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# On the boundary gauge dual of closed tensionless free strings in AdS

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ABSTRACT: We consider closed free tensionless strings in  $AdS_d$ , calculate exactly the boundary/boundary string evolution kernel and find the string dynamics to be completely frozen. We interpret therefore the boundary configurations as Wilson loop operators in a confining phase. This is taken as an argument in favor to the dual weakly coupled abelian gauge theory being that of (d-4)-forms in the (d-1) dimensional boundary Minkowski space.

KEYWORDS: AdS-CFT and dS-CFT Correspondence, Bosonic Strings, BRST Quantization, Duality in Gauge Field Theories.

### Contents

1.	Introduction	1
2.	A partial gauge fixing	3
3.	Two (conjugate) boundary pictures	5
	3.1 Singular Polyakov strings	5
	3.2 Condensed strings	6
	3.3 A gauge dual interpretation	7
4.	Open questions	9

## 1. Introduction

The tensionless limit of string theory revealed surprising features. Its study was started in flat space-time background [1] and was continued in [2]–[5]. In flat space the masslessness of all the higher spin excitation in the string spectrum actually generates an enhancement of the symmetries which reveals [5] in the zero-mode sector being a root concept to understand the tension parameter as the most natural length parameter driving a consistent maximal symmetry breaking by the mass spectrum lift. The dynamical description, in the form of a giant "pantagruelic" BEH-like phenomenon [6], of such a mechanism is of topmost importance to understand the very nature of length scales in string theory.

In AdS/CFT duality, the tensionless limit was conjectured to drive the dual gauge Yang-Mills theory to its abelian phase [7], but the first convincing plausibility arguments came with [8]. This issues attracted some attention [9] and are subject of spread interest [10]. The connection with higher spin gauge theories [11, 12] intertwines these two topics. For more recent results see also [13].

Since we control much better abelian gauge theories than non abelian ones, this might be a better side to study this supposed duality. The difficult part from the string theory side is not finished, since also string interactions are difficult to quantify in curved spaces. Therefore, it seems that the easiest corner to look for a confirmation of the string/gauge theory duality is in the tensionless and free limit for closed strings in AdS to be compared with some abelian gauge theory in flat Minkowski space. The aim of this letter is to focus on that particular narrow corner and to give some arguments on that conjectured duality. Actually, the tensionless limit of strings in AdS simplifies very much the theory since an

extra gauge symmetry appears while conformal anomalies are absent due to a particular contraction of the Virasoro algebra. This was developed in [14] while here we apply those results to AdS/CFT duality.

Our approach is quite simple and follows by considering the AdS space an hyperboloid in an higher dimensional flat auxiliary space. In such a picture we implement the restriction to the hyperboloid as a lagrangean constraint to the free theory in the ambient space. The natural set-up for the analysis of such a problem turns out then to be the constrained hamiltonian formalism [15] (see also [16] and [17]). It turns out that the constraint algebra structure simplifies in the massless/tensionless case in such a way that the arising of a larger gauge symmetry takes place. More specifically, it happens that the geometric constraints appear as second class for generic values of the tension parameter, but in the tensionless limit it is possible to single out one half of them which, together with the tensionless Virasoro generators, are first class. This property opens the way to a BRST covariant quantization of the system which was given in [14].

In few words, the main point is that massless free excitations on AdS can not probe the strength of the space-time curvature and therefore, in this case, the assignment of the value of a finite AdS radius, being a non physical datum, should be regarded as a gauge choice (the assignment of a zero radius being a degenerate gauge fixing condition). Let us underline again that this property is special of massless free excitations on AdS space-time.

For other approaches to the problem, see [18] and [19]. In particular, in [19] the covariantization of the string field equations to negative curvature spaces is obtained.

In this letter we describe explicitly the bulk/boundary correspondence for tensionless free closed strings. The AdS bulk theory is that of closed tensionless free strings which is argued to correspond to an abelian Yang-Mills gauge theory on the boundary Minkowski space. This is obtained by calculating the boundary-boundary evolution path-integral for a closed string and showing that it reproduces the correlation function of two charge conjugate Wilson loop operators for an abelian Yang-Mills gauge theory at strong coupling. By this we mean just that the correlation vanishes unless the two loops coincide showing those as carrying a magnetic monopole charge frozen in the confining phase and having therefore no dynamics. Its electromagnetic dual weakly coupled abelian theory (seen as the zero coupling limit of a non-abelian one) is than that of a (d-4)-form which gets indicated by our result, in agreement with [8], as the weakly coupled gauge dual of the tensionless free closed string in  $AdS_d$ . Since the technique is quite model independent, our argument can be naturally extended to supersymmetric strings.

Let us notice that our resulting picture partly resides on an earlier conjecture by Polyakov, pushed forward in [20] and in [21], about gauge/string theory duality.

In the next section we partially fix the gauge of the tensionless bosonic closed free strings in AdS in a way which is compatible with the boundary conditions on  $\partial AdS$  in the string path integral. In section three we describe two pictures of the resulting boundary theory and give a plausible weakly coupled abelian gauge theory dual. A last section is left for few comments.

# 2. A partial gauge fixing

Let us model the dimension d AdS space as an hyperboloid in a flat d+1 dimensional space. Labeling the coordinates in  $\mathbb{R}^{d+1}$  as  $x^{\mu}$ , as  $\mu=0,\ldots,d$ , the embedding equation is simply

$$x^{\mu}\eta_{\mu\nu}x^{\nu} = -x_0^2 + x_1^2 + \dots + x_{d-1}^2 - x_d^2 = -R^2$$

which defines the symmetric quadratic form  $\eta$ . We will usually write  $UV = U^{\mu}\eta_{\mu\nu}V^{\nu}$  for various (co)vectors.

Let us start reviewing the constrained system of closed strings in  $\mathbb{R}^{d+1}$  bound to stay on the hyperboloid  $x^2 = -R^2$  in the tensionless limit.

The starting point is the  $\alpha'$  expansion of the string coordinates and momenta. This is designed in the ambient space as

$$X^{\mu}(\sigma) = x^{\mu} + \sqrt{\alpha'} A^{\mu}(\sigma)$$
 and  $P_{\mu}(\sigma) = \frac{1}{2\pi} p_{\mu} + \frac{1}{\sqrt{\alpha'}} B_{\mu}(\sigma)$ , (2.1)

where (x, p) are the zero modes and (A, B) are the oscillator parts which are periodic in  $[0, 2\pi]$ , are dimensionless and independent on  $\alpha'$ .

The study of this system amounts to the extraction of the leading order terms in  $\alpha'$  out of the Virasoro constrains

$$\frac{1}{2} \left[ 2\pi \alpha' P^2 + \frac{1}{2\pi \alpha'} (X')^2 \right] = 0 \quad \text{and} \quad X'P = 0$$
 (2.2)

and the geometric constrains

$$\frac{1}{\alpha'}(X^2 + R^2) = 0$$
 and  $XP = 0$ . (2.3)

In [14] it was shown that one can extract one half of the geometric constrains such that, after the tensionless limit has been taken, together with the contracted Virasoro this full set is a choice of a complete set of first class constraints for the system. Their implementation in a quantum BRST charge to give a well defined quantum theory proves that the above procedure is well defined quantum mechanically.

The most interesting consequence of what we said above is that the left half of the geometric constraints in the tensionless limit plays the role of a gauge fixing condition rather than having a physical content. Because of the result obtained in [14], we can work safely with the constrained functional path-integral methods where potential anomalies (from normal ordering ambiguities) could have been be less transparent at a first stage.

In our expansion (2.1), the contracted Virasoro are equivalent to

$$p^2 = 0 \qquad \text{and} \qquad \oint A'B = 0 \tag{2.4}$$

as far as the zero modes are concerned, while for the oscillators we have

$$pA = 0 \quad \text{and} \quad pB = 0. \tag{2.5}$$

The chosen half of the geometric constraints (namely, the zero mode of XP and the fluctuation of  $X^2 + R^2$ ) are then

$$xp + \oint AB = 0$$
 and  $AA' = 0$  (2.6)

In our parameterization, to reach the AdS boundary, one has to choose a null hyperplane<sup>1</sup> at will, lets say  $X_+ + \rho = 0$ , intersect it with the hyperboloid and then let the hyperplane position  $\rho$  go to infinity.

Since our aim is to calculate the boundary-boundary string propagator, we can view the hyperplane placement as a partial gauge fixing, compatible with the boundary conditions, for our gauge theory. More completely, we partially gauge fix to<sup>2</sup>

$$X_{+} + \rho = 0$$
 and  $P_{+} + \eta = 0$ , (2.7)

where  $\rho$  and  $\eta$  are constants in  $\sigma$ . The above conditions (2.7) can be rephrased using (2.1) just as  $x_+ + \rho = 0$ ,  $A_+ = 0$ ,  $p_+ + \eta = 0$  and  $B_+ = 0$ .

As it is clear from a simple degrees of freedom counting, the conditions (2.7) do not fix the gauge symmetry completely. The important aspect of the above partial gauge choice is that it fixes completely the degrees of freedom in the two directions which are transverse to the boundary of the AdS space and let us with a string path integral in the relevant d-1 Minkowski space.

In order to see this in details, we consider the following subset of the gauge algebra generators

$$p^2$$
 and  $xp + \oint AB$  (2.8)

out of the zero modes and

$$pA$$
 and  $pB$   $(2.9)$ 

for the oscillators. Notice that we have chosen to fix all the contracted Virasoro generators but the level matching condition (which generates constant translations in  $\sigma$  and is too important to get rid of) and the bulk global dilatation generator.

The generators (2.8) and (2.9) get fixed completely by the gauge fixing conditions (2.7). Using the standard Faddeev formula for constrained path-integrals [16] (see also [17]), this can be verified by calculating the determinant of the matrix of Poisson brackets<sup>3</sup> to be non zero for generic values of  $(\eta, \rho)$  and to get canceled completely by the factors needed to solve the delta-functions for the gauge fixed constraints in  $(x_-, p_-)$  and  $(A_-, B_-)$ . Moreover, it turns out that the matrix of Poisson brackets of the left over gauge generators, i.e.  $\oint BA'$  and AA', with the partial gauge fixing conditions is zero once the gauge fixing conditions themselves are fulfilled. The reduced action for the gauge fixed directions, corresponding

<sup>&</sup>lt;sup>1</sup>We use adapted light-cone coordinates so that as usual  $UV = U_+V_- + U_-V_+ + uv$ .

<sup>&</sup>lt;sup>2</sup>At a first stage the reader might be worried by the impression that we are forgetting some important geometrical data about the AdS space. Actually we are just making a finite gauge transformation to the system which is nothing else than a change in the explicit gauge fixing conditions.

<sup>&</sup>lt;sup>3</sup>We refer to the standard Poisson bracket  $\{X^{\mu}(\sigma), P_{\nu}(\sigma')\}_{PB} = \delta^{\mu}_{\nu}\delta(\sigma, \sigma')$ , etc. expanded as in (2.1).

to the chosen boundary conditions in the path integral, is  $\int_{t_i}^{t_f} d\tau \oint (P^+ \dot{X}_+ - X^- \dot{P}_+)$  and vanishes once the gauge fixing conditions (2.7) are taken into account. Notice that the resulting path integral is completely independent on the actual values of  $(\eta, \rho)$ . Therefore, after this gauge fixing operation is completed, we are left with the string path-integral along the d-1 left coordinates and momenta with the first class constraints

$$\oint ba' = 0 \quad \text{and} \quad aa' = 0,$$
(2.10)

where we denoted with (a, b) the d-1 dimensional components of (A, B) others than  $(A_{\pm}, B_{\pm})$ . The scalar product is taken with the transverse minkowskian metric with signature (1, d-2).

# 3. Two (conjugate) boundary pictures

After the previous section, we can now recollect the left over string theory path integral from the boundary point of view.

The hamiltonian is made out of gauge generators only, and reads

$$H = \oint \left[ \frac{\lambda}{2\pi} ba' + \Lambda aa' \right],$$

where  $\lambda$  is a constant Lagrange multiplier while, conversely,  $\oint \Lambda = 0$ . There are actually two relevant pictures, corresponding to two different types of boundary conditions, to discuss. In them both the center of mass variables are completely blocked at zero hamiltonian and therefore have no evolution<sup>4</sup>.

#### 3.1 Singular Polyakov strings

This picture corresponds to the boundary conditions fixing b. Then, the action is

$$S_b = \int \left[ \frac{1}{2\pi} p\dot{x} - a\dot{b} - \frac{\lambda}{2\pi} ba' - \Lambda aa' \right]. \tag{3.1}$$

$$\int D[X,P] \exp\left\{i\int d\tau \left[\dot{X}_{+}P_{-} - \dot{P}_{+}X_{-} + \dot{x}p\right]\right\} \delta(P^{2})\delta(XP)\delta(P_{+} + \eta)\delta(X_{+} + \rho)[\det(*)],$$

where  $[\det(*)]$  is the determinant of the matrix of Poisson brackets of the gauge generators and the gauge fixings which gives  $2\eta^2$ . This factor is exactly the one needed to solve the  $\delta$ -functions in the light cone variables. The result is then independent on the value of  $(\rho, \eta)$  and gives just  $\delta(x_i - x_f)$ , where  $x_i$  and  $x_f$  are the initial and final transverse positions. The string case is a generalization of this approach. A careful reader might like to have a more rigorous proof of the gauge equivalence of the light-cone model to the one in which the hyperboloid constraint is manifest. This follows by considering the family of gauge fixing conditions  $(X^2 + R^2)(1 - t) + (P_+ + \eta)t = 0$  with  $t \in [0, 1]$  and  $X_+ + \rho = 0$  in the above path integral. Varying t, corresponds to the finite gauge transformation to the model we consider, where the calculations are much easier.

<sup>&</sup>lt;sup>4</sup>This is in fact the fate that a massless free particle would have. For the massless particle the relevant path integral is

We can fix  $\lambda = 2\pi$  just by rescaling  $\tau \to \frac{2\pi}{\lambda}\tau$  and  $\Lambda \to \frac{\lambda}{2\pi}\Lambda$ . Then, integrating over a we get for the reduced system the action

$$S_{SPS} = \frac{1}{2} \int (-\Lambda')^{-1} (\dot{b} + b')^2$$
 (3.2)

which is the Polyakov action for the string oscillations  $\frac{1}{2} \int \sqrt{g} g^{\alpha\beta} \partial_{\alpha} b \partial_{\beta} b$  with the singular world-sheet metric  $\sqrt{g} g^{\alpha\beta} = (-\Lambda')^{-1} \delta^{\alpha}_{+} \delta^{\beta}_{+}$ .

The relevance of singular metrics in the context of gauge/string dualities was already considered in [21]. Notice that the boundary gauge algebra is very simple and reads

$$\left\{ \oint ba', aa' \right\}_{P.B.} = (aa')' \tag{3.3}$$

and the other brackets vanishing. This can be easily shown to correspond to a definite contraction of the Virasoro algebra (inequivalent to the tensionless limit one). Expanding in Fourier modes  $aa'(\sigma) = \sum_{n\neq 0} l_n e^{in\sigma}$  and defining  $l_0 = \oint ba'$  (recall that this was already in the center of another Virasoro before the tensionless contraction and generates displacements in  $\sigma$ ), we get  $[l_0, l_n] = -nl_n$  and the others commuting. This algebra follows from the usual Virasoro  $[L_n, L_m] = (n-m)L_{n+m}$  by redefining  $L_m = \kappa l_m$  for  $m \neq 0$  and  $L_0 = l_0$  and then letting  $\kappa \to \infty$ . This contraction should correspond to the singular limit of the world-sheet metric. Let us notice that the gauge algebra commutation relations are realized in terms of the differential operator  $\partial_{\sigma}$  and the operator of multiplication by an arbitrary periodic function in  $[0, 2\pi]$  without zero mode. This acts on the space of the parameterization of the circle and somehow should be related to the zig-zag symmetry transformations.

#### 3.2 Condensed strings

This second picture is more convenient in order to compare with abelian gauge theories and corresponds in keeping the string boundary conditions at fixed a. Therefore the action is

$$S_a = \int \left[ \frac{1}{2\pi} p\dot{x} + b\dot{a} - \frac{\lambda}{2\pi} ba' - \Lambda aa' \right]. \tag{3.4}$$

As a first step we get rid of  $\Lambda$  with a gauge transformation. The action is in fact invariant under the gauge symmetry

$$\delta_K a = 0$$
  $\delta_K b = K' a - \frac{1}{2\pi} \oint K' a$   $\delta_K \Lambda = \dot{K} - \frac{\lambda}{2\pi} K'$   $\delta_K \lambda = 0$  (3.5)

which we can use to fix  $\Lambda = 0$ . The second term in  $\delta_K b$  is forced by the projection in the space of functions without zeromodes. Notice that with the same procedure we can not fix also the constant Lagrange multiplier  $\lambda$  since the gauge generator  $\oint ba'$  corresponds to the following invariance of  $S_a$ 

$$\delta_k a = ka' \qquad \delta_k b = kb' \qquad \delta_k \Lambda = k\Lambda' \qquad \delta_k \lambda = 0$$
 (3.6)

which are just constant shifts in  $\sigma$ . As in the previous case, we rescale the hamiltonian time  $\tau \to \frac{\lambda}{2\pi} \tau$  (and  $\Lambda \to \frac{2\pi}{\lambda} \Lambda$  if we want to do it equivalently before fixing  $\Lambda = 0$ ) and now the action reads

$$S_a = \int \left[ \frac{1}{2\pi} p\dot{x} + b\dot{a} - ba' \right]. \tag{3.7}$$

Now, integrating over b, we get the following familiar constraint on the string oscillations

$$\dot{a} - a' = 0 \tag{3.8}$$

which fixes them to be chiral. Eq. (3.8) means that the string path-integral is restricted to worldsheet such that the time evolution of the string profile is equivalent to a reparameterization of the string profile itself. Therefore the evolution kernel vanishes for all the boundary conditions incompatible with such a constrain, that is for all but the ones for which the initial and the final string configurations coincide up to a constant shift in  $\sigma$ , i.e. up to a gauge transformation — we have not fixed  $\lambda$ . This means that in this string theory there is no dynamical evolution at all for the string configurations on the boundary.

## 3.3 A gauge dual interpretation

The picture we obtained in the last subsection has to be compared with d-1 dimensional Yang-Mills gauge theory. By comparing with the results of [6]–[10] and [13] we expect the gauge dual interpretation to be given in terms of an abelian gauge theory seen as a zero coupling limit of a non-abelian one.

Our reasoning concerning the dual interpretation goes as follows. We start by a strong version of string/gauge theory duality by which we mean that we expect that there exists a well defined string dual picture for YM theory in an AdS-like (i.e. non conformal in general) background, the conformal cases being realized as conformal isometric (AdS× something, where "something" realizes extra global symmetries) backgrounds. This duality is understood to be realized by a string theoretic expression for the Wilson loops of the gauge theory as string worldsheet path integrals with fixed boundary conditions on the loops locations.

In particular, therefore, we ask if also closed free tensionless strings in AdS have a gauge theory dual.

Since the theory is conformal invariant by construction (the BRST charge commutes with all the generators of the conformal group), we expect the gauge dual to be conformal too. Since the string theory is non supersymmetric, we believe that the gauge dual is neither and that, since the string theory is the simplest conceivable, free and does not contain any length-scale, the gauge dual has to be at a free point. This already selects out abelian gauge theories, seen as zero coupling limits of non-abelian ones, as unique candidates.<sup>5</sup>

<sup>&</sup>lt;sup>5</sup>We assume that pure non-abelian YM theory has no fixed point at intermediate couplings. This situation in principle could be produced by adding some matter not necessarly in a supersymmetric way. In this case, anyway, one expects some further global symmetry which would have no counterpart in the free closed tensionless string.

Therefore, our aim is to find if the boundary-boundary correlator for closed free tensionless strings has any gauge dual interpretation as loop-operators expectation in an abelian gauge theory. By the calculations done in the paper, the object at hand is just the  $\delta$ functional in the loop space between the initial and the final string boundary.

We identify the string (degenerate) propagator as the correlator between two charge conjugate Wilson loops in the compact version at the infinite coupling point. Showing this is simple once the model of monopole condensation by Polyakov is taken in consideration. I will shortly review it here for convenience (see [20]). The effective action describing the monopole resummation is given by

$$S = \int \left[ \frac{1}{4e^2} B^2 + i\phi dB + e^2 m^2 V(\phi) \right] + i \int_{\Sigma} B, \qquad (3.9)$$

where  $V(\phi) = 1 - \cos \phi$  and  $m^2 = m_0^2 e^{-e_0^2/e^2}$ . We take  $\Sigma$  being a Riemann surface interpolating between two closed paths. In the path integral the sum over the surfaces is done as well as the integration over B and  $\phi$ .

It is clear that the perturbative expansion at small coupling  $(e \ll e_0 \Rightarrow e^2 m^2 \to 0)$  reproduces the standard perturbative abelian Wilson loop just by integration over  $\phi$  which is enforcing the Bianchi identity on the 2-form B. This means that B = dA and the sum over the surfaces factorises since the coupling is effectively only to the fixed boundary, that is  $\int_{\Sigma} B = \int_{\partial \Sigma} A$ .

At intermediate couplings, the exact path integration reproduces the total monopole sum of the compact abelian theory on top of the perturbative term.

At infinite coupling the potential term dominates and frezes  $\phi=2n\pi$ , where n is an arbitrary integer. This means that the integration on  $\phi$  gets reduced to a discrete sum imposing then a quantization condition on  $\int dB$ . The integration over the fluctuations of the B-field can be done by decomposing  $B=dA+*d^*V$ , where A is a 1-form and V is a 3-form. Notice that the quantization condition on  $\int dB$  only involves V and therefore the integration along A can be performed freely. This implies that the (d-3)-form  $\omega_{\Sigma}$  dual to the surface  $\Sigma$  (that is such that  $\int_{\Sigma} B = \int B \wedge \omega_{\Sigma}$ ) is closed. This condition can be satisfied only if the two boundary components of  $\Sigma$  coincide. So the surface sum reduces to closed surfaces such that  $d\omega_{\Sigma}=0$  and the path integral keeps V only and all closed surfaces to give an overall factor.

Therefore, we see that the tensionless free string boundary-boundary correlator exactly coincides with the correlator of two conjugated Wilson loops in the compact abelian gauge theory in the limit of infinite coupling.

Notice that the above Polyakov model strictly speaking has been worked out in 3 dimensions and generalized to 4. The dual interpretation of the strongly coupled theory as a weakly coupled abelian (d-4)-form theory in any dimension d-1 is claimed to hold in [20] and in [21]. Let us read it again for completeness starting again from the action (3.9) and lets keep the perturbative term  $\frac{1}{4e^2}B^2$  while still freezing the dynamic of  $\phi$ . The effective theory around the strong coupling point now is just

$$S = \int \left[ \frac{1}{4e^2} B^2 + iB \wedge \omega_{\Sigma} \right] \tag{3.10}$$

and has to be compared with the first order formalism of an abelian (d-4)-form theory with the small gauge coupling  $\tilde{e}^2 = \frac{1}{4e^2}$ . As we concluded above, the summation over the surfaces is limited in this regime to closed ones, that is such that  $d\omega_{\Sigma} = 0$ . This means that locally  $\omega_{\Sigma} = dF$ , where F is a (d-4)-form. Substituting in the action (3.10) we get in fact

$$S = \int \left[ \tilde{e}^2 B^2 + iB \wedge dF \right] \tag{3.11}$$

and the summation over the closed surfaces gets replaced by the integral over the local potential F. The action (3.11) is exactly the first order formalism action of an abelian (d-4)-form theory in d-1 dimensions with the (small) gauge coupling  $\tilde{e}$ .

Notice that the last point, that is the equivalence of (3.10) and (3.11), has been proposed by Polyakov as a conjecture and we do not claim we prove it here. We notice that if that Polyakov picture is true, then it gives a natural gauge dual picture to the tensionless string results.

Notice that for d = 5, we find the usual (compact) Maxwell theory on  $M_4$ . For d = 4 we find free scalar fields in three dimensions. These were shown in [8] to be dual to a minimal higher spin theory in  $AdS_4$ .

# 4. Open questions

Let us first comment about conformal invariance. This got apparently broken by our boundary conditions to Poincare' which is generated by the SO(2, d-1) generators fixing the null-direction we have chosen to implement the boundary reduction. Actually it has not been broken and its action can be calculated after the partial gauge fixing has been performed. The point is that it is just "non manifest" in the boundary picture.

Once this narrow corner (free and tensionless) is understood, one might pose the questions about how to resort the tension and how to switch on the string interactions. It is clear that our technique seems specific of the tensionless limit of strings in AdS and its generalization to the tensile case is not given for granted to be doable. Moreover, the hamiltonian formalism is difficult to extend to the interacting first quantized string case in general and it might be that one has to formulate the interacting theory in the second quantization, i.e. for string fields.

Linking our results with higher spin gauge theory is not too difficult and naturally passes by the second quantization of the model. Using the BRST charge for closed tension-less free strings in  $AdS_d$  built in [14], one can consider the string field action  $S = \frac{1}{2} \langle \Psi | \Omega \Psi \rangle$  who's higher spin symmetry can be spectrally analyzed by means of its global symmetries. Notice that the BRST charge for the closed tensionless strings, unlike the open string case, has the right (odd) ghost modes amount in order to the above action to be well defined. The partial gauge fixing procedure that we performed here can be understood as the fixation of a huge set of auxiliary fields along the path of [22]. It would be very interesting to see how the theory looks like in detail after that procedure has been carried out. This is based on the BRST charge implementing the remnant gauge symmetry. The higher spin symmetries of the resulting boundary reduced theory should be compared with the

analogous ones in the dual abelian gauge theory to further check the string/gauge correspondence for free tensionless strings. The second quantized tensionless theory seems to be the natural starting point for the introduction of string interactions.

Let us conclude by noticing that we realized the boundary reduction as a (partial) gauge fixing procedure. It would be interesting to understand if this mechanism is a peculiarity of the massless free case or if it could be a way realize the holographic mechanism in more general cases.

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