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THE EFFECT OF RADIATION PRESSURE ON ACCRETION DISKS AROUND BLACK HOLES

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ABSTRACT

Stationary disk accretion onto a black hole is studied for high accretion rates $\dot{M} \gtrsim \dot{M}_c = 2r_0L_E/GM$ (L_E is the Eddington luminosity) for which the dynamic effect of radiation pressure is important. The rotation of the disk is not assumed to be Keplerian but is considered as an unknown in the Newtonian dynamic equation. The problem is reduced to a set of two differential equations which are solved numerically. It is found that stationary solutions without mass-outflow exist for $\dot{M} > \dot{M}_c$. The radiated luminosity, however, is always of the order of the Eddington luminosity. For increasing accretion rates, the kinetic energy swallowed by the hole and the size of the radiating region increase.

Subject headings: black holes — stars: accretion — X-rays: sources

I. INTRODUCTION

Accretion of a black hole by matter endowed with angular momentum has been treated in connection with X-ray sources by Pringle and Rees (1972), and Shakura and Sunyaev (SS) (1973). Novikov and Thorne (1973) have considered general-relativistic corrections, Lightman (1974*b*) has studied the time-dependent problem. For further references see Lamb (1974).

It was noted by several authors (Salpeter 1972; Dilworth, Maraschi, and Reina 1973; Margon and Ostriker 1973) that the luminosity function of X-ray sources has a cutoff above 10^{38}–10^{39} ergs s^{-1}, which is of the same order of the Eddington luminosity $L_E = 1.2 \times 10^{38}$ ergs s^{-1} of a 1 $M_\odot$ star. This indicates that, in the brightest X-ray sources, radiation pressure plays an important role.

When radiation pressure is important with respect to gravitation, the approximation adopted so far, that the rotation of the gas is Keplerian, needs to be reconsidered. In this paper we take the angular velocity as an unknown of the problem and show that in the inner region of the disk the Keplerian approximation becomes inadequate.

The flow of gas through the disk is treated with a number of simplifying assumptions. Thermal pressure is neglected compared with radiation pressure. Radiative transfer is described as a diffusive process, dominated by Thomson scattering. The consistency of these approximations is discussed *a posteriori*. Angular momentum transfer is treated as in SS, assuming that the shear is proportional to the total energy density. Newtonian dynamics is used throughout. Although this is not adequate near the inner boundary, relativistic corrections should not alter the qualitative features of the results (Novikov and Thorne 1973).

The resulting set of equations can be reduced to two coupled differential equations for the angular and radial velocities of the matter, which are solved numerically.

The main result of the paper is that a steady solution exists for every value of the accretion rate $\dot{M}$. However, for high values of $\dot{M}$ the luminosity radiated by the disk tends to an upper bound of the order of L_E, being no longer proportional to the accretion rate. The reason is that, under these conditions, the kinetic energy of the gas at the inner boundary is greater than in the Keplerian case. This energy is then swallowed by the hole.

II. EQUATIONS

The relevant equations for our problem are: (1) mass conservation; (2) Euler equation; (3) hydrostatic equilibrium in the vertical direction; (4) conservation of angular momentum; (5) definition of tangential stress; (6) energy dissipation; (7) energy conservation; (8) photon diffusion.

With respect to the set considered by SS, equations (2), (7), and (8) have been added. The analytic expression of the equations is:

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{\partial}{\partial z}(\rho v_r) = 0\,, \tag{1}$$

$$v_r\frac{\partial v_r}{\partial r} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{\partial}{\partial r}\frac{\epsilon}{3} + r\omega^2\,, \tag{2}$$

$$0 = -\frac{GM}{r^3}z - \frac{1}{\rho}\frac{\partial}{\partial z}\left(\frac{\epsilon}{3}\right)\,, \tag{3}$$

$$\rho v \frac{\partial}{\partial r}(r^2\omega) = -\frac{1}{r}\frac{\partial}{\partial r}(r^2 w_{r\varphi}) , \quad (4)$$

$$w_{r\varphi} = -\alpha\epsilon , \quad (5)$$

$$q = w_{r\varphi} r \frac{\partial \omega}{\partial r} , \quad (6)$$

$$\nabla \cdot \boldsymbol{F} = q , \quad (7)$$

$$\boldsymbol{F} = -\frac{c m_p}{\sigma_T \rho} \nabla\left(\frac{\epsilon}{3}\right) , \quad (8)$$

where ρ is the density, v_r is the inward velocity, ω is the angular velocity, ϵ is the photon energy density, w is the shear stress, q is the power dissipated per unit volume, F is the radiative flux, c is the velocity of light, m_p is the proton mass, σ_T is the Thomson cross section, and α is a constant describing the viscosity.

The previous equations can be simplified by integrating over the vertical coordinate (see, e.g., Lightman 1974*a*). Introducing the new variables:

$$\Sigma = \int_0^\infty \rho dz , \qquad \epsilon = \int_0^\infty \epsilon dz ,$$

$$W_{r\varphi} = \int_0^\infty w_{r\varphi} dz , \qquad Q = \int_0^\infty q dz ,$$

and the scale height

$$h = \left(\frac{\epsilon}{3\Sigma}\frac{r^3}{GM}\right)^{1/2} ,$$

assuming that v_r is independent of z and $v_z \ll v_r = v$, the equations become:

$$4\pi r \Sigma v = \dot{M} , \quad (1a)$$

$$v\frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\Sigma}\frac{d}{dr}\left(\frac{\epsilon}{3}\right) + r\omega^2 , \quad (2a)$$

$$W_{r\varphi} = -\frac{\dot{M}}{4\pi}\left[\omega - \omega_0\left(\frac{r_0}{r}\right)^2\right] , \quad (4a)$$

$$W_{r\varphi} = -\alpha\epsilon , \quad (5a)$$

$$Q = W_{r\varphi} r \frac{d\omega}{dr} , \quad (6a)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(-v\frac{dv}{dr} - \frac{GM}{r^2} + r\omega^2\right)\right] - \frac{GM}{r^3} = \frac{\sigma_T}{c m_p}\frac{Q}{h} , \quad (7a)$$

where $\dot{M}$ is the accretion rate and r_0 is the radius at which the shear stress $w_{r\varphi}$ is null. This boundary condition allows one to obtain (4a) from integration of (4). In the case treated by SS, r_0 corresponds to the radius of the last stable orbit, inside which the flow becomes essentially radial. Equation (7a) has been obtained from equations (2), (7), and (8), assuming $q = Q/h$.

The system (1)–(7) can be reduced to two differential equations for ω and v which read

$$\frac{\partial\omega}{\partial r} = -\frac{3\alpha}{rv}\left(v\frac{dv}{dr} + \frac{GM}{r^2} - \omega^2 r\right) - 2\omega_0\frac{r_0^2}{r^3} , \quad (1)$$

$$\frac{\partial}{\partial r}\left(r^2\omega^2 - rv\frac{dv}{dr}\right) = \frac{\sigma_T}{c m_p}\frac{\dot{M}}{4\pi}(3\alpha GM)^{1/2}\left(\frac{\omega r^2 - \omega_0 r_0^2}{r^2 v^2}\right)^{1/2}\frac{d\omega}{dr} . \quad (2)$$

Introducing nondimensional variables

$$x = r/r_0 , \qquad A = v(GM/r_0)^{-1/2} , \qquad B = \omega(GM/r_0^3)^{-1/2} ,$$

the equations become

$$\frac{dB}{dx} = -\frac{3\alpha}{Ax}\left(A\frac{dA}{dx} + \frac{1}{x^2} - xB^2\right) - \frac{2B}{x^3} \tag{1}$$

$$\frac{d^2A}{dx^2} = \frac{1}{xA}\left[\frac{d}{dx}(x^2B^2) - A\frac{dA}{dx} - x\left(\frac{dA}{dx}\right)^2 - 2(3\alpha)^{1/2}\frac{\dot{M}}{\dot{M}_c}\left(\frac{B - B_0x^{-2}}{A}\right)\frac{dB}{dx}\right], \tag{2}$$

where $B_0 = B(1)$, $\dot{M}_c = 2r_0L_E/GM$, and $L_E = 4\pi GMm_pc/\sigma_T$ is the Eddington luminosity.

When the effects of radiation pressure are negligible, i.e., $\dot{M} \ll \dot{M}_c = 10^{-8}\ M_\odot\ \text{yr}^{-1}$, one has $v \ll \omega r$ and on the last stable orbit $\omega_0 = \omega_{0k} = (GM/r_0^3)^{1/2}$. In this case the energy released by the infalling gas is $GM\dot{M}/2r_0$.

III. SOLUTION OF THE EQUATIONS

We have solved the system numerically starting from an asymptotic solution for large values of x. The dominant terms in the asymptotic solution are $B \propto x^{-1.5}$, $A \propto Hx^{-2.5}$, where $H = 27\alpha(\dot{M}/\dot{M}_c)^2$.

In order to expand the right-hand term of equation (2) to the relevant order ($x^{-9/2}$), the asymptotic solution must be determined for B up to terms $O(x^{-5.5})$ and for A up to terms $O(x^{-6.5})$.

The resulting expansions are

$$B \propto x^{-1.5} - \frac{H}{4\alpha}x^{-3.5} + \frac{7}{12}\frac{HB_0}{\alpha}x^{-4} - \frac{1}{3}\frac{HB_0^2}{\alpha}x^{-4.5} - \left(\frac{9}{32}\frac{H^2}{\alpha^2} + \frac{5}{4}H^2\right)x^{-5.5} + o(x^{-5.5}),$$

$$A \propto H\left[x^{-2.5} - B_0x^{-3} + \frac{19}{12}\frac{H}{\alpha}x^{-4.5} - \frac{227}{36}\frac{H}{\alpha}B_0x^{-5} + \frac{145}{18}\frac{H}{\alpha}B_0^2x^{-5.5} - \frac{10}{3}\frac{H}{\alpha}B_0^3x^{-6} + \left(-\frac{125}{12}H^2 + \frac{1667}{288}\frac{H^2}{\alpha^2}\right)x^{-6.5} + o(x^{-6.5})\right].$$

The standard finite-difference method has been used for constructing numerical solutions. For $\dot{M} > \dot{M}_c$, solutions are obtained which are insensitive to variations of the initial point and step of the integration. B_0, which appears

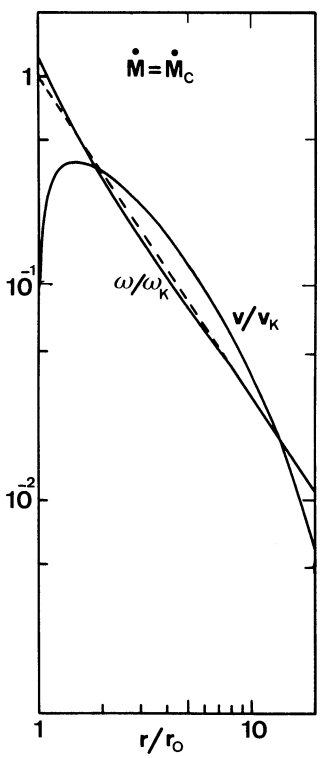

FIG. 1a

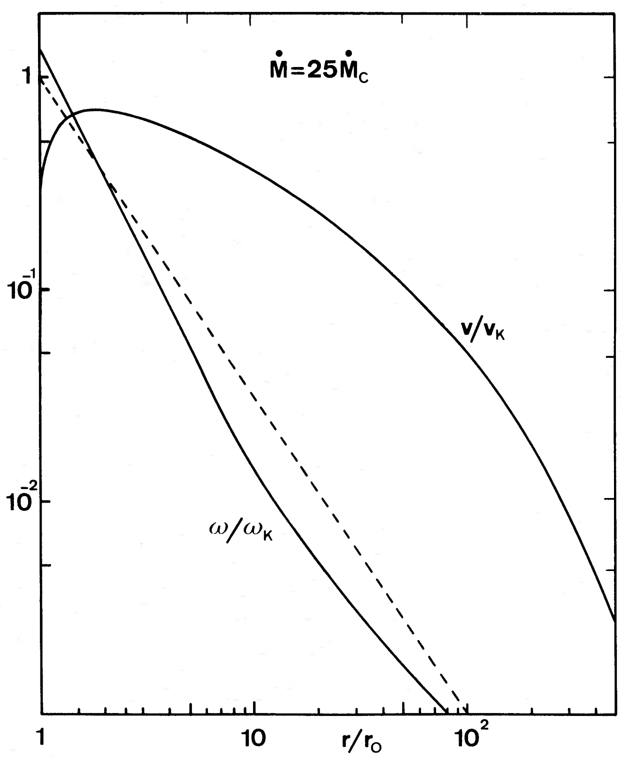

FIG. 1b

FIGS. 1a, 1b.—Angular velocity ω, and radial velocity v, of the infalling gas, versus distance for different values of the accretion rate $\dot{M}$. $\omega_k = (GM/r_0^3)^{1/2}$, $v_k = (GM/r_0)^{1/2}$, and $\dot{M}_c = 2r_0L_E/GM$. The dotted line corresponds to the Keplerian angular velocity.

as an eigenvalue, is determined by an iterative process. For $\dot{M} < \dot{M}_c$, numerical instabilities at large values of x prevent the construction of solutions by this crude method. The same happens for values of $\alpha < 1$.

In the following we discuss results for $\alpha = 1$ and $1 < \dot{M}/\dot{M}_c < 25$, the upper limit being imposed by computing time considerations. We think that this range of values is sufficiently representative to allow interesting conclusions.

IV. RESULTS AND DISCUSSION

In Figure 1 the angular and radial velocities are shown for $\dot{M}/\dot{M}_c = 1$ and 25. For large values of r the numerical solutions tend to the Keplerian values. Approaching the black hole, the angular velocity is found to be significantly smaller than in the Keplerian approximation. For increasing accretion rates the deviations become larger and start from larger values of r. In the very vicinity of the hole the tendency is opposite. The angular velocity grows very rapidly, approaching the law $\omega \propto r^{-2}$, and for $r \to r_0$ it becomes larger than the Keplerian value $\omega_{0k} = \sqrt{(GM/r_0^3)}$. The eigenvalues for $\dot{M}/\dot{M}_c = 2, 5, 25$ are $\omega_0 = 1.2\omega_{0k}$, $1.3\omega_{0k}$, $1.35\omega_{0k}$, respectively.

The physical reason underlying this behavior is that, since no energy is dissipated beyond the boundary r_0, the pressure gradient is directed inward for $r \sim r_0$, while for larger radii it is directed outward.

The total energy radiated by the infalling gas is

$$L = \dot{M}GM/r_0 - \tfrac{1}{2}(\dot{M}\omega_0^2 r_0^2 + \dot{M}v^2)\,,$$

where the term in brackets represents the kinetic energy at the last stable orbit, which is swallowed by the hole and therefore does not contribute to the luminosity. In the Keplerian approximation $\omega_0 = \omega_{0k}$, $v \approx 0$, and therefore the radiated energy is $\frac{1}{2}(\dot{M}GM/r_0)$. In the case considered here, since $\omega_0 > \omega_{0k}$ and $v \neq 0$, the luminosity is smaller and does not depend linearly on the accretion rate. In Figure 2, L is given versus the accretion rate. The saturation effect is clearly visible. It seems that the curve is upper-bounded, but from the limited range of accretion rates for which the solution has been computed it is not possible to determine the value of the limiting luminosity.

In Figure 3 the thickness h and the mean density $\rho = \Sigma/h$ of the disk are given. For large values of x a larger thickness corresponds to a larger accretion rate in agreement with the asymptotic expressions (see SS). Near the hole the thickness decreases rapidly and, for fixed values of r, the disk is thicker for smaller values of the accretion rate. Note that h/r is always less than one; i.e., no region of spherization is found, as was suggested by SS. The energy radiated by the disk within a radius r,

$$L(r) = \int_{r_0}^{r} 4\pi r Q dr\,,$$

is given in Figure 4. It is apparent that the region where most of the luminosity is produced increases with increasing accretion rate.

On the basis of the numerical results we can discuss the consistency of our approximations. The disk is thick to Thomson scattering; and free-free opacity, calculated at the minimum blackbody temperature, is indeed negligible with respect to Thomson opacity, therefore justifying the approximations used in the treatment of the radiation transfer.

We now consider the assumption that radiation pressure dominates with respect to kinetic pressure. It turns out that this is the case if $T \leqslant 10^{10}$ K. In the case $\dot{M} = 25\,\dot{M}_c$ this is always true, as in the optically thick regions, $(\tau_{es}\tau_{ff})^{1/2} \geqslant 1$, the blackbody temperature is less than 10^7 K and in the intermediate optically thin region, $10 \leqslant r/r_0 \leqslant 100$, the free-free temperature is less than 10^9 K. In the case $\dot{M} = \dot{M}_c$ the disk is almost everywhere

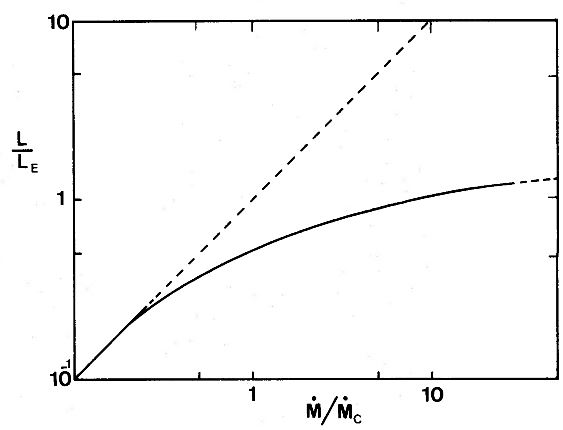

FIG. 2.—Total luminosity radiated by the disk versus accretion rate, $L_E = 4\pi GMm_p c/\sigma_T$, $\dot{M}_c = 2r_0 L_E/GM$

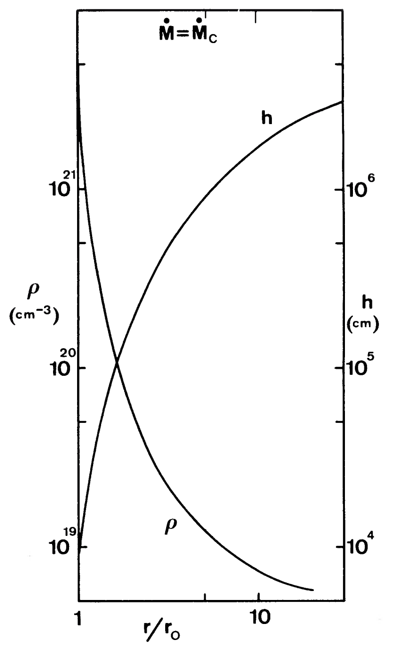

FIG. 3a

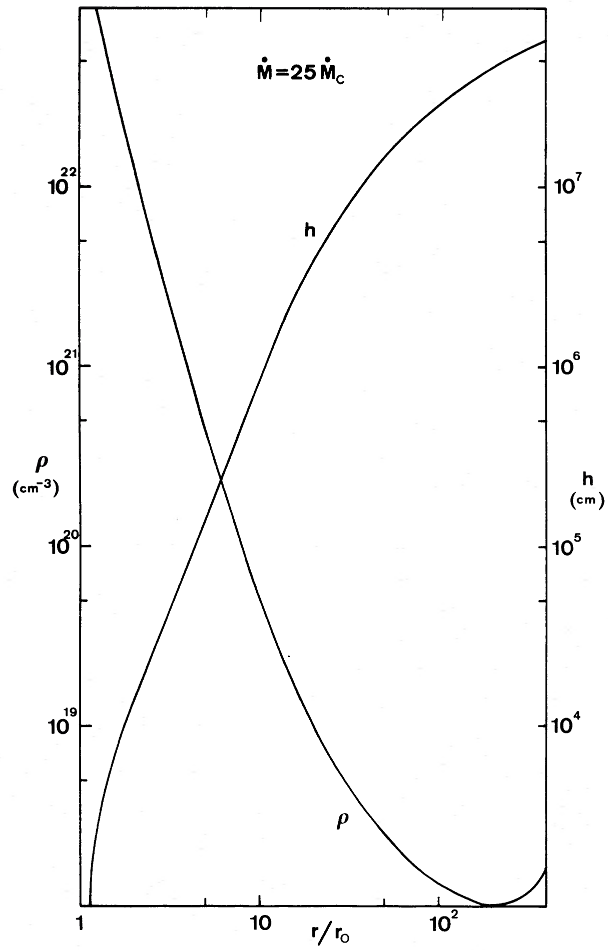

FIG. 3b

FIGS. 3a, 3b.—Half-thickness h and mean density ρ of the disk versus distance for different values of the accretion rate $\dot{M}$

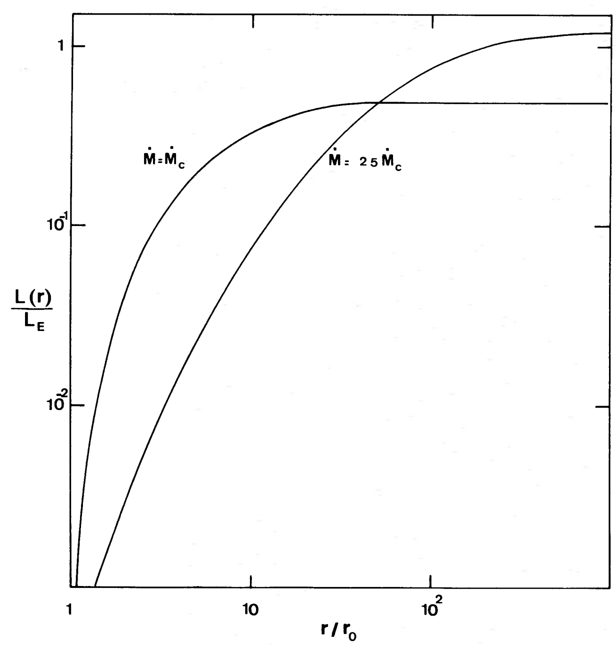

FIG. 4.—Luminosity radiated by the disk within distance r for different values of the accretion rate M

optically thin in the region of interest; and if one calculates the free-free temperature, one gets values between 10^{10} and 10^{12} K for $1.5 \leq r/r_0 \leq 10$. However, since Thomson scattering opacity is appreciable ($\tau_{es} > 1$), at such high temperatures energy losses by Compton scattering will be important, resulting in a substantial reduction of the temperature (by a factor 10^2–10^4) as discussed by SS. Therefore also in the case $\dot{M} = \dot{M}_c$ the solution is consistent with the assumption that the disk region under consideration is radiation pressure dominated.

A final point is the discussion of the actual value of the inner radius of the disk r_0, which in the numerical estimates has been assumed to be $r_0 = 6GM/c^2$. This value, which corresponds to the last stable orbit of a test particle in a Schwarzschild field, is generally taken as the inner boundary of accretion disks around black holes (SS). In the present case the effect of radiation pressure, calculated in the Newtonian approximation, is that of increasing the angular momentum of the stable orbits near the inner boundary, and therefore the value of r_0 could conceivably be altered. However, a boundary, where the shear stress $w_{r\varphi} \simeq 0$, should exist at $r \geqslant 2GM/c^2$, and the main results of our treatment depend only on the existence of such a boundary. In particular the dependence of the radiated luminosity L on the accretion rate $\dot{M}$ should not change essentially, even if r_0 is a function of $\dot{M}$. In fact, given a black boundary, radiation pressure would add to gravitation, allowing more kinetic energy to be swallowed by the hole than in the case where radiation pressure is neglected. This argument would remain valid even if relativistic corrections were taken into account.

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