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On Spherical Accretion near the Eddington Luminosity

L. Maraschi, C. Reina and A. Treves

Istituto di Fisica dell' Università and Laboratorio di Fisica Cosmica e Tecnologie Relative del C.N.R., Milano

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Summary. The problem of steady accretion of a compact star in spherical symmetry is considered taking into account the radiation pressure due to Thomson scattering and neglecting the internal pressure of the infalling gas and its heating by the outgoing radiation. The transfer of photons is treated as a diffusive-convective process. The electric field deriving from the difference of forces acting on electrons and protons is also given. The problem leads to a quasilinear second order equation for the velocity field, which can be solved analytically at large distances from the star where convection is negligible. The velocity, density, and luminosity profiles in the vicinity of the star are obtained numerically for different values of the total luminosity. In particular it is found that at the surface of the star the infall velocity tends to about one tenth of the keplerian velocity for luminosities approaching the Eddington limit.

Key words: accretion — X-ray sources

I. Introduction

The problem of steady spherical accretion of a fluid was solved by Bondi (1952), when the only acting forces are gravitation and internal pressure. Bondi's solution becomes inadequate when the power radiated by the accreting source is of the order of the Eddington limit

$$L_E = \frac{4\pi G M \mu_e m_p c}{\sigma_T} \simeq 1.2 \times 10^{38}\, \mu_e \left(\frac{M}{M_\odot}\right) \text{erg/s}$$

where M is the mass of the star, μ_e is the atomic weight per electron, m_p is the proton mass and σ_T is the Thomson cross section.

In this paper we study the effect of radiation pressure on the dynamics of spherical accretion with approximations that allow the problem to be treated mainly by analytical methods. In particular we assume that the flow is supersonic so that thermal pressure can be neglected.

The interest in studying critical accretion is stimulated by the discovery that typical luminosities of galactic X-ray sources are $\sim 10^{37} - 10^{38}$ erg/s (Salpeter, 1973). In this context Davidson (1973) has considered critical accretion within a cylinder, assuming that in the vicinity of the star the flow is completely decelerated by radiation pressure and the gravitational acceleration is negligible. This treatment approximates the symmetry of the flow within a magnetic funnel. However the limited region of validity of the cylindrical approximation does not allow to fix boundary conditions at infinity so that the condition of total deceleration has to be imposed a priori.

Critical accretion with spherical symmetry was considered by De Gregoria (1974) in the case where the central star is a white dwarf. The time dependent problem was studied taking into account thermal and radiation pressure. A solution was obtained within 1 stellar radius by numerical techniques dividing the gas into shells and adding successively new shells.

The simplified spherical problem considered here, although not directly applicable to realistic models of X-ray sources, leads to on analytic asymptotic solution which allows to derive a consistent solution everywhere.

II. Critical Flow

For simplicity we consider a completely ionized hydrogen plasma ($\mu_e = 1$) of zero temperature, and suppose that it can be treated as a one fluid. If the plasma were treated as a two fluid, an electric field would arise coupling electrons and protons but the dynamics of the infalling gas is practically unchanged (as shown in Section III).

In our hypothesis the relevant equations are, (1) Euler's equation, (2) the continuity equation, (3) the energy balance between photons and the gas and (4) the transfer equation for photons, that is,

$$\frac{\partial E}{\partial r} = -\frac{1}{\varrho}\frac{\partial}{\partial r}\left(\frac{U}{3}\right), \tag{1}$$

$$4\pi r^2 \varrho v = \dot{m}\,, \tag{2}$$

$$\frac{\partial}{\partial r}(4\pi r^2 F) = \dot{m}\frac{\partial E}{\partial r}, \tag{3}$$

$$F = -\frac{cm_p}{\sigma_T}\frac{1}{\varrho}\frac{\partial}{\partial r}\left(\frac{U}{3}\right) - Uv, \tag{4}$$

where v is the inward flow velocity, $\dot{m}$ is the accretion rate, $E = \frac{v^2}{2} - \frac{GM}{r}$ is the total energy per unit mass of the infalling gas, F is the radiation flux, and U is the radiation energy density.

Since $\dot{m}$ is independent of r, Eq. (3) is immediately integrated giving

$$F = \frac{\dot{m}}{4\pi r^2}(E + \psi). \tag{5}$$

The integration constant ψ is determined by the condition that the total energy E at infinity is null, which gives $\psi = GM/r_s$, if the gravitational power is completely converted into radiation.

The first term in Eq. (4) describes the diffusion of photons and the second term their convection by the infalling gas. Note that assuming that $\partial U/\partial r$ is of the order of U/r, which can be verified a posteriori, one has that the diffusive term dominates for

$$\frac{r}{r_s} \gg \frac{\dot{m}\sigma_T}{4\pi c m_p r_s} = \frac{P}{L_E}$$

where $P = \frac{GM}{r_s}\dot{m}$ is the radiated power.

Therefore convection must be taken into account only for $r \simeq r_s$ and $P \simeq L_E$. In order to obtain an asymptotic solution (i.e. for $r \gg r_s$) one can then substitute Eq. (4) with

$$F = -\frac{cm_p}{\sigma_T}\frac{1}{\varrho}\frac{\partial}{\partial r}\left(\frac{U}{3}\right).$$

From Eq. (5) one has

$$-\frac{1}{\varrho}\frac{\partial}{\partial r}\left(\frac{U}{3}\right) = \frac{P}{L_E}\frac{r_s}{r^2}\left(E + \frac{GM}{r_s}\right) \tag{6}$$

and inserting Eq. (6) into Eq. (1) one obtains,

$$\frac{\partial E}{\partial r} - \frac{P}{L_E}\frac{r_s}{r^2}E = \frac{P}{L_E}\frac{r_s}{r^2}\frac{GM}{r_s} \tag{7}$$

whose general solution reads

$$E = -\frac{GM}{r_s}\left[1 + C\exp\left(-\frac{P}{L_E}\frac{r_s}{r}\right)\right].$$

The integration constant C is determined by the boundary condition $E \to 0$ for $r \to \infty$, yielding $C = -1$. Therefore it follows

$$E = \frac{GM}{r_s}\left[\exp\left(-\frac{P}{L_E}\frac{r_s}{r}\right) - 1\right]. \tag{8}$$

This expression gives an asymptotic condition for our problem[1]).

In order to decouple the complete set of Eqs. (1), (2), (3) and (4), one can insert Eq. (1) into Eq. (4) yielding

$$U = \frac{1}{r}\left[\frac{cm_p}{\sigma_T}\frac{\partial E}{\partial r} - \frac{\dot{m}}{4\pi r^2}\left(E + \frac{GM}{r_s}\right)\right] \tag{9}$$

and introducing this expression into Eq. (1) one obtains

$$\begin{aligned}\frac{\partial^2 E}{\partial r^2} = \frac{1}{2\left(E + \frac{GM}{r}\right)}\left(\frac{\partial E}{\partial r} - \frac{GM}{r^2}\right) &\cdot \left[\frac{\partial E}{\partial r} - \frac{P}{L_E}\frac{r_s}{r^2}\left(E + \frac{GM}{r_s}\right)\right] \\ &- \frac{2P}{L_E}\frac{r_s}{r^2}\left[\frac{\partial E}{\partial r} + \frac{2}{r}\left(E + \frac{GM}{r_s}\right)\right]\end{aligned} \tag{10}$$

where we put

$$v = \sqrt{2\left(E + \frac{GM}{r}\right)}. \tag{11}$$

Equation (10) is easily solved numerically from the asymptotic behaviour (8). From Eqs. (11), (2) and (5) one obtains the radial profiles $v(r)$, $\varrho(r)$ and $L(r) = 4\pi r^2 F$, given in Figs. 1, 2 and 3 for different values of the total power P.

III. Electric Field Produced by Critical Accretion

In the case that the plasma is treated as a two fluid an electric field arises, due to the fact that radiation pressure acts predominantly on electrons as shown by Schwartzman (1970). The relevant equations read,

$$m_e\frac{\partial E_e}{\partial r} = -\frac{1}{n_e}\frac{\partial}{\partial \tau}\left(\frac{U}{3}\right) - e\mathscr{E}, \tag{1a}$$

$$m_p\frac{\partial E_p}{\partial r} = e\mathscr{E}, \tag{1b}$$

$$4\pi r^2 m_p n_p v_p = \dot{m}_p, \tag{2a}$$

$$\frac{\partial}{\partial r}(4\pi r^2 F) = -4\pi r^2 v_e\frac{\partial}{\partial r}\left(\frac{U}{3}\right), \tag{3a}$$

$$F = -\frac{c}{\sigma_T n_e}\frac{\partial}{\partial r}\left(\frac{U}{3}\right) - Uv_e, \tag{4a}$$

$$n_e v_e = n_p v_p \tag{12}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\mathscr{E}) = 4\pi(n_p - n_e)e, \tag{13}$$

[1]) It was pointed out to us by an anonymous referee of Astronomy and Astrophysics that this solution of the simplified system of equations was obtained by N.J. Shakura in his unpublished thesis.

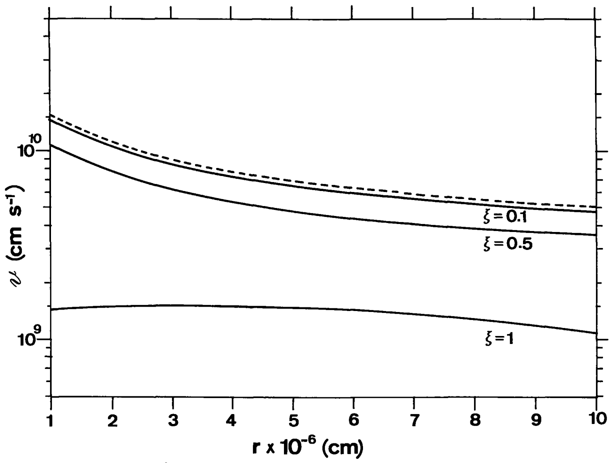

Fig. 1. Infall velocity $vs_{\odot}$ radius for different values of the total power $P=\xi L_E$. The dashed curve represents the Keplerian velocity $v=v_s \cdot \left(\frac{r_s}{r}\right)^{\frac{1}{2}}$. All the curves refer to a star with $M=1\,M_{\odot}$ and $r_s=10^6$ cm

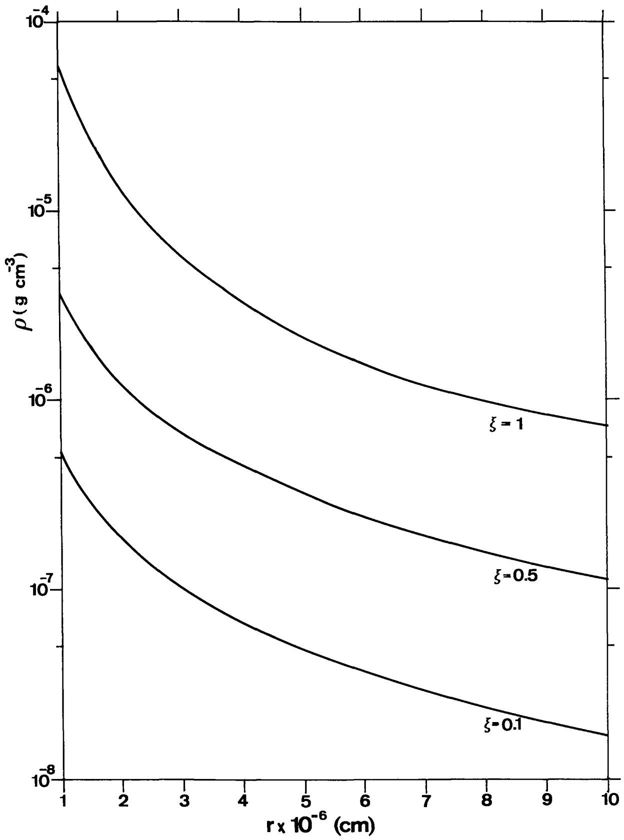

Fig. 2. Density of the infalling gas $vs_{\odot}$ radius for different values of the total power $P=\xi L_E$

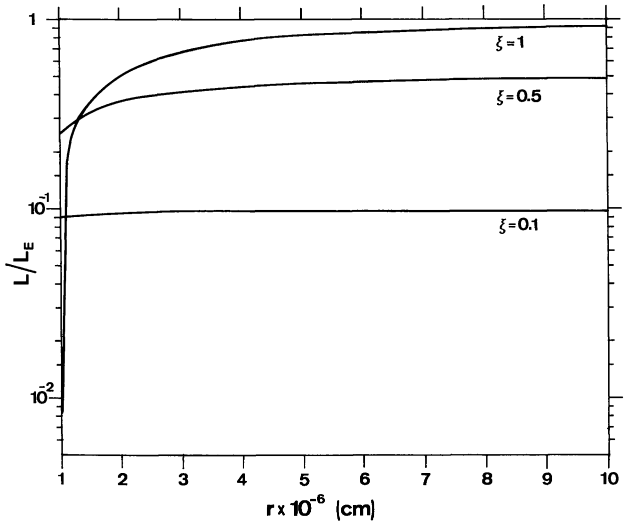

Fig. 3. Luminosity produced within the radius r for different values of the total power at infinity $P=\xi L_E$

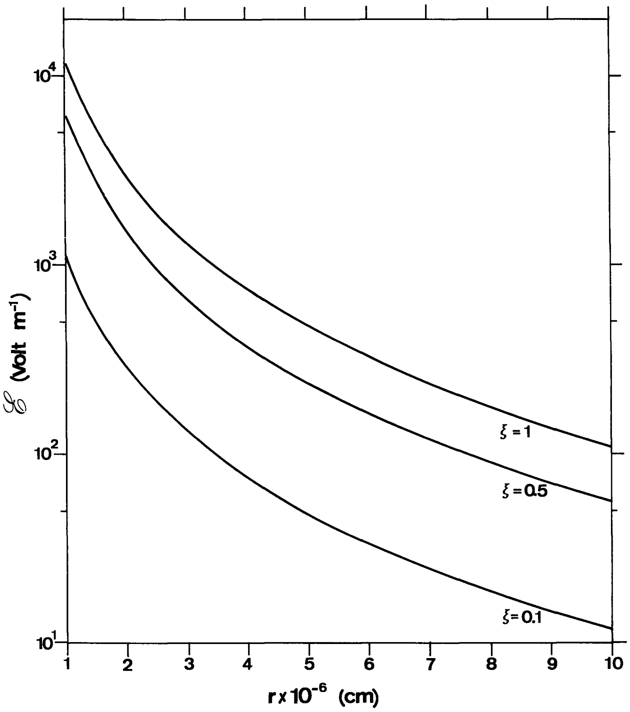

Fig. 4. Electric field $vs_{\odot}$ radius for different values of the total power $P=\xi L_E$

where $\mathscr{E}$ is the electric field and the subscripts e, p label quantities which refer to electrons and protons respectively. Equation (12) derives from the requirement that the charge distribution is stationary and Eq. (13) is Poisson's equation.

Neglecting m_e with respect to m_p and assuming $v_e \simeq v_p$ and consequently $n_e \simeq n_p$, one can derive the system of equations discussed in the previous section. In fact with the previous approximations summing (1 a) and (1 b) one has Eq. (1). Equations (2a), (3a) and (4a) become respectively Eqs. (2), (3) and (4).

From Eq. (1 b) one can derive the electric field which is given in Fig. 4. Note that for $P<L_E$, or $P\sim L_E$ and $r \gg r_s$, the electric field $\mathscr{E}$ can be calculated by means of the asymptotic solution (8) which yields

$$\mathscr{E} \simeq \frac{m_p}{e}\frac{GM}{r^2}\frac{P}{L_E}\exp\left(-\frac{P}{L_E}\frac{r_s}{r}\right). \tag{14}$$

Since the numerical solution gives a spatial charge density outside the star lower than that obtained from Eq. (14), an upper limit to the net charge density can be obtained from this equation. This yields

$$\frac{n_p - n_e}{n} \sim \frac{v_p - v_e}{v} < 10^{-17}$$

for every value of P and r, which is negligible consistently with our assumptions.

IV. Discussion

The results obtained in the previous sections rely on the assumption that the only interaction between electrons and photons is Thomson scattering. From this it follows that a part of the bulk kinetic energy of the plasma is transferred to photons, whilst the heating of the plasma by radiation is neglected. This picture can be expected to apply in the region where the flow is supersonic. The heating of the infalling gas by radiation has been considered by Buff and McCray (1974) neglecting effects of radiation pressure. It was shown that when $P \simeq L_E$ the region of supersonic flow is strongly reduced with respect to the Bondi case. However the extent of this region is still several orders of magnitude larger than the radius of the star. Therefore we consider that the present solution applies within a few stellar radii.

The most interesting feature of the solution obtained here is that the deceleration produced by radiation pressure does not stop the flow of the gas. Nevertheless, approaching the critical condition, the bulk of the luminosity is generated by the gas before hitting the surface of the star.

Although the computation of the spectrum of the accreting source is beyond the scope of this paper, we note that the present solution leads to expect, for luminosities close to the Eddington limit, (a) lower mean photon energies due to the reduced infall velocities, (b) a reduced effective blackbody temperature with respect to the non critical case in which all the energy is produced at the star surface, (c) a reduced range of infalling particles in the star crust yielding higher trasparency and large deviations from the black body spectral distribution (see Alme and Wilson, 1973). These effects tend to modify the spectrum in opposite ways so that a qualitative estimate of the distorsion with respect to the spectra computed neglecting radiation pressure (Zeldovich and Shakura, 1969; Alme and Wilson, 1973) would be hazardous. However we note that observation of X-ray sources has suggested the existence of an anticorrelation between luminosity and temperature for high luminosity variable sources in the Magellanic Clouds and near the Galactic Centre (Dilworth *et al.*, 1973). This may suggest that effects (a) and (b) are dominant with respect to (c).

Also in the case of critical spherical accretion of a white dwarf treated by De Gregoria (1974) it is found that for P approaching L_E the luminosity tends to be emitted at lower frequencies. For instance for $P/L_E = 0.4$ the effective temperature is reduced by a factor of 100 with respect to the non critical case. However this seems to be due to the large opacity implied by the high critical accretion rate for a white dwarf, rather than to the effect of radiation pressure on the gas flow.

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L. Maraschi
C. Reina
A. Treves
Istituto di Fisica
Via Celoria, 16
I-20133 Milano, Italy