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DARK MATTER AND THE SEVEN DWARFS

Thesis submitted for the degree of

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To Licia

"Giunti in una radura il corteo s'arrestò e la principessa vide innanzi a sé una strana casetta: era piccola, ma così piccola ...
'Chi abita qui?'
'I sette nani,' cinguettarano i passeri."

⋮

"Frattanto, alla casetta dei nani, Biancaneve aveva organizzato una festicciola in onore dei suoi amici che l'avevano ospitata tanto di buon grado."

(una fiaba)

"As it reached a clearing the procession stopped and the princess found herself looking at a strange little house: but it was so small, oh so very small ...
'Who lives here?'
'The seven dwarfs,' chirped the sparrows."

⋮

"Meanwhile, in the house of the dwarfs, Snow White had organised a party in honour of her friends who had shown her such warm hospitality."

(a fairy tale)

Abstract

There are seven ghost–like objects, known as dwarf spheroidal galaxies, which orbit the Milky Way. We consider the question of whether dark matter can be identified with the seven dwarfs. The former may or may not exist in any environment, and this is discussed in chapter 1. If they *are* associated with each other, then the former's identity is almost certainly *not* that of the only non–baryonic dark matter candidate actually known to exist — we discuss this issue in chapters 2 and 4. Somewhat larger dwarfs provide an intermediate case (chapter 3), however we find that neither theory nor observation is yet sufficiently developed to answer these questions.

This review is based on information which was known to be available in July '88.

ACKNOWLEDGEMENTS

I would like to thank Professor D. Sciama for having brought to my attention the inter–relation between phase–space constraints, dwarf galaxies and neutrinos, which stimulated my interest in this topic. Roberto Scaramella deserves my thanks for pointing out that 'dark matter in general' is too vast an argument for a review thesis whose deadline is less than a non–negligible amount of time away. Thanks also to Vincenzo Antonuccio for the occasional technical discussion or comment. Various people were kind enough to help a naïve layman operate a computer terminal: Marco Bruni, José Acosta, Antonio Lanza, Mauro Orlandini, and Michel Rieutord. I take this opportunity to also express my appreciation to John Miller, who helped me so much at the beginning of my stay in Italy.

Last but not least, I wish to acknowledge the index finger of my right hand, which with only the occasional complaint consented to digitate the entirety of this thesis.

Galaxy	M_V *(mag)*	*d (kpc)*	r_c *(kpc)*	r_t/r_c	$\mu_0(V)(mag/\square'')$	*HB type*
Fornax	−12.6	140	0.5	6	23.3	red
Leo I	−11.4	220	0.3	3	23.5	red?
Sculptor	−11.1	80	0.2	6	23.9	red
Leo II	−10.2	220	0.2	4	23.9	red
Carina	−9.4	100	0.2	3	24.9	red
Draco	−8.5	75	0.15	3	25.4	red
Ursa Minor	−8.5	65	0.15	6	26.1	blue(!)

Table 1: Some Dwarf Spheroidal Properties

Candidate/particle	Approximate mass	Predicted by	Astrophysical effects
$G(R)$	—	Non-Newtonian gravitation	Mimics DM on large scales
Λ (cosmological constant)	—	General relativity	Provides $\Omega = 1$ without DM
Axion, majoron, goldstone boson	10^{-5} eV	QCD; PQ symmetry breaking	Cold DM
Ordinary neutrino	10–100 eV	GUTs	Hot DM
Light higgsino, photino, gravitino, axino, sneutrino[b]	10–100 eV	SUSY/SUGR	Hot DM
Para-photon	20–400 eV	Modified QED	Hot/warm DM
Right-handed neutrino	500 eV	Superweak interaction	Warm DM
Gravitino, etc.[b]	500 eV	SUSY/SUGR	Warm DM
Photino, gravitino, axino, mirror particle, simpson neutrino[b]	keV	SUSY/SUGR	Warm/cold DM
Photino, sneutrino, higgsino, gluino, heavy neutrino[b]	MeV	SUSY/SUGR	Cold DM
Shadow matter	MeV	SUSY/SUGR	Hot/cold (like baryons)
Preon	20–200 TeV	Composite models	Cold DM
Monopoles	10^{16} GeV	GUTs	Cold DM
Pyrgon, maximon, perry pole, newtorites, Schwarzschild	10^{19} GeV	Higher-dimension theories	Cold DM
Supersymmetric strings	10^{19} GeV	SUSY/SUGR	Cold DM
Quark nuggets, nuclearites	10^{15} g	QCD, GUTs	Cold DM
Primordial black holes	10^{15-30} g	General relativity	Cold DM
Cosmic strings, domain walls	$10^{8-10}\ M_\odot$	GUTs	Promote galaxy formation, but cannot contribute much to Ω

[a] Abbreviations: DM, dark matter; QCD, quantum chromodynamics; PQ, Peccei & Quinn; GUTs, grand unified theories; SUSY, supersymmetric theories; SUGR, supergravity; QED, quantum electrodynamics.

[b] Of these various supersymmetric particles predicted by assorted versions of supersymmetric theories and supergravity, only one, the lightest, can be stable and contribute to Ω, but the theories do not at present tell us which one it will be or the mass to be expected.

Figure 1: Dark Matter Candidates — from Trimble '87

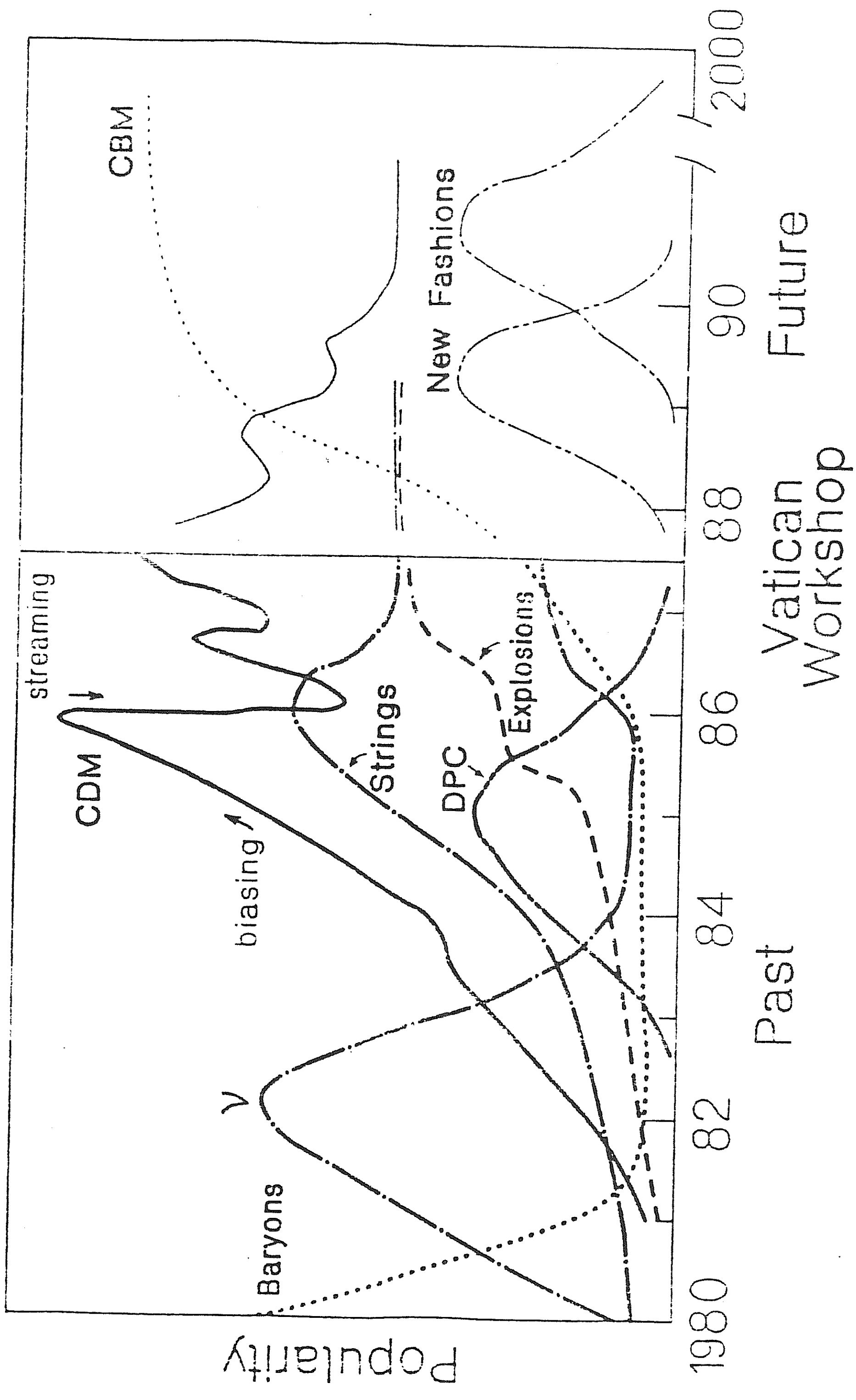

Figure 2: candidate popularity graph — from Dekel '88 (preprint)

Contents

Chapter 1

INTRODUCTION

1.1 HISTORY OF DARK MATTER

The so–called dark matter problem began back in 1932 when Oort [158] calculated that roughly one half of the mass in the solar neighbourhood inferred from stellar velocities had to be in a form unaccounted for by the visible stars. One year later, Zwicky [222] came to the conclusion that the luminous galaxies in Virgo could only account for 1–10% of the mass inferred from the measured velocity dispersions. The same problem was reinforced by Smith in 1936 [188], and in the following year the Coma cluster was found to have the same property [223].

During the subsequent three and a half decades, relatively little appeared in the literature on this problem, with one of the major works being that of Kahn and Woltjer [112], who in 1959 stated that necessary for the dynamical stability of the Local Group was the existence of an 'appreciable amount of intergalactic matter', which they concluded must be comprised mainly of ionised hydrogen. Two years after that, more evidence was found [154] that the total masses of individual galaxies in clusters determined from optical rotation curves did not add up to sufficiently high values to bound these clusters.

It will have already been noticed that the dark matter problem, right from the beginning, was one that existed on a variety of scales, from the solar neighbourhood up to the Virgo cluster. Indeed it will be seen later on in this account that the problem exists, at least in the 'majority view' on *all* scales ranging from the neighbourhood of the Sun right up to the Universe as a whole.

The modern era of dark matter investigation can be conveniently designated as having begun in 1973 when Ostriker and Peebles [160] postulated the existence of massive halos around spiral galaxies in order to provide a mechanism for heating up the disk components so that they would no longer be unstable to bar–making modes. This was a totally different kind of motivation for dark matter from those that had previously arisen. The following year, two independent sets of researchers [61, 161] tabulated galaxy masses as a function of their respective radii, to find the result $M(R) \propto R$ for $R \leq 100$ kpc and $M \leq 10^{12}M_{\odot}$, both for spirals and ellipticals. This was interpreted by these authors as strong evidence for dark halos, being a very convenient and natural mass distribution for them. It was at this stage, however, that the first strong doubts on the existence of substantial amounts of dark matter were expressed: in 1975, Burbidge [33] heavily criticised these works, pointing out their implicit assumptions that all the systems studied should be both *physical* systems (as opposed to, for example, chance projections on the sky) and bound ones — assumptions on which the result $M(R) \propto R$ was entirely dependent.

Another motivation for dark halos around galaxies found by both Ostriker et. al. [161] and Einasto et. al. [61] was the observation of flat rotation curves in nearby galaxies at large galactocentric distances where the neutral hydrogen HI extended well beyond the optical boundaries. This implied the presence of substantial mass past the visible boundaries of galaxies, since otherwise a more Keplerian–type fall–off would have been expected in these outer regions. But it was pointed out in the same paper by Burbidge that non–circular motions in these regions could by themselves be sufficient to explain the rotation curves. At roughly the same time, Woltjer [216], using globular clusters in the Milky Way 'halo', found a mass for the Galaxy substantially less than that expected from the Ostriker et. al. 'superhalo' scenario.

All of these problems, which we must in principle be in a position to resolve before confidently inferring the existence of large quantities of dark matter, have unfortunately remained until today along with numerous other problems, so that even though with time more evidence for dark matter has accumulated, so have the doubts on this evidence and its interpretation.

1.2 MASS TO LIGHT RATIOS

The most common way of investigating whether dark matter exists on many scales has been, and still is, the use of the mass to light ratios of the objects in question, such as globular clusters, spiral galaxy disks, spiral and elliptical galaxies, and clusters of galaxies. If life were considerably more simple and convenient than it actually is, we would be able to say for example that if the mass to light ratio of a spiral galaxy exceeded a certain well–defined value then this would unambiguously imply the existence of dark matter. This would be because our well–defined value of M/L would be the value obtained by dividing the total mass of all the galaxy's constituents known to us (stars, gas, dust etc.) by the total luminosity emitted from these constituents. Hence any M/L found to exceed this value would reveal a contribution which added to the mass but not the luminosity, i.e. a *dark* component. Unfortunately, things aren't quite so simple. We find for example that mass to light ratios of clusters are greater than those of smaller groups of galaxies, apparently indicating that there is proportionally more dark matter in larger clusters than in smaller ones. This is in fact the usual conclusion reached by most researchers, and may well be true. However, a large part of the discrepancy is due to stellar population differences between the early type galaxies in clusters and the spirals in small groups (the earlier type galaxies being, roughly speaking, less luminous than later types). In their classic review on the subject of dark matter in galaxies, Faber and Gallagher '79 (see [67]), showed that the mass to light ratios for the stellar population *alone* in spirals is substantially less than that in ellipticals and lenticulars. They thus concluded that 'the reality of excess unseen mass in great clusters relative to small groups must therefore still be considered uncertain at the present time.'

Mass to light ratios are measured in solar units, so that $M/L_B = X$ means X solar masses per solar luminosity, and the subscript B refers to the fact that the luminosity is usually measured in the *blue* band. When measuring mass to light ratios of distant objects like clusters, the distance, and hence the Hubble constant, usually comes into the calculation, so when comparing values given by different authors it is worth noting that the value for $H = 50$ will be one half of the value that would have been obtained if $H = 100$ had been used. We see then that one of the main difficulties that is encountered in the calculation of M/L ratios is the large uncertainty in the Hubble constant. We also note that when different luminosity bands are used they can be approximately intercompared by $M/L_V \simeq 0.7M/L_B$ and $M/L_{bol} \simeq 0.5M/L_V$. Another problem is how to correct for absorption in our Galaxy, since M/L can by this adjustment drop by a factor of two [197]. Moreover, normalising to face–on galaxies will also lower the M/L value.

A virtue of the M/L analysis is that it enables us to compare results found by many varied techniques. Bearing in mind all the inherent problems, we proceed with our criterion for the existence of dark matter to be when obtained M/L values are significantly higher than certain well-defined 'luminous' values, which will depend on the system in question. For the Galactic disk this will be around 7, for globular clusters 2–3, for spiral galaxies 6, elliptical galaxies 8. These values, in the blue band, and for $H = 50$, are from Faber and Gallagher '79. They are somewhat approximate, and should be understood as order of magnitude indicators only.

1.3 DARK MATTER TODAY

As has already been mentioned, there is circumstantial evidence for dark matter on nearly all astrophysical scales. Each one of these needs to be treated separately. There isn't room here for anything like a comprehensive account, and the interested reader is referred to the superb and elegant review by Trimble '87 (see [197]) and the references contained therein (all 777 of them), in particular the classic paper by Faber and Gallagher '79 (see [67]).

We begin with the *solar neighbourhood*, whose dynamics have recently been studied by J. Bahcall [18, 19]. He calculated a ratio of dark to luminous matter of 0.5 to 1.5 by mass, with the former distributed in the disk with scale height less than 0.7 kpc, as is the old disk population (Bahcall and Soneira '80 — see [17]). The main problem with this analysis however is a possible overestimation of the stellar brightnesses involved and an underestimation of distances, which results in an overestimate of the local mass density. This is because, roughly speaking, if a given tracer star at a given velocity travels further from the plane than thought, there must correspondingly be less mass in the plane holding it back than thought [197]. In fact, a more recent study extending to 2kpc from the plane requires no dark matter in the disk, be it thick or thin ([197], p.428). The current situation, then, for the solar neighbourhood, or more generally the Milky Way disk, is one which inspires a good deal of reservation for a separate dark component.

Measuring rotation curves for our *Galaxy* is an exceedingly difficult task, due to the problems of accurate distance determination and the conversion of heliocentric velocities to galactocentric ones, to the extent that probably a more accurate way of measuring the Milky Way mass is the use of probes such as globular clusters, field stars and dwarf galaxies which orbit in the outer regions. The two main approaches to estimate the mass contained interior to these objects is either from their velocities or from tidal interaction theory, the premise in the latter being that the observed sizes of the clusters and dwarf galaxies represent tidal truncation by the Milky Way, and thus reflect the amount of mass in the Milky Way because this determines the strength of the tide. Hodge and Michie [100] showed in 1969 that these objects *are* tidally influenced, however Seitzer [183] has more recently made calculations which *suggest* they are not tidally relaxed, so that the formula(e) used should give misleading results. Much more will be said about this question at a later stage, where it will prove a major consideration. Other problems are the assumption of circular orbits for the halo objects (not justified, and giving excess masses if incorrect), and similarly the assumption of a locally isotropic velocity distribution. Moreover the old problem of the correlations of ages and chemical compositions with dynamics makes everything still more complicated [40]. On the other hand, since instead of a sudden truncation in the outer regions of the satellite globular clusters and dwarf galaxies, the stars should leak away gradually, masses derived from the tidal method, according to a recent study [162], should be scaled up by a factor of about 2.75. We note here in passing that in the calculations, the masses assumed for the satellite dwarf galaxies enter linearly, a very interesting consequence of which is that either both the Milky Way and the dwarfs come out very massive or neither do. This will also be a relevant idea later on. Returning to the question of circular orbits, one argument recently put forward [167] *for* circular orbits (or at least orbits of low elongation) of the outer halo objects goes as follows: these objects are too weakly bound to have even survived *one* close approach to the Galactic centre where they would have been destroyed by a combination of bulge or disk shocking or dynamical friction, hence they must be on orbits which do not take them close to the central regions. To summarise the current situation for our Galaxy: from rotation curve analyses the mass within the solar radius is probably about half dark (and probably spheroidal in form), while the mass exterior to the Sun's orbit is some 2—10 times as much [197], although a recent study by Tremaine [196] points to a dark halo of only moderate extent ($\leq$ 50kpc). The results however are very sensitive to the unknown velocity distribution of the halo objects used.

As for other *spiral galaxies*, the universal method is that of rotation curve analysis, using either optical or 21cm observations. The fact that these rotation curves do not display Keplerian fall–offs and that this probably implies the existence of large quantities of gravitating mass outside the optical regions was noticed by Freeman [75] in 1970. Subsequent to the 1974 paper by Ostriker et.al. [161], the best accepted theoretical interpretation of this result was a picture in which M/L increased monotonically with R, consistent with massive dark halos. However in 1983, Kalnajs [113] showed that at least the *optical* rotation curves could be equally well fit by a disk *alone* which had constant mass to light ratio. The HI data extends considerably further out than the optical data, and once the rotation curve is seen to be still flat past about three exponential scale lengths [78], the method of Kalnajs can no longer account for the extra mass, which is thus identified as dark matter. A good example is NGC 3198 (see [208]), traced out by HI observations to 11 disk scale lengths, where the integral M/L_B is 24h and the average halo/disk ratio at least 4. As for the shape of the dark matter distribution in spiral galaxies, the evidence does seem to point towards a spheroidal one, although this is by no means conclusive [197]. The main reasons for believing halos are spheroidal are provided by HI flaring, warps, and the famous disk stability arguments first given by Ostriker and Peebles [160]. See, however, the recent claim by Kalnajs [114] that spheroidal halos are not necessary for disk stability. Finally it must again be remembered that the possibility of non–circular motions must be taken into account (although in this case there is evidence for non–circular velocity effects causing derivations of low rather than high masses [35]). In summary, then, there seems to be very strong evidence for dark matter halos in spiral galaxies, particularly when the HI observations extend past three scale lengths. It can moreover be argued that halo models give more natural fits to the data than the purely disk models of Kalnajs, even when it would in principle be possible to account for the data without a halo [176].

Probably the best example to date of evidence for dark matter in an *elliptical galaxy* is that of M87. Two separate analyses of the X–ray temperature profile of this galaxy [70, 191] obtain M(r) as linearly increasing with r out to at least 300kpc, a result which gives a mass within this radius of about $3 \times 10^{13} M_\odot$ and mass to light ratio of around 750, whereas the luminous contribution for such a system should be of the order of 8 or so. This result seems to imply that over 99% of the mass in this galaxy is composed of dark matter. We have to be careful with this interpretation however, because M87 is situated at the centre of the Virgo cluster — hence the result could be a manifestation of the potential well of the cluster as a whole, and not of the galaxy at the centre of it. We point out though that the core radius found to give the best fit to the observations by Stewart et.al. is around 25kpc, a value characteristic of a galaxy and not a cluster. Most other galaxies are not strong enough in X-rays for a sufficiently accurate temperature determination to calculate the mass of the galaxy, which is given by [27]:

$$M(r) = \frac{k_B T r}{G \mu m_p} \left(-\frac{\mathrm{d} \ln \rho}{\mathrm{d} \ln \mathrm{r}} - \frac{\mathrm{d} \ln \mathrm{T}}{\mathrm{d} \ln \mathrm{r}} \right) \tag{1.1}$$

Here μ is the mean molecular weight and m_p the proton mass. So for other elliptical galaxies we must turn to the methods based on velocity dispersions, namely the global Virial theorem, King models, or observations of test particles in circular motion about the spheroidal component. A review of these three methods is given by Faber and Gallagher [67], and a more recent investigation [16] gives average mass to blue light ratios for the *visible* regions of a large sample of normal ellipticals of about 13, subject to a Hubble constant $H \simeq 95$.

The next scale up is that of *binary galaxies*, a field of study plagued with problems for finding mass to light ratios. The main difficulty is that of actually identifying binary galaxies, since there is no way of telling whether apparent binaries are only brief encounters, optical doubles, or in fact real binaries. Therefore the investigations are based on statistical studies of the relative line of sight velocities, which itself has proved to be a tremendous problem. The situation has been aptly summed up by Binney and Tremaine [27], in that 'the mass to light ratio of binary galaxies is probably large, but not so large as the ratio of the mass of papers on this subject to the light they have shed on it.' See also Sharp '85 [186] for reasons to be cautious with calculations of binary galaxy masses. One of the more meaningful studies was done by White et.al. [211], who found no correlation of the line of sight velocity differences with the projected separations, concluding that point mass models for binary galaxies do not work. This is regarded as evidence for dark halos that extend well beyond the optical boundaries. There are, however, serious doubts on the stability of such configurations, in the sense that the halos could be truncated by tidal forces, and their survival against dynamical friction has to be questioned (failing which the binaries could merge within an orbit or two). Moreover, the models to date which seem to approach consistency with the data seem to require roughly isotropic velocity ellipsoids. This raises the question of how these galaxies ever acquired their orbital angular momentum [27]. For isotropic orbits the bulk of the most recent data for binary galaxies gives mass to light ratios of 30 ± 10 [197], which would more than double for circular orbits, but decrease for radial ones.

The *Local Group* (dynamically dominated by the Milky Way and Andromeda M31) and other *small groups* are subject to the same problems as binaries — the Local Group may not for example even be bound. A typical M/L ratio found for small groups is about 170h [197].

Larger *clusters* are modelled in much the same way as individual elliptical galaxies, that is by the use of the Virial theorem, or by King models. Once corrected for absorption (in our Galaxy), normalised to face–on values and calculated with $H_0 = 50$, Virgo, Coma, Perseus and other rich clusters have resultant Virial M/L's of 100 or more, whereas other methods tend to yield slightly larger values (see [197] for references). Uncertainties due to the average luminosity functions used could change M/L by factors of 2—4. The usual problem with the Virial analysis in general also applies here — if the outer parts of clusters are not relaxed, then the masses thus derived could well be too large by factors of 3—5 [43, 153]. It is finally worth noting that contamination by foreground and background objects will push up the measured velocity dispersion to increase M/L (see e.g. [72]), an effect that could also be caused by the existence of substructures within the cluster. The X–ray emission method used for M87, when applied to clusters, gives approximately $M/L = 200h$ [46, 71, 201]. In passing we note that gravitational lensing calculations have yielded M/L's of $10^{3.3}$, for the cluster that lenses the QSO 0857+561 [84, 171], however the method, which has also been applied to single galaxy lenses, is at this early stage poorly developed [34, 151], and no more will be said about it here.

Once we have moved up to the scale of *superclusters*, the results are usually given in terms of the cosmological parameter Ω. A measured value of (M/L)h will contribute $\Omega' = (M/L)h/1000h$ towards closure of the Universe. We note here that the obtained Ω is conveniently independent of the Hubble constant. On the assumption that all superclusters have the same M/L, measurements of 'Virgo–centric infall' (i.e. the phenomenon of our *recession* from Virgo being less than it would be in an unperturbed Hubble flow) give $\Omega = 0.2 \pm 0.1$, as do measurements of other superclusters [197]. This would at first sight seem like pretty strong evidence for ruling out the commonly favoured $\Omega = 1$, a question which will be discussed shortly. The 'Cosmic Virial Theorem' (i.e. the use of the correlation function [164]) yields on average about the same value [56, 165]. However a recent study [45] casts considerable doubt upon the validity of the correlation function's basic property of fundamental scale lengths, at least in the CfA slice of the Universe.

On even larger scales, Lahav [125] has found $\Omega = 0.3$ from galaxies at an average distance of $50h^{-1}$Mpc, while two recent studies of IRAS galaxies [144, 217] have yielded $\Omega = 0.5$ and 0.85 ± 0.15 respectively. These uncertain measurements need confirmation, and more of the IRAS galaxies need measured distances, but it is not impossible that the closure density, or something extremely close to it, has already been measured.

In summary, a belief in an $\Omega = 1$ Universe requires a belief in substantial amounts of dark matter (see section 1.4), whereas it could be consistent for an $\Omega = 0.1$—0.2 Universe to contain essentially no dark matter. Some of the alternative explanations to the combination of observations and calculations, which avoid dark matter, are given below for the various scales involved (from Trimble '87 [197]):

Solar neighbourhood Tracer stars brighter than assumed.

Rotation curves of spirals Outer gas in non circular and probably impermanent orbits owing to effects of recent arrival, companions etc; luminosity at large radii underestimated because sky background brightness overestimated.

Velocities of galaxy satellites Outer high–speed objects not in permanent bound orbits.

X–ray emission from ellipticals Gas temperature distribution declines steeply towards galactic centres.

Velocity dispersion calculations Preponderance of circular orbits at large radii.

Binary galaxies Preponderance of radial orbits or isotropic distribution, or spurious pairs.

Small groups Many unbound, or bound only as parts of larger structures.

Rich clusters Not yet relaxed; interacting subsystems; X–ray gas polytropic rather than isothermal; dynamics dominated by central massive core.

1.4 DARK MATTER CANDIDATES

The individual constituents which make up the dark matter can either be baryonic or non–baryonic. It is necessary, before discussing the various possibilities, however, to discuss the value of the density parameter Ω.

It was pointed out by Dicke and Peebles '79 [60] that if Ω is within a factor ten of one now (as we in fact measure), then it must have fallen within about one part in 10^{15} of unity during the nucleosynthesis epoque ($T \simeq (0.1$—$10)$MeV) [224], and one part in 10^{49} at the time of the GUT phase transition ($T \simeq (10^{-3}$—$10^{15})$GeV) [185]. This is strongly suggestive of a value of exactly unity for Ω. A more popular reason for believing $\Omega = 1$ is provided by inflation [88], which does not necessarily require a value of one exactly, although it is generally believed that it will force Ω today so close to 1 that the deviation will not be measurable [182]. We note however a recent study by Ellis '88 [64], which shows that inflation is not inconsistent with Ω today having *any* value, low or high. Not without its problems [28], inflation is the best theory to date of the early Universe — one of its main attractions is its ability to explain the correlation in density and temperature between mutually distant regions of the Universe without violating the principle of causality. Another motivation for a critical density Universe is the large degree of homogeneity we observe in the microwave background, since otherwise it is hard to form galaxies while retaining such a regular background [202]. An argument based on the Copernican Principle and leading to the same conclusion can be found in Binney and Tremaine '87 ([27], p.627). Now if Ω is indeed one, the obvious question is why have observations only yielded values around 0.2 ? The most immediate answer is that galaxy formation is biased: galaxies form preferentially in high density regions [59] so that luminous matter does not trace the overall mass distribution of the Universe. This will shortly be seen to be an important idea in connection with models of galaxy formation. It has to be borne in mind, however, despite these arguments which are attractive to many, that, as put by Turner '85 [200]: 'theoretical prejudice aside, there is no convincing evidence (or even unconvincing evidence for that matter) that Ω is any larger than about 0.2 ± 0.1'.

Pertinent to the Ω problem, we now turn to nucleosynthesis theory, which provides both lower and upper bounds on the *baryonic* contribution to Ω. The standard model [29, 57] gives limits:

$$0.015 \leq \Omega_B h^2 \leq 0.15$$

from which we conclude that all the matter in clusters and superclusters could consist entirely of baryonic matter. The above upper limit can be raised towards unity by, other than lowering H_0 to 25, allowing non–zero lepton number [204], or inhomogeneities in density or temperature [175], such that $\Omega = 1$ in baryons is not inconceivable, and has perhaps been ruled out rather too readily in the past. At the other end of the spectrum, of course, for high Hubble constant and modulo the standard model, an upper limit for Ω_B of around 0.15 could well require non–baryonic matter even in very large proportions in clusters and superclusters, and even for a low overall value of Ω.

It is now time to list the various possibilities for the individual components or particles that make up the dark matter, starting with the baryonic kind. We have to consider brown dwarfs, white dwarfs, black holes (small primordial ones, stellar mass ones, and very massive ones), gravitational radiation, and gas.

A closure density of intergalactic cold or warm gas would produce detectable emission and/or absorption lines which are not detected [163]. For very hot gas to close the Universe would require more than 10% of all available nuclear energy in the Universe to heat the gas, which is quite a problem [85] — moreover the spectrum cannot be fit by thermal radiation [80].

Not much can yet be said about gravitational radiation, since it could close the Universe without us having detected it [11, 172]. Perhaps a factor contributing to its attractiveness is that it cannot be clustered, in which case the galaxies wouldn't trace the overall mass distribution

As for primordial black holes, again little can be said, although they could close the Universe [42, 152], and their clustering properties are also unknown.

Brown dwarfs, substellar objects whose only energy source is contraction, could for example account for the Oort limit without having yet been detected. It remains an open question whether they exist in substantial quantities, and if so just how dynamically important they would be [197].

White dwarfs, the remnants of $(0.5\text{—}8)\pm 2M_\odot$ stars could also exist in our Galaxy for example without having been detected, and once again are very poorly constrained both by theory and observation [197].

Black holes of solar masses in our Galaxy can more or less be ruled out since they would accrete interstellar gas to produce X–rays which are not seen, and more heavy elements would have been detected in the gas [197].

Finally we consider massive black holes. Their existence in the halo *could* provide the explanation for the long known increase of stellar velocity dispersion with age in the disk; this would favour a $10^{12}M_\odot$ halo made up of $10^6M_\odot$ black holes [108, 115, 124].

Non–baryonic dark matter may well be needed in conjunction with baryonic contributions, since if dark matter were just one thing it would be somewhat difficult to see how it could be sufficiently 'cold' to settle down into the Galactic disk and simultaneously 'hot' enough to remain less clustered than superclusters over a Hubble time or so.

There are a tremendously large number of (mostly hypothetical only) non–baryonic candidates, obtained mainly from various different theories of particle physics. For an exquisite account of this subject the reader is referred to the review by Turner '85 [200]. A useful classification of these candidates is into either *cold dark matter*, *warm dark matter* or *hot dark matter*. Defined properly, hot dark matter is relativistic when galaxies form, which wipes out small perturbations and promotes large scale structure. Cold dark matter is non–relativistic when galaxies form, and this promotes small scale structure. Usually this can be parametrised by the individual particle mass, where the heavier the particle the colder it is. This is not however always the case. For example, a very light candidate particle, the axion, at around 10^{-5}eV, is cold.

Before going through a list of some of the more popular candidates, it should be noted that if the Universe is indeed closed by some form of non–baryonic matter this will present us with another fine–tuning problem, where the densities of nucleons and non–baryons are approximately the same (to within one order of magnitude or so), so that a *particular* value of the energy scale of a symmetry breaking (or some other process) is required [200]. Bearing this problem in mind we proceed with a very brief and non–exhaustive account of these 'objects', knowing all along that the vast majority of them, if not all, are incorrect as solutions of the Ω problem and/or do not exist.

One of the oldest such candidates is the *neutrino*, proposed by Cowsik and M_cClelland '72 [48], who concluded that a rest mass of 10—100eV for the neutrino could result in an $\Omega = 1$ Universe. Although neutrinos are known to exist (the *only* such candidate for which this can be said), it is still unknown whether they have non–zero rest masses or not. If they are massless, they will not make a significant contribution to Ω, whereas this latter parameter is a direct function of the rest mass if it is non–zero, so that only certain ranges of neutrino mass are allowed, as we will see later. One of the most popular tests of the respective hot and cold dark matter scenarios is the use of N–body simulations of structure formation, the results of which have perhaps been credited with too much importance in the past, since implicit in these simulations are separate processes such as the initial perturbation spectra, biasing mechanisms, post-formation clustering and relaxation, plus the effects of dust exploding stars or active nuclei (not to mention the standard problems with N–body simulations of error domination and non–physical softening parameters). Modulo this approach, cold dark matter has fared rather better than its hot opponents, to the extent that by 1985, during the IAU Symposium entitled '*Dark Matter In The Universe*' [119], hot dark matter in general and neutrinos in particular, on the strength of a simulation by White, had for all intents and purposes been ruled out. The other main constraint on the neutrino hypothesis, which forms the crux of chapter 2, was also pointing in this direction. The worst problem the neutrinos had with structure formation was that small scale structure such as galaxies was made too late [212] — the so called 'timing problem'. This problem has recently been shown to be resolvable [32], subject to a mechanism of galaxy formation called '*antibiasing*', which requires that galaxies form preferentially in flat 'sheets' rather than 'filaments', possibly finding themselves *less* clustered than the neutrinos. The neutrino scenario may well see a rise in popularity over the next few years (see 'popularity graph' on p. vi)

We note from this graph that at the time of lowest popularity for neutrinos, cold dark matter was at its highest, with White's simulations proving very persuasive. At the same time, a theoretical model of galaxy formation with biased cold dark matter was constructed by Dekel and Silk [58], which along with its bearing on dwarf galaxies (see chapter 4) seemed somewhat successful. However, the cold dark matter scenario has been dealt a serious blow (from past experience, we will not say 'fatal') by the recognition of its inability to account for large scale structure in general and streaming motions in particular [209, 214]. The neutrino picture on the other hand does seem equipped to explain these motions [32, 50]. We briefly discuss two more candidates, while most of the rest are tabulated on page v.

First, there is a semi–baryonic form of matter coined '*quark nuggets*', which was proposed in 1984 by Witten [215]. These are large globs of quark matter, which are formed during the quark/hadron phase transition if we have very large baryon number and if quark matter rather than nuclear matter is the main stable configuration (these are suppositions which may or may not be correct). These nuggets would not participate in primordial nucleosynthesis and would contribute about 0.9 to Ω. Witten himself concluded in the same paper that this strange matter candidate is indeed not a very likely one, as have the authors of several more recent studies [9, 12]. A compelling feature of quark nuggets however is that they seem to be the only non–baryonic candidate which is not subject to the aforementioned fine–tuning problem associated with symmetry breaking.

Secondly, we have *strings*, either in 'super' or 'cosmic' variety. A nice feature of string theories is that they automatically incorporate gravity, as do theories of supersymmetry (the point like, or field theory, limit of a superstring theory is a supersymmetric or supergravity grand unified theory—SUSY/SUGR GUT). This offers the hope of unifying gravity with the other forces [200]. Observationally, not much can yet be said about strings, and theoretically, relevant constraints do not exist. Recently, however, a model of galaxy formation with cosmic strings and massive neutrinos in conjunction has been proposed [23]. This is an example of what Trimble [197] coins rather skeptically as a scenario with 'two tooth fairies', and the point that thirty–odd single candidates is already enough without considering all the possible pairs among them is well taken. This scenario is fundamentally different to hot dark matter ones without cosmic strings, as the effect of the strings is to develop the cosmological structure in a hierarchical rather than fragmentary fashion. The theory is at the very least an improvement on previous attempts to combine cosmic strings with cold dark matter [22, 146].

Finally, we note that instead of playing with particle physics we can do the same with either Newtonian or Einsteinian physics. Firstly, modifications to Newtonian physics by for example allowing the gravitational 'constant' G to vary with separation or acceleration [148] could account for flat rotation curves and large velocity dispersions without the existence of dark matter. But apart from a general reluctance to forsake Newtonian gravity in favour of a less elegant theory, it is also possible that dark matter models do match the observations better [94]. Secondly, the old phantom possibly–non–zero cosmological constant Λ, which for a flat Universe would have to be presently of order $\pm 10^{-35} s^{-2}$ [197], gives us a fine–tuning problem if inflation is correct. This is in essence because during the inflationary epoque Λ is briefly enormous [31]. The likelihood of Λ being non–zero is a matter of taste, with most people preferring dark matter as a more compelling option.

As a final word on dark matter in general, substantial amounts of it on whatever scale may or may not exist. Some of the evidence is very persuasive to most, although there is still room for a good deal of doubt (see, for example [26]). Dark matter is at the very least an attractive (to some) single solution to a large number of problems, and a decent working hypothesis worthy of further investigation.

Chapter 2

PHASE SPACE CONSTRAINTS AND DWARF GALAXIES

2.1 THE TREMAINE GUNN CONSTRAINT

The reader will have noticed that there was no mention of dwarf galaxies in the brief introductory survey. The discussion of these objects will be further delayed until towards the end of this chapter, which lays the foundation for the main discussion of this thesis. It will be seen that it is the extreme nature of dwarf galaxies which makes them so relevant to the question of dark matter.

As we saw in the previous chapter, very little is known regarding what the dark matter might actually be, because we have very little either from observation or theory to guide us in favour of or against any particular candidate. Theoretical models and N–body simulations of galaxy formation, which discriminate between hot, warm and cold dark matter, have constituted one of the few ways of obtaining at least a hint or indication as to which *kind* of dark matter might be more likely than the others. As already noted, however, N–body simulations are not to be taken simply at face value, as other, separate, effects enter into these simulations, which are very difficult to separate away again, because they depend on certain model assumptions. More useful perhaps would be a rather more model–free constraint of some kind — in fact such a constraint exists. Nine years ago, Tremaine and Gunn [195], produced a result, based mainly on kinematics, which provided a limit on the mass that an individual hot, neutral, light lepton such as a neutrino could have if it were to settle down into configurations such as galactic halos. These authors actually went as far in their original paper as to conclude that the dark matter in isolated galaxies, binary galaxies, groups and clusters could not be made of *any* stable, neutral lepton less massive than 1MeV. This ruled out hot, light neutrinos, one of the main dark matter candidates to provide an $\Omega = 1$ Universe. However, such a conclusion has proved to have been somewhat too hasty, and the application of this mass constraint today will be seen to be a far less unambiguous proposition, to the extent that the possibility of ruling out light neutrinos on these grounds remains an open question.

Before entering into the details of the present day dilemma, in which the work of Tremaine and Gunn remains *in essence* unaltered, it is instructive to examine the original derivation. The calculation was performed explicitly for neutrinos, although in fact the result applies to any hypothetical non–interacting Maxwell–Boltzmann particle, but not bosons because of the nature of the statistics involved. The essence of the argument is to find an expression for the maximum coarse–grained phase space density D_0 of the neutrinos soon after they decoupled in the early Universe and before they began to cluster; an expression for the value of this quantity D_1 once they have settled into a galactic halo (*if* they settle into galactic halos, that is), and then to simply require that $D_0 > D_1$, since this quantity must decrease with time. The expression thus obtained will yield, on rearrangement, a lower limit to the mass of the neutrino, which will depend on the assumptions made about the relative masses of the different neutrino types, their masses being extremely poorly constrained by particle physics.

The parameter D_0 is simply $2g_\nu h^{-3}$, since the momentum distribution while the neutrinos are relativistic and still in thermal equilibrium is, according to the standard model of the early Universe [210]:

$$n_\nu(\vec{p})d\vec{p} = \left(\frac{g_\nu}{h^3}\right) \frac{d\vec{p}}{1 + \exp(p/kT_\nu(z))} \tag{2.1}$$

The same equation applies to all types of neutrino as well as their anti–particles. Here g_ν is the number of allowed helicity states, which has been left in because not enough is known about processes in the very early Universe which determine g_ν, which could either be 1 or 2. We introduce the notation ν_e for electron–neutrinos and ν_μ for muon–neutrinos, to be used later. Here however, Tremaine and Gunn made the simplifying assumption that both types were equal in mass and helicity, so that for the moment the notation ν will do. Since two types of neutrino were then known, this made a four–fold contribution to (2.1), whose maximum value gives D_0.

In obtaining D_1, rather more assumptions were made. Arbitrarily taking the central regions of the bound systems formed by the neutrinos to resemble isothermal gas spheres implied a Maxwellian velocity distribution [27]:

$$n_\nu(\vec{p})d\vec{p} = \frac{\rho_0 d\vec{p}}{m_\nu^4(2\pi\sigma^2)^{3/2}} \exp(-v^2/2\sigma^2) \tag{2.2}$$

from which could be read off the value:

$$D_1 = \rho_0 m_\nu^{-4}(2\pi\sigma^2)^{-3/2} \tag{2.3}$$

where ρ_0 is the central density and σ the one–dimensional velocity dispersion.

Eliminating ρ_0 in favour of the *core radius* (see Appendix C for a discussion of core radii and their various definitions), defined by:

$$r_c^2 = \frac{9\sigma^2}{4\pi G\rho_0} \tag{2.4}$$

and putting $D_0 > D_1$ one obtains:

$$m_\nu^4 > \frac{9h^3}{4(2\pi)^{5/2} g_\nu G\sigma r_c^2} \tag{2.5}$$

which, when written in more astronomical terms becomes:

$$m_\nu > (101eV)\left(\frac{100kms^{-1}}{\sigma}\right)^{1/4}\left(\frac{1kpc}{r_c}\right)^{1/2} g_\nu^{-1/4} \tag{2.6}$$

This is the upper limit on the neutrino mass as found by Tremaine and Gunn.

Ever since it was shown in 1967 by Lynden–Bell [132], it has been well known that for collisionless systems the maximum coarse–grained distribution function (that is, averaged over some finite region of space) will decrease with time, as a consequence of phase mixing or violent relaxation which cause the fluid in phase space to become 'frothy', allowing low density or empty regions to enter. This provides a simple justification for the statement $D_0 > D_1$, used to derive (2.6).

In passing it is interesting to note that a remarkably similar constraint would have been derived from the Pauli exclusion principle, which would have differed from (2.5) merely by a factor of $2^{1/4}$, making it less severe. The former states that the occupancy $f = (1 + e^{p/kT})^{-1}$ must be less than unity, whereas the argument used by Tremaine and Gunn forces f to be less than a half. This important point was noted by these authors in their original paper.

To obtain a numerical value from (2.6), they then considered a typical galactic halo, with the values $r_c = 20$kpc and $\sigma \simeq 150$km/s. This yielded:

$$m_\nu \geq (20eV) g_\nu^{-1/4} \tag{2.7}$$

What does this lower limit tell us? The very least we can say is that if neutrinos are to be a viable candidate for dark matter in halos, then this lower limit must for consistency be lower than any upper limits that exist. There are three ways of obtaining the latter. Firstly, we obtain an upper limit, for the electron neutrino only, of around 30eV from the recent supernova 1987A [181]. This is the best *model–independent* limit we can probably obtain from this event, although lower, model–dependent, values have in the past been claimed. Secondly, laboratory experiments provide upper limits on the neutrino mass [48], and for the electron neutrino the latest value is around 20eV [122]. This renders the supernova–based constraint irrelevant, except for the curiosity, noted by Schramm [181], that the occurence of 1987A in the Large Magellanic Cloud causes the value to come out so closely to the laboratory limit. Bearing these values in mind, we will prefer to concentrate on the upper limits provided by cosmology, which gives a bound on the sum of the various neutrino masses. The standard theory of the early Universe [210] leads to the following results...

At the point when the Universe has cooled to about 1—2MeV, most of the muons and pions have annihilated away. During cooling, the average neutrino collision rate decreases, due to the decrease in the weak interaction cross–section and the drop in number density. At around this temperature the time between collisions exceeds the expansion time, so that the neutrinos drop out of thermal equilibrium. Since both electron and muon neutrinos are known to be less massive than 1MeV they are still relativistic when they decouple. During the subsequent cooling and expansion the neutrinos are redshifted to lower energies according to $p \propto 1 + z$, and form a uniform background. Hence equation (2.1) continues to hold, as $T_\nu \propto 1 + z$ as well, so that the neutrinos behave as if they were still in thermal equilibrium even though they are no longer interacting (except by gravity). The temperature of the *photons* redshifts in the same fashion, preserving $T_\nu(z) = T_\gamma(z)$ which previously held when the neutrinos were still in equilibrium with the photons. An important property of the standard model of the thermal history of the early Universe, easily derivable from the approximation that all chemical potentials are zero (enabling energy densities and pressures of various particles to be functions of temperature only) is that the entropy per unit volume, s, is constant. After e^+e^- annihilation virtually only the photons are left in thermal equilibrium. Equating the expressions for s both before and after this event, one obtains:

$$T_{1\gamma}/T_{2\gamma} = (4/11)^{1/3}$$

where $T_{1\gamma}$ is the photon temperature before e^+e^- annihilation and $T_{2\gamma}$ the value afterwards. This phenomenon does *not* affect the neutrino temperature T_ν since the neutrinos had already decoupled. Therefore one obtains:

$$T_\gamma(z) = (11/4)^{1/3} T_\nu(z)$$

which holds from the end of e^+e^- annihilation up until the present. Now, since the present value $T_\gamma(0)$ is known to be 2.7K we obtain:

$$T_\nu(0) = (4/11)^{1/3} T_\gamma(0) = 1.9K$$

This calculation was originally done for zero–mass neutrinos, but since electron and muon neutrinos were relativistic straight after dropping out of thermal equilibrium, the calculation remains unchanged. Using this result it is easy to show that the relation between the number densities of neutrinos and photons is simply:

$$n_{\nu_i} = 3/11 n_\gamma g_{\nu_i}$$

where the 'i' labels each species of neutrino. Therefore the present mass density would be:

$$\rho_\nu = 3/11 n_\gamma (2 g_\nu m_\nu)$$

recalling the assumption that $g_{\nu_\mu} = g_{\nu_e}$ and $m_{\nu_\mu} = m_{\nu_e}$. The last expression follows because the neutrinos today should be non–relativistic. The number density of photons is found from the black–body radiation formula:

$$n_\gamma = \frac{1}{\pi^2 \hbar^3} \int_0^\infty \frac{p^2 dp}{\exp(p/kT_\gamma) - 1}$$

which yields

$$n_\gamma = 16\pi\zeta(3)(kT_\gamma(0)/h)^3$$

where $\zeta(3) = 1.202$ is the Riemann zeta function.

The critical density of the Universe is $\rho_c = 3H^2/8\pi G$, which implies that the contribution to Ω from neutrinos is predicted to be:

$$\Omega_\nu := \rho_\nu/\rho_c = 0.04\left(\frac{50}{H}\right)^2 \frac{2m_\nu g_\nu}{1eV} \tag{2.8}$$

From this equation there are various ways of obtaining upper limits to m_ν. Firstly, and most conservatively, we can simply put $\Omega_\nu \leq 1$, certainly in accordance with observations. This yields:

$$m_\nu g_\nu < (12.5eV)(H/50)^2 = \begin{cases} 12.5eV & , H = 50 \\ 50eV & , H = 100 \end{cases} \tag{2.9}$$

This is consistent with (2.7) for $H \geq 64$.

Secondly, the approach adopted by Tremaine and Gunn was to consider a quantity Ω^*, the ratio of the density distributed like the galaxies to ρ_c. On the assumption that the neutrinos participate in the galaxy clustering, ('justified' by the order of magnitude argument $v_\nu \sim k_B T_\nu(0)/m \leq 50$km/s for $m \geq 1$eV, a velocity smaller than the typical velocity dispersion within galaxies or clusters), we can put $\Omega_\nu < \Omega^*$. This latter quantity was estimated, subject to considerable errors, to be around 0.05 [83]. This gave a limit 20 times more stringent than that obtained from $\Omega_\nu < 1$:

$$m_\nu g_\nu \leq \begin{cases} 0.625eV & , H = 50 \\ 2.5eV & , H = 100 \end{cases} \tag{2.10}$$

So if we were to believe that $\Omega_\nu < 0.05$, the upper and lower limits were in contradiction, irrespective of the values of the two unknowns H and g_ν, where $50 \leq H \leq 100$ Km/s/Mpc and $1 \leq g_\nu \leq 2$. Moreover, relaxing the assumptions $m_{\nu_e} = m_{\nu_\mu}$ and $g_{\nu_e} = g_{\nu_\mu}$, and taking instead without loss of generality $m_{\nu_\mu} \gg m_{\nu_e}$ merely doubles the right hand side of (2.10) while increasing (2.7) by $2^{1/4}$. Hence these particles were ruled out as being the dark material in galactic halos, and *by extension* in clusters of galaxies where the dark matter is believed to be in the form of halos which have been tidally stripped from their galaxies to form a more uniform background.

As mentioned earlier, a standard working assumption in the cosmological theory is that of zero muon– and electron– lepton number, which amounts to the vanishing of the corresponding chemical potentials. Allowing for the more general case where this is not true, conservation of lepton number (both electron and muon type) yields $\mu \propto 1/R$, unchanging the form of the distribution given by (2.1), with the addition of μ, and the contradiction obtained above becomes worse. This is because the maximum phase space density of neutrinos plus antineutrinos is independent of μ_ν so that the lower limit is unchanged, whereas $\mu_\nu \neq 0$ increases their spatial density, requiring a lower mass and hence more restrictive upper limit.

The problem would appear to be solved: light neutrinos cannot form the dark matter in galaxies or clusters of galaxies, and (if one is attracted by the most economical theory, that the dark matter is the same on all scales) the same conclusion can be applied to the cosmological 'missing mass'.

However, we need to update the values of various quantities that were used in the original paper, consider possible generalisations which need weaker assumptions, and finally consider a case where this mass constraint would in effect be rendered irrelevant.

Firstly, we have to account for the fact that a third type of neutrino is now known to exist, the tau neutrino ν_τ. Before supposing that there might be some unlimited number of neutrino species in existence but not yet discovered, we note the result that the abundance of ^{4}He constrains the number of light ($m \leq 1$MeV) neutral leptons to $n \leq 3$—4 [189]. As regards the (unknown) value of g_ν, it is noted that there are two possibilities for massive neutrinos: Majorana neutrinos ($\nu \equiv \bar{\nu}$) which have $g_\nu = 1$; and Dirac neutrinos ($\nu \neq \bar{\nu}$) for which we put $g_\nu \simeq 1$ (due to the early decoupling of the right–handed neutrinos [185]). It is suggested [180], that we have the Majorana type, in which case $g_{\nu_i} = 1$ exactly for $i = 1, 2, 3$. (We note that Majorana right– handed neutrinos may or may not exist, whereas Dirac right–handed neutrinos *must* exist [224]. Combined with the fact that we *only* observe left–handed neutrinos, this is the justification for *supposing* that it is more likely we have Majorana neutrinos). Next, a more general version of (2.6), which allows for the number of neutrino species $n = 1$—4 explicitly is:

$$m_\nu > (120eV)\left(\frac{100km/s}{\sigma}\right)^{1/4}\left(\frac{1kpc}{r_c}\right)^{1/2}(ng_\nu)^{-1/4} \tag{2.11}$$

which reduces to (2.6) for $n = 2$. More importantly, however, is how seriously can we take the cosmological upper bound $\Omega_\nu < \Omega^* \simeq 0.05$? The observational *value* for Ω^* is not so much the question as is the line of argument leading to this inequality, which uses the rough relation $v_\nu \sim kT_\nu(0)/m_\nu \simeq 50/m_\nu$ km/s. Since the velocity dispersion in galactic halos is of the order of 100—200km/s, a mass $m_\nu \geq 0.5$eV will give a lower random velocity today for the neutrinos, suggesting that they could settle down into galaxies. However, the process of galaxy formation is far more complicated than that, and until we know a lot more about structure formation in general we do not know *a priori* the predominant clustering scale of the neutrinos. Therefore a substantial fraction of Ω_ν could still remain clustered on scales larger than a few hundred kiloparsecs characteristic of large galaxies. This, as noted by Schramm and Steigman [180], provides a natural explanation for the observation that there is proportionally more dark matter on larger scales. We abandon the Ω^* approach, considering it much safer and less contrived, certainly to put $\Omega_\nu < 1$, or, failing inflation (the classical interpretation thereof) but still believing observations on the largest scales, to put $\Omega_\nu \leq 0.2$. An interesting point is that we can also obtain $\Omega_0 h_0^2 \leq 1$ from the ages of globular clusters [180], which yields:

$$\sum_{i=1}^{3} m_{\nu_i} \leq 100eV$$

for Majorana neutrinos. However this will be ignored for it contains further uncertainties from stellar evolution theory. Hence we prefer the upper limits given most conservatively by $\Omega_\nu < 1$:

$$\sum_{i=1}^{3} m_{\nu_i} < \begin{cases} 25eV & , H = 50 \\ 100eV & , H = 100 \end{cases} \tag{2.12}$$

for Majorana particles. Thus we no longer have a contradiction with the Tremaine–Gunn constraint, which for one species of Majorana particle applied to a typical galaxy is $m_\nu \geq 24$eV.

Looking at the mathematical form of our phase space constraint (2.5) we note that it would be strengthened for smaller core radius r_c and smaller velocity dispersion σ. We are therefore led to consider systems which have much smaller characteristics, with the possibility of obtaining a lower bound which would be in conflict with equation (2.12). Hence the need to study these smaller systems, namely the dwarf galaxies, because if we are able to show in them the existence of substantial amounts of dark matter then we can apply equation (2.5) to them, with the possibility of being able to conclude something very fundamental about what that dark matter may not be. Before going on to examine these galaxies though, it is worth taking a slightly closer look at equation (2.5), which is dependent on the assumption that galactic halos are isothermal.

2.2 MORE GENERAL CONSTRAINTS ON THE NEUTRINO MASS

In the derivation of equation (2.5) we recall that the neutrino configuration around galaxies was taken to be that of an isothermal sphere. A justification for this could be the observational result obtained by Ostriker et.al. [161] that $M(r) \propto r$, appropriate for isothermal spheres. However this result has only been shown to apply in the *outer* regions, whereas it is the inner regions we are interested in. Lynden–Bell [132] showed that if the violent relaxation subsequent to galaxy formation is *complete* then the stellar distribution function will tend asymptotically to an isothermal. However, it is simply not known to what extent this relaxation is attained. So such an assumption is in fact a rather unjustified one. A method which could avoid this problem would lead to a far more general and reliable lower mass limit for neutrinos. One might suppose that the results are not very sensitive to the isothermality assumption, in view of the fact that Ruffini and Stella '83 [174] obtained a minimum m_ν which differed by only 5% or so by using a King model for the neutrinos. But this result is not very surprising in view of the fact that King models were devised so as to resemble the isothermal sphere at small radii, and that the family of King models forms a one–parameter sequence parametrised by, for example, the concentration c which is the logarithm of the ratio of two characteristic radii, and the isothermal sphere is approached asymptotically as $c \to \infty$.

As pointed out by Madsen and Epstein '84 [136], nothing is known about the dark halo configuration, so a *true* mass limit can only be obtained from a model–independent analysis which is not subject to the uncertainties obtained by assuming a specific configuration for the neutrinos in halos. Two such approaches were given by these authors. Firstly, the maximally compact configuration $M_{\nu,max}(r; m_\nu, M_{\nu,tot})$, where $M_{\nu,tot}$ is the total neutrino mass, was put to satisfy $M_{\nu,max}(r) \geq M_{dark}(r)$, where $M_{dark}(r)$ is the observational distribution of the dark matter for a galaxy. Their second method was to simply use the constraint that the halo neutrino pressure must exceed the value given by the limiting case of 'half–degeneracy', occupation number $f = 0.5$ for the relic neutrinos, the value used in the Tremaine–Gunn analysis, which is consistent with the requirement of a non–increasing coarse–grained distribution function.

For $g_\nu = 1$ and only one non–negligible neutrino mass, the maximally compact configurations were modelled by placing the neutrinos with $f = 0.5$ at the centre, with the others in outer layers in order of decreasing occupation number. This gives $f(r)$ as a monotonically decreasing function of r. A complicated differential equation for f is obtained, the solution of which (obtained numerically) will correspond to maximally compact neutrino galaxies which had the initial phase space distribution of equation (2.1). From this is obtained, for a pure neutrino galaxy, the relation:

$$m_\nu = 32.9 \left(\frac{10kpc}{R_{\nu,1/2}}\right)^{3/8} \left(\frac{10^{12}M_\odot}{M_{\nu,tot}}\right)^{1/8} eV \tag{2.13}$$

where $R_{\nu,1/2}$ is the half–mass radius. It is found, incorporating the effect of the baryonic matter of the galaxy, that for most cases of astrophysical interest this has only a modest effect, as do preselection effects where the neutrinos with the highest phase space densities may preferentially form galaxies. A lower bound on m_ν is then found once an upper bound for $M_{\nu,tot}$ is known. The latter can be found from considerations of interactions between galaxies (e.g. as in the local group), or by using mass to light ratios. Our main problem with this entire approach, however, is that in order to apply it we need to be able to determine the dark matter distribution.

The second method, one not requiring numerical solutions to equations, and with different observational demands, gives a possibly stronger lower bound on m_ν, and is based on pressure considerations. There are two requirements:

1. The existence of some radius r_0, beyond which $M(r)$ increases more slowly than r^2

2. Beyond this radius the mass density of baryons is negligible.

In mathematical language:

$$M(r) \leq r^\beta \frac{M(r_0)}{r_0^\beta} \tag{2.14}$$

for $r > r_0$ and $\beta < 2$.

$$\Rightarrow \rho_{dark}(r) = \frac{1}{4\pi^2}\frac{dM}{dr} \leq \frac{\beta M(r_0)}{4\pi r_0^3}\left(\frac{r}{r_0}\right)^{\beta-3} \tag{2.15}$$

for $r > r_0$, and equality at $r = r_0$.

Integrating the equation of hydrostatic equilibrium,

$$P_\nu(r_0) = \int_{r_0}^{\infty} \frac{GM(r)}{r^2} \rho_\nu(r) dr$$

and using the inequalities (2.14, 2.15) then yields:

$$P_\nu(r_0) \leq \frac{GM^2(r_0)}{8\pi r_0^4} \left(\frac{\beta}{2-\beta} \right) \tag{2.16}$$

An alternative expression for the pressure of these nonrelativistic neutrinos is:

$$P_\nu(r) = \frac{8\pi}{3h^3} \int_0^{p_{max}(r)} dp f(r) \frac{p^4}{m_\nu} \tag{2.17}$$

where $p_{max}(r)$ is the maximum neutrino momentum at radius r. Since the density $\rho_\nu(r)$ is given by:

$$\rho_\nu(r) = 2m_\nu h^{-3} f(r) \int_0^{p_{max}(r)} 4\pi p^2 dp = (8\pi m_\nu / 3h^3) f(r) p_{max}^3(r) \tag{2.18}$$

we have:

$$P_\nu(r) = \frac{8\pi}{15h^3 m_\nu} p_{max}^5(r) f(r) = \frac{8\pi h^2}{15 m_\nu^{8/3}} \left(\frac{3}{8\pi} \right)^{5/3} \frac{\rho_\nu(r)^{5/3}}{f(r)^{2/3}} \tag{2.19}$$

by eliminating $p_{max}(r)$. But the occupancy f must be less than 0.5, so we obtain from (2.19):

$$P_\nu(r) \geq \frac{8\pi h^2}{15 m_\nu^{8/3}} \left(\frac{3}{8\pi} \right)^{5/3} \frac{\rho_\nu(r)^{5/3}}{(f = 0.5)^{2/3}} \tag{2.20}$$

Now putting $r = r_0$ in (2.20) and using (2.15) and (2.16):

$$\frac{GM(r_0)^2}{8\pi r_0^4} \left(\frac{\beta}{2-\beta} \right) \geq \frac{2^{2/3} 8\pi h^2}{15 m_\nu^{8/3}} \left(\frac{3\beta M(r_0)}{32\pi^2 r_0^3} \right)^{5/3} \tag{2.21}$$

which on rearrangement yields:

$$m_\nu \geq \left(\frac{3\beta}{2\pi^2} \right)^{1/4} \left(\frac{2-\beta}{10 G r_0} \right)^{3/8} \frac{h^{3/4}}{M(r_0)^{1/8}} \tag{2.22}$$

or, in more astronomical language:

$$m_\nu \geq 13.5 \beta^{1/4} (2-\beta)^{3/8} \left(\frac{10kpc}{r_0} \right)^{3/8} \left(\frac{10^{12} M_\odot}{M(r_0)} \right)^{1/8} eV \tag{2.23}$$

Both equations (2.13) and (2.23) were applied by Madsen and Epstein to the Virgo cluster galaxy M87. Using $M_{\nu,tot} < 3 \times 10^{14} M_{\odot}$ from mass to light ratios consistent with the deduced $M_{dark}(r)$ if $R_{\nu,1/2} < 230$kpc in equation (2.13) gave:

$$m_\nu(M87) > 4.4eV \tag{2.24}$$

while the values $r_0 \sim 20kpc$, $M(r_0) \leq 7 \times 10^{12} M_{\odot}$, $\beta \sim 1$ and equation (2.23) gave:

$$m_\nu(M87) > 8.2eV \tag{2.25}$$

The pressure–based method is preferred for its mathematical convenience. It gives a higher $m_{\nu,min}$ because the maximum compactness configuration is not one that is actually very likely to be formed by the neutrinos, it is a somewhat artificial extreme case, or so we suspect [136]. Moreover, the requirement of knowing the dark matter distribution $M_{\nu,dark}(r)$ is a difficult one to realise. It is interesting to compare these values for M87 with that which would be obtained from the Tremaine–Gunn constraint (2.11) with $n = 1 = g_\nu$. Stewart et.al. '84 [191] find a probable core radius for M87 of 25kpc and core density $1.5 \times 10^{-2} M_{\odot} pc^{-3}$. Using the standard relation (2.4) gives velocity dispersion parameter $\sigma = 240$km/s. Using these values in (2.11) yields:

$$m_\nu > 19.2eV \tag{2.26}$$

Hence we see clearly that the more general constraints derived by Madsen and Epstein can lower the bounds on the neutrino mass by factors as great as 3—4.

We finally remark that the m_ν limits given by (2.13) and (2.23) are still not completely general, as an isotropic velocity distribution was implicit in the derivations. In a subsequent paper [137], Madsen and Epstein allowed for departures from isotropy via the parameter α, where:

$$< v_\theta^2 > = < v_\phi^2 > = (1 - \alpha) < v_r^2 >$$

The halo is assumed to be spherical and non–rotating so that:

$$< v_\theta > = < v_\phi > = < v_r > = 0$$

From Jeans' hydrodynamical equations one obtains:

$$\frac{dP_\nu^{rr}}{dr} + 2\alpha \frac{P_\nu^{rr}}{r} = -\frac{GM_r \rho_\nu}{r^2} \tag{2.27}$$

where $P_\nu^{rr} := \rho_\nu < v_r^2 >$ and M_r is the neutrino plus baryon mass within r. For $r > r_*$, a radius characteristic of the extent of the stellar component, α is assumed constant, and once again:

$$M_r(r) \leq M_r^* \left(\frac{r}{r_*}\right)^\beta \Rightarrow \rho_\nu(r) \leq \rho_\nu^* \left(\frac{r}{r_*}\right)^{\beta-3}$$

with equality at $r = r_*$, but $\beta < 2-\alpha$ this time (this is necessary for non-divergence on integration). Integrating the hydrodynamical equation and using these inequalities, we obtain:

$$P_\nu^{rr}(r_*) \leq \frac{GM_r^*\rho_\nu^*}{2(2-\alpha-\beta)r_*} \tag{2.28}$$

which reduces to (2.16) for isotropy $\alpha = 0$, since $\rho_\nu^* = \beta M_r^*/(4\pi r_*^3)$. The analogue of (2.20) is then:

$$P_\nu^{rr}(r) \geq \frac{2^{2/3}8\pi h^2}{15m_\nu^{8/3}(1-\alpha)^{2/3}} \left(\frac{3}{8\pi}\right)^{5/3} \rho_\nu(r)^{5/3} \tag{2.29}$$

which finally yields the mass limit:

$$m_\nu > \frac{13.5\beta^{1/4}(2-\alpha-\beta)^{3/8}}{(1-\alpha)^{1/4}} \left(\frac{10kpc}{r_*}\right)^{3/8} \left(\frac{10^{12}M_\odot}{M_r^*}\right)^{1/8} eV \tag{2.30}$$

Conservatively taking $r_* \simeq R_{25}$ for a sample of sixty spiral galaxies, where the rotational velocities were constant ($\beta = 1$) or slightly increasing ($\beta > 1$), Madsen and Epstein found, for $\alpha = 0$, lower limits on m_ν in the range $(20\text{—}35)h^{1/2}$eV, while reasonable values of the anisotropy parameter $\alpha \leq 0.4$ gave limits which were weaker by an average 13% or so. The values used for α were based on collapse simulations of hot collisionless systems [143, 207], which suggest isotropy ($\alpha = 0$) in the central regions, and radial dispersion ($\alpha > 0$) in the outer regions such that α gradually increases to about 0.4 at the radius enclosing 80% of the total mass.

A stronger limit would be obtained for less conservative estimates of r_* than R_{25}, and equation (2.30) is to be regarded as the most general, model–independent route to $m_{\nu,min}$ we have, subject to our ability to accurately determine the parameters β, r_* and M_r^*. We note that the inclusion of anisotropy lowers the mass limit still further, our most general version being substantially weaker than the Tremaine–Gunn constraint.

We conclude this chapter from a non–standard perspective. The conventional interpretation of flat or gently rising rotation curves is that galaxies possess dark, spheroidal halos, and it is upon this assumption that we hope to use our phase space constraints to make a strong statement about neutrino masses. But perhaps such is not the case, and the neutrinos are instead distributed uniformly throughout clusters. This is a possibility we have to consider, since the core radius of equation (2.6) would then be so large as to render $m_{\nu,min}$ so small that there would be no hope of ruling out massive neutrinos as the dark matter (if this is what we are hoping for, that is).

2.3 A NON–HALO NEUTRINO MODEL

Quite recently, Cowsik, one of the first people to speculate that neutrinos may close the Universe, and Ghosh [50] proposed a scenario, the essence of which is that neutrinos are distributed on the scale of a few Mpc, that is of rich clusters. A consequence of this is that the galaxies and intergalactic baryonic matter are *embedded* in a cluster–sized neutrino cloud. Pertinent to the main subject of this thesis is the question of $m_{\nu,min}$ obtained from the neutrino distribution in the smallest systems. The fact is, the core radius r_c of the neutrinos that appears in equation (2.6) would in this scenario not be of the order of a few kpc, characteristic of a galactic halo, but instead of the order of a few Mpc, characteristic of the cluster. The $m_{\nu,min}$ thus obtained will clearly not be large enough to challenge any independently obtained upper limits. In this case the phase space arguments, although interesting in the context of theoretical physics, will be of no use for obtaining definitive conclusions about the nature of the dark matter.

Until we know far more about the process of galaxy formation it is not possible to say whether the premise of this scenario is right or wrong. It does appear however to provide a rather more natural explanation than the halo picture for the flat and gently rising rotation curves which seem to continue indefinitely.

Cowsik and Ghosh use a modified version (from [49]) of the well–known mass–radius relation [126] for a degenerate non–relativistic fermion gas:

$$m_\nu^8 = \frac{91.9h^6}{G^3 g^2 \alpha^2 R_c^3 M_c} \tag{2.31}$$

where R_c is the core radius, M_c the core mass, and α is the 'effective filling factor' $\alpha \simeq 2g_{flavour}g_{hel}s^{-3/2} \simeq 2 \times 10^{-3}$, where s is the ratio of the magnitude of the initial density fluctuations to R_c.

From (2.32) a neutrino mass of around 10eV is consistent with neutrino clustering on scales of clusters of galaxies if the limiting radius is $R_L \simeq 20R_c$. The error in this value will not be too serious, thanks to the high power m_ν enters as:

$$5eV \leq m_\nu \leq 20eV$$

This mass range for the neutrino is consistent with an $\Omega = 1$ Universe [49]. The mathematical formalism which describes the embedding of the baryonic matter within the neutrino clouds amounts to the simultaneous solution of the collisionless Boltzmann and Poisson equations, satisfied by both the stellar and neutrino components. The simplest approach, and that taken by Cowsik and Ghosh, is to assume isotropy for both components. Further generality is lost on the assumption of Maxwellian distribution functions:

$$f_i \sim \exp\left[\frac{-m_i}{kT_i}(\frac{v^2}{2} + \varphi_g + \varphi_\nu)\right] \tag{2.32}$$

for $i = 1, 2$, where $i = 1$ refers to the non–neutrino and $i = 2$ the neutrino component. The assumption of a Maxwellian is probably a reasonable zero–order approximation to reality, and has the virtue of being easy to handle mathematically. The numerical solutions to the self–consistent coupled Boltzmann–Poisson equations are then shown to reproduce the following:

1. the distribution of galaxies in a cluster;
2. high stellar velocities in dwarf galaxies;
3. tidal stability of galaxies in general and dwarf galaxies in particular;
4. surface density profiles for *all* types of galaxy;
5. flat and gently rising rotation curves.

Points (2) and (3) will be discussed in chapter 4 as they are vitally important to the dwarf spheroidal debate which is the main subject of that chapter. The above successes of the embedding picture, particularly point (5), provide persuasive weight to the credibility of this scenario, which, if correct, will not yield lower limits on m_ν of more than 10eV or so, even in dwarf galaxies. So the existence of dark matter in these systems is, modulo this scenario, perfectly consistent with neutrinos being that dark matter.

As regards the validity of this scenario, which opposes the conventional picture that dark matter coagulates around individual galaxies, there may be at least two observational ways of finding which of the two, if either, is correct, as noted by Cowsik and Ghosh. For the conventional view, the fluctuations in the gravitational potential throughout the cluster will be far larger than in the embedding scenario. One probe of this is gravitational lensing, for which detailed calculations remain to be done. The second probe concerns the question of tidal distortion of individual galaxies in clusters, the shapes of these disrupted galaxies being substantially different in the two scenarios. For this phenomenon also, little detail is yet known.

Chapter 3

DARK MATTER IN DWARF SPIRALS

3.1 THE EVIDENCE FOR HALOS

A dwarf galaxy may be defined as a galaxy with magnitude $M_B > -18$. Amongst the various types of dwarf galaxy are *spirals, irregulars, ellipticals* and *spheroidals*. The spheroidals are devoid of HI gas and in general show no evidence of recent star formation. The ellipticals are somewhat larger and sometimes do possess young stars: a discussion of these two types of dwarf galaxy is deferred to chapter 4. The irregulars and spirals are slightly larger and more luminous than the ellipticals and spheroidals, and also exhibit recent star formation. The traditional view of their morphology has identified dwarf ellipticals as relatives of larger ellipticals, and dwarf spirals as relatives of their larger optical versions. However a rival picture has come to light in recent years, which identifies the various dwarf types together as sharing a similar heredity, as it has gradually been discovered that they share many common properties. This issue will be discussed again in chapter 4, as it bears on the question of whether dark matter exists in one type only or perhaps *all* types necessarily. The reader is referred to the review article by Aaronson '87 [6] for a comprehensive discussion.

As in ordinary spirals, the best evidence for dark matter in their smaller optical counterparts comes from extended HI rotation curves. In short, the method consists of measuring this rotation curve $V(r)$ as far out as possible, to r_{max}. *Assuming* a constant M/L for the disk component and using the surface brightness distribution, one calculates the rotation curve that would obtain in the presence of the disk alone.

The difference in these two curves will correspond to the contribution of the dark matter halo. Fortunately dwarf spirals do not usually contain central bulges, which makes the difficult matter of rotation curve decomposition substantially less of a problem than for the case of normal sized spirals which in general do have bulges. The disks are reasonably well approximated by the exponential law:

$$I(R) = I_0 \exp(-R/R_d) \tag{3.1}$$

where I(R) is the surface brightness profile and R_d the so–called disk scale length. Detailed discussions of the mathematics involved can be found in the articles by Carignan '85 [36] and Kalnajs '83 [113]. As the decomposition of the observed rotation curve is not determined uniquely, two extreme cases are computed, one using the disk to explain as much of the curve as possible, the other using the halo to fit the most it can. These are called the maximum disk and minimum disk models respectively (or, equivalently, minimum halo and maximum halo). The true situation should then be bracketed by these two extremes. Moreover, a narrower range can be obtained by considering spiral structure constraints, where minimum halo disks may be bar–unstable and maximum halo models inhibit the density waves required for the spiral pattern. See, for example, Athanassoula et.al. '87 [14].

Before consulting the best evidence for dark matter in dwarf spirals, we should bear in mind the considerable *ensemble* of difficulties that are involved in the analysis (see [120]):

1. as with normal spirals, the assumption of circular motion for the HI gas might not be correct;

2. the assumption that M/L is independent of radius is at best a very uncertain one;

3. H_2 contributions have to be taken into account, which is a more severe problem for the lower luminosity systems;

4. especially for edge–on galaxies, corrections for internal absorption are poorly known and important;

5. for non edge–on galaxies, corrections for velocity projection involve errors;

6. HI warps disrupt the velocity data;

7. the density distribution of the halo has to be guessed *a priori* and arbitrarily;

8. these models treat the disk and halo as *superposed* systems. They are not self–consistent because they neglect the gravitational effect of one upon the other. In the smallest systems, if the dark matter component is found to dominate the visible one (as it sometimes is), the corrections will be small;

9. by around $M_B \geq -14$ the rotation velocity has decreased to the same order as the velocity dispersion, so the errors can be large and we need corrections for this pressure support;

10. at this low luminosity, the disk fails to be flat, by amounts described by the usual $(V_{max}/\sigma, \epsilon)$ diagram [104], where ϵ is ellipticity, V_{max} the maximum rotation velocity and σ the velocity dispersion;

11. at this low luminosity, irregularities in the HI distribution and velocity field become greater;

12. at these low luminosities, it is harder to reach the $V = const$ part of the rotation curve.

Despite these difficulties, there is still extremely good evidence for dark matter in some very faint dwarf spirals. As a general rule, rotation curves which are flat out to about three exponential scale lengths can be modelled adequately and fully explained by pure disk systems or disks plus visible bulges, as found by Kalnajs [113]. If the rotation curve data extends beyond this radius and the curve remains flat or slightly rising, then this is no longer the case: the maximum disk model does not provide enough matter to account for the rotation curve, and an invisible component in the outer regions (a halo) is needed. An excellent example of this is provided by the faint dwarf spiral DDO 127, which has $M_B = -14.5$ and a rotation curve measured out to 7.5 scale lengths where it is still rising gently. It has a well defined inclination of 54° (problem (5) resolved), with regular HI distribution and velocity field (problem (11)), and the exponential fit is a good one. The results indicate a reasonably strong gravitational domination by the dark component (since the minimum and maximum amounts of allowed disk do not differ by much), so that the lack of self–consistency in the fitting will not be too serious. This in turn greatly narrows down our uncertainties in the halo parameters (problem(7)). It is thus very difficult to avoid the conclusion that DDO 127 contains dark matter in substantial quantities, and that it has a halo [120]. This is by no means the only dwarf spiral for which such a conclusion can be reached: Carignan and Freeman '85 [37], Carignan et.al. '85 [38] and van Albada et.al. '85 [208] find halo to disk ratios in the *minimum halo* case of 2—7 for five dwarf spiral galaxies. It thus seems pretty clear that dwarf spirals contain dark matter halos.

3.2 NEUTRINOS IN DWARF SPIRALS?

We want to apply equation (2.11), the Tremaine–Gunn constraint, to these small spiral systems. For this we need to determine the parameters r_c and σ for the dark matter distribution. For the three nearby 'pure disk' systems NGC 247, 300 and 3109, Carignan and Freeman '85 [37] assumed an isothermal sphere for the dark halo, consistent in the sense of it later being appropriate to apply equation (2.11). Using Kalnajs' procedure to calculate the rotation curve $V_{calc}(r)$ expected from the surface photometry distribution I(r), fitting $V_{calc}(r)$ to $V_{obs}(r)$ in the inner parts where the disk is expected to dominate the halo gravitationally, the M/L value, assumed constant, is calculated for the disk. This gives the maximum $(M/L)_{disk}$ possible and so we are already dealing with the minimum halo case. The (non self–consistent) next step is to put:

$$V_{obs}^2(r) = V_{calc}^2(r) + V_{iso.h}^2(r) \tag{3.2}$$

where $V_{iso.h}(r)$ is the circular velocity of the isothermal halo. At large r, $V_{obs}(r) \simeq V_{iso.h}(r) \rightarrow 2^{1/2}\sigma$ and r_c is related to σ by equation (2.4).

Carignan et.al. '85 [38] used the same procedure for UGC 2259, whereas van Albada et.al. '85 [208] used for NGC 3198 the modified law:

$$\rho(r) = \frac{\rho_0}{1 + (r/a)^\gamma} \tag{3.3}$$

with $\gamma \simeq 2$, which gives a better fit than the isothermal law in the inner regions.

The assumptions of isothermality or the use of equation (3.3), and the lack of self–consistency are two weaknesses in the above analysis. For example, they describe systems of infinite extent (since for large r, $\rho \propto r^{-2} \Rightarrow M(r) \propto r \rightarrow \infty$ as $r \rightarrow \infty$), and the velocity dispersion (which is constant in the isothermal model) in a finite system is expected to decrease in the outer regions where the escape velocity is lower. Thus the value calculated for σ could well be a factor of a few less than the true core value.

The values of r_c and σ calculated in this manner, taken from Kormendy '85 [120], are given below, along with the corresponding values of $m_{\nu,min}$ calculated from the Tremaine–Gunn constraint (2.11) with $g_\nu = n = 1$:

Galaxy	r_c *(kpc)*	σ *(km/s)*	$m_{\nu,min}$
NGC 247	22	90	26.3eV
NGC 3198	12	105	34.3eV
NGC 300	12	60	39.4eV
NGC 3109	10.5	40	46.6eV
UGC 2259	8.7	57	46.8eV
DDO 127	2.3	27	109.5eV

Table 3.1: $m_{\nu,min}$ for dwarf spirals

An inconsistency with the cosmological upper limit $m_\nu < 100h^2$ is seen to exist marginally for DDO 127. But this is probably still within the error bounds because of the various sources of inaccuracy already listed, as well as the fact that a model–independent limit rather than the Tremaine–Gunn constraint should be used. We have already seen that the more general phase space constraint can lower $m_{\nu,min}$ by a factor of three to four. In fact, for NGC 3198 we can use the values $r_0 = r_{25} = 11.2$ kpc, $M(r_0) = 0.057 \times 10^{12} M_\odot$ and $\beta = 1$ (from [176]), to obtain $m_{\nu,min} = 18.5$eV for anisotropy parameter $\alpha = 0$. We see then that this has *halved* the value we obtained from the Tremaine–Gunn constraint. Moreover, Kormendy [120] describes the rotation curve of DDO 127 as rising out to r_{25} and flat thereafter. From this, we can take $\beta = 1$ and $r_0 = r_{25}$. We take the value $r_{25} = 1.76kpc$, as also given by Kormendy. Combined with a maximum rotational velocity of 34km/s, we compute a value for the mass within this radius of $M(r_0) = 4.74 \times 10^8 M_\odot$, using the equation for spherical symmetry:

$$v_c^2(r) = \frac{GM(r)}{r} \tag{3.4}$$

which is probably a good enough approximation. Putting these values into equation (2.23) we obtain a revised value of $m_\nu > 51eV$ for DDO 127 from the more general constraint. Once again the value is halved, and we find a result which is still compatible with cosmology. We stress again that these more general, weaker limits cannot be ignored in the absence of a convincing reason to believe that halos are isothermal — either due to *complete* violent relaxation or some other mechanism. We note that, as expected, the values of $m_{\nu,min}$, compared consistently, are increasing with decreasing scale size, so that the natural next step is to consider the smallest type of galaxy known to exist — the dwarf spheroidals.

Chapter 4

DARK MATTER IN DWARF SPHEROIDALS

4.1 INTRODUCTION

The smallest and faintest galaxies known to us, dwarf spheroidals are by nature exceedingly difficult objects to observe. Being as faint as $M_V > -13$ or so means that integrated light measurements are virtually impossible. In compensation it is fortunate that there are seven (known) such galaxies in the immediate vicinity of the Milky Way. Their relative closeness is an advantage, although we are still unable to pin down with particular confidence either their distances or absolute magnitudes. The fact that we happen to find ourselves in a galaxy which is surrounded by so many of these dwarf galaxies is not in fact believed to be a matter of luck — they are probably the most common galaxies in the Universe. This alone makes them objects worthy of study. Moreover, their apparent structural simplicity makes these objects suitable for testing evolutionary models [6]. We mention in passing that an offshoot of some of the recent observations of the velocities of satellite dwarfs of the Milky Way, originally performed to investigate dark matter in these systems, has been the application to determinations of the Milky Way's mass. Regarding the dwarfs as test particles, their Galactocentric radial velocities and distances can be used to constrain the distribution of dark matter in the Galactic halo. This is a question which could in turn be very important for the masses of the dwarfs themselves (see §4.2). For references, see [91, 135, 157, 167].

We have been led to consider dwarf spheroidals, with the question of whether they contain dark matter. The possible consequences that an affermative conclusion would have on the viability of light, stable, neutral neutrinos do not provide the only motivation for such an investigation. If these, the smallest visible galaxies, have substantial or dominating halos, do completely dark halos exist by extension? A related speculation is that the visible matter in the smallest dwarfs could become non self–gravitating, due to the domination of the dark component. This, in turn, could turn off star formation, and there would be some natural lower limit to the sizes of small galaxies. The question of the existence of dark matter in dwarf galaxies is also of crucial importance to the problems of galaxy formation and evolution. For example, a recent theoretical model of galaxy formation [58] is successful in predicting various correlations that have been observed in dwarfs between absolute magnitude and M/L ratio, velocity dispersion, surface density and abundance. This picture *requires* that the dwarfs possess dark halos, and thus its success depends on whether we find good evidence for dark matter in dwarfs. We note here in passing that the scenario has dwarfs as realisations of primordial 1σ density fluctuations, while other 'normal' galaxies arise from the far less common 2σ and 3σ fluctuations. We will return to this question quite soon. As an alternative explanation for the observed trend in absolute magnitude with mass to light ratio, it may be simply that the initial mass function depends sharply on abundance, albeit far more sharply than for any other stellar system known to us. The dark matter would then be present either in the form of stellar remnants or brown dwarf stars [6]. This too is postponed to a later stage of our discussion. We are led back to the question of the *origin* of the dwarf spheroidals. Since only those that orbit the Milky Way are well studied, attention here is restricted to these, the local dwarfs. There are essentially two, rival, pictures of their formation, describable as 'disruptive' and 'isolated' respectively. In the former, the local dwarf spheroidals are products of the debris of tidal encounters between the Milky Way and the Magellanic Clouds. This could explain the roughly coplanar distribution of the spheroidals on the sky [102, 123, 134], and would avoid the difficulty of how star formation actually proceeded (we will encounter this difficulty in §4.3) in such low density conditions, as the galaxies could have formed in this way already containing stars which were born in the Magellanic Clouds. On the other hand, the stellar content of the spheroidals would be very difficult to account for, as would the high mass to light ratios which we shall see later, if they are correct. The $([Fe/H], M_V)$ relation is hard to explain [6], and the 'tight' nature of the observed ([Fe/H], age) relation [220] is not what might be expected to result from the separation event. It is moreover difficult to understand the apparent very old age and large quantity of the globular clusters which have been observed in Fornax [220].

The alternative scenario has the spheroidals forming from the infall of gas around large–sized seed galaxies, and then losing the majority of their original mass adiabatically, via some process such as supernovae–driven winds [187, 203]. The problem with this is that a system like Draco would have had to begin its existence a hundred times smaller and a million times more dense than it is now — not particularly compelling. An interesting point is that within this scenario, the observed $([Fe/H], M_V)$ and $([Fe/H]$, surface brightness) relations cannot be explained, according to Dekel and Silk '86 [58] unless there is a dark halo. If star formation is cut short sooner in smaller systems by gas removal, this would lead us to expect an increase in M/L with decreasing luminosity. They calculate $M/L \propto L^{-0.37}$, which as we see from figure (4.1) [6], seems to agree rather well with the observations.

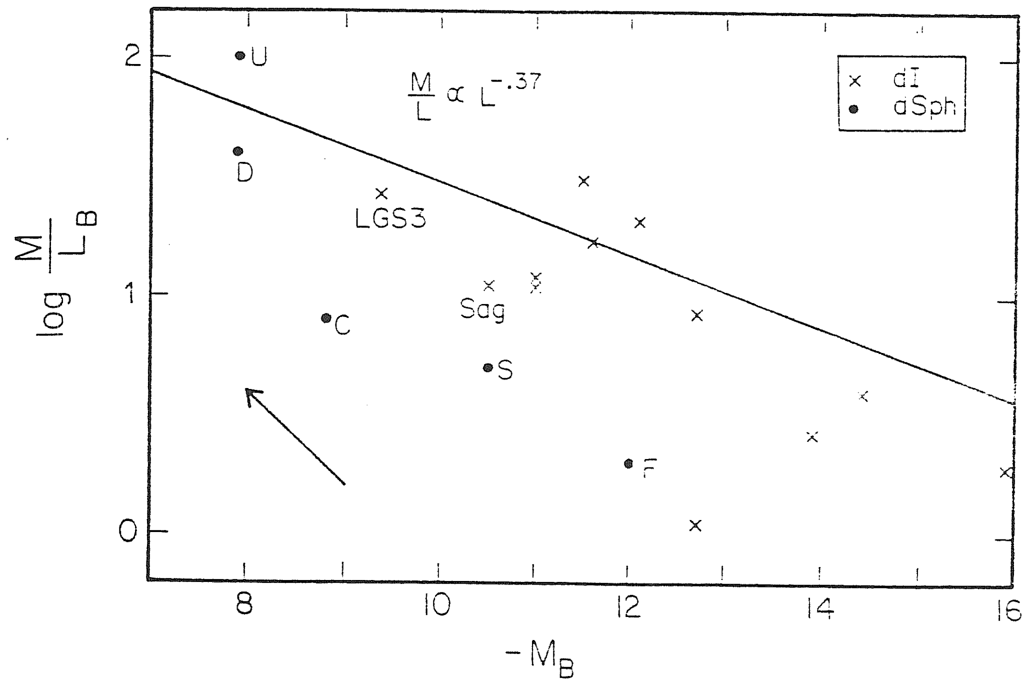

Figure 4.1: M/L as a function of L for dwarf spheroidals

We are not forced to accept the scenario of Dekel and Silk, even if we believe in the isolated type of formation. Figure (4.1) is also explicable in terms of the initial mass function, where the dark matter would either be predominantly in the form of stellar remnants or else brown dwarfs. For stellar remnants, we require an IMF biased towards high mass stars [145]. This has difficulties with enrichment and excess supernova activity [5]. For low mass stars, the IMF has to be sufficiently steep at low masses. This question is discussed in more detail in §4.4. Moreover, the prospects for the Dekel and Silk scenario do not at present look too good. It is a model of biased cold dark matter galaxy formation. As mentioned in chapter 1, cold dark matter encounters serious difficulties in explaining very large scale structure and streaming motions. Secondly, this model predicts a uniform distribution of dwarf galaxies throughout the Universe, in the sense that the dwarfs would be true tracers of the mass. In particular then, dwarfs would be found in the voids. However, a recent study by Thuan et.al. '87 [193] of the redshifts of 58 dwarf galaxies in the cfA slice of the Universe has led to the conclusion that they do *not* occupy the voids. See also Binggeli '88 [24] for a more detailed discussion of this question, and a similar conclusion. Whether or not we do reject this model is a matter of taste, especially as some doubts on the interpretation of the findings of Thuan and others can be raised, as discussed by Binggeli. The main point we wish to make here however is that the origin of dwarf galaxies is of vital importance for the question of dark matter, which is in turn relevant to our main topic.

We come back to a point raised in chapter 3, that of the relation between dwarf spirals and dwarf spheroidals. If the traditional view that dwarf spirals are just small spirals, and dwarf spheroidals are just small ellipticals is, as is increasingly believed, wrong, it may well be that these two kinds of dwarf are in origin related to each other. The first suggestions of this possibility [1, 62], based in part on the similarity in brightness distributions, had that the spheroidals formed from the spirals and irregulars through ram–pressure stripping of their gas. The relevance of this idea to the question of dark matter is obvious: the simplest approach would be to say that if we know that dark matter exists in the spirals (which we on the whole do believe to be the case), then by extension it exists in their gas–stripped descendants, the spheroidals. More quantitatively, Lin and Faber '83 [120] considered the value of M_*/M_{tot}, the ratio of visible to total mass of the dwarf spiral before stripping, using a model stellar mass to light ratio of 0.4 based on UBV colours [127]. The ratios M_{gas}/M_{total} and $(M/L_V)_{total}$ were known from 21cm studies. Data for dwarf irregular galaxies from the sources [73, 192, 199] gave nearly two thirds of the total mass within the Holmberg radius in non–luminous form. Allowing for fading, the disappearance of all the gas, they then calculated the mass to light ratios of the resultant spheroidals. These turned out to be consistent with the values they had found from their independent tidal analysis (see §4.2). Their results are illustrated in the following table.

Galaxy	M_{tot}	M_{gas}	M_*	M_{non}	L_V	$(M/L_V)_*$	$(M/L_V)_{tot}$
d. irr.	1.00	0.22	0.069	0.71	0.158	0.40	5.8
d. sph.	0.78	0.00	0.069	0.71	0.028	2.5	27.9

Table 4.1: Predicted M/L_V ratios of dwarf spheroidal galaxies

This is an interesting result, but we need to consider the whole question in somewhat more detail. Alternative to *ram pressure* stripping, the mechanism could be an *internal* one [58, 79, 203], or one whereby all of the gas makes stars. What is the evidence for this evolutionary picture for dwarf galaxies? Here we briefly list the observations which support it, based on the review by Kormendy '86 [121]:

1. similar surface brightness profiles (they are both equally well fit by exponentials);

2. similar correlations between various core parameters;

3. the discovery of huge, very low surface–brightness dwarfs in Virgo [177] is consistent with what we'd expect from galaxies that have lost great amounts of mass;

4. similar intrinsic shapes [39, 155]. The explanation for this is very simple [25]: as L decreases, so does V_{max}/σ by the Tully–Fisher relation, because the internal velocity dispersion remains constant at about 10km/s due to local processes like supernova heating. Thus the originally spiral, disk galaxy becomes less and less flattened (the ellipticity is given approximately by $\epsilon = v^2/(1+v^2)$, where $v := V_{max}/\sigma$), and eventually it becomes elliptical–shaped;

5. since we find intermediate age stellar populations in spheroidals, this implies stars were still forming for some time after the formation of the galaxies [5];

6. if Ursa Minor, one of the local dwarf spheroidals, was once an irregular, this would explain the observed clumpiness in the distribution of stars therein [156];

7. the metallicity–luminosity relation for dwarf spheroidals is as continuous with that of dwarf spirals and irregulars as it is with normal ellipticals [5];

8. the unusually large number of globular clusters in the spheroidal Fornax indicates that this galaxy was once brighter [90];

9. the high mass to light ratios calculated for the local spheroidals Draco and Ursa Minor [5] are consistent with fading after stripping;

10. the original suggestion of stripping [62] came from the observation that the type of satellite dwarf depends on distance from the parent galaxy. The spheroidals are found closer in, irregulars further out, and the closest irregulars are the most luminous ones, which have been able, with their deeper potential wells, to survive the process of stripping caused by the host's halo;

11. the spheroidals in Virgo are concentrated towards the centre where tidal stripping effects will be greatest, whereas the spirals and irregulars are distributed almost uniformly [178];

12. bright disk galaxies in Virgo contain less HI than similar galaxies in the field [81, 93].

As regards the possibility of the spheroidals being related to larger ellipticals, we first note that the existence of a metallicity–luminosity relation for the latter suggests that M/L should *increase* with L [194], whereas for dwarfs we observe a *decrease* (see figure 4.1, p. 40). Secondly, as we immediately see from figure (4.2)[121], the various core parameter correlations, mentioned in (2), imply that the normal ellipticals seem to be distinct objects from the dwarfs, as do globular clusters. This will be an important point to consider shortly.

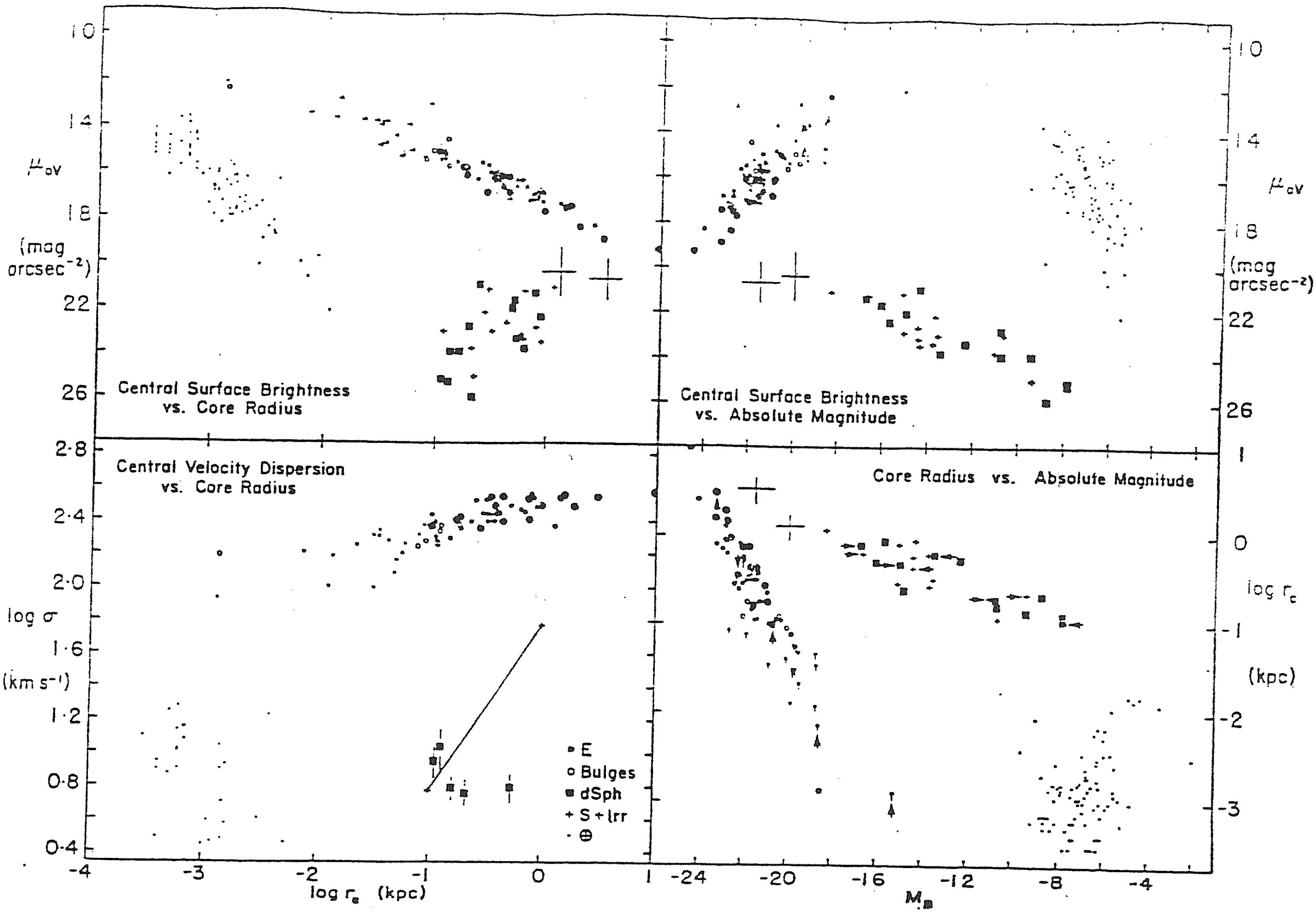

Figure 4.2: core parameter correlations

The evidence for a common heredity between the dwarfs is thus quite good (although quantitative analysis is needed), while the evidence that larger ellipticals are distinct objects from dwarf spheroidals is better. The two lowest luminosity local dwarf irregulars could provide evidence of objects which are transitional between irregular and spheroidal types: the Sagittarius dwarf and LGS3. These both seem to have absolute magnitudes of the same order as for spheroidals, $M_B \sim -10.5$ and -9.4 respectively. They also seem to have quite large M/L ratios [6]. The Sagittarius dwarf seems to be populated almost exclusively by carbon stars, very much like the spheroidal Fornax. As for LGS3, star formation has probably not occurred in this galaxy for the last 10^8 years. It seems that the only difference between LGS3 and a typical spheroidal is the existence of a small amount of HI in the former. So we may well be seeing here examples of galaxies in the process of transition from irregulars to spheroidals.

The last point we make about this transitional picture for the origin of dwarf spheroidals concerns the physical viability of the sweeping of the gas. Frank and Gisler '76 [74] made a simple quantitative model of this mechanism. Using, from this study, the equation for the rate of mass loss from irregular galaxies in and by the halo of the Milky Way, Lin and Faber '83 [129] calculated the required number density of gas in the halo. For a progenitor galaxy of initial mass $2.4 \times 10^7 M_\odot$, gas radius $\sim$1kpc, they found that for the gas to be stripped within a Hubble time, the gas in the Milky Way halo has to have an ambient number density of $\sim 10^{-6} cm^{-3}$ at a typical distance of about 100kpc. Unfortunately, we can not take the next step to see if this is the case, because such a density is too low to be detected. A density of this order though does seem plausible [129]. A more recent study of the plausibility of the ram–pressure stripping hypothesis has been made by Eskridge '88 [65]. The usual mechanism supposed responsible for the stripping is one where a nearby parent galaxy posesses a halo which sweeps out the gas from the dwarfs. Eskridge however considers the system of dwarf spheroidals recently discovered [30] in the outer regions of the M81 group where *no* nearby parent galaxy appears to exist. The 'stripping agent' for these galaxies could thus be the inter–cluster medium. It is found that the ratio of the mean free path of this medium, with respect to the dwarf interstellar medium, to the dwarf galaxy radius, is of the order of 10^{-4}. This is a successful test of viability since if the ratio had been greater than one then the particles would have typically travelled distances greater than the physical extent of the galaxy before interacting, rendering the stripping completely ineffective. Next, Eskridge considered the peculiar velocities of these dwarfs, calculated from the equation of Frank and Gisler. We require that these peculiar velocities not exceed the escape velocities of the galaxies from their cluster environment. Subject to this requirement, he finds that, apart from the case where the dwarfs contain significant amounts of dark matter while the cluster as a whole does not (which we suggest is not the most appealing of possibilities), the ram pressure stripping hypothesis is within reason. This means that such an origin for these dwarfs is consistent with their being bound to the group, although it is perfectly conceivable that they are not bound, since we observe them now in the outer regions. Uncertainty in the value of the intercluster density makes a definite conclusion impossible. If this density turns out to be extremely low, $\ll 1.7 \times 10^{-29} gcm^{-3}$, then the peculiar velocities of the dwarf galaxies would be so much greater than the cluster escape velocity as to rule out the stripping argument. The main result of this investigation then is that large parent galaxies do not seem necessary for the ram–pressure stripping argument.

Having described the main reasons for studying dwarf spheroidal galaxies and their relevance to our principal question of whether they contain dark matter, it is time to proceed and see what kind of an answer we can find. The most common measurement is of mass to light ratios, values above a certain fixed number being considered good evidence for dark matter, as this fixed value is the M/L that would be accounted for by the visible matter alone in the system under question. This fixed value will clearly vary amongst the various types of galaxy, as different kinds of galaxy have different stellar populations and varying proportions of dust and gas. The most usual way of approaching this problem (i.e. of deciding what this fixed value should actually be) for dwarf spheroidals has been, and still is, to consider globular clusters. The simplest picture is this: dwarf spheroidals and globular clusters are in stellar content extremely similar systems — the latter probably do not contain dark matter and have an average M/L of around 2. Hence any M/L reasonably greater than 2 that should be found for dwarf spheroidals would be strong evidence for dark matter in these galaxies. We must, however, be more cautious than to simply reason in this way without a certain amount of reservation, for two main reasons. In recent years it has been found that significant differences do exist in the composition of these two respective systems, and moreover the possibility of dark matter in globular clusters should not be completely ruled out. We list some of the main differences that we should bear in mind when comparing these systems, as given by Da Costa '84 [54]:

1. all dwarf spheroidals studied exhibit a spread in their heavy element abundance [190, 219], whereas all but one globular clusters (ω Cen) do not;

2. anomalous Cepheids seem to be far more common in dwarf spheroidals than globular clusters. An anomalous Cepheid is a variable star which does not obey the period–luminosity relation for either type II or classical Cepheids, and seems to have a mass 2—3 times those of RR Lyrae variables [218];

3. the dwarf spheroidals do not follow the conventional relation be-

tween horizontal branch type and metal abundance for galactic globular clusters;

4. carbon stars seem to be far more common in dwarf spheroidals;

5. at least some of the dwarf spheroidals contain intermediate age stars, which are not found in globular clusters [1].

These findings constitute at least a modification of the former simplified picture, which states: 'no gross differences in spectral appearance exist among stars in the dwarf spheroidal galaxies, the outlying globular clusters, and the inner halo clusters' [47]. Next, there is the question of dark matter in globular clusters, generally believed not to exist. There are two reasons to think again. Da Costa and Freeman '85 [55], from line of sight velocity measurements of stars in the globular cluster '47 Tucanae', concluded that this system must have missing mass to explain the observed velocity dispersion values, unlike the globular cluster M3. This dark matter is reckoned to provide 30—40% of the total mass. The result is however somewhat model–dependent: it is modulo the 'thermal equilibrium' multimass models that substantial amounts of dark matter are necessary (see Appendix D). Secondly, Peebles '84 [227] has constructed a cosmological model in which globular clusters tend to form with extended dark halos. In this cosmological model, the Universe is dominated by weakly interacting cold dark matter particles. We merely make the point here then that we cannot yet say that dark matter does or does not exist in globular clusters, at the very least until we know more about the radial behaviour of the velocity ellipsoid (v_r roughly constant in the outer regions would give $\rho(r) \propto r^{-2}$, consistent with a halo configuration). For our purposes however, the most conservative assumption that globular clusters do not contain dark matter is the best to use, in the sense that an eventual discovery based on this that dark matter exists in dwarf spheroidals would still hold if it also existed in globulars. As for the fixed M/L that we should consider as typical for globular clusters, we note that Illingworth '76 [103] found an average value of 1.6 from measured central velocity dispersions for ten clusters, using King models. Higher values, as high as 7 for M15 and 3.5 for NGC 6388 and 47 Tuc, have been calculated from thermal equilibrium multi–mass models by Illingworth and King '77 [105], where 30—60% of the mass in these systems is supposed to be in white dwarfs. We take an average and approximate M/L_V ratio of 2, and at the same time bear in mind that the differences (1)—(5) in stellar composition between dwarf spheroidals and globular clusters will have a certain effect on this value.

The younger, brighter populations in the dwarfs could make their luminous contribution to M/L *lower* than for globular clusters, although the escape of low mass stars from the latter by evaporation would work in the opposite sense. We now examine the methods that one uses for actually calculating these M/L ratios, which we hope will tell us whether dwarf spheroidal galaxies contain dark matter.

4.2 THE TIDAL APPROACH

It turns out that determining the M/L of dwarf spheroidals is a particularly thorny problem. As before, attention is restricted to the seven known local spheroidals, in the hope that they are typical such systems. Crucial for the credibility of mass to light ratios obtained from considerations of tidal theory is the question of whether the local spheroidals are tidally limited by the gravitational field of the Milky Way — and if they are tidally limited, are they tidally relaxed? Both these key questions will be discussed quite shortly, and we note here that the answer to them tells us whether or not we can actually apply the standard tidal formula (or some modification thereof) legitimately. Faber and Lin '83 [69] were the first to calculate dwarf spheroidal masses using this method. We consider here some of the details.

Amongst other things, it was shown by Hodge (e.g. [95—99]) in his extensive studies in the 60's of the local dwarf spheroidals that their luminosity profiles are well fitted by King models, which are models for tidally truncated systems. This is a good reason to believe that these systems are tidally truncated by the Milky Way. Approximating both the Milky Way and its satellites as *point masses*, and identifying the *tidal radius* r_t of the satellite (defined as the radius at which the satellite's density falls to zero) with the 'Jacobi limit', one can show to first order in the small quantity r_t/D:

$$r_t = D\left(\frac{m}{M(3+m/M)}\right)^{1/3} \simeq D\left(\frac{m}{3M}\right)^{1/3} \tag{4.1}$$

where D is the distance between the two systems, m and M are the dwarf and Milky Way masses respectively, and the orbits are assumed circular [27]. The Jacobi limit is defined as the distance between the dwarf and the saddle point of the effective potential Φ_{eff} which lies between the two galaxies, where $\Phi_{eff} := E_J - \frac{v^2}{2}$ and E_J is Jacobi's integral, constant along any orbit in a steadily rotating potential ([27], p.135). The problems with using this equation are obvious. The point mass approximation will fail badly if the satellites are actually orbiting within the Milky Way halo. The identification of tidal radius with the Jacobi limit is not unambiguous, because the zero–velocity surface is not spherical ([27], p.485), and the assumption of circular orbits is not justified for the spheroidals. A slightly alternative relation between the masses of the two respective galaxies and tidal radius of the satellite, and that used by Faber and Lin, was derived by King '62 [117]:

$$r_t = d_p \left(\frac{m}{(3+e)M} \right)^{1/3} \tag{4.2}$$

(see Appendix B for the derivation). A point mass potential is still assumed here, although the orbit is not necessarily circular: d_p is the distance of *closest* approach and e the orbital eccentricity. The point here is that the tidal limit is supposed to be determined by conditions at pericentre, an argument that might seem physically reasonable, though it is far from proven. The assumption of a point mass Milky Way will yield an underestimate of r_t and hence an underestimate of the dwarf spheroidal mass. It is underestimated still further by the fact that at a given perigalactic passage there will not be enough time for all the stars capable of reaching r_t to be stripped away; and a third effect works the other way, that a dwarf will shrink as it moves away from closest approach as its stars will feel the attracting gravitational force of the Milky Way less. These details are hard to model. An equally severe problem, an observational one this time, is the uncertainty in d_p: Faber and Lin used present heliocentric distances, which of course overestimate d_p and so underestimate m. As for the unknown nature of the orbits, they used an eccentricity $e = 0.5$ for each system in the hope that this value would average out the differences. The final unknown in equation (4.2), which is inverted to find the dwarf spheroidal mass m, is the Milky Way mass M. Faber and Lin assumed that the Galaxy has a spherical isothermal halo with constant circular velocity 225km/s, based on the study by Gunn et.al. '79 [87]. A weakness in this method then is that we need to know the mass distribution of the Milky Way in order to find the masses of the dwarf galaxies. Since m increases linearly with M in equation (4.2), a high mass for the Galaxy (i.e. the existence of a massive Galactic halo) automatically implies a high dwarf spheroidal mass, perhaps due to dark matter. This is an important point to be borne in mind, if the assumptions leading to equation (4.2) are to be believed.

Using equation (4.2) for the outer globular clusters of the Galactic halo, with data from Harris and Racine '79 [89], Faber and Lin obtained an average M/L_V of 1.34 ± 0.4. This seems a remarkably good result in that it agrees so well with more conventional evaluations of globular cluster mass to light ratios, and as such provides a certain amount of support for the method. We will do well to remember though that the method was actually designed to apply to globular clusters, and that dwarf spheroidal galaxies are somewhat different systems in their response to the tidal field of the Milky Way. Applying the same equation for the dwarf spheroidals that orbit our Galaxy, with data from [2, 3, 98, 101], Faber and Lin found M/L_V values as follows.

Galaxy	$d_{hel}(kpc)$	M/L_V	$L_V/10^6 L_\odot$
Ursa Minor	67	126	0.18
Draco	67	13	0.19
Sculptor	84	6.8	2.0
Carina	85	54	0.065
Fornax	188	1.0	24
Leo I	220	0.21	3.1
Leo II	220	0.30	0.71

Table 4.2: M/L_V(tidal) For The Dwarf Spheroidals

The mean M/L calculated in this fashion was 30.3 ± 19.3. This is a good order of magnitude higher than that for globular clusters, strongly indicating large quantities of dark matter in these systems. We note that M/L is lower for the more distant dwarfs and that tidal mass calculations for these objects, where the tidal action is less effective, could be systematically underestimated. Faber and Lin then went on to note that the dwarfs' luminosity profiles seem to be fit as well by exponentials as by Hubble laws or King models with cutoffs, if not better. If they are true exponentials, or more generally not truncated, then their masses and mass to light ratios derived from (4.2) will have been systematically underestimated, this effect being more pronounced for the more distant objects. We need to know more about this, and are led back to our initial questions as to whether the local dwarf spheroidals are indeed tidally truncated and, if so, have they had time to reach a state of tidal relaxation? It is moreover possible that, particularly the closer systems which yield the highest M/L's, are actually in the process of being *disrupted* by the tidal field of the Milky Way.

Regarding the first of these questions, we can go back to 1969 and a classic paper by Hodge and Michie [100]. Plotting the quantity $|T_z/V_z|$ against ellipticity, they obtained a correlation for Sculptor, Draco and Ursa Minor. The quantities T_z and V_z are defined by:

$$T_z = \int \frac{z}{\rho} F_z dM, \qquad V_z = -\int z \frac{\partial \Phi}{\partial z} dM \tag{4.3}$$

where the z–axis joins the centre of the Milky Way with the centre of the dwarf system, M is the mass of the system, and F_z the z–component of the tidal force exerted by the Milky Way. The ratio T_z/V_z should then be a measure of the importance of the tidal force in determining the internal structure of the dwarf, and thus a correlation of this quantity with ellipticity would be evidence for the tidal force as the principal determinant of the outer structure of the dwarfs. The trend was not obeyed by Leo I, II which are so distant that it is doubtful they would feel the Galactic tide to any real extent. A second test was a plot of $|T_z/V_z|$ against the ratio of limiting to core radius, and *all* the galaxies were found to obey the same trend this time: the stronger the tidal force, the lower the stellar density in the outer regions, so we expect and observe large values of $|T_z/V_z|$ to correspond with large central concentrations. We can thus conclude quite confidently that the local dwarf spheroidals are tidally limited by, and dependent in their outer properties on, the Milky Way.

The second question is more subtle, the question of whether these systems are tidally relaxed. Seitzer '85 [183] has studied this problem in the case of local globular clusters, numerically following 1000 test particles in a Plummer potential cluster. He found that about 50 orbits were necessary for tidal relaxation: the dwarf galaxies have probably completed less than 10. The point is, that this non–authentic N–body treatment of self gravitating systems can only be applied to very centrally concentrated systems like globular clusters, and probably *not* to the loosely bound dwarf galaxies. Freeman [77] has pointed out an example of globular clusters for which a large number of orbits may not be necessary to attain a state of tidal relaxation. The young LMC globular clusters are younger than one orbital time around the LMC. Measuring their masses however from their tidal radii yields results which agree with the more conventional methods of mass determination via luminosity functions and radial velocity measurements (see §4.3). He points out that these clusters have more or less circular orbits: we recall the fact that the Jacobi limit can not even be defined unless the orbit is circular

[27], as well as the uncertainty involved in using perigalacticon distances rather than apogalacticon or something in between for eccentric orbits. Freeman concludes that the tidal radii of these young globular clusters have more to do with their formation conditions than their dynamical evolution. This idea is relevant to the dwarf spheroidals: if they have orbits of low eccentricity, then it could be reliable to use equation (4.2) to determine their masses.

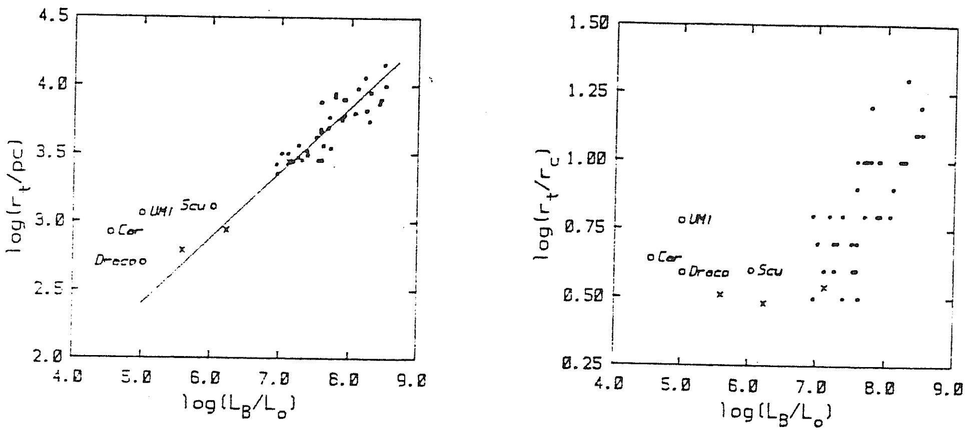

Figure 4.3: Tidal Disruption in Dwarf Spheroidals?

Related to the question of tidal relaxation is the even more serious concern (particularly for the galaxies themselves) that they may be in the process of tidal disruption. Hodge and Michie '69 [100] concluded that if Ursa Minor is currently on an approach passage, then the tidal field of the Galaxy will actually tear it apart. Ursa Minor's *observed* limiting radius was about three times larger than its computed tidal one [98], which indicates that a great deal of its mass is being pulled out. They calculated a dynamical time–scale for the disruption as short as 6×10^8 years, which is less than the system's transit time past the Milky Way. We now know that Ursa Minor *is* approaching the Galactic centre, and some authors (e.g. Lynden–Bell [134]) have concluded that the Milky Way must now be tearing this system apart. An interesting study of this problem was made by Joeveer '85 (*cited by* Einasto [63]). He plotted the tidal radii of Virgo Cluster dwarf spheroidal galaxies and local spheroidals against absolute blue luminosities L_B (see figure 4.3). On a logarithmic scale, the Virgo dwarfs scatter about a straight line, as do the local dwarfs Leo I, II and Fornax — these are the three most distant spheroidals. Relative to this straight line, the four nearer spheroidals have larger tidal radii. His interpretation of this result is that the nearest satellites are actually *expanded* rather than limited, thanks to the process of disruption. Einasto [63] concludes that the four nearest spheroidals — Ursa Minor, Draco, Carina and Sculptor — are all strongly *disturbed* by tidal forces, so that estimating their masses via an equation such as (4.2) is not possible.

We note however that some of the local dwarf spheroidals are considerably less luminous than the majority of those in the Virgo Cluster, so perhaps we ought not to expect the straight line to continue to apply in such an extreme regime.

The difference between the globular clusters and dwarf spheroidals which orbit our Galaxy, as regards susceptibility to tidal disruption, is apparent from the numerical experiments performed by Miller '86 [149]. He has shown, subject to the usual problems of N–body simulations, that the former are essentially in no such danger: a cluster with $M/L = 1$ could get as close as 100pc to the Galactic centre before perishing. It is the small size of these systems which protects them. For the dwarf spheroidals, the story is a different one: if these systems have $M/L = 1$ then their closest approaches consistent with survival were obtained as follows.

Galaxy	$d_p^{(1)}(kpc)$	$d_p^{(2)}(kpc)$	$d_{g.c.}(kpc)$	$(M/L)^{(1)}$	$(M/L)^{(2)}$
Fornax	44	56	188	0.013	0.088
Sculptor	48	63	84	0.18	0.57
Leo I	28	29	220	0.0021	0.017
Leo II	32	35	220	0.0031	0.025
Draco	41	50	67	0.23	0.56
Ursa Minor	95	176	67	2.8	6.9
Carina	97	183	85	1.5	4.7

Table 4.3: Dwarf Spheroidals: Tidal Disruption Properties

Here the superscripts (1), (2) correspond to a Galactic $1/r$ and logarithmic potential respectively, d_p^i are the distances of safe closest approach if $M/L = 1$, $d_{g.c.}$ is the present Galactocentric distance, and the last two columns list the M/L required to avoid disruption *at the present Galactocentric distances.* We see that for unit M/L, Ursa Minor and Carina would surely perish, while Draco and Sculptor would also be in some danger. We of course suspect somewhat higher M/L ratios for these systems. The values of closest approach should increase by factors of 1.26 and 1.41 for more than one orbit, while the M/L's should double. The M/L ratios required to avoid tidal disruption in the last two columns are considerably lower for all systems than those derived by Faber and Lin, from which we are able to conclude that the assumptions involved in the use of equation (4.2) yield M/L ratios that are consistent with these assumptions. However we so far have no real evidence that these galaxies are not being disrupted, and the high M/L values found, particularly for the closer systems, are also consistent with this possibility, where (4.2) would have then been used to yield spuriously high masses.

In view of the evidence that has been presented in this section, we cannot do otherwise than to admit we just don't know to what extent the application of equation (4.2) is legitimate. Even for systems that have tidally relaxed and are not subject to disruption, there are shortcomings in the method *per se* as well as observational uncertainties. It has to be admitted that the evidence seems to indicate that the dwarf spheroidal galaxies have not completed enough orbits around the Milky Way to achieve a state of relaxation in their outer regions, and that tidal expansion rather than truncation seems a possibly more likely phenomenon for the inner four systems. The apparent success of the tidal method for determining globular cluster masses is encouraging but tainted by the fact that the globulars are virtually immune to tidal disruption, and at least the inner globulars have probably completed enough Galactic orbits to have relaxed in their outer, tidal, regions. For the dwarf spheroidals, as we have seen, the story is a different and rather more complicated one. As Faber and Lin in their original paper admitted, the spirit of the investigation was a speculative one. We are forced by the lack of reliability of this approach to adopt another one, for which we hope the limitations will be less. It is the more conventional method of using stellar radial velocities, an approach which for observational reasons has only proved possible for the dwarf spheroidals over the last five years or so. Will we be able, once and for all, to confirm the high mass to light ratios obtained by Faber and Lin, and hence be in a position to rule out one of the classic dark matter candidates in dwarf spheroidal galaxies?

4.3 STELLAR VELOCITY APPROACH

There are various standard techniques for calculating the masses of galaxies from their velocity dispersions, which are obtained from measurements of stellar velocities. Radial velocities are obtained from the stellar spectra, but this can unfortunately only be done in the faint dwarf spheroidals for the stars at the tip of an old giant branch, where $V \geq 17mag$. High precision is also required to measure low velocity dispersions, precision as good as $\pm 1km/s$.

Aaronson '83 [2] was the first to use this approach for dwarf spheroidals. He measured the radial velocities, via cross–correlation techniques, of three carbon stars in Draco, to an accuracy of about 1km/s. It would clearly be absurd to calculate a dispersion from just three objects. Aaronson showed instead that the minimum velocity dispersion allowed by a χ^2 test at the 5% level with two degrees of freedom was $6.5km/s$. An isotropic King model was then used for these one–dimensional dispersions. A simple expression for the mass of the galaxy is given by Illingworth '76 [103] (see Appendix C):

$$M = 167 r_c \mu < v_r^2 > \quad (4.4)$$

where r_c is the core radius in parsecs and μ a dimensionless mass parameter which depends on the geometry of the galaxy. Using previously obtained values of r_c, μ and L then gave for Draco:

$$\frac{M}{L_V} = 0.72 < v_r^2 > \quad (4.5)$$

which gave a value of 31 at the 95% confidence level. We note that this is more than double the value obtained by Faber and Lin: Aaronson then used their equation with a higher Galaxy mass and lower perigalacticon distance to obtain $M/L_V = 29$ for Draco, and concluded that the tidal method could well be legitimate. We now encounter the problems inherent in this approach, and follow the lengthy debate which has sprung from these questions.

Aaronson [2] acknowledged these problems. The first is due to the tiny number of stellar velocities and the unreliability of the statistics involved. Secondly, there are suspicions that these carbon stars have large atmospheric motions, so that the velocities measured reflect the motions of the atmospheres rather than centre of mass velocities. Thirdly, their apparently high dispersion could be due to binary membership and thus totally misleading as to the motions that would have been induced by the smooth potential of the galaxy as a whole. Finally, we have our old question of tidal disruption, which would also affect the velocity dispersion, particularly for a system as close as Draco. As shown by Hodge and Michie [100], the time scale for tidal disruption would be on the order of 2×10^8 years, which is such a short time that Aaronson argued there is a small probability that we would actually see a system in this state. We at this stage should also bear in mind that the carbon stars may not actually give a good guide to the overall velocity dispersion, averaged over different stellar types with presumably different orbital behaviour. On this point Aaronson as a post–script cited a later measurement of a non–carbon stellar velocity in Draco which was of the same order as for the carbon stars. He thus concluded that there did not seem to be any significant zero–point difference between the carbon and non–carbon velocity systems. However the statistical significance of this result is even less.

Cohen '83 [44] promptly responded to these claims by measuring the radial velocities of four globular clusters in Fornax. She concluded that M/L_V was less than 8 at the 95% confidence level. This is interesting because the velocity dispersion of the clusters she found to be 6km/s, a factor of 5 or so less than that predicted roughly by Faber and Lin if Fornax were to have a mass to light ratio similar to that of Draco. However, the accuracy of the radial velocity measurements was only $\pm 10km/s$ for three and $\pm 20km/s$ for the fourth of the clusters, not sufficiently accurate for any meaningful determination of the dispersion. Moreover, we probably shouldn't expect to obtain high M/L ratios from data in the core regions in these galaxies where we do not expect the dark matter to dominate over the visible matter. This is a point we shall come back to later.

M_cClure '84 [141] addressed the question of binarism. He obtained 3—4 year span velocity measurements for CH stars in our Galaxy and some globular clusters. It is *believed* that the carbon stars in Draco, Ursa Minor and Sculptor are similar to these stars, and it is upon this belief that the interpretation of the results depends. We note however the recent discovery that dwarf spheroidals and globular clusters have a lot less in common regarding stellar populations than was once thought, as discussed in §4.1. M_cClure found two main effects among these cluster CH stars: systematic long term velocity variations, and random short term variations. Such behaviour would support binarism in these stars. Moreover, it is noted that the CH stars resemble the population I Ba II stars in their spectral features, the latter probably *all* being binary stars [140]. So perhaps the CH are binaries as well. This does not of course follow necessarily, particularly if we note that population II stars (CH stars are population II) are believed to be on the whole deficient in binary systems [21]. The short term velocity fluctuations could be explained by atmospheric motions (because CH stars are population II giants of low mass and so have low surface gravity), while the long term variations seem more consistent with binary membership. Furthermore, we should note the suggestion by M_cClure and Norris '77 [139] that globulars containing CH stars tend to be systems of lower concentration. This suggests that, if CH stars do indeed tend to be binaries, then binaries survive longer in systems of lower concentration. Dwarf spheroidals are considerably less concentrated than globular clusters, increasing the likelihood of binarism in them. We have to conclude that, at least to some extent, dwarf spheroidals probably contain binary systems among their carbon stars. The observations by Aaronson were not multiple epoque ones for two of the stars: these were what was needed to determine whether binarism or velocity variability in general was at work among the dwarf galaxy stellar components.

Seitzer and Frogel '85 [184] used the same method as Aaronson, obtaining radial velocities for six carbon stars in Carina, three in Sculptor, and five in Fornax. They obtained velocity dispersions of around 6km/s for each system and respective M/L_V ratios of $9.7, 3.4$ and 0.5. In spite of the considerable doubts on carbon stars as truly representing galactic velocity dispersions, they were chosen for the fact that they are the most luminous stars in those galaxies, and because accurate radial velocities could be obtained even for low signal–to–noise ratios due to the strong features in their spectra. Unfortunately these authors only observed these galaxies on one night each, so that nothing can be said about long–term variability of the velocities. On the bright side though, the velocity dispersion for Carina of 6km/s agreed with that obtained by Cook et.al. '83 [225], who had measured the same carbon stars a couple of years earlier. This should be regarded as weak circumstantial evidence for these stars *not* being members of binaries. Since equation (4.4) only gives the mass of the galaxy, in order to calculate M/L the total luminosities are needed separately. These are determined by the use of uncertain luminosity functions, often assumed to be the same as for some suitable globular cluster, an assumption which does not seem very safe. Moreover the core radii, tidal radii and distances of the galaxies, which are needed for the application of equation (4.4), are all considerably uncertain. With regard to the problems of the small number statistics involved here, the errors in the M/L_V were well over 50% for all three galaxies. The statistical error in the observed $< v^2 >$ is probably proportional to $(2/N)^{1/2}$ [53], where N is the number of stars involved. This means that in order to have a statistical error less than 30% in the mass requires at least 20 stellar radial velocities. These errors due to statistics could be even worse, as we will see later. Due to the small number of known carbon stars in the dwarf spheroidals it was therefore necessary to obtain velocities for other types of stars, both for this reason and because of the possibility of variability in the carbon stars.

An improvement in the situation was made by Aaronson and Olszewski '85 [4], who studied 10 stars in Ursa Minor and 11 in Draco. Apart from the improvement in number was that most stars were observed at two or more epoques, and that 16 of these 21 stars were K giants. Velocity dispersions as high as 10km/s were found for both systems. It is interesting to note that two of the Draco stars and three in Ursa Minor were velocity variables over the 1—3 year periods they were observed for. These authors argued against the high velocity dispersions being due to tidal effects from our Galaxy in the following way: the crossing time scale, and hence time scale for disruption, for Ursa Minor and Draco is about 10^8 years, while their stars are about 10^{10} years old. This gives something like a 1% probability of us seeing one such galaxy, and lower still for two. It is not clear how seriously this kind of argument should be taken. Perhaps more convincingly, Aaronson [226] pointed out that in the absence of dark matter the predicted dispersions in Draco and Ursa Minor would be of the order of $0.5 km/s$. Simple calculations have shown that tidal disruption is likely to only double this value (we should be careful in accepting this claim in the absence of a decent theory for such a phenomenon), which would still be an order of magnitude down on the dispersions observed. As for the binary question, we first note that 5 out of 21 stars showed velocity variations of the type expected for binaries (*none* however showed the kind of 'jitter' associated with atmospheric motions [138]). Secondly, most stars in this survey were K giants, as opposed to carbon stars. Carney and Latham '85 [41] found a binary proportion of 15% in the halo giants — a low fraction, although of course binary formation processes in the dwarfs could be very different from those in our Galaxy. As a further test of binarism, Aaronson and Olszewski performed Monte Carlo simulations to conclude that a 50% presence of binaries in a normal distribution in the dwarfs would not affect their observations to any significant degree.

These authors thus believed that their findings of high velocity dispersions were due to self gravitation and interpreted literally their results, obtained from equation (4.4) that $M/L_V = 40$ and 100 for Draco and Ursa Minor respectively.

Returning to the question of atmospheric motions in carbon stars, Jura '86 [110] noted that some of these stars are surrounded by CO envelopes [221]. The (radio–determined) CO radial velocity has been shown to probably be the actual centre of mass velocity of the star, to within 1km/s [150]. Using this result, Jura compared the *optical* radial velocities to CO radial velocities for carbon stars in our Galaxy, finding an intrinsic dispersion due to non centre of mass optically determined motions of about 5km/s. This result would have serious consequences for the carbon star measurements in dwarf spheroidals, *if* they have the same properties as their Milky Way counterparts. The dwarf galaxy carbon stars probably have lower metallicities and so different pulsational properties, effective temperatures and mass–loss rates [111], so it is not clear how far one should carry the results across to these systems — although for red giants in general there is good reason to believe in the existence of unstable atmospheres with fluctuating velocity fields [131]. As regards the K stars, similar effects are possible but unknown, although it *is* known that in metal–poor globular clusters these effects do *not* exist [168].

The most numerous radial velocities for any single galaxy were found for 16 K giants in Sculptor by Armandroff and Da Costa '86 [13]. They obtained a one dimensional velocity dispersion of $6.3km/s$ and $M/L_V = 6 \pm 3.1$ for this galaxy. The analysis of these authors differed in a couple of respects from previous ones. Firstly, they noted that since for some of the stars only there were multiple observations, then for these stars the velocities were significantly more certain than for the others. To take this effect into account, they used weighted calculations for the mean velocity and velocity dispersion. Instead of the standard formulae:

$$\overline{v} = \sum_{i=1}^{n} v_i/n \tag{4.6}$$

$$< v^2 >= \sum_{i=1}^{n} \frac{(v_i - \overline{v})^2}{n-1} \tag{4.7}$$

where the denominator $(n-1)$ is needed for these small number statistics, since the velocities follow a χ^2 distribution and $var(\chi^2) = (\frac{n}{n-1})var(v)$

(see[130]), these authors used:

$$\overline{v} = \sum_{i=1}^{n} \omega_i v_i / \sum_{i=1}^{n} \omega_i \tag{4.8}$$

$$< v^2 >= \left[\sum_{i=1}^{n} \left(\frac{\omega_i (v_i - \overline{v})^2}{\sum_{i=1}^{n} \omega_i} \right) - \left(\frac{n}{\sum_{i=1}^{n} \omega_i} \right) \right] \frac{n}{n-1} \tag{4.9}$$

where the weights ω_i were taken to be the inverse squares of the individual velocity uncertainties, which were in turn $4.7/N^{1/2}$ km/s, where N =number of observations and the factor 4.7 is the single observation uncertainty. The dispersion of the instrument is the second term in the square bracket. Secondly, they calculated the *central* M/L as well as the global value (see discussion on p.62), obtaining a value of 6. Given the error they quoted as ±3.1, combined with other problems, this is not strong evidence for dark matter in Sculptor.

As a passing remark, it is interesting to note that the mean stellar velocity, interpreted as the systemic velocity for Sculptor, turns out from these observations to be +107.4 km/s, a value very much higher than that of +20 km/s obtained by Richer and Westerlund '83 [169]. This has two interesting consequences. Firstly, it supports a relatively high Milky Way mass, in contrast to the studies of Lynden–Bell et.al. '83 [135] and Olszewski et.al. '86 [157], who used Richer and Westerlund's value. Secondly, the predicted radial velocity of Sculptor if its origin is of tidal debris from the Magellanic Clouds is -100 ± 20 km/s [133]. The very different observed velocity of +107.4 km/s thus provides good evidence against such a theory for the origin of the dwarf spheroidals (or at least for Sculptor, if we don't insist on a *common* origin), which as such lends support, by elimination, to the theory that the dwarf spheroidals are evolved versions of dwarf spirals and irregulars.

The most recent observations to date are measurements of seven carbon stars, once again in Sculptor, by Aaronson and Olszewski '87 [7]. With regard to the last point made, they obtained a systemic velocity for this galaxy of 109.2 ± 4.5 km/s, in remarkably good agreement with that of Armandroff and Da Costa. The main motivation for this work, apart from the question of the systemic velocity of Sculptor and its important consequences, was to investigate the reliability of measurements obtained from carbon stars. On this point, the results of Jura [110] were questioned for two main reasons: the radio–determined CO velocities were not known for *sure* to be indicative of centre of mass motion; and the very existence of CO envelopes implies the occurrence of mass–loss and instability. In fact, all but one of the stars considered by Jura has photometric amplitude variation $> 1.5mag$, the exception having a difference of $0.1km/s$ in the optical and CO velocities. Regarding temperature (or *colour*) as more reliable than luminosity as a gauge of the presence of atmospheric instability, they pointed out that the carbon stars in Draco, Ursa Minor and Sculptor are all much bluer than the carbon stars in the globular cluster 47 Tuc which were found to be velocity variables. This leads to the expectation that these carbon stars would *not* exhibit the pulsational instabilities of the latter. Virtually no amplitude–variable carbon type stars in Draco, Ursa Minor or Sculptor were found in extensive searches [15, 205, 206]. Hence Aaronson and Olszewski concluded that, based on this evidence with regard to colour and variability, the atmospheres of the C stars in Draco, Ursa Minor and Sculptor are not substantially unstable and so do not bias velocity measurements based on them. All the evidence is circumstantial however, and the question remains an open one.

From the seven C stars measured, the velocity dispersion obtained was 11.9 ± 3.4 km/s. Excluding the one star suspected to be a binary, brings the value down to 7.2 ± 2.3 km/s. From this, two points are immediately apparent. Firstly, it agrees reasonably well with the analysis of a larger number of K stars performed by Armandroff and Da Costa, which yielded a value 6.3 ± 1.2 km/s. Secondly, it shows the substantial effect the presence of even one binary can have in such a small sample. The *central* M/L ratio, they obtained as 7.7, although only two of their stars were within the core region. Taking the results at face value, that is ignoring all the various uncertainties present, this is more than twice the *global* value for globular clusters, and hence provides weak evidence for dark matter (it is more appropriate to compare the *central* M/L ratios of dwarf spheroidals to the *global* values for globular clusters, because in the latter systems the effects of mass segregation are probably important, so that the more massive stars would be expected to dominate the core regions instead of an unbiased mixture [13]). We now discuss some of the more important theoretical difficulties, and later go on to consider the question of the *nature* of the dark matter in dwarf spheroidals, if it exists.

As we have noted, the usual method of obtaining the mass or mass to light ratio of spheroidal or elliptical galaxies from the velocity dispersion is to use an isothermal or King model. This effectively amounts to using the simplest form of the Virial theorem $M \sim 3\sigma^2 r/G$, but with a particular choice of the geometrical constants omitted in the above.

The *global* galactic mass can be measured, using equation (4.4) and dividing by the total luminosity (a quantity which is unfortunately not directly measurable for these faint systems, so that further uncertainties are introduced by the use of a relatively arbitrary luminosity function, usually assumed to be the same as for some chosen globular cluster). Alternatively, one can directly calculate the *central* mass to light ratio. This is a process which avoids the need for L_{total} and minimises the dependence on the assumption of a King model, a model which was in fact designed for globular clusters, which are, more than conceivably, fundamentally different stellar systems (with regard to origin, stellar population and dynamics) from dwarf spheroidals. This latter approach, advocated by Kormendy '85 [120], considers the parameters I_0, Σ_0 and ρ_0, the central light density, central surface brightness and central mass density respectively. We then have:

$$I_0 = \frac{\Sigma_0}{p r_c} \tag{4.10}$$

$$\rho_0 = 166 \frac{\sigma^2}{r_c^2} \tag{4.11}$$

$$\Rightarrow \frac{\rho_0}{I_0} = 166 p \frac{\sigma^2}{\Sigma_0 r_c} \tag{4.12}$$

Equation (4.12) thus gives the central mass to light ratio, which is also a function of p, a geometric factor depending on concentration $\log r_t/r_c$, which is tabulated in Peterson and King '75 [166].

Despite the improvement that we obtain from this method, many inherent problems still abound. To begin with, the approach is not self–consistent, as was the case for rotation curve decompositions for spiral galaxies. The dark matter halo, if it exists, may well not be describable by a King model. The core radius r_c is measured for the *visible* matter and implicitly assumed to apply also to the dark matter, but the latter could in principle have *any* value $r_c(d.m.)$. So the assumption is almost certainly an incorrect one. In spiral galaxies for example, $r_c(d.m.)$ is substantially greater than $r_c(vis)$. It is therefore quite conceivable that $\rho_0(vis) > \rho_0(d.m.)$, so that the *central* velocity dispersion contains virtually no information on the dark matter which lies predominantly in the outer regions. The majority of stars whose radial velocities have been measured to date have been stars within one core radius: $\sigma(r)$ needs to be measured at least out to a radius where the dark matter contribution becomes important — which can not at present, for technological reasons, be realised in dwarf spheroidals. Kormendy points out that the various corrections to σ needed for self–consistency show that the simple calculations usually done have systematically underestimated M/L. More than one definition of core radius exists: modulo the model assumed, it is better to use the one defined dynamically by King '66 [118] rather than the empirical fitting function definition of King '62 [117] (see Appendix C for details). Since the dwarf spheroidals are actually elliptical in shape (at least for the visible matter), the use of mean radius would be a better approximation than the major–axis radius usually adopted. Since $\sigma(r)$ decreases with increasing r in a King model, σ should be corrected for the mean radius of the stars measured. Projection effects should be taken into account, as well as the phenomenon that the measured dispersion will be a systematic underestimate to the σ used in (4.11) and (4.12), by the amounts given by King '66 [118]. These corrections, taken together, increase the M/L ratio by a factor of around three, which is a very important result if King models are to be believed. It could turn out for example though that dwarf spheroidals have exponential rather than King–model brightness profiles, in which case we could no longer make the above statement. We note again though that in this case the *tidal* mass estimates would be too small. So it seems that in either case the estimates we have for the masses of dwarf spheroidals should be pushed *upwards* by improving the models.

We now take a closer look at the central densities of the five spheroidals for which this parameter has been measured, to find an interesting property. We tabulate, from Kormendy '85 [120], the central density of visible matter if this component has $M/L = 2$ (globular cluster type population), its value if $M/L_{vis} = 7$ (old disk populations), the total central density and central mass to light ratio:

Galaxy	$\rho_0(vis)$: $M/L_{vis} = 2$	$\rho_0(vis)$: $M/L_{vis} = 7$	ρ_0	ρ_0/I_0
Fornax	0.04	0.14	0.028 ± 0.018	1.4 ± 0.9
Sculptor	0.07	0.24	0.19 ± 0.16	5.5 ± 4.9
Carina	0.028	0.1	0.11 ± 0.07	7.8 ± 6.0
Ursa Minor	0.01	0.04	0.91 ± 0.51	175 ± 131
Draco	0.026	0.09	0.64 ± 0.34	48 ± 35

Table 4.4: Central densities of dwarf spheroidals (in $M_\odot/pc^3$)

We note immediately that in the three galaxies for which low mass to light ratios have been computed, we have the property:

$$\rho_0(d.m.) \leq \rho_0(vis)$$

Hence we should not be surprised when we find a low central mass to light ratio — dark matter could still exist in substantial quantities in these galaxies, but further out than a core radius or so. In stark contrast, we have for Ursa Minor and Draco extremely high central densities, $\rho_0(d.m.) \sim 100\rho_0(vis)$. This means that the central density is about an order of magnitude greater than in dwarf spirals, from which we suspect they may have evolved. But such a difference in density may be difficult to explain within such a picture. It also means that the luminous matter may not be self–gravitating, which raises the question of how the stars actually formed there, because self–gravitation is surely important for star formation. If we can solve the problem of how dwarf spirals evolve into dwarf spheroidals while their central densities increase by a factor of 10 or so, then the star formation problem would also be solved since formation would have occurred while the galaxy was in its spiral stage. Perhaps this is an even worse problem than it at first sight seems: it would require different kinds of dark halos in the two systems, since $\rho_0(d.m.) \leq \rho_0(vis)$ in dwarf spirals. Based on these considerations, Kormendy [120] concludes that σ has probably been seriously overestimated for Draco and Ursa Minor, so that he regards the existence of dark matter in dwarf spheroidals as still very insecure. Aaronson and Olszewski '87 [7] however do not agree — they prefer to regard the situation as quite natural that extreme systems should have extreme properties.

If σ has indeed been seriously overestimated, it may well be that the small number statistics involved, always notorious, have conspired to produce this effect. Such a possibility has been examined in detail for the case of Carina by Godwin and Lynden–Bell '87 [82]. They collect the observations of radial velocities for this galaxy from three independent sources. Although these sources show good agreement for the velocity dispersions of $6 \pm 3, 6 \pm 2$ and 8 ± 4 km/s, there is no correlation among the stars as to which move more quickly or slowly than the mean. Intercomparing the respective data, they conclude that the errors in the velocity dispersion had previously been underestimated by a factor of 2 or so. Using the method of maximum likelihood, they determine the best estimate $\sigma = 1.1$—3.2 km/s, which is certainly a low enough value to be explicable by a purely stellar content. This is a disturbing conclusion (for those who prefer to believe the dark matter hypothesis). It does not of course prove an absence of dark matter in Carina since the stellar velocities need to be measured in regions further from the centre, where the effects of such a component would become more important. However, it does point more in the direction away from dark matter than towards it, and underlines the problems involved with the errors which arise when statistics are applied to such small numbers, particularly when the errors themselves are of the same order as the dispersion. Intercomparisons of stellar velocities relative to the mean need to be performed for the other dwarf spheroidals — particularly Draco and Ursa Minor, the two galaxies which provide the strongest evidence for dark matter.

The hope that measuring the internal velocity dispersions of the local dwarf spheroidals would provide the answer as to whether they contain dark matter has not yet been realised. It is furthermore even debatable whether this method is more reliable than the tidal approach described in §4.2. What is interesting is the observation that these two independent methods give results which are in excellent qualitative agreement with each other. They both predict extremely high mass to light ratios for Draco and Ursa Minor, and it is also encouraging to note that they both put the five measured systems in the same 'weighing' order of mass to light ratio: Ursa Minor, Draco, Carina, Sculptor, Fornax. There are at least three possible explanations for this situation. Firstly, both approaches could be essentially correct, in which case we have convincing evidence for dark matter in the first two of these systems (and by Occam's rasor or a taste for simplicity therefore also in the others). Secondly, perhaps this is just a pure coincidence, improbable though it may seem. Thirdly, there may be some basic fallacy that both approaches have in common, which has systematically managed to produce this effect. We regard the most likely explanation for the third eventuality to be that the tidal field of the Milky Way acts on these galaxies not in the way we would like it to (giving nice tidally limited luminosity profiles), but in an unpleasant way (causing disruption of one or more of the galaxies). The correlation of M/L with distance supports this view, because of course the closer a spheroidal is to the Milky Way, the more it will be in danger of disruption by the tidal field. We at present do not have a decent theory of this type of interaction, so we do not know just how the velocity dispersions of these systems would be affected by the Milky Way — whether the effect would be great enough to explain the high values that have been measured. Since these dwarf spheroidals are loosely bound systems, the suspicion is that even their inner regions will be substantially affected, unlike the more tightly bound globular cluster systems which orbit our Galaxy. We exhibit in table (4.5) the weighing order by M/L for the dwarf spheroidals, according to the two independent methods of calculation (from [6, 69, 120]), and the correlation with distance from the Galactic centre which could well be the explanation.

Galaxy	*d(kpc)*	$(M/L_V)^{(1)}$	$(M/L_V)^{(2)}$ *global*	$(M/L_V)^{(2)}$ *central*
Ursa Minor	65	126	100	175
Draco	75	13	40	48
Carina	100	7.2 (corrected [6])	8	7.8
Sculptor	80	6.8	5	5.5
Fornax	140	1	2	1.4

Table 4.5: M/L's in weighing order for dwarf spheroidals

4.4 SUMMARY

Before giving a final evaluation of the argument, we will initially assume that dwarf spheroidals *do* contain dark matter. We are interested in the consequences of such a situation, particularly with regard to the constraints on the mass of the neutrino, if such a particle is supposed to constitute that dark matter. The strongest constraint will be obtained for the galaxy with smallest core radius and velocity dispersion, namely *Draco*, for which we use the values $\sigma = 9km/s$ and $r_c = 0.15kpc$, from Aaronson '87 [6].

Applying equation (2.11) in its mildest form, namely with $n = 3, m_\nu = m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau}, g_\nu = g_{\nu_e} = g_{\nu_\mu} = g_{\nu_\tau} \doteq 1$ (Majorana neutrinos), we obtain:

$$m_\nu = (120eV)(100/9)^{1/4}(1/0.15)^{1/2}3^{-1/4} = 430eV \qquad (4.13)$$

The limit will be stronger if only one kind of neutrino exists, by a factor $3^{1/4}$, giving $m_\nu > 566eV$. It will however be weakened by allowing for the very likely possibility that $r_c(d.m.) > r_c(vis)$, and the related uncertainty in the assumption $\sigma(d.m.) = \sigma(vis)$. Secondly, the neutrino halo is not necessarily an isothermal one, as assumed in the derivation of equation (2.6). As we have noted before, if the violent relaxation is not complete (and there is no way of knowing whether or not this is so), we should be very careful about the distibution we assume. We have also seen that the more general constraints derived by Madsen and Epstein [136, 137] can lower $m_{\nu,min}$ by a factor of two or three, so that a non–isothermal configuration for three types of neutrino can in principle provide a limit as low as around $m_\nu > 140eV$. If for example we further have $r_c(d.m.) \simeq 2r_c(vis)$, then we are reduced still lower, to $m_\nu \geq 100eV$. The upper limit from cosmology in this situation is, from equation (2.12):

$$m_\nu < \begin{cases} (25/3)eV & , \text{H=50} \\ (100/3)eV & , \text{H=100} \end{cases} \tag{4.14}$$

so that $m_\nu < 33eV$ is the best we can do towards compatibility with $m_\nu > 100eV$. At the other extreme, if only *one* neutrino species is present, the latter will become $m_\nu > 133eV$ while (4.14) becomes:

$$m_\nu < \begin{cases} 25eV & , \text{H=50} \\ 100eV & , \text{H=100} \end{cases} \tag{4.15}$$

Hence even for the extreme case $H = 100km/s/Mpc$, we obtain a contradiction between the upper and lower limits, although clearly the effects of using the constraints derived by Madsen and Epstein should be considered more quantitatively. This is not an easy task observationally, to determine the quantities β, r_* and $M(r_*)$. If the halo distribution is close to an isothermal in the inner regions [179], the discrepancy between the upper and lower limits is far stronger, even if we retain $r_c(d.m.) \simeq 2r_c(vis)$:

$$m_\nu \geq 305eV, \qquad m_\nu < H^2/300eV \qquad ; n = 3, \ all\ m_{\nu_i}\ equal \tag{4.16}$$

$$m_\nu \geq 400eV, \qquad m_\nu < H^2/100eV \qquad ; n = 1 \tag{4.17}$$

We thus conclude, to a reasonably high level of confidence, that if dwarf spheroidal galaxies contain dark matter, consistent with the results taken at face value for two of these systems, then this dark matter can *not* be made of light neutrinos.

Continuing with our assumption that the data and their interpretation for Ursa Minor and Draco are correct, we use the principle of Occam's rasor to infer that *all* non–pathological cases of dwarf spheroidal galaxies contain dark matter (probably as halos). Therefore light neutrinos are not the dark matter in dwarf spheroidals. The next step in the argument is usually to say that therefore neutrinos are not the dark matter on larger scales, and that in particular they therefore do not make up ordinary–sized galactic halos, the dark matter in clusters, and the cosmological dark matter which closes the Universe (if the Universe is closed, that is). This step is though by *no* means necessary. It is motivated in part by a taste for economy of theory, that the dark matter is the same on all scales, supported perhaps by the observation that the ratio of dark to visible mass seems approximately scale independent [68]. The information is not yet good enough to test this hypothesis on the smallest galactic scale for dwarf spheroidals. Moreover, it seems increasingly more likely that dwarf spheroidals are distinct objects from their larger namesakes, as discussed in §4.1 — if this is true, there is no reason to expect the constancy of the above ratio to carry over to these systems, and, equally, no reason to expect the dark matter to be made of the same particles or objects in dwarf galaxies as on larger scales.

We discuss briefly, in this light, the possibility that the dark matter in dwarf spheroidals is baryonic. The two main candidates are dull, low mass stars and heavy stellar remnants. This question has been discussed by numerous authors, in particular within the context of dwarf galaxies by Armandroff and Da Costa '86 [13], and by Aaronson and Olszewski '87 [7]. The former regard the high values of M/L in Draco and Ursa Minor as evidence that it is unlikely that the IMF be squeezed to the extremes required for baryonic dark matter in these systems. They instead consider the case of Sculptor, for which they calculated a central $M/L_V = 6$. Suppose that star formation occurred in a single burst in *globular clusters*, and that accordingly the initial mass function can be represented by the simple power–law expression:

$$dN \propto m^{-(1+x)} dm \tag{4.18}$$

with $0.2M_\odot \leq m \leq 50M_\odot$. If $m > 1M_\odot$ implies a $0.7M_\odot$ remnant, and no stars escape, then $x \simeq 2$ gives a global $M/L \simeq 2.8$, the mean value for the globular clusters in a sample they considered. Ignoring any differences that may occur when carried over to Sculptor, this gives for a M/L of 6 values $x \simeq 2.8$ and $x \simeq -0.5$ for low and high mass biasing respectively. Neither of these values is implausible [128]. Melnick and Terlevich '86 [145] constructed models for which the IMF is a function of chemical composition, in the sense that more metal–poor systems would yield smaller values of x. This supports the idea that stellar remnants form the dark matter in dwarf spheroidals. Using the same method for Ursa Minor and Draco, Aaronson and Olszewski obtained $x \simeq -1.3$ for remnants and $x \simeq 3.7$ for low mass stars if one allows $m \geq 0.1M_\odot$ instead of $0.2M_\odot$. They went on to argue that in the case of high mass biasing, one would expect considerable amounts of self–enrichment to occur, inconsistent with the extent to which dwarf spheroidals are metal–poor. M$_c$Clure et.al. '86 [142] find for globular clusters a correlation between abundance and mass function parameter: the most metal–rich have $x \sim -0.5$ and the most metal–poor $x \sim 2.5$. Since dwarf spheroidals are metal–poor, this supports a low mass biasing, provided a comparison between these two types of system can be reliably made. To sum up this situation, arguments exist for both alternatives — perhaps that based on abundance is the more convincing, indicating that the metal–poor spheroidals might be more likely to have their dark matter in low mass stars than in stellar remnants.

We now consider another consequence of taking the M/L calculations for dwarf spheroidals at face value. Firstly, the Tully–Fisher relation '77 [198] seems to be satisfied: σ increases with decreasing M_B [120]. This perhaps provides encouragement to believe that the results are correct, at least qualitatively. As for the relation derived by Faber and Jackson '76 [66], that $M/L_B \propto L_B^{1/2}$ for elliptical galaxies, we have a different situation. These authors realised that this relation would fail to hold exactly for fainter ellipticals. What we notice for the dwarf spheroidals however is a far greater discrepancy — the trend is actually reversed, namely M/L *decreases* with increasing luminosity for the spheroidals. This is good evidence that dwarf spheroidals are *not* a morphological continuation of larger elliptical galaxies, and as such lends weight to the idea that they are evolved from dwarf spirals.

Having convinced ourselves of the profound consequences of the existence of dark matter in dwarf spheroidal galaxies, we sum up the various pieces of evidence and arguments both in favour of and against such a hypothesis, which we conclude is a question which still has to be answered firmly. The following list is intended more as a reminder than some kind of balance scale, bearing in mind that the quality and not the quantity of argument is more important.

FOR:

1. The tidal calculations of Faber and Lin give mass to light ratios about one order of magnitude greater than for globular clusters;
2. the velocity dispersion measurements, pioneered by Aaronson, predict high mass to light ratios for dwarf spheroidals;

3. the strikingly good agreement between these two independent methods (in particular, they both put the spheroidals in the same ranking order of mass to light ratio);

4. the reproduction of the Tully–Fisher relation by the observations supports a belief in the observations;

5. Monte Carlo simulations suggest that binaries do not account for the velocity dispersions observed;

6. the strong evidence for dark halos in dwarf spirals, coupled with increasingly persuasive indications that spheroidals are evolved versions of these galaxies, implies dark matter in the spheroidals;

7. extended halos are *necessary* for dwarf galaxies in the theory of Dekel and Silk, which neatly accounts for various correlations that are observed in these galaxies.

AGAINST:

1. The tidal analysis might be flawed due to the non–attainment of tidal relaxation (local spheroidals not having completed enough orbits around the Milky Way);

2. the dwarf spheroidals for which we measure the highest M/L ratios from equation (4.2) are also those most likely to be tidally disrupting;

3. the velocity dispersion analysis might be flawed for the same reason;

4. the small number statistics involved may be too serious a difficulty, and error estimates in the past have been too conservative;

5. measured velocity dispersions may be manifestations of binary or atmospheric motions instead of self–gravity;

6. the King models used may not be applicable to dwarf spheroidals, which are in many ways different from globular clusters and might for example be described better by exponential rather than truncated profiles.

There is of course at least one scenario where dwarf spheroidals do contain dark matter, but this makes absolutely no difference to the viability of light neutrinos as their constituents. This is the model of Cowsik and Ghosh '87 [50], which has a critically closed Universe with $\sim 10eV$ neutrinos which settle down on the scale of a few Mpc and *not* of individual galactic halos. The lower limit on the neutrino mass derived from the Tremaine–Gunn constraint in this situation is around $m_\nu > 3eV$, in no contradiction with cosmological upper limits in a high Ω Universe.

In view of the large number of uncertainties in virtually all aspects of the argument, it seems appropriate to finish with a brief enumeration of some of the more obvious directions in which future work could be fruitfully aimed.

1. A question of importance to the origin of, and degree of homogeneity among, the local dwarf spheroidals is why Fornax is, subject to the observations, the only such system which contains globular clusters.

2. Of crucial importance to the credibility of velocity dispersion measurements would be a quantitative investigation into the effects of the Galactic tidal field on the internal dynamics of the dwarf spheroidal satellites. In particular, is the effect strong enough to produce the observed $\sigma \sim 10km/s$ as opposed to the $\sigma \sim 0.5km/s$ that would be expected if they were isolated systems without dark matter?

3. Attempts to model dwarf spheroidals self–consistently with dark matter components would ease the present uncertainties on the values of the core radius and velocity dispersions of the dark component, so important for the accurate application of the phase space constraints. The feeling is that the effects of $r_c(d.m.) > r_c(vis)$ will probably not be great enough to avoid a smaller upper limit than lower limit on m_ν, but this is completely untested. If the luminosity profiles are better fit by exponentials than tidally truncated King profiles, then the basic philosophy of King modelling for such systems may have to be abandoned in favour of some other approach.

4. If the data on central densities in Draco and Ursa Minor are correct, and if dwarf spheroidals are evolved versions of dwarf spirals,

then as a consequence of such a transition the final central density will have to be around a factor of 10 greater than its initial value. Could ram pressure stripping of gas or some other process account for such an effect, and if so, would this mean that halos in spheroidals are of a different nature to those in spirals, for which we have $\rho_0(d.m.) \leq \rho_0(vis)$?

5. How does the function $M(d.m.)/M(vis)$ behave for the smallest galaxies?

6. Can the Cowsik–Ghosh hypothesis be tested by, for example, gravitational lensing observations and calculations?

7. The theory of tidal limitation of satellites orbiting around a host galaxy is still in a rudimentary state. The most recent study, by Allen and Richstone '88 [10] shows that for circular orbits the tidal equation for r_t derived by King '62 [117] and a generalisation by Innanen et.al. '83 [107] are fairly well in agreement with numerical simulations, although they fail badly for elongated orbits. Allen and Richstone derive a first order theory for such orbits in much better agreement with the simulations, but improvements could still be made.

8. The effects of (two–body–encounter–induced) evaporation, dynamical friction, disk shocking and bulge shocking on globular clusters in orbits around our Galaxy (as regards the respective rates of destruction) are currently being investigated by Aguilar et.al. '88 [8]. A similar treatment would be interesting for the dwarf spheroidals, particularly with regard to the possibility that the seven dwarfs we observe are the only survivors from an initially larger population. Hence they would not be on orbits which are particularly elongated, since otherwise they would have already been destroyed by bulge shocking, suspected by Aguilar et.al. to have been the most effective destruction mechanism in the past. It has already been noted on p.52 that if the dwarf spheroidals are on low eccentricity orbits then the use of the tidal equation to determine their masses could be reliable.

Appendix A

THE ISOTHERMAL SPHERE

Consider the function f defined by:

$$f(\epsilon) = \frac{\rho_1}{(2\pi\sigma^2)^{3/2}} \exp(\epsilon/\sigma^2) = \frac{\rho_1}{(2\pi\sigma^2)^{3/2}} \exp\left(\frac{\psi - \frac{v^2}{2}}{\sigma^2}\right) \tag{A.1}$$

identified with the distribution function of some stellar dynamical system.

By integrating over velocity space we obtain:

$$\rho = \rho_1 \exp(\psi/\sigma^2) \tag{A.2}$$

Poisson's equation is:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = -4\pi G\rho \tag{A.3}$$

hence by (A2):

$$\frac{d}{dr}\left(r^2\frac{d\ln\rho}{dr}\right) = -\frac{4\pi G r^2\rho}{\sigma^2} \tag{A.4}$$

An *isothermal gas* has equation of hydrostatic support:

$$\frac{dp}{dr} = \frac{k_B T}{m}\frac{d\rho}{dr} = -\rho\frac{GM(r)}{r^2} \tag{A.5}$$

where k_B is Boltzmann's constant, p, T are the gas pressure and temperature, m is the mass per particle, and $M(r)$ is the total mass within radius r ([27], p.226). This yields, on differentiation:

$$\frac{d}{dr}\left(r^2\frac{d\ln\rho}{dr}\right) = -\frac{Gm}{k_B T}4\pi r^2\rho \tag{A.6}$$

Comparison of (A4) with (A6) suggests the identification:

$$\sigma^2 = k_B T/m \tag{A.7}$$

subject to which we can say that the distribution function f given by (A1) describes a stellar dynamical system which has the same structure as a self-gravitating isothermal gas sphere. Hence the so-called isothermal model of stellar dynamics.

Integrating (A1) over configuration space, we obtain for the distribution of velocities:

$$F(v) = \lambda \exp(-v^2/2\sigma^2) \tag{A.8}$$

for some λ subject to normalisation. Thus the isothermal sphere has a Maxwellian velocity distribution. It is interesting to note that this is for a *collisionless* system, while kinetic theory gives that a Maxwellian distribution of velocities also obtains for particles which bounce elastically off each other. We thus note that for the isothermal sphere it is of no importance whether the system is collisionless.

It is trivial to check that:

$$\overline{v^2} = 3\sigma^2 \tag{A.9}$$

so the mean square speed of the stars at a point is independent of position. Moreover, the distribution function depends only on ϵ, and so is isotropic. Therefore the one-dimensional velocity dispersion is equal to σ.

Setting a power law density profile in (A4) yields the simple solution:

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2} \tag{A.10}$$

known as the *singular isothermal sphere*. This solution suffers from an infinite central density. To avoid this problem, one uses dimensionless variables $\tilde{\rho}$ and $\tilde{r}$, defined by:

$$\tilde{\rho} := \rho/\rho_0, \qquad \tilde{r} := r/r_0 \tag{A.11}$$

where ρ_0 is the central density and r_0 the 'King radius' (see Appendix C):

$$r_0 = \left(\frac{9\sigma^2}{4\pi G\rho_0}\right)^{1/2} \tag{A.12}$$

Using equations (A2) and (A4) we obtain:

$$\frac{d}{d\tilde{r}}\left(\tilde{r}^2\frac{d(\psi/\sigma^2)}{d\tilde{r}}\right) = -9\tilde{r}^2 \exp\left(\frac{\psi(r)-\psi(0)}{\sigma^2}\right) \tag{A.13}$$

This equation can be solved numerically ([27], p.229), and approaches the singular isothermal sphere by around $r \sim 15r_0$, which can thus be used accurately in the outer regions. For spherical symmetry, the circular speed $v_c(r)$ is given by:

$$v_c^2 = \frac{GM(r)}{r} \tag{A.14}$$

which yields $v_c = \sigma\ 2^{1/2}$ at large r, a constant value in accordance with observed flat rotation curves.

Appendix B

LIMITING RADIUS AND THE TIDAL EQUATION

In 1962, King [117] obtained the following empirical law for the surface density f, derived from star counts in the outer regions of globular clusters:

$$f = f_1 \left(\frac{1}{r} - \frac{1}{r_t} \right)^2 \tag{B.1}$$

where the surface density drops to zero at $r = r_t$, and f_1 is a constant. This was the first definition of tidal radius r_t. King identified this abrupt cutoff, as the name suggests, with the tidal forces of the Galaxy, and performed the following simple calculation for a globular cluster in an elliptical orbit around the Galactic centre. An important point is that the tidal or *limiting* radius is determined at *perigalacticon* in this calculation, with justification that this is the point where the cluster is cut back most severely, the tidal field being at its strongest there. King argued that internal relaxation is too slow to increase the size of the cluster between successive perigalacticon passages. For the dwarf spheroidals which are of interest to us, internal relaxation will be slower, although these galaxies will also spend more time in completing any one orbit than the globular clusters since they are further out. It is unclear how accurate the taking of perigalactic distance actually is. The result of the calculation is that which was used by Faber and Lin '83 [69] to calculate the masses of the local dwarf spheroidals.

Let R, θ be polar coordinates of the cluster centre as measured from the centre of the Galaxy. Consider the line connecting the centre of the cluster to the centre of the Galaxy. The tidal limit is then defined as the distance from the point P to the cluster centre, where at P the acceleration on a star is zero with respect to the cluster centre, at the moment of perigalactic passage.

If ω is the angular velocity of the cluster with respect to the Galactic centre and $\Psi(R)$ is the Galactic potential, then:

$$\frac{d^2R}{dt^2} = R\omega^2 - \frac{d\Psi}{dR} \tag{B.2}$$

is the acceleration of the cluster with respect to the Galactic centre. Let R_s be the distance of some star measured from the same point. Then the acceleration of this star with respect to the Galactic centre will be:

$$\frac{d^2R_s}{dt^2} = R_s\omega^2 - \left(\frac{d\Psi}{dR}\right)_{R_s} - \frac{GM_c(R_s - R)}{|R_s - R|^3} \tag{B.3}$$

where M_c is the mass of the cluster, assumed a point mass. Hence the required relative acceleration is the difference between these two values:

$$\begin{aligned} \frac{d^2}{dt^2}(R_s - R) &= (R_s - R)\omega^2 - \left(\frac{d\Psi}{dR}\right)_{R_s} + \left(\frac{d\Psi}{dR}\right) - \frac{GM_c(R_s - R)}{|R_s - R|^3} \\ &\simeq \left(\omega^2 - \frac{d^2\Psi}{dR^2} - \frac{GM_c}{|R_s - R|^3}\right)(R_s - R) \end{aligned} \tag{B.4}$$

to first order. This will vanish when $|R_s - R| = r_{lim}$, which yields:

$$r_{lim}^3 = \frac{GM_c}{\omega^2 - d^2\Psi/dR^2} \tag{B.5}$$

For the sake of simplicity, the Galaxy is now supposed to be adequately represented by a point mass M_G, which yields:

$$\frac{d^2\Psi}{dR^2} = -\frac{2GM_G}{R^3} \tag{B.6}$$

and the angular velocity in the elliptical orbit is given by:

$$\omega^2 = \frac{GM_g}{R_p^4}a(1 - e^2) \tag{B.7}$$

where perigalacticon is given by:

$$R_p = a(1 - e) \tag{B.8}$$

Hence (B5) with this further loss of generality becomes:

$$r_{lim} = R_p \left(\frac{M_c}{M_G(3 + e)}\right)^{1/3} \tag{B.9}$$

which is the familiar equation used in chapter 4. The assumption of a point mass Galaxy underestimates M_c for fixed M_G and hence will not be responsible for any high values of M_c that may be found.

King found good agreement between r_t and r_{lim} for a sample of globular clusters around our Galaxy.

Appendix C

ANALYTIC KING MODELS

The analytic King model of 1966 [118] was designed specifically for globular clusters. At the centre of such a system, the relaxation time is a small fraction of its age [159]. Hence it is suspected that stellar encounters are important. An old version of the collisionless Boltzmann equation, corrected for collisions, is the Fokker-Planck equation [27], for which the encounters are predominantly weak. Assuming for simplicity an *unmixed* stellar system (or equivalently one in which all stars have the same mass) which is isotropic everywhere, one can begin by considering the function:

$$f(0,v) = k(\exp(-j^2v^2) - \exp(-j^2v_e^2)) \qquad (C.1)$$

at the cluster centre, where v_e is the escape velocity.

The potential $\Psi(r) = E - \frac{v^2}{2}$ is taken to be zero at the surface of the cluster. Hence a star with zero energy will just be able to reach the surface. The cut-off is incorporated into (C1), since for $v < v_e$, $f > 0$ and for $v > v_e$, $f < 0$ and so must be taken to be zero. Another approximation is the neglect of the fact that the shape of the cluster will be distorted by the tidal force, which pulls more strongly on the near side than the far side.

Since $v_e^2 = -2\Psi = -2\Psi(0) := -2\Psi_0$ at the centre, (C1) can be written in terms of energy:

$$f(0,v) = k\exp(2j^2\Psi_0)[\exp(-2j^2E) - 1] \qquad (C.2)$$

By Jeans' theorem, the distribution function is the same function of E at all points, so that at any other point we have:

$$f(\underline{r},v) = k\exp\left[-2j^2(\Psi - \Psi_0)\right]\left[\exp(-j^2v^2) - \exp(-j^2v_e^2)\right] \qquad (C.3)$$

We note that the velocity distribution has exactly the same form as (C1). Within an approximation made by Michie '63 [147], this function solves the steady-state Fokker-Planck equation, so stellar encounters are taken into account everywhere, not just in the central regions.

Having obtained an expression for the distribution function in this way from the velocity distribution, the next step is to derive the density profile. This is done as usual by integrating over velocity space which is isotropic in this case:

$$\rho = \int_0^{v_e} f(\underline{r}, v) 4\pi v^2 dv \tag{C.4}$$

For convenience the substitution $y = -2j^2\Psi$, $x = j^2v^2$ can be made to yield:

$$\rho = 2\pi k j^{-3} \exp(y - y_0) \int_0^y (\exp(-x) - \exp(-y)) x^{1/2} dx \tag{C.5}$$

$$\Rightarrow \rho = \frac{4}{3}\pi k j^{-3} \exp(y - y_0) \int_0^y \exp(-x) x^{3/2} dx \tag{C.6}$$

obtained by integrating the first term in the integrand by parts. The density is thus proportional to the quantity:

$$D(y) = \exp(y) \int_0^y \exp(-x) x^{3/2} dx \tag{C.7}$$

and so is given as a function of Ψ. To obtain ρ as a function of r, one must solve the Poisson equation:

$$\nabla^2\Psi = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Psi}{dr}\right) = \frac{d^2\Psi}{dr^2} + \frac{2}{r}\frac{d\Psi}{dr} = 4\pi G\rho \tag{C.8}$$

In terms of y and a dimensionless radius $R := r/r_c$ we have:

$$\frac{d^2y}{dR^2} + \frac{2}{R}\frac{dy}{dR} = -8\pi G j^2 r_c^2 \rho \tag{C.9}$$

The scale factor r_c will be very close to the value where the surface brightness drops to one half of its central value (this is the empirical definition of core radius from King '62 [117]), provided that the left hand side of (C9) is -9 at the centre. This is found by power series substitution. Hence:

$$8\pi G j^2 r_c^2 \rho_0 = 9 \tag{C.10}$$

Identifying (C2) with a so-called 'lowered isothermal sphere', which is in fact what a King model is [27], we note:

$$2j^2 = 1/\sigma^2 \tag{C.11}$$

Therefore:

$$r_c^2 = \frac{9\sigma^2}{4\pi G\rho_0} \tag{C.12}$$

is the analytic definition of core radius (some prefer to call it the King radius: see [27], p.233), where σ is the one-dimensional velocity dispersion of the isothermal sphere, constant by definition.

King models actually exist in families, because in solving the Poisson equation it is necessary to *choose* a value for the central potential, amounting to a choice of $y(r = 0)$ in equation (C9). The usual central boundary condition, $d\Psi/dr = 0$ is taken (a particle at the centre of a spherical system feels no force). Integrating Poisson's equation outwards, we find that the surface on which Ψ, y and therefore ρ vanish (this corresponds to the tidal radius r_t) increases with the chosen value of Ψ_0. The central escape velocity increases directly with $y(0)$: it is easy to see that as $y(0) \rightarrow \infty$ the family of King models goes over to the isothermal sphere. King models form a one-parameter family, characterised by $y(0)$, or equally by the concentration parameter:

$$c := \log_{10}(r_t/r_c) \tag{C.13}$$

We can now justify equation (4.4) quite simply, the equation commonly used to calculate the total mass of a cluster:

$$M = 167 r_c \mu < v_r^2 >_0 \tag{C.14}$$

The cluster is a finite sphere of radius r_t and hence:

$$M = \int_0^{r_t} \rho \; 4\pi r^2 dr = r_c^3 \int_0^{R_t} \rho \; 4\pi R^2 dR \tag{C.15}$$

where $R := r/r_c$ and the second integral is ρ_0 times μ, which defines μ. Therefore:

$$M = r_c^3 \; \rho_0 \; \mu \tag{C.16}$$

But by (C12) we have that:

$$\rho_0 \propto \sigma^2/r_c^2 \tag{C.17}$$

and thus:

$$M \propto r_c \; \mu \; \sigma^2 \tag{C.18}$$

The numerical factor of 167 comes from an adjustment to the units of pc, km/s, and solar masses [103].

As we have mentioned before, in a King model σ is a related parameter, but *not* the velocity dispersion. The latter quantity decreases away from the centre due to the decrease in escape velocity. This means that calculations of velocity dispersions from radial velocities of stars that have been observed away from the centre have to be corrected *upward* to obtain the σ of equation (C18). This correction will always, in the lack of extensive data, be a highly uncertain one, and it turns out for reasons discussed in §4.3 to be far more convenient and accurate to calculate the *central* value of M/L. This process is known as *core fitting*, and equation (4.12) gives the central M/L in terms of central one–dimensional velocity dispersion, central surface brightness and 'core radius'. Richstone and Tremaine '86 [170] have pointed out that on dimensional grounds alone one can always write the central mass to light ratio in the form:

$$\frac{M}{L} = \eta \frac{9\sigma^2}{2\pi G I(0) R_{hb}} \qquad \text{(C.19)}$$

where R_{hb} is the half-brightness radius, $I(R_{hb}) = 0.5\ I(0)$, and η is some constant. The interesting point is that η turns out to be very close to unity for a wide variety of models, although it seemingly never equals one exactly. For the isothermal sphere we have $\eta = 1.013$; for King models in the range $3.89 \leq r_t/r_c < 353.2$, we have $0.962 \leq \eta < 1.009$; for polytropes of finite mass, η varies from 0.903 to 0.971 as the polytropic index n ranges from 0.5 to 5 ($n \to \infty$ corresponds to the isothermal sphere); and η is equally close to unity for a number of other models [170]. It appears then that η is very nearly unity in any isotropic, spherical system with constant mass to light ratio and a well-defined, flat, central core. Richstone and Tremaine define *King's method* to consist of the application of (C19), with $\eta = 1$ exactly. There is confusion regarding the characteristic or 'core' or 'King' radius which may be used, different authors having used different definitions but not necessarily with a consistent nomenclature. For instance, King '62 [117] himself first gave an empirical definition of core radius r_c which amounted to:

$$I(R) = k_1 \left(\frac{1}{[1 + (R/r_c)^2]^{1/2}} - k_2 \right)^2 \qquad \text{(C.20)}$$

where k_1 and k_2 are constants. This core radius is close, but not equal to, the half-brightness radius:

$$0.919 < R_{hb}/r_c < 1 \ for \ r_t/r_c > 10 \qquad \text{(C.21)}$$

The second definition King gave is (C12), based on his analytic models. In this case too we have an approximate half-brightness radius:

$$0.883 < R_{hb}/r_c < 1.004 \; for \; r_t/r_c > 10.7 \tag{C.22}$$

For the isothermal sphere, $R_{hb} = 1.004 \; r_c$. In a related model of Rood et.al. '72 [173], the core radius is the half-brightness radius *exactly*, since the surface brightness is described by:

$$I(R) = \frac{I_0}{1 + (R/r_c)^2} \tag{C.23}$$

This model, incidentally, has $\eta = 0.999$

A heuristic derivation of $\eta = 1$ can be made as follows [170]. Make the *fallacious* assumption that the velocity dispersion is independent of radius and call it σ. The density profile in the Rood et.al. model is:

$$\rho(r) = \frac{\rho_0}{[1 + (r/r_c)^2]^{3/2}} \tag{C.24}$$

The Taylor expansion near the origin is:

$$\rho(r) = \rho_0 \left(1 - \frac{3r^2}{2r_c^2} + O(r^4)\right) \tag{C.25}$$

The equation of hydrostatic equilibrium for isotropy is:

$$\frac{d}{dr}(\rho\sigma^2) = -\frac{GM(r)}{r^2}\rho \tag{C.26}$$

Substituting (C25) into (C26) yields:

$$-3\rho_0 r/r_c^2 = -4\pi G\rho_0^2 r/3\sigma^2 \Rightarrow 9\sigma^2 = 4\pi G r_c^2 \rho_0 \tag{C.27}$$

Now using:

$$\frac{M}{L} = \frac{2}{I(0)}\, \rho_0 R_{hb} \tag{C.28}$$

which applies exactly for the Rood et.al. model, and putting (C27) into (C28) with $r_c = R_{hb}$ as already noted, we obtain (C19) with $\eta = 1$ exactly. This is in principle flawed of course since the velocity dispersion is not constant. For King models however, when $r_t/r_c \geq 10$, the velocity distribution near the centre is very nearly Gaussian, which means that the dispersion is *nearly* constant in this region, and η is nearly one.

Finally, it is interesting to note that a general expression for η, calculated from the hydrostatic equation, is:

$$\eta = \frac{4\pi}{9} R_{hb} \frac{\left[\int_0^\infty \rho(r) dr\right]^2}{\int_0^\infty M(r)\rho(r) dr / r} \tag{C.29}$$

a quantity which is manifestly different for each density profile, but which conspires to lie within a small neighbourhood of unity for a variety of models.

Appendix D

KING MODELS EXTENDED

King models are probably the most simple way of treating galaxies which are tidally truncated at their boundary — they are isotropic, and are single-mass models (i.e. all masses are treated as equal). The relaxation times of globular clusters are somewhat smaller than their ages. This suggests that mass segregation has occurred in these systems. The usual way of pinning down the structural parameters r_c, r_t and $I(0)$ is by fitting the observed $I(R)$ to that calculated from a King model. However, it is mainly the massive stars that are measured as contributors to $I(R)$, so if there is mass segregation this will not accurately represent the true profile. As regards isotropy, it is noted that the outermost stars in globular clusters are measured to be in almost *radial* orbits. Therefore King models are a first approximation only to real globular clusters.

In 1976, Da Costa and Freeman [51] made a multi-mass model for M3. The distribution function was essentially unchanged, although the dispersion parameter σ now depended on mass m. The crucial point is that *thermal equilibrium* was assumed, and represented as:

$$m\sigma^2(m) = constant \tag{D.1}$$

Thermal equilibrium leads directly to mass segregation, but it is not known and is difficult to check observationally, whether such a state has been fully reached, or if we only have *partial* equilibrium. There are reasons for which full thermal equilibrium may not be attained. Inagaki '84 [106] has found for instance that for equipartition at the cluster centre we need $m_{max}/m_{min} \leq 2.8$, whereas in real globular clusters the individual stellar masses will range from about $0.1M_\odot$ to $1M_\odot$. It therefore seems unlikely that full thermal equilibrium is reached [76]. But it does appear that at least some globular clusters have evolved some way towards equipartition: Da Costa '77 [52] (cited by Freeman [76]) has shown that in 47 Tuc the low mass stars ($\sim 0.45 M_\odot$) are less centrally concentrated than the more massive ones ($\sim 0.87 M_\odot$). One of his models, which corresponds to *partial* thermal equilibrium:

$$m^{1/2}\sigma^2 = constant \tag{D.2}$$

gave a M/L of about 4 for this globular cluster. The mass stratification was a continuous one, with a mass function which allowed for moderate proportions of both stellar remnants and white dwarfs, which would be the 'dark matter' responsible for such a M/L, high for globular cluster standards.

Illingworth and King '77 [105] used full thermal equilibrium models for M15, 47 Tuc, and NGC 6388, requiring as much as 30% to 60% by mass of white dwarfs to fit the σ and $I(r)$ data.

Anisotropy was first included in these models by Gunn and Griffin '79 [86]. A weak amount of anisotropy was sufficient to fit the data for M3 without invoking surprisingly large numbers of remnants. But apart from the neglect of both anisotropy and the equipartition process, King models perhaps have another serious shortcoming. This problem is associated with the nature of the cutoff. The crucial point is as follows: King models, and all related treatments, have distribution functions with truncation in *energy*. Consequently, only stars with negative energy, with respect to the boundary potential, are included in the system. Kashlinsky '88 [116] has demonstrated three unphysical consequences of such an effect: stars can be both bound to the system and at the same time excluded from it; no circular orbits are allowed in substantial regions; and the *spatial* boundary is poorly defined. As an alternative to such a truncation process, he instead presents a distribution function for a spherically symmetric anisotropic system with a truncation in *radius*. This function allows for *all* orbits bound to the galaxy. By normalising the potential to zero at the boundary $r = R$, the maximal energy of any *bound* star with angular momentum J is:

$$\epsilon \leq \epsilon_{max} = \frac{J^2}{2R^2} \tag{D.3}$$

With this value as the cutoff (it is positive, hence stars which are excluded by the King-model cutoff $\epsilon < 0$ can actually be bound to the system), Kashlinsky obtains the distribution function:

$$f = \frac{1}{(2\pi\sigma_0^2)^{3/2}}\left[\exp(-\epsilon/\sigma_0^2) - \exp(-J^2/2R^2\sigma_0^2)\right] \tag{D.4}$$

Two important points are to be noted. Firstly, this actually coincides with the King-model distribution function for purely radial orbits, where $J = 0$. Secondly, the distribution function is *automatically* anisotropic, since $f = f(\epsilon, J^2)$.

Although interesting as a step forward in principle for finite system modelling, further details are not really relevant for our purposes, for the following reasons. Firstly, tedious algebra shows that for this distribution function the velocity distribution is nearly isotropic in the central regions, as are King models everywhere (since only stars with low J can reach the central regions, equation (D4) makes this evident). Secondly, numerical integrations show that the f given by (D4) and a King model have similar total extents and total mass to light ratios. These are the important properties which were needed for the previous investigations. We note finally that for this new model the velocity distribution is tangential in the outer regions, the projected velocity dispersion is flatter than for a King model, as is the projected density; and that the new model is easily generalisable to axisymmetric flattening and rotation, producing rotation curves which are indeed flat. However, the observation of predominantly *radial* orbits in globular clusters, and other problems concerning the cutoff surface and absence of a third integral, indicate that further modification is required.

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