ISAS - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

Baryon and Lepton number violation in Gauge Unified Theories

Thesis submitted for the degree of
"Magister Philosophiæ"

CANDIDATE
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SUPERVISOR
Prof. Antonio Masiero

October 1990

TRIESTE

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A Consuelo

Es un gusto aprender en los autores
que tratan de las ciencias naturales,
por qué de las semillas nacen flores,
cómo hacen para andar los animales,
para qué fin hay rayos y temblores,
o de qué se componen los metales.
Cosas que cada día estoy leyendo,
que siempre admiro y que jamás entiendo.

José Batres Montúfar, **El Relox**, (1836)

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1. Introduction and Motivations

1.1 Motivations

The conservation of baryon and lepton numbers (B/L-number) is founded on experimental grounds, but not on theoretical ones. In fact in the Standard Model (SM, based on the group $G_{sm} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$) both of them are only accidental global symmetries. On the other hand the electric charge conservation based on experimental grounds, can be founded, theoretically, on the local $U(1)_{em}$ gauge symmetry which remains after the spontaneous symmetry breaking (SSB) of the electroweak group $G_{ew} = SU(2)_L \otimes U(1)_Y$. From the Noether Theorem we know that every symmetry in the physical laws has associated a conserved quantity. Conversely if we find a conserved quantity we search for the symmetry associated with it. Within the Standard Model, the B/L-number conservation can be related only with a global symmetry (*Not local*). But according to the Gauge Dogma *"Only the local gauge symmetries are exact in nature"*; so baryon and lepton number conservation which in the SM is not related with any local gauge symmetry are expected to be violated. On the contrary if we associate the baryon number with a gauge symmetry, the associated long range force would generate an apparent difference between the gravitational and the inertial mass [EPF, RKD, BP]. This could be detected in the Eötvos experiment unless its gauge coupling constant is incredibly small [LY, Oku].

There are more reasons, that suggest the non-conservation of baryon number:

1. The baryon-antibaryon asymmetry in the universe [KT].
2. The fact that the non-conservation of the baryon number can be incorporated, in an extension of the Standard Model, without destroying the renormalizability of the theory.
3. In the Standard Model, the B/L number are not strictly conserved due to non-perturbative effects; which by the way, are tremendously small being exponentially suppressed by a factor $e^{-(2\pi/\alpha_w)} \approx 10^{-86}$ [t'Ho].
4. In the interaction of a particle with a (real) Black Hole it should be possible to transform a proton in a positron yielding a violation of B and L [Zel,HPP]. Zeldovich has shown that is possible to obtain this effect with virtual black holes and found a decay time of the order

$\tau_B \approx 10^{45} - 10^{50}$ years [Zel].

Due to these reasons, it is of great interest to search for possible ways to obtain the baryon and lepton number violation, not only within the Standard Model but in GUT's and Supersymmetric Theories.

1.2 Brief Historical Remarks

The most important historical steps, first towards the conservation laws of Baryon and Lepton number and subsequently to the violation of these laws, are:

1929 - Herman Weyl proposed a conservation principle for the electron and the proton; at that time the positron had not yet been discovered [Wey].

1938 - Stückelberg propose the conservation of the "Heavy Charge", based on the observation that no heavy particles (neutrons and protons) transform into light particles (electrons and neutrinos) [Stü].

1949 - The conservation law of nucleon was reformulated by Wigner, who gave also a possible explicit decay scheme for the proton. In fact, he pointed out that without this conservation law, the proton could decay into a positron plus a photon ($p \longrightarrow e^+ + \gamma$) [Wig].

1953 - Marx, Zeldovich, Konopinski and Mahnoud [Mar,Zel1,KMa] proposed the conservation of lepton number, according with the experimental data of that time, (only one kind of neutrino was know) and Zeldovich alone considered the violation of lepton number through the $\beta\beta$ decay without neutrinos [Zel1,KMa].

1954 - Goldhaber, Reines and Cowan proposed that *"We cannot conceive an experiment which would prove the absolute stability of nucleon".* The laws of conservation of nucleon can be used with considerable confidence in discussions of *practically observable* nuclear reactions. Their proposition remains valid until now [Gol,RCG].

1957 - Pontecorvo considered the violation of flavour lepton number through the neutrino oscillations [Pon].

1967 - Kuzmin and Sakharov consider the baryon number violation in order to explain the baryon-antibaryon asymmetry in the universe [Sak, Kuz].

1973-74 - In Grand Unified Theories it is possible to arrive in a natural way to the Baryon and Lepton numbers violation [GG].

Finally the present status of the conservation laws in the Standard Model: There are three independent conserved lepton numbers L_μ, L_ν, L_τ associated with each of the three known generations of leptons. This obviously leads to the conservation of the total lepton number $L = L_\mu + L_\nu + L_\tau$. Furthermore, there is only one instead of three conserved baryon number, due to the mixing of the generations in the Kobayashi-Maskawa mass matrix.

On the other hand the experimental status about baryon number violation is based principally on the IMB, Kamioka and Frejus [Bio, Kos, Ern] experiments that give the lower limits for the ratio of the lifetime of the proton to branching ratio, $\tau/B = 10^{31} - 3 \times 10^{32}$ years for modes with charged leptons and $\tau/B = 10^{31} - 6 \times 10^{31}$ for modes with neutrinos. On the other hand the proposed 30 kT Super-Kamiokande water Cherenkov detector, could reach to a limit of 10^{34} years. It is more difficult to make measurements for leptons than it is for baryons and therefore the lifetime limits for electrons are poorer ($> 2 \times 10^{22}$ years [PD88]) than those for proton.

2. Generalities of B/L Non-conservation

Since the phenomenology of the Standard Model is well known, we will concentrate on possible ways to obtain the baryon and lepton number violation, without violating the angular momentum conservation, the Lorentz invariance , the electromagnetic charge conservation and the color charge conservation.

2.1 The Standard Model

Let's begin with a short review of the Standard Model [SM]. The standard model is based on the symmetry group $G_{sm}{=}SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ with three gauge couplings g_i (where $i = 1,2,3$) corresponding respectively to the three factor groups. This symmetry is broken spontaneously by the Higgs doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix},$$

Which transforms under the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ group as a $\phi \sim (1,2,+\frac{1}{2})$ multiplet.

The gauge bosons of this model are: $A^I_\mu \sim (1,3,0)$ $(I = 1,2,3)$ and $B_\mu \sim (1,1,0)$ corresponding to the electroweak interaction, and $G^A_\mu \sim (8,1,0)$ $(A = 1,...,8)$ corresponding to strong interaction. The SSB occurs due to the non zero value of the vacuum expectation value (VEV) of the scalar Higgs potential ϕ ($<\phi^0>= v/\sqrt{2}$). Obviously the SU(3) group remains unbroken and only the other two groups are broken to $U(1)_{em}$ group generated by $Q = T_{3L} + Y$. After this SSB we obtain the three massive gauge fields $W^\pm_\mu = (A^1_\mu \pm iA^2_\mu)/\sqrt{2}$, $Z_\mu = B_\mu \sin\theta_w - A^3_\mu \cos\theta_w$ and one massless gauge field $A_\mu = B_\mu \cos\theta_w + A^3_\mu \sin\theta_w$, where θ_w is the Weinberg angle.

The fermion constituents of the Standard Model are the following multiplets:

	$\begin{pmatrix} u^{0\alpha}_{aL} \\ d^{0\alpha}_{aL} \end{pmatrix}$	$\begin{pmatrix} \nu^{0}_{aL} \\ e^{0}_{aL} \end{pmatrix}$	$u^{0\alpha}_{aR}$	$d^{0\alpha}_{aR}$	e^{o}_{aR}
$SU(3)_C$	3	1	3	3	1
$SU(2)_L$	2	2	1	1	1
$U(1)_Y$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$	-1

Table 1

The index a ($a = 1, 2, 3$) is the family index, the superscript 0 means that the fermions are the current eigenstates, the index α ($\alpha = 1, 2, 3$) represents the colour degree of freedom, and R (L) represents the Right (Left) component of the fermion.

The baryon and lepton number are defined as:

$L(q) = 0$ and $B(q) = \frac{1}{3}(N_q - N_{\bar{q}})$ for quarks;

$B(B) = L(B) = 0$ for ordinary bosons;

$B(L) = 0, \;\; L_e = N_{e^-} + N_{\nu_e} - N_{e^+} - N_{\bar{\nu}_e}, \quad L_\mu = N_{\mu^-} + N_{\nu_\mu} - N_{\mu^+} - N_{\bar{\nu}_\mu},$

$L_\tau = N_{\tau^-} + N_{\nu_\tau} - N_{\tau^+} - N_{\bar{\nu}_\tau}$ and $L = L_e + L_\mu + L_\tau$ for leptons

After this brief review of the Standard Model we are ready to begin with a general discussion of the baryon and lepton number violation processes at low energies.

2.2 B/L non-conserving processes

After the discussion of the conservation of baryon and lepton numbers within the Standard Model, at least at non-perturbative level, we will now look for the possible operators that can violate these conserved quantities in some extensions of the Standard Model. We will start with a general analysis, with no reference to a particular model, in which the B/L-number violating operator should satisfy the Standard Model gauge and Lorentz invariance. So we need to check:

a) The charge or hypercharge conservation.

b) The colour charge conservation.

c) The invariance under $SU(2)_L$ group transformation.

d) The invariance under Lorentz transformation.

To check the invariance under this last two transformations, we can use the "F-parity" introduced by Weinberg [Wei].

	A	B	T	$A+T$	F
q_L, l_L	$\frac{1}{2}$	0	$\frac{1}{2}$	1	*even*
q_R, l_R	0	$\frac{1}{2}$	0	0	*even*
$\bar{q}_R, \bar{l}_R$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	*odd*
$\bar{q}_L, \bar{l}_L$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	*odd*
$W_\pm, A, \partial$	$\frac{1}{2}$	$\frac{1}{2}$	0, 1	$\frac{1}{2}, \frac{3}{2}$	*odd*
ϕ	0	0	$\frac{1}{2}$	$\frac{1}{2}$	*odd*

Table 2

in Table 2 (A, B) is the irreducible representation $SU(2)_R \otimes SU(2)_L$ of the $SO(3,1)$ Lorentz group and (T) is the irreducible representation of the $SU(2)_L$ group of the SM.

The F-parity is defined as $F = (-1)^{2A+2T}$, then F-parity *even* gives allowed interactions, but F-parity *odd* gives forbidden interactions. On the other hand the electric and colour charges conservation, can be checked easily.

To preserve the $SU(3)_C$ invariance we need at least three quark fields. Due to the Lorentz invariance, we can only have operators of the form:

$$2n_R + 2n_L \qquad n_R, n_L = 0, 1, 2, ... \tag{1}$$

We can have several operators that obey these restrictions. Considering B number violation, we begin with the operator minimum dimension (in power of mass), $d = 6$ for which we can have operators of the form $qqql$ or $qqql^c$. Both lead to electromagnetic and colour charge conserved processes. However using the F-parity we obtain F=*even* for the first process and F=*odd* for the second, which means that the later process is forbidden. By this method we can construct other types of interactions, like those shown in the following table.

$\Delta B = 0$	$\longrightarrow$	$\mid \Delta L \mid = 1$	$\longrightarrow$	$d = 5$	$\phi\phi ll$
	$\mid$	$\mid \Delta B = 1 \mid$	$\Delta B = \ \Delta L \ = -1$	$d = 6$	$qqql$
	$\mid$	$\mid \Delta B = 1 \mid$	$\Delta B = -\Delta L = -1$	$d = 7$	$Dqqql^c$
	$\mid$	$\mid \Delta B = 1 \mid$	$\Delta B = \frac{1}{3}\Delta L = -1$	$d = 9$	$qqqlll$
$\Delta B \neq 0$	$\mid$	$\mid \Delta B = 1 \mid$		$d = 11$	$qqqlll\phi\phi$
	$\mid$	$\mid \Delta B = 1 \mid$	$\Delta B = -\frac{1}{3}\Delta L = -1$	$d = 10$	$qqql^cl^cl^c\phi$
	$\mid$	$\mid \Delta B = 1 \mid$		$d = 12$	$qqql^cl^cl^c\phi\phi\phi$
	$\mid$	$\mid \Delta B \mid = 2$	$\Delta L = 0$	$d = 9$	$qqqqqq$
	$\mid$	$\mid \Delta B \mid = 2$	$\mid \Delta B = \Delta L \mid = 2$	$d = 12$	$qqqqqqll$

Table 3

where D represents a derivative D_μ or a boson field ϕ. Besides the operators showed in table 3, their hermitian conjugates are permitted too (*e.g.* $qqql \longrightarrow q^cq^cq^cl^c$ etc.).
Now we can do a detailed analysis of the $qqql$ operators. First according to (1) we are restricted only to the following combinations: $O^{(2)}_{abcd} = q_Lq_Lq_Rl_R$, $O^{(3)}_{abcd} = q_Lq_Lq_Ll_L$, $O^{(1)}_{abcd} = q_Rq_Rq_Ll_L$ and $O^{(5)}_{abcd} = q_Rq_Rq_Rl_R$ (a, b, c, d are generation indices) The whole previous analysis was made assuming the existence of only one generation of leptons and quarks, but as we known, there are three. Weinberg made [Weil] the complete analysis for more than one family and found the following operators:

$$O^{(1)}_{abcd} = \epsilon_{\alpha\beta\gamma}\epsilon_{ij}[\overline{(d_R)_a}^{c\alpha} u^\beta_{Rb}][\overline{(q_L)_c}^{c\gamma i} l^j_{Ld}]$$
$$O^{(2)}_{abcd} = \epsilon_{\alpha\beta\gamma}\epsilon_{ij}[\overline{(q_L)_a}^{c\alpha i} q^{\beta j}_{Lb}][\overline{(u_R)_c}^{c\gamma} e_{Rd}]$$
$$O^{(3)}_{abcd} = \epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{kl}[\overline{(q_L)_a}^{c\alpha i} q^{\beta j}_{Lb}][\overline{(q_L)_c}^{c\gamma k} l^l_{Ld}]$$
$$O^{(4)}_{abcd} = \epsilon_{\alpha\beta\gamma}(\bar{\tau}\epsilon)_{ij} \times (\bar{\tau}\epsilon)_{kl}[\overline{(q_L)_a}^{c\alpha i} q^{\beta j}_{Lb}][\overline{(q_L)_c}^{c\gamma k} l^l_{Ld}]$$
$$O^{(5)}_{abcd} = \epsilon_{\alpha\beta\gamma}[\overline{(d_R)_a}^{c\alpha} u^\beta_{Rb}][\overline{(u_R)_c}^{c\gamma} e_{Rd}]$$
$$O^{(6)}_{abcd} = \epsilon_{\alpha\beta\gamma}[\overline{(u_R)_a}^{c\alpha} u^\beta_{Rb}][\overline{(d_R)_c}^{c\gamma} e_{Rd}]$$

There are two new operators besides our four previous ones; these are clearly different from zero only for the case of more than one generation.

From the general form $q_Lq_Lq_Rl_R$ it is possible to construct only the $u_Ld_Lu_Re_R$ operator, because the operator with ν_R doesn't exist. Moreover since the hypercharge of q_L (u_L, d_L) is $\frac{1}{6}$ and by hypercharge conservation we have, $\Sigma Y(f) = -1 + \frac{1}{6} + \frac{1}{6} + x = 0$; then the other factor must be u_R, (it has a hypercharge of $x = \frac{2}{3}$). By the same procedure we can obtain from the operator $q_Lq_Lq_Ll_L$, two possibilities: $u_Ld_Ld_L\nu_L$, and $u_Ld_Lu_Le_L$;

from the operator $q_Rq_Rq_Ll_L$ other two possibilities $u_Rd_Ru_Le_L$ and $u_Rd_Rd_L\nu_L$, and finally from the operator $q_Rq_Rq_Rl_R$ only one: $u_Rd_Ru_Re_R$. The other two new operators $O^{(4)}_{abcd}$ and $O^{(6)}_{abcd}$ are not important for proton decay, because they involve at least fermions of the second generation.

The most important non-conserving baryon number process is proton decay. The decay amplitude in the simplest case ($qqql$) is proportional to $A \propto e^2/M^2$, the proton decay rate is proportional to $\Gamma \propto \mid A \mid^2 \propto e^4/M^4$ and finally the proton decay lifetime (over branching ratio) is proportional to $\tau_B \propto M^4/e^4$. Since the mass-energy dimension of τ_B is -1, we need a factor of $1/m_N^5$; where m_N is the mass of the nucleon. So for $\tau_B \propto M^4/m_N^5 e^4$ within the experimentally interesting region of $10^{31} - 10^{34}$ years, we need a mass M for the new heavy vector or scalar bosons, between $7 \times 10^{14} - 4 \times 10^{15}$ GeV.

Now let us consider the B and/or L violating operators of any dimension. In general for an operator of any dimension the constant coupling can be estimated, as:

$$g_{d,n} \approx e^{n-2} M^{4-d} G_F^{-m/2} \tag{2}$$

where e is the electromagnetic coupling (really the coupling constant depends on the model, but we can take e^{n-2} as a good approximation), n is the number of external fields (lines), M is the mass of the heavy particle, d is the dimensionality of the operator in powers of mass, G_F the Fermi's constant, (that should be present when there are VEV's of scalars fields), and m is the number of scalar fields.

Following the order of Table 3, we will show the results of the same analysis for the different processes. The mass (M) of the heavy bosons will be always estimated by requiring that the proton lifetime lies in the region $10^{31} - 10^{34}$ years. Then we proceed, with a brief description of some of them:

a) Processes with $\Delta B = \Delta L = -1$: The operators are of the form $qqql$, its dimension is $d = 6$. Their coupling is of the order e^2/M^2, that gives $M \approx (7 \times 10^{14} - 4 \times 10^{15})$ GeV. This operator leads to processes like:

$$p \longrightarrow e^+\pi^0, \quad p \longrightarrow \mu^+K^0, \quad p \longrightarrow \bar{\nu}K^+, \quad n \longrightarrow e^+\pi^-$$

b) Processes with $\Delta B = -\Delta L = -1$: The operators are of the form $Dqqql^c$, its dimension is $d = 7$. Their coupling constants are, in the case $D = \phi$ of the order $e^3 G_F^{-1/2}/M^3$ and in the case $D = D_\mu$ of the order e^2/M^3. $M \approx (3 \times 10^{10} - 10^{11})$ GeV and $(5 \times 10^9 - 2 \times 10^{10})$ GeV respectively. These operators leads to processes like:

$$n \to e^-\pi^+, \quad n \to e^-K^+, \quad p \to e^+\pi^+\pi^+, \quad p \to e^-\pi^+K^+$$

c) Processes with $\Delta B = \frac{1}{3}\Delta L = -1$: The operators are of the form $qqqlll$ of dimension $d = 9$ (They are irrelevant for proton decay since can be shown [CZ] that at least one of the three quarks q must belong to the second generation) and $qqqlll\phi\phi$ operators of dimension $d = 11$, with a coupling constant of the order $e^6 G_F^{-1/2}/M^7$, that gives $M \approx 2 - 5 \times 10^4$ GeV. Some of the possible processes mediated by this operator are:

$$p \to \pi^+\nu\nu\nu, \quad n \to \nu\nu e^-\pi^+$$

d) Processes with $\Delta B = -\frac{1}{3}\Delta L = -1$: The operator are of the form $qqql^cl^cl^c\phi$, of dimension $d = 10$, with coupling constant of the order $e^5 G_F^{-1/2}/M^6$, that gives $M \approx 3 - 7 \times 10^4$; and $qqql^cl^cl^c\phi\phi\phi$ operators of dimension $d = 12$, with coupling constant of the order $e^7 G_F^{-3/2}/M^8$, that gives $M \approx (8 \times 10^3 - 1.5 \times 10^4)$ GeV. Some of the possible processes mediated by these operators are:

$$p \to e^+\nu\nu, \quad p \to \mu^+\bar{\nu}e^+\pi^-$$

e) Processes with $\mid \Delta B \mid= 2$ and $\Delta L = 0$: The operators are of the form $qqqqqq$, of dimension $d = 9$, with coupling constant of the order e^4/M^5; that leads to non-leptonic processes like:

$$nn \to \pi^0\pi^0, \quad ,np \to \pi^0\pi^+ \quad n \leftrightarrow \bar{n}$$

There are semileptonic processes too, like:

$$np \to e^+\nu_e \text{ through the decay } n \to p^- e^+\nu_e.$$

The neutron-antineutron oscillations ($n \leftrightarrow \bar{n}$ Oscillations) involve $qqqqqq$ operators. These oscillations are similar to $K^0 \leftrightarrow \bar{K}^0$ oscillations. The mass matrix corresponding to $n \leftrightarrow \bar{n}$ oscillations is:

$$M = \begin{pmatrix} m & \delta m \\ \delta m & m \end{pmatrix}$$

its corresponding mass eigenstates are:

$$\mid n_{1,2} >= \frac{\mid n > \pm \mid \bar{n} >}{\sqrt{2}},$$

the mass eigenvalues are: $m_{1,2} = m \pm \delta m$. Starting with a neutron n at $t = 0$ the amplitude $n(t)$ at the time t is:

$$n(t) = (1/\sqrt{2}) e^{-\frac{1}{2}\gamma t} \{ n_1(0) e^{-im_1 t} + n_2(0) e^{-im_2 t} \},$$

The probability of finding at time t an antineutron $\bar{n}$ is:

$$P_{\bar{n}}(t) = e^{-\gamma t} \sin^2 (\delta m t) \qquad (3).$$

From (3) we can see that, when $t \ll \frac{1}{\delta m}$, $P_{\bar{n}} \approx (\delta m t)^2$ (provided that we neglect the exponential $e^{\gamma t}$. We can define: $\tau_{n\bar{n}} = 1/\delta m$. Using dimensionality arguments and the experimental bound $\tau_{n\bar{n}} > 10^6$ sec, from $\delta m \approx e^4 m_p^6 / M^5$, we obtain $M \geq 4 \times 10^5$ Gev.

f) Processes with $\mid \Delta B = \Delta L \mid = 2$: The operators are of the form $qqqqqqll$, of dimension $d = 12$, with coupling constant of the order $e^6 M^8$. Then produce processes like hydrogen-antihydrogen oscillations ($H \equiv pe^- \leftrightarrow p^- e^+ \equiv \bar{H}$) , double proton decay ($pp \rightarrow e^- e^+$) or semileptonic decay ($n \rightarrow pe^+ \nu_e$).

g) Processes with $\Delta B = 0$ and $\mid \Delta L \mid = 2$: The operators are of the form $\phi\phi ll$, its dimension is $d = 5$, its coupling constant is $e^2 G_F^{-1}/M$. The processes of this type can produce neutrino oscillations, neutrinoless double beta decay and others less important from the experimental point of view.

Processes where there is only lepton number violation, without baryon number violation, can be classified in two types: Those that involve violation of the total lepton number L, and those that involve violation of the different family lepton number (L_e, L_μ, L_τ). In the Standard Model, there is mixing between families only in the quark sector, while in the lepton sector there is no mixing between families, because the neutrino is massless.

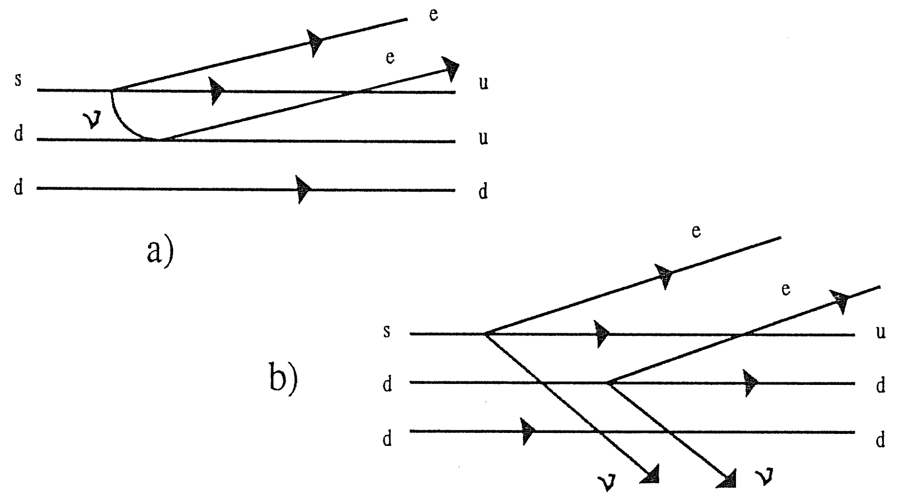

Fig. 1: $\beta\beta$ -Decay processes: a) neutrinoless, b) 2-neutrinos.

However in the extensions of the Standard Model that we will consider, we suppose the existence of a small neutrino mass and a right component of the neutrino field. In this way we have two types of oscillations: Oscillations between matter and anti-matter (e.g. $\nu_e \leftrightarrow \bar{\nu}_e$) or oscillations between families (e.g. $\nu_e \leftrightarrow \nu_\mu$). The first case is similar to $n \leftrightarrow \bar{n}$ oscillations, where we change the neutron with the neutrino, while the second case will be studied in the following paragraph.

If we call ν_a $(a = e,\ \mu,\ \tau, ...)$ the neutrino weak-eigenstates and ν_i $(i = 1,\ 2,\ 3, ...)$ the neutrino mass-eigenstate, the mixing matrix between different types of flavour in the neutrino sector can be defined as: $\nu_a = \sum_i U_{ai}\nu_i$. Since $\nu_i(t)$ is not a stationary state it will evolve according to the laws of Quantum Mechanics. Thus at $t > 0$ it will look like:

$$\nu_i(t) = \nu_i(0)e^{i(k_i x - w_i t)}$$

where k_i is the momentum, x the space coordinate and w_i the energy of the neutrino mass-eigenstate. For $w_i \gg m_i$, for every i, with m_i being the mass eigenvalue of ν_i, we set $w_i = E$ and we also set $k_i x - w_i t \approx \frac{m_i^2 t}{2E}$. Therefore we have $\nu_i(t) \approx \exp\left[-i\frac{m_i^2 t}{2E}\right]$, and

$$\nu_a(t) \approx \sum_i U_{ai}\nu_i(0)\exp\left[-i\frac{m_i^2 t}{2E}\right] ;$$

and then the probability to find the neutrino which initially was in the state b later in a state a, is:

$$P(\nu_a \to \nu_b) \approx \mid \sum_i U_{ai} \exp\left[-i\frac{m_i^2 t}{2E}\right] U_{bi}^* \mid^2 .$$

In the simplest case when we consider only two families, we can take the mixing matrix U_{ai} as being:

$$U_{ai} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix},$$

and we obtain the following equation:

$$P(\nu_e \to \nu_\mu) = \frac{1}{2}\sin^2 2\theta \left[1 - \cos\frac{\Delta m^2}{2E} t\right],$$

where $\Delta m^2 = \mid m_1^2 - m_2^2 \mid$.

For the moment there is no experimental evidence for neutrino oscillations, but there are limits obtained on the parameters θ ($\sin^2\theta > 0.25$, for large Δm^2) and Δm ($\Delta m^2 < 0.2\,(eV)^2$ with $\sin^2 2\theta = 1$). These experimental limits which were obtained for the $\bar{\nu}_e \to \bar{\nu}_x$ mode in the Gösgen experiment [Zac]; they are the best limits up to date.

Another important phenomenon involving the lepton number violation, is the double beta-decay. This is second order process, which is possible to be observed, only in the case when the normal beta-decay is forbidden by the conservation of energy. The double beta-decay ($\beta\beta_{2\nu}$-decay) can be produced within the SM (conserving the lepton number) in a reaction of the form:

$$n_1 + n_2 \to (p_1 + W_1) + (p_2 + W_2) \sim u_1 + (e_1 + \bar{\nu}_{e1}) + u_2 + (e_2 + \bar{\nu}_{e2}).$$

This type of decay was observed experimentally in 1987. Using a geochemical test; like the one used in β-decay; we obtain the following lifetime for Se and Te: $^{82}Se > 1.1 \times 10^{20}$ years and $^{128}Te > 8 \times 10^{24}$ years. On the other hand the direct search for the $\beta\beta_{2\nu}$-decay gives at the present for ^{128}Ge the limits of: $1.7 \times 10^{21} - 3.5 \times 10^{24}$ years. But it is possible in the extension of Standard Model to have the same reaction but without final neutrinos ($\beta\beta_{0\nu}$-decay). Of course this is possible only if neutrino is a Majorana particle or a superposition of Majorana eigenstates, in other words if the charge conjugate of the

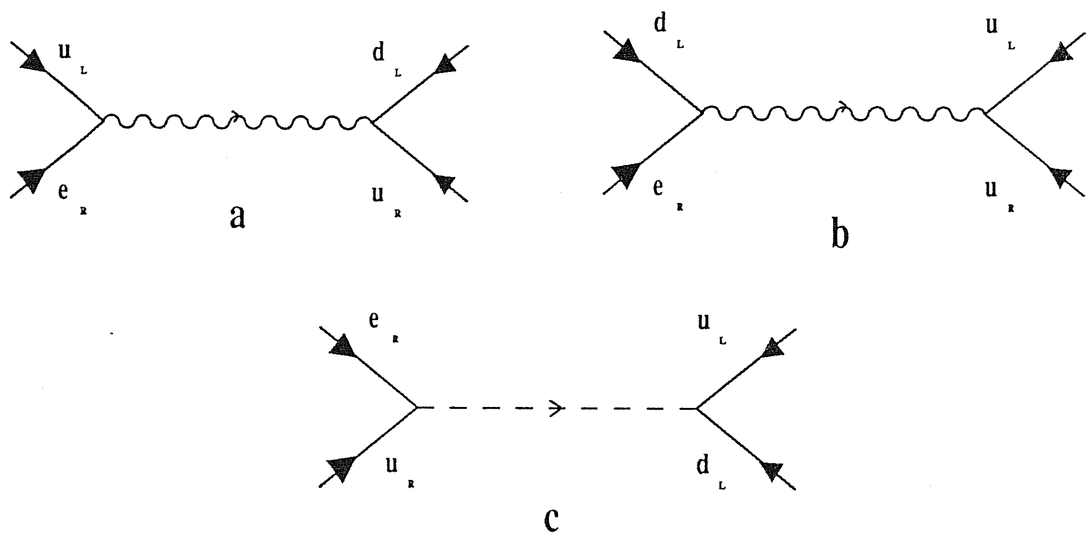

Fig. 2: Feynman diagrams for $u_L d_L u_R e_R$ operators.

neutrino is its antiparticle. Another way to obtain this kind of reaction is if a leptonic current j^l_μ contains right handed admixture:

$$j^l_\mu = \bar{e}\gamma_\mu[(1+\gamma^5)+\eta(1-\gamma^5)]\nu_0$$

with a non zero value of the parameter η. Of course this right handed component is not possible either within the SM, or within $SU(5)$ model; but is possible in other extensions of SM. Experimentally for the moment there is no evidence of this kind of decay, but there are several experiments in course.

The figure number 9 shows two different types of $\beta\beta$-decay processes; with (a) two neutrinos and (b) neutrinoless.

After the discussion about the possible values of the mass M of the heavy bosons, we turn onto the simplest $d = 6$ operator in order to analyse the behaviour of these heavy bosons under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ transformations. Our analysis begin with the $u_L d_L e_R u_R$ operator. We note that it is possible to construct, the following three different graphs, using Fierz rearrangements

We start by considering the graph a of Fig. 2. First, we observe that in the a vertex we have the boson field and two fermion fields: $u_L \sim (3, 2, \frac{1}{6})$ and $e_R \sim (1, 1, -1)$. For the moment we do not know what kind of n-plet is the boson field, under the $SU(3)$ group. Therefore we try the simplest n-plets that could give a singlet in the vertex. Thus

$$\mathbf{3}\otimes\mathbf{1}\otimes\mathbf{1}=\mathbf{3},\qquad \mathbf{3}\otimes\mathbf{1}\otimes\mathbf{3}=\bar{\mathbf{3}}\oplus\mathbf{6},\qquad \mathbf{3}\otimes\mathbf{1}\otimes\bar{\mathbf{3}}=\mathbf{1}\oplus\mathbf{8}$$

and therefore we obtain a singlet if the boson field is an anti-triplet. Now we will make the same analysis for the invariance under the $SU(2)$ group,

$$\mathbf{2}\otimes\mathbf{1}\otimes\mathbf{1}=\mathbf{2},\qquad \mathbf{2}\otimes\mathbf{1}\otimes\mathbf{2}=\mathbf{1}\oplus\mathbf{3}$$

and therefore we obtain a singlet if the boson field is a doublet. The hypercharge sum of the fermion fields in the vertex is: $\sum Y_a=\frac{1}{6}-1=-\frac{5}{6}$, and in order to conserve the hypercharge we need a value for the boson field of $Y=\frac{5}{6}$.

All the previous results must be consistent with the results in the b vertex. In the b vertex we have the fermion fields $u_L\sim(3,2,\frac{1}{6})$ and $u_R\sim(3,1,\frac{2}{3})$ plus the boson field; then we obtain:

Under $SU(3)\longrightarrow \mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\mathbf{10}$, which means that the boson field is an anti-triplet from a vertex to b vertex.

Under $SU(2)\longrightarrow \mathbf{2}\otimes\mathbf{1}\otimes\mathbf{2}=\mathbf{1}\oplus\mathbf{3}$, which means that effectively the boson field is a doublet.

Under $U(1)\longrightarrow \sum Y_b=\frac{1}{6}+\frac{2}{3}=\frac{5}{6}$, and then the boson field carries a hypercharge $Y=-\frac{5}{6}$ from a vertex to b vertex.

Our final result is that the boson field, behaves under $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$group as $X^i_{v1\alpha}\sim(\bar{3},2,-\frac{5}{6})$.

Making the same analysis on graph **b** of Fig. 2, we obtain a similar result $X^i_{v1\alpha}\sim(\bar{3},2,-\frac{5}{6})$.

In graph **c** of Fig. 2 we have $X^*_{1\alpha}\sim(\bar{3},1,\frac{1}{3})$; note that under $SU(2)_L$ it is a singlet, because in the a vertex we have $\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1}=\mathbf{1}$ and in b vertex, we have $\mathbf{2}\otimes\mathbf{2}\otimes\mathbf{1}=\mathbf{1}\oplus\mathbf{3}$. The hypercharge in the a vertex is: $\frac{2}{3}-1=-\frac{1}{3}$ and in the b vertex is $\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$; then the $X^*_{1\alpha}$ boson carries a $Y=\frac{1}{3}$ hypercharge.

In the case of the operator $d_Lu_Lu_Le_L$, we can draw two diagrams: in graph **d** of Fig. 3 we can observe that under $SU(2)_L$ that in both vertex:

$$\mathbf{2}\otimes\mathbf{2}\otimes\mathbf{1}=\mathbf{1}\oplus\mathbf{3},\quad \mathbf{2}\otimes\mathbf{2}\otimes\mathbf{2}=\mathbf{2}\oplus\mathbf{2}\oplus\mathbf{4}\quad \text{and } \mathbf{2}\otimes\mathbf{2}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{3}\oplus\mathbf{3}\oplus\mathbf{5},$$

then in this case we can have a singlet $X^*_{1\alpha}\sim(\bar{3},1,-\frac{1}{3})$ and a triplet $X^*_{2\alpha}\sim(\bar{3},3,-\frac{1}{3})$.

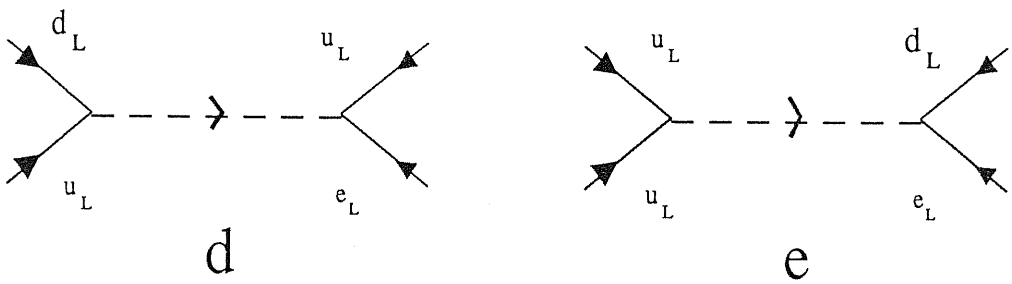

Fig. 3: Feynman diagrams for $d_L u_L u_L e_L$ operators.

Notice that in this case it is not possible to Fierz rearrange the operator so that vector bosons mediate the interaction. We can analyze the graphs from $d_L \nu_L u_R d_R$, $e_L u_L e_R u_R$, etc. in the same way and find all the possible bosons. Doing this we obtain the following five bosons:

$$X^i_{v1\alpha} \sim (\bar{3}, 2, \tfrac{5}{6}), \quad X^i_{v2\alpha} \sim (\bar{3}, 2, -\tfrac{1}{6}), \quad X^*_{1\alpha} \sim (3, 1, -\tfrac{1}{3})$$
$$X^*_{3\alpha} \sim (\bar{3}, 1, -\tfrac{4}{3}), \quad X^*_{2\alpha} \sim (\bar{3}, 3, -\tfrac{1}{3}).$$

If we take into account operators of dimension different than 6 it is possible to obtain more scalar and vector bosons, that would be gauge vector bosons. Following Costa and Zwirner [CZ] we present in the following tables the list of these scalar and vector bosons, (they carry color, weak and hyper- charge), that includes six or more fermions fields.

Scalar boson	Representation	q_{em}	$B-L$
Δ_1^*	$(1,3,-1)$	$-2,0,1$	-2
Δ_2^*	$(1,1,-1)$	-1	-2
Δ_3^*	$(1,1,-2)$	-2	-2
$X_{1\alpha}^*$	$(\bar{3},1,\frac{1}{3})$	$\frac{1}{3}$	$\frac{2}{3}$
$X_{2\alpha}^*$	$(\bar{3},3,\frac{1}{3})$	$-\frac{2}{3},\frac{1}{3},\frac{4}{3}$	$\frac{2}{3}$
$X_{3\alpha}^*$	$(\bar{3},1,\frac{4}{3})$	$\frac{4}{3}$	$\frac{2}{3}$
K_α^*	$(\bar{3},1,-\frac{2}{3})$	$-\frac{2}{3}$	$\frac{2}{3}$
$H_1^{\alpha i}$	$(3,2,\frac{1}{6})$	$-\frac{1}{3},\frac{2}{3}$	$\frac{4}{3}$
$H_2^{\alpha i}$	$(3,2,\frac{7}{6})$	$\frac{2}{3},\frac{5}{3}$	$\frac{4}{3}$
$Y_1^{\alpha\beta}$	$(6,1,\frac{1}{3})$	$\frac{1}{3}$	$\frac{2}{3}$
$Y_2^{\alpha\beta}$	$(6,3,\frac{1}{3})$	$-\frac{2}{3},\frac{1}{3},\frac{4}{3}$	$\frac{2}{3}$
$Y_3^{\alpha\beta}$	$(6,1,\frac{4}{3})$	$\frac{4}{3}$	$\frac{2}{3}$
$Y_4^{\alpha\beta}$	$(6,1,-\frac{2}{3})$	$-\frac{2}{3}$	$\frac{2}{3}$

Table 5

Vector boson	Representation	q_{em}	$B-L$
Δ_{v1}^1	$(1,2,-\frac{3}{2})$	$-2,-4$	-2
Δ_{v2}^1	$(1,2,-\frac{1}{2})$	$-1,0$	-2
$X_{v1\alpha}^i$	$(\bar{3},2,\frac{5}{6})$	$\frac{1}{3},\frac{4}{3}$	$\frac{2}{3}$
$X_{v2\alpha}^i$	$(\bar{3},2,-\frac{1}{6})$	$-\frac{2}{3},\frac{1}{3}$	$\frac{2}{3}$
H_{v1}^α	$(3,1,\frac{2}{3})$	$\frac{2}{3}$	$\frac{4}{3}$
H_{v2}^α	$(3,3,\frac{2}{3})$	$-\frac{1}{3},\frac{2}{3},\frac{5}{3}$	$\frac{4}{3}$
H_{v3}^α	$(3,1,\frac{5}{3})$	$\frac{5}{3}$	$\frac{4}{3}$
H_{v4}^α	$(3,1,-\frac{1}{3})$	$-\frac{1}{3}$	$\frac{4}{3}$
$Y_{v1}^{\alpha\beta i}$	$(6,2,\frac{5}{6})$	$\frac{1}{3},\frac{4}{3}$	$\frac{2}{3}$
$Y_{v2}^{\alpha\beta i}$	$(6,2,-\frac{1}{6})$	$-\frac{2}{3},\frac{1}{3}$	$\frac{2}{3}$

Table 6

Not all the scalar and vector bosons listed are useful for violating the B/L number in a second order Feynman diagram. Because there are some of them, that can not have different B/L number in their vertices. In these lists the existence of a singlet $\nu_{aR} \sim (1,1,0)$ under G_{sm} group (corresponding to the right component of the neutrino) is presupposed.

In table 5, (6) we present the designation of the scalar (vector) bosons in the first column, their representation under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ group in the second column, their electromagnetic charges in the third column and their $B-L$ quantum number in the fourth.

Note that besides the scalar and vector bosons listed in both tables, their corresponding hermitian conjugate partners exists as well. With this paragraph, we finish our general discussion of $B-L$-violating operators, and in the following we search for a gauge theory that contain operators like those, as generators.

At this point we have an idea about the order of magnitude of the mass of the hypothetical heavy bosons and some of the possible new processes mediated by these bosons. But, to go further with the predictions, we need to introduce these bosons within the context of a specific theory. This is possible in Grand Unified Theories, the theories that we will discuss in the next section.

3. B/L number violation in Grand Unified Theories

3.1 Generalities

As previously mentioned, the baryon and lepton number violation leads naturally to Grand Unified Theories. In order to understand how, we will make a short presentation of these theories.

The SM is a gauge theory based on the $G_{sm} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ group, but due to the fact that it is made of three different groups, we also have three different coupling constants. The hypothesis in the Grand Unified theories is to introduce a large simple or semi-simple gauge group G which contains as the effective low energy group G_{sm} as a subgroup. The theory based on the larger group G must be spontaneously broken at least at two hierarchically different scales.

$$G \xrightarrow{M} \left[G_{sm} + O\left(\frac{1}{M^2}\right)\right] \xrightarrow{m_w} \left[G_{es} + O\left(\frac{1}{m_w^2}\right)\right]$$

where G_{es} denotes the $SU(3)_C \otimes U(1)_{em}$ group.

But there are other restrictions that the theory must obey, in order to be a consistent theory. One of these restrictions is that the representation D_L and D_R (to which the f_L left-handed fermions and f_R right-handed fermions respectively, are assigned) must be complex with respect to the G_{sm} group and real with respect to the $G_{es} = SU(3)_C \otimes U(1)_{em}$ group. The complexity of the representations which contains the ordinary fermions is required to enforce the chiral protection which avoids large direct fermion masses in the lagrangian. Moreover the theory must be free of Adler-Bell-Jackiw Anomalies and must reproduce the known phenomenology of the SM.

We know that the three coupling *"constants"* of the Standard Model are different, but fortunately they are not really constants. They change with the energy according to the Renormalization Group Equations. The one loop approximation reads [BEGN,GQW,CEG]:

$$\frac{1}{\alpha_i(Q^2)} = \frac{1}{\alpha_i(M^2)} + \frac{b_i}{4\pi} ln \frac{M^2}{Q^2}, \tag{4}$$

where $\alpha_i(Q^2)$ are the running coupling constants for momentum transfer Q^2 and b_i are:

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + n_G \frac{4}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + n_H \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix}. \tag{5}$$

n_G is the number of fermion generations and n_H is the number of Higgs doublets. At an energy of the order m_w, the couplings α_i are different and they grow (or decrease) logarithmically. Since we want to embed $G_{sm} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ into a simple group G all the generators of G_{sm} must be renormalized in the same way, *i.e.* $\mathrm{Tr}(T^a T^b) = N\delta^{ab}$ for all the T^a generators of the Standard Model. For instance requiring that $\mathrm{Tr}(T_3^2) = \mathrm{Tr}(Y'^2)$ over a fermionic generation we obtain a new suitably normalized definition of the hypercharge

$$Y' = \sqrt{\frac{3}{5}} Y,$$

where Y had been defined in Section 2 as $Y = Q_3 - T_{3L}$. We require that at the value M, all the coupling constants are equal according to:

$$\alpha_1(M^2) = \alpha_2(M^2) = \alpha_3(M^2) = \alpha_{GUT} = \frac{8}{3}\alpha_{em}(M^2)$$

To obtain a first numerical value of the mass M, we use the one-loop approximation. The running coupling constant α_i are defined as:

$$\alpha_i = g_i^2/4\pi, \quad \text{where } i = 1, 2, 3$$

In the SM, we need the following relation to be valid.

$$e^2 = \frac{3}{5} g_1^2 \cos^2\theta_W = g_2^2 \sin^2\theta_W \tag{6}$$

From (6) we have:

$$\sin^2\theta_W + \cos^2\theta_W = \frac{5}{3}\frac{\alpha_{em}}{\alpha_1} + \frac{\alpha_{em}}{\alpha_2}$$

and

$$\frac{3}{\alpha_{em}(Q)} = \frac{5}{\alpha_1(Q)} + \frac{3}{\alpha_2(Q)},$$

From (4) we get:

$$\frac{4\pi}{\alpha_{em}(m_w)} = \frac{4\pi}{\alpha_{em}(M)} + b^0_{em} \ln \frac{M^2}{m_w^2};$$

with $b^0_{em} = \frac{5b^0_1+3b^0_2}{3}$ [using (5) and neglecting the contribution of the Higgs fields n_H]. Making combinations of the equations in (4), for $i = 1, 2, 3$, we have:

$$\frac{1}{\alpha_3(Q^2)} - \frac{1}{\alpha_2(Q^2)} = \frac{(b_3 - b_2)}{4\pi} \ln \frac{M^2}{Q^2}$$

$$\frac{1}{\alpha_3(Q^2)} - \frac{1}{\alpha_1(Q^2)} = \frac{(b_3 - b_1)}{4\pi} \ln \frac{M^2}{Q^2}. \tag{7}$$

Using, the previous equations, (4) and the minimal GUT version with three generations of fermions ($n_f = 3$) and now taking one Higgs doublet ($n_H = 1$), we obtain our final result [KMNO]:

$$M = m_w \mathrm{exp}\left[\frac{16\pi}{67}\left(\frac{3}{8}\frac{1}{\alpha_{em}(m_w)} - \frac{1}{\alpha_3(m_w)}\right)\right]. \tag{8}$$

using the experimental values: $m_w \approx 80$ GeV, $\alpha_{em}(m_w) \approx \frac{1}{128}$ and $\alpha_3(m_w) \approx \frac{1}{10}$, we can finally obtain, for the boson mass a value of:

$$M \approx 10^{15} \quad GeV \qquad \Longrightarrow \tau \approx 10^{31} \text{ years.}$$

It is of interest to consider how much the second loop approximation for the renormalization group equations can modify the above one loop approximation. At the two loop approximation the renormalization group equations take the following form [GRo]:

$$\beta_i = \mu^2 \frac{\partial}{\partial \mu^2} (\frac{\alpha_i}{4\pi})^2 \left[b^0_i + \sum_{j=1}^{3} b^1_{ij} (\frac{\alpha_i}{4\pi}) \right] \tag{9}$$

where the indices $i = 1, 2, 3$ correspond to $U(1)$, $SU(2)$, and $SU(3)$ groups respectively and the numerical coefficients are given by:

$$\begin{pmatrix} b^1_{11} & b^1_{12} & b^1_{13} \\ b^1_{21} & b^1_{22} & b^1_{23} \\ b^1_{31} & b^1_{32} & b^1_{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{136}{3} & 0 \\ 0 & 0 & 102 \end{pmatrix} - \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{13}{16} & 0 \\ 0 & 0 & 0 \end{pmatrix} n_H - \begin{pmatrix} \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\ \frac{1}{5} & \frac{49}{3} & 4 \\ \frac{11}{30} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} n_G \tag{10}$$

The value of the coupling constant at momentum Q is given by:

$$\frac{4\pi}{\alpha_i(Q)} = \frac{4\pi}{\alpha_i(M)} + b^0_i ln \frac{Q^2}{M^2} + \sum_{j=1}^{3} \frac{b^1_{ij}}{b^0_j} \ln \frac{\alpha_j(M)}{\alpha_j(Q)} + O(\alpha_i^2) \tag{11}$$

using the above equations, we can obtain [KMNO]:

$$M^1 = m_w \exp\left[\frac{2\pi}{b_3^0 - b_{em}^0}\left(\frac{3}{8}\frac{1}{\alpha_{em}(m_w)} - \frac{1}{\alpha_3(m_w)}\right)\right]\sum_{i=1}^{3}\left(\frac{\alpha_i(M)}{\alpha_i(m_w)}\right)^{\gamma_i}$$

where

$$\gamma_i = \frac{1}{2b_i^0}\left(\frac{b_{3i}^1 - \frac{3b_{2i}^1 - 5b_{1i}^1}{3}}{b_3^0 - \frac{3b_2^0 - 5b_i^0}{3}}\right)$$

using the numerical coefficients in (10) we have:

$$\gamma_1 \approx -0.01, \qquad \gamma_2 \approx -0.01, \qquad \gamma_3 \approx -0.31$$

finally we obtain the relation between the one-loop approximation and two-loops approximations:

$$M^{2-loops} \approx 0.64 M^{1-loop}$$

The effect of this correction is important to the proton lifetime, which decreases by a factor of the order ten.

$$\tau_p \sim M^4 \Rightarrow \tau_p^{2-loops} \approx 0.13 \tau_p^{1-loop} \approx 10^{30} \text{ years}$$

Another important source of modification for τ_p is the presence of threshold effects in treating the renormalization group equations at energy scales close to M. But I considered the so called *Step Approximation* where these effects are not taking into account, however the corrections introduced when we considered more complex approximation not modify substantially the limit for the proton lifetime [EGNR]. Therefore the GUT's in their minimal version (1 Higgs doublet) are in conflict with the experimental limit.

3.2 The SU(5) Model

We begin our search for the unification group among those groups of rank $r = 4$ because the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$group is of this rank. The groups of rank 4 are: $[SU(4)]^4$, $[O(5)]^2$, $[SU(3)]^2$, $[G_2]^2$, $O(8)$, $O(9)$, $Sp(8)$, F_4, and $SU(5)$.

We eliminate the groups $[SU(4)]^4$ and $[O(5)]^2$ because they do not contain $SU(3)$ as a subgroup. The group $[SU(3)]^2$ contain $SU(3)$ but since $SU(2) \otimes U(1)$ must be embedded in the remaining $SU(3)$ group is not good because, the sum of the charges of the quarks is zero due to the traceless charge generator. The groups $[G_2]^2$, $O(8)$, $O(9)$, $Sp(8)$ and

F_4 do not have the complex representation necessary to accommodate chirality protected fermions . Therefore only the $SU(5)$ group remains and we will study the model based on this group.

The adjoint representation of the $SU(5)$ group is 24-dimensional; its decomposition under G_{sm} group is:

$$\mathbf{24} \rightarrow (8,1,0) \oplus (1,3,0) \oplus (1,1,0) \oplus (\bar{3},2,\tfrac{5}{6}) \oplus (3,2,-\tfrac{5}{6}).$$

These gauge fields corresponds to:

a) The eight gluons of Quantum Chromodynamics, $G^{\alpha}_{\mu\beta} \sim (8,1,0)$.

b) The four vector gauge bosons of the electroweak theory, W^i_μ, $B_0 \sim (1,3,0)$, $(1,1,0)$.

c) The twelve electromagnetic and colour charged vector bosons, that can mediates the B/L violating processes, $X^i_{v1\alpha} \sim (\bar{3},2,\frac{5}{6})$ and $X^{i\dagger}_{v1\alpha} \sim (3,2,-\frac{5}{6})$.

To complete the know phenomenology of the standard model we need the fermions fields to be accommodated in a representation of the $SU(5)$ group. This is possible in the $\bar{\mathbf{5}} \oplus \mathbf{10}$ $(\psi_{La} \oplus \psi^{ab}_L)$ representation. In this representation, the decomposition under G_{sm} is:

$$\bar{\mathbf{5}} \oplus \mathbf{10} \rightarrow \left[(\bar{3},1,\frac{1}{3}) \oplus (1,2,-\frac{1}{2})\right] \oplus \left[(3,2,\frac{1}{6}) \oplus (\bar{3},1,-\frac{2}{3}) \oplus (1,1,1)\right] \tag{12}$$

with the following identification of the fermion fields:

$$(\bar{3},1,\frac{1}{3}) \sim (d^c)_L, \quad (1,2,-\frac{1}{2}) \sim (e^-_L,\nu_L), \quad (\bar{3},1,-\frac{2}{3}) \sim (u^c)_L \tag{13}$$

$$(3,2,\frac{1}{6}) \sim (u_L,d_L) \text{ and } \quad (1,1,1) \sim (\mathrm{e}^+)_{\mathrm{L}}. \tag{14}$$

In this way, we can construct the interaction terms in the $SU(5)$ theory, using as covariant derivatives the following relations:

$$(D_\mu\psi)^a = \left[\partial_\mu\delta^a_b - i\frac{g}{\sqrt{2}}(A_\mu)^a_b\right]\psi^b,$$

$$(D_\mu\psi)^{ab} = \partial_\mu\psi^{ab} - i\frac{g}{\sqrt{2}}(A_\mu)^a_c\psi^{cb} - i\frac{g}{\sqrt{2}}(A_\mu)^d_b\psi^{ad}$$

where A is the matrix:

$$A = \begin{pmatrix} G^1_1 - \sqrt{\frac{2}{15}}B & G^1_2 & G^1_3 & \bar{X}^1_{1v\alpha} & \bar{X}^{1\dagger}_{1v\alpha} \\ G^1_2 & G^2_2 - \sqrt{\frac{2}{15}}B & G^2_3 & \bar{X}^2_{1v\alpha} & \bar{X}^{2\dagger}_{1v\alpha} \\ G^3_1 & G^3_2 & G^3_3 - \sqrt{\frac{2}{15}}B & \bar{X}^3_{1v\alpha} & \bar{X}^{3\dagger}_{1v\alpha} \\ X^1_{1v\alpha} & X^2_{1v\alpha} & X^3_{1v\alpha} & \frac{W^3}{\sqrt{2}} + \frac{3}{2}\sqrt{\frac{2}{15}}B & W^+ \\ X^{1\dagger}_{1v\alpha} & X^{2\dagger}_{1v\alpha} & X^{3\dagger}_{1v\alpha} & W^- & -\frac{W^3}{\sqrt{2}} + \frac{3}{2}\sqrt{\frac{2}{15}}B \end{pmatrix}$$

and $a, b, ... = 1, 2, ..., N$ are the $SU(N)$ indices.

Using (12)-(16), the interaction terms take the form:

$$\mathcal{L}^{\bar{5}}_{int} = g[(\bar{d}^c_L\gamma^\mu G_\mu \frac{\lambda}{2} d^c_L) + (\bar{\nu}_e\ \bar{e})_L\gamma^\mu W_\mu \frac{\tau}{2}\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L + \sqrt{\frac{3}{5}}(-\frac{1}{2}\bar{\nu}_L\gamma^\mu B_\mu \nu_L - \frac{1}{2}\bar{e}_L\gamma^\mu B_\mu e_L$$

$$+\frac{1}{3}\bar{d}^c_L\gamma^\mu B_\mu d^c_L) + [\frac{1}{\sqrt{2}}(-\bar{e}_L\gamma^\mu \bar{X}^\alpha_\mu d^c_{\alpha L}) + \frac{1}{\sqrt{2}}(-\bar{\nu}_L\gamma^\mu \bar{X}^{\dagger\alpha}_\mu d^c_{\alpha L}) + h.c.]] \tag{17}$$

and

$$\mathcal{L}^{10}_{int} = g[(\bar{u}_L\gamma^\mu G_\mu \frac{\lambda}{2} u_L + \bar{d}_L\gamma^\mu G_\mu \frac{\lambda}{2} d_L + \bar{u}^c_L\gamma^\mu G_\mu \frac{\lambda}{2} u^c_L) + (\bar{u}\ \bar{d})_L\gamma^\mu W_\mu \frac{\tau}{2}\begin{pmatrix}u \\ d\end{pmatrix}_L$$

$$+\sqrt{\frac{3}{5}}(-\frac{1}{6}\bar{u}_L\gamma^\mu B_\mu u_L - \frac{1}{6}\bar{d}_L\gamma^\mu B_\mu d_L - \frac{2}{3}\bar{u}^c_L\gamma^\mu B_\mu u^c_L + \frac{1}{2}\bar{e}^c_L\gamma^\mu B_\mu e^c_L)$$

$$+[\frac{1}{\sqrt{2}}(-\bar{u}_{L\alpha}\gamma^\mu \bar{X}^{\dagger\alpha}_\mu d^c_L - \epsilon_{\alpha\beta\gamma}\bar{u}^{c\gamma}_L\gamma^\mu \bar{X}^{\dagger\alpha}_\mu d^{c\beta}_L) + h.c.]]. \tag{18}$$

In the relations (17) and (18) we have the interactions terms of the SM reproduced, but moreover we also have new interaction terms. These new interaction terms involve lepton-quark currents. We can note that, in an interaction at a second order, we obtain operators of dimension $d = 6$, similar to those studied in the previous section. In second order approximation an effective theory at low energies has a factor $g^2/2M^2$. Then the discussion about the operators made in the precedent section is valid also for the heavy bosons of the $SU(5)$-Model.

Due to the fact that at low energies we need to reproduce the Standard Model with little corrections, the heavy bosons must acquire their mass in the symmetry breaking from $SU(5)$ to G_{sm}-group. The $SU(5)$ group is broken in two steps:

$$SU(5) \longrightarrow G_{sm} \longrightarrow G_{es},$$

by the Higgs fields ϕ and H. The first step, break of the $SU(5)$ symmetry to the G_{sm}, is performed by a 24-plet ($SU(3) \otimes SU(2)_L$):

$$\phi \longrightarrow \mathbf{24} = (8,1) \oplus (3,2) \oplus (\bar{3},2) \oplus (1,3) \oplus (1,1), \tag{19}$$

and the most general potential, that we can associate with it, is:

$$V(\phi) = -\frac{\mu^2}{2}\mathrm{Tr}(\phi^2) + \frac{1}{4}a\{\mathrm{Tr}(\phi^2)\}^2 + \frac{1}{2}b\mathrm{Tr}(\phi^4) + \frac{1}{3}c\mathrm{Tr}(\phi^3). \tag{20}$$

If we introduce the symmetry $\phi = -\phi$, thus $c = 0$; without loss of generality. The second step, break of the G_{sm} group to G_{es} group, is performed by the 5-plet ($SU(3) \otimes SU(2)_L$):

$$\mathbf{5} \longrightarrow (3,1) \oplus (1,2) \quad \sim \quad H = (H^\alpha) \oplus (H^r) \tag{21}$$

where H^α, ($\alpha = 1,2,3$) is a $SU(3)$ color triplet and H^r ($r = 1,2$) is a $SU(2)$ doublet equivalent to that of the Standard Model. The most general potential that we can associate with H, is:

$$V(H) = -\frac{\mu_5^2}{2}H^\dagger H + \frac{\lambda}{4}\left(H^\dagger H\right)^2, \tag{22}$$

Besides the potentials $V(\phi)$ and $V(H)$, we need to add to the lagrangian the cross-term potential:

$$V(H,\phi) = \alpha H^\dagger H \mathrm{Tr}\phi^2 + \beta H^\dagger \phi^2 H + \delta H^\dagger \phi H \tag{23}$$

in order to avoid that the colour triplet H^α remains at the same mass of ϕ (Because H^α possesses the correct quantum numbers to mediate a proton decay from the 6-dim $qqql$ operator therefore H^α must be heavier than m_w). Using another time the symmetry $\phi = -\phi$, we obtain $\delta = 0$.

The vacuum expectation values that yield the correct symmetry breaking pattern is:

$$< V(\phi) >= \begin{pmatrix} \nu & 0 & 0 & 0 & 0 \\ 0 & \nu & 0 & 0 & 0 \\ 0 & 0 & \nu & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2}\nu - \frac{\epsilon}{2} & 0 \\ 0 & 0 & 0 & 0 & -\frac{3}{2}\nu + \frac{\epsilon}{2} \end{pmatrix}, \tag{24}$$

$$< V(H) >= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{\nu_0}{\sqrt{2}} \end{pmatrix}. \tag{25}$$

From the minimization of the potential we arrive to:

$$\mu^2 = \frac{15}{2} a\nu^2 + \frac{7}{2} b\nu^2 + \alpha\nu_0^2 + \frac{9}{30}\beta\nu_0^2, \tag{26}$$

$$\mu_5^2 = \frac{1}{2}\lambda\nu_0^2 + 15\alpha\nu^2 + \frac{9}{2}\beta\nu^2 - 3\epsilon\beta\nu^2. \tag{27}$$

$$\epsilon = \frac{3}{20}\frac{\beta\nu_0^2}{b\nu^2} + O\left(\frac{\nu_0^4}{\nu^4}\right). \tag{28}$$

Since we require that $\nu \sim O(M)$ and $\nu_0 \sim O(m_w)$, we get $\epsilon\nu \ll \nu_0$. Equation (27) clearly illustrates the gauge hierarchy problem. For $\alpha = \beta = 0$ we must recover the well know Standard Model relation $\mu_5^2 = \frac{1}{2}\lambda\nu_0^2$, implying that $\mu_5 \sim \nu_0$ for $\lambda \leq 1$. When we switch on the couplings α and β we produce a drastic modification to such relations. Indeed, the new terms proportional to α and β in the right hand side of equation (27) are of $O(M^2)$. For $\lambda \leq 1$, the only way to obtain a consistent result is that in their sum an incredibly accurate cancellation takes place so that they add a term of $O(m_w^2)$ to the previous $\frac{1}{2}\lambda\nu_0$. We need a fine tuning of the 24th decimal figure and this is obviously very unnatural.

The Higgs mechanism is useful not only for the symmetry breaking but also to give mass to the fermions. The mass terms in the lagrangian involve tensor products between the fermions fields. The possible tensor products are:

$$\psi_{La}^T C\psi_{Lb} \longrightarrow \bar{\mathbf{5}} \otimes \bar{\mathbf{5}} = \overline{\mathbf{10}} \oplus \overline{\mathbf{15}} \tag{30}$$

$$\psi_{La}^T C\psi_L^{bc} \longrightarrow \bar{\mathbf{5}} \otimes \mathbf{10} = \mathbf{5} \oplus \overline{\mathbf{45}} \tag{31}$$

$$\psi_L^{Tab} C\psi_L^{cd} \longrightarrow \mathbf{10} \otimes \mathbf{10} = \bar{\mathbf{5}} \oplus \mathbf{45} \oplus \mathbf{50} \qquad (32).$$

It is easy to see that it is possible give mass to the fermions only through the **5** and **45** representations (and their hermitian conjugates). Therefore the fermions masses can be obtained from:

$$\mathbf{10} \otimes \mathbf{10} \otimes \bar{H} \longrightarrow u - \text{quarks masses}$$

$$\bar{\mathbf{5}} \otimes \mathbf{10} \otimes H \longrightarrow d - \text{quark and } e \text{ masses}$$

In this so called minimal version (used only the 5 representation), the lagrangian Yukawa term takes the form:

$$\mathcal{L}_{Yuk} = g_{1mn}\psi^T_{Lma} C\psi^{ab}_{Ln} H^\dagger_{5b} + g_{2mn}\psi^{Tab}_{Lm} C\psi^{cd}_{Ln} H^e_5 \epsilon_{abcde} + \text{h.c.} \tag{33}$$

where g_{1mn} and g_{2mn} are the Yukawa couplings and C the charge conjugate matrix,

$$\bar{\mathbf{5}} = \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ e^- \\ -\nu_e \end{pmatrix}_L \tag{34}$$

and

$$\mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^c_3 & -u^c_2 & -u^1 & -d^1 \\ -u^c_3 & 0 & u^c_1 & -u^2 & -d^2 \\ u^c_2 & -u^c_1 & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{pmatrix}_L \tag{35}$$

The signs in (34) and (35) are just to reproduce the assignment to the fermions in the Standard Model. They are in any case unessential because we are so far in the current eigenstate basis. It is useful to represent (34) and (35) in the following way:

$$\bar{\mathbf{5}} = \begin{pmatrix} \nu_e & \\ & d^c_\alpha \\ e^- & \end{pmatrix}_L \tag{36}$$

$$\mathbf{10} = \begin{pmatrix} & u^\alpha & \\ e^+ & & u^c_\alpha \\ & d^\alpha & \end{pmatrix}_L \tag{37}$$

Taking as VEV for the Higgs field

$$< 0 \mid H^a \mid 0 >= \frac{i}{\sqrt{2}}\nu_0 \delta^5_a$$

we obtain:

$$\begin{aligned} \mathcal{L}_{Yuk} &= -\frac{\nu_0}{2} g_{1mn}(\bar{d}_{mR} d_L + \bar{e}^+_{mR} e^+_{nL}) + 4\frac{\nu_0}{\sqrt{2}} g_{2mn}\bar{u}_{mR} u_{nL} + \text{h.c.} \\ &= -\bar{d}_R M_d d_L - \bar{e}^+_R M^e e^+_L - \bar{u}_R M^u u_L + \text{h.c.} \end{aligned} \tag{38}$$

where $M^d = M^e = \frac{\nu_0}{2} g_{1mn}$ and $M^u = \frac{4\nu_0}{\sqrt{2}} g_{2mn} = M^{uT}$.

Due to $\psi^{55} = 0$, no neutrino mass is possible. A serious theoretical problem created by this mechanism is that it leads to the following relation between the masses of fermions (due to the presence in the same multiplet of d and e, s and μ and b and τ), at any energy:

$$\frac{m_d}{m_s} = \frac{m_e}{m_\mu} \tag{39}$$

But experimentally the following relations are valid $\frac{m_d}{m_s} = \frac{1}{24}$, and $\frac{m_e}{m_\mu} = \frac{1}{207}$, that are not in accord with (39).

For solving this problems there are two ways:

1) The introduction of non-renormalizable effective interactions giving rise to contributions in the MeV range [EGa].

2) The addition of the another Higgs field (the other only one permitted **45**). But this addition cut the power prediction of the theory, because leaves more free parameters [FNS, GJa].

To transform the fermion fields from the interaction basis to physical basis, we need to diagonalize the mass matrix through the relations:

$$\left(A_R^d\right)^\dagger M^d A_L^d = M_D^d \tag{40}$$

$$\left(A_R^u\right)^\dagger M^u A_L^u = M_D^u \tag{41}$$

$$\left(A_R^e\right)^\dagger M^e A_L^e = M_D^e \tag{42}$$

where $A_{R,L}^x$ are unitary matrices and $M_D^d = \mathrm{diag}(m_d, m_s, m_b, ...)$, etc. The fermion fields change from the current eigenstates to the mass eigenstates as:

$$d_{L,R}^0 = \left(A_L^d\right)^\dagger d_{L,R}$$

If we take a basis such that: e_L^- and d_L are diagonal; the $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations, in terms of mass eigenstates are [using the notation of (36) and (37)]:

$$\bar{\mathbf{5}} = \begin{pmatrix} \nu_m & \\ & (d^c A_R^{d\dagger})^m \\ e_m^- & \end{pmatrix}_L \tag{43}$$

and

$$\mathbf{10} = \begin{pmatrix} & (A_L^u u)_m & \\ (A_L^{e^+} e^+)_m & & (u^c A_R^{u\dagger})_m \\ & d_m & \end{pmatrix}_L \tag{44}$$

The matrices $A_{L,R}$ are determined up to the phase matrices $K_{L,R}$ by the condition that M_D^2 be real and diagonal. The phases of K_L and K_R^* are determined by the reality and positivity of M_D. The phases in K_L can be chosen to put the A_L matrices in the following convenient form. $2n_G - 1$ of the $3n_G$ phases in the K_L^d, K_L^u and $K_L^{e^+}$ (K are diagonal matrices of phases) matrices may be chosen to put the Kobayashi-Maskawa matrix (KM) A_L^u into the standard $(n_G - 1)^2$ parameter form. n_G phases may be chosen to simplify $A_L^{e^+}$, which therefore has only $n_G^2 - n_G$ observable parameters. The final phase may be chosen to simplify A_R^u so that A_R^u and A_R^d have $n_G^2 - 1$ and n_G^2 observable parameters, respectively.

In the general case, the matrices $A_{L,R}$ are arbitrary. It would be possible to choose mass matrices so that, for example, the u and d quarks are associated in multiplets with the τ lepton, which would greatly suppress proton decay [Jar]. However, the situation simplifies enormously if only 5 dimensional Higgs are included [EGN, Moh, BEGN]. One then has $A_{L,R}^d = A_{L,R}^{e^+} = I$ (in the basis being used). Moreover, the symmetry $M^u = M^{uT}$ implies (for a given A_L^u)

$$A_R^{\bar{u}} = A_L^u \bar{K}$$

where K is a diagonal matrix of phases (assuming no degeneracy of the eigenvalues) which is uniquely determined by the condition that M_D^u be real and positive. Then the $\bar{\mathbf{5}}$ and $\mathbf{10}$ fields are:

$$\bar{\mathbf{5}} = \begin{pmatrix} \nu_m & \\ & (d_m^c) \\ e_m^- & \end{pmatrix}_L \tag{45}$$

and

$$\mathbf{10} = \begin{pmatrix} & U_{mn} u_n & \\ e_m^+ & & U_{mn} e^{-i\alpha_n} u_n^c \\ & d^m & \end{pmatrix}_L \tag{46}$$

where U is the generalized Cabibbo matrix and $e^{-i\alpha_n}$ is the nth diagonal entry of $\bar{K}$ (only $F-1$ of these phases are observable; the last corresponds to an arbitrary phase of all fields in the theory). Hence except for the extra phases all of the mixing matrices are determined

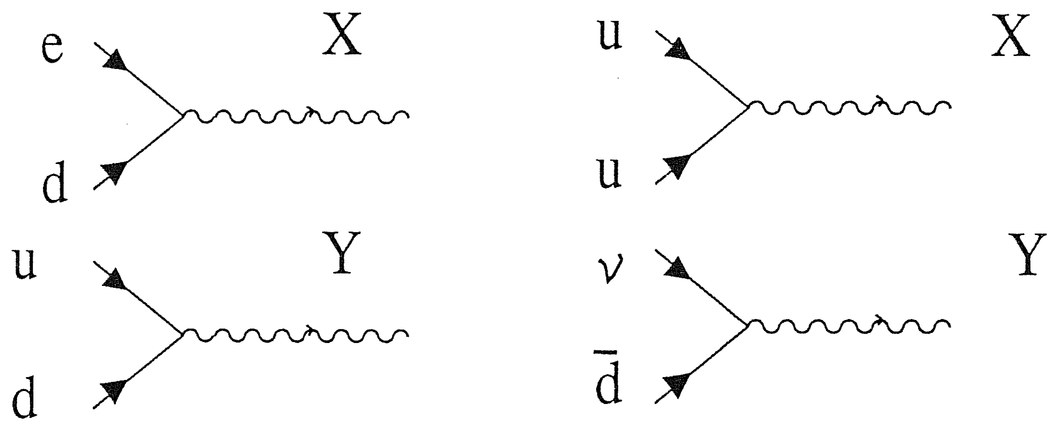

Fig. 4: Tree diagrams for B/L-number operators in $SU(5)$.

by the Cabibbo matrix, the light quarks are associated with the leptons, and proton decay cannot be *rotated away* [EGN,Moh,BEGN].

3.3 Predictions of SU(5)

It is possible to make several predictions using the $SU(5)$ model. For us the most important is the lifetime of the proton decay.

We begin the discussion about the proton lifetime within the $SU(5)$ model remembering the interaction terms, that involve the heavy bosons, that can violate the B/L-number. They are contain in the final part of equations (17) and (18).

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}}\bar{X}^i_\mu \left(\bar{d}_{iR}\gamma^\mu e^+_R + \epsilon_{ijk}\bar{u}^{ck}_L\gamma^\mu u^j_L + \bar{d}_{iL}\gamma^\mu e^+_L\right)$$

$$+\frac{g}{\sqrt{2}}\bar{X}^{i\dagger}_\mu \left(\bar{d}_{iR}\gamma^\mu \nu^c_R + \epsilon_{ijk}\bar{u}^{cj}_L\gamma^\mu d^k_L + \bar{u}_{iL}\gamma^\mu e^+_L\right) + \text{h.c.} \qquad (47)$$

From the equation (47) we obtain the tree diagrams of figure 4.

In all the graphs $\Delta(B-L) = \frac{2}{3}$. From this basic tree diagrams, we can construct the second order interactions, with $\Delta B = 1$, showed in the graph 5:

The effective Lagrangian ($E \ll M$) obtained from the precedent graphs is:

$$\frac{1}{4}\mathcal{L}_{\Delta B=1} = \frac{g^2}{8M^2}(\epsilon_{ijk}\bar{u}_L^{ck}\gamma^\mu u_L^j)(2\bar{e}_L^+\gamma^\mu d_L^i + \bar{e}_R^+\gamma^\mu d_R^i)$$

$$+\frac{g^2}{8M^{\dagger 2}}(\epsilon_{ijk}\bar{u}_L^{ck}\gamma^\mu d_L^j)(\bar{\nu}_{eR}^c\gamma^\mu d_R^i) + \text{h.c.} \tag{48}$$

Taking into account the mixing between families the interaction term of the heavy bosons in the lagrangian (47) can be written as:

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}}\bar{X}_\mu^i\left(\bar{d}_{iR}\gamma^\mu e_R^+ + \epsilon_{ijk}\bar{u}_L^{ck}K\gamma^\mu u_L^j + \bar{d}_{iL}\gamma^\mu e_L^+\right)$$

$$+\frac{g}{\sqrt{2}}\bar{X}_\mu^{i\dagger}\left(\bar{d}_{iR}\gamma^\mu\nu_R^c + \epsilon_{ijk}\bar{u}_L^{ck}KA_C^\dagger\gamma^\mu d_L^k + \bar{u}_{iL}U^\dagger\gamma^\mu e_L^+\right) + \text{h.c.} \tag{49}$$

Then the effective lagrangian (48) for the case with only two generations, becomes:

$$\frac{1}{4}\mathcal{L}_{\Delta B=1} = e^{i\phi}\frac{g^2}{8M^2}(\epsilon_{ijk}\bar{u}_L^{ck}\gamma^\mu u_L^j)\{[(1+\cos^2\theta_c)\bar{e}_L^+ + \sin\theta_c\cos\theta_c\bar{\mu}_L^+]\gamma_\mu d_L^i$$

$$+[(1+\cos^2\theta_c)\bar{\mu}_L^+ + \sin\theta_c\cos\theta_c\bar{e}_L^+]\gamma_\mu s_L^i + \bar{e}_R^+\gamma_\mu d_R^i) + \bar{\mu}_R^+\gamma_\mu s_R^i\}$$

$$+\frac{-g^2}{8M^{\dagger 2}}(\epsilon_{ijk}\bar{u}_L^{ck}\gamma^\mu(d_L^j\cos\theta_c + s_L^j\sin\theta_c)][\bar{\nu}_{eR}^c\gamma_\mu d_R^i + \bar{\nu}_{\mu R}^c\gamma_\mu s_R^i + \text{h.c.}. \tag{50}$$

we can take $M = M^\dagger$. The equation (50) give the following quantitative predictions for relative decay rates:

$$\frac{\Gamma(N \longrightarrow \mu^+\text{non}-\text{strange})}{\Gamma(N \longrightarrow e^+\text{non}-\text{strange})} = \frac{\sin^2\theta_W\cos^2\theta_W}{(1+\cos^2\theta_W)^2+1} \tag{51}$$

$$\frac{\Gamma(N \longrightarrow e^+\text{strange})}{\Gamma(N \longrightarrow \mu^+\text{strange})} = \frac{\sin^2\theta_W\cos^2\theta_W}{(1+\sin^2\theta_W)^2+1} \tag{52}$$

The effective lagrangians (48) or (50) are derived in tree approximation from the exchange of a single boson. The next higher order corrections (and quantitatively the most important) are those shown in the graphs of fig 6.

These corrections due to gluon exchange can be summed using standard renormalization group techniques, with the result that $\mathcal{L}_{\Delta B=1}$ (50) is multiplied by anomalous dimension enhancement factor [BEGN]

$$A_3 = \left(\frac{\alpha_3(m_w^2)}{\alpha_{GUT}}\right)^{\frac{2}{[11+(4/3)n_G]}} \tag{53}$$

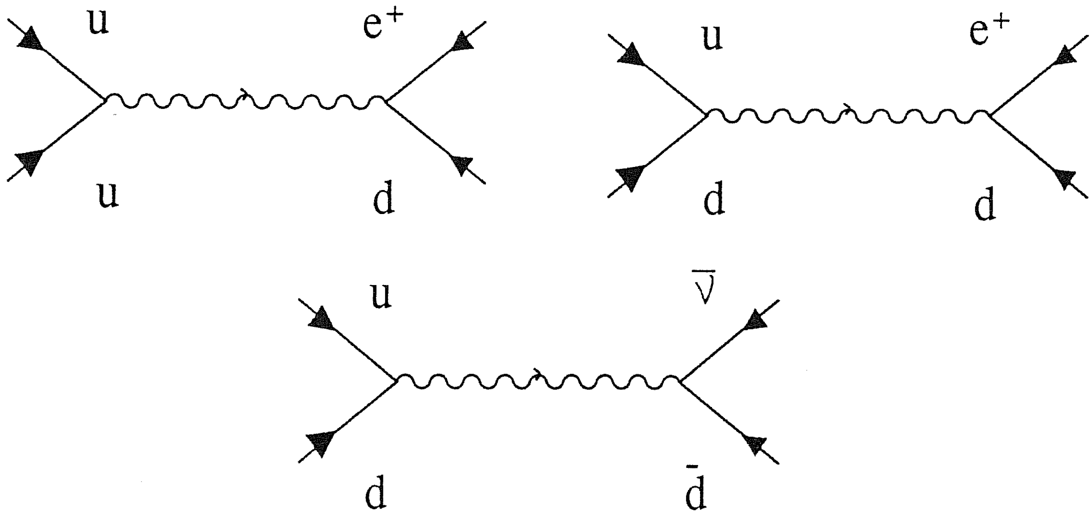

Fig. 5: B/L-violationg processes in $SU(5)$.

The contribution due to the exchange of the bosons of the electroweak group [EGN, WZe] give the factor (for the case of only one Higgs 5-plet)

$$A^1_{ew} = \left(\frac{\alpha_1(m_w^2)}{\alpha_{GUT}}\right)^{\frac{-33}{[6+80n_G]}} \left(\frac{\alpha_2(m_w^2)}{\alpha_{GUT}}\right)^{\frac{-27}{[86+16n_G]}} \tag{54}$$

$$A^2_{ew} = \left(\frac{\alpha_1(m_w^2)}{\alpha_{GUT}}\right)^{\frac{-69}{[6+80n_G]}} \left(\frac{\alpha_2(m_w^2)}{\alpha_{GUT}}\right)^{\frac{-27}{[86+16n_G]}} \tag{55}$$

where A^1_{ew} applies to the O_1 operator and A^2_{ew} to the O_2 operator, where:

$$O_1 = \epsilon_{ijk}\bar{u}^{ck}_L\gamma^\mu u^j_L \bar{e}^+_L\gamma^\mu d^i_L \tag{56}$$

$$O_2 = \epsilon_{ijk}\bar{u}^{ck}_L\gamma^\mu u^j_L \bar{e}^+_R\gamma^\mu d^i_R - \epsilon_{ijk}\bar{u}^{ck}_L\gamma^\mu d^j_L \bar{\nu}^c_{eR}\gamma^\mu d^i_R + \text{h.c..} \tag{57}$$

Therefore the decay rates are enhance by the factor $A^2_3(A^1_{ew})^2$ and $A^2_3(A^2_{ew})^2$ for O_1 and O_2 respectively.

Then finally, we mention the better results obtained for the minimal $SU(5)$ model, taking into account these corrections, plus the corrections due to the uncertainty in the hadronic matrix elements, as found by Vergados [Ver]:

$$\tau(p \longrightarrow e_+\pi^0) \approx 2.9 \times 10^{30} \left(\frac{M}{4\times 10^{14}}\right) \text{ years}$$

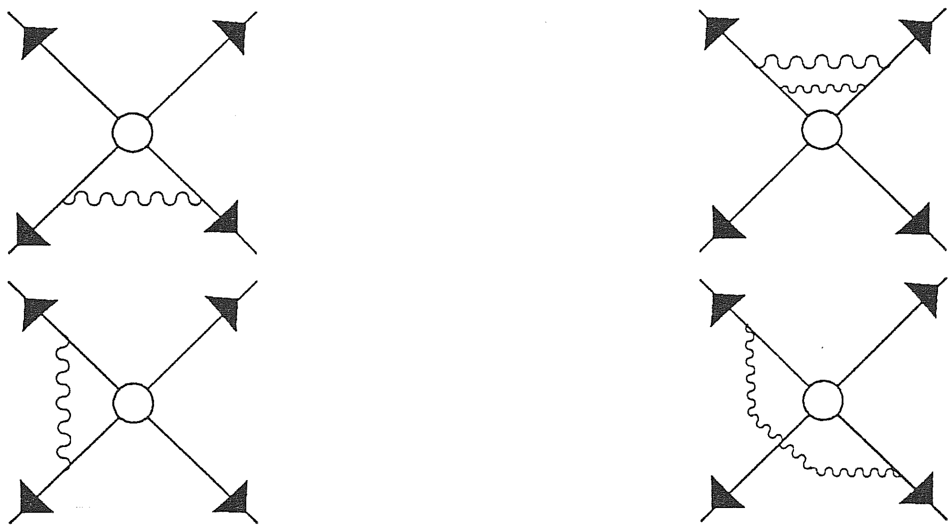

Fig. 6: Higher order correction to four-fermions vertices.

Using the better value for the unification mass, the result is:

$$\tau(p \longrightarrow e_+\pi^0) = 2.2 \times 10^{29} - 2.2 \times 10^{31} \text{ years}$$

for the mode with the higher branching ratio. The result taking into account all possible decay channels, in terms of the unification mass is [Ver]:

$$\tau(p \longrightarrow l^c m) = 6.9 \times 10^{28} - 6.9 \times 10^{30} \text{ years} \qquad (58)$$

Unfortunately this result is not in accord with the experimental limit. But if we introduce new parameters in the theory it is possible to be in accord with the experimental limit, however losing predictive power in the theory.

3.4 Conclusions

The $SU(5)$ model solves some questions that the Standard Model doesn't answer, but unfortunately not all of them. Moreover it has some theoretical problems principally in its minimal version. We present a list of the positive and negative features of this model.

The positive ones are:

1) The $SU(5)$ group has the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ group as a maximal subgroup (i.e. the rank of them are equal) and is the only group that give an acceptable theory with this property (The others are of larger rank).

2) The electric charge is quantized [i.e. $Q(\text{quark}) = -\frac{1}{3}Q(e^-)$] because the electric charge generator is a $SU(5)$ generator.

3) Due to the mixing of quarks and leptons within the same representations, **10** and $\bar{\mathbf{5}}$, their masses and isospin are correlated.

4) The prediction of $\sin^2\theta_W$ is in good agreement with the actual experimental value.

5) The predictions about the charged weak currents reproduces the good agreement with the experiments of the Standard Model.

6) $B - L$ is conserved and then automatically leads to massless ν's.

7) The simplest version has no flavour changing neutral currents as the Standard Model.

8) The anomalies of **10** and $\bar{\mathbf{5}}$ are equal and opposite, thus we obtain an anomaly free theory. In this structure there is no place for ν_R (unless we add it as a singlet. This doesn't destroy the anomaly cancellation because a singlet does not contribute to the anomaly).

9) In this model it is possible to solve the baryon-antibaryon asymmetry of the universe, because we can construct baryon number violation processes.

10) The prediction of $\sin^2\theta_W$ and $\tau_{p,n}$ are unique.

The negative ones are:

1) The $p \longrightarrow e^+\pi^0$ decay channel gives $\tau_p < 10^{31}$ years in the minimal version. The only way not to be in contrast with the experimental bound is to take more than only one **5** Higgs representation.

2) The $\mathbf{10} \oplus \bar{5}$ representation is reducible. In larger groups as in $SO(10)$ is reducible.

3) As in the Standard Model the number of generations is open. There are not theoretical restrictions about its number.

4) The wrong prediction of the minimal model about $\frac{m_d}{m_s}$. That can be resolved only adding another free parameters and consequent loss predictive power.

5) The production of superheavy 't Hooft-Polyakov monopoles causes cosmological problems.

6) The gravity is not included and the mass scales M and M_{Plank} are close.

7) The great desert between M and M_{Plank}, where no new physics occur.

8) The large arbitrariness connected with the presence of the Higgs Scalars . It is worse than in the Standard Model.

9) The hierarchy problem. We need a fine tuning of $\frac{m_w^2}{M^2} \approx 10^{-24}$.

4. B/L-violation in Partial Unified Theories

4.1 The Left-Right Model

The original motivation for the construction of the $L - R$ model is to reestablish at higher energies the symmetry of the Left and Right components of the fermion fields, and to explain the physical meaning of this lack of symmetry, at lower energies.

The model is based on the group [PSa,MPa,SMo]:

$$G_{L-R} = SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{(B-L)},$$

in order to preserve the $L - R$ invariance we need that the coupling constants g_L and g_R are equals ($g_L = g_R$). Therefore we have three coupling constants as in the Standard Model.

The quark and lepton fields are assigned to representations under G_{L-R} group as:

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3, \frac{1}{2}, 0, \frac{1}{3}), \qquad q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \sim (3, 0, \frac{1}{2}, \frac{1}{3}),$$

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1, \frac{1}{2}, 0, -1) \qquad \text{and} \qquad l_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \sim (1, 0, \frac{1}{2}, 1).$$

The electromagnetic charge is defined within this model as [MMa]:

$$Q = T_{3L} + T_{3R} + \frac{1}{2}(B - L),$$

then the $\frac{1}{2}(B - L)$ number becomes a gauge symmetry. Breaking this new local symmetry to the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry, we can have a violation of the form:

$$\Delta T_{3L} = -\Delta \frac{1}{2}(B - L).$$

The breaking of this symmetry can be made following the pattern:

$$G_{L-R} \longrightarrow G_{sm} \longrightarrow G_{es}$$

we made the first step of this breaking, taking a VEV of a Higgs field $<\Delta_R>\sim(1,0,2)$ at a scale M_R. The second step at a scale m_w by mean of the VEV's of the fields $<\Delta_L>\sim(0,1,2)$ and $<\phi>\sim(\frac{1}{2},\frac{1}{2},0)$. This second step is similar to that of the Standard Model; in the sense that we give mass to the fermions by mean of $<\phi>$; on the other hand $<\Delta_L>$ can give a Majorana mass to neutrino.

Let us write the Higgs fields as:

$$\Delta_{L,R} = \begin{pmatrix} \frac{1}{2}\delta^+ & \delta^{++} \\ \delta^0 & \frac{1}{\sqrt{2}}\delta^+ \end{pmatrix}_{L,R}$$

and

$$\phi = \phi_1 = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^1 & \phi_2^0 \end{pmatrix}, \quad \tilde{\phi} = \phi_2 = \begin{pmatrix} \phi_1^{0*} & -\phi_2^+ \\ -\phi_2^- & \phi_1^{0*} \end{pmatrix}$$

then the most general potential, which do not break explicitly the lepton number, is:

$$V = -\sum_{i,j=1}^{2}\mu_{ij}^2\mathrm{tr}(\phi_i^\dagger\phi_j) + \sum_{i,j,k,l=1}^{2}\lambda_{ijkl}\mathrm{tr}(\phi_i^\dagger\phi_j)\mathrm{tr}(\phi_k^\dagger\phi_l) + \sum_{i,j,k,l=1}^{2}\lambda'_{ijkl}\mathrm{tr}(\phi_i^\dagger\phi_j\phi_k^\dagger\phi_l)$$
$$-\mu^2\mathrm{tr}(\Delta_L^\dagger\Delta_L + \Delta_R^\dagger\Delta_R) + \rho_1[(\mathrm{tr}(\Delta_L^\dagger\Delta_L))^2 + (\mathrm{tr}(\Delta_R^\dagger\Delta_R))^2] + \rho_2(\mathrm{tr}(\Delta_L^\dagger\Delta_L\Delta_L^\dagger\Delta_L +$$
$$\Delta_R^\dagger\Delta_R\Delta_R^\dagger\Delta_R) + \rho_3\mathrm{tr}(\Delta_L^\dagger\Delta_L\Delta_R^\dagger\Delta_R) + \sum_{i,j=1}^{2}\alpha_{ij}^2\mathrm{tr}(\phi_i^\dagger\phi_j)\mathrm{tr}(\Delta_L^\dagger\Delta_L + \Delta_R^\dagger\Delta_R)$$
$$+\sum_{i,j=1}^{2}\beta_{ij}\left[\mathrm{tr}(\Delta_L^\dagger\Delta_L\phi_i^\dagger\phi_j) + \mathrm{tr}(\Delta_R^\dagger\Delta_R\phi_i^\dagger\phi_j)\right] + \sum_{i,j=1}^{2}\gamma_{ij}\mathrm{tr}(\Delta_L^\dagger\phi_i\Delta_R\phi_j^\dagger). \qquad (59)$$

The VEV's of the Higgs field which minimize this potential are:

$$<\Delta_L> = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \qquad <\Delta_R> = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

$$<\phi> = \begin{pmatrix} k & 0 \\ 0 & k'e^{i\alpha} \end{pmatrix}, \qquad <\tilde{\phi}> = \begin{pmatrix} k'e^{i\alpha} & 0 \\ 0 & k \end{pmatrix}$$

and to be in accord with the know phenomenology we need [MSe,MMa1] to take $v_Lv_R = \gamma k^2$, where

$$\gamma = \frac{2\gamma_{12}}{4(\rho_1+\rho_2) - 2\rho_3}$$

and $v_R \gg v_L$.

In first stage of the symmetry breaking by the $< \Delta_R >$, the boson fields $W_R^{\pm}$ acquire the mass

$$m_{W_R} = g v_R$$

and the combination $Z_R = \sqrt{\cos 2\theta_W}(\sec\theta_W)W_{3R} - \tan\theta_W B$ acquires the mass

$$m_{Z_R}^2 = m_{W_R}^2 \frac{2\cos^2\theta_W}{\cos 2\theta_W}.$$

In the second stage of the symmetry breaking by $< \phi >$ and $< \Delta_L >$, a mass is given also to the boson fields $W_L^{\pm}$. The relation $v_L \ll k, k'$ is suggested by the experimental data. Thus the major contribution is from $< \phi >$, but since ϕ transforms non-trivially under the left and right handed gauge groups it mixes the $W_L^{\pm}$ and $W_R^{\pm}$ fields. In fact we obtain the following eigenstates:

$$W_L \approx W_1 = \cos\zeta W_L + e^{i\alpha}\sin\zeta W_R$$

$$W_R \approx W_2 = -\sin\zeta e^{i\alpha} W_L + \cos\zeta W_R$$

with

$$\tan\zeta = \frac{kk'}{k^2 + k'^2 + 8v_R}.$$

The masses are:

$$m_{W_L}^2 \approx \frac{g^2}{2}(k^2 + k'^2) \equiv m_{W_1}^2$$

and

$$m_{Z_L}^2 \approx m_{Z_0}^2 \left[1 - \eta\frac{\cos 2\theta_W}{2}(1 - \frac{1}{4}\tan^4\theta_W)\right] + O(\eta^2).$$

$$m_{A_W} = 0$$

where Z_0 is the neutral boson field of the Standard Model and $\eta = \left(\frac{m_{W_L}}{m_{W_R}}\right)^2$.

The charged weak current

$$\mathcal{L}_{wk}^{CC} = \frac{g}{2}(j_{\mu L} \cdot W_{\mu L} + j_{\mu R} \cdot W_{\mu R})$$

and the neutral weak current

$$\mathcal{L}_{wk}^{NC} = ig(j_{\mu L}^3 W_{\mu L}^3 + j_{\mu R}^3 W_{\mu R}^3 + j_\mu^{B-L} B_\mu)$$

at low energies leads to the effective hamiltonians:

$$\mathcal{H}_{wk}^{CC} = \frac{G_F}{\sqrt{2}}[(\cos^2\zeta + \sin^2\zeta)j_{\mu L}^{+}j_{\mu L}^{-} + (\eta\cos^2\zeta + \sin^2\zeta)j_{\mu L}^{+}j_{\mu R}^{-}$$

$$+e^{i\alpha}\cos\zeta\sin\zeta(1-\eta j_{\mu L}^{+}\cdot j_{\mu R}^{-} + \text{h.c.}]$$

and

$$\mathcal{H}_{wk}^{NC} = \frac{4G_F}{\sqrt{2}}[j_L^Z j_L^Z + \eta\frac{\cos 2\theta_W}{\cos^4\theta_W}j_L^Z(j_L^Z\sin^2\theta_W + j_R^Z\cos^2\theta_W)$$

$$+\frac{1}{2\cos^4\theta_W}(j_L^Z\sin^2\theta_W + j_R^Z\cos^2\theta_W)]$$

where

$$A_\mu = \sin\theta_W(W_{L\mu}^3 + W_{R\mu}^3) + \sqrt{\cos 2\theta_W}B_\mu$$

$$Z_{L\mu} \approx (\sin\theta_W W_{L\mu}^3 - \sin\theta_W\tan\theta_W W_{R\mu}^3) - \tan\theta_W\sqrt{\cos 2\theta_W}B_\mu$$

$$Z_{R\mu} \approx \sqrt{\cos 2\theta_W}\sec\theta_W W_{R\mu}^3) - \tan\theta_W B_\mu.$$

and $j_{L,R}^Z = j_{L,R}^3 - A\sin^2\theta_W$ and $\tan\theta_W = \frac{g'}{g^2+g'^2}$. Therefore the modification to the H_{wk}^{NC} of the Standard Model is of order $\eta = (\frac{m_{W_L}}{m_{W_R}})^2$. Barnes *et. al.* [Bar] have done a systematic analysis of the neutral currents predictions of the $L-R$ symmetric model. Using their results in our case where $g_L = g_R$ we obtain $m_{Z_R} \geq 4m_{Z_L}$, which implies $m_{Z_R} \geq 220$ GeV.

We can have different type of process in the $L-R$ model. If we choose $\Delta T_{3R} = 1 \longrightarrow \Delta(B-L) = 2$ and we can have two cases:

i) $\Delta B = 0$, $\Delta L = 2$. With this kind of processes we give a majorana mass to the neutrinos.

ii) $\Delta B = 2$, $\Delta L = 0$. With this kind of processes we obtain neutron-antineutron oscillations.

After the SSB we construct the Yukawa term for leptonic sector for 1-generation

$$\mathcal{L}_{Yuk}^{Lepton} = -\left[h_1\bar{l}_{Li}\phi_j^i l_R^j + h_2\bar{l}_{Li}\tilde{\phi}_j^i l_R^i + h_5\left(\bar{l}_L^{ci}\epsilon_{ij}\Delta_{Rk}^i l_R^k\right)\right] + \text{h.c.}$$

where $\tilde{\phi} = \tau_2\phi^*\tau_2$ and we assume real Yukawa couplings. After the SSB we have:

$$\mathcal{L}_{mass}^{\nu} = -[(h_1k + h_2k')\bar{\nu}_L\nu_R + h_5 v_L\bar{\nu}_L^c\nu_L + h_5 v_R\bar{\nu}_R^c\nu_R] + \text{h.c.} \quad (60)$$

we can rewrite (60), taking $h_1 = h_2 = h$ as:

$$\mathcal{L}^{\nu}_{mass} = MN^TCN + hk\nu^TCN + h_5 v_L \nu^T C\nu + \text{h.c.}$$

$$= (\,\nu^T \quad N^T\,)\mathcal{M}C\begin{pmatrix}\nu\\ N\end{pmatrix} + \text{h.c.} \tag{61}$$

where

$$\mathcal{M} = \begin{pmatrix} h_5 v_l & \frac{1}{2}hk \\ \frac{1}{2}hk & M \end{pmatrix}, \tag{62}$$

$\nu = \nu_L$, $N = (\nu_R)^c = C(\nu_R)^T$, and $M = h_5 v_R$. After the diagonalization of the matrix (62), the two eigenstates are two Majorana spinors with masses

$$m_\nu \approx h_5 v_L + \frac{1}{4}h\frac{k^2}{v_R} = \left(h_5\gamma + \frac{1}{4}\frac{h^2}{h_5^2}\right)\frac{k^2}{v^R} \quad \text{and} \quad m_N \approx h_5 v_R \tag{63}$$

corresponding to the mass eigenstates:

$$\nu = \nu_L \cos\xi + \nu_R \sin\xi$$

$$N = -\nu_L \sin\xi + \nu_R \cos\xi$$

with $\xi \approx \left[\frac{m_\nu}{m_N}\right]^{\frac{1}{2}}$ and we can write the leptonic charged currents as :

$$\begin{pmatrix} \nu\cos\xi + N\sin\xi \\ e^- \end{pmatrix}_L$$

and

$$\begin{pmatrix} -\nu\sin\xi + N\cos\xi \\ e^- \end{pmatrix}_R .$$

If we take the limit $v_R \longrightarrow \infty$ in (63) thus $m_\nu \longrightarrow 0$ ($m_N \longrightarrow \infty$) and we recover the $V - A$ structure. Then the smallness of light-neutrino is related with the suppression of the right-handed weak interactions at low energies.

4.2 Lepton number violation process

Due to the Dirac plus Majorana character of the neutrino mass matrices we have the violation of lepton number ($\Delta L \neq 0$) and the presence of flavour changing neutral weak effect in the leptonic sector. The lepton number violation can be obtained also through

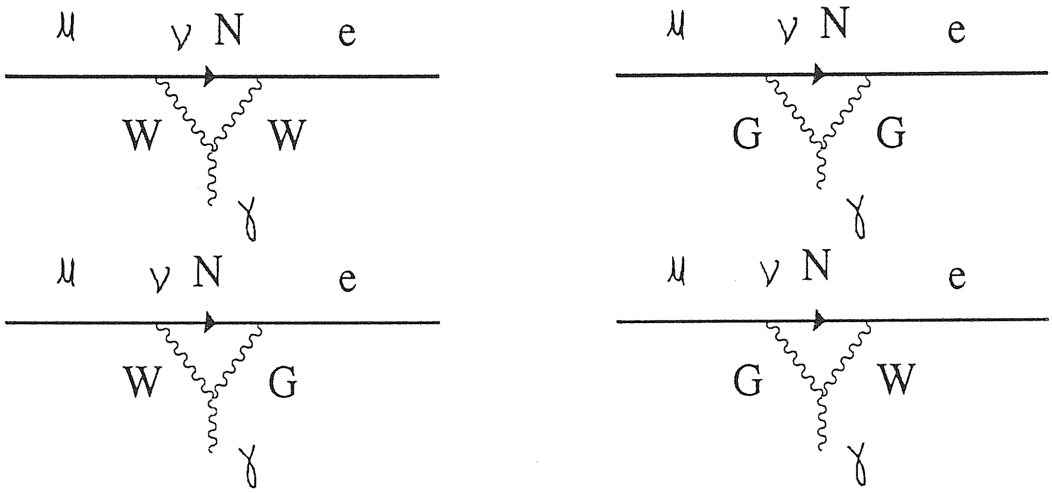

Fig. 7: Feynman diagrams for $\mu \longrightarrow e\gamma$ process.

the radiative corrections involving right handed neutrinos as intermediate states. The last sources of lepton number violation give processes like:

$$\nu_\mu \longrightarrow 3\nu_e \qquad \mu \longrightarrow 3e \qquad \mu \longrightarrow e\gamma \qquad \text{etc.}$$

The absence of $\mu \longrightarrow e\gamma$ decay has been a crucial argument in the lepton flavour assignment [GMF,Fei] and in the conclusion that $\nu_\mu \neq \nu_e$ in the Standard Model. But it is possible to obtain this kind of process in a simple extension of the Standard Model, adding a right handed component of the neutrino field to the usual field content and making radiative corrections at least at one loop. The amplitude of this process is [Rah]:

$$m(\mu \longrightarrow e\gamma) = \epsilon^\mu < e \mid j_\mu \mid \mu > \tag{64}$$

where ϵ^μ is the polarization 4-vector of the photon and j^μ is the electromagnetic current associated with the $\mu - e$ vertex which can be put in the form:

$$j_\mu = (f_0 + \gamma_5 f_{05})(q^\mu q^\nu - g^{\mu\nu} q^2)\gamma_\nu + (f + f_5\gamma_5)\frac{i}{2}\sigma^{\mu\nu} q_\nu \tag{65}$$

where $\sigma_{\mu\nu} = \frac{1}{2i}[\gamma_\mu, \gamma_\nu]$ and q_μ is the momemtum transfer. The dominating diagrams for $\mu \longrightarrow e\gamma$ are in figure 7.

where $G_{L,R}$ are the Goldstone bosons and we consider $m_H \gg m_w$, thus the contribution of the physical Higgs particles are ignored.

From (65) and taking into account that the photon is on-shell ($q^2 = 0$ and $\epsilon^\mu q_\mu = 0$) we obtain:

$$m(\mu \longrightarrow e\gamma) = \epsilon^\mu \bar{e}(f + f_5\gamma_5) i m_\mu \sigma_{\mu\nu} q^\nu \mu \tag{66}$$

From (66) we obtain [MS80] the decay width

$$\Gamma(\mu \longrightarrow e\gamma) = \frac{m_\mu^5}{8\pi}\left(| f |^2 + | f_5 |^2\right) \tag{67}$$

where $f = f_\nu$ when we take the ν propagator, $f = f_N$ when we take the N propagator and

$$f_\nu = f_{5\nu} \approx \frac{e}{16\pi^2}\frac{g^2}{8m_{W_L}^2}\delta_\nu,$$

$$f_N = f_{5N} \approx \frac{e}{16\pi^2}\frac{g^2}{8m_{W_R}^2}\delta_N$$

and

$$\delta_\nu = \sum_i U_{Li1}^* U_{Li2}\frac{m_{\nu_i}^2}{m_{W_R}^2}$$

$$\delta_N = \sum_i U_{Ri1}^* U_{Ri2}\frac{m_{N_i}^2}{m_{W_R}^2}$$

where $i = e, \mu, \tau$ and $U_{L,R}$ are the analogous in the leptonic sector the Cabibbo-Kobayashi-Maskawa matrix.

For two generations we have:

$$\delta_\nu = \sin\theta_L \cos\theta_L \frac{m_{\nu_e}^2 - m_{\nu_\mu}^2}{m_{W_L}^2}$$

$$\delta_N = \sin\theta_R \cos\theta_R \frac{m_{N_e}^2 - m_{N_\mu}^2}{m_{W_R}^2}$$

since in this case

$$U_{L,R} = \begin{pmatrix} \cos\theta_{L,R} & \sin\theta_{L,R} \\ -\sin\theta_{L,R} & \cos\theta_{L,R} \end{pmatrix}$$

The usual lepton number changing muon decay is:

$$\Gamma(\mu \longrightarrow e\nu_\mu\nu_e) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

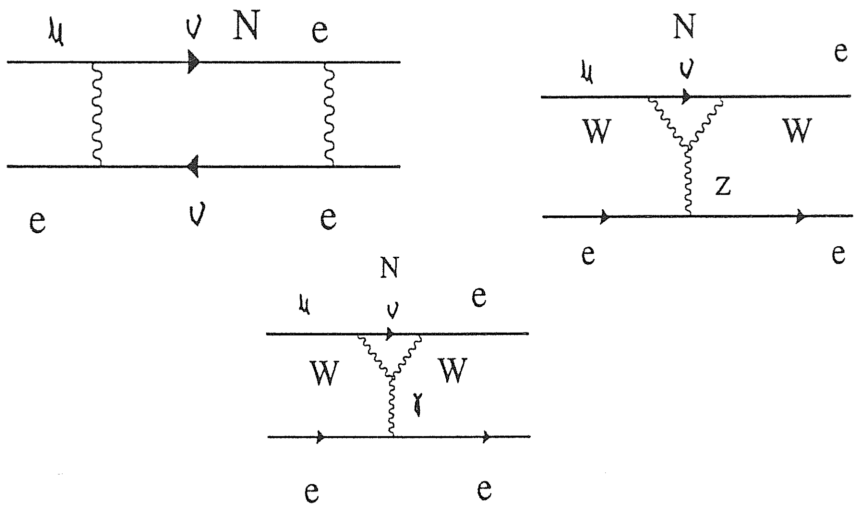

Fig. 8: Feynman diagrams for $\mu \longrightarrow ee\bar{e}$ process.

and we define the branching ratio

$$B(\mu \longrightarrow e\gamma) = \frac{\Gamma(\mu \longrightarrow e\gamma)}{\Gamma(\mu \longrightarrow e\nu_\mu\nu_e)} = B^\nu(\mu \longrightarrow e\gamma) + B^N(\mu \longrightarrow e\gamma) + B^{\nu N}$$

$$\approx \frac{\alpha}{\pi}\left(\delta_\nu^2 + \left(\frac{m_{W_L}}{m_{W_R}}\right)^4 \delta_N^2 + \left(\frac{m_{W_L}}{m_{W_R}}\right)^4 \delta_N \delta_\nu\right) \qquad (68)$$

We have that $\delta_\nu \ll \delta_N$ thus in (68) we have that δ_ν^2 and $\delta_\nu\delta_N$ are negligible, then (5x) becomes:

$$B(\mu \longrightarrow e\gamma) = \frac{\alpha}{\pi}\left(\frac{m_{W_L}}{m_{W_R}}\right)^4 \delta_N^2$$

If we take the value $\delta_N \approx 10^{-2} - 10^{-3}$ we have:

$$\frac{\alpha}{\pi}\left(\frac{m_{W_L}}{m_{W_R}}\right) = \frac{1}{100} \quad \text{thus} \quad B(\mu \longrightarrow e\gamma) = 10^{-11} - 10^{-13}$$

This is well below the best experimental limit [PD88,Kin]:

$$B(\mu \longrightarrow e\gamma) < 5 \times 10^{-11}.$$

Another interesting phenomenon that violates the lepton number is the $\Gamma(\mu \longrightarrow ee\bar{e})$ process. The Feynman diagrams that contributes to $\Gamma(\mu \longrightarrow ee\bar{e})$ are in figure 8.

the branching ratio obtained in this case is [SMo]:

$$B(\mu \longrightarrow ee\bar{e}) = \frac{\Gamma(\mu \longrightarrow ee\bar{e})}{\Gamma(\mu \longrightarrow e\nu_\mu\bar{\nu}_e)}$$

the final result is that:

$$\frac{B(\mu \longrightarrow ee\bar{e})}{B(\mu \longrightarrow e\gamma)} \approx \frac{\alpha}{\sin\theta_W} \approx (1 - 10\%)$$

the experimental limit in this case is [Kor,PD88]:

$$B(\mu \longrightarrow ee\bar{e}) < 1 \times 10^{-12}.$$

and consequently is easier to detect experimentally the first process.

4.3 Conclusions

The Left-Right model has like the $SU(5)$ model, positive and negative features. Among the positive ones we have:

1) It is possible to generate neutrino mass. Experimentally is not clear whether the mass of the neutrino vanishes.

2) It generates parity violation.

3) It gives physical meaning to the $U(1)$ generator, because it is the $B - L$ quantum number.

4) It correlates the smallness of CP violation with the suppression of the $V + A$ currents.

5) The suppression of the $V + A$ currents is correlated too with the smallness of neutrino mass.

The negative features can be resumed in only one. The model is not a unified theory, and its predictive power is limited. It should be incorporated in a more complete theory.

Beyond SU(5) and Left-Right Models

5.1 The SO(10) Model

The basic assumption in Unified Gauge Theories is to propose a gauge group in which we can accommodate all the known phenomenology. After the $SU(5)$ group, the next unification group that permits to accommodate such known phenomenology is the $SO(10)$ group [Geo,FMi]. On the other hand this group has several advantages over the $SU(5)$ group. For example it is possible to accommodate all fermions of one generation only in the 16-d spinor representation (which is automatically anomaly free). The $SO(10)$ conserves parity. Another advantage is that it is the minimal L-R symmetric gauge unified theory that gauges the $B-L$ symmetry.

In order to simplify the discussion of the $SO(10)$ group, we discuss it in terms of a $SU(5)$ basis.

Let χ_i $(i=1,...,n)$ and their hermitian conjugate $\chi_i^\dagger$ be operators that satisfy the anticommutation relations:

$$\{\chi_i, \chi_j^\dagger\} = \delta_{ij} \qquad \text{and} \qquad \{\chi_i, \chi_j\} = 0.$$

We can construct the operators $T^i_j \equiv \chi_i^\dagger \chi_j$, that satisfy the algebra of the $U(N)$ group (i.e. $[T^i_j, T^k_l] = \delta^k_j T^i_l - \delta^i_l T^k_j$). Now we can define the $2N$ operators Γ_μ $(\mu = 1,...,2N)$

$$\Gamma_{2j-1} = -i(\chi_j - \chi_j^\dagger) \qquad \text{and} \qquad \Gamma_{2j} = (\chi_j + \chi_j^\dagger) \quad (j=1,...,N)$$

we can verify that the operators Γ_μ form a Clifford algebra of rank $2N$ ($\{\Gamma_\nu, \Gamma_\mu\} = \delta_{\mu\nu}$). Finally we can construct the generators of the $SO(2N)$ group as follows:

$$\Sigma_{\mu\nu} = \frac{1}{2i}[\Gamma_\mu, \Gamma_\nu]$$

or written in terms of χ_j and $\chi_j^\dagger$ as:

$$\Sigma_{2j-1,2k-1} = \frac{1}{2i}[\chi_j, \chi_k^\dagger] - \frac{1}{2i}[\chi_k, \chi_j^\dagger] + i(\chi_j\chi_k + \chi_j^\dagger\chi_k^\dagger)$$

$$\Sigma_{2j,2k-1} = \frac{1}{2}[\chi_j, \chi_k^\dagger] + \frac{1}{2}[\chi_k, \chi_j^\dagger] - (\chi_j\chi_k + \chi_j^\dagger\chi_k^\dagger)$$

$$\Sigma_{2j,2k} = \frac{1}{2i}[\chi_j, \chi_k^\dagger] - \frac{1}{2i}[\chi_k, \chi_j^\dagger] - i(\chi_j\chi_k + \chi_j^\dagger\chi_k^\dagger).$$

In our case for $N = 5$, the spinor representation of $SO(10)$ is $2^5 = 32$ dimensional. To write it in terms of the $SU(5)$ basis let us define a *vacuum* state $\mid 0 >$ which is $SU(5)$ invariant.

We can split this representation into 2^{N-1} dimensional $(16-d)$ representation under a chiral projection operator:

$$\psi_\pm = \frac{1}{2}(1 \pm \Gamma_0)\psi$$

$$\Gamma_0 = i^N \Gamma_1\Gamma_2 ... \Gamma_N \qquad (\Gamma_0 = \Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5 \text{ in our case})$$

that has the following property (as the Dirac γ_5 matrix):

$$[\Sigma_{\mu\nu}, \Gamma_0] = 0$$

Thus we can represent all the left handed fermions of one generation in the basis

$$\mid \psi > = \mid 0 > \psi_0 + \chi_j^\dagger \mid 0 > \psi_j + \frac{1}{2}\chi_j^\dagger\chi_k^\dagger \mid 0 > \psi_{jk} + \frac{1}{12}\chi_k^\dagger\chi_l^\dagger\chi_m^\dagger \mid 0 > \bar{\psi}_{ji}$$

$$+ \frac{1}{24}\epsilon^{jklm}\chi_k^\dagger\chi_l^\dagger\chi_m^\dagger\chi_n^\dagger \mid 0 > \bar{\psi}_j + \chi_1^\dagger\chi_2^\dagger\chi_3^\dagger\chi_4^\dagger\chi_5^\dagger \mid 0 > \bar{\psi}_0$$

where $\bar{\psi}$ are independent of ψ,

$$\psi = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix}$$

and

$$\psi^+ = \begin{pmatrix} \psi_0 \\ \psi_{ij} \\ \bar{\psi}_j \end{pmatrix}, \qquad \psi^- = \begin{pmatrix} \bar{\psi}_0 \\ \bar{\psi}_{ij} \\ \psi_j \end{pmatrix}.$$

The decomposition of the **16** representation of the $SO(10)$ to the $SU(5)$ is:

$$\mathbf{16} = \mathbf{10} \oplus \bar{\mathbf{5}} \oplus \mathbf{1}$$

thus making the identification

$$\bar{\psi}_i \longrightarrow \bar{\mathbf{5}} \qquad \psi_{ij} \longrightarrow \mathbf{10} \qquad \psi_0 \longrightarrow \mathbf{1}$$

we finally obtain:

$$\psi_+ = \begin{pmatrix} \nu \\ u_1 \\ u_2 \\ u_3 \\ e^- \\ d_1 \\ d_2 \\ d_3 \\ -d_1^c \\ +d_2^c \\ +d_3^c \\ -e^+ \\ +u_3^c \\ -u_2^c \\ -u_1^c \\ \nu^c \end{pmatrix}_L . \tag{69}$$

$i.e$ all the fermions belongs to one generations can be accommodated in ψ_+. Now we can define the covariant derivative as:

$$(D_\mu)^\kappa = \partial_\mu \psi_+^\kappa + \frac{1}{2} ig \Sigma_{\kappa\lambda}^{\sigma\nu} A_\mu^{\sigma\nu} \psi_+^\kappa$$

where $A^{\sigma\nu}$ are the gauge bosons belong to the adjoin **45** dimensional representation of $SO(10)$. This gauge bosons can be identified if we decompose the **45** representation with respect to $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R$.

$$\mathbf{45} = (8,1,1) + (1,3,1) + (1,1,3) + (1,1,1) + (\bar{3},2,2) + (3,2,2) + (3,1,1) + (\bar{3},1,1)$$

thus the identification of the gauge bosons of the Standard Model, the $SU(5)$ Model and the $L-R$ Model is obviously:

$$\mathbf{45} = G_\beta^\alpha + W_L^{1,2,3} + W_R^{1,2,3} + B'_\mu + \begin{pmatrix} X & \bar{Y}' \\ Y & \bar{X}' \end{pmatrix} + \begin{pmatrix} X' & \bar{Y} \\ Y' & \bar{X} \end{pmatrix} + X_3 + \bar{X}_3 .$$

The electromagnetic charge operator is:

$$Q = \frac{1}{2}\Sigma_{78} - \frac{1}{6}(\Sigma_{12} + \Sigma_{34} + \Sigma_{56})$$

In general the mass terms in the lagrangian are of the form:

$$\tilde{\psi} BC^{-1}\Gamma_\mu \psi \phi_\mu, \quad \tilde{\psi} BC^{-1}\Gamma_\mu \Gamma_\nu \psi \phi_{\mu\nu} \quad \text{or} \quad \tilde{\psi} BC^{-1}\Gamma_\mu \Gamma_\nu \Gamma_\lambda \psi \phi_{\mu\nu\lambda}$$

$$\tilde{\psi}BC^{-1}\Gamma_\mu\Gamma_\nu\Gamma_\lambda\Gamma_\sigma\psi\phi_{\mu\nu\lambda\sigma} \quad \text{or} \quad \tilde{\psi}BC^{-1}\Gamma_\mu\Gamma_\nu\Gamma_\lambda\Gamma_\sigma\Gamma_\alpha\psi\phi_{\mu\nu\lambda\sigma\alpha} \qquad \text{etc...} \qquad (70)$$

where $\tilde{\psi}$ is the transpose of ψ, B is the equivalent to the charge conjugation matrix in the $SO(10)$ Model, C the usual charge conjugation matrix and ϕ_μ, $\phi_{\mu\nu}$, $\phi_{\mu\nu\lambda}$, $\phi_{\mu\nu\lambda\sigma}$ and $\phi_{\mu\nu\lambda\sigma\alpha}$ are Higgs mesons.

The spinor $\tilde{\psi}$ does not transform like a conjugate spinor representation of $SO(2N)$, but if we define the B matrix in such a way that:

$$B^{-1}\tilde{\Sigma}_{\mu\nu}B = -\Sigma_{\mu\nu}$$

thus $\tilde{\psi}B$ behaves like the conjugate of ψ and is possible to show that [Moh1]:

$$B \cdot \begin{pmatrix} \bar{\psi}_0 \\ \bar{\psi}_{ij} \\ \psi_j \\ \psi_0 \\ \psi_{ij} \\ \bar{\psi}_j \end{pmatrix} = \begin{pmatrix} \bar{\psi}_0 \\ -\bar{\psi}_{ij} \\ \psi_j \\ \psi_0 \\ \psi_{ij} \\ -\bar{\psi}_j \end{pmatrix}.$$

Counting the degrees of freedom in the mass terms in (70). The first term (Γ_μ) give 10, the second term ($\Gamma_\mu\Gamma_\nu$) give 45, the third term ($\Gamma_\mu\Gamma_\nu\Gamma_\lambda$) give 120, the fourth term ($\Gamma_\mu\Gamma_\nu\Gamma_\lambda\Gamma_\sigma$) give 210 and the last term ($\Gamma_\mu\Gamma_\nu\Gamma_\lambda\Gamma_\sigma\Gamma_\alpha$) give 252 (really $126 = \frac{252}{2}$ because this last operator is totally antisymmetric).

According with (69) all the fermions are in a **16** representation, then the mass term are of the form

$$\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126}$$

thus ϕ_μ, $\phi_{\mu\nu\lambda}$ and $\phi_{\mu\nu\lambda}$ corresponds to the **10**, **120** or **126** dimensional Higgs fields, respectively. The terms with an even number of Γ's are not useful because corresponds to the tensor product

$$\overline{\mathbf{16}} \otimes \mathbf{16} = \mathbf{1} \oplus \mathbf{45} \oplus \mathbf{210}$$

that includes a mix between the $\overline{\mathbf{16}}$ and **16** representations. The decomposition of these representations to the $SU(5)$ representation are:

$$\mathbf{10} = \mathbf{5} \oplus \bar{\mathbf{5}},$$

$$\mathbf{126} = \mathbf{1} \oplus \bar{\mathbf{5}} \oplus \mathbf{10} \oplus \overline{\mathbf{1}}\mathbf{5} \oplus \mathbf{45} \oplus \bar{\mathbf{5}}\mathbf{0},$$

$$\mathbf{120} = \mathbf{5} \oplus \bar{\mathbf{5}} \oplus \mathbf{10} \oplus \bar{\mathbf{10}} \oplus \mathbf{45} \oplus \bar{\mathbf{45}},$$

then we can construct the following mass terms:

$$\tilde{\psi}_L BC^{-1}\Gamma\psi_L\phi_{10} = \phi_{10}(5)(\bar{u}_R u_L + \bar{\nu}_R \nu_L) + \phi_{10}(\bar{5})(\bar{d}_R d_L + \bar{e}_R e_L)$$

$$\tilde{\psi}_L BC^{-1}\Gamma\psi_L\phi_{126} = \phi_{126}(1)\nu_R^T\sigma_2\nu_R + \phi_{126}(15)\nu_L^T\sigma_2\nu_L$$
$$+\phi_{126}(5)(\bar{u}_R u_L - 3\bar{\nu}_R\nu_L) + \phi_{126}(\bar{4}5)(\bar{d}_R d_L - 3\bar{e}_R e_L)$$

$$\tilde{\psi}_L BC^{-1}\Gamma\psi_L\phi_{120} = \phi_{120}(5)\bar{u}_R u_L + \phi_{120}(45)\bar{\nu}_R\nu_L$$
$$+\phi_{120}(\bar{5})(\bar{d}_R d_L + \bar{e}_R e_L) + \phi_{120}(\overline{45})(\bar{d}_R d_L - 3\bar{e}_R e_L)$$

These are all the possible terms that can be obtained. The choice of one or more of them is conditioned by phenomenological requirements.

There are different ways to break the $SO(10)$ symmetry (almost 20). We will shortly discuss now some of them.

The first pattern is:

$$\begin{array}{c} SO(10) \\ M_U \quad \downarrow \quad \{16\} \\ SU(5)\otimes U(1) \\ M_X \quad \downarrow \quad \{45\} \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\ M_X \quad \downarrow \quad \{10\},\{120\},\{126\} \\ SU(3)_C \otimes U(1)_{em} \end{array}$$

the first step the breaking can be realized by a 16-d Higgs field, the second by a 45-d Higgs field and the last by a 10-d Higgs field in its minimal version (That presents the same problems as the original $SU(5)$ Model. It is possible to make the last breaking too with the 126-d or with the 120-d Higgs Field. But this symmetry breaking is not so interesting, because the proton lifetime predicted is less that of the $SU(5)$ Model, as we will show forward [CMP].

The second pattern is:

$$\begin{array}{c} SO(10) \\ M_U \quad \downarrow \quad \{45\} \\ SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \\ M_R \quad \downarrow \quad \{126\} \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\ m_w \quad \downarrow \quad \{10\}, \{120\}, \{126\} \\ SU(3)_C \otimes U(1)_{em} \end{array}$$

in this second pattern we the break of $SO(10)$ through a 45-d Higgs field we obtain the $L-R$ Model discussed previously. Then we can reproduce all its principal features, with the advantage that now is part of a unified theory. The second symmetry breaking was made with $\Delta_R \sim$126-d and the last to the exact symmetry by mean of a 10-d, a 120-d or a 126-d (different from that of the previous step) Higgs fields as in the first pattern.

The third pattern is:

$$\begin{array}{c} SO(10) \\ M_U \quad \downarrow \quad \{54\} \\ SU(4)_{ps} \otimes SU(2)_L \otimes SU(2)_R \\ M_R \quad \downarrow \quad \{126\} \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\ m_w \quad \downarrow \quad \{10\}, \{120\}, \{126\} \\ SU(3)_C \otimes U(1)_{em} \end{array}$$

the first breaking of the symmetry can be performed by mean of a 54-d Higgs Field (obtaining the so called Pati-Salam group, which we will discuss forward in this section), the second step to the G_{sm} group through a 126-d Higgs field and the last in the same form of the previous pattern and the last as in the previous patterns.

The fourth pattern is:

$$
\begin{array}{c}
SO(10) \\
M_U \qquad \downarrow \qquad \{54\} \\
SU(4)_{ps} \otimes SU(2)_L \otimes SU(2)_R \\
M_C \qquad \downarrow \qquad \{45\} \\
SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1) \\
M_R \qquad \downarrow \qquad \{126\} \\
SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\
m_w \qquad \downarrow \qquad \{10, 120, 126\} \\
SU(3)_C \otimes U(1)_{em}
\end{array}
$$

in this pattern the symmetry breaking is performed by a 54-d and a 45-d Higgs Fields for the first two steps, meanwhile the breaking of the last two symmetries proceeds in a way similar to that of the previous two cases.

In order to choose the more convenient (from the phenomenological point of view) path of the symmetry breaking we will use the following boundary conditions and the renormalization group equations.

If a unified group G is broken into G_1 as an intermediate step, and then in G_2. If M_1 and M_2 are their respective mass scales, in the limit $M_1 \to M_2$ when the mass scales coincides, thus both the mass scale and the value of $\sin^2\theta$ are given by the values obtained when G is broken directly into G_2.

Consider the following patterns

Fifth pattern:

$$\begin{array}{c} SO(10) \\ M_X \quad \downarrow \\ SU(4)_{ps} \otimes SU(2)_L \otimes SU(2)_R \\ M_C \quad \downarrow \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L} \\ M_R \quad \downarrow \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\ m_w \quad \downarrow \end{array}$$

and

Sixth pattern:

$$\begin{array}{c} SO(10) \\ M_X \quad \downarrow \\ SU(4)_{ps} \otimes SU(2)_L \otimes SU(2)_R \\ M_R \quad \downarrow \\ SU(4) \otimes SU(2)_L \otimes U(1)_R \\ M_C \quad \downarrow \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L} \\ M' \quad \downarrow \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\ m_w \quad \downarrow \\ SU(3)_C \otimes U(1)_{em} \end{array}$$

If $M_C \rightarrow M_X$ and $M_R \rightarrow M_X$ in the left and right paths respectively thus the $SU(4)_{ps} \otimes SU(2)_L \otimes SU(2)_R$ is eliminated. Now using the renormalization group equations and the above boundary conditions was found [TBM] that the intermediate scales obeys:

$$A\ln(M_X/M_5) + B\ln(M_5/M_j) + C(\sin^2\theta - \sin^2\theta_5) = 0,$$

where M_j denotes the intermediate mass scales A, B, C are numerical constants M_5 and θ_5 are the mass and the Weinberg angle respectively, calculated in the minimal $SU(5)$ Model. Finally neglecting the contribution of the Higgs bosons, we obtain:

$$\ln\left(\frac{M_X}{M_5}\right) + \frac{1}{2}\ln\left(\frac{M_5}{M_C}\right) = \frac{3\pi\Delta}{11\alpha} \tag{76}$$

$$\ln\left(\frac{M_5}{M_C}\right) + \ln\left(\frac{M_5}{M_R}\right) = \frac{6\pi\Delta}{11\alpha} \tag{77}$$

where $\Delta = \sin^2\theta - \sin^2\theta_5$. From these two equations (76) and (77) we obtain:

$$\ln\left(\frac{M_X}{M_5}\right) = \frac{1}{2}\ln\left(\frac{M_5}{M_R}\right) \Rightarrow M_X = M_5\left(\frac{M_5}{M_R}\right)^{\frac{1}{2}}$$

then the proton lifetime is corrected by a factor:

$$\tau_{10} = \tau_5\left(\frac{M_5}{M_R}\right)^2 \tag{78}$$

which is even independent of the scale M_C. From (78) we conclude that if $M_R < M_5$ thus the unification scale M_X for $SO(10)$ becomes higher, then the proton lifetime τ_p becomes longer. Therefore, a longer proton lifetime in the context of $SO(10)$ grand unification implies that there must exist an intermediate $L-R$ symmetric scale. After the elimination of the first pattern the choice of one of the remaining three patterns is more difficult, because there is no enough experimental information for that choice. And exhaustive study of the different patterns of symmetry breaking was made [CMGMP] including besides the groups discussed above the normal P-parity and the so called D-parity, defined in the following paragraph.

There exists an element of the $SO(10)$ that we call $D = \Sigma_{23}\Sigma_{67}$ $\Sigma_{\mu\nu}$ $\mu, \nu = 1, ..., 10$ are the 45 totally anti symmetry generators of the $SO(10)$ group which make

$$f_L \longrightarrow f_L^c = (C\bar{f}^T)_L \tag{80}$$

behaves almost like the parity operator but in general cannot be identified with the parity or charge conjugation operator; however, under special circumstance when all couplings in the lagrangian are real, it become the same as the parity operator.

After the study made by Chang, Mohapatra, Gipson, Marshak and Parida it was found that the most convenient symmetry breaking pattern is:

Seventh pattern

$$\begin{array}{c} SO(10) \\ M_U \quad \downarrow \quad \{54\} \\ SU(4)_{ps} \otimes SU(2)_L \otimes SU(2)_R \otimes P \\ M_P \quad \downarrow \quad \{210\} \\ SU(4)_{ps} \otimes SU(2)_L \otimes SU(2)_R \\ M_C \quad \downarrow \quad \{210\} \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L} \\ M_D \quad \downarrow \quad \{126\} \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\ m_w \quad \downarrow \quad \{10\}, \{120\}, \{126\} \\ SU(3)_C \otimes U(1)_{em} \end{array}$$

where P is the parity or charge conjugate operator.

It is favorable for low energy phenomenology *i.e.* leads to $M_R \approx M_C \approx 10^5$ GeV, $M_D \approx 1$ TeV for $M_U \approx 10^{16}$ GeV and $\sin^2\theta_w \approx 0.227$; and give testable predictions for events like $K_L^0 \to \mu\bar{\mu}$ decay, $N - \bar{N}$ oscillations and for proton lifetime.

In order to break the symmetry pattern in (79) we can choose the following Higgs fields: a 54-d for break the $SO(10)$ symmetry at a mass scale M_U, a 210-d for break the D-parity at a mass scale M_P, another 210-d for break the G_{ps} group at a mass scale M_C, a 126-d to go to G_{sm} at a mass scale M_{R^0} and the final step to G_{es}, with one or more of the Higgs fields that can be coupled to the fermions (10-d, 120-d or 126-d). As was shown in references [CMP] and [CMGMP] this pattern of symmetry breaking leads to $M_R \approx M_C \approx 10^5$ GeV, $M_{R^0} \approx 1$ TeV for a mass unification $M_U \approx 10^{16}$ GeV and $\sin^2\theta \approx 0.227$ for a D-parity breaking scale of 10^{14} GeV. The proton lifetime predicted at two loops [CMGMP] is:

$$\tau_P \approx 10^{35\pm2} \text{ years.}$$

Therefore there is no enough experimental information for to decide if the $SO(10)$ Model is right.

5.2 The $\mathbf{SU}(4)_{ps}\otimes$ $\mathbf{SU}(2)_L\otimes$ $\mathbf{SU}(2)_R$ Model

The $SO(10)$ group can be broken to the Pati-Salam group [PSa] by means of the different patterns previously presented. Choosing the third we found that the intermediate symmetry can be broken by the Higgs fields:

$$\phi \sim (0,2,2), \quad \Delta_L \sim (10,3,1) \quad \text{and} \quad \Delta_R \sim (10,1,3)$$

In the first step we need a VEV $< \Delta_R > \neq 0$ for breaks the right symmetry. Meanwhile in the second step is performed using $< \Delta_L > \neq 0$, and $< \phi > \neq 0$. We can accommodate the fermions as follows

$$\begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}_L \sim (4,2,0)$$

$$\begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}_R \sim (4,0,2)$$

Thus we can write the G_{ps} invariant Yukawa terms

$$\mathcal{L}^Y = f(\psi^T_{La} C^{-1}\tau_2\tau\psi_{Lb}\cdot\Delta^\dagger_{Lab} + L \to R) + \text{h.c.}$$

where $a,b = 1,2,3,4$ are the $SU(4)$ indices and f the Yukawa coupling constant. In this notation the $SU(2)_R$ breaking occurs due to

$$< \Delta^{1+i_2}_{R^{44}} > = v_R \neq 0,$$

to obtain B/L violation processes, note that, the most general Higgs potential contains a term of the form:

$$V' = \lambda\epsilon^{a_1a_2a_3a_4}\epsilon^{b_1b_2b_3b_4}\epsilon^{pq}\epsilon^{p'q'}\epsilon^{rs}\epsilon^{r's'} \times \Delta^{pp'}_{a_1b_1}\Delta^{qq'}_{a_2b_2}\Delta^{rr'}_{a_3b_3}\Delta^{ss'}_{a_4b_4} + \text{all permutation}$$

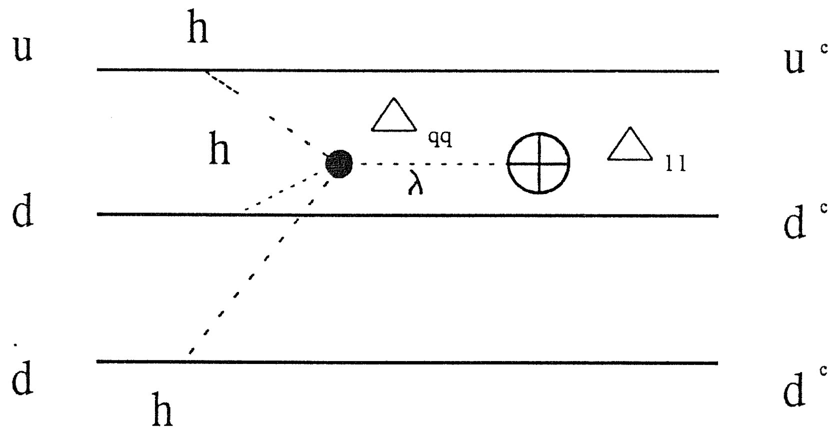

Fig. 9: Feynman diagram for $\bar{N} - N$ oscillations.

this term can produce $N \leftrightarrow \bar{N}$ oscillations according with the Feynman diagram of figure 9.

this graph leads to a estimate strength of the oscillations [MMa] of:

$$G_{\Delta B=2} \approx \frac{f^3 \lambda v_R}{m^6_{\Delta_{qq}}}$$

where f is the gauge coupling. Introducing renormalization group corrections to its strength; and then taking account of the hadronic wave functions [CMS,RSh,SMi,Che] it was found that is corrected by a factor 10^{-4} thus is reasonable to say that

$$\delta m \approx G_{\Delta B=2} \times 10^{-4} \text{ GeV}$$

if we choose $m_{\Delta_{qq}} \approx v_R \approx 30$ TeV as a typical value and $f \approx \lambda \approx 10^{-1}$ we obtain

$$\delta m \approx 10^{-31} \text{ GeV}$$

This corresponds to

$$\tau_{N-\bar{N}} \approx 10^7 \text{ sec}$$

just close to the experimental limit $\tau_{N-\bar{N}} \leq 10^7$ sec.

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