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Penrose limit and duality between string and gauge theories

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Abstract. We give a brief introduction to the Penrose limit and its use in the AdS/CFT correspondence. Related developments on the relationship between gauge theories and integrable systems are discussed also for non–conformal theories.

In General Relativity there is a remarkably simple argument, due to Penrose [1], which shows that any space–time has a plane wave as a limit. The Penrose limit has been recently generalized to string theory [2], and has attracted a fair amount of activity since then. The basic motivation is that this limiting procedure yields new supersymmetric *curved* backgrounds with non-trivial fluxes, on which the string spectrum can be computed *exactly* in the (inverse) string tension α'. In this note we will focus on the maximally supersymmetric plane wave in type IIB string theory, which can be obtained as a Penrose limit of the $\mathrm{AdS}_5\times\mathrm{S}^5$ space [3]. Berenstein, Maldacena and Nastase proposed a very concrete description of string theory in this background in terms of a particular sector of the $\mathscr{N}=4$ Super Yang-Mills theory [4]. This is the first example in which the world–sheet of a well defined string theory is reproduced starting from gauge theory amplitudes, and provides at the moment the best theoretical laboratory to study and test the gauge/string duality. Let us start by recalling the $\mathrm{AdS}_5\times\mathrm{S}^5$ solution of IIB supergravity in global coordinates

$$\begin{aligned} ds^2 &= R^2\left[-\cosh^2 r\,dt^2+dr^2+\sinh^2 r\,d\Omega_3^2+\cos^2\theta\,d\psi^2+d\theta^2+\sin^2\theta\,d\Omega_3'^2\right], \\ F_5 &= \frac{1}{R}\left(dV_{AdS_5}+dV_{S^5}\right) \quad , \quad \phi=\text{constant}\,, \end{aligned} \tag{1}$$

where ϕ is the dilaton field and F_5 the Ramond–Ramond five form. The solution (1) preserves the maximal number of supersymmetries and is believed to be exact to all orders in α'. The Penrose limit consists in focusing on the neighborhood of a null geodesic and rescaling coordinates and metric in order to blow up this neighborhood to the whole space. In supergravity also the other forms have to be rescaled properly in the limit: the homogeneity of the action under these rescalings ensures that the space obtained in the Penrose limit is still an exact solution [2]. For the $\mathrm{AdS}_5\times\mathrm{S}^5$ space, there are essentially two different Penrose limits, according to the different choices of the null geodesic [3]: if this lies entirely in the AdS space, one gets a limiting flat space. By choosing instead the geodesic of a particle rotating around the sphere, *i.e.* $t=\psi$ and

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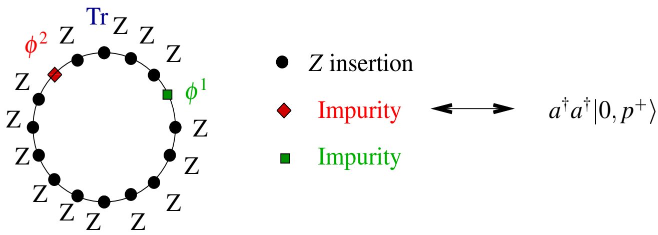

$r = \theta = 0$, and going to the light–cone coordinates

$$x^+ = \frac{t+\psi}{2\mu}\ , \quad x^- = \mu R^2 \frac{t-\psi}{2}\ , \quad \hat{r} = Rr\ , \quad y = R\theta\ , \tag{2}$$

one gets in the Penrose limit $R \to \infty$ with $x^\pm, \hat{r}, y$ fixed

$$\begin{aligned} ds^2 &= -4dx^+dx^- - \mu^2(x_I)^2 dx^{+\,2} + dx^{I\,2}\ , \\ F_{+1234} &= F_{+5678} = 2\mu\ , \quad \phi = \text{constant}. \end{aligned} \tag{3}$$

The key features of the above plane-wave solution are that *(i)* it is a curved background $(R_{++} \sim \mu)$ with non-trivial Ramond–Ramond flux, and *(ii)* it preserves all 32 supersymmetries of type IIB string theory. This last feature results from a general property of the Penrose limit, according to which the number of (super)symmetries cannot decrease in the limit [3]. The physical interpretation of the Penrose limit is a *truncation* of the string spectrum to the states describing small oscillations around a point-like string spinning along the five–sphere. The geometry (3) seen by these states is greatly simplified, so that their spectrum can be computed *exactly*. In fact the world–sheeet action in the light–cone gauge turns out to be free [5]. The light–cone Hamiltonian at fixed light–cone momentum p^+ is a sum of eight bosonic and eight fermionic harmonic oscillators, all with the same frequency

$$\frac{H}{\mu} = \sum_{n=-\infty}^{+\infty} \omega_n \left(a_n^{I\dagger} a_n^I + b_n^{\alpha\dagger} b_n^\alpha \right)\ , \quad \omega_n = \sqrt{1 + \frac{n^2}{(\mu\alpha' p^+)^2}}\ , \tag{4}$$

with $I = 1,\ldots,8$, $\alpha = 1,\ldots,8$ [5]. The basic idea of [4] was to regard some gauge-invariant operators of the $\mathcal{N} = 4$ Super Yang–Mills theory containing a large number J of fields as a discretised version of the physical type IIB string on the plane-wave background. The BMN operators are single trace operators formed by a long chain of one of the elementary scalar fields of $\mathcal{N} = 4$, with the insertion of a few other fields and covariant derivatives (called *impurities*), each of them corresponding to a different excitation of the string (see figure).

The anomalous dimensions of these operators are expected to coincide with the mass of the corresponding string state. On the gauge theory side, the Penrose limit corresponds to take the 't Hooft coupling λ to infinity while keeping the ratio $\lambda' = \lambda/J^2$ fixed. It is thus different from the usual 't Hooft expansion. The remarkable feature of the spectrum (4) is that, when using the AdS/CFT dictionary $\lambda' = 1/\mu p^+ \alpha'$, one gets an *analytic* function of the (effective) gauge coupling λ'. This makes it possible to directly compare the string spectrum with the perturbative expansion of the gauge theory, finding agreement [4]! Notice that this comparison is based on the extrapolation of the perturbative gauge theory results to the strong coupling regime $\lambda \to \infty$, λ' fixed. A definite proof of the validity of this extrapolation is still lacking, and would certainly improve our understanding of the duality in the large J limit.

A lot of work has been done in order to include the string interaction in the duality. In [6], we constructed a supersymmetric 3-string interaction vertex on the plane-wave background. At the leading order in λ', the resulting amplitudes are in agreement with that extracted from three-point correlators of BMN operators. However, the extension of this correspondence at the subleading order remains an open problem (for a review, see [7]). The main obstacle is that in the Penrose limit the boundary of the AdS space is washed away (recall that the null geodesic (2) sits at the center of AdS space), and this makes it rather problematic to define a clear holographic principle. See however [8] for some promising progress.

The study of the AdS/CFT correspondence in the plane-wave limit has posed the difficult problem to compute the anomalous dimensions of composite operators of the $\mathcal{N} = 4$ SYM containing a large number of fields $J \to \infty$. A very useful result in this context [9] is that, in the large-N limit, the matrix of one-loop anomalous dimensions can be mapped into the Hamiltonian of an integrable system. This intriguing remark applies not only to the BMN operators, but also to a wider class of operators containing a large number of impurities $J_i \sim J$. In particular, for the (closed) subsector of composite operators of two scalar fields, one gets the Hamiltonian of an XXX Heisenberg spin chain [9]. In [10] we have analysed the analogous sector for the $\mathcal{N} = 2$ SYM theory, finding an XXZ spin-chain. From the dual string theory point of view, the presence of the anisotropy in one of the spin directions should be related to the non-trivial fluxes which break the supersymmetry from $\mathcal{N} = 4$ to $\mathcal{N} = 2$. One interesting lesson from the $\mathcal{N} = 2$ case is that the breaking of conformal invariance by virtue of the non-vanishing beta function $\beta \sim -g^3$ does *not* affect the renormalization-group equations for these operators at one-loop order. This is a general property of gauge theories which is not related to supersymmetry. These features make it particularly interesting to investigate whether some relation can be found between the integrability of some subsectors of gauge theories in the large N limit and the existence of a dual string theory description for them.

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