as a first approximation we can assume a uniform field B_o in dealing with plasma waves.

To summarize we can say that the solar corona can be considered as a plasma characterized by the following features:

a) homogeneity on a large scale;

b) low density;

c) high temperature;

d) it is isothermal;

e) high electric conductivity;

f) low - β.

TABLE I

Characteristic macroscopic length	(L)	10^9 cm
Electron number density	(n_e)	10^8 cm^{-3}
Mass density	(ϱ)	10^{-17} g cm^{-3}
Electron temperature	(T_e)	10^6 K
Thermal velocity	(v_{Te})	10^8 cm s^{-1}
	(v_{Ti})	10^7 cm s^{-1}
Sound speed	(v_s)	10^7 cm s^{-1}
Alfvèn speed	(v_A)	10^7 cm s^{-1}
Electrical conductivity	(σ)	10^{16} s^{-1}

Magnetic induction	(B)	10^{-4} T
Beta parameter	(β)	10^{-6}
Mean free path	(l_e)	10^{9} cm
	(l_i)	10^{9} cm
Relaxation time	(τ_e)	10 s
	(τ_i)	10^{2} s
Collision frequency	(ν_{pe})	10^{-1} s^{-1}
Plasma frequency	(ω_{pe})	10^{8} s^{-1}
	(ω_{pi})	10^{6} s^{-1}
Gyrofrequency	(Ω_e)	10^{7} s^{-1}
	(Ω_i)	10^{3} s^{-1}
Debye length	(λ_D)	10^{-1} cm
Plasma parameter	(N_D)	10^{11}

I. 2 The Solar Corona as a Propagating Medium

An analytic model for the coronal electron density is that by Allen (1947) and Baumbach (1937)

$$n_e = 10^8 \, [1.55 \, (r/R_\odot)^{-6} + 2.99 \, (r/R_\odot)^{-16}] \quad cm^{-3}$$

which gives the electron number density n_e as a function of the distance from the Sun center r. Other commonly used models are the one by Newkirk (1967) and the one by Saito (1970). In fact, all these models underestimate the density of the active corona: in presence of coronal condensations and coronal streamers, the densities derived from the above models are usually multiplied by a factor which ranges from 2 to 10 and sometimes more.

The electrons density is decreasing with increasing height and so does also the plasma frequency $\omega_p \propto (n_e)^{1/2}$. The theory states that an electromagnetic wave of frequency ω can propagates in the plasma only if its frequency is greater or equal to the plasma frequency ω_p ($\omega \geq \omega_p$), because if it is lower ($\omega < \omega_p$) it is immediately reabsorbed. This peculiarity allows a selective study of the solar corona by means of a radiotelescope: in the plasma emission hypothesis a radio wave of frequency ω_1 such that $\omega_1 \sim \omega_{p1} = (4 \pi n_e(r_1) e^2/m)^{1/2}$ is produced at a height r_1 corresponding to the density $n_e(r_1)$ according to the chosen density model $n_e = n_e(r)$.

In such a way we can realize that higher frequencies come from lower (i.e. closer to the Sun) coronal levels and viceversa lower frequencies come from higher (i.e. farther from the Sun) levels.

CHAPTER II

THE PLASMA RADIATION MECHANISM

In Figure 1 it is represented a schematization of the commonly accepted mechanism, according to which a plasma can originate electromagnetic transverse waves (radio waves) in a wide range of astrophysical conditions. Such a mechanism is called "plasma radiation mechanism" (PRM) and it is invoked to explain many types of radio emissions which can be detected on the Sun as well as on other objects (e.g. flare stars, dwarf novae, magnetic cataclysmic variables, etc.).

A basic review on the PRM for different astrophysical applications can be found for instance in Kaplan and Tsytovich (1969), the extended theory in many text-books (e.g. Krueger, 1979; Melrose, 1980; McLean and Labrum (eds.), 1985) and in compact form in the updated review by Dulk (1986). However for our purposes it will be sufficient to outline the main points by applying the theory to the particular situation under study.

By following Dulk (1986) we can say that the PRM is a coherent radio-emitting process and the produced radio waves occur at the "plasma frequency"

$$\omega_{p\alpha} = \left[\frac{4 \pi n_\alpha q_\alpha^2}{m_\alpha} \right]^{1/2}$$

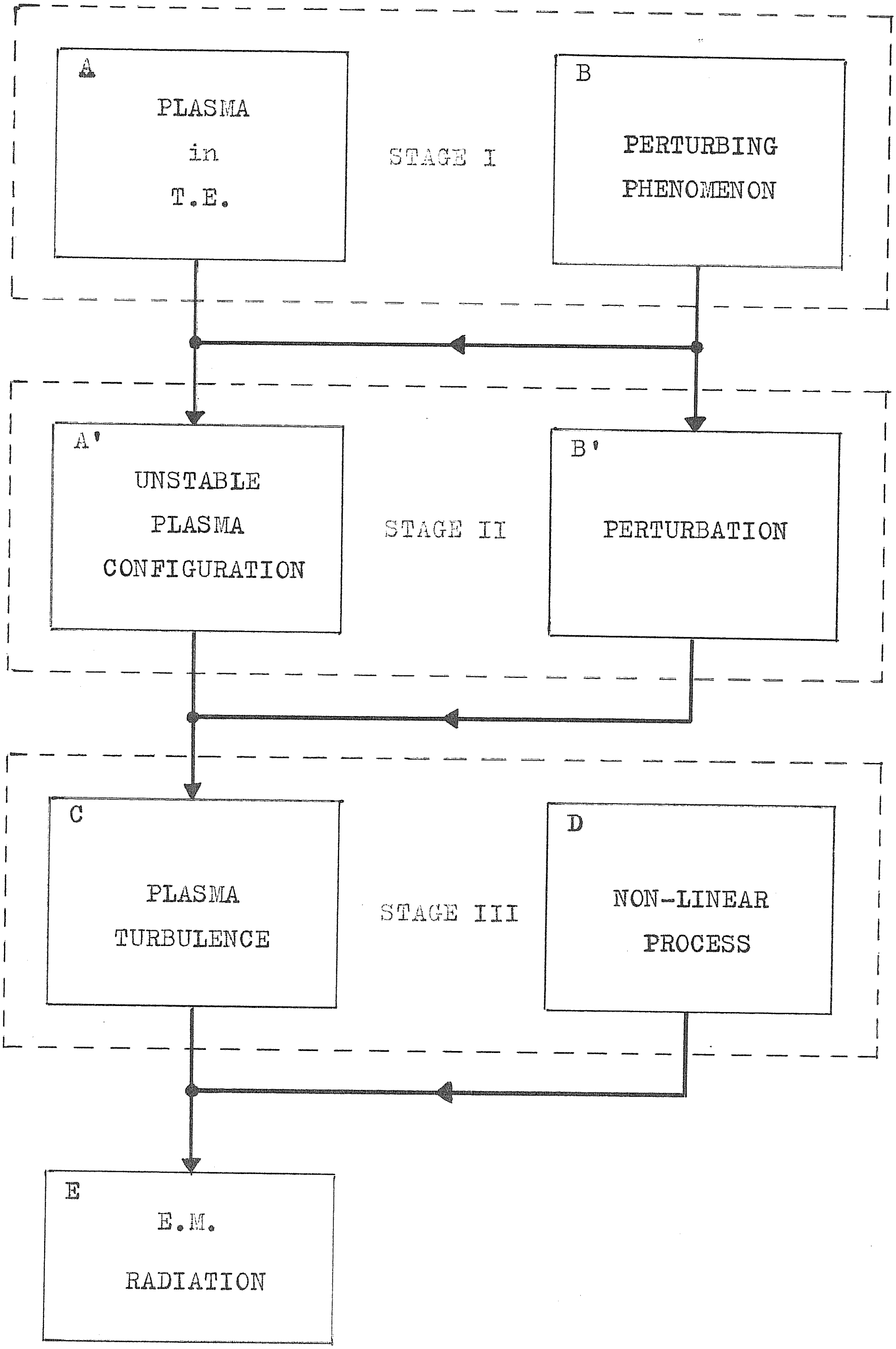


FIGURE 1 : GENERAL SCHEME OF THE PLASMA RADIATION MECHANISM

(where the subscript " α " refers to the particle species (electrons or ions), n_α is the particle number density, q_α is the particle electric charge and m_α is the particle rest mass) or at its second harmonic ($2 \cdot \omega_{p\alpha}$) and in very particular conditions at higher harmonics.

As a general trend, derived from some observations on the Sun and theoretical considerations which can possibly hold for the other objects, it can be said that the PRM is dominant at longer wavelengths whereas the gyrosynchrotron emission, due to the spiralling of charged particles along the magnetic field lines, prevails at shorter wavelengths.

Coming back to the proposed scheme for PRM (Figure 1), in the following we will comment the three stages which can lead to the radioemission.

Stage I: in this stage there must be:

(A) a plasma in thermodynamic equilibrium, which is a stable configuration;

(B) a perturbing phenomenon, which originates a source of free energy.

After the action of the perturbing phenomenon the plasma acquires some form of free energy and enters the Stage II.

Stage II: in this stage we find:

(A') an unstable plasma configuration (marginal stability) due to the presence of the source of free energy;

(B') a perturbation in the form of a wave.

Under suitable conditions the perturbation can grow by extracting energy from the plasma and the Stage III is entered. If the perturbation is damped, i.e. it gives energy to the plasma, the process is stopped.

Stage III: in this stage it happens the following:

- a plasma turbulence (C), generated by the growing instability, is formed, namely a set of plasma waves within a certain range of wavenumbers in permitted modes;
- it can interact with other waves and/or particles via a non-linear process (D).

The result of this interaction is the conversion of longitudinal plasma waves into electromagnetic transverse waves (E). The final energy of the em waves is related to the energy of the plasma turbulence and to the efficiency of the conversion process.

This scheme is quite rough but all the fundamental principles of the PRM are considered.

CHAPTER III

RADIOEMISSION FROM BEAMS OF PARTICLES

III.1 Electron Beams

One of the best interpreted solar radio events is the type III burst, a drifting frequency radioemission lasting from 1 to a few seconds with decreasing receiving frequency.

The theoretical explanation is based on the plasma radiation mechanism and the observations suggest that an electron beam is the exciting agent (e.g. Melrose, 1980 and references therein).

A. Emission Process

Let us heuristically consider the various stages of the process by making reference to the boxes of Figure 1, which schematizes the PRM, and contemporaneously summing up the inferences from the observations. The scheme is the following:

1. The solar corona represents the unperturbed ambient plasma (box A), which can be associated to a Maxwellian distribution corresponding to a temperature $T = 2x10^6$ K in the statistical description of the kinetic plasma theory;
2. a chromospheric flare occurs, due to magnetic field

annihilation in the reconnection region, and a great amount of energy is released (10^{30} erg on the average) in various form. (For a review of the flare process see eg. Dulk et al., 1985);

3. an acceleration region is active at coronal levels (possibly at the top of a magnetic loop), where electrons are accelerated to relativistic velocities (0.2 - 0.6 c), corresponding to non-thermal energies (10 - 100 keV): electron beams are formed and injected upwards into the corona along open magnetic field lines.

   The beam can be represented by a drifting Maxwellian distribution.

   A beam-plasma system is formed: the injected beam represents a source of free energy added to the plasma, which total energy is augmented. Such a plasma configuration is unstable (box A');

4. a minimum level of background fluctuations always exists in the plasma in the form of waves (box B') and some of such perturbations can interact with the beam-plasma system at each height during the travel of the beam upwards in the corona;

5. the theory states that if the phase velocity of the perturbation is "adequately" near the streaming velocity of the beam (resonance condition) and, within a certain range of particle velocities, the number of particles slower than the perturbation is smaller than the number of faster ones, the oscillation can gain energy at the latter ones' expense, i.e. at the expense of the beam

energy. Its amplitude will grow in time: a plasma instability set in.

The distribution function of the beam-plasma system is composed by the sum of the background plasma Maxwellian and the beam drifting Maxwellian, such that the deviation from the thermodynamic equilibrium is represented by a small bump in the tail of the stationary background distribution.

The fact that the instability is related to a peculiarity of the statistical distribution function makes it classified as "kinetic" instability. The shape of the distribution gives it the name of "bump-on-tail" instability, also called "two-stream" instability if we consider that the beam is moving with respect to the stationary background;

6. due to the setting in of the instability a set of electrostatic longitudinal plasma oscillations with wavevectors within a permitted range will be generated: such is the plasma turbulence indicated in box C. A basic review on the plasma turbulence can be found in Tsytovich (1973), but in the present work no quantitative details will be given on this phenomenon;
7. a non-linear interaction can occur at each plasma level between plasma waves and/or plasma particles (cf. box D), satisfying some requirements on the conservation of the wavevector (e.g. Tsytovich, 1967; Melrose, 1980).

The result is the conversion of longitudinal plasma waves into electromagnetic transverse waves (radio waves) at the local plasma frequency ω_p and/or at its second harmonic $2\cdot\omega_p$. These em waves are received by the radiotelescopes at the Earth.

As the beam is travelling upwards, higher and higher coronal levels will be excited to emit at their local plasma frequency, which is decreasing with height. The result is that the frequency of the radioemission will decrease as the time elapses; see Figure 2, which represents a radiospectrogram of a type III burst received by the ETH, Ikarus Spectrometer.

The observed frequency drift (-df/dt) of a typical type III burst is quite high (-150 MHz/s) and from that it is possible to deduce the electron beam velocity.

B. Difficulties in the Interpretation

The observations agree rather well with the theory, but some points remain still obscure and many improvements to the model have been proposed to overcome the impasse. Here we will cite a few remarkable points, which are important in the frame of an analysis of a beam-plasma system:

a. up to now it is not clear the way the electron beam is formed: one possibility for the formation of a bump in the distribution is that faster electrons overtake slower ones in velocity space after some travel time, but the problem is still open;

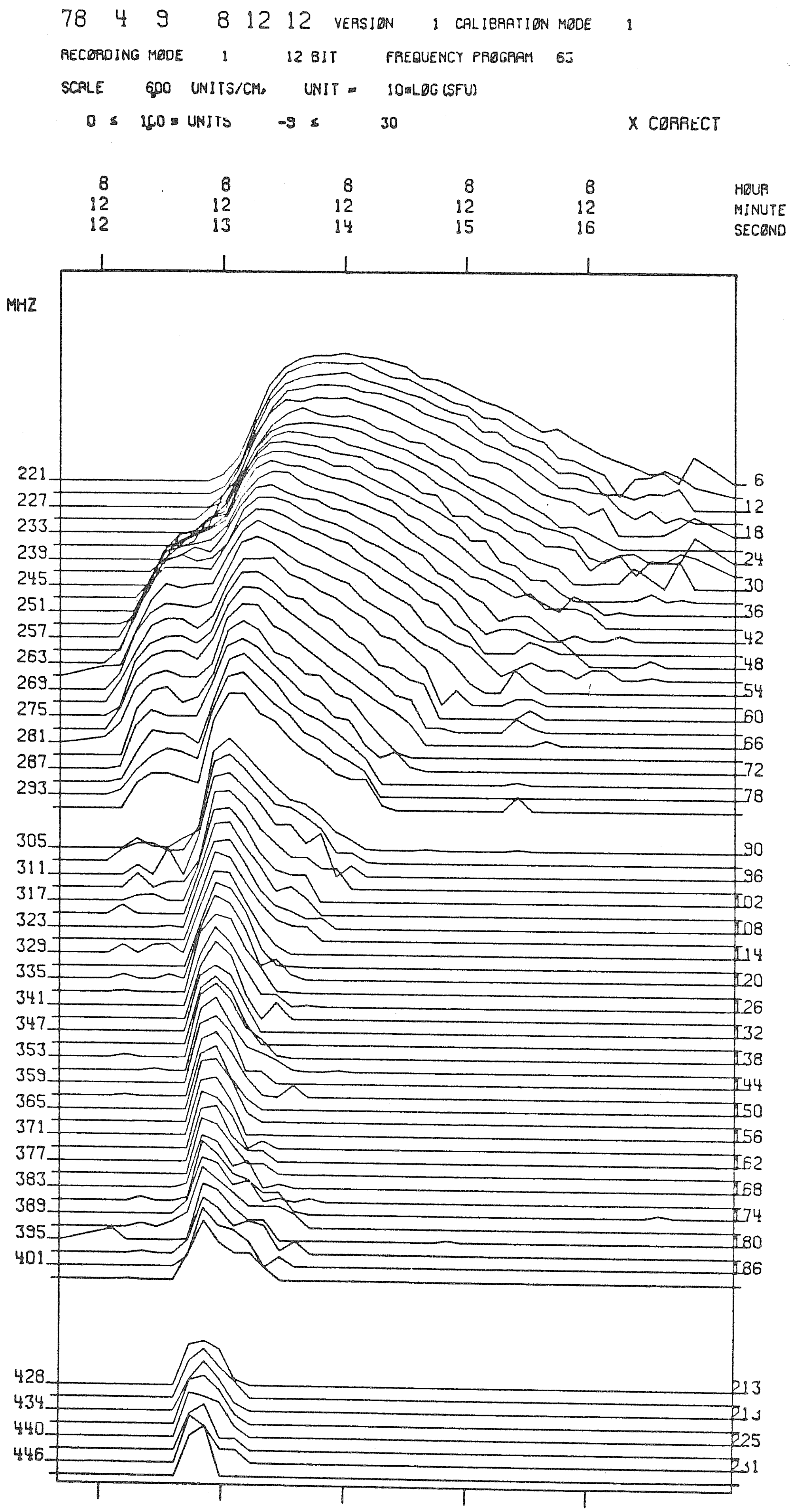


FIGURE 2 : RADIOSPECTROGRAM OF A TYPE III RADIO BURST

(Ikarus Spectrometer - ETH, Zurich)

b. the duration of the radio events (1 s in the metric range), i.e. the time interval during which radio waves are emitted, is directly related to:

1) the travel time of the beam, which must preserve its identity at least for the relevant time without being disrupted (e.g. fanned out or collisionally disrupted).

Observations of escaped electrons indicate a travel distance of 2 - 50 solar radii;

2) the endurance of the instability condition, which must be maintained for the relevant time to produce wave growth, but the instability must be saturated, i.e. the growth limited. In the opposite case all the beam energy is transferred to the growing wave in a short time much smaller than the observed event duration (10^{-5} s corresponding to a travel distance of 1 km for V_b = 0.3 c) leading to the beam disruption.

This problem requires a detailed study of the time-space evolution of the beam-plasma distribution function, which has been done for type III bursts (e.g. Grognard, 1985) leading to the acceptance of the quasi-linear relaxation theory for the beam velocity distribution;

c. the conversion process of plasma waves into electromagnetic ones is very inefficient. The estimated efficiency expressed as the ratio em-wave energy-to-plasma wave energy is of the order of 10^{-7}-10^{-5} and whence a high level of plasma waves is required to justify the observed radioemission flux density.

III.2 Proton Beams

A. Possible Experimental Evidence

According to both theoretical and experimental evidencies it is actually widely accepted that an electron beam is the exciter of type III bursts. However the possibility that a proton beam could be the exciter has been considered by different authors (Wild et al., 1954; Friedman et al., 1969; Smith, 1970).

Recent observations, in fact, suggest that low -drift type III-like bursts can be due to the radioemission by proton beams (Benz and Simnett, 1986) and this fact opens again the possibility for proton beams to be considered as exciting agents.

The cited authors observed 34 cases of low-drift radio bursts and estimated the following average features, which can be compared with those of a typical type III:

	Slow drift burst		Type III burst
Drift rate	-20 MHz/s		-150 MHz/s
Streaming velocity	13×10^{8} cm/s		10^{10} cm/s
Duration	< 2 s		< 2 s
Collision time	0.6 s	(electrons)	30 s
	5 m	(protons)	-

A radiospectrogram of such a slow drift burst is visible in Figure 3 a, whereas that of a typical type III burst is reproduced in Figure 3 b. Both events were recorded by the Ikarus Spectrometer running at the E.T.H. in Zürich.

The remarkable points are:

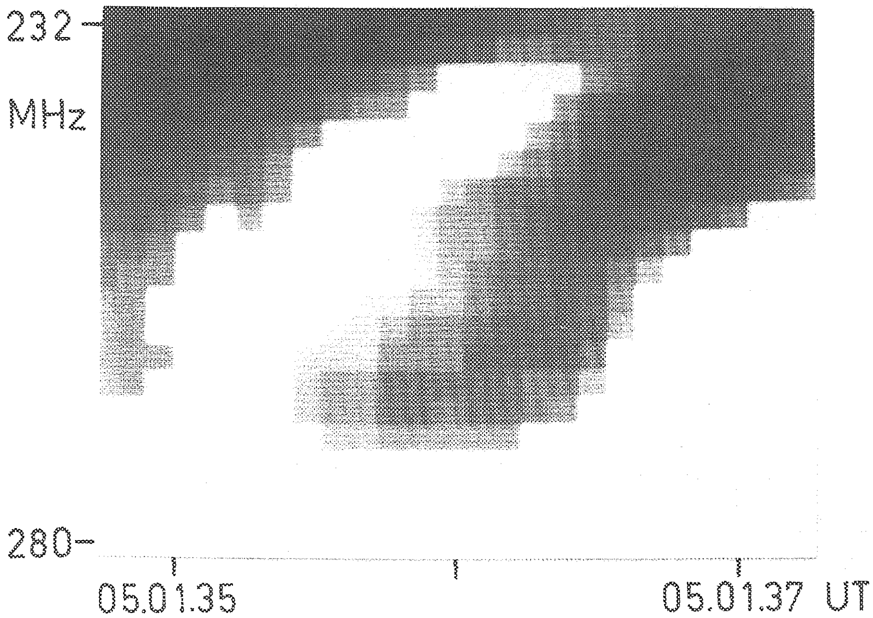
(i) the observed drift rate is 1 order of magnitude smaller than that of a typical type III;

(ii) the streaming velocity is 1 order of magnitude greater than the maximum velocity estimated for a traveling shock which originates a type II burst: it is not a sort of type II burst.

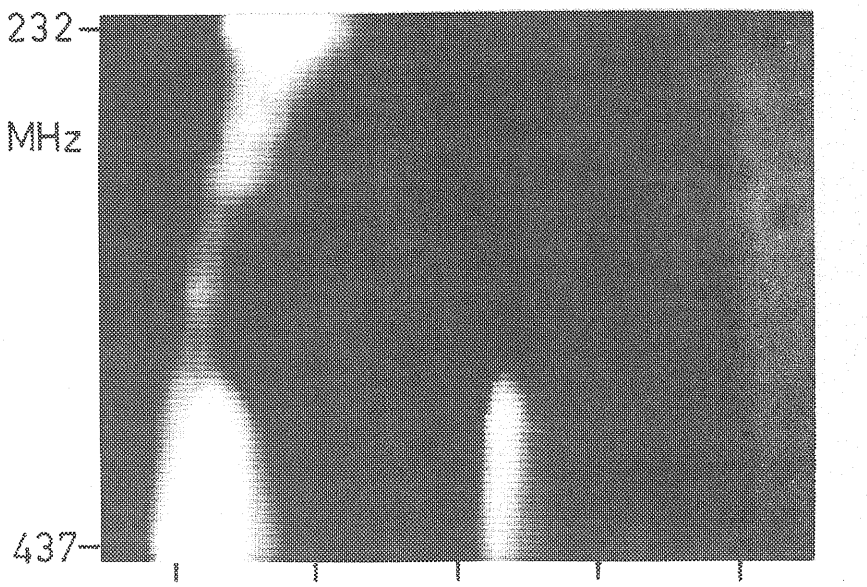
Moreover it ranges from 1 to 2 times the thermal velocity in corona: the two stream instability is strongly Landau-damped, if we assume the plasma radiation mechanism;

(iii) if we consider an electron beam as exciter, the collision time for electrons is too short compared to the event duration: the beam would be disrupted after a short time.

Based on this considerations Benz and Simnett suggest that the exciter can be a beam of thermal protons with energies in the range 0.56 ÷ 3.7 MeV.



(a)



(b)

FIGURE 3 : (a) RADIOSPECTROGRAM OF A LOW-DRIFT TYPE III RADIO BURST

(b) RADIOSPECTROGRAM OF A HIGH-DRIFT TYPE III RADIO BURST

(Ikarus Spectrometer - ETH, Zurich)

CHAPTER IV

THEORETICAL APPROACH TO PROTON BEAM PHYSICS

If we accept the above observations as an experimental evidence of the radioemission by proton beams, it remains to give a suitable theoretical interpretation which can explain the observed features. A self-consistent theory must account for:

a. the beam formation and b. evolution;

c. the plasma instability mechanism;

d. the radioemission process.

In the following we will consider the points a., b. and c., neglecting the point d. which is beyond the aims of the present work and will be matter of a further analysis.

IV. 1 The Maxwellian Distribution

When a plasma is in thermodynamic equilibrium, the statistical mechanics states that the most probable velocity distribution of the particle of species α is the Maxwellian distribution, which is isotropic and can be written in 3 dimensions as:

$$f_\alpha(\overline{v}) = \frac{n_\alpha}{\pi^{3/2}\ v_{T\alpha}{}^3}\ \exp\left[-\left(\frac{\overline{v}}{v_{T\alpha}}\right)^2\right]$$

with $\overline{v} \equiv (v_x, v_y, v_z)$ is the particle velocity, n_α is the particle number density and $v_{T\alpha}$ is the thermal velocity defined as

$$v_{T\alpha} = \left[\frac{2kT_{\alpha}}{m_{\alpha}} \right]^{1/2}$$

with k - Boltzmann constant, m_{α} - particle mass and T_{α} - absolute temperature.

The quantity $f_{\alpha}(\overline{v})$ dv is just the number density of the particles with velocities in the range ($\overline{v} \div \overline{v} + d\overline{v}$) and the multiplying factor comes out from the normalization condition

$$n_{\alpha} = \int_{-\infty}^{+\infty} f_{\alpha}(\overline{v})\, d\overline{v}.$$

The temperature T_{α} controls the width of the distribution.

Another important parameter is the average kinetic energy of the particles, given by

$$\langle E_{KIN} \rangle_{\alpha} = \frac{\int_{-\infty}^{+\infty} 1/2\ m_{\alpha}\ \overline{v}^2\ f_{\alpha}(\overline{v})\ d\overline{v}}{\int_{-\infty}^{+\infty} f_{\alpha}(\overline{v})\ d\overline{v}} = 3/2\ kT.$$

It is obvious that a plasma can have more than one temperature, because each particle species α can have its own temperature and the resulting velocity distribution is in fact a sum of Maxwellian's:

$$F(\overline{v}) = \Sigma_{\alpha} f_{\alpha}(\overline{v}).$$

Without loss of generality, for our purposes it can be used the "reduced form" of the Maxwellian distribution, which is a 1-dimensional distribution obtained by integrating along two coordinates the complete form. As we will deal with electrostatic longitudinal oscillations only, if we adopt a geometry such that the wavevector $\overline{k}$ is parallel to the x-axis of a Cartesian reference frame ($\overline{k} = k_x \hat{x}$), it will be convenient to integrate the distribution in the directions orthogonal to k, i.e. the y and the z one. We can write:

$$f_o(v_x) = \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} dz \; F(\overline{v}) \qquad \overline{v} \equiv (v_x, v_y, v_z)$$

$$= \frac{n_{\alpha}}{(\pi)^{3/2} v_{T\alpha}{}^3} \int_{-\infty}^{+\infty} dy \; \exp\left[- \frac{y}{v_{T\alpha}}^2\right] \int_{-\infty}^{+\infty} dz \; \exp\left[- \frac{z}{v_{T\alpha}}^2\right] \exp\left[- \frac{v_x}{v_{T\alpha}}^2\right]$$

whence

$$f_o(v_x) = \frac{n_{\alpha}}{\sqrt{\pi} \; v_{T\alpha}} \exp\left[- \frac{v_x}{v_{T\alpha}}^2\right] .$$

From now on we will drop the subscript "x" understanding that v will not represent the total velocity $\overline{v}$ but its x-component v_x . Such a procedure simplifies considerably the notations.

It is important to determine the velocity at which a Maxwellian presents the maximum value (v_{MD}) of the first derivative with respect to the velocity, as this result will be used in the instability analysis. It is straightforward to get:

$$f_{o\alpha}'(v) = -\frac{2\,n_\alpha}{\sqrt{\pi}\,v_{T\alpha}^3}\,v\,\exp\left[-\frac{v}{v_{T\alpha}}^2\right]$$

and

$$f_{o\alpha}''(v) = -\frac{2\,n_\alpha}{\sqrt{\pi}\,v_{T\alpha}^3}\left[1-\frac{2}{v_{T\alpha}^2}\,v^2\right]\exp\left[-\left(\frac{v}{v_{T\alpha}}\right)^2\right] = 0\ ,$$

whence

$$v_{MD\alpha} = \pm\frac{v_{T\alpha}}{\sqrt{2}}\ .$$

Another useful distribution function is the "drifting" Maxwellian, which can be used to represent a beam of particles with a streaming velocity $\overline{V}_b = V_{bx}\,\hat{x}$ (with the usual convention). It has the form:

$$f_{o\alpha}^D(v) = \frac{n_\alpha}{\sqrt{\pi}\,v_{T\alpha}}\exp\left[-\left(\frac{v - V_b}{v_{T\alpha}}\right)^2\right]$$

where the maximum derivative velocity becomes:

$$v_{MD\alpha}^D = V_b \pm \frac{v_{T\alpha}}{\sqrt{2}}\ .$$

In the limit of vanishing temperature ($T_\alpha \rightarrow 0$), the

Maxwellian and the drifting distributions are transformed into delta-functions:

$$f_{o\alpha}(v) \rightarrow n_\alpha\, \delta(v)$$

$$f_{o\alpha}^{D}(v) \rightarrow n_\alpha\, \delta(v - V_b) \quad .$$

This represents the connection between the kinetic (statistical) description of the plasma (microscopic) for a non-zero temperature and the hydrodynamic description (macroscopic).

IV. 2 The Beam-Plasma Velocity Distribution

In the frame of the kinetic theory a beam-plasma system can be represented by the sum of two reduced Maxwellian distributions for the background electrons and ions and a drifting reduced Maxwellian with a parallel streaming velocity V_b for the ion beam:

$$f_{oe}(v) = \frac{n_e}{\sqrt{\pi}\, v_{Te}} \exp\left[-\left(\frac{v}{v_{Te}}\right)^2\right] \qquad \text{(background electrons)}$$

$$f_{oi}(v) = \frac{n_i}{\sqrt{\pi}\, v_{Ti}} \exp\left[-\left(\frac{v}{v_{Ti}}\right)^2\right] \qquad \text{(background ions)}$$

$$f_{ob}(v) = \frac{n_b}{\sqrt{\pi}\, v_{Tb}} \exp\left[-\left(\frac{v - V_b}{v_{Tb}}\right)^2\right] \qquad \text{(beam ions)}$$

with n_e , n_i , n_b - number densities,

v_{Te}, v_{Ti}, v_{Tb} - thermal velocities,

T_e , T_i , T_b - absolute temperatures

of background electrons, ions and beam ions respectively. The number densities and the temperatures are the six free parameters, which completely define the total distribution function:

$$F_o(v) = f_{oe}(v) + f_{oi}(v) + f_{ob}(v) \quad .$$

In regard with that it is convenient to consider as fundamental parameters only the electron temperature (T_e) and density (n_e) by expressing all the other ones as a function of them. This is a common practice introductory to successive numerical computations (cf. e.g. Cuperman, 1981; Birdsall and Langdon, 1985).

For the number densities we must consider two situations dependent of the way the beam-plasma system is formed:

- Case 1. During the evolution of the background ion Maxwellian a bump in the tail is formed because faster ions overtake slower ones. The charge quasi-neutrality requires:

$$n_e \simeq n_i + n_b \simeq n_o$$

and defining

$$n_b = \varepsilon\, n_o$$

where ε much less than unity means a weak beam (ε<<1) and ε equal to unity is a non-realistic limiting situation of strong beam with almost all the backgroung ions concurring to form the beam. We get:

$$n_e \simeq n_o$$

$$n_i \simeq (1 - \varepsilon)\ n_o$$

$$n_b \simeq \varepsilon\ n_o \quad .$$

As the beam ions are a fraction of the background ions, the charge neutrality is automatically realized.

- Case 2. An ion beam is injected into the background from outside and its presence will alter the charge equilibrium:

$$n_e \simeq n_i \simeq n_o$$

$$n_b = \varepsilon\ n_o$$

and

$$n_i + n_b = (1 + \varepsilon)\ n_o \quad .$$

However the ion charge excess will be neutralized by the background electrons on a short time scale.

For the present analysis we prefer the second possibility and all the following considerations will be based on that.

The ion and beam temperature are defined as:

$$T_i = \alpha\, T_e \qquad \alpha \in [0,1]$$

and

$$T_b = \beta\, T_e \qquad \beta \in [0,1]$$

with the two limiting cases: 1) α, β vanishing (α, $\beta \to 0$) means cold and 2) α, β unitary (α, $\beta \to 1$) means warm ions and beam.

The thermal velocities are:

$$v_{Ti} = \left[\frac{2kT_i}{M} \right]^{1/2} = \left[\frac{\alpha}{MR} \right]^{1/2} v_{Te}$$

and

$$v_{Tb} = \left[\frac{2kT_b}{M} \right]^{1/2} = \left[\frac{\beta}{MR} \right]^{1/2} v_{Te}$$

where M is the ion (proton) mass and MR is the ion-to-electron rest mass ratio: MR = (M/m) $\simeq$ 1836.

The streaming velocity of the beam is expressed as:

$$V_b = \gamma\, v_{Te} \qquad \gamma \in (0, \gg 1).$$

If γ is less or equal to unity ($\gamma \lesssim 1$) the beam is "thermal" in the sense that the streaming velocity V_b is less than or equal to the electron thermal velocity ($V_b < v_{Te}$), whereas a value of γ greater than unity ($\gamma > 1$) characterizes a "non-thermal" beam ($V_b > v_{Te}$).

If the thermal spread of the beam is small ($T_b \to 0$;

quasi-monoenergetic or cold beam), we can assume the maximum kinetic energy:

$$E_{KM} = \frac{1}{2} MV_b^2$$

as representative of the beam particle kinetic energy, otherwise ($T_b > 0$; hot beam) the average kinetic energy must be used:

$$E_{KA} = \frac{1}{2} kT_b \quad .$$

After this definition of the parameters and in the case of a proton beam injected into an ion + electron plasma (case 2. above) the final form of the total velocity distribution becomes:

$$F_o(v) = \frac{n_o}{\sqrt{\pi}\, v_{Te}} \exp\left[-\left(\frac{v}{v_{Te}}\right)^2\right] +$$

$$+ \frac{n_o}{\sqrt{\pi}\, v_{Te}} \left[\frac{MR}{\alpha}\right]^{1/2} \exp\left[-\frac{MR}{\alpha}\left(\frac{v}{v_{Te}}\right)^2\right] +$$

$$+ \frac{\varepsilon\, n_o}{\sqrt{\pi}\, v_{Te}} \left[\frac{MR}{\beta}\right]^{1/2} \exp\left[-\frac{MR}{\beta}\left(\frac{v - v_{Te}}{v_{Te}}\right)^2\right]$$

with $MR - 1836$, $\alpha = T_i / T_e$, $\beta = T_b / T_e$, $\gamma = V_b / v_{Te}$.

IV. 3 Beam Formation

The role of protons in solar flares has been reconsidered in the light of the recent observations. The commonly accepted idea is that most of the flare energy is carried by non-thermal electrons with energies above 20 keV. However by analyzing a limb flare, Simmett and Strong (1984) suggested that the energy bulk is to be attributed to suprathermal protons (100 - 1000 keV) during the impulsive phase. In a recent work Simnett (1985) proposed that in general the energy transfer mechanism in a flare is realized by means of non-thermal protons in the range (100 - 1000 keV). This interpretation can be questionable (see de Jager, 1986), but it is a signature that the physics of protons in the solar environment needs to be better developed and understood.

However it is well recognized that protons are accelerated during flares as well as electrons. We have a direct proof of that from the detection of interplanetary protons accelerated a) during the rare "cosmic ray flares", with energy greater than 500 MeV and b) during the more common "proton flares", with energies from 10 up to 100 MeV (cf. Dodson-Prince and Bruzek, 1977 and references therein). An indirect proof of accelerated protons is the observation of γ-ray lines in the energy range 4-7 MeV (Chupp, 1983), due to nuclear interactions of accelerated protons with the ambient plasma (Ramaty et al., 1983). Usually protons are accelerated within 2s of energetic electrons, but sometimes 20-100 keV electrons, protons and highly relativistic electrons emit simultaneously within the instrumental time resolution (Vlahos et al., 1985). The physics of protons in solar flares is extensively treated in Smith and Stephen (1986).

The long-time storage of high-energy protons (accelerated by a flare) in coronal magnetic traps is considered in Meerson and Rogachevskii (1983).

Emslie and Brown (1985) analyzed the possibility that hard X-ray bremsstrahlung is produced by high energy (~ 20 MeV) proton beams, giving arguments in favour of this hypothesis.

We can conclude that all the physical conditions are satisfied which lead to the formation and propagation of proton beams as it is for electron beams. In fact, due to the higher proton mass the velocity of the beam will be surely subrelativistic and perhaps thermal as it seems to be proved by the radio observations we will consider in the following.

IV. 4 Beam Evolution

We know from the theory that to get an observable radioemission the beam must maintain its identity along its propagation path for a sufficient time interval. The problem of the evolution of the beam velocity distribution is quite a complex topic and in the present work we will limit the treatment to a simple theory which can be the starting point for a future development.

Assume for simplicity:

- longitudinal propagation in the x-direction: we can use a reduced Maxwellian representing the initial beam

$$f_o(v) = \frac{n_b}{\sqrt{\pi}\ v_{Tb}} \exp\left[-\left(\frac{v}{v_{Tb}}\right)^2\right]$$

with n_b - beam density, $v_{Tb} = (2kT_b/M)^{1/2}$ - thermal velocity;

- no collisions;
- acceleration as partial heating.

In the acceleration region (spatial coordinate $x = x_o = 0$) the distribution will be:

$$f_o(x=0,v,t) = \frac{n_b}{\sqrt{\pi}\ v_{Tb}} \exp\left[-\left(\frac{v}{v_{Tb}}\right)^2\right] \exp\left[\frac{t}{\tau}\right]$$

with t - time and τ - characteristic acceleration time.

After some travel time the beam will have covered some distance and will be located at $x = x_1 \neq 0$. The particles will arrive in $x = x_1$ at different times

$$t_1(v) = \frac{x_1 - x_o}{v}$$

depending on their own velocities, the fastest ones arriving first. The distribution will be changed and can be expressed from the original one but computed at the time $(t - t_1)$:

$$f_o(x=x_1,v,t) = f_o(x=0,v,t-t_1)$$

$$= \frac{n_b}{\sqrt{\pi}\, v_{Tb}} \exp\left[-\left(\frac{v}{v_{Tb}}\right)^2\right] \exp\left[\frac{t}{\tau} - \frac{x_1 - x_o}{v}\frac{1}{\tau}\right]$$

$$= \frac{n_b}{\sqrt{\pi}\, v_{Tb}} \exp\left[\frac{t}{\tau}\right] \exp\left[-\left(\frac{v}{v_{Tb}}\right)^2 - \frac{\Delta x_1}{v\,\tau}\right]$$

and generalizing for the coordinate x_1 and normalizing, one gets:

$$\tilde{f}_o(\tilde{x}_1,\tilde{v},t) = A(t)\exp\left[-\tilde{v}^2 - \frac{\tilde{x}_1}{\tilde{v}}\right]$$

with $\tilde{v} = v/v_{Tb}$ and $\tilde{x}_1 = \Delta x_1 / (\tau\, v_{Tb})$ respectively the normalized velocity and the normalized distance from the acceleration region.

In Figure 4a the function $\tilde{f}_o$ is plotted as a function of $\tilde{v}$ for six different distances $\tilde{x}_1 = 1,..,6$ and one can realize that as the beam travels it is preserved even if there is a tendency to spread in velocity, because a tail is formed on the side of higher velocities.

It can be seen also that the streaming velocity is augmented as the time elapses: the beam is accelerated. It is straightforward to obtain analytically this result. In fact, the velocity corresponding to the maximum of the distribution is

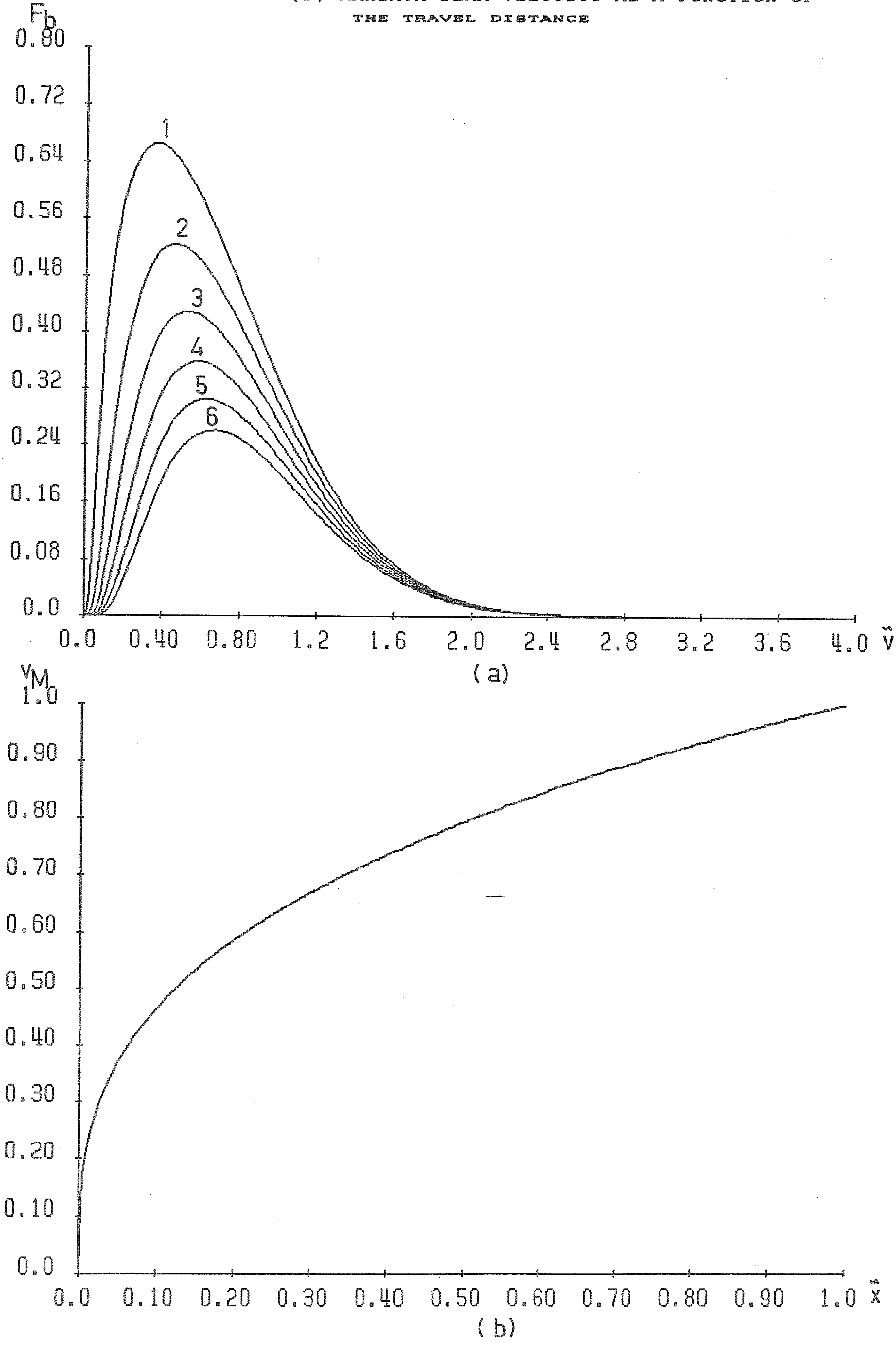
$$\tilde{v}_M = (\tilde{x}_1)^{1/3}$$

i.e.

$$v_M = v_{Tb} \left[\frac{\Delta x_1}{\tau \; v_{Tb}} \right]^{1/3} .$$

In Figure 4b it is plotted $\tilde{v}_M$ as a function of $\tilde{x}_1$.

FIGURE 4 : (a) EVOLUTION OF THE BEAM DISTRIBUTION FOR INCREASING (1 -> 6) TRAVEL DISTANCE
(b) MAXIMUM BEAM VELOCITY AS A FUNCTION OF THE TRAVEL DISTANCE



IV. 5 The Plasma Instability Mechanism

A. Overview of the Literature

It is resonable to suppose that the plasma radiation mechanism is working for proton beams as well as for electron beams. The task is now to find a suitable kind of plasma instability, which can lead to the generation of a consistent level of growing waves.

By reviewing the existent literature one realizes that a comprehensive theory about the stability of ion beams, suitable of application in the frame of the radioemission, has not been developed yet.

Different kinds of streaming instabilities has been widely treated (e.g. Briggs, 1964; Fainberg, 1968; Nezlin, 1971; Mikhailovskii, 1974), but usually for the electron case. Even if the ion case is included, no emphasis is given to possible astrophysical applications and mainly the laboratory plasma experiments are considered.

The longitudinal ion oscillations in a hot plasma were investigated by Fried and Gould (1961) and the stability limits for longitudinal waves in an ion-beam plasma interaction were graphically and semi-analytically determined by Fried and Wong (1966).

Chernov et al. (1970) made an experimental investigation on high frequency instability of a proton beam injected into a background electron plasma. They found that coherent oscillations can be amplified in the direction of the beam motion at all frequencies below the electron plasma frequency.

Perkins (1976) treated the linear stability of ion-cyclotron waves in a plasma with ion beams and considered some basic research experiments such as collisionless shocks and two component thermonuclear burning devices. Successively Hauck et al. (1978a,b) analyzed the ion-cyclotron waves excited by ion beams in Q-machine plasmas.

Da Jornada et al. (1979, 1981) examined the electrostatic instabilities excited by an energetic ion beam, claiming that energetic ion beams are the most successful techniques for heating magnetically confined plasmas.

Some astrophysical applications are considered with regard to the interplanetary environment in recent papers.

In Gary (1985) the electromagnetic ion beam instabilities are considered for hot beams at the interplanetary shocks; in Andre' (1985) the complete dispersion relation is numerically solved for two models of ion-beam plasma system related to some satellite observations.

B. The Research Project

The cited papers are representative of the work which has been done in the field of the theoretical instability analysis of ion beams.

In principle some of the above results could be applied to the problem of solar ion beams, but a great care must be used because in almost all cases the starting conditions are different.

Taking that into account we decided to analyze "ex novo" the possible instabilities an ion beam can undergo by using the general methods of plasma physics and then referring the application to our particular case, which is the solar environment.

Such an investigation is a big task due to the great variety of instabilities which can occur and hence it cannot be concluded in the present work. In fact, as a first approach, we will limit our attention to some sort of electrostatic kinetic instabilities in an unmagnetized plasma, leaving the further analysis to future researches.

The choiche is not unreasonable if one considers that the electrostatic instabilities can be more effective than other kinds under suitable conditions of the beam-plasma system. Secondarily it is known that the particles are escaping along open magnetic field lines in the corona, where the field intensity is low, and the assumption of vanishing static magnetic field is acceptable.

CHAPTER V

KINETIC ELECTROSTATIC INSTABILITIES

V. 1 Linear Theory

The kinetic theory is a statistical theory and each particle species α in the plasma is represented by a probability distribution function in the space-velocity hyperspace:

$$f_\alpha = f_\alpha(\bar{r}, \bar{v}, t)$$

with $\bar{r}$-spatial coordinate, $\bar{v}$-velocity and t-time. The normalization condition must hold:

$$\int f_\alpha \, d^3\bar{r} \, d^3\bar{v} = N_\alpha$$

where N_α is the total number of particles of species α .

The behaviour of the system is governed by the kinetic Boltzmann equation, for each particle species (eg. Akhiezer, 1975; Schmidt, 1979), which becomes

$$\left[\frac{\partial}{\partial t} + \bar{v} \cdot \frac{\partial}{\partial \bar{r}} + \frac{q_\alpha}{m_\alpha} (\bar{E} + \bar{v} \times \bar{B}) \cdot \frac{\partial}{\partial \bar{v}}\right] f_\alpha(\bar{r},\bar{v},t) = \left[\frac{\partial f_\alpha}{\partial t}\right]_{coll}$$

if we consider only electromagnetic forces. The term on the right side represents the rate of change of the distribution due to collisions between particles.

If the plasma is collisionless, as we will assume, that term vanishes and we get the Vlasov equation:

$$\left[\frac{\partial}{\partial t} + \bar{v} \cdot \frac{\partial}{\partial \bar{r}} + \frac{q_\alpha}{m_\alpha} (\bar{E} + \bar{v} \times \bar{B}) \cdot \frac{\partial}{\partial \bar{v}}\right] f_\alpha(\bar{r},\bar{v},t) = 0 \quad .$$

The electric and the magnetic fields are the sum of the external field (if any) and the self-consistent field, generated by the particles themselves:

$$\bar{E} = \bar{E}(\bar{r},t) = \bar{E}^{(e)}(\bar{r},t) + \bar{E}^{(p)}(\bar{r},t)$$

$$\bar{B} = \bar{B}(\bar{r},t) = \bar{B}^{(e)}(\bar{r},t) + \bar{B}^{(p)}(\bar{r},t)$$

and they must satisfy the Maxwell equations:

$$\bar{\nabla} \cdot \bar{E} = 4 \pi (\varrho^{(e)} + \varrho^{(p)}) \qquad \bar{\nabla} \times \bar{E} = - \frac{1}{c} \frac{\partial \bar{B}}{\partial t}$$

$$\bar{\nabla} \cdot \bar{B} = 0 \qquad \bar{\nabla} \times \bar{B} = \frac{1}{c} \frac{\partial \bar{E}}{\partial t} + \frac{4\pi}{c} (\bar{j}^{(e)}+\bar{j}^{(p)})$$

where $\varrho^{()}$ and $\bar{j}^{()}$ are the charge and current densities respectively (external and self-consistent). The latter ones are given by:

$$\varrho^{(p)}(\bar{r},t) = \Sigma_\alpha q_\alpha \int f_\alpha d^3\bar{v} \qquad \bar{j}^{(p)}(\bar{r},t) = \Sigma_\alpha q_\alpha \int \bar{v} f_\alpha d^3\bar{v}$$

Let us suppose that the system is in thermal equilibrium at the time $t < t_o = 0$ and it is perturbed at the time

$t = t_0 = 0$. We wish to investigative the response of the system at the time $t = t_1 > t_0$, i.e. we want to determine which kind of oscillations are sustained in the plasma.

For simplicity consider one single species of particles and assume:

1) small perturbation amplitude:

$$f(\overline{r},\overline{v},t) = f_0(\overline{v}) + f_1(\overline{r},\overline{v},t)$$

with f_0 - unperturbed initial distribution and f_1 - small perturbation such that $|f_1| << |f_0|$. This assumption is valid for all the involved quantities (e.g. the electric field $\overline{E} = \overline{E}_0 + \overline{E}_1$, the number density $n = n_0 + n_1$ and so on);

2) electrostatic oscillations only: $\overline{\nabla} \times \overline{E} = 0$.

Under these assumptions the Vlasov equation can be linearized and one gets:

$$\frac{\partial f_1}{\partial t} + \overline{v} \cdot \frac{\partial f_1}{\partial \overline{r}} + \frac{q}{m} \overline{E}_1 \cdot \frac{\partial f_0}{\partial \overline{v}} = 0 \quad .$$

The electric field can be calculated by means of the Poisson equation:

$$\overline{\nabla} \cdot \overline{E}_1 = 4 \pi q \int f_1 \, d^3\overline{v} \quad .$$

The last three equations constitute a set of self-consistent equations, sufficient to determine the unknowns $f_1(\overline{r},\overline{v},t)$ and $\overline{E}_1(\overline{r},t)$, provided the boundary conditions are given.

If we search for traveling plane wave solutions of the form

$$f_1 \, , \, \bar{E}_1 \sim \exp\left[i \, (\bar{k} \cdot \bar{r} - \omega t) \right]$$

with $\bar{k}$-wavevector and ω-oscillation frequency, the three equations become

$$f_1 = \frac{i}{\bar{k} \cdot \bar{v} - \omega} \frac{q}{m} \bar{E}_1 \cdot \frac{\partial f_o}{\partial \bar{v}}$$

$$i \, \bar{k} \cdot \bar{E}_1 = 4 \pi q \int f_1 \, d^3\bar{v}$$

$$\bar{k} \times \bar{E}_1 = 0 \quad .$$

From the last one it is evident that the oscillation is longitudinal, i.e. $\bar{E}_1 \parallel \bar{k}$; this fact will justify the use of reduced distribution functions, i.e. integrated along the directions orthogonal to the motion. Moreover, we can assume the motion along the x-axis to deal with a 1-D problem and drop the vector signs.

By integrating the first equation and using the Poisson one, we get a DISPERSION RELATION, which relates the wave numbers to the frequencies of the waves:

$$F(\omega, \bar{k}) = \frac{4\pi q^2}{m} \frac{\bar{k}}{k^2} \cdot \int \frac{\partial f_o}{\partial \bar{v}} \frac{1}{\bar{k} \cdot \bar{v} - \omega} \, d^3\bar{v} = 1 \quad .$$

The pole of the integrand is related to the presence of particles, which have a velocity v nearly equal to the phase velocity ω/k of the oscillation:

$$v = \frac{\omega}{k} \qquad \text{(resonance condition)}.$$

Such particles can easily exchange energy with the wave after being trapped in the wave potential wells: the wave will give energy to the slower particles and will get energy from the faster ones. The oscillation will be damped if the number of slower particles exceeds that of the faster ones (Landau damping) or it will grow in the opposite situation. This effect is a characteristic kinetic effect, which depends on the features of the distribution function as it will be better specified in the following.

V. 2 The Weak Instability Approximation

The ambiguity about the way to integrate the dispersion relation around the pole is overcome by the Fourier-Laplace method, which we will not outline in detail.

For our purposes it is sufficient to write the form of the dispersion relation for electrostatic instabilities in the weak instability approximation, which is called Landau dielectric function (cf. Galeev and Sudan, 1983):

$$D(\omega, k) = 1 + \Sigma_\alpha \frac{4\pi q_\alpha^2}{m_\alpha} \frac{1}{k} \int \frac{\partial f_{o\alpha}}{\partial v} \frac{1}{\omega - kv} dv = 0$$

for $\mathrm{Im}\,\omega > 0$ and $\omega = \omega_r + i\gamma$ ($\omega_r = \mathrm{Re}(\omega)$, $\gamma = \mathrm{Im}(\omega)$), the complex oscillation frequency.

The imaginary part γ is called damping rate if it is negative ($\gamma < 0$) or growth rate if it is positive ($\gamma > 0$), in which case the wave amplitude is growing and we speak of instability.

Defining

$$D = D_r + i\, D_i = 0$$

with

$$D_r = \mathrm{Re}\, D(\omega_r + i\gamma, k)$$

$$D_i = \mathrm{Im}\, D(\omega_r + i\gamma, k)$$

and assuming that the instability is weak, i.e.

$$|\gamma| \ll |\omega_r| \quad , \quad |D_i| \ll |D_r|$$

we can expand in Taylor series the dispersion relation to get:

$$D_r(\omega_r, k) + i \left[\gamma \frac{\partial}{\partial \omega_r} D_r(\omega_r, k) + D_i(\omega_r, k) \right] + \ldots = 0$$

By using the relation

$$\lim_{\gamma \to 0^+} \frac{1}{\omega_r - kv + i\gamma} = \frac{P}{\omega_r - kv} - i\,\pi\,\delta(\omega_r - kv)$$

with P - Cauchy principal value, we obtain:

$$D_r(\omega_r,k) = 1 + \Sigma_\alpha \frac{4\pi q_\alpha^2}{m_\alpha} \frac{1}{k} P \int \frac{\partial f_{o\alpha}}{\partial v} \frac{1}{\omega_r - kv} dv = 0$$

which determines the real oscillation frequency ω_r, and

$$D_i(\omega_r,k) = -\pi \Sigma_\alpha \frac{4\pi q_\alpha^2}{m_\alpha} \frac{1}{k} \int \frac{\partial f_{o\alpha}}{\partial v} \delta(\omega_r - kv)\, dv \quad .$$

Whence from the Taylor expansion we get an approximate expression for the damping rate in the case of weak instability:

$$\gamma = \mathrm{Im}\,\omega = - \frac{D_i(\omega_r,k)}{\dfrac{\partial D_r(\omega_r,k)}{\partial \omega_r}} \quad .$$

From the expression of D_i and γ we can immediately realize that if $\partial f_o / \partial v < 0$ then $\gamma < 0$ and the wave is damped (stability), whereas if $\partial f_o / \partial v > 0$ then $\gamma > 0$: the wave is growing (instability).

CHAPTER VI

THE COLD CASE

VI. 1 The Resonant Instability

In the limit of vanishing temperature for all the components of the beam-plasma system (electrons, ions and beam-ions) we stated in a previous paragraph that the distribution functions become delta-functions:

$$f_{oe} = n_o\ \delta(v) \qquad f_{oi} = n_o\ \delta(v) \qquad f_{ob} = n_b\ \delta(v - V_b)$$

where n_o is the plasma number density, $n_b = \varepsilon\, n_o$ is the beam density and V_b is the streaming velocity taken in the x-direction.

In such a case the electrostatic dispersion relation can be easily integrated to yield:

$$D(\omega, k) = 1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_{pb}^2}{(kV_b - \omega)^2} = 0$$

with $\omega = \omega_r + i\gamma$

$$\omega_p = \omega_{pe} + \omega_{pi} = \left[\frac{4\pi n_o e^2}{m}\right]^{1/2} + \left[\frac{4\pi n_o e^2}{M}\right]^{1/2}$$

$$\omega_{pb} = \left[\frac{4\pi n_b e^2}{M}\right]^{1/2} .$$

We can refer the beam plasma frequency to the background plasma frequency as

$$\omega_{pb} = \left[\frac{\varepsilon}{MR + 1} \right]^{1/2} \omega_p = (\varepsilon^*)^{1/2} \quad \omega_p$$

if we define $MR = M/m = 1836.15$

$$\varepsilon^* = \frac{\varepsilon}{(MR + 1)} \quad .$$

The dispersion relation becomes:

$$1 - \frac{\omega_p^2}{\omega^2} - \frac{\varepsilon^* \quad \omega_p^2}{(kV_b - \omega)^2} = 0 \quad .$$

This is a fourth order algebraic equation in ω and its solutions $\omega_i = \omega_i(k)$ $(i = 1,..,4)$ are the permitted oscillations in the beam-plasma system. If $\omega = \omega_r + i\gamma$ has a non vanishing imaginary part the waves can be damped ($\gamma < 0$) or growing ($\gamma > 0$).

An effective analytical analysis of the solutions can be found in Mikhailovskii (1974) in the weak beam approximation ($\varepsilon \ll 1$) and we will extend those results to our case, in which also the background ions are considered.

For kV_b not too near ω_p, the following roots are obtained:

$${}_1\omega_2 = \pm \quad \omega_p \qquad \text{(plasma oscillations)}$$

$$ {}_{3}\omega_{4} = kV_b \pm (\varepsilon^*)^{1/2} \frac{\omega_p}{\left[1 - \left(\frac{\omega_p}{kV_b}\right)^2 \right]^{1/2}} \qquad \text{(beam oscillations)} $$

The group velocity of the latter ones is $v_g \sim V_b$ and hence they are called "beam oscillations". The solutions are complex for $kV_b < \omega_p$ (wavelength not too short) and one of them corresponds to instability with growth rate

$$ \gamma = (\varepsilon^*)^{1/2} \frac{\omega_p}{\left[\left(\frac{\omega_p}{kV_b}\right)^2 - 1 \right]^{1/2}} $$

which tends to be infinite for $kV_b \rightarrow \omega_p$ (resonance between plasma oscillations and beam oscillations → resonant instability).

At the zero-th order in ε^* we get:

$$ \omega^{(0)} \approx \omega_p $$

$$ \omega^{(1)} >> | \omega_p - kV_b | $$

and

$$ \text{Re} \quad \omega^{(1)} = - \omega_p \frac{(\varepsilon^*)^{1/3}}{2^{4/3}} $$

$$ \text{Im} \quad \omega^{(1)} = \gamma = \omega_p \frac{\sqrt{3}}{2^{4/3}} (\varepsilon^*)^{1/3} = \gamma_{MAX} $$

which is of the same order of the previous expression at the limit of its applicability $(\omega_p - kV_b)^2 - 1 \approx (\varepsilon^*)^{1/3}$.

Such is the maximal growth rate of oscillations excited in a plasma by a low-density beam.

It is immediately clear that electron beams have a growth rate 1 order of magnitude greater, because in the case of proton beams it is reduced by a factor $(MR)^{1/3} \sim 12$.

VI. 2 Stability Analysis

It is useful to determine the range of unstable wavevectors and in doing that we will parallel the work by Vedenov et al. (1961).

We can write the dispersion relation as

$$F(\omega, k) = \frac{\omega_p^2}{\left[\frac{\omega}{k}\right]^2} + \frac{\omega_{pb}^2}{\left[\frac{\omega}{k} - V_b\right]^2} = k^2$$

There exists just one critical value of k, i.e. k_{CR} such that if $k < k_{CR}$ the dispersion relation has 2 real roots and 2 conjugate complex (instability); if $k > k_{CR}$ one gets 4 real roots (stability).

This value k_{CR} corresponds to a minimum of the functional $F(\omega/k)$, which can be determined by setting to zero the first derivative and solving for (ω/k). This procedure leads to the relation:

$$k_{CR} = \frac{\omega_p}{V_b}\left[1 + \frac{\omega_{pb}}{\omega_p}^{2/3}\right]^{3/2} = \frac{\omega_p}{V_b}\left[1 + (\varepsilon^*)^{1/3}\right]^{3/2}$$

corresponding to an oscillation frequency

$$\omega_{CR} = \omega_p \; [\; 1 + (\varepsilon^*)^{1/3} \;]^{1/2} \quad .$$

Also from this relation it is evident that the instability range is reduced for proton beams compared to that of electron beams.

VI. 3 Numerical Analysis of the Dispersion Relation

The derived dispersion relation can be easily solved numerically, if we use the normalizations:

$$\tilde{\omega} = \omega \; / \; \omega_p \qquad\qquad \tilde{k} = k \; V_b \; / \; \omega_p$$

and write it in the form

$$\tilde{\omega}^4 - 2\tilde{k} \; \tilde{\omega}^3 + (\tilde{k}^2 - 1 - \varepsilon^*) \; \tilde{\omega}^2 + 2\tilde{k} \; \tilde{\omega} - \tilde{k}^2 = 0 \quad .$$

This is a fourth order algebraic equation of complex variable $\tilde{\omega} = \tilde{\omega}_r + i \; \tilde{\gamma}$ and real coefficients, which depends only on the free parameter ε related to the beam density ($\varepsilon = n_b \; / \; n_o$).

Formally the solutions are the branches of the dispersion relation, i.e. the permitted oscillations (real):

$$\tilde{\omega}_{ri} = \tilde{\omega}_{ri}(\tilde{k}) \qquad\qquad (i = 1,..,4) \quad .$$

The correspondent damping rates (negative, zero or positive) are computed from the approximate expressions, where the values of $\tilde{\omega}$ for each $\tilde{k}$ is deduced from the numerical solutions of the above real dispersion relation,

$$\tilde{\gamma}_i = \tilde{\gamma}_i(\tilde{k}) \qquad (i = 1,..,4) \quad .$$

We considered three cases of beam density:

1) strong beam $\varepsilon = 1$;

2) weak beam $\varepsilon = 10^{-2}$;

3) weak beam $\varepsilon = 10^{-6}$;

choosing in 2) and 3) the extreme values estimated for the electron beams in type III bursts.

For each case the numerical results are visible in Figure 5, 6 and 7 respectively, where the different oscillation branches identified by digits are plotted in panels (a) and the corresponding damping rates in panels (b) for growing solutions only.

In all cases it is easy to identify the high-frequency plasma branch (1) and the resonance branch (2), which represents the unstable beam-plasma oscillations ($\tilde{\gamma}_2 > 0$), as it is stated by the theory.

As the beam density decreases, the range of wavevectors corresponding to an appreciable value of the growth rate $\tilde{\gamma}_2$ is reduced and such is also the maximum growth rate.

FIGURE 5 : NUMERICAL SOLUTIONS OF THE DISPERSION RELATION FOR THE COLD CASE (ε : 1)
(a) OSCILLATION BRANCHES; (b) GROWTH RATE

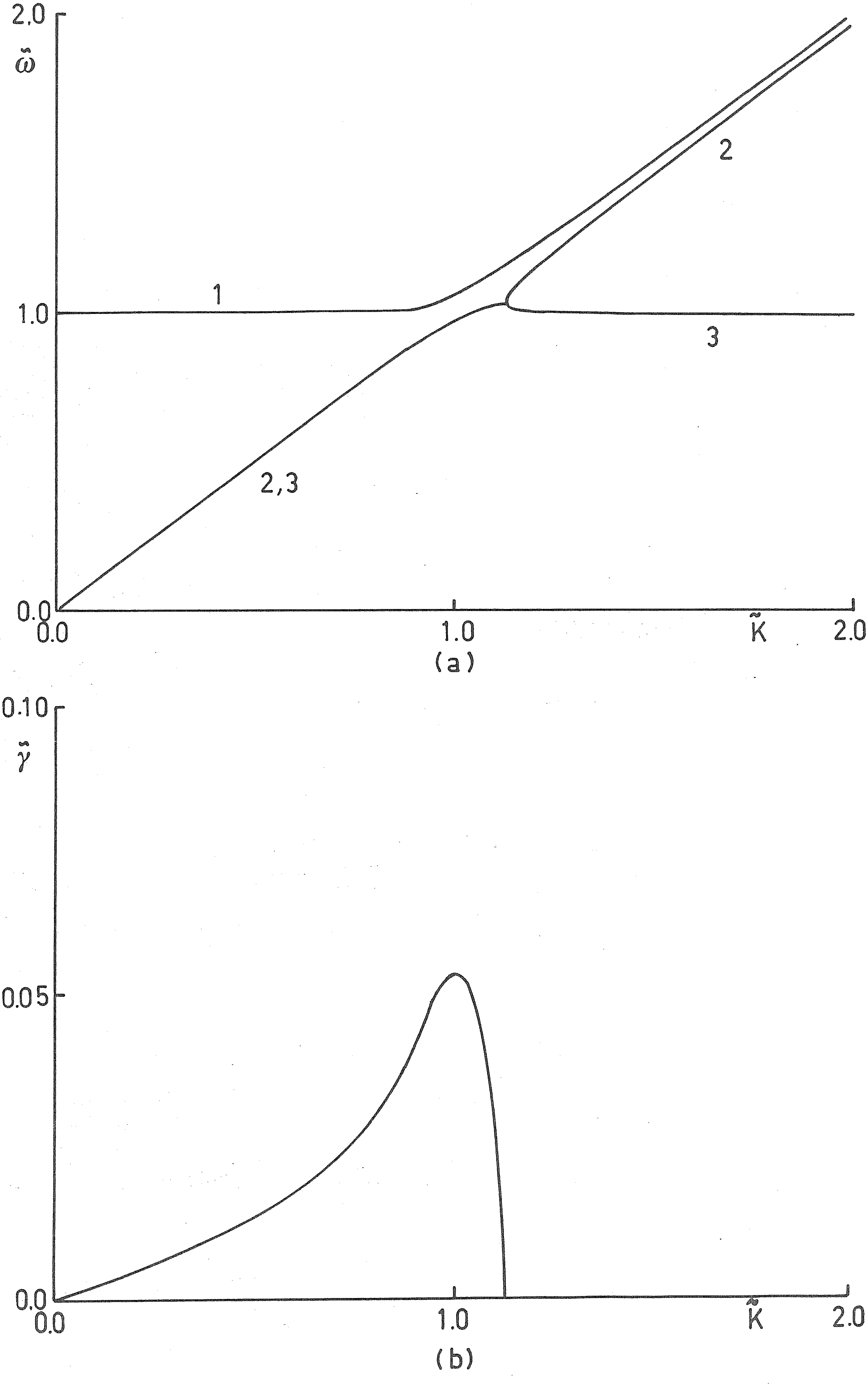
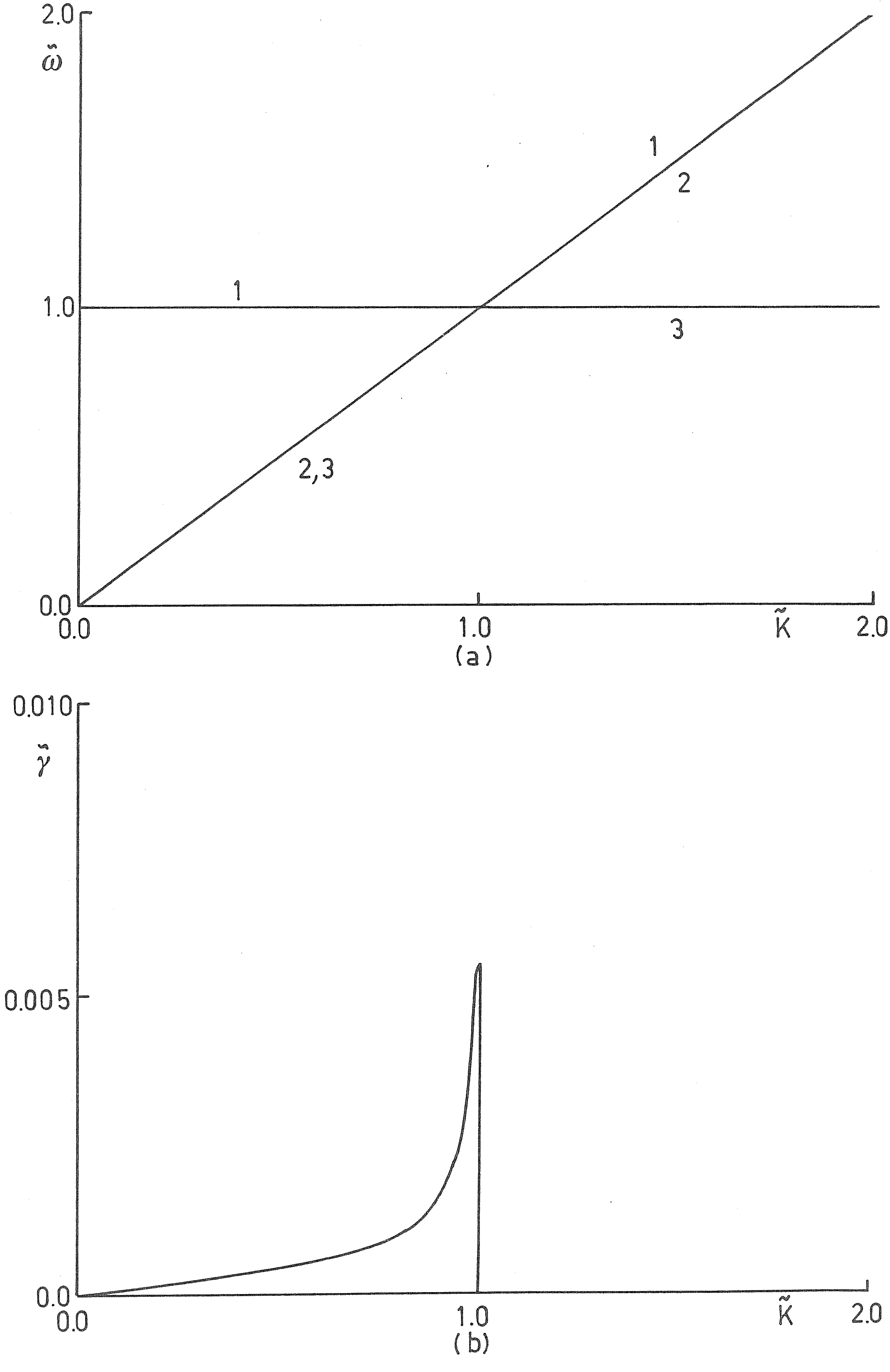
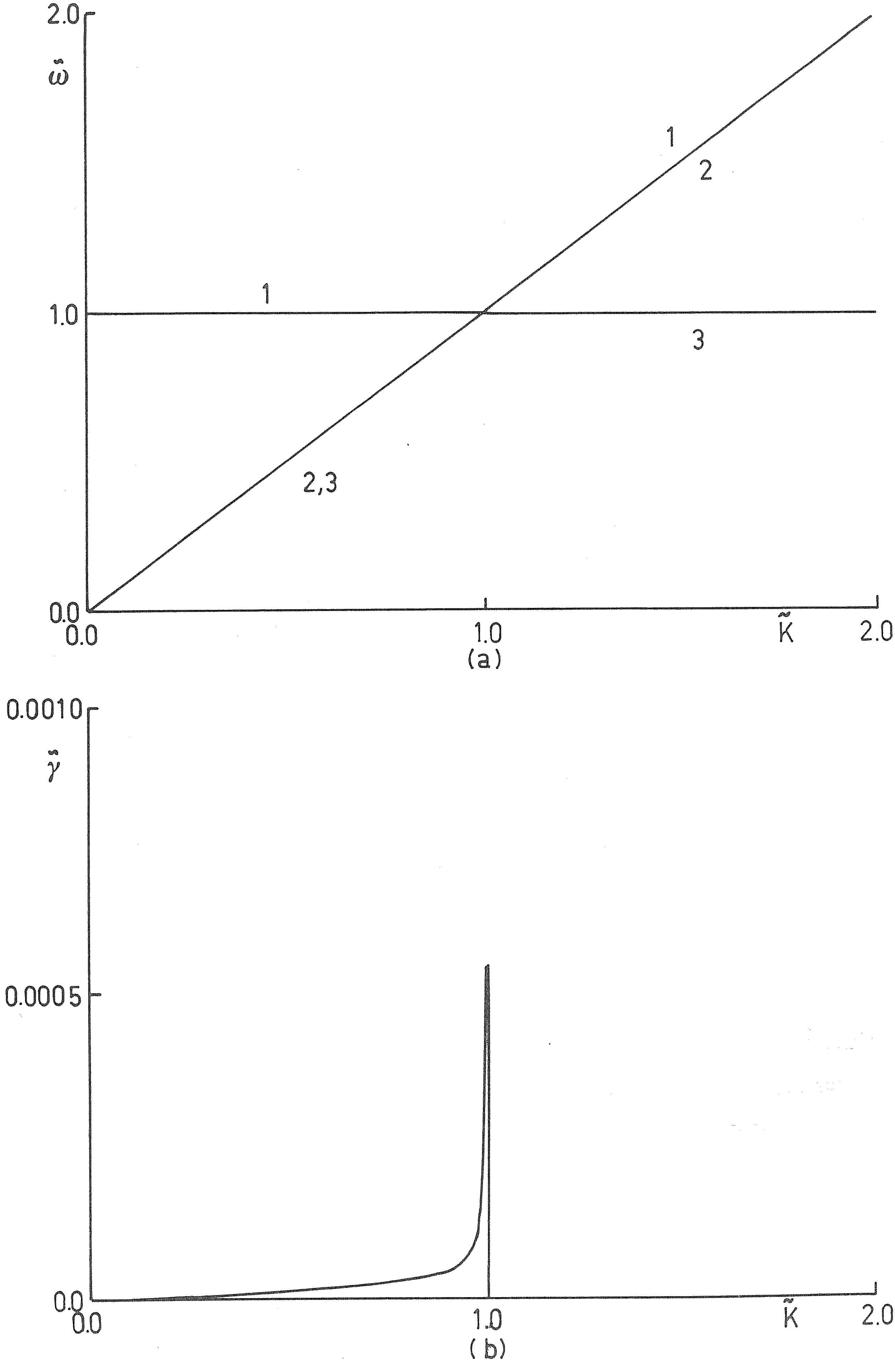


FIGURE 6 : NUMERICAL SOLUTIONS OF THE DISPERSION RELATION FOR THE COLD CASE (ε : 10^-3)
(a) OSCILLATION BRANCHES; (b) GROWTH RATE



NUMERICAL SOLUTIONS OF THE DISPERSION RELATION
FOR THE COLD CASE (ε : 10^-6)
(a) OSCILLATION BRANCHES; (b) GROWTH RATE



CHAPTER VII

THE WARM CASE

VII.1 Dispersion Relation

In the case of non-vanishing temperature, the background and the beam will be represented by Maxwellian distributions and the dielectric function reads (e.g. Mikhailovskii, 1974):

$$D(\omega,k) = 1 + \Sigma_\alpha \frac{1}{(kd_\alpha)^2}\left[1 + i\sqrt{\pi}\,\frac{\omega}{k\,v_{T\alpha}}\,W\!\left(\frac{\omega}{k\,v_{T\alpha}}\right)\right] = 0$$

for arbitrary complex $\omega = \omega_r + i\gamma$ and $v_{T\alpha}$ - thermal velocity of particle species α , d_α - Debye length.

The function

$$W(x) = \exp(-x^2)\left[1 + \frac{2i}{\sqrt{\pi}}\int_0^x e^{t^2}\,dt\right] \qquad x \in \mathbb{C}$$

is the probability integral (or error function) for complex argument.

For the ion-beam plasma system the relation becomes:

$$\begin{aligned} D(\omega,k) = 1 &+ \frac{1}{(kd_e)^2}\left[1 + i\sqrt{\pi}\,\frac{\omega}{k\,v_{Te}}\,W\!\left(\frac{\omega}{k\,v_{Te}}\right)\right] + \\ &+ \frac{1}{(kd_i)^2}\left[1 + i\sqrt{\pi}\,\frac{\omega}{k\,v_{Ti}}\,W\!\left(\frac{\omega}{k\,v_{Ti}}\right)\right] + \\ &+ \frac{1}{(kd_b)^2}\left[1 + i\sqrt{\pi}\,\frac{\omega - kV_b}{k\,v_{Tb}}\,W\!\left(\frac{\omega - kV_b}{k\,v_{Tb}}\right)\right] . \end{aligned}$$

As it is not a trivial task to integrate this terms analytically, we will consider a numerical approach to the solution.

In fact, the probability integral W(x) can be asymptotically expanded depending on the magnitude of its argument as

$$W(x) = \frac{i}{\sqrt{\pi}\, x}\left[1 + \frac{1}{2x^2} + \frac{3}{4x^4} + \ldots \right] + \exp(-x^2) \qquad |x| >> 1$$

$$W(x) = 1 + \frac{2ix}{\sqrt{\pi}} + \ldots \qquad |x| << 1$$

and a suitable analysis of the arguments in the context of the physical conditions will lead to an asymptotic but simplified form of the dispersion relation valid in the limit of the initial assumptions.

In the following we will consider two cases:

1) the generation of longitudinal beam-plasma oscillations by a non-thermal proton beam ($V_b >> v_{Te}$), i.e. a bump-on-tail instability;

and

2) the generation of ion sound oscillations by a thermal proton beam ($V_b < v_{Te}$), i.e. a ion acoustic instability.

However to get the normalized form of the dispersion relation in the two cases, suitable to be numerically solved, a detailed analysis of the conditions on the parameters, which determine the two different situations, is needed.

VII.2 Bump-on-tail Instability

A. Analysis of the Distribution

To privilege the excitation of beam-plasma oscillations the following conditions in the velocity space must hold:

I. $\omega/k = v_{ph} > 3\, v_{Te}$ to avoid strong Landau damping of the oscillations by plasma electrons. This means that: $v_{ph} / v_{Te} > 3$.

But this is just an extreme condition and, as it is done in practice, we will assume $v_{ph} \gg 3\, v_{Te}$, which means $|v_{ph} / v_{Te}| \gg 1$ and we can use the corresponding asymptotic approximation for the dispersion relation.

II. $v_{ph} \gg v_{Ti}$ is a corollary of the first assumption as the inequality $v_{Te} \gg v_{Ti}$ is always realized even if $T_i = T_e$ (case of isothermal corona), which gives $v_{Ti} \simeq 1/43\, v_{Te}$ if we assume that the background ions are mostly protons; usually in active regions $T_e > T_i$ and the strong inequality is still better satisfied). In such a case: $|v_{ph} / v_{Ti}| \gg 1$ is always valid, leading to the choice of the same asymptotic approximation as in point I.

If we consider a parametrized distribution function, where $T_i = \alpha\, T_e$ this leads to the constraint on the parameter $\alpha = T_i / T_e$: $\alpha \in [0,1]$ where $\alpha = 0$ means "cold" ion background (strong temperature anisotropy in the background) and $\alpha = 1$ means isothermal background.

III. (v_{Tb} i.e. $T_b \to 0$) can be a valid assumption if we admit that the injected beam is quasi-monoenergetic, i.e. "cold" in this limit with respect to the background but not absolutely (it must have a small thermal spread in the particle velocities). If we consider the parametrized $v_{Tb} = (\beta / MR)^{1/2} v_{Te}$ form with $T_b = \beta T_e$, this means that it must be $\beta \ll 1$ and it is not unreasonable to assume $\beta = \alpha / 10$. In fact, in the aforesaid isothermal background case it leads to $v_{Tb} \simeq 1/135 \; v_{Te}$.

Whence $\beta \in [0, \alpha / 10]$ where $\beta = 0$ means cold (monoenergetic) beam and $\beta = \alpha / 10$ means quasi-monoenergetic beam.

IV. $V_b - 2 v_{Tb} < v_{ph} < V_b$ must hold for getting the excitation of oscillations. This range of phase velocities corresponds to the range of positive values of the derivative of the distribution function ($\partial F / \partial v > 0$), condition which is required for the growth of electrostatic perturbations. The lower limit $V_b - 2 v_{Tb}$ is a rough estimate coming from a numerical analysis of the distribution function for some real values of the parameters as it is not trivial to give an analytical expression which represents the exact velocity coordinate corresponding to the initial point of the positive derivative range for the tri-Maxwellian distribution. As it comes out from the numerical analysis it can be located at $v_{MIN} < V_b - 2\ v_{Tb}$ or even at $v_{MIN} > V_b - v_{Tb}$ and according to that the chosen limit must be considered as an upper limit. Let it be: $V_b = \gamma\ v_{Te}$. As condition I. must be satisfied to avoid electron Landau damping, the following inequality is needed:

$$v_{ph} > V_b - 2\ v_{Tb} \gg 3\ v_{Te}$$

i.e.

$$v_{ph} > \gamma\ v_{Te} - 2\ (\beta\ /\ MR)^{1/2}\ v_{Te} \gg 3\ v_{Te}$$

but $\beta = \alpha\ /\ 10$, whence

$$- 2 \left[\frac{\alpha}{10\ MR} \right]^{1/2} \gg 3 \quad \Longrightarrow \quad \gg 3 + 2 \left[\frac{\alpha}{10\ MR} \right]^{1/2} .$$

Mathematically speaking we can conclude that

$$\gamma \in \left(3 + 2 \left(\frac{\alpha}{10\ MR} \right)^{1/2} ,\ + \infty \right]$$

but obviously there is a finite upper limit of γ due to the finite maximum beam velocity (beam particle kinetic energy), which can be found in nature. For instance, if we assume as upper limit of the beam velocity that estimated for the fastest electrons in a type III-exciting beam

$$V_{bMAX} = 0.6\ c = 1.8 \times 10^{10}\ \text{cm/s} \qquad (E_K = 92\ \text{keV}),$$

where c is the velocity of light in vacuum, and as electron thermal velocity that of the isothermal corona

$$v_{Te} = 7.8 \times 10^{8} \text{ cm/s}$$

we get

$$\gamma_{MAX} = 23.$$

We must remind that it is not easy to find a mechanism which accelerates protons to relativistic velocities even in the perturbed corona. In fact a beam velocity

$$V_b = 0.6 \ c$$

would correspond to proton kinetic energies of the order of E_K = 169 MeV.

However, we can assume the estimated γ_{MAX} as an upper limit in some particular situations. In conclusion, the consistent range of values for γ can be set as $\gamma \in [4, 23]$.

Consistently with the hypothesis that the beam is quasi-monoenergetic, it holds

$$\left| \frac{v_{ph} - V_b}{v_{Tb}} \right| \gg 1$$

for the argument of the related plasma dispersion function.

Noteworth mentioning is the fact that large arguments of the plasma dispersion function $x \gg 1$ realize the cold case, i.e. the limit of the hydrodynamic approximation.

Summarizing, the ranges of the parameters are as follows:

$\varepsilon \in [0,1]$ $\alpha \in [0,1]$ $\beta = (\alpha / 10)$ $\gamma \in (3,23]$

The situation chosen for the numerical computation refers to the following parameter values:

		$\varepsilon = 10^{-3}$
$T_e = 2.0x10^6$ K	$v_{Te} = 7.80\ x10^8$ cm/s	
$T_i = 1.0\ T_e$	$v_{Ti} = 0.18\ x10^8$ cm/s	$\alpha = 1.0$
$T_b = 0.10\ T_e$	$v_{Tb} = 0.058x10^8$ cm/s	$\beta = 0.10$
$V_b = 5.0\ v_{Te}$	$V_b = 39\ x10^8$ cm/s	$\gamma = 5.0$ (E_{Kp} = 80 MeV)

In such a case the velocity distribution function is plotted (normalized) in Figure 8b (2-D form) and in Figure 9 (reduced 1-D form) and it represents what is called a "gap" distribution. This denomination is due to the fact that the high streaming velocity of the beam (non-thermal beam) makes its location in velocity space far from velocities where the background Maxwellian has an appreciable value and apparently a gap divides the background from the beam.

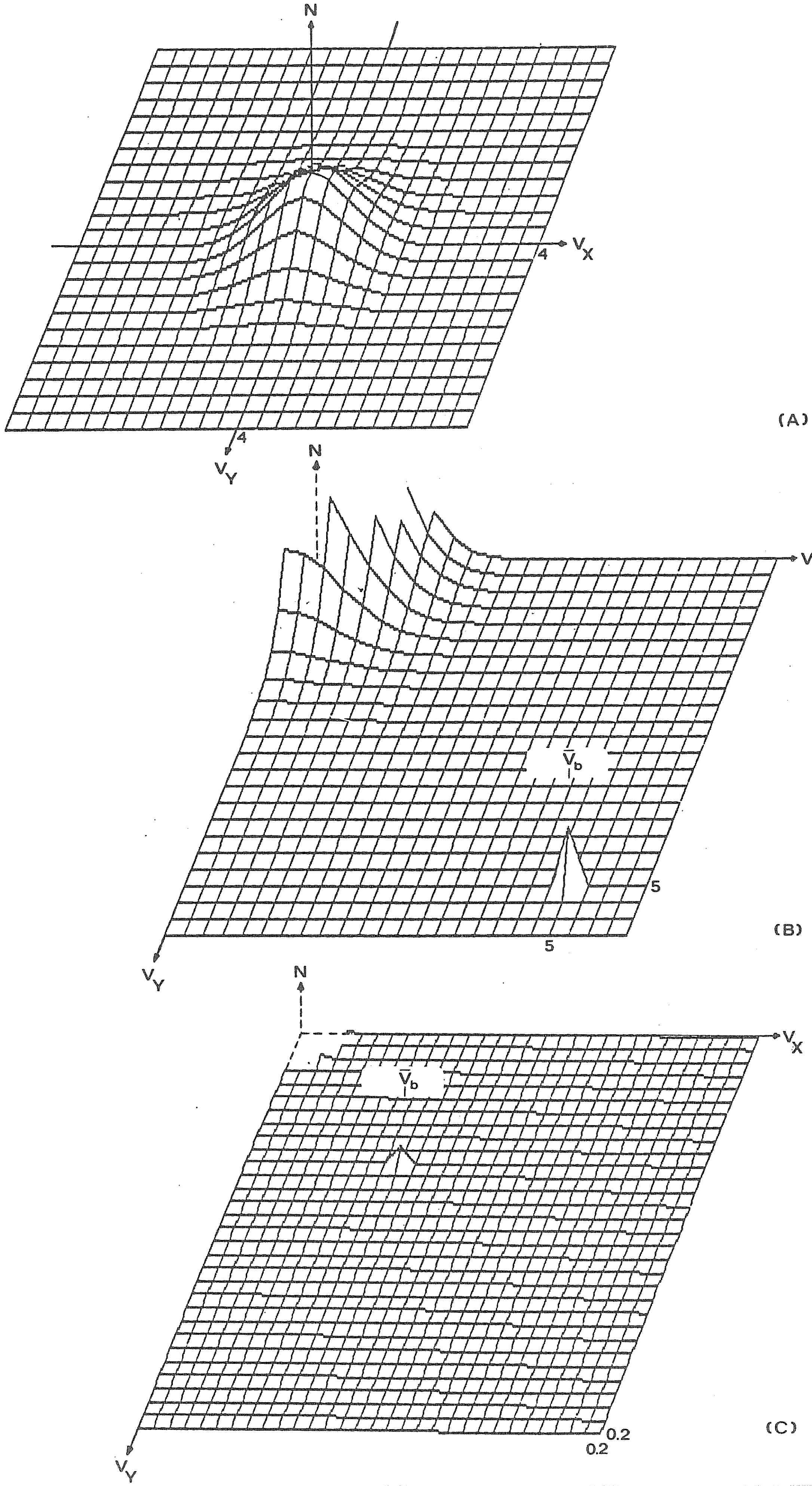


FIGURE 6 : 2-D REDUCED FORM OF THE BEAM-PLASMA DISTRIBUTION
(a) BACKGROUND PLASMA MAXWELLIAN DISTRIBUTION
(b) BACKGROUND + NON-THERMAL BEAM DISTRIBUTION
(c) BACKGROUND + SUB-THERMAL BEAM DISTRIBUTION

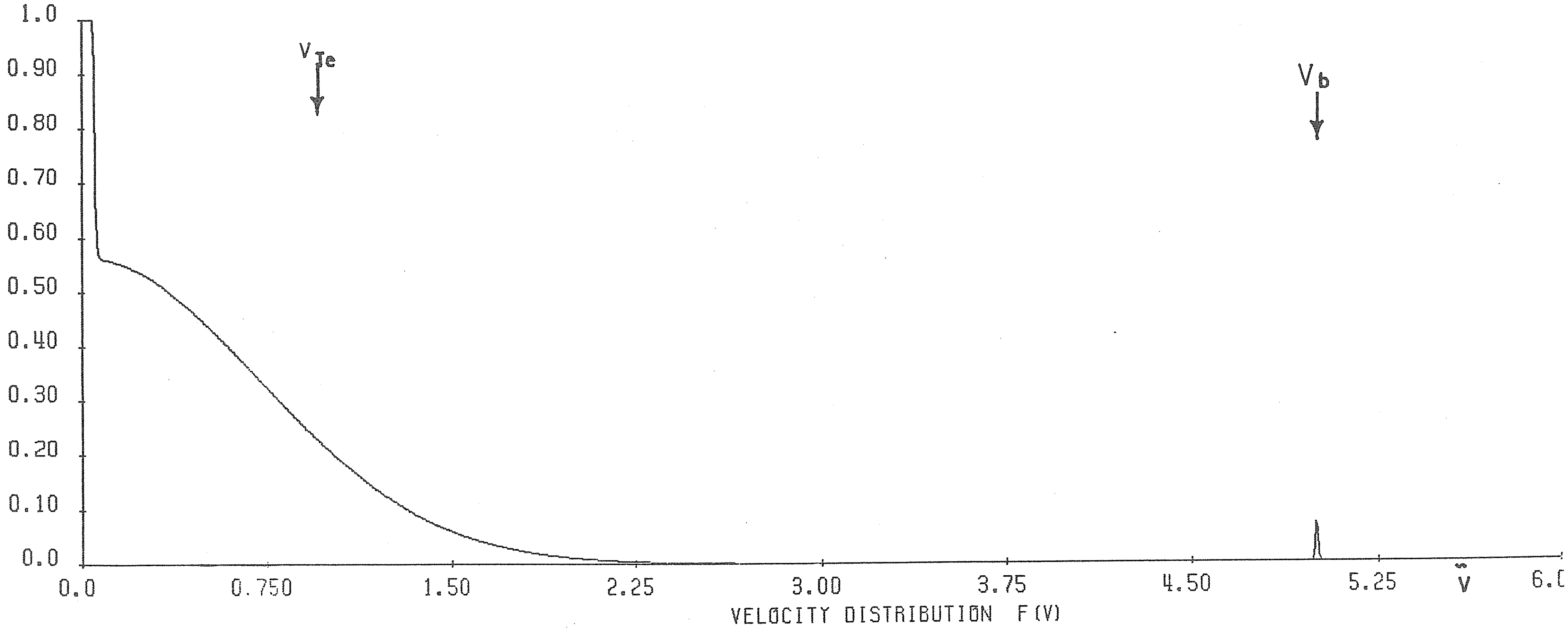


FIGURE 9 : 1-D REDUCED FORM OF THE BEAM-PLASMA DISTRIBUTION FOR A NON-THERMAL BEAM

B. Numerical Analysis of the Dispersion Relation

From the analysis in A. we get the three arguments of the plasma dispersion function for the electrons, the ions and the beam protons satisfying the relations:

$$\left| \frac{\omega}{k\, v_{Te}} \right| << 1 \qquad \left| \frac{\omega}{k\, v_{Ti}} \right| >> 1 \qquad \left| \frac{\omega - kV_b}{k\, v_{Tb}} \right| >> 1$$

and we can substitute the correspondent asymptotic expansions of W(x) in the dispersion relation. By the way we stress that large arguments of the function W realizes the limit of vanishing temperature.

Using normalized variables as in the cold case and applying the weak instability approximation, we obtain the asymptotic form of the dispersion relation for a non-thermal beam as:

$$\tilde{D}_r(\tilde{\omega}_r,\tilde{k}) = -\frac{1}{2}\frac{1}{\tilde{\omega}^2} - \frac{1}{2}\frac{1}{MR}\frac{1}{\tilde{\omega}^2} - \frac{1}{2}\frac{\varepsilon}{MR}\frac{1}{(\tilde{\omega}-\tilde{k})^2} - \frac{3}{4}\frac{1}{\gamma^2}\frac{\tilde{k}^2}{\tilde{\omega}^4} -$$

$$-\frac{3}{4}\frac{\alpha}{MR^2}\frac{1}{\gamma^2}\frac{\tilde{k}^2}{\tilde{\omega}^4} - \frac{3}{4}\frac{\beta}{MR^2}\frac{\varepsilon}{\gamma^2}\frac{\tilde{k}^2}{(\tilde{\omega}-\tilde{k})^4} + 1 = 0$$

which is an 8th order algebraic equation in $\tilde{\omega}$ and can be solved numerically to yield the oscillation branches.

The damping rate is obtained by means of the formula:

$$\tilde{\gamma} = -\frac{\tilde{D}_i(\tilde{\omega}_r,\tilde{k})}{\partial \tilde{D}_r / \partial \tilde{\omega}_r}$$

and its explicit form is reported in Figure 10.

$$\tilde{\gamma} = \tilde{\gamma}_{Le} + \tilde{\gamma}_{Li} + \tilde{\gamma}_{Lb}$$

$$\tilde{\gamma}_{Le} = \frac{-\sqrt{\pi}\ \gamma^3 \dfrac{\tilde{\omega}}{\tilde{k}^3} \exp\left\{-\left[\gamma \dfrac{\tilde{\omega}}{\tilde{k}}\right]^2\right\}}{\partial \tilde{D}_r / \partial \tilde{\omega}_r}$$

$$\tilde{\gamma}_{Li} = \frac{-\sqrt{\pi}\ \sqrt{MR}\ \dfrac{\gamma^3}{\alpha^{3/2}} \dfrac{\tilde{\omega}}{\tilde{k}^3} \exp\left\{-\left[\gamma \left(\dfrac{MR}{\alpha}\right)^{1/2} \dfrac{\tilde{\omega}}{\tilde{k}}\right]^2\right\}}{\partial \tilde{D}_r / \partial \tilde{\omega}_r}$$

$$\tilde{\gamma}_{b} = \frac{-\sqrt{\pi}\ \sqrt{MR}\ \varepsilon\ \dfrac{\gamma^3}{\beta^{3/2}} \dfrac{(\tilde{\omega}-\tilde{k})}{\tilde{k}^3} \exp\left\{-\left[\gamma \left(\dfrac{MR}{\beta}\right)^{1/2} \dfrac{(\tilde{\omega}-\tilde{k})}{\tilde{k}}\right]^2\right\}}{\partial \tilde{D}_r / \partial \tilde{\omega}_r}$$

$$\frac{\partial \tilde{D}_r}{\partial \tilde{\omega}_r} = \frac{1}{\tilde{\omega}^3} + \frac{1}{MR}\frac{1}{\tilde{\omega}^3} + \frac{\varepsilon}{MR}\frac{1}{(\tilde{\omega}-\tilde{k})^3} + \frac{3}{\gamma^2}\frac{\tilde{k}^2}{\tilde{\omega}^5} +$$

$$+ 3\frac{\alpha}{MR^2}\frac{1}{\gamma^2}\frac{\tilde{k}^2}{\tilde{\omega}^5} + 3\frac{\beta}{MR^2}\frac{\varepsilon}{\gamma^2}\frac{\tilde{k}^2}{(\tilde{\omega}-\tilde{k})^5}$$

FIGURE 10: GROWTH RATE FOR THE RESONANT INSTABILITY

It is composed by 3 terms: the first two terms represent the Landau damping by electrons and ions and are negative; the third term is due to the presence of the beam and can lead to growth ($\tilde{\gamma}_b > 0$) when $\tilde{\omega} < \tilde{k}$ i.e. $v_{ph} < V_b$ as it is known.

The maximum growth in the case of a gap distribution is obtained for

$$V_b - v_{ph} = \frac{v_{Tb}}{\sqrt{2}}$$

whence in our notation:

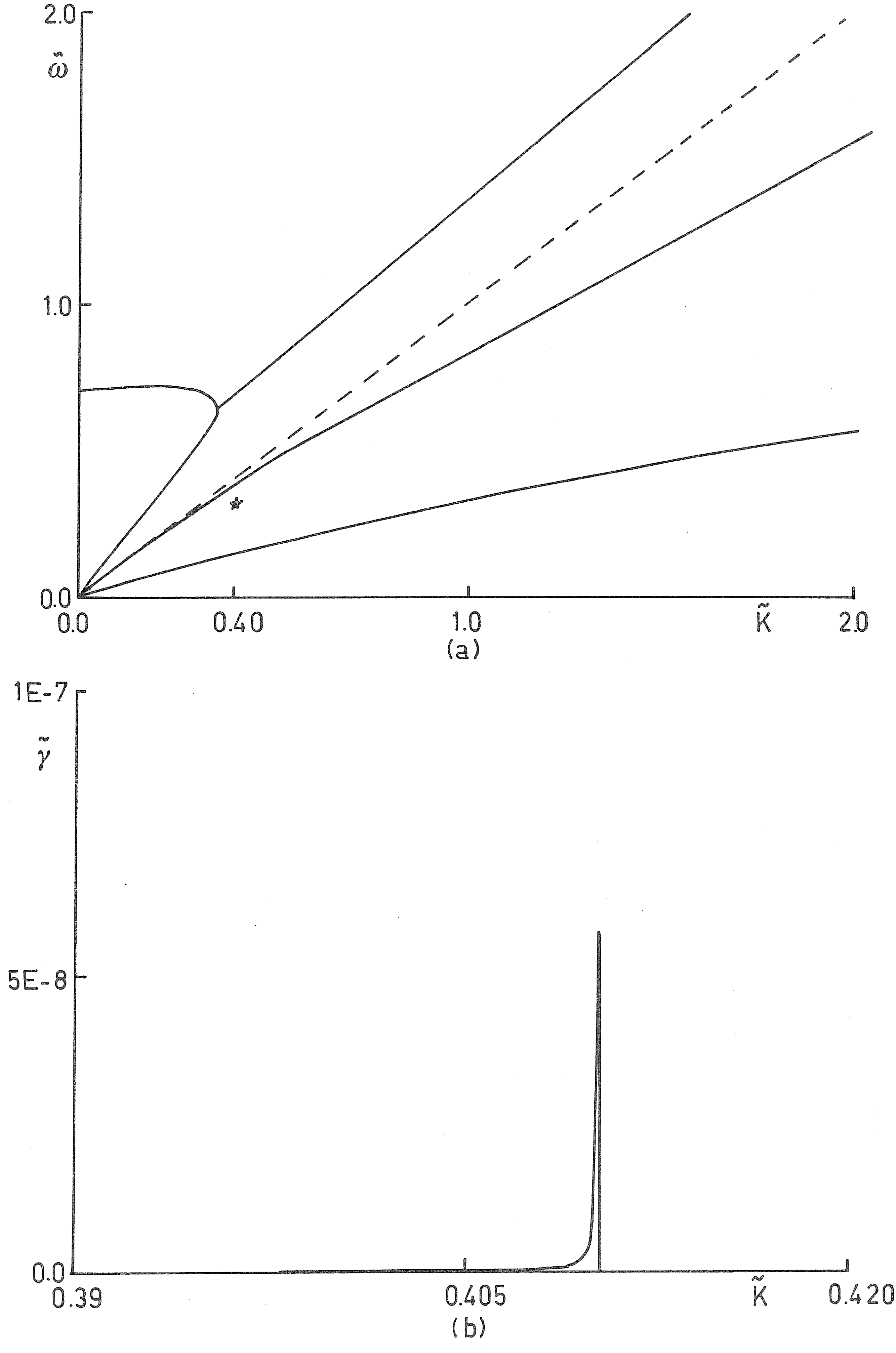
$$\tilde{k}_{MAX} = \frac{\tilde{\omega}_G(\tilde{k}_{MAX})}{1 - \dfrac{1}{\gamma}\left[\dfrac{\beta}{MR}\right]^{1/2}} \qquad \tilde{\gamma}_{MAX} = \tilde{\gamma}\,(\tilde{\omega}_G, \tilde{k}_{MAX})$$

is the wavevector corresponding to maximum growth and $\tilde{\omega}_G$ is the growing oscillation given by the dispersion relation.

We solved numerically the equation $\tilde{D}_r = 0$ for parameter values as in point A., obtaining the real oscillation branches reported in Figure 11a in the form $\tilde{\omega}_i = \tilde{\omega}_i(\tilde{k})$. Then the growth rates were computed using the formula of Figure 10 for each real solution and the only positive one is plotted in Figure 11b as $\tilde{\gamma}_i = \tilde{\gamma}_i(\tilde{k})$.

The unstable solution is marked with a star (*) in panel (a) and the growing oscillation is limited to a very restricted range of wavenumbers in the proximity of the resonance branch. Moreover the maximum growth has a very small magnitude. In this particular case it seems that the generation of beam-plasma oscillations requires very specialized condition to be effective.

FIGURE 11: NUMERICAL SOLUTIONS OF THE DISPERSION RELATION FOR A NON-THERMAL BEAM IN THE WARM CASE (a) OSCILLATION BRANCHES; (b) GROWTH RATE



VII.3 Ion-Acoustic Instability

A. Analysis of the Distribution

The basic requirement for the excitation of ion sound oscillations is: a) the presence of a strong temperature anisotropy $T_i << T_e$ in the background and b) a range for the perturbation phase velocity as

$$v_{Ti} << v_{ph} << v_{Te} \quad .$$

As a first approximation we will assume that the dispersion relation for ion sound waves is $\omega \sim c_s k$ at long wavelengths ($k^2 d_e^2 << 1$, $d_e = (2\,k\,T_e / 4\,n_o\,e^2)^{1/2}$ Debye length of electrons), where c_s is the ion sound velocity

$$c_s = \left[\frac{2\,k\,T_e}{M} \right]^{1/2} = \left[\frac{1}{MR} \right]^{1/2} v_{Te}$$

and neglecting the thermal corrections.

I. According to the latter statement it holds

$$v_{ph} = \frac{\omega}{k} \quad c_s = \left[\frac{1}{MR} \right]^{1/2} v_{Te} \sim 0.023\ v_{Te}$$

whence $v_{ph} << v_{Te}$ and the right part of the inequality is satisfied, leading to the validity of $| v_{ph}/v_{Te} | <<1$ for the argument of the plasma dispersion function related to the electrons. It can be shown that the Landau damping by plasma electrons is very small in this range of phase velocities even if the resonance electrons are augmemted. In fact the difference between the number of slow and fast electrons (which is $\propto -\partial F/\partial v$) is small when $v_{ph} = c_s << v_{Te}$ and such is also the damping rate.

II. The left part of the inequality reads

$$v_{Ti} << v_{ph} = (1/MR)^{1/2}\, v_{Te}$$

but to avoid ion Landau damping, it must be at least

$$3\, v_{Ti} < v_{ph}$$

i.e.

$$3 \left[\frac{\alpha}{MR} \right]^{1/2} < \left[\frac{1}{MR} \right]^{1/2} \quad ==> \quad \alpha < \frac{1}{9} = 0.\overline{11}$$

and this introduces a new constraint on the parameter α , which range is now $\alpha \in [0, 0.\overline{11})$.
In this case we can argue that $|v_{ph}/v_{Ti}| >> 1$ is the condition for the argument of the plasma dispersion function related to the ions.

III. It appears evident from I. and II. that it must be

$$v_{Ti} < V_b < v_{Te}$$

$$(\alpha\, /\, MR)^{1/2} < \gamma < 1$$

but this inequality will be better specified in the following.

IV. For having growth ($\partial F / \partial v > 0$) it must be:

$$V_b > v_{ph} > V_b - v_{Tb} > 3\, v_{Ti}$$

where the extreme right part of the inequality prevents the oscillations from being Landau damped by the ions. The other left and right parts of the inequality refers to the range of positive derivative of the distribution function. In such a case the lowest boundary was estimated as $V_b - v_{Tb}$ instead of $V_b - 2v_{Tb}$ (cf. beam-plasma oscillation approximation) as in the actual situation the range of positive derivative is restricted by the low streaming velocity of the beam, which put the location of the beam in velocity space at velocity coordinates where the value of the background distribution function is no more negligible.

With the usual conventions on the parameters we can write:

$$\gamma > \left[\frac{1}{MR} \right]^{1/2} > \gamma - \left[\frac{\beta}{MR} \right]^{1/2} > 3 \left[\frac{\alpha}{MR} \right]^{1/2}$$

but in the monoenergetic beam approximation we can assume, as it was done previously, $\beta = \alpha / 10 \ll 1$ and whence:

$$\underbrace{\gamma > \overbrace{\left[\frac{1}{MR} \right]^{1/2}}^{3}}_{} \; \overbrace{>}^{1}\; \gamma - \left[\frac{\alpha}{10\; MR} \right]^{1/2} > 3 \left[\frac{\alpha}{MR} \right]^{1/2}$$

(Labels under the inequality: 1 spans $\left[\frac{1}{MR}\right]^{1/2} > \gamma - \left[\frac{\alpha}{10\,MR}\right]^{1/2}$; 3 spans $\gamma > \left[\frac{1}{MR}\right]^{1/2}$; 2 spans $\gamma - \left[\frac{\alpha}{10\,MR}\right]^{1/2} > 3\left[\frac{\alpha}{MR}\right]^{1/2}$.)

i.e. from (1)

$$\left[\frac{1}{MR}\right]^{1/2} + \frac{1}{\sqrt{10}}\left[\frac{\alpha}{MR}\right]^{1/2} > \gamma \qquad \frac{1}{\sqrt{MR}}\left[1 + \frac{1}{\sqrt{10}}\sqrt{\alpha}\right] > \gamma$$

and from (2)

$$\gamma > 3\frac{\sqrt{\alpha}}{\sqrt{MR}} + \frac{1}{\sqrt{10}}\frac{\sqrt{\alpha}}{\sqrt{MR}} = \left[3 + \frac{1}{\sqrt{10}}\right]\frac{\sqrt{\alpha}}{\sqrt{MR}}$$

and from (3)

$$\gamma > \frac{1}{\sqrt{MR}} \sim 0.023 \quad .$$

This leads to the following constraint on γ :

$$\alpha < \left(3 + \frac{1}{\sqrt{10}}\right)^{-2} \qquad \gamma \in \left(\frac{1}{\sqrt{MR}}, \frac{1}{\sqrt{MR}}\left[1 + \frac{\sqrt{\alpha}}{\sqrt{10}}\right]\right)$$

$$\alpha > \left(3 + \frac{1}{\sqrt{10}}\right)^{-2} \qquad \gamma \in \left(\left[3 + \frac{1}{\sqrt{10}}\right]\frac{\sqrt{\alpha}}{\sqrt{MR}}, \frac{1}{\sqrt{MR}}\left[1 + \frac{\sqrt{\alpha}}{\sqrt{10}}\right]\right)$$

From the assumption that the thermal spread of the beam ions is small ($\beta \ll 1$) it follows that the argument of the plasma dispersion function related to the beam satisfies the inequality

$$\left|\frac{\omega - kV_b}{k\, v_{Tb}}\right| \gg 1 \quad .$$

To summarize the parameters' range in the ion sound approximation we can write:

$$\alpha \in \left(0\ ,\ \left[3+\frac{1}{\sqrt{10}}\right]^{-2}\right] \qquad \left[\left[3+\frac{1}{\sqrt{10}}\right]^{-2}\ ,\ 0.\overline{11}\right)$$

$$\beta \in \left(0,\ 9.1\ x\ 10^{-3}\right] \qquad \left[9.1\ x\ 10^{-3}\ ,\ 1.1\ x\ 10^{-2}\right)$$

$$\gamma \in \left(\frac{1}{\sqrt{MR}}\ ,\ \frac{1}{\sqrt{MR}}\left[1+\frac{\sqrt{\alpha}}{\sqrt{10}}\right]\right] \quad \left[\left[3+\frac{1}{\sqrt{10}}\right]\frac{\sqrt{\alpha}}{\sqrt{MR}}\ ,\ \frac{1}{\sqrt{MR}}\left[1+\frac{\sqrt{\alpha}}{\sqrt{10}}\right]\right)$$

For the numerical computation the following parameter values were considered:

$$\varepsilon = 10^{-3}$$

$$\alpha = 0.08$$

$$\beta = 0.008$$

$$\gamma = 0.068$$

leading to the real values:

T_e = $25x10^6$ K (2.15 keV) v_{Te} = $2.7x10^9$ cm/s

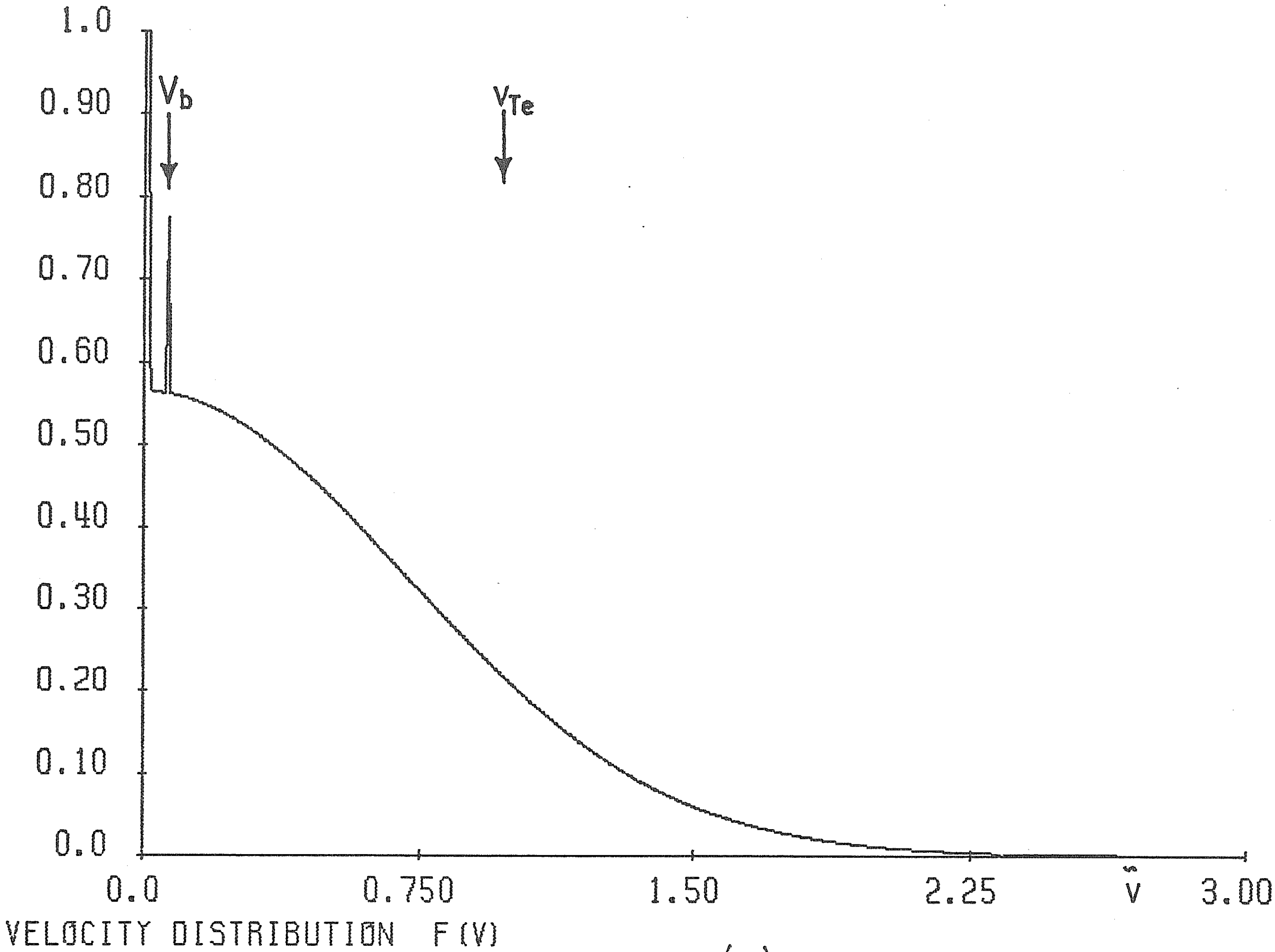
T_i = $2x10^6$ K v_{Ti} = 18 $x10^6$ cm/s

T_b = $2x10^6$ K v_{Tb} = $5.7x10^6$ cm/s

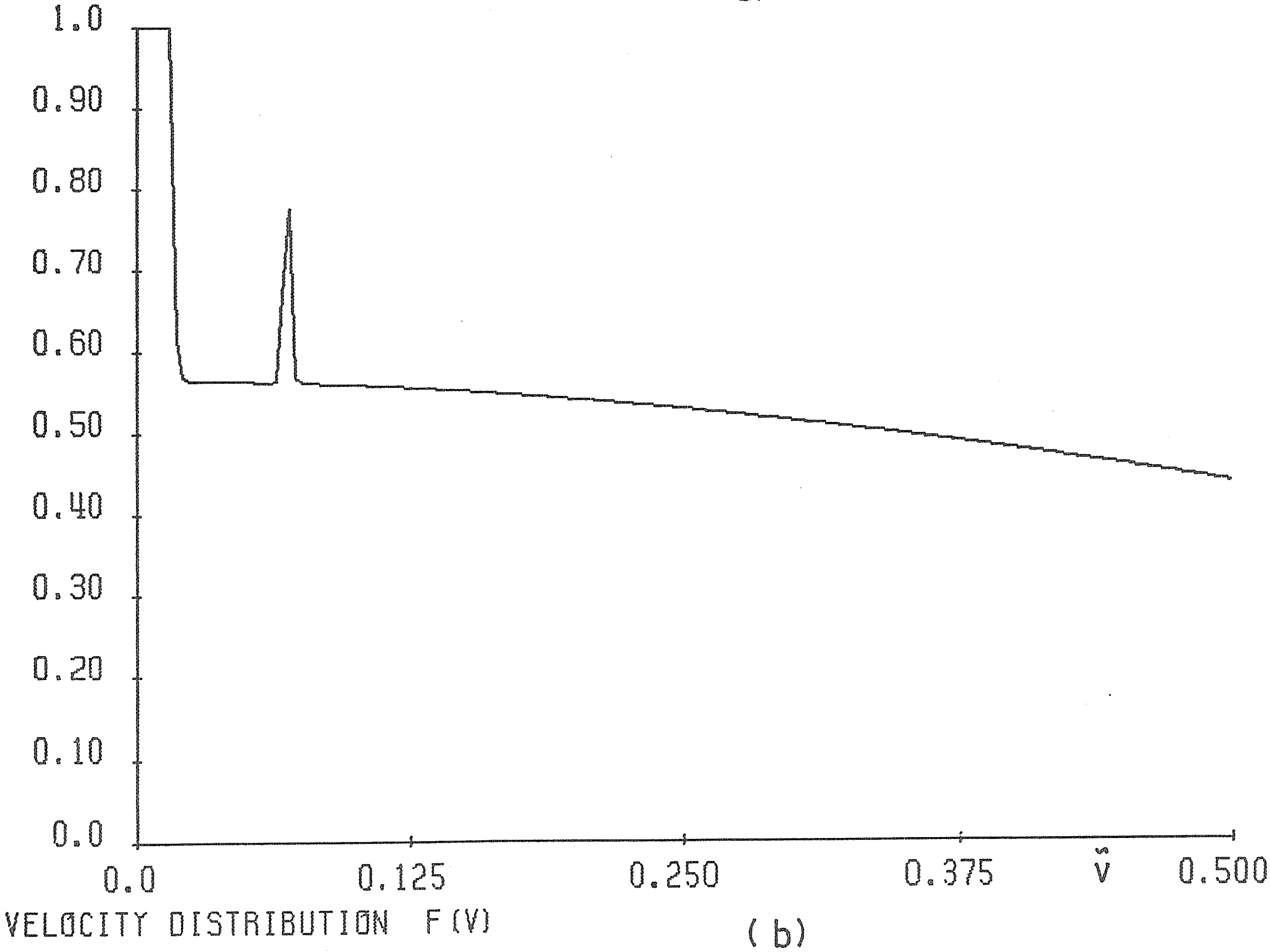
V_b = $187x10^6$ cm/s E_K = 18.2 keV

This leads to a distribution function which is no more a gap distribution but it appears (normalized) as in Figure 8c (2-D form) and as in Figure 12a,b (reduced 1-D form).

FIGURE 12: 1-D REDUCED FORM OF THE BEAM-PLASMA DISTRIBUTION FOR A SUB-THERMAL BEAM



(a)



(b)

B. Numerical Analysis of the Dispersion Relation

From the previous analysis it follows that the arguments of the plasma dispersion function satisfy the relations:

$$\left| \frac{\omega}{k\, v_{Te}} \right| << 1 \qquad \left| \frac{\omega}{k\, v_{Ti}} \right| >> 1 \qquad \left| \frac{\omega - kV_b}{k\, v_{Tb}} \right| >> 1$$

and we will use the correspondent asymptotic expansions for the function W(x).

Paralleling the procedure followed in the paragraph VII.2.B we get the asymptotic form of the dispersion relation for a thermal beam:

$$\tilde{D}_r(\tilde{\omega}_r, \tilde{k}) = \frac{\gamma^2}{\tilde{k}^2} - 2 \quad \gamma^4 \frac{\tilde{\omega}^2}{\tilde{k}^4} - \frac{1}{2\,MR} \frac{1}{\tilde{\omega}^2} - \frac{3}{4} \frac{\alpha}{\gamma^2} \frac{1}{MR^2} \frac{\tilde{k}^2}{\tilde{\omega}^4} -$$

$$- \frac{1}{2} \frac{\varepsilon}{MR} \frac{1}{(\tilde{\omega} - \tilde{k})^2} - \frac{3}{4} \frac{\beta}{\gamma^2} \frac{1}{MR^2} \varepsilon \frac{\tilde{k}^2}{(\tilde{\omega} - \tilde{k})^4} + 1 = 0$$

which is a 12th order algebraic equation in $\tilde{\omega}_r$.

The growth rate is reported in Figure 13. and the same considerations are valid as in VII.2.B.

The solution related to ion sound oscillations is plotted in Figure 14a for the parameter values considered in VII.3.A and the corresponding damping rate is presented in Figure 14 b.

For this choice of the parameters the ion sound waves seem to be damped and not growing for all the range of wavevectors.

$$\tilde{\gamma} = \tilde{\gamma}_{Le} + \tilde{\gamma}_{Li} + \tilde{\gamma}_{Lb}$$

$$\tilde{\gamma}_{Le} = \frac{-\sqrt{\pi}\ \gamma^3 \dfrac{\tilde{\omega}}{\tilde{k}^3}}{\partial \tilde{D}_r \ / \ \partial \tilde{\omega}_r}$$

$$\tilde{\gamma}_{Li} = \frac{-\sqrt{\pi}\ \sqrt{MR}\ \dfrac{\gamma^3}{\alpha^{3/2}} \dfrac{\tilde{\omega}}{\tilde{k}^3} \exp\left\{ - \left[\gamma \left(\dfrac{MR}{\alpha}\right)^{1/2} \dfrac{\tilde{\omega}}{\tilde{k}} \right]^2 \right\}}{\partial \tilde{D}_r \ / \ \partial \tilde{\omega}_r}$$

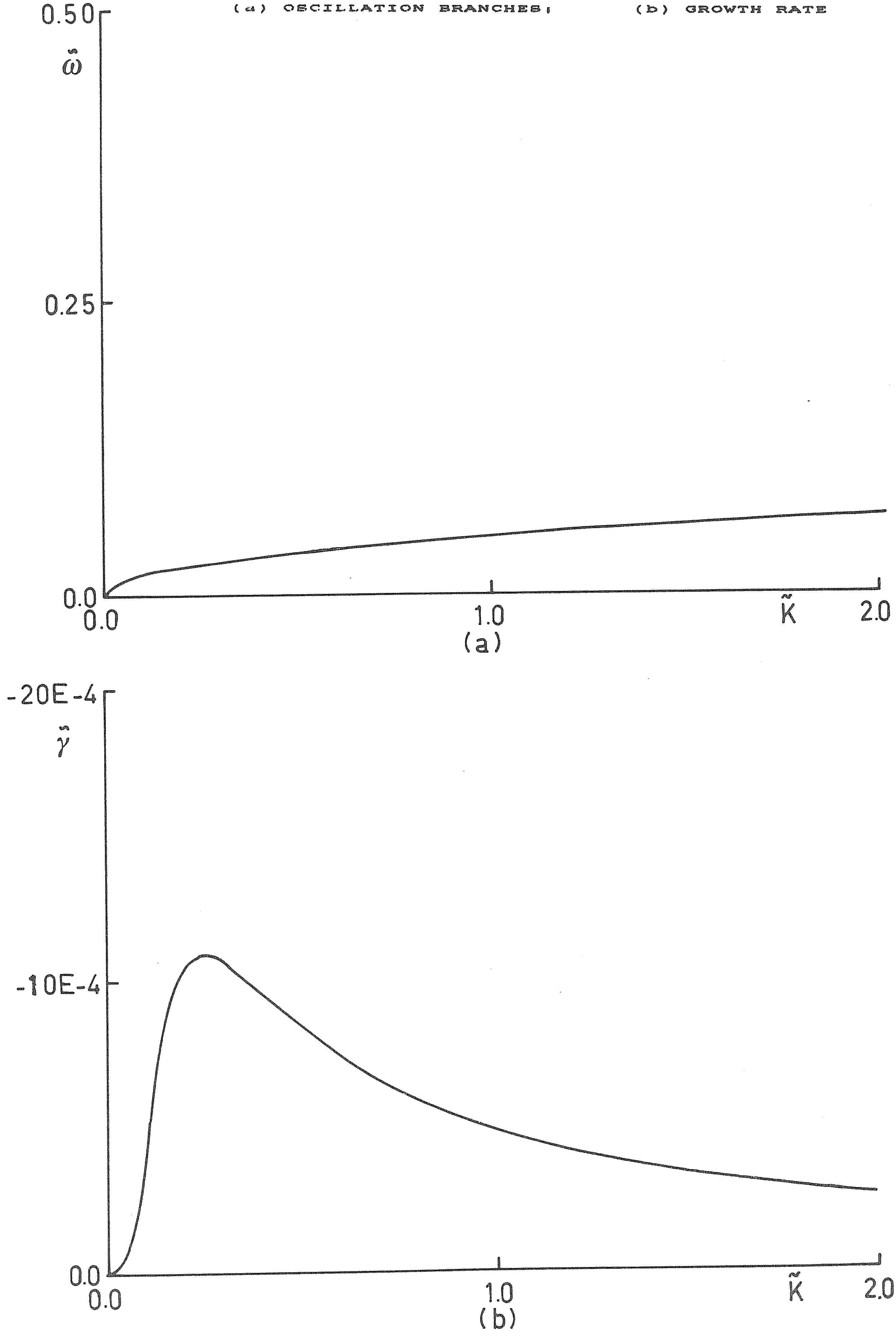
$$\tilde{\gamma}_{b} = \frac{-\sqrt{\pi}\ \sqrt{MR}\ \varepsilon \dfrac{\gamma^3}{\beta^{3/2}} \dfrac{(\tilde{\omega}-\tilde{k})}{\tilde{k}^3} \exp\left\{ - \left[\gamma \left(\dfrac{MR}{\beta}\right)^{1/2} \dfrac{(\tilde{\omega}-\tilde{k})}{\tilde{k}} \right]^2 \right\}}{\partial \tilde{D}_r \ / \ \partial \tilde{\omega}_r}$$

$$\frac{\partial \tilde{D}_r}{\partial \tilde{\omega}_r} = -4 \ \gamma^4 \frac{\tilde{\omega}}{\tilde{k}^4} + \frac{1}{MR} \frac{1}{\tilde{\omega}^3} + 3 \frac{\alpha}{\gamma^2 \ MR^2} \frac{\tilde{k}^2}{\tilde{\omega}^5} +$$

$$+ \frac{\varepsilon}{MR} \frac{1}{(\tilde{\omega}-\tilde{k})^3} + 3 \frac{\beta}{MR^2} \frac{\varepsilon}{\gamma^2} \frac{\tilde{k}^2}{(\tilde{\omega}-\tilde{k})^5}$$

FIGURE 13: GROWTH RATE FOR THE ION-ACOUSTIC INSTABILITY

FIGURE 14: NUMERICAL SOLUTIONS OF THE DISPERSION RELATION FOR A SUB THERMAL BEAM IN THE WARM CASE
(a) OSCILLATION BRANCHES; (b) GROWTH RATE



VII.4 Remarks

The analysis of the instability range via numerical solutions of the dispersion relation was used for just two cases of the parameter values to indicate the methodology the future work will be based on and so the obtained results are not exhausting the problem.

What is of interest is to find out the range of parameter values which lead to instability, stating the boundaries between different kind of instabilities, and this requires a sensible amount of computing time.

Another numerical way to determine the instability range of a velocity distribution in the electrostatic case is the application of the Penrose criterion and a suitable software is under development.

CONCLUSION

The physics of proton beams is a quite complex topic and in this thesis we just scratched the surface by outlining some of the relevant problems.

As a concluding remark we can say that from the necessarily limited analysis performed it comes out that the proton beams have a restricted range of unstable electrostatic oscillations. It is however evident that the analysis have to be extended to many different values of the beam-plasma system parameters.

If it were true that the electrostatic bump-on-tail instability is not so effective to generate a suitable level of plasma turbulence and such also the ion acoustic instability, many other kinds of instability can be investigated and we guess that for instance the ion-cyclotron instability could have a dominant role.

It is understood that many things have to be done in the subject and actually the work is in progress as:

a) the development of an improved analytical-numerical analysis of the electrostatic instabilities, based on the Penrose criterion;

b) the solution of the complete dispersion relation in the electrostatic and electromagnetic case;

c) the extension of the analysis to other kinds of instabilities, which we are considering;

d) the approach to the non-linear theory of the radioemission process.

The project is stimulating and very new results can come out from the theory, never considered before or maybe considered but not with the due emphasis to the proton beam physics.

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