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Revisiting leptogenesis in a SUSY $SU(5) \times T'$ model of flavour

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ABSTRACT

We investigate the generation of the baryon asymmetry of the Universe within a SUSY $SU(5) \times T'$ model of flavour, which gives rise to realistic masses and mixing patterns for quarks and leptons. The model employs the see-saw mechanism for generation of the light neutrino masses and the baryon asymmetry is produced via leptogenesis. We perform detailed calculations of both the CP violating lepton asymmetries, originating from the decays of the heavy Majorana neutrinos operative in the see-saw mechanism, and of the efficiency factors which account for the lepton asymmetry wash-out processes in the Early Universe. The latter are calculated by solving numerically the system of Boltzmann equations describing the generation and the evolution of the lepton asymmetries. The baryon asymmetry in the model considered is proportional to the J_{CP} factor, which determines the magnitude of CP violation effects in the oscillations of flavour neutrinos. The leptogenesis scale can be sufficiently low, allowing to avoid the potential gravitino problem.

1. Introduction

In the present Letter we consider the generation of the baryon asymmetry of the Universe in the SUSY model of flavour based on the $SU(5) \times T'$ symmetry, which was developed in [1,2]. The model possesses a number of appealing features which makes it worthwhile to investigate whether it provides also a viable scenario for the baryon asymmetry generation.

The group T' is the double covering of the symmetry group of the tetrahedron A_4 (see, e.g., [3]). It was realised by a number of authors (see, e.g., [4]) that the T' symmetry can be used for the description of masses and mixing of both leptons and quarks. The $SU(5) \times T'$ model of flavour of interest accounts successfully for the pattern of quark masses and mixing, including the CP violation in the quark sector [2]. It is free of discrete gauge anomalies [5] and gives rise to realistic masses and mixing of the leptons as well.

The $SU(5) \times T'$ model proposed in [1,2] we will discuss in the present Letter, includes three right-handed (RH) neutrino fields N_{lR}, $l = e, \mu, \tau$, which possess a Majorana mass term. The light neutrino masses are generated by the type I see-saw mechanism and are naturally small. The light neutrino mass spectrum is predicted [6] to be with normal ordering and is hierarchical (throughout this Letter we use the definitions and the conventions given in [7]). The neutrino masses m_j, $j = 1, 2, 3$, are functions of two real parameters of the model [2,6]. The latter can be determined by using the values of the two neutrino mass squared differences, Δm^2_{21} and Δm^2_{31}, or of Δm^2_{21} and the ratio $r = \Delta m^2_{21}/\Delta m^2_{31}$, obtained in the global analyses of the neutrino oscillation data. Using the best fit values of $\Delta m^2_{21} = 7.58 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} = 2.35 \times 10^{-3}$ eV^2, found in the analysis performed in [8], we have [6]:

$$m_1 = 1.14 \times 10^{-3}\ \text{eV}, \qquad m_2 = 8.78 \times 10^{-3}\ \text{eV}, \qquad m_3 = 4.867 \times 10^{-2}\ \text{eV}. \tag{1}$$

The values of m_1, m_2 and m_3 are essentially fixed: the uncertainties corresponding to the 3σ ranges of allowed values of Δm^2_{21} and r are remarkably small [6].

The part of the Pontecorvo, Maki, Nakagawa and Sakata (PMNS) neutrino mixing matrix (see [7]), resulting from the diagonalisation of the Majorana mass term of the left-handed flavour neutrino fields $\nu_{lL}(x)$, $l = e, \mu, \tau$, which is generated by the see-saw mechanism, is

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of the tri-bimaximal form [9]. The latter is "corrected" by the unitary matrix originating from the diagonalisation of the charged lepton mass matrix M_e (for a general discussion of such corrections see, e.g., [10–12]). Since the model is based on the $SU(5)$ GUT symmetry, the charged lepton mass matrix is related to the down-quark mass matrix M_d. The model exploits the Georgi–Jarlskog approach for obtaining viable relations between the masses of the muon and the s-quark [1,2]. The Cabibbo angle is given by the "standard" expression: $\theta_c \cong \sqrt{m_d/m_s}$, m_d and m_s being the masses of the d- and s- quarks. As a consequence, in particular, of the connection between M_e and M_d, the smallest angle in the neutrino mixing matrix θ_{13}, is related to the Cabibbo angle [2]:

$$\sin\theta_{13} \cong \frac{1}{3\sqrt{2}} \sin\theta_c. \tag{2}$$

Here we implicitly assumed the "standard" parametrisation of the PMNS matrix [7]:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \operatorname{diag}\left(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}\right), \tag{3}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, $0 \leqslant \theta_{ij} \leqslant \pi/2$, δ and α_{j1}, $j = 2, 3$, are the Dirac and the two Majorana CP violation phases [13], $0 \leqslant \delta \leqslant 2\pi$ and, in general [14], $0 \leqslant \alpha_{j1}/2 \leqslant 2\pi$. We will use this parametrisation in the discussion which follows.

The CP violation, predicted by the model, can entirely be geometrical in origin [2]. This interesting aspect of the $SU(5) \times T'$ model we will consider is a consequence of one of the special properties of the group T', namely, that its group theoretical Clebsch–Gordan (CG) coefficients are intrinsically complex [15]. The only dominant source of CP violation in the lepton sector of the model is the Dirac phase δ. The two Majorana phases present in the PMNS neutrino mixing matrix, α_{21} and α_{31}, are predicted to leading order to have CP conserving values. In the standard parametrisation of U_{PMNS} we have: $\alpha_{21} \cong 0$ and $\alpha_{31} \cong \pi$. Higher order corrections induce small CP violating deviations of the order of few degrees from these CP conserving values of the two phases [6].

The Dirac phase δ in the PMNS matrix is induced effectively by the complex CG coefficients of the group T'. As we shall see in Section 2, δ can take two values in the model considered. One was identified in [2] and is equal approximately to $\delta \cong 5\pi/4 = 225°$, the precise value being

$$\delta \cong 226.9°. \tag{4}$$

The second possible value of δ is given to leading order, as will be discussed in Section 2, by

$$\delta \cong \frac{\pi}{4} = 45°. \tag{5}$$

The tri-bimaximal mixing value of the solar neutrino mixing angle θ_{12}, which corresponds to $\sin^2\theta_{12} = 1/3$, is corrected by a quantity which, as it follows from the general form of such corrections [10–12], is determined by the angle θ_{13} and the Dirac phase δ:

$$\sin^2\theta_{12} \cong \frac{1}{3} + \frac{2\sqrt{2}}{3} \sin\theta_{13}\cos\delta. \tag{6}$$

In the $SU(5) \times T'$ model considered, θ_{13} is related to the Cabibbo angle, Eq. (2).

The rephasing invariant associated with the Dirac phase δ [16], J_{CP}, which determines the magnitude of CP violation effects in neutrino oscillations [17], predicted by the model to leading order reads [6,10,11]:

$$J_{CP} \cong \frac{1}{3\sqrt{2}} \sin\theta_{13}\sin\delta \cong \frac{1}{18}\sin\theta_c\sin\delta. \tag{7}$$

For $\delta \cong 5\pi/4$, Eq. (4), the correction to the TBM value of $\sin^2\theta_{12}$ given in Eq. (6), is negative and $\sin^2\theta_{12} \cong 0.299$, where we have used Eq. (2) and $\sin\theta_c = 0.22$. This value lies within the 1σ allowed range, found in the global data analysis [8]. We also have, including the higher order corrections [2]: $J_{CP} \cong -9.66 \times 10^{-3}$. If $\delta \cong \frac{\pi}{4}$, Eq. (5), the correction to the TBM value of $\sin^2\theta_{12}$ is positive and $\sin^2\theta_{12} \cong 0.37$. According to the analyses performed in [8] and in [18], the current neutrino oscillation data imply respectively $\sin^2\theta_{12} \lesssim 0.36$ and $\sin^2\theta_{12} \lesssim 0.374$ at 3σ. Thus, the case of $\delta \cong \frac{\pi}{4}$ is disfavoured by the data. For the J_{CP} factor in this case we get: $J_{CP} \cong +9.95 \times 10^{-3}$.

Since the neutrino masses, the neutrino mixing angle and the CP violating phases in the PMNS matrix have essentially fixed values, the model provides also specific predictions [6] for the sum of the three neutrino masses,

$$m_1 + m_2 + m_3 \cong 5.9 \times 10^{-2}\ \text{eV}, \tag{8}$$

as well as for the effective Majorana mass in neutrinoless double-beta decay (see, e.g., [19]):

$$\left|\langle m\rangle\right| \cong 3.4 \times 10^{-3}\ \text{eV}. \tag{9}$$

It should be clear from the preceding discussion that the $SU(5) \times T'$ model of flavour of interest is remarkably predictive: the values of the neutrino masses, the type of the neutrino mass spectrum, the values of the neutrino mixing angles and the CP violating phases in the neutrino mixing matrix, as well as the effective Majorana mass in neutrinoless double beta decay, obtained in the model are essentially free of ambiguities. The predictions for $\sin\theta_{13}$, $\sin^2\theta_{12}$, δ and J_{CP} can be tested directly in the upcoming neutrino oscillation experiments. The value of $\sin\theta_{13}$ one gets in the model, for instance, is relatively small, $\sin\theta_{13} \cong 0.058$.[2] It lies outside the 2σ, but within the 3σ,

[2] A larger value of θ_{13} can, in principle, be obtained along the lines discussed in Ref. [12].

ranges of allowed values of $\sin\theta_{13}$, determined in the global analyses of the current neutrino oscillation data [8,20]. The results of the three reactor $\bar{\nu}_e$ experiments on θ_{13}, Double Chooz [21], RENO [22] and Daya Bay [23], which are currently taking data, can provide a critical test of the model.

In the present Letter we investigate the prediction of the $SU(5)\times T'$ model of flavour proposed in [1,2] for the baryon asymmetry of the Universe. The latter is generated in the model via the leptogenesis mechanism [24,25]. The dominant source of CP violation in the lepton sector and in leptogenesis is the Dirac phase δ.[3] Therefore there is a direct connection between the baryon asymmetry of the Universe and the CP violation in neutrino oscillations.

The generation of the baryon asymmetry in the $SU(5)\times T'$ model of interest was studied in [27]. However, the authors of [27] limited the discussion of the baryon asymmetry generation to the calculation of the CP asymmetries in the additive lepton charges, ϵ_i^ℓ, generated in the heavy Majorana neutrino decays, $\ell = e,\mu,\tau$, $i = 1,2,3$. They based their conclusions on the results obtained for these asymmetries. In the present Letter we perform a complete calculation of the baryon asymmetry, i.e., we calculate not only the asymmetries ϵ_i^ℓ, but also the corresponding efficiency factors which account for the effects of the CP asymmetry wash-out processes, taking place in the Early Universe. The efficiency factors are computed by solving numerically the Boltzmann equations, which describe the evolution of the CP violating asymmetries in the Early Universe. The results we obtain for the lepton asymmetries ϵ_i^ℓ do not agree with those found in [27] and our results for the baryon asymmetry contradict the claims made in [27].

2. Ingredients

In the L–R convention in which the neutrino mass terms are written with the RH neutrino fields on the right, the superpotential of the model leads [1,2] to the following neutrino Dirac mass matrix,

$$M_D = \begin{pmatrix} 2\xi_0+\eta_0 & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & -\xi_0+\eta_0 \\ -\xi_0 & -\xi_0+\eta_0 & 2\xi_0 \end{pmatrix} \zeta_0\zeta_0' v_u \equiv \tilde{Y}_\nu v_u, \tag{10}$$

and to the following Majorana mass matrix of the RH neutrinos,

$$M_{RR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} s_0 \Lambda. \tag{11}$$

In Eqs. (10) and (11), ξ_0, η_0, ζ_0, ζ_0' and s_0 are dimensionless real parameters, Λ is the scale above which the T' symmetry is exact, $\tilde{Y}_\nu$ is the matrix of the neutrino Yukawa couplings in the basis in which the charged lepton and the RH neutrino mass matrices are not diagonal, and v_u is the vacuum expectation value of the "up" Higgs doublet field of the SUSY extensions of the Standard Model. Thus, the neutrino Dirac mass matrix in the model, M_D, is real and symmetric. As can be easily shown, it is diagonalised by the tri-bimaximal mixing (TBM) matrix:

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}. \tag{12}$$

We have:

$$U_{\mathrm{TBM}}^T M_D U_{\mathrm{TBM}} = M_D^{\mathrm{diag}} = \mathrm{diag}(3\xi_0+\eta_0, \eta_0, 3\xi_0-\eta_0)\zeta_0\zeta_0' v_u \tag{13}$$

where all elements in the diagonal matrix M_D^{diag} are real.

The RH neutrino Majorana mass matrix M_{RR} is diagonalised by the unitary matrix S:

$$S^T M_{RR} S = D_N = \mathrm{diag}(M_1, M_2, M_3) = s_0\Lambda\,\mathrm{diag}(1,1,1), \quad M_j > 0, \quad j = 1,2,3, \tag{14}$$

where

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -i/\sqrt{2} \\ 0 & 1/\sqrt{2} & i/\sqrt{2} \end{pmatrix}. \tag{15}$$

and M_j are the masses of the heavy Majorana neutrinos N_j (possessing definite masses),

$$N_j = S^\dagger_{jl} N_{lR} + S^T_{jl} C(\bar{N}_{lR})^T = C(\bar{N}_j)^T, \quad j = 1,2,3, \tag{16}$$

C being the charge conjugation matrix. Thus, to leading order, the masses of the three heavy Majorana neutrinos N_j coincide, $M_j = s_0\Lambda \equiv M$, $j = 1,2,3$. It follows from Eq. (14) that $S^*S^\dagger$ is a real matrix, so $S^*S^\dagger = SS^T$.

The effective Majorana mass matrix of the left-handed (LH) flavour neutrinos, M_ν, which is generated by the see-saw mechanism,

$$M_\nu = -M_D M_{RR}^{-1} M_D^T, \tag{17}$$

[3] The Casas–Ibarra matrix [26], which can be an additional source of CP violation in leptogenesis, is real in the model under discussion [6].

($\mathcal{L}^M_{\nu_L} = -\frac{1}{2}\bar{\nu}_L M_\nu \nu^c_R + h.c.$) is also diagonalised by the TBM matrix (12),

$$U^T_{TBM} M_\nu U_{TBM} = \mathrm{diag}\big((3\xi_0+\eta_0)^2, \eta_0^2, -(-3\xi_0+\eta_0)^2\big)\frac{(\zeta_0\zeta'_0 v_u)^2}{s_0\Lambda} = Q\,\mathrm{diag}(m_1, m_2, m_3)\,Q^T. \tag{18}$$

Here $Q = i\,\mathrm{diag}(1, 1, \pm i)$ is the matrix which determines, as we shall see, the leading order values of the two Majorana phases in the PMNS matrix, and $m_k > 0$, $k = 1, 2, 3$, are the masses of the three light Majorana neutrinos,

$$m_1 \equiv \frac{(X+3Z)^2}{M}, \qquad m_2 \equiv \frac{X^2}{M}, \qquad m_3 \equiv \frac{(X-3Z)^2}{M}, \tag{19}$$

where $X \equiv \eta_0(\zeta_0\zeta'_0 v_u)$ and $Z \equiv \xi_0(\zeta_0\zeta'_0 v_u)$. In what follows we will ignore the overall unphysical factor i in Q. The values of m_j given in Eq. (1) correspond to [6] $X = \pm 1.71 \times 10^{-2} v_u$, and $Z = \mp 7.74 \times 10^{-3} v_u$.

The charged lepton mass matrix M_e is not diagonal; it is diagonalised by a bi-unitary transformation: $M_e = V_{eR} M^d_e U^\dagger_e$, where V_{eR} and U_e are unitary matrices and $M^d_e = \mathrm{diag}(m_e, m_\mu, m_\tau)$, m_ℓ being the mass of the charged lepton ℓ, $\ell = e, \mu, \tau$. The matrix U_e, which enters into the expression for the PMNS matrix, $U = U^\dagger_e U_\nu$, diagonalises the matrix $M^\dagger_e M_e$. The charged lepton mass matrix M_e (with the corresponding mass term written in the R–L convention in terms of the chiral charged lepton fields l'_{aR} and l'_{aL}) has the following form [2]:

$$M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi'_0 & \phi_0\psi'_0 \\ (1+i)\phi_0\psi'_0 & -3\psi_0\zeta'_0 & \phi_0\psi'_0 \\ 0 & 0 & \zeta_0 \end{pmatrix} y_d v_d \phi_0. \tag{20}$$

It is related to the down-type quark mass matrix M_d via the well-known $SU(5)$ relation: $M_e = M^T_d$, with the factor (-3) in M_e replaced by 1 in M_d. The up-type quark mass matrix in the model has the form [2]:

$$M_u = \begin{pmatrix} i\phi'^3_0 & (\frac{1-i}{2})\phi'^3_0 & 0 \\ (\frac{1-i}{2})\phi'^3_0 & \phi'^3_0 + (1-\frac{i}{2})\phi^2_0 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u. \tag{21}$$

In Eqs. (20) and (21), ϕ_0, ψ'_0, ϕ'_0, ψ_0, y_d and y_t are real dimensionless parameters, y_d and y_t are Yukawa couplings and v_d is the vacuum expectation value of the "down" type Higgs doublet of the SUSY extension of the Standard Model.

Fitting the quark sector observables and charged lepton masses one finds that [2] two of the three angles, present in the "standard-like" parametrisation of the matrix U_e, are extremely small, $\sin\theta^e_{13} \cong 1.3 \times 10^{-5}$, $\sin\theta^e_{23} \cong 1.5 \times 10^{-4}$, while the third satisfies:

$$\sin\theta^e_{12} = \frac{1}{3}\sin\theta_c. \tag{22}$$

Thus, to a very good approximation one can set $\theta^e_{13} = \theta^e_{23} = 0$, and in this approximation U_e takes the form [6]: $U_e = \Phi R_{12}(\theta^e_{12})$, where $\Phi = \mathrm{diag}(1, e^{i\varphi}, 1)$ and

$$R_{12}(\theta^e_{12}) = \begin{pmatrix} \cos\theta^e_{12} & \sin\theta^e_{12} & 0 \\ -\sin\theta^e_{12} & \cos\theta^e_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{23}$$

It follows from the above discussion that in the basis in which the charged lepton and the RH neutrino mass matrices are diagonal, the matrix of neutrino Yukawa couplings Y_ν has the form:

$$Y_\nu = U^\dagger_e \tilde{Y}_\nu S = \frac{1}{v_u} U^\dagger_e M_D S. \tag{24}$$

In the same basis, the Majorana mass term for the LH flavour neutrinos, generated by the see-saw mechanism, is given by:

$$M_\nu = -v_u^2 Y_\nu D_N^{-1} Y_\nu^T = U D_\nu U^T, \tag{25}$$

where $D_N \equiv \mathrm{diag}(M_1, M_2, M_3) = M\,\mathrm{diag}(1, 1, 1)$, $D_\nu \equiv \mathrm{diag}(m_1, m_2, m_3)$ and U is the PMNS matrix,

$$U = U^\dagger_e U_{TBM} Q. \tag{26}$$

Using the approximate expression for $U_e = \Phi R_{12}(\theta^e_{12})$, with $\Phi = \mathrm{diag}(1, e^{i\varphi}, 1)$ and $R_{12}(\theta^e_{12})$ given by Eq. (23), we get:

$$U \cong \begin{pmatrix} \sqrt{2/3}c^e_{12} + \sqrt{1/6}s^e_{12}e^{-i\varphi} & \sqrt{1/3}(c^e_{12} - s^e_{12}e^{-i\varphi}) & \sqrt{1/2}s^e_{12}e^{-i\varphi} \\ \sqrt{2/3}s^e_{12} - \sqrt{1/6}c^e_{12}e^{-i\varphi} & \sqrt{1/3}(s^e_{12} + c^e_{12}e^{-i\varphi}) & -\sqrt{1/2}c^e_{12}e^{-i\varphi} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix} Q, \tag{27}$$

where $c^e_{12} = \cos\theta^e_{12}$, $s^e_{12} = \sin\theta^e_{12}$.

As was shown in [6,12], the phase φ in Eq. (27) and the Dirac phase δ in Eq. (3) are related as follows:

$$\delta = \varphi + \pi. \tag{28}$$

Comparing the expressions in the left-hand and right-hand sides of the equation $M_e^\dagger M_e = U_e (M_e^d)^2 U_e^\dagger$ and assuming first following [2] that $\phi_0\psi_0'\psi_0\zeta_0' < 0$ and $y'\psi_0\zeta_0 < 0$ (with $\cos\theta_{12}^e \sin\theta_{12}^e > 0$), one finds [6]:

$$\varphi = \frac{\pi}{4}, \qquad \delta = \frac{5}{4}\pi. \tag{29}$$

The choice $\phi_0\psi_0'\psi_0\zeta_0' < 0$ together with the choice $y'\psi_0\zeta_0 < 0$ (see Eq. (21)) allows to get the best description of the quark masses and mixing, possible in the model considered. However, one gets similar description also in the case of $\phi_0\psi_0'\psi_0\zeta_0' > 0$ and $y'\psi_0\zeta_0 > 0$.[4] In this latter case we get for φ and δ:

$$\varphi = \frac{\pi}{4} \pm \pi, \qquad \delta = \frac{\pi}{4}. \tag{30}$$

Numerically, for $\varphi = \pi/4$ and $s_{12}^e = 0.22/3$ (see Eq. (22)), the PMNS matrix, Eq. (27), reads:

$$U \simeq \begin{pmatrix} 0.836e^{-i1.452^\circ} & 0.546e^{i3.139^\circ} & 0.0518e^{-i45.000^\circ} \\ 0.367e^{i173.380^\circ} & 0.607e^{i2.829^\circ} & -0.705 \\ -0.408 & 0.577 & 0.707 \end{pmatrix} Q. \tag{31}$$

Taking into account the corrections due to the non-zero values of the angles θ_{13}^e and θ_{23}^e in $U_e^\dagger$ on finds [2]:

$$U \simeq \begin{pmatrix} 0.838e^{-i1.626^\circ} & 0.543e^{i3.551^\circ} & 0.0582e^{-i45.000^\circ} \\ 0.362e^{i172.463^\circ} & 0.610e^{i3.160^\circ} & -0.705 \\ -0.408 & 0.577 & 0.707 \end{pmatrix} Q. \tag{32}$$

Obviously, the differences between the approximate and the “exact” matrices (31) and (32) are negligibly small.

The leading order predictions of the $SU(5)\times T'$ model for $\sin\theta_{13}$, $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ were given in the Introduction (see Eqs. (2), (4), (6) and the related discussions). They can be obtained by comparing Eqs. (3) and (27) and using Eq. (22).

Eqs. (31) and (32) allow to determine the values of the Majorana phases α_{21} and α_{31}. In the parametrisation in which the PMNS matrix is written in Eqs. (27), (31) and (32) they are fixed by the matrix $Q = \mathrm{diag}(1,1,\pm i)$ and read $\alpha_{21}/2 = 0$ and $\alpha_{31}/2 = \pi/2$ or $3\pi/2$. Thus, α_{21} and α_{31} are CP conserving. Note, however, that the parametrisation of the PMNS matrix in Eqs. (27), (31) and (32) does not coincide with the standard one. Thus, in order to get the values of the Dirac and Majorana phases δ and $\alpha_{21}/2$ and $\alpha_{31}/2$ of the standard parametrisation of the PMNS matrix, one has to bring the expressions (31) or (32) in a form which corresponds to the “standard” one in Eq. (3). This can be done by using the freedom of multiplying the rows of the PMNS matrix with arbitrary phases and by shifting some of the common phases of the columns to a diagonal phase matrix $\tilde{Q}$. The results for the “approximate” and “exact” numerical matrices, Eqs. (31) and (32), is:

$$U \simeq \begin{pmatrix} 0.836 & 0.546 & 0.0518e^{-i226.69^\circ} \\ -0.367e^{-i3.48^\circ} & 0.607e^{i1.38^\circ} & 0.705 \\ 0.408e^{i3.14^\circ} & -0.577e^{-i1.45^\circ} & 0.707 \end{pmatrix} \tilde{Q}_a Q, \tag{33}$$

and [6]

$$U \simeq \begin{pmatrix} 0.838 & 0.543 & 0.0582e^{-i226.93^\circ} \\ -0.362e^{-i3.99^\circ} & 0.610e^{i1.53^\circ} & 0.705 \\ 0.408e^{i3.55^\circ} & -0.577e^{-i1.63^\circ} & 0.707 \end{pmatrix} \tilde{Q}_e Q, \tag{34}$$

where $\tilde{Q}_a = \mathrm{diag}(e^{-i3.14^\circ}, e^{i1.45^\circ}, -1)$ and $\tilde{Q}_e = \mathrm{diag}(e^{-i3.55^\circ}, e^{i1.63^\circ}, -1)$. Now comparing Eq. (33) and Eq. (34) with Eq. (3) we can obtain the “approximate” and “exact” values of the Dirac and the two Majorana phases of the standard parametrisation of the PMNS matrix, predicted by the model. For the Dirac phase, for instance, we find, respectively, $\delta \cong 226.7^\circ$ and [2] $\delta \cong 226.9^\circ$. Note that the Majorana phases $\alpha_{21}/2$ and $\alpha_{31}/2$ in the standard parametrisation are not CP conserving [6]: due to the matrix $\tilde{Q}_a$ (or $\tilde{Q}_e$) they get small CP violating corrections to the CP conserving values 0 and $\pi/2$ or $3\pi/2$.

The high precision provided by the expression (27) for the PMNS matrix is more than sufficient for the purposes of our investigation and we will use it in our further analysis. This allows to get simple analytic results for the CP violating asymmetries, relevant in leptogenesis, which in turn makes transparent and easy to interpret the results we are going to obtain.

Eq. (25), as is well known, allows to express Y_ν in terms of U, D_ν, D_N and an orthogonal (in general, complex) matrix [26] R, $R^T R = R R^T = \mathbf{1}$:

$$Y_\nu = \frac{1}{v_u} U \sqrt{D_\nu} R \sqrt{D_N}. \tag{35}$$

From Eqs. (24)–(35) and (13), we obtain the following exact expression for the matrix R:

$$R = \left(\sqrt{D_\nu}\right)^{-1} Q^* M_D^{\mathrm{diag}} U_{TBM}^T S \left(\sqrt{D_N}\right)^{-1}. \tag{36}$$

Using the explicit forms of $Q = \mathrm{diag}(1,1,\pm i)$, M_D^{diag}, U_{TBM}, S and $D_N = M\,\mathrm{diag}(1,1,1)$ we get:

[4] This observation is based on numerical results obtained by M. Spinrath. We thank M. Spinrath for communicating to us the results of his numerical analysis.

$$R=\begin{pmatrix}-\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0\\ \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} & 0\\ 0 & 0 & -1\end{pmatrix}. \tag{37}$$

The same expression for the matrix R was obtained in [27]. Thus, in the $SU(5)\times T'$ model considered, the R matrix is real, i.e., CP conserving [28] (see also [29]), and symmetric, $R^*=R$, $R^T=R$, and the elements $R_{k3}=R_{3k}=0$, $k=1,2$. We note that the signs of the entries in the 1–2 sector of R depend on the signs of X and Z: the signs in Eq. (37) correspond to $X>0$ and $Z<0$ (see Eq. (19) and the related comments).

3. Radiatively induced leptogenesis

As we have seen, the three heavy Majorana neutrinos N_j are degenerate in mass at the scale M_X at which the Majorana mass matrix of the RH neutrinos is generated. We will assume that this scale does not exceed the GUT scale, $M_{GUT}=2\times 10^{16}$ GeV: $M_X\leqslant M_{GUT}$. Actually, in the SUSY $SU(5)\times T'$ model considered, we have $M_X=M_{GUT}$. Given the fact that the R matrix is real and CP conserving, the baryon asymmetry can only be generated in the regime of flavoured leptogenesis [30,31]. The regimes of 2-flavour and 3-flavour leptogenesis are realised, in general, for values of the masses $M_j\cong M$, $j=1,2,3$, of the heavy Majorana neutrinos satisfying [28] $M\lesssim T<(1+\tan^2\beta)\times 10^{12}$ GeV and $M\lesssim T<(1+\tan^2\beta)\times 10^{9}$ GeV, respectively, where T is the temperature of the Early Universe and $\tan\beta=v_u/v_d$ is the ratio of the vacuum expectation values of the two Higgs doublet fields, present in the SUSY theories, $v\equiv\sqrt{v_u^2+v_d^2}=174$ GeV. If the heavy Majorana neutrinos would be degenerate in mass at the scale (temperatures) at which the flavoured leptogenesis can take place, as is well known, no net baryon asymmetry would be generated. However, if leptogenesis takes place at a scale $M_{FLG}<(\ll)M_X$, higher order corrections accounted for by the renormalisation group (RG) equations describing the change of the masses M_j with the change of the energy scale from M_X to $M_{FLG}\lesssim(1+\tan^2\beta)\times 10^{12}$ GeV, lift the degeneracy of N_j [32–35], generating relatively small splittings between M_1, M_2 and M_3: $\Delta M_{ij}(M_{FLG})\equiv M_i(M_{FLG})-M_j(M_{FLG})\neq 0$, $i\neq j=1,2,3$. Since the mass splittings $|\Delta M_{ij}(M_{FLG})|$ thus generated are exceedingly small, we expect the baryon asymmetry to be generated in the regime of resonant flavoured leptogenesis [28,36].

In the case of resonant flavoured leptogenesis, the CP violating asymmetry in the lepton charge L_l, $l=e,\mu,\tau$, generated in the out of equilibrium decays of the heavy Majorana neutrino N_j taking place at the scale M_{FLG}, is given by [28]:

$$\epsilon_i^\ell\equiv\frac{\Gamma(N_i\to\ell^-H^+)+\Gamma(N_i\to\nu_\ell H^0)-\Gamma(N_i\to\ell^+H^-)-\Gamma(N_i\to\bar\nu_\ell\bar H^0)}{\Gamma(N_i\to\ell^-H^+)+\Gamma(N_i\to\nu_\ell H^0)+\Gamma(N_i\to\ell^+H^-)+\Gamma(N_i\to\bar\nu_\ell\bar H^0)}=-\frac{1}{8\pi}\sum_{j\neq i}S_{ij}\mathcal{I}_{ij}^\ell. \tag{38}$$

Here

$$S_{ij}=\frac{M_iM_j\Delta M_{ji}^2}{(\Delta M_{ji}^2)^2+M_i^2\Gamma_j^2},\qquad \mathcal{I}_{ij}^\ell=\frac{\mathrm{Im}[(Y_\nu^\dagger Y_\nu)_{ij}(Y_\nu)^*_{\ell,i}(Y_\nu)_{\ell,j}]}{(Y_\nu^\dagger Y_\nu)_{ii}}, \tag{39}$$

where Y_ν is defined in Eqs. (35),

$$\Gamma_j=\frac{1}{8\pi}\left(Y_\nu^\dagger Y_\nu\right)_{jj}M_j, \tag{40}$$

and

$$\Delta M_{ji}^2\equiv M_j^2-M_i^2\cong 2M_i^2\delta_{ji}^N,\quad \delta_{ji}^N=\frac{M_j}{M_i}-1,\quad j\neq i. \tag{41}$$

The parameter δ_{ji}^N describes the deviation from complete degeneracy of the masses of the heavy Majorana neutrinos N_j and N_i. All quantities which appear in Eqs. (38)–(41) should be evaluated at the leptogenesis scale M_{FLG}. The baryon asymmetry is generated in the regime of resonant leptogenesis if at M_{FLG} the following condition is fulfilled:

$$M_i\Gamma_j\cong\Delta M_{ji}^2,\quad i\neq j. \tag{42}$$

We have discussed above the asymmetry generated in the decays of the heavy Majorana neutrinos N_i into the Higgs and lepton doublets. A lepton flavour asymmetry $\epsilon_i^{\tilde\ell}$ is also generated from the out-of-equilibrium decays of N_i in the Higgsino and slepton doublets $\tilde\ell$. Similarly, the sneutrinos $\tilde N_i$ generate CP asymmetries $\epsilon_{\tilde i}^{\ell}$ and $\epsilon_{\tilde i}^{\tilde\ell}$ with, respectively, ℓ and $\tilde\ell$ in the final state. As can be shown, one has neglecting soft SUSY breaking terms: $\epsilon_i^\ell=\epsilon_i^{\tilde\ell}=\epsilon_{\tilde i}^{\ell}=\epsilon_{\tilde i}^{\tilde\ell}$.

It follows from Eq. (39) that the necessary conditions for a successful resonant flavoured leptogenesis include: (i) the presence of CP violating phases in the matrix of neutrino Yukawa couplings Y_ν; (ii) non-vanishing off-diagonal elements of the matrix $Y_\nu^\dagger Y_\nu$: $(Y_\nu^\dagger Y_\nu)_{ij}\neq 0$ for $i\neq j$; (iii) non-degeneracy of the heavy Majorana neutrino masses M_i: $\delta_{ji}^N\neq 0$, $i\neq j$. The first requirement is fulfilled by the presence of the CP violating phases in the neutrino mixing matrix U. The second and third general requirements are satisfied, as we are going to discuss next, owing to the RG corrections in the quantities M_i and Y_ν, which have to be included when the latter are evaluated at the leptogenesis scale M_{FLG}.

The RG running of the heavy Majorana neutrino masses M_i depends on the quantity $Y_\nu^\dagger Y_\nu$ [33]. It proves convenient to work at the scale M_X in a basis of the heavy Majorana neutrino fields in which the matrix $Y_\nu^\dagger Y_\nu$ is diagonal. This can be achieved by performing an

Table 1
Values of the heavy Majorana mass splitting parameter δ^N_{ij}.

	$M_X/M = 2\times 10^6$	$M_X/M = 2\times 10^5$	$M_X/M = 2\times 10^4$
δ^N_{21}	-9.28×10^{-7}	-7.81×10^{-6}	-6.33×10^{-5}
δ^N_{31}	-5.80×10^{-6}	-4.88×10^{-5}	-3.96×10^{-4}
δ^N_{32}	-4.87×10^{-6}	-4.10×10^{-5}	-3.33×10^{-4}

orthogonal transformation of N_j. The latter can be done without affecting the heavy Majorana neutrino mass term since at the scale of interest the heavy Majorana neutrinos N_j are degenerate in mass. The change of basis, $N_j = O^T_{jk} N'_k$, where O is an orthogonal matrix, implies the following change of the matrix of neutrino Yukawa couplings: $Y'_\nu = Y_\nu O$. Using Eq. (35) and the facts that $D_N = M\,\mathrm{diag}(1,1,1)$ and the matrix R is real and orthogonal, there always exists an orthogonal matrix O such that RO, and correspondingly $Y'^\dagger_\nu Y'_\nu$, are diagonal matrices. Taking into account the explicit form of the matrix R in the model considered, Eq. (37), in what follows we will use

$$O \equiv \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{43}$$

It is easy to verify that $RO = \mathrm{diag}(-1,1,-1) \equiv I$, $I_{jj} = \eta_j$, with $\eta_2 = -\eta_{1,3} = 1$. The matrix of neutrino Yukawa couplings Y'_ν is given by:

$$Y'_\nu \equiv Y_\nu O = \frac{\sqrt{M}}{v_u} U \sqrt{D_\nu} I. \tag{44}$$

In the new basis we have $(Y'^\dagger_\nu Y'_\nu)_{ij} = 0$ for $i \neq j$, and

$$\left(Y'^\dagger_\nu Y'_\nu\right)_{ii} = \frac{M_i}{v_u^2} m_i, \qquad \Gamma'_i = \frac{M_i^2}{8\pi v_u^2} m_i, \quad M_i \cong M, \tag{45}$$

where Γ'_i is the N'_i total decay width.

The expression for the CP violating asymmetry ϵ^ℓ_i in the new basis in which $Y'^\dagger_\nu Y'_\nu$ is diagonal at M_X can be obtained from Eqs. (38)–(40) by replacing Y_ν and Γ_i with Y'_ν and Γ'_i, respectively. Note, however, that in the new basis we have $\mathcal{I}^\ell_{ij} = 0$. Thus, the CP violating asymmetries ϵ^ℓ_i will be zero unless non-diagonal elements of $Y'^\dagger_\nu Y'_\nu$ are radiatively generated at the leptogenesis scale $M_{FLG} < (\ll) M_X$.

As will be shown later, in the model considered a non-zero baryon asymmetry can be produced only in the regime of 3-flavoured leptogenesis, i.e. for $M < (1+\tan^2\beta)\times 10^9$ GeV $\lesssim 4.9\times 10^{12}$ GeV, where we have used the constraint $\tan\beta \lesssim 70$ (see,[5] e.g., [37]). Taking into account that $m_i \lesssim 5\times 10^{-2}$ eV and $v = 174$ GeV, we get $|(Y'^\dagger_\nu Y'_\nu)_{ii}| \lesssim 8\times 10^{-3} \ll 1$.

In the new basis, the running of the heavy Majorana neutrino masses M_i is governed by the following equation [33]:

$$\frac{dM_i}{dt} = 4\left(Y'^\dagger_\nu Y'_\nu\right)_{ii} M_i, \quad t \equiv \frac{1}{16\pi^2} \ln\frac{\mu}{M_X}, \tag{46}$$

where the initial conditions are at the scale $\mu = M_X$ at which $M_i = M$, $i = 1,2,3$, and the masses M_i are evaluated at the scale $\mu = M_{FLG} < (\ll) M_X$. The latter coincides, up to negligibly small corrections, with M: $M_{FLG} \cong M$. The running of the masses M_i from M_X to $M_{FLG} \cong M$ induces the splitting between the masses of the heavy Majorana neutrinos, necessary for a potentially successful leptogenesis. The solutions of Eqs. (46) [34,35] lead for $|(Y'^\dagger_\nu Y'_\nu)_{ii}| \ll 1$ to the following expression for the mass splitting parameter δ^N_{ji}:

$$\delta^N_{ji} \cong -4\left[\left(Y'^\dagger_\nu Y'_\nu\right)_{jj} - \left(Y'^\dagger_\nu Y'_\nu\right)_{ii}\right]\tilde{t} \cong -4\frac{M}{v_u^2}(m_j - m_i)\tilde{t}, \quad j \neq i, \quad \tilde{t} = \frac{1}{16\pi^2}\ln\left(\frac{M_X}{M}\right). \tag{47}$$

For $M_X/M = 2\times 10^6$, 2×10^5 and 2×10^4, we get $\tilde{t} = 0.092, 0.077, 0.063$. The corresponding values of δ^N_{ji} are given in Table 1.

The elements of the matrix of neutrino Yukawa couplings Y'_ν also evolve with the scale μ when the latter diminishes from M_X to $M_{FLG} \cong M$. This change is governed by the RG equations for $(Y'_\nu)_{\ell i}$, whose general form was given in [32–35]. In the case considered by us we have at M_X: $(Y'^\dagger_\nu Y'_\nu)_{ij} = 0$, $i \neq j$, and[6] $|(Y'^\dagger_\nu Y'_\nu)_{ii}| \lesssim 8\times 10^{-3}$. In this case the RG equations for $(Y'_\nu)_{\ell i}$ [35] simplify considerably and read:

$$\frac{d(Y'_\nu)_{\ell i}}{dt} \cong \left[3\sum_{q=u,c,t} y_q^2 - \frac{3}{5}g_1^2 - 3g_2^2 + y_\ell^2\right](Y'_\nu)_{\ell i}, \quad \ell = e,\mu,\tau, \quad i = 1,2,3, \tag{48}$$

[5] In the calculation of the baryon asymmetry we will values of $\tan\beta \sim 10$, which are much smaller than the quoted maximal value.

[6] For a matrix of neutrino Yukawa couplings Y_ν such that $\mathrm{Re}[(Y^\dagger_\nu Y_\nu)_{ij}] \neq 0$, $i \neq j$, the RG equations for Y_ν have a singularity in the case of degenerate in mass heavy Majorana neutrinos [32,34,35]. As a consequence, the quantity $(Y^\dagger_\nu Y_\nu)_{ij}$, $i \neq j$, that enters into the expression for the CP violating asymmetry ϵ^ℓ_i, does not vary continuously with the scale when the latter changes from M_X to M_{FLG}. This fact was not taken into account in the calculation of the asymmetries ϵ^ℓ_i performed in [27]. Since in the basis in which we work we have $(Y'^\dagger_\nu Y'_\nu)_{ij} = 0$, $i \neq j$, at M_X at which $M_j = M$, $j = 1,2,3$, the indicated problem does not appear when we consider the RG evolution of $(Y'_\nu)_{\ell i}$ and of $(Y'^\dagger_\nu Y'_\nu)_{ij}$.

where y_q, $q = u, c, t$, and y_ℓ, $\ell = e, \mu, \tau$, are the charge 2/3 quark and charged lepton Yukawa couplings, $g_{1,2}$ are the $U(1)_Y$ and $SU(2)$ gauge couplings of the Standard Model and we have neglected terms $\propto Y'^{\dagger}_{\nu} Y'_{\nu}$. The quantities which appear in the square brackets in the r.h.s. of Eq. (48) evolve with the scale μ as it decreases from M_X, but the effects of their evolution are subdominant for the problem under study and we will neglect them. Thus, we will use their values at the scale M_X, which will be assumed to be close, or equal, to M_{GUT}.

We are interested in the quantities $(Y'^{\dagger}_{\nu} Y'_{\nu})_{ij}$, $i \neq j$, at the scale $M_{FLG} \cong M$, which enter into the expression for the CP violating asymmetry ϵ_i^ℓ. Since these quantities are zero at M_X, they can get non-zero values at M_{FLG} due only to the term involving the charged lepton Yukawa coupling y_ℓ^2 in the RG equation (48) [34,35]. The solutions of the RG equations (48) in the leading logarithmic approximation lead to the following result:

$$\left(Y'^{\dagger}_{\nu} Y'_{\nu}\right)_{ij} \cong -2 y_\tau^2 \left(Y'^{*}_{\nu}\right)_{\tau i} \left(Y'_{\nu}\right)_{\tau j} \tilde{t}, \quad i \neq j, \tag{49}$$

where $y_\tau = (m_\tau / v_d) \cong (m_\tau / v)\sqrt{1 + \tan^2\beta}$ is the τ Yukawa coupling, m_τ being the τ mass, and $v = \sqrt{v_u^2 + v_d^2} = 174$ GeV. Neglecting relatively small effects, the quantities in the r.h.s. of Eq. (49) can be taken at the scale M_X. Note that even though at M_X the off-diagonal elements of $Y'^{\dagger}_{\nu} Y'_{\nu}$ are zero, they have non-zero values at the leptogenesis scale M_{FLG} due to the radiative corrections.

Using the result obtained for $(Y'^{\dagger}_{\nu} Y'_{\nu})_{ij}$, Eq. (49), and Eqs. (38), (44) and (45), we get for the CP violating asymmetry:

$$\epsilon_i^\ell = +\frac{1}{8\pi} y_\tau^2 \tilde{t} \sum_{i \neq j} \frac{\delta_{ji}^N}{\left[(\delta_{ji}^N)^2 + \left(\frac{M_j m_j}{16\pi v_u^2}\right)^2\right]} \frac{M_j m_j}{v_u^2} \operatorname{Im}\left[U^*_{\tau i} U_{\tau j} U^*_{\ell i} U_{\ell j}\right]. \tag{50}$$

It follows from the expression (50) for ϵ_i^ℓ we have derived that $\epsilon_i^e + \epsilon_i^\mu + \epsilon_i^\tau = 0$, $i = 1, 2, 3$. This result is a consequence of the fact that the R matrix in the model considered is CP conserving (see, e.g., [28]). One can easily convince oneself using the explicit expression for the PMNS matrix (27) that we also have: $\epsilon_i^\tau = 0$, $i = 1, 2, 3$. The same conclusion is reached also if one uses the PMNS matrix in which the higher order corrections have been included,[7] Eq. (32) or (34). Thus, in the SUSY $SU(5) \times T'$ model of interest the baryon asymmetry can be generated only in the regime of 3-flavoured leptogenesis [31].

The requirement that the baryon asymmetry is generated in the 3-flavoured thermal leptogenesis regime combined with the upper limit on $\tan\beta$ implies: $M \lesssim 4.9 \times 10^{12}$ GeV. As is not difficult to show, we have for $M \lesssim 10^{13}$ GeV:

$$\left(\delta_{ji}^N\right)^2 \gg \left(\frac{1}{16\pi} \frac{M_j m_j}{v_u^2}\right)^2. \tag{51}$$

For $M = 10^{13}$ GeV, $(\delta_{ji}^N)^2$ is bigger by a factor of 10 than the term in the right-hand side of the above inequality. Neglecting the correction due to the latter, we get a rather simple expression for the asymmetry ϵ_i^ℓ:

$$\epsilon_i^\ell \cong -\frac{y_\tau^2}{32\pi} \sum_{i \neq j} \frac{m_j}{m_j - m_i} \operatorname{Im}\left[U^*_{\tau i} U_{\tau j} U^*_{\ell i} U_{\ell j}\right], \tag{52}$$

where we have used Eqs. (47) and (50).

Expression (52) for ϵ_i^ℓ does not depend explicitly on the masses of the heavy Majorana neutrinos and on the RG factor $\tilde{t}$. Thus, the CP-asymmetries ϵ_i^ℓ are entirely determined by the τ Yukawa coupling and the low-energy neutrino mixing parameters, i.e., the neutrino masses, the neutrino mixing angles and CP violating phases in the neutrino mixing matrix. They depend weakly on scales M_X and M, e.g., via the running of the τ Yukawa coupling. The asymmetries ϵ_i^ℓ depend quadratically on the τ Yukawa coupling and thus on $\tan^2\beta$. This dependence is crucial for having a viable thermal leptogenesis in the $SU(5) \times T'$ model of flavour under consideration.

From Eq. (52), using Eqs. (27) and (7), we obtain:

$$\epsilon_i^e \cong -\frac{y_\tau^2}{32\pi} J_{CP} \sum_{j \neq i} \frac{m_j}{m_j - m_i} \rho_{ji}, \qquad \epsilon_i^\mu = -\epsilon_i^e, \quad i = 1, 2, 3, \tag{53}$$

where $\rho_{ji} = -\rho_{ij}$, $i \neq j$, and $\rho_{21} = \rho_{31} = \rho_{23} = +1$. Thus, the CP violating asymmetries $\epsilon_i^{e,\mu}$, $i = 1, 2, 3$, are all proportional to the J_{CP} factor, which determines the magnitude of CP violation effects in the flavour neutrino oscillations.

The final lepton number asymmetry, which is partially converted into a non-zero baryon number asymmetry by the fast sphaleron interactions in the thermal bath in the Early Universe, receives a contribution from the out-of-equilibrium decays of the three heavy Majorana neutrinos (sneutrinos) N'_i ($\tilde{N}'_i$), which are quasi-degenerate in mass. The amount of matter-antimatter asymmetry predicted by the model is computed numerically by solving the corresponding system of Boltzmann equations. We report below the relevant set of Boltzmann equations in supersymmetric leptogenesis [38,39] for the lepton flavour (lepton charge) asymmetries $\hat{Y}_{\Delta_\ell} \equiv Y_{\Delta_\ell} + Y_{\Delta_{\tilde{\ell}}}$, with $\Delta_{\ell(\tilde{\ell})} \equiv B/3 - L_{\ell(\tilde{\ell})}$[8]:

[7] It is claimed in [27] that $\epsilon_i^\tau \neq 0$, which does not correspond to the result $\epsilon_i^\tau = 0$ we obtain. The latter is not difficult to verify.

[8] As was pointed out earlier, the CP asymmetries ϵ_i^τ $(i = 1, 2, 3)$ are equal to zero in the model we are discussing. Nonetheless, a source term for $\Delta_{\tau(\tilde{\tau})}$ is provided by non-zero $\hat{Y}_{\Delta_{e,\mu}}$, as is explicit from the flavoured Boltzmann equation (56).

$$\frac{dY_{N'_i}}{dz} = -\frac{z}{sH(M_{FLG})} 2\left(\gamma_D^i + \gamma_{S,\Delta L=1}^i\right)\left(\frac{Y_{N'_i}}{Y_{N'_i}^{\rm eq}} - 1\right), \tag{54}$$

$$\frac{dY_{\tilde{N}'_i}}{dz} = -\frac{z}{sH(M_{FLG})} 2\left(\gamma_D^{\tilde{i}} + \gamma_{S,\Delta L=1}^{\tilde{i}}\right)\left(\frac{Y_{\tilde{N}'_i}}{Y_{\tilde{N}'_i}^{\rm eq}} - 1\right), \tag{55}$$

$$\frac{d\hat{Y}_{\Delta_\ell}}{dz} = -\frac{z}{sH(M_{FLG})} \sum_{i=1}^{3}\Bigg[\left(\epsilon_i^\ell + \epsilon_i^{\tilde{\ell}}\right)\left(\gamma_D^i + \gamma_{S,\Delta L=1}^i\right)\left(\frac{Y_{N'_i}}{Y_{N'_i}^{\rm eq}} - 1\right) + \left(\epsilon_{\tilde{i}}^\ell + \epsilon_{\tilde{i}}^{\tilde{\ell}}\right)\left(\gamma_D^{\tilde{i}} + \gamma_{S,\Delta L=1}^{\tilde{i}}\right)\left(\frac{Y_{\tilde{N}'_i}}{Y_{\tilde{N}'_i}^{\rm eq}} - 1\right)$$
$$- \left(\frac{\gamma_D^{i,\ell} + \gamma_D^{i,\tilde{\ell}}}{2} + \gamma_{W,\Delta L=1}^{i,\ell} + \gamma_{W,\Delta L=1}^{i,\tilde{\ell}} + \frac{\gamma_D^{\tilde{i},\ell} + \gamma_D^{\tilde{i},\tilde{\ell}}}{2} + \gamma_{W,\Delta L=1}^{\tilde{i},\ell} + \gamma_{W,\Delta L=1}^{\tilde{i},\tilde{\ell}}\right)\frac{\sum_{\ell'} A_{\ell\ell'}\hat{Y}_{\Delta_{\ell'}}}{\hat{Y}_\ell^{\rm eq}}\Bigg]. \tag{56}$$

Here $Y_{N'_i}$ ($Y_{N'_i}^{\rm eq}$) is the N'_i (N'_i-equilibrium) abundance, $z \equiv M_{FLG}/T$, T being the temperature of the thermal bath, s is the entropy density and $H(T)$ is the expansion rate of the Universe. The quantity γ_D^i ($i = 1, 2, 3$) is the thermally averaged total decay rate of the Majorana neutrino N'_i into the SM lepton and Higgs doublets. Similarly, $\gamma_{S,\Delta L=1}^i$ is the corresponding $\Delta L = 1$ thermal scattering rate of N'_i with SM leptons, quarks and gauge bosons. The flavour-dependent wash-out processes involving N'_i inverse decays and the relative $\Delta L = 1$ scatterings are denoted as $\gamma_D^{i,\ell(\tilde{\ell})}$ and $\gamma_{W,\Delta L=1}^{i,\ell(\tilde{\ell})}$, respectively. Finally, the matrix elements of A in supersymmetric type I see-saw scenarios are [39]: $A_{\alpha\beta} = 16/2133$ for $\alpha \neq \beta$ and $A_{\alpha\alpha} = -221/2133$ ($\alpha = e, \mu, \tau$).

The entropy density, s, and the expansion rate of the Universe, $H(T)$, are given by:

$$s = \frac{g_* 2\pi^2 T^3}{45}, \qquad H(T) \simeq \frac{1.66\sqrt{g_*}T^2}{m_{Pl}}, \tag{57}$$

where $g_* = 228.75$ [31] and $m_{Pl} \simeq 1.22 \times 10^{19}$ GeV is the Planck mass.

In the case in which the soft SUSY breaking terms are negligible, the thermal rates in (54)–(56) satisfy the conditions [31]: $\gamma_X^i = \gamma_X^{\tilde{i}}$ and $\gamma_X^{i,\ell} = \gamma_X^{i,\tilde{\ell}} = \gamma_X^{\tilde{i},\ell} = \gamma_X^{\tilde{i},\tilde{\ell}}$. As a good approximation, supersymmetric leptogenesis proceeds as a manifest generalisation of the standard leptogenesis scenario of the type I see-saw extension of the SM. Indeed, new effects due to different supersymmetric equilibration mechanisms between particle and sparticle number densities provide typically only relatively small corrections [39], which can be safely neglected for the purposes of the present study.

The dominant contribution to the production and damping of the lepton asymmetries is generally provided by decays and inverse decays of N'_i [40], whose thermal averaged rates are

$$\gamma_D^i \simeq \frac{M^3}{\pi^2 z}\mathcal{K}_1(z)\Gamma'_i, \qquad \gamma_D^{i,\ell} = \gamma_D^i \frac{|(Y'_\nu)_{\ell i}|^2}{(Y_\nu'^\dagger Y'_\nu)_{ii}}, \tag{58}$$

where $\mathcal{K}_1(z)$ is a modified Bessel function of the second kind.

We neglect in (54)–(56), for simplicity, thermal corrections to the CP asymmetries and the decay/scattering rates [40]. We do not include either the $\Delta L = 2$ wash-out of the flavour lepton asymmetries in the Boltzmann equations listed above because they are subdominant at the temperatures at which the 3-flavoured leptogenesis takes place.[9]

The final baryon number density (normalised to the entropy density of the Universe) is:

$$Y_B = \frac{10}{31}\left(\hat{Y}_{\Delta_e} + \hat{Y}_{\Delta_\mu} + \hat{Y}_{\Delta_\tau}\right). \tag{59}$$

In order to have successful leptogenesis, the CP asymmetries ϵ_i^ℓ ($\ell = e, \mu$) should be sufficiently large and should have the correct sign. According to Eq. (53), the sign of $\epsilon_i^e = -\epsilon_i^\mu$ and, consequently, of Y_B, is fixed by the value of the rephasing invariant associated to the Dirac phase δ, J_{CP}. Numerically, from (53), we get for $\tan^2\beta \gg 1$:

$$\epsilon_1^e \simeq -2.3 \times 10^{-6} J_{CP}(\tan\beta)^2, \qquad \epsilon_2^e \simeq 1.3 \times 10^{-6} J_{CP}(\tan\beta)^2, \qquad \epsilon_3^e \simeq 2.1 \times 10^{-7} J_{CP}(\tan\beta)^2, \tag{60}$$

where we have used Eq. (1) and $y_\tau^2 \simeq 10^{-4}\tan^2\beta$. Taking, more explicitly, $\tan\beta = 10$, one easily obtains:

$$\epsilon_1^e \simeq -\,\mathrm{sgn}(\sin\delta)1.4 \times 10^{-6}, \qquad \epsilon_2^e \simeq \mathrm{sgn}(\sin\delta)7.0 \times 10^{-7} \qquad \epsilon_3^e \simeq \mathrm{sgn}(\sin\delta)1.3 \times 10^{-7}, \tag{61}$$

which, in general, is the right order of magnitude of the CP asymmetry in order to have a successful leptogenesis. Notice that $\mathrm{sgn}(\sin\delta)$ is equal either to (-1) or to $(+1)$, depending on whether the Dirac phase $\delta \cong 5\pi/4$ or $\delta \cong \pi/4$, which are the two approximate values δ can have in the model considered (see Eqs. (29) and (30)).

Taking into account Eq. (60), expression (59) can be recast in the form:

$$Y_B \approx J_{CP}(\tan\beta)^2 \epsilon\,\eta_B Y_{N'}^{\rm eq}(z \ll 1), \tag{62}$$

[9] As is well known, $\Delta L = 2$ scatterings mediated by N'_i ($\tilde{N}'_i$) can be safely neglected if $\Gamma'_i/H(T) \ll 10 \times M_i/(10^{14}$ GeV) [31].

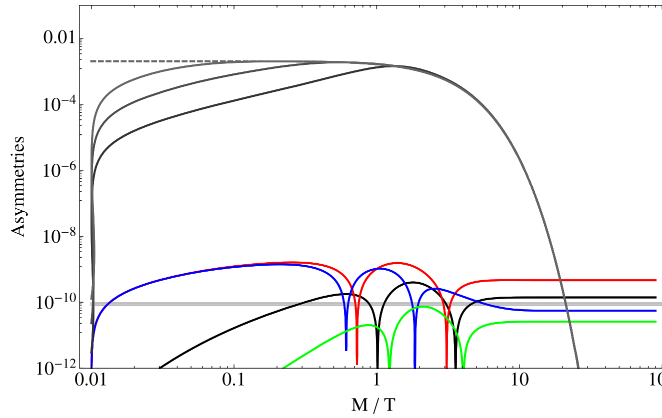

Fig. 1. Solution of the Boltzmann equations (54)–(56) for $\tan\beta = 10$ and $\delta = \pi/4$. See the text for details. (For interpretation of the references to colour, the reader is referred to the web version of this Letter.)

where $Y^{eq}_{N'} = 45/(\pi^4 g_*) \simeq 2 \times 10^{-3}$, $\epsilon \equiv 10^{-6}$ and $\eta_B > 0$ is, by definition, the efficiency factor of the asymmetry. It follows from Eqs. (7) and (62) that for $\delta = 226.93^\circ \simeq 5\pi/4$, the baryon asymmetry has the wrong sign. Thus, the observed value of the baryon asymmetry can be obtained in the model considered only for $\delta \simeq \pi/4$.

The efficiency factor η_B in Eq. (62) can be computed by solving the full system of Boltzmann equations (54)–(56). We note that in the model considered the parameter η_B does not depend on the leptogenesis scale $M_{FLG} \sim M$. This can be easily understood if one considers, for simplicity, the solution of the Boltzmann equations where only decay and inverse decay processes are included: as we have already mentioned, this is a good approximation in thermal flavoured leptogenesis. In this case, from Eqs (45), (57) and (58) one has:

$$\frac{z\gamma_D^i}{sH(M)} \propto \frac{m_i m_{Pl}}{v_u^2}. \tag{63}$$

Therefore, the Boltzmann equations do not explicitly depend on the heavy Majorana neutrino mass scale M within the indicated approximation. We verified numerically that the dependence of η_B and Y_B on M is relatively weak also if we take into account the scattering processes. This implies that, in the class of SUSY see-saw models of the type considered in this Letter, the leptogenesis scale M_{FLG} can be lowered sufficiently in order to avoid the potential gravitino problem.[10]

In Fig. 1, we report the solution of the full set of Boltzmann equations (54)–(56) for $\tan\beta = 10$ and $\delta = \pi/4$. The red, blue, green and black lines represent $|\hat{Y}_{\Delta_e}|$, $|\hat{Y}_{\Delta_\mu}|$, $|\hat{Y}_{\Delta_\tau}|$ and $|Y_B|$, respectively. The dashed line corresponds to $Y^{eq}_{N'_1}$, while the other three black lines are the RH neutrino abundances $Y_{N'_{1,2,3}}$. The gray horizontal band gives the 3σ interval of experimental values of Y_B: $Y_B^{obs} = (8.77 \pm 0.21) \times 10^{-11}$ [42], where we have quoted the 1σ error. In this numerical example, we get the final asymmetries:

$$\hat{Y}_{\Delta_e} \simeq 4.7 \times 10^{-10}, \qquad \hat{Y}_{\Delta_\mu} \simeq -5.8 \times 10^{-11}, \qquad \hat{Y}_{\Delta_\tau} \simeq 2.6 \times 10^{-11} \quad \text{and} \quad Y_B \simeq 1.4 \times 10^{-10}. \tag{64}$$

From Eq. (62) and the numerical value of Y_B thus computed, we get an efficiency factor $\eta_B \simeq 0.07$. Obviously, one can get a value of Y_B closer to the mean best fit value $\bar{Y}_B^{obs} = 8.77 \times 10^{-11}$ for a somewhat smaller value of $\tan\beta$.

We would like to conclude with the following remarks. As we have shown, the correct sign of the baryon asymmetry in the $SU(5) \times T' \times Z_{12} \times Z'_{12}$ model considered [1,2] can be obtained only in the case of $\delta \cong \pi/4$. As has already been discussed in the Introduction, for this value of the Dirac phase δ we have $\sin^2\theta_{12} \cong 0.37$, while the current neutrino oscillation data imply at 3σ $\sin^2\theta_{12} \lesssim 0.36$ [8], or $\sin^2\theta_{12} \lesssim 0.374$ [18], depending on the details of the analysis. For $\delta \cong 5\pi/4$, the value of $\sin^2\theta_{12} \cong 0.299$ predicted by the model lies within the 1σ interval of values suggested by the data, but the predicted baryon asymmetry of the Universe has the wrong sign[11] (see Eq. (62)). If $\sin^2\theta_{12} \cong 0.37$ would be definitely excluded by future data, one would have to modify the $SU(5) \times T'$ model of flavour we have considered in the present Letter. One possible “minimal” modification could be to lift the degeneracy in mass of the three heavy Majorana neutrinos (sneutrinos) at the scale M_X, at which the flavour symmetry is spontaneously broken. This could be achieved, e.g., by replacing the chiral superfield S in the $SU(5) \times T' \times Z_{12} \times Z'_{12}$ invariant superpotential of [2] with a new chiral supermultiplet χ, which is a Standard Model singlet and is charged only under the discrete group Z'_{12}, with charge ω^2. The model, therefore, has the same gauge and flavour symmetry groups and the same number of fields as the one discussed in [2]. In this new scenario, the flavour structure of the superpotential naturally generates a Majorana mass matrix (term) for the heavy RH neutrinos at the scale M_X. The latter is still diagonalised by the tri-bimaximal mixing matrix U_{TBM}, but has non-degenerate eigenvalues. The low energy phenomenology, as well as the generation of the baryon asymmetry of this class of models is therefore worthwhile investigating, but such an investigation lies outside the scope of the present work.

[10] The Davidson-Ibarra bound [41] does not apply in the radiative leptogenesis scenario discussed by us.

[11] Our result for the sign of the baryon asymmetry in the case of $\delta = 226.93^\circ \simeq 5\pi/4$ contradicts the claim made in [27].

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References

[1] M.-C. Chen, K.T. Mahanthappa, Phys. Lett. B 652 (2007) 34, arXiv:0710.2118.
[2] M.-C. Chen, K.T. Mahanthappa, Phys. Lett. B 681 (2009) 444;
M.-C. Chen, K.T. Mahanthappa, PoS ICHEP2010 (2010) 407, arXiv:1011.6364;
M.-C. Chen, K.T. Mahanthappa, arXiv:1012.1595.
[3] H. Ishimori, et al., Prog. Theor. Phys. Suppl. 183 (2010) 1, arXiv:1003.3552 [hep-th].
[4] P.H. Frampton, T.W. Kephart, Int. J. Mod. Phys. A 10 (1995) 4689, arXiv:hep-ph/9409330;
A. Aranda, C.D. Carone, R.F. Lebed, Phys. Rev. D 62 (2000) 016009;
F. Feruglio, C. Hagedorn, Y. Lin, L. Merlo, Nucl. Phys. B 775 (2007) 120;
P.H. Frampton, S. Matsuzaki, arXiv:0902.1140;
G.J. Ding, Phys. Rev. D 78 (2008) 036011.
[5] C. Luhn, Phys. Lett. B 670 (2009) 390.
[6] M.-C. Chen, K.T. Mahanthappa, A. Meroni, S.T. Petcov, arXiv:1109.0731 [hep-ph].
[7] K. Nakamura, S.T. Petcov, in: K. Nakamura, et al. (Eds.), J. Phys. G 37 (2010) 075021.
[8] G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, A.M. Rotunno, Phys. Rev. D 84 (2011) 053007.
[9] L. Wolfenstein, Phys. Rev. D 18 (1978) 958;
P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167;
P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 535 (2002) 163.
[10] P.H. Frampton, S.T. Petcov, W. Rodejohann, Nucl. Phys. B 687 (2004) 31;
A. Romanino, Phys. Rev. D 70 (2004) 013003.
[11] K.A. Hochmuth, S.T. Petcov, W. Rodejohann, Phys. Lett B 654 (2007) 177.
[12] D. Marzocca, S.T. Petcov, A. Romanino, M. Spinrath, JHEP 1111 (2011) 009.
[13] S.M. Bilenky, J. Hosek, S.T. Petcov, Phys. Lett. B 94 (1980) 495.
[14] E. Molinaro, S.T. Petcov, Eur. Phys. J. C 61 (2009) 93.
[15] J.-Q. Chen, P.-D. Fan, J. Math. Phys. 39 (1998) 5519.
[16] C. Jarlskog, Z. Phys. C 29 (1985) 491;
C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039.
[17] P.I. Krastev, S.T. Petcov, Phys. Lett. B 205 (1988) 84.
[18] M. Maltoni, in: Interactions, Rome, Italy, 5 December 2011.
[19] S.M. Bilenky, S.T. Petcov, Rev. Mod. Phys. 59 (1987) 671;
S.M. Bilenky, S. Pascoli, S.T. Petcov, Phys. Rev. D 64 (2001) 053010;
S. Pascoli, S.T. Petcov, Phys. Rev. D 77 (2008) 113003.
[20] T. Schwetz, M. Tortola, J.W.F. Valle, New J. Phys. 13 (2011) 109401.
[21] F. Ardellier, et al., Double Chooz Collaboration, arXiv:hep-ex/0606025.
[22] J.K. Ahn, et al., RENO Collaboration, arXiv:1003.1391.
[23] See, e.g., the Daya Bay homepage, http://dayawane.ihep.ac.cn/.
[24] M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45.
[25] V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B 155 (1985) 36.
[26] J.A. Casas, A. Ibarra, Nucl. Phys. B 618 (2001) 171.
[27] M.-C. Chen, K.T. Mahanthappa, arXiv:1107.3856 [hep-ph].
[28] S. Pascoli, S.T. Petcov, A. Riotto, Phys. Rev. D 75 (2007) 083511;
S. Pascoli, S.T. Petcov, A. Riotto, Nucl. Phys. B 774 (2007) 1.
[29] E. Molinaro, S.T. Petcov, Phys. Lett. B 671 (2009) 60.
[30] R. Barbieri, P. Creminelli, A. Strumia, N. Tetradis, Nucl. Phys. B 575 (2000) 61;
H.B. Nielsen, Y. Takanishi, Nucl. Phys. B 636 (2002) 305;
T. Endoh, T. Morozumi, Z. Xiong, Prog. Theor. Phys. 111 (2004) 123.
[31] A. Abada, et al., JCAP 0604 (2006) 004;
E. Nardi, Y. Nir, E. Roulet, J. Racker, JHEP 0601 (2006) 164;
A. Abada, et al., JHEP 0609 (2006) 010.
[32] P.H. Chankowski, W. Krolikowski, S. Pokorski, Phys. Lett. B 473 (2000) 109;
P.H. Chankowski, S. Pokorski, Int. J. Mod. Phys. A 17 (2002) 575.
[33] J.A. Casas, J.R. Espinosa, A. Ibarra, I. Navarro, Nucl. Phys. B 569 (2000) 82.
[34] R. Gonzalez Felipe, F.R. Joaquim, B.M. Nombre, Phys. Rev. D 70 (2004) 085009;
G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim, B.M. Nobre, Phys. Lett. B 633 (2006) 336.
[35] K. Turzynski, Phys. Lett. B 589 (2004) 135.
[36] A. Pilaftsis, Phys. Rev. D 56 (1997) 5431;
A. Pilaftsis, T.E.J. Underwood, Nucl. Phys. B 692 (2004) 303.
[37] W. Altmannshofer, D.M. Straub, JHEP 1009 (2010) 078.
[38] S. Antusch, S.F. King, A. Riotto, JCAP 0611 (2006) 011, arXiv:hep-ph/0609038.
[39] C.S. Fong, M.C. Gonzalez-Garcia, E. Nardi, J. Racker, JCAP 1012 (2010) 013, arXiv:1009.0003 [hep-ph].
[40] G.F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia, Nucl. Phys. B 685 (2004) 89, arXiv:hep-ph/0310123.
[41] S. Davidson, A. Ibarra, Phys. Lett. B 535 (2002) 25, arXiv:hep-ph/0202239.
[42] E. Komatsu, et al., WMAP Collaboration, Astrophys. J. Suppl. 192 (2011) 18, arXiv:1001.4538 [astro-ph.CO].