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The 10^{-3} eV frontier in neutrinoless double beta decay

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ABSTRACT

The observation of neutrinoless double beta decay would allow to establish lepton number violation and the Majorana nature of neutrinos. The rate of this process in the case of 3-neutrino mixing is controlled by the neutrinoless double beta decay effective Majorana mass $|\langle m\rangle|$. For a neutrino mass spectrum with normal ordering, which is favoured over the spectrum with inverted ordering by recent global fits, $|\langle m\rangle|$ can be significantly suppressed. Taking into account updated data on the neutrino oscillation parameters, we investigate the conditions under which $|\langle m\rangle|$ in the case of spectrum with normal ordering exceeds 10^{-3} (5×10^{-3}) eV: $|\langle m\rangle|_{\text{NO}} > 10^{-3}$ (5×10^{-3}) eV. We analyse first the generic case with unconstrained leptonic CP violation Majorana phases. We show, in particular, that if the sum of neutrino masses is found to satisfy $\Sigma > 0.10$ eV, then $|\langle m\rangle|_{\text{NO}} > 5\times10^{-3}$ eV for any values of the Majorana phases. We consider also cases where the values for these phases are either CP conserving or are in line with predictive schemes combining flavour and generalised CP symmetries.

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1. Introduction

Despite their elusiveness, neutrinos have granted us unique evidence for physics beyond the Standard Theory. Observations of flavour oscillations in experiments with solar, atmospheric, reactor, and accelerator neutrinos (see, e.g., [1]) imply both non-trivial mixing in the leptonic sector and above-meV masses for at least two of the light neutrinos. Neutrino oscillations, however, are blind to the absolute scale of neutrino masses and to the nature – Dirac or Majorana – of massive neutrinos [2,3].

In order to uncover the possible Majorana nature of these neutral fermions, searches for the lepton-number violating process of neutrinoless double beta ($(\beta\beta)_{0\nu}$-)decay are underway (for recent reviews, see e.g. [4,5]). This decay corresponds to a transition between the isobars (A, Z) and $(A, Z+2)$, accompanied by the emission of two electrons but – unlike usual double beta decay – without the emission of two (anti)neutrinos. A potential observation of $(\beta\beta)_{0\nu}$-decay is feasible, in principle, whenever single beta decay is energetically forbidden, as is the case for certain even–even nuclei. The searches for $(\beta\beta)_{0\nu}$-decay have a long history (see, e.g., [6]). The best lower limits on the half-lives $T^{0\nu}_{1/2}$ of this decay have been obtained for the isotopes of germanium-76, tellurium-130, and xenon-136: $T^{0\nu}_{1/2}(^{76}\text{Ge}) > 8.0\times10^{25}$ yr reported by the GERDA-II collaboration [7], $T^{0\nu}_{1/2}(^{130}\text{Te}) > 1.5\times10^{25}$ yr obtained from the combined results of the Cuoricino, CUORE-0, and CUORE experiments [8], and $T^{0\nu}_{1/2}(^{136}\text{Xe}) > 1.07\times10^{26}$ yr reached by the KamLAND-Zen collaboration [9], with all limits given at the 90% CL.

In the standard scenario where the exchange of three Majorana neutrinos ν_i $(i = 1, 2, 3)$ with masses $m_i < 10$ MeV provides the dominant contribution to the decay rate, the $(\beta\beta)_{0\nu}$-decay rate is proportional to the square of the so-called effective Majorana mass $|\langle m\rangle|$. Given the present knowledge of neutrino oscillation data, the effective Majorana mass is bounded from below in the case of a neutrino mass spectrum with inverted ordering (IO) [10], $|\langle m\rangle|_{\text{IO}} > 1.4\times10^{-2}$ eV. Instead, in the case of a spectrum with normal ordering (NO), $|\langle m\rangle|$ can be exceptionally small: depending on the values of the lightest neutrino mass and of the CP violation (CPV) Majorana phases we can have $|\langle m\rangle|_{\text{NO}} \ll 10^{-3}$ eV (see, e.g., [1]). Recent global analyses show a preference of the data for NO spectrum over IO spectrum at the 2σ CL [11,12]. In the latest analysis performed in [13] this preference is at 3.1σ CL.

New-generation experiments seek to probe and possibly cover the IO region of parameter space, working towards the $|\langle m\rangle| \sim 10^{-2}$ eV frontier. Aside from upgrades to the ones mentioned above, such experiments include (see, e.g., [4,5]): CANDLES (^{48}Ca), Majorana and LEGEND (^{76}Ge), SuperNEMO and DCBA (^{82}Se, ^{150}Nd), ZICOS (^{96}Zr), AMoRE and MOON (^{100}Mo), COBRA

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(^{116}Cd, ^{130}Te), SNO+ (^{130}Te), and NEXT, PandaX-III and nEXO (^{136}Xe). In case these searches produce a negative result, the next frontier in the quest for $(\beta\beta)_{0\nu}$-decay will correspond to $|\langle m\rangle| \sim 10^{-3}$ eV.

In the present article we determine the conditions under which the effective Majorana mass in the case of 3-neutrino mixing and NO neutrino mass spectrum exceeds the millielectronvolt value. We consider both the generic case, where the Majorana and Dirac CPV phases are unconstrained, as well as a set of cases in which the CPV phases take particular values, motivated by predictive schemes combining generalised CP and flavour symmetries. Our study is a natural continuation and extension of the study performed in [14].

2. The effective Majorana mass

Taking the dominant contribution to the $(\beta\beta)_{0\nu}$-decay rate, $\Gamma_{0\nu}$, to be due to the exchange of three Majorana neutrinos ν_i ($m_i < 10$ MeV; $i = 1, 2, 3$), one can write the inverse of the decay half-life, $(T^{0\nu}_{1/2})^{-1} = \Gamma_{0\nu}/\ln 2$, as

$$\left(T^{0\nu}_{1/2}\right)^{-1} = G_{0\nu}(Q, Z)\left|\mathcal{M}_{0\nu}(A, Z)\right|^2 |\langle m\rangle|^2\,, \tag{1}$$

where $G_{0\nu}$ denotes the phase-space factor, which depends on the Q-value of the nuclear transition, and $\mathcal{M}_{0\nu}$ is the nuclear matrix element (NME) of the decay. The former can be computed with relatively good accuracy whereas the latter remains the predominant source of uncertainty in the extraction of $|\langle m\rangle|$ from the data (see, e.g., [4,15]).

The effective Majorana mass $|\langle m\rangle|$ is given by (see, e.g., [16]):

$$|\langle m\rangle| = \left|\sum_{i=1}^{3} U_{ei}^2\, m_i\right|\,, \tag{2}$$

with U being the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) leptonic mixing matrix. The first row of U is the one relevant for $(\beta\beta)_{0\nu}$-decay and reads, in the standard parametrization [1],

$$U_{ei} = \left(c_{12}\,c_{13}\,,\quad s_{12}\,c_{13}\,e^{i\alpha_{21}/2}\,,\quad s_{13}\,e^{-i\delta}\,e^{i\alpha_{31}/2}\right)_i\,. \tag{3}$$

Here, $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$, where $\theta_{ij} \in [0, \pi/2]$ are the mixing angles, and δ and the α_{ij} are the Dirac and Majorana CPV phases [2], respectively ($\delta, \alpha_{ij} \in [0, 2\pi]$).

The most stringent upper limit on the effective Majorana mass was reported by the KamLAND-Zen collaboration. Using the lower limit on the half-life of ^{136}Xe obtained by the collaboration and quoted in the Introduction, and taking into account the estimated uncertainties in the NMEs of the relevant process, the limit reads [9]:

$$|\langle m\rangle| < (0.061 - 0.165)\ \text{eV}\,. \tag{4}$$

Neutrino oscillation data provides information on mass-squared differences, but not on individual neutrino masses. The mass-squared difference $\Delta m^2_\odot$ responsible for solar ν_e and very-long baseline reactor $\bar\nu_e$ oscillations is much smaller than the mass-squared difference $\Delta m^2_{\rm A}$ responsible for atmospheric and accelerator ν_μ and $\bar\nu_\mu$ and long baseline reactor $\bar\nu_e$ oscillations, $\Delta m^2_\odot/|\Delta m^2_{\rm A}| \sim 1/30$. At present the sign of $\Delta m^2_{\rm A}$ cannot be determined from the existing data. The two possible signs of $\Delta m^2_{\rm A}$ correspond to two types of neutrino mass spectrum: $\Delta m^2_{\rm A} > 0$ – spectrum with normal ordering (NO), $\Delta m^2_{\rm A} < 0$ – spectrum with inverted ordering (IO). In a widely used convention we are also going to employ, the first corresponds to the lightest neutrino being ν_1, while the second corresponds to the lightest neutrino being ν_3. Combined with the fact that in this convention $\Delta m^2_\odot \equiv \Delta m^2_{21} > 0$ we have:

- $m_1 < m_2 < m_3$, $\Delta m^2_{31} \equiv \Delta m^2_{\rm A} > 0$, for NO; and
- $m_3 < m_1 < m_2$, $-\Delta m^2_{23} \equiv \Delta m^2_{\rm A} < 0$, for IO,

where $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$. For either ordering, $|\Delta m^2_{\rm A}| = \max(|m_i^2 - m_j^2|)$, $i, j = 1, 2, 3$. We also define $m_{\rm min} \equiv m_1\ (m_3)$ in the NO (IO) case. A NO or IO mass spectrum is additionally said to be normal hierarchical (NH) or inverted hierarchical (IH) if respectively $m_1 \ll m_{2,3}$ or $m_3 \ll m_{1,2}$. In the converse limit of relatively large $m_{\rm min}$, $m_{\rm min} \gtrsim 0.1$ eV, the spectrum is said to be quasi-degenerate (QD) and $m_1 \simeq m_2 \simeq m_3$. In this last case, the distinction between NO and IO spectra is blurred and $\Delta m^2_\odot$ and $|\Delta m^2_{\rm A}|$ can usually be neglected with respect to $m^2_{\rm min}$.

Table 1
Ranges for the relevant oscillation parameters in the case of an IO neutrino spectrum, at the 3σ CL, taken from the global analysis of Ref. [13]. As in Table 2, $\Delta m^2_{\rm A}$ is obtained from the quantities defined in Ref. [13] using the best-fit value of Δm^2_{21}.

$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$\frac{\Delta m^2_{23}}{10^{-3}\ \text{eV}^2}$	$\frac{\sin^2\theta_{12}}{10^{-1}}$	$\frac{\sin^2\theta_{13}}{10^{-2}}$
6.92–7.91	2.38–2.58	2.64–3.45	1.95–2.43

In terms of the lightest neutrino mass, CPV phases, neutrino mixing angles, and neutrino mass-squared differences, the effective Majorana mass reads:

$$|\langle m\rangle|_{\rm NO} = \left|m_{\rm min}\,c_{12}^2\,c_{13}^2 + \sqrt{\Delta m^2_\odot + m^2_{\rm min}}\,s_{12}^2\,c_{13}^2\,e^{i\alpha_{21}} + \sqrt{\Delta m^2_{\rm A} + m^2_{\rm min}}\,s_{13}^2\,e^{i\alpha'_{31}}\right|, \tag{5}$$

$$|\langle m\rangle|_{\rm IO} = \left|\sqrt{|\Delta m^2_{\rm A}| - \Delta m^2_\odot + m^2_{\rm min}}\,c_{12}^2\,c_{13}^2 + \sqrt{|\Delta m^2_{\rm A}| + m^2_{\rm min}}\,s_{12}^2\,c_{13}^2\,e^{i\alpha_{21}} + m_{\rm min}\,s_{13}^2\,e^{i\alpha'_{31}}\right|, \tag{6}$$

where we have defined $\alpha'_{31} \equiv \alpha_{31} - 2\delta$.

It proves useful to recast $|\langle m\rangle|_{\rm NO}$ and $|\langle m\rangle|_{\rm IO}$ given above in the form

$$|\langle m\rangle| = \left|\widetilde{m}_1 + \widetilde{m}_2\,e^{i\alpha_{21}} + \widetilde{m}_3\,e^{i\alpha'_{31}}\right|, \tag{7}$$

with $\widetilde{m}_i > 0$ ($i = 1, 2, 3$). It is then clear that the effective Majorana mass is the length of the vector sum of three vectors in the complex plane, whose relative orientations are given by the angles α_{21} and α'_{31}.

For the IO case, taking into account the 3σ ranges of Δm^2_{32}, Δm^2_{21}, $\sin^2\theta_{12}$, and $\sin^2\theta_{13}$ summarised in Table 1, one finds that there is a hierarchy between the lengths of the three vectors, $\widetilde{m}_3 < 0.1\,\widetilde{m}_2$ and $\widetilde{m}_2 < 0.6\,\widetilde{m}_1$, which holds for all values of $m_{\rm min}$. In particular, $\widetilde{m}_3 = m_{\rm min}\,s_{13}^2$ can be neglected with respect to the other terms since $s_{13}^2 \ll \cos 2\theta_{12}$.[2] The above implies that extremal values of $|\langle m\rangle|_{\rm IO}$ are obtained when the three vectors are aligned ($\alpha_{21} = \alpha'_{31} = 0$, $|\langle m\rangle|_{\rm IO}$ is maximal) or when $\widetilde{m}_1$ is anti-aligned with $\widetilde{m}_{2,3}$ ($\alpha_{21} = \alpha'_{31} = \pi$, $|\langle m\rangle|_{\rm IO}$ is minimal). It then follows that there is a lower bound on $|\langle m\rangle|_{\rm IO}$ for every value of $m_{\rm min}$ [10]. This bound reads: $|\langle m\rangle|_{\rm IO} \gtrsim$

[2] It follows from the current data that $\cos 2\theta_{12} > 0.30$ at 3σ CL.

Table 2
Ranges for the relevant oscillation parameters in the case of a NO neutrino spectrum, at the $n\sigma$ $(n = 1, 2, 3)$ CLs, taken from the global analysis of Ref. [13] (cf. Table 1).

Parameter	1σ range	2σ range	3σ range
Δm^2_{21} / (10^{-5} eV^2)	7.20–7.51	7.05–7.69	6.92–7.91
Δm^2_{31} / (10^{-3} eV^2)	2.46–2.53	2.43–2.56	2.39–2.59
$\sin^2\theta_{12}$ / 10^{-1}	2.91–3.18	2.78–3.32	2.65–3.46
$\sin^2\theta_{13}$ / 10^{-2}	2.07–2.23	1.98–2.31	1.90–2.39

$\sqrt{|\Delta m^2_{\rm A}| + m^2_{\rm min}}\, c^2_{13} \cos 2\theta_{12} > \sqrt{|\Delta m^2_{\rm A}|}\, c^2_{13} \cos 2\theta_{12}$, $|\langle m\rangle|_{\rm IO} > 1.4 \times 10^{-2}$ eV, for variations of oscillation parameters in their respective 3σ ranges. In the limit of negligible $m_{\rm min}$ (IH spectrum), $m^2_{\rm min} \ll |\Delta m^2_{\rm A}|$, one has $|\langle m\rangle|_{\rm IO} \in [1.4, 4.9] \times 10^{-2}$ eV.

Before proceeding to the analysis of the NO case, let us comment on present constraints on the absolute neutrino mass scale. The "conservative" upper limit of Eq. (4), $|\langle m\rangle|^{\rm max}_{\rm exp} = 0.165$ eV, which is in the range of the QD spectrum, implies, as it is not difficult to show, the following upper limit on the absolute Majorana neutrino mass scale (i.e., on the lightest neutrino mass): $m_{\rm min} \simeq m_{1,2,3} < 0.60$ eV, with $m_{\rm min} \lesssim |\langle m\rangle|^{\rm max}_{\rm exp}/(\cos 2\theta_{12} - s^2_{13})$, taking into account the 3σ ranges of $\cos 2\theta_{12}$ and $\sin^2\theta_{13}$. Measurements of the end-point electron spectrum in tritium beta decay experiments constrain the combination $m_\beta \equiv \sum_i |U_{ei}|^2 m_i$. The most stringent upper bounds on m_β, $m_\beta < 2.1$ eV and $m_\beta < 2.3$ eV, both at the 95% CL, are given by the Troitzk [17] and Mainz [18] collaborations, respectively. The KATRIN experiment [19] is planned to either improve this bound by an order of magnitude, or discover $m_\beta > 0.35$ eV. Taking into account the 3σ ranges for the relevant mixing angles and mass-squared differences, the Troitzk bound constrains the lightest neutrino mass to be $m_{\rm min} < 2.1$ eV. Cosmological and astrophysical data constrain instead the sum $\Sigma \equiv \sum_i m_i$. Depending on the likelihood function and data set used, the upper limit on Σ reported by the Planck collaboration [20] varies in the interval $\Sigma < [0.34, 0.72]$ eV, 95% CL. Including data on baryon acoustic oscillations lowers this bound to $\Sigma < 0.17$ eV, 95% CL. Taking into account the 3σ ranges for the mass-squared differences, this last bound implies $m_{\rm min} < 0.05\,(0.04)$ eV in the NO (IO) case. One should note that the Planck collaboration analysis is based on the ΛCDM cosmological model. The quoted bounds may not apply in nonstandard cosmological scenarios (see, e.g., [21]).

3. The case of normal ordering

We henceforth restrict our discussion to the effective Majorana mass $|\langle m\rangle|_{\rm NO}$, for which there is no lower bound. In fact, unlike in the IO case, here the ordering of the lengths of the $\widetilde{m}_i$ depends on the value of $m_{\rm min}$ and cancellations in $|\langle m\rangle|_{\rm NO}$ are possible: one risks "falling" inside the "well of unobservability".[3]

We summarise in Table 2 the $n\sigma$ $(n = 1, 2, 3)$ ranges for the oscillation parameters relevant to $(\beta\beta)_{0\nu}$-decay in the NO case, obtained in the recent global analysis of Ref. [13]. Considering variations of oscillation parameters in the corresponding 3σ ranges, for $m_{\rm min} \leq 5 \times 10^{-2}$ eV there is an upper bound $|\langle m\rangle|_{\rm NO} \leq 5.1 \times 10^{-2}$ eV (obtained for $\alpha_{21} = \alpha'_{31} = 0$). In the limit of negligible $m_{\rm min}$, $m^2_{\rm min} \ll |\Delta m^2_{\rm A}|$, one has $|\langle m\rangle|_{\rm NO} \in [0.9, 4.2] \times 10^{-3}$ eV.

From inspection of Eqs. (5) and (7), the vector lengths explicitly read $\widetilde{m}_1 = m_{\rm min}\, c^2_{12} c^2_{13}$, $\widetilde{m}_2 = \sqrt{\Delta m^2_\odot + m^2_{\rm min}}\, s^2_{12} c^2_{13}$, and

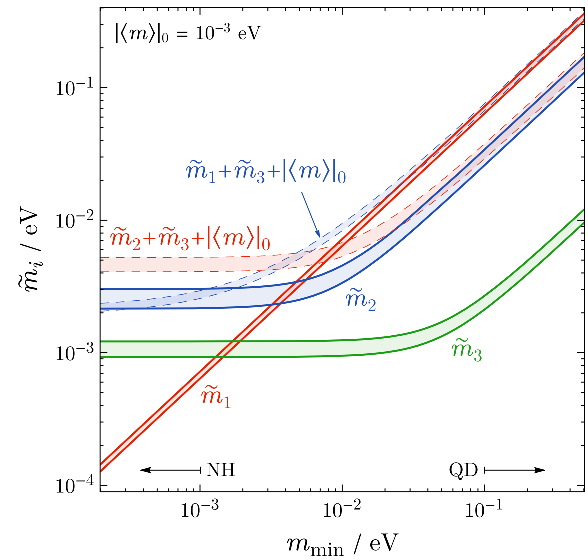

Fig. 1. Lengths $\widetilde{m}_i$ of the complex vectors entering the expression of $|\langle m\rangle|_{\rm NO}$ as a function of the lightest neutrino mass $m_{\rm min}$, for NO spectrum. For comparison, the sums $\widetilde{m}_1 + \widetilde{m}_3 + |\langle m\rangle|_0$ and $\widetilde{m}_2 + \widetilde{m}_3 + |\langle m\rangle|_0$ are also shown (see text), with $|\langle m\rangle|_0 = 10^{-3}$ eV. Bands are obtained by varying the mixing angles and mass-squared differences in their respective 3σ ranges (see Table 2). See text for details.

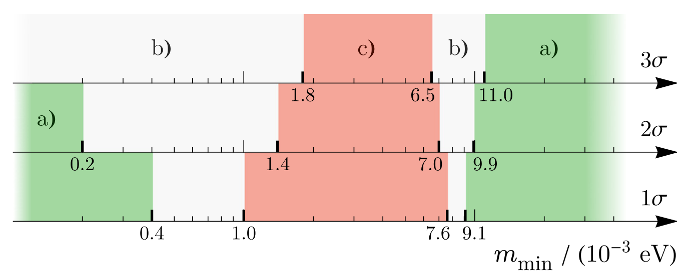

Fig. 2. Ranges of $m_{\rm min}$ for a NO spectrum and for oscillation parameters inside their $n\sigma$ $(n = 1, 2, 3)$ intervals (see Table 2) for which: in green, a) $|\langle m\rangle|_{\rm NO} > |\langle m\rangle|_0 = 10^{-3}$ eV for all values of θ_{ij}, Δm^2_{ij}, and $\alpha^{(\prime)}_{ij}$ from the corresponding allowed or defining intervals; in grey, b) there exist values of θ_{ij}, Δm^2_{ij} from the 1σ, 2σ and 3σ allowed intervals and values of $\alpha^{(\prime)}_{ij}$ such that $|\langle m\rangle|_{\rm NO} < |\langle m\rangle|_0 = 10^{-3}$ eV; and in red, c) for all values of θ_{ij} and Δm^2_{ij} from the corresponding allowed intervals there exist values of the phases α_{21} and α'_{31} for which $|\langle m\rangle|_{\rm NO} < |\langle m\rangle|_0 = 10^{-3}$ eV.

$\widetilde{m}_3 = \sqrt{\Delta m^2_{\rm A} + m^2_{\rm min}}\, s^2_{13}$. In Fig. 1, these lengths are plotted as functions of $m_{\rm min}$ for 3σ variations of oscillation parameters.

The requirement of having the effective Majorana mass above a reference value $|\langle m\rangle|_0$ is geometrically equivalent to not being able to form a quadrilateral with sides $\widetilde{m}_1$, $\widetilde{m}_2$, $\widetilde{m}_3$, and $|\langle m\rangle|_0$. This happens whenever one of the lengths exceeds the sum of the other three. If however $|\langle m\rangle|_0 > \sum_i \widetilde{m}_i$, it follows that $|\langle m\rangle| \leq \sum_i \widetilde{m}_i < |\langle m\rangle|_0$. Thus, for values of $m_{\rm min}$ and oscillation parameters for which $\widetilde{m}_2 > \widetilde{m}_1 + \widetilde{m}_3 + |\langle m\rangle|_0$ or $\widetilde{m}_1 > \widetilde{m}_2 + \widetilde{m}_3 + |\langle m\rangle|_0$ (see Fig. 1) one is guaranteed to have $|\langle m\rangle|_{\rm NO} > |\langle m\rangle|_0$ independently of the choice of CPV phases α_{21} and α'_{31}. There are instead values of $m_{\rm min}$ for which the conditions $\widetilde{m}_2 < \widetilde{m}_1 + \widetilde{m}_3 + |\langle m\rangle|_0$ and $\widetilde{m}_1 < \widetilde{m}_2 + \widetilde{m}_3 + |\langle m\rangle|_0$ hold independently of the values of oscillation parameters within a given range. In such a case, values of α_{21} and α'_{31} such that $|\langle m\rangle|_{\rm NO} < |\langle m\rangle|_0$ are sure to exist.

We summarise in Fig. 2 the ranges of $m_{\rm min}$ for which these different conditions apply (see caption). We vary oscillation parameters in their respective $n\sigma$ $(n = 1, 2, 3)$ intervals and focus on the millielectronvolt "threshold", $|\langle m\rangle|_0 = 10^{-3}$ eV. We find that, for 3σ variations of the $\sin^2\theta_{ij}$ and Δm^2_{ij}, one is guaranteed to have

[3] For the consequences of not observing $(\beta\beta)_{0\nu}$-decay with $|\langle m\rangle|_{\rm NO} \gtrsim 10^{-3}$ eV, see [22].

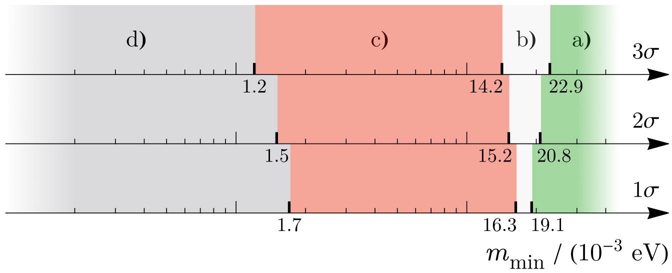

Fig. 3. The same as in Fig. 2, but for the reference value $|\langle m\rangle|_0 = 5 \times 10^{-3}$ eV: in green, a) $|\langle m\rangle|_{\rm NO} > |\langle m\rangle|_0 = 5 \times 10^{-3}$ eV for all values of θ_{ij}, Δm^2_{ij}, and $\alpha^{(\prime)}_{ij}$ from the corresponding allowed or defining intervals; in grey, b) there exist values of θ_{ij}, Δm^2_{ij} from the 1σ, 2σ and 3σ allowed intervals and values of $\alpha^{(\prime)}_{ij}$ such that $|\langle m\rangle|_{\rm NO} < |\langle m\rangle|_0 = 5 \times 10^{-3}$ eV; and in red, c) for all values of θ_{ij} and Δm^2_{ij} from the corresponding allowed intervals there exist values of the phases α_{21} and α'_{31} for which $|\langle m\rangle|_{\rm NO} < |\langle m\rangle|_0 = 5 \times 10^{-3}$ eV. In the darker grey ranges d) of $m_{\rm min}$, one has $|\langle m\rangle|_{\rm NO} < |\langle m\rangle|_0 = 5 \times 10^{-3}$ eV independently of the values of oscillation parameters and CPV phases.

$|\langle m\rangle|_{\rm NO} > 10^{-3}$ eV if $m_{\rm min} > 1.10 \times 10^{-2}$ eV. This corresponds to the lower bound $\Sigma > 0.07$ eV on the sum of neutrino masses. For 2σ variations, having $m_{\rm min} < 2 \times 10^{-4}$ eV or $m_{\rm min} > 9.9 \times 10^{-3}$ eV is enough to ensure $|\langle m\rangle|_{\rm NO} > 10^{-3}$ eV.

If one takes instead the higher value $|\langle m\rangle|_0 = 5 \times 10^{-3}$ eV and allows the relevant oscillation parameters to vary in their respective 3σ ranges, $|\langle m\rangle|_{\rm NO} > |\langle m\rangle|_0$ is guaranteed provided $m_{\rm min} > 2.3 \times 10^{-2}$ eV, which corresponds to the lower bound $\Sigma > 0.10$ eV on the sum of neutrino masses. This lower bound on Σ practically coincides with min(Σ) in the case of IO spectrum. Thus, if Σ is found to satisfy $\Sigma > 0.10$ eV, that would imply that $|\langle m\rangle|$ exceeds 5×10^{-3} eV, unless there exist additional contributions to the $(\beta\beta)_{0\nu}$-decay amplitude which cancel at least partially the contribution due to the 3 light neutrinos. If instead $m_{\rm min} < 1.4 \times 10^{-2}$ eV, for all (3σ allowed) values of oscillation parameters there is a choice of α_{21} and α'_{31} such that $|\langle m\rangle|_{\rm NO} < |\langle m\rangle|_0 = 5 \times 10^{-3}$ eV. These results are shown graphically in Fig. 3.

Let us briefly remark on the dependence of $|\langle m\rangle|_{\rm NO}$ on the CPV phases.[4] For the present discussion, 3σ variations of oscillation parameters are considered. For all values of α'_{31} and $\epsilon > 0$ there exist values of α_{21} and $m_{\rm min}$ such that $|\langle m\rangle|_{\rm NO} < \epsilon$, i.e. such that $|\langle m\rangle|_{\rm NO}$ is arbitrarily small. This is a consequence of the fact that, for any fixed oscillation parameters and α'_{31}, there is always a point $m^*_{\rm min}$ at which $|\widetilde{m}_1(m^*_{\rm min}) + \widetilde{m}_3(m^*_{\rm min})\, e^{i\alpha'_{31}}| = \widetilde{m}_2(m^*_{\rm min})$. Instead, there are values of α_{21} and $\epsilon > 0$ for which, independently of α'_{31} and $m_{\rm min}$, one has $|\langle m\rangle|_{\rm NO} > \epsilon$, i.e. for which $|\langle m\rangle|_{\rm NO}$ cannot be arbitrarily small. This conclusion may be anticipated from the graphical results of Ref. [28], where the structure of the $|\langle m\rangle|_{\rm NO}$ "well" has been studied as a function of $m_{\rm min}$ and α_{21}. In fact, we find that for $\alpha_{21} \lesssim 0.81\pi$ or $\alpha_{21} \gtrsim 1.19\pi$, $|\langle m\rangle|_{\rm NO}$ cannot be zero at tree-level since $|\widetilde{m}_1 + \widetilde{m}_2\, e^{i\alpha_{21}}| > \widetilde{m}_3$, strictly.

In Fig. 4 we highlight the region of the $(m_{\rm min}, \alpha_{21})$ plane in which $|\langle m\rangle|_{\rm NO}$ is guaranteed to satisfy $|\langle m\rangle|_{\rm NO} > 5 \times 10^{-3}$ eV, independently of α'_{31} and of variations of oscillation parameters inside their 3σ ranges.

4. CP and generalised CP

Given the strong dependence of $|\langle m\rangle|$ on α_{21} and α'_{31}, some principle which determines these phases are welcome. The re-

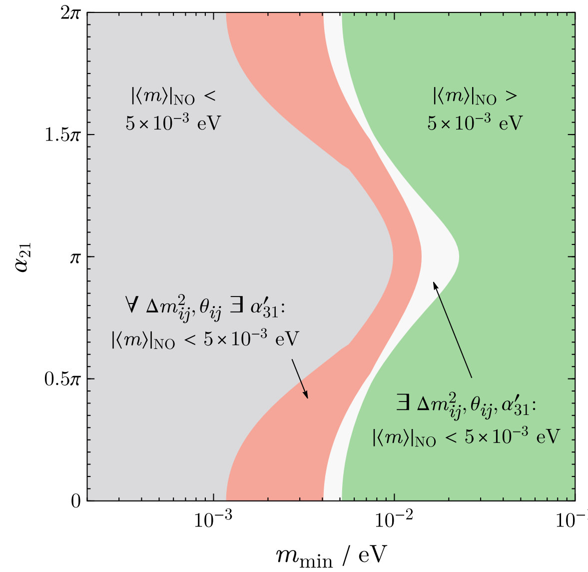

Fig. 4. Regions in the $(m_{\rm min}, \alpha_{21})$ plane where different conditions on $|\langle m\rangle|_{\rm NO}$ apply. In the green (dark grey) region, $|\langle m\rangle|_{\rm NO}$ satisfies $|\langle m\rangle|_{\rm NO} > 5 \times 10^{-3}$ eV ($|\langle m\rangle|_{\rm NO} < 5 \times 10^{-3}$ eV) for all values of θ_{ij}, Δm^2_{ij}, and α'_{13} from the corresponding 3σ or defining intervals. In the red and grey regions, conditions analogous to those described in the caption of Fig. 2 apply and are indicated. This figure is to be contrasted with Fig. 3, where the dependence on α_{21} is not explicit.

quirement of CP invariance constrains the values of the CPV phases α_{21}, α_{31}, and δ to integer multiples of π [29–31], meaning the relevant CP-conserving values are $\alpha_{21}, \alpha'_{31} = 0, \pi$. Non-trivial predictions for the leptonic CPV phases may instead arise from the breaking of a discrete symmetry combined with a generalised CP (gCP) symmetry. We focus on schemes with large enough residual symmetry such that the PMNS matrix depends at most on one real parameter θ [32] and realisations thereof where the predictions for the CPV phases are unambiguous, i.e. independent of θ. For symmetry groups with less than 100 elements, aside from the aforementioned CP-conserving values, the non-trivial values $\alpha_{21}, \alpha'_{31} = \pi/2, 3\pi/2$ are possible predictions [33–39].

In what follows, we analyse the behaviour of $|\langle m\rangle|_{\rm NO}$ and $|\langle m\rangle|_{\rm IO}$ for each of 16 different $(\alpha_{21}, \alpha'_{31})$ pairs, with the relevant phases taking gCP-compatible values: $\alpha_{21}, \alpha'_{31} \in \{0, \pi/2, \pi, 3\pi/2\}$.[5] As can be seen from Eq. (7), some pairs are redundant as they lead to the same values of $|\langle m\rangle|$. We are left with 10 inequivalent pairs of phases: $(\alpha_{21}, \alpha'_{31}) = (\pi/2, 0) \sim (3\pi/2, 0)$, $(\pi/2, \pi) \sim (3\pi/2, \pi)$, $(0, \pi/2) \sim (0, 3\pi/2)$, $(\pi, \pi/2) \sim (\pi, 3\pi/2)$, $(\pi/2, \pi/2) \sim (3\pi/2, 3\pi/2)$, and $(\pi/2, 3\pi/2) \sim (3\pi/2, \pi/2)$.

The 2σ-allowed values of the effective Majorana mass $|\langle m\rangle|$ are presented in Fig. 5 as a function of $m_{\rm min}$, for both orderings. Regions corresponding to different pairs $(\alpha_{21}, \alpha'_{31})$ with CP-conserving phases, $\alpha_{21}, \alpha'_{31} = 0, \pi$, are singled out. The predictions for the remaining pairs of fixed phases, containing at least one phase which is gCP-compatible but not CP-conserving, are shown in Fig. 6 for IO and in Figs. 7 and 8 for NO (for one CP-conserving phase and for no CP-conserving phases, respectively). Allowed values of $|\langle m\rangle|$ are found by constructing an approximate χ^2 function from the sum of the one-dimensional projections in Ref. [13], and varying mixing angles and mass-squared differences

[4] The Majorana phases α_{21} and α'_{31} here defined are related to parameterisation-independent charged-lepton rephasing invariant products $U^*_{ei}U_{ej}$ [23–27] through $\alpha_{21} = 2\arg U^*_{e1}U_{e2}$ and $\alpha'_{31} = 2\arg U^*_{e1}U_{e3}$.

[5] Given our scope and the available literature, we find that if $\alpha'_{31} = \pi/2, 3\pi/2$, then necessarily $\alpha_{21} = \pi/2, 3\pi/2$ is predicted. We nevertheless take all 16 pairs of phases into consideration.

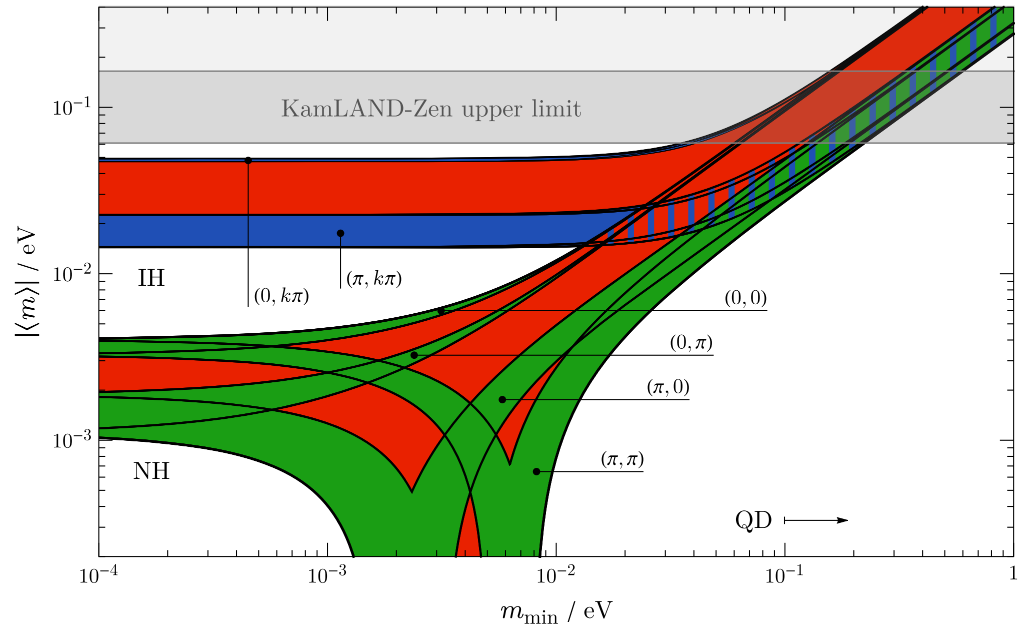

Fig. 5. The effective Majorana mass $|\langle m \rangle|$ as a function of m_{min}, for both orderings, allowing for variations of mixing angles and mass-squared differences at the 2σ CL (see text). The phases α_{21} and $\alpha'_{31} = \alpha_{31} - 2\delta$ are varied in the interval $[0, 2\pi]$. Blue and green bands correspond to (the indicated, with $k = 0, 1$) CP-conserving values of the phases $(\alpha_{21}, \alpha'_{31})$, for IO and NO neutrino mass spectra, respectively, while in red regions at least one of the phases takes a CP-violating value. Blue hatching is used to locate CP-conserving bands in the case of IO spectrum whenever IO and NO spectra regions overlap. The KamLAND-Zen bound of Eq. (4) is indicated. See also [1,14].

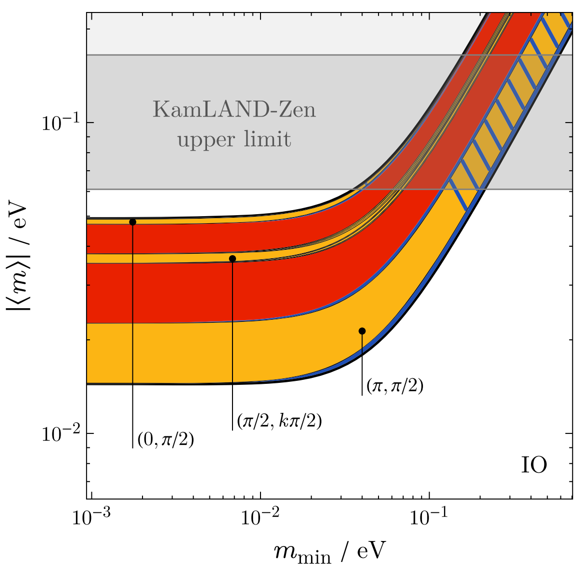

Fig. 6. The effective Majorana mass $|\langle m \rangle|$ as a function of m_{min}, for IO spectrum, allowing for variations of mixing angles and mass-squared differences at the 2σ CL (see text). Yellow bands correspond to (the indicated, $k = 0, 1, 2, 3$) gCP-compatible but not CP-conserving values of the phases $(\alpha_{21}, \alpha'_{31})$. Blue bands correspond to CP-conserving phases (see Fig. 5) and hatching indicates overlap with such regions, while red regions are not gCP-compatible (for the models under consideration, see text). The KamLAND-Zen bound of Eq. (4) is also indicated.

while keeping $\chi^2(\theta_{ij}, \Delta m^2_{ij}) \lesssim 9.72$ (2σ CL, for joint estimation of 4 parameters).

From Figs. 5–8, one sees that for each value of m_{min} there exist values of the effective Majorana mass which are incompatible with CP conservation. Some of these points may nonetheless be compatible with gCP-based predictive models. For IO, one sees there is substantial overlap between the bands with $(\alpha_{21}, \alpha'_{31}) = (0, 0)$ and $(0, \pi)$, between those of $(\pi, 0)$ and (π, π), and between the four bands $(\pi/2, k\pi/2)$, with $k = 0, 1, 2, 3$. In the case of NO, it is interesting to note that, for a fixed, gCP-compatible but not

Table 3
Lower bounds on $|\langle m \rangle|_{\text{NO}}$ given at the 3σ (2σ) CL, where applicable, for different fixed values of the phases α_{21} and α'_{31}. A tilde denotes equivalence between cases. All bounds are given in meV.

α_{21}	α'_{31}			
	0	$\pi/2$	π	$3\pi/2$
0	3.1 (3.3)	2.4 (2.4)	1.0 (1.1)	$\sim (0, \pi/2)$
$\pi/2$	2.4 (2.4)	3.1 (3.3)	2.1 (2.2)	$\sim (3\pi/2, \pi/2)$
π	no bound [a]	0.91 (0.95) [b]	no bound [c]	$\sim (\pi, \pi/2)$
$3\pi/2$	$\sim (\pi/2, 0)$	1.0 (1.1)	$\sim (\pi/2, \pi)$	$\sim (\pi/2, \pi/2)$

[a] $|\langle m \rangle|_{\text{NO}} > 1$ meV if $m_{\text{min}} > 5.8$ ($m_{\text{min}} \notin [0.1, 5.3]$) meV.
[b] Only bounded case where $|\langle m \rangle|_{\text{NO}}$ is not strictly at or above the meV value, for $m_{\text{min}} \in [2.9, 5.9]([3.2, 5.3])$ meV. $|\langle m \rangle|_{\text{NO}} > 1$ meV if e.g. $\sin^2\theta_{13} > 2.04 \times 10^{-2}$.
[c] $|\langle m \rangle|_{\text{NO}} > 1$ meV if $m_{\text{min}} \notin [3.1, 11.5]([3.4, 10.6])$ meV.

CP-conserving pair $(\alpha_{21}, \alpha'_{31})$, $|\langle m \rangle|_{\text{NO}}$ is bounded from below at the 2σ CL, with the lower bound at or above the meV value, $|\langle m \rangle|_{\text{NO}} \gtrsim 10^{-3}$ eV. We collect in Table 3 information on the lower bound on $|\langle m \rangle|_{\text{NO}}$ for each pair of phases.

5. Conclusions

The observation of $(\beta\beta)_{0\nu}$-decay would allow to establish lepton number violation and the Majorana nature of neutrinos. In the standard scenario of three light neutrino exchange dominance, the rate of this process is controlled by the effective Majorana mass $|\langle m \rangle|$. In the case of neutrino mass spectrum with inverted ordering (IO) the effective Majorana mass is bounded from below, $|\langle m \rangle|_{\text{IO}} > 1.4 \times 10^{-2}$ eV, where this lower bound is obtained using the current 3σ allowed ranges of the relevant neutrino oscillation parameters – the solar and reactor neutrino mixing angles θ_{12} and θ_{13}, and the two neutrino mass-squared differences Δm^2_{21} and Δm^2_{23}. In the NO case, the effective Majorana mass $|\langle m \rangle|_{\text{NO}}$, under certain conditions, can be exceedingly small, $|\langle m \rangle|_{\text{NO}} \ll 10^{-2}$ eV, suppressing the $(\beta\beta)_{0\nu}$-decay rate.

Currently taking data and next-generation $(\beta\beta)_{0\nu}$-decay experiments seek to probe and possibly cover the IO region of parameter space, working towards the $|\langle m \rangle| \sim 10^{-2}$ eV frontier. In case these

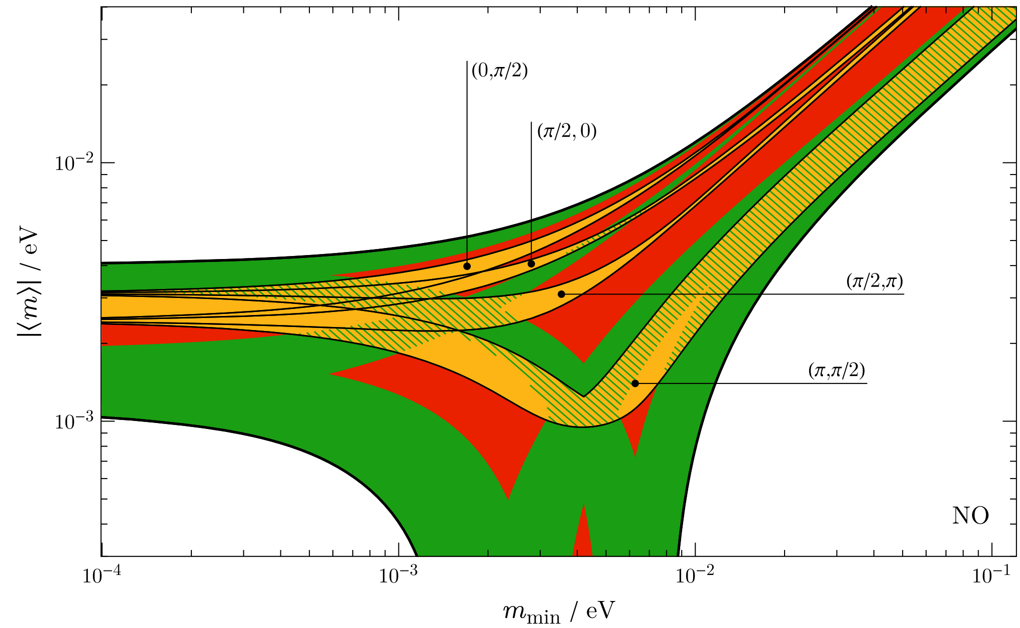

Fig. 7. The effective Majorana mass $|\langle m\rangle|$ as a function of $m_{\rm min}$, for NO spectrum, allowing for variations of mixing angles and mass-squared differences at the 2σ CL (see text). Yellow bands correspond to (the indicated) pairs $(\alpha_{21}, \alpha'_{31})$ of phases, with one CP-conserving, the other being gCP-compatible but not CP-conserving. Green bands correspond to CP-conserving phases (see Fig. 5) and hatching indicates overlap with such regions, while red regions are not gCP-compatible (for the models under consideration, see text).

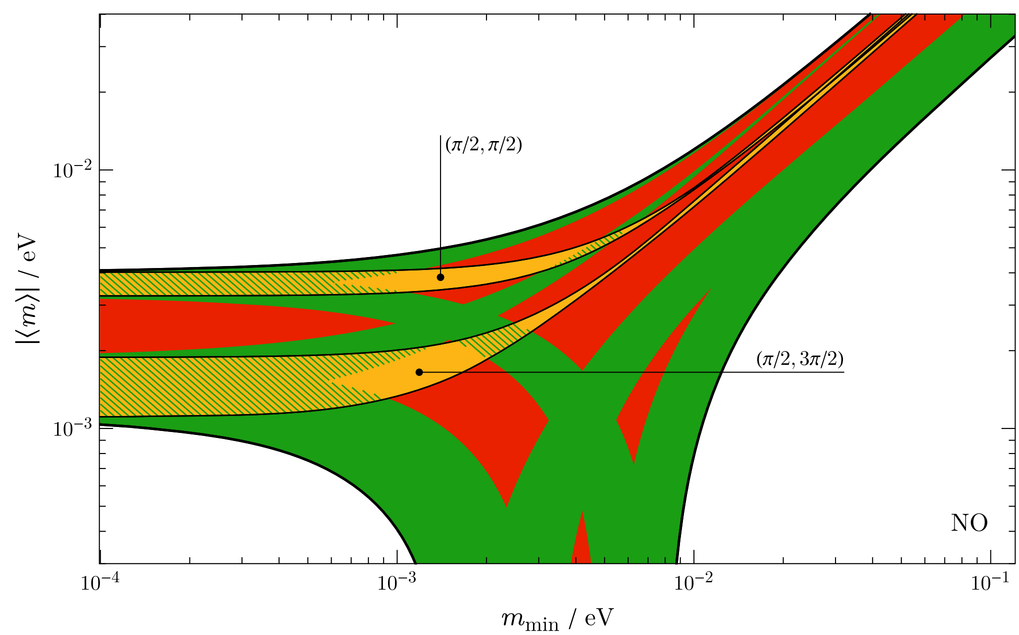

Fig. 8. The same as in Fig. 7, with yellow bands corresponding to pairs $(\alpha_{21}, \alpha'_{31})$ with both phases being gCP-compatible but not CP-conserving.

searches produce a negative result, the next frontier in the quest for $(\beta\beta)_{0\nu}$-decay will correspond to $|\langle m\rangle| \sim 10^{-3}$ eV.

Taking into account updated global-fit data on the 3-neutrino mixing angles and the neutrino mass-squared differences, we have determined the conditions under which the effective Majorana mass in the NO case $|\langle m\rangle|_{\rm NO}$ exceeds the 10^{-3} eV (5×10^{-3} eV) value. The effective Majorana mass $|\langle m\rangle|_{\rm NO}$ of interest, as is well known, depends on the solar and reactor neutrino mixing angles θ_{12} and θ_{13}, on the two neutrino mass-squared differences Δm^2_{21} and Δm^2_{31}, on the lightest neutrino mass $m_{\rm min}$ as well as on the CPV Majorana phase α_{21} and on the Majorana–Dirac phase difference $\alpha'_{31} = \alpha_{31} - 2\delta$. For variations of θ_{12}, θ_{13}, Δm^2_{21} and Δm^2_{31} in their $n\sigma$ $(n = 1, 2, 3)$ intervals, we have determined the ranges of the lightest neutrino mass $m_{\rm min}$ such that (see Figs. 2–4):

a) $|\langle m\rangle|_{\rm NO} > 10^{-3}$ (5×10^{-3}) eV independently of the values of α_{21} and α'_{31}; $|\langle m\rangle|_{\rm NO} > 5 \times 10^{-3}$ eV is fulfilled when $m_{\rm min} > 2.3 \times 10^{-2}$ eV (for 3σ variations),
b) for some values of the θ_{ij} and Δm^2_{ij} there are choices of the CPV phases α_{21} and α'_{31} such that $|\langle m\rangle|_{\rm NO} < 10^{-3}$ (5×10^{-3}) eV,

c) for all values of the θ_{ij} and Δm^2_{ij} there are choices of the CPV phases α_{21} and α'_{31} such that $|\langle m\rangle|_{\rm NO} < 10^{-3}$ (5×10^{-3}) eV, and
d) $|\langle m\rangle|_{\rm NO} < 5 \times 10^{-3}$ eV independently of the values of α_{21} and α'_{31}.

We have shown, in particular, that if the sum of the three neutrino masses is found to satisfy the lower bound $\Sigma > 0.10$ eV, that would imply in the case of NO neutrino mass spectrum $|\langle m\rangle|_{\rm NO} > 5 \times 10^{-3}$ eV for any values of the CPV phases α_{21} and α'_{31}, unless there exist additional contributions to the $(\beta\beta)_{0\nu}$-decay amplitude which cancel at least partially the contribution due to the 3 light neutrinos.

We have additionally studied the predictions for $|\langle m\rangle|_{\rm IO}$ and $|\langle m\rangle|_{\rm NO}$ in cases where the leptonic CPV phases are fixed to particular values, α_{21}, $\alpha_{31} - 2\delta \in \{0, \pi/2, \pi, 3\pi/2\}$, which are either CP conserving (see Fig. 5) or may arise in predictive schemes combining generalised CP and flavour symmetries (see Figs. 6–8, lower bounds on the effective mass $|\langle m\rangle|_{\rm NO}$ for such choices of phases are given in Table 3). We find that $|\langle m\rangle|_{\rm NO} \gtrsim 10^{-3}$ eV for all gCP-compatible but not CP-conserving pairs of the relevant phases.

The searches for lepton number non-conservation performed by the neutrinoless double beta decay experiments are part of the searches for new physics beyond that predicted by the Standard Theory. They are of fundamental importance – as important as the searches for baryon number non-conservation in the form of, e.g., proton decay. Therefore if current and next-generation $(\beta\beta)_{0\nu}$-decay experiments seeking to probe the IO region of parameter space produce a negative result, the quest for $(\beta\beta)_{0\nu}$-decay should continue towards the $|\langle m\rangle| \sim 5 \times 10^{-3}$ eV and possibly the $|\langle m\rangle| \sim 10^{-3}$ eV frontier.

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