An effective description of the landscape — II

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Diego Gallego[a,b] **and Marco Serone**[a,b]

[a]*International School for Advanced Studies (SISSA/ISAS)*
via Beirut 2-4, 34151 Trieste, Italy

[b]*INFN, Sezione di Trieste,*
Via Valerio, 2, I - 34127 Trieste, Italy

E-mail: gallego@sissa.it, serone@sissa.it

ABSTRACT: We continue our analysis of establishing the reliability of "simple" effective theories where massive fields are "frozen" rather than integrated out, in a wide class of four dimensional theories with global or local $\mathcal{N} = 1$ supersymmetry. We extend our previous work by adding gauge fields and $\mathcal{O}(1)$ Yukawa-like terms for the charged fields in the superpotential. For generic Kähler potentials, a meaningful freezing is allowed for chiral multiplets only, whereas in general heavy vector fields have to properly be integrated out. Heavy chiral fields can be frozen if they approximately sit to supersymmetric solutions along their directions and, in supergravity, if the superpotential at the minimum is small, so that a mass hierarchy between heavy and light fields is ensured. When the above conditions are met, we show that the simple effective theory is generally a reliable truncation of the full one.

Contents

1 Introduction

Effective field theories arising from string compactifications are notoriously difficult to study, due to the proliferation of fields that one generally gets. The first guiding principle to understand the impact of a given state on the low-energy dynamics is its mass. Whenever the latter is sufficiently heavy, we expect the effect of the related state to be small enough that we can safely neglect it. A very common and drastic simplification is to assume that string, Kaluza-Klein and winding resonances are all so heavy that they can be safely neglected. In so doing, we are left with a given supergravity (SUGRA) field theory, assumed from now on to be four-dimensional with $\mathcal{N} = 1$ supersymmetry (SUSY). Despite this huge simplification, an explicit study of such SUGRA theories is still a formidable task, containing typically hundreds of fields. Even just finding the vacua is often a hard task. The recent progress in string compactifications with fluxes has shown that many of these fields can get large masses,[1] so that one can integrate them out and simplify the low energy theory, in the spirit of effective theories. However, the multiplicity of these fields is such that even at the classical level integrating them out is practically very hard. As a matter of fact, the typical attitude taken in the literature consists in just neglecting these fields.

In a previous companion paper [1], we have started to study, in a general $\mathcal{N} = 1$ supersymmetric setting, under what conditions massive fields can be neglected rather than integrated out, in theories with chiral multiplets only.[2] Since the notion itself of heavy

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[1]By consistency of the construction, such masses should however be smaller than the lightest string, Kaluza-Klein and winding states.

[2]See [2–5] for previous related studies of this sort and [6–9] for studies in the context of the Kachru-Kallosh-Linde-Trivedi (KKLT) [10] and related scenarios.

and light fields manifestly depends on the vacuum, the requirement to be able to uniquely define heavy fields in a wide regime in field space for the remaining light states, forced us to study SUSY theories with a superpotential of the form

$$W(H, L) = W_0(H) + \epsilon\, W_1(H, L)\,, \tag{1.1}$$

with $\epsilon \simeq m_L/m_H \ll 1$ being the ratio of the typical light over heavy mass scale, H and L denoting schematically heavy and light chiral superfields. Our ansatz for W is also motivated by the fact that several recent string compactifications admit a superpotential of the form (1.1). For instance, in the well-known class of Type IIB flux compactification models [11], W_0 can be identified with the Gukov-Vafa-Witten superpotential [12], the heavy fields H are identified with the dilaton and the complex structure moduli of the underlying Calabi-Yau manifold, and the L with the Kähler structure moduli. The parameter ϵ is a schematic way to recall that all terms in W_1 are non-perturbatively generated and that in a wide region in moduli space are all suppressed.

As discussed in some detail in [1], assuming that all the eigenvalues of the Kähler metric are parametrically larger than ϵ, for any Kähler potential mixing term between H and L, it is always possible to define a canonically normalized field basis where the physical heavy field fluctuations are entirely given by linear combinations of the H. Hence the superpotential form (1.1) defines which are the heavy fields in the theory. It has shown in [1] that in theories with a superpotential of the form (1.1), heavy fields can be neglected, but provided they are "frozen" to approximate SUSY values H_0, defined as the solutions of $\partial_H W_0 = 0$ and that, of course, all of them effectively develop large physical mass terms from the term W_0 in eq. (1.1). In SUGRA theories the further condition $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$ is needed,[3] ensuring that curvature terms are small enough so that the hierarchy between heavy and light fields is retained, and the mass splitting induced by SUSY breaking negligible in the heavy field directions.[4]

Aim of the present paper is to extend the study of [1] by adding gauge fields. In order to further extend the class of theories falling in our analysis, we also generalize the form of the superpotential (1.1) by including $\mathcal{O}(1)$ field-dependent Yukawa couplings for the charged fields. Models of this sort necessarily imply various scales, possibly associated to different symmetry breaking mechanisms, SUSY breaking, gauge symmetry breaking, etc. In order to keep our analysis as simple as possible, and yet capture the essential features, we will assume the presence of just two kinds of light charged fields, characterized by having Vacuum Expectation Values (VEV) parametrically larger than ϵ and of $\mathcal{O}(\epsilon)$. We denote them respectively by Z and C. The remaining light fields are denoted by M. The latter can also transform under gauge transformations, but only non-linearly. They

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[3]The condition $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$ is not invariant under Kähler transformations. More precisely, the requirement is the existence of a Kähler gauge where $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$, $K \sim \mathcal{O}(1)$ and the Kähler metric has eigenvalues parametrically larger than ϵ.

[4] If two sectors in a theory are sufficiently screened, one can effectively study the dynamics of one of them, neglecting (freezing) the other, even in absence of a hierarchy in masses between the two sectors. The conditions when this can happen in SUGRA theories have been given in [13] (see also [5]) and have been also discussed in [1] in the context of some string compactifications [14] where they approximately apply. In that case W_0 can be arbitrary. We do not consider in this paper this alternative possibility.

would correspond, in explicit string constructions, to light moduli fields.[5] Like the light fields, the heavy fields can also be standard charged fields or moduli-like fields, but we will mostly focus on the case in which they are all moduli-like fields. The schematic form of the superpotential in this more complicated set-up is taken as follows:

$$W = W_0(H) + Y_N(H, M, Z)C^N + \epsilon\Big[W_1(H, M, Z) + \mu_M(H, M, Z)C^M\Big], \qquad (1.2)$$

where W_0, Y_N, W_1 and μ_M ($N \geq 3, M \geq 2$), are arbitrary holomorphic functions, constrained only by gauge invariance. No assumption on the dependence of the superpotential on the charged fields Z is required, so that our results can apply also in presence of non-perturbatively generated superpotential terms, non-polynomial in the Z's. The Kähler potential is arbitrary, with the only assumptions that admits a Taylor expansion in the charged fields C and that, as already mentioned, all the eigenvalues of the associated Kähler metric are parametrically larger than ϵ. Similarly, the gauge kinetic functions f are taken to be arbitrary, but regular, moduli dependent holomorphic functions.

First of all, we notice that, contrary to the fields H, massive gauge fields do not generally admit a freezing. More precisely, while superpotentials of the form (1.2) include a very wide class of known superpotentials arising from string compactifications, the class of Kähler potentials which would allow a freezing of the vector fields is quite limited and not very interesting, unless the vector field is heavy and decoupled, in which case one can trivially set it to zero. Hence we will not insist in freezing massive vector fields, but rather we will only show how the freezing of the heavy chiral fields is (not) affected in presence of heavy vector fields, the latter being always properly integrated out.

In general, the scalar condition $F_{H,0} = \partial_H W_0 = 0$ does not fix all the VEV of the heavy fields H, since gauge invariance constrains the form of W_0. Only gauge invariant combinations of the heavy moduli, being well-defined, can reliably be frozen. The orthogonal combinations will not appear in W_0, but possibly in other terms of the superpotential (1.2), in combination with light fields and/or in D-terms. Hence, these remaining gauge invariant combinations will typically be relevant in the low-energy dynamics and should properly be included among the light fields. More precisely, one can neglect non-neutral heavy moduli, assuming of having chosen a gauge-fixing where they are gauged away, but then one has to carefully take into account the dynamics of the associated massive vector-super field. In the context of SUSY breaking, the possible D-term SUSY breaking contributions hidden in the massive vector super field are generally non-negligible. We emphasize this point because, although already made present in the literature in various contexts [13, 15], it seems to have been overlooked in some string constructions, where gravity and moduli are neglected altogether. The impossibility of naively neglecting non-neutral moduli has nothing to do with gravity but is purely dictated by gauge invariance, so it remains also in the global limit with gravity decoupled.

Along the lines of [1], we compare the "full" effective theory obtained by classically integrating out the heavy fields to the "simple" one obtained by just "freezing" them. We

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[5]When the gauge symmetry is broken, a combination of the fields L actually get a heavy mass. Nevertheless, in order to distinguish them from the fields H appearing in eq. (1.2), with an abuse of language, we will keep calling them light.

use a manifestly SUSY approach in which one solves the super-field equation $\partial_H W = 0$ and then plug back the result in W, K and f to get the effective full quantities W_{full}, K_{full} and f_{full}. We find that in the charged field range $|C|/m_H \lesssim \mathcal{O}(\epsilon)$, for any values of the M and Z fields where the superpotential W has the form (1.2), the simple theory is a reliable effective field theory. More precisely W_{sim}, K_{sim} and f_{sim} differ by the corresponding full quantities only by operators which are sub-leading in an effective field theory sense and correct the coefficients of the already existing couplings by a small amount. The leading C-dependent part of the scalar potential is identical in both theories. On the other hand, cubic terms of the schematic form C^3 in K and C^6 terms in W (and higher) are not reliable in the simple theory, so care has to be taken in working with it anytime higher order operators are considered. We have also checked, by working in a component approach, that higher terms in the light auxiliary fields, not detected by just solving $\partial_H W = 0$, are always negligible in the scalar potential, namely they can only give rise to corrections of the same order of magnitude or smaller than the ones given by the manifestly SUSY integration of the heavy fields.

All the above results apply to global and local SUSY theories, provided that in the latter theories $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$, as mentioned above. This is in agreement with the recent results of [16], where it has been pointed out that the meaningful super-field equation that should be used to integrate out heavy fields is $\partial_H W = 0$ in both the global and local SUSY case. It has also been emphasized in [16] that the condition $W \ll 1$ is always necessary in order to ensure that the effective theory is still described by a standard SUGRA theory up to two derivative level. We emphasize here that the (generally) stronger condition $W \sim \mathcal{O}(\epsilon)$ allows a freezing of the heavy fields, under the conditions discussed above.

The structure of the paper is as follows. In section 2, after a brief review of the results of [1], we consider global and local SUSY theories with W as in eq. (1.2), in the limit of vanishing gauge couplings. In section 3 we include vector multiplets. We first consider theories with unbroken gauge group and then analyze theories with broken gauge group, where heavy vector fields have to be integrated out. We draw our conclusions in section 4. We report in appendix A a more detailed study in components of the comparison of the equations of motion (e.o.m.) in the full and simple theory. For illustrative purposes, we report in appendix B a numerical study of a "string-inspired" SUGRA model with an U(1) gauge symmetry where our considerations can be concretely applied. In order to simplify the notation, we set the the cut-off of the microscopic theory Λ to be the reduced Planck mass M_p and use units in which $\Lambda = M_p = 1$. In addition, in order to not introduce some additional hierarchy of scales, we assume that $m_H/M_p \gg \epsilon$ and that $m_V \sim m_H$, m_V being the scale of the heavy vectors.

2 Chiral multiplets only

Before turning our attention to full models with gauge interactions and vector multiplets, it is useful to work in absence of the latter, in the limit of vanishing gauge couplings. In the next subsection we first rederive the results of [1], valid in absence of $\mathcal{O}(1)$ Yukawa couplings and obtained working in field components, using a manifestly SUSY approach.

2.1 No $\mathcal{O}(1)$ Yukawa couplings

The study performed in [1], restricted to superpotentials of the form $W = W_0(H) + \epsilon W_1(H, L)$, with $\epsilon \ll 1$, established that up to $\mathcal{O}(\epsilon^2)$ the (full) effective scalar potential one gets by integrating out heavy moduli in a given vacuum is identical to the one obtained by simply freezing the heavy fields at their leading (in ϵ) VEV defined as $\partial_H W_0 = 0$. The results are valid in both global and local SUSY, where in the latter case the additional constraint $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$ is required. In this way, it has been shown in [1] that SUSY is broken at $\mathcal{O}(\epsilon)$ by the light fields, with a suppressed back-reaction of SUSY breaking on the heavy fields, $F^H \sim \mathcal{O}(\epsilon^2)$. In what follows, in order to use the same notation and conventions adopted in [1], we will take $m_H \lesssim 1$ so that m_H does not explicitly appear in the formulae. We will relax this condition in the next subsection.

Let us start by considering the global SUSY case. When SUSY is broken at a scale of $\mathcal{O}(\epsilon)$, parametrically smaller than the mass scale of the heavy fields, a manifestly SUSY approach to integrate out heavy fields is allowed. As well known, neglecting covariant and ordinary derivatives, this approach amounts in solving at the super-field level the equations

$$\partial_H W = 0 \tag{2.1}$$

and then plugging them back in W and K (see e.g. [17]). In this way, one automatically integrates out the heavy fields and gets the full effective Kähler and superpotential K_{full} and W_{full}. Strictly speaking, the effective scalar potential one obtains from W_{full} and K_{full} differs in general from that obtained by integrating out the heavy fields in components, since in the latter one is exactly resumming (at zero momentum) all the auxiliary components of the light fields. In the manifestly SUSY approach with covariant derivatives neglected, instead, by definition one considers all the auxiliary components up to quadratic order only. In other words, the effective K associated to the scalar potential computed in components, as in [1], is in general an infinite sum of terms involving covariant SUSY derivatives. In the situation at hand, however, the light fields of the F-terms are of $\mathcal{O}(\epsilon)$ and thus all cubic and higher F terms of the light fields are negligible for our purposes. We solve the equations $\partial_H W = 0$ perturbatively in ϵ:

$$H^i = H_0^i + \epsilon H_1^i(L) + \mathcal{O}(\epsilon^2) \tag{2.2}$$

where H_0^i are defined by $\partial_i W_0(H_0) = 0$ (notation as in [1]). The effective Kähler and superpotential read

$$W_{\text{full}} = W_{\text{sim}} + \epsilon^2 \left(\frac{1}{2} \partial_i \partial_j W_0 H_1^i H_1^j + \partial_i W_1 H_1^i \right) + \mathcal{O}(\epsilon^3), \tag{2.3}$$

$$K_{\text{full}} = K_{\text{sim}} + \epsilon \left(\partial_i K_{\text{sim}} H_1^i + \partial_{\bar{i}} K_{\text{sim}} \bar{H}_1^{\bar{i}} \right) + \mathcal{O}(\epsilon^2), \tag{2.4}$$

where $W_{\text{sim}} = W(H_0)$ and $K_{\text{sim}} = K(H_0, \bar{H}_0)$. The leading shift H_1^i equals

$$H_1^i = -W_0^{ij} \partial_j W_1, \tag{2.5}$$

with $W_0^{ij} \sim \mathcal{O}(1)$, the inverse of $W_{0,ij} \equiv \partial_i \partial_j W_0$, and gives

$$W_{\text{full}} = W_{\text{sim}} - \frac{1}{2} \epsilon^2 \partial_i W_1 W_0^{ij} \partial_j W_1 + \mathcal{O}(\epsilon^3). \tag{2.6}$$

Independently of the Kähler potential and its corrections, the ϵ^2 terms in W_{full} can only contribute to the effective scalar potential at $\mathcal{O}(\epsilon^3)$ and hence

$$V_{\text{full}} = V_{\text{sim}} + \mathcal{O}(\epsilon^3). \tag{2.7}$$

It has been recently shown that in SUGRA eq. (2.1) is still the correct super-field equation to be solved up to quadratic order in derivatives and auxiliary fields [16]. The latter approximation, however, also applies in the gravitational sector where, in the super-conformal approach, amounts in requiring that the F-term of the compensator field Φ is also suppressed: $F_\Phi \ll 1$. Modulo an overall rescaling,

$$F_\Phi = e^{K/2}\left(\overline{W} - \frac{1}{3}K_M F^M\right). \tag{2.8}$$

where $F^M = g^{\bar{M}M}[\partial_{\bar{M}}\overline{W} + (\partial_{\bar{M}}K)\overline{W}]$. When $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$, or more generally $\exp(K/2)W \sim \epsilon$, $F_\Phi \sim \epsilon$ and cubic or higher auxiliary fields of the compensator can safely be neglected. In this situation, the above analysis valid in global SUSY applies also in SUGRA with *no* extra complication.

Notice that the freezing of a heavy field has to be performed at the level of W and K and not of the scalar potential. Indeed, the structure of the SUSY lagrangian is such that several couplings appear to be proportional to the heavy field mass. In this case, the decoupling does not arise since all the heavy fields interactions proportional to these couplings are unsuppressed and have to be considered. As a matter of fact, such leading terms of the heavy field integration are automatically retained when freezing H in W and K. In order to explicitly see that, it might be useful to derive eq. (2.7) in a component formalism. The full effective scalar potential, once the heavy moduli have been integrated out, reads[6]

$$V_{\text{full}} = V(\langle H \rangle, L) + V_{\text{int}}(\langle H \rangle, L), \tag{2.9}$$

where

$$V = e^K(F^M F_M - 3|W|^2) \tag{2.10}$$

is the microscopical scalar potential and

$$V_{\text{int}} = -\frac{1}{2}V_I V^{IJ} V_J|_{H=\langle H \rangle} \tag{2.11}$$

is the potential term induced by a gaussian integration of the heavy moduli. Corrections to eq. (2.11), due to cubic or higher terms in the heavy field fluctuations are negligible. For simplicity, in eq. (2.11) we have collected the holomorphic and antiholomorphic indices in a single notation, $I = i, \bar{i}$, $V_I = \partial_I V$, and V^{IJ} is the inverse matrix of $V_{IJ} = \partial_I \partial_J V$. It is not difficult to see that $\partial_I V \sim \mathcal{O}(\epsilon)$, implying that the heavy field fluctuations $\hat{H}^i$ have to

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[6]For simplicity of notation, below and throughout the paper, we use the same notation to denote a chiral superfield and its lowest scalar component, since it should be clear from the context to what we are referring to.

be explicitly integrated out, giving a correction of $\mathcal{O}(\epsilon^2)$ to the effective scalar potential.[7] Given the form of eq. (2.11), the knowledge of the potential at $\mathcal{O}(\epsilon^2)$ requires to compute the $\mathcal{O}(\epsilon)$ terms in $\partial_I V$ and the $\mathcal{O}(1)$ in V^{IJ}, the latter arising from the inverse of $V_{i\bar{j}}$. One gets

$$\partial_i \partial_{\bar{j}} V|_0 = e^K \partial_{\bar{j}} \overline{F}_{\bar{k},0}\, g^{\bar{k}l}\, \partial_i F_{l,0}\,, \tag{2.12}$$

$$\partial_i V|_1 = e^K \partial_i F_{l,0} F^l\,, \tag{2.13}$$

where $F_{i,0} = \partial_i W_0$. Using eqs. (2.12) and (2.13), we find

$$V_{\text{int}} = -e^K F^i \tilde{g}_{i\bar{j}} \overline{F}^{\bar{j}}\,, \tag{2.14}$$

where $\tilde{g}_{i\bar{j}}$ is the inverse of $g^{\bar{j}i}$, not to be confused with $g_{i\bar{j}}$. The full effective scalar potential reads then

$$
\begin{aligned}
V_{\text{full}} &= e^K \left(g^{\bar{M}N} \overline{F}_{\bar{M}} F_N - 3|W|^2 \right) - e^K F^i \tilde{g}_{i\bar{j}} \overline{F}^{\bar{j}} + \mathcal{O}(\epsilon^3) \\
&= \epsilon^2 e^K \left(\tilde{g}^{\bar{\alpha}\alpha} F_\alpha \overline{F}_{\bar{\alpha}} - 3|W|^2 \right) + \mathcal{O}(\epsilon^3) = V_{\text{sim}} + \mathcal{O}(\epsilon^3)\,,
\end{aligned} \tag{2.15}
$$

where $\tilde{g}^{\bar{\alpha}\alpha}$ is the inverse of $g_{\alpha\bar{\alpha}}$, the metric appearing in the simple model where the heavy fields are frozen, and the following matrix identity has been used:

$$\tilde{g}^{\bar{\alpha}\alpha} = g^{\bar{\alpha}\alpha} - g^{\bar{\alpha}i} \tilde{g}_{i\bar{j}} g^{\bar{j}\alpha}\,. \tag{2.16}$$

Being the potential (2.15) of $\mathcal{O}(\epsilon^2)$, it is enough to keep the leading terms H_0^i for the position of the VEV's in the heavy directions, finally recovering eq. (2.7).

2.2 $\mathcal{O}(1)$ Yukawa couplings

A necessary generalization of the set up in [1] is the introduction of couplings of $\mathcal{O}(1)$ between charged fields, i.e., to allow non-suppressed couplings in the light field sector. This extension forces us to distinguish, among the light fields L, between the charged fields and the moduli. As is going to be clear below, it is also useful to distinguish between charged fields with $\mathcal{O}(\epsilon m_H)$ and $\mathcal{O}(m_H)$ VEV's. We denote by M^μ ($\mu = 1, \ldots, n_M$) the light moduli (including gauge singlets), by $Z^{\hat{\alpha}}$ ($\hat{\alpha} = 1, \ldots, n_Z$) the charged fields with $\mathcal{O}(m_H)$ VEV's and by C^α ($\alpha = 1, \ldots, n_C$) the charged fields with $\mathcal{O}(\epsilon m_H)$ VEV's. We use calligraphic letters $\mathcal{A}, \mathcal{B}, \ldots$, to collectively denote all the light field indices: $\mathcal{A} = (\alpha, \hat{\alpha}, \mu)$ and $M, N, \ldots$ to collect all fields indices, heavy and light, $M = (i, \mathcal{A}) = (i, \alpha, \hat{\alpha}, \mu)$. Finally, we denote by $L^{\mathcal{A}} = (C^\alpha, Z^{\hat{\alpha}}, M^\mu)$ and by $\phi^M = (H^i, C^\alpha, Z^{\hat{\alpha}}, M^\mu)$ the set of all light and of all light+heavy fields, respectively. The superpotential is taken as follows:

$$W = W_0(H^i) + \eta\, \widetilde{W}_0(H^i, M^\mu, Z^{\hat{\alpha}}, C^\alpha) + \epsilon\, W_1(H^i, M^\mu, Z^{\hat{\alpha}}, C^\alpha)\,. \tag{2.17}$$

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[7]One can neglect the heavy field fluctuations $\hat{H}^i$ even at the level of the scalar potential, if the light fields are kept dynamical in evaluating the vacuum shifts H_1^i. In other words, by computing $H_1^i(L)$, with $H^i = \langle H^i \rangle + \hat{H}^i = H_0^i + \epsilon H_1^i(\langle L \rangle + \hat{L}) + \mathcal{O}(\epsilon^2) + \hat{H}^i$. This was actually the procedure followed in [1] to establish the equivalence of the full and simple theories.

In eq. (2.17), $\widetilde{W}_0$ and W_1 are gauge-invariant polynomials in the charged fields C^α, with field-dependent couplings:

$$\widetilde{W}_0 = Y_{3,\alpha\beta\gamma}(H^i, M^\mu, Z^{\hat\alpha})C^\alpha C^\beta C^\gamma + \mathcal{O}(C^4),$$
$$W_1 = \widetilde{W}_1(H^i, M^\mu, Z^{\hat\alpha}) + \mu_{2,\alpha\beta}(H^i, M^\mu, Z^{\hat\alpha})C^\alpha C^\beta + \mathcal{O}(C^3). \tag{2.18}$$

The superpotentials $\widetilde{W}_0$ and W_1 do not need to admit a polynomial expansion in the $Z^{\hat\alpha}$ fields and can effectively be treated as moduli. The requirement that W_0 give a supersymmetric mass of $\mathcal{O}(m_H)$ to the heavy fields and that the fields C have a mass of $\mathcal{O}(m_L)$ fix $W_0, \widetilde{W}_1$ and $\mu_{2,\alpha\beta}$ to be of $\mathcal{O}(m_H)$. The parameter η is a dummy variable which will be useful in what follows, but that eventually will be taken to be equal to 1. The Kähler potential is of the form

$$K = K_0 + K_{1,\alpha\bar\beta}C^\alpha \bar{C}^{\bar\beta} + (K_{2,\alpha\beta}C^\alpha C^\beta + c.c) + \mathcal{O}(C^3), \tag{2.19}$$

with K_0, K_1 and K_2 arbitrary functions of H^i, M^μ and $Z^{\hat\alpha}$ and their complex conjugates, constrained only by gauge invariance.

Our aim is to compare the theory defined above by W and K (the full theory) with the simple effective one where the H^i are frozen at their leading VEV's H_0^i. As before, we solve eq. (2.1) perturbatively in ϵ and, at each order in ϵ, further expand in η:

$$H^i = H_0^i + \eta \delta H_0^i(L) + \mathcal{O}(\eta^2) + \epsilon\Big[H_1^i(L) + \mathcal{O}(\eta)\Big] + \dots \tag{2.20}$$

where, as usual, H_0^i are defined by $\partial_i W_0(H_0) = 0$. The effective Kähler and superpotential read

$$W_{\text{full}} = W_{\text{sim}} + \eta^2\Big(\frac{1}{2}\partial_i\partial_j W_0 \delta H_0^i \delta H_0^j + \partial_i\widetilde{W}_0 \delta H_0^i\Big)$$
$$+ \epsilon\eta\Big(\partial_i\partial_j W_0 \delta H_0^i H_1^j + \partial_i\widetilde{W}_0 H_1^i + \partial_i W_1 \delta H_0^i\Big)$$
$$+ \epsilon^2\Big(\frac{1}{2}\partial_i\partial_j W_0 H_1^i H_1^j + \partial_i W_1 H_1^i\Big) + \mathcal{O}(\eta^3, \eta^2\epsilon, \eta\epsilon^2, \epsilon^3), \tag{2.21}$$

$$K_{\text{full}} = K_{\text{sim}} + \eta\Big[\partial_i K_{\text{sim}}\delta H_0^i + \partial_{\bar i}K_{\text{sim}}\delta\bar{H}_0^{\bar i}\Big] + \mathcal{O}(\epsilon, \eta^2). \tag{2.22}$$

The leading shift δH_0^i is

$$\delta H_0^i = -W_0^{ij}\partial_j\widetilde{W}_0, \tag{2.23}$$

with $W_0^{ij} \sim 1/m_H$ and H_1^i as in eq. (2.5). By plugging back eqs. (2.5) and (2.23) in eqs. (2.21) and (2.22), one easily finds

$$W_{\text{full}} = W_{\text{sim}} - \frac{1}{2}\eta^2\partial_i\widetilde{W}_0 W_0^{ij}\partial_j\widetilde{W}_0 - \epsilon\eta\partial_i\widetilde{W}_0 W_0^{ij}\partial_j W_1 - \frac{1}{2}\epsilon^2\partial_i W_1 W_0^{ij}\partial_j W_1 +$$
$$\mathcal{O}(\eta^3, \eta^2\epsilon, \eta\epsilon^2, \epsilon^3), \tag{2.24}$$

$$K_{\text{full}} = K_{\text{sim}} - \eta\Big[\partial_i K_{\text{sim}}W_0^{ij}\partial_j\widetilde{W}_0 + \partial_{\bar i}K_{\text{sim}}\overline{W}_0^{\bar i\bar j}\partial_{\bar j}\widetilde{\overline{W}}_0\Big] + \mathcal{O}(\epsilon, \eta^2). \tag{2.25}$$

Let us now see the structure of the new induced operators and their possible relevance, recalling that the meaningful region we can explore in the light charged field directions is

defined to be $|C|/m_H \lesssim \mathcal{O}(\epsilon)$. We will assume that $H^i \sim \mathcal{O}(1)$ to simplify the scaling analysis that follows. The leading C-dependent terms of W_{full} and K_{full} (which are in W_{sim} and K_{sim}) are of $\mathcal{O}(\epsilon^3 m_H^3)$ and $\mathcal{O}(\epsilon^2 m_H^2)$ respectively, and it is easily shown that correspondingly the leading C-dependent terms $V(C)$ of the scalar potential are of $\mathcal{O}(\epsilon^4 m_H^4)$. Notice that it is crucial to take $\exp(K/2)W_0 = \mathcal{O}(\epsilon m_H)$ in SUGRA, otherwise terms of $\mathcal{O}(\epsilon^3 m_H^4)$ would appear in the scalar potential, invalidating the equivalence of the simple and full theories. It is straightforward to see that all the induced couplings appearing in W_{full} are at most of $\mathcal{O}(\epsilon^4 m_H^3)$ and those appearing in K_{full} of $\mathcal{O}(\epsilon^3 m_H^2)$, so that

$$V(C)_{\text{full}} = V(C)_{\text{sim}} + \mathcal{O}(\epsilon^5 m_H^5). \tag{2.26}$$

Eq. (2.26) implies that the e.o.m. of the light fields C^α are the same in both approaches up to $\mathcal{O}(\epsilon^3)$, which is the first non-trivial order for these fields, being by assumption $\langle C^\alpha \rangle \sim \mathcal{O}(\epsilon m_H)$. Of course, as far as the C-independent $\mathcal{O}(\epsilon^2)$ scalar potential is concerned, eq. (2.7) still applies, implying, in particular, that the leading e.o.m. of the fields Z and M are identical in the full and simple theories. Notice that the $\mathcal{O}(\epsilon^5 m_H^5)$ terms in eq. (2.26) arises only from $\mathcal{O}(\epsilon)$ corrections to coefficients of operators present in $V(C)_{\text{sim}}$ and not from the new higher derivative operators induced by the heavy field integration. The latter, as we will see below, are sub-dominant, being at most of $\mathcal{O}(\epsilon^7 m_H^6)$.

The structure of the higher dimensional operators which are generated by the heavy field integration is easily seen from W_{full} and K_{full}. The term proportional to η^2 in W_{full} gives rise to new induced couplings between the charged fields of the form $Y_{N_i} Y_{N_j} C^{N_i + N_j}$. Their coefficients scale as $1/m_H$, which is higher than the natural scale $\mathcal{O}(1)$ for such operators. It is easy to see that the coefficients of all the higher dimensional operators induced by the terms in W_{full} proportional to $\epsilon\eta$ and ϵ^2 are smaller than their natural values. Similarly, the terms proportional to η in the Kähler potential K_{full} give rise to C^N higher dimensional operators with coefficients of $\mathcal{O}(1/m_H)$, higher than their $\mathcal{O}(1)$ natural values, implying that holomorphic cubic terms in C are not reliable in the simple effective Kähler potential. One can also compute the structure of the lowest dimensional induced operators appearing in the scalar potential $V_{\text{full}}(C)$, but possibly not present in $V_{\text{sim}}(C)$. We get

$$\delta V(C) \sim \frac{Y_{N_i} Y_{N_j} Y_{N_k}}{m_H} C^{N_i + N_j + N_k - 2} + \frac{Y_{N_i} Y_{N_j} \mu_{M_k} \epsilon}{m_H} C^{N_i + N_j + M_k - 2}$$
$$+ \frac{Y_{N_i} \mu_{M_j} \mu_{M_k} \epsilon^2}{m_H} C^{N_i + M_j + M_k - 2} + \dots. \tag{2.27}$$

where we have schematically denoted by Y_{N_i} and μ_{M_k} couplings and their derivatives with respect to H, M and Z, we have omitted generic M and Z dependent coefficients and we have not distinguished between fields and their complex conjugates. The ellipsis contains terms which are of the same order or higher in ϵ. The lowest dimensional operators appearing in $\delta V(C)$ are of order C^5 but with suppressed couplings. The only operators with coefficients higher than their natural values are those appearing in the first term of eq. (2.27), of $\mathcal{O}(C^7)$. Notice that in the limit in which Y_N are of $\mathcal{O}(\epsilon)$, the superpotential (2.17) is effectively of the form (1.1) and the results of [1] apply, namely $\delta V(C) = \mathcal{O}(\epsilon^3)$. This explains why the minimum sum of powers of Y_N and ϵ in eq. (2.27) is three.

It is straightforward to explicitly check that cubic or higher order terms in the light auxiliary fields (including the compensator in SUGRA) are all at most of the same order as the correction terms appearing in eq. (2.27), so that they can safely be ignored. As a further check of this, we have also derived the scalar potential in an on-shell component approach, which amounts to sum all powers of the light auxiliary fields, including the compensator field, and found complete agreement with eq. (2.27). Needless to say, even when the approximate manifestly SUSY equation $\partial_H W = 0$ would give $\delta V(C) = 0$, such as in the case when Y_N, W_1 and μ_M are all independent of H, higher order terms in the auxiliary fields would still give rise to terms of the form (2.27), if Kähler mixing terms between H and the light fields are present.

3 Chiral and vector multiplets

We introduce in this section the vector multiplets by switching on the gauge couplings. We assume that at the vacuum a gauge group $\mathcal{G}$ is spontaneously broken to a subgroup $\mathcal{H}$ at a scale parametrically larger than ϵ. The gauge group $\mathcal{H}$ might further be broken to a subgroup, but only at scales of $\mathcal{O}(\epsilon m_H)$. We denote by X_A^M and $\overline{X}_A^{\bar{M}}$ the holomorphic and anti-holomorphic Killing vectors generating the (gauged) isometry group $\mathcal{G}$, defined as $\delta\phi^M = \lambda^A X_A^M$, $\delta\bar\phi^{\bar{M}} = \lambda^A \overline{X}_A^{\bar{M}}$, with λ^A infinitesimal real parameters. The corresponding D-terms are

$$D_A = iX_A^M G_M = -i\overline{X}_A^{\bar{M}}\overline{G}_{\bar{M}}\,, \tag{3.1}$$

where $G = K + \log|W|^2$ and $G_M = \partial_M G = F_M/W$ are the Kähler invariant function and its derivatives and $F_M = D_M W = \partial_M W + (\partial_M K)W$ is the Kähler covariant derivative.[8] We denote by capital latin letters $A, B, \ldots = 1, \ldots, \mathrm{adj}(\mathcal{G})$, the gauge group indices, not to be confused with the light field indices $\mathcal{A}, \mathcal{B}, \ldots$ introduced before. For simplicity of presentation, we take the holomorphic gauge kinetic functions f_{AB} diagonal in the gauge indices, so that

$$f_{AB} = \delta_{AB}f_A(H^i, M^\mu, Z^{\hat\alpha}, C^\alpha) \tag{3.2}$$

are generic holomorphic functions and $\mathrm{Re}\, f_A = 1/g_A^2$, with g_A the coupling constants of $\mathcal{G}$.

Contrary to the pure F-term case, the condition $\partial_i W_0 = 0$ may not fix all the fields H^i. Indeed, gauge invariance relates the derivatives of W_0 by

$$X_A^i \partial_i W_0 = 0\,, \tag{3.3}$$

so that in general they are not linearly independent and some of the fields may remain unfixed. Thus, if the X_A^i are not all vanishing, one can always choose a basis in which some of the fields do not appear at all in W_0. It is very simple to explicitly construct such a basis for the relevant case where the H^i are moduli shifting under a U(1) gauge symmetry. If $\delta H^i = i\xi^i\Lambda$, with Λ the chiral super-field associated to the U(1) transformation, one can

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[8]We have not included constant Fayet-Iliopoulos terms ξ_A in eq. (3.1) motivated by the fact that they do not seem to appear in string derived models of any sort. In SUGRA, their presence requires super-potentials which are gauge-invariant only modulo an R-symmetry shift, namely $X_A^i\partial_i W = \xi_A W$ for U(1) transformations.

always choose to parametrize W_0 in terms of, say, $M^1 \equiv H^1$ (with $\xi^1 \neq 0$) and the $n_H - 1$ gauge invariant operators $H^i_{GI} = \xi^1 H^i - \xi^i H^1$, with $i > 1$. One can invert this relation and use the fields H^i_{GI} in W_0. In this field basis, the only non-vanishing isometry component is X^1. From eq. (3.3) we immediately see that W_0 is independent of M^1, depending only on the $n_H - 1$ gauge invariant fields H^i_{GI}. Of course the same argument can be repeated for each U(1) generator independently. Hence there is no well defined meaning in freezing M^1. This is not even a meaningful gauge invariant statement, since $\delta M^1 \neq 0$ and thus M^1 will necessarily enter in W (if any) in a gauge invariant combination with light fields. One might get rid of M^1 by gauging it away, namely choosing a gauge (which is not the Wess-Zumino gauge) where it is a constant. This is possible, but then its dynamics will reappear in the U(1) vector super field, more precisely in the longitudinal component of the gauge field and the lowest auxiliary component of the U(1) vector super field, which is now dynamical. If the vector field is sufficiently massive and is integrated out, the effects of M^1 will eventually appear in new contributions to the Kähler potential of the remaining light fields. A direct relevant consequence of the above result is the impossibility of naively neglecting moduli fields responsible for field-dependent Fayet-Iliopoulos terms and then forgetting the implicit gauge-fixing taken behind this choice, as sometimes done in the literature in local string constructions, where all moduli dynamics is neglected altogether. Clearly, this obstruction has nothing to do with gravity and is purely dictated by gauge invariance. This observation actually dates back to [15] and was more generally expressed in [13]. In conclusion, for all proper heavy fields we must impose

$$X^i_A = 0 \,. \tag{3.4}$$

In addition to eq. (3.4), for simplicity we also assume that the remaining isometry components do not depend on H^i, $X^\mathcal{A}_A = X^\mathcal{A}_A(L^\mathcal{B})$.

Since vector multiplets do not appear in W, the manifestly SUSY integration of chiral multiplets is not naively affected at leading order by their presence. However, even at the level of two derivatives (quadratic auxiliary fields), terms with covariant derivatives in the Kähler potential may now appear, coming from terms of the form DF or D^2.[9] However, these terms are necessarily further suppressed by powers of $1/m_H$. This is easily seen by noticing that a D-term is the $\theta^2\bar{\theta}^2$ component of a vector multiplet as opposed to an F term of a chiral field, which is its θ^2 component. This implies that the DF and D^2 generated terms in K arises from terms with at least two and four covariant derivatives, respectively. They are then necessarily suppressed by ϵ or ϵ^2 with respect to the F^2 generated terms and can thus be neglected.

When the gauge group $\mathcal{G}$ is unbroken at $\mathcal{O}(1)$, namely when there are no Z fields and all the moduli M are neutral, the effective Kähler and superpotential are as in eqs. (2.24) and (2.25). By plugging eqs. (2.5) and (2.23) in f_{AB} we get the effective holomorphic gauge kinetic functions

$$f_{AB,\text{full}} = f_{AB,\text{sim}} + \partial_i f_{AB} \delta H^i_0 + \mathcal{O}(\epsilon, \eta^2) \,. \tag{3.5}$$

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[9]We thank the authors of [16] for discussions on this issue and sharing with us a preliminary draft of their paper.

Eq. (3.5) implies that all C-independent terms and C-dependent ones up to C^{N-1} (included) entering in the f_{AB} are reliable at $\mathcal{O}(1)$ in ϵ. The effective Kähler potential (2.25) gives rise to corrections to the D-terms of the schematic form

$$
\begin{aligned}
D_{A,\text{full}} - D_{A,\text{sim}} &= iX_A^\alpha \partial_\alpha \Big[\partial_i K_{\text{sim}} (\eta \delta H_0^i + \epsilon H_1^i) + \partial_{\bar{\imath}} K_{\text{sim}} (\eta \delta \bar{H}_0^{\bar{\imath}} + \epsilon \bar{H}_1^{\bar{\imath}}) \Big] + \mathcal{O}(\epsilon^2, \eta^2, \epsilon\eta) \\
&= \frac{Y_{N_i}}{m_H} C^{N_i+2} + \epsilon C^2 + \frac{\epsilon \mu_{M_i}}{m_H} C^{M_i+2} + \dots ,
\end{aligned}
\tag{3.6}
$$

where we have set $\eta = 1$ in the second row of eq. (3.6) and where, for concreteness, we have counted the powers of C (including their complex conjugates) by taking linear realizations of the gauge group (X_A^α proportional to C^α). Gauge invariance of $\widetilde{W}_0$ and W_1 has been used, constraining $X_A^\alpha \partial_\alpha \delta H_0^i$ and $X_A^\alpha \partial_\alpha H_1^i$ to vanish.

We can easily extract from eq. (3.6) the lower dimensional operators generated by the heavy field integration, appearing in the D-term scalar potential $V_{D,\text{full}}$ but not present in $V_{D,\text{sim}}$. They are of the form $g^2 Y_{N_i} C^{N_i+4}/m_H$ and $\epsilon \mu_{M_i} g^2 C^{M_i+4}/m_H$. Both are at most of $\mathcal{O}(\epsilon^7 m_H^6)$ and hence irrelevant. As already mentioned, in presence of vector multiplets, quadratic terms in the auxiliary fields of the form DF and D^2 are missed. It is useful to explicitly see how this discrepancy arises in our class of models. By construction, all the effective operators one obtains from the effective D-term scalar potential by a manifestly SUSY integration, where covariant derivatives are neglected, give rise to only operators proportional to g^2, as above. On the other hand, it is obvious that in presence of a Kähler mixing term between the charged fields and the heavy fields, higher order operators with coefficients proportional to $(g^2)^2/m_H^2$ should be expected. By explicitly computing the scalar potential in components, indeed, we find

$$
\delta V_D \supset \frac{g^2 Y_{N_i}}{m_H} C^{N_i+4} + \frac{\epsilon g^2 \mu_{M_i}}{m_H} C^{M_i+4} + \frac{(g^2)^2}{m_H^2} C^8 .
\tag{3.7}
$$

The first two terms are exactly those found in the manifestly SUSY case, whereas the latter arises from the undetected D^2 terms and, as expected, is negligible, being of $\mathcal{O}(\epsilon^8 m_H^6)$.

In presence of Z fields and non-neutral light moduli, the gauge group is spontaneously broken and the analysis is more involved, because extra states become massive, the real scalar fields associated to the would-be Goldstone bosons eaten by the heavy gauge fields $A_{\hat{a}}$ and the gauge fields $A_{\hat{a}}$ themselves, where we have splitted the gauge index $A = (a, \hat{a})$, $\hat{a} \in \mathcal{G}/\mathcal{H}$, $a \in \mathcal{H}$. Such fields cannot be frozen to their VEV's but they should be properly integrated out. A manifestly SUSY integration of a vector super field requires that $\langle D \rangle / m_V^2 \ll 1$, with m_V the gauge field mass. In our case this condition is always satisfied, as shown in appendix A, being $\langle D_{\hat{a}} \rangle = \mathcal{O}(\epsilon^2)$. A vector super field is supersymmetrically integrated out, neglecting covariant derivative terms coming from the holomorphic gauge field action, by setting [15]

$$
\partial_{V_{\hat{a}}} K = 0 ,
\tag{3.8}
$$

$V_{\hat{a}}$ being the vector-superfields associated to the broken generators. For typical charged field Kähler potentials, eq. (3.8) does not admit a simple constant solution, so that it is in general hopeless to freeze a heavy vector super field. Integrating out a vector super

field implies a choice of gauge fixing. The physical gauge where one gets rid of the eaten Goldstone bosons and their superpartners is the super-field version of the unitary gauge. On the other hand, it is practically easier to work in a gauge where a chiral field Z (or a non-neutral modulus M) is frozen at its VEV Z_0 (or M_0). The field choice is arbitrary, provided it has a non-vanishing component in the would-be Goldstone direction. This can be done for each broken generator so that $\dim \mathcal{G}/\mathcal{H}$ light chiral multiplets (or combinations thereof) are gauged away from the theory. Let us denote by $L^{\mathcal{A}'}$ the remaining directions, with $\mathcal{A}' = 1, \ldots, n_L - \dim \mathcal{G}/\mathcal{H}$, and by $V_{\hat{a}}^0$ the solution to eq. (3.8). By plugging back in the Lagrangian $V_{\hat{a}} = V_{\hat{a}}^0$, we get the SUSY effective theory with heavy vector fields integrated out. As far as the chiral fields are concerned, the correction terms one obtains from the holomorphic gauge kinetic terms are negligible, being suppressed by four covariant derivatives with respect to the corrections terms coming from the effective Kähler potential $K' = K(V_{\hat{a}} = V_{\hat{a}}^0)$.[10] The gauge fixing should also be plugged in W, giving rise to a superpotential W', which is a function of the H^i and $L^{\mathcal{A}'}$. The effective D_a' term now reads

$$D_a' = i X_a^{\mathcal{A}'} G_{\mathcal{A}'}' = -i \overline{X}_a^{\bar{\mathcal{A}}'} G_{\bar{\mathcal{A}}'}', \tag{3.11}$$

with $X_a^{\mathcal{A}'}$ the components of the pulled back isometry vectors on the Kähler invariant function G', defined by the gauge fixing. Since the latter is linear in the fields, the pull-back is trivial and $X_a^{\alpha'}$ are nothing else than the original isometry vector components along the non-gauged away directions $L^{\mathcal{A}'}$. After having integrated out the massive vector fields and their scalar partners, we get an intermediate effective theory given by the Kähler potential K', isometries $X_a^{\mathcal{A}'}$, superpotential W' and gauge kinetic functions f_a, with $\mathrm{Re}\, f_a = 1/g_a^2$. No field with non-vanishing VEV and charged under $\mathcal{H}$ appears in this theory. All the Z and the non-neutral M have been either gauged away or appear as gauge singlet combinations in the intermediate theory, effectively behaving as new gauge invariant fields. By expanding K' in powers of the charged fields we get a Kähler potential of the form (2.19) (where K_0', K_1' and K_2', in turn, can be expanded in powers of the heavy vector mass) and we are effectively back to the case discussed before of unbroken gauge group. We can now integrate out the H and compare the resulting theory with the simple effective theory $K_{\mathrm{sim}}' = K'(H_0)$, $W_{\mathrm{sim}}' = W'(H_0)$, $f_{a,\mathrm{sim}} = f_a(H_0)$. As expected, K_{sim}' and W_{sim}' coincide with the Kähler and superpotential terms obtained by integrating out the heavy vector fields in the simple theory, when the same gauge-fixing taken in the full theory is used.

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[10]If all the components of $V_{\hat{a}}^0$ are much smaller than the heavy vector mass to the appropriate power, an explicit expression for K' can be given by expanding the vector-super fields $V_{\hat{a}}$ up to quadratic order, in which case one has

$$V_{\hat{a}}^0 = -\widetilde{m}_{\hat{a}\hat{b}}^{-2} K_{\hat{b}}/2, \tag{3.9}$$

where $\widetilde{m}_{\hat{a}\hat{b}}^2 = 2\langle g_{M\bar{N}} X_{\hat{a}}^M \overline{X}_{\hat{b}}^{\bar{N}} \rangle$ is the non-canonically normalized mass matrix for the gauge fields and $K_{\hat{a}} = \partial_{V_{\hat{a}}} K|_{V=0}$. Plugging back in K, it gives

$$K' = K(V_{\hat{a}} = 0) - K_{\hat{a}} \widetilde{m}_{\hat{a}\hat{b}}^{-2} K_{\hat{b}}/4. \tag{3.10}$$

4 Conclusions

We have shown that in a wide class of SUSY theories, including vector fields and $\mathcal{O}(1)$ Yukawa couplings for charged fields, it is reasonable to neglect the dynamics of heavy chiral fields and yet retain a predictive simple effective field theory at low-energies. It is enough that the heavy fields sit at an approximately SUSY vacuum in their directions, even when SUSY is completely broken in the light field directions, and (in SUGRA) that W is sufficiently small so that curvature corrections and SUSY breaking mass splitting are much smaller than the heavy field masses. In our class of models, where a chiral field is heavy through a large superpotential term, by construction only gauge-invariant fields can be frozen. No constraint on the Kähler potential is assumed, provided that it is sufficiently regular. It is not difficult to extend the results of our paper to the situation where more mass hierarchies are present. As expected, new expansion parameters $\epsilon_i \equiv m_i/m_H$, where m_i are the new mass scales, appear and all the analysis still applies, being only more cumbersome.

One might wonder whether light moduli, with mass of $\mathcal{O}(m_L)$, can somehow also be frozen. The generic answer is clearly no, but it is interesting to see under what special conditions this may be done. The first obvious requirement to freeze the light moduli M is that $F^M \simeq 0$. This is a non-trivial condition since the backreaction induced by other SUSY breaking effects on them is no longer suppressed by a heavy mass. If it does not occur, not only the light moduli cannot be neglected, but it is generally also hard to integrate them out and the resulting effective theory cannot be described by a SUSY theory. The best one can hope is that their interactions are negligible and just retain the effect of their F-terms, in which case one can compute the associated soft terms [18]. Even if $F^M \simeq 0$, the condition $F_\Phi \ll m_L$ will not in general be satisfied, so that a description of the resulting effective theory in terms of a standard two-derivative SUGRA action will not be available. On the other hand, if $F^M \simeq 0$ and $F_\Phi \ll m_L$, the analysis of our paper can be repeated almost step by step. In the reasonable hypothesis that the Yukawa couplings Y do not depend on M, because, say, the same symmetry forbidding an $\mathcal{O}(1)$ superpotential term W_0 for them might also forbid M-dependent $\mathcal{O}(1)$ Yukawa terms, one can just replace W_0 with W_1 and the couplings Y_N with μ_M. In this set-up, the theory that so far we have called "simple" becomes the underlying theory and we call "super-simple" the resulting theory with the light moduli frozen. It is straightforward to see, properly adapting eqs. (2.24) and (2.25) to this situation, that the induced couplings appearing in W_{full} are at least of $\mathcal{O}(\epsilon^5 m_H^3)$, but those appearing in K_{full} are now of $\mathcal{O}(\epsilon^2 m_H^2)$, since C^2 terms in K may be induced with unsuppressed coefficients. The latter can be relevant for the computation of holomorphic soft mass terms. Hence, the super-simple theory can generally be reliable only if $F^M \simeq 0$, $F_\Phi \ll m_L$ and the induced C^2 terms in K are somehow suppressed.

A A detailed analysis of the vacuum

In this appendix, along the lines of [1], we show in some more detail how the e.o.m. of the light scalar fields agree at leading order in the full and simple models in presence of vector fields. Since non-trivial e.o.m. for the fields C appear only at $\mathcal{O}(\epsilon^3)$, for simplicity we set them to zero, which is always a solution to their e.o.m., and only study the e.o.m. for the remaining M and Z fields. In order to keep the notation as simple as possible, we omit in this appendix the subscript "full", being understood that any quantity with no specification arises in the full theory.

The new ingredient with respect to the analysis performed in [1] is the D-term scalar potential

$$V_D = \frac{1}{2}g_A^2 D_A^2\,. \tag{A.1}$$

We study the location of the vacuum in both theories in a series expansion in ϵ:

$$\langle \phi^M \rangle = \phi_0^M + \epsilon\phi_1^M + \epsilon^2\phi_2^M + \dots\,. \tag{A.2}$$

Although the D-term potential does not admit an expansion in ϵ, being governed by generally $\mathcal{O}(1)$ gauge couplings, at the vacuum the D-terms are related to F-terms and hence an expansion in ϵ is still possible.

At $\mathcal{O}(\epsilon^0)$, $\partial_i W_0(H_0) = 0$ solve the F-term e.o.m. for the heavy fields and trivialize the corresponding ones for the light fields: $(\partial_i V_F)_0 = (\partial_\mathcal{A} V_F)_0 = 0$. Of course, due to the presence of V_D, this is no longer a sufficient condition but it is still necessary. In this way, the leading order VEV's for the heavy fields are fixed. At $\mathcal{O}(\epsilon^0)$ and $H^i = H_0^i$, the e.o.m. for the light fields are entirely given by the D-term potential:

$$(\partial_\mathcal{A} V_D)_0 = \frac{1}{2}(\partial_\mathcal{A} g_A^2)D_A^2 + g_A^2 D_A \partial_\mathcal{A} D_A = 0\,, \tag{A.3}$$

evaluated at ϕ_0^M, which admit the simple solution

$$D_A(\phi_0^M) = 0\,. \tag{A.4}$$

When the gauge symmetry is unbroken, eq. (A.4) is a solution to all orders in ϵ. Indeed, from eq.(3.1) it is straightforward to deduce the following general bound,

$$\sqrt{2}g_A D_A \leq \sqrt{g^{\bar{N}N}K_N K_{\bar{N}}}\, m_{AA}\,, \tag{A.5}$$

with m_{AA} being the diagonal components of the gauge field mass matrix

$$m_{AB}^2 = 2g_A\, g_B\, g_{M\bar{N}}X_A^M \overline{X}_B^{\bar{N}}\,. \tag{A.6}$$

For spontaneously broken symmetries, another relation between F and D-terms is valid at the vacuum, of the form $\langle D \rangle \sim \langle F^2 \rangle / m_V^2$ (see eq.(A.17) below), where m_V is the typical scale of the heavy vector fields, parametrically larger than ϵ. Requiring the F-terms to be all at most of $\mathcal{O}(\epsilon)$, we conclude that at the vacuum $\langle D \rangle \lesssim \mathcal{O}(\epsilon^2)$. Eq. (A.4) is the only sensible solution to eq. (A.3) for vacua with no $\mathcal{O}(1)$ SUSY breaking. Eq. (A.4) also

ensures that at $\mathcal{O}(\epsilon^0)$ the e.o.m. of the heavy fields are automatically satisfied at H_0, since $(\partial_i V_D)_0 = 0$.

At $\mathcal{O}(\epsilon)$, the e.o.m. for the light fields are still given by the D-term potential only, so that

$$(\partial_{\mathcal{A}} V_D)_1 = g_A^2 \left[\partial_N D_A(\phi_0)\phi_1^N + \partial_{\bar{N}} D_A(\phi_0)\phi_1^{\bar{N}} \right] \partial_{\mathcal{A}} D_A(\phi_0) = 0 \,. \tag{A.7}$$

Two possible solutions can be taken. Either $\partial_{\mathcal{A}} D_A(\phi_0) = 0$, which implies $X_A^{\mathcal{A}}(\phi_0^M) = 0$, being $X_A^i = 0$, or $\partial_N D_A(\phi_0)\phi_1^N + \partial_{\bar{N}} D_A(\phi_0)\phi_1^{\bar{N}} = 0$. The two situations correspond respectively to unbroken and broken generators. Indeed, splitting the gauge index $A = (a, \hat{a})$ in eq. (A.6), with $a \in \mathcal{H}$, $\hat{a} \in \mathcal{G}/\mathcal{H}$, we have

$$m_{\hat{a}\hat{b}}^2 = \mathcal{O}(m_V^2), \quad m_{\hat{a}a}^2 = m_{a\hat{a}}^2 = 0, \quad m_{ab}^2 = 0 \,. \tag{A.8}$$

By taking g_A^2 and the Kähler metric parametrically of order one, eq. (A.8) gives

$$\langle X_a^M \rangle = 0, \quad \langle X_{\hat{a}}^M \rangle = \mathcal{O}(m_V) \,. \tag{A.9}$$

So, we have

$$X_{M,a}(\phi_0) = X_a^M(\phi_0) = 0, \quad a \in \mathcal{H} \,,$$
$$\partial_N D_{\hat{a}}(\phi_0)\phi_1^N + \partial_{\bar{N}} D_{\hat{a}}(\phi_0)\phi_1^{\bar{N}} = 0, \quad \hat{a} \in \mathcal{G}/\mathcal{H} \,. \tag{A.10}$$

Eqs. (A.10) imply that both D_a and $D_{\hat{a}}$ vanish at $\mathcal{O}(\epsilon)$, in agreement with our previous argument that $D_A \leq \mathcal{O}(\epsilon^2)$. When eq. (A.10) is satisfied, the e.o.m. for the heavy fields at $\mathcal{O}(\epsilon)$ are given by V_F only and fix H_1^i as in the pure F-term case studied in [1],

$$H_1^i = -(\hat{K}^{-1})_{\bar{j}}^i (\overline{G}^{\bar{j}})_0 \,, \tag{A.11}$$

with $\hat{K}_{\bar{j}}^{i\bar{i}} = g^{\bar{i}k}(\partial_k G_j)_{-1}$ and $(\overline{G}^{\bar{j}})_0 = (\overline{G}_{\bar{M}})_0 g^{\bar{M}i}$, notation like in [1], evaluated at $H^i = H_0^i$. At the shifted vacuum $H_0^i + \epsilon H_1^i$, we have

$$G_i = \mathcal{O}(1), \quad G_{\mathcal{A}} = \mathcal{O}(1), \quad G^i = \mathcal{O}(\epsilon), \quad G^{\mathcal{A}} = \mathcal{O}(1) \,, \tag{A.12}$$

showing that the matching of the F-term part of the e.o.m. of the light fields at $\mathcal{O}(\epsilon^2)$, $(\partial_{\mathcal{A}} V_F)_{\text{full}} = (\partial_{\mathcal{A}} V_F)_{\text{sim}} + \mathcal{O}(\epsilon^3)$, continues to hold in presence of D-terms. Hence we can just focus on V_D. The term $(\partial_{\mathcal{A}} V_D)_2$ can be written as follows:

$$(\partial_{\mathcal{A}} V_D)_2 = g_{\hat{a}}^2(\phi_0) D_{\hat{a}}(\phi_0 + \epsilon\phi_1 + \epsilon^2\phi_2) \partial_{\mathcal{A}} D_{\hat{a}}(\phi_0) \,, \tag{A.13}$$

where we have explicitly written the order at which the various quantities should in principle be known.

A similar expansion of the e.o.m. can be performed in the simple effective theory, where the D-term reads $V_{D,\text{sim}} = g_A^2(D_A)_{\text{sim}}^2/2$, with $(D_A)_{\text{sim}} = iX_A^{\mathcal{A}}\partial_{\mathcal{A}} K_{\text{sim}}$, and all quantities evaluated at the leading frozen vacuum H_0^i. At $\mathcal{O}(\epsilon^0)$ and $\mathcal{O}(\epsilon)$ we get

$$(D_A)_{\text{sim}}(\phi_0) = 0, \quad (X_a^{\mathcal{A}})_{\text{sim}}(\phi_0) = 0, \quad \partial_{\mathcal{A}} D_{\hat{a},\text{sim}}(\phi_0)\phi_{1,\text{sim}}^{\mathcal{A}} + \partial_{\bar{\mathcal{A}}} D_{\hat{a},\text{sim}}(\phi_0)\phi_{1,\text{sim}}^{\bar{\mathcal{A}}} = 0 \,, \tag{A.14}$$

which implies that $D_{\text{sim}}(\phi_0) \sim \mathcal{O}(\epsilon^2)$. The form of $(\partial_\mathcal{A} V_D)_{\text{sim}}$ at $\mathcal{O}(\epsilon^2)$ is the same as eq. (A.13), but written in terms of $(D_A)_{\text{sim}}$ and evaluated at H_0^i. Thus, the equivalence of the two descriptions requires that the following non-trivial relation holds:

$$D_{\hat{a}}(\langle \phi^M \rangle) = (D_{\hat{a}})_{\text{sim}}(H_0^i, \langle L^\mathcal{A} \rangle) + \mathcal{O}(\epsilon^3) \,, \tag{A.15}$$

with $\langle \phi \rangle$ and $\langle L \rangle$ expanded up to $\mathcal{O}(\epsilon^2)$. Luckily enough, we do not need to work out the vacuum up to $\mathcal{O}(\epsilon^2)$, since we can trade $\dim G$ e.o.m. to write $\dim G$ relations between the F and D-terms at the vacuum.[11] Indeed, by taking the combination of the e.o.m. $\text{Im}(X_A^M \partial_M V) = 0$ (the real part being identically vanishing by gauge invariance: $\delta_\lambda V = \lambda^A \text{Re}(X_A^M \partial_M V) = 0$), one easily derive the following equation (see e.g. [4, 19]:

$$q_{AM\bar{M}} F^M \overline{F}^{\bar{M}} - \frac{1}{2} D_B \left[m_{AB}^2 + \delta_{AB}(F^M F_M - m_{3/2}^2) \right] = 0 \,. \tag{A.16}$$

where $q_{AM\bar{M}} = \nabla_M \nabla_{\bar{M}} D_A$ and $m_{3/2} = \exp(K/2) W$.[12] When $A = a$, the second term in eq. (A.16) vanishes and the equations boil down to a set of constraints for the F-terms at the vacuum, dictated by gauge invariance. When $A = \hat{a}$, instead, we can invert eq. (A.16) to solve for $D_{\hat{a}}$:

$$D_{\hat{a}} = 2 m_{\hat{a}\hat{b}}^{-2} q_{\hat{b}M\bar{M}} F^M \overline{F}^{\bar{M}} + \mathcal{O}(\epsilon^4) \,. \tag{A.17}$$

A similar relation occurs in the simple effective theory upon replacing the indices M and $\bar{M}$ with $\mathcal{A}$ and $\bar{\mathcal{A}}$. As expected, eq. (A.17) tells us that $D_{\hat{a}} \sim \mathcal{O}(\epsilon^2)$, since the F-terms are all at most of $\mathcal{O}(\epsilon)$. In addition, since $F^i(\langle \phi^M \rangle) \sim \mathcal{O}(\epsilon^2)$, we see that eq. (A.15) is easily proved:

$$D_{\hat{a}}(\langle \phi^M \rangle) = 2 m_{\hat{a}\hat{b}}^{-2} q_{\hat{b}\mathcal{A}\bar{\mathcal{A}}} F^\mathcal{A} \overline{F}^{\bar{\mathcal{A}}}(\phi_0^M) + \mathcal{O}(\epsilon^3) = (D_{\text{sim}})_{\hat{a}}(H_0^i, L_0^\mathcal{A}) + \mathcal{O}(\epsilon^3) \,. \tag{A.18}$$

We have then established that even in presence of D-terms the location of the vacuum as computed by the simple effective theory is reliable.

B A numerical curved space model with U(1) gauge symmetry

In this appendix we apply the results of the paper to a string-inspired SUGRA toy model with $n_H = 2$, $n_M = 1$, $n_Z = 2$ and $n_C = 0$, where a Fayet-like SUSY breaking mechanism occurs (see [20, 21] where, building on [22], effective models of this sort have been analyzed in some detail). Although the model contains just five complex fields, instead of hundreds as in realistic string models, it is already sufficiently complicated to make an analytical study a formidable task. For this reason we opt here for a numerical analysis. In particular we will show how the vacuum and the scalar mass spectrum as given by the simple effective

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[11]Notice that, modulo global gauge transformations, the vacuum is uniquely determined since the "missing" $\dim G$ e.o.m. are provided by the $\dim G$ D-term constraints (A.4).

[12]If the gauge kinetic functions are not gauge-invariant (e.g. as required by anomaly cancellation), an extra term appears in eq. (A.17). Being of $\mathcal{O}(D^2) \sim \mathcal{O}(\epsilon^4)$, it is completely negligible for our purposes. See e.g. [4] for a more general formula including these terms.

model is in agreement with the full-fledged analysis. The Kähler and superpotential terms are taken as follows:

$$K = -2\log\left[(T + \bar{T} - \delta V_X)^{3/2} + \xi(S + \bar{S})^{3/2}\right] - \log(Z + \bar{Z}) - \log(S + \bar{S})$$
$$+ \frac{\bar{\phi}\,e^{-2V_X}\phi}{(Z + \bar{Z})^{n_\phi}} + \frac{\bar{\chi}\,e^{2V_X}\chi}{(Z + \bar{Z})^{n_\chi}}\,, \tag{B.1}$$

$$W = aZ^2 + bZ + S(cZ^2 + dZ + e) + mZ\phi\chi + \beta Z^2\phi^{\alpha\delta/2}e^{-\alpha T}\,. \tag{B.2}$$

The heavy fields S and Z mimic respectively the dilaton and a complex structure modulus of some IIB flux Calabi-Yau compactification, T represents the overall universal light Kähler modulus, ϕ and χ are two charged fields with $\mathcal{O}(1)$ VEV's and opposite U(1) charge with respect to a U(1) gauge field A_X. The holomorphic gauge kinetic function associated to $U(1)_X$ is taken to be $f_X = T$. The Kähler modulus T is not gauge invariant, but transforms as $\delta T = i\delta/2\Lambda$, where $\delta\phi = i\Lambda\phi$, $\delta\chi = -i\Lambda\chi$. The fields S and Z are gauge invariant. We refer the reader to the appendix B of [1], where a similar superpotential was considered, for an explanation of the various terms appearing in W and the choice of parameters that follows. The $U(1)_X$ D-term is equal to

$$D_X = iX^M G_M = iX^M\partial_M K = \frac{|\chi|^2}{(2Z_r)^{n_\chi}} - \frac{|\phi|^2}{(2Z_r)^{n_\phi}} + \frac{3\delta T_r^{1/2}}{4(T_r^{3/2} + \xi S_r^{3/2})}\,, \tag{B.3}$$

where the subscript r denotes the real part of the field. The essential dynamics of the system is driven by the D_X term. In order to minimize the energy carried by the effective field-dependent FI term, the third term in eq. (B.3), ϕ acquires a non-vanishing VEV. The mass term in the superpotential becomes then effectively a Polonyi-like term for the field χ, which breaks SUSY. We have taken[13]

$$\begin{aligned}
&a = -2.55 - 10^{-13}\,, &&b = 25.5 + 2\cdot 10^{-12}\,, &&c = 0.25\,, &&d = -2.45\,, &&e = -0.5 \\
&\alpha = 1\,, &&\beta = -0.5\,, &&\delta = 1\,, &&m = 2.44\times 10^{-12}\,, \\
&n_\chi = 1\,, &&n_\phi = 0\,, &&\xi = 0.1\,. &&&& \tag{B.4}
\end{aligned}$$

The smallness of m, required to get a dS vacuum with a sufficiently small cosmological constant, justifies the location of the $Z\phi\chi$ operator in W_1. The SUSY VEV's of S and Z, as given by $\partial_S W_0 = \partial_Z W_0$ are precisely $S_0 = Z_0 = 10$, with $W_0(S_0, Z_0) = 10^{-11}$. In table 1 we report the exact VEV's and F-terms of the fields as numerically computed in the full model and their relative shifts compared to those computed, again numerically, in the simple effective model, with S and Z frozen at their values S_0 and Z_0.[14] The model above belongs to the general class of models studied in the main text with an ϵ roughly

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[13]Notice that there is a typo in the values of a and b in eq. (B.3) of [1], the correct values being the ones reported in eq. (B.4).

[14]For simplicity of the numerical analysis, we have not integrated out the massive $U(1)_X$ gauge field, as it should be done in both theories. However, this is not going to affect the vacuum or the mass spectrum and hence the results that follow are reliable.

	$\langle X \rangle$	$\Delta \langle X \rangle$	F^X	ΔF^X
S	$10 + 2 \cdot 10^{-13}$	$2 \cdot 10^{-14}$	$-9.9 \cdot 10^{-28}$	—
Z	$10 + 2 \cdot 10^{-13}$	$2 \cdot 10^{-14}$	$5.6 \cdot 10^{-28}$	—
T	31.4	$1.1 \cdot 10^{-15}$	$-3.4 \cdot 10^{-15}$	$-1.4 \cdot 10^{-16}$
ϕ	0.15	$-1.3 \cdot 10^{-15}$	$1.4 \cdot 10^{-16}$	$-7.4 \cdot 10^{-14}$
χ	0.05	$-7.6 \cdot 10^{-14}$	$7.5 \cdot 10^{-15}$	$1.5 \cdot 10^{-14}$

Table 1. VEV's and $F^N = e^{K/2} g^{\bar{M}N} \bar{F}_{\bar{M}}$ terms for the fields and their relative shifts, as derived by a numerical analysis. Here and in the main text $\Delta X \equiv (X_{\text{full}} - X_{\text{sim}})/X_{\text{full}}$. All quantities are in reduced Planck units.

$\mathcal{O}(10^{-13})$. Keeping two significant digits, the physical masses are:

$$m_{H_1}^2 = 1.1 \cdot 10^{-2}, \qquad m_{H_2}^2 = 1.0 \cdot 10^{-2}, \qquad m_{\phi_r}^2 = 1.5 \cdot 10^{-3}, \tag{B.5}$$
$$m_{T_i}^2 = 4.0 \cdot 10^{-27}, \qquad m_{T_r}^2 = 3.7 \cdot 10^{-27}, \qquad m_{\chi_i}^2 = 2.3 \cdot 10^{-28}, \quad m_{\chi_r}^2 = 2.3 \cdot 10^{-28}.$$

where $H_1 \simeq Z + S$, $H_2 \simeq Z - S$ and the subscript r and i denote real and imaginary field components, respectively. The masses m_{H1}^2 and m_{H2}^2 refer to both components of the complex scalar fields, being SUSY breaking effects negligible. The imaginary component of ϕ is approximately the Goldstone boson eaten by A_X and hence is exactly massless. The relative mass shifts are

$$\Delta m_{\phi_r}^2 = -4.7 \cdot 10^{-15}, \qquad \Delta m_{T_i}^2 = -2.1 \cdot 10^{-14}, \quad \Delta m_{T_r}^2 = -2.6 \cdot 10^{-14},$$
$$\Delta m_{\chi_i}^2 = 1.4 \cdot 10^{-14}, \qquad \Delta m_{\chi_r}^2 = 1.8 \cdot 10^{-14}. \tag{B.6}$$

Finally, we also report the gravitino mass, the D_X-term, the cosmological constant and their relative shifts:

$$m_{3/2}^2 = 9.5 \cdot 10^{-31}, \quad \Delta m_{3/2}^2 = -2.7 \cdot 10^{-14},$$
$$D_X = 3.7 \cdot 10^{-27}, \qquad \Delta D_X = 1.5 \cdot 10^{-14},$$
$$V_0 = 1.2 \cdot 10^{-32}, \qquad \Delta V_0 = 8.2 \cdot 10^{-12}. \tag{B.7}$$

Notice that being the cosmological constant fine-tuned to be "small", namely of order $10^{-2} m_{3/2}^2$, its relative shift is larger. The latter is inversely proportional to the smallness of V_0.

The relative shifts of the various quantities considered are smaller than ϵ due to the fact that the Kähler mixing between the H and L fields coming from (B.1) in the above vacuum are relatively small. This example shows the excellent agreement between the full and the simple theory.

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