



Exact microstate counting for dyonic black holes in AdS₄



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ABSTRACT

We present a counting of microstates of a class of dyonic BPS black holes in AdS₄ which precisely reproduces their Bekenstein–Hawking entropy. The counting is performed in the dual boundary description, that provides a non-perturbative definition of quantum gravity, in terms of a twisted and mass-deformed ABJM theory. We evaluate its twisted index and propose an extremization principle to extract the entropy, which reproduces the attractor mechanism in gauged supergravity.

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1. Introduction

Supersymmetric black holes in string theory constitute important models to test fundamental questions about quantum gravity in a relatively simple setting. The main question we would like to address here is the origin of the black hole (BH) entropy, which statistically is expected to count the number of degenerate BH configurations. String theory provides a microscopic explanation for the entropy of a class of asymptotically flat black holes [1]. Much less is known about asymptotically AdS ones in four or more dimensions.

In principle AdS/CFT [2] provides a non-perturbative definition of quantum gravity in asymptotically AdS space, as a dual boundary quantum field theory (QFT). The BH microstates appear as particular states in the boundary description. The difficulty with this approach is the need to perform computations in a strongly coupled QFT, but the development of exact non-perturbative techniques makes progress possible. We recently reported [3] on a particular example of magnetically charged BPS black holes in AdS₄ [4] with a known field theory dual—topologically twisted ABJM theory. Using the technique of supersymmetric localization we were able to calculate in an independent way the (regularized) number of ground states of the theory and successfully match it with the leading macroscopic entropy of the black holes.

In this Letter we discuss a function $Z(u_a)$ that encodes the quantum entropies of static dyonic BPS black holes in AdS₄, com-

puted non-perturbatively from the dual QFT description, and show that its leading behavior reproduces the Bekenstein–Hawking entropy [5,6].

In particular we show that, at leading order, the entropy of BPS black holes with magnetic charges p_a , electric charges q_a and asymptotic to AdS₄ × S⁷ can be obtained by extremizing the quantity

$$\mathcal{I} = \log Z(u_a) - i \sum_a u_a q_a \quad (1)$$

with respect to a set of complexified chemical potentials u_a for the global U(1) flavor symmetries of the boundary theory. $Z(u_a)$ is the topologically twisted index [7] of the ABJM theory [8] which explicitly depends on the magnetic charges p_a (see [9,10] for other examples). The entropy is given by $S = \mathcal{I}(\hat{u})$ evaluated at the extremum, with a constraint on the charges that S be real positive.

As we will see, the extremization of \mathcal{I} is equivalent to the attractor mechanism for AdS₄ black holes in gauged supergravity. We also argue, generalizing [3], that the extremization of \mathcal{I} selects the exact R-symmetry of the superconformal quantum mechanics dual to the AdS₂ horizon region. We notice strong similarities between our formalism and those based on Sen's entropy functional [11] and the OSV conjecture [12].

2. The black holes

We consider dyonic BPS BHs that can be embedded in M-theory and are asymptotic to AdS₄ × S⁷. They are more easily described as solutions in the STU model, a four-dimensional $\mathcal{N}=2$ gauged supergravity with three vector multiplets, which is a consistent truncation

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cation both of M-theory on S^7 , and of the 4d maximal $\mathcal{N}=8$ $SO(8)$ gauged supergravity [13]. The model contains four Abelian vector fields (one is the graviphoton) corresponding to the $U(1)^4 \subset SO(8)$ isometries of S^7 .

In 4d $\mathcal{N}=2$ supergravities with n_V vector multiplets, one can use the standard machinery of special geometry [14–16]. The Lagrangian \mathcal{L} of the theory is completely specified by the prepotential $\mathcal{F}(X^\Lambda)$, which is a homogeneous holomorphic function of sections X^Λ , and the vector of Fayet–Iliopoulos (FI) terms $\mathcal{G} = (g^\Lambda, g_\Lambda)$. The symplectic index $\Lambda = 0, 1, \dots, n_V$ runs over the graviphoton and the n_V vectors in vector multiplets. The scalars z^i in vector multiplets, with $i = 1, \dots, n_V$, parametrize a special Kähler manifold \mathcal{M} and X^Λ are sections of a symplectic Hodge vector bundle on \mathcal{M} . The formalism is covariant with respect to symplectic $Sp(2n_V + 2)$ transformations. Indicating as (A^Λ, A_Λ) the $2n_V + 2$ components of a symplectic vector A , the scalar product is $\langle A, B \rangle = A_\Lambda B^\Lambda - A^\Lambda B_\Lambda$. One defines the covariantly-holomorphic sections

$$\mathcal{V} = e^{\mathcal{K}(z, \bar{z})/2} \begin{pmatrix} X^\Lambda(z) \\ \mathcal{F}_\Lambda(z) \end{pmatrix} \quad (2)$$

on \mathcal{M} , where \mathcal{K} is the Kähler potential and $\mathcal{F}_\Lambda \equiv \partial_\Lambda \mathcal{F}$. They satisfy $D_{\bar{i}} \mathcal{V} \equiv (\partial_{\bar{i}} - \frac{1}{2} \partial_{\bar{i}} \mathcal{K}) \mathcal{V} = 0$. The Kähler potential is then determined by $\langle \mathcal{V}, \bar{\mathcal{V}} \rangle = -i$.

The ansatz for dyonic black holes is of the form

$$ds^2 = -e^{2U(r)} dt^2 + e^{-2U(r)} (dr^2 + V(r)^2 ds_{\Sigma_g}^2) \quad (3)$$

where Σ_g is a Riemann surface of genus g , and the scalar fields z^i are assumed to only have radial dependence. We can write the metric on Σ_g locally as

$$ds_{\Sigma_g}^2 = d\theta^2 + f_\kappa^2(\theta) d\varphi^2, \quad f_\kappa(\theta) = \begin{cases} \sin \theta & \kappa = 1 \\ \theta & \kappa = 0 \\ \sinh \theta & \kappa = -1 \end{cases} \quad (4)$$

where $\kappa = 1$ for S^2 , $\kappa = 0$ for T^2 , and $\kappa = -1$ for Σ_g with $g > 1$. The scalar curvature is 2κ and the volume is

$$\text{Vol}(\Sigma_g) = 2\pi \eta, \quad \eta = \begin{cases} 2|g - 1| & \text{for } g \neq 1 \\ 1 & \text{for } g = 1. \end{cases} \quad (5)$$

The magnetic and electric charges of the black hole are

$$\int_{\Sigma_g} F^\Lambda = \text{Vol}(\Sigma_g) p^\Lambda, \quad \int_{\Sigma_g} G_\Lambda = \text{Vol}(\Sigma_g) q_\Lambda, \quad (6)$$

where $G_\Lambda = 8\pi G_N \delta(\mathcal{L} d\text{vol}_4) / \delta F^\Lambda$ and G_N is the Newton constant. This particular normalization ensures that the BPS equations are independent of g (besides a linear constraint). The charges are collected in the vector $\mathcal{Q} = (p^\Lambda, q_\Lambda)$. The vector \mathcal{G} of FI terms controls the gauging and determines the charges of the gravitini under the gauge fields. In a frame with purely electric gauging g_Λ , the lattice of electro-magnetic charges is

$$\eta g_\Lambda p^\Lambda \in \mathbb{Z}, \quad \frac{\eta}{4G_N g_\Lambda} q_\Lambda \in \mathbb{Z} \quad (7)$$

not summed over Λ . It turns out that the BPS equations fix the more stringent condition

$$\langle \mathcal{G}, \mathcal{Q} \rangle = -\kappa, \quad (8)$$

that we call the linear constraint.

It has been noticed in [17] that the BPS equations of gauged supergravity for the near-horizon geometry can be put in the form

of “attractor equations”. One defines the central charge of the black hole \mathcal{Z} and the superpotential \mathcal{L} :

$$\begin{aligned} \mathcal{Z} &= \langle \mathcal{Q}, \mathcal{V} \rangle = e^{\mathcal{K}/2} (q_\Lambda X^\Lambda - p^\Lambda \mathcal{F}_\Lambda) \\ \mathcal{L} &= \langle \mathcal{G}, \mathcal{V} \rangle = e^{\mathcal{K}/2} (g_\Lambda X^\Lambda - g^\Lambda \mathcal{F}_\Lambda). \end{aligned} \quad (9)$$

The BPS equations for the near-horizon geometry

$$ds_{\text{nh}}^2 = -\frac{r^2}{R_A^2} dt^2 + \frac{R_A^2}{r^2} dr^2 + R_S^2 ds_{\Sigma_g}^2 \quad (10)$$

with constant scalar fields z^i imply the following two equations [17]: $\mathcal{Z} - i R_S^2 \mathcal{L} = 0$ and $D_j (\mathcal{Z} - i R_S^2 \mathcal{L}) = 0$, where $D_j = \partial_j + \frac{1}{2} \partial_j \mathcal{K}$, besides $\langle \mathcal{G}, \mathcal{Q} \rangle = -\kappa$. These equations can be rewritten as

$$\partial_j \frac{\mathcal{Z}}{\mathcal{L}} = 0, \quad -i \frac{\mathcal{Z}}{\mathcal{L}} = R_S^2. \quad (11)$$

In other words, the scalars z^i at the horizon take a value such that the quantity $-i\mathcal{Z}/\mathcal{L}$ has a critical point on \mathcal{M} and then its value is proportional to the Bekenstein–Hawking black hole entropy.

Notice that a condition to have BHs with smooth horizon is that $-i\mathcal{Z}/\mathcal{L}$ be real positive at the critical point. Since the critical-point equations already fix the values of the scalars, this condition becomes a second (non-linear) constraint on the charges. Therefore the domain of allowed electro-magnetic charges has real dimension $2n_V$ (before imposing quantization). There are other inequalities to be satisfied by the charges, for instance to ensure that also R_A^2 be positive.

In the case of very special Kähler geometry, *i.e.* that the prepotential takes the form $\mathcal{F} = d_{ijk} X^i X^j X^k / X^0$ or symplectic transformations thereof, general solutions to the near-horizon BPS equations as well as full BH solutions have been found in [18–20]. That analysis guarantees that all near-horizon solutions can be completed into full BH solutions.

Our focus is on the STU model, which has $n_V = 3$ and prepotential

$$\mathcal{F} = -2i \sqrt{X^0 X^1 X^2 X^3}, \quad (12)$$

with purely electric gauging $g_\Lambda \equiv g$, $g^\Lambda = 0$. Then the AdS₄ vacuum has radius $L^2 = 1/2g^2$. Note that all dyonic BH solutions with electric charges have complex profiles for the scalars, *i.e.* the axions are turned on.

3. The dual field theory

M-theory on AdS₄ × S⁷ has a dual holographic description as a three-dimensional supersymmetric gauge theory, the ABJM theory [8], which provides a non-perturbative definition thereof. In $\mathcal{N}=2$ notation, the ABJM theory is a $U(N)_1 \times U(N)_{-1}$ Chern–Simons theory (the subscripts are the levels) with bi-fundamental chiral multiplets A_i and B_j , $i, j = 1, 2$, transforming in the (N, \bar{N}) and (\bar{N}, N) representations of the gauge group, respectively, and with superpotential $W = \varepsilon^{ik} \varepsilon^{jl} \text{Tr} A_i B_j A_k B_l$. The theory has $\mathcal{N}=8$ superconformal symmetry and $SO(8)$ R-symmetry. The identification between gravitational and QFT parameters is

$$\frac{L^2}{G_N} = \frac{1}{2g^2 G_N} = \frac{2\sqrt{2}}{3} N^{3/2}. \quad (13)$$

The “topologically twisted index” of an $\mathcal{N}=2$ three-dimensional theory is its supersymmetric Euclidean partition function on $S^2 \times S^1$ with a topological twist on S^2 [7]. Its higher-genus generalization, namely the twisted partition function on $\Sigma_g \times S^1$, has

been constructed as well [21,22]. They depend on a set of integer magnetic fluxes p_a and complex fugacities y_a , along the Cartan generators of the flavor symmetry group.

In the present case, to make the enhanced symmetry more manifest, we introduce an index $a = 1, 2, 3, 4$ that simultaneously runs over the four ABJM chiral fields and the four Abelian symmetries $U(1)^4 \subset SO(8)$. This is done by introducing a basis of four R-symmetries R_a , each acting with charge 2 on one of the chiral fields and zero on the others. Then the magnetic fluxes identify a $U(1)$ subgroup of $SO(8)$ used to twist, and are required by supersymmetry to satisfy $\sum_a p_a = 2g - 2$. The complex fugacities $y_a = \exp i u_a$ must satisfy $\prod_a y_a = 1$ ($\sum_a u_a \in 2\pi\mathbb{Z}$) and encode background values for the flavor symmetries. Writing $u_a = \Delta_a + i\beta\sigma_a$ (where β is the length of S^1), we can identify Δ_a with flavor flat connections and σ_a with real masses.

The Hamiltonian definition of the index is [7]

$$Z(u_a, p_a) = \text{Tr}(-1)^F e^{i \sum_{a=1}^3 \Delta_a J_a} e^{-\beta H}, \quad (14)$$

where $J_a = \frac{1}{2}(R_a - R_4)$ are the three independent flavor symmetries and H is the twisted Hamiltonian on S^2 , explicitly dependent on the magnetic charges p_a and the real masses σ_a . Due to the supersymmetry algebra $Q^2 = H - \sum_{a=1}^3 \sigma_a J_a$, the index $Z(u_a, p_a)$ is a meromorphic function of y_a . For simplicity, we will keep the dependence on p_a implicit and use the shorthand notation $\sigma J = \sum_{a=1}^3 \sigma_a J_a$. We stress that, in general, (14) is well-defined only for complex u_a while the index for $\sigma_a = 0$ is defined by analytic continuation. We would like to see how we can extract the BH entropies from Z .

4. Statistical interpretation

The partition function $Z(u)$ describes a supersymmetric ensemble which is canonical with respect to the magnetic charges (*i.e.* all states have the same, fixed, magnetic charges) but grand canonical with respect to the electric charges (*i.e.* it is a sum over all electric charge sectors, with fixed chemical potentials u_a). A similar viewpoint in BH physics is advocated in [23]. We can decompose Z as a sum over sectors with fixed charges q_a under $R_a/2$ (then the lattice of charges is such that both $J_a, R_a \in \mathbb{Z}$, up to a possible zero-point shift in the vacuum):

$$Z(u) = \sum_q e^{i \sum_{a=1}^3 u_a (q_a - q_4)} Z_q. \quad (15)$$

We would like to identify $S_q \equiv \mathbb{R}e \log Z_q$ with the leading entropy of a BH of fixed electric charges q_a . We take the real part to remove the effect of a possible overall sign. An important assumption is that $(-1)^F$ in the trace (14) does not cause dangerous cancellations at leading order.

We can Fourier transform the previous expression with respect to the three independent Δ_a to obtain

$$\sum'_{q_4} Z_q = \int \frac{d^3 \Delta_a}{(2\pi)^3} e^{-i \sum_{b=1}^3 \Delta_b (q_b - q_4)} Z(u), \quad (16)$$

where prime means that the sum is taken at fixed integer $q_a - q_4$. As we will see, for supergravity BHs both the electric charges q_a and $\log Z$ are of order $N^{3/2}$, therefore the previous expression can be evaluated at large N using a saddle point approximation:

$$\sum'_{q_4} Z_q = \exp \left[\log Z(\hat{u}) - i \sum_{a=1}^3 \hat{u}_a (q_a - q_4) \right] \quad (17)$$

at leading order, where \hat{u}_a is a solution for u_a to

$$\frac{\partial}{\partial u_a} \left[\log Z(u) - i \sum_{b=1}^3 u_b (q_b - q_4) \right] = 0 \quad (18)$$

with $a = 1, 2, 3$. This saddle point in general gives complex values for \hat{u}_a . The sum on the LHS of (17) will also be dominated by a specific value of q_4 , corresponding to the electric R-charge of the black hole. For that value:

$$S_q = \mathbb{R}e \left[\log Z(\hat{u}) - i \sum_{a=1}^3 \hat{u}_a (q_a - q_4) \right]. \quad (19)$$

We can restore the permutation symmetry between the charges, part of the Weyl group of $SO(8)$, by introducing

$$\mathcal{I}(u) \equiv \log Z(u) - i \sum_{a=1}^4 u_a q_a. \quad (20)$$

Eqn. (18) is equivalent to extremization of \mathcal{I} and the entropy is given by $S_q = \mathbb{R}e \mathcal{I}(\hat{u})$.

This argument does not determine the R-charge of the BH, essentially because the index $Z(u)$ lacks a chemical potential for it. However from the attractor equations (11) it follows that, for given magnetic charges p_a and flavor electric charges $q_a - q_4$, there is at most one value of q_4 leading to a large smooth BH. Our argument then gives an unambiguous prediction for the leading entropy of that BH.

5. RG flow interpretation

We can extract more information from the index if we interpret the BH as an holographic RG flow. The near-horizon geometry of BPS black holes contains an AdS_2 factor permeated by constant electric flux, where the super-isometry algebra is enhanced to $\mathfrak{su}(1, 1|1)$. Thus we can think of the BH solution as a holographic RG flow from the 3d theory on S^2 to an ensemble of $\mathfrak{su}(1, 1|1)$ -invariant states in a 1d system. The bosonic subalgebra is $\mathfrak{sl}(2, \mathbb{R}) \times \mathfrak{u}(1)_c$ where the second factor is the IR superconformal R-symmetry, which is some linear combination of $U(1)^4 \subset SO(8)$. In the near-horizon canonical ensemble this implies that all BH states have zero $U(1)_c$ charge (by an argument similar to that in [24,25]). We will assume that there are no other contributions outside the horizon.

The asymptotic behavior of electrically charged BH solutions with axions turned on suggests that the dual ABJM theory is also deformed by real masses σ_a . In general, they lift a possible vacuum degeneracy of the Hamiltonian H . The presence of AdS_2 with constant electric flux, though, indicates that there should be a large vacuum degeneracy for a modified Hamiltonian H_{nh} in which the energy of states gets an extra contribution linear in the charge: $H_{\text{nh}}(\sigma) = H(\sigma) - \sigma$. From the supersymmetry algebra $Q^2 = H_{\text{nh}}$ we conclude that $H_{\text{nh}} \geq 0$, and the index gets contribution only from its ground states. We can rewrite the index in (14) as

$$Z(\Delta, \sigma) = \text{Tr}' e^{i\pi R_{\text{trial}}(\Delta)} e^{-\beta\sigma J}, \quad (21)$$

where $\text{Tr}' = \text{Tr}_{H_{\text{nh}}=0}$. We introduced a trial R-current $R_{\text{trial}}(\Delta) \equiv R_0 + \Delta J/\pi$ that parametrizes the mixing of the R-symmetry with the flavor symmetries, with R_0 a reference R-symmetry such that $e^{i\pi R_0} = (-1)^F$.

We want to argue, generalizing [3], that the superconformal R-symmetry R_c of the Hamiltonian H_{nh} can be found by extremizing $Z(\Delta, \sigma)$ for fixed values of σ_a . Let $\hat{\Delta}_a$ be the value such that $R_{\text{trial}}(\hat{\Delta}) = R_c$. One computes $\partial \log Z / \partial \Delta_a |_{\hat{\Delta}_a} = i \langle J_a e^{-\beta\sigma J} \rangle / \langle e^{-\beta\sigma J} \rangle$, using that at zero temperature the density matrix is uniformly distributed over the ground states of H_{nh} , and that $R_c = 0$ in those states as argued above. The expression on the right is imaginary, implying that $\hat{\Delta}_a$ are determined by extremizing the index with respect to Δ_a at fixed σ_a :

$$\frac{\partial \mathbb{R}e \log Z(\Delta, \sigma)}{\partial \Delta_a} \Big|_{\hat{\Delta}} = 0. \quad (22)$$

This is the generalization of the \mathcal{I} -extremization principle proposed in [3]. Assuming the large N factorization $\langle J \rangle = \langle J \rangle (e^{-\beta\sigma J})$, we also have

$$\frac{\partial \text{Im} \log Z(\Delta, \sigma)}{\partial \Delta_a} \Big|_{\hat{\Delta}} = i \langle J_a \rangle \equiv i(q_a - q_4), \quad (23)$$

where $\langle J_a \rangle$ is the charge of the vacuum density matrix. This determines the relation between the flavor charges $q_a - q_4$ and σ_a . Since $Z(\Delta, \sigma)$ is a holomorphic function of $u_a = \Delta_a + i\beta\sigma_a$, we can summarize the result in the complex equation

$$\frac{\partial \log Z(u)}{\partial u_a} \Big|_{\hat{u}} = i(q_a - q_4), \quad (24)$$

which determines both $\hat{\Delta}_a$ and σ_a as functions of q_a .

From eqn. (21), at the critical point

$$Z(\hat{\Delta}, \sigma) = e^{-\beta\sigma \langle J \rangle} \text{Tr} 1 = e^{-\sum_{a=1}^4 \beta\sigma_a q_a} e^{S_q}. \quad (25)$$

The real part of the logarithm of this expression reproduces the result of the statistical argument, namely

$$S_q = \text{Re} \left[\log Z(\hat{u}) - i \sum_{a=1}^4 \hat{u}_a q_a \right]. \quad (26)$$

An advantage of this derivation is that we can argue, at least at leading order, that e^{S_q} is the number of ground states, without dangerous signs that could cause cancellations.

We can also write the entropy in a slightly different form and make a conjecture for the value of the fourth charge. Since \hat{u} only depends on the differences $q_a - q_4$ and $\sum_a u_a \in 2\pi\mathbb{Z}$, we can always shift the integer charges q_a and write the entropy in the permutationally symmetric and holomorphic form

$$S_q = \log Z(\hat{u}) - i \sum_{a=1}^4 \hat{u}_a q_a = \mathcal{I}(\hat{u}), \quad (27)$$

up to $\mathcal{O}(N^0)$ terms which are invisible in the large N limit. The determination of the logarithm is such that $\log Z$ is real for $\sigma_a = 0$ and extended by continuity. The requirement that (27) be real positive fixes the fourth charge. Interestingly, this is precisely the constraint (11) that comes from supergravity.

6. Explicit match for ABJM

The large N expression for the index of ABJM was found in [3, 21] for the case of real u_a , and we can extend it to the complex plane using holomorphy:

$$\log Z = \frac{N^{3/2}}{3} \sqrt{2u_1 u_2 u_3 u_4} \sum_{a=1}^4 \frac{p_a}{u_a}. \quad (28)$$

This is valid for $\sum_a u_a = 2\pi$ and $0 < \text{Re} u_a < 2\pi$. The \mathcal{I} -extremization principle (24) is equivalent to the extremization of

$$\mathcal{I}_{\text{QFT}} = \sum_{a=1}^4 \left(\frac{N^{3/2}}{3} \sqrt{2u_1 u_2 u_3 u_4} \frac{p_a}{u_a} - i q_a u_a \right). \quad (29)$$

Then the entropy is given by $S_q = \mathcal{I}_{\text{QFT}}(\hat{u})$, with the constraint on the charges that $\mathcal{I}_{\text{QFT}}(\hat{u})$ be positive.

In supergravity, the BH entropy is determined by

$$S_{\text{BH}} = \frac{\text{Area}}{4G_N} = -i \frac{\mathcal{Z}}{\mathcal{L}} \frac{2\pi\eta}{4G_N} \equiv \mathcal{I}_{\text{SUGRA}} \quad (30)$$

using (12), and $\mathcal{I}_{\text{SUGRA}}$ should be extremized with respect to X^Λ . We can identify the index $\Lambda = \{0, 1, 2, 3\}$ with $a = \{1, 2, 3, 4\}$, as

well as $2\pi X^a / \sum_b X^b = u_a$ since they have the same domain and constraint:

$$\mathcal{I}_{\text{SUGRA}} = \frac{\eta}{4gG_N} \sum_{a=1}^4 \left(\sqrt{u_1 u_2 u_3 u_4} \frac{p_a}{u_a} - i q_a u_a \right). \quad (31)$$

Identifying the integers in (7) with the charges p_a, q_a , respectively, and using (13) we obtain a perfect match $\mathcal{I}_{\text{QFT}} = \mathcal{I}_{\text{SUGRA}}$. The field theory extremization principle corresponds to the supergravity attractor mechanism: they lead to the same entropy and non-linear constraint on the charges.

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