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Towards Precision Cosmography with Cosmic Chronometers

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Antes de cerrar este capítulo de mi vida, es merecido echar la vista atrás para darme cuenta de todo lo que he vivido hasta llegar aquí. Desde luego, no estaba en la lista de cosas que tenía pensadas para mi futuro cuando empecé el Doble Grado en Física y Matemáticas en Uniovi, que con 22 años estaría aterrizando solo en Venecia para comenzar esta nueva etapa de mi vida. Esta etapa ahora concluye, felizmente, en un enorme sosiego y con unas ganas septembrinas (como en la vuelta al cole) de ver qué deparan los próximos años. En este echar la vista atrás me doy cuenta de lo enormemente feliz que soy de haber podido llegar aquí, porque sabiendo cómo me estoy sintiendo en este momento, si se me hubiera dado a elegir, habría escogido el mismo camino sin dudarlo. En resumen, lo que sigue es una forma de decir gracias a todas las personas de las que me acuerdo en las siguientes líneas -y seguro alguna que falta-, porque Trieste no ha sido una parte triste de mi vida, sino todo lo contrario, y de alguna forma, todos habéis contribuido a ello.

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"No. [The Universe can be] formidable, frightening. But not evil."

Dr. Amelia Brand (Interstellar)

Abstract

The Hubble tension, the growing discrepancy between early- and late-Universe determinations of H_0 , has renewed interest in late-time, model-independent probes of the cosmic expansion history. Cosmic chronometers (CCs), passively evolving early-type galaxies whose differential ageing directly traces the cosmic expansion rate, $H(z) \propto dz/dt$, offer one such probe, entirely decoupled from the distance ladder and from early-Universe physics. This thesis develops and applies a Lick-index-based CC pipeline, combining high signal-to-noise spectral stacking, stellar-population synthesis fitting, and a cosmographic (Taylor-expansion) reading of the resulting age-redshift relations, to two large spectroscopic samples: 90 396 SDSS Legacy galaxies at $0.05 \lesssim z \lesssim 0.35$ and 527 287 DESI-DR1 luminous red galaxies at $0.36 \lesssim z \lesssim 0.80$.

Applied to the SDSS sample, a classic, $z_0 = 0$ cosmographic fit of TMJ-modelled stellar ages yields $H_0 = 70.0^{+4.1}_{-7.6} \text{ km s}^{-1} \text{ Mpc}^{-1}$, comfortably compatible with both sides of the Hubble tension, together with a $w_0\text{CDM}$ extension fully consistent with a cosmological constant. Applied to the substantially larger and higher-redshift DESI sample, a pivotal-redshift cosmography anchored at $z_0 \approx 0.57$ delivers a fully covariant $H(z)$ reconstruction across $0.36 \lesssim z \lesssim 0.80$, together with two independent, differential $H(z)$ measurements at $z \approx 0.55$ and $z \approx 0.61$; every one of these results is individually consistent with the ΛCDM prediction of [Planck Collaboration et al. \(2020\)](#), taken today as the typical benchmark cosmology to compare with. As a by-product of the same stacks, age-metallicity- α -enhancement scaling relations are derived at $z \approx 0$ from SDSS and, indirectly, at $z \approx 0.57$ from DESI, both consistent with the downsizing scenario for the assembly of early-type galaxies.

Taken together, these results establish Lick-index cosmic chronometers, built on carefully stacked spectra and interpreted through a cosmographic rather than a purely finite-difference framework, as a competitive, genuinely model-independent probe of the low- and intermediate-redshift expansion history of the Universe. The thesis closes by identifying the main systematic limitations of the current pipeline, including an unexplained oscillatory pattern in the index-redshift relations, and by outlining concrete directions for future work on both the astrophysical and the cosmographic sides of the method.

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Introduction

The cosmological picture

Over the last two decades, cosmology has entered a precision era driven by increasingly large and sophisticated observational datasets. Measurements of the cosmic microwave background (CMB; [Planck Collaboration et al., 2020](#); [Louis et al., 2025](#)), baryon acoustic oscillations (BAO; [Eisenstein et al., 2005](#); [Adame et al., 2025](#); [Abdul Karim et al., 2025](#)), Type Ia supernovae (SNIa [Brout et al., 2022](#); [Abbott et al., 2024](#); [Rubin et al., 2025](#)), and the distance ladder ([Riess et al., 2022](#)), and large-scale structure surveys ([Alam et al., 2021](#); [Abbott et al., 2022](#)) have converged toward a concordance model, commonly described by the Λ CDM paradigm. In this framework, the expansion history of the Universe and the growth of cosmic structures are encoded in a small set of parameters that describe the dynamics of the Universe and the nature of the energies that populate it.

The empirical success of the model accommodating the observed phenomenology of cosmological probes made it a reference or "benchmark" for the scientific community. The main drawback of this framework is that the model needs a major contribution of an "exotic" type of energy that produces negative pressure and a type of cold matter not belonging to the known (discovered part of the) standard model of particle physics. Beyond that, there are also rising tensions in the measurement of the values of some of the aforementioned parameters that have motivated the introduction of less simple models that typically include more parameters.

One of those tensions appears when comparing the estimated value of the Hubble Constant, H_0 , computed with probes from the nearby (late) Universe (SNIa at $0 < z \lesssim 1$, calibrated through the Cepheid-based distance ladder, e.g. [Riess et al. \(2022\)](#)) and from the faraway (early) Universe (CMB, $z \sim 10^3$). This is known as the **Hubble Tension**, and the physical meaning of the involved parameter is the current time value of the expansion rate of the Universe, H . Fundamentally, this is the time derivative of the scale factor, a , of the universe normalised by its value: $H \equiv \dot{a}/a$ ($\dot{a} \equiv da/dt$), which enters the Friedmann-Lemaître-Robertson-Walker (FLRW) metric as a global factor for its spatial part, only dependent on the cosmic coordinate time. The importance of H is major in cosmology, as among other things, it is independent of the chosen model. More specifically, any content description of the Universe gets encapsulated within its functional form (i.e. $H(z)$), but the absolute value, H ,

at each time (or redshift) can be directly measured, without the need to introduce any dynamical model at all. This is the case study of **Cosmography**, which tries to constrain only the kinematical evolution of the Universe, through parameters that independently fix the shape of H (i.e. in time, $H(t)$, or in redshift, $H(z)$).

Cosmography was used by the community prior to the standardisation of cosmological descriptions like Λ CDM or the [Chevallier & Polarski \(2001\)](#) and [Linder \(2003\)](#) parametrisation as working tools. Two decades ago, modern cosmography based on the Taylor expansion of the Hubble parameter [Visser \(2004\)](#) was used to directly fit the luminosity distance data from Gamma Ray Bursts (GRBs) ([Capozziello, S. & Izzo, L., 2008](#)). In the decade of the 2010s issues related to the convergence and truncation of the Taylor series at high redshift prompted the community to abandon cosmography when the data would not live decisively "closer" to us than the cosmic noon. Some alternative parametrisation within cosmography, like the y -redshift ([Cattoën & Visser, 2007](#)) tried to mitigate the issue, but this only managed to move the problem a few fractions of a unit further in redshift. On top of that, some works ([Busti et al., 2015](#)) emphasised that higher-order parameters in cosmography tended to be poorly constrained at low redshift, especially when using the y -redshift parametrisation.

However, the last decade has seen a robust "rehabilitation" of cosmography, driven primarily by the escalating Hubble tension and the arrival of high-redshift data from DESI and Euclid. To overcome the mathematical limitations of Taylor expansions, recent works have shifted toward rational approximants, specifically Padé and Chebyshev polynomials, which demonstrate superior convergence properties and stability at $z > 1$ ([Mishra et al., 2026](#)). This "numerical cosmography" is no longer just a kinematic exercise but a rigorous diagnostic tool; by measuring the cosmic jerk, j and snap s (higher-order terms in the cosmographic Taylor expansion; see Sec. 1.1.1, eq. 1.29 and discussion therein) through model-independent reconstructions like Gaussian Processes (GP), researchers are actively testing for "New Physics" beyond Λ CDM. Furthermore, the integration of direct $H(z)$ measurements has allowed cosmography to bypass the distance ladder entirely, providing a purely late-time, model-agnostic counterweight to the early-universe constraints from Planck. Consequently, cosmography has transitioned from a discarded relic of the pre-precision era into a frontline defence against potential systematic biases in our standard cosmological model.

The astrophysical picture

Time is one of the most fundamental quantities through which physical reality is described, yet it is also among the most elusive to measure directly. In everyday human experience, time is inferred through change: biological ageing, geological stratification, or the accumulation of historical records. Across the human sciences, elapsed time is rarely measured in an absolute sense; instead, it is reconstructed from imprints left by irreversible processes: growth rings in trees, sediment layers, linguistic evolution, or demographic transitions. In this broader context, time is not observed but

encoded, and its passage is inferred by decoding persistent markers that accumulate as systems evolve.

In cosmology, we benefit from the fact that light has a constant speed, comparable to astrophysical and cosmological distances in times distinguishable by us humans. We all studied at school that it takes around eight minutes for the light emitted by the Sun to get to Earth. Thus, what we are seeing of the Sun is how it was eight minutes ago. Likewise, the aspect of the face of the Moon at the time we look at it is more than a second old. In greater astrophysical scales, this effect intensifies: some stars we see in the night sky are already dead: having become white dwarfs or gone supernova leaving a neutron star or a black hole behind. Well beyond our galaxy, and observable with the right telescope, we start talking about cosmological scales, where we are directly looking ages back in time, to when the Universe was much younger, to the time in which galaxies produced new stars at their fastest speed, or even when it was so dense that everything seemed homogeneous even at the smallest scales.

The struggle in cosmology was to find the observable present across vast periods of cosmic history, whose light encodes its age. In their seminal paper, [Jimenez & Loeb \(2002\)](#) pointed to Early-Type Galaxies (ETGs) as the right objects to examine, but statistically rather than individually: the average age of a well-selected sample of ETGs, whose light can be fairly approximated by a passively evolving single stellar population. In other words, these well-selected groups evolve passively with the Universe, and from their integrated light, we observe only the ageing due to the passage of cosmic time. This is the founding idea behind Cosmic Chronometers (CCs): ETGs whose differential ageing directly traces the cosmic expansion rate, $H(z)$.

Galaxies are complex systems in which age is not strictly an absolute quantity. In the case of our own galaxy, the Milky Way, it is anything but unambiguous. From a human perspective, our planet was born a bit later than our Solar System formed, which started to condensate from a Hydrogen overdensity more than 9 thousand million years later than the oldest stars in our Galaxy. To draw a comparison, our Sun is roughly 4.6 thousand million years old, which means that the oldest objects in our Galaxy are three times as old. In contrast, the stellar population (i.e. the stars) living in the most massive and passively evolving ETGs (which constitute the best CC candidates) are roughly coeval, as we know from fundamental studies in Galaxy Formation and Evolution ([Thomas et al., 2010](#)). This has brought the cosmology community to talk about "the age of ETGs" (or more broadly "galaxies") when physically referring to the age of their dominant stellar population, as this is what researchers are interested in regarding these objects.

The spectroscopic picture

Extracting a precise age (or relative age differences) from the integrated light of these coeval populations remains a non-trivial task. The traditional approach in the CC community to extract absolute age estimations has been full spectral fitting (FSF). However, relative age differences can be directly measured without the need for absolute estimations. In the literature, the characteristic 4000 Å break in the spectral energy distribution (SED) of ETGs (D4000; Moresco et al., 2012; 2015; 2016) has been widely used for this purpose, as it constitutes a high signal-to-noise indicator of relative age differences between populations, from which dt/dz can be estimated. It has recently been used to fit DESI data (Loubser et al., 2025; Loubser, 2025). Yet, the dependence of broadband features on the global spectral shape can introduce sensitivities to spectrophotometric calibration and the choice of the stellar population synthesis (SPS) model. Regarding the FSF method, which has yet to be applied to large datasets due to its computationally-intensive nature, it is able to extract the maximum amount of information from a spectrum and absolute ages, although the age-metallicity degeneracy is still a problem to be solved (Jiao et al., 2023). Recent works as Jiao et al. (2023) or Tomasetti et al. (2023; 2026) have put direct constraints on the dust reddening, the timescale for the formation of the stellar populations, the specific star-formation rate or the amount of stellar mass formed.

A third approach, pioneered in the CC context by Borghi et al. (2022b), focuses on narrow spectroscopic features that are primarily sensitive to the physical conditions of the stellar atmospheres rather than the global continuum. These absorption lines or indices are known as Lick indices, after the system originally developed in the 1970s and 80s at the Lick Observatory (Faber et al., 1985; Worthey et al., 1994) to quantify the narrow-band spectroscopic features of integrated starlight in galaxies. Historically, these indices were the primary means by which astronomers disentangled the chemical evolution of galaxies from their star-formation histories, providing the first systematic evidence for the downsizing scenario (see the dedicated section 1.3.3) in the formation of ETGs (Thomas et al., 2003b; 2010; Clemens et al., 2006; 2009). The inclusion of lines of the Balmer series ($H\beta$, $H\delta$, $H\gamma$) targets the temperature of the main-sequence turn-off (Worthey, 1994; Worthey et al., 1994), tightly correlated to the age of the population. However, the reliability of these Balmer indicators is inherently challenged by the age-metallicity-abundance degeneracy; a phenomenon where a younger, metal-rich population can mimic the spectroscopic signature of an older, metal-poor one. To break this degeneracy, the Lick system necessitates the simultaneous measurement of metal-sensitive indices, such as Iron (i.e. Fe4383 or Fe4531) and α -element abundance (i.e. Mgb) markers. By incorporating these additional features, one can construct a multi-dimensional diagnostic grid to isolate the temporal evolution, obtaining a clear reading of the dominant stellar population age.

Synthesis: Cosmographic analysis of large datasets

The central objective of this thesis is to leverage the "archaeological" information encoded in the spectra of ETGs to perform a model-independent reconstruction of the cosmic expansion history from direct spectroscopic observations. We have focused on addressing the historical limitations of direct $H(z)$ measurements by implementing a continuous, rather than discrete, approach, fitting the $t(z)$ relations with a cosmographic expansion of the cosmic time function. Motivated by the vast datasets from SDSS and the newly released DESI observations, as emphasised by [Moresco et al. \(2022\)](#), we aimed at testing the effective reduction of the statistical uncertainty in local ($z < 1$) direct observations of the Hubble parameters.

We started in [Alvarez et al. \(2025\)](#) (hereafter [A25](#)) by presenting a direct cosmographic fit of the ages of strictly selected ETGs from the SDSS Legacy Survey, which is well known and has been thoroughly analysed in the literature. We performed a cosmographic expansion of $H(z)$ centred at present time; this is, dependent on the current value of the kinematical parameters of the evolution of the Hubble function: Hubble constant, H_0 , current deceleration q_0 and current jerk j_0 .

Early in the present year, we finished treating the recently released Luminous Red Galaxy (LRG) data from the first data release of DESI (DESI-DR1). The full results are presented in [Alvarez et al. \(2026\)](#) (hereafter [A26](#) - submitted to A&A). Since DESI LRG data target a redshift window within 0.3 and 1.0 with a homogeneous distribution of sources ([Zhou et al., 2023](#)), our previously utilised cosmographic expansion was deemed inefficient. We drew inspiration from the work of [Fazzari et al. \(2025\)](#), in which an arbitrary pivotal redshift cosmography was introduced to assess independent $H(z)$ measurements at any redshift, and applied a similar expansion to obtain a reliable sampling of $H(z)$ around $z_0 = 0.57$, with which we are able to provide a number of interdependent measurements and the respective covariance matrix, for future use by the Cosmology community. Since absolutely independent and non-correlated measurements are sometimes preferred, we also leverage the archaeological groups to provide two local $H(z)$ measurements at different redshifts, which can be treated by the community in the exact same way other local measurements are.

1.1 Cosmological framework

The quest to understand the origin, evolution, and ultimate fate of the Universe has transitioned, over the last century, from purely philosophical speculation to a precision science. At the heart of modern cosmology lies the Λ CDM model, a theoretical framework that elegantly describes a universe dominated by Dark Energy (represented by the cosmological constant, Λ) and Cold Dark Matter (CDM). This model posits that the Universe began in a hot dense state (the Big Bang) and has been expanding ever since, and according to current observations, it does so in an accelerated manner.

Since the geometry of space-time is involved in this expansion, a suitable way to organise these words into concrete physics is via General Relativity. This framework allows us to describe the expansion of the Universe in a simple and self-contained manner, although some of the derivations that we present can be obtained from first principles as well.

At this point it is customary to introduce the cosmological principle, according to which the Universe is homogeneous and isotropic on large scales at any epoch, linking the properties of the primordial universe to those of the Universe today. This principle follows from extrapolating the observed isotropy on the largest scales around us, together with the assumption of homogeneity based on the Copernican principle, according to which we do not occupy a privileged position in the Universe. Given these prescriptions, the most general metric to describe the large space-time scales within which this Universe can exist is given by the Friedmann-Lemaître-Robertson-Walker metric (hereafter FLRW; [Friedmann, 1922](#); [Lemaître, 1927](#); [Robertson, 1935](#); [Walker, 1937](#)),

$$ds^2 = -c^2 dt^2 + a^2 d\Sigma^2 = -c^2 dt^2 + a^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (1.1)$$

where $a = a(t)$ is a scale factor for the spatial distances that can only depend on the coordinate time, t , in order to respect the isotropy and homogeneity conditions. $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ is the unit-sphere metric and k is the spatial curvature constant, which carries units of the inverse of a squared length due to a being dimensionless. Depending on whether k is positive, zero or negative, the geometry corresponds to a closed (three-spherical), flat (Euclidean) or open (hyperbolic) Universe respectively.

1.1.1 General Relativity: an expanding Universe draws $H(z)$

There are two quantities in eq. (1.1) that make the Universe we live in something particularly unique. These are the scale factor, $a(t)$, and the curvature of space, k . For the latter we know from many different observables that our Universe is close to be flat, or in case it is not, the value of the curvature parameter must be really small, giving a huge curvature radius as of today $R_c(t_0) = a(t_0)/\sqrt{|k|} \equiv |k|^{-1/2} \gtrsim 60 \text{ Gpc}$ ([Planck Collaboration et al., 2020](#)), which would put a lower limit of least 2 million Milky Way stellar discs aligned. This quantity is important since if the Universe was closed and an observer could travel almost instantaneously with respect to the expansion rate of the Universe (i.e. $\Delta t \approx 0$), they would need to travel 2π times that distance to find themselves in the same spatial point.

However, it is the scale factor that draws more attention to the community and will be particularly important in the research presented here, as it encodes the expansion of the Universe and, if fully understood, would predict its future. One important feature of the scale factor of the Universe is that its value is encoded in the light through which we observe the astrophysical objects contained in our Universe. The most mechanical way to see this is by looking at the equations of motion in the FLRW

space-time,

$$\frac{dp^\mu}{d\lambda} + \Gamma_{\nu\sigma}^\mu p^\nu p^\sigma = 0, \quad (1.2)$$

for a photon. From the previous equation, we are interested in solving the time component of the moment of the photon, this is, $p^0 = E/c$. From the Christoffel symbols, $\Gamma_{\nu\sigma}^\mu$, we are then only interested in those with $\mu = 0$; and from these, only $\Gamma_{ij}^0 = \dot{a}/a g_{ij}$ are non vanishing, where the subindices ij refer to the spatial components and $\dot{a}$ is the derivative of the scale factor, a , with respect to time. At this point, it is convenient to use the null condition of the massless particle, $p_\alpha p^\alpha = 0$, to solve $g_{ij}p^i p^j = -g_{00}(p^0)^2$. The FLRW is written in coordinates as $g_{00} = -c^2$ and $g_{ii} = a(t)$. Knowing this, we can finally solve the equation for p^0 :

$$\frac{dp^0}{d\lambda} = -\frac{\dot{a}}{a}(p^0)^2 \Rightarrow p^0 \frac{dp^0}{dt} = -\frac{da}{dt} \frac{1}{a}(p^0)^2 \Rightarrow \frac{dp^0}{p^0} = -\frac{da}{a} \Rightarrow p^0 = \frac{E(t)}{c} \propto \frac{1}{a(t)} \quad (1.3)$$

In the first step we have applied the chain rule and used the definition of the four-momentum: $p^\mu = dx^\mu/d\lambda$. In the second step we have profited from the fact that the sole dependence is time. Finally, we solve the differential equation (third step) to obtain the fourth and final expression. Given that $E(t) = hc/\lambda(t)$, one can easily divide the solution above at different epochs, to find the useful relation

$$\frac{E(t_1)}{E(t_2)} = \frac{\lambda(t_2)}{\lambda(t_1)} = \frac{a(t_2)}{a(t_1)}. \quad (1.4)$$

At the present cosmological time we set $a(t = \text{now}) = 1$. In cosmology, it is customary to work with the quantity *redshift*, which is defined as the ratio between the scale factor today and the scale factor at any cosmological time; that is,

$$1 + z(t) = \frac{a(t = \text{now})}{a(t)}. \quad (1.5)$$

Taking this into account, we can rewrite the right-hand side of eq. (1.4) as

$$\lambda_{\text{obs}} = \lambda_{\text{em}} \frac{a_{\text{obs}}}{a_{\text{em}}} = \lambda_{\text{em}}(1 + z), \quad (1.6)$$

where "obs" stands for "observed" and "em" for "emitted"; for any light observed today coming from any source at any redshift, z . Note that at present time $z = 0$, and it tends to infinity as $a \rightarrow 0$.

The cosmological redshift serves as a fundamental metric in our ability to observe the expansion (scale) of the Universe, as it anchors observable astrophysical phenomena to distinct cosmological epochs. In particular, it becomes physically imprinted on the emission and absorption features of galaxies, which is the observational core of this thesis. But even beyond the scope of this work, and as a general cosmological practice, galactic redshifts are precisely determined by comparing the observed wavelengths of well-known spectral features against their laboratory or rest-frame values measured in the local Universe.

In traditional observational cosmology, however, the kinematic history and the dynamic evolution of this expanding spacetime are treated in the context of the dynamics imposed by the material or

energetic content of the Universe. Cosmological models arise naturally from these descriptions and imply different solutions for the functional description of the scale factor, $a(t)$. A way to see this is by working out the solution to the Einstein's Field Equations (EFE),

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (1.7)$$

where $G_{\mu\nu}$ is the Einstein tensor, $g_{\mu\nu}$ represents the components of the FLRW metric tensor (eq. 1.1), Λ is a cosmological constant, and $T_{\mu\nu}$ is the energy-momentum tensor describing the physical sources of matter and radiation. Lovelock's theorem (Lovelock, 1971) states that $G_{\mu\nu} + \Lambda g_{\mu\nu}$ is the most general symmetric, divergence-free tensor constructible in a four-dimensional spacetime from the metric tensor and its first two derivatives.

When we say that we aim to find the functional form of $a(t)$ from a certain matter-energy content of the Universe, we mean that we want to find the geometry, $G_{\mu\nu}$, that the specific form of the stress-energy tensor, $T_{\mu\nu}$, imposes. The latter can be defined simply by enforcing the symmetries previously imposed on the spacetime geometry via the Cosmological Principle on $T_{\mu\nu}$. Therefore, under the axioms of large-scale homogeneity and isotropy that led to the FLRW metric, the energy-momentum tensor cannot possess any privileged spatial directions or local overdensities. This restricts its structure to that of a perfect fluid evaluated in its comoving frame, which takes the diagonal form $T^\mu{}_\nu = \text{diag}(-\rho c^2, p, p, p)$, where $\rho \equiv \rho(t)$ is the mass density and $p \equiv p(t)$ is the isotropic pressure. Then, from the non-vanishing components of eq. (1.7), we derive the fundamental equations of modern cosmology, known as the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \quad \text{and} \quad (1.8)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}. \quad (1.9)$$

In the first Friedmann equation, k is the curvature of the spatial part of the metric (eq. 1.1). Also, it is natural to introduce the Hubble parameter,

$$H \equiv \dot{a}/a, \quad (1.10)$$

which quantifies the instantaneous expansion rate of the Universe. In fact, the description of this quantity is the main goal of Cosmic Chronometer (CC) studies in the literature (see Sec. 1.2) and the work that we present in the following chapters. The value of this parameter at present time is the Hubble constant, $H_0 \equiv H(t_0)$, which has recently garnered considerable attention due to the persistent and growing tension between early-Universe cosmological observables and local, model-independent measurements.

To see how the evolution of the Hubble parameter is determined by the properties of the cosmological fluid, we start by applying the covariant conservation of the stress-energy tensor ($\nabla_\mu T^{\mu\nu} = 0$). This

results in the continuity equation,

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0, \quad (1.11)$$

which can be solved for the different equations of state relative to different fluid components. If we write $p_i = w_i \rho_i c^2$ for any fluid (i) the continuity equation will have a functional solution dependent on the scale factor, as it becomes a differential equation that can be separated for ρ and a . In the equation of state, w_i is typically referred to as *the barotropic parameter* of the equation of state, and is associated with a different type of matter. For example, $w_m \equiv w_{\text{cold matter}} = 0$ refers to massive particles whose energy approximately corresponds to their comoving mass (i.e. non relativistic particles); $w_r \equiv w_{\text{radiation}} = 1/3$ refers to photons, neutrinos and relativistic matter in which the dominant portion of the energy comes from its kinetic energy. The general solution for a constant w_i is $\rho_k(a) = \rho_{i,0} a^{-3(1+w_i)}$, and it is fully defined once one sets the current value of the density for each kind of fluid, $\rho_{k,0}$. For the two types of matter that we directly observe in our surroundings, *cold* matter and radiation, the equations become $\rho_m(a) = \rho_{m,0} a^{-3}$ and $\rho_r(a) = \rho_{r,0} a^{-4}$.

These two types of fluid can be added together ($\rho = \rho_m + \rho_r$) and included in the first Friedmann equation, yielding

$$H^2 = \frac{8\pi G}{3} \left(\rho_{r,0} a^{-4} + \rho_{m,0} a^{-3} \right) - kc^2 a^{-2} + \frac{\Lambda c^2}{3} a^0. \quad (1.12)$$

We can realise that this expression can be simplified by eliminating the units on the right hand-side of the equation by factorising a H_0^2 out. Then, we define a critical density $\rho_{\text{crit},0} \equiv 3H_0^2 / (8\pi G)$ that is also factored out, and finally one makes up an *adimensional density* parameter for the $\propto a^{-2}$ term and another one for the $\propto a^0$ term. This is, we summarise the *densities* as

$$\Omega_m \equiv \rho_{m,0} \frac{8\pi G}{3H_0^2}, \quad \Omega_r \equiv \rho_{r,0} \frac{8\pi G}{3H_0^2}, \quad \Omega_\Lambda \equiv \frac{\Lambda c^2}{3H_0^2}, \quad \Omega_k \equiv -\frac{kc^2}{a_0^2 H_0^2}; \quad \sum_i \Omega_i = 1 \quad (1.13)$$

where we note that these Ω_i terms are constants, and all the time dependence is included in the scale factor. Both the eventual cosmological constant and the curvature of space take a density-interpretation in this formalism, and therefore have an impact in the shape of H . On the right hand side of (1.13) we can see the closure relation for the density parameters. From this point, we can vary our dependence from t to z , identify $a = 1/(1+z)$, and find the usual expression for the Hubble parameter in a Λ CDM cosmology:

$$\begin{aligned} H(a) &= H_0 \sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda} \quad \Leftrightarrow \\ H(z) &= H_0 \sqrt{\Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda}, \end{aligned} \quad (1.14)$$

which has been used as the reference model for cosmology for much of the last century. This form of the first Friedmann equation will be mostly referred to as *the H equation*, for its remarkable simplicity. However, for the current discourse, and in order to show its physical interpretation, redshift does not have the clear physical interpretation that time has for us. The exact analytical form of the scale

factor as a function of cosmic time in this Universe comes as the solution to the following differential equation,

$$dt = \frac{1}{H_0} \frac{da}{a \sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda}}, \quad (1.15)$$

which is a rewriting of eq. (1.14) and is far from simple. Indeed, what the community does is separating the different epochs of domination of each component, as in each case the resulting power law takes a different power. That is, as time passes, for a certain current status of the rate between the density constants, there will be clearly differentiated epochs in the expansion of the universe characterised by an $a(t)$ that depends on the dominating type of cosmic fluid. Actually, for all powers, p of a in eq. (1.15) that are different from 0, the general solution for a single-fluid universe takes the form

$$a \propto t^{-2/p} = t^{2/(3+3w)}, \quad (1.16)$$

where in the last step we have made $p = -3(1+w)$, as w corresponds to the parameter of the equation of state described right after the presentation of the continuity equation (1.11). That equation holds for whatever w different from -1 , with $w > -1$ representing a time power law in which the fluid with w closest to -1 ends up dictating the exact power. For a Λ -dominated Universe, that is $p = 0$ and $w = -1$, the solution is different, taking the form of an exponential function:

$$a \propto e^t. \quad (1.17)$$

Before acknowledging the tensions that observational cosmology has placed onto the cosmological model, it is instructive to see how various types of matter dominating this Universe-model imply different perspectives for its future evolution. For an illustrative purpose, in Fig. 1.1 several combinations of the density parameters draw different shapes for the dimensionless Hubble parameter, $E \equiv H/H_0$, and the scale factor, a , which has been obtained by numerically integrating eq. (1.15). We have chosen a unit system for the cosmic time such that the current age of the Universe, t_U , equals 1.

The current *benchmark* (Ryden, 2003) cosmology (red line), which predicts the fastest expansion, consists of a Universe that is compatible with being flat ($\Omega_k \sim 0$) where at present day cold matter and dark energy dominate ($\Omega_m \sim 0.3$, $\Omega_\Lambda \sim 0.7$), and where radiation, which would be the component that would further tame the growth of the scale factor, is negligible with respect to matter ($\Omega_r \ll \Omega_m$) today. The *Big Crunch* scenario (light blue line), also known as closed Friedmann Universe (Friedmann, 1922), on the other hand, predicts a Universe that eventually reaches a maximum expansion, $a_{\max}$, and then contracts to $a = 0$. This scenario is defined by a positively curved space ($k > 0$; three-sphere) in the presence of cold matter. Other single component models shown for comparison are the radiation-dominated Universe (Tolman, 1934), the matter-dominated Einstein-de Sitter Universe (Einstein & de Sitter, 1932) and the empty and curved Universe (Milne, 1932). These Universe models are currently interpreted with $\Omega_r = 1$, $\Omega_m = 1$ and $\Omega_k = 1$ within the cosmological framework that we have defined. All of these models share the same curve in the upper panel in Fig. 1.1, a

coincidence that comes from the derivation of eq. (1.14). From eq. (1.15), $a = (1+z)^{-1} = t^{-2/p}$ -for p the power of a in (1.14)-, which results in the powers cancelling in eq. (1.14) and leading to $E \propto t^{-1}$ in any single component Universe.

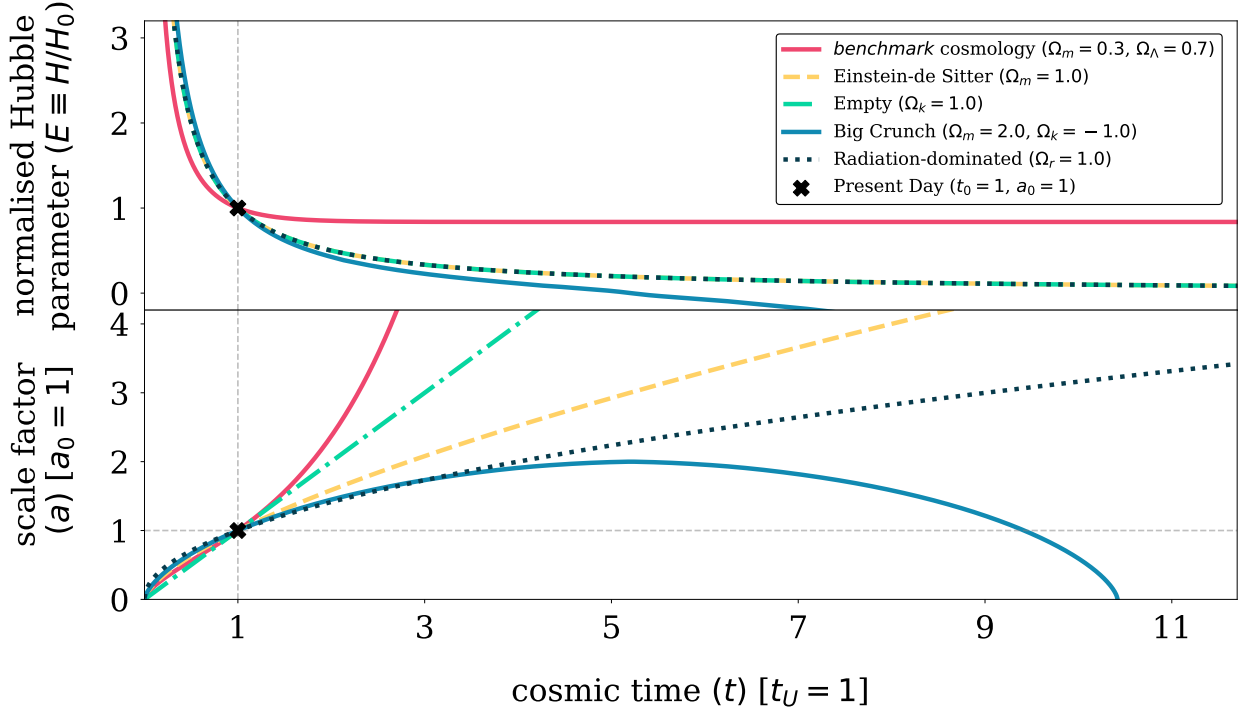


Figure 1.1: Sketch of the different types of Universe for different combinations of the density parameters. In the upper panel we show the normalised Hubble parameter $E \equiv H/H_0$, and in the lower panel the scale factor, a . Both are represented as a function of cosmic time, which we have normalised so that the age of the Universe today equals 1, $t_0 = 1$. Solid lines represent the *benchmark* cosmology (red) and a Big Crunch Universe (blue). Single component universes are shown as dashed (Einstein-de Sitter), dotted-dashed (empty) and dotted lines (radiation-dominated).

The upper panel is cut slightly below $E = 0$, and negative E values are only attained by Big Crunch models or more exotic models among which we can find oscillating Universes. For all the other Universes shown in Fig. 1.1, E becomes asymptotically flat (0 in single-component cases and $\sqrt{\Omega_\Lambda}$ for the *benchmark* cosmology) at $t \rightarrow \infty$.

1.1.2 The *benchmark* model and precision cosmology

The consolidation of the Λ CDM paradigm as the standard or *benchmark* model of cosmology stands as one of the most remarkable achievements of modern astrophysics. This consensus did not emerge from a single breakthrough, but rather through the historical convergence of independent, highly complementary observational probes over the past three decades. Up until the late 1990s, the prevailing cosmological consensus favoured an Einstein-de Sitter model ($\Omega_m = 1, \Omega_\Lambda = 0$) or an open, low-density matter universe ($\Omega_m \sim 0.3, \Omega_\Lambda = 0$). By the end of the decade, this view became

abruptly overturned by the seminal measurements of distant Type Ia Supernovae (SNIa) by the *Supernova Cosmology Project* (Perlmutter et al., 1999) and the *High-Z Supernova Search Team* (Riess et al., 1998).

SNIa act as excellent standardized candles due to the empirical relation between their peak luminosity and the width of their light curves (Phillips, 1993). By measuring the luminosity distance, $d_L \propto \sqrt{L/F}$, which is related to the proper distance, $d_p(t) = c a(t) \int_0^t dt' / a(t')$, by a $(1+z)$ factor (Etherington, 1933), and can be written in terms of the deceleration parameter,

$$q(z) \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} \sum_i \Omega_i E^{-2} (1+z)^{3(1+w_i)} (1+3w_i), \quad (1.18)$$

both teams found unambiguous evidence for an accelerating expansion. Notably, this signature is independent of any assumed cosmological model, as can be derived from the kinematic definition of the Hubble parameter (third term in eq. (1.18)). From the perspective of the cosmological models visited before, the observed acceleration of the Universe required the presence of a smooth, non-clustered energy component with negative pressure ($w \approx -1$), consistent with Einstein's cosmological constant, Λ . Conversely, introducing an exotic fluid in the Einstein equations with $p = -\rho$ recreates the effect of the cosmological constant in the Friedmann equation. For this reason, it has been given the name of dark energy, a hypothesised type of *fluid* with the exotic property of producing negative pressure.

While SNIa provided direct kinematic evidence for acceleration, the overall geometry and total energy budget of the Universe were firmly anchored by measurements of the Cosmic Microwave Background (CMB) anisotropies. The angular power spectrum of the CMB temperature fluctuations provides a snapshot of the acoustic oscillations in the baryon-photon fluid at the epoch of recombination ($z \approx 1100$). The physical scale of the sound horizon at that epoch, $r_s(z_*)$, constitutes a standard ruler calibrated by primordial plasma physics, and its angular size on the sky manifests as the position of the first acoustic peak in the power spectrum. Successive space missions, from the pioneering observations of WMAP (Bennett et al., 2013) to the definitive, high-precision maps from the *Planck* satellite (Planck Collaboration et al., 2014, 2020), constrained the spatial curvature to be incredibly tight around flatness ($\Omega_k = 0.0007 \pm 0.0019$), and the Planck Collaboration et al. (2020) TT+TE+EE+lensing results fixed $\Omega_m = 0.315 \pm 0.007$ and $\Omega_\Lambda = 0.685 \pm 0.007$.

The third pillar of the benchmark model is the measurement of Baryon Acoustic Oscillations (BAO) in the large-scale clustering of galaxies. The same acoustic waves that imprinted features on the CMB left a statistical signature in the distribution of matter, frozen at the drag epoch ($z_d \approx 1060$). This reveals itself as a preferred clustering scale at a comoving separation of $r_s(z_d) \approx 147.0$ Mpc. As a standard ruler in the late-time Universe, BAO measurements allow astronomers to isolate the expansion rate both along and across the line of sight. This maps out both the Hubble parameter $H(z)$ and the angular diameter distance $D_A(z)$ simultaneously via the Alcock-Paczynski distortion

effect (Alcock & Paczyński, 1979). Over the past two decades, large spectroscopic surveys such as the *Sloan Digital Sky Survey* (SDSS; Eisenstein et al., 2005), its subsequent iterations like BOSS and eBOSS (Alam et al., 2017; Zarrouk et al., 2021), and the ongoing *Dark Energy Spectroscopic Instrument* (DESI; Abdul Karim et al., 2026), have traced this standard ruler across a wide range of redshifts ($0.1 < z < 2.5$). These measurements broke structural degeneracies between Ω_m and Ω_Λ that were inherent to early supernova-only or CMB-only analyses.

However, in spite of its extraordinary explanatory power using only six free parameters, the standard Λ CDM framework is currently facing significant conceptual and observational challenges. These issues have forced the cosmological community to look critically at the model's foundational assumptions (Di Valentino, 2022; Perivolaropoulos & Skara, 2022; Abdalla et al., 2022; Peebles, 2025; Efstathiou, 2025). A persistent challenge in interpreting cosmological data is the presence of mathematical degeneracies, where different combinations of cosmological parameters yield identical observational signatures. For example, the "geometric degeneracy" in the CMB allows variations in Ω_m , Ω_Λ , and spatial curvature k to produce the exact same angular position θ_* for the acoustic peaks. This degeneracy can only be lifted by combining the CMB with late-time probes like BAO or weak gravitational lensing. Similarly, in uncalibrated standard candle observations (such as SNIa without a fixed absolute magnitude M), a strong degeneracy exists between the calculated value of H_0 and the intrinsic luminosity of the source.

The most severe empirical challenge is the ongoing *Hubble tension*, a statistically significant discrepancy between the value of H_0 inferred from early-Universe probes assuming Λ CDM and the value measured directly via the local distance ladder. For illustrative purposes, we show a snapshot of the situation of the discrepancy in the year 2019 in Figure 1.2 (adapted from Wong et al. (2020)), which highlights that the tension is robust and not driven by any single experiment. The early-Universe results from CMB (Planck Collaboration et al., 2020) are supported by the combination of Baryon Acoustic Oscillations (BAO; Alam et al., 2017), Dark Energy Survey (DES) clustering, and Big Bang Nucleosynthesis (BBN Cooke et al., 2018) data (Abbott et al., 2018), which yields an independent and equally low estimate of $H_0 = 67.4^{+1.1}_{-1.2} \text{ km s}^{-1} \text{ Mpc}^{-1}$. The late-Universe measurements, up to then starred by the distance ladder of cepheids (SH0ES; Riess et al., 2019), received the contribution of gravitational time-delay cosmography from the H0LiCOW collaboration (Wong et al., 2020). By measuring the time delays between lensed images of distant quasars, they obtained a high value of $H_0 = 73.3^{+1.7}_{-1.8} \text{ km s}^{-1} \text{ Mpc}^{-1}$, in excellent agreement with the SH0ES results.

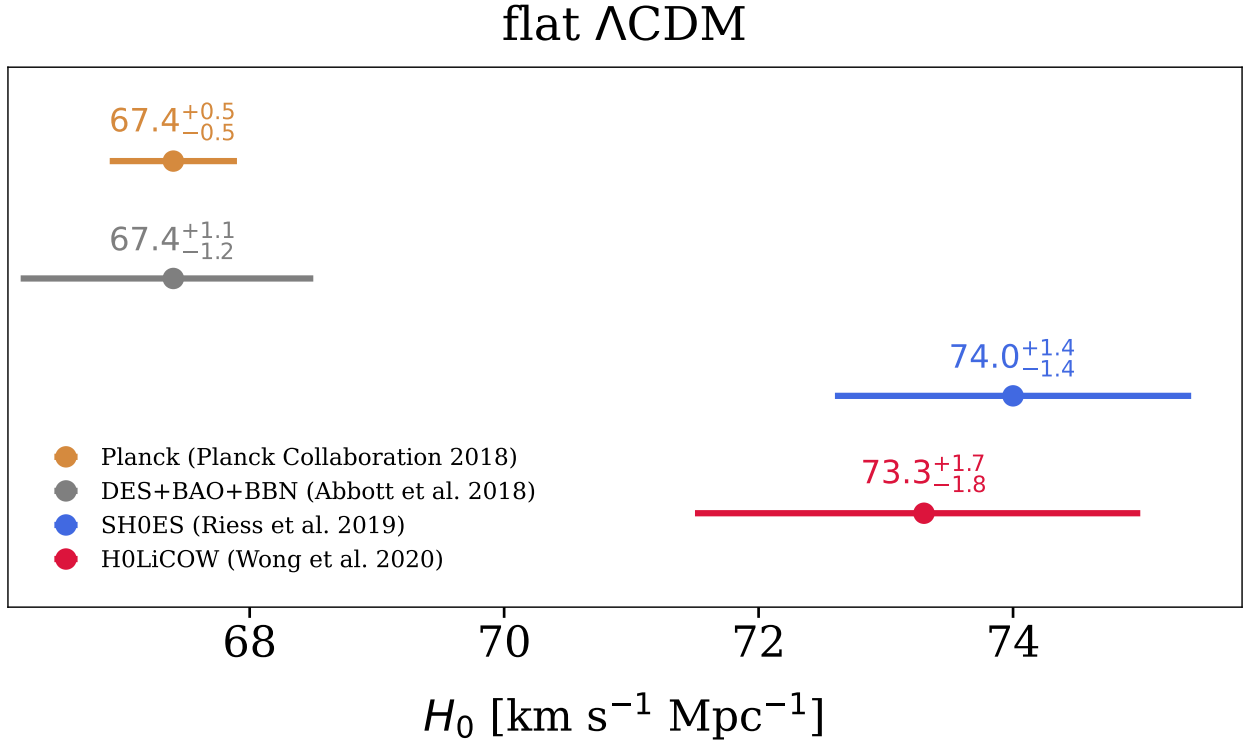


Figure 1.2: Summary of the most cited measurements of H_0 , including CMB (Planck Collaboration et al., 2020), DES (Abbott et al., 2018), distance ladder (Riess et al., 2019) and time delay cosmography (Wong et al., 2020). This figure is a reconstruction of Figure 12 from Wong et al. (2020).

As of today, the *Planck* collaboration result in Fig. 1.2 still stands, while local measurements using Cepheid-calibrated SNIa by the SH0ES collaboration yield $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Riess et al., 2022). With the statistical significance of this disagreement exceeding the 5σ discovery threshold, the cosmology community increasingly points towards either unidentified, deeply entrenched systematic uncertainties or new physics operating beyond the baseline Λ CDM model.

1.1.3 Going beyond the Λ CDM model and independent observables

If these tensions are not caused by undetected systematic errors, they point directly to new physics, including Early Dark Energy acting before recombination (Poulin et al., 2019; Kamionkowski & Riess, 2023), modifications to neutrino physics (Kreisch et al., 2020; Efstathiou, 2021), or dynamical dark energy models with a time-varying equation of state $w = w(z)$ (Di Valentino et al., 2021). For a recent, comprehensive ranking of the viability of these and other proposed solutions to the Hubble tension, see Schöneberg et al. (2026).

A classical parametrisation of the time-varying equation of state comes from the seminal works by Chevallier & Polarski (2001) and Linder (2003), taking the phenomenological form

$$w(z) = w_0 + w_a \frac{z}{1+z}, \quad (1.19)$$

which is also known as w_0w_a CDM model. One particular solution that will be useful further into this work is what happens when w_a is identically or at least significantly close to 0. The resulting flat w CDM cosmology can be linked to a purely kinematic approximation of $H(z)$ using the deceleration (eq. 1.18) and jerk,

$$j \equiv \frac{\ddot{a} \dot{a}^2}{\dot{a}^3} = \frac{\ddot{a}}{aH^3}, \quad (1.20)$$

parameters. The full flat w_0w_a CDM model can be linked, in turn, to a further kinematic expansion including the snap, s , defined analogously to q (eq. 1.18) and j (eq. 1.20).

This complex landscape underscores the crucial need for a diverse set of direct, model-independent measurements of the expansion history. To overcome current systematic barriers, the cosmological community has actively explored a rich array of emerging, non-standard probes. As detailed in the comprehensive review by Moresco et al. (2022), several alternative techniques are currently being leveraged to exploit large datasets from currently available and upcoming surveys.

Among these, new standard candle candidates have gained significant traction. These include the non-linear relation between ultraviolet and X-ray luminosities in Quasars (QSOs) (Risaliti & Lusso, 2015, 2019; Salvestrini et al., 2019; Solomon & Stojkovic, 2022); the correlation between the luminosity and the velocity dispersion of Balmer lines in HII galaxies and giant HII regions (Chávez et al., 2014, 2025; Terlevich et al., 2015); and various distance-dependent correlations operating in Gamma-Ray Bursts (GRBs) (Yonetoku et al., 2004; Ghirlanda et al., 2005, 2008, 2010; Amati & Della Valle, 2013).

Beyond standard candles, the cosmological community has harnessed independent geometric and structural probes to break parameter degeneracies. Among these, gravitational wave observations of compact binary coalescences have emerged as standard sirens, offering a completely novel and calibration-free anchor for the cosmic distance scale (Schutz, 1986; Holz & Hughes, 2005; Abbott et al., 2017), attracting a lot of attention in recent years. Another probe that has recently attracted considerable attention is time-delay cosmography, able to produce competitive measurements of H_0 as shown in Fig. 1.2 (Wong et al., 2020). Other probes are also reviewed, including the use of cosmic voids, through their void size function and void density profile; and the intensity mapping of neutral Hydrogen (HI), through its 3D power spectrum.

So far, we have reviewed several emerging observables that can complement the traditional probes utilized by large-scale cosmological collaborations. Many of these novel techniques, much like their classic counterparts, are anchored by a physical attribute designated as a *standard*. For example, we have introduced *standard candles*, which rely on an intrinsic luminosity or a predictable flux ratio that is physically well-understood, and from it, a luminosity distance can be directly inferred. Likewise, we have talked about BAOs as *standard rulers*, a primordial spatial scale to map the angular diameter distance across cosmic time. We have also mentioned *standard sirens* to refer to the absolute

amplitude of the gravitational signal that emerges from the merging of massive compact objects. Any of these *standards* are used to measure cosmological distances, whether these be luminosity distances or angular diameter distances. However, there is a quantity that appears directly in eq. (1.14) and more explicitly in eq. (1.15) and has not been reviewed yet: cosmic time.

Properly speaking, it is not absolute time that enters into those equations but differential time. Also, directly measuring differential times has the advantage of putting a direct constraint on the Hubble parameter, which can be written in a very useful differential form,

$$H \equiv \frac{\dot{a}}{a} = \frac{1}{a} \frac{da}{dt} = (1+z) \frac{-1}{(1+z)^2} \frac{dz}{dt} = -\frac{1}{1+z} \frac{dz}{dt}, \quad (1.21)$$

in contrast to distance measurements that are an integrated quantity of H^{-1} . For these reasons, an object whose age can be measured at different redshifts can be called as a *standard chronometer*, although the community has preferred *cosmic chronometer* as a name. In terms of measurements, the community has obtained several direct estimations of $H(z)$ during the past two decades using this approach. In chronological order we can cite [Simon et al. \(2005\)](#); [Stern et al. \(2010\)](#); [Moresco et al. \(2012\)](#); [Zhang et al. \(2014\)](#); [Moresco \(2015\)](#); [Moresco et al. \(2016\)](#); [Ratsimbazafy et al. \(2017\)](#); [Borghi et al. \(2022b\)](#); [Jiao et al. \(2023\)](#); [Tomasetti et al. \(2023\)](#); [Jimenez & Loeb \(2002\)](#); [Loubser et al. \(2025\)](#); [A25](#); [Loubser \(2025\)](#) and [A26](#).

Cosmic chronometers should not be confused with cosmic clocks, which would work as lower bounds to the age of the Universe, t_U , as they would represent the age of the oldest astrophysical objects in the Universe. This quantity can be directly measured and would put a constraint on the integral of eq. (1.15) that would help break degeneracies between the dynamical part of the Hubble parameter, $E(z)$, and the Hubble constant, H_0 . The absolute age of the Universe has been constrained recently with data from local globular clusters (GCs) in our vicinity (e.g. [O'Malley et al., 2017](#); [Valcin et al., 2025](#)), but also at high redshift thanks to resolved lensed clusters and progenitors of globular clusters (e.g. [Adamo et al., 2023, 2024](#); [Tomasetti, Elena et al., 2025](#)).

1.2 Cosmic Chronometers

The seminal paper of [Jimenez & Loeb \(2002\)](#) is widely seen as the kick-off for the research with CCs as it established a clear pathway to use spectroscopically dated passively evolving galaxies. That is, using the spectra of passively evolving galaxies to estimate their ages, which would then be used to put direct constraints on the Hubble parameter. More specifically, it consisted in using N statistically representative groups of passively evolving ETGs at N somewhat different redshifts. For illustrative purposes, we consider here the simplest case of just two samples, drawn from two disjoint redshift intervals; we will refer to these as the higher- z and lower- z sample of the pair, keeping in mind that the method generalises to any number $N > 2$ of samples, as extensively used in later chapters. This statistical procedure can produce several independent measurements of H . Indeed, following the

first order discretisation of eq. (1.21), we see that

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt} \approx -\frac{1}{1+z} \frac{\Delta z}{\Delta t} = -\frac{1}{1+(z_1+z_2)/2} \frac{z_1-z_2}{t_1-t_2} \equiv H_{12}(z=(z_1+z_2)/2), \quad (1.22)$$

where for each pair of reliable statistical samples of CCs living at average redshifts z_1 and z_2 , with average ages of t_1 and t_2 , we obtain a single H measurement at $z=(z_1+z_2)/2$, defined as H_{12} . We note that by this definition we have changed the notation t , used in cosmology when referring to cosmic time (i.e. Fig. 1.1), to galaxy ages. Sometimes this causes some unrest in the community but the lack of a symbol that better represents age than t and its simplicity make its use convenient. This standard is kept in both A25 and A26. The reason why the differential relation can be applied for passively evolving galaxies as CCs is the hypothesis that on average (statistically speaking), these systems formed at the same cosmic time, which is an additive factor that cancels out in Δt : $\Delta t = t_1 - t_2 = \text{age}_1 + t_{\text{form}} - (\text{age}_2 + t_{\text{form}}) = \text{age}_1 - \text{age}_2 = \Delta \text{age}$. For this, we will speak of ages using the notation t .

The strength of using passively evolving stellar populations as CC candidates is a central discussion point in Jimenez & Loeb (2002). They argue that relative ages, which is the quantity that enters in eq. (1.22), avoid the systematics related to absolute age determinations, which is a well known problem in the dating of passively evolving stellar populations (Worthey, 1994; Ferreras et al., 1999). Also, they test the resilience of the method against contamination with recently quenched galaxies without a counterpart in the higher-redshift group (progenitor bias; van Dokkum & Franx, 1996) and the exit, from the lower-redshift sample, of some counterparts of the higher-redshift sample due to spontaneous bursts of star-formation, which would be in theory triggered by some wet merger. This analysis can be observed in Fig. 1.3, reproduced from Jimenez & Loeb (2002), where they explained the results from the Kolmogorov-Smirnov (KS) test that they applied to recover the inferred age shift after Poissonian alterations of the low redshift distributions were applied. The original distribution of ages at high redshift, shown on the top panel of the figure, was previously shifted 0.15 and 0.30 Gyr, emulating the effect of the passive evolution. Then, on these two distributions the Poissonian noise is included (the effect on the 0.3 Gyr shifted distribution is what is shown on the middle panel). Finally, the KS test returns an inferred age shift, and by repeatedly doing the whole process multiple times a probability for getting a certain inferred age can be retrieved, which is what is shown in the lower panel of Fig. 1.3. They also added shaded regions to illustrate the uncertainty with respect to the *real* shift that is allowed and still different dark energy models could be distinguished using CC data.

Notably, the former example used groups of 75 galaxies, which is a particularly reduced number of sources. Although they assume high individual signal-to-noise (S/N) levels (≈ 30), this cannot be guaranteed for all datasets that would represent good targets for CC studies. In particular, this is the case of the ones presented in this thesis. Moreover, the reduced number of sources could have an important impact if it comes paired with an inhomogeneous sampling of galaxies living in

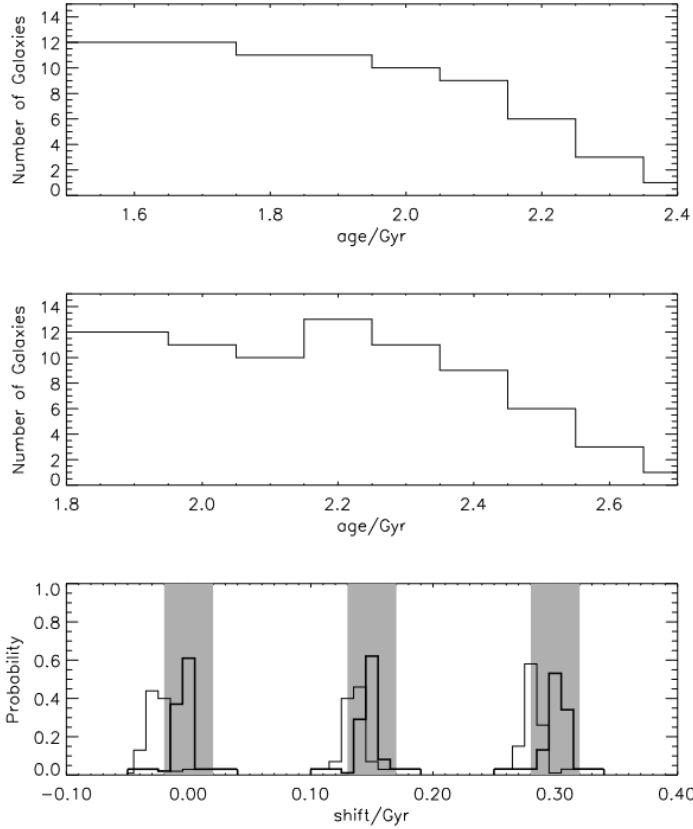


Figure 1.3: View of Figure 4 from Jimenez & Loeb (2002). In the upper panel we observe a simulated distribution of ages for a group of passively evolving galaxies at a given redshift. The middle panel shows the result of a Poissonian modification of the age bins after the whole distribution has been transported +0.3 Gyr. The bottom panel shows the probability of detecting 0.00, 0.15 and 0.30 Gyr age shifts after the Poissonian modification. Individual checks are obtained performing a KS test and repeated Monte-Carlo realisations produce the probability distribution. Shaded regions represent age estimations that would help distinguish between different dark energy models.

dense/sparse environments in the different redshift groups (Clemens et al., 2006; Clemens et al., 2009; Thomas et al., 2010). However, recent works (Liu et al., 2016) have converged to the observation that when only selecting the *reddest envelope* of passively evolving galaxies (the most massive ones) the effect of the environment becomes less dominant.

Beyond mitigating environmental biases, the selection of massive, passively evolving galaxies addresses a more fundamental requirement of the CC method: the validity of the assumption, central to eq. (1.22), that the galaxies contributing to a given redshift bin formed their stellar populations at (approximately) the same cosmic time, t_{form} , and subsequently evolved passively. This assumption is best approximated by galaxies whose star formation was both early and short-lived, i.e. those whose stellar populations most closely resemble a single stellar population. As shown by Thomas et al. (2010), more massive early-type galaxies host systematically older stellar populations that formed over shorter timescales and with higher formation redshifts than their lower-mass counterparts, which instead display more extended star-formation histories and a larger spread in t_{form} (see also the review by Moresco (2024) for an updated discussion in the context of CCs). Selecting the high-mass end of the passively evolving population therefore reduces the intrinsic scatter in formation time within each redshift bin, bringing the sample closer to the idealised, coeval population implicitly assumed in the derivation of eq. (1.22), and consequently improving the reliability of the individual CC candidates used to build H_{12} .

This mass-based selection, which, in practice, will be controlled via a constraint in stellar velocity dispersion (see Sec. 2.2.1), following the known relation between this quantity and the dynamical mass of the galaxy (Faber & Jackson, 1976), also plays a complementary role in controlling progenitor bias. Because the most massive ETGs are, by construction, the oldest and least likely to have recently transitioned from a star-forming phase, restricting the sample to this population reduces the risk of contamination by recently quenched galaxies lacking a counterpart at higher redshift. This effect is further suppressed when CC subsamples are constructed in narrow, closely spaced redshift bins, as adopted throughout this thesis: the shorter the time baseline Δt spanned between the bins, the smaller the cosmic time available for a galaxy to undergo quenching (or a merger-triggered starburst) and thus escape or enter the passively evolving population between the two epochs being compared (Carollo et al., 2013; Belli et al., 2015).

The strength of Cosmic Chronometers does not only lie in direct estimates of $H(z)$, but also extends to their potential in constraining the nature of dark energy. As discussed in recent literature, cosmological analyses often consider dynamical dark energy models (e.g. eq. 1.19) to alleviate some of the tensions within the Λ CDM model. As shown by Jimenez & Loeb (2002), cosmic chronometers are particularly sensitive to an evolving equation of state $w(z)$, since derivatives of the expansion history encode local information about $w(z)$ without requiring additional line-of-sight integrations beyond those already contained in $H(z)$. In this context, the second derivative of redshift with respect to cosmic time can be expressed in terms of $w(z)$ and dz/dt , effectively recasting the dependence on the integrated expansion history into local observables.

This relation is made explicit in their equation (4), which reads

$$\frac{d^2z}{dt^2} = \frac{(dz/dt)^2}{(1+z)} \left(\frac{5}{2} + \frac{3}{2}w(z) \right) - \frac{3}{2}\Omega_m(1+z)^4w(z), \quad (1.23)$$

in our notation. This expression can be understood as a purely kinematic consequence of the Friedmann equations, in which the dynamics of dark energy enter through $w(z)$. It is closely related in spirit to the concept of redshift drift, first proposed by Sandage (1962) and later formalised by Loeb (1998), although here the emphasis is not on the direct temporal evolution of the observed redshift, but rather on rewriting cosmological dynamics in terms of derivatives with respect to cosmic time. Even if this is not an intended aim of this thesis, we emphasise the power of this somewhat less considered part of the work by Jimenez & Loeb (2002), in what refers to the measurements and estimations derived from their ideas, as we believe that the cosmographic approach that we are about to introduce can be used to put direct constraints on $w(z)$.

Our approach consists in leveraging the large spectroscopic datasets that are already available (Abdul Karim et al., 2026, i.e. DESI) to obtain a redshift-extended estimation of $H(z)$, and the actual observable are the inferred $t - z$ relations for different mass regimes of CCs. Then, $\hat{t}(z)$ becomes the theoretical model to fit, and therefore one could directly reconstruct d^2t/dz^2 or provide a local

estimation, i.e. $d^2t/dz^2|_{z=z_0}$.

Fig. 1.4 compiles all CC-based $H(z)$ measurements published to date, reaching redshifts of $z \gtrsim 2.0$ (results presented later in this thesis are deliberately excluded from this compilation). The different colours encode the diverse methodologies that have been employed in the literature to estimate the ages, or differential ages, of the CC samples: full spectral fitting (FSF; Simon et al., 2005; Stern et al., 2010; Zhang et al., 2014; Ratsimbazafy et al., 2017; Jiao et al., 2023; Tomasetti et al., 2023, 2026), the D4000 break strength (Moresco et al., 2012; Moresco, 2015; Moresco et al., 2016; Loubser et al., 2025; Loubser, 2025), and, more recently, the direct modelling of the intensity of individual spectral absorption lines (Lick-index based methods). Borghi et al. (2022a) remains, to date, the only work that has exploited this latter approach to derive an $H(z)$ measurement from CCs. A further point was added to the figure by Jimenez et al. (2023), exploiting a neural network, trained with spectroscopic ages from the LEGA-C survey (van der Wel et al., 2021), to estimate ages from independent photometric data from the COSMOS2015 catalog (Laigle et al., 2016). A Λ CDM $H(z)$ from the Planck Collaboration et al. (2020) results has been plotted in the background to show how independent CC data match the benchmark cosmology.

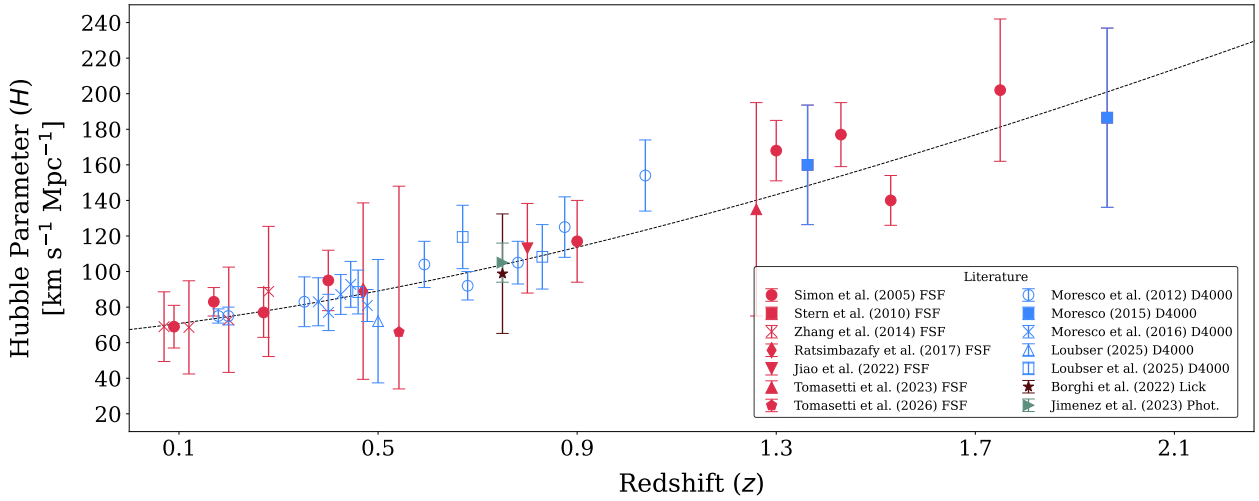


Figure 1.4: Compilation of CC-based $H(z)$ measurements available in the literature up to $z \gtrsim 2.0$, excluding the results presented later in this thesis. Marker colours indicate the method used to estimate ages or differential ages: full spectral fitting (FSF, red; Simon et al., 2005; Stern et al., 2010; Zhang et al., 2014; Ratsimbazafy et al., 2017; Jiao et al., 2023; Tomasetti et al., 2023, 2026), the D4000 break (blue; Moresco et al., 2012; Moresco, 2015; Moresco et al., 2016; Loubser et al., 2025; Loubser, 2025), and Lick-index line-intensity fitting (dark red star; Borghi et al. 2022a), together with the photometric estimate of Jimenez et al. (2023) (green). The dashed line shows a reference Λ CDM expansion history.

We note that observations of CCs from the SDSS ($z < 0.5$) and DESI ($0.3 < z < 1.2$) spectroscopic

datasets would contribute to the *low*-redshift end of the diagram ($z \lesssim 1.0$). Even if DESI data formally extend up to $z \approx 1.2$, we restrict our analysis to $z \approx 0.9$, due to limitations on the number of observed spectral lines at higher redshifts. Looking beyond the redshift range currently accessible with DESI, upcoming Euclid spectroscopic data (Scaramella et al., 2022) hold significant potential to further populate the CC Hubble diagram at $z \gtrsim 1$, where measurements remain comparatively sparse and reliant on FSF or D4000-based approaches (see the review by Moresco et al. (2022) for a comprehensive discussion of current and forthcoming CC prospects).

1.2.1 Cosmography

The finite difference approximation of the Hubble parameter (eq. 1.22) does not intend to directly provide continuous functional information for a sample. In order to do so without introducing the programmatic bias of a predefined parametric background cosmology (such as Λ CDM), a more general mathematical approach is required. A powerful alternative lies in the cosmographic expansion of the cosmological parameters, which relies solely on the assumption of the cosmological principle and a kinematic description of the spacetime metric.

Historically, standard cosmography has relied on a Taylor expansion of the Hubble parameter centred around the local universe ($z = 0$), a framework that successfully facilitated early model-independent mappings of distance indicators at low redshifts (e.g. Capozziello, S. & Izzo, L., 2008). However, as expanded upon by Capozziello et al. (2011) and Bamba et al. (2012), extending these standard expansions out to intermediate and high redshifts introduces severe mathematical challenges, primarily involving the slow convergence or absolute divergence of the Taylor series when $z \geq 1$. While alternative reparametrizations, such as the y -redshift variable, $y = z/(1+z)$, were developed to improve series performance over extended intervals (Cattoën & Visser, 2007), higher-order kinematic terms still suffer from severe parameter degeneracies and remain poorly constrained when forced to anchor at $z = 0$.

Since the works presented in this thesis aimed at being useful to the cosmology community, we ended up choosing a more refined cosmography for intermediate redshifts, $0.4 \leq z \leq 1.0$, where DESI provided LRG spectra (Zhou et al., 2023). This cosmography consists of a pivotal redshift concept inaugurated by Fazzari et al. (2025), which is suitable for extended redshift CC works at any redshift. However, classic cosmography, even for the typical redshift parametrisation, can provide a good approximation to any fiducial form of the Hubble function at low redshifts, as shown in Fig. 1.5, which is a reconstruction of Figures 1 and 2 from Capozziello et al. (2011). Therefore, the classical Taylor series around $z = 0$ is appropriate when working with SDSS spectra A25.

As observed, while higher-order series can better mimic the true behaviour of the underlying physics at very low redshifts, they diverge significantly faster than lower-order expansions once the local

regime is exceeded. This discrepancy is particularly pronounced when the true analytical $H(z)$ stems from a framework more complex than a Λ CDM cosmology (e.g., a CPL parametrisation), yielding residuals that fast exceed 20% in the $z \gtrsim 1$ regime. This pathological divergence highlights the mathematical risk of over-fitting high-order kinematic parameters when the series expansion is decoupled from its physical radius of convergence. In general, the $z \lesssim 0.5$ is well-reproduced by the [Capozziello et al. \(2011\)](#) expansion for either underlying cosmology (Λ CDM or CPL), while the $0.5 < z < 1.0$ interval can suffer from approximation defects of up to a 20% with an hypothetical underlying CPL physics model. This fact supports the choice of the [Fazzari et al. \(2025\)](#) inspired pivotal-redshift cosmography to operate in this redshift range (see Figs. 1.6 and 1.7 and the discussion therein).

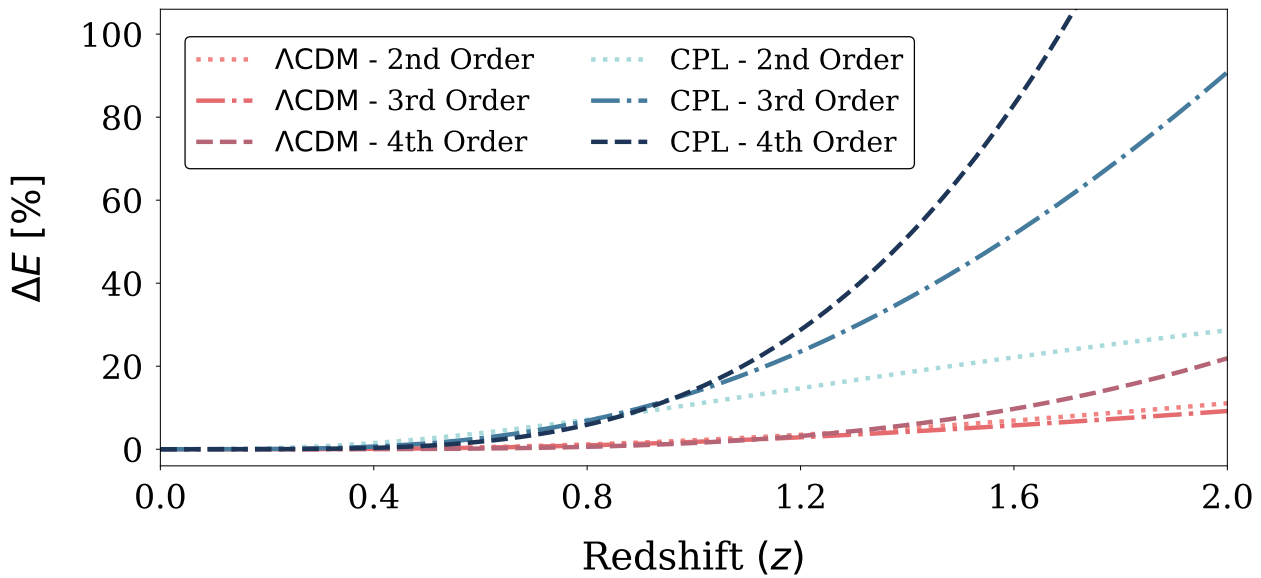


Figure 1.5: Reconstruction of Figures 1 and 2 from [Capozziello et al. \(2011\)](#). We present the residuals from the Taylor series expansion of the Hubble function at different truncation orders (2nd: dotted line, 3rd: dashed-dotted line, 4th: dashed line) for the Λ CDM (red) and CPL (blue) parametrisations of $H(z)$ as underlying physics.

The objective of the cosmographic framework adopted in this thesis is to reconstruct the absolute expansion history of the universe, parametrised by the Hubble function $H(z)$, in a strictly model-independent manner. However, within our observational pipeline, the fundamental physical observables are not direct cosmic expansion rates, but rather the absolute stellar ages, t , extracted from the absorption spectra of passively evolving galaxies.

To bridge this gap, we assume that the entire cosmological dependence is encoded within the differential evolution of cosmic time with redshift, which can be obtained by solving eq. (1.10) for t . Our formulation also takes into account the downsizing scenario of elliptical galaxy formation, further detailed in section 1.3.2. This baseline assumes that the underlying physics encoded in

the $t(z)$ map can be separated in two independent components: a vertical relation that accounts for the scaling relations of age with respect to galaxy velocity dispersion, $t_{\text{archaeology}}(\sigma)$, and the passive ageing of the stellar population $t_{\text{cosmology}}(z)$, which we will call Δt , since it corresponds to the difference in age between two redshifts¹. After this, we can model our age estimations for a galaxy (population) at redshift z and with stellar velocity dispersion σ as

$$t(z, \sigma) = t_{\text{arch}}(\sigma) + \Delta t(z) = t_{\text{arch}}(\sigma) - \int_0^z \frac{dz'}{(1+z')H(z')} , \quad (1.24)$$

where on the right side we have integrated eq. (1.10) as previously mentioned, and $t_{\text{arch}}(\sigma)$ is known at present time, this is, at $z = 0$. However, for any cosmographic model developed around any redshift other than zero (i.e. z_0), there will be a pivotal redshift scaling relation such that $t_{\text{arch}}(\sigma) \rightarrow t_{\text{arch}}(\sigma; z_0)$ and, coherently, $\Delta t(z) \rightarrow \Delta t(z; z_0) \equiv t(z) - t(z_0)$. Note that for $z > z_0$ we have $\Delta t < 0$, while for $z < z_0$, $\Delta t > 0$.

The cosmographic approach is then implemented in the Hubble function that appears in the integral in eq. (1.24). In A25 we applied a numerical integration to obtain $\Delta t(z)$, while in A26 we followed the idea put forward in Fazzari et al. (2025), integrating the Taylor series of $((1+z)H(z))^{-1}$ in order to obtain an analytical polynomial model for $t(z)$. The cosmographic model used in A25 was taken as the y -redshift parametrisation of H as in Capozziello et al. (2011) and taken to second order in $y \equiv z/(1+z)$; this is,

$$E(z) \equiv \frac{H(z)}{H_0} \approx 1 + (1+q_0) \frac{z}{1+z} + \left(1 + \frac{j_0}{2} + q_0 - \frac{q_0^2}{2}\right) \left(\frac{z}{1+z}\right)^2 . \quad (1.25)$$

Working with higher redshift data (A26) can put stress on the validity of the Taylor series, forcing the inclusion of an estimation of the residuals of the approximation as a non-negligible contribution to the error budget. In order to avoid this, the pivotal zero-redshift technique proposed by Fazzari et al. (2025) offers an interesting and practical alternative. The most significant drawback of this concept is the loss of a direct estimation of H_0 , a quantity that, if directly measured, would put CCs in direct contention with other well known observables (see Fig. 1.2). Indeed, a central idea behind the $[z_0 = 0]$ -anchored cosmography applied in A25 was providing a direct estimation of H_0 , among other objectives. In exchange for this, the constraining power of the cosmographic fit is extended to both sides around the pivotal redshift. Actually, Fazzari et al. (2025) emphasise the reduction in statistical uncertainty around the pivot, a quality that is amplified when the pivot is chosen to lie near the centre of the redshift distribution of the CCs, providing less stringent confidence intervals for growing $\Delta z \equiv z - z_0$. This means that a well-chosen pivotal redshift can provide optimised confidence intervals extended to both redshift sides ($\Delta z \gtrless 0$), as opposed to the power of the cosmographic fit. A perfect visual example of this is shown in Figure 4 from Fazzari et al. (2025), brought here as Fig. 1.6, where we can see how a $z_0 = 0$ pivotal cosmography, equivalent

¹Same notation as in A26.

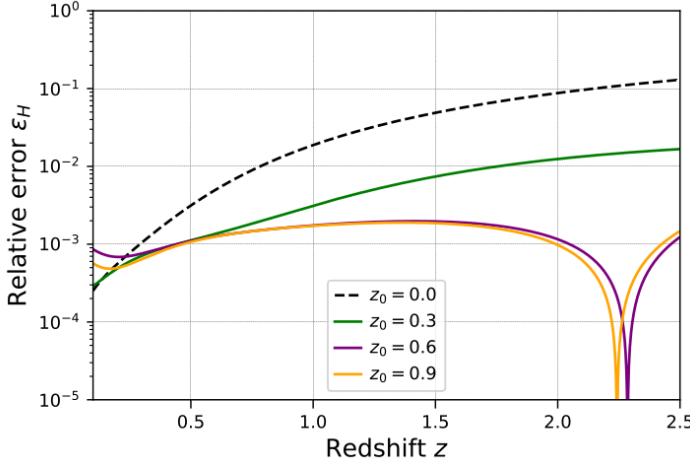


Figure 1.6: Partial view of Figure 4 from [Faz-zari et al. \(2025\)](#) (relevant part for the Taylor approximation of the Hubble function). On the vertical axis, the percentual residual of the cosmography with respect to the fiducial cosmology (Λ CDM). Colour solid lines represent the pivotal cosmography at $z_0 \neq 0$, while the dashed black line represents the classic cosmography at $z_0 = 0$.

to a classic model centred in $z = 0$, obtains the biggest residuals with respect to a Λ CDM cosmology. $z_0 \neq 0$ cosmographies improve the performance and residuals stay well below 1% across all redshifts, particularly for $z_0 \gtrsim 0.5$, although this is highly dependent on the redshift distribution of the data. One can easily observe that the dashed black line representing the classic cosmography corresponds to the fourth order Λ CDM Taylor series represented as a red dashed curve in Fig. 1.5².

In the following lines, which constitute a thorough extension of the theoretical derivation presented in [A26](#), we build the full cosmographic derivation of a $t(z)$ model for any object that evolves passively with the Universe. The boundary condition, as previously discussed, will be that at $z = z_0$ the age of the object is t_{z_0} . In practice, the main difficulty lies in expressing the integrand on the right-hand side of Eq. (1.24) as a Taylor series. In other words, we aim at finding the coefficients b_m in

$$\frac{1}{1+z} \frac{1}{H(z)} = \sum_m b_m (z - z_0)^m. \quad (1.26)$$

Since the full parametrisation will depend on derivatives of the Hubble function, namely q, j, s, l (i.e. see eqs. (1.18) and (1.20)), it all becomes much less cumbersome if one develops a Taylor series for H^3 . Therefore, we seek expressions for the coefficients a_n in terms of q, j, s and l in

$$H(z) = \sum_n a_n (z - z_0)^n, \quad (1.27)$$

so that we can write $b_m(q, j, s, l)$. However, this is straightforward, as it is the Taylor series of H ,

$$H(z) = \sum_n \frac{H^{(n)}(z_0)}{n!} (z - z_0)^n \Rightarrow a_n = \frac{H^{(n)}(z_0)}{n!}, \quad (1.28)$$

where we have written $^{(n)}$ to refer to the n -th derivative of H with respect to z . Knowing this, and using the definitions of q (eq. 1.18), j (eq. 1.20) and the immediate extension of these to s and l , we

²Note the difference in the scale, in Fig. 1.5 is linear while in Fig. 1.6 is logarithmic.

³We adopt some changes in the notation with respect to [A26](#) for better clarity. In particular, the a_n notation used here for the coefficients of the Taylor series of H is used in [A26](#) for the coefficients of the Taylor series of $(1+z)H$.

can write a closed form for the a_n . We note that

$$\zeta^k \equiv \frac{1}{aH^k} \frac{d^k a}{dt^k}, \quad (1.29)$$

with $\zeta^k \equiv \{j \ (k=3), s \ (k=4), l \ (k=5) \dots\}$. The derivatives of H evaluated at an arbitrary redshift z are

$$\frac{dH}{dz} = H \frac{1+q}{1+z}, \quad (1.30)$$

$$\frac{d^2 H}{dz^2} = H \frac{j-q^2}{(1+z)^2}, \quad (1.31)$$

$$\frac{d^3 H}{dz^3} = H \frac{3q^3 + 3q^2 - (4q+3)j - s}{(1+z)^3}, \text{ and} \quad (1.32)$$

$$\frac{d^4 H}{dz^4} = H \frac{l + (8+7q)s + (12+25q^2+32q)j - 4j^2 - 7qj - 15q^3 - 15q^4 - 25q^2j}{(1+z)^4}. \quad (1.33)$$

When we evaluate these at $z = z_0$ we obtain the a_n terms, which consist in dividing (1.30), (1.31), (1.32) and (1.33) by $1! = 1$, $2! = 2$, $3! = 6$ and $4! = 24$ respectively. For simplicity, it is also customary to perform the following identifications: $H_{z_0} \equiv H(z_0)$, $q_{z_0} \equiv q(z_0)$, $j_{z_0} \equiv j(z_0)$, $s_{z_0} \equiv s(z_0)$ and $l_{z_0} \equiv l(z_0)$. Given the presence of $(1+z_0)^n$ in the denominator of each of the derivatives, it becomes convenient to adopt the x -redshift, $x \equiv (z - z_0)/(1 + z_0)$, so that we can write

$$H(x) = H_{z_0} \left[1 + \tilde{a}_1 x + \tilde{a}_2 x^2 + \tilde{a}_3 x^3 + \tilde{a}_4 x^4 + \mathcal{O}(x^5) \right], \quad (1.34)$$

where

$$\tilde{a}_1 = 1 + q_{z_0}, \quad (1.35)$$

$$\tilde{a}_2 = j_{z_0} - q_{z_0}^2, \quad (1.36)$$

$$\tilde{a}_3 = 3q_{z_0}^3 + 3q_{z_0}^2 - (4q_{z_0} + 3)j_{z_0} - s_{z_0}, \text{ and} \quad (1.37)$$

$$\tilde{a}_4 = l_{z_0} + (8 + 7q_{z_0})s_{z_0} + (12 + 25q_{z_0}^2 + 32q_{z_0})j_{z_0} - 4j_{z_0}^2 - 7q_{z_0}j_{z_0} - 15q_{z_0}^3 - 15q_{z_0}^4 - 25q_{z_0}^2j_{z_0}. \quad (1.38)$$

Knowing this, we can explicitly write the denominator on the left-hand side of eq. (1.26) in terms of known terms by doing

$$\begin{aligned} (1+z)H &= (1+z_0) \left(1 + \frac{z-z_0}{1+z_0} \right) H = (1+z_0)(1+x)H \\ &= H_{z_0}(1+z_0) \left[1 + (\tilde{a}_1 + 1)x + (\tilde{a}_2 + \tilde{a}_1)x^2 + (\tilde{a}_3 + \tilde{a}_2)x^3 + (\tilde{a}_4 + \tilde{a}_3)x^4 + \mathcal{O}(x^5) \right]. \end{aligned} \quad (1.39)$$

We retain this result and then apply the same procedure as we did to find the relation between the a_n and the derivatives of H , but now for $\tilde{b}_m \equiv b_m(1+z_0)^m$. The latter is merely a convenient change now that we took the $x \equiv (z - z_0)/(1 + z_0)$ parametrisation. After introducing the Taylor series expansion of $P(z) \equiv (1+z)H(z)$, we see that

$$\sum_m \tilde{b}_m x^m \equiv \sum_m \tilde{b}_m \left(\frac{z-z_0}{1+z_0} \right)^m = \frac{1}{1+z} \frac{1}{H(z)} \equiv P^{-1}(z) = \sum_m \frac{d^m P^{-1}(x)}{dx^m} \Big|_{x=0} \frac{x^m}{m!}, \quad (1.40)$$

so that

$$\tilde{b}_m = \frac{d^m P^{-1}(x)}{dx^m} \Big|_{x=0} \frac{1}{m!}, \quad (1.41)$$

where we only need to evaluate the derivatives of P^{-1} in $x = 0$ (equivalently, $z = z_0$). After straightforward algebra, we obtain the following results,

$$\tilde{b}_0 = P^{-1}(0) = \frac{1}{H_{z_0}(1+z_0)}, \quad (1.42)$$

$$\tilde{b}_1 = \frac{dP^{-1}(x)}{dx} \Big|_{x=0} = -\frac{\tilde{a}_1 + 1}{H_{z_0}(1+z_0)}, \quad (1.43)$$

$$\tilde{b}_2 = \frac{1}{2} \frac{d^2 P^{-1}(x)}{dx^2} \Big|_{x=0} = \frac{(\tilde{a}_1 + 1)^2 - (\tilde{a}_2 + \tilde{a}_1)}{H_{z_0}(1+z_0)}, \quad (1.44)$$

$$\tilde{b}_3 = \frac{1}{6} \frac{d^3 P^{-1}(x)}{dx^3} \Big|_{x=0} = \frac{-(\tilde{a}_1 + 1)^3 + 2(\tilde{a}_1 + 1)(\tilde{a}_2 + \tilde{a}_1) - (\tilde{a}_3 + \tilde{a}_2)}{H_{z_0}(1+z_0)}, \text{ and} \quad (1.45)$$

$$\tilde{b}_4 = \frac{1}{24} \frac{d^4 P^{-1}(x)}{dx^4} \Big|_{x=0} = \frac{(\tilde{a}_1 + 1)^4 - 3(\tilde{a}_1 + 1)^2(\tilde{a}_2 + \tilde{a}_1) + 2(\tilde{a}_1 + 1)(\tilde{a}_3 + \tilde{a}_2) + (\tilde{a}_2 + \tilde{a}_1)}{H_{z_0}(1+z_0)}, \quad (1.46)$$

which depend only on H_{z_0} , q_{z_0} , j_{z_0} , s_{z_0} and l_{z_0} (see eqs. 1.35-1.38). The (absolute) interval of cosmic time can be integrated easily in the current polynomial form,

$$|\Delta t(z, z_0)| = \int_{z_0}^z \frac{dz'}{(1+z')H(z')} = (1+z_0) \sum_m \tilde{b}_m \int_0^x x'^m dx' = \sum_m \frac{(1+z_0)\tilde{b}_m}{m+1} x^{m+1}, \quad (1.47)$$

where in the first step we have performed a variable change $\{z \rightarrow x; dz = (1+z_0)dx\}$ and in the second we have performed the integration. At this point the problem is resolved for an arbitrary cosmographic expansion, and a generic interval of cosmic time from z_0 reads

$$t(z, \sigma) = t_{\text{arch}}(\sigma, z_0) - \sum_m \frac{(1+z_0)\tilde{b}_m}{m+1} \left(\frac{z - z_0}{1+z_0} \right)^{m+1}. \quad (1.48)$$

However, in practical applications, the utility of this cosmographic expansion heavily relies on the quality of available observational data. Higher-order Taylor series might lead to overfitting when used to estimate a confidence interval for H , as most CC data produce a noisy $t(z)$ relation, making high-order derivatives of $H(z)$ hard to constrain. To illustrate this trade-off, we compare the theoretical accuracy of the cosmographic truncation against empirical uncertainties in Fig. 1.7. We show the relative error ($|\Delta H|/H$) introduced by truncating the z_0 -pivotal cosmographic expansion at second and fourth order within a benchmark Λ CDM universe as background, alongside the statistical ($\sigma_{\text{stat.}}$) and systematic ($\sigma_{\text{syst.}}$) relative errors from the $H(z)$ reconstruction achieved in A26.

As can be seen, across the entire redshift range where the reconstruction is performed ($0.36 \lesssim z \lesssim 0.80$), both the statistical and systematic observational uncertainties from A26 are of order 10^{-1} (10%). In contrast, the error inherent to the cosmographic approximation itself is several orders of magnitude smaller. For the second-order expansion, which is the truncation order selected as the reference framework throughout A26, the truncation error remains well below 10^{-2} within the data window, dropping even lower near the pivotal redshift, which is chosen to be the one

in A26, $z_0 = 0.57$. Extending the expansion to fourth order suppresses this theoretical error even further, pushing it down to the range of 10^{-3} to 10^{-4} . Consequently, the current observational data cannot resolve the mathematical nuances of these higher-order derivatives. The total error budget is overwhelmingly dominated by the statistical and systematic noise of the CC data rather than the mathematical truncation of the model.

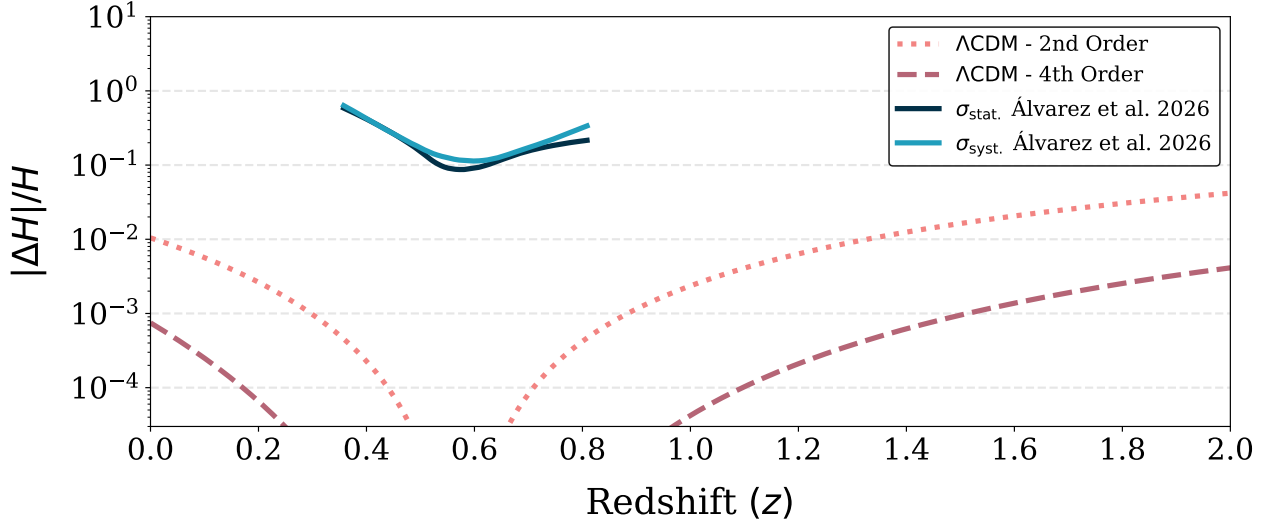


Figure 1.7: Truncation residuals of the cosmographic approximation for a Λ CDM underlying cosmology (dotted and dashed lines). For comparison, in blue we find the statistical (darker blue) and systematic (lighter blue) contributions to the overall uncertainty of the H reconstruction in A26.

While the gap between theoretical precision and empirical uncertainty might shrink when evaluating cosmological models more complex or rapidly evolving than Λ CDM, which seems reasonable following the conclusions from Capozziello et al. (2011) (see Fig. 1.5), a second-order cosmographic expansion remains an exceptionally accurate and well-justified compromise for current available data. It minimises the risk of overfitting while introducing a completely negligible systematic bias compared to the current observational uncertainties.

So far, we have set the theoretical foundations of cosmography as to provide a robust framework when interpreting the observed $t(z)$ trends that were derived in A25 and A26. Cosmography offers a strictly kinematic and model-independent description of the spacetime expansion, agnostic to the physical nature of the clocks used to map it. The mathematical elegance of eq. (1.48) is only as reliable as our empirical capacity to isolate $\Delta t(z)$ from the complex baryonic processes that govern galaxy evolution, as can be deduced from the above discussion about accuracy limits. Indeed, statistical and astrophysical systematic uncertainties dominate the error budget (Fig. 1.7), a few orders of magnitude above the expected residuals of the cosmographic approximation. Crucially, cosmic chronometers are massive, unresolved ensembles of stars deeply affected by their formation history, chemical enrichment, and structural properties. Therefore, transforming the raw spectroscopic observations

of passively evolving galaxies into a pristine cosmological tool requires moving away from pure geometric kinematics and diving into the astrophysical mechanisms that dictate stellar ageing. The foundational principles governing these stellar populations, their synthetic reconstruction, and the systematic indicators used to decode their evolutionary stages are thoroughly examined in the next section.

1.3 Astrophysical framework

The fundamental premise of using passively evolving galaxies as cosmic chronometers relies on our ability to model their integrated starlight as a function of time. In the ideal case, the stellar content of these massive systems can be approximated by a Single Stellar Population (SSP). An SSP is defined as a coeval ensemble of stars that share a unique initial chemical composition, which is characterised by its metallicity $[Z/H]$ and abundance patterns, and are formed from the same parental molecular cloud during a single, instantaneous starburst event.

Consequently, the temporal evolution of an SSP is purely deterministic, governed by the stellar lifetimes of its individual constituents. As time progresses, high-mass stars rapidly exhaust their nuclear fuel and depart from the Main Sequence (MS), leaving progressively lower-mass stars to dictate the integrated energy distribution of the population. This systematic migration across the Hertzsprung-Russell (HR) diagram implies that the age of an SSP is tightly linked to the precise location of its Main Sequence Turn-Off (MSTO) point (Iben, 1967; Renzini, 2006), which serves as the primary astrophysical indicator of the time elapsed since the starburst.

Empirically, the cleanest manifestations of SSPs in the universe are Galactic globular clusters (GCs). Because their constituent stars are gravitationally bound, coeval, and chemically homogeneous, their observed Colour-Magnitude Diagrams (CMDs) provide an empirical snapshot of stellar evolution at a fixed age. As exemplified in Fig. 1.8, which is a view of Figure 1 from Sarajedini et al. (2007), the MS is progressively depleted from the top down as time elapses. This particular diagram represents the stellar population of NGC 5466, a globular cluster in the Milky Way, displaying a classical CMD layout: the magnitude in the Hubble Space Telescope (HST) F606W filter ($\sim$ optical V band; Holtzman et al., 1995; Sirianni et al., 2005) on the vertical axis (brightness increases upwards) against the F606W–F814W colour index on the horizontal axis (temperature decreases from left to right).

Our objects of science, however, are not GCs but passively evolving elliptical galaxies, which constitute much more complex systems. Indeed, the chemical simplicity of a single SSP starts to diverge from the physical reality of ellipticals due to the broad metallicity distribution born from galactic self-enrichment. A simple rule of thumb to quantify this is the relation between the stellar lifespan of a typical massive star and the average timescale for the assembly of the overall stellar population. The former can be computed using the well-known relation between the duration

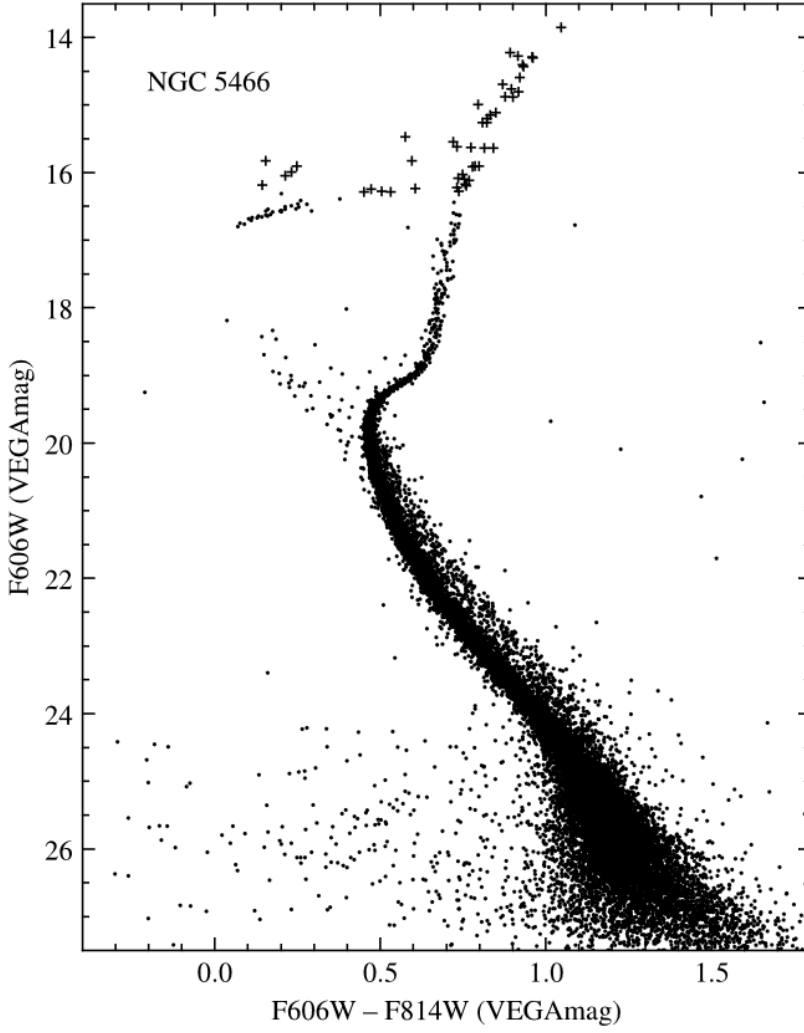


Figure 1.8: View of Figure 1 from [Sarajedini et al. \(2007\)](#). We see the CMD (F606W vs. F606W-F814W) of the NGC 5466 globular cluster of the Milky Way. The most relevant features are the Main Sequence (MS), the Main Sequence Turn-Off (MSTO), the Horizontal Branch (HB), the subgiants (SGB) and the Red Giant Branch (RGB). Many HB and RGB stars appear represented as crosses, corresponding to saturated pixels in the photometric bands.

of the MS phase and the Zero-Age MS mass, $\tau_{\text{MS}} \propto M^{-2.5}$ ([Eddington, 1924](#); [Kuiper, 1938](#)). The average lifetime of a $\sim 10 M_{\odot}$ star, which constitutes the lower mass threshold required to undergo a core-collapse supernova (CCSN) explosion and promptly return enriched material to the surrounding interstellar medium (ISM) formed via α -capture processes ([Woosley et al., 2002](#); [Truran & Heger, 2003](#)), is on the order of 10^7 years (~ 0.01 Gyr). In contrast, the timescale for the starburst that builds a passively evolving elliptical galaxy is significantly longer, on the order of 0.1 to 1 Gyr ([Jiao et al., 2023](#); [Bosi et al., 2025](#)). This timescale decoupling implies that the continuous star-formation process within these systems takes place in an ISM that is actively and progressively contaminated by the debris of successive CCSNe. Consequently, unlike the chemically homogeneous environment of a globular cluster, the resulting stellar population in an early-type galaxy inevitably exhibits a multi-generation, continuous spread in metallicity ([Pagel, 1997](#); [Harris et al., 1999](#)), moving the system away from the ideal single-metallicity framework of a pure SSP.

On top of this inherent chemical complexity, individual stars are not resolvable at cosmological scales. While for Milky Way GCs the spatial resolution allows us to isolate individual stars and directly pinpoint the geometric position of the MSTO, distant galaxies are completely unresolved,

appearing effectively as single, blended points of integrated light. When all the light from a galaxy's stellar population is integrated, the subtle spectroscopic signatures of the MSTO become deeply entangled with the dominating flux of evolved stars and the underlying metallicity distribution. Within this blended SED, a trade-off occurs: stars undergoing the Turn-Off phase dominate the high-energy blue and UV regions ($\lambda \lesssim 4500 \text{ \AA}$), whereas the highly luminous RGB and HB stars dominate the low-energy optical and near-infrared continuum ($\lambda \gtrsim 5500 \text{ \AA}$). Consequently, any observational tracer extracted from the integrated flux is a luminosity-weighted average of these competing components, which is why ages derived from unresolved systems are conventionally referred to as *luminosity-weighted ages*.

In order to untangle these degeneracies and decode the differential age information locked within this integrated light, we must evaluate how the evolutionary stages of SSPs manifest spectroscopically. In other words, we aim at understanding how age and metallicity variations alter the integrated continuum and absorption features, paving the way to interpreting the empirical spectra retrieved from modern surveys.

1.3.1 Inspecting a Single Stellar Population

Some Stellar Population Synthesis models (a review on SPS is provided in Sec. 1.4) offer an easy visualisation of the SED for variations of input parameters. The model of Knowles et al. (2023) is one of the most recent examples, offering a wide parameter space in age, metallicity and abundance of α -elements, which correlates with the duration of the star-formation episode of an elliptical galaxy (Matteucci, 1994; Thomas et al., 1999). In Fig. 1.9 we represent the predicted SED of a single stellar population at fixed metallicity ($[Z/H] = +0.15$) and α -enhancement ($[\alpha/Fe] = +0.20$), values typical of massive ETGs (Worthey et al., 1992; Thomas et al., 2005, 2010; Clemens et al., 2006; Clemens et al., 2009; Graves et al., 2009; Graves & Faber, 2010), for a set of ages spanning $t \in [3, 12]$ Gyr.

Several features of the SED respond visibly to this variation in age. First, the overall continuum reddens with age: the flux ratio between the red and blue ends of the optical range steadily increases as older populations are dominated by progressively cooler, lower-mass stars at the main-sequence turn-off (see Sec. 1.3), while the hot, blue stars that dominate young populations disappear on nuclear timescales. Second, the discontinuity at $\lambda_{\text{RF}} \approx 4000 \text{ \AA}$ known as D4000 break, becomes markedly more pronounced with age, as it is fundamentally caused by the accumulation of metal absorption lines (predominantly of Ca and Fe-peak elements) in the atmospheres of the increasingly abundant low-mass, high surface-gravity stars (Worthey et al., 1994). Indeed, it has motivated its use as indicator of differential age differences in the field of CCs (Moresco et al., 2011, 2012; Moresco, 2015; Moresco et al., 2016; Loubser, 2025). Third, essentially every absorption feature in the spectrum deepens with age, most notably the Ca H&K doublet, the G-band (which defines the G4300 index), and the Mgb and Fe features around $5100 - 5400 \text{ \AA}$, while the Balmer lines (β , δ , γ) evolve in

the opposite sense, becoming progressively shallower as the main-sequence turn-off cools and the temperature-sensitive Balmer absorption weakens.

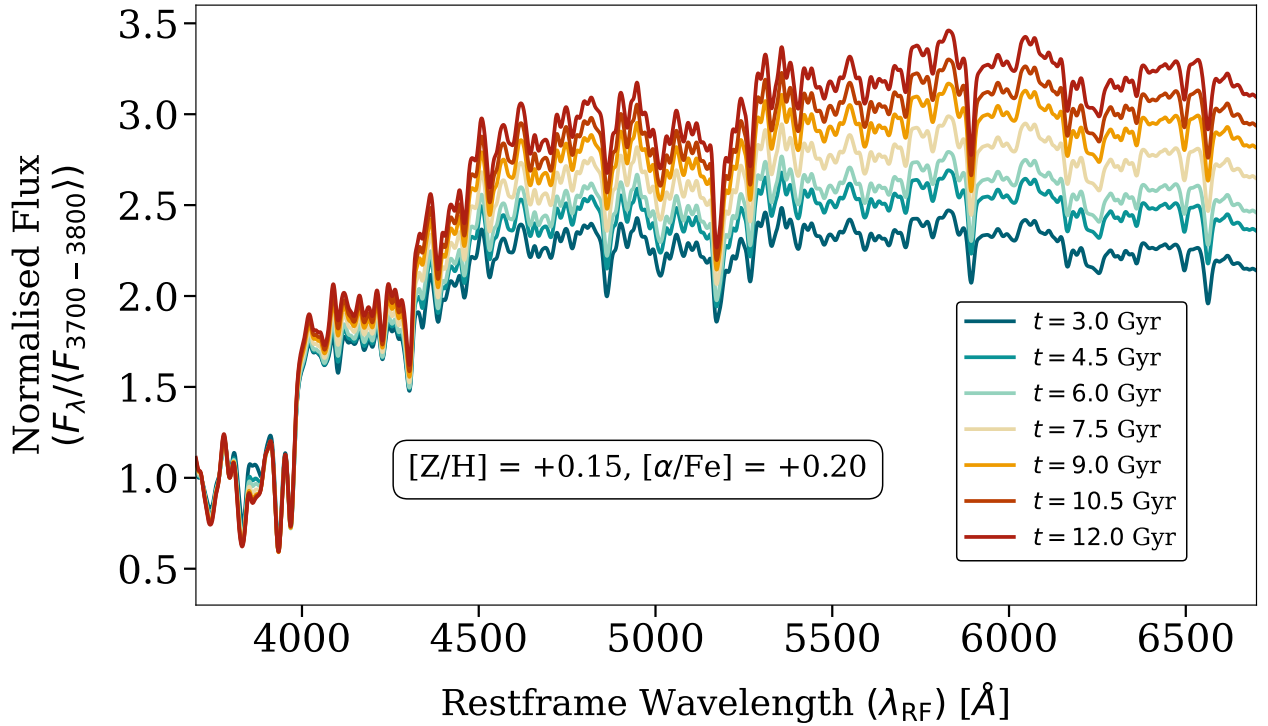


Figure 1.9: Spectral energy distribution of SSPs from the [Knowles et al. \(2023\)](#) SPS model. Ages vary from 3 Gyr to 12 Gyr in steps of 1.5 Gyr. Metallicity and abundance of α -elements are fixed to the typical values expected for old passively evolving stellar populations. To observe the differences in the SED, the flux is normalised to the median flux in the 3700 – 3800 Å range.

Finally, we note that the spacing between consecutive age tracks in Fig. 1.9 visibly shrinks in relative terms towards the oldest ages: the SED evolves comparatively faster between $t = 3$ and $t = 6$ Gyr than between $t = 9, 10.5$ and 12 Gyr. This effect was already noted by [Worthey et al. \(1994\)](#) and revisited in works that inspect passively evolving stellar populations in long redshift intervals ([A26](#)), where the variation of features with age undergoes different behaviour regimes. This has direct consequences for CC studies, as it implies that the statistical uncertainty on the inferred age necessarily grows for the oldest, most massive ETGs, exactly the population that CC studies target for its passive, quiescent nature. On top of this loss of sensitivity at old ages, the features described above do not evolve independently of metallicity either. Indeed, an increase in $[Z/H]$ produces a very similar qualitative effect on the SED as an increase in age: the continuum reddens, the D4000 break deepens, and the metallic absorption features strengthen. This near-equivalence gives rise to the well-known age-metallicity degeneracy ([Worthey, 1994](#); [Chavez & Bertone, 2011](#); [Barone et al., 2018](#)), whereby a young, metal-rich population and an old, metal-poor one can produce almost indistinguishable SEDs. This behaviour was first quantified by [Worthey \(1994\)](#) through the so-called

3/2 rule: the change in age required to compensate a given change in metallicity, at fixed index strength, follows approximately $\Delta \log t / \Delta [Z/H] \approx -3/2$.

We illustrate this in Fig. 1.10, where a fiducial SSP at $t = 7.5$ Gyr and $[Z/H] = +0.15$ fixing $[\alpha/\text{Fe}] = +0.20$ (both metallicity and α -enhancement are written here and in the remainder of this thesis in the logarithm of solar units) is compared against a younger, more metal-rich SSP ($t = 6.0$ Gyr, $[Z/H] = +0.26$) and an older, more metal-poor one ($t = 9.0$ Gyr, $[Z/H] = +0.06$). Note that, quantitatively, this follows the *3/2 relation* introduced before. The three SEDs are visually indistinguishable over the full optical range, and the residuals with respect to the fiducial spectrum remain below $\sim 2\%$ at all wavelengths, well within the level of observational noise for typical CC spectra. Regarding the residuals, both compensated spectra depart from zero more noticeably around the Balmer-line regions (most visibly $H\gamma$ and $H\beta$, at $\lambda_{\text{RF}} \approx 4300 - 4900 \text{ \AA}$), and do so in opposite directions. The metal-poorer and older SSP falls systematically below it, while the metal-rich and younger one lies slightly above. This is precisely the residual signature that a purely broadband or continuum-based diagnostic would struggle to exploit, but that narrow-band index measurements, particularly of Balmer lines relative to the metallic species, are designed to capture.

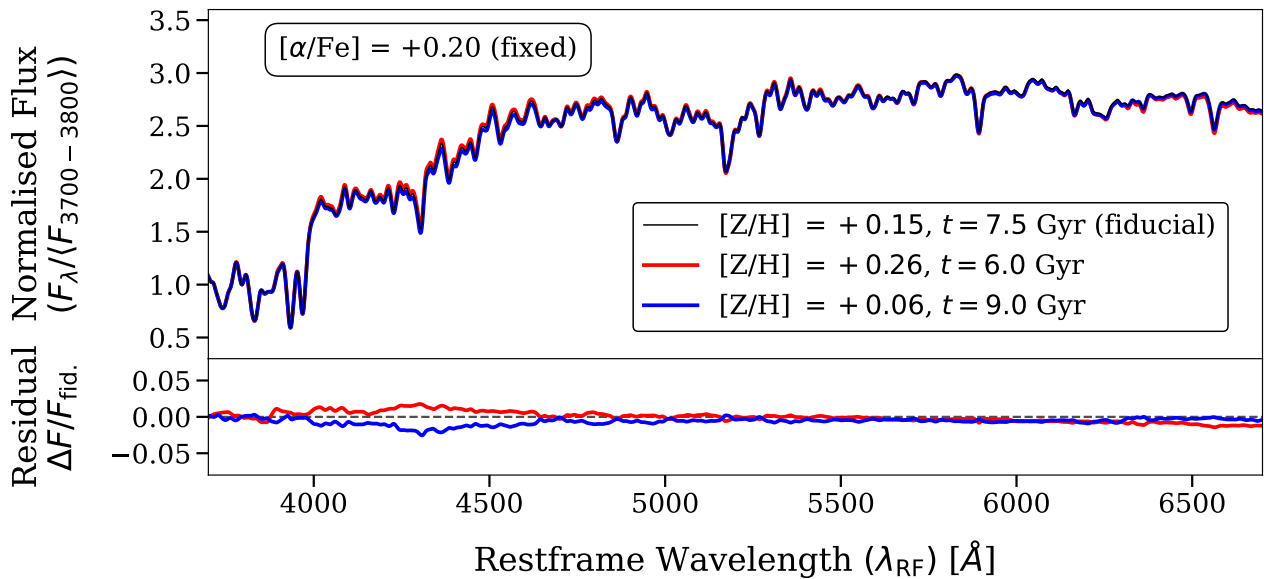


Figure 1.10: Age-metallicity degeneracy using Knowles et al. (2023) SSPs. In the upper panel, the SED of a fiducial SSP ($t = 7.5$ Gyr, $[Z/H] = +0.15$, black) compared against two SSPs in which age and metallicity are shifted in opposite directions (blue is metal poorer and older, while red is more metal rich and younger) satisfying the *3/2 rule* of Worthey (1994). Abundance of α -elements is fixed to $[\alpha/\text{Fe}] = +0.20$. *Bottom*: fractional residual of each compensated SED with respect to the fiducial spectrum.

Since the late 1980s, the community has measured the strength of a restricted set of such features, the Lick indices (Faber et al., 1985; Worthey et al., 1994), defined over specific bandpasses centred on

the absorption lines discussed above, together with adjacent pseudo-continuum windows used for local normalisation. Suitable combinations of these can reduce the age-metallicity degeneracy.

1.3.2 Lick indices and other spectroscopic features

The Lick system, originally developed at the Lick Observatory (Faber et al., 1985; Worthey et al., 1994; Worthey & Ottaviani, 1997; Trager et al., 1998), defines each atomic index strength as the equivalent width over a central band, $\lambda \in [\lambda_{\text{cb}}, \lambda_{\text{cr}}]$, where cb and cr stand for central-blue and central-red limits,

$$I_{\text{atomic}} \equiv \int_{\lambda_{\text{cb}}}^{\lambda_{\text{cr}}} \left(1 - \frac{F(\lambda)}{F_{\text{p-c}}(\lambda)} \right) d\lambda; \quad (1.49)$$

and each molecular index as a magnitude,

$$I_{\text{molecular}} \equiv -2.5 \log \left(\frac{1}{\lambda_{\text{cr}} - \lambda_{\text{cb}}} \int_{\lambda_{\text{cb}}}^{\lambda_{\text{cr}}} \frac{F(\lambda)}{F_{\text{p-c}}(\lambda)} d\lambda \right), \quad (1.50)$$

computed over a central bandpass flanked by two adjacent pseudo-continuum windows, blueward and redward of the central region. The median fluxes in these bands, $\langle F_{\text{b}} \rangle$ and $\langle F_{\text{r}} \rangle$ respectively, are used to establish a local linear continuum,

$$F_{\text{p-c}}(\lambda) = \langle F_{\text{b}} \rangle \frac{\bar{\lambda}_{\text{r}} - \lambda}{\bar{\lambda}_{\text{r}} - \bar{\lambda}_{\text{b}}} + \langle F_{\text{r}} \rangle \frac{\lambda - \bar{\lambda}_{\text{b}}}{\bar{\lambda}_{\text{r}} - \bar{\lambda}_{\text{b}}}, \quad (1.51)$$

where we have used a superscore, $\bar{\lambda}_i$, to refer to the central wavelength of each band $i \equiv \{\text{b}, \text{r}\}$. Table 1.1 summarises the central and pseudo-continuum bandpasses of the 25 classical Lick indices, together with a set of complementary features that, although not part of the original Lick compilation, are routinely measured alongside it and discussed throughout this thesis. Both groups are described in detail below.

Each Lick index is primarily sensitive to a specific physical property of the underlying stellar population. In Fig. 1.11 we show their location over a representative optical spectrum, grouped accordingly. The Balmer-line indices ($\text{H}\delta_{\text{A}}$, $\text{H}\delta_{\text{F}}$, $\text{H}\gamma_{\text{A}}$, $\text{H}\gamma_{\text{F}}$, $\text{H}\beta$) trace the temperature of the main-sequence turn-off and are therefore the primary age indicators, as already discussed in section 1.3.1. The iron-peak indices ($\text{Fe}4383$, $\text{Fe}4531$, $\text{Fe}5015$, $\text{Fe}5270$, $\text{Fe}5335$, $\text{Fe}5406$, $\text{Fe}5709$, $\text{Fe}5782$) are instead governed by the iron abundance $[\text{Fe}/\text{H}]$, and the magnesium features (Mg_1 , Mg_2 , Mgb) trace the abundance of α -elements. Magnesium is singled out among the α -elements as the preferred tracer of $[\alpha/\text{Fe}]$ because its nucleosynthesis is dominated almost exclusively by core-collapse supernovae, on the short timescales discussed in Sec. 1.3, whereas iron receives a substantial delayed contribution from Type Ia events (e.g. Tinsley, 1979; Nomoto et al., 1984; Woosley & Weaver, 1995; Thomas et al., 2003b, 2005). The Mg/Fe ratio is therefore a direct clock for the duration of the star-formation episode that built the population (Greggio & Renzini, 1983; Matteucci & Tornambe, 1987). Interestingly, this dominance of magnesium as the reference α -tracer is not shared by all α -elements. For example, calcium, despite coming from an analogous nucleosynthetic origin, is observed to systematically

		Index	Units	$\lambda_{\text{cb}} - \lambda_{\text{cr}} [\text{\AA}]$	$\lambda_{\text{bb}} - \lambda_{\text{br}} [\text{\AA}]$	$\lambda_{\text{rb}} - \lambda_{\text{rr}} [\text{\AA}]$
Lick Indices	Balmer	H δ_A	$\text{\AA}$	4083.500 – 4122.250	4041.600 – 4079.750	4128.500 – 4161.000
		H δ_F	$\text{\AA}$	4091.000 – 4112.250	4057.250 – 4088.500	4114.750 – 4137.250
		H γ_A	$\text{\AA}$	4319.750 – 4363.500	4283.500 – 4319.750	4367.250 – 4419.750
		H γ_F	$\text{\AA}$	4331.250 – 4352.250	4283.500 – 4319.750	4354.750 – 4384.750
		H β	$\text{\AA}$	4847.875 – 4876.625	4827.875 – 4847.875	4876.625 – 4891.625
	Iron	Fe4383	$\text{\AA}$	4369.125 – 4420.375	4359.125 – 4370.375	4442.875 – 4455.375
		Fe4531	$\text{\AA}$	4514.250 – 4559.250	4504.250 – 4514.250	4560.500 – 4579.250
		Fe5015	$\text{\AA}$	4977.750 – 5054.000	4946.500 – 4977.750	5054.000 – 5065.250
		Fe5270	$\text{\AA}$	5245.650 – 5285.650	5233.150 – 5248.150	5285.650 – 5318.150
		Fe5335	$\text{\AA}$	5312.125 – 5352.125	5304.625 – 5315.875	5353.375 – 5363.375
		Fe5406	$\text{\AA}$	5387.500 – 5415.000	5376.250 – 5387.500	5415.000 – 5425.000
		Fe5709	$\text{\AA}$	5696.625 – 5720.375	5672.875 – 5696.625	5722.875 – 5736.625
		Fe5782	$\text{\AA}$	5776.625 – 5796.625	5765.375 – 5775.375	5797.875 – 5811.625
	Mg	Mg ₁	mag	5069.125 – 5134.125	4895.125 – 4957.625	5301.125 – 5366.125
		Mg ₂	mag	5154.125 – 5196.625	4895.125 – 4957.625	5301.125 – 5366.125
		Mgb	$\text{\AA}$	5160.125 – 5192.625	5142.625 – 5161.375	5191.375 – 5206.375
	Other Lick Indices	CN ₁	mag	4142.125 – 4177.125	4080.125 – 4117.625	4244.125 – 4284.125
		CN ₂	mag	4142.125 – 4177.125	4083.875 – 4096.375	4244.125 – 4284.125
		Ca4227	$\text{\AA}$	4222.250 – 4234.750	4211.000 – 4219.750	4241.000 – 4251.000
		G4300	$\text{\AA}$	4281.375 – 4316.375	4266.375 – 4282.625	4318.875 – 4335.125
		Ca4455	$\text{\AA}$	4452.125 – 4474.625	4445.875 – 4454.625	4477.125 – 4492.125
		C ₂ 4668	$\text{\AA}$	4634.000 – 4720.250	4611.500 – 4630.250	4742.750 – 4756.500
		NaD	$\text{\AA}$	5876.875 – 5909.375	5860.625 – 5875.625	5922.125 – 5948.125
		TiO ₁	mag	5936.625 – 5994.125	5816.625 – 5849.125	6038.625 – 6103.625
		TiO ₂	mag	6189.625 – 6272.125	6066.625 – 6141.625	6372.625 – 6415.125
Other Indices		[OII] λ 3727	$\text{\AA}$	3717.000 – 3737.000	3660.000 – 3708.000	3736.000 – 3764.000
	CaII K	$\text{\AA}$	—	3845.000 – 3880.000	3950.000 – 3954.000	
	CaII H	$\text{\AA}$	—	3950.000 – 3954.000	3983.000 – 3993.000	
	D4000	dex	—	3750.000 – 3950.000	4050.000 – 4250.000	
	D4000 _n	dex	—	3850.000 – 3950.000	4000.000 – 4100.000	
	H β ₀	$\text{\AA}$	4839.275 – 4877.097	4821.175 – 4838.404	4897.445 – 4915.845	
	[OIII] λ 5007	$\text{\AA}$	5001.000 – 5012.000	4975.000 – 4991.000	5020.000 – 5045.000	
	H α	$\text{\AA}$	6553.000 – 6573.000	6505.000 – 6538.000	6600.000 – 6688.000	

Table 1.1: Spectral indices used throughout this thesis, grouped by primary physical sensitivity. *Lick Indices*: 25 features of the classical Lick/IDS system (Worthey et al., 1994; Worthey & Ottaviani, 1997; Trager et al., 1998). *Other Indices*: complementary features, including the D4000/D4000_n continuum breaks (Bruzual A., 1983; Hamilton, 1985; Balogh et al., 1999), the CaII H&K doublet (Fanfani, 2019), H β ₀ (Cervantes & Vazdekis, 2009) and the emission lines whose absence is required for the CC sample selection (Tremonti et al., 2004).

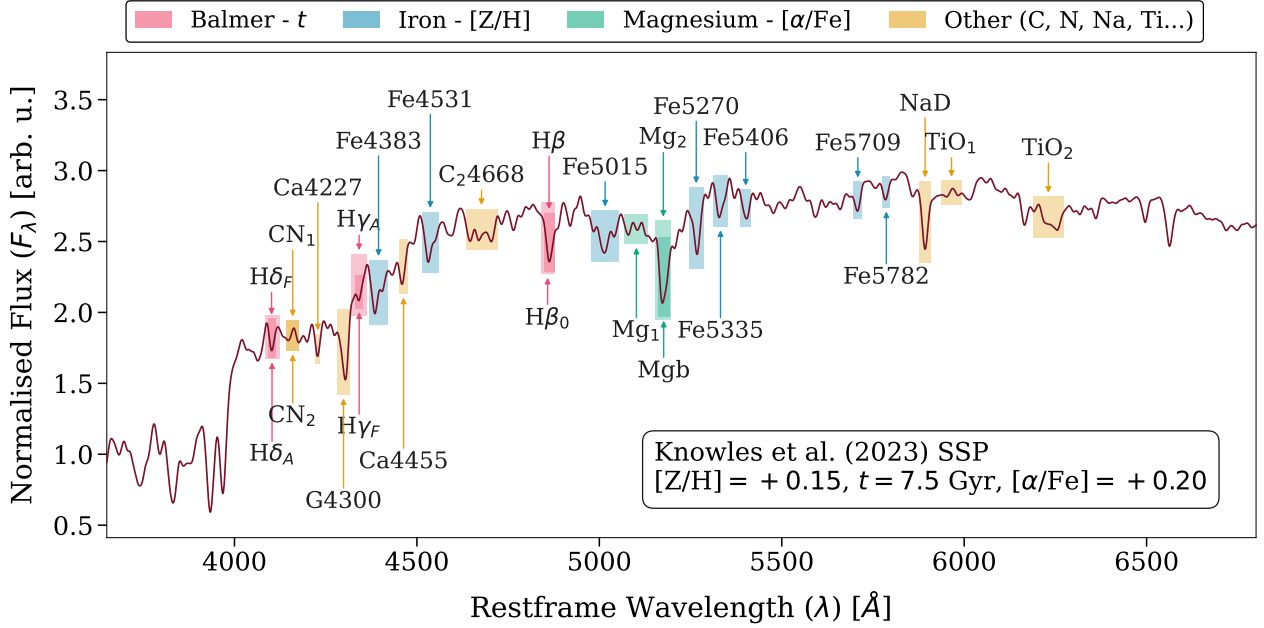


Figure 1.11: The 25 classical Lick indices overlaid on a representative optical spectrum of a passively evolving ETG (same as in Fig. 1.10). Shaded regions mark the central bandpass of each index.

under-track the magnesium abundance in early-type galaxies, a long-standing discrepancy (Vazdekis et al., 1996, 1997; Thomas et al., 2003a).

Together, these three families are the ones most commonly combined in seminal works of stellar population fits through Lick indices (e.g. Johansson et al. 2012). Their complementary sensitivities are what allow the age-metallicity degeneracy discussed in Sec. 1.3.1 to be broken. The remaining Lick indices trace more specific physical conditions: G4300 measures the CH molecular band, present in the atmospheres of cool stars (both at the main-sequence turn-off of old populations and along the giant branch), and correlates with both age and carbon abundance (Worthey et al., 1994); CN₁ and CN₂ are similarly sensitive to carbon and nitrogen abundances; Ca4227 and Ca4455 trace calcium, although the discussion provided in the previous paragraph stands; and NaD, TiO₁ and TiO₂ are markedly sensitive to the low-mass end of the initial mass function (e.g. Conroy & van Dokkum, 2012; La Barbera et al., 2013), which, together with their proximity to telluric absorption and interstellar contamination, is why they are typically excluded from cosmic chronometer index fits.

Beyond the classical Lick indices, we have included in Tab. 1.1 the H β_0 index (Cervantes & Vazdekis, 2009), a redefinition of H β with narrower pseudo-continuum bandpasses, designed specifically to minimise contamination from neighbouring metallic lines and to maximise sensitivity to age while keeping sensitivity to $[\alpha/\text{Fe}]$ low. As shown in Knowles et al. (2023), a combination of only two indices can break the age-metallicity degeneracy by means of an index-index diagram. It is the exact case of H β_0 when combined with the secondary [MgFe] index, which is defined as the geometric

average of Mgb and the arithmetic mean of Fe5270 and Fe5335,

$$[\text{MgFe}] \equiv \sqrt{\text{Mgb} \cdot \frac{\text{Fe5270} + \text{Fe5335}}{2}}. \quad (1.52)$$

A further requirement is the previous estimation of the abundance of α -elements, as the $t - [\text{Z}/\text{H}]$ map within the $\text{H}\beta_0 - [\text{MgFe}]$ gets shifted with variations of $[\alpha/\text{Fe}]$. This can be observed in Fig. 1.12, inspired in colour and shape in figures 6 and 7 from Knowles et al. (2023), built with their models and set for a Chabrier (2003) IMF, where in the top panels the optimised two-index maps are overlaid for several $[\alpha/\text{Fe}]$ combinations. We note that for $[\alpha/\text{Fe}] > 0.2$ the map changes little, but can produce suboptimal and biased fits of age and metallicity if α -abundances are not properly taken into account. Fortunately, another two index-index maps resulting from the combination of the magnesium Mgb and each of the two iron components of the $[\text{MgFe}]$ index, Fe5270 and Fe5335, fully separate the parameter space formed by $t \times [\text{Z}/\text{H}]$ for the typical values expected of these quantities in CC works. This is portrayed in the lower panels of the aforementioned figure, where the $[4.0 < t [\text{Gyr}] < 14.0] \times [-0.35 < [\text{Z}/\text{H}] < 0.26]$ regions are well separated for reasonably small steps in $[\alpha/\text{Fe}]$, allowing to easily choose a single $t - [\text{Z}/\text{H}]$ map in the upper panels.

When the lower limit of ages is extended to 3, and especially, 2 Gyr, the regions start being connected, which puts a limit on the age range for which this methodology is applicable. Towards higher ages (lower redshift) the methodology also finds some limitations, as the progenitor of this idea (Vazdekis et al., 2015, figures 28 and 29), through the SDSS galaxy spectra stacks of La Barbera et al. (2013), showed that both model estimated Balmer- β (wide and narrow definition) fail to reproduce the observed data. That is, the observed indices tend to be less strong than what the model predicts.

This methodology has also been used more recently, coming across some of these limitations. In particular, one can see in figure 3 in Tomasetti et al. (2024) an $\text{H}\beta - \text{Mgb}$ diagram in which the observed ages of Milky Way GCs exceed the limits of the SPS model they used (Thomas et al., 2011, TMJ). In the text of the referred article, the emphasis is given to the lack of a proper account for the Horizontal Branch component of GCs in SPS models, systematically estimating a less metallic and younger stellar population. However, exceedingly high ages are observed, with respect to the model limits, for all metallicities. Measurements performed in the articles on which this thesis is based (A25, A26) can be used to perform index-index plane analyses like the ones presented above and in Fig. 1.12. Although these are not included in any of the articles, the reader may find a review in Section 4.1.

Another two indices included in table 1.1 that have been thoroughly used in CC studies to produce independent Hubble parameter measurements are the D4000 break and its narrower variant D4000_n (Bruzual A., 1983; Hamilton, 1985; Balogh et al., 1999). Unlike the Lick indices, the two variants of this feature are not defined over a narrow central bandpass but as the ratio between two broad

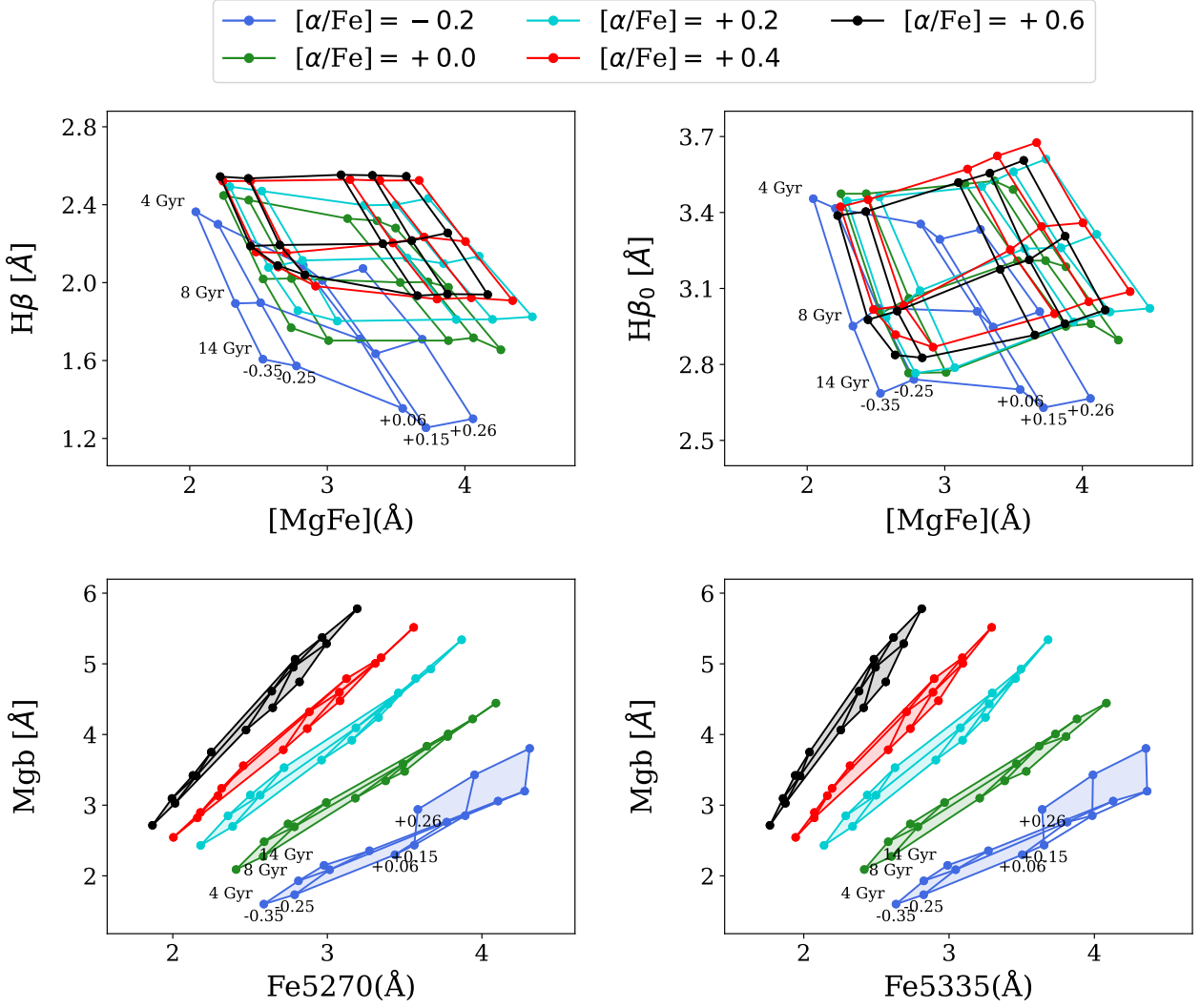


Figure 1.12: Index-index planes to break the age-metallicity degeneracy (upper panels) and finding the α -abundance regime (lower panels). This figure is inspired by figures 6 and 7 from [Knowles et al. \(2023\)](#), representing their model predictions using a [Chabrier \(2003\)](#) IMF. Here we shaded the whole parameter space $t \times [Z/H]$ shown in the lower panel to highlight the separation between different α -abundance regimes.

continuum windows straddling the 4000 Å discontinuity, a spectral feature described in Sec. 1.3.1,

$$D4000 = \frac{(\lambda_{\text{br}} - \lambda_{\text{bb}}) \int_{\lambda_{\text{rb}}}^{\lambda_{\text{rr}}} \lambda^2 F(\lambda) d\lambda}{(\lambda_{\text{rr}} - \lambda_{\text{rb}}) \int_{\lambda_{\text{bb}}}^{\lambda_{\text{br}}} \lambda^2 F(\lambda) d\lambda}, \quad (1.53)$$

where the limits of the red ($\lambda_{\text{rb}}, \lambda_{\text{rr}}$) and blue ($\lambda_{\text{bb}}, \lambda_{\text{br}}$) bands vary depending on the definition⁴.

Indeed, this discontinuity arises from the blended opacity of a large number of ionised metal lines (i.e. the CaII K line, which is clearly marked in Fig. 1.11 as the second absorption at the left of the 4000Å drop, is included in the blue band of these spectral lines) rather than from any single

⁴Note that in the text around eq. (1.53), and later eq. (1.54), we use D4000 to refer to either the wide and narrow definition.

transition, making it a broadband ratio and a comparatively high signal-to-noise age indicator.

The strength of this spectral feature for CC studies lies in its linearity with age, and the even more simplified relation of its differential value with differential ages (Hamilton, 1985; Moresco et al., 2011). This is,

$$-\frac{1}{H(1+z)} = \frac{dt}{dz} = \frac{1}{A(Z, \text{SFH})} \frac{dD4000}{dz}. \quad (1.54)$$

The Balogh et al. (1999) definition, which accounts for the narrower bands of 100Å each (see Fig. 1.1), was shown to respond better to dust reddening effects, rendering it the preferred choice in the literature (Moresco et al., 2011, 2012; Moresco, 2015; Moresco et al., 2016; Loubser, 2025). The evolution of the spectral index with ages can be observed in Fig. 1.13 for the SSPs shown in Fig. 1.9. On the left we can see a renormalisation of the SED performed, dividing by $\langle F_b \rangle$, to see better the increase in the 4000 Å discontinuity mentioned in Sec. 1.3.1; while on the right, how the variation of the SSP age produces an almost linear change in the $D4000_n$ strength.

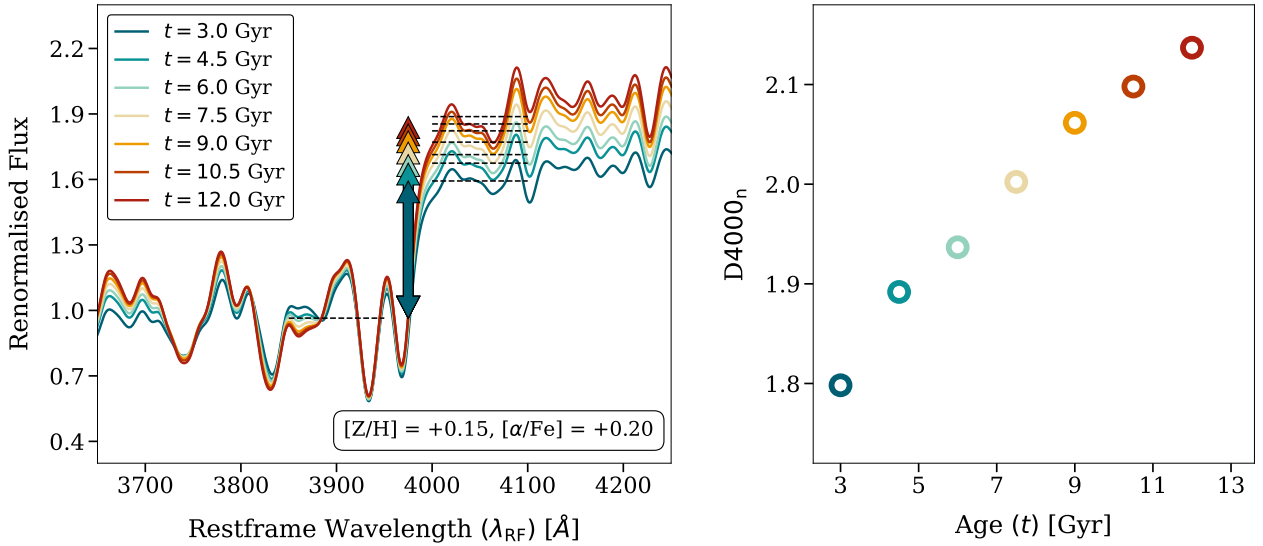


Figure 1.13: Evolution of the $D4000_n$ spectral index for the SSPs shown in Fig. 1.9. On the left, a view of the 4000 Å discontinuity. The flux of the SED has been renormalised in order to put the median flux, $\langle F_b \rangle$, at the same level for all the SSPs. On the right, the resulting $D4000_n - t$ relation.

This methodology, however, needs an additional independent estimation of the metallicity in order to produce direct estimations of H , as can be directly read from eq. (1.54), typically named as $A(Z, \text{SFR})$. Indeed, being a single observable, $D4000_n$ is intrinsically degenerate with metallicity, so studies relying on $D4000_n$ alone introduce an auxiliary calibration -typically an assumed or independently constrained metallicity relation- to translate the observed break strength into an age.

In practice, Moresco et al. (2011, 2012) address this degeneracy not by solving for metallicity inde-

pendently for each galaxy, but by restricting the sample to a population for which the star formation history (SFH) and metallicity can be reasonably bracketed a priori. Selecting massive, passively evolving early-type galaxies (ETGs) from SDSS, they show that these systems are consistent with a rapidly quenched SFH, with an exponentially declining star formation rate, $\text{SFR}(t) \propto (t/\tau^2) \exp(-t/\tau)$, and a best-fit timescale $\tau \lesssim 0.3$ Gyr for the majority of the sample (Moresco et al., 2011). Under this assumption, the SPS-predicted $\text{D4000}_n - t$ relation, and hence $A(Z, \text{SFH})$, becomes only weakly sensitive to the precise SFH, leaving stellar metallicity as the dominant remaining source of degeneracy. We recall here that SFH and α -abundances, which we have discussed for the SPS model of Knowles et al. (2021); Knowles et al. (2023), are tightly related. Thomas et al. (2005) calibrated an approximate relation between $[\alpha/\text{Fe}]$ and the width of a Gaussian SFH, $[\alpha/\text{Fe}] \approx 1/5 - 1/6 \log(\tau)$. This has since been used in both assuming a short τ to predict α -enhancement (as effectively done in Moresco et al. 2011, 2012), or, conversely, using measured $[\alpha/\text{Fe}]$ to infer formation timescales (e.g. Bosi et al. 2025).

To address the metallicity degeneracy, Moresco et al. (2011) calibrate the $\text{D4000}_n - t$ relation directly from SPS models (Bruzual & Charlot (2003) and Maraston & Strömbäck (2011)) over a grid of metallicities and delayed-exponential SFHs, consistent with the short formation timescales independently inferred for their ETG sample from SED fitting. They find that, at fixed metallicity, D4000_n is well approximated by a linear function of age, $\text{D4000}_n \propto t$, but only if the relation is fit separately for two index regimes: a *low* D4000_n and a *high* D4000_n (roughly $\text{D4000}_n \lesssim 1.95$ and $\gtrsim 1.95$, respectively). In other words, a single global slope fails to capture the mild curvature of the relation across the full range probed by the data. Within each regime, the dependence of the fitted slope A on the assumed τ is found to be small compared to its dependence on Z . This can be seen in Fig. 1.14, a direct take of figure 4 from Moresco et al. (2011). The width of each colour trend, portrayed in different line styles, represents the dependence on the SFH, which is clearly subdominant. In the lower panel, we observe the average slope found for each metallicity and regime.

In general, for a model that can offer a wide parameter space of metallicities and SFH (alternatively α -enhancement), these $A(Z_i, \text{SFH}_j)$ can be computed. The next step taken by Moresco et al. (2011) consists in fitting the $A(Z)$ function after these individual points. They use the width produced by the SFH at each metallicity as the uncertainty, so an SFH-averaged slope $\langle A(Z) \rangle$ is adopted for each metallicity and regime, $\langle A_{(\text{low D} / \text{high D})}(Z_i) \rangle$, and subsequently fit to a quadratic function to build the model $A(Z)$. Alternatively, if counting on different SPS, the $A(Z_i)$ sampling can be used as the uncertainty tracer, reducing the credible interval given by the fit to a systematic uncertainty based on the model choice. At the same time, such a methodology would allow for a two-parameter direct dependence of the A function, removing the SFH (or α -element abundance) from being a contributor to the systematics. In Moresco et al. (2011) they choose not to use the model variability, which they find small for A , as the source of uncertainty, but the variability with the SFH. They found, however, that the two sampled stellar libraries produce different zero-points for the linear tendencies, which,

fortunately, is irrelevant for differential age studies (eq. 1.54).

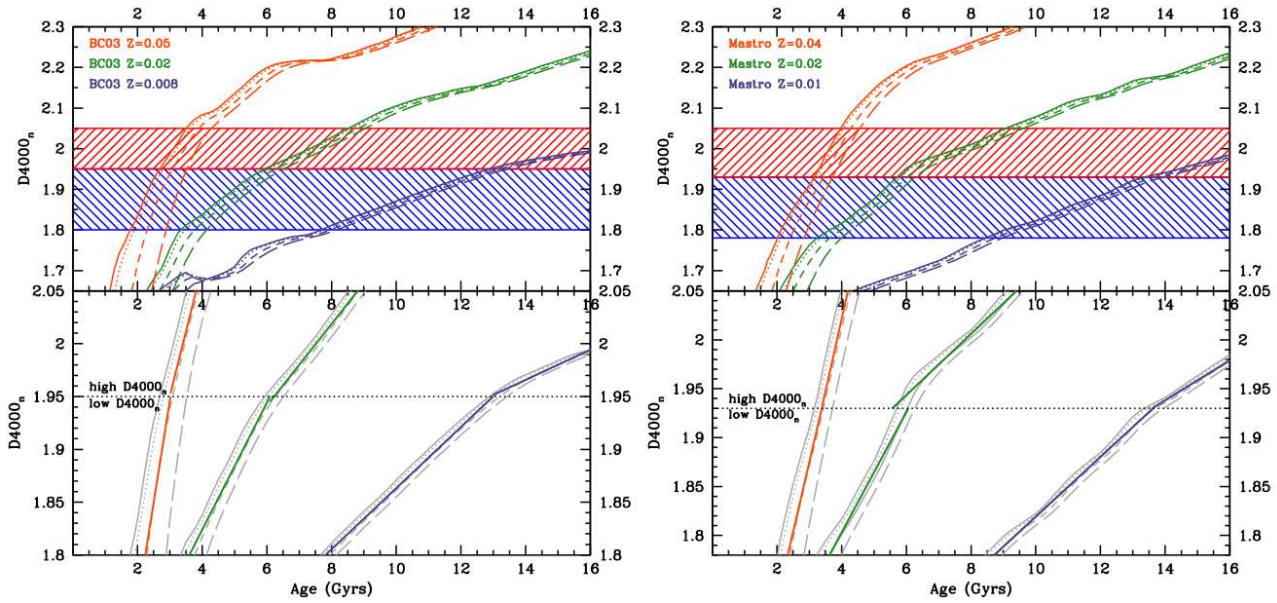


Figure 1.14: Direct view of figure 4 from Moresco et al. (2011). On the left results coming from the Bruzual & Charlot (2003) models; on the right, from Maraston & Strömbäck (2011). Upper and lower panels display the $D4000_n - t$ relation; while on the upper panels the observed SPS behaviour is shown including all the possible variability (Z and SFH), in the lower panel the piecewise linear fit is depicted, conserving two slopes (A) per metallicity, one for high- $D4000_n$ values and another for low- $D4000_n$ values.

This still leaves the practical problem of assigning a metallicity to the observed sample. Rather than fitting Z from $D4000_n$ itself, which would simply reintroduce the original degeneracy, Moresco et al. (2011) adopt an independent estimate for each SDSS ETG from Gallazzi et al. (2005), obtained through a Bayesian fit of a combination of age-sensitive ($D4000$, $H\beta$, $H\delta_A + H\gamma_A$) and metallicity-sensitive ($[MgFe]'$, $[Mg_2Fe]$) Lick indices, chosen for their negligible dependence on $[\alpha/Fe]$ against BC03 models. This estimate is cross-checked against the independent, prior-free SFH code VESPA (Tojeiro et al., 2007), finding no significant systematic offset. The ETG sample is then divided into narrow stellar-mass bins, each characterised by an approximately constant, independently measured median metallicity. This median Z is used to read off the corresponding $A(Z)$, which is applied to the observed $D4000_n - z$ evolution of that bin via eq. (1.54) to obtain $H(z)$. As a further test, one mass bin is split into low- and high-metallicity halves and reanalysed with the appropriate $A(Z)$ for each. The resulting H_0 and w agree within 1σ , which Moresco et al. (2011) take as evidence that the calibration itself does not bias the result. Moresco et al. (2012) apply the same machinery: SPS-calibrated $A(Z)$, mass-binned Gallazzi et al. (2005)-type metallicities, piecewise-linear slopes to a larger, higher-redshift ($z \lesssim 1.1$) sample, and explicitly propagate the residual metallicity uncertainty into the systematic error budget of $H(z)$, alongside the smaller contributions from the SFH and SPS

model choice. This combination of an SPS-based $A(Z)$ calibration, an independent spectroscopic metallicity estimate and mass-binned consistency checks constitutes the standard methodology adopted in subsequent D4000_n-based CC studies (Moresco, 2015; Moresco et al., 2016).

Finally, there are 5 more indices in table 1.1 which are not included in the previous discussion. These are the two ionised calcium lines, CaII K and CaII H, the one and two times ionised oxygen lines, [OII] λ 3727 and [OIII] λ 5007, and the Balmer- α line, H α . We note that the one time ionised oxygen line is a doublet that, even if typically resolvable with the spectroscopic abilities of the SDSS and DESI spectrographs as two separate peaks at 3726 Å and 3729 Å, it cannot be well measured as two separate EWs due to the overlapping of their red and blue tails respectively. However, the latter three of these lines are actually not to be observed in CC candidates. They are included in the upper table since, for the sample selection methodology that will be presented in Section 2.2.2, their EWs are measured, and candidates with non-negligible emission are eliminated from the final sample of objects used as CCs. Even a modest residual population of young, hot stars produces nebular emission that fills these lines and reveals ongoing or very recent star-formation, contaminating the assumption of a single, passively ageing stellar population on which the whole CC method rests. H β occupies a peculiar dual role in this respect: while it is one of the Lick indices used in the age fit, it is also one of the most easily contaminated by nebular emission, which fills in the stellar absorption trough from within and biases the measured strength towards artificially young ages if left unaccounted for. Indeed, this can become a systematic limitation of the CC methodology if the H β line is partially filled but remains observed in absorption. It can destabilise the relation between this spectral feature and other Lick indices, biasing the age estimation in Lick index studies. If we recall the discussion around the index-index planes, this behaviour could match the systematically low H β strength observed in the literature with respect to the one predicted by different models.

Much of the latter is addressed in Concas et al. (2017), where they thoroughly analyse the response of the H β line in CC studies. They realise that the Balmer- β can be in absorption while ionised gas indicators ([OII], [OIII], H α ; [NII] λ 6584, [SII] λ 6717, [SII] λ 6731) clearly point to a non-negligible contribution of a young stellar population. Naively, this should be reassuring of a thorough spectroscopic cut as a sufficient demand to avoid disturbing effects on H β ; yet the disagreement between the observed and predicted strength of this spectral feature remains. As of today, the reasons behind this gap are still uncertain.

The calcium lines come as a further diagnostic of residual star-formation, complementary to the emission-line cuts above. The indicator comes as the ratio between the CaII H and CaII K line strengths. Although both features originate from the same ionic species and are expected to scale together in a purely old, passively evolving population, the CaII H line sits at 3968 Å, essentially coincident with the Balmer He transition at 3970 Å. In the presence of young, hot stars, this blending

fills in and deepens the CaII H feature relative to CaII K, which remains largely unaffected. [Moresco et al. \(2018\)](#) observed that regardless of the age of the stellar population, the inversion between the depths of the K and H lines occurs rapidly after the introduction of only a very low fraction of young stars. [Borghi et al. \(2022b\)](#) extended this analysis to define the H/K (EW) ratio as an effective veto against residual star-formation. Previous works in the literature ([Rose, 1985](#); [Caldwell et al., 1996](#); [Leonardi & Rose, 1996](#); [Longhetti et al., 1999](#); [Trager et al., 2005](#); [Lonoce et al., 2014](#); [Pimblet et al., 2019](#)) had seen that the inversion in the relative deepness of the lines ($(H/K)_{F_{\min}} > 1$) implied the presence of hot stars among the overall stellar population. By requiring $H/K < 1.2$ for a source to be considered reliably passive [Borghi et al. \(2022b\)](#) established a straightforward way to include the EWs of these two lines as a criterion to guarantee the purity of the CC sample. We adopt this in the works that represent the basis for this thesis ([A25](#), [A26](#)). This argument is thoroughly inspected in [Sec. 2.2.2](#).

1.3.3 The downsizing scenario for galaxy formation

The Lick indices presented in the previous section have been extensively used ([Thomas et al., 2003b](#); [Heavens et al., 2004](#); [Cimatti et al., 2006](#); [Thomas et al., 2010](#); [Clemens et al., 2006](#); [Clemens et al., 2009](#); [Johansson et al., 2012](#)) to study the galaxy parameter-mass relation, widely named in the literature as scaling relations. These studies established the *Downsizing Scenario* for the galaxy formation of elliptical galaxies, which was possibly one of the most successful and renowned theories developed via the use of Lick indices. We now examine this scenario in some depth, both because it constitutes one of the most influential organising principles to emerge from three decades of stellar population synthesis applied to ETGs, and because it supplies the physical model for the dependence of age on host-galaxy mass, $t_{\text{arch}}(\sigma)$, that we left unspecified in [eq. \(1.24\)](#) when constructing our cosmographic $t(z)$ solution.

Naively, one would expect the mass assembly of galaxies to mirror that of the dark matter haloes that host them. In a hierarchical, bottom-up Λ CDM Universe, low-mass haloes collapse first and subsequently merge to build up more massive structures ([Press & Schechter, 1974](#); [White & Rees, 1978](#); [Blumenthal et al., 1984](#); [White & Frenk, 1991](#); [Springel et al., 2005](#)); an extrapolation of this picture to the baryonic component would therefore predict that the most massive galaxies observed today should also be, on average, the youngest, having assembled the bulk of their final mass through late-time merging of smaller, older sub-units. The empirical evidence accumulated since the early 2000s points, however, in the opposite direction. Using population synthesis models with variable element-abundance ratios applied to Lick-index measurements of a sample of 124 field and cluster ETGs, [Thomas et al. \(2005\)](#) demonstrated that the light-weighted age, metallicity $[Z/H]$ and α -enhancement $[\alpha/Fe]$ of the dominant stellar population are all positively correlated with the central velocity dispersion, σ , of the host galaxy. Velocity dispersion is a long-established independent dynamical proxy for the virial mass of a galaxy ([Faber & Jackson, 1976](#); [Cappellari et al., 2006](#)),

generalised within the Fundamental Plane of elliptical galaxies (Djorgovski & Davis, 1987; Dressler et al., 1987). For this reason, it is quite common to refer to velocity dispersion, a directly measurable quantity (Sec. 2.1.2), in scaling relation works.

The physical driver most commonly invoked to explain this trend is a mass-dependent quenching efficiency: feedback from active galactic nuclei (AGN) (Silk & Rees, 1998; Di Matteo et al., 2005; Croton et al., 2006; Bower et al., 2006; Fabian, 2012) and/or the virial shock-heating of infalling gas (Birnboim & Dekel, 2003; Dekel & Birnboim, 2006; Kereš et al., 2005) act more rapidly and more completely in massive haloes, suppressing further star formation on a short timescale, whereas lower-mass systems retain a reservoir of cold gas capable of sustaining star formation over an extended period. Renzini (2006) summarises this scenario within a broader review of stellar population diagnostics: the star-formation history of an ETG can be described, to first approximation, by a decaying exponential, $\text{SFR}(t) \propto (t/\tau^2) \exp(-t/\tau)$, already introduced in Sec. 1.3.2 in the context of the D4000_n calibration of Moresco et al. (2011), with a characteristic timescale τ that anti-correlates with the final stellar (or dynamical) mass of the galaxy. Massive ellipticals are thus not only older on average, but also formed their stars over a narrower window of cosmic time, a fact directly imprinted in their enhanced $[\alpha/\text{Fe}]$ ratios (Sec. 1.3.2), since a shorter formation timescale leaves less time for the delayed iron production by Type Ia supernovae to dilute the α -element enrichment inherited from core-collapse events, which remains imprinted in the light of lower mass stars that today make up the majority of the flux coming from these systems.

Fig. 1.15, a direct take from Thomas et al. (2010), reproduces the possibly best and most concise summary of the status of the downsizing scenario in a single picture. The authors describe throughout their work two separate populations of ETGs, reflected in bimodal distributions for the relevant galaxy parameters: t , $[Z/H]$ and $[\alpha/\text{Fe}]$. These make up a purely old and passively evolving stellar population, dominant at higher redshifts, $z \gtrsim 0.2$, and a rejuvenation phase in which minor mergers and recent bursts of star formation take place. However, the latter is only relevant at very low redshifts.

For what concerns the formation phase at higher redshifts, we can clearly see how the SFH of the most massive systems peaks earlier in time and extends over much shorter timescales (notice that the figure is presented linearly distributed in lookback time, adopting a Λ CDM cosmology: $\Omega_m = 0.24$, $\Omega_\Lambda = 0.76$, $H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$). According to their estimation, systems with $\log M_{\text{dyn}}/M_\odot \sim 12.0$ form their stellar population around $z \sim 4$ and in $\sim 1 \text{ Gyr}$, while $\log M_{\text{dyn}}/M_\odot \sim 11.0$ systems, only one order of magnitude less, do so around $z \sim 1$ and in $\sim 3 \text{ Gyr}$, a timescale three times larger. We note here how this observation is critical and must be taken into account when analysing elliptical galaxies at high redshift (i.e. $z = 1$), where, according to this picture, the expected progenitors of less massive systems may have formed too recently or still be forming their stellar population.

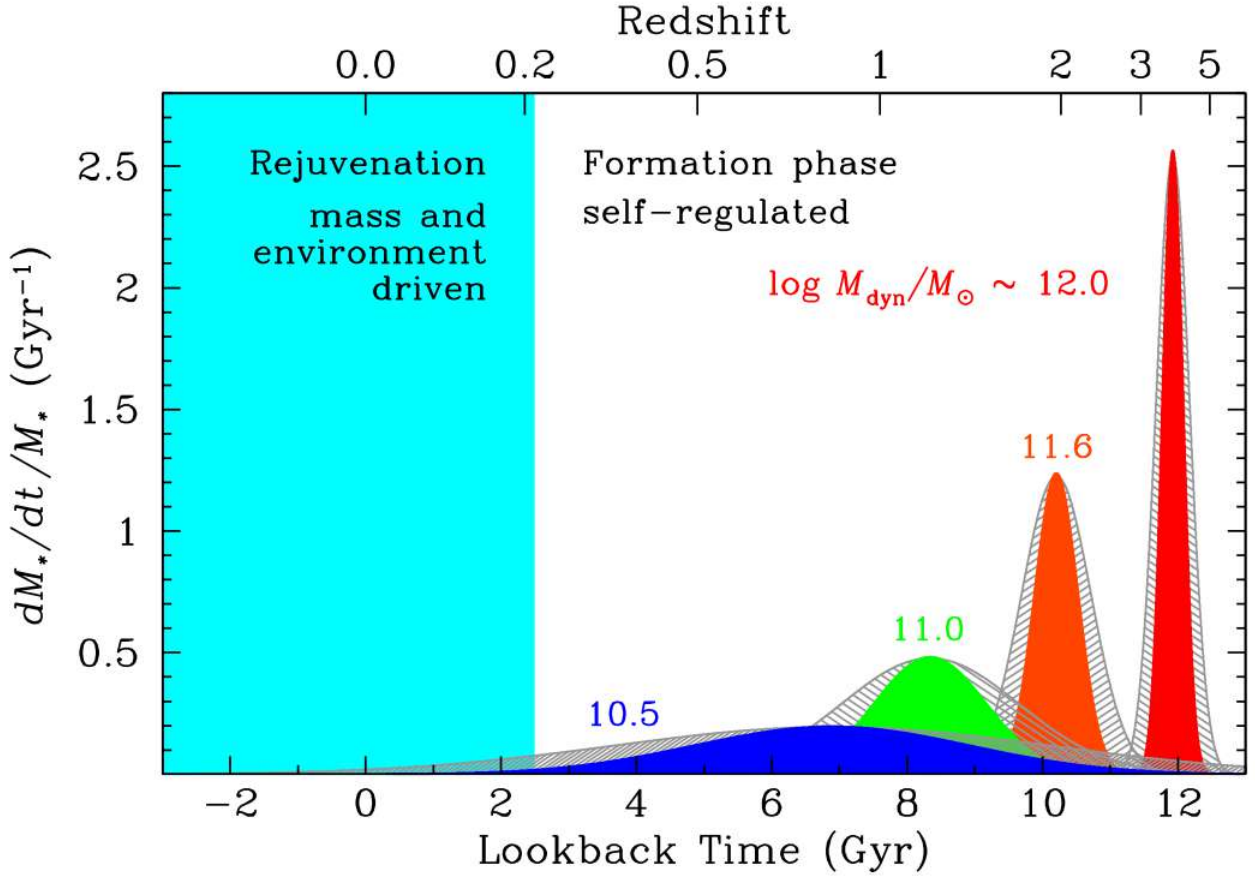


Figure 1.15: View of Figure 9 from [Thomas et al. \(2010\)](#), illustrating the two-phase scenario for the build-up of the stellar mass of ETGs. Coloured curves show the specific star-formation rate, $\dot{M}_*/M_*$, as a function of lookback time (bottom axis) and redshift (top axis), for four bins of dynamical mass, $\log(M_{\text{dyn}}/M_{\odot}) = 10.5$ (blue), 11.0 (green), 11.6 (orange) and 12.0 (red). The cyan-shaded region marks the low-redshift ($z \lesssim 0.2$) rejuvenation phase, driven by mass- and environment-dependent late episodes of star formation superimposed on an otherwise old population; the white region marks the earlier, self-regulated formation phase during which the bulk of the stellar mass of each bin was assembled.

In the literature, another source of ageing for elliptical galaxies made up of passively evolving stellar populations is considered: the effect of the local density. At fixed mass, the environment in which a galaxy resides leaves a measurable imprint on its stellar population. [Thomas et al. \(2005\)](#) already noted that the age- σ , $[Z/H]$ - σ and $[\alpha/\text{Fe}]$ - σ relations derived independently for their high- and low-density sub-samples are offset from one another: massive ETGs in low-density (also named as *field*-like) environments are found to be, on average, ~ 2 Gyr younger and mildly ($\sim 0.05 - 0.1$ dex) more metal-rich than their counterparts of the same mass residing in high-density (cluster) environments, a result that has since been corroborated by numerous independent studies using larger, more homogeneously selected samples (e.g. [Clemens et al., 2006](#); [Clemens et al., 2009](#); [Carson & Nichol, 2010](#)). Physically, this is naturally interpreted within the same quenching framework

invoked above: the denser the environment, the more efficiently the available gas supply is either consumed, stripped, or heated at early times, truncating star formation earlier and more completely than would occur for an equal-mass galaxy evolving in isolation. [Clemens et al. \(2006\)](#); [Clemens et al. \(2009\)](#) extend this analysis using far-infrared and radio continuum tracers of residual star formation and dust content, and reach a fully consistent picture: not only are the light-weighted ages of ETGs systematically younger in the field, but a non-negligible fraction of nominally quiescent, morphologically early-type systems retain measurable amounts of cold gas and dust capable of sustaining low-level star formation well into the present day, fuel for the rejuvenation episodes seen in Fig. 1.15.

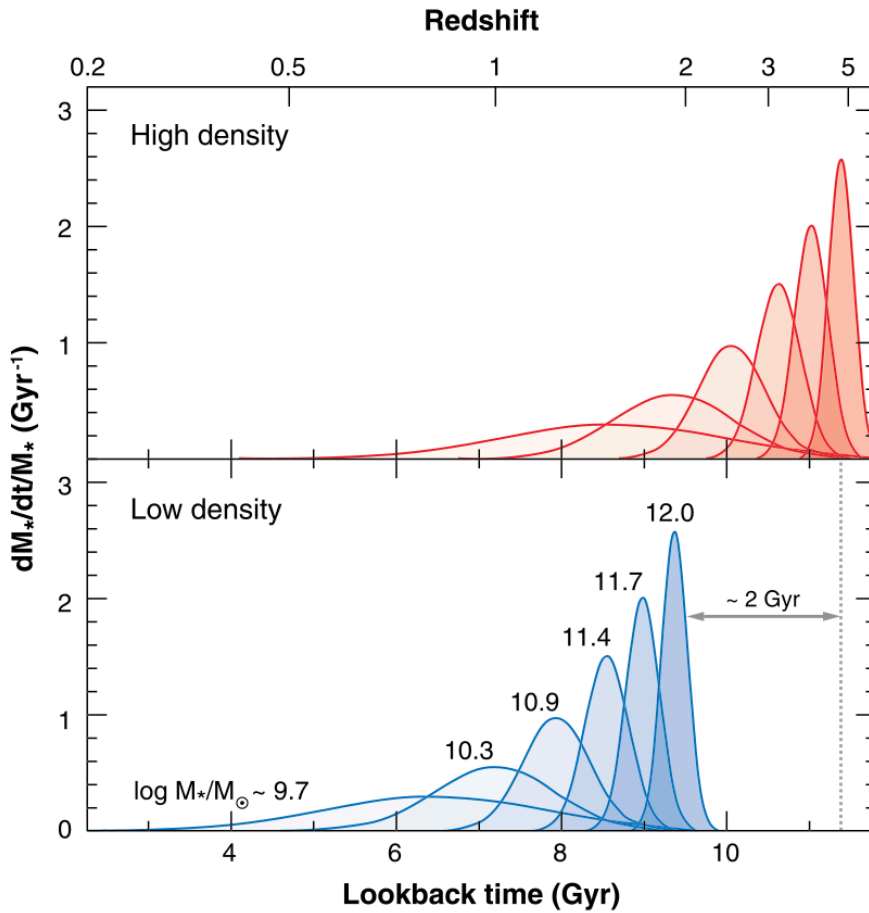


Figure 1.16: View of Figure 6 from [Renzini \(2006\)](#), reproducing the specific star-formation rate as a function of lookback time and redshift of ETGs split by dynamical/stellar mass (labels, in $\log M_*/M_\odot$) and by environment: high density (top panel) and low density (bottom panel).

Figure 1.16, reproduced from [Renzini \(2006\)](#), makes this environmental offset explicit by directly comparing the star-formation histories of ETGs of equal mass but in contrasting environments. For every mass bin shown, the peak of the specific star-formation rate occurs systematically earlier (larger lookback time, higher redshift) in the high-density panel than in the low-density one, by an amount fully consistent with the ~ 2 Gyr age offset quoted above and marked explicitly in the figure. Combining this environmental view with the pure mass-downsizing trend of Fig. 1.15, [Thomas et al. \(2005\)](#) were able to place statistical constraints on the redshift range within which the bulk of the stellar mass of present-day ETGs was assembled, finding that most star-formation activity took place

between $z \sim 3$ and $z \sim 5$ in high-density environments, and between $z \sim 1$ and $z \sim 2$ in low-density ones, with at least half of the total present-day stellar mass density already in place by $z \sim 1$, in agreement with independent estimates from the evolution of the cosmic SFR density.

For the works included in this thesis, a thorough three-dimensional ($\text{dec} \times \text{ra} \times z$) analysis of the sources' location in order to match high-density regions, or the application of density selection procedures like the one described in [Clemens et al. \(2006\)](#), is not applied. Instead, the spectrophotometric cuts summarised in [Moresco et al. \(2018\)](#) should be enough to isolate objects in dense environments ([Crawford et al., 2010](#); [Liu et al., 2016](#)).

Coming back to the general downsizing scenario, the scaling relations admit a compact, quantitative description between the stellar population parameters $\{t, [Z/H], [\alpha/\text{Fe}]\}$ and σ , the latter acting as an observationally robust surrogate for the galaxy mass ([Faber & Jackson, 1976](#); [Cappellari et al., 2006, 2013](#); [Cappellari, 2016](#)), $\sigma \propto M_{\star}^{0.3}$ ([Zahid et al., 2016](#)). Working in $\log \sigma$, both [Thomas et al. \(2005\)](#) and, on a substantially larger and more homogeneously selected sample drawn from SDSS, [Johansson et al. \(2012\)](#) find that age, $[Z/H]$ and $[\alpha/\text{Fe}]$ are each well described, at fixed formation epoch, by an approximately linear relation of the form

$$\log t = b_t + a_t \log \left(\sigma \text{ [km s}^{-1}] \right), \quad (1.55)$$

$$[Z/H] = b_{[Z/H]} + a_{[Z/H]} \log \left(\sigma \text{ [km s}^{-1}] \right), \quad (1.56)$$

$$[\alpha/\text{Fe}] = b_{[\alpha/\text{Fe}]} + a_{[\alpha/\text{Fe}]} \log \left(\sigma \text{ [km s}^{-1}] \right), \quad (1.57)$$

where the three slopes, a_t , $a_{[Z/H]}$ and $a_{[\alpha/\text{Fe}]}$, quantify the strength of the downsizing trend in age, metallicity and α -enhancement respectively. The age- σ slope found for the red sequence of galaxies in [Thomas et al. \(2010\)](#) is found to be $a_t \approx 0.52$; that is, an increase in the velocity dispersion by a factor of two, which would be the case of comparing galaxies with $\sigma = 200 \text{ km s}^{-1}$ and $\sigma = 400 \text{ km s}^{-1}$, represents a 40% increase in the age of the stellar population at fixed redshift. This is a simple computation that makes the point for separating CC candidates in velocity dispersion groups. The $[Z/H] - \sigma$ and $[\alpha/\text{Fe}] - \sigma$ relations are comparably steep and, importantly, the zero-point, slope and scatter of the $[\alpha/\text{Fe}] - \sigma$ relation are found to be indistinguishable between cluster and field galaxies. This is because $[\alpha/\text{Fe}]$ retains a memory of the duration of star formation regardless of when, cosmically, that star formation took place, precisely the behaviour expected if $[\alpha/\text{Fe}]$ is set by the competition between core-collapse and Type Ia enrichment timescales (Sec. 1.3.2) and not directly by environment. It is the $t - \sigma$ and $[Z/H] - \sigma$ relations, by contrast, that carry the environmental offset discussed in the previous subsection, entering eqs. (1.55) and (1.56) as a shift in the zero-point b_t ($b_{[Z/H]}$) between high- and low-density sub-samples, rather than in the slope itself.

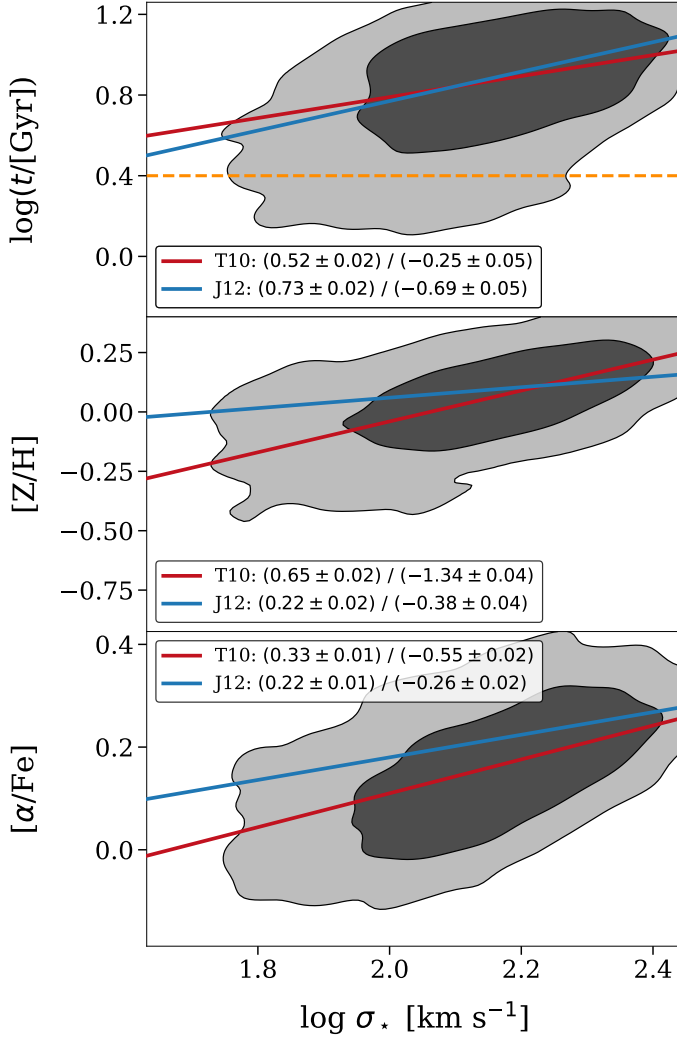


Figure 1.17: Reconstruction of the $t - \sigma$ (top), $[Z/H] - \sigma$ (middle) and $[\alpha/\text{Fe}] - \sigma$ (bottom) scaling relations for ETGs, illustrating the trends reported by [Thomas et al. \(2010\)](#) and [Johansson et al. \(2012\)](#). Contour plots represent the density of sources of the MOSES catalog in the range $0.05 < z < 0.06$, and we have plotted in the upper panel the separation between the purely passive and rejuvenated populations, set at $\log(t) = 0.4$, with an orange dashed line. We refer to figure 2 in [Thomas et al. \(2010\)](#) for the distribution of sources in age, where the bimodality due to the fraction of rejuvenated galaxies is depicted.

Figure 1.17 shows the best-fit scaling relations from [Thomas et al. \(2010\)](#) (red line) and [Johansson et al. \(2012\)](#) (blue line), superposed over the contour levels indicating the density of sources in the plane parameter- σ . The galaxies used come from the [Schawinski et al. \(2007\)](#) MOSES (MOrphologically Selected Ellipticals in SDSS) catalog, a subsample of SDSS made up of $\sim 16\,000$ galaxies at $0.05 < z < 0.10$. In the figure, only the $z < 0.06$ objects are represented in the density contours.

In the age panel the rejuvenated fraction of ETGs is shown as the off-linear overdensity below $\log(t) = 0.4$. We note that the best-fit scaling relations in [Thomas et al. \(2010\)](#) are obtained including only sources with $\log(t) > 0.4$, with the fraction of rejuvenated galaxies increasing in less dense environments. A slight non homogeneous behaviour can also be spotted for the metallicity panel. We refer to figure 2 in [Thomas et al. \(2010\)](#) to observe the parametrisation of the purely passive and rejuvenated populations in the distribution of ages.

Coupling the archaeological and cosmographic descriptions

We are now in a position to return to the cosmographic $t(z, \sigma)$ model of Sec. 1.2.1 and to give an explicit, physically motivated form to its archaeological term, $t_{\text{arch}}(\sigma, z_0)$, which so far had been left

as an unspecified boundary condition of eq. (1.24). We recall that this term encodes the age of the dominant stellar population, at the pivotal redshift z_0 , as a function of the velocity dispersion, σ . Since our CC samples are, by construction, restricted to the old, quiescent, reddest envelope of the ETG population (Sec. 1.2), and since we further impose a purity cut on residual star formation via the H/K criterion of Borghi et al. (2022b) (Sec. 1.3.2), we may reasonably neglect the low-amplitude rejuvenation component (see the age distribution in figure 2 in Thomas et al. (2010)) and adopt the single power-law downsizing relation as our working archaeological model.

Explicitly, we take eq. 1.55, which can be rewritten in the more compact form $\log(t) = a_t \log(\sigma/\sigma_{\text{ref}})$, where $b_t = -a_t \log(\sigma_{\text{ref}} [\text{km s}^{-1}])$, with σ_{ref} being a free parameter necessarily. This form can be more customary for central black-hole relations, where σ_{ref} is set manually (Tremaine et al., 2002; Gültekin et al., 2009; McConnell & Ma, 2013); but doing so would reduce a freedom degree that we need, so unavoidably one would have to introduce another free parameter once again, taking us back to eq. (1.55). However, it will be convenient to adopt this form, giving a physical meaning to b_t and avoiding fractional powers of the velocity dispersion, as we will instead raise the ratio $\sigma/\sigma_{\text{ref}}$ to a power. Indeed, we work with the original form of the scaling relation (eq. 1.55) and a reference velocity dispersion σ_{ref} introduced as a scale. We do so by defining $t_{\text{ref}} = t(\sigma_{\text{ref}}, z_0) \equiv 10^{b_t + a_t \log(\sigma_{\text{ref}})}$, the age of a typical galaxy of $\sigma = \sigma_{\text{ref}}$ at $z = z_0$. Then, we remove the logarithm from both sides of the scaling relation and combine it with 1.48 to find

$$t(z, \sigma; z_0, \sigma_{\text{ref}}; t_{\text{ref}}, a_t; \text{cosmo}) = t_{\text{ref}} \left(\frac{\sigma}{\sigma_{\text{ref}}} \right)^{a_t} - \sum_m \frac{(1+z_0)\tilde{b}_m}{m+1} \left(\frac{z-z_0}{1+z_0} \right)^{m+1}, \quad (1.58)$$

where by *cosmo* we refer to the cosmographic parameters $\{H_{z_0}, q_{z_0}, j_{z_0}, s_{z_0}, l_{z_0}\}$ that enter in each of the $\tilde{b}_i$. We remind that $x = (z - z_0)/(1 + z_0)$. This combined form allows for individual CCs to be used without unavoidably grouping them in velocity dispersion (mass) bins, introducing a zero (or pivotal redshift)-age free parameter per group, leveraging the full sample of passively evolving galaxies in σ . We remark that the number of free parameters in eq. (1.58) gets fixed in the moment that a truncation limit is chosen (i.e. x^3 in A26), which is particularly convenient when working with large CC datasets that allow for several σ groups without inefficiently extending the parameter space.

1.4 Stellar Population Synthesis

The description of Cosmic Chronometers relies on the ability to translate an observed set of spectroscopic indices into a physical age, as commented when introducing SSPs in Sec. 1.3.1. This translation is only possible once a model exists that predicts how those indices, or the full SED of a passively evolving stellar population, depend on its fundamental parameters: age, metallicity, and detailed chemical abundance pattern. Building such models is the domain of Stellar Population Synthesis (SPS), comprehensively reviewed by Conroy (2013), whose general structure we follow in this section.

The starting point of any SPS model is the SSP, already introduced in Sec. 1.3, a coeval ensemble of stars sharing a single initial metallicity and abundance pattern. Following [Conroy \(2013\)](#), the spectrum of an SSP can be written as

$$f_{\text{SSP}}(t, Z) = \int_{m_{\text{lo}}}^{m_{\text{up}}(t)} f_{\text{star}}(T_{\text{eff}}(M), \log(g(M)) | t, Z) \Phi(M) dM, \quad (1.59)$$

where M is the initial (zero-age main sequence) stellar mass, $\Phi(M)$ is the initial mass function (IMF), f_{star} is the spectrum of an individual star as a function of effective temperature, T_{eff} , and (logarithmic) surface gravity, $\log(g)$, and f_{SSP} is the resulting age and metallicity-dependent SSP spectrum. The upper limit of integration, $m_{\text{up}}(t)$, is fixed by stellar evolution: it corresponds to the turn-off mass at age t , i.e. the physical quantity behind the MSTO discussed in Sec. 1.3. The lower limit, m_{lo} , is conventionally set at the hydrogen-burning limit ($0.08 - 0.1 M_{\odot}$), since stars below it contribute negligibly to the integrated light.

Equation (1.59) makes explicit the three physical ingredients that any SPS model must specify: a mapping between $\{M, t, Z\}$ and the atmospheric parameters $\{T_{\text{eff}}, \log(g)\}$, i.e. a set of isochrones (Sec. 1.4.2); a library of stellar spectra f_{star} as a function of those atmospheric parameters, i.e. a stellar library (Sec. 1.4.1); and a functional form for $\Phi(M)$, the initial mass function (Sec. 1.4.3). Once combined, the resulting SSP grid can be integrated over a star-formation history to build a Composite Stellar Population (CSP; Sec. 1.4.4), or evaluated directly, or under a short-formation-timescale approximation, to construct the SSP-equivalent models routinely used in CC studies (Sec. 1.4.5).

A visual summary of this pipeline is given in Fig. 1.18, a classic view quite popular in the community reproduced from [Conroy \(2013\)](#). It shows how the three ingredients just introduced combine into an SSP, and how an SSP is, in turn, combined with additional ingredients to build a CSP. The upper row of the figure illustrates representative examples of each SSP ingredient: several characteristic forms of the IMF ([Salpeter, 1955](#); [Chabrier, 2003](#); [van Dokkum, 2008](#)), a set of isochrones spanning $10^6 - 10^{10}$ yr, and the spectra of a handful of individual stars covering a wide range in effective temperature and luminosity. Combining these three ingredients through eq. (1.59) produces the SSP spectra shown in the central panel of the middle row, plotted here for several ages between 10^6 and 10^{10} yr; the systematic reddening and steepening of the spectrum with age offers a first, qualitative illustration of the same age sensitivity already discussed for individual spectral features in Sec. 1.3.1.

The middle row also introduces the two additional ingredients required to move from an SSP to a CSP: a star-formation history (SFH) and chemical-enrichment history (left panels), and a dust attenuation and emission model (right panels). Their combination yields the dust-free and dust-attenuated CSP spectra shown in the bottom panel. While a detailed treatment of dust and of extended, complex star-formation histories lies largely outside the scope of this thesis, restricted, as motivated in Sec. 1.3.2, to old, passively evolving and effectively dust-free populations, Fig. 1.18 remains a useful map of the full SPS machinery.

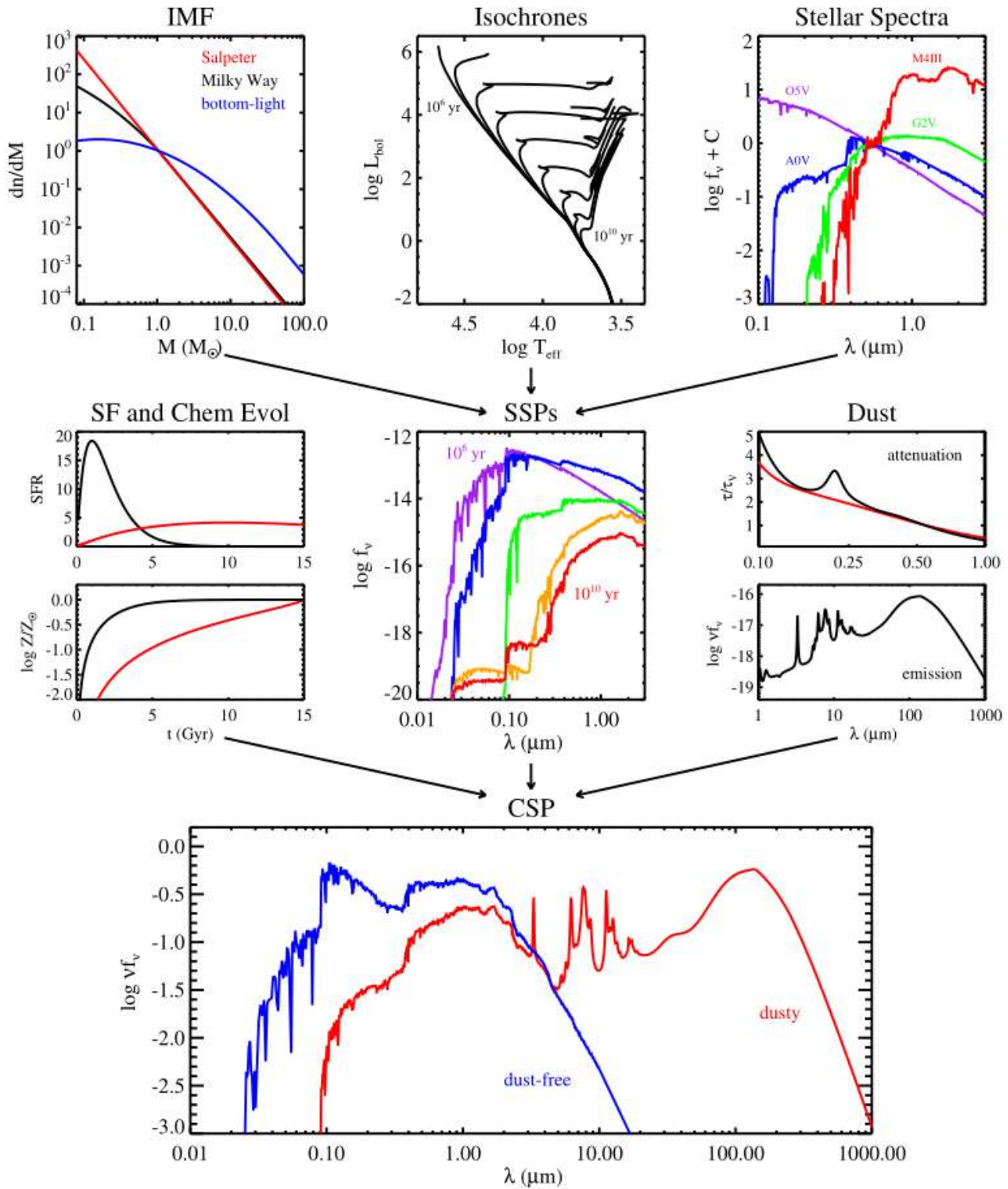


Figure 1.18: Direct view of Figure 1 from [Conroy \(2013\)](#). Overview of the stellar population synthesis technique. The upper row illustrates the three basic ingredients needed to build an SSP: IMF ([Salpeter, 1955](#); [Chabrier, 2003](#); [van Dokkum, 2008](#)), a set of isochrones spanning $10^6 - 10^{10}$ yr (centre), and individual stellar spectra (right). The middle row shows the resulting SSP spectra at several representative ages (centre), together with the two additional ingredients required to build a CSP: a SFH and its associated chemical enrichment (left), and a dust attenuation and emission model (right). The bottom panel shows the resulting CSP spectrum, both before (blue) and after (red) a dust model is applied.

1.4.1 Stellar libraries

Populating eq. (1.59) requires a spectrum f_{star} for every combination of T_{eff} , $\log g$ and chemical composition sampled by the isochrones. These are known as libraries and there are two largely complementary strategies to build them. Theoretical libraries compute f_{star} from first principles by solving the radiative transfer equations in a model atmosphere; they offer dense, arbitrarily fine coverage of parameter space (Lejeune et al., 1997, 1998; Westera et al., 2002; Martins et al., 2005; Munari et al., 2005; Coelho et al., 2005), but are only as reliable as the input atomic and molecular line lists, which remain notably incomplete for cool stars (Kitamura et al., 2017; Kurucz, 2017). Empirical libraries instead assemble observed stellar spectra (e.g. Pickles, 1998; Prugniel & Soubiran, 2001; Le Borgne et al., 2003; Valdes et al., 2004; Sánchez-Blázquez et al., 2006; Rayner et al., 2009; Falcón-Barroso et al., 2011); they reproduce the real physics of stellar atmospheres by construction, but are limited to the stars that exist in the solar neighbourhood, where hot, low-metallicity stars are intrinsically rare, and to the fixed Galactic abundance pattern of the sampled stars.

For the optical range and moderate spectral resolution relevant to Lick-index work, empirical libraries have historically dominated. The MILES library (Sánchez-Blázquez et al., 2006; Falcón-Barroso et al., 2011) is the reference case: nearly 1000 stars spanning a wide range in T_{eff} , $\log g$ and $[\text{Fe}/\text{H}]$ (Cenarro et al., 2007), observed at a common resolution ($\text{FWHM} = 2.51 \text{ \AA}$) that improves the resolution of the Lick/IDS system (Sec. 1.3.2), matches that of the SDSS spectrograph, and is degraded with respect to that of the DESI spectrographs. Its main limitation, common to any empirical library, is the fixed abundance pattern of its constituent stars: at low $[\text{Fe}/\text{H}]$, Galactic disc and halo stars are systematically α -enhanced (Sec. 1.3.2), so using MILES spectra directly to model extragalactic populations at arbitrary $[\alpha/\text{Fe}]$ would implicitly fix the abundance pattern to that of the Milky Way. This limitation motivated the semi-empirical extension sMILES (Knowles et al., 2021), in which the base MILES spectra are perturbed through theoretical response functions (Sec. 1.4.5) to populate a genuine $t \times [\text{Z}/\text{H}] \times [\alpha/\text{Fe}]$ grid, rather than the $t \times [\text{Z}/\text{H}]$ grid of the base library. It is this sMILES library that underlies the Knowles et al. (2023) models already used in Sec. 1.3.1 to illustrate the age-metallicity- $[\alpha/\text{Fe}]$ diagnostics of Figs. 1.9 and 1.12. For a direct comparison of how the library choice propagates into derived ages, see Koleva et al. (2008).

1.4.2 Isochrones

Isochrones supply the mapping between the fundamental parameters $\{M, t, Z\}$ and the atmospheric parameters $\{T_{\text{eff}}, \log(g)\}$ required by eq. (1.59). In practice, an isochrone is built by interpolating a grid of stellar evolutionary tracks (individual solutions of the stellar structure equations, followed from the pre-main sequence through the most advanced evolutionary phases a star of a given mass and composition will reach within a Hubble time) at fixed age t (Conroy, 2013). The resulting $T_{\text{eff}}(M, t, Z)$ and $\log g(M, t, Z)$ relations are directly sensitive to the input physics of the underlying

evolutionary code: the treatment of convective overshooting, the efficiency of mass-loss, the nuclear reaction network and the adopted opacities all leave an imprint on the exact placement of the isochrone in the HR diagram, and in particular on the temperature of the MSTO that CC ages ultimately rely upon (Sec. 1.3).

Fig. 1.19, reproduced from Bressan et al. (2012), illustrates this mapping explicitly for a sequence of PARSEC isochrones at fixed, solar metallicity: as age increases, the progressive depletion of the upper main sequence and the corresponding growth of the giant branch trace out the same physical picture already discussed for the MSTO in Sec. 1.3. Here, it is made explicit as a continuous family of curves in the HR diagram rather than as a single evolving SED. We note how the diagram includes pre-MS phases (upper-right region to the MS) in the youngest isochrones.

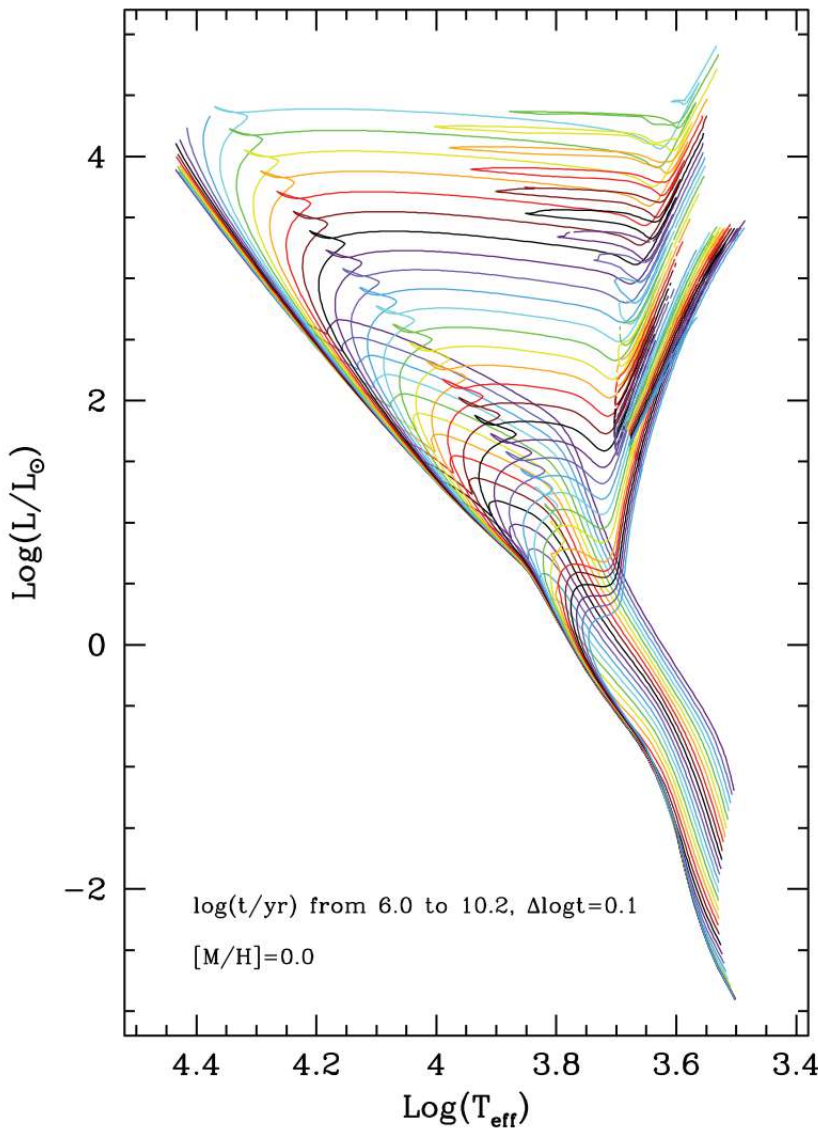


Figure 1.19: View of the left panel in figure 15 from Bressan et al. (2012). Age dependence of the isochrones: a sequence of PARSEC isochrones at fixed, solar metallicity, for ages from $\log(t [\text{yr}]) = 6$ to 10.2 in equally spaced steps of $\Delta \log(t) = 0.1$. Each sequence is completed down to $0.1 M_{\odot}$, with the presence of the pre-main-sequence phase in the youngest isochrones.

No single isochrone set covers the full range of mass, age and metallicity relevant to old, passively evolving populations without some degree of splicing between evolutionary codes, or successive

updates that extend the coverage or physics of a given database (Schaller et al., 1992; Cassisi et al., 1997; Yi et al., 2001; Vandenberg et al., 2006; Marigo & Girardi, 2007; Marigo et al., 2008; Dotter et al., 2008; Bressan et al., 2012; Pietrinferni et al., 2021). Two families dominate the literature relevant to this thesis. The Padova isochrones (Girardi et al., 2000) cover a broad range in mass and metallicity with a standard treatment of convective overshooting, and constitute, together with the STELIB empirical library, the backbone of the Bruzual & Charlot (2003) models. This database has itself been repeatedly updated: Marigo & Girardi (2007) and Marigo et al. (2008) revised the thermally-pulsating AGB phase and recalibrated it against resolved clusters in the Magellanic Clouds, while Bressan et al. (2012) (PARSEC), shown in Fig. 1.19, recomputed the whole grid with updated opacities and nuclear reaction rates, extending the coverage down to the pre-main sequence and up to higher masses. Within a single family of models, it illustrates the continuous process of splicing and patching noted above.

The BaSTI database (Pietrinferni et al., 2004), extended to α -enhanced compositions by Pietrinferni et al. (2006) and more recently updated by Pietrinferni et al. (2021) with revised input physics and a finer sampling in mass, age and metallicity, additionally provides a self-consistent treatment of the horizontal branch (HB) and of its dependence on mass-loss efficiency along the red-giant branch. This is the isochrone set adopted by the MILES/sMILES-based models (Vazdekis et al., 2010, 2015; Knowles et al., 2023), and it is this explicit HB treatment that makes BaSTI the preferred choice whenever an SPS model is meant to be applied to CC index measurements. The treatment of the HB is not a minor detail.

As already discussed in Sec. 1.3.2, an unmodelled population of blue HB stars can masquerade as a younger population when Balmer-line indices alone are used to estimate age (Thomas et al., 2011; Concas et al., 2017). Together with the low-metallicity coverage gaps illustrated by dedicated intermediate-age, metal-deficient isochrone calculations (Cassisi et al., 1997), these are the isochrone-level assumptions that propagate directly into the systematic error budget of any CC age estimate.

1.4.3 Initial Mass Function

The initial mass function, $\Phi(M)$, completes eq. (1.59) by weighting the contribution of each stellar mass to the integrated light. Its functional form has been characterised in the solar neighbourhood since Salpeter (1955), with more recent parametrisations such as that of Chabrier (2003). For old, passively evolving populations of the kind targeted by CC studies (Sec. 1.2), the role of the IMF in eq. (1.59) is comparatively minor: because the integrated light of an SSP older than a few Gyr is dominated by stars clustered tightly around the turn-off mass, the logarithmic slope of $\Phi(M)$ mostly rescales the overall mass-to-light ratio rather than reshaping the SED itself (Conroy, 2013). Spectral indices, being ratios of fluxes measured within the same spectrum, are essentially insensitive to this rescaling, in clear contrast with what happens for stellar-mass estimates (Sec. 1.3.3), where the IMF

normalisation is a dominant source of systematic uncertainty.

This near-degeneracy between IMF slope and index strength does not mean that the choice of $\Phi(M)$ is irrelevant for SPS models in general. A separate line of research uses precisely the residual sensitivity of gravity-sensitive dwarf/giant features (e.g. the Na I doublet, the Ca II triplet, or the FeH Wing-Ford band) to test for a non-universal, "bottom-heavy" IMF in the cores of massive early-type galaxies (Conroy & van Dokkum, 2012). To accommodate this kind of study, several SPS codes allow the IMF slope itself to vary as a free parameter, rather than fixing it to a single functional form. This is the case of the unimodal and bimodal parametrisations introduced by Vazdekis et al. (1996), characterised by a single logarithmic slope Γ (unimodal) or Γ_b (bimodal, which additionally tapers the low-mass end), and later adopted by the MILES/sMILES model family (Vazdekis et al., 2010, 2015; Knowles et al., 2023). A steep bimodal slope ($\Gamma_b \sim 3.0$) is markedly more bottom-heavy than a Salpeter IMF and is the one typically invoked to explain the observed IMF gradients and mass-to-light excesses of the most massive ETGs, whereas $\Gamma_b \sim 1.3$ closely reproduces a standard, Kroupa-like IMF.

The Knowles et al. (2023) sMILES grid used in this thesis (Sec. 1.4.1, Sec. 1.4.5) is computed for five such IMF choices: a Universal and a Revised Kroupa IMF (Kroupa, 2001), thirteen unimodal and thirteen bimodal slopes spanning $\Gamma, \Gamma_b = 0.3 - 3.5$ following Vazdekis et al. (1996); Vazdekis et al. (2015), and a Chabrier IMF with a high-mass segment slope of 1.3 (Chabrier, 2003), all computed between fixed mass cut-offs of 0.1 and 100 $M_\odot$. This breadth of coverage is what makes the grid suitable both for dwarf-sensitive IMF studies of the kind described above and, more relevantly for this work, for explicitly checking that the CC ages derived in the following chapters are insensitive to this choice, rather than simply assuming so a priori.

That the IMF is indeed a subdominant systematic for CC studies based on spectral indices has been explicitly verified by Moresco et al. (2020), in their systematic assessment of the SPS-related uncertainty budget of the D4000_n-based CC method. Varying the SPS ingredients entering the models (stellar library, isochrones, spectral resolution, and IMF), they find that the resulting systematic covariance of the derived $H(z)$ is dominated by the choice of SPS code and by the treatment of metallicity and star-formation history (Sec. 1.4.4), while IMF variations contribute at a comparatively negligible level, consistent with the theoretical expectation of eq. (1.59) discussed above. This result ultimately justifies why CC oriented models can adopt a single, fixed IMF (e.g. Knowles et al., 2023) without this choice contributing appreciably to the systematic budget of the derived ages.

1.4.4 Composite Stellar Populations

Real galaxies are not SSPs but Composite Stellar Populations (CSPs), built from stars formed continuously, or in multiple episodes, over an extended star-formation history (SFH). A CSP spectrum

is obtained by integrating the SSP grid of eq. (1.59) against a SFH and a metallicity distribution function,

$$f_{\text{CSP}}(t) = \int_{t'=0}^{t'=t} \int_{Z=0}^{Z_{\text{max}}} \text{SFR}(t-t') P(Z, t-t') f_{\text{SSP}}(t', Z) dt' dZ, \quad (1.60)$$

where dust attenuation and emission, negligible for the quiescent systems considered in this thesis, have been dropped from the original expression for clarity. The SFH, $\text{SFR}(t)$, is most commonly parametrised as an exponentially declining τ -model, $\text{SFR}(t) \propto \exp(-t/\tau)$, its delayed variant, $\text{SFR}(t) \propto t \exp(-t/\tau)$, or a double-power-law, $\text{SFR}(t) \propto ((t/\tau)^\alpha + (t/\tau)^{-\beta})^{-1}$. The metallicity distribution function $P(Z, t)$ is, in the vast majority of applications, simplified to a Dirac delta function, i.e. a single metallicity for the whole CSP (Conroy, 2013).

In practice, fitting a CSP model to data means treating eq. (1.60) itself as the model to be fit, rather than reading off an SSP-equivalent age from a small number of pre-computed Lick indices (Sec. 1.3.2). A CSP-fitting code builds a library of model spectra by integrating the SSP grid against a chosen SFH parametrisation, together with a dust attenuation law and a fixed or free metallicity, and compares the resulting synthetic spectra to the full observed spectrum, or to the spectrum combined with broadband photometry, through a χ^2 or Bayesian likelihood. Full spectral fitting (FSF) codes of this kind include STARLIGHT (Cid Fernandes et al., 2005), VESPA (Tojeiro et al., 2007), whose non-parametric age binning already appeared in Sec. 1.3.2, and BAGPIPES (Carnall et al., 2018), which combines flexible parametric SFHs with a nested-sampling exploration of the full posterior on age, τ , metallicity, dust attenuation and stellar mass formed.

Works that have directly constrained the timescale for the SFH, agree on $\tau \lesssim 1.0$ Gyr (Jiao et al., 2023; Tomasetti et al., 2023, ; note that this is as a very conservative upper limit). In this regime, it is justified treating CC candidates as quasi-SSPs: the star-formation timescale short enough that eq. (1.60) collapses, to good approximation, back onto a single-metallicity SSP evaluated at a mass-dependent formation epoch (Bower et al., 1992; Kodama & Arimoto, 1997; Jimenez & Loeb, 2002; Renzini, 2006; Thomas et al., 2010).

1.4.5 Stellar Population Synthesis models

The combination of the aforementioned elements into a working, publicly available code is what is generically referred to as an SPS model. For the purposes of this thesis it is useful to distinguish two broad, only partially overlapping families, which differ not only in their treatment of the abundance pattern but, as will become clear below, in what they actually predict as their primary output.

The first family targets full spectrophotometric or broadband modelling of galaxies of any type, star-forming or quiescent. Bruzual & Charlot (2003) (BC03) combines Padova isochrones (Girardi et al., 2000) with the STELIB and BaSeL libraries; Maraston (2005), and its higher-resolution counterpart Maraston & Strömbäck (2011) (MaStro), instead adopt the fuel-consumption theorem and place

particular emphasis on the thermally-pulsating AGB phase, which dominates the near-infrared flux of intermediate-age populations. Neither of these models natively allows the elemental abundance pattern to vary independently of the total metallicity, which makes them well suited to broadband SED fitting (Sec. 1.3) and to the D4000_n calibration of Moresco et al. (2011) (Sec. 1.3.2), but poorly suited to the index-based methodology adopted in this thesis.

The second family is built specifically to predict Lick indices as a function of an independently varying abundance pattern. These models do not synthesise a spectrum at all: their direct output is the predicted strength of each Lick index (Table 1.1) as a function of $(t, [Z/H], [\alpha/Fe])$, obtained by perturbing a base SSP index grid through the response-function technique (Tripicco & Bell, 1995). The reference flux-calibrated model of this family, adopted as the necessary link between observations and age estimations in most of the analyses presented in this thesis, is Thomas et al. (2011) (TMJ), which extended the earlier index grids of Thomas et al. (2003b) and Thomas et al. (2004) to a finer sampling in age, metallicity and $[\alpha/Fe]$. Indeed, TMJ is the model adopted by Borghi et al. (2022b) in the work that pioneered the use of Lick indices in the CC context (Sec. 1.3.2).

Lick-index models

Index-models are built around the concept of an index response function: the fractional change in the strength of a given Lick index produced by enhancing the abundance of a single chemical element, at fixed age and total metallicity, computed from model atmosphere and line formation calculations for a small number of stars representative of the different evolutionary phases (typically dwarfs, turn-off stars and giants) populating the isochrone at that age and metallicity. Once tabulated, a response function can be applied multiplicatively to any base SSP index grid to synthesise the effect of an arbitrary, non-solar abundance pattern, without ever having to recompute a full theoretical spectrum.

This technique was pioneered by Tripicco & Bell (1995), who computed response functions for ten elements at solar metallicity from a fixed, 5 Gyr solar-metallicity isochrone. Trager et al. (2000b) developed the formalism needed to incorporate these response functions into a working SSP index model, which Thomas et al. (2003b) (TMB) used to build the first full grid of Lick indices as a function of age, total metallicity and $[\alpha/Fe]$, based on the evolutionary synthesis code of the predecessor of Maraston (2005) and on the stellar evolutionary tracks of Cassisi et al. (1997), adopting a Salpeter (1955) IMF throughout. Thomas et al. (2004) (TMK) extended this framework to the higher-order Balmer indices $H\delta_A$, $H\delta_F$, $H\gamma_A$ and $H\gamma_F$ (Table 1.1), showing that these become increasingly sensitive to individual element abundances, and not only to age, as metallicity increases.

A significant limitation of the Tripicco & Bell (1995) response functions is that they were computed at a single solar metallicity, and were then applied, as a fixed multiplicative correction, across the full metallicity range of the SSP grid. Korn et al. (2005) addressed this directly by recomputing response

functions over a wide range of metallicities and evolutionary phases, confirming the general validity of the [Tripicco & Bell \(1995\)](#) results while providing a more self-consistent, metallicity-dependent correction, and by applying the response functions to the flux within each absorption line, rather than to the index itself, which is always positive and therefore numerically more robust than an index that can, in some cases, take on negative values (Sec. 1.3.2). This is the response-function set incorporated into TMJ, which otherwise follows the same architecture as TMB and TMK, but is based on the updated [Maraston \(2005\)](#) evolutionary code and calibrated directly against the flux-calibrated MILES library (Sec. 1.4.1), while retaining a [Salpeter \(1955\)](#) IMF and the [Cassisi et al. \(1997\)](#) tracks of its predecessors.

The parameter space upon which Lick indices are predicted in the TMJ models is $\{0.1 < t \text{ [Gyr]} < 15.0\} \times \{-2.25 < [Z/H] < 0.67\} \times \{0.0 < [\alpha/\text{Fe}] < 0.5\}$, following a sampling that is not linear and neither is logarithmic, although low ages are more thoroughly sampled than higher ages, addressing the different regimes that could be of interest to the users of the models ([Thomas et al., 2011](#), [A25](#), [A26](#)). The given output is a grid of values (20 ages, 6 metallicities, 4 α -abundances), for which, depending on the required precision needed, some sort of interpolation is needed. For both [A25](#) and [A26](#) a three dimensional linear interpolation is performed (in the fashion of [Borghi et al. 2022b](#)) using a classical NumPy routine. At different stages of the analysis the width of the parameter space was reduced as required by the scientific purpose in particular (i.e. see Sec. 2.6.2 and the discussion around Fig. 2.24). As the interpolation did not produce relevant scientific results other than what is commented in Sec. 2.6.2 about the metallicity, a dedicated section for the model grid is not presented in this thesis, and the relevant aspects are distributed along the text where necessary.

The discussion about the model interpolation should not be concluded without introducing the possibility of a functional reconstruction. At the beginning of the research project, the production of an interpolator that responded at the precision level required for the data was considered. However, the associated computational complexity along with the onset of other systematics led to the simpler choice of a fixed fine linear interpolation of the base models.

Recent models

The response-function technique itself has since been generalised beyond the fixed set of 25 classical Lick indices. [Conroy & van Dokkum \(2012\)](#) and [Conroy et al. \(2018\)](#) compute the response of the entire optical-to-near-infrared spectrum, rather than of a handful of pre-defined bandpasses, to variations in the abundance of individual elements, at a spectral resolution high enough to resolve individual absorption features well beyond the classical Lick system. This generalisation is conceptually the bridge between the index philosophy and the full spectrum output of the [Knowles et al. \(2023\)](#) sMILES models introduced in Sec. 1.4.5, and it is, in the spirit described there, the strategy followed to build the sMILES library itself (Sec. 1.4.1).

The Knowles et al. (2023) sMILES models can be used to predict index strengths as a function of an independently varying abundance pattern. However, their primary output is full synthetic spectra. They are constructed by applying differential corrections (Knowles et al., 2021) directly to the empirical MILES spectra, combined with BaSTI α -enhanced isochrones (Pietrinferni et al., 2004; Pietrinferni et al., 2006) and a fixed IMF (Sec. 1.4.3). The resulting grid spans a parameter-space similar to that of TMJ: $\{0.03 < t \text{ [Gyr]} < 14.0\} \times \{-1.79 < [Z/H] < 0.26\} \times \{-0.2 < [\alpha/\text{Fe}] < 0.6\}$, on which the evaluated spectra can be subsequently integrated over the bandpasses of Table 1.1, producing a set of predicted Lick indices.

The spectral output makes possible, in principle, to redefine indices, define new ones and even try full or partial spectral fitting procedures, on top of the index-index diagnostic diagrams of Sec. 1.3.1 (Figs. 1.9-1.12) and Lick index analyses. A closely related, independent effort is the MILES-based $[\alpha/\text{Fe}]$ -variable model of Vazdekis et al. (2010, 2015), which reaches α -enhanced compositions through an analogous strategy and is routinely cross-checked against TMJ-type predictions in the CC and ETG literature.

More recently, Park et al. (2025) presented a new generation of α -enhanced stellar population models specifically aimed at recovering accurate stellar-population parameters for massive quiescent galaxies at high redshift, extending this model comparison into the regime where DESI and Euclid-selected CC candidates (Moresco et al., 2022) are expected to be found.

1.5 This Thesis

The remainder of this thesis is structured in three chapters, covering the methodology (Chap. 2), results (Chap. 3) and future prospects (Chap. 4) that the works presented have produced. We note that, from this point onwards, almost all of the sections in which the text is structured include (scientific) results, and in the section that receives that particular name only covers the *cosmographic* results that constitute the central aim of A25 and A26.

In Sec. 2.1 the surveys from which we have taken the raw data are presented, including a particular analysis on the generic anatomy shared by modern spectroscopic surveys and on the specific resolution, redshift and velocity-dispersion determinations of SDSS and DESI, the two facilities underlying A25 and A26 respectively. Sec. 2.2 then describes how a pure sample of massive, passively evolving early-type galaxies is built out of each parent catalogue, from the photometric and morphological cuts applied at the target-selection level to the spectroscopic vetoes, emission lines and the CaII H/K ratio, that remove any residual contamination from ongoing or recent star formation. Sec. 2.3 details the corrections that need to be applied to the raw Lick-index measurements before they can be compared to any stellar population model, including the instrumental resolution and velocity-dispersion corrections, together with the aperture correction that was ultimately not adopted

in either work. Sec. 2.4 presents the stacking procedure through which individual, low-S/N spectra are combined into the high-S/N composites that make a reliable estimation of stellar-population parameters possible, and that, incidentally, also solves the choice of redshift binning in a data-driven way. Sec. 2.5 covers the Bayesian machinery used throughout, from the fit of stellar-population parameters to individual Lick-index measurements to the cosmographic fit of the resulting $t - z$ relations. The chapter closes with Sec. 2.6, which separates the systematic uncertainties considered in this thesis into those folded directly into the statistical budget, those explicitly propagated as a dedicated contribution, and those merely reported and discussed, among which the still unexplained index-redshift oscillations uncovered in the course of this work stand out.

Chap. 3 presents the cosmological results, organised around the two cosmographic strategies developed in A25 and A26. Sec. 3.1 covers the classic, $z_0 = 0$ cosmography applied to the SDSS sample: the baseline Hubble constant obtained from the TMJ-modelled ages, its translation into a w_0 CDM reading, the comparatively poor behaviour of the Knowles model, and the resulting reconstruction of the Hubble diagram. Sec. 3.2 turns to the pivotal-redshift cosmography applied to the substantially larger DESI sample, presenting the joint fit of the $t - z$ relations across velocity-dispersion groups, the resulting $H(z)$ reconstruction together with its covariance matrix, and two independent, local measurements of the Hubble parameter obtained as a by-product of the same dataset. Sec. 3.3 steps back from the cosmographic use of the data to its archaeological content, presenting the scaling relations of stellar age, metallicity and α -enhancement with velocity dispersion, obtained directly at low redshift from SDSS and, indirectly, through the pivotal-redshift ages of each velocity-dispersion group, at $z \approx 0.57$ from DESI. Sec. 3.4 closes the chapter with a summary of the main numerical results.

Chap. 4 closes the thesis by turning to the open questions that the previous two chapters could largely only flag rather than resolve, and is accordingly written in a more exploratory register than the rest of the text. Sec. 4.1 revisits the estimation of galaxy ages itself, discussing the compatibility between full spectral fitting and the Lick-index route followed throughout this thesis, an adaptive, per-stack selection of the index set inspired by the iterative pruning schemes of Thomas et al. (2010) and Johansson et al. (2012), and a first application, to the DESI stacks of A26, of the index-index diagnostic diagrams introduced in Sec. 1.3.2. Sec. 4.2 turns instead to the cosmographic machinery on the other end of the pipeline, presenting a pivotal-redshift cosmography applied directly to the SDSS sample, a joint cosmographic-archaeological fit that lets the zero-age and the kinematic parameters be constrained simultaneously, and a direct comparison of the SDSS and DESI $t - z$ relations in their region of redshift overlap. Finally, Sec. 4.3 reports on a preliminary, partially inconclusive follow-up in which the index-redshift oscillations of Sec. 2.6.2 were sought in an independent survey, GAMA, closing the thesis on the open question about the nature of these features.

Circumstantial note about A26

Given the timing for the presentation of this thesis, the references to the A26 figures are given with respect to the submitted version of the article in <https://arxiv.org/abs/2608.13178>. The numeration might change according to the request of the referee of the article in altering one figure present in the submitted version. Also, new figures, similar to some included here, but not taken from the submitted version of the article, might get included in the corrected version. The status of the paper in the date of submission of this thesis is in correction process, after a positive assessment given by the referee asking for changes in the manuscript.

Instruments, Methodology and Systematics

Throughout this chapter, we present the methodological steps that have been taken in the pipeline. These serve to bring us from catalog data of spectroscopically observed galaxies (CC candidates) to direct estimations of the Hubble parameter.

Beyond the routine steps that can be directly read in [A25](#); [A26](#), some additional corrections and in-depth analyses that have not been portrayed in the articles are introduced. For example, the correction of the velocity dispersion effect (Sec. [2.3.1](#)), which needs to be applied to properly match observed spectral properties with the predictions from SPS models, has undergone alternative construction tests, like one based on manually degrading galaxy spectra to extrapolate the original indices at zero velocity dispersion. Moreover, originally, the aperture correction (Sec. [2.3.2](#)) considered in [Clemens et al. \(2006\)](#); [Clemens et al. \(2009\)](#) was assessed in [A25](#), but was finally excluded as the overall effect on the indices was small and the correction limited to being applied only over single-galaxy index measurements. That is, not directly applicable to the stacks that we use as tracers of our observable, the $t - z$ relations. Moreover, the strength of the correction is only important at very low redshift, far from the target of current CC works.

Other parts of the methodology are also more thoroughly analysed than reported in the articles. Alternative selection criteria (Sec. [2.2.1](#)) to enhance the purity of the CC candidates were considered at the initial stages, but could only be used as later checks of the quality of the selection criteria, rather than as flags themselves. Similarly, the spectrum stacking procedure that was presented in [A25](#) and refined in [A26](#) underwent small unreported trials, and was only implemented after the observed limitations of individual galaxy stellar parameter estimations were unavoidable (some of this is reported in [A25](#)).

Finally, the importance of a reliable estimation of systematic uncertainties, and of their impact on the final estimation of $H(z)$, will be highlighted, as it is a central point in CC studies that will determine their utility as cosmological observables. For this reason, Sec. [2.6](#) is included here. We separate the different kinds of uncertainty that have appeared in the analysis according to whether they are incorporated directly into the statistical error budget, explicitly quantified and propagated as a dedicated systematic contribution, or merely reported and discussed without being numerically

accounted for. We also try to give some insight about which systematic effects should be thoroughly addressed in upcoming works.

2.1 Spectroscopic surveys

It was already commented at the beginning of Sec. 1.2, as a central idea coming from [Jimenez & Loeb \(2002\)](#), that a good sample of galaxies with corresponding spectra is needed to produce CC measurements. At low redshift ($z \lesssim 1$), two big surveys have taken spectra from hundreds of thousands of CC candidates, these are the SDSS ([Eisenstein et al., 2005](#)), which includes its later extension to the BOSS and eBOSS ([Alam et al., 2017](#); [Zarrouk et al., 2021](#)), and the DESI survey ([Abdul Karim et al., 2026](#)). Before detailing the particular flags and cuts that we applied in [A25](#) and [A26](#), it is worth pausing on the generic anatomy shared by any modern wide-field spectro-photometric survey, since the vocabulary and the order in which decisions are made are common to both SDSS and DESI, and indeed to essentially any survey capable of feeding a CC pipeline.

Broadly, one can separate the operation of such a survey into four stages. First, a photometric imaging campaign covers a given footprint of the sky in a handful of broad bands (e.g. *ugriz* ([Fukugita et al., 1996](#)) for SDSS, *grz* complemented by the unresolved WISE ([Wright et al., 2010](#)) W1/W2 bands for the DESI Legacy Imaging Surveys), producing a catalog of detected sources with positions, fluxes, and morphological (structural) parameters. Second, a target selection algorithm is run on top of this photometric catalog, assigning each source to one or more scientific target classes: galaxies, quasars, stars, or, in the case of galaxy targets, finer sub-classes such as luminous red galaxies (LRGs), emission-line galaxies (ELGs) or a magnitude-limited bright galaxy sample. This is performed through a combination of magnitude, colour and morphology cuts designed to isolate the population of interest while keeping the fibre budget (and thus the observing time) under control. Third, the actual spectroscopic follow-up takes place: a multi-object, fibre-fed spectrograph is pointed at the selected targets, and the resulting one-dimensional spectra are processed through a reduction and classification pipeline that delivers, for every fibre, a redshift, a spectral classification (star, galaxy, or QSO) and an estimate of the reliability of that redshift. Finally, once a data release becomes public, a number of value-added catalogs (VACs) are typically produced by collaborating teams, cross-matching the reduced spectra with stellar-population or emission-line fitting codes to deliver higher-level physical quantities (e.g. velocity dispersions, stellar masses, star-formation rates) that are not part of the core reduction pipeline but that are extremely convenient for a downstream user. It is at this last stage, rather than at the raw spectral level, that we typically begin our own selection, since VACs already package the redshift, its quality flag, and a first morphological/spectral classification that we can query directly.

The SDSS Legacy Survey (SDSS-I/II; [York et al., 2000](#); [Abazajian et al., 2009](#)), on which [A25](#) is based,

followed exactly this scheme: photometric imaging with the dedicated 2.5-m telescope at Apache Point Observatory in the *ugriz* system (Stoughton et al., 2002), followed by spectroscopic observations with a pair of double fibre-fed spectrographs (with 3 arcsec diameter fibres) covering the observed range $\sim 3800 - 9200 \text{ \AA}$ at a resolving power $R \sim 1800 - 2200$ (York et al., 2000; Stoughton et al., 2002). The reduced spectra and their derived redshifts, together with a first-order spectral classification, are made public, and their relevant physical information can be retrieved through the `specObj` view of the SDSS Catalog Archive Server (CAS; Szalay et al., 2002; Thakar et al., 2008), whose `Class` (GALAXY, STAR, QSO) and `subClass` (e.g. STARFORMING, STARBURST, AGN, BROADLINE) columns already provide a coarse, if generic, discrimination between quiescent and star-forming or active systems.

Crucially, and unlike DESI (see below), the SDSS Legacy Survey imaging did not carry a dedicated, purpose-built target class isolating old, passively evolving ellipticals: the spectroscopic MAIN galaxy sample was selected on the basis of a single *r*-band Petrosian magnitude limit, essentially agnostic to the star-formation state of the target (Strauss et al., 2002), while the SDSS LRG sample of Eisenstein et al. (2001), though colour-selected to preferentially pick out massive red ellipticals out to higher redshift, was optimised for the study of large-scale structure and BAO rather than for the archaeological purity required by the CC method. For this reason, the selection of massive and passively evolving ETGs that we require in A25 had to be constructed from scratch on top of the generic GALAXY class, combining morphological, kinematic and, later, spectroscopic criteria (Secs. 2.1.1 and 2.2), rather than being inherited directly from any pre-existing SDSS target flag. Fortunately, the literature includes works (e.g. Clemens et al., 2006; Clemens et al., 2009) that tried to select passively evolving galaxies while avoiding a thorough spectroscopic selection, which would require substantial computational time. Thanks to such works, some secondary flags using SDSS photometric flags became standard.

The situation is markedly different for DESI (Abdul Karim et al., 2026), whose imaging component, the DESI Legacy Imaging Surveys (Dey et al., 2019), was explicitly designed from the outset to support a dedicated LRG target class (Zhou et al., 2023). Robotic fibre positioners feed ten spectrographs covering $360 - 980 \text{ nm}$ at $R \sim 2000 - 5500$, and every one of the nearly 5000 fibres available per pointing is assigned, through the bitmask stored in the `DESI_TARGET` column, to one (or more) of the main target classes of the survey: the Bright Galaxy Survey (BGS), LRGs, ELGs, and quasars (QSOs), on top of calibration and secondary programs. The LRG selection of Zhou et al. (2023) is, by design, already a strong pre-selection towards the kind of object a CC study needs: it combines a lower limit on a $z - W1$ colour proxy for the strength of the $4000 \text{ \AA}$ break, a $r - W1$ versus $W1$ sliding cut that tracks the stellar mass-to-light ratio and is tuned to keep the comoving number density of targets approximately constant with redshift, and a set of quality and star-galaxy separation flags, all calibrated to maximise the yield of massive, evolved systems while suppressing the low-mass, star-forming contaminants that would otherwise dilute the sample. In a sense, when working with

DESI data in [A26](#), we start from a catalog that has already undergone one full layer of the kind of purification that we had to build by hand in [A25](#): our subsequent cuts (Sec. [2.1.1](#)) are, to a good extent, a refinement of an already LRG-oriented sample, rather than a full construction of one starting from a generic magnitude-limited galaxy catalog.

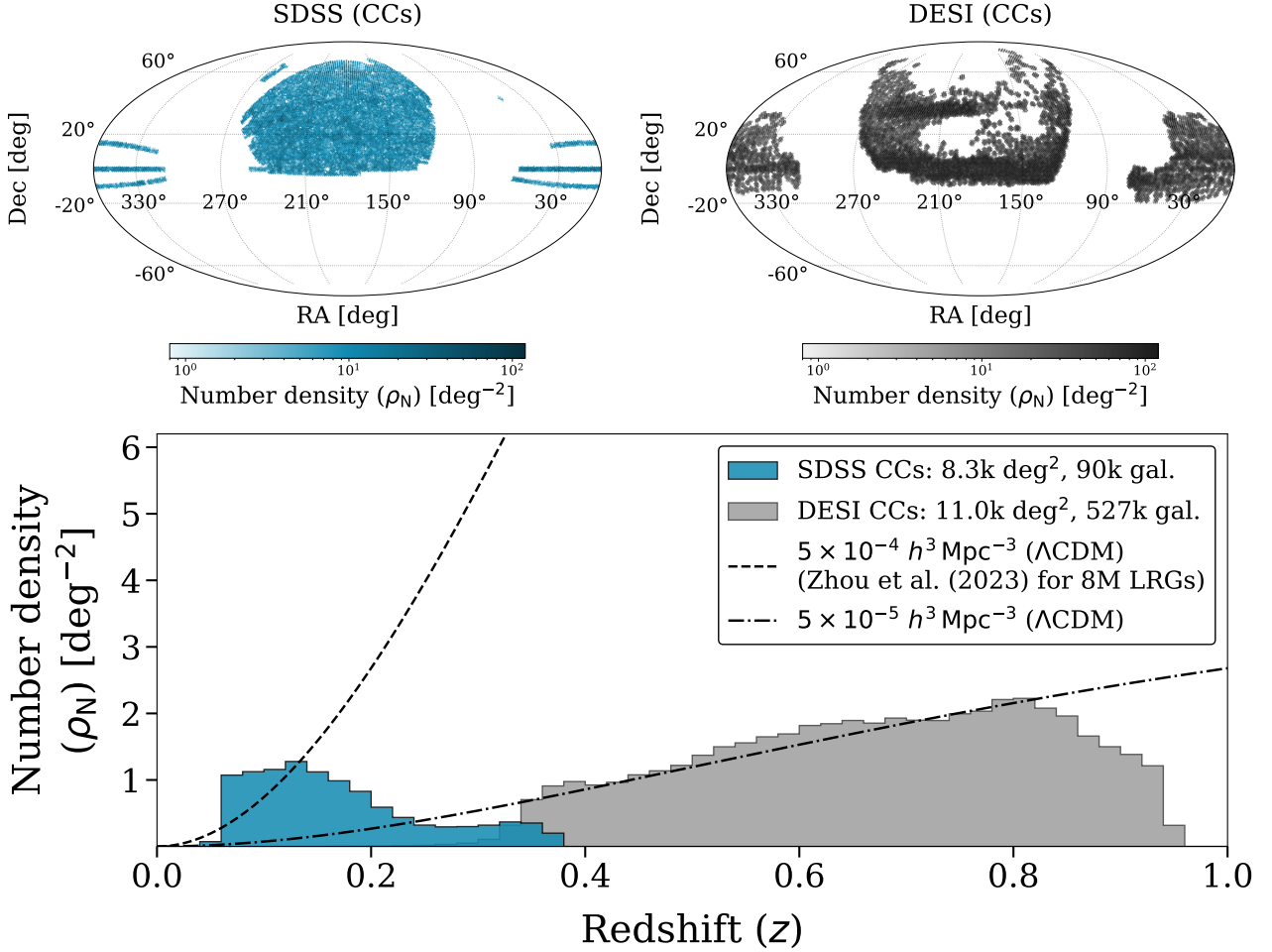


Figure 2.1: Sky density, ρ_N , of the cosmic chronometer (CC) samples used in [A25](#) (SDSS; blue) and [A26](#) (DESI; grey), shown on the Mollweide projection (equatorial coordinates, logarithmic colour scale) in the upper panels and as a function of redshift in the lower panel. The latter is adapted from figure 1 of [Zhou et al. \(2023\)](#). For reference, the dashed line in the lower panel shows a Λ CDM comoving number density of $5 \times 10^{-4} h^3 \text{Mpc}^{-3}$ that [Zhou et al. \(2023\)](#) associate with the full, five-year DESI LRG sample (~ 8 million objects), while the dot-dashed line shows a curve one order of magnitude lower, $5 \times 10^{-5} h^3 \text{Mpc}^{-3}$, matching the DESI CCs density.

To complement the discussion above, Fig. [2.1](#) brings together, in a single figure, both the spatial footprint and the redshift distribution of the two CC samples used throughout this thesis. The top row shows the angular number density of the spectroscopically selected sources, in a Mollweide projection of equatorial coordinates, for the SDSS-based sample of [A25](#) (upper left panel) and the DESI-DR1-based sample of [A26](#) (upper right panel). The SDSS footprint is the classic single,

contiguous North Galactic Cap characteristic of the Legacy Survey, complemented by three narrow equatorial stripes towards the South Galactic Cap; the DESI-DR1 footprint, by contrast, is both wider in right ascension and reaches further south, reflecting the larger and more homogeneous area already covered after only the first 13 months of the Main Survey (Sec. 2.1), consistent with the total sky areas of 8.3k deg^2 and 11.0k deg^2 for the SDSS and DESI CC samples, respectively.

The bottom panel reproduces the sky density as a function of redshift, ρ_N , for both samples, based on figure 1 of Zhou et al. (2023). As discussed there, the DESI CC sample reaches a sky density that already traces, at the correct order of magnitude, the Λ CDM curve corresponding to the comoving number density that Zhou et al. (2023) associate with the full, five-year DESI LRG sample ($5 \times 10^{-4} h^3 \text{Mpc}^{-3}$, ~ 8 million objects), despite our spectroscopically selected sample being $\gtrsim 10\times$ smaller (about half a million galaxies) than that eventual target, a consequence of DESI-DR1 covering only a fraction of the survey’s final footprint and integration time (Sec. 2.1), rather than of any additional restriction of our own.

2.1.1 Resolution

When comparing line index strengths between observations and models, one needs to be particularly careful with the resolution at which they are measured. This is relevant since, due to the definition as EWs, a certain instrumental resolution works as a natural smoothing that flattens the SED, in particular in the region around a feature, thereby reducing its measured strength (in absolute value). In other words, index strengths are not resolution-invariant observables. On top of this, the complexity of the instrumental resolution does not end here, as, typically, spectrographs do not observe light at a constant resolution but it becomes a function of the wavelength: $\sigma_{\text{IR}}(\lambda)$.

This is not a peculiarity of our own datasets: it is the very reason the original Lick/IDS system (Burstein et al., 1984; Faber et al., 1985), on which the definitions of all 25 classical indices rest, was tied from the outset to a specific instrumental configuration. The Image Dissector Scanner (IDS; Robinson & Wampler, 1972) has a low resolution (Vazdekis et al., 2010), typically quoted as FWHM $\sim 8 - 11 \text{ \AA}$, rising towards both the blue and red ends of the $4000\text{--}6400 \text{ \AA}$ range (Worthey et al., 1994; Worthey & Ottaviani, 1997). Any spectrum obtained with a higher-resolution instrument must therefore be deliberately degraded, through convolution with a wavelength-dependent Gaussian kernel, before its indices can be compared to values calibrated on the Lick/IDS system or to any model built from it (Carson & Nichol, 2010).

Moreover, this wavelength-dependent curve could not be established with full confidence, given the somewhat unpredictable response of the instrument over time (Worthey et al., 1994; Trager et al., 1998). Compounding this, the original IDS spectra were never properly flux-calibrated, so that the (slightly variable) instrumental response curve of the IDS itself is imprinted on the shape

of the continuum against which every index is measured (Worthey et al., 1994). As a consequence, no purely analytic prescription, consisting of only a wavelength-dependent Gaussian degradation, suffices to bring an independent dataset onto the Lick/IDS system: the residual continuum-shape and zero-point offsets left behind by these two effects can only be calibrated empirically, by observing a common set of Lick standard stars and deriving correction terms per index (e.g. Worthey et al., 1994; Trager et al., 1998; Puzia, T. H. et al., 2002).

Modern, carefully flux-calibrated stellar libraries such as MILES (Sánchez-Blázquez et al., 2006; Falcón-Barroso et al., 2011) were designed to overcome this problem. The TMJ and Knowles models used in the article are anchored to the MILES resolution and flux scale, requiring no standard-star zero-point correction of their own (Thomas et al., 2011). This is an example of how, in general, recent stellar population models have moved the target resolution away from the Lick/IDS system itself and onto the resolution of whichever empirical stellar library underlies them. The resolution-matching step described below, by contrast, remains unavoidable in either case, and is the one genuinely universal ingredient shared by any Lick-index analysis. Both the TMJ (Thomas et al., 2011) and Knowles (Knowles et al., 2023) models used in this thesis are built from the MILES library (Sánchez-Blázquez et al., 2006; Falcón-Barroso et al., 2011), whose rms wavelength resolution is $\sigma_{\text{IR}} \approx 1.06 \text{ \AA}$ (FWHM $\approx 2.5 \text{ \AA}$), substantially higher than the original Lick/IDS system but still requiring the same conceptual treatment: for a fair comparison, index measurements must be brought to the same resolution as the models.

The pipelines in SDSS, BOSS and DESI produce a wavelength-dependent instrumental resolution for each observed spectrum, varying statistically for different objects, so having a particular and recognisable form, as depicted in Fig. 2.2. Notably, the DESI resolution stays below the MILES resolution at all wavelengths, requiring us to slightly degrade the spectra (see Fig. 2.3 - discussion later on) to take it to the MILES resolution and the compare with the models based on it (TMJ, Knowles). The SDSS and BOSS resolution is at the level of MILES for the blue arm of the spectrograph ($\lambda \lesssim 6000 \text{ \AA}$), while the red arm ($\lambda \gtrsim 6000 \text{ \AA}$) takes a more functionally complex function of wavelength, overall jumping up to $\sigma_{\text{IR}} \sim 1.50$. The treatment of the instrumental resolution is, in that case, much more complex. Regarding the comparison between the dispersion of the observed resolution in SDSS and DESI spectra, we can conclude that the newest spectroscopic survey is much more robust than its predecessor when it comes to producing a whole sample of spectra with a similar quality.

We note that, at high redshift, Lick indices, which fall on the wavelength range covered by the MILES library (dashed line in Fig. 2.2), are observed at redder wavelengths. Therefore, an index that should be found in the $\sigma_{\text{SDSS}} \sim \sigma_{\text{MILES}} \equiv 1.06 \text{ \AA}$ wavelength region in a low-redshift object, can be found with a degraded resolution $\sim 1.50 \text{ \AA}$ in a high-redshift object. The colours in Fig. 2.2 are

chosen to match Fig. 2.1 and figure 1 of Zhou et al. (2023); and the galaxies used to extract the median functional form of the resolution are taken from the CC sample in the three cases. For the BOSS sample we followed the same spectroscopic requirements of the DESI sample (Sec. 2.2.2); however, the subsequent cosmographic analysis was left aside on the expectation of much better and significant results coming from the recently released DESI data (A26).

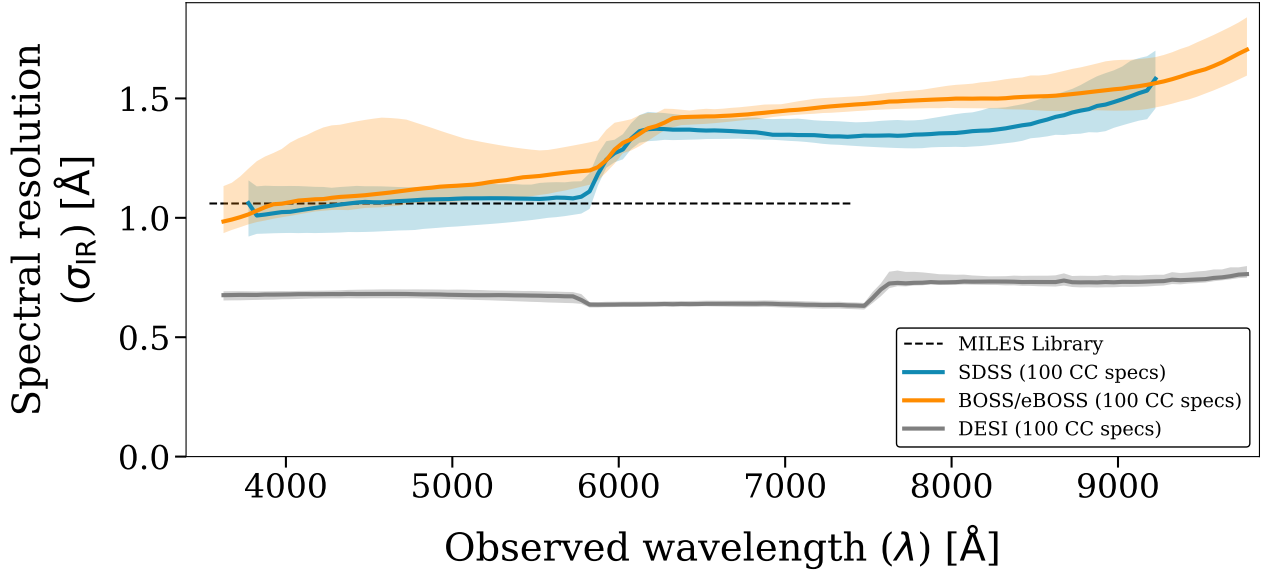


Figure 2.2: Instrumental resolution of the SDSS (blue), BOSS (orange) and DESI (grey) spectrographs. In a dashed black line we represent the MILES stellar library resolution. The median and dispersion of the resolution comes from averaging the specific resolution from 100 selected objects belonging to a set of CC candidates in all three cases.

The effect of instrumental resolution changes between $\sigma_{\text{IR}} \approx 0.5 \text{ \AA}$ and $\sigma_{\text{IR}} \approx 1.5 \text{ \AA}$ is only mild. Indeed, as anticipated in the previous paragraphs, Fig. 2.3 is the exact example of this, where we have represented the effect of degrading the resolution on the SED around the $\text{H}\beta$ feature (left panel), the $\text{H}\beta$ index strength (middle panel) and the SPS estimated age (right panel). The object there represented is the 50th stack from the $250 < \sigma [\text{km s}^{-1}] < 280$ velocity dispersion¹ group coming from the final CC DESI sample.

The first conclusion that one may draw is the statistical compatibility of the three age estimates among themselves, which, contrary to the observed trend in the index strength of the $\text{H}\beta$ shown in the middle panel, do not follow a monotonic trend. The middle panel, however, shows the expected effect of a smoothing on the SED over a Lick index, a monotonic decrease in the index strength with increasing Gaussian-kernel width values: none (orange), $\sqrt{1.06^2 - \sigma_{\text{DESI}}^2}$ (red) and $\sqrt{1.50^2 - \sigma_{\text{DESI}}^2}$ (dark red). We have anticipated here the computation of a matching resolution between the aimed

¹One should not confuse $\sigma [\text{km s}^{-1}]$, which denotes a velocity (dispersion) value, with σ_{IR} or σ_{VD} (to be presented later), both of which denote the resolution (instrumental and the one due to the velocity dispersion) in wavelength units.

resolution and the observed resolution (eq. 2.3), and the resolution at which we have taken the observed SDSS spectra to in A25 and the observed resolution, in order to observe all indices with a constant resolution (greater detail in the following paragraphs and in Sec. 2.3.1). As introduced at the beginning of the current section, the smoothing of the flux dilutes the prominence of a feature (even if seemingly imperceptibly, as could be thought by looking at the SED in the left panel of Fig. 2.3) with respect to its surroundings, thereby reducing the strength of the EW ($F(\lambda)/F_{p-c}(\lambda) \rightarrow 1 \Rightarrow \text{EW} \downarrow$ in eq. 1.49).

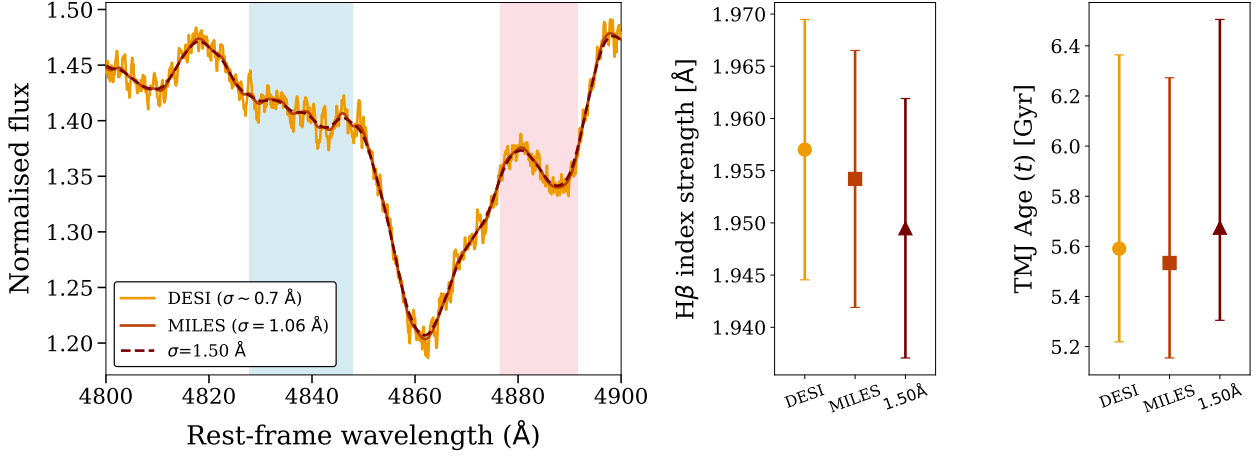


Figure 2.3: Effect of degrading the (instrumental) resolution on the SED (around the $H\beta$ feature, left panel), the $H\beta$ index strength (middle panel) and the overall SPS (TMJ) estimated age (see details in Sec. 2.6). The object-spectrum is the 50th stack from the $250 < \sigma \text{ [km s}^{-1}] < 280$ velocity dispersion group from the DESI final sample. The stacked spectrum at DESI resolution can be observed in orange, the same spectrum at MILES resolution (1.06 Å) is plotted in red, and the same spectrum at $\sigma = 1.50 \text{ Å}$ with a dashed dark red line. We shadow the regions belonging to the blue and red pseudocontinuum of the $H\beta$ line for reference.

Following the anticipated comments in the previous paragraph, we address now the procedure to match degraded resolutions. We note here that this can be generalised to the effect of a velocity dispersion (discussed in the following section, 2.1.2) imprinted in the light coming from a galaxy. The effect of that resolution will be named velocity dispersion effect resolution, σ_{VD} , and it can be computed (note the wavelength units of σ_{VD} and the velocity units of σ) as $\sigma_{VD} = \sigma \lambda / c$. Together with the instrumental resolution imprinted by the spectrograph on the data, they constitute the overall resolution of the object,

$$\sigma_R = \sqrt{\sigma_{IR}^2 + \sigma_{VD}^2}. \quad (2.1)$$

Given a flux density, F_{R_0} , at resolution R_0 ($\sigma_R = \sigma_{R_0}$), it can be taken to any degraded resolution R_f

($\sigma_R = \sigma_{R_f}$) by performing the following convolution,

$$F_R(\lambda) = \frac{1}{\sqrt{2\pi\sigma_{\text{match}}^2}} \int dk F_{R_0}(k) \exp \left[-\frac{1}{2} \left(\frac{\lambda - k}{\sigma_{\text{match}}} \right)^2 \right], \quad (2.2)$$

where the final flux is F_R and we have introduced a matching resolution which is defined as

$$\sigma_{\text{match}} \equiv \sqrt{\sigma_R^2 - \sigma_{R_0}^2}. \quad (2.3)$$

We note that, in particular, this can be applied only to instrumental resolutions (getting rid of the velocity dispersion induced degradation in the resolution).

It was mentioned before that the SDSS spectrograph splits the incoming light into a blue and a red channel through a dichroic beamsplitter at $\sim 5500 - 6000 \text{ \AA}$ (observed frame), each fed to its own CCD, so that $\sigma_{\text{IR}}(\lambda)$ shows a sharp, step-function-like jump at that wavelength rather than a smooth trend. The solution that is considered when working with those spectra, avoiding individual checks on which indices fall on which resolution regime for each galaxy or stack, consists in degrading all the observed data to $\sigma_{\text{IR}} = 1.50 \text{ \AA}$. Since MILES-based models are computed at the MILES library resolution, a correction needs to be applied to the observed indices. However, as presented in eq. (2.1), the smoothing effect of velocity dispersion is indistinguishable from that produced by the instrument. Therefore, the correction is applied altogether in A25, with a dedicated discussion considered in Sec. 2.3.1.

The situation is comparatively simpler for DESI, whose three spectrograph arms (blue, $3600 - 5930 \text{ \AA}$; red, $5660 - 7720 \text{ \AA}$; and near-infrared, $7470 - 9824 \text{ \AA}$, with a dichroic split analogous to the single SDSS jump but producing a much more gentle step-like resolution function) deliver a resolving power at all wavelengths finer than the MILES library. This means that, unlike the SDSS case, no data-side instrumental resolution correction of the kind described above is strictly required: the raw DESI spectra are already at a higher resolution than the target, and the appropriate operation is a straightforward degradation. However, the smoothing of the spectra due to the velocity dispersion effect is unavoidable, for which dedicated correction functions (Carson & Nichol, 2010; Clerici et al., 2024) need to be developed regardless.

2.1.2 Redshift and velocity dispersion

Both the redshift and the velocity dispersion of every source in our samples are quantities we inherit directly from the survey pipelines and their associated VACs, rather than measuring them ourselves. It is nonetheless worth reviewing how each survey obtains them, since the logic behind the velocity-dispersion measurement in particular reappears in our own treatment of the resolution correction (Sec. 2.3.1). In both surveys, the redshift and the coarse spectral classification (GALAXY, STAR, QSO) are determined jointly, through a template-fitting pipeline that is entirely independent of the Lick-index measurements we perform downstream.

For SDSS Legacy data (A25), this is the role of a spectroscopic classification pipeline (SubbaRao et al., 2002; Adelman-McCarthy et al., 2007) which represents each candidate class (star, galaxy, quasar) as a small set of principal-component eigenspectra and, for every object, evaluates χ^2 over a dense grid of trial redshifts for each class, reporting the best-fit redshift, class, sub-class and a quality flag at the global χ^2 minimum. This is conceptually very similar to the `idlspec2d` pipeline developed for BOSS (Bolton et al., 2012), which was the working spectroscopic pipeline for BOSS and eBOSS data. For DESI, the analogous role is played by the `redrock` pipeline (Guy et al., 2023), which follows the same conceptual scheme and delivers, per target, the best-fit redshift, its uncertainty, the SPECTYPE classification, and the ZWARN quality bitmask that we require to be zero in A26 (Sec. 2.2.1).

The measurement of the stellar velocity dispersion, σ , follows a related but distinct philosophy in the two surveys, built around the same underlying operation in both cases: broadening a set of template spectra by a trial velocity dispersion and comparing the result to the observed galaxy spectrum, repeating this over a grid of trial values until the best match is found. For SDSS, this is the so-called *direct-fitting method*, adopted from Data Release 6 onwards after an earlier Fourier-fitting approach was found to be biased at low dispersions ($\sigma \lesssim 150 \text{ km s}^{-1}$; Bernardi et al. 2003). A principal-component basis of a handful of stellar eigenspectra originally derived from 32 K and G-type giant stars in M67², and later replaced by eigenspectra built from the ELODIE library (Prugniel & Soubiran, 2001), is redshifted to the galaxy’s own frame, broadened by successively larger trial velocity dispersions, and fit to the observed galaxy spectrum in pixel space via a linear least-squares combination at each grid point. The $\chi^2(\sigma)$ curve so obtained is then used to locate the best-fit dispersion and its uncertainty. The fit is performed over the rest-frame range where the strongest stellar absorption features (essentially Ca H&K, the G4300, Mgb and the Fe lines) are found. This is only attempted for spectra with a spectral type consistent with an early-type, non-emission-line galaxy, which is, incidentally, why a valid σ measurement is itself usable, as in A25, as an implicit quality flag on the passivity of a source.

For DESI, the velocity dispersion is not part of the core `redrock` output but is instead delivered by the `FastSpecFit` VAC (Moustakas et al., 2023), which we already introduced in Sec. 2.1 as our primary source of metadata. `FastSpecFit` fits the observed spectrum (together with the available broadband photometry) at the fixed redshift provided by `redrock`, using a stellar continuum built from a linear combination of synthetic simple stellar population templates drawn from the C3K library within the flexible stellar population synthesis (FSPS) framework (Conroy & Gunn, 2010), broadened by a single Gaussian line-of-sight velocity distribution whose width is a free parameter of the fit, jointly with the emission-line and dust-attenuation components of the model. The stellar velocity dispersion is then read off directly from the best-fit broadening of this continuum model. The operation is thus conceptually the same direct-fitting logic used by SDSS.

²<https://classic.sdss.org/dr5/algorithms/veldisp.php>

We stress that this broaden-and-compare operation, applied by both pipelines, to extract a value for the velocity dispersion, maintains a certain similarity with the broadening process applied to similar stellar or synthetic spectra when deriving the velocity dispersion correction functions $C_I(\sigma)$ of Sec. 2.3.1. These corrections also include the instrumental resolution effect for the cases in which the observed spectra are partially or totally degraded with respect to the model spectra.

2.2 Samples

For the sake of a clear comparison and continuity in the reading of the articles that serve as basis to this thesis, we have adopted a common language to refer to the different layers of the selection process of CC candidates. We note that the first submitted version of A26 does not follow the notation that we comment in the following lines, but is being revised in order to be consistent with A25 and this thesis. We will refer to objects tagged as galaxies as the **Parent** sample, following the notation introduced in A25: ‘We began from a parent sample, constructed by requiring that the objects be galaxies and satisfy the wavelength coverage condition [...]’. However, we note that the wavelength coverage condition is not applied to the DESI sample of *only galaxies*. The reason for this is simple: the SDSS sample beyond the redshift at which the H α line can no longer be observed represented a very limited number of sources; therefore, we decided to put a prior cut in redshift asking for H α to be observed. In A26, however, we dropped this condition since an indirect redshift cut would be implicit in the condition that the fixed set of Lick indices (Sec. 2.6) used to derive the SPS parameters would be completely covered.

The use of VAC and other catalog columns, either from a photometric and/or spectroscopic analysis, allows us to produce a cleaner sample. We emphasise that any cut performed here includes parameters and measurements not performed by us. In A25 we called this *query sample*, since it came from a SQL query to the CAS. In A26, this sample came as a subsample of the LRGs sample, where the latter was mainly built through photometric criteria (Zhou et al., 2023). It was complemented with further conditions, but the most stringent cut relative to the *only-galaxies* sample (hereafter parent³ also for A26) came from the selection of LRG targets that were tagged as not being QSOs or ELGs. We note that the photometric criteria used to select LRGs in DESI are not applied in the selection of the *query sample* in SDSS, and the opposite is also true, as we could apply some cuts to the SDSS sample (i.e. the concentration parameter cut: Shimasaku et al., 2001; Clemens et al., 2006), based on photometric values, that were not possible for the DESI sample. We note that the spectroscopic flags addressed in the building of these samples exclude emission lines and the CaII ratio. For the remainder of this thesis, we will refer to this sample as the **LRGs** sample. However, if

³We remind that the currently available version of A26 can be misleading as it calls *parent* sample to what here we refer to as LRGs sample. This is being corrected at the moment following the recommendation of the referee and for consistency with this thesis.

speaking more precisely, the SDSS galaxies should rather be called *elliptical galaxies* (EGs), based on the selection criteria that define the group.

The final spectroscopic selection, including cuts in emission lines, tracers of recent star-formation activity and very young stellar population components, will receive the name of final CCs sample. We note that this does not mean that all the sources within this sample end up contributing to the cosmographic results. Indeed, some post $t - z$ plane sampling selection occurs in both A25 and A26 for different reasons, as explained in Sec. 2.6.1.

2.2.1 Targets and photometric flags

In the following lines, we present the photometric and spectroscopic cuts that were applied to the parent sample but do not include the emission-line cuts and H/K ratio, as these are performed after the spectra are retrieved using the pyLick code (Borghi et al., 2022b), which is used to measure Lick indices as well (along the lines of what was presented in table 1.1). Nevertheless, the Catalog Archive Server (CAS) in SDSS contains measurements of emission lines (Kauffmann et al., 2003; Tremonti et al., 2004; Brinchmann et al., 2004) that agree on a good level with our own measurements as will be presented in Sec. 2.2.2. Indeed, these are not limited to emission lines, but also absorption lines, including the Lick indices. Beyond the use of these indices as proofs of the reliability of the pyLick algorithm when working with spectra of this kind, we also used them eventually for the strict spectroscopic selection, producing a markedly reliable set of CC candidates.

Given the complexity in describing homogeneously the separate selection criteria for SDSS and DESI at this level, we will divide the remainder of this section into subsections. The aim here is to introduce cuts and observations that were ultimately not included in the articles for various reasons. We note that, in some cases, maps and diagrams show a slicing in the three levels of selection (parent/LRGs/final) to draw conclusions and behaviours, even if the full selection process of the final sample has not been thoroughly explained yet.

SDSS

Starting from the parent sample already defined above (GALAXY-classified objects satisfying the rest-frame wavelength coverage condition; 669 475 sources), the query sample of A25 was assembled through a single SQL statement combining three further conditions. First, the subClass column was required to show no signature of ongoing star formation, discarding sources tagged, e.g., as STARFORMING, STARBURST, AGN or BROADLINE. Second, the velocity dispersion was restricted to $100 < \sigma [\text{km s}^{-1}] < 400$, selecting massive galaxies with stellar masses $\gtrsim 10^{10.5} M_{\odot}$ while excluding rotation-dominated, typically star-forming systems (Di Teodoro et al., 2016; Veale et al., 2017). Third, an elliptical morphology was imposed through the concentration (or compactness) parameter of Shimasaku et al. (2001), requiring the ratio of the Petrosian radii enclosing 50% and 90% of the flux

to satisfy $R_{50}/R_{90} < 0.33$. Of these three, the velocity dispersion cut alone brings the parent sample down to 506 475 objects, while the combination of the star-formation flag and the concentration parameter further reduces it to the 135 554 galaxies of the query sample – the concentration parameter being, on its own, by far the most restrictive criterion, cutting the sample by roughly two thirds.

In practice, both the parent and the query sample were retrieved directly from the CAS through `astroquery.sdss` (Ginsburg et al., 2019), issuing a single SQL statement per sample via `SDSS.query_sql` rather than downloading intermediate catalogs and cross-matching them offline. The parent sample follows from a minimal join between the `photoObj` and `specObj` tables, requiring only a `GALAXY` classification, membership in the original SDSS-I/II LEGACY survey (`s.survey = 'sdss'`, as opposed to later BOSS/eBOSS spectra sharing the same CAS), and a redshift such that the $H\alpha$ -line region is completely seen, translated directly into a cut on the tabulated `wavemin/wavemax` of each spectrum:

```
SELECT s.specobjid, p.objid, s.plate, s.mjd, s.fiberid, s.ra, s.dec, s.z
FROM photoobj as p, specobj as s
WHERE
    s.bestobjid = p.objid and
    s.fluxobjid = p.objid and
    s.class = 'GALAXY' and
    s.survey = 'sdss' and
    s.wavemin < 3600*(1.0+s.z) and
    s.wavemax > 6700*(1.0+s.z)
ORDER BY s.z
```

The query sample is built by extending this same join to the `galSpecIndx` table (the MPA/JHU-derived catalog of Lick and related spectral indices; Kauffmann et al., 2003; Tremonti et al., 2004; Brinchmann et al., 2004) and tightening the `WHERE` clause with the three conditions described above, directly expressed as CAS columns: `s.subClass`, `s.velDisp`, and the concentration parameter as the ratio of Petrosian radii, `p.petroR50_r/p.petroR90_r`. A representative instance of this query reads:

```
SELECT s.specobjid, s.plate, s.mjd, s.fiberid, s.ra, s.dec,
    p.u, p.g, p.r, p.i, p.z, s.z,
    gidx.d4000, gidx.d4000_n, gidx.lick_hd_a, gidx.lick_hd_f,
    gidx.lick_cn1, gidx.lick_cn2, gidx.lick_ca4227, gidx.lick_g4300,
    gidx.lick_hg_a, gidx.lick_hg_f, gidx.lick_fe4383, gidx.lick_ca4455,
    gidx.lick_fe4531, gidx.lick_c4668, gidx.lick_hb, gidx.lick_fe5015,
    gidx.lick_mg1, gidx.lick_mg2, gidx.lick_mgb, gidx.lick_fe5270,
    gidx.lick_fe5335, gidx.lick_fe5406, gidx.lick_fe5709, gidx.lick_fe5782,
    gidx.lick_nad, gidx.lick_tio1, gidx.lick_tio2
FROM photoobj as p, specobj as s, galspecindx as gidx
WHERE
    s.bestobjid = p.objid and
    s.fluxobjid = p.objid and
    gidx.specobjid = s.specobjid and
```

```

s.class = 'GALAXY' and
s.subClass <> 'STARFORMING' and
s.subClass <> 'STARBURST' and
s.survey = 'sdss' and
p.petroR50_r/p.petroR90_r < 0.33 and
s.wavemin < 3600*(1.0+s.z) and
s.wavemax > 6700*(1.0+s.z) and
s.velDisp > 100 and
s.velDisp < 400
ORDER BY s.velDisp, s.z

```

In practice, the full query sample was assembled by looping this statement over a set of narrow, contiguous redshift shells (rather than issuing it once over the full range), a choice dictated by the CAS's own row-return limits on large joins rather than by any property of the selection itself. We note that this query already returns the `SUBCLASS`-based exclusion of `STARFORMING`/`STARBURST` sources together with the morphological and kinematic cuts in a single pass, so that, unlike the parent sample, the query sample is retrieved with all three catalog-level conditions of Sec. 2.1 applied simultaneously; the CAS-provided `galSpecIndx` columns retrieved alongside it are not used for the selection itself, but serve, as commented above, as an independent cross-check of our own `pyLick` index measurements.

This last criterion deserves a closer look than it received in A25, given how central it turns out to be. The concentration index is a purely photometric proxy for the shape of a galaxy's light profile, requiring no 2D profile fitting and no spectroscopic information whatsoever: a small R_{50}/R_{90} ratio (equivalently, a large R_{90}/R_{50}) indicates a centrally concentrated, bulge-dominated system well described by a de Vaucouleurs profile (de Vaucouleurs, 1948), whereas a disk-dominated system with a shallower, exponential profile yields a systematically larger ratio. Its appeal, and the reason it became a standard secondary flag in works aiming to assemble large samples of passively evolving galaxies without the computational cost of a full spectroscopic vetting (Clemens et al., 2006; Clemens et al., 2009), is precisely that it is already tabulated for every source in the SDSS photometric pipeline, at the cost of only two aperture-photometry measurements. In this sense, it is the closest photometric analogue that a catalog-level query can offer to a genuine Sérsic-index (Sérsic, 1963) or bulge-to-total (Kent, 1985; Simien & de Vaucouleurs, 1986) cut: concentration is known to correlate, albeit with non-negligible scatter, with both quantities (Blanton et al., 2003; Conselice, 2003; Weinzirl et al., 2009).

Beyond the criteria actually adopted, a photometric colour-colour selection was considered (although it was finally relegated to a subsequent cross-check or validation), at different stages of A25; as a possible complement or alternative to the morphological and kinematic cuts above. Indeed, many recent works with cosmic chronometers have performed such selection cuts (e.g. Borghi et al., 2022b; Tomasetti et al., 2023). The bimodal distribution of galaxy colours has long been exploited to separate

the red sequence from the blue cloud, from the original $u - r = 2.22$ separator of [Strateva et al. \(2001\)](#) on the $u - g$ versus $g - r$ plane, to more recent, CC-oriented refinements such as the $g - r$ versus $r - i$ cut ([Eisenstein et al., 2011](#); [Masters et al., 2011](#); [White et al., 2011](#); [Maraston et al., 2013](#)), applied to BOSS spectra by [Moresco et al. \(2016\)](#), the $(\text{NUV} - r)_0 / (r - J)_0$ plane ([Ilbert, O. et al., 2013](#)), and the $(U - V)_0 / (V - J)_0$ plane ([Williams et al., 2009](#); [McLure et al., 2018](#)). The latter two were never a realistic option for our SDSS sample: both require rest-frame near-ultraviolet or near-infrared photometry that would have to come from GALEX and 2MASS/UKIDSS/VISTA cross-matches, neither of which reaches a depth ([Martin et al., 2005](#); [Skrutskie et al., 2006](#)) or sky coverage ([Lawrence et al., 2007](#); [Edge et al., 2013](#)) comparable to SDSS over our full redshift range. Imposing either cut would have discarded a large, and non-random, fraction of otherwise perfectly usable CC candidates simply for lacking a UV or NIR counterpart. The $(u - r)_0$ versus $(r - z)_0$ polygon of [Holden et al. \(2012\)](#) was, by contrast, directly accessible from native SDSS *ugriz* photometry, and was indeed tested as a subsequent consistency check (figure 2 of [A25](#)). This was a natural choice given its intrinsic $\sim 18\%$ missing fraction of purely passively evolving galaxies ([Holden et al., 2012](#)). That threshold, however, was calibrated close to $z \sim 0$ by the authors, and our own tests indicate that it can leave a greater fraction of sources out at higher redshift. This might happen as the value of the K-corrections ([Chilingarian et al., 2010](#); [O'Mill et al., 2011](#)) needed to take observed colours to rest-frame colours grows with z , expanding the induced effect of the dispersion associated with the corrections. Indeed, we show in Fig. 2.4 how in the $0.26 < z < 0.27$ subgroup of SDSS selected CCs (right panel) the cut would exclude a $\approx 23\%$ of CCs, while it works as expected at low redshift ($0.05 < z < 0.06$), with a missing fraction of less than a 16%.

Just by looking at the right panel in the figure, one might come to the conclusion that the K-correction for the u band could be responsible for the mismatch, something that is reinforced by looking at the dispersion of the related correction for high redshift ($z \sim 0.3 - 0.4$) in figure 3 (upper left panel) of [Chilingarian et al. \(2010\)](#). Coming back to our plane, this idea is further supported by the fact that the r band, which is the other element of the $(u-r)_0$ colour, is used for the colour in the horizontal axis, $(r-z)_0$, but this one appears to work well in the region drawn by [Holden et al. \(2012\)](#). The comparison between the individual distribution of both colours might point in the same direction, as in the left panel (low redshift) the CCs sample is clearly placed at the reddest part (greater colour value) of the already redder mode. Regardless of this, we note that, if the bimodality in $(u-r)_0$ is to be physically observed at all redshifts, the smaller budget of the $(u-r)_0 \sim 1.5$ mode (starforming systems) with respect to the parent sample at $0.26 < z < 0.27$ points towards the intrinsic cut in $r < 17.77$ of the SDSS possibly having worked as a selector of passively evolving galaxies at higher redshifts. Not only this, but also in the redder mode, $(u-r)_0 \sim 2.5$, the total budget of CCs grows with respect to the parent sample at higher redshift.

By looking at the map of uncorrected colours (Fig. 2.5), one can guess that a more sophisticated

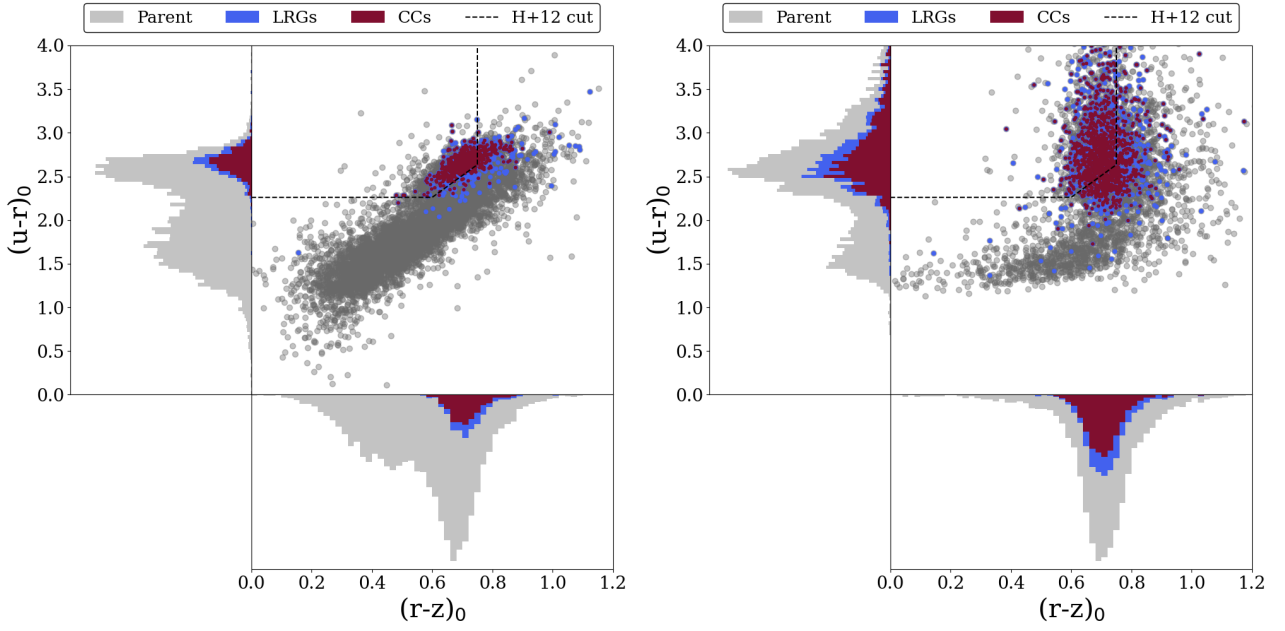


Figure 2.4: The left panel shows a view of figure 2 from A25, representing low redshift ($0.05 < z < 0.06$) data after removing the sample only cut in velocity dispersion, leaving the Parent (grey), LRG (dark blue) and final CC (maroon) samples with respect to the original figure in the article. The right panel is constructed in the same way the left panel is, but for higher redshift sources ($0.26 < z < 0.27$).

cut could be developed, expanding the Holden et al. (2012) cut to observed colour terms. Holden et al. (2012) set a sharp cut at $(r-z)_0 = 0.75$, which might be physically motivated to avoid mistaking starforming systems obscured by dust for purely red galaxies. Indeed, dust would push galaxies to higher $(r-z)_0$ values faster than it would on the $(u-r)_0$ axis (Cardelli et al., 1989; Calzetti et al., 2000). However, an extension of the diagonal slope of the cut to higher $(r-z)_0$, would be enough to extend this photometric cut to high redshift sources observed in the *ugriz* system.

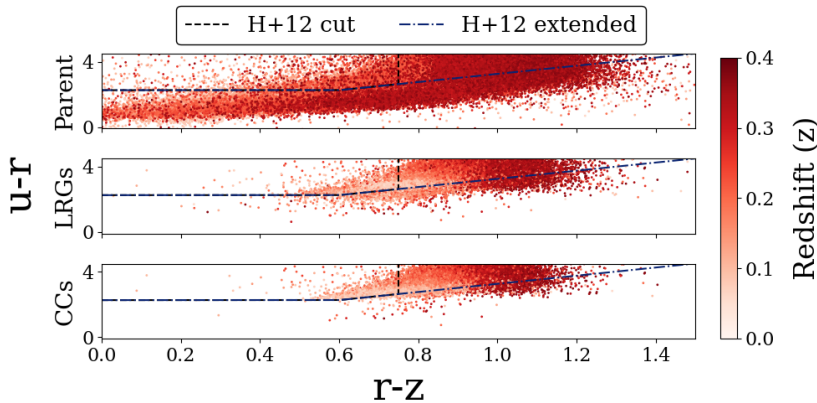


Figure 2.5: *u-r* vs *r-z* map of observed colours. Parent, LRG and CC samples from upper to lower panels. The Holden et al. (2012) cut is presented, making of this a full-redshift version of Fig. 2.4. We have also depicted an extended proposal of the Holden et al. (2012) cut for higher *r-z* colour values.

Higher redshifts are depicted in Fig. 2.5 with redder tonalities. For clarity, we have separated the

three samples (Parent, LRGs, CCs) to better observe the effect of extending (blue dot-dashed line) the [Holden et al. \(2012\)](#) criterion to higher $r-z$ colour values. A remarkable feature that we observe in the figure is how without the need for a K-correction, low redshift sources stay within the [Holden et al. \(2012\)](#) box.

Nonetheless, other observed colours have also been used to draw selection criteria in the literature. A few paragraphs above, we had mentioned the standards set by [Strateva et al. \(2001\)](#) and [Maraston et al. \(2013\)](#); the latter of which was specialised in BOSS photometric data, so that, in principle, it could work suboptimally for objects at redshift windows different from those of BOSS. To check the consistency of our CC sample with the hypothesis of a set of purely passively evolving galaxies, we show in Fig. 2.6 the $u-r$ to $g-i$ plane for the $z > 0.35$ subsample. On the vertical axis, we draw the [Strateva et al. \(2001\)](#) criterion, $u-r > 2.22$; and on the horizontal axis, the [Maraston et al. \(2013\)](#) criterion, $g-i > 2.35$. The shadowed regions represent the forbidden regions for purity according to each of the two cuts.

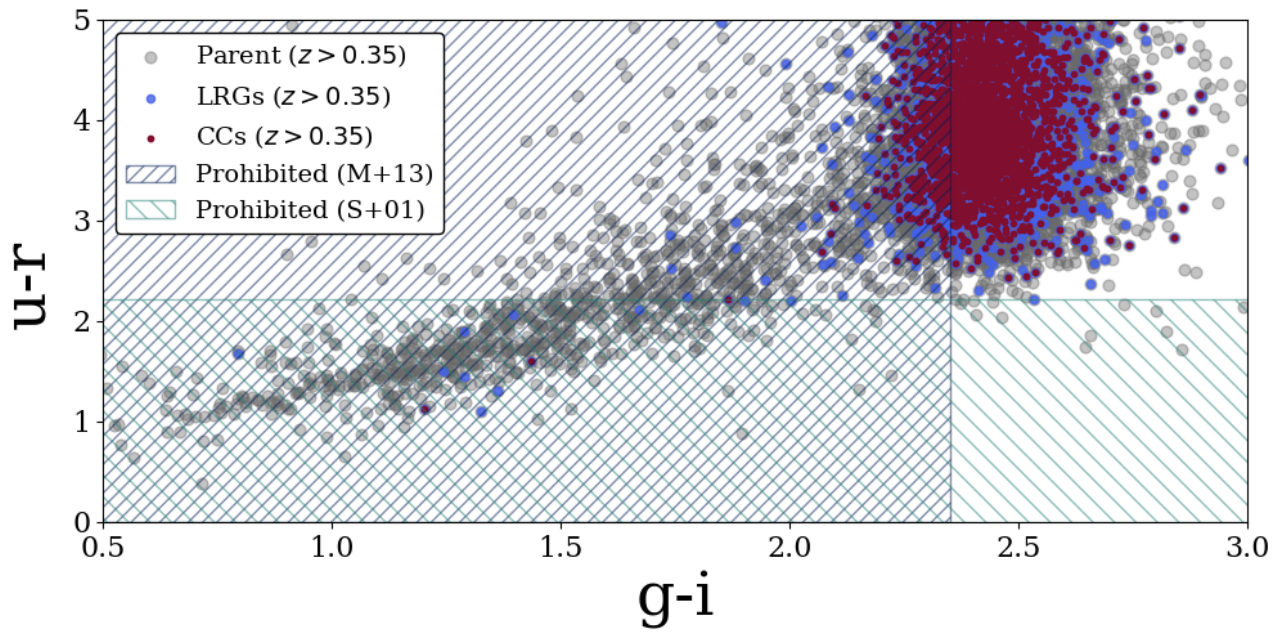


Figure 2.6: $u-r$ vs $g-i$ map of observed colours in the high redshift group ($z > 0.35$) of the SDSS analysed galaxies. The colour legend for Parent, LRG and CC samples is taken from Fig. 2.4. The [Strateva et al. \(2001\)](#) criterion, $u-r > 2.22$, and the [Maraston et al. \(2013\)](#) criterion, $g-i > 2.35$, leave the respectively prohibited areas dashed with diagonal lines.

If our selection criteria of CCs truly select a pure sample of passively evolving galaxies, the missing fraction following the [Strateva et al. \(2001\)](#) cut is almost 0, while for the [Maraston et al. \(2013\)](#) cut, it goes up to almost 18%; still less than the alleged average missing fraction following the [Holden et al. \(2012\)](#) criterion.

On a different note, some common patterns can be noticed in the figures shown in this section. The starforming clump is clearly observable as separate clouds in Figs. 2.4 and 2.6, placed towards bluer values of both colours in each map. Then, we can see how the selection of ellipticals (LRGs) using catalog parameters already does a good job at eliminating most objects with photometric values compatible with recent or ongoing starforming activity. Nevertheless, this makes the spectroscopic cut necessary to guarantee the purity of the CC sample and avoid biases in the derivation of ages, particularly in stacking-based analysis in which removing galaxies is no longer possible once ages are estimated.

DESI

The catalog selection of A26 starts from the FastSpecFit VAC built on the iron spectroscopic production. On top of the Parent, GALAXY-classified sample (8 490 171 galaxies), the clean LRG sample was assembled by, first, discarding through the DESI_TARGET bitmask any source additionally flagged as an emission-line galaxy (ELG) or quasar (QSO) target, retaining only sources carrying the LRG target bit; second, requiring a reliable redshift ($Z_{\text{WARN}} = 0$) and performing a consistency check between the independent redshift estimates reported in the METADATA and FASTSPEC units of the catalog, discarding sources for which the two disagreed significantly; and, third, requiring a valid velocity dispersion measurement restricted to $200 < \sigma \text{ [km s}^{-1}] < 400$. The lower bound follows the Faber-Jackson relation (Faber & Jackson, 1976) in selecting the most massive, and thus statistically the oldest, passively evolving galaxies (Moresco, 2024, 2026), shifted upwards relative to the SDSS case to compensate for the higher typical redshift (and hence younger apparent age) of the DESI LRG population; the upper bound reflects the observationally established physical ceiling for the velocity dispersions of elliptical galaxies (Bernardi et al., 2003; Sheth et al., 2003; Shen et al., 2003; Taylor et al., 2010; Bezanson et al., 2011; Trujillo et al., 2011; Bolton et al., 2012; Thomas et al., 2013).

It is worth noting that the most stringent selection cut in A26 comes from the LRG target bit itself, which in combination with the exclusion of possible ELG and QSO targets leaves only 2 735 943 exclusively targeted LRGs from the almost 8.5 million GALAXY-classified objects of the Parent sample. The LRG bit actually encodes a fairly elaborate photometric selection, described in detail by Zhou et al. (2023) that we briefly summarise in the following lines. Built entirely from g , r , z optical photometry (from the DESI Legacy Imaging Surveys; Dey et al. 2019) complemented by the unresolved WISE W1 band (Wright et al., 2010), the DESI LRG selection combines four ingredients. First, a colour cut of the form $z\text{-}W1 > 0.8(r - z) - 0.6$ exploits the $1.6 \mu\text{m}$ rest-frame opacity minimum ("bump") present in the spectra of evolved stellar populations but absent in stellar spectra, efficiently rejecting Galactic stars (down to a stellar contamination of $\sim 0.5\%$) without any morphological cut whatsoever (the DESI selection relies purely on colour and, for bright sources, on *Gaia* photometry to veto stars), unlike the SDSS concentration-based approach of Sec. 2.1.1. Second, a colour cut in $r\text{-}z$ (and, in the north/south photometric systems, $g\text{-}W1/g\text{-}r$) removes lower-redshift objects while preserving high

completeness for the massive, $z \gtrsim 0.4$ population that DESI targets. Third, and most distinctively, a *sliding* colour-magnitude cut in the r-W1 versus W1 plane acts as an effective luminosity threshold: since r-W1 is itself a good proxy for redshift for a passively evolving population with a strong 4000 Å break, letting the W1 magnitude limit slide with r-W1 selects, at every redshift, the most luminous galaxies in the observed W1 band, which is precisely the mechanism that keeps the resulting comoving number density close to $5 \times 10^{-4} h^3 \text{Mpc}^{-3}$ over $0.4 < z < 0.8$ (Sec. 2.1, lower panel of Fig. 2.1) rather than falling steeply with redshift as a fixed apparent-magnitude limit would. Finally, a faint-magnitude limit is imposed not on a standard total or Petrosian magnitude but on the DESI *fibre* magnitude (the flux expected to fall within the 1.5 arcsec aperture actually collected by a DESI fibre; Dey et al., 2019) ensuring that the cut tracks the quantity that determines spectroscopic S/N rather than the total light of the galaxy. Together, these four ingredients yield a target density of $\sim 605 \text{ objects deg}^{-2}$ for the Main Survey selection we build upon, roughly double the surface density achieved by the BOSS/eBOSS LRG programmes at comparable redshifts.

Together, these catalog-level flags, applied entirely on top of quantities already tabulated in the VAC with no additional photometric measurement required from us, leave 1 855 845 sources with a well-defined redshift, classification and velocity dispersion. Retrieving the corresponding spectra from the SPARCL database and confirming their validity narrows this down to the working sample of 1 728 366 sources that A26 itself refers to as LRGs sample, and from which the Lick-index-based spectroscopic selection of Sec. 2.2.2 proceeds.

It is worth noting how the LRG selection (Zhou et al., 2023) already delivers a stringent cut in most of the typical diagnostics (including colour) that we commented on for the SDSS selection process. In this way, some distribution shapes used as quality checks and tested in the CC literature (i.e. CaII K, D_n4000 , $H\delta_A$, G4300; Borghi et al., 2022b) (i.e. sSFR, $\log(M_\star/M_\odot)$; Tomasetti et al., 2023; Loubser, 2025) look, at this stage (LRGs), already very similar to those of the final, selected CC sample. We illustrate this progression in Fig. 2.7, where we show, for the Parent, LRG and CC samples, the distribution of D_n4000 , g-r colour and specific star-formation rate (sSFR); the latter was obtained through a cross-match with the v3.0 VAC, since the v2.1 VAC, from which the rest of the spectro-photometric data used in this thesis was drawn, offered unreliable sSFR figures that disagreed with the typical range expected for the different DESI target classes.

Following the style introduced by Jiao et al. (2023), Borghi et al. (2022b) and Tomasetti et al. (2023) to visualise the effect of successive quality cuts on a CC sample, each panel of Fig. 2.7 marks the median of the corresponding distribution with an arrow of the same colour as the sample it refers to, allowing the shift induced by each selection stage to be read off directly rather than inferred from the (heavily overlapping) histograms alone. Both D_n4000 and $\log(\text{sSFR})$ already show the expected bimodality at the Parent-sample level, reflecting the mix of star-forming and quiescent galaxies

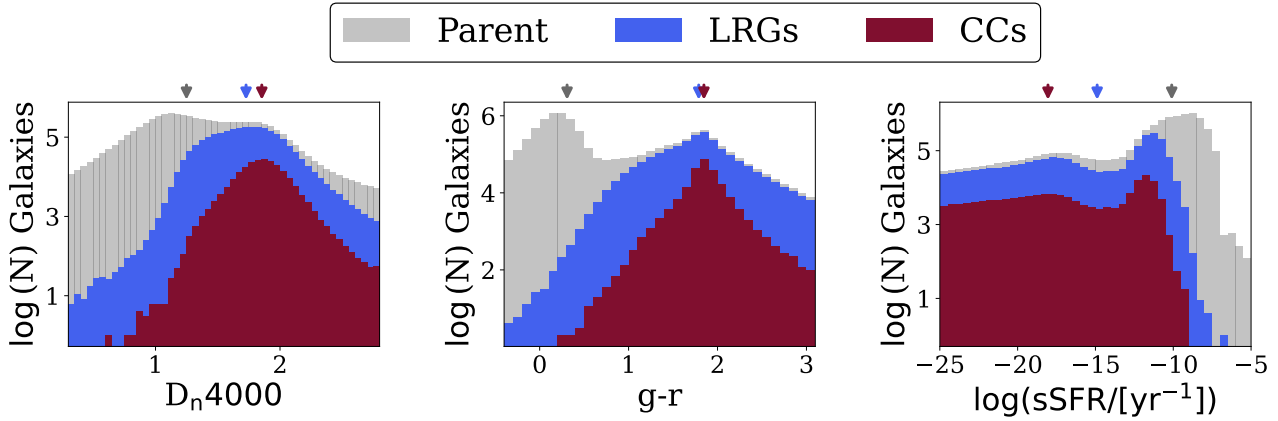


Figure 2.7: Distribution of D_n4000 (left), observed-frame $g-r$ colour (centre) and specific star-formation rate, $\log(sSFR/[yr^{-1}])$ (right), for the Parent (grey), LRG (blue) and CC (dark red) samples of A26. Downward arrows mark the median of each distribution, following the visualisation style introduced by Jiao et al. (2023), Borghi et al. (2022b) and Tomasetti et al. (2023) to track the effect of successive selection cuts on a CC sample.

that any generic GALAXY-classified catalog necessarily contains; the $g-r$ colour, less sensitive to the specifics of recent star formation, shows a broader but comparably asymmetric distribution instead. In all three panels, the LRG cut moves the median markedly towards the passively evolving end (higher D_n4000 , redder $g-r$, lower $\log(sSFR)$) essentially forming one of the two modes of the Parent distribution, which confirms quantitatively the qualitative picture already anticipated in Sec. 2.1. The subsequent spectroscopic cut (Sec. 2.2.2) that isolates the final CC sample produces a further shift in the three diagnostics presented in Fig. 2.7, although its amplitude varies among them. Notably, the shift of the median is only minor in the $g-r$ distribution, and the greater difference between the LRG and CC samples lies in the tails. Similarly, the shift of the median D_n4000 value is less enhanced from the LRGs to the CCs than it was from the Parent to the LRGs, although the tails tell a very different story, with a non-negligible amount of LRGs (bad CCs) penetrating well below $D_n4000 \sim 1.5$. These observations underline that residual star formation is the dimension along which the spectroscopic cut is most discriminating at this stage.

A complementary view of the same three-sample progression is offered by the observed-frame $g-r$ versus $r-W1$ colour-colour plane, shown in Fig. 2.8. Unlike the rest-frame diagnostics of Fig. 2.7, this observed-frame plane is precisely the one used by Zhou et al. (2023) to define part of the LRG selection itself. We show it for a small redshift range ($0.57 < z < 0.58$) centred around the median of our final CCs distribution, together with the corresponding Zhou et al. (2023) boundary overlaid explicitly. The Parent sample spans the full extent of the plane, including a prominent low $(g-r)$ -colour cloud populated by star-forming galaxies that is entirely absent from the LRG and CC clouds. The LRG sample occupies a well-defined, elongated placement lying entirely on the red side of the Zhou et al. (2023) boundary, as expected since the cut is, by construction, one of the criteria

that defines LRG membership; while the CC sample forms a markedly tighter, redder sub-region nested within the upper end of the LRGs, consistent with it being a purity-refined subset of the same underlying population rather than a distinct one.

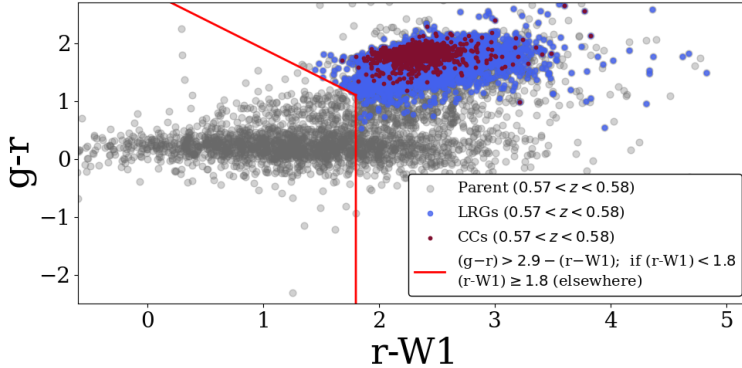


Figure 2.8: Observed-frame $g-r$ versus $r-W1$ colour-colour diagram for the Parent (grey), LRG (blue) and CC (dark red) samples of A26, restricted to the narrow redshift shell $0.57 < z < 0.58$. We included the cut performed by Zhou et al. (2023) in this colour-colour plane.

This figure is, in essence, a partial reconstruction of the upper-right panel of figure 3 in Zhou et al. (2023), extended here to also include the Parent sample alongside the LRG and CC targets that the original figure did not illustrate. We note, in passing, that $W1$ is centred further into the infrared than any of the $ugriz$ bands, so that the $r-W1$ axis is primarily sensitive to the infrared excess of the evolved stellar populations that dominate LRGs, rather than to the strength of the $4000 \text{ \AA}$ break traced by purely optical colours such as $g-r$.

2.2.2 Spectroscopic selection

The catalog-level cuts of Sec. 2.2.1 already isolate a photometrically and kinematically clean population of candidate massive ellipticals, but, as anticipated, they carry no direct spectroscopic information on residual star formation: neither the SDSS concentration parameter nor the DESI LRG target bit can, by construction, distinguish a genuinely quiescent galaxy from one whose optical light is dominated by an old population but that still hosts a faint, spatially unresolved star-forming component. Indeed, the DESI LRG target was not designed to isolate purely passively evolving galaxies that could be CC candidates, as can be directly observed in figure 7 of Abdul Karim et al. (2026), where the average spectrum of 75 – 100 LRGs contains clear emission patterns, for example in [OII] ($\lambda 3726, 29$). The layer of purification here presented is common in design to A25 and A26: both works measure the same set of diagnostics with the same pyLick machinery (Borghi et al., 2022b) (table 1.1), and apply them in almost the same way. For this reason, and unlike Secs. 2.1.1 and 2.2.1, we do not split the discussion below by survey: the emission-line and H/K criteria are presented here as a single procedure, noting the handful of local points at which some divergent treatment of the SDSS and DESI data was made.

Emission lines

The first and most direct spectroscopic diagnostic of residual star formation is the equivalent width (EW) of a small set of nebular emission lines known to be excited by young, hot stars: [OII] λ 3726 – 29 (this is actually a doublet, but a single EW is measured. We refer to the discussion provided in Sec. 1.3.2 for further detail), H β , [OIII] λ 5007 and, whenever the observed wavelength range allows it, H α (Sec. 1.3.2 and table 1.1). All four EWs are measured directly by us from the retrieved, rest-frame spectra with `pyLick`, rather than adopted from any externally tabulated emission-line catalog, so that the same pipeline that later delivers the Lick indices used in the age fit also delivers the quantities on which the sample is purified.

The numerical criterion applied to each of these EWs (by convention, negative for emission - see eq. 1.49) is essentially the same in both articles, refined only in its details: a source is rejected outright if any of the lines shows $\text{EW} \leq -5 \text{ \AA}$, an unambiguous emission by any standard adopted in the CC literature (Moresco et al., 2011, 2012; Borghi et al., 2022b; Tomasetti et al., 2023). Sources with $-5 < \text{EW} \leq -2 \text{ \AA}$ are, however, not automatically discarded, since a marginal negative EW of this size can equally well be a noise fluctuation on an intrinsically flat continuum, particularly if the pseudocontinuum level or the spectral S/N are very low. To break this degeneracy, both works additionally require a minimum significance for the line to be treated as a genuine detection: A25 retains sources in this intermediate range provided the associated $\text{S/N} < 3$, while A26 imposes the (marginally stricter) condition $\text{S/N} \leq 2$.

These conditions caused some trouble in understanding during the review process of A25, for which we consider it useful to give a more thorough interpretation: a line with a small enough S/N can be accepted as noise (and not as a small emission) as long as the overall EW value is low enough to be regarded as an effective noise fluctuation. This can be re-written from the point of view of the emission: given a small but measurable emission ($-5 < \text{EW} < -2$), this will be ignored if the S/N ratio of the line is low enough for it to be considered a random noise fluctuation. The threshold of the S/N also created some confusion, so we clarify it in the following: putting an upper limit of acceptance at a lower bar restricts more sources from being accepted (i.e. the criterion is an upper limit indeed). Both choices sit within the range already explored in the literature, from the $\text{S/N} < 2$ threshold of Moresco et al. (2016) to the $\text{S/N} < 3$ adopted by Tomasetti et al. (2023), and both are considerably more conservative than simply following the maximum-emission threshold of $-5 \text{ \AA}$ used across essentially all CC works to date. We note that, even if emission remnants in any of these lines are typically associated with emission remnants on all others, individual line restrictions serve as a purity check. For illustrative purposes, Fig. 2.9 represents two examples stacks of 100 selected LRGs that are (top panel) good CC and (lower panel) bad CC candidates with residual star-formation activity and/or younger stellar populations. We marked at the sides of the central panels a zoom in the [OII] and H α lines. We notice that H α is apparently observed in absorption in the pure CC sample

stack. The second Balmer line, a Lick index itself, is zoomed in the overlapping panel. This feature shows a refilling of the absorption line in the discarded LRGs case (blue, lower panel); however, we notice that in both cases the feature is measured as an overall absorption ($H\beta = 1.26 \pm 0.06$ in the contaminated stack and $H\beta = 1.92 \pm 0.04$ in the pure CCs stack), highlighting the need for a strict spectroscopic selection to avoid biases in the final estimation of ages. The effect of the inversion on the CaII K and CaII H lines can also be perceived in Fig. 2.9, a full comment on this is given in the H/K cut section below, paired with a zoom in of the affected region as a separate figure (2.11).

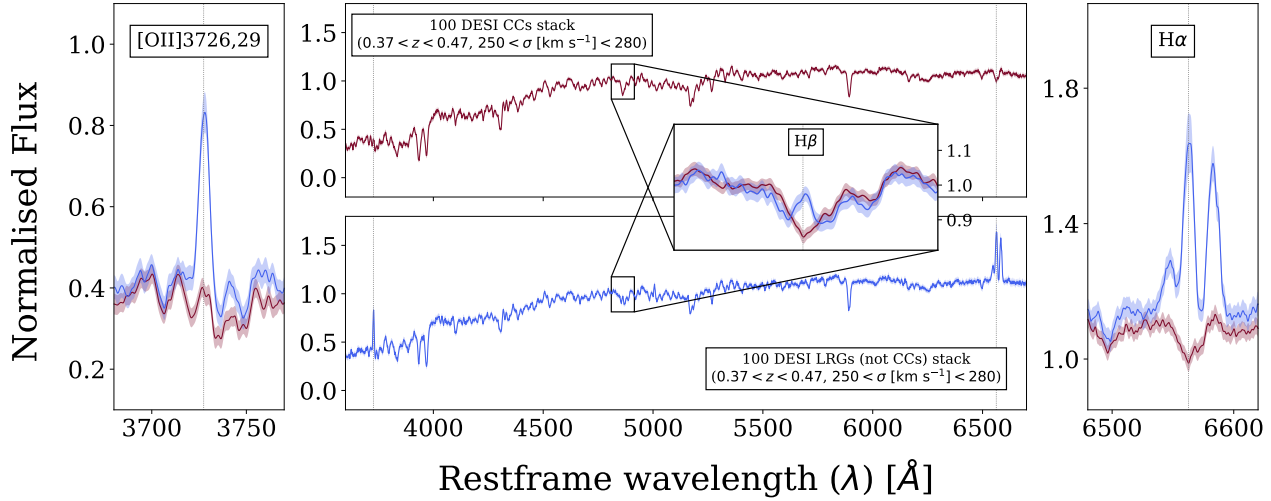


Figure 2.9: Representative SEDs of a stack of 100 good CC candidates (upper panel) and a stack of 100 bad CC candidates (lower panel) that are, however, classified as LRGs. The side panels show the difference observed in the [OII] (left) and $H\alpha$ (right) regions. In the central panel we zoom in the $H\beta$ region, which shows residual emission and refilling in the not pure CCs sample. The colours try to keep the convention of Figs. 2.4, 2.6 and 2.7 for LRGs (blue) and selected CCs (maroon).

Another methodological difference that separates the two articles lies in the use of corrected or uncorrected indices for the selection process. In A26 the EW cut is explicitly applied to the spectral features after they have been corrected for the effect of velocity dispersion. In A25, by contrast, the cut was performed on the EWs as directly measured on the observed, uncorrected spectra. As discussed at length in Sec. 2.1.1, any resolution broadening (whether instrumental or due to the internal velocity dispersion of the galaxy) acts as a smoothing of the flux around a feature and, by the very definition of the EW (eq. 1.49), systematically dilutes its measured strength, pushing $|EW|$ towards lower values. An emission line is no exception to this rule: a weak nebular component that would exceed a given $|EW|$ threshold once deconvolved from the galaxy’s own velocity dispersion can appear slightly below it if measured directly on the (still broadened) observed spectrum. Cutting on the corrected EW is therefore both the more demanding and the more physically motivated choice of the two: it removes the trivial dependence of the measured emission strength on σ itself (recall that our samples span the fairly wide range $100 - 400 \text{ km s}^{-1}$ for SDSS and $200 - 400 \text{ km s}^{-1}$ for

DESI, Sec. 2.2.1), and guarantees that two galaxies with the same intrinsic star-formation content are treated identically by the cut regardless of their velocity dispersion or the instrument that observed them.

From the 135 554 galaxies reported above as SDSS LRGs, a total of 90 396 survive the spectroscopic cuts that we set (including the ones not yet covered) and constitute the CCs SDSS sample used in A25 (a full description of the cuts and number figures at each step can be read from table 2.1). If these were performed on the velocity-dispersion corrected indices, this number would have been reduced to 89 258. In some sense, the tiny fraction of sources that are rejected with the cut based on corrected indices ($\sim 1\%$) tells us that the contamination introduced by sources on the verge of having some remote remnant of star-formation is only marginal. Indeed, the stacking process subsequently undertaken would dilute the effect of these sources as only very contaminating cases (such as the one presented in blue in Fig. 2.9) would contribute to a real bias. Nevertheless, a small effect of the velocity dispersion correction is expected for the emission lines of not so massive objects, since up to $\sim 300 \text{ km s}^{-1}$ the corresponding index correction function (Fig. 2.13) stays below 1.5 (50% change). We further observe that the spectra used to derive the corrections in A26, in particular for the emission lines that we are referring to, would be portrayed in absorption $\{\text{H}\alpha, \text{H}\beta\}$ or as unclear features $\{[\text{OII}], [\text{OIII}]\}$ in the model spectra used for the derivation. This is because the referred spectra come from SPS synthetic SEDs that are representative of passively evolving systems as CCs, and therefore, it can happen that the value of the correction is underestimated with respect to how it would affect emission patterns. Indeed, absorption and unclear features are less strong than emission lines (differences reported in Fig. 2.9), so that a smoothing would have a smaller effect on the integrated index.

Even if the SDSS galaxies were cut on the uncorrected lines side, a second robustness cut was contemplated at the time, by performing the same cut with the indices measured by Tremonti et al. (2004) and stored in the CAS galSpecIndx table. Lick EWs are also measured and stored in the galSpecLine table (hereafter we will refer to these external measurements, both Lick and emission lines, as MPA/JHU catalog). We note that in most works that perform a spectroscopic selection of the sources they use as CCs, indices are typically taken from databases, and sometimes uncorrected for resolution effects, for which a thorough visual inspection is needed to guarantee that no residual starforming galaxies are included in the final sample and bias the results. We note that this comparison sample is measured from the same spectra with a conceptually different code, so it offers a natural external check on our own pipeline, one that is particularly valuable here since any pipeline-driven bias in the emission-line EWs would propagate directly into the purity of the final CC sample. We find good overall agreement between the two sets of measurements, both for the emission lines and for the absorption indices, including the Lick indices that MPA/JHU also tabulates.

We illustrate this consistency explicitly in Fig. 2.10, where we show the logarithm of the relative discrepancy, $\Delta EW \equiv (EW_{A25} - EW_{MPA/JHU})/EW_{MPA/JHU}$, between our pyLick and MPA/JHU measurements on the CCs sample for three indices $\{H\gamma_F, \text{Mgb}, \text{Fe5270}\}$ that are present in our SPS fits and strongly correlate with age, α -enhancement and metallicity respectively (see Sec. 1.3.3 and the discussion around Fig. 1.12). For an easy interpretation, we show the 1σ (most solid) and 2σ (lightest) contours in the $\log(\Delta EW) - z$ panel. In all cases, the whole central 95% of the distribution falls below the 10% relative discrepancy limit, delimited by the black dot-dashed line, while the central 68% distributions stay fully (metallic lines) or mostly ($H\gamma_F$) below the 1% discrepancy threshold, drawn as a black dashed line.

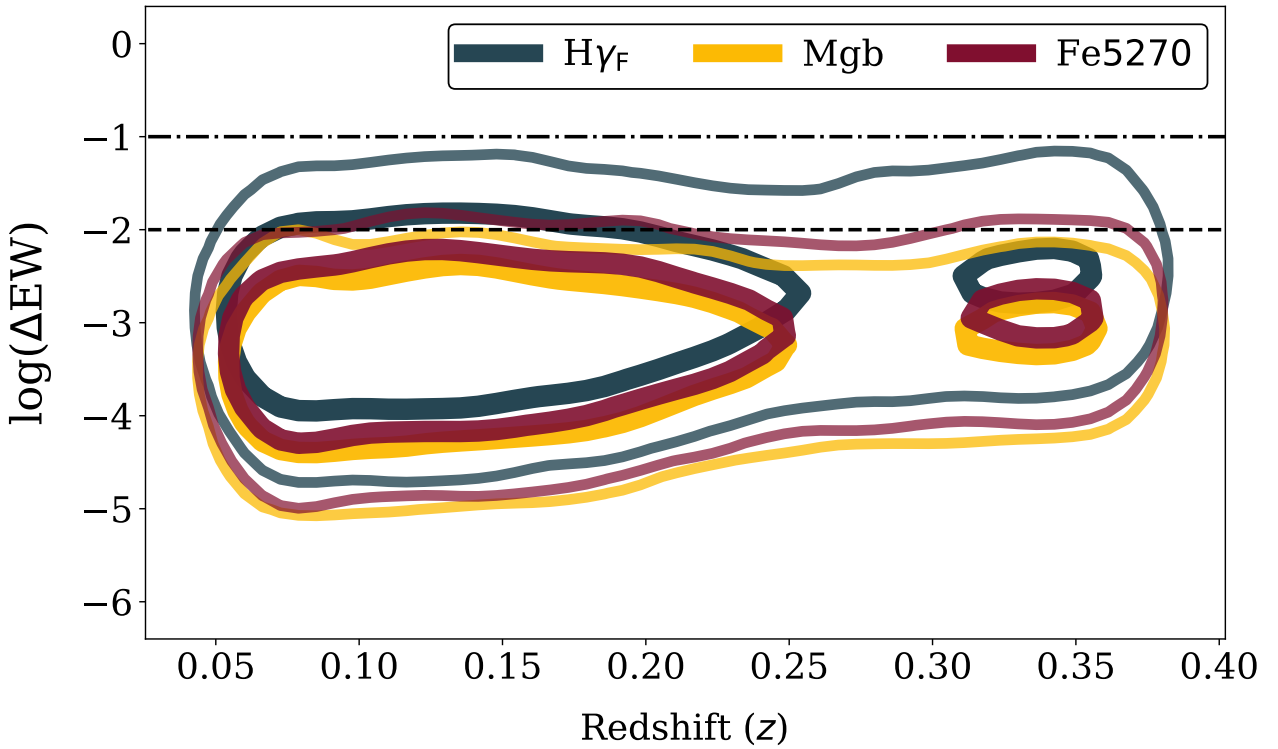


Figure 2.10: Logarithmic relative discrepancy, $\log(\Delta EW) \equiv \log |EW_{A25} - EW_{MPA/JHU}| - \log |EW_{MPA/JHU}|$, for three relevant Lick indices $\{H\gamma_F, \text{Mgb}, \text{Fe5270}\}$ measured on the A25 CCs sample used in the SPS fit procedure, plotted against redshift. For visualisation purposes, the 1% (dashed line) and 10% (dot-dashed line) thresholds are overlaid on the plot.

The same exercise can, in fact, be extended to the BOSS sources. Professor C. Tremonti kindly provided us (private communication) with an analogous, MPA/JHU-style table of spectral indices for a set of BOSS galaxies, for which we had already carried out the full catalog and spectroscopic selection described in Sec. 2.1.1 in anticipation of a possible BOSS-based CC analysis, eventually shelved once the first DESI-DR1 data became available and A26 took priority. Even though this sample never made it into a dedicated article, it offers a useful, independent replication of the SDSS

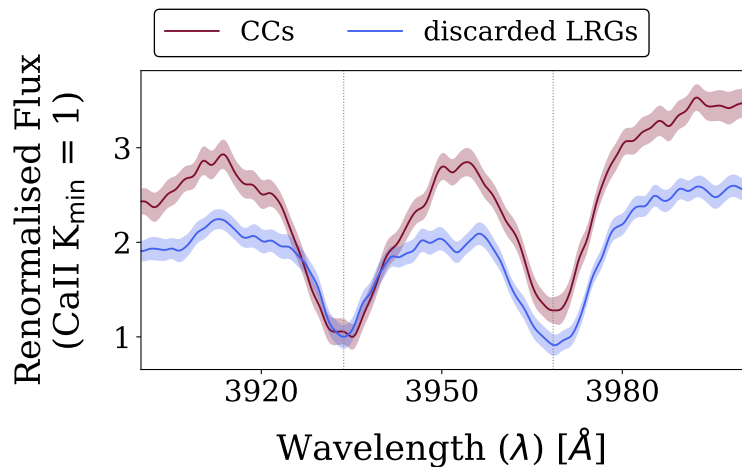
test above at the bridging redshift interval between SDSS and DESI. The provided table⁴ included all the required emission features that serve to discriminate between pure CCs and starforming LRGs, but only the $H\beta$ line (since it is a test-emission line shared with the Lick set) can be compared to our pyLick measurements when it comes to Lick indices.

The H/K ratio

The emission-line cuts above are, by design, only sensitive to residual star formation that is recent and vigorous enough to produce measurable nebular emission in the handful of lines considered; a low-level, more evolved star-forming or post-starburst component, with too weak an ionising continuum to leave a clear emission signature, can in principle survive this first filter unnoticed. Moresco et al. (2018) introduced the H/K ratio in the CC context for this reason, and following Borghi et al. (2022b), both A25 and A26 complement the emission-line selection using this diagnostic. In particular, we use the ratio between the equivalent widths of the respective absorption features, which is useful as it is a quantity that can be directly measured using pyLick.

Because the CaII H line at 3968 Å is blended with the Balmer H ϵ transition at 3970 Å, while CaII K remains essentially unaffected, even a modest population of young, hot stars is enough to deepen the CaII H relative to CaII K, inflating the ratio above the value expected for a purely old population. This can be directly spotted in the full spectra shown in Fig. 2.9 for the stack of LRGs contaminated with active star-formation (lower panel). We zoom in on the CaII absorption features region in Fig. 2.11, where we have renormalised the relative spectra so that the minimum of the CaII K line matched in both cases and equal to 1. In this way, the inversion of the relative position of the CaII H minimum can be easily noticed, with the CCs sample CaII H being less deep than CaII K, while for the contaminated LRGs the CaII H minimum is below 1.

Figure 2.11: Zoom of the CaII region from Fig. 2.9. The flux was renormalised to guarantee that the minimum of the CaII K feature matched in both the CCs stack and the SF contaminated LRGs stack and is equal to 1.



Both $EW(CaII H)$ and $EW(CaII K)$ are measured with the same pyLick pipeline used for the Lick

⁴https://data.sdss.org/datamodel/files/BOSS_SPECTRO_REDUX/RUN2D/PLATE4/RUN1D/spZline.html

indices and the emission lines above, so that the H/K diagnostic is obtained entirely from our measurements, with no external catalog analogue: unlike the emission lines, CaII H and CaII K are not part of the standard MPA/JHU output, so the cross-check against [Kauffmann et al. \(2003\)](#); [Tremonti et al. \(2004\)](#); [Brinchmann et al. \(2004\)](#) performed above for the emission lines cannot be repeated here, and the reliability of the H/K cut rests on the internal consistency of our own measurement chain alone. Following [Borghi et al. \(2022b\)](#), we require $H/K < 1.2$ for a source to be retained, a threshold adopted unchanged in both [A25](#) and [A26](#) and applied at the same stage of the pipeline, after (and independently of) the emission-line cuts described above. As with the emission-line criterion, this cut is deliberately conservative: [Borghi et al. \(2022b\)](#) note that their own final samples rarely reach H/K values above ~ 3 , so that the 1.2 threshold discards a comparatively modest, but not negligible, tail of otherwise passing sources, for the benefit of a markedly cleaner final sample.

We summarise the full funnel of cuts described throughout this section, from each survey’s underlying galaxy catalog down to the final cosmic chronometer sample, in Table 2.1. We include a middle row accounting for how the BOSS and eBOSS data would be pipelined to the final CCs sample. For this selection we have applied the most current version of the spectroscopic selection, the one applied in [A26](#) for DESI data, where the involved spectral features are corrected for the resolution smoothing effects (note that these indices were not accounted for in the corrections developed in [A25](#), for which we cannot apply the full correction that would better suit (e)BOSS data). Since this is an informative row for the completion of this section, we warn that these numbers (spectroscopic selection) may suffer some slight variations in the future if correction functions are developed for the (e)BOSS dataset in particular, including the $1.06\text{\AA} \rightarrow 1.50\text{\AA}$ spectral resolution jump from the DESI corrections (here applied) to the observed resolution of the single (e)BOSS spectra.

	Parent	LRGs	CCs		
			No Emission	H/K < 1.2	
A25 SDSS	669 475	135 554	91 816	126 898	90 396
— (e)BOSS	1, 511 608	204 355	143 297	172 227	123 934
A26 DESI	8 490 171	1 728 366	699 372	1 220 942	527 287

Table 2.1: Summary of the sample sizes at each stage of the selection pipeline for [A25](#) (first row), [A26](#) (third row) and the current status of the unfinished (e)BOSS CCs work (middle row). *Parent* refers to the raw galaxy catalog. *LRGs* gives the number of sources passing the catalog-level, photometric and kinematic cuts of Sec. 2.2.1. Under *CCs*, *No Emission* and $H/K < 1.2$ give the number of *LRGs* sources passing each spectroscopic criterion individually, while the last column (below *CCs*) gives the size of the final, combined spectroscopically selected sample.

Read together, the numbers make explicit a point that was only argued qualitatively above: the H/K criterion is, indeed, the gentler of the two spectroscopic cuts. Table 2.1 confirms this: H/K would retain a larger fraction of the *LRGs* population than the emission-line criterion in all three samples. In A26, the emission-line cut alone keeps only 40.5% of the 1 728 366 DESI *LRGs* (699 372 sources), against 70.6% for H/K alone (1 220 942); the gap is narrower, but points the same way, in A25, where 67.7% of the 135 554 SDSS *LRGs* survive the emission-line cut (91 816) against 93.6% for H/K alone (126 898), and again in the still unused (for cosmographic purposes) (e)BOSS dataset, where the corresponding fractions are 70.1% (143 297 of 204 355) and 84.3% (172 227). In every case the combined cut is close to, but somewhat more restrictive than, the emission-line criterion on its own: 30.5% for DESI, 66.7% for SDSS and 60.6% for (e)BOSS. This confirms that H/K is best understood as a complementary veto, one that removes a further, comparatively small tail of contaminated sources on top of the emission-line cut, rather than as a primary purity criterion in its own right.

The overall restrictiveness of the combined cut over the *LRGs* sample, however, differs markedly between the SDSS/(e)BOSS pair and DESI (60 – 67% against 30.5%), a rather large gap that can be explained by the fact that the *LRGs* selection criteria are different for SDSS/(e)BOSS and DESI. We note that the concentration parameter restriction, based on the Petrosian radii ratio, R_{50}/R_{90} , is a morphological cut that is absent in the DESI selection of *LRGs*; and remark that, taken together, the emission-line and H/K diagnostics are the only two criteria in the whole selection pipeline that are genuinely common in definition across the SDSS, (e)BOSS and DESI analyses of this thesis.

Apart from these selection cuts relative to spectral features in the SED of our CC candidates, further selection cuts after the stacking (Sec. 2.4) and the estimation of SPS parameters take place in both A25 and A26. These cuts do not respond to a strict CC quality selection as the ones explained above, but rather to the possible bias that could be induced in the final $H(z)$ estimation if the corresponding $t - z$ datapoints were included in the cosmographic fit. A dedicated discussion on this is provided in Sec. 2.6.1.

2.3 Index corrections

Section 2.1.1 already laid out the generic physical picture behind any resolution mismatch between data and models: instrumental and velocity-dispersion broadening act as a smoothing of the flux around a feature, systematically diluting its measured strength, and the overall resolution of an object is the quadrature sum of the two effects (eq. 2.1). What remains to be specified is how that generic picture is turned, in practice, into the actual numerical corrections applied index-by-index in A25 and A26. This is the subject of Sec. 2.3.1 below. We then close this section with a related secondary correction for the fixed angular aperture of the fibre-fed spectrographs used by SDSS that was seriously considered at an early stage of A25. Unlike the resolution correction, this was

ultimately not carried over into the final pipeline.

2.3.1 Resolution effects: Instrument and velocity dispersion

The goal of the correction is to bring every measured index, irrespective of the velocity dispersion of the galaxy it comes from or of the survey (and hence the specific instrumental resolution) it was observed with, onto a common reference frame: zero velocity dispersion and the MILES instrumental resolution ($\sigma_{\text{IR}} = 1.06 \text{ \AA}$) at which the TMJ and Knowles models are themselves defined (Sec. 2.1.1). Only once this is done can indices measured across the full σ range spanned by our samples (100–400 km s^{-1} for SDSS, 200–400 km s^{-1} for DESI) be fitted with the same model grid, or combined into the velocity-dispersion stacks of Sec. 2.4, without an artificial σ -dependent bias contaminating the resulting ages.

Because the TMJ model is itself only defined as a direct prediction of index strengths at MILES resolution and zero velocity dispersion, it offers no natural way to be degraded to match an individual object; the correction must instead be applied to the data once the indices have already been measured. For SDSS spectra, this is done in two conceptually distinct steps, motivated by the sharp, step-function-like jump in instrumental resolution at the blue/red arm dichroic split ($\sim 5500\text{--}6000 \text{ \AA}$ observed frame; Fig. 2.2). First, all science spectra (and, correspondingly, all stacks) are degraded to a common, fixed instrumental resolution of $\sigma_{\text{IR}} = 1.5 \text{ \AA}$, using the matching-resolution formalism of eq. (2.3); this value is not itself a target resolution in any physical sense, but simply the coarsest instrumental resolution reached by any SDSS spectrum in the sample, chosen so that *every* object can be safely degraded down to it (rather than up), turning a source- and wavelength-dependent $\sigma_{\text{IR}}(\lambda)$ into a single, uniform number to which the same correction function can afterwards be applied indiscriminately across the whole sample. Note that at different redshifts, observed indices can be found either at the low or high resolution regimes of the SDSS spectrograph, for which different corrections would be needed for each index at different redshift; a procedure that would be obviously suboptimal. Second, and only once every object shares this common $\sigma_{\text{IR}} = 1.5 \text{ \AA}$ baseline, a correction function is applied to take the resulting indices, measured at the galaxy’s own velocity dispersion σ , down to the zero-VD, MILES-resolution ($\sigma_{\text{IR}} = 1.06 \text{ \AA}$) values that the TMJ model predicts. In A25 these two steps are folded together into a single empirical function $C_I(\sigma)$, calibrated, as described below, directly between $(\sigma_{\text{VD}}, \sigma_{\text{IR}} = 1.5 \text{ \AA})$ and $(\sigma_{\text{VD}} = 0 \text{ \AA}, \sigma_{\text{IR}} = 1.06 \text{ \AA})$.

The situation is simpler for DESI: unlike SDSS, the DESI spectrographs deliver a resolution finer than that of MILES over essentially the whole wavelength range of interest (Sec. 2.1.1), so that the intermediate, common-resolution step described before has no equivalent here: every DESI spectrum is, by construction, already better resolved than the $1.06 \text{ \AA}$ MILES target, and a single degradation directly to $\sigma_{\text{IR}} = 1.06 \text{ \AA}$ suffices. As a direct consequence, the correction functions tabulated in A26 encode the velocity-dispersion smoothing alone, calibrated between $(\sigma_{\text{VD}}, \sigma_{\text{IR}} = 1.06 \text{ \AA})$ and

($\sigma_{\text{VD}} = 0 \text{ \AA}$, $\sigma_{\text{IR}} = 1.06 \text{ \AA}$), with no residual instrumental offset folded into them.

This correction function, $C_I(\sigma)$, is calibrated empirically rather than derived analytically, following the general approach of [Carson & Nichol \(2010\)](#): a set of high-S/N stellar spectra of known, zero intrinsic broadening is progressively smoothed by trial velocity dispersions, and the resulting change in each index is tracked and averaged over the whole set of stars, yielding a sampled function $I_0/I(\sigma)$ (for atomic indices) or $I_0 - I(\sigma)$ (for molecular indices), which is then fit with a low-order polynomial in σ . Depending on the index type, the correction is applied as

$$I_0 = \begin{cases} I(\sigma) \cdot C_I(\sigma), & \text{atomic,} \\ I(\sigma) + C_I(\sigma), & \text{molecular,} \end{cases} \quad (2.4)$$

with $C_I(\sigma) = \sum_{n=0}^3 a_n \sigma^n$ in both [A25](#) and [A26](#). In [A25](#), the calibrating stars were SDSS spectra of the giant stars of the open cluster M67 (NGC 2682), already used by the survey itself to derive its own velocity-dispersion templates (Sec. [2.1.2](#)); this choice made the correction directly tied to the SDSS instrumental setup, but also meant it inherited whatever (small) departures from a purely Gaussian instrumental response those particular spectra might carry. In [A26](#) we tried to rely on surveyed spectra; however, the resulting correction functions were comparatively disappointing, perhaps because they were not intentionally optimised spectra for velocity dispersion correction purposes. Instead, the functions were re-derived directly from the continuous, high-resolution MILES v9.1 library spectra ([Falc3n-Barroso et al., 2011](#)), extending the calibration from the original 25 Lick indices to the full set of 33 spectral features measured by pyLick in [A26](#). For each of these correction functions, an uncertainty budget is derived numerically as the average dispersion of the sampling performed from the fiducial values and covariance matrix obtained in the fitting procedure.

Table [2.2](#) reproduces the polynomial coefficients derived in [A25](#) (left-side columns) and in [A26](#) (right-side columns). The functional forms reported in this table can be visually seen in figures A.3 in [A25](#) and A.1 in [A26](#), here portrayed as Figs. [2.12](#) and [2.13](#). The plots of the [A25](#) corrections are not compared with previous correction functions that can be found in the literature. This is because the functions are defined to be applied on spectra measured at $\sigma_{\text{IR}} = 1.5 \text{ \AA}$ instrumental resolution to be brought to the MILES resolution, something that only these particular corrections do (to our knowledge). A point worth making explicit here, since it is easy to read past it, is that the zero-point coefficient a_0 in this table is therefore not the trivial value (1 for atomic indices, 0 for molecular ones) that a bare velocity-dispersion correction would have at $\sigma = 0$.

This reasoning might sound valid to exclude the [Carson & Nichol \(2010\)](#) corrections (designed to be applied onto IDS/Lick resolution spectra) from Fig. [2.13](#) as well; however, the final destination (in instrumental resolution) of the 2010 corrections is the same as the initial one, although this does

			A25 (M67 stars)				A26 (MILES library)			
		Index	a_0	a_1 ($\times 10^{-4}$)	a_2 ($\times 10^{-6}$)	a_3 ($\times 10^{-8}$)	a_0	a_1 ($\times 10^{-3}$)	a_2 ($\times 10^{-6}$)	a_3 ($\times 10^{-9}$)
Lick Indices	Balmer	H δ_A	1.007	-0.297	1.893	0.137	1.000	-0.026	1.970	-1.324
		H δ_F	0.938	-9.164	-9.720	1.396	1.003	-0.127	-1.595	3.262
		H γ_A	0.996	-0.263	-0.951	0.221	1.001	-0.042	-1.458	3.056
		H γ_F	1.010	-0.641	2.561	-0.362	1.000	-0.033	1.845	-2.392
		H β	1.004	1.498	0.347	0.023	0.991	0.676	-1.346	0.501
	Iron	Fe4383	1.002	-2.635	3.658	-0.202	1.007	-0.446	3.059	-0.722
		Fe4531	1.015	1.501	2.133	-0.083	1.000	-0.007	2.286	-0.754
		Fe5015	1.021	1.398	5.114	-0.491	0.999	0.017	5.589	-6.137
		Fe5270	1.010	0.100	3.823	-0.309	0.999	0.040	4.521	-3.836
		Fe5335	1.020	0.661	6.097	0.083	0.999	0.029	5.586	2.173
		Fe5406	1.004	-2.339	4.860	0.341	1.002	-0.216	3.796	4.855
		Fe5709	1.008	2.145	1.200	0.300	0.996	0.304	2.449	0.923
		Fe5782	1.002	-1.793	4.609	0.654	1.007	-0.622	5.548	4.299
	Mg	Mg ₁	0.000	0.026	0.078	-0.010	0.000	0.000	0.093	-0.098
		Mg ₂	0.000	0.077	0.068	-0.008	0.000	0.026	-0.053	0.067
		Mgb	0.997	-2.877	2.714	-0.023	1.007	-0.496	2.785	0.275
	Other Lick Indices	CN ₁	0.004	0.380	0.164	-0.029	-0.001	0.095	0.089	-0.245
		CN ₂	0.003	-0.020	0.455	-0.051	0.000	0.033	0.402	-0.426
		Ca4227	1.011	2.267	-1.146	-1.621	0.995	0.249	2.644	10.311
		G4300	1.002	-0.254	1.653	-0.172	1.002	-0.154	2.049	-2.290
		Ca4455	1.036	0.737	7.826	-0.191	0.999	-0.037	7.939	-1.469
		C ₂ 4668	0.998	-1.769	1.087	0.031	1.003	-0.174	0.223	2.401
		NaD	1.002	1.619	0.061	0.182	0.998	0.116	1.331	-0.280
		TiO ₁	0.000	-0.026	0.009	0.000	0.000	-0.007	0.035	-0.026
		TiO ₂	0.000	0.031	-0.013	0.002	0.000	0.019	-0.064	0.085
Other Indices	[OII] λ 3727		—	—	—	—	0.998	-0.064	10.154	-3.786
	CaII K		—	—	—	—	0.998	0.103	0.610	2.598
	CaII H		—	—	—	—	0.999	-0.001	1.599	1.167
	D4000		—	—	—	—	0.000	0.005	0.139	-0.147
	D4000 _n		—	—	—	—	-0.001	0.051	0.200	-0.238
	H β_0		—	—	—	—	0.999	0.086	1.182	-1.323
	[OIII] λ 5007		—	—	—	—	1.002	-0.310	2.144	-2.165
	H α		—	—	—	—	1.002	-0.181	1.862	2.375

Table 2.2: Coefficients of the velocity-dispersion correction function to take measured indices to MILES instrumental resolution, $C_I(\sigma, \sigma_{\text{IR}}) = a_0 + a_1\sigma + a_2\sigma^2 + a_3\sigma^3$, for every spectral feature measured in this thesis. Corrections developed in A25 (left block) are to be applied on sources at $\sigma_{\text{IR}} = 1.50\text{\AA}$, while the ones developed in A26 (right block) are to be applied on sources at $\sigma_{\text{IR}} = 1.06\text{\AA}$ (MILES resolution). Since emission-line and continuum-break features were not corrected for velocity dispersion in A25 (Sec. 2.3.1), the corresponding cells in the *Other Indices* block are left blank (—).

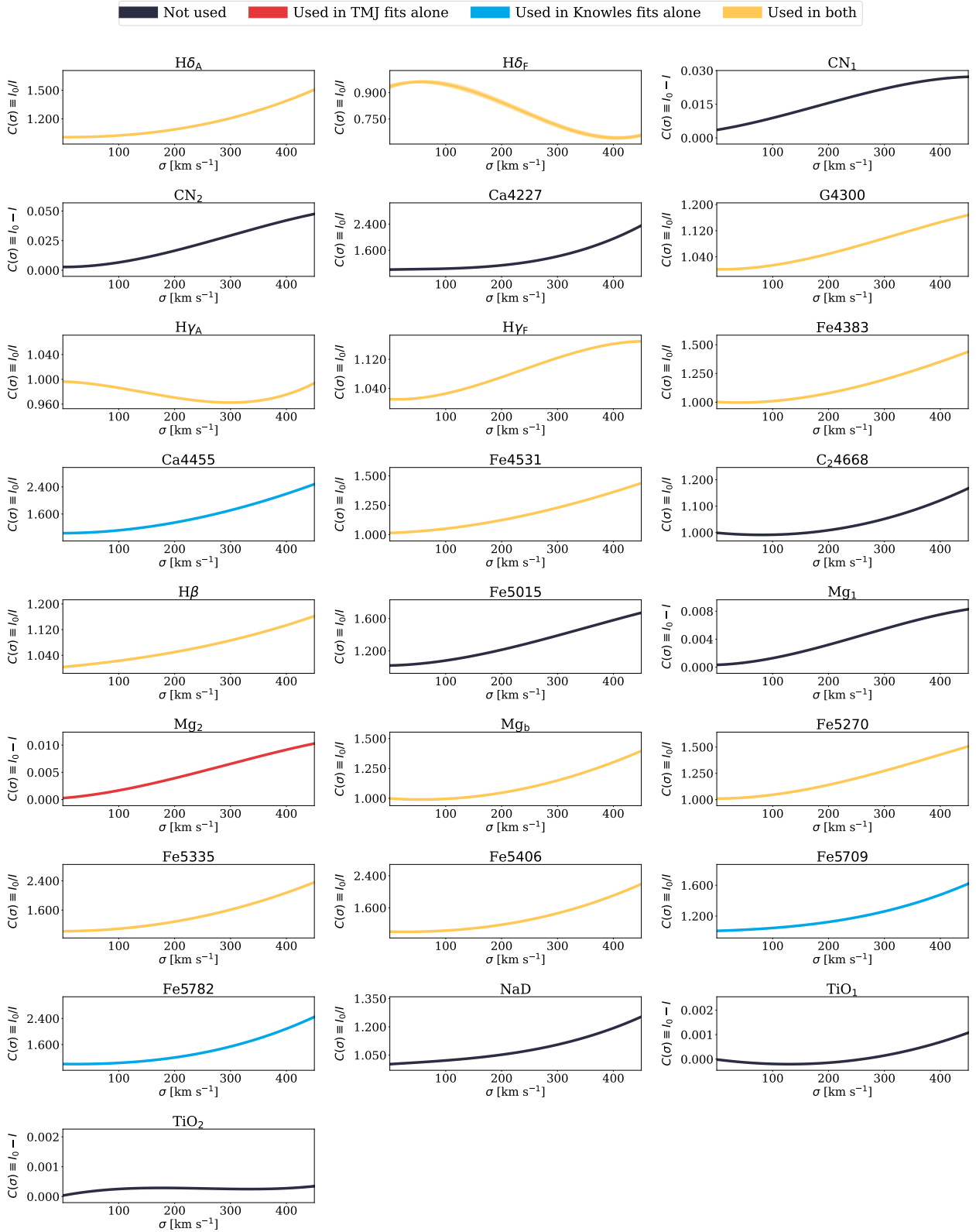


Figure 2.12: Velocity-dispersion correction functions, $C(\sigma)$, derived in A25, designed to be applied to indices measured at $\sigma_{\text{IR}} = 1.50 \text{ \AA}$ instrumental resolution. In dark blue, the indices not used in any of the SPS model fits performed in A25, in red those used only for TMJ model fits, in light blue those only used in Knowles model fits and in yellow those used in both.

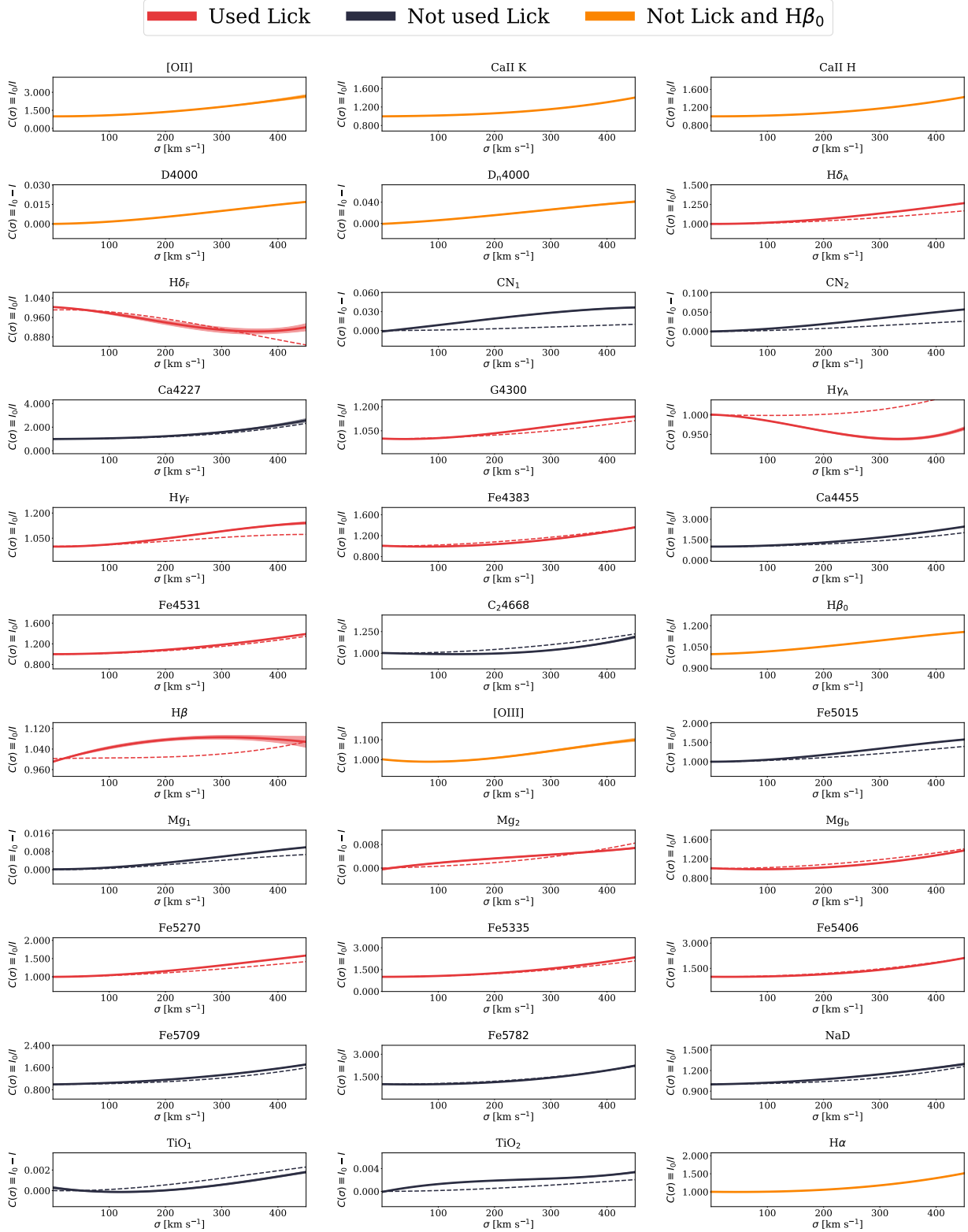


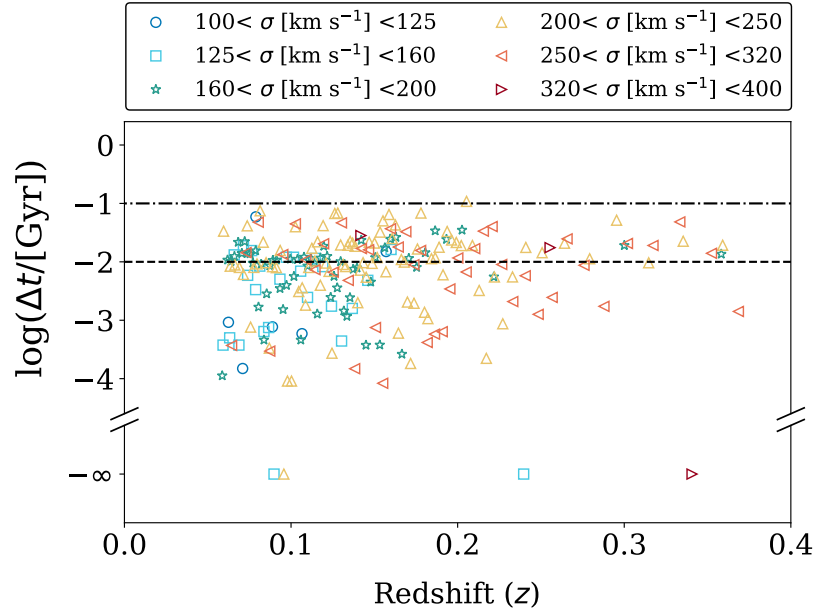
Figure 2.13: Velocity-dispersion correction functions, $C(\sigma)$, derived in A26, designed to be applied to indices measured at $\sigma_{\text{IR}} = 1.06\text{\AA}$ instrumental resolution (MILES resolution). In dark blue, the indices not used in any of the SPS model fits performed in A26, in red those used and in yellow those indices that belong to the extended list of indices measured with pyLick but do not belong to the Lick set. Dashed lines represent the corrections by Carson & Nichol (2010).

not make the corrections equivalent. This is so because the effect of the smoothing over an already smoothed signal (to the IDS instrumental resolution) is smaller than doing so over a less degraded spectrum. All things considered, the comparison with the [Carson & Nichol \(2010\)](#) corrections could be used to check the validity of ours (at least in order of magnitude) and attest to the reproduction of expectations described in the previous discussion. Indeed, we observe that overall the functions seem to behave very similarly, and we can spot a slightly steeper behaviour in our computations for some critical indices (i.e. $H\delta_A$, G4300, $H\gamma_F$, Fe5270). However, this is not something generalised, since we do observe some indices for which our corrections are flatter. Overall, the computation of the [A26](#) corrections seems justified by the fact that, first, eight additional indices (useful for selection and parallel purposes) are included, and second, the exact physical application of these differs from those available in the literature.

The correction functions above are only one of the two philosophies used in this thesis to deal with the velocity-dispersion effect. When fitting with the Knowles model, which predicts SEDs, we instead degraded the model spectra themselves, on a per-object basis, to the specific velocity dispersion and instrumental resolution of the galaxy (or stack) being fitted, using the same `pyLick` machinery employed for the data. This effectively turns velocity dispersion into a further configuration parameter of the model, and removes the need for any correction function of the form of eq. (2.4) altogether, at the cost of having to repeat the model-to-index step for every distinct value of σ (and σ_{IR}) present in the dataset rather than once and for all. Already in [A25](#) the availability of this in-model version for Knowles SPS fits offered a natural internal consistency check on the correction-function approach itself: instead of (or in addition to) degrading the model, one can equally well apply the very same a posteriori $C_I(\sigma)$ correction functions, calibrated for TMJ, to the data before fitting them with Knowles, and compare the resulting ages against those obtained from the properly degraded Knowles model. In Fig. 2.14 we represent the logarithm of the discrepancy between the best-fit ages for the [A25](#) stacks estimated using the Knowles model set at the object spectra velocity dispersion resolution and those using the traditional correction functions. We observe that most stacks have discrepancies below the 1% level, and at most, we find 10% disagreements. We note that, given the grid setting of the model, linearly interpolated in steps of 0.01 Gyr (see Secs. 1.4.5 and 2.6.2), some sources have identical age estimations (numerical $\log(\Delta t) = -\infty$).

In the end, the similarity found between the σ -tuned model and the corrected indices technique in the derivation of stellar ages, supports continuing to use the correction functions for their simplicity in computation and application. Tuning the SPS model to accommodate it to the data resolution can become somewhat cumbersome if one looks for a good match of the estimated velocity dispersion of the object or tries to apply it to single galaxies rather than stacks with similar average velocity dispersion values.

Figure 2.14: Logarithm of the relative discrepancy in the estimation of A25 stacks ages resulting from the use of σ -tuned Knowles SPS models or the traditional velocity dispersion corrections technique. Colours are taken from the choice made in A25.



In the incipient phases of the work with SDSS sources in A25, a different approach was tried. It consisted of using the same galaxy data that served for the following analysis to build the corrections themselves. To do so, galaxy spectra were smoothed from their observed values up to a resolution equivalent to 450 km s^{-1} . Then, a functional extrapolation was used to infer the rest index measurements for each galaxy. At this point the uncertainty of the extrapolation was uneven for low velocity dispersion galaxies and high velocity dispersion data, for which, through the use of a thin binning in velocity dispersion, a sampling of I_0/I_σ was found. In the end, a third-order polynomial was used to extrapolate the final correction function $C_I(\sigma)$ for each index. The results were, however, somewhat disappointing, as the uncertainty and noise of the correction function for some indices rendered the functions useless; particularly when compared with the already available Carson & Nichol (2010) corrections or the ones that we ended up developing using the SDSS stars (as detailed above). We show in Fig. 2.15 some corrections computed using this methodology overlaid over the actual functions derived using stellar spectra. The top panels show two indices whose corrections are similar enough to those derived via stars, although, at high velocity dispersion, the associated uncertainties grow particularly fast. In the lower panel we have the case of two indices, $H\delta_A$ and Fe5335, for which the divergence is so big that the corrections become useless and rather a further bias than a solution to the velocity dispersion effect. The four indices shown are critical in SPS fits, for which the need for valid corrections for all of them is critical.

Finally, it is worth noting that the uncertainty derived, as explained above, for the correction functions shown in the figures, is included indirectly within the statistical uncertainty budget. This is so because the uncertainties associated with the corrected indices are accordingly propagated,

$$\delta I_0 = \begin{cases} \delta I(\sigma) \cdot |C_I(\sigma)| + |I(\sigma)| \delta C_I(\sigma), & \text{atomic,} \\ \delta I(\sigma) + \delta C_I(\sigma), & \text{molecular.} \end{cases} \quad (2.5)$$

Then, the SPS fit translates all of this information, along with the fiducial value for the corrected indices, to SPS parameters and their associated uncertainties.

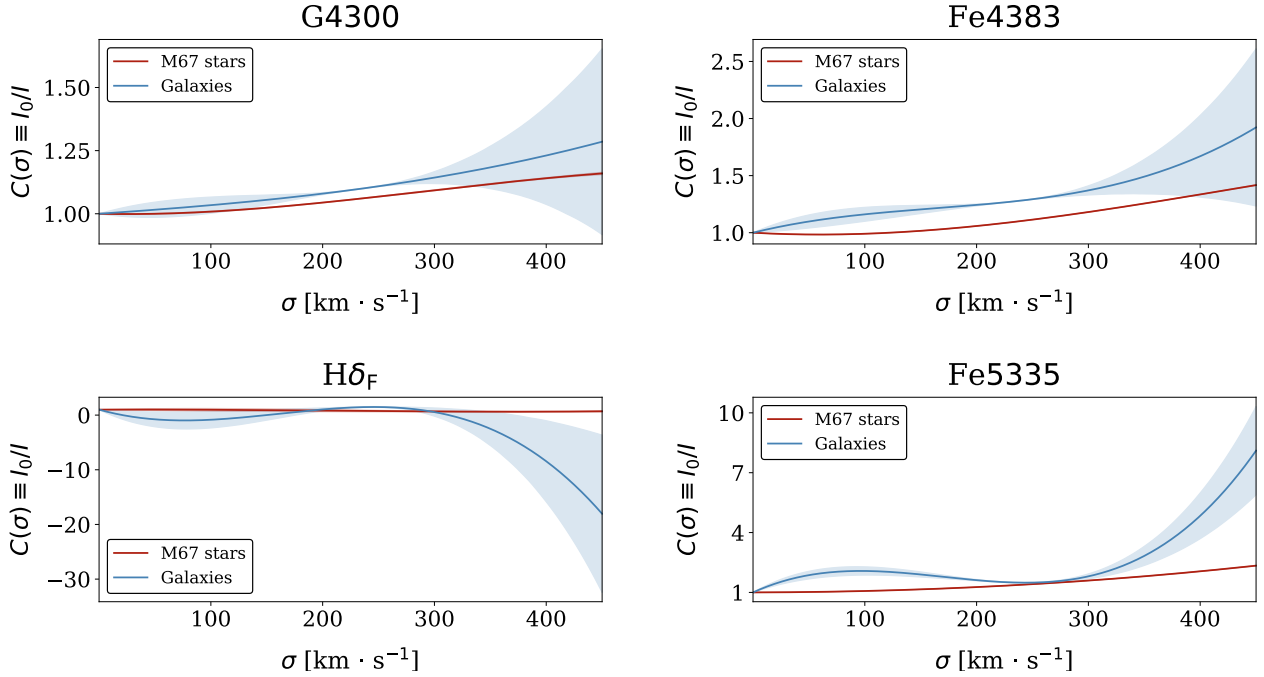


Figure 2.15: Correction functions developed using galaxy spectra (blue) overlaid over the star corrections from Fig. 2.12, for 4 critical indices in SPS fits.

2.3.2 Secondary corrections: Aperture correction

A further, in principle non-negligible (at low redshift), systematic effect follows from the fixed angular aperture of a fibre-fed spectrograph: since the SDSS spectroscopic fibres subtend a fixed angular diameter of 3 arcsec on the sky, the *physical* radius of a galaxy actually sampled by the fibre depends on its angular size, and hence, for a fixed physical size, on its redshift. This would be of no consequence for a genuinely homogeneous stellar population, but early-type galaxies are known to host negative radial gradients in most Lick indices (Davies et al., 1993; Mehlert et al., 2003; Rampazzo, R. et al., 2005) and, correspondingly, in luminosity-weighted age and metallicity, so that a fibre-based index measurement carries an implicit, redshift-dependent bias towards the light-weighted properties of an increasingly small central region as redshift decreases. The same galaxy, observed at two different redshifts, would in principle yield two different index measurements purely as a consequence of this aperture effect, entirely independently of any genuine evolution.

The standard treatment of this bias in the CC literature is that of Clemens et al. (2006) and Clemens et al. (2009), who built on the radial index-gradient measurements of 50 elliptical galaxies by Rampazzo, R. et al. (2005). In short, the mean fractional change of each index between two characteristic physical radii (an inner one and an outer one, expressed in units of the galaxy’s own

effective radius r_e) is parametrised, for each index, as a linear function of $\log \sigma$, calibrated on the [Rampazzo, R. et al. \(2005\)](#) sample; given the (photometrically measured) r_e of a target galaxy and the physical radius subtended by the fibre at that source’s redshift, this radial-gradient relation can then be used to extrapolate the fibre-measured index to a common, standard aperture of $r_e/10$, under the working assumption of a linear trend of index strength with $\log(\text{radius})$ between the two calibration points. Applied to the SDSS 1.5 arcsec fibre radius, this yields a per-index, per-object correction factor (or offset, depending on whether the index is atomic or molecular) that depends on both σ and r_e .

This aperture correction was seriously considered, and implemented, at an early stage of [A25](#), but was ultimately excluded from the pipeline actually used in both articles, for two related reasons. First, its overall effect on the measured indices was found to be small, subdominant with respect to both the velocity-dispersion correction of [Sec. 2.3.1](#) and the statistical uncertainties already present in the individual index measurements, so that including it did not meaningfully change the derived stellar-population parameters. Second, and more decisively for the DESI extension of the analysis, the correction of [Clemens et al. \(2006\)](#); [Clemens et al. \(2009\)](#) requires a reliable photometric measurement of r_e for every object, a quantity that, while available for the SDSS `photoObj` sample used in [A25](#), is not propagated as a standard column of the FastSpecFit VAC used for DESI in [A26](#) ([Sec. 2.2.1](#)); applying the correction only to the SDSS-based sample, and not to its DESI counterpart, would have introduced an asymmetry in the treatment of the two datasets that we preferred to avoid altogether, rather than accept for the sake of a correction of only marginal impact. Moreover, when working with stacked data (see [Sec. 2.4](#)), this correction becomes impractical even if we had a recording of r_e in our sample, as different galaxy sizes (r_e) are included in each stack. To perform such a correction, a third grouping of similar r_e galaxies for each velocity dispersion and redshift stack would be necessary, at the expense of the viability of the whole stacking procedure. Even though the correction was not adopted, the fact that we tested it in detail is worth illustrating explicitly. [Fig. 2.16](#) shows the change from the aperture and resolution corrected EWs relative to the only-resolution corrected EWs for the three Lick indices presented in [Fig. 2.10](#). These results should be considered and compared against the resolution correction functions for [A25](#) in [Fig. 2.12](#).

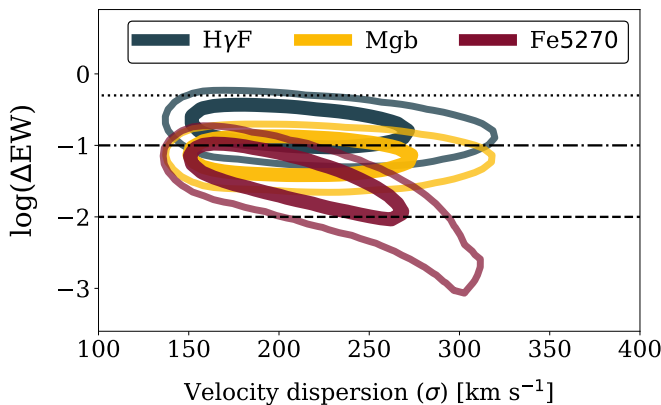


Figure 2.16: Logarithmic relative change in the EW of three Lick indices of the SDSS sample against velocity dispersion. The change from resolution corrected to resolution and aperture corrected index strengths is computed as $\Delta\text{EW} = (\text{EW}_{\text{APC}} - \text{EW}_{\text{CVD}}) / \text{EW}_{\text{CVD}}$. The colours imitate those of [Fig. 2.10](#). 50% (dotted), 10% (dot-dashed) and 1% (dashed) levels are shown.

Along with the 1% (dashed) and 10% (dot-dashed) thresholds represented also in Fig. 2.10, the 50% level is also represented as a dotted line. When compared with the effects of the resolution alone (Fig. 2.12), we see that a 10% level correction is attained for the metallic indices over $\sim 200 \text{ km s}^{-1}$ in velocity dispersion, while a 50% change is never reached for the resolution correction of $\text{H}\delta_{\text{F}}$. For metallic indices, the aperture correction could be almost as important as the resolution correction at low mass, while at high mass the latter would easily dominate.

2.4 Stacking spectra

The case for stacking was already made on physical grounds in Sec. 2.3: high S/N spectra are a prerequisite for reliable Lick index measurements, and single-galaxy spectra rarely reach it. The consequences of ignoring this are not subtle. When the TMJ model is fit directly to single-galaxy indices, a large fraction of the sample is pushed to the edges of the allowed parameter space. That is, metallicities saturate at the prior boundary, and a spurious population of extremely young ($\sim 3 \text{ Gyr}$) ages appears, alongside implausibly old ones, rather than tracing the tight and physically sensible trend expected for massive, passively evolving ETGs. Stacking a set of galaxies of similar velocity dispersion and redshift is precisely what suppresses this behaviour: it is the noise in the individual spectra, not any genuine diversity in the underlying stellar populations, that drives single-galaxy fits into these pathological regions. We portray in Fig. 2.17 figure 5 in A25. There, we show the derived ages, metallicities and α -abundances for each galaxy, using the set of indices that was taken as a baseline for TMJ model fits in both A25 and A26 and for Knowles model fits in A25.

The most notable feature is the absence of extremely low ($\lesssim 4 \text{ Gyr}$) and extremely high ($\gtrsim 12 \text{ Gyr}$) stellar ages for the stacked galaxies, particularly eliminating the excess of sources hitting the upper bound of the prior. A second observation is the anticipation of a discrepancy between TMJ and Knowles modelled parameters, with the TMJ distribution showing an average age of $\sim 7 \text{ Gyr}$ and the Knowles distribution one of $\sim 9 - 10 \text{ Gyr}$. Metallicities are also constrained naturally towards the super-solar end of the distribution ($[\text{Z}/\text{H}] \gtrsim 0$). Overall, the stacking approach seems justified as it helps to avoid unreliable age estimations that can affect the eventual average age for redshift bins. Note that this does not come at the index level but at the SPS parameter level, as shown in the index-redshift trends plotted in figures 13, D.1, D.2, D.3 and D.4 in A25. When it comes to comparing the averaging of individual galaxy ages with the estimation of stacked galaxy ages, the latter is also preferable in order to avoid further manual choices.

The cosmic chronometer method, in its original formulation (Sec. 1.2), is built around the idea that two well-defined populations of passively evolving galaxies, observed at different redshifts, are statistically the same population at different times, so that the relevant quantity is always a population-average age, never the age of any individual object. A stack, by construction, returns

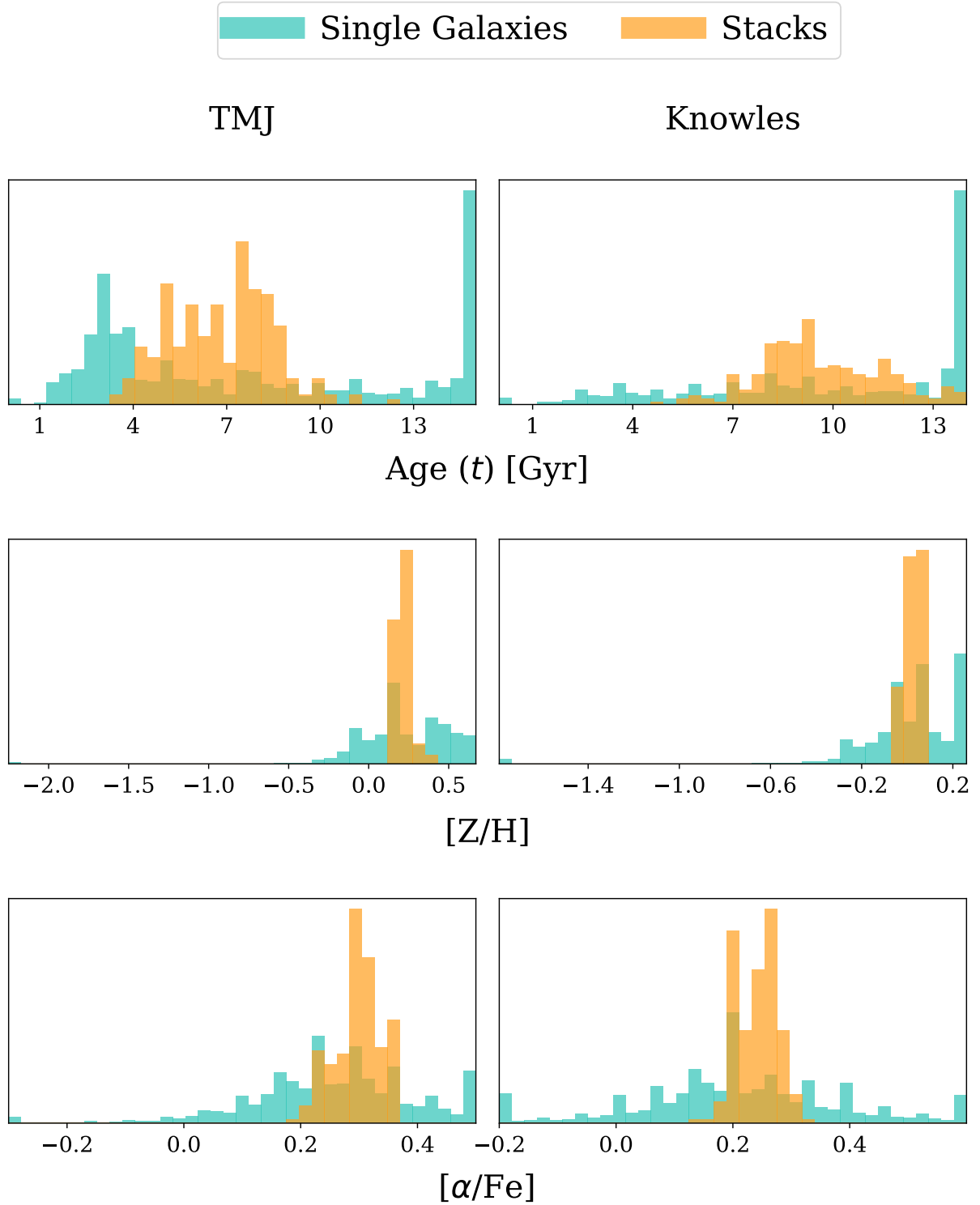


Figure 2.17: View of figure 5 from [A25](#). In light blue, the derived SPS parameters for single galaxies, and in yellow, those for stacked galaxies. The set of indices used for the respective fits are the ones finally selected for the science results for both TMJ model and Knowles model fits in [A25](#).

exactly this average age for the velocity-dispersion (mass) bin it represents. This is relevant for galaxies that formed later than average in low-mass groups (see Fig. 1.15), where individual galaxies could show ages as low as 2 Gyr and still be good CC candidates. The stacking technique allows for average ages to be retrieved from the first moments, which also reduces the parameter space upon which the SPS models are required to work well.

Turning to the methodological description of the process, both A25 and A26 share the same basic stacking ingredients, already introduced in Sec. 2.1.1. First, single spectra are shifted to the rest frame, normalised on a stable window between 4200 and 4400 Å, resampled onto a common 0.1 Å grid, and combined through the pixel-by-pixel median, with the uncertainty of the stacked flux estimated via the normalised median absolute deviation (NMAD; Hoaglin et al., 1983) rather than the standard deviation, for its robustness to outlier spectra within a stack. Velocity-dispersion stratification is always fixed, since it separates physically distinct populations, while the redshift grouping is adaptive, tuned to reach a target median S/N per stack.

It is in the specific mechanics of this redshift grouping that the two articles diverge, reflecting the very different scale of the SDSS and DESI-DR1 samples. In A25, the target S/N (set to 300 as the baseline) is reached indirectly: an auxiliary set of stacks is first built on a fixed 2D grid in velocity dispersion and redshift, and used to empirically sample the function $N(z, \langle S/N \rangle)$, the number of galaxies needed at a given redshift to reach a given median S/N; this function is then interpolated to obtain $N(z)$ at the fixed reference $\langle S/N \rangle = 300$, which in turn drives a sequential procedure that opens a stack, adds galaxies one by one in order of redshift, and closes it once the local galaxy count exceeds the value that $N(z)$ prescribes at the running average redshift of the stack. Figure 3 from A25, here portrayed as Fig. 2.18, shows the $\log(S/N) - N - z$ space needed for this construction, along with the $z - \sigma$ plane for density of sources.

In A26, given the much larger DESI parent sample (Sec. 2.2.1), this two-pass, auxiliary-grid approach was replaced by a direct adaptive search: for each new stack, a bisection algorithm (following an initial doubling phase to bracket the solution) searches directly for the smallest number of consecutive, redshift-ordered galaxies whose composite spectrum reaches the target S/N, evaluated on a fast, undegraded composite; only once this membership is fixed is the final stack rebuilt in full, applying the resolution degradation of Sec. 2.3.1 and measuring its Lick indices. This removes the need for the intermediate empirical calibration of $N(z, \langle S/N \rangle)$ altogether, at the cost of repeating the (fast) trial composites several times per stack during the bisection search itself. This is not a minor issue and has some implications for the attained S/N level expectations. This is why A26 targets a lower reference S/N of 150 rather than the 300 of A25. Each individual stack is built from arrays of size $N \times M$, where N is the number of galaxies being combined and M the wavelength sampling of the (not yet reducible) flux array; keeping M fine enough to preserve the individual SEDs caps the

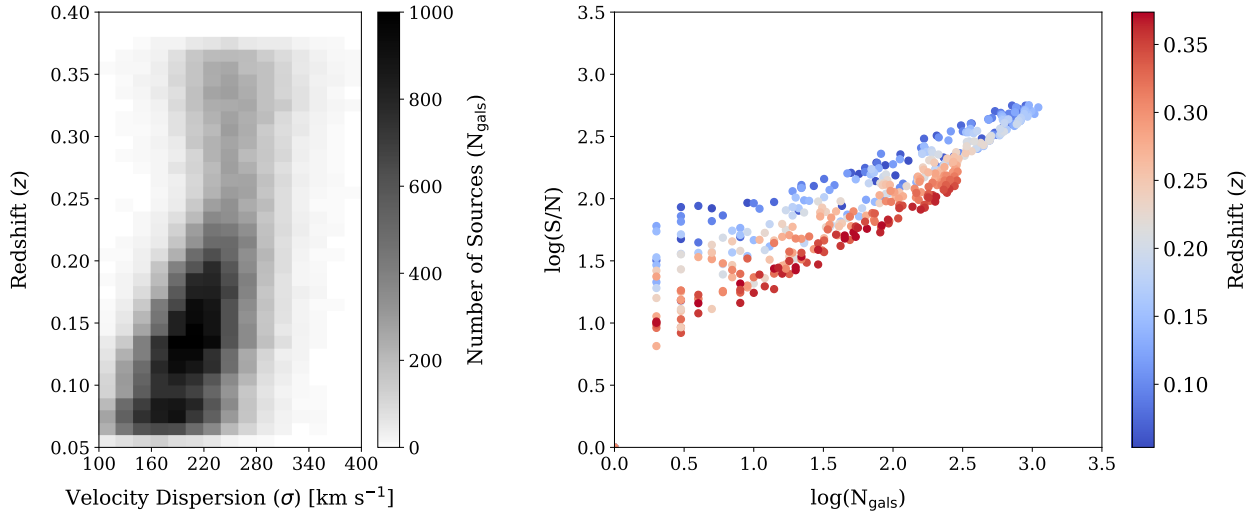


Figure 2.18: View of figure 3 from A25. On the left panel, the density of sources in fixed bins (pixels) of $\{\Delta\sigma = 20 \text{ km s}^{-1} \times \Delta z = 0.01\}$. On the right panel, the dispersion map of the averaged S/N and z and total N taken from the pixels of the left panel.

number of galaxies that can be practically stacked together at any one time to a few thousand, which limits the growth of the S/N level.

Beyond the reference S/N level actually adopted for the baseline analysis, both articles construct additional sets of support stacks at several other target S/N values, precisely to assess how sensitive the final results are to this particular methodological choice; this is discussed together with the rest of the systematic uncertainty budget in Sec. 2.6. Likewise, a final quality selection applied directly to the resulting $t - z$ stacks (excluding, for instance, stacks with an excessive internal spread in redshift or with insufficiently significant age determinations) is deferred to the same section, since it is more naturally discussed alongside the other systematic effects it is designed to control.

2.5 Bayesian Statistics

Both stages of the data treatment that turn a set of measured spectral features into a cosmographic constraint, the stellar-population fit that maps Lick indices onto $(t, [Z/H], [\alpha/Fe])$ (Sec. 2.3), and the cosmographic fit, which maps the resulting $t - z$ relation onto (H_0, q_0, j_0) or their pivotal-redshift analogues (Sec. 1.2), are non-linear and multi-parameter fits. In the first case, the fit is performed against a model that is only available on a discrete, interpolated grid. In both cases we are not simply after a single best-fit point, but after the full, possibly asymmetric or correlated, probability distribution of the parameters: the age-metallicity degeneracy (Sec. 2.6) and the correlation between q_0 (q_{z_0}) and H_0 (H_{z_0}) (Sec. 1.2) are two examples, encountered repeatedly in A25 and A26, of exactly the kind of feature that a simple point estimate and a symmetric error bar would fail to capture. This is the reason why a Bayesian, Markov chain Monte Carlo (MCMC) approach to both fits, rather than

a direct χ^2 minimisation, was adopted in both articles.

In the Bayesian framework, the posterior probability of the set of parameters θ from a given model, given the data $\mathbf{x}$ is

$$\mathcal{P}(\theta|\mathbf{x}) = \frac{\mathcal{L}(\mathbf{x}|\theta)\mathcal{P}(\theta)}{\int \mathcal{P}(\mathbf{x}|\theta)\mathcal{P}(\theta)d\theta} , \quad (2.6)$$

where $\mathcal{L}(\mathbf{x}|\theta)$ is the likelihood of the data given the parameters and $\mathcal{P}(\theta)$ is the prior probability for the parameters. The denominator is called the *evidence*, and it acts as a normalisation factor. The prior encodes whatever is known about the parameters before the data are considered. In both fitting stages, uniform (flat) priors are adopted over literature-motivated functional forms for every parameter: t , $[Z/H]$ and $[\alpha/Fe]$ for the SPS fit; H_0 (or H_{z_0}), q_0 and j_0 , together with a per-velocity-dispersion-group zero-age $t_{0,v}$, for the cosmographic fit. This reflects the absence of any strong external constraint we wished to encode, beyond keeping the sampled range wide enough that the resulting posterior is genuinely constrained by the data itself rather than by an artificially narrow prior boundary. This is precisely why, for instance, the priors on q_0 and j_0 were deliberately set far wider than the range of values expected under Λ CDM (Sec. 1.2).

However, the underwhelming outcome of constraints at several points of the fitting procedure ended up motivating some choices to guarantee convergence and the utility of the results. Regarding the SPS fit, the full range of metallicities allowed in TMJ had to be limited to $[Z/H] \lesssim 0.3$ (effectively 0.26, which represents $Z \sim 2Z_\odot$), in order to avoid an observed bimodal behaviour (see Sec. 2.6.2) and guarantee that metallicity values aligned with the well-grounded literature (Trager et al., 2000b,a; Gallazzi et al., 2005, 2006; Thomas et al., 2005; Conroy et al., 2014; McDermid et al., 2015), where only partially super-solar metallicities are expected in good CC candidates. The effect of removing this limitation, which on the other hand was designed to physically compare the results from using the TMJ and Knowles models for the stellar parameter estimations, is addressed in Sec. 2.6. Then, in A25 the constraint $dH/dz > 0$ needed to be applied to prevent the model's low sensitivity to q_0 from translating, through its degeneracy with H_0 , into a biased estimate of the latter. This was not considered, at the time, as a source of systematic uncertainty but rather as a limitation itself. Fortunately, the stability of the $t - z$ sampling derived from the DESI LRG sample and the increase in the sensitivity to q allowed us to eliminate this requirement in A26.

Within the Markov chain Monte Carlo approach, the posterior distribution is sampled using *emcee* (Foreman-Mackey et al., 2013), the Python implementation of the affine-invariant ensemble sampler of Goodman & Weare (2010). Rather than propagating a single chain through parameter space with a hand-tuned step size, an ensemble of many *walkers* explores the space simultaneously, each proposing its next position based on the current positions of the other walkers in the ensemble. As is standard practice, an initial *burn-in* segment of each chain, during which the ensemble has not yet forgotten its (essentially arbitrary) starting positions, is discarded before any inference is

drawn from the remaining, stationary part of the chain. Convergence of the chains is assessed using the so-called mean autocorrelation time, τ . In particular, the emcee run stops once the number of iterations exceeds 50 times the mean autocorrelation time, following the recommendations in the emcee documentation.

Under the working assumption that the uncertainties on the individual measurements (index strengths in the SPS fit, stack ages in the cosmographic fit) are independent and Gaussian-distributed,

$$\mathcal{P}(y_i|\boldsymbol{\theta}) = \frac{e^{-(y_i - \hat{y}_i)^2 / 2\sigma_i^2}}{\sqrt{2\pi}\sigma_i} , \quad (2.7)$$

the likelihood function becomes the product of the probability of the individual observations. Then, one takes logarithms to work with sums instead and finds

$$\log \mathcal{L}(\{y_1 \dots y_N\}|\boldsymbol{\theta}) = \mathcal{N}(\sigma_1 \dots \sigma_N) - \frac{1}{2} \sum_{i=1}^N (y_i - \hat{y}_i)^2 / \sigma_i^2 \propto -\chi_\nu^2 , \quad (2.8)$$

where the N observations represent the N indices used in the SPS fit or the number of datapoints in the $t - z$ plane in the cosmographic fit, and $\hat{y}$ represents the SPS model based on $\boldsymbol{\theta} \equiv \{t, [Z/H], [\alpha/\text{Fe}]\}$ or the cosmographic function parametrised through $\boldsymbol{\theta} \equiv \{t_v, H, q, j\}$ respectively. Note that t_v is the zero age for each velocity dispersion group and $\{H, q, j\}$ are measured at the zero-redshift value in each article ($z = 0$ in A25 and $z_0 \approx 0.57$ in A26). Finally, we showed how this logarithmic likelihood is proportional to a χ^2 distribution with $\nu = N - m$ degrees of freedom, where m is the number of parameters of the model (3 for the SPS fit and 8 for the cosmographic fit).

A further, more specific feature is applied in the particular cosmographic fit of the Knowles-modelled ages in A25. It is the case of an additional f_v -factor (Hogg et al., 2010) per velocity-dispersion group to address the observation that its nominal age uncertainties appeared implausibly small given the intrinsic scatter of the data (Sec. 3.1.3). Rather than manually inflating the quoted uncertainties, the likelihood variance itself was augmented as $\sigma_{i,v}^2 \rightarrow \sigma_{i,v}^2 + t_{i,v}^2 \cdot f_v^2$, with $\log f_v$ given a wide, uniform prior and sampled jointly with the rest of the parameters; this is a standard way of letting the data inform how much additional (otherwise unmodelled) variance is required directly within the likelihood.

Finally, both articles summarise their posteriors through the median and the 16th/84th percentiles, quoted as an asymmetric credible interval whenever the posterior itself is visibly skewed, as is routinely the case for q_0 (or q_{z_0}), whose posterior is found to pile up against the $q_0 = -1$ boundary imposed by $dH/dz > 0$ in A25. Also, the corresponding corner plots of the full joint posterior are presented to visualise correlations between different parameters. A26 additionally reports the maximum-a-posteriori (MAP) estimate as a complementary point summary for its pivotal-redshift $H(z)$ reconstruction, and explicitly notes that the MAP and the median-based curve can differ perceptibly whenever the underlying posterior is not close to Gaussian, which is itself a useful diagnostic of how much a single point estimate can be trusted over the full sampled distribution.

2.6 Systematic uncertainty

Throughout the previous sections of this chapter, four qualitatively different kinds of uncertainty have appeared, and it is worth separating them explicitly before going any further. First, there are effects that are formally systematic in origin but that both articles fold directly into the statistical error budget, because the fitting procedure itself makes it impossible to disentangle the two once the fit has been performed; the clearest example is the uncertainty of the velocity-dispersion correction functions, δC , already discussed in Sec. 2.3.1. Second, there are genuine methodological choices whose impact on the final $H(z)$ estimate is explicitly quantified and propagated into a dedicated systematic uncertainty budget in both A25 and A26. The first part of this section will cover these systematics properly reported in the articles.

Then, there are systematic effects that are addressed and reported but that have not been included in the systematic error budget either because the overall effect is small or because the tractability was well beyond the scope of the articles, an example of this would be the oscillations in the index- z plane or the effect of shortening the metallicity prior to $[Z/H] < 0.26$ in the TMJ model fits. The latter, together with the effects that are estimated in the articles from external literature results, can be grouped together as SPS model systematics. These include a wide range of variabilities that could be properly addressed in a dedicated work, such as the set of Lick indices used to estimate the stellar parameters or the effect of the components of the SPS model itself. We organise this last part starting from the most relevant SPS effects that have been partially or totally discussed in the articles and concluding it with the discussion around the index- z plane oscillations, as these merit a separate discussion for the relevance given in A25 and the repeated partial observation in A26.

2.6.1 Uncertainties included in the budget

The general strategy in both works is the same: the full pipeline, from the stacking stage onward, is repeated under a discrete set of alternative, individually defensible methodological choices, and the spread of the resulting $H(z)$ curves across these alternatives is taken as the systematic contribution, to be combined in quadrature with the statistical uncertainty from the MCMC sampling ($\sigma_{\text{total}}^2 = \sigma_{\text{stat}}^2 + \sigma_{\text{syst}}^2$).

In A25, two choices are tested this way: the target S/N of the stacks (eleven support-stack sets, from 200 to 400 in steps of 20, around the baseline of 300), and the post-stacking threshold in σ_z used to exclude stacks with an excessively wide internal redshift spread (six values, from no cut at all to $\sigma_z = 0.002$, around the adopted baseline of $\sigma_z < 0.005$). This additional cut was introduced at the end of Sec. 2.2.2 as a further reduction in the number of sources that end up contributing to the $H(z)$ estimate.

For each redshift, the systematic contribution is computed as the median, over the MCMC chain, of the standard deviation of $H(z)$ across the alternative choices, $\sigma_{H,\text{syst}}(z) = \text{med}_k \left[\sqrt{\text{Var}_i(H_i(z; H_{0,k}, q_{0,k}, j_{0,k}))} \right]$, where i indexes the alternative choices and k indexes the MCMC samples. The resulting budget is roughly twice the statistical uncertainty in size, with the S/N-level choice dominating at low redshift and the σ_z -selection threshold becoming comparatively more important towards higher redshift.

A26 extends this same logic by including two further selections onto the final CCs sample. These take place in the pre-SPS fit phase, contemporary to the S/N choice, and the pre-cosmographic fit phase, once the SPS fit has already provided a measurement for the average stellar population age. The first incorporation accounts for a further cut in the final sample that had to be designed after the observation that the application of the (back then not yet optimised) pipeline developed for A25 produced an unexpected systematic effect on the $t - z$ plane⁵. This effect was particularly enhanced at the lowest velocity dispersion groups; in fact, the $160 < \sigma \text{ [km s}^{-1}] < 200$ sources were actually considered in the first stages of the work but had to be eliminated afterwards. In the upper panel in Fig. 2.19 one can observe the result of the application of the standard pipeline on the full final sample (grey) as defined in Sec. 2.2.2, and also the further cut final sample (blue) on the $t - z$ plane. For the reasons that will be detailed in the following lines, this cut was named *archaeological coherence cut*. We represent only the $200 < \sigma \text{ [km s}^{-1}] < 225$ (lighter colours) and $225 < \sigma \text{ [km s}^{-1}] < 250$ (darker colours) stacks (see Sec. 2.4). Theoretical Λ CDM $t(z)$ trends using the Planck Collaboration et al. (2020) results and the estimated pivotal-redshift ages for these groups (Sec. 3.2) in the central cosmographic fit in A26 are plotted in the background, marking the expected behaviour of each group at different redshifts. The residual⁶ between the estimated age and the Λ CDM trend, $\Delta t \equiv (t_{\text{T MJ}} - t_{\Lambda\text{CDM}})/t_{\Lambda\text{CDM}}$, is shown in the lower panel.

The first thing to notice is the superposition of the blue dataset with the grey dataset in perfect agreement in the common redshift window. This is expected, since the archaeological coherence cut only limits the maximum redshift up to which we *should* include sources for each velocity dispersion group. Indeed, the full set of sources including the spectroscopic cuts will still be called the *final* sample, as, strictly speaking, there is no direct observational quantity that disqualifies any of the LRGs within from being good CC candidates. What motivates this further cut is the behaviour of the trends at high redshift, departing from the (natural) noisy trend around Λ CDM at somewhat different redshifts (marked by the change in colour from blue to grey in each case). The discrepancy with Λ CDM is monotonic with redshift and particularly enhanced at the highest redshifts, overcoming the 100% threshold. We also observe that the trends actually converge in the upper panel, something that should not occur, and back in preliminary stages, when the $160 < \sigma \text{ [km s}^{-1}] < 200$ dataset was

⁵This is the first time in this thesis that the *real* observable in CC works, the $t - z$ relations, are presented in a figure. We remark that we use t to refer to *ages*, as absolute cosmic time is excluded from the interests of the works presented in this thesis.

⁶ Δt here should not be confused with the interval in cosmic time between two different redshifts used in Sec. 1.2.1.

also included, the convergence also affected these and the departure from Λ CDM happened at ever earlier redshifts.

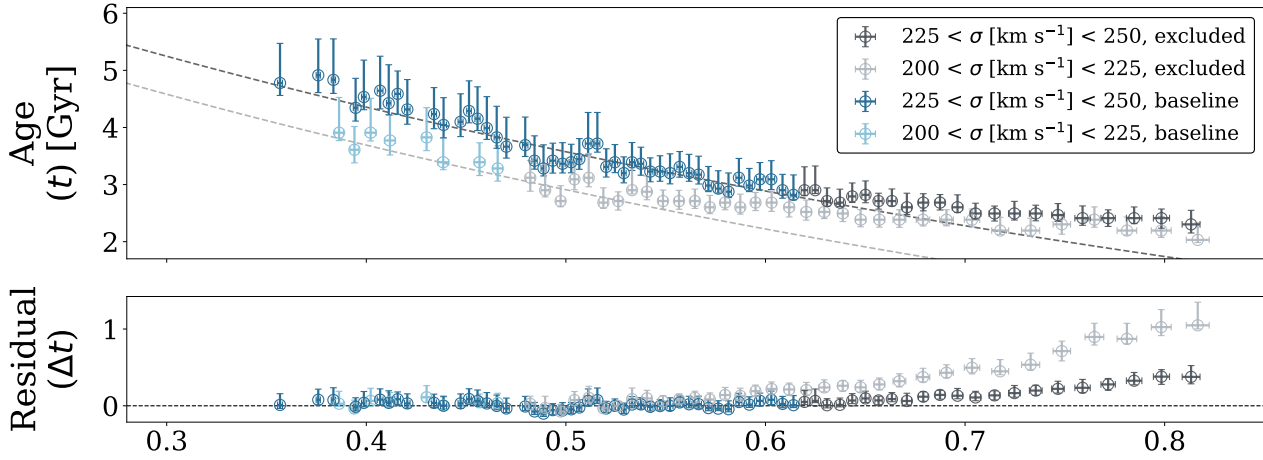


Figure 2.19: In the upper panel the estimated ages for the DESI sample using the TMJ model. Darker colours are used for the $225 < \sigma \text{ [km s}^{-1}] < 250$ stacks and lighter colours for the $200 < \sigma \text{ [km s}^{-1}] < 225$ stacks. These are overlaid forward to the general trend for these two groups based on the Λ CDM results from [Planck Collaboration et al. \(2020\)](#), where the pivotal-redshift age was taken from the baseline cosmographic fit in [A26](#). In the lower panel, the relative residuals between the two. The grey dataset represents the final sample (including the H/K condition) while the blue represents the sample after the further archaeological coherence cut.

We realised that this pattern was produced by the discrepancy between observed indices and the indices predicted by the TMJ model at low ages. We observed this for critical indices, like the δ and γ Balmer lines (Fig. [2.20](#); view of figure B.1 in [A26](#)). For the representation, we applied a scaling relation to define the present age of each galaxy in the $200 < \sigma \text{ [km s}^{-1}] < 250$ range and a Λ CDM cosmology to translate observed redshifts into ages, then we made bins of 100 galaxies age-ordered and plotted the average index strength against the estimated ages. One can easily notice how somewhere between 1 and 2 Gyr in estimated age the trends of the TMJ predicted (for expected metallicity, $[Z/H] > -0.33$, and α -enhancement, $[\alpha/Fe] > 0.0$, values) and observed index strength depart from each other. The effect gets enhanced for lower ages.

As a compromise, we decided to make a prior cut in redshift in each velocity dispersion group applying the aforementioned procedure. First, one chooses a scaling relation to set the zero-redshift ages, $t_0(\sigma)$, for each velocity dispersion bin. Then, we set a conservative limit of $\delta t \equiv 2.5$ Gyr as the minimum age at which the TMJ model responds *well* to the observed indices, and translate look-back times to a redshift cut, $z_{\text{max}} \equiv z_{\text{cosmo}}(t_0(\sigma) - \delta t)$, by applying a model cosmology. Obviously, these include systematic choices that, contrary to what one might naively think, do not introduce a cosmological bias. That is, the model cosmology used does not set the shape of H or any prior on the

cosmological (cosmographic) parameters; instead, it only limits the redshift up to which we can *look*. An indirect proof of this is again the overlapping between the blue and grey stacks in Fig. 2.19 in the common redshift window; that is, no source was removed (thereby eventually causing a cosmological bias) from the *allowed* redshift window. The assessment of these choices, which consist of three possible archaeological (scaling relation) rules (Thomas et al., 2010; Johansson et al., 2012; A25) and two possible cosmologies (Planck Collaboration et al., 2020; Riess et al., 2022), is what is included in the systematic pre-SPS error budget. In the future, a more dedicated analysis around the limitation of the TMJ models to reliably reproduce low ages with the set of Lick indices as we chose and in the grid we used can be worthwhile. Moreover, the threshold we set at 2.5 Gyr was a conservative choice, to avoid including biased age estimations in the baseline results in A26. However, with more time this choice can be inspected in depth.

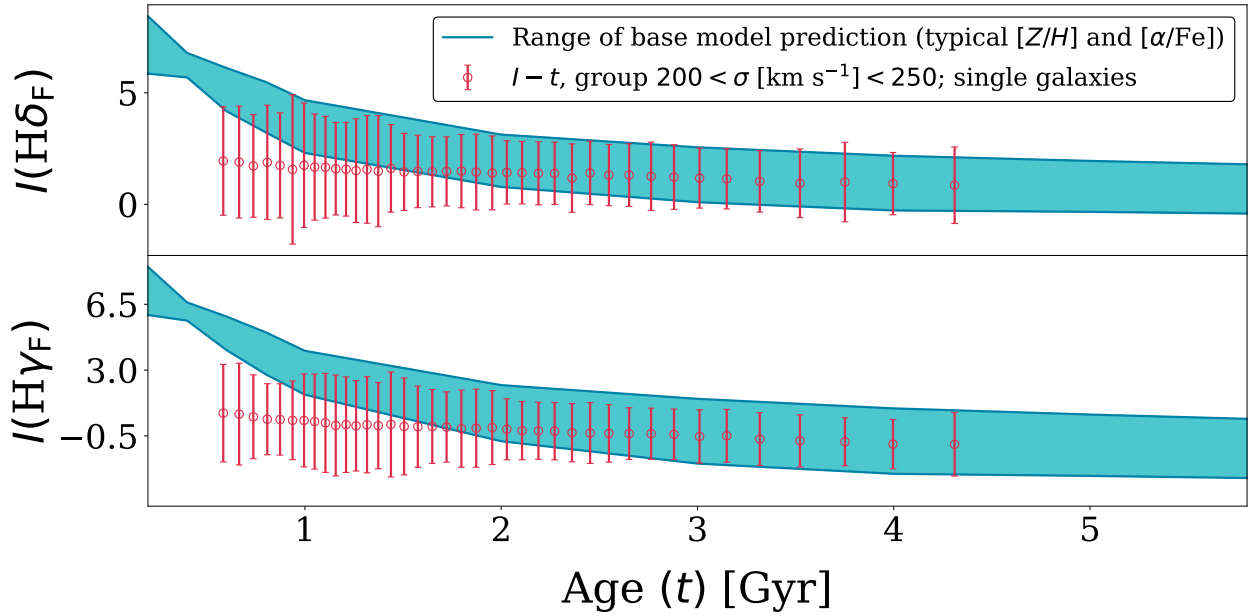


Figure 2.20: Index strength (upper panel $H\delta_F$, lower panel $H\gamma_F$) versus (estimated) age. On the background, the range of values predicted by the TMJ model for typical metallicity ($[Z/H] > -0.33$) and α -enhancement ($[\alpha/Fe] > 0.0$) values in CC candidates. Overlaid, bins of 100 galaxies ordered by estimated age using the zero-age scaling relation prescriptions in A25 and a Λ CDM cosmology with the Planck Collaboration et al. (2020) constraints to translate observed redshifts to ages.

Besides this selection particularly necessary for the DESI data, the S/N-level choice (now eleven levels from 50 to 150 in steps of 10, around a baseline of 150) is also maintained as a source of systematic uncertainty. Then, on top of these pre-SPS cuts, four pre-cosmographic fit quality flags are included, expanding the σ_z cut performed in A25 and detailed above. These consist of a minimum S/N threshold for the estimated ages and the averaged redshifts of 5 and 50 respectively, and a maximum σ_t of 1 Gyr along with the maximum σ_z threshold as well. The effect on the overall $t - z$

relation is presented in Fig. 2.21, which is a view of figure C.1 in A26.

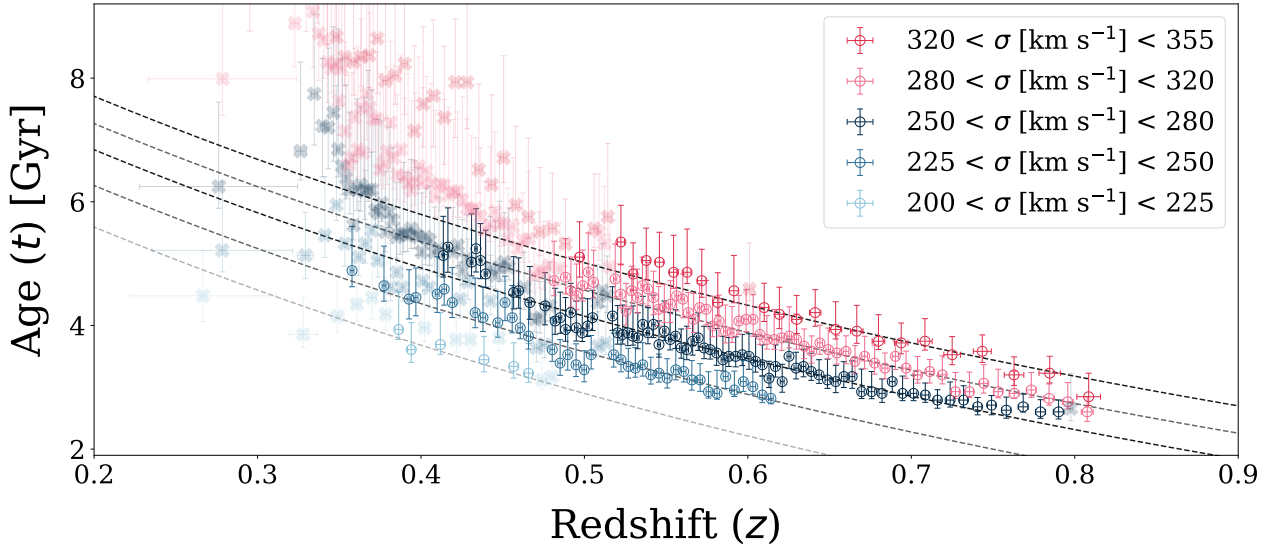


Figure 2.21: $t - z$ relations in A26, including the stacks discarded following the pre-cosmographic fit flags: $S/N(t) > 5$, $S/N(z) > 50$, $\sigma_z < 0.005$ and $\sigma_t < 1$ Gyr. The Λ CDM expected relations are presented in the background, using the pivotal-redshift ages derived in the cosmographic fit as intercept.

This figure constitutes the central observable in A26, including the stacks that contribute to the baseline cosmographic fit results (more vivid colours with circle markers) and the stacks that were discarded following the quality flags mentioned (bold crosses with faded colours). The structure of the figure is equivalent to the upper panel in Fig. 2.19. The effect of the flags is clearly centred at low redshifts, where the slope of the trends departs heavily from Λ CDM and the $S/N(t)$ drops well below 5. A spike around $z \approx 0.5$ can be perceived, and will be addressed separately in the following section.

The resulting overall systematic uncertainty is computed through an analogous construction as the one in A25, and is found to be of the same order as the statistical uncertainty across the whole redshift range covered, with the pre-fit contributions (S/N level and archaeological/cosmological prescriptions, combined) slightly larger than the post-fit quality-flag contribution, and, within the former, the S/N -level choice outweighing the archaeological prescription by roughly a 3 : 2 ratio on average. We bring both systematic uncertainty budgets together in Fig. 2.22, combining the results of figure 10 of A25 and figure 6 of A26 into a single, directly comparable plot. The upper panel gives the systematic uncertainty budget in units of the total statistical uncertainty at each z , while the lower panel gives the statistical uncertainty itself, expressed as a fraction of $H(z)$. Colours follow the same SDSS/DESI convention already used in Figs. 2.1 and 2.2 (blue and grey tonalities, respectively), and, to keep the terminology consistent between the two works, we relabel the choices made prior to

the SPS fit (stacking-related, in both cases) as *Pre SPS fit*, and those made on the already-fitted $t - z$ plane, prior to the cosmographic fit itself, as *Pre Cosmo fit*.

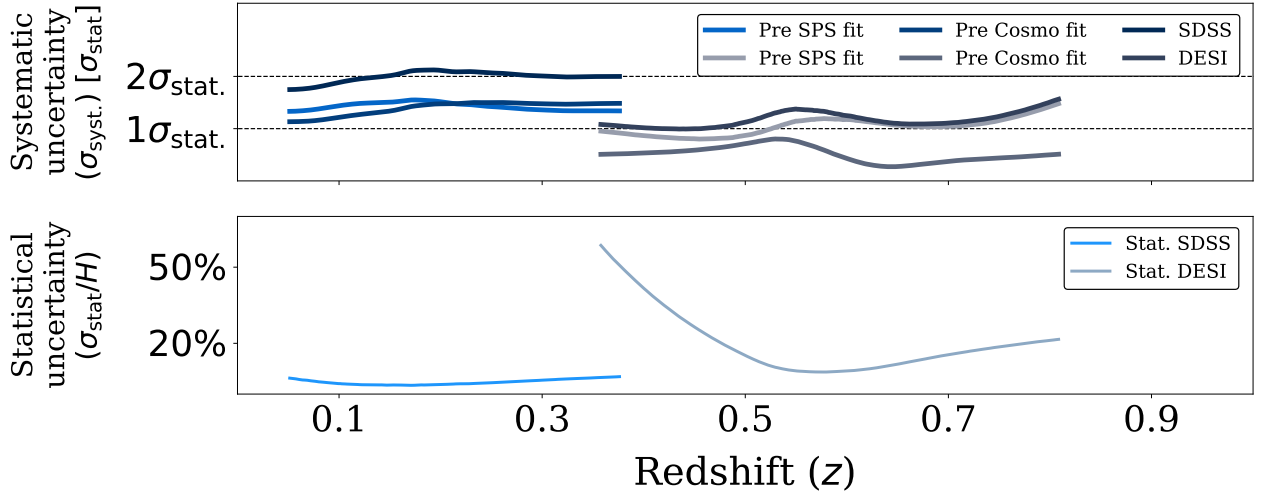


Figure 2.22: Comparison of the systematic and statistical uncertainty budgets of [A25](#) (SDSS, blue tonalities) and [A26](#) (DESI, grey tonalities). On the top panel, the systematic uncertainty, $\sigma_{\text{syst.}}$, in units of the statistical uncertainty, $\sigma_{\text{stat.}}$, at each redshift, split into the *Pre SPS fit* and *Pre Cosmo fit* contributions, together with their quadrature sum (darkest lines, labelled *SDSS/DESI*). On the bottom panel, the statistical uncertainty, $\sigma_{\text{stat.}}$, as a fraction of $H(z)$, for both works.

Read side by side, the two panels of Fig. 2.22 tell a somewhat counter-intuitive story. In relative (statistical-uncertainty) terms, [A25](#) is the more systematics-dominated of the two works: its total budget sits close to $2\sigma_{\text{stat}}$ across essentially the whole covered redshift range, against a comparatively milder $1 - 1.5\sigma_{\text{stat}}$ for [A26](#). Yet, in absolute terms, it is the SDSS analysis that reaches by far the tighter statistical precision on $H(z)$ (bottom panel, $\lesssim 15\%$ throughout), while DESI only approaches a comparable precision in a narrow window around its pivotal redshift, z_0 (taken as the median redshift of the used stacks sample, ≈ 0.57). The SDSS sample is comparatively compact and concentrated at low redshift, so a handful of essentially arbitrary methodological choices can shift the whole $t - z$ relation by an amount that is a non-negligible fraction of its already small statistical error. The much larger and more extended DESI sample, by contrast, has enough flexibility given by the statistical uncertainty, so no single methodological choice can move the fit by nearly as much in relative terms.

The relative weight of the two DESI contributions is equally instructive. The *Pre SPS fit* term tracks the total budget closely and clearly dominates over the *Pre Cosmo fit* term (the post-SPS fit quality flags on the $t - z$ plane) across almost the entire range, particularly at high redshift. This makes sense when one looks at the behaviour at the edges of both the systematic and statistical uncertainty. At high redshift, the statistical uncertainty grows as fewer velocity dispersion groups remain available in the $t - z$ plane: the $200 < \sigma [\text{km s}^{-1}] < 225$ and $225 < \sigma [\text{km s}^{-1}] < 250$ groups are limited in

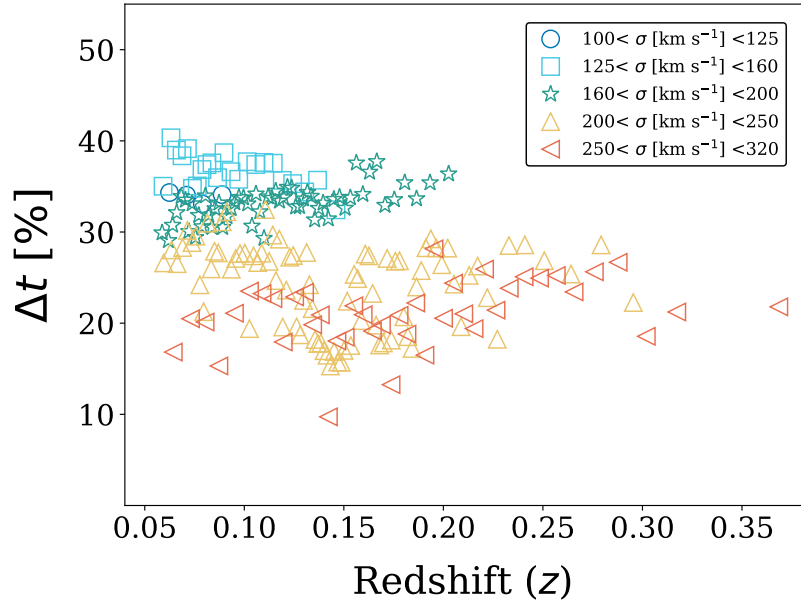
redshift by the restriction imposed by the archaeological coherence cut. In this regime, it is the *Pre SPS fit* term that dominates, which comes naturally if different prescriptions for the archaeological coherence cut include or exclude sources that hit the limit of the TMJ model sensitivity (as reported in the discussion around Fig. 2.20). At the low redshift end, the statistical uncertainty blows up, naturally caused by the lack of sources and sparsity of those remaining due to the $t - z$ plane quality flags, particularly the restriction in $S/N(t) > 5$. On this side, the *Pre Cosmo fit* term almost converges with the *Pre SPS fit* term, which seems reasonable as the loosening of the restrictions for datapoints entering into the fitting algorithm drives up the divergence with the baseline cosmographic fit. Overall, the *Pre SPS fit* term dominates in absolute terms all throughout but particularly at high redshift, while the *Pre Cosmo fit* flags modulate the total systematic uncertainty budget at lower redshift.

2.6.2 Other systematic effects

The most relevant systematic effect that is alluded to in recent CC works (Borghi et al., 2022a; Jiao et al., 2023; Tomasetti et al., 2023; Loubser, 2025) and also in the articles that this thesis reviews is the effect of the SPS model itself. Fortunately, an expensive in-depth analysis in every CCs article is avoidable thanks to the dedicated work of Moresco et al. (2020). The work provided a thorough assessment of the impact on the final H measurement of the different available SPS models and their constituents (IMF, stellar library, etc.) separately. They include five SPS models, two of them available for several IMF choices and one of them for two stellar libraries. The results show that the IMF has a marginal bias in the H estimation, accounting for, at most, a $\sim 0.5\%$ budget. On the other hand, the stellar library and all other constituents of the SPS model dominate the systematic error budget, which can produce up to a 15% bias in the final measurement, particularly below $z \approx 0.5$. However, this study was specialised in the D4000 spectral feature, meaning that the systematic effects of the different SPS models on the predicted strength of the Lick indices can only be guessed from these results. The sole CC work in literature with Lick indices (Borghi et al., 2022a) provides a dedicated analysis of the systematic uncertainty, comparing the baseline results using the TMJ model with those obtained using the SED-based Vazdekis et al. (2015) ones (note that this is a predecessor of Knowles et al. (2023)), which allowed for the assessment of the SFH as a separate contributor to the systematic error budget.

In A25 a dedicated analysis of the results using both the TMJ and Knowles models on the stacked data was tried. The Knowles model, however, ended up being problematic in terms of the reliability of the estimated ages. We observed far too widely scattered ages in each velocity dispersion group with unrealistically small uncertainties (very peaked posteriors) from the SPS fit (figure 8 in A25). This was a stark contrast with the TMJ derived ages, which as shown in Fig. 2.21, had associated uncertainties and a scattering pattern within which a benchmark cosmology can be locally fit by-eye (and results in loose enough cosmographic parameter estimations). At the time, several tests were

Figure 2.23: Residuals from the age estimation using the TMJ and Knowles models on the baseline set of stacks employed in A25. Different colours represent the different velocity dispersion groups, respecting the choice made in the article.



performed, but eventually the sole working solution to stretch the posterior distributions of the cosmographic parameters was the introduction of the auxiliary f_v functions reported in Sec. 2.5. In Fig. 2.23 the disagreement in the estimation of ages is reported as the relative residual with respect to the TMJ age estimations, which we take as more reliable. We note that this figure includes also the effects of differences in the underlying scaling relation, as can be noted for different groups of velocity dispersion scattering along a different Δt level. In particular, less massive groups are predicted much different ages than the most massive ones; and the internal discrepancy within each group can spread for a 10 – 20%.

These differences have been measured for ages estimated using a relatively similar set of Lick indices for the two SPS models, the ones reported in A25. The TMJ set consists of $H\delta_A$, $H\delta_F$, Fe4383, Fe4531, $H\beta$, Mgb, Fe5270, Fe5335, Fe5406 (the baseline used in Johansson et al. (2012)) plus G4300, $H\gamma_A$, $H\gamma_F$, and Mg_2 (which reproduced well the globular cluster data as reported in Thomas et al. (2011)); and the Knowles set consists of $H\delta_A$, $H\delta_F$, G4300, $H\gamma_A$, $H\gamma_F$, Fe4383, Ca4455, Fe4531, $H\beta$, Mgb, Fe5270, Fe5335, Fe5406, Fe5709 and Fe5782 (after discarding those that, as referred to in Knowles et al. (2023), were too broad or sensitive to constituents of the model). In both cases the 5 Balmer-line indices are included, all the irons included in the TMJ set are also included in the Knowles set, with this one reaching up to two redder irons: Fe5709 and Fe5782. The G4300 is common in both sets and the Ca4455 is only included for Knowles. Finally, the Thomas et al. (2011) positive assessment of Mg_2 led to its inclusion in the TMJ set, while it is absent for Knowles.

In Borghi et al. (2022b,a) several sets of Lick indices are considered for the baseline fit with the TMJ model, finding similar slopes in their $t - z$ relations but noticeable differences in the overall age estimation of up to ~ 2 Gyr, which on the other hand is not so different from the overall effect we found by comparing the TMJ and Knowles results overall before. For that work, the 7 combinations

tried for the TMJ model were applicable only to some different CCs subset in each case, due to the wavelength coverage of the SED. A full combinatorial assessment of different sets of Lick indices on a large dataset is lacking in the literature, and the sole similar references, [Thomas et al. \(2010\)](#) and [Johansson et al. \(2012\)](#) take an approach in which the set of indices does not remain stable for the whole dataset, but varies depending on the compatibility of the minimum χ^2 value of the overall SPS fit with the hypothesis of the SPS model being accurate to give a description of the parameters. The central idea here is that each galaxy was approached independently, and the failure to meet the previous criterion resulted in the progressive removal of indices (according to the impact of each of them in the minimum χ^2 value of each galaxy), producing a final distribution of estimated ages, metallicities and α -abundances from fits with a variable number of indices. Given the results of the dedicated analysis in [Borghi et al. \(2022b\)](#), we estimated it would be more correct to use a common set of indices for the whole sample. No further analysis was considered deep or valuable enough to be reported here; however, we refer to Sec. 4.1 for ideas on how an article regarding the systematics associated with the SPS models (including the Lick indices set) can be structured and which aims it can have.

A third effect related to the SPS model itself comes from the metallicity range adopted as prior for the fits. As commented in Sec. 2.5, the metallicity range for the TMJ model had to be reduced to $[Z/H] < 0.26$, which was initially a choice naively made to sample a common parameter space with TMJ and the Knowles model. The encouraging results of the fit were the ones reported in [A25](#) and, at the time, other sources of uncertainty or ad hoc choices (as for example the $dH/dz > 0$ requirement) seemed more relevant than a metallicity that was aligned with the expected results. Works in the literature were also consistent with the expected figures (i.e. [Gallazzi et al., 2006](#); [Conroy et al., 2014](#); [McDermid et al., 2015](#)), and we had little to compare to but low-redshift works aimed at constraining the scaling relations ([Thomas et al., 2010](#); [Johansson et al., 2012](#)), where only in the middle-to-high velocity dispersion groups the metallicity hit the upper edge of the prior (figure 11 in [A25](#)), which was natural for very old systems and not far from the published results. A later insight into the corner plot for the SPS parameters in the TMJ-modelled ages of the [A26](#) stacks raised concerns about a persistent systematic effect on the metallicity, as all sources hit the upper bound of the prior. Actually, for the upper end of the redshift interval, we did have other published CC ages and metallicities to compare to. We observed that the derived ages matched the ones obtained by [Borghi et al. \(2022b\)](#), but the metallicities could not have been more different, with us getting readouts of $[Z/H] > 0.2$ (only the lower limit constrained), while [Borghi et al. \(2022b\)](#) found unguided constraints at $[Z/H] \sim 0.1$ with competitive error bars. The following analysis pushed us to eliminate the metallicity prior and observe the effect on the ages, as well as to attempt to manipulate the fit by taking Gaussian priors centred on estimations available in the literature ([Jiao et al., 2023](#)).

In Fig. 2.24 these trials are shown for the $250 < \sigma \text{ [km s}^{-1}] < 280$ velocity dispersion stacks. The four solid datapoints represent the Borghi et al. (2022b) redshift bins for the massive sample (velocity dispersion above the median). The dark blue trend represents the baseline fit performed in the article (with the prior in $[Z/H] < 0.26$). The green scatter represents the metal-free fit, with sources taking the preferred spot in the parameter space without restriction other than the natural limits of the model. Then, the pink $\{[Z/H] \sim \mathcal{N}(0.1, 0.02)\}$ and yellow $\{[Z/H] \sim \mathcal{N}(0.84, 0.41)\}$ distributions show the result of applying Gaussian priors of different kinds (onto the full original grid). The former represents an arbitrary choice that ensures that the $[Z/H]$ estimation remains within the range estimated by Borghi et al. (2022b). The latter represents the direct adoption as prior of the median and dispersion of the metallicity distribution found by Jiao et al. (2023).

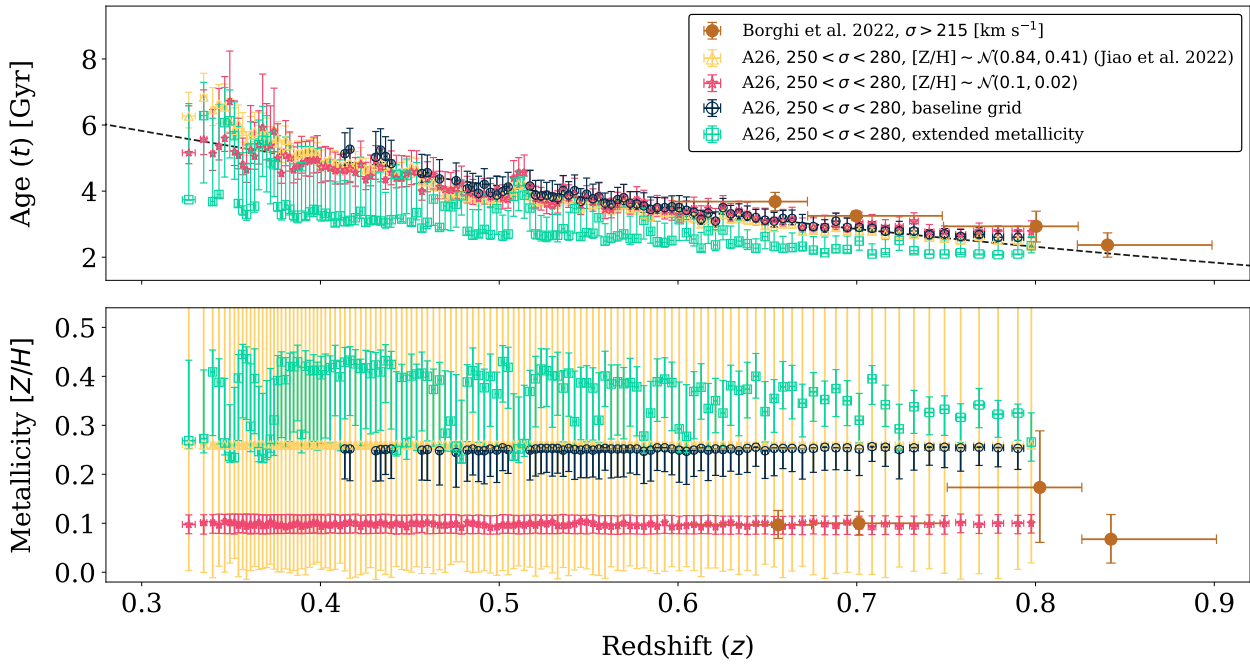


Figure 2.24: SPS age (upper panel) and metallicity (lower panel) estimations following different rules for the prior in the latter parameter. In dark blue, the $t - z$ relation that is taken as the baseline in A26. In green, the result of removing this prior. In yellow and pink, two Gaussian priors, $[Z/H] \sim \mathcal{N}(0.84, 0.41)$ and $[Z/H] \sim \mathcal{N}(0.1, 0.02)$. Finally, the Borghi et al. (2022b) estimations for the above-average velocity dispersion group are presented as the four filled-brown datapoints.

Several interesting patterns can be observed in the upper panel, which is the most relevant in the CCs framework. First, the narrow Gaussian prior plotted in pink produces indistinguishable ages from those of the baseline grid with the particular improvement of a more stable trend at low redshift (with respect to the observations in Fig. 2.21). Similarly, a wider Gaussian prior based on the Jiao et al. (2023) results ends up finding a stable pattern for the best-fitting metallicities around the upper bound set in the baseline grid, producing a $t - z$ relation similar to that of the tighter Gaussian prior.

Then, we see that the metal-free case produces an uneven pattern that, when it was inspected in detail by looking at the MCMC chains, was found to be driven by a bimodality in the $t - [Z/H]$ plane. In essence, two *preferred tracks* in the $t - z$ plane are formed, with an easy jump between the two being possible although the (problematic) younger option is preferred by the fitting algorithm; in physical terms, a return to the classical age-metallicity degeneracy (Worthey et al., 1994; Chavez & Bertone, 2011; Barone et al., 2018). Finally, we see that the Borghi et al. (2022b) ages converge to our ages as expected and the metallicities agree when the strict Gaussian prior is imposed.

Even if several of these alternatives could have been tried in order to partially assess some of the systematic uncertainty due to the SPS model (in this particular case, TMJ), we opted to leave this pathology as a commented observation rather than a numerically accounted for contribution to the error budget. Moreover, the $H(z)$ estimation in the green case could produce an unwanted strong bias with respect to the other cases. It remains to be seen in the future whether this metallicity issue is related, and to what extent, to the divergence in $t - z$ observed in Fig. 2.21.

Index- z relations

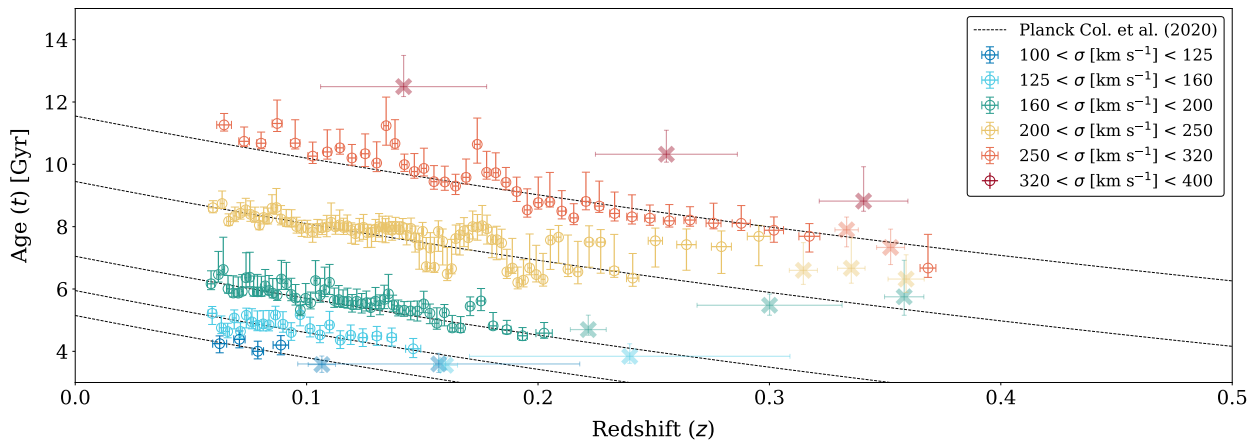


Figure 2.25: Anticipation of the $t - z$ relations for the TMJ-modelled ages of A25 (right panel of figure 6 in the article). The synchronous departure from the smooth $t - z$ trend, common to all three central velocity-dispersion groups with data at these redshifts, is most clearly visible around $z \sim 0.17$, with smaller features around $z \sim 0.08$ and $z \sim 0.13$.

At the end of Sec. 2.6.1 we introduced the spike around $z \approx 0.5$ visible in the DESI $t - z$ relation of Fig. 2.21, responsible for the sharp increase in age uncertainty (and hence for the removal of data points by the $S/N(t) > 5$ quality flag) in that narrow redshift window. This is not an isolated curiosity of A26, an entirely analogous phenomenon was already identified and studied in much greater depth in A25. Actually, it is one of the central results of the article, and it is worth anticipating here the panel that first reveals it, even though the full cosmographic fit it is associated with will only be presented properly in the results chapter of this thesis (Sec. 3.1). We portray it here as Fig. 2.25,

where the different colours represent the velocity dispersion groups (same criterion as in the original figure 6 in A25). We have cleaned the figure of unnecessary features for the present representation.

Once this pattern was noticed, we set out to determine whether it was driven by any particular subset of the indices entering the TMJ fit. Fig. 2.26 shows the age deviation from the Planck Collaboration et al. (2020) Λ CDM prediction, $t - t_{\text{Planck}}$, plotted against the strength of individual indices for every stack in the three central velocity-dispersion groups at $0.16 < z < 0.19$, where the main feature of Fig. 2.25 sits. Two indices, $H\gamma_F$ and Fe4383, show a tight correlation with the age deviation that is conspicuously absent for other, comparably age-sensitive indices such as $H\beta$ or Fe5335; this does not mean the latter are insensitive to age in general, only that they are not the ones responsible for this particular, localised fluctuation.

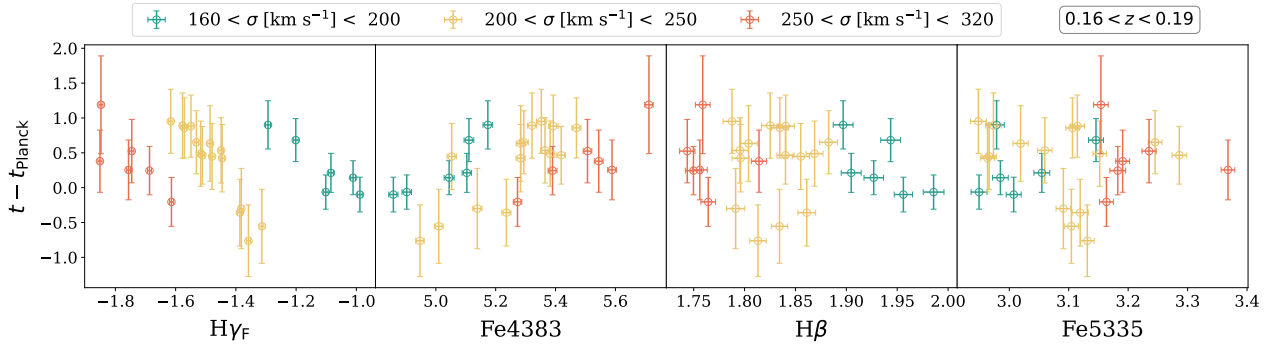


Figure 2.26: View of figure 12 of A25. Age deviation from the Planck Collaboration et al. (2020) Λ CDM trend, $t - t_{\text{Planck}}$, against the strength of $H\gamma_F$ (left) and Fe4383 (right), for the three central velocity-dispersion groups with data at $0.16 < z < 0.19$.

Having identified the $H\gamma_F$ index as one of the clearest tracers of the effect, in A25 we tracked its behaviour directly against redshift (Fig. 2.27 - direct view of figure 13 in the article), revealing a synchronous, roughly 10% oscillation with respect to the smooth global trend, shared by every velocity-dispersion group with data in the affected window. Three possible origins were tested and, in each case, ruled out. First, the stacking procedure itself: the mean index strength measured directly on the individual, unstacked galaxy spectra reproduces the oscillation just as clearly, so it cannot be an artefact introduced by the act of combining spectra. Second, the pyLick measurement pipeline: the SDSS archive independently provided the MPA/JHU measurements, and these track our own measurements almost perfectly for $z < 0.25$, ruling out a bug or bias specific to our own pipeline. Third, the well-documented non-linear jump in SDSS instrumental resolution around 5500 Å observed frame (Sec. 2.3.1): at the redshifts where the oscillation peaks, the $H\gamma_F$ and Fe4383 bandpasses fall safely within the region where σ_{IR} is still close to the MILES value, so this cannot be the direct cause either, even though other indices that do straddle the resolution jump at this redshift (Fe5015, Mg_1 , Mg_2 , Mgb , Fe5270, Fe5335) were checked and show no correlation with the

age deviation.

$H\gamma_F$ is far from the only index affected, nor is it where the effect is strongest: we reported analogous fluctuations, at varying levels of significance and at somewhat different redshifts, in $H\delta_A$, $H\beta$ and Fe4531 (Appendix D in A25, figures D.1-D.2), and, most strikingly of all, in the D4000/D4000_n break and in C₂4668 and TiO₁ (figures D.3-D.4), where the oscillation is enhanced, periodic and stable over a much wider redshift range than for any of the Balmer or iron indices. The D4000_n in particular shows a sharp, non-monotonic drop around $z \sim 0.10$ that runs directly counter to the smooth decline expected from its role as an age indicator. Having exhausted the explanations under our own control, in A25 we could only point to two remaining candidates rooted in the SDSS data reduction pipeline itself (Stoughton et al., 2002): a residual bias in the flux calibration, introduced through the interpolation of the continuum of the hot calibration stars across their own (very deep) Balmer lines; or uncorrected telluric absorption or emission features. Neither was found, in preliminary tests, to plausibly account for the bulk of the effect, and the question was explicitly left open for future work.

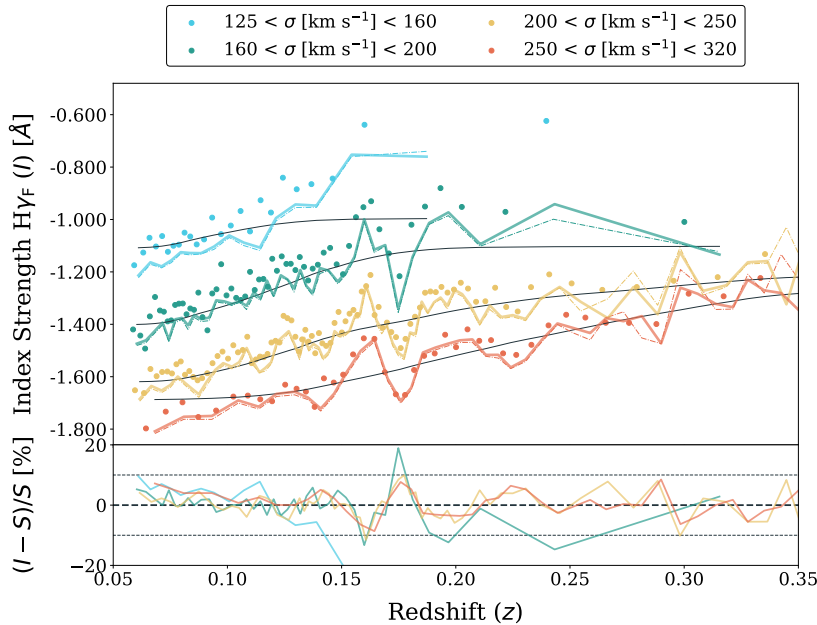


Figure 2.27: View of figure 13 in A25. $H\gamma_F$ vs z for the four central σ groups. Coloured points are the stack measurements. Solid lines of the same colour are the corresponding single-galaxy averages. Dashed lines are the SDSS MPA/JHU measurements. The bottom panel shows the fractional deviation from a smoothed moving mean, highlighting the $\sim 10\%$ synchronous oscillation around $z \sim 0.17$.

This is precisely the point at which the DESI-side spike of Fig. 2.21 becomes relevant. In A26 we performed the same diagnostic exercise used above for $H\gamma_F$ on an independent dataset, this time looking at the wiggle located around $z \approx 0.5$, and found a similar pattern (Fig. 2.28).

Taken at face value, this is a genuinely striking coincidence: two independent surveys, observed with entirely different spectrographs and reduced through entirely different pipelines (Sec. 2.1.1), each show an unexplained, synchronous oscillation in their $t - z$ relation, and in both cases the feature is traceable back to the same index. It is tempting to read this as evidence against a purely SDSS-specific origin (residual flux-calibration bias, telluric contamination), and in favour of something

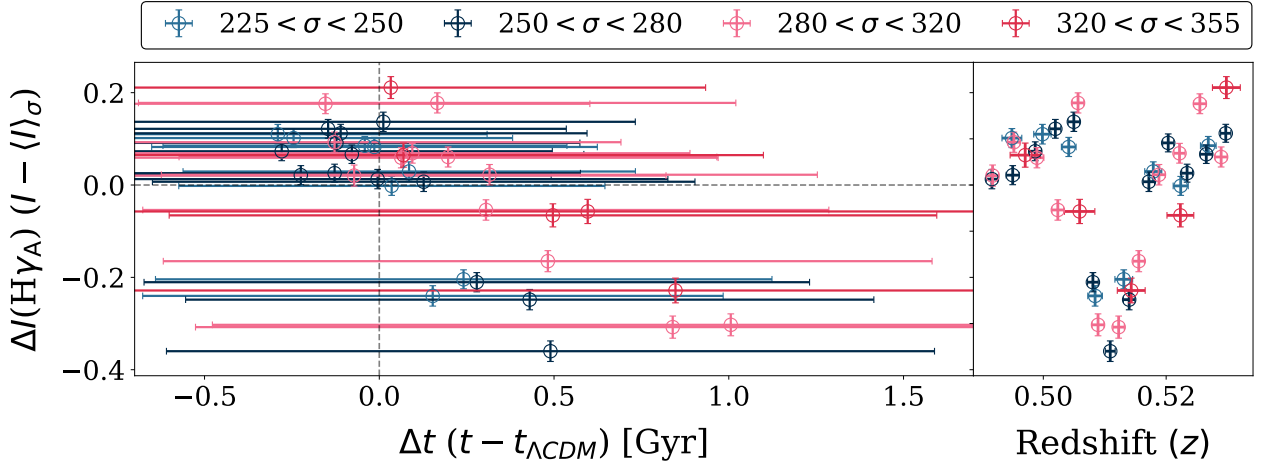


Figure 2.28: Relation of the $t - z$ wiggle at $z \sim 0.5$ with the $H\gamma_F$ from the final DESI stacks. On the left panel, the fractional deviation of $H\gamma_A$ from its local average, $(I - \langle I \rangle)/\langle I \rangle$, against the age deviation from the Λ CDM prediction, $\Delta t \equiv t - t_{\Lambda\text{CDM}}$, for the $225 < \sigma [\text{km s}^{-1}] < 250$, $250 < \sigma [\text{km s}^{-1}] < 280$, $280 < \sigma [\text{km s}^{-1}] < 320$ and $320 < \sigma [\text{km s}^{-1}] < 355$ groups. On the right panel, the same quantity against redshift, isolating the wiggle around $z \sim 0.50\text{--}0.52$.

more fundamental, whether astrophysical or tied to a more generic limitation of ground-based optical spectroscopy in this particular part of the spectrum; but we want to be careful not to overstate this. The redshift at which each survey's main (detected) oscillation appears is different ($z \sim 0.17$ for SDSS, $z \sim 0.5$ for DESI) and it happens for several indices. We regard the parallel as suggestive for future works (see Secs. 4.1 and 4.3), and we ask ourselves whether the DESI $z \sim 0.5$ wiggle, the associated metallicity pathology, and the legacy SDSS oscillation of A25 are three faces of the TMJ model limitations and (or) of the Lick-index-based CC methodology more broadly.

We close the section by noting that this systematic effect was taken into account, although not quantitatively, in both A25 and A26. We performed tests in which we could see that the overall posterior sampling of the cosmographic parameters did not perceptibly change by the (un)masking of this region. In particular, the oscillations are included within the baseline in A25, but the removal of the most affected redshift interval does not affect the cosmographic fit. In A26, on the other hand, the affected redshift window was democratically masked as the wiggle was associated with a drop in $S/N(t)$, but the by-hand inclusion of the affected data did not change the overall results of the cosmographic fit either.

Results

The previous two chapters built, piece by piece, the machinery that turns a catalog of galaxy spectra into a cosmological observable: a survey and a target selection (Sec. 2.2), a set of index measurements and their velocity-dispersion and instrumental-resolution corrections (Sec. 2.3), a stacking strategy designed to guarantee a stable, high-S/N sampling of the $t - z$ plane (Sec. 2.4), a stellar-population model that turns those indices into ages (Sec. 1.3.2, 2.5), and, finally, a cosmographic framework that turns the resulting $t - z$ relation into a constraint on $H(z)$ (Sec. 1.2.1). This chapter closes the loop: it presents the actual outcome of that pipeline as applied to the SDSS Legacy sample of A25 and to the DESI-DR1 LRG sample of A26, and it does so figure by figure, following, in essence, the results sections of the two articles.

We organise the chapter as follows. Sec. 3.1 covers the classical, $z_0 = 0$ cosmography applied to the SDSS data, including the baseline H_0 measurement, its extension to a w_0 CDM readout, the comparatively poor behaviour of the Knowles-modelled ages, and the reconstructed Hubble diagram. Sec. 3.2 moves to the pivotal-redshift cosmography applied to the DESI data, presenting the joint $\{H_{z_0}, q_{z_0}, j_{z_0}\}$ posterior, the resulting $H(z)$ sampling and its covariance matrix, and two independent, discrete $H(z)$ measurements obtained as a by-product. Sec. 3.3 turns to the archaeological (scaling relations) side of the results. At the beginning, these were motivated as an update to the existing results in the literature, using stacked galaxies (Sec. 2.4) as opposed to single galaxies (common in the literature), and after the motivation provided by the review (Knowles et al., 2023) of the stacks performed by La Barbera et al. (2013) as ideal target data. Then, we realised that, in a single computation (eq. 1.48), both the cosmography and the age scaling relation could be put together at any redshift and for any sample, and we show the results in the aforementioned section.

A word of caution on scope. Nearly every systematic effect that shapes the uncertainty budgets quoted below, from the choice of S/N level in the stacking to the archaeological and cosmological prescriptions used to cap the redshift window in A26, has already been introduced and dissected in Sec. 2.6. We do not repeat that discussion here; instead, we point back to the relevant subsection whenever a systematic contribution enters a quoted uncertainty, and reserve this chapter for the results themselves and for the physical reading of what they mean.

3.1 Classic cosmography at $z = 0$: the SDSS CCs

A25 anchors the Taylor expansion of $H(z)$ at the local Universe, $z_0 = 0$ (eq. 1.25), which is an appropriate choice given that the final spectroscopically selected SDSS sample of 90 396 sources spans $0.05 \lesssim z \lesssim 0.35$ (Sec. 2.2), safely within the radius of convergence discussed in Sec. 1.2.1. Stellar ages were derived from Lick-index fitting with two SPS models, TMJ (Thomas et al., 2011) and Knowles (Knowles et al., 2023), on stacks built to a target $\langle S/N \rangle = 300$ and organised into six logarithmically spaced velocity-dispersion bins, $[100, 125)$, $[125, 160)$, $[160, 200)$, $[200, 250)$, $[250, 320)$ and $[320, 400)$ km s^{-1} (Sec. 2.4), the last of which was excluded from the cosmographic fit once the σ_z quality cut removed every one of its data points.

3.1.1 The baseline TMJ fit

The central result of A25 comes from the joint cosmographic fit of the final five $t - z$ relations obtained with the TMJ-modelled ages, under the physical prior $dH/dz > 0$ (Sec. 1.2.1), which translates into $q(z) > -1$ at every redshift and, in particular, into $q_0 > -1$. Fig. 3.1 reproduces the outcome of this fit: on the left, the corner plot of the joint posterior for $\{H_0, q_0, j_0\}$; on the right, the five $t - z$ relations together with the 1σ band of the cosmographic model sampled from the same posterior.

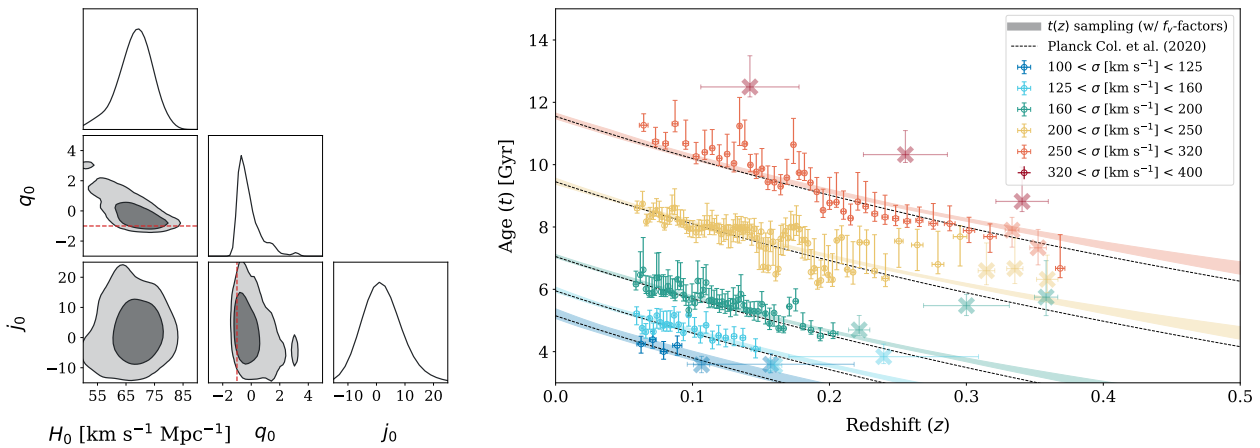


Figure 3.1: View of figure 6 in A25. On the left panel, the corner plot of the joint posterior probability distribution of H_0 , q_0 and j_0 from the cosmographic fit of the TMJ-modelled ages, with 1σ and 2σ contours. The dashed red $q_0 = -1$ lines mark the boundary imposed by $dH/dz > 0$. On the right panel, the corresponding $t - z$ dispersion, with the velocity-dispersion groups in different colours, the 1σ sampling of the posterior as shaded bands, and the Λ CDM prediction of Planck Collaboration et al. (2020) as dashed black lines, anchored at the zero-redshift age of each group.

The Hubble constant is constrained to a mean value of $H_0 = 70.0^{+4.1}_{-7.6}$ $\text{km s}^{-1} \text{Mpc}^{-1}$ (statistical uncertainty only), a credible interval that comfortably includes both the CMB-inferred value of Planck Collaboration et al. (2020) and the distance-ladder value of Riess et al. (2022) (see Fig. 3.2,

where we have added this value to Fig. 1.2, where we had presented the current status of the H_0 tension), i.e. it does not, on its own, favour either side of the Hubble tension. The constrained value is far from competitive with other sources, and it has a number of conditionings and systematic effects that should be better assessed; however, it shows that CCs can still have a big impact in the field of cosmology. This is, to our knowledge, the first H_0 estimate obtained directly from a cosmographic fit to Lick-index-derived cosmic chronometer ages, as opposed to the finite-difference approach that dominates the literature (Sec. 1.2).

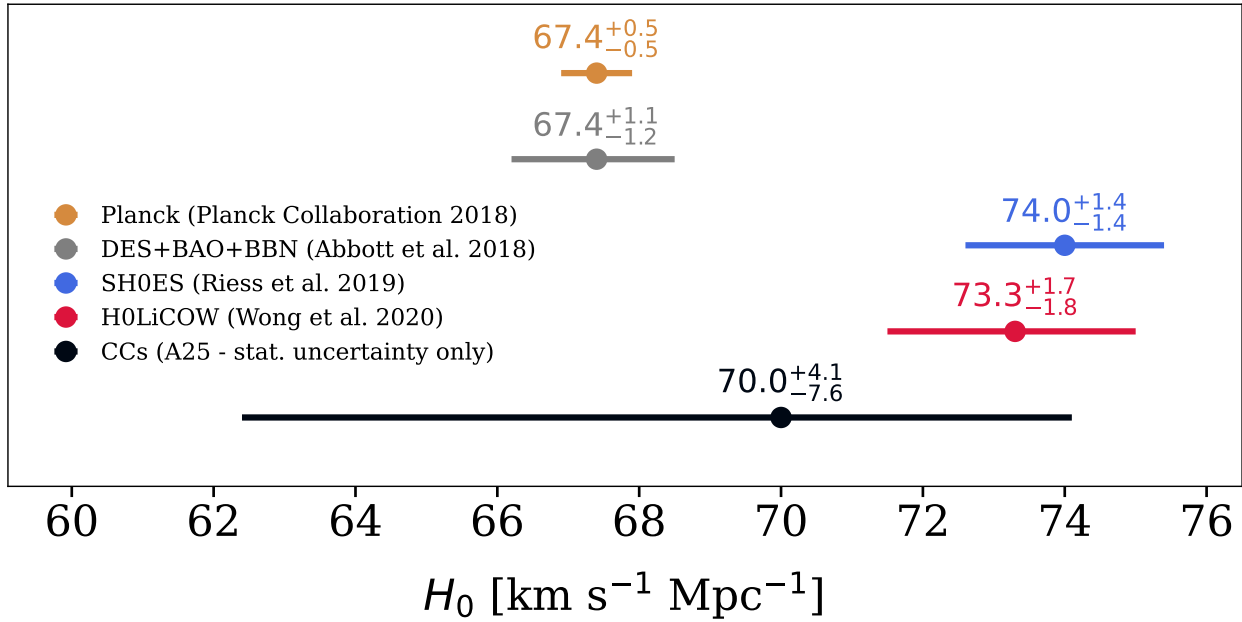


Figure 3.2: Addition of the statistical credible interval for H_0 derived from TMJ-modelled ages for SDSS stacks (A25 baseline) to the current status of the H_0 tension (Fig. 1.2).

The deceleration parameter tells a more nuanced story. Its posterior is markedly skewed against the $q_0 = -1$ boundary: the modal value is $\text{mod}(q_0) = -0.95$, to the left of the 16th percentile, $P_{16}(q_0) = -0.85$, while the 84th percentile sits at $P_{84}(q_0) = 0.62$. A direct inspection of the MCMC chain shows that 99.91% of the samples satisfy $q_0 \geq -1$, with only 0.09% straying below the boundary, a fraction attributable to the residual of the second-order Taylor truncation (Sec. 1.2.1) rather than to any genuine preference for $q_0 < -1$. We read this as evidence that the second-order expansion in $y = z/(1+z)$ is an adequate approximation over the redshift range spanned by the SDSS sample, consistent with the quantitative assessment of the truncation error presented in Fig. 1.7. The jerk parameter, by contrast, is essentially unconstrained by the fit: the posterior only closes into an approximately Gaussian shape once the prior is widened well beyond the range of physical interest, and the resulting estimate, $j_0 = 0.9^{+5.5}_{-3.7}$, is compatible with the Λ CDM value $j_0 = 1$ but carries almost no discriminating power; the window required in the prior to obtain a closed posterior spans more than ten times the Λ CDM value. Both features, the skew in q_0 and the low sensitivity to j_0 , were

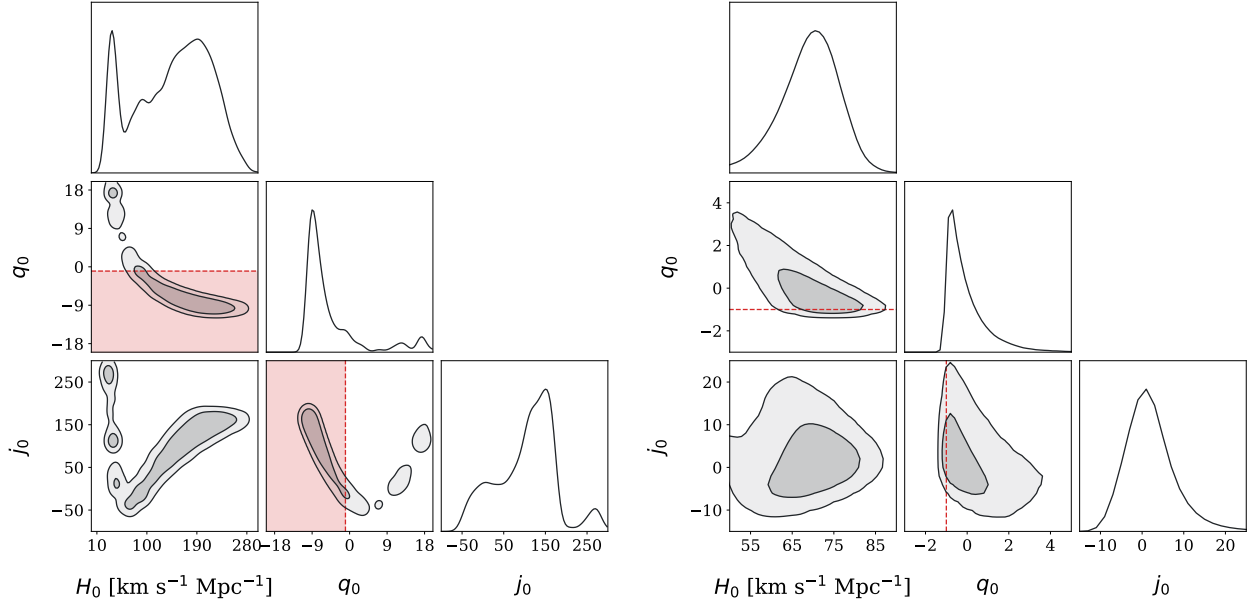


Figure 3.3: View of figure B.1 in [A25](#) (rotated: top figure in the article is now on the left, bottom figure in the article is here on the right). On the left panel, the corner plot for the cosmographic parameters after the removal of the $dH/dz > 0$ condition. The dashed red $q_0 = -1$ lines mark the forbidden region in the baseline fit. On the right panel, the corner plot for the baseline fit after the oscillating part of the $t - z$ relations ($0.15 < z < 0.20$) is masked.

anticipated in the discussion of the pivotal-redshift cosmography of Sec. [1.2.1](#), and they motivated the choice, in [A26](#), of a pivotal redshift removed from $z = 0$ (Sec. [3.2](#)).

Apart from the baseline fit, several more trials were considered at the time. In appendix B of the article, the corner plots (figure B.1 in the article; here portrayed in Fig. [3.3](#)) relative to the case in which the $dH/dz > 0$ condition is removed (left panel) and the case in which the most noticeable oscillation pattern in the $t - z$ relations present at $0.15 < z < 0.20$ is masked (right panel) are shown. The corner plots are structured as the one in Fig. [3.1](#) (left panel); but here, for clarity, in the free dH/dz case (left panel), the whole $dH/dz < 0$ region is shadowed in red, highlighting the part of the sampling forbidden by the condition on the derivative introduced in the baseline fit.

Out of all the tests considered, these two have great relevance for different reasons. The corner plot in which we removed the requirement for a growing H shows how the CCs data combined with the classic cosmographic approach anchored in $z = 0$ is not enough to break the degeneracy between q_0 and H_0 . To understand the importance of the degeneracy, a substantially widened parameter space, $0 < H_0$ [km s⁻¹ Mpc⁻¹] < 300 , $-20 < q_0 < 20$, $-100 < j_0 < 300$, was considered. The resulting marginals widen considerably, in some cases well beyond the baseline limits, and a clear anti-correlation between H_0 and q_0 appears that extends into the $q_0 < -1$ region; the minimum χ^2 of this unconstrained fit occurs at $H_0 \sim 220$ km s⁻¹ Mpc⁻¹. Nonetheless, the values found across all the

regions explored that remain compatible with Λ CDM ($H_0 \sim 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $q_0 \sim -0.5$, $j_0 \sim 1$) are consistent with the baseline result of Fig. 3.1, showing a χ^2 value only marginally higher. The lack of sensitivity could come from the lack of sources at both sides of the pivotal redshift (in this case 0), as a further test applying the pivotal redshift cosmography (Sec. 3.2) on the SDSS data managed to return a modest improvement in the sensitivity to the cosmographic parameters (Sec. 4.2). This is a further check that supports the idea that the pivotal-redshift cosmography introduced by Fazzari et al. (2025) and applied in A26 is a suitable theoretical framework for CC studies.

The corner plot on the right side tells a different story, which is a proof of the little impact the oscillations in the $t - z$ pattern have on the estimation of the cosmographic parameters. This idea was put forward in Sec. 2.6.2, but the actual proof that supports it is this figure. In A25 we alluded to the lack of sensitivity of a third order polynomial on z to fit such a localised feature as the reason why the overall fit is insensitive to it, something that was again supported in A26 by the eventual test performed by unmasking the $z \sim 0.5$ region (as mentioned in the discussion around Fig. 2.28). We can conclude that the observed $t - z$ oscillations do not have an impact on the cosmographic results in general. In other words, until the nature of the index oscillations that produce the $t - z$ wiggles is understood, data can still be used for cosmographic purposes without big concerns about the effect of these oscillations, which is a very positive prospect for future works.

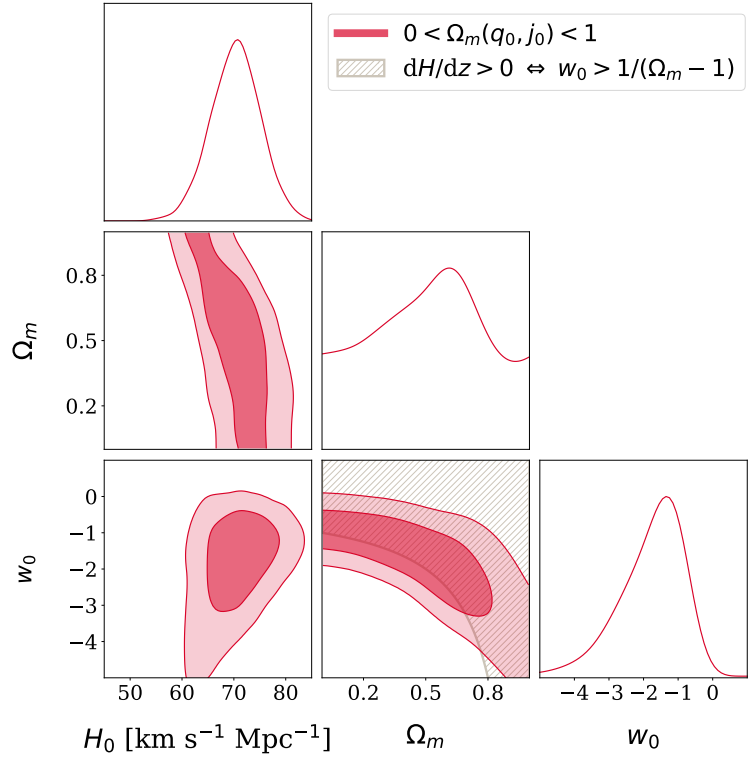
3.1.2 The w_0 CDM readout

Although the cosmographic fit itself makes no reference to any specific dark-energy model, the pair $\{q_0, j_0\}$ can be mapped onto the matter density Ω_m and the dark-energy equation-of-state parameter w_0 of a w_0 CDM cosmology, following the algebraic relations given by Capozziello et al. (2011) (see the discussion of the pivotal cosmographic framework in Sec. 1.2.1). Because the unrestricted posterior of Fig. 3.1 populates, at some level, the regions $\Omega_m > 1$ and $\Omega_m < 0$, an additional MCMC run was performed enforcing $0 < \Omega_m(q_0, j_0) < 1$, whose outcome is shown in Fig. 3.4.

Under this condition, H_0 is essentially unaffected, $H_0 = 70.7^{+4.3}_{-5.1} \text{ km s}^{-1} \text{ Mpc}^{-1}$, confirming that restricting Ω_m to the physical range mostly removes low-probability tails rather than reshaping the bulk of the posterior. Ω_m itself remains largely unconstrained, with a mild preference for central values, while the dark-energy equation of state reads $w_0 = -1.01^{+0.03}_{-1.75}$: fully compatible with a cosmological constant, but with a markedly asymmetric credible interval, tightly bounded at the edge of the phantom side ($w_0 = -1$) by the $dH/dz > 0$ condition (equivalently $w_0 > 1/(\Omega_m - 1)$, the brown hatched region of Fig. 3.4), and essentially unbound towards $w_0 \ll -1$. Only 0.19% of the MCMC steps populate the region formally excluded by the derivative condition, a fraction comparable to the 0.09% found for $q_0 < -1$ in Sec. 3.1.1 and, again, attributable to the residual of the Taylor truncation. We highlight how the requirement of a growing $H(z)$ limits the number of phantom dark-energy states available, and the failure to do so forces the fit to choose combinations

of the parameter space that represent unphysical (as a readout within the current understanding of Cosmology) solutions.

Figure 3.4: View of figure 7 in A25. Posterior probability distribution for H_0 , Ω_m and w_0 , obtained by converting the sampled q_0 and j_0 into w_0 CDM parameters through the $\Omega_m(q_0, j_0)$ and $w_0(q_0, j_0)$ relations, and enforcing $0 < \Omega_m < 1$. The brown hatched region in the $\Omega_m - w_0$ plane marks the area allowed by the requirement of $dH/dz > 0 \Leftrightarrow w_0 > 1/(\Omega_m - 1)$.



Taken together, these results show that the TMJ-modelled SDSS cosmic chronometers are, at the current level of statistical precision, blind to any meaningful departure from a cosmological constant, while still delivering a competitive and fully model-independent constraint on H_0 . Nevertheless, this is proof that the cosmographic approach is built correctly and the fitting algorithm is sensitive to kinematic parameters describing the expansion rate of the Universe, but knows nothing about dynamical interpretations.

3.1.3 The Knowles model: a cautionary tale

The same cosmographic machinery was applied to the ages estimated with the sMILES-based Knowles model (Knowles et al., 2023), under otherwise identical conditions. Fig. 3.5 shows the outcome, both for the direct application of the fit (purple) and for a modified likelihood in which an additional, per-velocity-dispersion-group scatter parameter, f_v , is introduced to account for a possibly underestimated age variance (Sec. 2.5; grey).

The direct fit yields $H_0 = 41.7^{+0.6}_{-0.6} \text{ km s}^{-1} \text{ Mpc}^{-1}$, $q_0 = -0.99^{+0.08}_{-0.01}$ and $j_0 = 55.4^{+2.6}_{-2.4}$: a value of H_0 incompatible with essentially every other probe in the literature, and posteriors on H_0 and j_0 that are implausibly narrow, several σ away from their expected values. The origin of this pathology could not be traced to the treatment of the velocity-dispersion effect (Sec. 2.3.1), which was explicitly ruled

out by refitting the Knowles model at zero velocity dispersion using the same correction-function methodology applied to TMJ, with negligible changes in the resulting ages and uncertainties. In the writing of this thesis, this was anticipated in Fig. 2.14 and the discussion therein. We could only possibly allude to the intrinsic narrowness of the Knowles-modelled age uncertainties themselves, found to be systematically smaller than the TMJ ones by a factor of $\sim 6 - 7$, coupled with an intrinsic scatter of the ages that the cosmographic model reacts to as an anomaly: the minimum χ^2 of the fit is three orders of magnitude larger than in the TMJ case (Fig. 3.1).

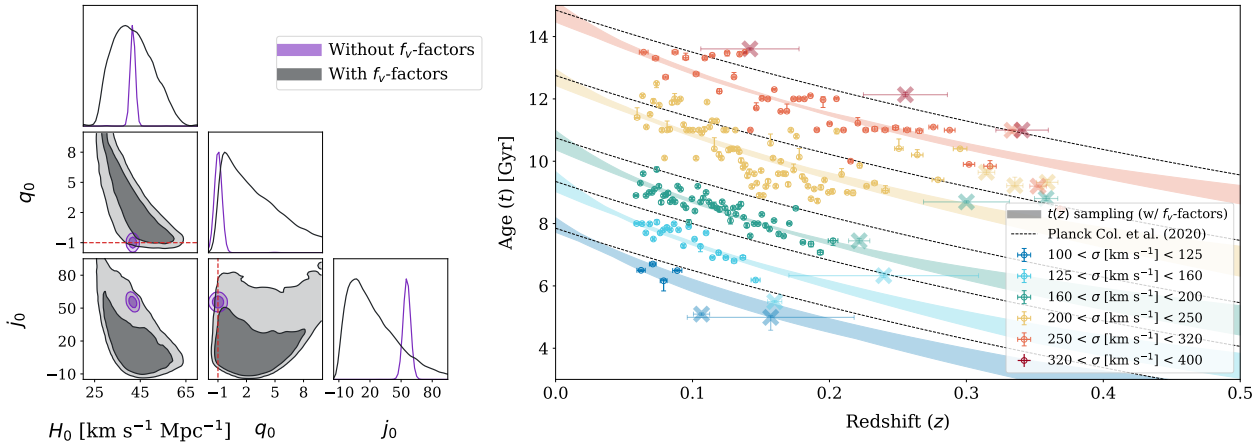


Figure 3.5: View of figure 8 in A25. $t - z$ relations for the Knowles-modelled ages, with the same structure as Fig. 3.1. The purple contours and posteriors correspond to the direct fit; the grey contours and posteriors include the f_v factors (Sec. 2.5) that inflate the per-group variance to account for the underestimated Knowles age uncertainties.

Introducing the f_v factors partially, but not fully, remedies the situation: $H_0 = 38.2^{+12.9}_{-5.1}$ km s⁻¹ Mpc⁻¹ remains incompatible with other probes at the 2σ level, although the widened posterior now reaches a 3σ agreement with $H_0 \sim 70$ km s⁻¹ Mpc⁻¹; the average minimum χ^2 over the last thousand MCMC steps returns to the order of magnitude found for the TMJ fit. The deceleration parameter remains skewed towards $q_0 = -1$ ($\text{mod}(q_0) \sim -0.9$, $P_{16}(q_0) = -0.17$), now with the positive tail extending to the prior boundary, while the jerk parameter becomes only formally constrained, $j_0 = 5.8^{+42.6}_{-5.1}$, compatible with $j_0 = 1$ within 1σ thanks to the extra freedom afforded by f_v . We conclude, in agreement with A25, that the Knowles-modelled ages should be used with caution for cosmographic purposes until the source of this narrow-uncertainty, high-scatter behaviour is better understood (Sec. 2.6.2); for the remainder of the chapter, unless explicitly stated, all quoted A25 results refer to the TMJ baseline.

The physical interpretation that we give several months after these results is the need for an in-depth analysis of the set of Lick indices that should be considered for fits with the Knowles model. We observe that our measurement does not incur in catastrophic errors in the selection of indices or the

overall performance. If we try to give credit to what we see, it might be reasonable to find a low H_0 value given that locally close to $z = 0$ the slope of the $t - z$ relation seems steeper than Λ CDM. From the several tests that were performed prior to the publication of A25, no better answer was found, other than the fact that an artificial (unphysical) resizing of the uncertainties could alleviate the tension.

In A25, the SPS model fit was performed independently of the only published parameter estimations available, the Knowles et al. (2023) fit on the La Barbera et al. (2013) stacks for the purpose of estimating scaling relations. Results for this are shown in figure 11 (right panel) in A25 (here portrayed as Fig. 3.11), where we observe no relevant differences in the fiducial values, although no uncertainties to compare with were provided by the authors. In parallel with the selection of Lick indices, a refined version of the index-index diagrams proposed by the authors of the model (Fig. 1.12 and the corresponding discussion in the main text) could represent a solution to the problems described earlier; in particular under the hypothesis that fewer indices to fit against could produce wider age posteriors. In Sec. 4.1 a sketch idea is described.

3.1.4 The reconstructed Hubble diagram

The posterior of Fig. 3.1 can be propagated directly into a continuous sampling of $H(z)$ through eq. (1.25), providing a direct functional reconstruction of the cosmographic Hubble diagram built from cosmic chronometers, and using only Lick indices in the process. Fig. 3.6 shows the result, together with the systematic uncertainty contribution discussed in Sec. 2.6.1 and a compilation of model-independent $H(z)$ measurements from the literature up to $z = 0.9$.

Over the redshift range where our cosmographic reconstruction is best sampled, $z \lesssim 0.35$, the statistical and combined uncertainty bands are competitive with, and in some cases narrower than, the FSF-based points, though somewhat wider than the handful of D4000_n-based local measurements available at similar redshifts (Sec. 1.2). The systematic uncertainty, shown in Sec. 2.6.1 (Fig. 2.22), is stable across the redshift range and roughly twice the size of the statistical uncertainty, with the target S/N level of the stacks dominating at low redshift and the σ_z exclusion threshold becoming relatively more important towards the high-redshift end of the sample. The reconstruction naturally loses constraining power beyond $z \gtrsim 0.3$, where the number of contributing sources (encoded in the opacity of the band in Fig. 3.6) drops. This is precisely the regime that motivated the extension of the same methodology to DESI data in A26 (Sec. 3.2).

Notably, the Λ CDM prediction of Planck Collaboration et al. (2020) sits comfortably within the 1σ credible interval of our reconstruction across the whole redshift range probed. This underlines once again that, while the SDSS cosmic chronometers do not yet have the statistical leverage to discriminate between competing cosmologies, they provide an entirely independent and consistent

cross-check of the standard model over $0.05 \lesssim z \lesssim 0.35$.

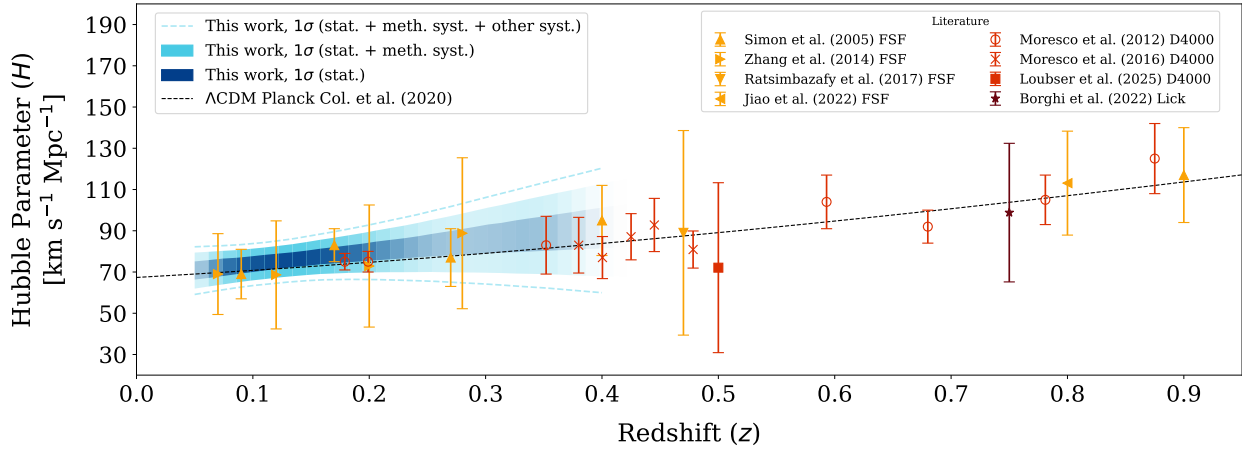


Figure 3.6: View of figure 9 in A25. Reconstruction of $H(z)$ from the posterior of Fig. 3.1, shown in shades of blue: the darker band gives the 16th–84th percentile statistical interval, the lighter band adds the systematic contribution of Sec. 2.6.1 in quadrature. The dashed black line is the Λ CDM prediction of Planck Collaboration et al. (2020). Coloured points are independent literature measurements obtained through full spectral fitting (yellow; Simon et al. 2005; Zhang et al. 2014; Ratsimbazafy et al. 2017; Jiao et al. 2023), the D4000_n feature (orange; Moresco et al. 2012, 2016; Loubser et al. 2025) and Lick indices (dark red; Borghi et al. 2022a).

3.2 Pivotal-redshift cosmography at $z_0 \approx 0.57$: the DESI cosmic chronometers

A26 repeats the exercise of Sec. 3.1 on a substantially larger and higher-redshift dataset: 527 287 spectroscopically selected DESI-DR1 LRGs, stacked into five logarithmically spaced velocity-dispersion groups, [200, 225), [225, 250), [250, 280), [280, 320) and [320, 355) km s^{−1} (the [355, 400) km s^{−1} group was dropped after showing an anomalously small age offset with respect to the [320, 355) km s^{−1} group; Sec. 2.2), stacked to a target S/N ≈ 150 (Sec. 2.4), and cleaned of the low-age pathology of the TMJ model through the archaeological coherence cut introduced in Sec. 2.6.1 (Fig. 2.19). Only the TMJ model was used for the stellar-parameter fit, given the reservations raised for the Knowles model in Sec. 3.1.3.

3.2.1 The $t - z$ relations across velocity-dispersion groups

Fig. 3.7 shows the final $t - z$ relations that enter the cosmographic fit, after the quality flags of Sec. 2.6.1 have been applied ($S/N(z) > 50$, $\sigma_z < 0.01$, $S/N(t) > 5$, $\sigma_t < 1$ Gyr).

The five groups trace an approximately parallel, monotonically decreasing sequence of ages with

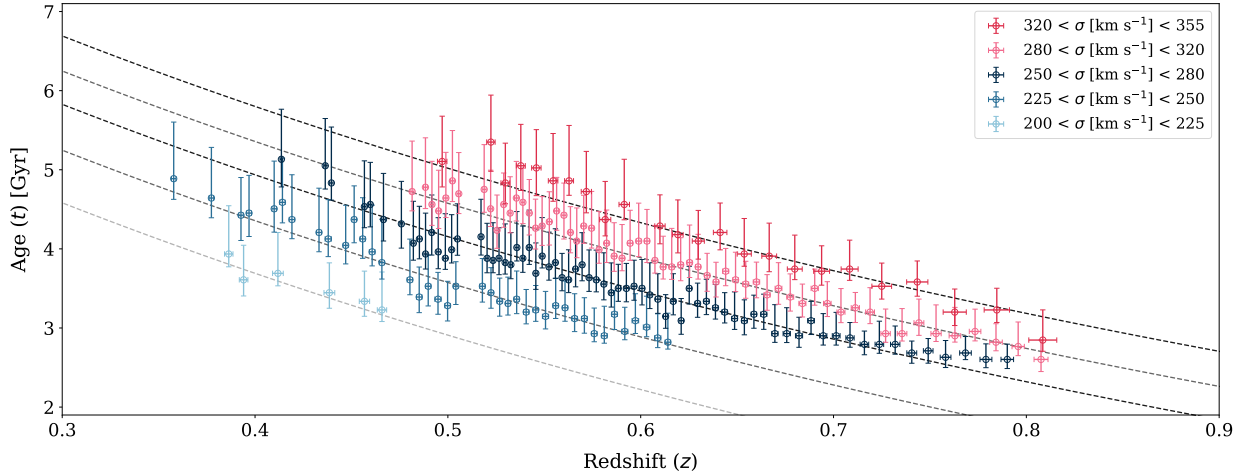


Figure 3.7: View of figure 3 in A26. Final $t - z$ relations for the five velocity-dispersion groups of the DESI sample, after all quality flags of Sec. 2.6.1 have been applied. Dashed lines show the Λ CDM prediction of Planck Collaboration et al. (2020), anchored at the pivotal-redshift age of each group as recovered by the baseline cosmographic fit (Fig. 3.8). Blue and red tonalities mark the super-groups later combined into the two local, finite-difference $H(z)$ measurements of Sec. 3.2.4.

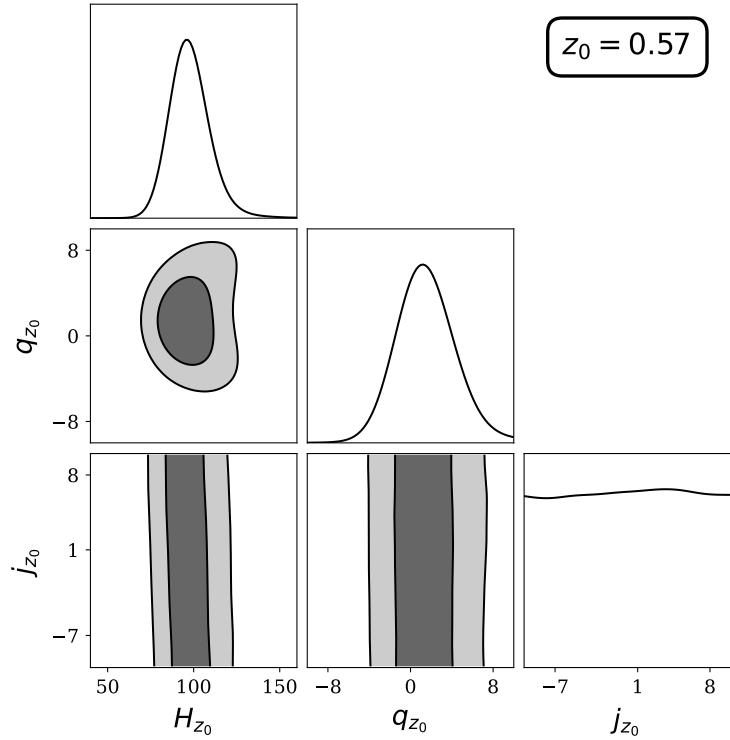
redshift, in line with the cosmic-chronometer premise, and are visibly offset from one another according to velocity dispersion: more massive galaxies are systematically older at any given redshift, precisely the downsizing signature already discussed in Sec. 1.3.3 and quantified further in Sec. 3.3. The theoretical Λ CDM curves of Planck Collaboration et al. (2020), anchored at the pivotal-redshift age of each group, run comfortably through the data over the whole $0.36 \lesssim z \lesssim 0.80$ window, already anticipating the compatibility of the cosmographic $H(z)$ reconstruction with the standard cosmology (Sec. 3.2.3). As discussed in Sec. 2.6.2, the removal of the quality-flagged data leaves a conspicuous gap in the $0.49 \lesssim z \lesssim 0.53$ window, directly traceable to the $z \sim 0.5$ index-strength oscillation (Fig. 2.28). Additional tests (as commented before) have shown that the cosmographic results would remain unchanged even if all sources in this gap were to be included by-hand (contrary to the democratic choice by which we have decided to remove them). We note that such a case would be the one symmetric to the baseline in A25, where the biggest oscillating patterns were included in the fit.

3.2.2 The joint cosmographic fit

Given the higher-redshift coverage of the DESI sample, the pivotal-redshift cosmography of Fazzari et al. (2025) (eq. 1.48, Sec. 1.2.1) is adopted instead of the classic, $z_0 = 0$ expansion of Sec. 3.1. The pivot is fixed to the median redshift of the sample of stacks, $z_0 \approx 0.57$, and the joint fit of the five $t - z$ relations of Fig. 3.7 to the pivotal cosmographic model (eq. 1.48 truncated at second order in $z - z_0$) yields the posterior shown in Fig. 3.8.

The Hubble parameter at the pivot is constrained to $H_{z_0=0.57} = 95.1^{+10.9}_{-6.0}$ (stat.) $\text{km s}^{-1} \text{Mpc}^{-1}$, with

Figure 3.8: View of figure 4 in A26. Posterior probability distribution for the pivotal-redshift cosmographic parameters $\{H_{z_0}, q_{z_0}, j_{z_0}\}$ from the joint fit of all velocity-dispersion groups in Fig. 3.7, at $z_0 = 0.57$.



an additional systematic-level uncertainty of $\pm 11.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Sec. 2.6.1) if the entirety of the systematic budget in $H(z = z_0)$ is attributed to H_{z_0} alone. Unlike in A25 (Sec. 3.1.1), no $dH/dz > 0$ prior is imposed here. At the time of writing, it was argued that the increased stability of the DESI $t - z$ relations and their improved sensitivity to q_{z_0} (Sec. 2.5) possibly allowed this constraint to be dropped altogether; however, we discovered recently (as commented before) that having sources on both sides could be the real driver in improving the utility of the cosmographic approach applied to CCs (see Sec. 4.2). The deceleration parameter offers modest, but genuine, constraining power with respect to its prior, $q_{z_0=0.57} = 1.0^{+2.7}_{-1.8}$, a value that, taken together with the visibly quasi-linear behaviour of the $t - z$ relations in Fig. 3.7, already hints that higher-order kinematic terms are only weakly probed by the data. The jerk parameter offers no information at all under the standard prior; only when the prior is artificially widened to $\sim \pm 200$ does the posterior close into an approximately Gaussian shape, with a 1σ credible interval between -67 and 51 and a maximum-a-posteriori (MAP) estimate that varies erratically, from ~ 0 to ~ 5 , across different runs of the MCMC algorithm, underscoring that j_{z_0} should be regarded as effectively unconstrained by this dataset, exactly as was the case for j_0 in A25 (Sec. 3.1.1).

3.2.3 The reconstructed $H(z)$ and its covariance

Propagating the posterior of Fig. 3.8 through the pivotal cosmographic expansion produces the continuous $H(z)$ reconstruction of Fig. 3.9, populated over the entire redshift window sampled by the DESI cosmic chronometers.

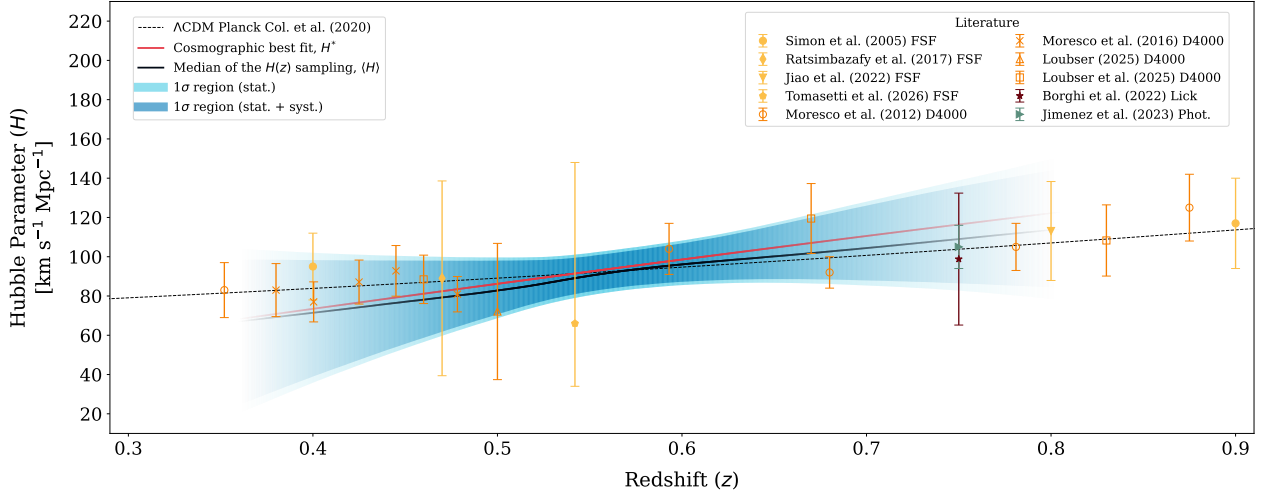


Figure 3.9: View of figure 5 in A26. Hubble diagram populated with the $H(z)$ reconstruction from the pivotal-redshift cosmographic fit (Fig. 3.8): darker blue shows the statistical 1σ region, lighter blue adds the systematic contribution of Sec. 2.6.1. The solid navy line is the median of the posterior sampling at each redshift, $\langle H \rangle$; the solid red line is the maximum-a-posteriori (MAP) solution, $H^*(z) \equiv H(z; z_0; H_{z_0}^*, q_{z_0}^*, j_{z_0}^*)$. The dashed line shows the ΛCDM prediction of Planck Collaboration et al. (2020). Literature points are shown as in Fig. 3.6, adding the recent measurements by Loubser et al. (2025) and Loubser (2025), the photometric measurement of Jimenez et al. (2023) and the full-spectral-fitting point of Tomasetti et al. (2026).

The credible band is, unsurprisingly, tightest in the vicinity of the pivotal redshift, $z_0 \simeq 0.57$, where its width is comparable to that of precision-level D4000-based measurements (Moresco et al., 2012; Loubser et al., 2025), and it fades towards both the low- and high-redshift edges of the sample, as an increasingly small number of stacks contributes to the fit there (the opacity of the shaded band in Fig. 3.9 is scaled accordingly). The only other CC measurement in this redshift range derived from Lick-index observations, Borghi et al. (2022a) at $z \sim 0.7$, has a credible interval of comparable width to our own reconstruction when extrapolated to that redshift, a useful external cross-check of the precision level attainable from this methodology. As anticipated already by the good visual agreement between the $t - z$ relations and the ΛCDM curves in Fig. 3.7, the dashed ΛCDM prediction of Planck Collaboration et al. (2020) runs consistently within the 1σ band of Fig. 3.9 across the whole redshift range, again providing no evidence against the standard cosmology from an entirely independent, late-time, model-agnostic probe. We also note that the median curve, $\langle H \rangle$, is differentially flatter than the MAP curve, $H^*(z)$, at $z \gtrsim z_0$, which is a signature of the residual, mild non-Gaussianity in the joint posterior (in particular in j_{z_0} ; Sec. 3.2.2) rather than of any pathology in the fit itself.

Statistical and systematic uncertainty budgets are of comparable order across the sampled redshift range (Sec. 2.6.1), which limits the increase in the total uncertainty (light blue band) with respect

to the purely statistical one (dark blue band); the pre-SPS systematic contributors (target S/N level and the archaeological/cosmological prescriptions, Sec. 2.6.1) are found to be somewhat more significant than the post-SPS quality-flag contributors, with the S/N-level choice dominating over the archaeological prescription by a ratio of roughly 3 : 2.

Because the pivotal-redshift reconstruction is intrinsically coupled through the three global parameters $\{H_{z_0}, q_{z_0}, j_{z_0}\}$, rather than providing independent point estimates at each redshift, A26 supplements the continuous band of Fig. 3.9 with a discrete, ready-to-use data product: an array of 45 $H(z)$ values sampled from $z = 0.36$ to $z = 0.80$ in steps of $\Delta z = 0.01$, together with its full covariance matrix $\mathbf{C}_H = \mathbf{C}_{\text{stat}} + \mathbf{C}_{\text{syst}}$, where $\mathbf{C}_{\text{stat}}$ is built directly from the MCMC chain (Sec. 2.5) and $\mathbf{C}_{\text{syst}}$ models the systematic contribution as fully correlated between redshift bins, $C_{\text{syst},ij} = \sigma_{\text{syst}}(z_i)\sigma_{\text{syst}}(z_j)$. This product is, to our knowledge, the first fully covariant $H(z)$ dataset released by a Lick-index cosmic chronometer study. It is meant to allow future analyses to fold the correlated nature of the reconstruction directly into a matrix-based likelihood ($\chi^2 = (\mathbf{H}_{\text{test}} - \mathbf{H}_{\text{obs}})^T \mathbf{C}_H^{-1} (\mathbf{H}_{\text{test}} - \mathbf{H}_{\text{obs}})$), to be used together with other independent measurements for cosmological reconstruction purposes (e.g. Zheng et al., 2016; Verde et al., 2017; Sultana et al., 2022; Gómez-Valent et al., 2024) or the calibration of other observables (e.g. Favale et al., 2023, 2026).

3.2.4 Local, discrete $H(z)$ measurements

The same quality flags that shape the $t - z$ relations of Fig. 3.7 incidentally leave each velocity-dispersion group covering a slightly different window of the overall redshift range (a direct consequence of the archaeological coherence cut of Sec. 2.6.1, Fig. 2.19), so that velocity dispersion becomes, in practice, an indirect tracer of redshift. A26 exploits this to build two independent, physically distinct measurements of $H(z)$, obtained not from the pivotal cosmographic model but from the classic finite-difference approximation, $H(z_0) \approx -\Delta z / [(1 + z_0)\Delta t]$ (eq. 1.22), applied separately to two super-groups: the three lower-mass bins, [200, 225), [225, 250) and [250, 280) km s^{-1} (blue in Fig. 3.7), and the two higher-mass bins, [280, 320) and [320, 355) km s^{-1} (red). Fig. 3.10 shows the outcome, compared with the same literature compilation used in Fig. 3.9. We note that three (blue groups) and two (red groups) average ages at the respective average redshifts had to be computed as nuisance parameters, and the whole cosmographic variability is encoded into a single parameter, the (blue and red) slope(s) of the $t - z$ relations, that is the inverse of the local Hubble parameter.

The two measurements read $H(z \approx 0.55) = 104.5_{-7.6}^{+13.2}$ (stat.) ± 22.4 (syst.) $\text{km s}^{-1} \text{Mpc}^{-1}$ and $H(z \approx 0.61) = 88.5_{-12.6}^{+6.7}$ (stat.) ± 8.1 (syst.) $\text{km s}^{-1} \text{Mpc}^{-1}$, with standard deviations of the underlying redshift distributions of $\sigma_{z;z_0 \approx 0.55} = 0.10$ and $\sigma_{z;z_0 \approx 0.61} = 0.09$, comparable to the redshift intervals typically used in finite-difference CC measurements from the literature (e.g. Ratsimbazafy et al., 2017; Borghi et al., 2022b; Jiao et al., 2023; Loubser et al., 2025). Both posteriors are mildly skewed towards lower H values, a consequence of the non-linear mapping between the prior on the slope $m \equiv dt/dz$

and the sampled H , although this might not be clear in plain sight. The systematic budget for these local measurements includes a third contribution, absent from the continuous reconstruction of Sec. 3.2.3, that accounts for the departure of the true $t - z$ trend from linearity across the (comparatively broad) redshift window of each super-group. This contribution is subdominant in both cases, of order $\sim 3 \text{ km s}^{-1} \text{ Mpc}^{-1}$, confirming that the linear approximation implicit in the finite-difference estimator is adequate given the size of the redshift bins involved. Note that even if this could seem inconsistent with respect to the choice of three cosmographic parameters in the joint cosmographic fit, the redshift range of the total sample is somewhat larger than each of these two.

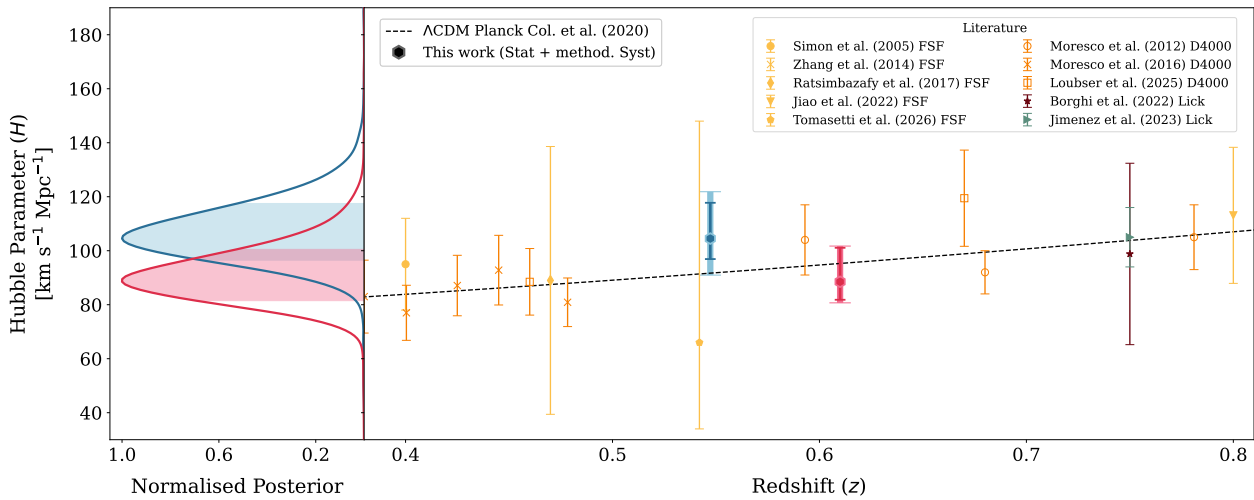


Figure 3.10: View of figure 7 in A26. Literature Hubble diagram (as in Fig. 3.9, restricted to $z > 0.38$) with the two local, finite-difference $H(z)$ measurements of this work added: in blue, the joint fit of the $[200, 225)$, $[225, 250)$ and $[250, 280) \text{ km s}^{-1}$ groups; in red, the joint fit of the $[280, 320)$ and $[320, 355) \text{ km s}^{-1}$ groups. The left-hand side of the panel shows the shape of the normalised posterior of H for each measurement, with the shaded region marking its 1σ statistical interval. On the right-hand side the total uncertainty estimated is represented in lighter colours after the quadratic addition of the systematic uncertainty.

We stress, following A26, that the two measurements are statistically independent of one another, but come from physically different populations (massive versus super-massive passively evolving ETGs). Therefore, they should not be read as two points along a redshift evolution of the same observable. We developed this local estimation as a further check that the reason why the j_{z_0} parameter was unconstrained in Fig. 3.8 is that possibly a third order polynomial has more degrees of freedom than those needed to explain the $t - z$ data. Then, we realised that a point-like measurement (in the style of those provided in the literature) could be of great interest to more complex works in Cosmology that make use of independent CCs measurements (as, for example, those cited at the end of the previous section).

We draw the attention of prospective users of this result to the fact that the two points represent physically different objects. Therefore, according to the hypothesis that the best CC candidates are the reddest and most massive passively evolving galaxies, the measurement derived from the red $t - z$ relations constitutes the best example of an independent CC-based H estimate.

3.3 Stellar archaeology: scaling and dispersion relations

Beyond their cosmographic use, the stellar ages, metallicities and α -enhancements estimated along the way (Sec. 2.5) carry independent astrophysical information about the assembly history of massive ETGs, encapsulated in the well-known scaling relations between these quantities and the velocity dispersion, σ , a proxy for the depth of the galactic potential well and, ultimately, for stellar mass (Sec. 1.3.3). In A25 we exploit the low-redshift data to derive these relations, which then became useful for works on galaxy evolution like Bosi et al. (2025). In that work, there was a need for a dispersion estimation of the relations, for which we performed a bootstrap algorithm on the moving mean. Ideally, more sophisticated ideas regarding the stacking procedure and variations of it could also help obtain an estimation of the dispersion relation.

In A26 this side-project was left aside as DESI does not target low-redshift LRGs. However, we realised that the pivotal-redshift intercepts of each velocity dispersion average age could be leveraged to obtain a scaling relation. This was not the main objective of the work, so the whole cosmographic theoretical model architecture was not designed for an optimised study of the age scaling relation. Only at the final steps of the work we did notice that a combined cosmographic and archaeological approach could be undertaken, in the form presented in eq. (1.58). Incidentally, this form actually reduces the number of parameters of the model, bringing it down to a fixed number of two archaeological parameters (a single pivotal-redshift age for a selected velocity dispersion group, t_{ref} , and the slope of the relation, a_t) and the number of cosmographic parameters wanted.

3.3.1 SDSS scaling relations

In A25 we isolated the $0.05 < z < 0.10$ end of its sample, 18955 galaxies ($\sim 21\%$ of the final sample), and restacked them in groups of 400 galaxies ordered by velocity dispersion and regardless of redshift. In this way, we traced a smooth relation from $\sigma \sim 100$ to $\sigma \sim 350 \text{ km s}^{-1}$, finer than the six bins used for the cosmographic fit. Fig. 3.11 shows the resulting age, metallicity and α -enhancement trends for both the TMJ (left) and Knowles (right) models.

All three parameters run as expected with velocity dispersion, more massive ETGs are systematically older, more metal-rich and more α -enhanced, the classic downsizing signature already introduced qualitatively in Sec. 1.3.3, and the log-log linear fits summarised in Table 3.1 quantify this behaviour for both SPS models. A caveat applies at the massive end, $\log \sigma \gtrsim 2.3$: the metallicity posteriors

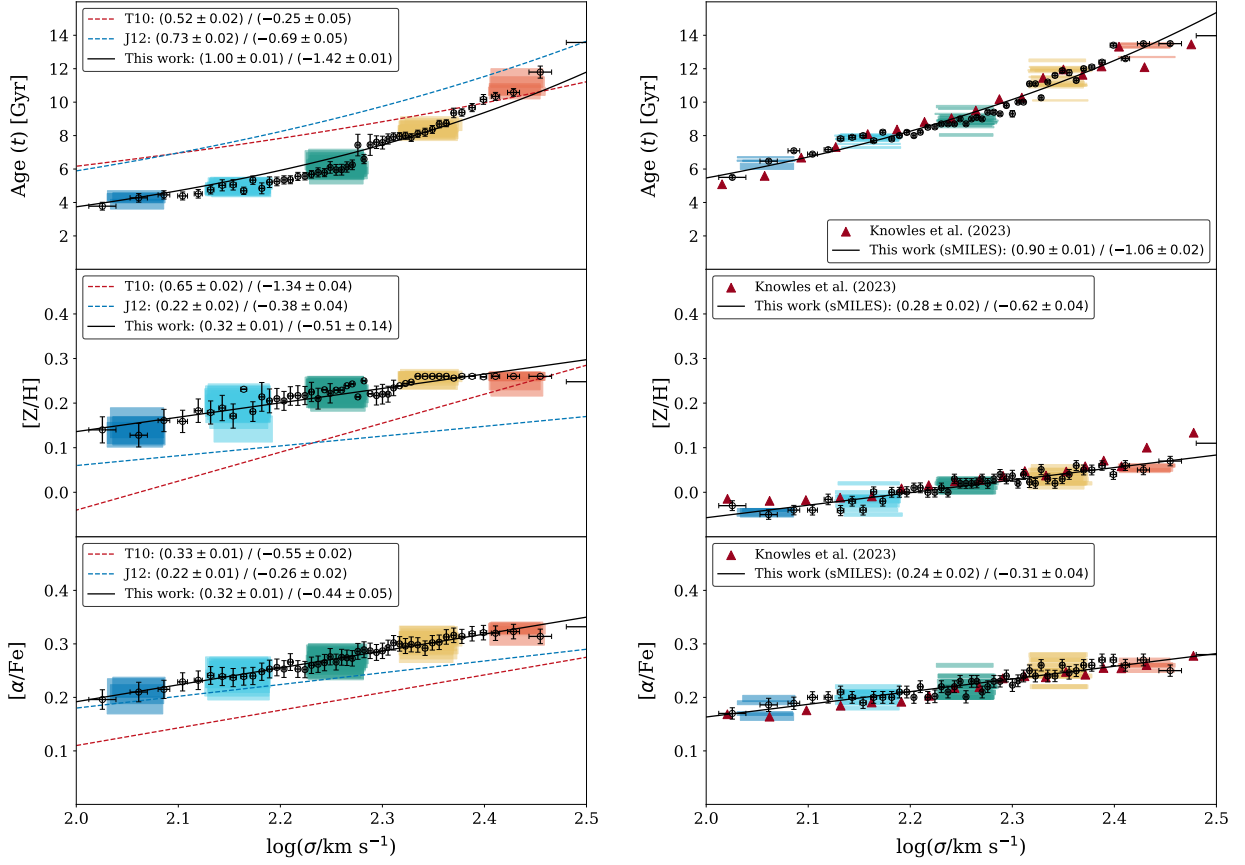


Figure 3.11: View of figure 11 in A25. Age, metallicity and α -enhancement trends with velocity dispersion at low redshift ($0.05 < z < 0.10$), for the TMJ (left) and Knowles (right) models. Black points are the results from the fine-binned (400-galaxy) stacks; coloured rectangles reproduce, for comparison, the results from the coarser velocity-dispersion bins used in the cosmographic analysis (Fig. 3.1) for stacks with average $z < 0.10$. Dashed lines in the left panel reproduce the trends of Thomas et al. (2010) (red) and Johansson et al. (2012) (blue); the right panel compares to Knowles et al. (2023) results over La Barbera et al. (2013) stacks (red triangles).

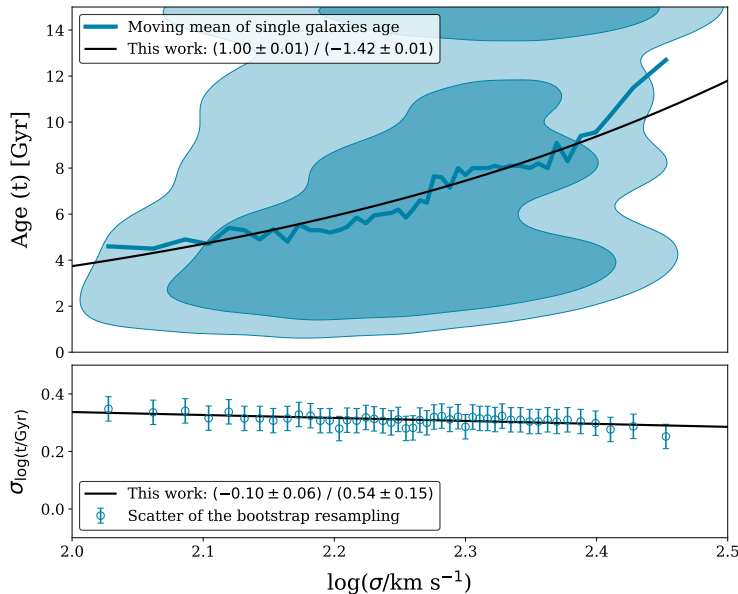


Figure 3.12: View of the upper left panel in figure C.1 in A25, representing the scaling (upper subpanel) and dispersion (lower subpanel) age relations. In the upper panel the central 68% and 95% density contours for the single galaxies show the $t - \sigma$ plane pathologies presented in Fig. 2.18, the moving mean and the best-fit scaling relation are shown overlaid to the contours.

saturate against the upper edge of the prior imposed in the SPS fit ($[Z/H] < 0.26$, Sec. 2.5, 2.6.2), and relaxing this bound reintroduces the classic age-metallicity degeneracy (Worthey et al., 1994; Chavez & Bertone, 2011; Barone et al., 2018), breaking the otherwise smooth increase of age with σ ; the hard prior is therefore retained throughout.

We observe that the best fit scaling relation (thick black line in both cases) almost perfectly matches the trend for the average ages of the single galaxies obtained from the minimum χ^2 estimation of the parameters. This can be observed in figure C.1 in A25, where we had presented the scaling and dispersion trends for the parameters and the two models, and where in the upper panel we show the moving mean in the parameter- σ plane along with the functional shape of the corresponding relation. In this thesis, we show one of these subplots for reference in Fig. 3.12: the one relative to the $t - \sigma$ relation. In the background of the figure, one can see the aspect of the single-galaxy age-estimation contour plots, representing the 68% and 95% central densities. Some aspects mentioned in the previous sections are present and merit a comment, in particular the number of sources hitting the upper limit of the prior or the young $\sim 2 - 4$ Gyr mode (see Fig. 2.18 and discussion therein).

	TMJ	Knowles
$\log(t)$	$(-1.42 \pm 0.01) + (1.00 \pm 0.01) \log(\sigma)$	$(-1.06 \pm 0.02) + (0.90 \pm 0.01) \log(\sigma)$
$[Z/H]$	$(-0.51 \pm 0.14) + (0.32 \pm 0.01) \log(\sigma)$	$(-0.62 \pm 0.04) + (0.28 \pm 0.02) \log(\sigma)$
$[\alpha/Fe]$	$(-0.44 \pm 0.05) + (0.32 \pm 0.01) \log(\sigma)$	$(-0.31 \pm 0.04) + (0.24 \pm 0.02) \log(\sigma)$
$\sigma_{\log(t)}$	$(0.54 \pm 0.15) + (-0.10 \pm 0.06) \log(\sigma)$	$(1.82 \pm 0.01) + (-0.73 \pm 0.01) \log(\sigma)$
$\sigma_{[Z/H]}$	$(0.87 \pm 0.07) + (-0.28 \pm 0.03) \log(\sigma)$	$(1.12 \pm 0.03) + (-0.43 \pm 0.01) \log(\sigma)$
$\sigma_{[\alpha/Fe]}$	$(0.52 \pm 0.03) + (-0.19 \pm 0.01) \log(\sigma)$	$(0.72 \pm 0.04) + (-0.27 \pm 0.02) \log(\sigma)$

Table 3.1: Scaling (upper block) and dispersion (lower block) relations for the stellar population parameters as a function of velocity dispersion, for the TMJ and Knowles models, reproduced from Table 1 of A25.

Quantitative differences with respect to the earlier, single-galaxy-based studies of Thomas et al. (2010) and Johansson et al. (2012) are apparent in Fig. 3.11: our TMJ ages are systematically lower than those of Johansson et al. (2012), by a roughly constant offset of ~ 2 Gyr, and lower than those of Thomas et al. (2010) at the low- σ end, while our metallicities and α -enhancements run somewhat higher. A25 attributes this, at least in part, to three methodological differences: the older TMJ precursor model used by Thomas et al. (2010) (Thomas et al., 2003b; Thomas et al., 2004), which was not flux-calibrated; their reliance on morphologically, rather than spectroscopically, selected single-galaxy spectra, as opposed to our high- $\langle S/N \rangle$ stacks of a spectroscopically purified sample (Sec. 2.2.2). Also, there is an important difference in the set of Lick indices employed in both Thomas et al. (2010) and Johansson

et al. (2012) SPS fits (Sec. 2.5). The qualitative downsizing picture, however, remains unchanged across all these choices. The Knowles-model comparison with Knowles et al. (2023) (right panel of Fig. 3.11), obtained on an independent stack of massive ETGs originally assembled by La Barbera et al. (2013), shows remarkable quantitative agreement, even as the Knowles-modelled ages themselves run systematically ~ 2 Gyr higher than the TMJ ones across the full velocity-dispersion range, mirroring the tension already identified in the cosmographic context of Sec. 3.1.3.

Two conclusions follow. First, despite the individually unreliable, highly scattered nature of single-galaxy Lick-index fits, already anticipated in Sec. 2.5 and quantified in Fig. 3.12 by a broad, occasionally bimodal density of points, their median tracks the stack-based scaling relation remarkably well, an independent confirmation that the stacking procedure of Sec. 2.4 recovers the correct population-level behaviour rather than an artefact of the averaging itself. Second, the intrinsic dispersion in age (and hence in formation time) decreases with velocity dispersion for both SPS models, consistent with the physical expectation that the most massive local quiescent galaxies assembled in a more synchronous, shorter burst of star formation than their lower-mass counterparts (Sec. 1.3.3); the Knowles-modelled age dispersion, however, drops more abruptly at the high- σ end, a direct consequence of the systematically higher (and less variable) ages that the model assigns to the most massive galaxies. These relations, reported for the first time in this context, provide an analytical recipe for the formation-time distribution of the progenitors of local quiescent galaxies that can feed, for instance, studies of the assembly history of massive ellipticals (Bosi et al., 2025).

3.3.2 An indirect age scaling relation at $z \approx 0.57$ from DESI

A26 does not repeat a dedicated stellar-archaeology analysis at low redshift, since its LRG sample does not extend there (Sec. 2.2); instead, the full parameter space of the joint pivotal-redshift cosmographic fit (Fig. 3.8) already includes, as nuisance parameters, the pivotal-redshift age of each velocity-dispersion group, $t_{z_0, v}$, which can be read off directly as an indirect measurement of the age-scaling relation as seen by an observer at $z \approx 0.57$, rather than at $z = 0$. Fig. 3.13 shows the posterior of these pivotal ages on the left panel, and the sampling of the scaling relation, a and b , of $t(\sigma) = 10^{a \log(\sigma) + b}$, on the right panel. The corner plot on the left shows no correlation between the pivotal age of different velocity-dispersion groups, confirming that their relative offset, and hence the downsizing signature, is independent of the passive cosmological evolution captured by $\{H_{z_0}, q_{z_0}, j_{z_0}\}$. The posterior of the least massive group, $[200, 225) \text{ km s}^{-1}$, is moderately wider than the rest, a consequence of the small number of stacks (6) surviving the quality flags for that bin (Sec. 2.6.1).

Using only the statistical uncertainties of the pivotal ages, the fit yields $\log t = -2.1_{-0.6}^{+0.7} + (1.1_{-0.3}^{+0.3}) \log \sigma$: the slope agrees, within uncertainties, with the low-redshift determinations of Thomas et al. (2010), Johansson et al. (2012) and A25 (Sec. 3.3.1), a further, independent confirmation that the underlying

physics of the downsizing scenario for galaxy assembly remains consistent across cosmic epochs. The intercept, b , is instead shifted towards smaller values with respect to the local-Universe determinations, which in A26 we interpret as evidence that the population of massive ETGs observed at $z \approx 0.57$ appears more compressed in age range than a naive extrapolation from local-Universe scaling relations would suggest, a plausible signature of the progressive build-up of the age spread of the ETG population as cosmic time advances (Sec. 1.3.3). The statistical uncertainties on a and b are, as expected, considerably larger than those quoted for the direct SDSS relations of Table 3.1, reflecting the fact that these parameters are extracted here only indirectly, as a by-product of a fit whose primary target is the cosmographic reconstruction.

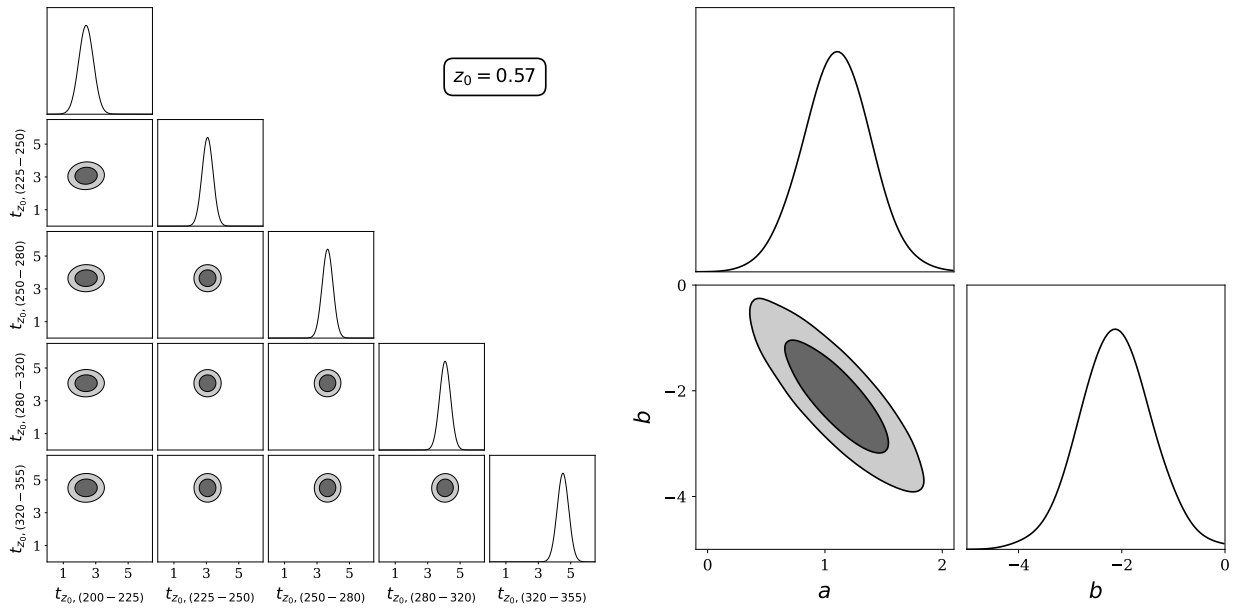


Figure 3.13: On the left panel the pivotal-redshift ages part of the joint cosmographic fit, shown for the cosmographic parameters alone in Fig. 3.8. On the right, the resulting contour plot and marginal posteriors of the two archaeological parameters that appear in the scaling relation fit.

3.4 Summary

This chapter has presented the two central cosmographic results of this thesis. From the SDSS sample of A25, a classic, $z_0 = 0$ cosmographic fit of TMJ-modelled stellar ages yields $H_0 = 70.0^{+4.1}_{-7.6} \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Sec. 3.1.1), a value compatible with both sides of the Hubble tension, together with a w_0 CDM readout fully consistent with a cosmological constant (Sec. 3.1.2) and a reconstructed Hubble diagram competitive with existing FSF-based measurements out to $z \lesssim 0.35$ (Sec. 3.1.4). The Knowles model, in contrast, was shown to produce implausibly narrow posteriors for the same data (Sec. 3.1.3), a cautionary result that motivated the exclusive use of TMJ in the subsequent DESI analysis. From the substantially larger DESI-DR1 sample used in A26, a pivotal-redshift cosmography anchored at $z_0 \approx 0.57$ delivered a tighter, fully covariant $H(z)$

reconstruction across $0.36 \lesssim z \lesssim 0.80$ (Sec. 3.2.3), together with two independent, local $H(z)$ measurements, $H(z \approx 0.55) = 104.5_{-7.6}^{+13.2}$ (stat.) ± 22.4 (syst.) $\text{km s}^{-1} \text{Mpc}^{-1}$ and $H(z \approx 0.61) = 88.5_{-12.6}^{+6.7}$ (stat.) ± 8.1 (syst.) $\text{km s}^{-1} \text{Mpc}^{-1}$ (Sec. 3.2.4); every one of these results is individually consistent with the ΛCDM prediction of [Planck Collaboration et al. \(2020\)](#).

Using the low-redshift sample from the SDSS dataset we produced scaling relations (Sec. 3.3.1) at the present time for the three SPS parameters: age, metallicity and α -enhancement. The results are compatible with the downsizing scenario for the formation of ETGs, which is directly visible in the $t - z$ planes that serve as our observable for the cosmographic analysis (Sec. 3.1.1). This particular structure of the $t - z$ relations allows for an indirect measurement of the age scaling relation alone using the pivotal-redshift average ages of each velocity dispersion group. In [A26](#) we used these nuisance parameters to compute the scaling relations as observed at the pivotal-redshift of the sample, obtaining results compatible with what was directly computed in [A25](#): using only the statistical uncertainties of the pivotal ages, the fit at $z \approx 0.57$ yields $\log t = -2.1_{-0.6}^{+0.7} + (1.1_{-0.3}^{+0.3}) \log \sigma$ (Sec. 3.3.2), with a slope in agreement with the local-Universe determinations and an intercept shifted towards smaller values, as discussed above.

Taken together, these results establish Lick-index cosmic chronometers, built on carefully stacked, high-S/N spectra and interpreted through a cosmographic rather than a purely finite-difference framework, as a competitive, genuinely model-independent probe of the low- and intermediate-redshift expansion history of the Universe. The systematic effects and the accurate assessment of the uncertainty budget associated with them remain significant limitations of this work. Future works could benefit from the establishment of a comprehensive and robust guideline. In the works presented here, the systematic effects produced by data management were the ones mainly assessed, in an attempt to trace all the further biases that could be introduced by the methodology presented therein. The observation of oscillations in the continuous $t - z$ plane, provoked by the consistent (at all velocity dispersions) irregularity of the trend of several indices with redshift is a thoroughly reported issue in [A25](#). The DESI data ([A26](#)) shows a similar pattern, although we could clearly spot it only at one redshift; in contrast, the SDSS data presents these wiggles at various redshifts.

The Future of Cosmic Chronometers

The previous chapters presented a complete, self-contained pipeline, from spectrum to sample selection, from index measurement to stellar age, and from a $t - z$ relation to a cosmographic constraint on $H(z)$, together with the results it produced for the SDSS and DESI-DR1 cosmic chronometer samples. Along the way, several choices were made that, while defensible and thoroughly tested within the scope of the two published articles on which this thesis is based, were never meant to be the last word: a single, fixed set of Lick indices, a single SPS model taken as baseline, a cosmographic expansion anchored at one specific redshift, an index- z oscillation flagged but left formally unexplained. This closing chapter collects those loose threads and proposes a discussion about possible ways forward in research. We include some partial or preliminary results that serve as illustrative previews.

The threads fall into two broad groups. The first concerns the age-estimation step itself: whether full spectral fitting and the Lick-index route can be made to talk to each other, either as a cross-check or as a source of external priors. Whether an adaptive, per-stack selection of which indices to trust, in the spirit of the pruning schemes already used by [Thomas et al. \(2010\)](#) and [Johansson et al. \(2012\)](#), could replace the single baseline combination used throughout this thesis. And whether a purely geometric, model-grid-based reading of age, metallicity and $[\alpha/\text{Fe}]$ through index-index diagrams, applied here for the first time to real DESI stacks, could complement or even substitute part of the SPS MCMC fits; with an eye, in particular, on the still poorly understood behaviour of the Knowles-modelled ages.

The second group concerns the cosmographic machinery on the other end of the pipeline, and the possibility, hinted at but not tested in [A25](#) and [A26](#), that the classic $z_0 = 0$ expansion for SDSS is not optimal and that the pivotal-redshift one for DESI can still be refined. We report here a handful of additional fits that were actually run in preparation for this thesis but left out of the two articles: a pivotal-redshift cosmography applied directly to the SDSS sample, a joint cosmographic-archaeological fit that lets the zero-age and the kinematic parameters be constrained simultaneously, and a first, direct comparison of the SDSS and DESI $t - z$ relations in their region of redshift overlap. This last comparison leads naturally into a short, preliminary follow-up in which the index- z oscillations were sought, with mixed success, in an independent survey, GAMA, which closes the chapter on the open question of this pattern, unveiled in [A25](#).

4.1 Estimation of galaxy ages

The age-redshift relations that ultimately feed $H(z)$ are only as reliable as the individual age estimates obtained for each of the stacks. The systematic uncertainties assessed and considered in [A25](#) and [A26](#) avoid, for practical reasons, a thorough analysis of the different methods used to compute stellar ages from the different available SPS models. In other words, it would be worth asking whether the age estimation step itself, as implemented throughout this thesis via the Lick-index MCMC of [Sec. 2.5](#), can be technically improved. Two broad directions seem promising and are explored below. On the one hand, we consider refining the age estimation from within the Lick-index framework itself, through a smarter selection of which indices are trusted for a given fit or the use of index-index planes. On the other hand, we propose complementing the SPS model estimation beyond the use of Lick indices, bringing in information from full spectral fitting (FSF), which exploits the spectrum in a fundamentally different way.

The three subsections that follow take up these directions. [Sec. 4.1.1](#) discusses the compatibility between FSF and the Lick-index route, both as a cross-check and as a potential source of external priors for the MCMC. [Sec. 4.1.2](#) stays within the Lick-index method and proposes an adaptive, per-stack selection of the index set, inspired by the iterative χ^2 pruning of [Thomas et al. \(2010\)](#) and [Johansson et al. \(2012\)](#), as an alternative to the single fixed baseline combination used so far. Finally, [Sec. 4.1.3](#) takes a step back from the likelihood-based fit altogether and explores index-index diagnostic diagrams as a purely geometric, model-grid-level way of reading off age, metallicity and $[\alpha/\text{Fe}]$, applying it for the first time to the DESI stacks of [A26](#).

4.1.1 Compatibility of different methods

[Sec. 1.2](#) already stressed that the CC literature has settled on three largely disjoint ways of dating a spectrum: full spectral fitting (FSF), the $\text{D4000}_{(\text{n})}$ break, and the Lick-index route followed throughout this thesis, each carrying a distinct systematic fingerprint because each uses a different portion of the spectral information. FSF ([Simon et al. 2005](#); [Zhang et al. 2014](#); [Ratsimbazafy et al. 2017](#); [Jiao et al. 2023](#); [Tomasetti et al. 2023](#); [2026](#)) exploits every pixel across the full optical range, at the price of the well-known age-dust-metallicity degeneracy and a full dependence on the continuum shape and flux calibration over several thousand Å. $\text{D4000}_{(\text{n})}$ ([Moresco et al. 2011, 2012](#)) can only provide relative age variations, sacrificing absolute ages for a single, high-S/N broadband ratio, at the price of an external, model-dependent $A(Z, \text{SFH})$ calibration ([Sec. 1.3.2](#)). Lick indices instead normalise each bandpass against its own local pseudo-continuum ([eq. 1.51](#)), which should make them comparatively immune to broadband calibration issues, an advantage hard to reconcile with the still-unexplained index- z oscillations of [Sec. 2.6.2](#), with the question of which set of Lick indices and SPS model to use still open. We ask ourselves if a joint analysis of various elements could help constrain the systematic effects related to the nature of the age estimation (so far disregarded to our knowledge) and, at best,

provide a more universal method to work with CCs.

The most direct test would be to run an FSF pipeline of the kind used by [Jiao et al. \(2023\)](#) or [Tomasetti et al. \(2023; 2026\)](#) on the very same SDSS and DESI stacks assembled in [A25](#) and [A26](#), and to compare, group by group, the resulting $t - z$ relation against the Lick-based one. Then, a systematic error budget from the comparison could be drawn, computing the overall difference injected in the final H credible interval estimation. This idea would require using the same SPS model in both techniques, something that right now is not possible with the TMJ model, which strictly produces Lick index strengths and not overall spectra varying as a function of the typical parameters that FSF algorithms are built for.

Beyond a diagnostic comparison, FSF and Lick indices could also be combined constructively. [Borghi et al. \(2022b\)](#) already uses full-spectrum information indirectly, through the emission-line and H/K vetoes of Sec. 1.3.2, to reject star-forming contaminants before any index is fit. A natural extension is to let an FSF code supply, per stack, an independent estimate of residual dust reddening and of the (small) young-population fraction that a purely EW-based veto cannot always catch (Sec. 1.3.2, on the residual filling of $H\beta$; [Concas et al. 2017](#)), and feed it into the Lick-index MCMC of Sec. 2.5 as an informative prior on the star-formation-history width, which the [Thomas et al. \(2005\)](#) relation already links to $[\alpha/\text{Fe}]$ (Sec. 1.3.2).

Along these lines, a more radical idea came to mind: tailoring pieces of FSF and Lick indices into a single estimation. This idea was developed after reading [Jiao et al. \(2023\)](#), where a combined photometric and spectroscopic FSF was developed to reanalyse the data used in [Borghi et al. \(2022b\)](#). This idea is so far not developed much further, but could be interesting and draw attention in the coming years if the expertise from groups that worked recently with FSF and Lick indices is put together.

4.1.2 Set of Lick Indices

A second, more narrowly scoped avenue concerns the index combination itself rather than the dating technique as a whole. Throughout this thesis (as in [Borghi et al. 2022a](#)) a single, fixed baseline set of indices is fit to every stack, chosen a priori to maximise the number of usable objects across the full redshift and wavelength range; [Borghi et al.](#)'s own appendix inspection of alternative combinations shows that this choice can shift absolute ages by up to ~ 1 Gyr and still contributes about a third of the total systematic budget in $H(z)$ (Sec. 2.6.2), even though the differential H itself remains stable within 0.4σ . This robustness of the slope against a poor absolute calibration is reassuring, but it also means that a fixed index set is a somewhat blunt instrument, since it cannot tell, stack by stack, whether a particular bandpass is being badly reproduced by the model; it only reports the net effect once the fit is already completed.

A natural refinement, and one already validated on Lick-index galaxy samples, is to let the fit itself decide which indices to trust. [Thomas et al. \(2010\)](#) and [Johansson et al. \(2012\)](#) both implement an iterative χ^2 scheme in which, after an initial fit to the full index set, the probability $P(\chi^2)$ of the best-fit solution is tested against a fixed threshold; if $P > 0.999$ the single index contributing most to χ^2 is discarded and the fit repeated, until an acceptable solution is reached. [Johansson et al. \(2012\)](#) go further and structure the whole index set into an inner/outer loop keyed to individual elements (Fe-peak, Mg, Ca, Ti, ...), so that each abundance ratio is constrained only by the bandpasses genuinely sensitive to it, and report which indices are discarded most often (G4300 in 3.8% of objects, Fe5270 in 1.1%, all others below 1%) as a diagnostic of the index set's overall health. Porting a version of this adaptive pruning into the MCMC of Sec. 2.5, flagging, per stack, whichever baseline index is the largest outlier relative to the TMJ grid and either removing weight from it in the fit or dropping it before the age posterior is drawn, would replace the current all-or-nothing baseline choice with a data-driven one. This idea could be implemented together with those of the previous section and the one that follows.

4.1.3 Index-Index planes

Sec. 1.3.2 introduced the index-index diagnostic diagrams of [Knowles et al. \(2023\)](#), built from their sMILES SSP grid, as a graphical way of separating age, metallicity and α -enhancement without relying on a full likelihood fit: a Mgb-Fe5270 or Mgb-Fe5335 plane to isolate $[\alpha/\text{Fe}]$, and, once that abundance is fixed, a $\text{H}\beta_0$ -[MgFe] plane (eq. 1.52) that separates age and metallicity almost orthogonally (Fig. 1.12). That discussion, however, was entirely illustrative of the model itself, in the spirit of the demonstrative figures of [Vazdekis et al. \(2015\)](#), and made no contact with either of the two datasets analysed in this thesis. Here we take a first, deliberately rudimentary step in that direction, applying the same machinery directly to one of the DESI stacks of A26, as a proof of concept for the kind of graphical, model-grid-based analysis that a dedicated future work could develop into a full methodology.

Fig. 4.1 shows the outcome for the $250 < \sigma [\text{km s}^{-1}] < 280$ velocity-dispersion group, chosen simply because it sits well within the regime where the age-metallicity degeneracy of the TMJ fit is most delicate (Sec. 3.3.1). We proceeded in two steps. First, the individual stack measurements of Mgb, Fe5270 and Fe5335 were overlaid on the Knowles sMILES grid (Sec. 1.4.5) for a [Chabrier \(2003\)](#) IMF, at five values of $[\alpha/\text{Fe}]$ between -0.2 and $+0.6$ (lower panels), to identify which single α -enhancement value the group as a whole is most consistent with. The points cluster tightly and almost exclusively within the $[\alpha/\text{Fe}] = +0.2$ cell (shaded), a value in good agreement with the scaling relations of Sec. 3.3.1 for this range of σ . Then, having fixed $[\alpha/\text{Fe}] = +0.2$, the same stacks were placed on the corresponding $\text{H}\beta$ -[MgFe] and $\text{H}\beta_0$ -[MgFe] grids (upper panels), where they again cluster in a comparatively narrow region of the metallicity nodes shown, offering an entirely independent, purely geometric reading of the age-metallicity locus of the group, obtained without ever running

the MCMC fit of Sec. 2.5.

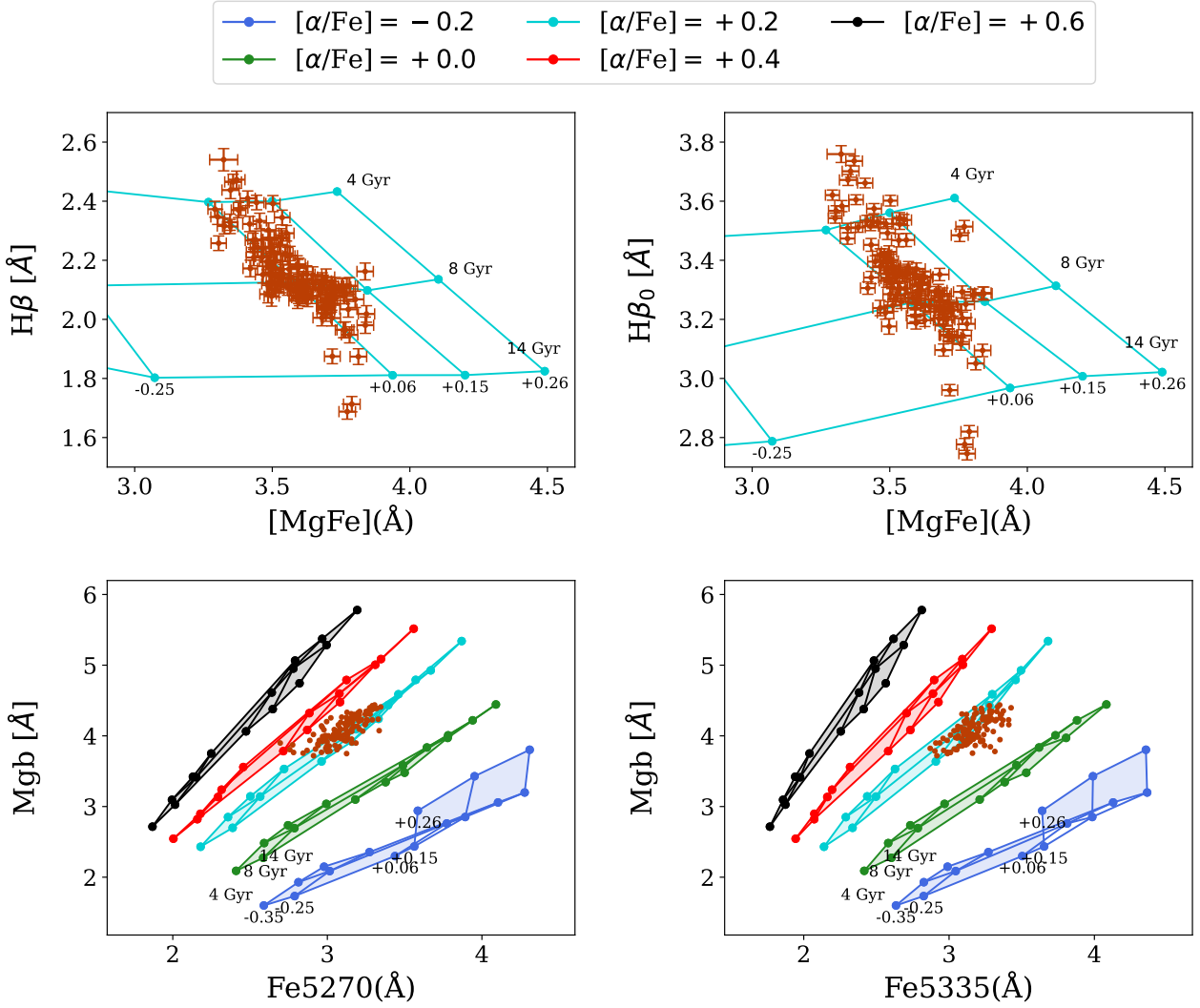


Figure 4.1: Index-index planes for the $250 < \sigma [\text{km s}^{-1}] < 280$ stacks of [A26](#), built on the Knowles sMILES grid (Sec. 1.4.5) for a [Chabrier \(2003\)](#) IMF, in the style of Fig. 1.12. On the lower panels, Mg b against Fe5270 (left) and Fe5335 (right), for five values of $[\alpha/\text{Fe}]$; the stacks (brown points) single out $[\alpha/\text{Fe}] = +0.2$ (shaded) as the group-representative abundance. On the upper panels, $H\beta$ (left) and $H\beta_0$ (right) against $[\text{MgFe}]$ (eq. 1.52), fixing $[\alpha/\text{Fe}] = +0.2$ (from the lower panels observations), with metallicity and age nodes labelled.

Rudimentary as it is, this exercise hints at genuine possibilities beyond a mere visual cross-check. Reading $[\alpha/\text{Fe}]$ off the lower panels first, and only then reading age and metallicity off the upper ones, effectively decomposes the three-parameter fit of Sec. 2.5 into two, lower-dimensional and largely decoupled problems, where the first of them can be safely inspected by eye rather than through more complex statistical tools. A systematic version of this two-step procedure, applied stack by stack rather than group by group, could serve either as an independent prior on $[\alpha/\text{Fe}]$ ahead of the full fit, or, at the very least, as a fast, model-grid-level sanity check able to flag a stack

whose posterior falls outside the region its own index measurements actually occupy on the grid. We observe that this happens for some points in the age-metallicity planes that exceed the Knowles model limit of 14 Gyr.

Also, these index-index diagrams make the true scatter of a group of stacks across the grid directly visible, geometrically, in a way that is largely independent of how the likelihood of eq. (2.6) happens to be shaped in the neighbourhood of its minimum. This can be very valuable in order to fully understand and mitigate the pathologies of the Knowles model age estimations addressed in A25 (here presented in Fig. 3.5 and the corresponding discussion).

A methodological caveat, already anticipated in Sec. 1.4.5, restricts this kind of diagram to the Knowles model for the time being. Because Knowles is built around full synthetic spectra, any bandpass definition, including one entirely absent from the original Lick/IDS system such as $H\beta_0$ (Cervantes & Vazdekis, 2009), can simply be integrated directly over the model SED with the same `pyLick` machinery used on real data (Sec. 2.3.1). TMJ, by contrast, is a direct, tabulated prediction of a fixed set of 25 Lick-index strengths as a function of $(t, [Z/H], [\alpha/Fe])$, with no underlying spectrum for us to measure anything from; consequently, an $H\beta_0$ -based plane simply cannot be built for TMJ, however useful it might be, since $H\beta_0$ was never part of the grid it was calibrated on. The Mgb-Fe5270 and Mgb-Fe5335 planes of the lower panels of Fig. 4.1, however, use only classical Lick indices that are part of the TMJ grid, and an equivalent, TMJ-based version of at least this half of the diagnostic would be entirely feasible. Given the systematic, ~ 2 Gyr age offset and the order-of-magnitude discrepancy in formal uncertainties between the two models documented throughout this thesis (Secs. 3.1.3, 3.3.1), repeating this exercise with TMJ, group by group and directly alongside the Knowles version presented here, is in our view one of the more immediately actionable and informative follow-ups this chapter can point to.

4.2 Extended Cosmography

This section addresses possible improvements to the cosmographic machinery that converts $t - z$ relations into constraints on $H(z)$ and its derivatives (Sec. 1.2.1). We wonder whether the specific choices made in A25 and A26, namely a classic $z_0 = 0$ expansion for SDSS and a pivotal-redshift expansion for DESI, are in fact the most natural or the most powerful ones available, or whether the two frameworks could be exchanged, combined, or extended beyond what either paper explored on its own.

Three directions are pursued below. First, we test what happens when the pivotal-redshift cosmography of A26 is applied to the SDSS sample instead of the classic expansion used in A25, to see whether anchoring the fit near the bulk of the data, rather than at $z_0 = 0$, can deliver a competitive local H estimate without relying on the growing- H prior. Second, we revisit the combined cosmographic-

archaeological parametrisation of eq. (1.48), left untested in both papers, and fit it directly to the data as a single, joint model. Third, we take a step back from any single dataset and compare the SDSS and DESI $t - z$ relations directly in the region where they overlap in redshift, as a first, exploratory look at whether the index- z oscillations of Sec. 2.6.2 are intrinsic to the age-estimation method or instrument-dependent.

4.2.1 Revisiting SDSS with pivotal-redshift cosmography

Sections 3.1 and 3.2 present two cosmographic reconstructions built with the same underlying philosophy, Lick-index stellar ages fit with the TMJ model and interpreted through a Taylor expansion of $H(z)$, but anchored at different ends of the redshift range accessible to low-redshift spectroscopic surveys. The classic, $z_0 = 0$ cosmography (A25) delivers a direct measurement of the present-day Hubble constant, $H_0 = 70.0^{+4.1}_{-7.6} \text{ km s}^{-1} \text{ Mpc}^{-1}$, which can be placed on the same footing as the CMB and distance-ladder values (Fig. 3.2). This is, by construction, unavailable to the pivotal-redshift approach of A26, whose reconstruction is instead anchored at $H_{z_0=0.57} = 95.1^{+10.9}_{-6.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$, a value that is only meaningful once translated back through a specific cosmological model. In exchange, the pivotal approach yields a markedly tighter reconstruction in the vicinity of its own pivot than the classic expansion would achieve if extrapolated to the same redshift (Fig. 1.6), and it does so while keeping the truncation error of the Taylor series many orders of magnitude below both the statistical and systematic observational uncertainties (Fig. 1.7), a comfort that the SDSS analysis, confined to $z \lesssim 0.35$, did not strictly require, but that becomes essential once the redshift baseline of the DESI sample is exploited in full.

The two datasets are also complementary in the shape of their constraints on the higher-order kinematic parameters. In both cases j_0 (respectively j_{z_0}) is essentially unconstrained by the standard prior and only closes into a Gaussian-like posterior once artificially widened, a direct consequence of the intrinsically low sensitivity of a smooth, monotonic $t - z$ relation to third-derivative information (Sec. 1.2.1). The deceleration parameter, by contrast, behaves rather differently in the two works: in A25 the posterior of q_0 is dramatically shaped by the $dH/dz > 0$ prior, piling up against the $q_0 = -1$ boundary, whereas in A26 q_{z_0} is genuinely, if loosely, constrained by the data without any such prior being imposed. We read this improvement as a direct consequence of the larger, more homogeneous DESI dataset (Sec. 2.1) and of the reduced susceptibility of the pivotal expansion to the exact redshift at which the second-order truncation starts to bite (Sec. 1.2.1). However, if a pivotal-redshift cosmographic approach improved the robustness of the SDSS fit and allowed it to provide competitive estimations of H_{z_0} without the need for the growing H restriction, this methodology could easily become a game-changer for CC works and the strength of the estimations derived from them. To test this idea, we performed a pivotal-cosmographic fit over the final sample of SDSS stacks, and we realised that we could obtain a perfectly well constrained and absolute $H_{z_0 \approx 0.13}$ estimation, proving that pivotal cosmography apparently works better than classical $z_0 = 0$

models. The corresponding corner plot can be seen in Fig. 4.2, structured in the same way as the corner plots shown in the previous chapter.

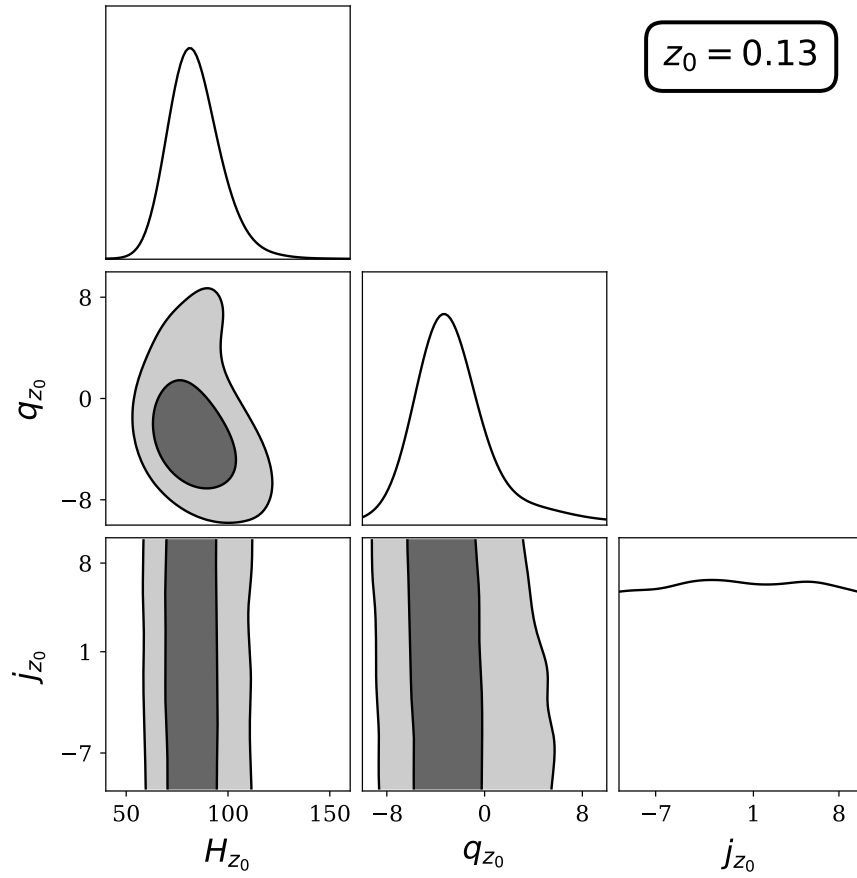


Figure 4.2: Posterior distributions for the cosmographic parameters of the pivotal-redshift fit applied on the SDSS final stacks (they can be observed in Fig. 3.1, right panel).

Obviously, the trade-off between a direct constraint on H_0 and the strength of the result is clear. However, the proximity between the pivotal redshift and the present time makes this measurement a valuable contribution to the H_0 tension, either through a direct extrapolation to $z = 0$ or by the imposition of an upper limit (if the $dH/dz > 0$ is considered, not within the fit as an imposed constraint, but to justify the upper limit quality of the H_{z_0} measurement of this fit). The values of the two constrained cosmographic parameters are $H_{z_0} = 77.5^{+16.6}_{-3.9}$ and $q_{z_0} = -4.1^{+3.7}_{-0.8}$ (only statistical uncertainties considered). We note that the posteriors are notably skewed towards the left in both cases, showing a slight degeneracy that prefers lower H_{z_0} values for close-to-expected (high) q_{z_0} values. Knowing that $z_0 \approx 0.13$, a direct extrapolation to estimate H_0 is safer than at higher redshift; however, the continuous $H(z)$ reconstruction from this fit is still conflicting with the established cosmology, producing a decreasing trend with redshift. This means that we observe $H_0 > H_{z_0}$, indicating that the pivotal-cosmography is good at producing a local H (H_{z_0}) estimation, but can confuse statistical noise for spurious curvature or anomalous values for higher-order derivatives.

4.2.2 Combining cosmography and archaeology

The full combined cosmographic-archaeological fit introduced in eq. (1.48) constitutes another possible improvement to the methodology if implemented as presented there. That means using a fixed set of parameters, 2 for archaeology $\{t_{\text{ref}}, a_t\}$ and a variable number of cosmographic ones (we keep the convention of the articles by which we chose to truncate the Taylor expansion at the j term). For a practical test of this possibility, we performed a modified MCMC algorithm in which we substituted the pivotal-redshift average ages for the archaeological parametrisation presented. The sampling of the posterior probability associated with the parameters can be observed in Fig. 4.3. It is structured in the same way as previous corner plots, with the important difference that the whole parameter space is here presented (in this case, 5 parameters in total).

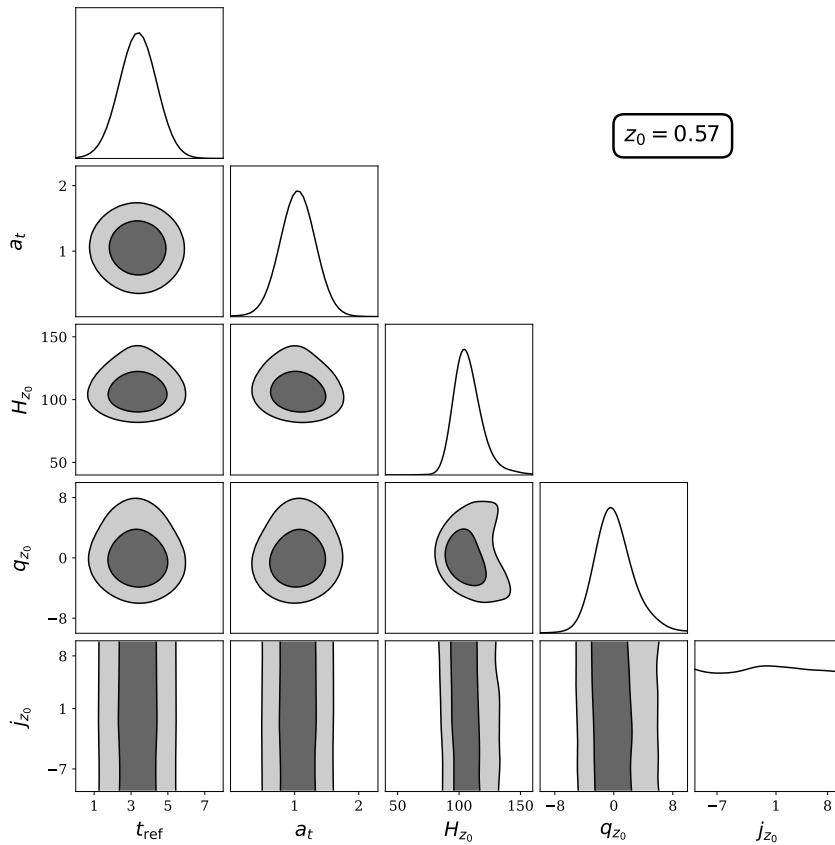


Figure 4.3: Full posterior probability distribution on the parameters of the combined cosmographic and age-archaeological fit.

A relevant check of the reliability of this method is the match between the a_t parameter here and the a parameter in the scaling relation constrained in the right panel in Fig. 3.13. In both cases, the value of the slope of the (logarithmic) scaling relation is close to 1. In the current fit, we chose 250 km s^{-1} as the reference velocity dispersion, so that the sole age estimated is that referred to the average age of a typical CC candidate of that velocity dispersion at z_0 . We observe that the posterior peaks slightly above 3 Gyr, similar to the estimated average age for the 250 – 280 group in the baseline cosmographic fit in A26 (Fig. 3.13, left panel). The correlation between the archaeological and cosmographic parameters is, at most, very mild, in line with the observed pivotal-redshift average

group ages in the baseline fit. These results motivate the use of this rich cosmographic-archaeological model for cosmographic CC studies in the future.

4.2.3 Extended $t - z$ relations

Given the wealth of spectroscopic surveys currently available and future prospects beyond SDSS and DESI (e.g. 4MOST-CRS, [Verdier et al. 2026](#); WST [Bacon et al. 2024](#), [Mainieri et al. 2024](#)), it could be an interesting practice to look at the $t - z$ relations that can be derived from galaxies at overlapping redshifts. Doing so, one can inspect differences and systematic effects produced by the spectrographs, which, so far, have not been reported. In Fig. 4.4 we show a view of the $t - z$ relations for the red envelope of CCs, that is, the best CC candidates among ETGs. To do so, we took the $250 < \sigma \text{ [km s}^{-1}] < 320$ stacks from Figs. 3.1 and 3.7, and put them together in a single $t - z$ map. We note that, given the finer binning in σ performed in [A26](#), two groups are included; therefore, in the figure, the expectation should be the division of the SDSS trend into two DESI subgroups at higher redshift.

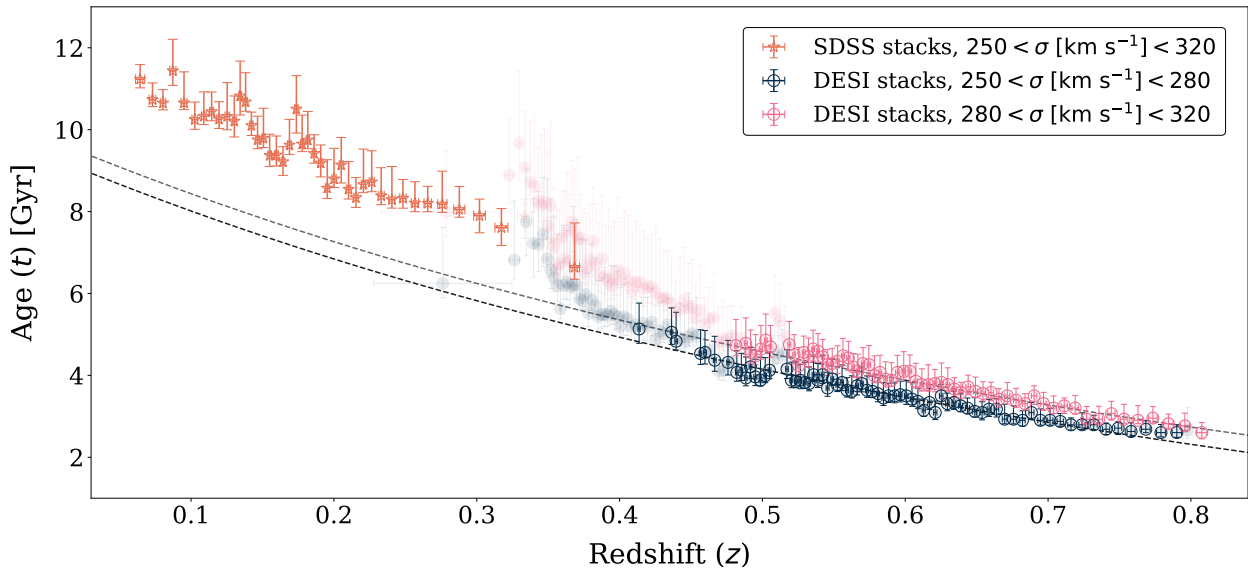


Figure 4.4: $t - z$ plane for the red envelope ($250 < \sigma \text{ [km s}^{-1}] < 320$) of galaxies. The colour legend is kept from figures 3.1 and 3.7.

We observe that the continuity is good only when the discarded objects from the post-SPS fit quality flags (faded colour) are reconsidered. This points towards the hypothesis that, rather than an instrumental effect, the differences come from the response of the TMJ model in general or in particular to the set of Lick indices used. For both [A25](#) and [A26](#) the same set of indices (when working with the TMJ model) was used, as explained in the articles and before in Sec. 2.6.2; but this exact set of indices can have a different joint sensitivity at different age regimes (i.e. $2.5 < t \text{ [Gyr]} \lesssim 6$, $6 \lesssim t \text{ [Gyr]} \lesssim 8$ and $t \gtrsim 8 \text{ Gyr}$). This can also be followed from Fig. 2.20, which apart from presenting the discrepancy between expected and predicted ages below $\sim 2.5 \text{ Gyr}$, shows how Balmer lines are

typically much more sensitive to age variations below ~ 4 Gyr, and flatten beyond ~ 5 Gyr. In some way, this supports the idea that upcoming work along the lines of what was discussed in Sec. 4.1.2 could be welcome in the field of CCs and mean a serious improvement of the methodology.

Apart from issues with the SPS model, on the more local level, the junction between the SDSS stacks and the DESI discarded stacks is not perfectly clean. It could point towards the idea that another wiggle of the $t - z$ relation is present at $z \approx 0.33$. If that was the case, the overall $t - z$ trends, discarding the oscillation regions, remain relatively close to Λ CDM overall, although the increased slope at low redshift would still push down the local H value regardless.

A further test that can be performed is the inclusion of BOSS data into the overall picture, as well as data from complementary surveys such as GAMA (Hopkins et al., 2013, ; more about this in the following section). This could also be done by including crossed-stacks, inspecting the particular systematic effects that could be directly associated with the different spectroscopic instruments.

4.3 Index oscillations

Just prior to the preliminary results of the A26 work, we planned a follow-up of the Lick-index oscillations by analysing the GAMA (Hopkins et al., 2013) spectra of CC candidates, chosen in a similar way to SDSS and BOSS CCs. By the time the work with the DESI data overtook the main work pipeline, we had performed measurements of Lick indices of the GAMA galaxies and tried comparing them with the SDSS results. The outcome of the work was, as can be observed in Fig. 4.5, disappointing in some aspects. In the picture we have zoomed in around the coordinated oscillation observed for the $H\gamma_F$ index (see Fig. 2.27) on the left panel, and shown the whole redshift coverage for the C_24668 index on the right panel.

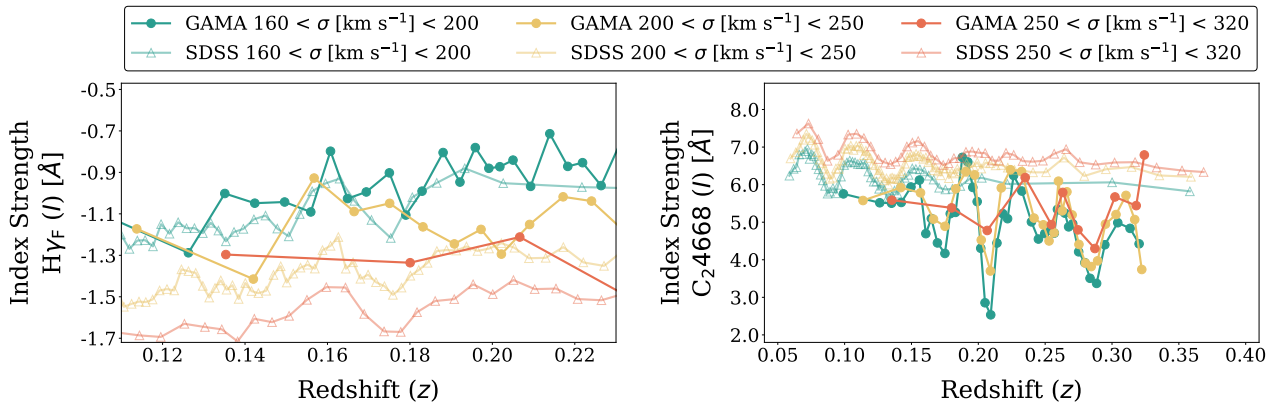


Figure 4.5: Index-redshift relation for $H\gamma_F$ (left panel) and for C_24668 (right panel). On the $H\gamma_F$ panel we zoomed in around the coordinated oscillation described in A25 as the tracer of the clearest $t - z$ oscillation. For C_24668 the whole redshift coverage is visible.

The first thing that can be observed is the fact that the oscillation that causes the $t - z$ oscillation in the SDSS sample, the one at $z \approx 0.17$, is not present in the GAMA sample (left panel, H_{γ_F}). Obviously, this can be caused by the averaging of sources produced by the stacking, since the density of sources at that redshift is so low that a feature like that can appear to be smoothed out. We experimented with more detailed samplings (that is, averaging fewer sources each time) and random noise was the result at $0.14 \lesssim z \lesssim 0.20$.

We tried other indices with clearer redshift patterns across the whole redshift range in order to assess the presence of the oscillations in the GAMA sample. The clearest oscillating pattern that we observed in SDSS came from an index that was never included in the set of indices used for the SPS fit, the C_{24668} (right panel). We note that this index also suffers from important oscillations in the GAMA sample, although the biggest oscillations do not appear at the same redshift at which they appeared in the SDSS sample.

As things stand, it is hard to tell whether redshift oscillations depend exclusively on the intrinsic properties of the spectroscopic instrument. We highlight the fact that these oscillations are imprinted (although not directly visible) in the spectra themselves. For this reason, alternatives to classical Lick indices such as the ones presented in Sec. 4.1 or even the D4000 method suffer from these. In particular, the latter was tested and reported in [A25](#) and partially in appendices A and B in [Moresco et al. \(2016\)](#).

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