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Mass modelling from rotation curves.

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We construct a generic mass model of a disk galaxy by combining two mass components, an exponential thin stellar disk and a pseudo-isothermal dark halo. We construct its generic rotation curve, with two free parameters linked to its shape and one to its amplitude, namely the halo core radius, the fractional amount of Dark Matter and the total mass at one disk scale lenght.
We conduct a statystical analysis of the fitting procedure performed with the model on a family of rotation curves, and we conclude that an observed rotation curve fulfilling well defined quality requirements can be uniquely and properly decomposed in its dark and luminous components; through a suitable mass modelling together with a maximum-likelihood fitting method, in fact, the disentangling of the mass components is accomplished with a very high resolution.

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The rotation curves of spiral galaxies are determined by the mass distribution, including its luminous component (the disk) and its dark component (the halo). While the radial distribution of the luminous matter is observed for spiral disks, the disk mass itself is unknown. Regarding the Dark Matter, both the amount and the distribution are unknown, and since these physical quantities affect the rotation velocity profile in a different way at different radii, the study of the rotation curves becomes a very important tool to investigate the structure of galaxies.
But how well does a rotation curve yield the true mass model of a galaxy? Moreover, is the information we obtain from the observational data unique, and what is the resolution of our result? In other words, can we distinguish between different Dark Matter amounts and distributions by analyzing a rotation curve?
We model the rotation curves of a galaxy from the distribution of the luminous and Dark Matter. We assume a fixed mass density distribution of the Dark Matter, *i.e.* a pseudo-isothermal (PI) halo, which agrees with a number of observations ([6]; notice that the Universal Rotation Curve and the individual RCs are well fitted by a $PI + disk$ model, [1]; [3]), and we investigate the capability of the model to fit simulated "observed" rotation curves. In section (1) we present the mass model, while in section (2) we investigate the possibility of uniquely deriving, from an high-quality rotation curve, the mass distribution. In section (3) we finally conclude.

1. The model

The total mass of a disk galaxy can be divided into two independent components, namely the luminous disk and the Dark Matter halo (neglecting the bulge is irrelevant in our study). These components contribute separately to the total rotation velocity:

$$V^2(\tilde{r}) = V_d^2(\tilde{r}) + V_h^2(\tilde{r}) \tag{1.1}$$

For the luminous matter, we consider the stellar disk, whose surface density profile is found as:

$$\Sigma(r) = \Sigma_0 e^{-R/R_D} \tag{1.2}$$

where Σ_0 is the central surface density, and R_D is the disk scale lenght. The disk component of the velocity is [2]:

$$V_d^2(\tilde{r}) = \frac{GM_D}{R_D}\,\nu(\tilde{r}) \tag{1.3}$$

in which M_D is the disk mass and $\nu(\tilde{r}) = \tilde{r}^2/2 \cdot (I_0K_0 - I_1K_1)|_{\tilde{r}/2}$, with I_0K_0 and I_1K_1 being the Bessel functions of order 0 and 1, evaluated at $\tilde{r}/2$. The contribution of the luminous matter to the total rotation velocity is determined by two parameters: the disk scale lenght R_D, measured in each galaxy, and the total mass M_D, that can be obtained, at least in principle, from the disk luminosity, but since its determination is affected by large uncertainties, it is usually considered as an unknown quantity.
The DM density distribution of *pseudo-isothermal* (PI) halos is given by:

$$\rho(r) = \frac{\rho_0}{1 + (r/R_C)^2} \tag{1.4}$$

For convenience we define: $\tilde{r} \equiv r/R_D$ as the galactocentric distance, and $\tilde{R}_C \equiv R_C/R_D$ as the core radius of the halo, both in units of the disk scale lenght. We take $\tilde{R}_C$ as a free parameter and we consider its possible variation range between 0 and infinity. $\tilde{R}_C < 1$ implies a mass distribution very similar to that of the NFW profile (see subsection 2.2). On the other hand, cases with $\tilde{R}_C > 3$ are scarcely distinguishable from one another, since most of the observational data do not reach regions of disks beyond $\tilde{r} = 3$.
The halo velocity component is:

$$V_h^2(\tilde{r}) = \frac{GM_H(\tilde{r})}{\tilde{r}\,R_D} \tag{1.5}$$

and it has two free parameters, the central halo density and the halo characteristic scale lenght, that fully determine the DM distribution. The mass of the dark halo as a function of the normalized radius is obtained as: $M_H(\tilde{r}) = 4\pi G\rho_0\, \tilde{R}_C^3 R_D^3 [\tilde{r}/\tilde{R}_C - tan^{-1}(\tilde{r}/\tilde{R}_C)]$, and we define: $\lambda(\tilde{r}) \equiv (\tilde{R}_C^3/\tilde{r})\, [\tilde{r}/\tilde{R}_C - tan^{-1}(\tilde{r}/\tilde{R}_C)]$. It is useful to perform the following transformation:

$$\alpha \equiv \frac{GM_D}{R_D\, V_1^2}, \qquad \beta \equiv \frac{4\pi G\rho_0\, R_D^3}{R_D\, V_1^2}, \qquad \lambda_1 \equiv \tilde{R}_C^3 \left[\frac{1}{\tilde{R}_C} - tan^{-1}\left(\frac{1}{\tilde{R}_C}\right)\right]$$

The parameters α and β are directly proportional to the fraction, at R_D, of the luminous and dark mass respectively. We define $V_1^2 \equiv V^2(\tilde{r}=1))$. The radial profile of the normalized rotation velocity is then given by:

$$\frac{V^2(\tilde{r})}{V_1^2} = \alpha\, \nu(\tilde{r}) + \beta\, \lambda(\tilde{r}) \tag{1.6}$$

α, β and λ determine the shape of the curve, while V_1^2 determines its amplitude. Let's stress that, as concerning the results of this section, the actual value of the constant V_1 is irrelevant since, as a normalization parameter, it doesn't affect the RC shape. From (1.6) it follows that: $\alpha\, \nu_1 + \beta\, \lambda_1 = 1$, where $\nu_1 = \nu(\tilde{r}=1)$. We notice that $\alpha\nu_1$ and $\beta\lambda_1$ represent the fractional contributions to the total (normalized) velocity at R_D due to the luminous and Dark Matter respectively; for simplicity we define $f_{DM} \equiv \beta\lambda_1$.
We now rewrite the radial profile of the normalizated rotation velocity as

$$\frac{V^2(\tilde{r})}{V_1^2} = (1 - f_{DM})\, \frac{\nu(\tilde{r})}{\nu_1} + f_{DM}\, \frac{\lambda(\tilde{r})}{\lambda_1} \tag{1.7}$$

under the following ranges of variations: $0 \le f_{DM} \le 1$ and $0 \le \tilde{R}_C \le 3$. In this way, we split the original three free parameters into a 2+1 combination: 2 determine the RC profile and 1 sets the RC amplitude. We allow the free parameters related with the profile to vary over a very wide range of values, and we plot the corresponding curves as functions of radius $\tilde{r}$.

2. Disentangling a high-quality RC

We now estimate the precision with which we can resolve the luminous and the Dark Matter distribution from the rotation curve. We build a family of 25 reference curves that simulate "observed" RCs of real galaxies, each one consisting of 25 data points, evenly spread between 0 and 4 disk scale lenghts, and is characterized by a set of two parameters in the ensemble: $f_{DM} = \{0.1, 0.3, 0.5, 0.7, 0.9\} \otimes \tilde{R}_C = \{0.1, 1.0, 2.0, 3.0, 4.0\}$. To each point of these curves we assign "observational" errors of the order to those of high-quality published RCs [5]: $\varepsilon_V = 0.02$ is the averaged uncertainty in $V(\tilde{r})$, and $\varepsilon_D = 0.05$ in the RC slope.
We investigate each of the "observed" RCs with a mass model, fitting the data to obtain their structural parameters. Regarding the parameter V_1, if the observed RC is of high quality, then it's possible to set its value directly from the data. In this case, $V_1 \equiv V(1) \pm O(10^{-2})$. Let us caution that this cannot always be done: we define a high-quality RC as a set of kinematical data for which such a parameter reduction is possible. Therefore, the reference "observed" RCs are fitted with the following mass model (see eq. 1.7):

$$V_{mod}^2(\tilde{r}) = V^2(1)\, [(1 - f_{DM})\, \gamma_L(\tilde{r}) + f_{DM}\, \gamma_{DM}(\tilde{r}; \tilde{R}_C)] \tag{2.1}$$

where $\gamma_L(\tilde{r}) \equiv \nu(\tilde{r})/\nu_1$ is known, and $\gamma_{DM}(\tilde{r}; \tilde{R}_C) \equiv \lambda(\tilde{r}; \tilde{R}_C)/\lambda_1(\tilde{R}_C)$.
The measure of the distance between a model curve and an "observed" RC is given by a likelihood parameter, that we define as the sum of the χ^2 computed on the velocity and on the RC slope:

$$\chi^2_{TOT} = \chi^2[V] + \chi^2[D] \tag{2.2}$$

The need of considering the contribution of the gradient of the velocity in the computation of the likelihood parameter is well explained by the examples shown in figure (1). In the top panels we show the value of the χ^2_V only, as a function of the values of the model parameters, for three different reference curves. The 1σ

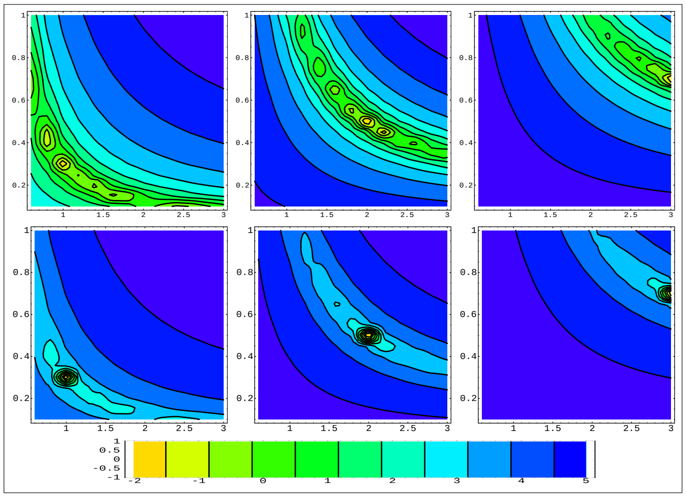

Figure 1: Top panels: $\chi^2(V)$ for 3 reference curves, as a function of the model parameters: $\tilde{R}_C \in [0.2, 4]$ (x-axis) and $f_{DM} \in [0.05, 1]$ (y-axis). The "true" parameters are: ($\tilde{R}_C = 1.0, f_{DM} = 0.3$) (left), ($\tilde{R}_C = 2, f_{DM} = 0.5$) (middle), ($\tilde{R}_C = 3, f_{DM} = 0.7$) (right). Bottom panels: χ^2_{TOT} for the same cases.

contour of χ^2_V includes also values of the parameters far from the "true" ones, unveiling a certain inability of the model to disentangle the mass components. With the information given by the gradient of the circular velocity and by the χ^2_{TOT} as defined in equation (2.2), the degeneracy of the different model curves largely disappears. In the bottom panels we show the resolution reached now: we find that the χ^2_{TOT} rises sharply to a value of $\sim 10^2$ in an interval of $\Delta f_{DM} \sim 0.05$ and of $\Delta \tilde{R}_C < 0.25$.

For each one of the 25 "observed" reference curves, we perform the fit with the model given by equation (2.1), considering a very large range for the values of its parameters. We quantify the "success" of the fit by the computation of χ^2_{TOT} for each model curve considered. We now perform a general analysis of the behaviour of χ^2_{TOT}, computed for 10.000 couples *reference "observed" RC - model curve*. It is worth to consider χ^2_{TOT} as a function of the following suitable "distance" in the paramater space: $D \equiv (\Delta^2(\tilde{R}_C/4) + \Delta^2(f_{DM}))^{1/2}$, with $\Delta(x) = x_{mod} - x_{obs}$ (notice that, since the range of variation of R_C is four times that of f_{DM}, we normalize the former by a factor of 4 to make the two contributions to D comparable). The result is shown in figure (2a). The red circles represent cases in which the distance is dominated by the variation of the core radius ($\Delta^2(\tilde{R}_C/4) > \Delta^2(f_{DM})$), the blue triangles cases in which the distance is mainly due to the variation of the amount of Dark Matter. The vertical line defines a distance in the parameter space of $D = 0.25$. The points in the range $D \leq 0.25$ and laying below the 3σ limit, although representing cases in which the model fails in fitting uniquely a RC, can be considered irrelevant and will not be counted in the statystics: in fact, the parameters defining the curves are so close to each other, that even if a RC has multiple fits, they differ one from the other by a negligible amount.

The horizontal lines represent the limit value of χ^2_{TOT} corresponding to 3σ and 1σ. Most of the points lay over the 3σ limit, indicating that the two curves of each couple are well resolved and therefore distinguishable. In the interval $1-3\sigma$ we find about 0.9% of the total cases, and below the 1σ limit about 0.23%. The cases below the 1σ limit have to be considered as failures of the model in uniquely distinguishing the Dark Matter distribution, and those below the 3σ limit as partial failures: therefore, studying our simulated "observed" RCs we find that there is a total possibility of only about 1% not to resolve (completely or partially) the mass model, with the adopted ε_V, ε_D: these errors are typical of about the top 50% of the RCs observed today, for which, through the application of this procedure, there are very good possibilities to disentangle the mass distribution. However, it is possible to obtain an observed RC with an error even smaller than the

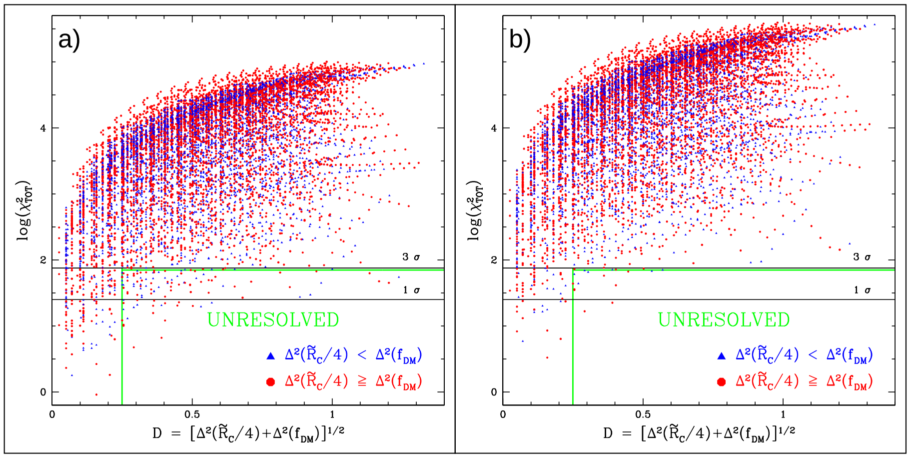

Figure 2: χ^2_{TOT} of the 25 reference "observed" curves mapped with 400 model curves, as a function of the distance in the parameter space. Red points (circles) are for distances dominated by the variation of $\tilde{R}_C$, blue points (triangles) are for distances dominated by the variation of f_{DM}. The straight lines represent the 1σ and 3σ limits. Panel *a)* $\varepsilon_V = 0.02$ and $\varepsilon_D = 0.05$. Panel *b)* $\varepsilon = 0.01$ and $\varepsilon_D = 0.03$.

one we adopted: for about the top 10% of the RCs available today we can consider observational errors of $\varepsilon_V = 0.01$ and $\varepsilon_D = 0.03$, and the number of these curves is going to increase in the future. The same analysis performed with these errors yields a χ^2_{TOT} *vs* D relation as shown in (2b). Now we have only about the 0.15% of cases in the interval $1-3\sigma$, and only the 0.02% is below 1σ; the worst case among these sets the lower limit for the resolution of our model: $(\Delta(f_{DM}))_{MAX} = 0.25$ and $(\Delta R_C)_{MAX} = 1.2 R_D$, which is still a reasonable uncertainty. Therefore, considered the very small amount of cases below the 3σ limit, we can state that the average resolution that we reach corresponds to the step with which we select the model parameters, *i.e.* $(\Delta(f_{DM})) = 0.05$ and $(\Delta R_C) = 0.2 R_D$.

3. Conclusions

We performed a mass modelling of the rotation curve of a spiral galaxy, representing it as a function of two parameters that govern its shape and one that governs its amplitude, namely the dark halo core radius, the Dark Matter contribution to the total rotation velocity and the total velocity at one disk scale lenght. Under the hypothesis of investigating a high-quality RC, we show that the model is able to disentangle the mass components of the galaxy and to determine the DM distribution.

Our results refer to the adopted halo profile, without any attempt to investigate whether this is the actual profile of DM in spirals. Nevertheless, the same analysis could be conducted with the NFW halo [4], for instance, and in this case the same results would apply, since the NFW profile, having one free parameter less than the PI case and featuring a known radial behaviour of the velocity profile, is easier to reconstruct from a rotation curve.

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