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On black hole temperature in Horndeski gravity

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ABSTRACT

It has been observed that for black holes in certain family of Horndeski gravity theories Wald's entropy formula does not lead to the correct first law for black hole thermodynamics. For this family of Horndeski theories speeds of propagation of gravitons and photons are in general different and gravitons move on an effective metric different than the one seen by photons. We show that the temperature of the black hole should be modified from surface gravity over 2π to include effects of this effective metric. The modified temperature, with the entropy unambiguously computed by the solution phase space method, yields the correct first law. Our results have far reaching implications for the Hawking radiation and species problem, going beyond the Horndeski theories.

Theories of beyond Einstein general relativity, with either theoretical [1] or dark energy or dark matter and cosmological model building motivations [2–8], have been very extensively discussed in the literature. These theories while generally covariant, typically have fourth order field equations and hence have propagating ghost degrees of freedom and are pathological. There are, nonetheless, special classes of theories which are ghost free, like the so-called $f(R)$ theories [9], Lovelock theories [10] and the class of scalar-tensor theories first formulated and classified by Horndeski [11]. In the last two decades, Horndeski family has also been extended further [12–16].

Black holes are ubiquitous solutions to generally covariant gravity theories, Einstein gravity and beyond, and recent observations of gravity-waves [17–19] have put them at forefront of fundamental physics research [4–6]. Theoretically, black hole are typically specified by having an event horizon, yet to be confirmed observationally. At the theoretical level, once quantum effects are also taken into account, black holes behave as a thermodynamical system with an entropy and temperature associated with the horizon and satisfy laws of thermodynamics [20].

In the "standard picture", the temperature $T_{\rm H}$ for black holes with a Killing horizon, is given by the surface gravity κ at the horizon as $T_{\rm H} = \kappa/(2\pi)$. This is the same temperature as the blackbody radiation emitted by the black hole, Hawking radiation [21]. The black hole entropy $S_{\rm BH}$, on the other hand, for Einstein gravity theory is given by Bekenstein-Hawking area law [22], $S_{\rm BH} = A_{\rm H}/(4G_{\rm N})$, where $A_{\rm H}$ is area of horizon and $G_{\rm N}$ is the Newton constant. The other black hole charges appearing in the first law of black hole thermodynamics, like mass, angular momentum and the electric charge, are then typically computed in the asymptotic region and defined e.g. by the ADM method or its extensions [23,24].

Thermodynamic description of black holes is quite universal and applies also to black holes in beyond Einstein gravity; as shown in two seminal papers [25,26] they are a result of general invariance of the theory. Black hole entropy, however, depends on the theory and in general is not given by the area law. Despite its elegance and very wide success, it has been observed that Wald's entropy formula [25] and/or the first law does not work for a family of solutions to certain Horndeski theories, e.g. see [27]. This is the problem we will address in this work.

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A similar violation of the first law was reported for some black holes in Einstein-dilaton theories which have a scalar field ϕ with shift symmetry [28] and a similar suggestion was put forward: One may introduce a new term in the first law, associating a chemical potential and an ad hoc conserved charge to the scalar field [27,28]. This proposal, while fixing the issue with the first law, has the problem that the (Noether) conserved charge associated with the scalar shift symmetry is zero and the charge associated with the scalar is "ad hoc". Moreover, for Einstein-dilaton theory black holes, this is in contrast with the no-hair theorems and the absence of independent conserved charges associated with such scalar fields. Indeed, this proposal was refuted by proving that the dilaton moduli are redundant parameters and that there cannot be a term associated with variation of asymptotic value scalar fields in the first law [29].

To tackle the problem with Horndeski black holes we carefully revisit Wald's derivation of the first law and his definition of the entropy. Wald's entropy is based on the standard Noether method and postulates $\kappa/(2\pi)$ as the black hole temperature. Noether's theorem and Wald formula have ambiguities which should be carefully dealt with. As we will briefly discuss, these ambiguities do not all vanish in the case of Horndeski theories. Fortunately, there is another method for calculation of charges (called covariant formulation [30–32]) which is free of the ambiguity in Wald's formula. To put this method into its full computational power, we use its version introduced in [33,34], the solution phase space method (SPSM).

To read the entropy in SPSM we need to provide the surface gravity and/or the black hole temperature. The key point of the current Letter comes from the fact that in Horndeski theories gravitons do not move with the speed of light [6,35]; they effectively propagate on a spacetime which is especially different than the black hole metric close to the horizon. Therefore, they feel a different surface gravity and hence a temperature different than the usual Hawking temperature $\kappa/(2\pi)$. This modified temperature, together with the correspondingly defined entropy, results in the correct first law.

We first introduce Horndeski theories and the formulae for speed of gravitons in them and show why Wald entropy does not in general work for Horndeski black holes. We then very briefly review the solution phase space method (SPSM) [34] (which is based on covariant phase space formulation of charges [30–32]) for computing the entropy in generally covariant theories and apply it to Horndeski black holes. The SPSM takes surface gravity for gravitons as an input to compute the entropy. We provide this input through effective near horizon metric as seen by gravitons and verify in two examples how our modified temperature restores the first law for the Horndeski black holes. Finally, we discuss the deep implications our analysis and results can have for better understanding of Hawking radiation and black hole dynamics.

1. Review of Horndeski gravity

Horndeski theories are a class of scalar-tensor theories with the action [11,16,36–38]

$$S_{\text{Horn.}} = \frac{1}{16\pi G_N}\int d^n x\sqrt{-g}\,\mathcal{L}_{\text{Horn.}} \tag{1}$$

$$\begin{aligned}\mathcal{L}_{\text{Horn.}} &= \mathcal{G}_2(\phi,\mathcal{X}) - \mathcal{G}_3(\phi,\mathcal{X})\Box\phi + \mathcal{G}_4(\phi,\mathcal{X})R\\ &+ \mathcal{G}_4'(\phi,\mathcal{X})\left((\Box\phi)^2-(\partial_{\mu\nu}\phi)^2\right) - \mathcal{G}_5(\phi,\mathcal{X})G^{\mu\nu}\partial_{\mu\nu}\phi\\ &- \frac{\mathcal{G}_5'(\phi,\mathcal{X})}{6}\left((\Box\phi)^3+2(\partial_{\mu\nu}\phi)^3-3\Box\phi(\partial_{\mu\nu}\phi)^2\right)\end{aligned} \tag{2}$$

where $g_{\mu\nu}$ is the spacetime metric, R is Ricci scalar, $G_{\mu\nu}$ is the Einstein tensor, $\partial_{\mu\nu}\phi = \nabla_\mu\nabla_\nu\phi$, $\Box\phi = g^{\mu\nu}\partial_{\mu\nu}\phi$, $\mathcal{X} := -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ and $\mathcal{G}_i' = d\mathcal{G}_i/d\mathcal{X}$. We are adopting the conventions that $\mathcal{G}_4(\phi=0,\mathcal{X}=0)=1$. This is how we define the Newton constant G_N.

For our analysis below we restrict ourselves to a large class of models with $\mathcal{G}_4, \mathcal{G}_5$ whose Lagrangian up to some total derivatives, takes the form [37]

$$\mathcal{L}_{\text{Horn.}} = \mathcal{G}_2 + (\mathcal{G}-\mathcal{G}'\mathcal{X})R + \mathcal{G}'G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \tag{3}$$

In our analysis below we assume $\mathcal{G}'\neq 0$. For $\mathcal{G}'=0$ cases we recover the usual Brans-Dicke type theory which is not the subject of our analysis here, as Wald entropy formula works for them properly.

Let us consider the "$\phi+3$" decomposition of the Horndeski Lagrangian (3) along the constant ϕ surfaces by taking

$$g_{\mu\nu} = h_{\mu\nu} + \sigma\phi_\mu\phi_\nu, \qquad \phi_\mu := \frac{\partial_\mu\phi}{|\partial\phi|}, \tag{4}$$

σ is sign of $\phi_\mu\phi^\mu$, it is -1 for cosmological backgrounds and $+1$ for black holes, and $h_{\mu\nu}$ is the metric along constant ϕ surface, $h_{\mu\nu}\phi^\nu=0$. The details of the analysis may be found in [37] and the result for the "$\phi+3$" decomposed Lagrangian is

$$\begin{aligned}\mathcal{L} &= \mathcal{G}_2 + \mathcal{G}\,{}^{(3)}R + (\mathcal{G}-2\mathcal{X}\mathcal{G}')(K_{\mu\nu}K^{\mu\nu}-K^2)\\ &+ 2\sqrt{-2\mathcal{X}}\mathcal{G}_{,\phi}K + \text{total derivative terms}\end{aligned} \tag{5}$$

where ${}^{(3)}R$ is the scalar curvature of $h_{\mu\nu}$, K is the extrinsic curvature of our constant ϕ surfaces, $K_{\mu\nu} = h_\mu^\alpha\nabla_\alpha\phi_\nu$, $K = K^\mu{}_\mu$ and $\mathcal{G}_{,\phi} = d\mathcal{G}/d\phi$.

2. Speed of gravitons on black hole backgrounds

To compute the speed of gravitons, one should systematically study linearized field equations around a given background, in our case a black hole. While this can be done, see e.g. [39,40], the above "$\phi+3$" decomposition provides a shortcut.

For a black hole ϕ_μ is typically along the "radial direction" and is normal to the horizon and, the time direction is in the "3" part, along $h_{\mu\nu}$ metric and normal to ϕ_μ. From (5) one may then directly read the speed of gravitons which is now direction dependent and for the case of black holes[1] is [40]:

$$c_g^2 = \begin{cases}\frac{\mathcal{G}-2\mathcal{X}\mathcal{G}'}{\mathcal{G}} & \text{for gravitons moving along } \phi_\mu\\ \frac{\mathcal{G}}{\mathcal{G}}=1 & \text{for gravitons moving normal to } \phi_\mu\end{cases} \tag{6}$$

3. Wald entropy formula and Horndeski theory

Consider a covariant gravitational theory described by the Lagrangian $\mathcal{L} = \mathcal{L}(g_{\mu\nu}, R_{\mu\nu\alpha\beta}, \nabla_\rho R_{\mu\nu\alpha\beta}, \cdots)$, where $g_{\mu\nu}$ is the spacetime metric, which we take to be n dimensional, $R_{\mu\nu\alpha\beta}$ is its Riemann curvature and ∇_ρ is its covariant derivative. Horndeski theory (3) is an example of such theories. The Wald entropy for a black hole solution to this theory is defined as [25]

$$S_{\text{BH}}^{\text{W}} := 2\pi\int_{\text{H}} \mathbf{X}^{\mu\nu}\boldsymbol{\epsilon}_{\mu\nu}, \tag{7}$$

in which $\boldsymbol{\epsilon}_{\mu\nu}$ is the binormal tensor to the $(n-2)$-surface H associated to the black hole horizon, normalized as $\boldsymbol{\epsilon}_{\mu\nu}\boldsymbol{\epsilon}^{\mu\nu}=-2$, and satisfying the identity

[1] Note that in most of the Horndeski literature which deals with cosmological background, ϕ_μ is timelike. For this case (5) still holds but then leads to $c_g^2 = \frac{\mathcal{G}}{\mathcal{G}-2\mathcal{X}\mathcal{G}'}$ for all gravitons [35].

$$(d\xi_{\rm H})_{\mu\nu} = 2\kappa \boldsymbol{\epsilon}_{\mu\nu}, \tag{8}$$

where $\xi_{\rm H}$ is the horizon Killing vector and κ is surface gravity of the black hole, and $\mathbf{X}^{\mu\nu}$ is

$$(\mathbf{X}^{\mu\nu})_{\mu_3\dots\mu_n} = -\frac{\delta\mathcal{L}}{\delta R_{\alpha\beta\mu\nu}}\boldsymbol{\epsilon}_{\alpha\beta\mu_3\dots\mu_n}, \tag{9}$$

where $\boldsymbol{\epsilon}_{\mu_1\mu_2\mu_3\dots\mu_n}$ is the spacetime volume form [25,26].

The Wald entropy formula has been extremely successful in providing the correct entropy in black hole literature, nonetheless it suffers from an ambiguity which can yield wrong entropy in special cases. As we show below Horndeski theories are among these special cases. The Horndeski Lagrangian (3) has a $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ term. Since $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$, we have terms like $R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, which can be rewritten as

$$R^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi = -(\nabla_\rho\nabla_\sigma\phi)^2 + (\Box\phi)^2 + \nabla_\mu\mathcal{W}^\mu, \tag{10}$$

with

$$\mathcal{W}^\mu = (\nabla_\nu\phi\nabla^\nu\nabla^\mu\phi - \Box\phi\nabla^\mu\phi). \tag{11}$$

The explicit dependence on the Ricci tensor $R_{\mu\nu}$ can hence be removed in favor of terms with ϕ derivatives. A careful analysis [41] reveals that this yields an ambiguity in $\mathbf{X}^{\mu\nu}$ (9), a "**W** ambiguity" in Wald's terminology [25,26],

$$(\mathbf{X}^{\mu\nu})_{\mu_3\dots\mu_n} \to (\mathbf{X}^{\mu\nu})_{\mu_3\dots\mu_n} + \lambda\frac{\delta R^{\rho\sigma}}{\delta R_{\alpha\beta\mu\nu}}\partial_\rho\phi\partial_\sigma\phi\boldsymbol{\epsilon}_{\alpha\beta\mu_3\dots\mu_n}, \tag{12}$$

where λ is an arbitrary number. The last term above can be nonzero and contribute to the Wald formula (7) (see Appendix B for details). To circumvent this problem in Horndeski gravity, we use the solution phase space method which is free of this ambiguity.

4. Entropy in solution phase space method

Consider an n dimensional generally covariant theory described by a Lagrangian $\mathcal{L}$ and denote the dynamical fields collectively by Φ and its generic solutions by $\bar\Phi$. Let the n-form $\mathbf{L}$ be the Hodge dual of the Lagrangian. The Noether current $(n-1)$-form $\mathbf{J}$ associated with a smooth vector ξ^μ is then

$$\mathbf{J}_\xi = \boldsymbol{\Theta}(\delta_\xi\Phi) - \xi\cdot\mathbf{L}, \tag{13}$$

where $\delta_\xi\Phi$ are Lie-derivative of fields along ξ and the $\boldsymbol{\Theta}$ term is the standard surface $(n-1)$-form which is read by the variation of the Lagrangian $\delta\mathbf{L} = \mathbf{E}\delta\Phi + d\boldsymbol{\Theta}(\delta\Phi)$; d denotes exterior derivative on the space-time, and $\mathbf{E}$ represents the equations of motion. Using the identity $d(\xi\cdot\mathbf{L}) = \delta_\xi\mathbf{L}$, then $d\mathbf{J}_\xi = \mathbf{E}\delta_\xi\Phi$. This is the celebrated Noether theorem: by the on-shell condition $\mathbf{E}=0$, $d\mathbf{J}_\xi = 0$. As a result, by the Poincarè Lemma $\mathbf{J}$ is an exact form on-shell, i.e. $\mathbf{J}_\xi = d\mathbf{Q}_\xi$. Noether charge density $\mathbf{Q}$ is an $(n-2)$-form which is locally built out of Φ and ξ. One can define an $n-2$ dimensional form [32]

$$\boldsymbol{k}_\xi(\delta\Phi,\bar\Phi) := \delta\mathbf{Q}_\xi - \xi\cdot\boldsymbol{\Theta}(\delta\Phi) \tag{14}$$

where $\delta\Phi$ is a generic variation of the fields satisfying linearized field equations.

The variation of the Hamiltonian generator associated with flows of $\xi_{\rm H}$ is given by [26,34] $\delta H_{\xi_{\rm H}} = \int_{\rm H}\boldsymbol{k}_\xi(\delta\Phi,\bar\Phi)$. The entropy variation $\delta S_{\rm BH}$ which satisfies a consistent first law is then defined as $\delta H_{\xi_{\rm H}} := T_{\rm BH}\delta S_{\rm BH}$, i.e.

$$\delta S_{\rm BH} := \frac{1}{T_{\rm BH}}\int_{\rm H}\boldsymbol{k}_{\xi_{\rm H}}(\delta\Phi,\bar\Phi), \tag{15}$$

where $T_{\rm BH}$ is the black hole temperature, which should be a purely geometric quantity and a constant over H. The definition (15) yields an entropy variation free of **W** ambiguity [34,42–45].

The key question in this approach is to how fix $T_{\rm BH}$. In usual cases [25,26] $T_{\rm BH} = \frac{\kappa}{2\pi} = T_0$, where κ is the horizon surface gravity and T_0 is the Hawking temperature [21]. Below we identify $T_{\rm BH}$ in Horndeski theories.

5. Effective Metric for Gravitons (EMG) and effective surface gravity

Given (6) and the direction dependence of the speed of gravitons, one may ask what is the "effective" metric $\mathfrak{g}_{\mu\nu}$ whose null rays, $\mathfrak{g}^{\mu\nu}k_\mu k_\nu = 0$, the gravitons move on. From (6) it is easy to write this metric:

$$\begin{aligned}\mathfrak{g}_{\mu\nu} &= (\mathcal{G} - 2\mathcal{X}\mathcal{G}')g_{\mu\nu} - \mathcal{G}'\partial_\mu\phi\partial_\nu\phi\\ &= (\mathcal{G} - 2\mathcal{X}\mathcal{G}')h_{\mu\nu} + \mathcal{G}\phi_\mu\phi_\nu\end{aligned} \tag{16}$$

where ϕ_μ is defined in (4) and to avoid having a singular effective metric we assume $\mathcal{G}', \mathcal{G} - 2\mathcal{X}\mathcal{G}' \neq 0$. In the terminology of Horndeski gravity literature, the above is a "disformal map" [6,35,46] from original spacetime metric to the EMG.

In order to compute the surface gravity as seen by gravitons using (16), we note that the horizon generating Killing vector $\xi_{\rm H}^\mu$ is normal to ϕ_μ at the horizon and one may hence use this to compute the surface gravity,

$$d\xi_{\rm H} = 2\kappa c_g\boldsymbol{\mathcal{E}} \quad \text{at the horizon}, \tag{17}$$

where κ is the surface gravity in the matter metric $g_{\mu\nu}$, $c_g^2 = \frac{\mathcal{G}-2\mathcal{X}\mathcal{G}'}{\mathcal{G}}$ and $\boldsymbol{\mathcal{E}}$ is the bi-normal tensor to the bifurcation surface H, normalized as $\boldsymbol{\mathcal{E}}_{\mu\nu}\boldsymbol{\mathcal{E}}^{\mu\nu} = -2$. In particular, note that to lower and raise indices on $\xi_{\rm H}^\mu, \boldsymbol{\mathcal{E}}$ we should use $\mathfrak{g}_{\mu\nu}$ as in (16).

To relate to the entropy formula (15), however, we need to rewrite the above in terms of $\boldsymbol{\epsilon}$, the volume two form of the original metric $g_{\mu\nu}$. Recalling (16) we have

$$\boldsymbol{\mathcal{E}} = \sqrt{\mathcal{G}(\mathcal{G} - 2\mathcal{X}\mathcal{G}')}\,\boldsymbol{\epsilon}. \tag{18}$$

Inserting this into (17) we obtain

$$d\xi_{\rm H} = 2\kappa(\mathcal{G} - 2\mathcal{X}\mathcal{G}')\boldsymbol{\epsilon}, \tag{19}$$

implying

$$T_{\rm graviton} = (\mathcal{G} - 2\mathcal{X}\mathcal{G}')T_0, \tag{20}$$

where $T_0 = \frac{\kappa}{2\pi}$ is the ordinary Hawking temperature and we still use (15) to compute the entropy.

The key point is, it is only by identifying $T_{\rm graviton} = T_{\rm BH}$ that we get a consistent first law using (15). That is, the SPSM indicates that the integrable entropy is the charge of the vector,

$$\zeta_{\rm H} := \frac{1}{T_{\rm graviton}}\xi_{\rm H} = \frac{2\pi}{\kappa\cdot(\mathcal{G} - 2\mathcal{X}\mathcal{G}')}\xi_{\rm H}. \tag{21}$$

We note that the arguments of exponential peeling of graviton null rays close to the horizon [47], leads to the same temperature as (20), once we consider the appropriate scaling of units (18).

6. Examples

To show how our modified temperature resolves the first law issue for Horndeski black holes we discuss two examples investigated in [27,48–50]. Three more examples have been discussed in the Appendix A.

Example 1. In our first example, we study a spherically symmetric black hole in the Horndeski gravity,

$$\mathcal{L} = \frac{1}{16\pi G_{\rm N}}\Big(R - F_{\mu\nu}F^{\mu\nu} + 2\gamma G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\Big) \tag{22}$$

This action corresponds to $\mathcal{G}_2 = 0, \mathcal{G}_4 = 1, \mathcal{G}_5 = 2\gamma\phi$ in (2), which yields $\mathcal{G} = 1 + 2\gamma\mathcal{X}$ in (3), and $F = dA$ is the electromagnetic field strength. This theory has a charged black hole solution [27],

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{23}$$

where

$$h = 1 - \frac{2m}{r} + \frac{q^2}{r^2} - \frac{q^4}{12r^4}, \qquad f = \frac{4r^4 h}{(2r^2 - q^2)^2}. \tag{24}$$

The gauge and scalar fields can also be written as

$$A = \left(\frac{q}{r} - \frac{q^3}{6r^3}\right)dt, \qquad d\phi = \sqrt{\frac{-q^2}{2\gamma r^2 f}}dr. \tag{25}$$

To have a real ϕ we take $\gamma < 0$. This solution is asymptotically flat and horizon is at $h = f = 0$,

$$r_{\rm H} - 2m + \frac{q^2}{r_{\rm H}} - \frac{q^4}{12r_{\rm H}^3} = 0. \tag{26}$$

Note that, while the derivative of the scalar field diverges at the horizon, $\gamma\partial^\mu\phi\partial_\mu\phi = \frac{-q^2}{2r^2}$ is finite at the horizon.

The standard methods for calculating conserved charges yields $M = \frac{m}{G_{\rm N}}$ is the mass and $Q = \frac{q}{G_{\rm N}}$ is the electric charge of the black hole. Moreover, the surface gravity and horizon electric potential are [27],

$$\kappa = \frac{2r_{\rm H}^2 - q^2}{4r_{\rm H}^3}, \qquad \Phi_{\rm H} = \frac{q}{r_{\rm H}} - \frac{q^3}{6r_{\rm H}^3}. \tag{27}$$

With the Hawking temperature $T_0 = \frac{\kappa}{2\pi}$ together the Wald entropy, which yields the usual area law for this example $S = \pi r_{\rm H}^2/G_{\rm N}$ (see the appendix for more details of the computation), the first law $\delta S = \frac{1}{T_0}(\delta M - \Phi_{\rm H}\delta Q)$ does not hold. Moreover, with $T_{\rm BH} = T_0$ the entropy obtained in the SPSM is not even integrable over parameters m and q of the solution [27]. This can be easily seen by replacing all terms in RHS of δS above in terms of m, q and observe that it is not variation of some $S(m, q)$.

However, the new temperature in (20),

$$\begin{aligned} T_{\rm BH} &= (\mathcal{G} - 2\mathcal{X}\mathcal{G}')\Big|_{\rm H} T_0 \\ &= \left(1 - \frac{q^2}{2r_{\rm H}^2}\right)T_0 = \frac{\left(1 - \frac{q^2}{2r_{\rm H}^2}\right)^2}{4\pi r_{\rm H}}, \end{aligned} \tag{28}$$

makes the entropy computed using (15) integrable, which for this example is given by the usual Bekenstein-Hawking entropy $S_{\rm BH} = \pi r_{\rm H}^2/G_{\rm N}$. With this entropy and temperature (28), it is immediate to verify that the first law,

$$T_{\rm BH}\delta S_{\rm BH} = \delta M - \Phi_{\rm H}\delta Q$$

is also satisfied. Note also that $T_{\rm BH} \leq T_0$.

Example 2. Recalling (20), the class of models with $\mathcal{G} - 2\mathcal{G}'\mathcal{X} = 1$ seem to be special. That is, for $\mathcal{G} = 1 + 2\beta\sqrt{-\mathcal{X}}$ we do not expect a temperature shift. This is in fact confirmed from the black hole solution discussed in [51]. The Lagrangian of the theory is

$$\mathcal{L} = \frac{1}{16\pi G_{\rm N}}\Big((1 + \beta\sqrt{-\mathcal{X}})R - 2\Lambda + \eta\mathcal{X} - \frac{\beta}{2\sqrt{-\mathcal{X}}}\boldsymbol{\Psi}\Big) \tag{29}$$

where $\boldsymbol{\Psi} := (\Box\phi)^2 - (\partial_{\mu\nu}\phi)^2$ and β, η are constants. We consider the black hole solution [51] with the metric of the form (23) and

$$h = f = 1 - \frac{2m}{r} - \frac{\beta^2}{2\eta r^2} - \frac{\Lambda r^2}{3}, \quad d\phi = \frac{\sqrt{2}\beta}{\eta r^2\sqrt{h}}dr. \tag{30}$$

The horizon is sitting at $h = 0$,

$$r_{\rm H} - 2m - \frac{\beta^2}{2\eta r_{\rm H}} - \frac{\Lambda r_{\rm H}^3}{3} = 0. \tag{31}$$

The mass and surface gravity for this solution are

$$M = \frac{m}{G_{\rm N}}, \qquad \kappa = \frac{\beta^2 + 2\eta(r_{\rm H}^2 - \Lambda r_{\rm H}^4)}{4\eta r_{\rm H}^3}. \tag{32}$$

For this case (20) becomes,

$$T_{\rm BH} = T_0 = \frac{\kappa}{2\pi}. \tag{33}$$

For the entropy we need to apply the SPSM formula (15), which together with (33), after lengthy but straightforward algebra yields the area law again, $S_{\rm BH} = \pi r_{\rm H}^2/G_{\rm N}$. One can then readily observe that the first law $T_{\rm BH}\delta S_{\rm BH} = \delta M$ is also satisfied.

7. Discussion and outlook

We discussed that due to the presence of non-vanishing ambiguities, Wald entropy formula (7) does not necessarily yield the correct entropy for black holes in Horndeski theories. This matches with the previous observations that Wald entropy does not yield a well-defined first law of thermodynamics for such black holes [27]. Our main result here is that the resolution is in assigning a new temperature to these black holes corresponding to the surface gravity for gravitons, which together with the entropy variation computed using SPSM formulation yields the correct first law.

The black hole temperature, as one expects both from Hawking [21] or Unruh [52] analysis should be a quantity determined only by near horizon geometry. In ordinary cases all different light species in the problem near the horizon see a similar geometry and move with the same speed, in accordance with Einstein's equivalence principle. The key point in our analysis is that gravitons move on a different metric than the one the matter fields see, (16), as in many other beyond Einstein gravity theories see e.g. [53,54]. Our results suggest that the relevant geometry for black hole thermodynamics is the one seen by gravitons. Among other things, this will provide a resolution to the species problem [55,56].

That the effective metric for gravitons is the one relevant to black hole temperature and thermodynamics, may be checked further by repeating in more detail Hawking's process for these black holes and/or analyzing the Euclidean on-shell action [57] for our black hole solutions. The κ-peeling argument like those carried out in [47] for gravitons suggests that Hawking analysis should yield the same temperature as ours in (20). Carrying out these analysis more closely could be illuminating.

Due to presence of a profile of the scalar field we have a spontaneous breaking of Lorentz symmetry in the near horizon geometry, and this yields direction-dependent speed of gravitons (6). A similar feature is also shared by Einstein-Æther theories [58]. It would hence be interesting to apply our ideas and results here to this framework and verify if they resolve similar issues about the black hole thermodynamics in those cases [59,60].

Assigning a temperature different than $\kappa/(2\pi)$ may raise the question about generalized second law of thermodynamics [22].

Consider lump of gas of photons of energy $\delta E \ll m_{\mathrm{BH}}$ at temperature T_γ falling into the hole. The first law implies $T_{\mathrm{BH}}\Delta S_{\mathrm{BH}} = \delta E = \frac{3}{4}T_\gamma S_\gamma$, where ΔS_{BH} is the change in the entropy of the hole due to the fall and S_γ is the entropy of the photon lump. The second law then requires, $\Delta S_{\mathrm{BH}} \geq S_\gamma$ or $T_{\mathrm{BH}} \leq \frac{3}{4}T_\gamma$. For a photon to be absorbed into the hole its wave-length should be smaller than $4r_{\mathrm{H}}$ and hence $T_\gamma \sim (4r_{\mathrm{H}})^{-1}$. Therefore, a sufficient but not necessary condition for the second law is $T_{\mathrm{BH}} \leq \frac{3}{16 r_{\mathrm{H}}}$, which is satisfied for our examples.

Finally, we note that with the temperature (20) in hand, one can fix the ambiguity in the Wald entropy analysis and provide a refined Wald entropy formula [41].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Three more examples

Example 3. As our third example we analyze a 4-dimensional black brane with AdS_4 asymptotics which is a solution of the Lagrangian

$$\mathcal{L} = \frac{1}{16\pi G_{\mathrm{N}}}\Big(R - 2\Lambda - F_{\mu\nu}F^{\mu\nu} - 2(\alpha g^{\mu\nu} - \gamma G^{\mu\nu})\partial_\mu\phi\partial_\nu\phi\Big) \tag{34}$$

with arbitrary Λ and α. Comparing with Lagrangian (3), we find $\mathcal{G}_2 = 4\alpha\mathcal{X} - 2\Lambda$ and $\mathcal{G} = 1 + 2\gamma\mathcal{X}$. Instead of Λ, α, one can use two other constants ℓ, β [27]

$$\Lambda = -\frac{3(1+\frac{\beta}{2})}{\ell^2}, \qquad \alpha = \frac{3\gamma}{\ell^2}. \tag{35}$$

In this convention, an electrically charged black brane solution in the coordinate (t, r, x, y) is

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2), \tag{36}$$

where

$$h = \frac{r^2}{\ell^2} - \frac{m}{r} + \frac{4q^2}{(4+\beta)r^2} - \frac{4q^4\ell^2}{15(4+\beta)^2r^6},$$
$$f = \frac{(4+\beta)^2r^8h}{\left(\frac{2q^2\ell^2}{3} - (4+\beta)r^4\right)^2},$$
$$d\phi = \sqrt{\frac{\beta - \frac{2q^2\ell^2}{3r^4}}{4\gamma f}}dr, \quad A = \Big(\frac{q}{r} - \frac{2q^3\ell^2}{15(4+\beta)r^5}\Big)dt. \tag{37}$$

The brane is situated at r_{H} where $f(r_{\mathrm{H}}) = h(r_{\mathrm{H}}) = 0$. Mass and electric charge "densities" for this solution are

$$M = \frac{(4+\beta)m}{32\pi G_{\mathrm{N}}}, \qquad Q = \frac{q}{4\pi G_{\mathrm{N}}}. \tag{38}$$

By densities it is understood that the charges are calculated without performing the integration over the x and y coordinates. Horizon surface gravity and electric potential are

$$\kappa = \frac{3r_{\mathrm{H}}}{2\ell^2} - \frac{q^2}{(4+\beta)r_{\mathrm{H}}^3}, \qquad \Phi_{\mathrm{H}} = \frac{q}{r_{\mathrm{H}}} - \frac{2q^3\ell^2}{15(4+\beta)r_{\mathrm{H}}^5}. \tag{39}$$

Insisting on the Hawking temperature $T_0 = \frac{\kappa}{2\pi}$, the first law is not satisfied and the charge of the vector $\frac{1}{T_0}\xi_{\mathrm{H}}$ is not integrable [27]. On the other hand, the new temperature in (20) can be calculated to be

$$T_{\mathrm{graviton}} = \left(\frac{3(4+\beta)r_{\mathrm{H}}^4 - 2q^2\ell^2}{12r_{\mathrm{H}}^4}\right)T_0. \tag{40}$$

This is exactly the T_{BH} which makes the entropy integrable. The entropy can be calculated using SPSM to be found the Bekenstein-Hawking entropy density $r_{\mathrm{H}}^2/(4G_{\mathrm{N}})$. It is easy to verify that the first law is also satisfied by the charge densities as

$$T_{\mathrm{BH}}\delta S_{\mathrm{BH}} = \delta M - \Phi_{\mathrm{H}}\delta Q\,.$$

Example 4. As the fourth example, we study a rotating neutral BTZ-like black hole in 3 dimensional space-times which is a solution to the Lagrangian (34), and it is [61,62]

$$ds^2 = -hdt^2 + \frac{dr^2}{h} + r^2(d\varphi - \frac{j}{r^2}dt)^2,$$
$$h = -m + \frac{\alpha r^2}{\gamma} + \frac{j^2}{r^2}, \quad d\phi = \sqrt{\frac{-(\alpha+\gamma\Lambda)}{2\alpha\gamma h}}\,dr \tag{41}$$

where $\gamma < 0$ and (m, j) are free parameters in the solution. Mass, angular momentum, horizon angular velocity, surface gravity, and horizon radii for this solution are

$$M = \frac{(\alpha - \Lambda\gamma)m}{16\alpha G_{\mathrm{N}}}, \qquad J = \frac{(\alpha - \Lambda\gamma)j}{8\alpha G_{\mathrm{N}}},$$
$$\kappa_\pm = \frac{\alpha(r_+^2 - r_-^2)}{\gamma r_\pm}, \qquad \Omega_\pm = \frac{j}{r_\pm^2},$$
$$r_\pm^2 = \frac{\gamma m \mp \sqrt{\gamma^2m^2 - 4\gamma\alpha j^2}}{2\alpha}. \tag{42}$$

Notice that $\alpha < 0$ in order to have positive horizon radii. By the new temperature in (20), one finds that

$$T_{\mathrm{BH}} = \left(\frac{\alpha - \Lambda\gamma}{2\alpha}\right)T_0, \tag{43}$$

where $T_0 = \frac{\kappa}{2\pi}$. Using this, (15) yields $S_{\mathrm{BH}} = 2\pi r_{\mathrm{H}}/(4G_{\mathrm{N}})$ as the entropy of this black hole which satisfied the first law for each one of the horizons

$$T_{\mathrm{BH}}\delta S_{\mathrm{BH}} = \delta M - \Omega_{\mathrm{H}}\delta J.$$

Example 5. The last example we present is a spherically symmetric neutral black hole solution of the Horndeski theory (34). In 4-dimensional space-time the black hole solution is in the form of (23) in which [63]

$$h = 1 - \frac{2m}{r} + \frac{\alpha(4\alpha - \lambda)}{3\gamma(4\alpha+\lambda)}r^2 + \frac{\lambda^2\sqrt{\frac{\gamma}{\alpha}}}{(16\alpha^2 - \lambda^2)r}\tan^{-1}\Big(\frac{r}{\sqrt{\frac{\gamma}{\alpha}}}\Big),$$
$$f = \frac{(\gamma + \alpha r^2)h}{\gamma(rh)'}, \qquad d\phi = \sqrt{\frac{-\lambda(r^2h^2)'r}{8(\gamma+\alpha r^2)^2h^2}}dr, \tag{44}$$

where $\lambda = 2\alpha + 2\gamma\Lambda$. For this solution, the mass M and the surface gravity κ are equal to

$$M = \frac{\sqrt{16\alpha^2 - \lambda^2}}{4\alpha G_{\rm N}} m, \; \kappa = \frac{\alpha(4\gamma + (4\alpha - \lambda)r_{\rm H}^2)}{2\gamma r_{\rm H}\sqrt{16\alpha^2 - \lambda^2}}, \tag{45}$$

where $r_{\rm H}$ is the root of $h(r)$. The Hawking temperature $T_0 = \frac{\kappa}{2\pi}$ has the same problems as the other examples in this paper, i.e. the first law is not satisfied and entropy is not an integrable charge. The new temperature (20),

$$T_{\rm BH} = T_{\rm graviton} = \left(\frac{4\gamma + (4\alpha - \lambda)r_{\rm H}^2}{4(\gamma + \alpha r_{\rm H}^2)}\right) T_0, \tag{46}$$

resolves these problems and reproduces the Bekenstein-Hawking entropy $S_{\rm BH} = \pi r_{\rm H}^2/G_{\rm N}$, which satisfies the first law. It is worth mentioning that in the presence of Λ, the first law can be extended to include a Volume-Pressure term. One can find out [64] how this extension is possible by considering the Λ as a conserved charge associated with a global gauge transformation [65]. The bottom-line is that this extension is possible, and is compatible with the new temperature.

Appendix B. Details of ambiguity in Wald entropy

Considering the extra term appearing in (12) in Wald formula (7), and using (8) for the Killing vector $\xi_{\rm H}$, we find

$$\begin{aligned}
&\frac{\delta\big(R^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi\big)}{\delta R_{\alpha\beta\mu\nu}}\boldsymbol{\epsilon}_{\mu\nu} = \frac{\delta\big(R^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi\big)}{\delta R_{\alpha\beta\mu\nu}}\frac{\nabla_{[\mu}\xi_{\nu]}}{\kappa}\\
&= (g^{\alpha\mu}\nabla^\beta\phi\nabla^\nu\phi)\frac{\nabla_{[\mu}\xi_{\nu]}}{\kappa}\\
&= \frac{1}{\kappa}\Big(\nabla_\mu\Big((g^{\alpha\mu}\nabla^\beta\phi\nabla^\nu\phi)\xi_\nu\Big) - \nabla_\mu(g^{\alpha\mu}\nabla^\beta\phi\nabla^\nu\phi)\xi_\nu\Big)\\
&= \frac{-1}{\kappa}\nabla^\alpha(\nabla^\beta\phi\nabla^\nu\phi)\xi_\nu = \frac{-1}{\kappa}\nabla^\alpha(\nabla^\nu\phi)\nabla^\beta\phi\xi_\nu
\end{aligned} \tag{47}$$

in which we have used isometry condition $\xi_\mu\nabla^\mu\phi = 0$.

One may check that pull-back of the result to the bifurcation surface of horizon (which for our examples means choosing indices $(\alpha, \beta) = (t, r)$ and multiplying by $\sqrt{-g}$) is non-zero on the horizon. This contribution from (47) for the Example 1 and for $\xi_{\rm H} = \partial_t$ is

$$\oint_{\rm H} \frac{-\sqrt{-g}\, d\theta d\varphi}{\kappa}\nabla^t(\nabla^\nu\phi)\nabla^r\phi\, \xi_{{\rm H}\nu} = \frac{-\pi q^2}{\gamma}. \tag{48}$$

Therefore, there is a non-vanishing ambiguity in the Wald entropy.

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